Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

6-Hyperbolic-functions/6.7-Miscellaneous/185-6.7.1-Hyperbolic-functions

Nasser M. Abbasi

December 9, 2023 Compiled on December 9, 2023 at 6:34am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [1059]. This is test number [185].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	99.62 (1055)	0.38 (4)
Rubi	99.15 (1050)	0.85 (9)
Fricas	93.67 (992)	6.33 (67)
Maple	88.67 (939)	11.33 (120)
Giac	76.68 (812)	23.32 (247)
Maxima	71.95 (762)	28.05 (297)
Mupad	69.88 (740)	30.12 (319)
Sympy	32.48 (344)	67.52 (715)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

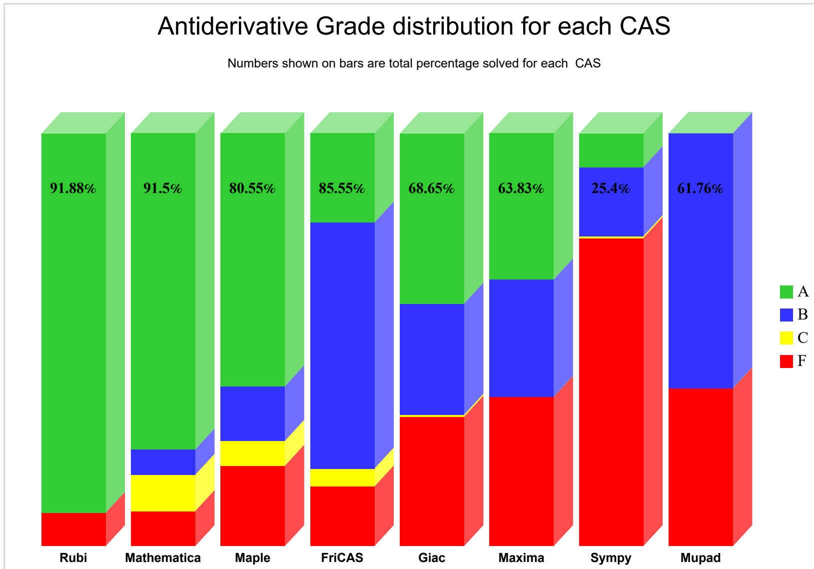
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

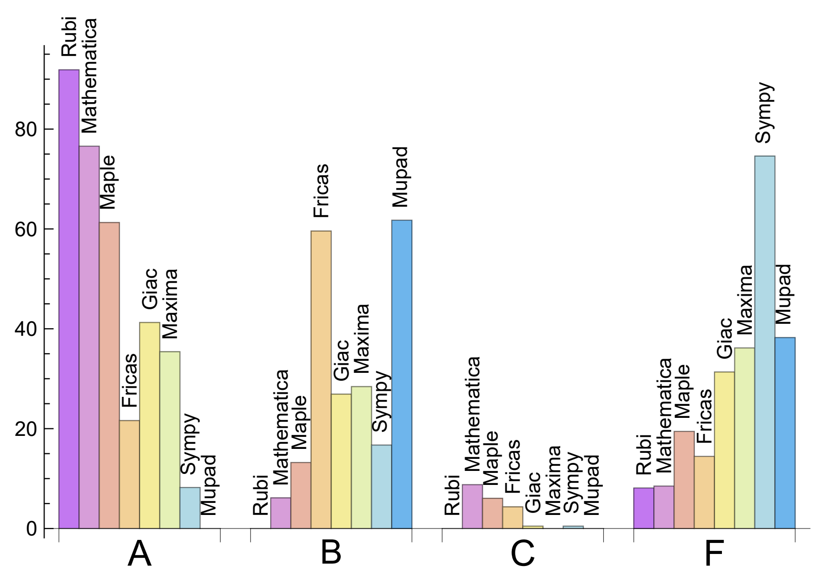
System	% A grade	% B grade	% C grade	% F grade
Mathematica	76.582	6.138	8.782	8.499
Rubi	72.899	0.000	18.130	8.971
Maple	61.284	13.220	6.043	19.452
Giac	41.265	26.912	0.472	31.350
Maxima	35.411	28.423	0.000	36.166
Fricas	21.624	59.585	4.344	14.448
Sympy	8.215	16.714	0.472	74.599
Mupad	0.000	61.756	0.000	38.244

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	4	0.00	100.00	0.00
Rubi	9	100.00	0.00	0.00
Fricas	67	49.25	0.00	50.75
Maple	120	99.17	0.83	0.00
Giac	247	91.09	1.21	7.69
Maxima	297	61.62	0.00	38.38
Mupad	319	0.00	100.00	0.00
Sympy	715	75.94	23.92	0.14

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.25
Fricas	0.27
Rubi	0.38
Giac	0.48
Mupad	1.74
Mathematica	2.89
Sympy	8.02
Maple	8.40

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	73.05	1.08	51.50	1.00
Giac	89.39	1.73	54.00	1.44
Maxima	99.11	2.55	70.00	1.64
Mupad	99.59	2.14	51.00	1.46
Maple	157.66	1.59	51.00	1.11
Mathematica	163.02	2.06	52.00	1.00
Sympy	324.97	5.05	59.50	1.89
Fricas	591.30	6.27	164.00	3.35

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

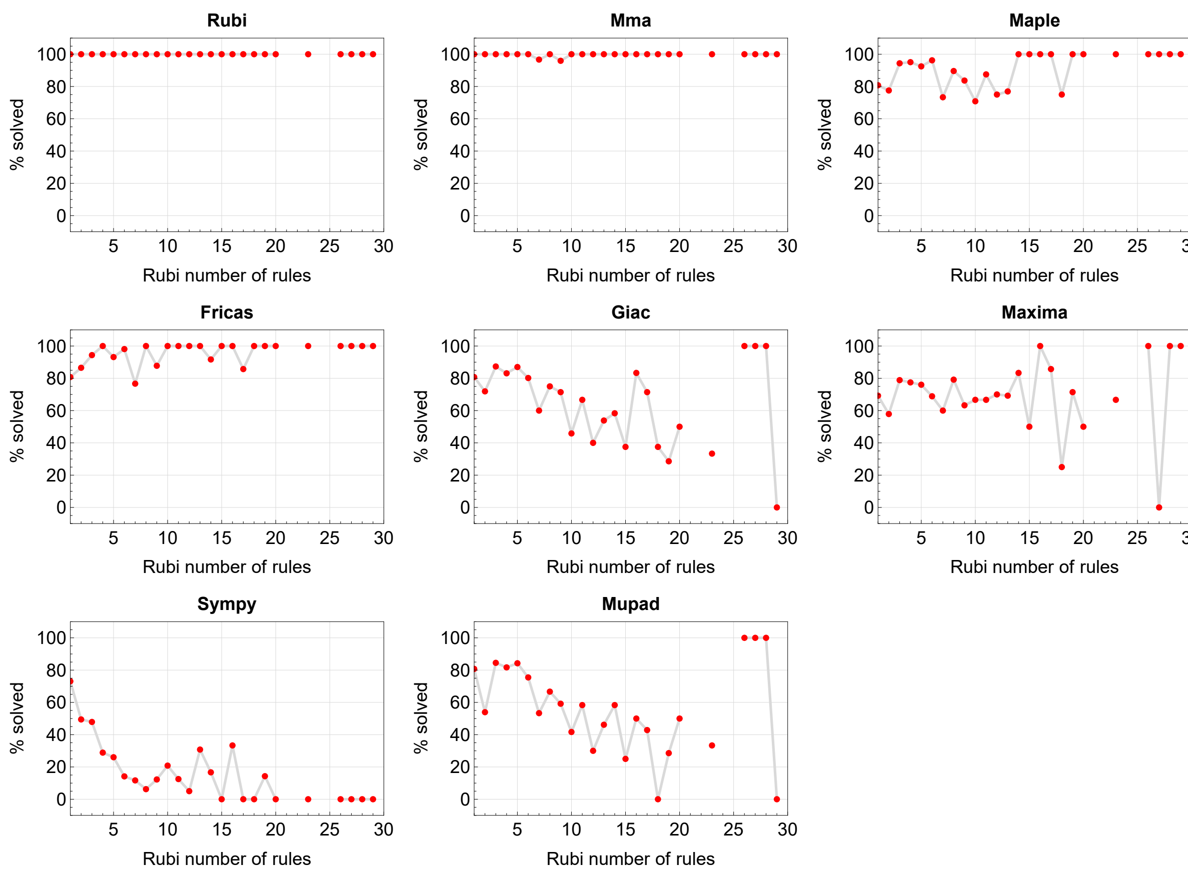


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

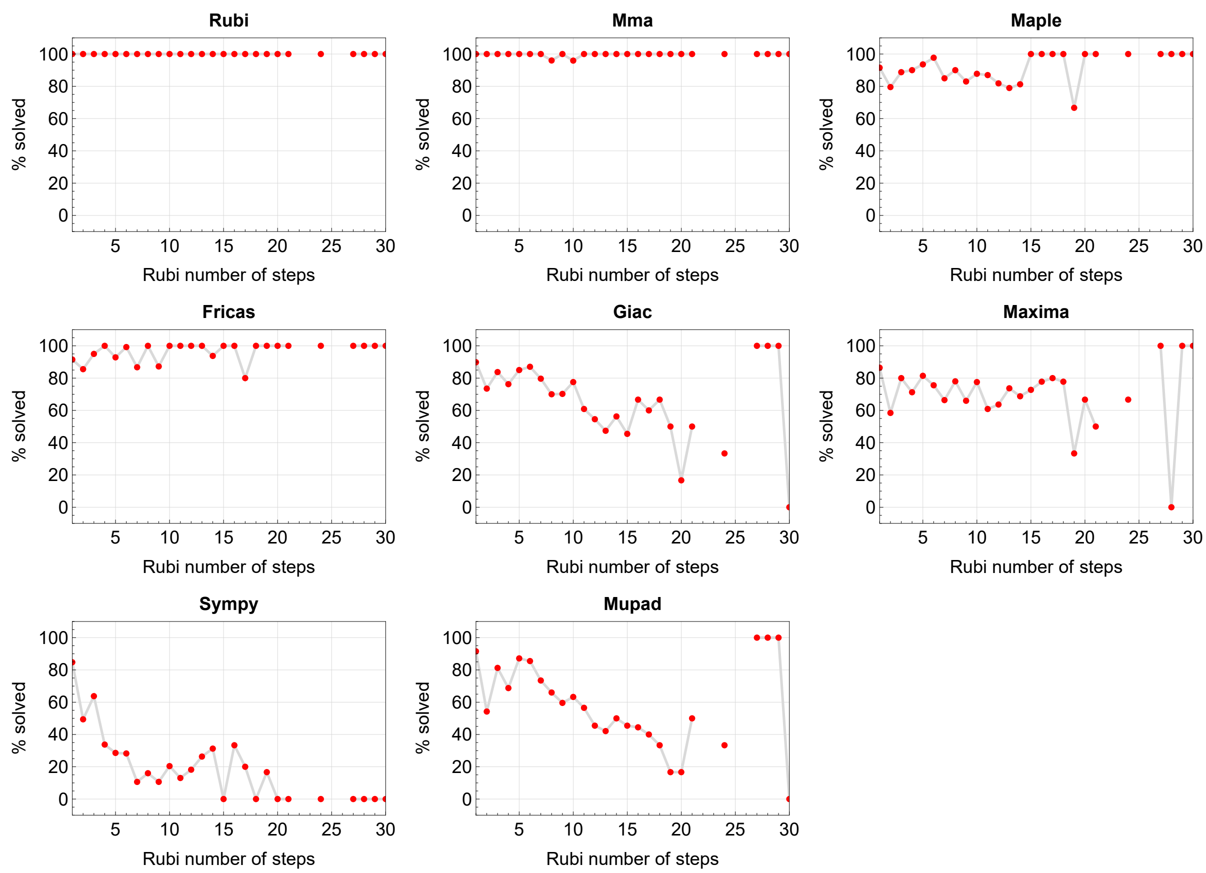


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

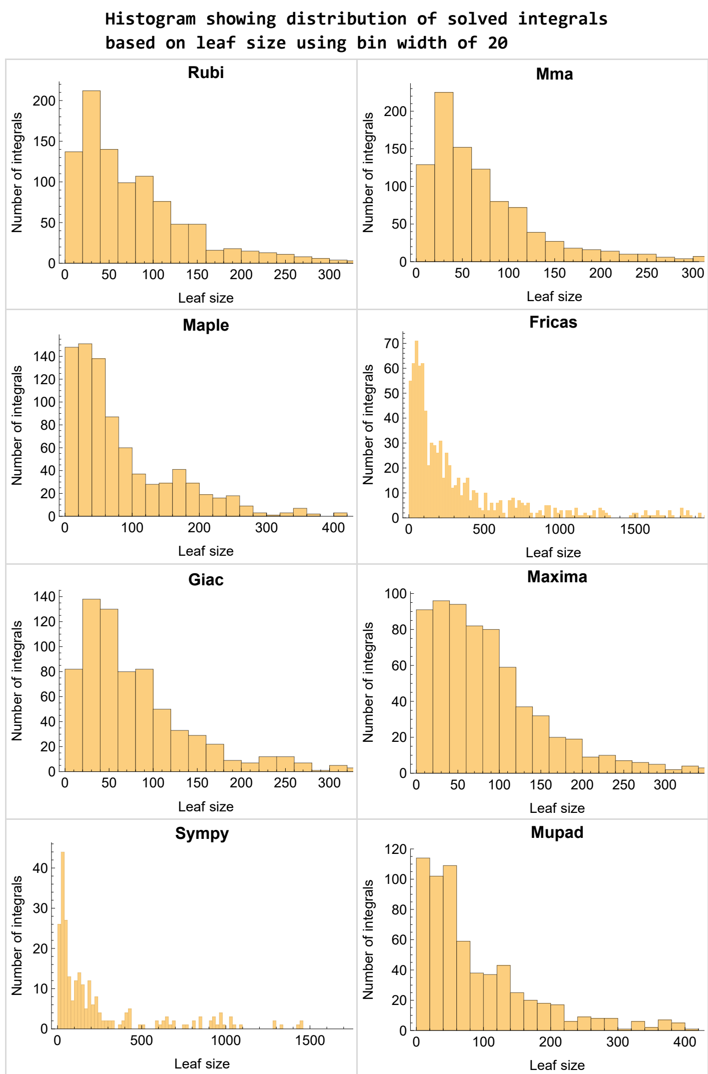


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

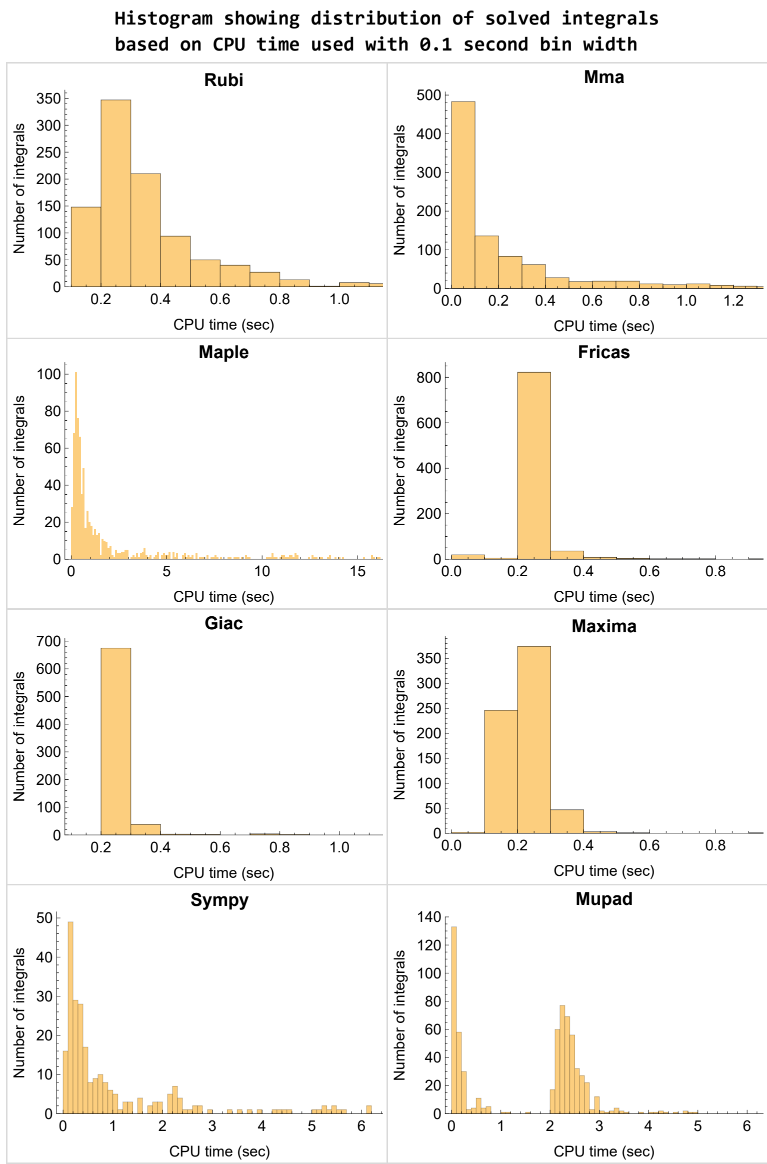


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

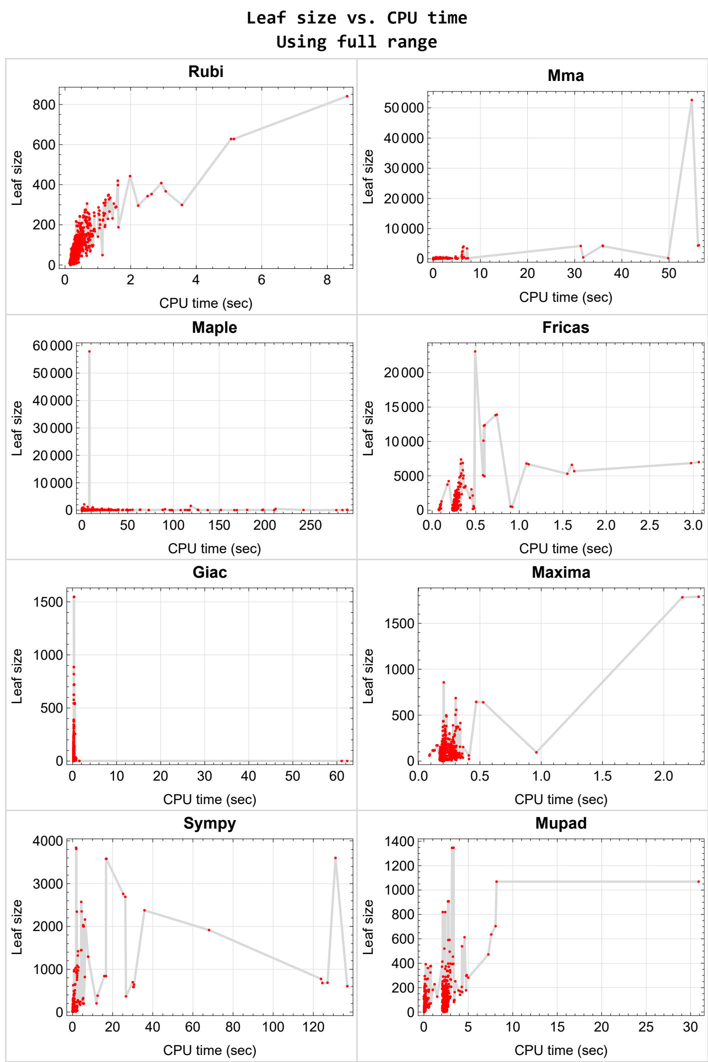


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{334, 339, 340, 341, 346, 347, 348, 353, 354, 355, 360, 361, 362, 367, 368, 369, 374, 375, 376, 381, 382, 383, 388, 389, 390, 395, 396, 397, 402, 403, 404, 409, 410, 411, 416, 417, 424, 429, 430, 431, 436, 437, 438, 443, 444, 445, 450, 451, 452, 457, 458, 459, 464, 465, 466, 471, 472, 473, 478, 479, 480, 485, 486, 487, 492, 493, 494, 499, 500, 501, 505, 506, 507, 512, 513, 514, 519, 520, 521, 526, 527, 870, 1014, 1015, 1016, 1017}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {28, 36, 38, 44, 46, 48, 57, 58, 61, 62, 65, 66, 76, 107, 111, 525, 858, 859, 860, 873, 902, 907, 909, 914, 919, 931, 933, 934}

Mathematica {622, 761, 762, 763, 765, 766, 767, 768, 769, 774, 775, 776, 777, 819}

Maple $\{89, 120, 765, 766, 767\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

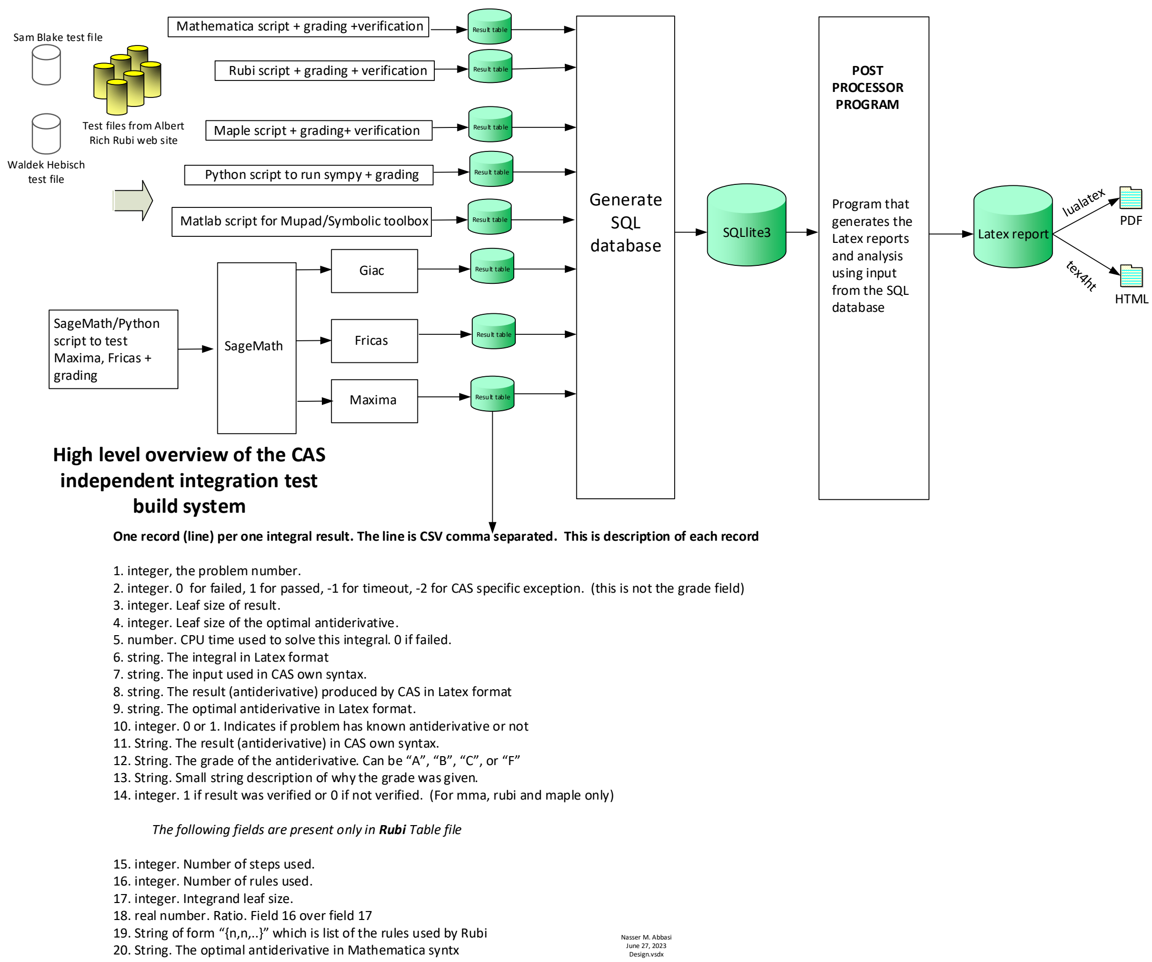
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 27, 35, 37, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 101, 103, 108, 110, 112, 113, 114, 115, 116, 117, 120, 125, 131, 132, 133, 134, 135, 136, 139, 140, 143, 144, 145, 146, 147, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 259, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 280, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 338, 342, 343, 344, 345, 350, 351, 352, 356, 357, 358, 359, 365, 366, 370, 371, 372, 373, 380, 384, 385, 387, 408, 418, 427, 428, 435, 441, 448, 449, 474, 475, 476, 477, 481, 482, 483, 488, 489, 490, 502, 503, 508, 509, 510, 515, 516, 517, 518, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 563, 566, 567, 568, 569, 570, 571, 572, 574, 575, 576, 577, 578, 580, 581, 583, 586, 588, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 680, 681, 682, 683, 691, 693, 697, 700, 701, 702, 705, 730, 731, 732, 733, 735, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 784, 785, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059 }

B grade { }

C grade { 24, 26, 28, 29, 30, 31, 32, 33, 34, 36, 38, 39, 40, 41, 42, 43, 44, 46, 48, 90, 91, 92, 100, 102, 104, 105, 106, 107, 109, 111, 118, 119, 121, 122, 123, 124, 126, 127, 128, 129, 130, 137, 138, 141, 142, 148, 152, 160, 164, 196, 197, 198, 221, 222, 223, 250, 255, 256, 257, 258, 260, 262, 278, 279, 281, 282, 302, 335, 336, 337, 349, 363, 364, 377, 378, 379, 386, 391, 392, 393, 394, 398, 399, 400, 401, 405, 406, 407, 412, 413, 414, 415, 419, 420, 421, 425, 426, 432, 433, 434, 439, 440, 442, 446, 447, 453, 454, 455, 456, 460, 461, 462, 463, 467, 468, 469, 470, 484, 491, 495, 496, 497, 498, 504, 511, 522, 523, 524, 525, 561, 562, 564, 565, 573, 579, 582, 584, 585, 587, 589, 620, 622, 678, 679, 684, 685, 686, 687, 688, 689, 690, 692, 694, 695, 696, 698, 699, 703, 704, 706, 707, 708, 709, 710, 711, 712, 713, 715, 716, 718, 724, 725, 726, 727, 728, 729, 734, 736, 737, 738, 782, 783, 786, 787, 842, 843, 844, 845, 846, 847, 994, 1043 }

F normal fail { 422, 423, 714, 717, 719, 720, 721, 722, 723 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 30, 32, 34, 35, 36, 37, 38, 40, 42, 44, 45, 46, 47, 48, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 145, 148, 149, 151, 152, 154, 157, 160, 161, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 235, 237, 239, 240, 241, 242, 243, 244, 246, 248, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 342, 343, 344, 345, 349, 350, 351, 352, 356, 357, 358, 359, 363, 364, 365, 366, 370, 371, 372, 373, 377, 378, 379, 380, 384, 385, 386, 387, 391, 392, 393, 394, 398, 399, 400, 401, 405, 406, 407, 408, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 426, 428, 433, 434, 439, 440, 441, 442, 446, 447, 448, 449, 453, 454, 455, 462, 463, 467, 468, 469, 474, 475, 476, 477, 483, 484, 488, 489, 497, 498, 502, 503, 510, 511, 515, 516, 517, 518, 522, 523, 524, 525, 528, 530, 532, 533, 534, 535, 536, 537, 538, 539, 541, 543, 544, 546, 548, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 561, 562, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 585, 586, 587, 588, 589, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 615, 617, 618, 619, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 671, 673, 675, 676, 678, 679, 680, 681, 682, 683, 684, 685, 686, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 703, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 746, 747, 748, 751, 753, 754, 755, 756, 757, 758, 759, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 809, 810, 811, 812, 813, 814, 815, 816, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 912, 913, 914, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 936, 938, 941, 942, 945, 946, 947, 948, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 981, 982, 986, 987, 988, 989, 990, 991, 992, 993, 995, 996, 997, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1018, 1019, 1020, 1021, 1023, 1024, 1025, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1054, 1057, 1058, 1059 }

B grade { 8, 24, 100, 121, 122, 123, 127, 143, 144, 150, 155, 156, 162, 189, 234, 245, 254, 427, 432, 456, 460, 461, 470, 481, 482, 490, 495, 496, 508, 509, 563, 575, 584, 614, 616, 648, 658, 668, 674, 677, 687, 702, 704, 745, 749, 750, 752, 760, 808, 882, 949, 980, 983, 984, 994, 998, 999, 1000, 1001, 1002, 1026, 1027, 1053, 1055, 1056 }

C grade { 29, 31, 33, 39, 41, 43, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 146, 147, 153, 158, 159, 165, 211, 213, 215, 229, 230, 231, 232, 233, 236, 238, 247, 249, 435, 491, 504, 529, 531, 540, 542, 545, 547, 556, 558, 576, 590, 591, 592, 593, 594, 595, 620, 622, 670, 672, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 774, 775, 776, 777, 817, 818, 819, 911, 918, 930, 935, 937, 939, 940, 943, 944, 985, 1022 }

F normal fail { }

F(-1) timedout fail { 772, 773, 778, 779 }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 151, 154, 163, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 192, 193, 194, 195, 196, 197, 198, 199, 200, 206, 207, 212, 216, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 234, 235, 245, 246, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 345, 349, 350, 351, 352, 359, 363, 364, 365, 366, 373, 377, 378, 379, 380, 387, 391, 392, 393, 394, 401, 405, 406, 408, 412, 413, 415, 418, 419, 420, 421, 422, 423, 425, 428, 434, 435, 439, 440, 441, 442, 447, 448, 449, 453, 454, 456, 463, 467, 470, 476, 477, 481, 482, 484, 491, 498, 504, 508, 509, 511, 517, 518, 522, 525, 566, 567, 572, 573, 575, 576, 578, 579, 580, 581, 582, 583, 584, 585, 586, 588, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 620, 621, 623, 625, 626, 627, 628, 629, 630, 631, 632, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 650, 651, 652, 653, 654, 655, 656, 657, 658, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 680, 681, 682, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 762, 764, 771, 777, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 792, 795, 798, 801, 802, 803, 804, 805, 806, 807, 809, 810, 811, 812, 813, 814, 815, 816, 818, 819, 822, 823, 824, 825, 832, 833, 834, 835, 836, 837, 848, 855, 856, 857, 864, 866, 872, 873, 874, 875, 876, 877, 878, 879, 880, 885, 886, 887, 893, 894, 897, 898, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 941, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 968, 969, 973, 974, 975, 979, 982, 986, 987, 991, 992, 993, 994, 995, 996, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1018, 1019, 1020, 1021, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1038, 1039, 1041, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1055, 1056 }

B grade { 5, 7, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 152, 153, 158, 159, 160, 161, 164, 165, 211, 214, 236, 237, 238, 247, 248, 249, 343, 358, 372, 385, 398, 399, 400, 407, 414, 426, 427, 432, 433, 446, 455, 460, 461, 462, 468, 469, 483, 490, 495, 496, 497, 503, 510, 523, 524, 531, 540, 547, 556, 560, 563, 568, 569, 570, 571, 574, 577, 587, 589, 619, 622, 624, 634, 649, 659, 677, 683, 704, 743, 744, 745, 761, 763, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 776, 778, 779, 790, 791, 793, 794, 796, 797, 799, 800, 817, 820, 821, 830, 838, 839, 842, 843, 844, 845, 846, 847, 851, 852, 853, 858, 859, 860, 861, 862, 863, 865, 867, 868, 869, 983, 984, 990, 997, 1022, 1023, 1040 }

C grade { 2, 3, 4, 6, 89, 120, 143, 144, 145, 150, 155, 156, 157, 162, 201, 202, 203, 204, 208, 209, 210, 213, 215, 217, 218, 219, 220, 231, 232, 233, 240, 241, 242, 243, 244, 344, 386, 808, 826, 827, 828, 829, 831, 840, 841, 937, 938, 939, 940, 942, 943, 944, 980, 981, 985, 988, 989, 1037, 1052, 1053, 1054, 1057, 1058, 1059 }

F normal fail { 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 188, 189, 190, 191, 205, 239, 250, 259, 268, 280, 289, 298, 307, 316, 325, 342, 356, 357, 370, 371, 384, 474, 475, 488, 489, 502, 515, 516, 528, 529, 530, 532, 533, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 548, 549, 550, 551, 552, 553, 554, 555, 557, 558, 559, 561, 562, 564, 565, 678, 679, 684, 685, 849, 850, 854, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 970, 971, 972, 976, 977, 978, 998, 1042 }

F(-1) timedout fail { 97 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 71, 75, 78, 102, 106, 109, 167, 173, 179, 182, 193, 195, 196, 199, 216, 224, 225, 226, 228, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 276, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 293, 294, 295, 298, 299, 300, 301, 303, 306, 307, 308, 309, 310, 312, 313, 315, 316, 317, 318, 319, 321, 322, 324, 325, 326, 330, 387, 442, 572, 580, 581, 585, 596, 597, 599, 600, 603, 604, 605, 606, 608, 609, 610, 612, 613, 617, 625, 627, 628, 629, 630, 632, 635, 637, 638, 639, 640, 642, 647, 657, 658, 660, 667, 668, 669, 670, 675, 681, 688, 691, 694, 695, 724, 727, 730, 732, 733, 736, 739, 740, 741, 742, 746, 747, 748, 771, 777, 780, 781, 782, 783, 784, 785, 786, 787, 789, 792, 795, 798, 802, 803, 804, 805, 806, 807, 811, 812, 813, 814, 815, 816, 823, 824, 825, 838, 839, 842, 855, 856, 857, 864, 872, 879, 880, 886, 887, 893, 894, 901, 903, 910, 915, 920, 922, 927, 948, 955, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 989, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1035, 1036, 1043, 1044, 1055 }

B grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 183, 184, 185, 186, 187, 192, 194, 197, 198, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 263, 272, 275, 277, 284, 290, 291, 292, 296, 297, 302, 304, 305, 311, 314, 320, 323, 327, 328, 329, 331, 332, 333, 338, 342, 343, 344, 345, 350, 351, 352, 356, 357, 358, 359, 365, 366, 370, 371, 372, 373, 380, 384, 385, 386, 394, 398, 399, 400, 401, 405, 406, 407, 408, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 432, 433, 434, 435, 439, 440, 441, 446, 447, 448, 449, 453, 454, 455, 456, 460, 461, 462, 463, 470, 474, 475, 476, 477, 484, 488, 489, 490, 491, 497, 498, 502, 503, 504, 511, 515, 516, 517, 518, 525, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 579, 582, 583, 584, 586, 587, 588, 589, 598, 601, 602, 607, 611, 614, 615, 616, 618, 619, 620, 621, 622, 623, 624, 626, 631, 633, 634, 636, 641, 643, 644, 645, 646, 648, 649, 650, 651, 652, 653, 654, 655, 656, 659, 661, 662, 663, 664, 665, 666, 671, 672, 673, 674, 676, 677, 678, 679, 680, 682, 683, 684, 685, 686, 687, 689, 690, 692, 693, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 725, 726, 728, 729, 731, 734, 735, 737, 738, 743, 744, 745, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 768, 769, 770, 772, 773, 774, 775, 776, 778, 779, 788, 790, 791, 793, 794, 796, 797, 799, 800, 801, 808, 809, 810, 817, 818, 819, 820, 821, 822, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 840, 841, 843, 844, 845, 846, 847, 848, 849, 850, 858, 859, 860, 865, 866, 867, 868, 869, 873, 874, 875, 876, 877, 878, 885, 897, 898, 902, 904, 905, 906, 907, 908, 909, 911, 912, 913, 914, 916, 917, 918, 919, 921, 923, 924, 925, 926, 928, 929, 930, 931, 932, 933, 934, 935, 936, 941, 942, 943, 944, 945, 946, 947, 952, 953, 954, 959, 960, 961, 962, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1037, 1038, 1039, 1040, 1041, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1056, 1057, 1058, 1059 }

C grade { 335, 336, 337, 349, 363, 364, 377, 378, 379, 391, 392, 393, 467, 468, 469, 481, 482, 483, 495, 496, 508, 509, 510, 522, 523, 524, 590, 591, 592, 593, 594, 595, 761, 762, 763, 764, 765, 766, 767, 851, 852, 853, 937, 938, 939, 940 }

F normal fail { 188, 189, 190, 191, 205, 239, 854, 861, 862, 863, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965 }

F(-1) timedout fail { }

F(-2) exception fail { 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 966, 1042 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 6, 8, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 29, 30, 42, 70, 75, 77, 79, 82, 84, 85, 86, 106, 113, 115, 116, 117, 139, 140, 143, 149, 150, 161, 162, 206, 216, 235, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 330, 331, 332, 333, 335, 336, 337, 338, 344, 349, 352, 359, 363, 364, 366, 377, 378, 379, 386, 391, 392, 393, 398, 399, 400, 405, 406, 407, 412, 413, 414, 423, 425, 426, 432, 433, 435, 439, 440, 446, 449, 453, 454, 455, 460, 467, 468, 469, 476, 481, 482, 483, 491, 496, 498, 508, 509, 510, 517, 522, 523, 524, 575, 576, 580, 581, 583, 586, 596, 599, 600, 601, 602, 603, 604, 605, 608, 609, 610, 611, 612, 613, 617, 618, 620, 627, 628, 629, 630, 631, 632, 633, 637, 638, 640, 641, 642, 647, 648, 658, 660, 662, 668, 669, 670, 671, 672, 674, 675, 680, 681, 688, 690, 691, 693, 697, 700, 707, 709, 711, 713, 715, 717, 719, 721, 723, 730, 731, 732, 733, 739, 740, 741, 746, 747, 748, 753, 754, 755, 756, 783, 786, 802, 803, 804, 805, 806, 807, 809, 811, 812, 813, 814, 815, 816, 823, 824, 825, 842, 843, 844, 845, 846, 847, 848, 851, 852, 853, 855, 856, 857, 858, 859, 872, 873, 874, 875, 876, 877, 893, 894, 897, 898, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 967, 968, 969, 973, 974, 975, 979, 981, 982, 986, 993, 994, 1013, 1019, 1020, 1026, 1027, 1028, 1031, 1033, 1034, 1037, 1038, 1043, 1044, 1045, 1046, 1050, 1051, 1055, 1056 }

B grade { 5, 7, 10, 11, 13, 14, 24, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 67, 68, 71, 72, 73, 74, 76, 78, 80, 81, 83, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 144, 145, 146, 147, 148, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 192, 193, 194, 196, 197, 198, 200, 201, 207, 208, 211, 212, 217, 218, 221, 222, 223, 225, 226, 227, 229, 230, 234, 236, 240, 241, 245, 246, 247, 262, 277, 283, 302, 320, 329, 345, 350, 351, 365, 373, 380, 387, 394, 401, 408, 415, 421, 422, 427, 428, 434, 441, 442, 447, 448, 456, 461, 462, 463, 470, 477, 484, 495, 497, 504, 511, 518, 525, 560, 561, 562, 563, 564, 565, 566, 567, 570, 573, 574, 577, 579, 582, 584, 588, 597, 598, 606, 607, 614, 615, 616, 619, 621, 622, 623, 624, 625, 626, 634, 635, 636, 639, 643, 644, 645, 646, 649, 651, 653, 654, 655, 656, 657, 659, 661, 663, 664, 665, 666, 667, 673, 676, 677, 678, 679, 682, 683, 684, 685, 686, 687, 702, 703, 704, 705, 735, 749, 750, 751, 752, 768, 769, 770, 774, 775, 776, 808, 810, 817, 818, 819, 820, 821, 822, 830, 838, 839, 840, 841, 860, 980, 983, 984, 987, 990, 991, 992, 995, 996, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1018, 1021, 1022, 1023, 1024, 1025, 1030, 1032, 1040, 1048, 1049, 1053, 1054, 1057, 1058 }

C grade { }

F normal fail { 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 188, 189, 190, 191, 202, 203, 204, 205, 209, 210, 213, 214, 215, 219, 220, 231, 232, 233, 237, 238, 239, 242, 243, 244, 248, 249, 342, 343, 356, 357, 358, 370, 371, 372, 384, 385, 474, 475, 488, 489, 490, 502, 503, 515, 516, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 590, 591, 592, 593, 594, 595, 761, 762, 763, 764, 765, 766, 767, 771, 772, 773, 777, 778, 779, 826, 827, 828, 829, 831, 832, 833, 834, 835, 837, 849, 850, 854, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 970, 971, 972, 976, 977, 978, 985, 988, 989, 997, 998, 999, 1000, 1001, 1002, 1029, 1039, 1041, 1042, 1047, 1052, 1059 }

F(-1) timedout fail { }

F(-2) exception fail { 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 195, 199, 224, 228, 418, 419, 420, 568, 569, 571, 572, 578, 585, 587, 589, 650, 652, 689, 692, 694, 695, 696, 698, 699, 701, 706, 708, 710, 712, 714, 716, 718, 720, 722, 724, 725, 726, 727, 728, 729, 734, 736, 737, 738, 742, 743, 744, 745, 757, 758, 759, 760, 780, 781, 782, 784, 785, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 836, 878, 879, 880, 885, 886, 887, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 1035, 1036 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 6, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 30, 32, 33, 40, 42, 43, 47, 48, 70, 71, 73, 77, 78, 86, 87, 88, 90, 95, 97, 101, 108, 109, 117, 118, 119, 143, 145, 149, 150, 151, 152, 154, 155, 157, 161, 162, 163, 164, 166, 170, 171, 176, 177, 182, 183, 184, 197, 202, 203, 204, 206, 209, 216, 217, 219, 222, 235, 237, 241, 243, 251, 252, 253, 255, 256, 257, 258, 260, 261, 262, 264, 265, 266, 267, 269, 270, 271, 273, 274, 275, 276, 281, 282, 283, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 303, 304, 305, 306, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 324, 326, 327, 328, 330, 331, 332, 333, 359, 366, 387, 408, 435, 498, 566, 567, 568, 569, 572, 573, 575, 576, 578, 579, 585, 586, 587, 588, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 645, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 660, 661, 662, 663, 664, 665, 666, 667, 669, 670, 671, 672, 673, 680, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 740, 742, 743, 746, 747, 754, 755, 756, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 793, 795, 796, 798, 799, 801, 802, 803, 804, 805, 806, 807, 808, 809, 811, 812, 813, 814, 815, 816, 819, 822, 823, 824, 825, 826, 828, 829, 830, 831, 832, 834, 835, 837, 838, 839, 840, 841, 855, 856, 857, 858, 859, 860, 872, 873, 874, 875, 876, 877, 878, 879, 880, 885, 886, 887, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 968, 969, 973, 974, 975, 980, 981, 985, 986, 987, 988, 989, 995, 1020, 1033, 1034, 1035, 1036, 1037, 1043, 1044, 1046, 1052, 1053, 1055, 1056, 1058, 1059 }

B grade { 5, 7, 8, 10, 11, 13, 14, 24, 25, 26, 27, 28, 29, 31, 34, 35, 36, 37, 38, 39, 41, 44, 45, 46, 69, 72, 74, 75, 76, 79, 80, 81, 82, 84, 85, 91, 93, 94, 96, 98, 99, 100, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 115, 116, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 146, 147, 148, 153, 156, 158, 159, 160, 165, 167, 168, 169, 172, 173, 174, 175, 178, 179, 180, 181, 185, 186, 187, 192, 193, 194, 195, 196, 198, 199, 200, 201, 207, 208, 210, 211, 212, 213, 214, 215, 218, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 238, 240, 242, 244, 245, 246, 247, 248, 249, 254, 263, 272, 277, 278, 279, 284, 302, 311, 320, 329, 338, 344, 345, 350, 351, 352, 365, 373, 380, 386, 394, 401, 415, 418, 427, 428, 434, 441, 442, 447, 448, 449, 456, 463, 470, 477, 484, 491, 497, 504, 511, 518, 525, 570, 571, 574, 577, 580, 581, 582, 583, 584, 589, 596, 605, 614, 616, 644, 646, 648, 658, 659, 668, 674, 675, 676, 681, 682, 686, 687, 702, 703, 704, 705, 739, 741, 744, 745, 748, 749, 750, 751, 752, 753, 768, 769, 770, 771, 774, 775, 776, 777, 791, 794, 797, 800, 810, 817, 818, 820, 821, 836, 893, 894, 979, 982, 983, 984, 990, 991, 992, 993, 994, 996, 997, 998, 999, 1001, 1002, 1003, 1006, 1007, 1010, 1011, 1012, 1013, 1018, 1019, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1030, 1031, 1032, 1040, 1045, 1047, 1054, 1057 }

C grade { 897, 898, 1000, 1050, 1051 }

F normal fail { 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 83, 89, 92, 114, 120, 126, 188, 189, 190, 191, 205, 239, 250, 259, 268, 280, 289, 298, 307, 316, 325, 335, 336, 337, 342, 343, 349, 356, 357, 358, 363, 364, 370, 371, 372, 377, 378, 379, 384, 385, 391, 392, 393, 398, 399, 400, 405, 406, 407, 412, 413, 414, 419, 420, 421, 422, 423, 425, 426, 432, 433, 439, 440, 446, 453, 454, 455, 460, 461, 462, 467, 468, 469, 475, 476, 481, 482, 483, 488, 489, 490, 495, 496, 502, 503, 508, 509, 510, 515, 517, 522, 523, 524, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 590, 591, 592, 593, 594, 595, 677, 678, 679, 683, 684, 685, 761, 762, 763, 764, 765, 766, 767, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 867, 868, 869, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 970, 971, 972, 976, 977, 978, 1004, 1005, 1008, 1009, 1029, 1039, 1041, 1042, 1048, 1049 }

F(-1) timedout fail { 474, 827, 833 }

F(-2) exception fail { 493, 516, 544, 757, 758, 759, 760, 772, 773, 778, 779, 861, 862, 863, 864, 865, 866, 1028, 1038 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 149, 150, 152, 153, 155, 156, 158, 159, 161, 162, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 260, 261, 262, 263, 269, 270, 271, 272, 281, 282, 283, 284, 290, 291, 292, 293, 299, 300, 301, 302, 308, 309, 310, 311, 317, 318, 319, 320, 326, 327, 328, 329, 338, 344, 345, 350, 351, 352, 359, 365, 366, 373, 380, 386, 387, 394, 401, 408, 415, 418, 427, 428, 434, 435, 441, 442, 447, 448, 449, 456, 463, 470, 477, 484, 491, 497, 498, 504, 511, 518, 525, 560, 563, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 680, 681, 682, 683, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 746, 747, 748, 749, 753, 754, 755, 756, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 792, 795, 798, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 830, 836, 838, 839, 840, 841, 855, 856, 857, 858, 859, 872, 873, 874, 875, 876, 877, 878, 879, 880, 885, 886, 887, 893, 894, 897, 898, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 968, 969, 973, 974, 975, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1040, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059 }

C grade { }

F normal fail { }

F(-1) timedout fail { 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 145, 148, 151, 154, 157, 160, 163, 166, 188, 189, 190, 191, 205, 239, 250, 255, 256, 257, 258, 259, 264, 265, 266, 267, 268, 273, 274, 275, 276, 277, 278, 279, 280, 285, 286, 287, 288, 289, 294, 295, 296, 297, 298, 303, 304, 305, 306, 307, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 330, 331, 332, 333, 335, 336, 337, 342, 343, 349, 356, 357, 358, 363, 364, 370, 371, 372, 377, 378, 379, 384, 385, 391, 392, 393, 398, 399, 400, 405, 406, 407, 412, 413, 414, 419, 420, 421, 422, 423, 425, 426, 432, 433, 439, 440, 446, 453, 454, 455, 460, 461, 462, 467, 468, 469, 474, 475, 476, 481, 482, 483, 488, 489, 490, 495, 496, 502, 503, 508, 509, 510, 515, 516, 517, 522, 523, 524, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 561, 562, 564, 565, 590, 591, 592, 593, 594, 595, 621, 622, 623, 651, 652, 653, 678, 679, 684, 685, 743, 744, 745, 750, 751, 752, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 790, 791, 793, 794, 796, 797, 799, 800, 801, 826, 827, 828, 829, 831, 832, 833, 834, 835, 837, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 970, 971, 972, 976, 977, 978, 998, 999, 1000, 1001, 1002, 1037, 1038, 1039, 1041, 1050, 1051 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 8, 82, 96, 97, 113, 132, 193, 194, 196, 197, 198, 199, 222, 223, 224, 225, 226, 227, 251, 252, 253, 254, 260, 261, 262, 263, 269, 270, 271, 272, 281, 282, 283, 284, 299, 300, 301, 302, 308, 309, 310, 311, 317, 318, 319, 320, 329, 580, 596, 599, 600, 605, 606, 608, 609, 618, 628, 638, 648, 668, 674, 680, 730, 731, 732, 739, 740, 741, 746, 747, 748, 749, 755, 756, 855, 857, 1003, 1018, 1026, 1027, 1030, 1031, 1032, 1043, 1045, 1055 }

B grade { 2, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 81, 83, 85, 87, 88, 105, 112, 114, 131, 133, 134, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 192, 195, 221, 228, 290, 291, 292, 293, 326, 327, 328, 568, 569, 574, 575, 576, 577, 581, 582, 583, 584, 585, 586, 597, 598, 601, 602, 607, 610, 611, 619, 621, 623, 629, 631, 633, 639, 641, 643, 658, 688, 689, 691, 692, 697, 700, 703, 705, 706, 715, 724, 727, 733, 736, 753, 754, 783, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 838, 839, 840, 841, 856, 872, 873, 874, 878, 879, 880, 885, 886, 887, 893, 894, 897, 898, 901, 902, 903, 907, 908, 909, 913, 914, 915, 919, 920, 921, 925, 926, 927, 931, 932, 933, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 968, 969, 973, 974, 975, 983, 984, 1028, 1044, 1050, 1051 }

C grade { 566, 567, 1052, 1053, 1054 }

F normal fail { 3, 4, 5, 6, 7, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 59, 60, 61, 62, 63, 64, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 84, 86, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 188, 189, 190, 191, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 264, 265, 266, 267, 268, 273, 274, 275, 276, 277, 278, 279, 280, 285, 286, 287, 288, 289, 294, 295, 296, 297, 298, 303, 304, 305, 306, 307, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 330, 331, 332, 333, 335, 336, 337, 338, 342, 343, 344, 345, 349, 350, 351, 352, 356, 357, 358, 359, 363, 364, 365, 366, 370, 371, 372, 373, 377, 378, 379, 380, 384, 385, 386, 387, 392, 393, 394, 398, 399, 400, 401, 405, 406, 407, 408, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 433, 434, 435, 441, 442, 447, 448, 449, 455, 456, 463, 467, 468, 469, 470, 474, 475, 476, 477, 481, 482, 483, 484, 488, 489, 490, 491, 495, 496, 497, 498, 502, 503, 504, 508, 509, 510, 511, 515, 516, 517, 518, 522, 523, 524, 525, 530, 531, 532, 540, 541, 542, 543, 545, 546, 547, 548, 549, 556, 557, 558, 560, 561, 563, 572, 573, 578, 579, 590, 591, 593, 594, 603, 604, 612, 613, 614, 615, 616, 617, 620, 622, 624, 625, 626, 627, 630, 632, 634, 635, 636, 637, 640, 642, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 670, 671, 672, 673, 675, 676, 677, 681, 682, 683, 684, 686, 687, 694, 695, 762, 763, 764, 765, 769, 770, 771, 772, 773, 775, 776, 777, 778, 780, 781, 784, 785, 786, 787, 788, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 862, 863, 864, 865, 866, 868, 869, 871, 875, 876, 877, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 904, 922, 937, 938, 939, 940, 941, 942, 943, 944, 949, 966, 970, 971, 972, 976, 977, 978, 979, 980, 981, 982, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1046, 1047, 1048, 1049, 1056, 1057, 1058, 1059 }

F(-1) timedout fail { 49, 50, 55, 56, 57, 58, 65, 66, 67, 68, 69, 369, 383, 390, 391, 404, 411, 412, 424, 431, 432, 438, 439, 440, 445, 446, 452, 453, 454, 459, 460, 461, 462, 528, 529, 533, 534, 535, 536, 537, 538, 539, 544, 550, 551, 552, 553, 554, 555, 559, 562, 564, 565, 570, 571, 587, 588, 589, 592, 595, 678, 679, 685, 690, 693, 696, 698, 699, 701, 702, 704, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 725, 726, 728, 729, 734, 735, 737, 738, 742, 743, 744, 745, 750, 751, 752, 757, 758, 759, 760, 761, 766, 767, 768, 774, 779, 782, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 854, 858, 859, 860, 861, 867, 905, 906, 910, 911, 912, 916, 917, 918, 923, 924, 928, 929, 930, 934, 935, 936, 950, 951, 956, 957, 958, 963, 964, 965, 1013, 1029, 1042 }

F(-2) exception fail { 250 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	21	34	19	21	21
N.S.	1	1.00	1.00	0.77	0.95	1.55	0.86	0.95	0.95
time (sec)	N/A	0.176	0.067	0.203	0.261	0.253	0.134	0.278	0.110

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	44	21	47	185	21	21
N.S.	1	1.00	1.00	2.00	0.95	2.14	8.41	0.95	0.95
time (sec)	N/A	0.200	0.090	2.945	0.269	0.245	3.537	0.270	0.095

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	44	21	47	0	21	21
N.S.	1	1.00	1.00	2.00	0.95	2.14	0.00	0.95	0.95
time (sec)	N/A	0.220	0.383	2.888	0.269	0.249	0.000	0.269	2.203

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	44	21	47	0	21	21
N.S.	1	1.00	1.00	2.00	0.95	2.14	0.00	0.95	0.95
time (sec)	N/A	0.223	0.271	0.584	0.269	0.265	0.000	0.261	2.151

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	40	69	89	0	51	57
N.S.	1	1.00	1.00	1.82	3.14	4.05	0.00	2.32	2.59
time (sec)	N/A	0.224	0.294	0.545	0.264	0.262	0.000	0.269	0.466

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	44	21	47	0	21	21
N.S.	1	1.00	1.00	2.00	0.95	2.14	0.00	0.95	0.95
time (sec)	N/A	0.227	0.348	0.627	0.266	0.256	0.000	0.261	0.166

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	40	38	89	0	48	57
N.S.	1	1.00	1.00	1.82	1.73	4.05	0.00	2.18	2.59
time (sec)	N/A	0.223	0.525	0.608	0.264	0.257	0.000	0.266	2.506

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	13	22	19	29	13
N.S.	1	1.00	2.47	0.93	0.87	1.47	1.27	1.93	0.87
time (sec)	N/A	0.181	0.017	0.257	0.173	0.246	0.079	0.260	0.060

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	68	49	35	19
N.S.	1	1.00	1.00	1.05	1.00	3.58	2.58	1.84	1.00
time (sec)	N/A	0.193	0.007	6.837	0.176	0.258	0.334	0.264	2.192

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	37	39	54	373	175	638	327	135
N.S.	1	0.95	1.00	1.38	9.56	4.49	16.36	8.38	3.46
time (sec)	N/A	0.233	0.047	197.020	0.307	0.262	1.214	0.334	2.423

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	54	49	82	686	379	2574	722	255
N.S.	1	0.92	0.83	1.39	11.63	6.42	43.63	12.24	4.32
time (sec)	N/A	0.245	0.138	0.144	0.303	0.267	4.323	0.379	2.637

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	68	49	35	19
N.S.	1	1.00	1.00	1.05	1.00	3.58	2.58	1.84	1.00
time (sec)	N/A	0.200	0.007	4.891	0.172	0.257	0.371	0.297	2.207

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	39	44	55	293	189	648	325	132
N.S.	1	0.98	1.10	1.38	7.32	4.72	16.20	8.12	3.30
time (sec)	N/A	0.241	0.215	0.132	0.294	0.266	1.216	0.319	0.169

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	54	77	82	558	407	2351	721	254
N.S.	1	0.92	1.31	1.39	9.46	6.90	39.85	12.22	4.31
time (sec)	N/A	0.249	0.256	0.071	0.307	0.262	4.536	0.365	2.372

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	51	23	33	39	40	92	32	18
N.S.	1	1.11	0.50	0.72	0.85	0.87	2.00	0.70	0.39
time (sec)	N/A	0.277	0.069	3.213	0.173	0.242	0.162	0.274	0.101

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	79	40	61	88	90	136	88	43
N.S.	1	1.14	0.58	0.88	1.28	1.30	1.97	1.28	0.62
time (sec)	N/A	0.373	0.066	50.433	0.178	0.254	0.329	0.284	2.254

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	107	52	79	110	138	189	116	53
N.S.	1	1.16	0.57	0.86	1.20	1.50	2.05	1.26	0.58
time (sec)	N/A	0.482	0.085	0.049	0.177	0.238	0.682	0.278	2.397

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	77	40	56	88	90	136	88	42
N.S.	1	1.15	0.60	0.84	1.31	1.34	2.03	1.31	0.63
time (sec)	N/A	0.342	0.035	22.627	0.177	0.248	0.340	0.272	2.259

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	105	33	74	66	97	189	60	32
N.S.	1	1.17	0.37	0.82	0.73	1.08	2.10	0.67	0.36
time (sec)	N/A	0.459	0.063	0.347	0.175	0.253	0.690	0.266	0.197

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	133	62	92	132	197	231	144	65
N.S.	1	1.18	0.55	0.81	1.17	1.74	2.04	1.27	0.58
time (sec)	N/A	0.580	0.117	0.056	0.176	0.252	1.304	0.284	2.394

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	103	52	66	110	138	189	116	53
N.S.	1	1.17	0.59	0.75	1.25	1.57	2.15	1.32	0.60
time (sec)	N/A	0.430	0.059	127.172	0.174	0.261	0.674	0.283	2.416

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	131	62	84	132	195	231	144	65
N.S.	1	1.18	0.56	0.76	1.19	1.76	2.08	1.30	0.59
time (sec)	N/A	0.546	0.081	0.046	0.178	0.250	1.313	0.278	0.251

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	159	43	102	86	179	277	88	42
N.S.	1	1.19	0.32	0.76	0.64	1.34	2.07	0.66	0.31
time (sec)	N/A	0.698	0.108	0.085	0.172	0.250	2.623	0.281	2.687

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	15	31	12	50	60	0	41	30
N.S.	1	1.36	2.82	1.09	4.55	5.45	0.00	3.73	2.73
time (sec)	N/A	0.183	0.024	0.388	0.262	0.239	0.000	0.282	2.149

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	20	42	23	61	155	0	64	52
N.S.	1	0.87	1.83	1.00	2.65	6.74	0.00	2.78	2.26
time (sec)	N/A	0.197	0.119	0.893	0.177	0.243	0.000	0.274	0.069

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	28	36	23	88	371	0	93	78
N.S.	1	1.04	1.33	0.85	3.26	13.74	0.00	3.44	2.89
time (sec)	N/A	0.208	0.027	2.802	0.252	0.263	0.000	0.272	2.140

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	32	57	33	108	697	0	88	133
N.S.	1	0.84	1.50	0.87	2.84	18.34	0.00	2.32	3.50
time (sec)	N/A	0.221	0.089	7.685	0.178	0.261	0.000	0.265	2.162

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	41	46	33	131	1073	0	122	169
N.S.	1	1.02	1.15	0.82	3.28	26.82	0.00	3.05	4.22
time (sec)	N/A	0.222	0.065	21.980	0.265	0.256	0.000	0.280	2.181

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	29	29	25	43	103	0	54	48
N.S.	1	1.21	1.21	1.04	1.79	4.29	0.00	2.25	2.00
time (sec)	N/A	0.208	0.014	0.677	0.260	0.241	0.000	0.264	2.159

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	28	13	32	18	81	0	18	18
N.S.	1	1.22	0.57	1.39	0.78	3.52	0.00	0.78	0.78
time (sec)	N/A	0.231	0.051	2.356	0.171	0.248	0.000	0.291	0.080

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	60	29	47	90	511	0	102	107
N.S.	1	1.22	0.59	0.96	1.84	10.43	0.00	2.08	2.18
time (sec)	N/A	0.228	0.012	6.325	0.261	0.263	0.000	0.287	2.245

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	42	46	44	94	230	0	60	152
N.S.	1	1.11	1.21	1.16	2.47	6.05	0.00	1.58	4.00
time (sec)	N/A	0.226	0.130	16.523	0.178	0.250	0.000	0.280	2.206

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	91	29	60	136	1183	0	124	210
N.S.	1	1.30	0.41	0.86	1.94	16.90	0.00	1.77	3.00
time (sec)	N/A	0.245	0.015	39.874	0.258	0.258	0.000	0.279	0.079

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	29	34	25	91	379	0	93	78
N.S.	1	1.04	1.21	0.89	3.25	13.54	0.00	3.32	2.79
time (sec)	N/A	0.218	0.031	1.220	0.257	0.251	0.000	0.267	0.079

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	52	86	43	106	709	0	110	111
N.S.	1	1.06	1.76	0.88	2.16	14.47	0.00	2.24	2.27
time (sec)	N/A	0.224	0.120	4.115	0.175	0.260	0.000	0.282	2.212

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	61	43	102	774	0	96	96
N.S.	1	0.95	1.42	1.00	2.37	18.00	0.00	2.23	2.23
time (sec)	N/A	0.231	0.060	11.823	0.268	0.263	0.000	0.279	2.311

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	64	101	53	149	1573	0	128	192
N.S.	1	0.97	1.53	0.80	2.26	23.83	0.00	1.94	2.91
time (sec)	N/A	0.247	0.113	30.047	0.186	0.252	0.000	0.281	2.228

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	53	54	53	181	2103	0	171	187
N.S.	1	0.91	0.93	0.91	3.12	36.26	0.00	2.95	3.22
time (sec)	N/A	0.244	0.351	97.623	0.256	0.258	0.000	0.277	0.081

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	43	33	33	90	515	0	80	129
N.S.	1	1.16	0.89	0.89	2.43	13.92	0.00	2.16	3.49
time (sec)	N/A	0.213	0.011	2.813	0.267	0.249	0.000	0.269	0.080

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	42	45	45	90	229	0	60	153
N.S.	1	1.14	1.22	1.22	2.43	6.19	0.00	1.62	4.14
time (sec)	N/A	0.224	0.110	9.110	0.173	0.240	0.000	0.287	2.130

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	74	33	65	132	1176	0	124	187
N.S.	1	1.12	0.50	0.98	2.00	17.82	0.00	1.88	2.83
time (sec)	N/A	0.236	0.012	22.996	0.263	0.252	0.000	0.294	2.123

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	56	43	45	90	330	0	31	31
N.S.	1	1.06	0.81	0.85	1.70	6.23	0.00	0.58	0.58
time (sec)	N/A	0.227	0.042	72.999	0.172	0.230	0.000	0.277	0.063

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	105	33	78	178	2092	0	148	291
N.S.	1	1.18	0.37	0.88	2.00	23.51	0.00	1.66	3.27
time (sec)	N/A	0.257	0.013	151.886	0.265	0.264	0.000	0.281	2.161

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	41	46	33	133	1082	0	122	169
N.S.	1	1.05	1.18	0.85	3.41	27.74	0.00	3.13	4.33
time (sec)	N/A	0.220	0.088	5.368	0.265	0.259	0.000	0.284	0.066

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	82	123	61	155	1591	0	130	214
N.S.	1	1.17	1.76	0.87	2.21	22.73	0.00	1.86	3.06
time (sec)	N/A	0.234	0.130	15.399	0.176	0.265	0.000	0.286	2.134

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	53	56	61	179	2114	0	171	187
N.S.	1	0.91	0.97	1.05	3.09	36.45	0.00	2.95	3.22
time (sec)	N/A	0.236	0.132	36.280	0.265	0.272	0.000	0.283	2.202

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	94	139	71	195	2802	0	152	295
N.S.	1	1.06	1.56	0.80	2.19	31.48	0.00	1.71	3.31
time (sec)	N/A	0.266	0.132	115.248	0.203	0.267	0.000	0.282	0.098

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	65	91	70	150	2231	0	124	205
N.S.	1	0.94	1.32	1.01	2.17	32.33	0.00	1.80	2.97
time (sec)	N/A	0.245	0.059	242.163	0.267	0.255	0.000	0.280	2.203

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	111	59	0	0	997	0	0	0
N.S.	1	1.05	0.56	0.00	0.00	9.41	0.00	0.00	0.00
time (sec)	N/A	0.456	0.048	0.000	0.000	0.273	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	86	59	0	0	591	0	0	0
N.S.	1	1.06	0.73	0.00	0.00	7.30	0.00	0.00	0.00
time (sec)	N/A	0.336	0.034	0.000	0.000	0.269	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	84	59	0	0	310	0	0	0
N.S.	1	1.06	0.75	0.00	0.00	3.92	0.00	0.00	0.00
time (sec)	N/A	0.322	0.037	0.000	0.000	0.257	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	58	59	0	0	142	0	0	0
N.S.	1	1.07	1.09	0.00	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.227	0.024	0.000	0.000	0.260	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	58	57	0	0	144	0	0	0
N.S.	1	1.07	1.06	0.00	0.00	2.67	0.00	0.00	0.00
time (sec)	N/A	0.225	0.020	0.000	0.000	0.253	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	84	57	0	0	311	0	0	0
N.S.	1	1.06	0.72	0.00	0.00	3.94	0.00	0.00	0.00
time (sec)	N/A	0.320	0.022	0.000	0.000	0.253	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	86	59	0	0	598	0	0	0
N.S.	1	1.06	0.73	0.00	0.00	7.38	0.00	0.00	0.00
time (sec)	N/A	0.313	0.024	0.000	0.000	0.275	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	111	59	0	0	1001	0	0	0
N.S.	1	1.05	0.56	0.00	0.00	9.44	0.00	0.00	0.00
time (sec)	N/A	0.430	0.027	0.000	0.000	0.270	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	137	59	0	0	1042	0	0	0
N.S.	1	0.88	0.38	0.00	0.00	6.72	0.00	0.00	0.00
time (sec)	N/A	0.412	0.040	0.000	0.000	0.277	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	137	59	0	0	751	0	0	0
N.S.	1	0.88	0.38	0.00	0.00	4.85	0.00	0.00	0.00
time (sec)	N/A	0.405	0.042	0.000	0.000	0.280	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	59	0	0	1003	0	0	0
N.S.	1	1.00	0.24	0.00	0.00	4.13	0.00	0.00	0.00
time (sec)	N/A	0.450	0.039	0.000	0.000	0.275	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	218	215	59	0	0	727	0	0	0
N.S.	1	0.99	0.27	0.00	0.00	3.33	0.00	0.00	0.00
time (sec)	N/A	0.346	0.027	0.000	0.000	0.278	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	109	59	0	0	572	0	0	0
N.S.	1	0.85	0.46	0.00	0.00	4.47	0.00	0.00	0.00
time (sec)	N/A	0.306	0.023	0.000	0.000	0.279	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	109	59	0	0	578	0	0	0
N.S.	1	0.85	0.46	0.00	0.00	4.52	0.00	0.00	0.00
time (sec)	N/A	0.305	0.021	0.000	0.000	0.269	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	218	217	57	0	0	723	0	0	0
N.S.	1	1.00	0.26	0.00	0.00	3.32	0.00	0.00	0.00
time (sec)	N/A	0.339	0.018	0.000	0.000	0.281	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	241	57	0	0	1013	0	0	0
N.S.	1	0.99	0.23	0.00	0.00	4.17	0.00	0.00	0.00
time (sec)	N/A	0.450	0.023	0.000	0.000	0.275	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	137	59	0	0	749	0	0	0
N.S.	1	0.88	0.38	0.00	0.00	4.83	0.00	0.00	0.00
time (sec)	N/A	0.400	0.022	0.000	0.000	0.273	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	137	59	0	0	1056	0	0	0
N.S.	1	0.88	0.38	0.00	0.00	6.81	0.00	0.00	0.00
time (sec)	N/A	0.418	0.024	0.000	0.000	0.272	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	61	93	0	0	6
N.S.	1	1.00	1.00	0.00	3.81	5.81	0.00	0.00	0.38
time (sec)	N/A	0.194	0.008	0.000	0.278	0.276	0.000	0.000	2.341

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	61	93	0	0	0
N.S.	1	1.00	1.00	0.00	3.81	5.81	0.00	0.00	0.00
time (sec)	N/A	0.190	0.012	0.000	0.281	0.254	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	0	54	0	17	31
N.S.	1	1.00	1.00	0.70	0.00	5.40	0.00	1.70	3.10
time (sec)	N/A	0.189	0.007	2.121	0.000	0.246	0.000	0.259	2.189

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	21	23	21	41	86	0	32	49
N.S.	1	0.91	1.00	0.91	1.78	3.74	0.00	1.39	2.13
time (sec)	N/A	0.193	0.011	0.273	0.273	0.253	0.000	0.275	2.245

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	22	21	33	54	31	0	41	49
N.S.	1	1.05	1.00	1.57	2.57	1.48	0.00	1.95	2.33
time (sec)	N/A	0.210	0.093	0.271	0.192	0.253	0.000	0.274	2.387

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	52	62	91	463	0	97	107
N.S.	1	1.00	1.06	1.27	1.86	9.45	0.00	1.98	2.18
time (sec)	N/A	0.210	0.015	0.892	0.270	0.252	0.000	0.295	2.492

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	31	37	51	98	93	0	67	131
N.S.	1	0.84	1.00	1.38	2.65	2.51	0.00	1.81	3.54
time (sec)	N/A	0.218	0.099	0.389	0.187	0.260	0.000	0.313	2.494

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	25	25	25	56	197	0	60	48
N.S.	1	0.89	0.89	0.89	2.00	7.04	0.00	2.14	1.71
time (sec)	N/A	0.206	0.014	0.342	0.272	0.252	0.000	0.276	0.086

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	51	31	39	64	54	0	71	50
N.S.	1	1.28	0.78	0.98	1.60	1.35	0.00	1.78	1.25
time (sec)	N/A	0.220	0.166	0.338	0.184	0.238	0.000	0.295	2.241

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	35	35	41	103	742	0	99	97
N.S.	1	0.81	0.81	0.95	2.40	17.26	0.00	2.30	2.26
time (sec)	N/A	0.231	0.037	0.432	0.276	0.259	0.000	0.324	2.240

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	32	38	33	71	290	0	55	77
N.S.	1	0.84	1.00	0.87	1.87	7.63	0.00	1.45	2.03
time (sec)	N/A	0.205	0.013	0.608	0.275	0.256	0.000	0.271	0.096

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	33	40	52	79	63	0	63	78
N.S.	1	0.87	1.05	1.37	2.08	1.66	0.00	1.66	2.05
time (sec)	N/A	0.217	0.110	0.618	0.191	0.256	0.000	0.302	2.186

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	62	78	81	116	851	0	117	136
N.S.	1	0.94	1.18	1.23	1.76	12.89	0.00	1.77	2.06
time (sec)	N/A	0.237	0.019	1.212	0.273	0.257	0.000	0.297	2.170

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	39	34	33	81	457	0	84	77
N.S.	1	0.98	0.85	0.82	2.02	11.42	0.00	2.10	1.92
time (sec)	N/A	0.214	0.025	1.497	0.277	0.267	0.000	0.277	0.122

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	23	54	17	23	24
N.S.	1	1.00	1.00	1.09	2.09	4.91	1.55	2.09	2.18
time (sec)	N/A	0.175	0.007	0.195	0.197	0.245	0.095	0.261	2.085

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	84	22	27	13
N.S.	1	1.00	1.00	0.93	0.87	5.60	1.47	1.80	0.87
time (sec)	N/A	0.183	0.009	0.554	0.196	0.243	0.130	0.274	0.081

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	36	115	39	0	31
N.S.	1	1.00	1.00	1.06	2.25	7.19	2.44	0.00	1.94
time (sec)	N/A	0.198	0.070	1.228	0.314	0.273	0.158	0.000	2.225

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	138	0	31	31
N.S.	1	1.00	1.00	0.93	0.87	9.20	0.00	2.07	2.07
time (sec)	N/A	0.198	0.005	0.928	0.188	0.233	0.000	0.277	0.080

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	208	44	37	230
N.S.	1	1.00	1.00	0.93	0.87	13.87	2.93	2.47	15.33
time (sec)	N/A	0.197	0.004	1.452	0.186	0.240	0.251	0.305	0.123

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	69	0	38	42
N.S.	1	1.00	1.00	1.05	1.00	3.63	0.00	2.00	2.21
time (sec)	N/A	0.204	0.014	6.535	0.190	0.269	0.000	0.294	2.205

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	25	27	24	148	172	41	45	48
N.S.	1	0.93	1.00	0.89	5.48	6.37	1.52	1.67	1.78
time (sec)	N/A	0.201	0.092	0.453	0.193	0.240	0.182	0.275	0.085

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	29	31	26	214	345	46	45	251
N.S.	1	0.94	1.00	0.84	6.90	11.13	1.48	1.45	8.10
time (sec)	N/A	0.216	0.144	3.870	0.195	0.248	0.397	0.291	2.319

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	36	34	32	209	345	219	0	0	101
N.S.	1	0.94	0.89	5.81	9.58	6.08	0.00	0.00	2.81
time (sec)	N/A	0.232	0.272	6.181	0.336	0.244	0.000	0.000	2.382

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	36	56	26	276	304	0	53	270
N.S.	1	1.16	1.81	0.84	8.90	9.81	0.00	1.71	8.71
time (sec)	N/A	0.219	0.115	5.951	0.199	0.235	0.000	0.286	0.137

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	40	29	26	352	551	0	148	168
N.S.	1	1.14	0.83	0.74	10.06	15.74	0.00	4.23	4.80
time (sec)	N/A	0.225	0.187	0.412	0.308	0.253	0.000	0.304	2.464

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	46	73	55	504	180	0	0	115
N.S.	1	1.15	1.82	1.38	12.60	4.50	0.00	0.00	2.88
time (sec)	N/A	0.239	0.637	117.675	0.300	0.266	0.000	0.000	2.326

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	43	66	269	0	76	82
N.S.	1	1.00	1.00	1.26	1.94	7.91	0.00	2.24	2.41
time (sec)	N/A	0.259	0.013	1.013	0.273	0.256	0.000	0.275	0.097

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	60	74	75	112	814	0	102	186
N.S.	1	1.09	1.35	1.36	2.04	14.80	0.00	1.85	3.38
time (sec)	N/A	0.350	0.017	1.245	0.304	0.250	0.000	0.278	2.231

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	60	55	58	110	808	0	98	215
N.S.	1	1.09	1.00	1.05	2.00	14.69	0.00	1.78	3.91
time (sec)	N/A	0.343	0.013	2.919	0.288	0.277	0.000	0.291	2.194

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	191	185	29	35	129
N.S.	1	1.00	1.00	0.86	9.10	8.81	1.38	1.67	6.14
time (sec)	N/A	0.191	0.020	2.926	0.221	0.240	0.289	0.245	2.237

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	A	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	25	0	371	634	34	35	520
N.S.	1	1.00	1.00	0.00	14.84	25.36	1.36	1.40	20.80
time (sec)	N/A	0.205	0.025	180.000	0.198	0.246	2.418	0.265	2.304

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	48	48	46	85	925	0	73	200
N.S.	1	1.26	1.26	1.21	2.24	24.34	0.00	1.92	5.26
time (sec)	N/A	0.372	0.008	16.046	0.281	0.252	0.000	0.268	0.061

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	46	36	36	85	925	0	73	206
N.S.	1	1.28	1.00	1.00	2.36	25.69	0.00	2.03	5.72
time (sec)	N/A	0.363	0.008	169.339	0.279	0.251	0.000	0.265	2.166

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	45	67	26	857	778	0	66	820
N.S.	1	1.36	2.03	0.79	25.97	23.58	0.00	2.00	24.85
time (sec)	N/A	0.210	0.037	0.090	0.205	0.234	0.000	0.262	2.130

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	21	42	21	59	113	0	44	53
N.S.	1	0.91	1.83	0.91	2.57	4.91	0.00	1.91	2.30
time (sec)	N/A	0.195	0.050	0.298	0.195	0.247	0.000	0.258	0.075

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	28	22	33	56	31	0	45	49
N.S.	1	1.27	1.00	1.50	2.55	1.41	0.00	2.05	2.23
time (sec)	N/A	0.211	0.011	0.311	0.196	0.254	0.000	0.282	2.089

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	51	85	62	108	612	0	105	112
N.S.	1	1.04	1.73	1.27	2.20	12.49	0.00	2.14	2.29
time (sec)	N/A	0.212	0.104	0.975	0.203	0.291	0.000	0.293	0.082

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	42	37	51	100	89	0	71	131
N.S.	1	1.14	1.00	1.38	2.70	2.41	0.00	1.92	3.54
time (sec)	N/A	0.211	0.016	0.556	0.196	0.247	0.000	0.288	2.122

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	28	25	23	70	203	105	63	49
N.S.	1	1.04	0.93	0.85	2.59	7.52	3.89	2.33	1.81
time (sec)	N/A	0.210	0.017	0.499	0.185	0.253	0.826	0.273	0.078

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	62	31	39	66	60	0	71	50
N.S.	1	1.55	0.78	0.98	1.65	1.50	0.00	1.78	1.25
time (sec)	N/A	0.225	0.175	0.552	0.188	0.252	0.000	0.279	0.090

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	35	43	120	743	0	102	97
N.S.	1	0.95	0.81	1.00	2.79	17.28	0.00	2.37	2.26
time (sec)	N/A	0.229	0.036	0.772	0.196	0.264	0.000	0.302	2.100

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	33	60	31	87	357	0	69	81
N.S.	1	0.87	1.58	0.82	2.29	9.39	0.00	1.82	2.13
time (sec)	N/A	0.218	0.088	0.891	0.193	0.258	0.000	0.287	2.071

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	42	38	52	79	63	0	67	78
N.S.	1	1.11	1.00	1.37	2.08	1.66	0.00	1.76	2.05
time (sec)	N/A	0.225	0.015	1.293	0.192	0.249	0.000	0.288	0.106

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	63	103	81	133	1077	0	123	140
N.S.	1	0.95	1.56	1.23	2.02	16.32	0.00	1.86	2.12
time (sec)	N/A	0.244	0.126	2.700	0.189	0.282	0.000	0.318	2.119

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	41	35	33	95	457	0	85	77
N.S.	1	1.05	0.90	0.85	2.44	11.72	0.00	2.18	1.97
time (sec)	N/A	0.221	0.024	2.310	0.199	0.257	0.000	0.275	0.112

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	25	56	17	25	24
N.S.	1	1.00	1.00	1.09	2.27	5.09	1.55	2.27	2.18
time (sec)	N/A	0.188	0.008	0.144	0.209	0.232	0.779	0.281	0.061

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	86	22	27	13
N.S.	1	1.00	1.00	0.93	0.87	5.73	1.47	1.80	0.87
time (sec)	N/A	0.199	0.010	0.164	0.186	0.247	1.558	0.278	0.077

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	53	115	36	0	31
N.S.	1	1.00	1.00	1.06	3.31	7.19	2.25	0.00	1.94
time (sec)	N/A	0.203	0.016	0.324	0.349	0.254	2.192	0.000	2.096

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	139	0	31	31
N.S.	1	1.00	1.00	0.93	0.87	9.27	0.00	2.07	2.07
time (sec)	N/A	0.198	0.003	0.191	0.187	0.247	0.000	0.309	2.065

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	208	0	37	231
N.S.	1	1.00	1.00	0.93	0.87	13.87	0.00	2.47	15.40
time (sec)	N/A	0.200	0.004	0.214	0.197	0.245	0.000	0.318	2.086

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	70	0	39	43
N.S.	1	1.00	1.00	1.05	1.00	3.50	0.00	1.95	2.15
time (sec)	N/A	0.213	0.017	2.002	0.196	0.255	0.000	0.293	2.137

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	32	27	24	148	171	0	49	48
N.S.	1	1.19	1.00	0.89	5.48	6.33	0.00	1.81	1.78
time (sec)	N/A	0.208	0.011	0.212	0.199	0.250	0.000	0.306	2.082

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	36	31	27	214	343	0	49	252
N.S.	1	1.16	1.00	0.87	6.90	11.06	0.00	1.58	8.13
time (sec)	N/A	0.218	0.018	0.356	0.191	0.243	0.000	0.330	2.147

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	37	35	34	479	414	216	0	0	100
N.S.	1	0.95	0.92	12.95	11.19	5.84	0.00	0.00	2.70
time (sec)	N/A	0.236	0.050	0.855	0.340	0.263	0.000	0.000	2.175

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	42	75	45	84	387	0	84	87
N.S.	1	1.24	2.21	1.32	2.47	11.38	0.00	2.47	2.56
time (sec)	N/A	0.260	0.088	0.915	0.186	0.258	0.000	0.279	0.083

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	70	113	58	129	1109	0	106	219
N.S.	1	1.27	2.05	1.05	2.35	20.16	0.00	1.93	3.98
time (sec)	N/A	0.361	0.125	1.004	0.191	0.262	0.000	0.288	0.114

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	70	113	74	133	1114	0	110	190
N.S.	1	1.27	2.05	1.35	2.42	20.25	0.00	2.00	3.45
time (sec)	N/A	0.369	0.110	0.968	0.191	0.264	0.000	0.318	2.050

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	25	27	14	149	164	0	30	144
N.S.	1	1.47	1.59	0.82	8.76	9.65	0.00	1.76	8.47
time (sec)	N/A	0.208	0.035	0.476	0.191	0.248	0.000	0.263	2.074

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	139	222	0	29	210
N.S.	1	1.00	1.00	1.29	8.18	13.06	0.00	1.71	12.35
time (sec)	N/A	0.211	0.010	0.500	0.196	0.244	0.000	0.255	2.228

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	35	30	33	368	114	0	0	87
N.S.	1	1.35	1.15	1.27	14.15	4.38	0.00	0.00	3.35
time (sec)	N/A	0.226	0.080	34.492	0.321	0.268	0.000	0.000	2.243

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	60	95	46	98	1260	0	71	214
N.S.	1	1.58	2.50	1.21	2.58	33.16	0.00	1.87	5.63
time (sec)	N/A	0.397	0.028	1.077	0.208	0.247	0.000	0.268	2.090

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	35	47	20	431	430	0	48	413
N.S.	1	1.40	1.88	0.80	17.24	17.20	0.00	1.92	16.52
time (sec)	N/A	0.221	0.036	1.867	0.202	0.249	0.000	0.276	2.084

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	39	29	24	191	250	0	47	40
N.S.	1	1.34	1.00	0.83	6.59	8.62	0.00	1.62	1.38
time (sec)	N/A	0.203	0.014	0.279	0.199	0.254	0.000	0.285	2.094

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	45	33	26	435	442	0	54	372
N.S.	1	1.36	1.00	0.79	13.18	13.39	0.00	1.64	11.27
time (sec)	N/A	0.223	0.012	1.022	0.200	0.244	0.000	0.268	2.040

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	58	87	58	71	23
N.S.	1	1.00	0.96	0.89	2.15	3.22	2.15	2.63	0.85
time (sec)	N/A	0.202	0.028	0.427	0.207	0.260	0.174	0.271	0.166

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	59	75	61	78	23
N.S.	1	1.00	1.00	0.89	2.19	2.78	2.26	2.89	0.85
time (sec)	N/A	0.206	0.032	0.414	0.208	0.243	0.174	0.265	2.145

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	58	89	58	69	23
N.S.	1	1.00	0.96	0.89	2.15	3.30	2.15	2.56	0.85
time (sec)	N/A	0.203	0.013	0.424	0.199	0.250	0.165	0.267	2.119

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	59	77	58	74	23
N.S.	1	1.00	0.96	0.89	2.19	2.85	2.15	2.74	0.85
time (sec)	N/A	0.196	0.014	0.389	0.202	0.251	0.171	0.280	2.122

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	46	29	151	83	259	0	95	115
N.S.	1	1.24	0.78	4.08	2.24	7.00	0.00	2.57	3.11
time (sec)	N/A	0.323	0.347	0.237	0.285	0.261	0.000	0.285	2.641

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	42	30	149	87	216	0	86	121
N.S.	1	1.17	0.83	4.14	2.42	6.00	0.00	2.39	3.36
time (sec)	N/A	0.311	0.332	0.235	0.286	0.266	0.000	0.280	2.653

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	53	29	155	157	259	0	97	115
N.S.	1	1.43	0.78	4.19	4.24	7.00	0.00	2.62	3.11
time (sec)	N/A	0.317	0.324	0.247	0.216	0.253	0.000	0.281	0.524

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	50	32	153	160	216	0	90	121
N.S.	1	1.39	0.89	4.25	4.44	6.00	0.00	2.50	3.36
time (sec)	N/A	0.310	0.313	0.247	0.216	0.266	0.000	0.277	2.560

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	77	68	184	0	79	266
N.S.	1	1.00	0.75	2.14	1.89	5.11	0.00	2.19	7.39
time (sec)	N/A	0.243	0.150	0.431	0.284	0.251	0.000	0.288	3.044

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	75	67	156	0	70	268
N.S.	1	1.00	0.82	2.27	2.03	4.73	0.00	2.12	8.12
time (sec)	N/A	0.239	0.151	0.460	0.282	0.262	0.000	0.272	2.976

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	44	28	79	133	184	0	81	266
N.S.	1	1.22	0.78	2.19	3.69	5.11	0.00	2.25	7.39
time (sec)	N/A	0.250	0.157	0.203	0.204	0.256	0.000	0.285	2.332

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	41	29	77	129	156	0	74	269
N.S.	1	1.24	0.88	2.33	3.91	4.73	0.00	2.24	8.15
time (sec)	N/A	0.245	0.141	0.247	0.215	0.270	0.000	0.290	0.321

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	86	167	57	327	0	49	133
N.S.	1	1.00	2.97	5.76	1.97	11.28	0.00	1.69	4.59
time (sec)	N/A	0.260	0.041	0.343	0.286	0.249	0.000	0.267	2.288

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	102	205	105	902	0	97	173
N.S.	1	1.00	2.27	4.56	2.33	20.04	0.00	2.16	3.84
time (sec)	N/A	0.421	0.076	0.363	0.285	0.270	0.000	0.264	0.222

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	86	70	240	149	1737	0	120	0
N.S.	1	1.19	0.97	3.33	2.07	24.12	0.00	1.67	0.00
time (sec)	N/A	0.713	0.232	0.396	0.292	0.277	0.000	0.266	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	93	155	94	439	0	93	139
N.S.	1	1.00	3.21	5.34	3.24	15.14	0.00	3.21	4.79
time (sec)	N/A	0.258	0.042	0.327	0.210	0.247	0.000	0.260	0.158

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	110	197	140	1237	0	136	181
N.S.	1	1.00	2.39	4.28	3.04	26.89	0.00	2.96	3.93
time (sec)	N/A	0.418	0.076	0.330	0.212	0.267	0.000	0.261	2.281

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	94	70	230	186	2372	0	169	0
N.S.	1	1.29	0.96	3.15	2.55	32.49	0.00	2.32	0.00
time (sec)	N/A	0.737	0.238	0.387	0.197	0.269	0.000	0.266	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	148	49	87	0	49	65
N.S.	1	1.00	1.00	5.69	1.88	3.35	0.00	1.88	2.50
time (sec)	N/A	0.230	0.086	0.416	0.208	0.266	0.000	0.261	0.242

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	83	181	70	405	0	68	150
N.S.	1	1.00	2.37	5.17	2.00	11.57	0.00	1.94	4.29
time (sec)	N/A	0.289	0.068	1.136	0.291	0.258	0.000	0.265	2.245

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	42	120	246	0	51	0
N.S.	1	1.00	0.92	1.11	3.16	6.47	0.00	1.34	0.00
time (sec)	N/A	0.314	0.119	0.988	0.204	0.257	0.000	0.269	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	30	26	150	84	86	0	51	65
N.S.	1	1.15	1.00	5.77	3.23	3.31	0.00	1.96	2.50
time (sec)	N/A	0.233	0.080	0.186	0.206	0.272	0.000	0.268	2.243

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	90	172	103	617	0	104	156
N.S.	1	1.00	2.50	4.78	2.86	17.14	0.00	2.89	4.33
time (sec)	N/A	0.295	0.058	0.311	0.203	0.257	0.000	0.269	0.216

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	57	131	246	0	53	0
N.S.	1	1.00	0.90	1.46	3.36	6.31	0.00	1.36	0.00
time (sec)	N/A	0.320	0.126	0.554	0.199	0.242	0.000	0.274	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	86	167	59	327	0	53	133
N.S.	1	1.00	2.97	5.76	2.03	11.28	0.00	1.83	4.59
time (sec)	N/A	0.260	0.040	0.361	0.276	0.262	0.000	0.267	0.173

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	102	207	103	902	0	97	173
N.S.	1	1.00	2.27	4.60	2.29	20.04	0.00	2.16	3.84
time (sec)	N/A	0.401	0.075	0.392	0.287	0.260	0.000	0.266	0.213

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	86	115	238	149	1737	0	122	0
N.S.	1	1.19	1.60	3.31	2.07	24.12	0.00	1.69	0.00
time (sec)	N/A	0.714	0.230	0.444	0.303	0.274	0.000	0.270	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	93	155	90	439	0	91	139
N.S.	1	1.00	3.21	5.34	3.10	15.14	0.00	3.14	4.79
time (sec)	N/A	0.255	0.039	0.270	0.201	0.256	0.000	0.273	0.159

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	110	195	144	1237	0	142	183
N.S.	1	1.00	2.39	4.24	3.13	26.89	0.00	3.09	3.98
time (sec)	N/A	0.411	0.073	0.312	0.221	0.263	0.000	0.268	2.234

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	94	70	228	184	2372	0	167	0
N.S.	1	1.29	0.96	3.12	2.52	32.49	0.00	2.29	0.00
time (sec)	N/A	0.721	0.221	0.427	0.203	0.277	0.000	0.277	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	146	51	86	0	50	64
N.S.	1	1.00	1.00	5.62	1.96	3.31	0.00	1.92	2.46
time (sec)	N/A	0.228	0.081	0.434	0.197	0.250	0.000	0.263	0.221

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	83	183	70	405	0	68	148
N.S.	1	1.00	2.37	5.23	2.00	11.57	0.00	1.94	4.23
time (sec)	N/A	0.287	0.069	1.126	0.300	0.253	0.000	0.264	2.230

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	26	119	248	0	49	0
N.S.	1	1.00	0.92	0.68	3.13	6.53	0.00	1.29	0.00
time (sec)	N/A	0.313	0.124	0.954	0.202	0.249	0.000	0.264	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	30	26	152	80	87	0	50	66
N.S.	1	1.15	1.00	5.85	3.08	3.35	0.00	1.92	2.54
time (sec)	N/A	0.233	0.080	0.281	0.204	0.256	0.000	0.260	2.233

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	90	170	105	617	0	106	156
N.S.	1	1.00	2.50	4.72	2.92	17.14	0.00	2.94	4.33
time (sec)	N/A	0.292	0.061	0.441	0.206	0.250	0.000	0.289	2.248

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	36	132	243	0	51	0
N.S.	1	1.00	0.90	0.92	3.38	6.23	0.00	1.31	0.00
time (sec)	N/A	0.320	0.124	0.685	0.206	0.244	0.000	0.269	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	0	72	153	85	42
N.S.	1	1.00	1.00	0.93	0.00	1.67	3.56	1.98	0.98
time (sec)	N/A	0.223	0.156	0.442	0.000	0.245	0.323	0.288	0.169

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	0	120	405	120	76
N.S.	1	1.00	1.11	0.92	0.00	1.94	6.53	1.94	1.23
time (sec)	N/A	0.259	0.515	0.816	0.000	0.259	0.722	0.261	0.270

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	86	84	0	218	918	179	182
N.S.	1	1.00	0.95	0.92	0.00	2.40	10.09	1.97	2.00
time (sec)	N/A	0.287	0.326	1.496	0.000	0.263	1.911	0.278	0.554

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	106	89	0	192	1027	156	152
N.S.	1	1.00	1.20	1.01	0.00	2.18	11.67	1.77	1.73
time (sec)	N/A	0.276	0.501	1.815	0.000	0.265	1.557	0.294	2.695

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	0	414	2001	260	337
N.S.	1	1.00	1.10	0.92	0.00	2.88	13.90	1.81	2.34
time (sec)	N/A	0.353	1.165	4.585	0.000	0.248	5.481	0.282	2.614

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	177	190	0	731	3580	373	906
N.S.	1	1.00	0.91	0.97	0.00	3.75	18.36	1.91	4.65
time (sec)	N/A	0.399	1.082	10.555	0.000	0.273	16.688	0.282	2.725

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	0	71	153	85	42
N.S.	1	1.00	1.00	0.93	0.00	1.65	3.56	1.98	0.98
time (sec)	N/A	0.225	0.117	0.381	0.000	0.247	0.317	0.265	0.152

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	0	115	408	120	68
N.S.	1	1.00	1.11	0.92	0.00	1.85	6.58	1.94	1.10
time (sec)	N/A	0.246	0.484	0.808	0.000	0.255	0.703	0.265	0.247

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	0	217	921	179	180
N.S.	1	1.00	0.93	0.92	0.00	2.38	10.12	1.97	1.98
time (sec)	N/A	0.277	0.285	1.477	0.000	0.287	1.881	0.272	2.533

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	105	89	0	192	1027	156	115
N.S.	1	1.00	1.19	1.01	0.00	2.18	11.67	1.77	1.31
time (sec)	N/A	0.275	0.465	1.715	0.000	0.241	1.582	0.270	2.617

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	0	397	2008	260	337
N.S.	1	1.00	1.10	0.92	0.00	2.76	13.94	1.81	2.34
time (sec)	N/A	0.327	1.084	4.770	0.000	0.253	5.288	0.277	2.691

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	176	190	0	726	3582	373	908
N.S.	1	1.00	0.90	0.97	0.00	3.72	18.37	1.91	4.66
time (sec)	N/A	0.370	1.109	11.099	0.000	0.267	16.835	0.281	2.717

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	0	71	153	85	42
N.S.	1	1.00	1.00	0.93	0.00	1.65	3.56	1.98	0.98
time (sec)	N/A	0.226	0.154	0.461	0.000	0.255	0.327	0.264	0.150

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	0	119	408	120	68
N.S.	1	1.00	1.11	0.92	0.00	1.92	6.58	1.94	1.10
time (sec)	N/A	0.250	0.469	0.836	0.000	0.255	0.707	0.267	0.246

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	0	213	921	179	182
N.S.	1	1.00	0.93	0.92	0.00	2.34	10.12	1.97	2.00
time (sec)	N/A	0.284	0.314	1.619	0.000	0.263	1.956	0.274	2.565

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	74	63	0	114	408	124	76
N.S.	1	1.00	1.09	0.93	0.00	1.68	6.00	1.82	1.12
time (sec)	N/A	0.244	0.512	0.782	0.000	0.260	0.742	0.260	2.297

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	107	89	0	192	1027	156	135
N.S.	1	1.00	1.22	1.01	0.00	2.18	11.67	1.77	1.53
time (sec)	N/A	0.269	0.461	1.965	0.000	0.262	1.558	0.261	2.615

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	0	398	2001	260	337
N.S.	1	1.00	1.10	0.92	0.00	2.76	13.90	1.81	2.34
time (sec)	N/A	0.323	1.013	4.817	0.000	0.264	5.549	0.285	2.618

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	90	90	0	243	935	183	183
N.S.	1	1.00	0.93	0.93	0.00	2.51	9.64	1.89	1.89
time (sec)	N/A	0.286	0.371	1.326	0.000	0.249	1.941	0.286	0.552

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	0	443	2030	256	337
N.S.	1	1.00	1.11	0.92	0.00	3.21	14.71	1.86	2.44
time (sec)	N/A	0.335	1.126	4.009	0.000	0.262	5.342	0.278	2.645

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	176	190	0	729	3580	373	908
N.S.	1	1.00	0.90	0.97	0.00	3.74	18.36	1.91	4.66
time (sec)	N/A	0.405	1.044	11.361	0.000	0.268	16.865	0.300	2.795

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	103	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.331	0.717	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	240	0	0	0	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	2.150	0.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	99	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.797	0.000	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	103	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.319	0.520	0.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	15	15	12	27	17	20	25	6
N.S.	1	1.88	1.88	1.50	3.38	2.12	2.50	3.12	0.75
time (sec)	N/A	0.181	0.026	0.322	0.194	0.253	0.127	0.266	0.071

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	22	20	27	13
N.S.	1	1.00	1.00	0.82	1.59	1.29	1.18	1.59	0.76
time (sec)	N/A	0.180	0.044	0.416	0.197	0.261	0.133	0.268	0.061

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	36	20	27	14
N.S.	1	1.00	1.00	0.82	1.59	2.12	1.18	1.59	0.82
time (sec)	N/A	0.182	0.025	0.361	0.193	0.328	0.131	0.263	0.058

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	42	78	59	26
N.S.	1	1.00	0.71	0.80	0.00	1.20	2.23	1.69	0.74
time (sec)	N/A	0.213	0.038	0.324	0.000	0.244	0.253	0.261	0.097

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	23	15	12	27	19	20	25	11
N.S.	1	1.53	1.00	0.80	1.80	1.27	1.33	1.67	0.73
time (sec)	N/A	0.190	0.031	0.315	0.194	0.246	0.128	0.270	2.187

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	25	17	14	27	33	20	26	11
N.S.	1	1.47	1.00	0.82	1.59	1.94	1.18	1.53	0.65
time (sec)	N/A	0.184	0.030	0.399	0.187	0.244	0.129	0.258	0.060

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	25	17	14	27	38	20	27	15
N.S.	1	1.47	1.00	0.82	1.59	2.24	1.18	1.59	0.88
time (sec)	N/A	0.183	0.030	0.413	0.196	0.258	0.127	0.256	2.187

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	42	37	59	26
N.S.	1	1.00	0.71	0.80	0.00	1.20	1.06	1.69	0.74
time (sec)	N/A	0.213	0.046	0.324	0.000	0.254	0.265	0.270	2.204

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	27	19	16	53	115	0	36	47
N.S.	1	1.42	1.00	0.84	2.79	6.05	0.00	1.89	2.47
time (sec)	N/A	0.203	0.012	0.177	0.277	0.262	0.000	0.261	2.262

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	60	46	76	0	43	23
N.S.	1	1.00	1.00	3.16	2.42	4.00	0.00	2.26	1.21
time (sec)	N/A	0.242	0.021	0.236	0.284	0.252	0.000	0.269	2.183

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	80	69	42	0	213	0	71	71
N.S.	1	1.16	1.00	0.61	0.00	3.09	0.00	1.03	1.03
time (sec)	N/A	0.297	0.097	0.261	0.000	0.271	0.000	0.342	0.788

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	60	0	306	0	81	82
N.S.	1	1.00	0.93	0.69	0.00	3.52	0.00	0.93	0.94
time (sec)	N/A	0.409	0.148	0.306	0.000	0.267	0.000	0.288	3.388

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	93	87	84	0	300	0	100	98
N.S.	1	1.07	1.00	0.97	0.00	3.45	0.00	1.15	1.13
time (sec)	N/A	0.370	0.093	0.293	0.000	0.280	0.000	0.273	3.442

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.268	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	14	10	9	16	42	0	16	16
N.S.	1	1.40	1.00	0.90	1.60	4.20	0.00	1.60	1.60
time (sec)	N/A	0.201	0.008	0.199	0.271	0.257	0.000	0.256	0.055

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	49	118	0	36	47
N.S.	1	1.00	1.00	0.85	2.45	5.90	0.00	1.80	2.35
time (sec)	N/A	0.203	0.017	0.200	0.272	0.263	0.000	0.269	0.082

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	30	28	72	60	128	0	54	52
N.S.	1	1.07	1.00	2.57	2.14	4.57	0.00	1.93	1.86
time (sec)	N/A	0.266	0.023	0.244	0.274	0.257	0.000	0.263	2.153

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	76	42	0	293	0	75	141
N.S.	1	1.00	0.93	0.51	0.00	3.57	0.00	0.91	1.72
time (sec)	N/A	0.369	0.165	0.256	0.000	0.254	0.000	0.280	4.069

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	42	38	102	0	164	0	68	56
N.S.	1	1.11	1.00	2.68	0.00	4.32	0.00	1.79	1.47
time (sec)	N/A	0.291	0.043	0.243	0.000	0.270	0.000	0.274	2.325

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	42	39	42	35	0	38	35
N.S.	1	1.00	2.62	2.44	2.62	2.19	0.00	2.38	2.19
time (sec)	N/A	0.189	0.061	1.183	0.270	0.252	0.000	0.258	0.100

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	27	17	26	45	52	0	41	27
N.S.	1	1.29	0.81	1.24	2.14	2.48	0.00	1.95	1.29
time (sec)	N/A	0.207	0.008	1.229	0.268	0.253	0.000	0.260	2.196

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	110	40	0	215	0	115	251
N.S.	1	1.00	1.55	0.56	0.00	3.03	0.00	1.62	3.54
time (sec)	N/A	0.252	0.038	1.228	0.000	0.261	0.000	0.342	2.120

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	80	57	101	0	182	0	118	100
N.S.	1	1.29	0.92	1.63	0.00	2.94	0.00	1.90	1.61
time (sec)	N/A	0.311	0.057	1.320	0.000	0.246	0.000	0.291	2.172

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	269	78	0	250	0	154	288
N.S.	1	1.00	3.16	0.92	0.00	2.94	0.00	1.81	3.39
time (sec)	N/A	0.304	0.071	1.254	0.000	0.264	0.000	0.283	2.440

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	6	0	3	3
N.S.	1	1.00	1.00	0.86	1.00	0.86	0.00	0.43	0.43
time (sec)	N/A	0.213	0.001	0.233	0.269	0.249	0.000	0.269	0.048

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	40	39	31	0	19	19
N.S.	1	1.00	1.00	2.67	2.60	2.07	0.00	1.27	1.27
time (sec)	N/A	0.205	0.021	0.251	0.273	0.253	0.000	0.299	0.068

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	62	50	76	0	44	42
N.S.	1	1.00	1.00	2.38	1.92	2.92	0.00	1.69	1.62
time (sec)	N/A	0.208	0.017	0.272	0.269	0.257	0.000	0.280	0.074

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	87	84	41	0	243	0	68	282
N.S.	1	1.16	1.12	0.55	0.00	3.24	0.00	0.91	3.76
time (sec)	N/A	0.278	0.091	0.295	0.000	0.258	0.000	0.283	4.994

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	38	30	92	0	107	0	58	41
N.S.	1	1.06	0.83	2.56	0.00	2.97	0.00	1.61	1.14
time (sec)	N/A	0.241	0.018	0.271	0.000	0.250	0.000	0.265	0.206

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	23	15	12	27	19	20	25	6
N.S.	1	2.88	1.88	1.50	3.38	2.38	2.50	3.12	0.75
time (sec)	N/A	0.181	0.003	0.321	0.198	0.254	0.130	0.249	0.056

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	25	17	14	27	33	20	26	11
N.S.	1	1.47	1.00	0.82	1.59	1.94	1.18	1.53	0.65
time (sec)	N/A	0.179	0.006	0.428	0.180	0.243	0.140	0.258	2.097

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	25	17	14	27	36	20	27	14
N.S.	1	1.47	1.00	0.82	1.59	2.12	1.18	1.59	0.82
time (sec)	N/A	0.181	0.004	0.405	0.180	0.237	0.130	0.266	2.089

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	42	42	59	26
N.S.	1	1.00	0.71	0.80	0.00	1.20	1.20	1.69	0.74
time (sec)	N/A	0.212	0.025	0.338	0.000	0.247	0.248	0.259	0.083

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	27	17	20	25	9
N.S.	1	1.00	1.00	0.80	1.80	1.13	1.33	1.67	0.60
time (sec)	N/A	0.178	0.005	0.326	0.185	0.248	0.125	0.265	0.057

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	20	20	27	20
N.S.	1	1.00	1.00	0.82	1.59	1.18	1.18	1.59	1.18
time (sec)	N/A	0.178	0.004	0.434	0.186	0.263	0.130	0.259	2.129

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	34	20	27	15
N.S.	1	1.00	1.00	0.82	1.59	2.00	1.18	1.59	0.88
time (sec)	N/A	0.176	0.004	0.411	0.189	0.238	0.129	0.274	2.151

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	42	56	59	26
N.S.	1	1.00	0.71	0.80	0.00	1.20	1.60	1.69	0.74
time (sec)	N/A	0.202	0.020	0.312	0.000	0.251	0.255	0.256	0.070

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	27	51	16	52	73	0	45	48
N.S.	1	1.42	2.68	0.84	2.74	3.84	0.00	2.37	2.53
time (sec)	N/A	0.203	0.042	0.194	0.283	0.252	0.000	0.263	2.132

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	55	17	153	82	0	45	53
N.S.	1	1.00	2.75	0.85	7.65	4.10	0.00	2.25	2.65
time (sec)	N/A	0.198	0.053	0.196	0.287	0.255	0.000	0.259	2.145

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	80	113	42	0	213	0	119	133
N.S.	1	1.16	1.64	0.61	0.00	3.09	0.00	1.72	1.93
time (sec)	N/A	0.282	0.033	0.304	0.000	0.260	0.000	0.351	0.078

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	249	42	0	293	0	127	141
N.S.	1	1.00	3.04	0.51	0.00	3.57	0.00	1.55	1.72
time (sec)	N/A	0.317	0.033	0.272	0.000	0.262	0.000	0.303	0.093

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	93	281	79	0	258	0	157	170
N.S.	1	1.07	3.23	0.91	0.00	2.97	0.00	1.80	1.95
time (sec)	N/A	0.356	0.068	0.303	0.000	0.259	0.000	0.287	0.100

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	14	25	16	29	52	0	26	29
N.S.	1	1.40	2.50	1.60	2.90	5.20	0.00	2.60	2.90
time (sec)	N/A	0.194	0.021	0.313	0.191	0.249	0.000	0.254	2.121

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	55	47	50	57	104	0	55	57
N.S.	1	1.22	1.04	1.11	1.27	2.31	0.00	1.22	1.27
time (sec)	N/A	0.295	0.027	0.257	0.276	0.251	0.000	0.263	0.060

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	30	73	63	70	101	0	67	71
N.S.	1	1.07	2.61	2.25	2.50	3.61	0.00	2.39	2.54
time (sec)	N/A	0.264	0.051	0.248	0.275	0.266	0.000	0.255	0.071

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	133	190	0	272	0	157	143
N.S.	1	1.00	1.21	1.73	0.00	2.47	0.00	1.43	1.30
time (sec)	N/A	0.411	0.105	0.276	0.000	0.264	0.000	0.296	0.097

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	42	95	87	0	157	0	89	101
N.S.	1	1.11	2.50	2.29	0.00	4.13	0.00	2.34	2.66
time (sec)	N/A	0.298	0.056	0.285	0.000	0.261	0.000	0.254	0.089

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	75	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.321	0.000	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	44	43	68	0	39	32
N.S.	1	1.00	1.00	2.93	2.87	4.53	0.00	2.60	2.13
time (sec)	N/A	0.191	0.006	1.274	0.278	0.252	0.000	0.272	2.180

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	40	114	31	0	19	19
N.S.	1	1.00	1.00	2.67	7.60	2.07	0.00	1.27	1.27
time (sec)	N/A	0.197	0.020	1.375	0.277	0.248	0.000	0.259	2.129

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	40	0	215	0	135	126
N.S.	1	1.00	0.94	0.56	0.00	3.03	0.00	1.90	1.77
time (sec)	N/A	0.239	0.072	1.448	0.000	0.259	0.000	0.359	1.501

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	89	84	41	0	243	0	68	297
N.S.	1	1.19	1.12	0.55	0.00	3.24	0.00	0.91	3.96
time (sec)	N/A	0.293	0.088	1.189	0.000	0.253	0.000	0.282	4.779

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	83	0	282	0	177	206
N.S.	1	1.00	0.95	0.98	0.00	3.32	0.00	2.08	2.42
time (sec)	N/A	0.267	0.055	1.282	0.000	0.266	0.000	0.266	4.266

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	21	8	19	19	0	16	19
N.S.	1	1.00	3.00	1.14	2.71	2.71	0.00	2.29	2.71
time (sec)	N/A	0.210	0.010	0.255	0.204	0.264	0.000	0.256	0.060

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	27	21	24	47	52	0	40	29
N.S.	1	1.29	1.00	1.14	2.24	2.48	0.00	1.90	1.38
time (sec)	N/A	0.197	0.008	0.271	0.267	0.252	0.000	0.281	0.070

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	67	53	60	54	0	57	61
N.S.	1	1.00	2.58	2.04	2.31	2.08	0.00	2.19	2.35
time (sec)	N/A	0.213	0.061	0.271	0.275	0.250	0.000	0.276	0.062

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	80	57	101	0	180	0	108	104
N.S.	1	1.29	0.92	1.63	0.00	2.90	0.00	1.74	1.68
time (sec)	N/A	0.309	0.068	0.290	0.000	0.259	0.000	0.280	2.217

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	37	91	77	0	101	0	79	91
N.S.	1	1.03	2.53	2.14	0.00	2.81	0.00	2.19	2.53
time (sec)	N/A	0.267	0.058	0.280	0.000	0.255	0.000	0.264	0.089

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	82	66	0	59	88	0	0	0
N.S.	1	1.17	0.94	0.00	0.84	1.26	0.00	0.00	0.00
time (sec)	N/A	0.336	0.094	0.000	0.089	0.076	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	104	50	74	86	74	119	73	64
N.S.	1	1.11	0.53	0.79	0.91	0.79	1.27	0.78	0.68
time (sec)	N/A	0.386	0.067	0.883	0.190	0.256	0.306	0.260	2.206

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	66	39	58	64	62	75	57	46
N.S.	1	1.03	0.61	0.91	1.00	0.97	1.17	0.89	0.72
time (sec)	N/A	0.283	0.045	0.652	0.195	0.257	0.233	0.273	2.162

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	49	28	42	46	42	56	41	28
N.S.	1	1.11	0.64	0.95	1.05	0.95	1.27	0.93	0.64
time (sec)	N/A	0.241	0.041	0.499	0.204	0.256	0.201	0.258	0.066

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	13	22	19	29	13
N.S.	1	1.00	2.47	0.93	0.87	1.47	1.27	1.93	0.87
time (sec)	N/A	0.184	0.004	0.288	0.184	0.252	0.088	0.272	2.136

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	33	25	26	23	37	0	23	0
N.S.	1	1.22	0.93	0.96	0.85	1.37	0.00	0.85	0.00
time (sec)	N/A	0.362	0.014	0.882	0.245	0.250	0.000	0.259	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	49	42	55	27	65	0	52	0
N.S.	1	1.26	1.08	1.41	0.69	1.67	0.00	1.33	0.00
time (sec)	N/A	0.433	0.051	1.133	0.254	0.257	0.000	0.291	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	77	61	89	30	104	0	86	0
N.S.	1	1.28	1.02	1.48	0.50	1.73	0.00	1.43	0.00
time (sec)	N/A	0.534	0.101	1.205	0.250	0.246	0.000	0.261	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	97	77	121	31	115	0	120	0
N.S.	1	1.14	0.91	1.42	0.36	1.35	0.00	1.41	0.00
time (sec)	N/A	0.628	0.093	1.542	0.253	0.248	0.000	0.265	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	114	0	113	162	0	0	0
N.S.	1	1.00	0.85	0.00	0.84	1.21	0.00	0.00	0.00
time (sec)	N/A	0.400	0.180	0.000	0.119	0.082	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	152	86	141	160	135	146	140	108
N.S.	1	1.30	0.74	1.21	1.37	1.15	1.25	1.20	0.92
time (sec)	N/A	0.679	0.279	2.872	0.210	0.257	0.410	0.271	2.229

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	91	65	109	122	105	105	108	69
N.S.	1	1.10	0.78	1.31	1.47	1.27	1.27	1.30	0.83
time (sec)	N/A	0.446	0.152	2.421	0.213	0.271	0.308	0.274	2.205

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	51	46	56	84	74	61	76	41
N.S.	1	1.13	1.02	1.24	1.87	1.64	1.36	1.69	0.91
time (sec)	N/A	0.259	0.085	1.899	0.207	0.258	0.254	0.280	0.099

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	38	20	54	13
N.S.	1	1.00	1.00	0.93	0.87	2.53	1.33	3.60	0.87
time (sec)	N/A	0.188	0.003	0.757	0.189	0.244	0.100	0.278	2.147

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	39	47	42	67	0	42	0
N.S.	1	1.00	0.83	1.00	0.89	1.43	0.00	0.89	0.00
time (sec)	N/A	0.309	0.015	1.376	0.262	0.266	0.000	0.271	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	70	95	50	124	0	90	0
N.S.	1	1.00	0.88	1.19	0.62	1.55	0.00	1.12	0.00
time (sec)	N/A	0.365	0.125	1.773	0.272	0.254	0.000	0.268	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	105	159	58	196	0	156	0
N.S.	1	1.00	0.88	1.34	0.49	1.65	0.00	1.31	0.00
time (sec)	N/A	0.429	0.187	2.607	0.276	0.263	0.000	0.273	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	138	226	58	223	0	223	0
N.S.	1	1.00	0.90	1.47	0.38	1.45	0.00	1.45	0.00
time (sec)	N/A	0.487	0.217	3.464	0.273	0.250	0.000	0.271	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	110	0	117	172	0	0	0
N.S.	1	1.00	0.79	0.00	0.84	1.24	0.00	0.00	0.00
time (sec)	N/A	0.449	0.135	0.000	0.115	0.089	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	207	91	146	171	191	226	145	125
N.S.	1	1.34	0.59	0.94	1.10	1.23	1.46	0.94	0.81
time (sec)	N/A	0.771	0.420	7.582	0.200	0.264	0.688	0.291	2.348

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	111	70	114	127	154	150	113	89
N.S.	1	1.10	0.69	1.13	1.26	1.52	1.49	1.12	0.88
time (sec)	N/A	0.392	0.162	5.505	0.214	0.248	0.442	0.273	2.240

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	75	50	69	91	108	110	81	57
N.S.	1	1.15	0.77	1.06	1.40	1.66	1.69	1.25	0.88
time (sec)	N/A	0.328	0.091	3.924	0.206	0.261	0.302	0.268	0.120

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	54	20	57	13
N.S.	1	1.00	1.00	0.93	0.87	3.60	1.33	3.80	0.87
time (sec)	N/A	0.186	0.002	2.097	0.183	0.253	0.146	0.271	2.128

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	50	45	73	0	45	0
N.S.	1	1.00	0.89	0.94	0.85	1.38	0.00	0.85	0.00
time (sec)	N/A	0.315	0.040	2.880	0.286	0.251	0.000	0.260	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	80	105	53	139	0	100	0
N.S.	1	1.00	0.90	1.18	0.60	1.56	0.00	1.12	0.00
time (sec)	N/A	0.388	0.126	4.392	0.294	0.244	0.000	0.259	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	112	173	60	227	0	168	0
N.S.	1	1.00	0.90	1.38	0.48	1.82	0.00	1.34	0.00
time (sec)	N/A	0.457	0.371	6.585	0.327	0.246	0.000	0.263	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	241	59	261	0	236	0
N.S.	1	1.00	0.89	1.43	0.35	1.54	0.00	1.40	0.00
time (sec)	N/A	0.507	0.334	9.278	0.288	0.247	0.000	0.261	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	13	13	0	13	0
N.S.	1	1.00	1.00	0.88	1.62	1.62	0.00	1.62	0.00
time (sec)	N/A	0.221	0.004	0.361	0.233	0.245	0.000	0.262	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	26	16	15	15	24	0	30	0
N.S.	1	1.62	1.00	0.94	0.94	1.50	0.00	1.88	0.00
time (sec)	N/A	0.284	0.005	0.406	0.252	0.242	0.000	0.266	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	40	27	24	13	43	0	48	0
N.S.	1	1.48	1.00	0.89	0.48	1.59	0.00	1.78	0.00
time (sec)	N/A	0.355	0.006	0.451	0.241	0.254	0.000	0.251	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	114	0	113	162	0	0	0
N.S.	1	1.00	0.85	0.00	0.84	1.21	0.00	0.00	0.00
time (sec)	N/A	0.397	0.216	0.000	0.125	0.085	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	150	84	141	160	135	146	140	119
N.S.	1	1.28	0.72	1.21	1.37	1.15	1.25	1.20	1.02
time (sec)	N/A	0.683	0.275	3.878	0.251	0.270	0.411	0.253	0.167

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	102	66	109	122	104	105	108	82
N.S.	1	1.23	0.80	1.31	1.47	1.25	1.27	1.30	0.99
time (sec)	N/A	0.436	0.304	2.984	0.215	0.251	0.330	0.306	2.178

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	43	38	56	84	76	61	76	44
N.S.	1	0.96	0.84	1.24	1.87	1.69	1.36	1.69	0.98
time (sec)	N/A	0.258	0.094	2.505	0.216	0.281	0.217	0.254	2.153

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	32	20	54	13
N.S.	1	1.00	1.00	0.93	0.87	2.13	1.33	3.60	0.87
time (sec)	N/A	0.182	0.003	1.153	0.201	0.257	0.099	0.258	2.091

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	42	67	0	42	0
N.S.	1	1.00	0.87	1.00	0.89	1.43	0.00	0.89	0.00
time (sec)	N/A	0.288	0.015	1.663	0.287	0.266	0.000	0.265	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	68	96	50	126	0	91	0
N.S.	1	1.00	0.85	1.20	0.62	1.58	0.00	1.14	0.00
time (sec)	N/A	0.343	0.141	1.881	0.309	0.242	0.000	0.268	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	107	159	58	195	0	156	0
N.S.	1	1.00	0.90	1.34	0.49	1.64	0.00	1.31	0.00
time (sec)	N/A	0.407	0.153	2.733	0.272	0.251	0.000	0.262	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	138	229	58	224	0	222	0
N.S.	1	1.00	0.90	1.49	0.38	1.45	0.00	1.44	0.00
time (sec)	N/A	0.472	0.174	3.822	0.289	0.248	0.000	0.255	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	106	0	71	122	0	0	0
N.S.	1	1.00	1.25	0.00	0.84	1.44	0.00	0.00	0.00
time (sec)	N/A	0.324	0.121	0.000	0.091	0.088	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	58	79	91	140	250	78	70
N.S.	1	1.00	0.73	1.00	1.15	1.77	3.16	0.99	0.89
time (sec)	N/A	0.305	0.110	11.058	0.214	0.247	0.553	0.274	2.284

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	48	63	69	110	204	62	52
N.S.	1	1.00	0.80	1.05	1.15	1.83	3.40	1.03	0.87
time (sec)	N/A	0.282	0.099	7.806	0.250	0.263	0.440	0.249	2.207

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	51	88	131	46	36
N.S.	1	1.00	1.00	1.15	1.24	2.15	3.20	1.12	0.88
time (sec)	N/A	0.228	0.154	4.937	0.229	0.270	0.311	0.268	0.108

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	51	23	33	39	40	92	32	18
N.S.	1	1.11	0.50	0.72	0.85	0.87	2.00	0.70	0.39
time (sec)	N/A	0.260	0.011	3.158	0.211	0.255	0.157	0.256	0.071

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	30	27	41	0	27	0
N.S.	1	1.00	0.97	0.91	0.82	1.24	0.00	0.82	0.00
time (sec)	N/A	0.255	0.262	3.394	0.252	0.255	0.000	0.271	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	45	54	32	88	0	55	0
N.S.	1	1.00	0.87	1.04	0.62	1.69	0.00	1.06	0.00
time (sec)	N/A	0.280	0.058	4.190	0.265	0.250	0.000	0.266	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	65	92	36	140	0	89	0
N.S.	1	1.00	0.97	1.37	0.54	2.09	0.00	1.33	0.00
time (sec)	N/A	0.318	0.069	5.999	0.253	0.266	0.000	0.271	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	79	122	36	172	0	123	0
N.S.	1	1.00	0.86	1.33	0.39	1.87	0.00	1.34	0.00
time (sec)	N/A	0.360	0.130	8.790	0.271	0.263	0.000	0.269	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	209	175	0	171	248	0	0	0
N.S.	1	1.00	0.84	0.00	0.82	1.19	0.00	0.00	0.00
time (sec)	N/A	0.526	0.261	0.000	0.154	0.102	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	125	213	245	274	253	212	167
N.S.	1	1.00	0.62	1.05	1.21	1.36	1.25	1.05	0.83
time (sec)	N/A	0.481	0.780	24.677	0.238	0.253	0.753	0.263	0.548

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	105	165	187	209	182	164	123
N.S.	1	1.00	0.71	1.11	1.26	1.41	1.23	1.11	0.83
time (sec)	N/A	0.401	0.188	18.546	0.267	0.249	0.554	0.267	2.570

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	70	117	129	152	112	116	83
N.S.	1	1.00	0.74	1.24	1.37	1.62	1.19	1.23	0.88
time (sec)	N/A	0.291	0.114	13.598	0.227	0.249	0.420	0.273	0.159

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	36	27	26	78	64	44	82	26
N.S.	1	1.16	0.87	0.84	2.52	2.06	1.42	2.65	0.84
time (sec)	N/A	0.229	0.048	8.601	0.201	0.254	0.213	0.272	2.133

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	61	71	64	103	0	64	0
N.S.	1	1.00	0.84	0.97	0.88	1.41	0.00	0.88	0.00
time (sec)	N/A	0.370	0.060	8.326	0.308	0.253	0.000	0.271	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	104	151	76	214	0	144	0
N.S.	1	1.00	0.84	1.22	0.61	1.73	0.00	1.16	0.00
time (sec)	N/A	0.474	0.212	9.303	0.321	0.291	0.000	0.267	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	162	250	88	338	0	243	0
N.S.	1	1.00	0.88	1.36	0.48	1.84	0.00	1.32	0.00
time (sec)	N/A	0.555	0.304	14.241	0.303	0.254	0.000	0.282	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	212	349	88	397	0	342	0
N.S.	1	1.00	0.89	1.47	0.37	1.67	0.00	1.44	0.00
time (sec)	N/A	0.656	0.287	21.045	0.311	0.263	0.000	0.255	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	117	172	0	0	0
N.S.	1	1.00	0.79	0.00	0.83	1.22	0.00	0.00	0.00
time (sec)	N/A	0.423	0.078	0.000	0.121	0.087	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	207	95	146	171	191	226	145	126
N.S.	1	1.34	0.61	0.94	1.10	1.23	1.46	0.94	0.81
time (sec)	N/A	0.742	0.422	13.011	0.223	0.249	0.540	0.261	2.334

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	111	72	114	127	154	150	113	89
N.S.	1	1.10	0.71	1.13	1.26	1.52	1.49	1.12	0.88
time (sec)	N/A	0.383	0.140	10.400	0.238	0.261	0.410	0.266	0.220

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	75	50	69	91	108	110	81	55
N.S.	1	1.15	0.77	1.06	1.40	1.66	1.69	1.25	0.85
time (sec)	N/A	0.324	0.087	7.361	0.225	0.244	0.310	0.264	0.163

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	54	20	57	13
N.S.	1	1.00	1.00	0.93	0.87	3.60	1.33	3.80	0.87
time (sec)	N/A	0.181	0.002	4.709	0.224	0.250	0.136	0.262	0.069

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	50	45	73	0	45	0
N.S.	1	1.00	0.89	0.94	0.85	1.38	0.00	0.85	0.00
time (sec)	N/A	0.303	0.038	4.558	0.299	0.239	0.000	0.264	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	105	53	139	0	100	0
N.S.	1	1.00	0.88	1.18	0.60	1.56	0.00	1.12	0.00
time (sec)	N/A	0.356	0.155	5.328	0.305	0.247	0.000	0.265	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	113	173	60	229	0	168	0
N.S.	1	1.00	0.90	1.38	0.48	1.83	0.00	1.34	0.00
time (sec)	N/A	0.423	0.319	5.542	0.329	0.253	0.000	0.260	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	241	61	264	0	236	0
N.S.	1	1.00	0.89	1.43	0.36	1.56	0.00	1.40	0.00
time (sec)	N/A	0.478	0.319	8.566	0.286	0.247	0.000	0.258	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	209	174	0	171	248	0	0	0
N.S.	1	1.00	0.83	0.00	0.82	1.19	0.00	0.00	0.00
time (sec)	N/A	0.504	0.218	0.000	0.147	0.083	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	136	213	245	274	253	212	173
N.S.	1	1.00	0.67	1.05	1.21	1.36	1.25	1.05	0.86
time (sec)	N/A	0.473	0.315	39.141	0.221	0.257	0.770	0.265	0.289

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	98	165	187	214	182	164	112
N.S.	1	1.00	0.66	1.11	1.26	1.45	1.23	1.11	0.76
time (sec)	N/A	0.394	0.277	26.872	0.240	0.272	0.542	0.278	0.244

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	70	116	129	153	112	116	71
N.S.	1	1.00	0.74	1.23	1.37	1.63	1.19	1.23	0.76
time (sec)	N/A	0.290	0.116	19.315	0.221	0.259	0.401	0.266	0.156

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	27	26	78	79	44	82	26
N.S.	1	0.97	0.87	0.84	2.52	2.55	1.42	2.65	0.84
time (sec)	N/A	0.215	0.049	12.892	0.206	0.251	0.209	0.278	2.146

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	71	64	103	0	64	0
N.S.	1	1.00	0.86	0.97	0.88	1.41	0.00	0.88	0.00
time (sec)	N/A	0.362	0.054	11.172	0.307	0.251	0.000	0.315	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	106	147	76	203	0	140	0
N.S.	1	1.00	0.85	1.19	0.61	1.64	0.00	1.13	0.00
time (sec)	N/A	0.450	0.162	12.740	0.321	0.247	0.000	0.262	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	164	246	88	336	0	239	0
N.S.	1	1.00	0.89	1.34	0.48	1.83	0.00	1.30	0.00
time (sec)	N/A	0.530	0.241	13.449	0.309	0.258	0.000	0.292	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	212	349	88	392	0	342	0
N.S.	1	1.00	0.89	1.47	0.37	1.65	0.00	1.44	0.00
time (sec)	N/A	0.629	0.287	21.615	0.333	0.245	0.000	0.273	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	119	0	117	172	0	0	0
N.S.	1	1.00	0.77	0.00	0.75	1.11	0.00	0.00	0.00
time (sec)	N/A	0.457	0.130	0.000	0.131	0.085	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	90	146	171	248	314	145	126
N.S.	1	1.00	0.63	1.02	1.20	1.73	2.20	1.01	0.88
time (sec)	N/A	0.402	0.587	88.148	0.226	0.262	1.017	0.266	2.465

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	72	114	127	202	212	113	89
N.S.	1	1.00	0.69	1.09	1.21	1.92	2.02	1.08	0.85
time (sec)	N/A	0.339	0.151	63.257	0.219	0.279	0.771	0.261	0.347

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	50	82	91	148	148	81	55
N.S.	1	1.00	0.75	1.22	1.36	2.21	2.21	1.21	0.82
time (sec)	N/A	0.261	0.163	44.818	0.216	0.255	0.545	0.269	0.251

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	29	35	34	56	72	42	57	26
N.S.	1	0.94	1.13	1.10	1.81	2.32	1.35	1.84	0.84
time (sec)	N/A	0.220	0.050	31.923	0.206	0.243	0.309	0.269	0.109

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	50	45	73	0	45	0
N.S.	1	1.00	0.89	0.94	0.85	1.38	0.00	0.85	0.00
time (sec)	N/A	0.319	0.526	28.397	0.286	0.254	0.000	0.272	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	105	53	159	0	100	0
N.S.	1	1.00	0.88	1.18	0.60	1.79	0.00	1.12	0.00
time (sec)	N/A	0.385	0.143	30.955	0.288	0.245	0.000	0.280	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	118	173	61	274	0	168	0
N.S.	1	1.00	0.90	1.32	0.47	2.09	0.00	1.28	0.00
time (sec)	N/A	0.447	0.156	32.460	0.298	0.262	0.000	0.272	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	241	61	315	0	236	0
N.S.	1	1.00	0.89	1.43	0.36	1.86	0.00	1.40	0.00
time (sec)	N/A	0.506	0.212	49.265	0.278	0.250	0.000	0.272	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	16	100	18	17	18	20
N.S.	1	1.00	1.20	1.60	10.00	1.80	1.70	1.80	2.00
time (sec)	N/A	0.192	0.336	0.132	0.489	0.257	10.115	0.272	2.173

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	118	88	116	84	257	0	0	0
N.S.	1	1.30	0.97	1.27	0.92	2.82	0.00	0.00	0.00
time (sec)	N/A	0.603	0.178	0.468	0.262	0.273	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	88	66	94	63	207	0	0	0
N.S.	1	1.35	1.02	1.45	0.97	3.18	0.00	0.00	0.00
time (sec)	N/A	0.470	0.116	0.504	0.259	0.258	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	59	44	70	40	141	0	0	0
N.S.	1	1.31	0.98	1.56	0.89	3.13	0.00	0.00	0.00
time (sec)	N/A	0.329	0.088	0.470	0.263	0.257	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	13	21	37	0	24	11
N.S.	1	1.00	1.00	1.18	1.91	3.36	0.00	2.18	1.00
time (sec)	N/A	0.170	0.001	0.164	0.213	0.245	0.000	0.258	0.063

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	16	22	18	15	18	20
N.S.	1	1.00	1.20	1.60	2.20	1.80	1.50	1.80	2.00
time (sec)	N/A	0.184	9.075	0.225	0.267	0.248	1.006	0.250	2.189

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	16	29	18	17	18	20
N.S.	1	1.00	1.20	1.60	2.90	1.80	1.70	1.80	2.00
time (sec)	N/A	0.188	16.638	0.212	0.266	0.255	0.847	0.251	2.212

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	19	20	20
N.S.	1	1.00	1.12	1.12	1.25	1.25	1.19	1.25	1.25
time (sec)	N/A	0.464	48.176	0.123	0.361	0.252	35.580	0.277	2.214

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	125	130	0	0	672	0	0	0
N.S.	1	1.11	1.15	0.00	0.00	5.95	0.00	0.00	0.00
time (sec)	N/A	0.544	0.217	0.000	0.000	0.267	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	75	85	154	0	468	0	0	0
N.S.	1	1.09	1.23	2.23	0.00	6.78	0.00	0.00	0.00
time (sec)	N/A	0.354	0.223	0.751	0.000	0.255	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	32	59	37	116	0	70	49
N.S.	1	1.00	1.33	2.46	1.54	4.83	0.00	2.92	2.04
time (sec)	N/A	0.234	0.036	0.482	0.362	0.258	0.000	0.263	0.076

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	23	54	0	23	13
N.S.	1	1.00	1.00	1.09	2.09	4.91	0.00	2.09	1.18
time (sec)	N/A	0.180	0.003	0.245	0.200	0.241	0.000	0.272	0.059

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	60	20	17	20	20
N.S.	1	1.00	1.12	1.12	3.75	1.25	1.06	1.25	1.25
time (sec)	N/A	0.323	6.146	0.209	0.325	0.245	1.448	0.414	2.178

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	64	20	19	20	20
N.S.	1	1.00	1.12	1.12	4.00	1.25	1.19	1.25	1.25
time (sec)	N/A	0.344	6.594	0.212	0.331	0.256	1.520	0.684	2.202

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	19	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.06	1.11	1.11
time (sec)	N/A	0.560	39.727	0.120	0.358	0.252	110.413	0.290	2.259

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	102	86	121	110	1113	0	0	0
N.S.	1	1.23	1.04	1.46	1.33	13.41	0.00	0.00	0.00
time (sec)	N/A	0.559	0.978	2.531	0.284	0.269	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	47	55	73	94	378	0	142	102
N.S.	1	1.12	1.31	1.74	2.24	9.00	0.00	3.38	2.43
time (sec)	N/A	0.344	0.091	1.640	0.282	0.260	0.000	0.297	2.216

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	43	131	105	0	184	36
N.S.	1	1.00	1.00	1.43	4.37	3.50	0.00	6.13	1.20
time (sec)	N/A	0.251	0.181	1.152	0.216	0.247	0.000	0.278	2.203

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	84	0	27	13
N.S.	1	1.00	1.00	0.93	1.53	5.60	0.00	1.80	0.87
time (sec)	N/A	0.186	0.001	0.610	0.208	0.241	0.000	0.256	2.176

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	101	20	17	20	20
N.S.	1	1.00	1.11	1.00	5.61	1.11	0.94	1.11	1.11
time (sec)	N/A	0.345	22.239	0.230	0.305	0.245	3.862	0.273	2.209

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	100	20	19	20	20
N.S.	1	1.00	1.11	1.00	5.56	1.11	1.06	1.11	1.11
time (sec)	N/A	0.387	18.928	0.272	0.306	0.253	4.616	0.268	2.219

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	19	20	22
N.S.	1	1.00	1.12	1.12	1.25	1.25	1.19	1.25	1.38
time (sec)	N/A	0.414	14.706	0.200	0.375	0.246	38.334	0.263	2.203

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	226	211	0	0	609	0	0	0
N.S.	1	1.16	1.08	0.00	0.00	3.12	0.00	0.00	0.00
time (sec)	N/A	1.109	0.196	0.000	0.000	0.267	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	153	153	0	0	477	0	0	0
N.S.	1	1.13	1.13	0.00	0.00	3.53	0.00	0.00	0.00
time (sec)	N/A	0.767	0.144	0.000	0.000	0.267	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	93	162	0	328	0	0	0
N.S.	1	1.00	1.21	2.10	0.00	4.26	0.00	0.00	0.00
time (sec)	N/A	0.439	0.085	0.664	0.000	0.260	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	21	23	21	41	86	0	32	49
N.S.	1	0.91	1.00	0.91	1.78	3.74	0.00	1.39	2.13
time (sec)	N/A	0.190	0.003	0.322	0.321	0.252	0.000	0.266	0.063

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	17	20	22
N.S.	1	1.00	1.12	1.12	1.25	1.25	1.06	1.25	1.38
time (sec)	N/A	0.445	5.168	0.411	0.430	0.250	1.346	0.319	2.233

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	19	20	22
N.S.	1	1.00	1.12	1.12	1.25	1.25	1.19	1.25	1.38
time (sec)	N/A	0.559	4.607	0.335	0.400	0.266	1.407	0.348	2.199

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	144	22	20	22	22
N.S.	1	1.00	1.17	1.67	12.00	1.83	1.67	1.83	1.83
time (sec)	N/A	0.199	0.408	0.233	0.703	0.247	120.276	0.286	2.224

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	113	99	125	108	721	0	0	0
N.S.	1	1.27	1.11	1.40	1.21	8.10	0.00	0.00	0.00
time (sec)	N/A	0.609	1.069	1.003	0.284	0.262	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	84	77	99	84	515	0	0	0
N.S.	1	1.29	1.18	1.52	1.29	7.92	0.00	0.00	0.00
time (sec)	N/A	0.435	0.673	0.893	0.268	0.266	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	46	41	95	185	0	95	45
N.S.	1	1.00	1.48	1.32	3.06	5.97	0.00	3.06	1.45
time (sec)	N/A	0.254	0.115	0.954	0.238	0.257	0.000	0.285	0.106

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	23	17	25	33	0	24	20
N.S.	1	1.00	1.77	1.31	1.92	2.54	0.00	1.85	1.54
time (sec)	N/A	0.175	0.001	0.266	0.217	0.263	0.000	0.289	0.076

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	49	22	19	22	22
N.S.	1	1.00	1.17	1.67	4.08	1.83	1.58	1.83	1.83
time (sec)	N/A	0.198	17.056	0.380	0.297	0.246	3.313	0.284	2.211

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	68	22	20	22	22
N.S.	1	1.00	1.17	1.67	5.67	1.83	1.67	1.83	1.83
time (sec)	N/A	0.198	10.978	0.376	0.299	0.242	3.960	0.285	2.251

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	22	22	0	22	22
N.S.	1	1.00	1.11	1.11	1.22	1.22	0.00	1.22	1.22
time (sec)	N/A	0.291	52.673	0.213	0.392	0.253	0.000	0.293	2.282

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	420	245	0	0	2163	0	0	0
N.S.	1	1.75	1.02	0.00	0.00	9.01	0.00	0.00	0.00
time (sec)	N/A	1.685	2.006	0.000	0.000	0.293	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	259	180	0	0	1577	0	0	0
N.S.	1	1.81	1.26	0.00	0.00	11.03	0.00	0.00	0.00
time (sec)	N/A	1.246	0.588	0.000	0.000	0.287	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	147	116	178	0	1064	0	0	0
N.S.	1	1.62	1.27	1.96	0.00	11.69	0.00	0.00	0.00
time (sec)	N/A	0.673	0.638	1.395	0.000	0.287	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	43	66	269	0	76	82
N.S.	1	1.00	1.00	1.26	1.94	7.91	0.00	2.24	2.41
time (sec)	N/A	0.258	0.003	1.033	0.292	0.265	0.000	0.266	2.219

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	131	22	19	22	22
N.S.	1	1.00	1.11	1.11	7.28	1.22	1.06	1.22	1.22
time (sec)	N/A	0.288	15.276	0.351	0.357	0.257	8.664	1.766	2.245

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	133	22	20	22	22
N.S.	1	1.00	1.11	1.11	7.39	1.22	1.11	1.22	1.22
time (sec)	N/A	0.287	11.082	0.407	0.361	0.247	10.785	1.633	2.304

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	19	20	22
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.06	1.11	1.22
time (sec)	N/A	0.508	17.722	0.273	0.442	0.251	113.002	0.279	2.229

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	222	159	189	181	966	0	0	0
N.S.	1	1.20	0.86	1.02	0.98	5.22	0.00	0.00	0.00
time (sec)	N/A	1.204	0.286	2.301	0.267	0.282	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	154	122	152	138	789	0	0	0
N.S.	1	1.18	0.94	1.17	1.06	6.07	0.00	0.00	0.00
time (sec)	N/A	0.887	0.251	1.651	0.277	0.277	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	108	72	110	95	558	0	0	0
N.S.	1	1.21	0.81	1.24	1.07	6.27	0.00	0.00	0.00
time (sec)	N/A	0.595	0.299	1.242	0.266	0.252	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	25	25	25	56	197	0	60	48
N.S.	1	0.89	0.89	0.89	2.00	7.04	0.00	2.14	1.71
time (sec)	N/A	0.207	0.012	0.731	0.307	0.261	0.000	0.269	0.065

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	46	20	17	20	22
N.S.	1	1.00	1.11	1.00	2.56	1.11	0.94	1.11	1.22
time (sec)	N/A	0.539	12.197	0.419	0.357	0.242	2.823	0.268	2.205

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	53	20	19	20	22
N.S.	1	1.00	1.11	1.00	2.94	1.11	1.06	1.11	1.22
time (sec)	N/A	0.633	10.521	0.379	0.331	0.238	3.444	0.261	2.242

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	22	22	0	22	22
N.S.	1	1.00	1.11	1.11	1.22	1.22	0.00	1.22	1.22
time (sec)	N/A	0.558	45.958	0.277	0.440	0.248	0.000	0.281	2.305

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	201	196	0	0	1225	0	0	0
N.S.	1	1.24	1.21	0.00	0.00	7.56	0.00	0.00	0.00
time (sec)	N/A	1.125	0.878	0.000	0.000	0.281	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	125	132	205	0	879	0	0	0
N.S.	1	1.20	1.27	1.97	0.00	8.45	0.00	0.00	0.00
time (sec)	N/A	0.769	0.812	0.893	0.000	0.268	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	56	50	94	81	283	0	102	90
N.S.	1	1.22	1.09	2.04	1.76	6.15	0.00	2.22	1.96
time (sec)	N/A	0.456	0.223	0.717	0.355	0.257	0.000	0.301	0.113

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	22	21	33	54	31	0	41	22
N.S.	1	1.05	1.00	1.57	2.57	1.48	0.00	1.95	1.05
time (sec)	N/A	0.212	0.005	0.373	0.208	0.251	0.000	0.271	0.082

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	79	22	19	22	22
N.S.	1	1.00	1.11	1.11	4.39	1.22	1.06	1.22	1.22
time (sec)	N/A	0.561	8.597	0.350	0.352	0.248	8.511	0.876	2.278

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	87	22	20	22	22
N.S.	1	1.00	1.11	1.11	4.83	1.22	1.11	1.22	1.22
time (sec)	N/A	0.698	6.401	0.348	0.350	0.266	10.905	1.227	2.332

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	171	22	0	22	22
N.S.	1	1.00	1.17	1.67	14.25	1.83	0.00	1.83	1.83
time (sec)	N/A	0.211	0.687	0.250	0.735	0.253	0.000	0.312	2.328

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	230	249	234	236	2207	0	0	0
N.S.	1	1.26	1.36	1.28	1.29	12.06	0.00	0.00	0.00
time (sec)	N/A	1.361	1.539	0.669	0.254	0.276	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	146	196	164	183	1649	0	0	0
N.S.	1	1.26	1.69	1.41	1.58	14.22	0.00	0.00	0.00
time (sec)	N/A	0.856	1.354	0.661	0.265	0.278	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	100	69	111	131	1106	0	0	0
N.S.	1	1.22	0.84	1.35	1.60	13.49	0.00	0.00	0.00
time (sec)	N/A	0.495	0.596	0.575	0.266	0.270	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	36	27	23	61	339	0	65	25
N.S.	1	1.33	1.00	0.85	2.26	12.56	0.00	2.41	0.93
time (sec)	N/A	0.241	0.001	0.372	0.297	0.273	0.000	0.277	2.219

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	111	22	19	22	22
N.S.	1	1.00	1.17	1.67	9.25	1.83	1.58	1.83	1.83
time (sec)	N/A	0.200	13.219	0.405	0.286	0.249	19.856	0.280	2.285

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	143	22	20	22	22
N.S.	1	1.00	1.17	1.67	11.92	1.83	1.67	1.83	1.83
time (sec)	N/A	0.207	8.077	0.394	0.292	0.242	27.048	0.273	2.370

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	16	102	18	17	18	20
N.S.	1	1.00	1.20	1.60	10.20	1.80	1.70	1.80	2.00
time (sec)	N/A	0.202	5.997	0.120	0.502	0.255	108.551	0.263	2.203

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	142	91	200	130	216	0	0	0
N.S.	1	1.63	1.05	2.30	1.49	2.48	0.00	0.00	0.00
time (sec)	N/A	0.649	0.003	0.323	0.256	0.262	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	106	66	166	96	168	0	0	0
N.S.	1	1.68	1.05	2.63	1.52	2.67	0.00	0.00	0.00
time (sec)	N/A	0.492	0.003	0.313	0.251	0.268	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	71	47	122	58	112	0	0	0
N.S.	1	1.58	1.04	2.71	1.29	2.49	0.00	0.00	0.00
time (sec)	N/A	0.342	0.003	0.318	0.240	0.258	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	15	19	13	23	37	0	25	11
N.S.	1	1.36	1.73	1.18	2.09	3.36	0.00	2.27	1.00
time (sec)	N/A	0.167	0.003	0.147	0.203	0.247	0.000	0.266	0.070

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	16	35	18	15	18	20
N.S.	1	1.00	1.20	1.60	3.50	1.80	1.50	1.80	2.00
time (sec)	N/A	0.196	0.277	0.184	0.278	0.247	3.293	0.257	2.182

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	16	46	18	17	18	20
N.S.	1	1.00	1.20	1.60	4.60	1.80	1.70	1.80	2.00
time (sec)	N/A	0.192	0.717	0.242	0.251	0.251	3.826	0.267	2.245

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	0	20	22
N.S.	1	1.00	1.12	1.12	1.25	1.25	0.00	1.25	1.38
time (sec)	N/A	0.391	17.231	0.253	0.395	0.244	0.000	0.270	2.210

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	225	175	246	206	511	0	0	0
N.S.	1	1.36	1.06	1.49	1.25	3.10	0.00	0.00	0.00
time (sec)	N/A	1.086	0.166	0.602	0.266	0.267	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	151	125	196	152	391	0	0	0
N.S.	1	1.31	1.09	1.70	1.32	3.40	0.00	0.00	0.00
time (sec)	N/A	0.780	0.113	0.625	0.264	0.256	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	88	73	139	94	255	0	0	0
N.S.	1	1.33	1.11	2.11	1.42	3.86	0.00	0.00	0.00
time (sec)	N/A	0.441	0.054	0.629	0.256	0.253	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	21	42	21	59	113	0	44	53
N.S.	1	0.91	1.83	0.91	2.57	4.91	0.00	1.91	2.30
time (sec)	N/A	0.191	0.008	0.372	0.196	0.266	0.000	0.270	2.228

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	17	20	22
N.S.	1	1.00	1.12	1.12	1.25	1.25	1.06	1.25	1.38
time (sec)	N/A	0.447	8.899	0.353	0.400	0.252	9.383	0.432	2.230

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	19	20	22
N.S.	1	1.00	1.12	1.12	1.25	1.25	1.19	1.25	1.38
time (sec)	N/A	0.545	24.615	0.381	0.400	0.250	10.631	0.440	2.236

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	0	20	22
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.22
time (sec)	N/A	0.518	16.830	0.277	0.422	0.255	0.000	0.286	2.380

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	246	236	272	225	876	0	0	0
N.S.	1	1.37	1.31	1.51	1.25	4.87	0.00	0.00	0.00
time (sec)	N/A	1.252	0.457	1.139	0.281	0.259	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	172	178	222	171	697	0	0	0
N.S.	1	1.37	1.41	1.76	1.36	5.53	0.00	0.00	0.00
time (sec)	N/A	0.930	0.249	1.142	0.273	0.271	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	120	72	162	113	488	0	0	0
N.S.	1	1.36	0.82	1.84	1.28	5.55	0.00	0.00	0.00
time (sec)	N/A	0.617	0.302	1.018	0.266	0.266	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	28	25	23	70	203	0	63	49
N.S.	1	1.04	0.93	0.85	2.59	7.52	0.00	2.33	1.81
time (sec)	N/A	0.208	0.009	0.772	0.199	0.258	0.000	0.275	0.065

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	57	20	17	20	22
N.S.	1	1.00	1.11	1.00	3.17	1.11	0.94	1.11	1.22
time (sec)	N/A	0.560	7.144	0.396	0.323	0.263	21.801	0.281	2.285

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	72	20	19	20	22
N.S.	1	1.00	1.11	1.00	4.00	1.11	1.06	1.11	1.22
time (sec)	N/A	0.651	7.397	0.435	0.330	0.240	26.594	0.279	2.296

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	48	0	336	0	101	48
N.S.	1	1.00	1.00	1.45	0.00	10.18	0.00	3.06	1.45
time (sec)	N/A	0.354	0.038	0.327	0.000	0.272	0.000	0.266	0.130

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	98	64	87	0	617	0	0	0
N.S.	1	1.34	0.88	1.19	0.00	8.45	0.00	0.00	0.00
time (sec)	N/A	0.657	0.093	0.334	0.000	0.264	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	125	85	117	0	875	0	0	0
N.S.	1	1.23	0.83	1.15	0.00	8.58	0.00	0.00	0.00
time (sec)	N/A	0.802	0.098	0.361	0.000	0.279	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	141	56	82	146	916	0	0	0
N.S.	1	2.24	0.89	1.30	2.32	14.54	0.00	0.00	0.00
time (sec)	N/A	1.075	0.066	0.428	0.237	0.273	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	0	135	127	174	1512	0	0	0
N.S.	1	0.00	1.41	1.32	1.81	15.75	0.00	0.00	0.00
time (sec)	N/A	0.000	0.190	0.451	0.240	0.285	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	0	191	184	238	2067	0	0	0
N.S.	1	0.00	1.21	1.16	1.51	13.08	0.00	0.00	0.00
time (sec)	N/A	0.000	0.638	0.449	0.249	0.270	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	0	20	20
N.S.	1	1.00	1.12	1.12	1.25	1.25	0.00	1.25	1.25
time (sec)	N/A	0.462	15.271	0.117	0.337	0.249	0.000	0.274	2.291

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	117	109	174	121	551	0	0	0
N.S.	1	1.26	1.17	1.87	1.30	5.92	0.00	0.00	0.00
time (sec)	N/A	0.519	0.200	0.441	0.266	0.268	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	73	69	134	83	367	0	0	0
N.S.	1	1.24	1.17	2.27	1.41	6.22	0.00	0.00	0.00
time (sec)	N/A	0.355	0.199	0.407	0.262	0.251	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	114	54	64	169	0	93	53
N.S.	1	1.00	4.56	2.16	2.56	6.76	0.00	3.72	2.12
time (sec)	N/A	0.237	0.038	0.369	0.248	0.249	0.000	0.276	0.131

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	25	56	0	25	13
N.S.	1	1.00	1.00	1.09	2.27	5.09	0.00	2.27	1.18
time (sec)	N/A	0.182	0.003	0.155	0.189	0.253	0.000	0.265	2.273

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	79	20	17	20	20
N.S.	1	1.00	1.12	1.12	4.94	1.25	1.06	1.25	1.25
time (sec)	N/A	0.296	22.777	0.231	0.288	0.258	9.118	0.472	2.287

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	79	20	19	20	20
N.S.	1	1.00	1.12	1.12	4.94	1.25	1.19	1.25	1.25
time (sec)	N/A	0.341	32.498	0.232	0.293	0.286	11.472	0.748	2.404

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	145	22	0	22	22
N.S.	1	1.00	1.17	1.67	12.08	1.83	0.00	1.83	1.83
time (sec)	N/A	0.219	7.611	0.248	0.724	0.260	0.000	0.281	2.328

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	131	222	198	146	632	0	0	0
N.S.	1	1.51	2.55	2.28	1.68	7.26	0.00	0.00	0.00
time (sec)	N/A	0.635	0.606	0.990	0.267	0.269	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	96	117	156	108	453	0	0	0
N.S.	1	1.48	1.80	2.40	1.66	6.97	0.00	0.00	0.00
time (sec)	N/A	0.475	0.953	0.991	0.273	0.260	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	35	46	54	115	189	0	98	45
N.S.	1	1.13	1.48	1.74	3.71	6.10	0.00	3.16	1.45
time (sec)	N/A	0.260	0.088	0.862	0.236	0.257	0.000	0.272	2.720

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	27	18	25	33	0	24	20
N.S.	1	1.00	2.08	1.38	1.92	2.54	0.00	1.85	1.54
time (sec)	N/A	0.179	0.002	0.231	0.196	0.273	0.000	0.268	0.079

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	69	22	19	22	22
N.S.	1	1.00	1.17	1.67	5.75	1.83	1.58	1.83	1.83
time (sec)	N/A	0.208	0.126	0.330	0.310	0.260	21.531	0.280	2.362

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	91	22	20	22	22
N.S.	1	1.00	1.17	1.67	7.58	1.83	1.67	1.83	1.83
time (sec)	N/A	0.212	0.729	0.307	0.310	0.252	25.975	0.263	2.330

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	22	22	0	22	22
N.S.	1	1.00	1.11	1.11	1.22	1.22	0.00	1.22	1.22
time (sec)	N/A	0.513	26.427	0.255	0.435	0.258	0.000	0.275	2.377

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	184	219	241	216	1055	0	0	0
N.S.	1	1.29	1.53	1.69	1.51	7.38	0.00	0.00	0.00
time (sec)	N/A	1.100	0.317	0.612	0.279	0.275	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	121	152	185	157	731	0	0	0
N.S.	1	1.27	1.60	1.95	1.65	7.69	0.00	0.00	0.00
time (sec)	N/A	0.768	0.221	0.594	0.281	0.270	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	79	89	109	367	0	144	95
N.S.	1	1.00	1.68	1.89	2.32	7.81	0.00	3.06	2.02
time (sec)	N/A	0.438	0.321	0.563	0.272	0.260	0.000	0.301	0.100

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	28	22	33	56	31	0	45	22
N.S.	1	1.27	1.00	1.50	2.55	1.41	0.00	2.05	1.00
time (sec)	N/A	0.213	0.005	0.330	0.191	0.252	0.000	0.266	2.315

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	94	22	19	22	22
N.S.	1	1.00	1.11	1.11	5.22	1.22	1.06	1.22	1.22
time (sec)	N/A	0.534	18.292	0.308	0.332	0.261	52.124	0.750	2.391

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	102	22	20	22	22
N.S.	1	1.00	1.11	1.11	5.67	1.22	1.11	1.22	1.22
time (sec)	N/A	0.644	14.331	0.312	0.333	0.248	70.250	1.628	2.397

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	0	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.11
time (sec)	N/A	0.532	35.339	0.104	0.367	0.256	0.000	0.281	2.478

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	114	118	177	130	979	0	0	0
N.S.	1	1.37	1.42	2.13	1.57	11.80	0.00	0.00	0.00
time (sec)	N/A	0.575	1.103	0.535	0.264	0.270	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	52	55	72	107	383	0	139	101
N.S.	1	1.24	1.31	1.71	2.55	9.12	0.00	3.31	2.40
time (sec)	N/A	0.354	0.095	0.424	0.268	0.259	0.000	0.275	2.318

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	43	130	107	0	184	36
N.S.	1	1.00	1.00	1.43	4.33	3.57	0.00	6.13	1.20
time (sec)	N/A	0.246	0.178	0.381	0.208	0.250	0.000	0.270	2.367

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	25	86	0	27	13
N.S.	1	1.00	1.00	0.93	1.67	5.73	0.00	1.80	0.87
time (sec)	N/A	0.191	0.003	0.170	0.185	0.250	0.000	0.268	2.380

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	128	20	17	20	20
N.S.	1	1.00	1.11	1.00	7.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.320	15.930	0.213	0.306	0.258	21.179	0.273	2.455

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	127	20	19	20	20
N.S.	1	1.00	1.11	1.00	7.06	1.11	1.06	1.11	1.11
time (sec)	N/A	0.383	19.038	0.201	0.307	0.246	27.851	0.265	2.428

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	22	22	0	22	22
N.S.	1	1.00	1.11	1.11	1.22	1.22	0.00	1.22	1.22
time (sec)	N/A	0.301	164.808	0.201	0.406	0.258	0.000	0.275	2.387

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	398	280	340	262	1802	0	0	0
N.S.	1	1.98	1.39	1.69	1.30	8.97	0.00	0.00	0.00
time (sec)	N/A	1.712	3.309	1.093	0.282	0.284	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	256	222	210	197	1311	0	0	0
N.S.	1	2.08	1.80	1.71	1.60	10.66	0.00	0.00	0.00
time (sec)	N/A	1.272	1.835	1.052	0.289	0.279	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	153	116	156	124	842	0	0	0
N.S.	1	1.87	1.41	1.90	1.51	10.27	0.00	0.00	0.00
time (sec)	N/A	0.697	0.864	1.042	0.277	0.264	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	42	75	45	84	387	0	84	87
N.S.	1	1.24	2.21	1.32	2.47	11.38	0.00	2.47	2.56
time (sec)	N/A	0.260	0.008	0.779	0.200	0.253	0.000	0.266	2.333

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	157	22	19	22	22
N.S.	1	1.00	1.11	1.11	8.72	1.22	1.06	1.22	1.22
time (sec)	N/A	0.289	47.096	0.348	0.325	0.263	51.835	1.706	2.358

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	159	22	20	22	22
N.S.	1	1.00	1.11	1.11	8.83	1.22	1.11	1.22	1.22
time (sec)	N/A	0.298	47.665	0.373	0.326	0.253	61.924	2.492	2.398

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	173	22	0	22	22
N.S.	1	1.00	1.17	1.67	14.42	1.83	0.00	1.83	1.83
time (sec)	N/A	0.208	89.980	0.243	0.771	0.264	0.000	0.358	2.482

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	266	422	375	302	1985	0	0	0
N.S.	1	1.49	2.36	2.09	1.69	11.09	0.00	0.00	0.00
time (sec)	N/A	1.439	2.144	0.527	0.252	0.292	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	168	314	246	226	1467	0	0	0
N.S.	1	1.47	2.75	2.16	1.98	12.87	0.00	0.00	0.00
time (sec)	N/A	0.906	2.090	0.525	0.280	0.284	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	112	131	164	149	975	0	0	0
N.S.	1	1.37	1.60	2.00	1.82	11.89	0.00	0.00	0.00
time (sec)	N/A	0.520	0.937	0.518	0.265	0.279	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	40	34	23	79	346	0	66	25
N.S.	1	1.48	1.26	0.85	2.93	12.81	0.00	2.44	0.93
time (sec)	N/A	0.244	0.043	0.315	0.196	0.262	0.000	0.307	2.335

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	144	22	19	22	22
N.S.	1	1.00	1.17	1.67	12.00	1.83	1.58	1.83	1.83
time (sec)	N/A	0.209	0.262	0.332	0.310	0.260	118.960	0.284	2.560

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	175	22	20	22	22
N.S.	1	1.00	1.17	1.67	14.58	1.83	1.67	1.83	1.83
time (sec)	N/A	0.210	0.210	0.339	0.317	0.277	159.591	0.274	2.565

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	17	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	1.06	1.12	1.38
time (sec)	N/A	0.395	8.760	0.100	0.314	0.251	0.342	0.265	2.389

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	184	150	241	203	448	0	0	0
N.S.	1	1.24	1.01	1.63	1.37	3.03	0.00	0.00	0.00
time (sec)	N/A	0.698	0.255	0.844	0.208	0.259	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	124	108	186	148	351	0	0	0
N.S.	1	1.28	1.11	1.92	1.53	3.62	0.00	0.00	0.00
time (sec)	N/A	0.495	0.194	0.674	0.189	0.263	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	68	67	125	87	224	0	0	0
N.S.	1	1.17	1.16	2.16	1.50	3.86	0.00	0.00	0.00
time (sec)	N/A	0.338	0.164	0.529	0.197	0.275	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	15	31	12	50	60	0	41	30
N.S.	1	1.36	2.82	1.09	4.55	5.45	0.00	3.73	2.73
time (sec)	N/A	0.182	0.005	0.356	0.269	0.267	0.000	0.257	0.069

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.38
time (sec)	N/A	0.246	13.677	0.201	0.314	0.256	0.537	0.298	2.318

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	17	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	1.06	1.12	1.38
time (sec)	N/A	0.243	13.794	0.199	0.312	0.265	0.385	0.359	2.303

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	19	20	22
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.06	1.11	1.22
time (sec)	N/A	0.485	102.848	0.114	0.302	0.263	0.382	0.264	2.398

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	262	263	0	0	1280	0	0	0
N.S.	1	1.16	1.16	0.00	0.00	5.66	0.00	0.00	0.00
time (sec)	N/A	0.609	0.267	0.000	0.000	0.303	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	179	181	0	0	937	0	0	0
N.S.	1	1.23	1.24	0.00	0.00	6.42	0.00	0.00	0.00
time (sec)	N/A	0.479	0.175	0.000	0.000	0.278	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	78	95	90	401	0	0	0
N.S.	1	1.00	1.16	1.42	1.34	5.99	0.00	0.00	0.00
time (sec)	N/A	0.312	0.284	1.309	0.291	0.253	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	20	42	23	61	155	0	64	52
N.S.	1	0.87	1.83	1.00	2.65	6.74	0.00	2.78	2.26
time (sec)	N/A	0.200	0.006	0.822	0.189	0.254	0.000	0.277	0.067

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	99	20	17	20	22
N.S.	1	1.00	1.11	1.00	5.50	1.11	0.94	1.11	1.22
time (sec)	N/A	0.366	35.641	0.194	0.320	0.259	0.752	0.968	2.295

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	111	20	19	20	22
N.S.	1	1.00	1.11	1.00	6.17	1.11	1.06	1.11	1.22
time (sec)	N/A	0.365	22.311	0.196	0.341	0.268	0.525	1.229	2.354

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	19	20	22
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.06	1.11	1.22
time (sec)	N/A	0.543	80.004	0.116	0.313	0.252	0.367	0.276	2.311

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	260	524	359	329	3409	0	0	0
N.S.	1	1.08	2.18	1.50	1.37	14.20	0.00	0.00	0.00
time (sec)	N/A	0.706	6.395	7.581	0.211	0.312	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	380	256	229	2523	0	0	0
N.S.	1	1.00	2.57	1.73	1.55	17.05	0.00	0.00	0.00
time (sec)	N/A	0.485	2.611	5.470	0.213	0.291	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	87	166	142	1543	0	0	0
N.S.	1	1.00	0.92	1.75	1.49	16.24	0.00	0.00	0.00
time (sec)	N/A	0.329	0.379	3.822	0.231	0.288	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	28	36	23	88	371	0	93	78
N.S.	1	1.04	1.33	0.85	3.26	13.74	0.00	3.44	2.89
time (sec)	N/A	0.214	0.012	2.610	0.269	0.257	0.000	0.268	0.077

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	147	20	17	20	22
N.S.	1	1.00	1.11	1.00	8.17	1.11	0.94	1.11	1.22
time (sec)	N/A	0.397	47.503	0.204	0.329	0.255	0.731	0.456	2.436

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	155	20	19	20	22
N.S.	1	1.00	1.11	1.00	8.61	1.11	1.06	1.11	1.22
time (sec)	N/A	0.423	26.474	0.251	0.335	0.285	0.515	0.543	2.334

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	19	20	22
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.06	1.11	1.22
time (sec)	N/A	0.545	45.165	0.098	0.322	0.258	0.343	0.278	2.351

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	274	334	0	0	1309	0	0	0
N.S.	1	1.16	1.41	0.00	0.00	5.52	0.00	0.00	0.00
time (sec)	N/A	0.625	1.886	0.000	0.000	0.308	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	191	252	0	0	966	0	0	0
N.S.	1	1.22	1.61	0.00	0.00	6.15	0.00	0.00	0.00
time (sec)	N/A	0.489	1.033	0.000	0.000	0.277	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	184	179	0	567	0	0	0
N.S.	1	1.00	2.33	2.27	0.00	7.18	0.00	0.00	0.00
time (sec)	N/A	0.315	0.493	0.994	0.000	0.285	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	29	29	25	43	103	0	54	48
N.S.	1	1.21	1.21	1.04	1.79	4.29	0.00	2.25	2.00
time (sec)	N/A	0.198	0.003	0.615	0.280	0.252	0.000	0.263	2.265

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	106	20	17	20	22
N.S.	1	1.00	1.11	1.00	5.89	1.11	0.94	1.11	1.22
time (sec)	N/A	0.362	31.244	0.223	0.350	0.267	0.565	0.835	2.377

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	114	20	19	0	22
N.S.	1	1.00	1.11	1.00	6.33	1.11	1.06	0.00	1.22
time (sec)	N/A	0.386	24.083	0.256	0.350	0.254	0.403	0.000	2.439

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.655	5.218	0.140	0.334	0.264	0.386	0.272	2.454

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	135	307	263	180	1924	0	0	0
N.S.	1	1.59	3.61	3.09	2.12	22.64	0.00	0.00	0.00
time (sec)	N/A	0.721	1.477	6.984	0.208	0.286	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	96	216	199	118	1327	0	0	0
N.S.	1	1.50	3.38	3.11	1.84	20.73	0.00	0.00	0.00
time (sec)	N/A	0.537	0.906	4.993	0.209	0.286	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	41	26	62	87	292	0	72	43
N.S.	1	1.37	0.87	2.07	2.90	9.73	0.00	2.40	1.43
time (sec)	N/A	0.324	0.163	3.527	0.203	0.265	0.000	0.268	0.080

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	28	13	32	18	81	0	18	18
N.S.	1	1.22	0.57	1.39	0.78	3.52	0.00	0.78	0.78
time (sec)	N/A	0.214	0.003	2.729	0.194	0.271	0.000	0.288	2.455

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	101	22	19	22	22
N.S.	1	1.00	1.10	1.00	5.05	1.10	0.95	1.10	1.10
time (sec)	N/A	0.273	24.151	0.246	0.322	0.251	0.566	0.270	2.323

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	105	22	20	22	22
N.S.	1	1.00	1.10	1.00	5.25	1.10	1.00	1.10	1.10
time (sec)	N/A	0.279	18.241	0.241	0.321	0.267	0.390	0.264	2.312

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.703	60.430	0.124	0.325	0.262	0.363	0.280	2.334

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	206	310	0	0	3825	0	0	0
N.S.	1	1.00	1.50	0.00	0.00	18.57	0.00	0.00	0.00
time (sec)	N/A	0.682	4.856	0.000	0.000	0.333	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	205	232	0	2227	0	0	0
N.S.	1	1.00	1.71	1.93	0.00	18.56	0.00	0.00	0.00
time (sec)	N/A	0.376	3.162	8.926	0.000	0.288	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	60	29	47	90	511	0	102	107
N.S.	1	1.22	0.59	0.96	1.84	10.43	0.00	2.08	2.18
time (sec)	N/A	0.227	0.002	6.353	0.279	0.256	0.000	0.289	0.091

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	214	22	19	22	22
N.S.	1	1.00	1.10	1.00	10.70	1.10	0.95	1.10	1.10
time (sec)	N/A	0.461	49.821	0.254	0.393	0.245	0.573	2.065	2.309

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	215	22	20	22	22
N.S.	1	1.00	1.10	1.00	10.75	1.10	1.00	1.10	1.10
time (sec)	N/A	0.518	32.903	0.247	0.404	0.256	0.398	2.812	2.350

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	19	20	22
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.06	1.11	1.22
time (sec)	N/A	0.587	67.847	0.151	0.324	0.258	0.359	0.270	2.379

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	262	565	417	352	3394	0	0	0
N.S.	1	1.09	2.35	1.74	1.47	14.14	0.00	0.00	0.00
time (sec)	N/A	0.699	6.512	3.828	0.216	0.298	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	388	266	243	2562	0	0	0
N.S.	1	1.00	2.62	1.80	1.64	17.31	0.00	0.00	0.00
time (sec)	N/A	0.487	2.954	2.622	0.231	0.270	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	85	170	145	1578	0	0	0
N.S.	1	1.00	0.89	1.79	1.53	16.61	0.00	0.00	0.00
time (sec)	N/A	0.332	0.464	1.844	0.225	0.287	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	29	34	25	91	379	0	93	78
N.S.	1	1.04	1.21	0.89	3.25	13.54	0.00	3.32	2.79
time (sec)	N/A	0.215	0.018	1.244	0.275	0.256	0.000	0.268	2.491

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	167	20	17	20	22
N.S.	1	1.00	1.11	1.00	9.28	1.11	0.94	1.11	1.22
time (sec)	N/A	0.403	55.269	0.252	0.354	0.246	0.566	0.413	2.380

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	170	20	19	20	22
N.S.	1	1.00	1.11	1.00	9.44	1.11	1.06	1.11	1.22
time (sec)	N/A	0.430	23.517	0.257	0.362	0.254	0.406	0.505	2.293

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.747	60.007	0.124	0.333	0.252	0.359	0.277	2.286

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	317	346	466	0	0	5356	0	0	0
N.S.	1	1.09	1.47	0.00	0.00	16.90	0.00	0.00	0.00
time (sec)	N/A	1.326	6.438	0.000	0.000	0.334	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	197	197	341	0	0	3804	0	0	0
N.S.	1	1.00	1.73	0.00	0.00	19.31	0.00	0.00	0.00
time (sec)	N/A	0.748	6.336	0.000	0.000	0.321	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	139	148	166	1694	0	0	0
N.S.	1	1.00	1.28	1.36	1.52	15.54	0.00	0.00	0.00
time (sec)	N/A	0.383	1.368	5.888	0.306	0.283	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	52	86	43	106	709	0	110	111
N.S.	1	1.06	1.76	0.88	2.16	14.47	0.00	2.24	2.27
time (sec)	N/A	0.220	0.009	3.900	0.202	0.260	0.000	0.312	0.089

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	226	22	19	22	22
N.S.	1	1.00	1.10	1.00	11.30	1.10	0.95	1.10	1.10
time (sec)	N/A	0.475	69.138	0.267	0.378	0.248	0.564	2.356	2.336

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	226	22	20	3	22
N.S.	1	1.00	1.10	1.00	11.30	1.10	1.00	0.15	1.10
time (sec)	N/A	0.527	43.819	0.186	0.393	0.251	0.406	20.102	2.432

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.701	167.592	0.119	0.325	0.253	0.360	0.286	2.371

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	311	274	445	381	6764	0	0	0
N.S.	1	1.30	1.14	1.85	1.59	28.18	0.00	0.00	0.00
time (sec)	N/A	1.233	3.337	29.302	0.226	0.328	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	197	192	299	273	4779	0	0	0
N.S.	1	1.32	1.29	2.01	1.83	32.07	0.00	0.00	0.00
time (sec)	N/A	0.764	3.064	21.848	0.220	0.313	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	119	106	197	164	3025	0	0	0
N.S.	1	1.31	1.16	2.16	1.80	33.24	0.00	0.00	0.00
time (sec)	N/A	0.470	0.749	15.865	0.222	0.302	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	61	43	102	774	0	96	96
N.S.	1	0.95	1.42	1.00	2.37	18.00	0.00	2.23	2.23
time (sec)	N/A	0.231	0.006	11.704	0.276	0.258	0.000	0.278	0.091

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	208	22	19	22	22
N.S.	1	1.00	1.10	1.00	10.40	1.10	0.95	1.10	1.10
time (sec)	N/A	0.277	51.986	0.261	0.358	0.257	0.575	0.532	2.250

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	206	22	20	22	22
N.S.	1	1.00	1.10	1.00	10.30	1.10	1.00	1.10	1.10
time (sec)	N/A	0.276	38.386	0.257	0.373	0.247	0.401	0.641	2.271

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	100	77	0	0	0	0	0	0
N.S.	1	1.15	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.402	0.389	0.000	0.000	0.000	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	72	142	0	0	0	0	0	0
N.S.	1	1.12	2.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	2.332	0.000	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	72	56	0	0	0	0	0	0
N.S.	1	1.12	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	0.262	0.000	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	109	250	0	0	0	0	0
N.S.	1	1.00	2.95	6.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	1.286	0.261	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	0.278	0.000	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	68	57	0	0	0	0	0	0
N.S.	1	1.06	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.314	0.000	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	72	57	0	0	0	0	0	0
N.S.	1	1.12	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	0.328	0.000	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	96	69	0	0	0	0	0	0
N.S.	1	1.10	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	0.361	0.000	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	116	69	0	0	0	0	0	0
N.S.	1	1.08	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.517	0.418	0.000	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	92	65	0	0	0	0	0	0
N.S.	1	1.10	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.418	0.314	0.000	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	88	57	0	0	0	0	0	0
N.S.	1	1.05	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.411	0.300	0.000	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	46	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.319	0.235	0.000	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	100	250	0	0	0	0	0
N.S.	1	1.00	1.75	4.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	1.400	0.287	0.000	0.000	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	92	71	0	0	0	0	0	0
N.S.	1	1.10	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.407	0.322	0.000	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	92	125	0	0	0	0	0	0
N.S.	1	1.10	1.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.404	3.289	0.000	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	120	93	0	0	0	0	0	0
N.S.	1	1.12	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.506	0.421	0.000	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	134	103	0	0	0	0	0	0
N.S.	1	1.11	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.494	0.360	0.000	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	106	143	0	0	0	0	0	0
N.S.	1	1.08	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.406	3.954	0.000	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	106	77	0	0	0	0	0	0
N.S.	1	1.08	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	0.284	0.000	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	115	229	0	0	0	0	0
N.S.	1	1.00	1.62	3.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.316	1.665	0.232	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	56	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	0.277	0.000	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	102	66	0	0	0	0	0	0
N.S.	1	1.04	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.394	0.296	0.000	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	106	67	0	0	0	0	0	0
N.S.	1	1.08	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.392	0.357	0.000	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	130	89	0	0	0	0	0	0
N.S.	1	1.07	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.490	0.339	0.000	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	130	83	0	0	0	0	0	0
N.S.	1	1.07	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.548	0.550	0.000	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	106	75	0	0	0	0	0	0
N.S.	1	1.08	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.422	0.351	0.000	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	102	70	0	0	0	0	0	0
N.S.	1	1.04	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.419	0.288	0.000	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	56	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.243	0.000	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	100	229	0	0	0	0	0
N.S.	1	1.00	1.41	3.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	0.898	0.285	0.000	0.000	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	106	67	0	0	0	0	0	0
N.S.	1	1.08	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.419	0.457	0.000	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	106	111	0	0	0	0	0	0
N.S.	1	1.08	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.414	3.176	0.000	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	134	103	0	0	0	0	0	0
N.S.	1	1.11	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.524	0.495	0.000	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	42	35	53	0	0	35
N.S.	1	1.00	1.00	3.23	2.69	4.08	0.00	0.00	2.69
time (sec)	N/A	0.319	0.111	0.655	0.310	0.249	0.000	0.000	2.288

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	84	23	0	69	95	0	0	0
N.S.	1	2.71	0.74	0.00	2.23	3.06	0.00	0.00	0.00
time (sec)	N/A	0.442	0.107	0.000	0.335	0.262	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	114	29	0	103	253	0	0	0
N.S.	1	2.28	0.58	0.00	2.06	5.06	0.00	0.00	0.00
time (sec)	N/A	0.529	0.140	0.000	0.301	0.259	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	35	42	54	55	0	0	23
N.S.	1	1.00	2.69	3.23	4.15	4.23	0.00	0.00	1.77
time (sec)	N/A	0.357	0.106	0.640	0.299	0.245	0.000	0.000	2.234

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	74	21	0	109	97	0	0	0
N.S.	1	2.39	0.68	0.00	3.52	3.13	0.00	0.00	0.00
time (sec)	N/A	0.432	0.085	0.000	0.300	0.252	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	99	44	0	163	259	0	0	0
N.S.	1	1.98	0.88	0.00	3.26	5.18	0.00	0.00	0.00
time (sec)	N/A	0.542	0.287	0.000	0.308	0.266	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	60	50	122	167	585	87	178
N.S.	1	1.00	1.15	0.96	2.35	3.21	11.25	1.67	3.42
time (sec)	N/A	0.329	0.159	0.687	0.278	0.266	30.481	0.277	2.556

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	62	52	119	174	586	88	177
N.S.	1	1.00	1.17	0.98	2.25	3.28	11.06	1.66	3.34
time (sec)	N/A	0.332	0.233	0.714	0.297	0.258	30.303	0.275	0.342

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	56	104	0	289	840	60	198
N.S.	1	1.00	0.98	1.82	0.00	5.07	14.74	1.05	3.47
time (sec)	N/A	0.335	0.091	0.372	0.000	0.265	16.661	0.263	2.513

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	56	109	0	299	840	62	199
N.S.	1	1.00	0.95	1.85	0.00	5.07	14.24	1.05	3.37
time (sec)	N/A	0.334	0.125	0.413	0.000	0.277	15.822	0.259	2.486

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	58	62	134	0	96	103
N.S.	1	1.00	1.00	2.32	2.48	5.36	0.00	3.84	4.12
time (sec)	N/A	0.268	0.295	0.621	0.409	0.261	0.000	0.301	2.427

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	55	0	129	0	100	105
N.S.	1	1.00	1.00	2.20	0.00	5.16	0.00	4.00	4.20
time (sec)	N/A	0.278	0.115	0.556	0.000	0.274	0.000	0.281	2.427

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	63	59	0	249	0	53	636
N.S.	1	1.00	1.02	0.95	0.00	4.02	0.00	0.85	10.26
time (sec)	N/A	0.456	0.205	0.510	0.000	0.482	0.000	0.267	7.579

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	71	77	58	141	172	0	90	539
N.S.	1	1.22	1.33	1.00	2.43	2.97	0.00	1.55	9.29
time (sec)	N/A	0.467	1.195	0.680	0.284	0.467	0.000	0.286	4.291

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	39	64	70	238	41	56
N.S.	1	1.00	1.26	2.05	3.37	3.68	12.53	2.16	2.95
time (sec)	N/A	0.253	0.015	0.206	0.272	0.268	3.320	0.252	2.290

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	17	9	14	17	41	14	14
N.S.	1	1.00	2.12	1.12	1.75	2.12	5.12	1.75	1.75
time (sec)	N/A	0.287	0.022	0.251	0.191	0.246	0.449	0.242	0.046

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	18	9	14	17	12	14	14
N.S.	1	1.00	2.25	1.12	1.75	2.12	1.50	1.75	1.75
time (sec)	N/A	0.289	0.035	0.243	0.196	0.257	0.369	0.245	0.049

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	39	102	70	61	38	56
N.S.	1	1.00	1.26	2.05	5.37	3.68	3.21	2.00	2.95
time (sec)	N/A	0.248	0.061	0.540	0.287	0.263	1.042	0.255	0.119

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	79	127	84	0	598	0	71	704
N.S.	1	1.07	1.72	1.14	0.00	8.08	0.00	0.96	9.51
time (sec)	N/A	0.625	0.626	0.605	0.000	0.476	0.000	0.259	8.066

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	87	106	86	158	401	0	109	613
N.S.	1	1.26	1.54	1.25	2.29	5.81	0.00	1.58	8.88
time (sec)	N/A	0.646	1.624	0.720	0.295	0.480	0.000	0.270	4.577

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	8	23	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.89	2.56	1.00
time (sec)	N/A	0.143	0.036	0.238	0.189	0.257	0.081	0.259	2.321

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	37	46	42	78	74	39
N.S.	1	1.00	0.97	1.00	1.24	1.14	2.11	2.00	1.05
time (sec)	N/A	0.197	0.051	0.999	0.196	0.256	0.086	0.263	2.320

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	51	63	48	69	97	66	134	53
N.S.	1	1.46	1.80	1.37	1.97	2.77	1.89	3.83	1.51
time (sec)	N/A	0.209	0.117	5.914	0.195	0.259	0.116	0.262	0.101

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	75	87	90	103	168	265	208	143
N.S.	1	1.04	1.21	1.25	1.43	2.33	3.68	2.89	1.99
time (sec)	N/A	0.281	0.101	24.436	0.202	0.264	0.188	0.274	2.316

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	83	133	102	191	298	172	344	117
N.S.	1	1.36	2.18	1.67	3.13	4.89	2.82	5.64	1.92
time (sec)	N/A	0.250	0.201	180.655	0.210	0.248	0.249	0.266	0.137

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	47	46	39	0	148	105	35	35
N.S.	1	1.24	1.21	1.03	0.00	3.89	2.76	0.92	0.92
time (sec)	N/A	0.212	0.027	0.655	0.000	0.265	2.106	0.254	0.106

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	27	29	62	604	26	22
N.S.	1	1.00	1.00	1.59	1.71	3.65	35.53	1.53	1.29
time (sec)	N/A	0.185	0.029	5.051	0.228	0.249	136.944	0.259	2.281

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	85	96	146	0	1495	0	88	157
N.S.	1	1.10	1.25	1.90	0.00	19.42	0.00	1.14	2.04
time (sec)	N/A	0.313	0.359	32.794	0.000	0.286	0.000	0.260	2.336

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	64	46	498	527	0	53	47
N.S.	1	1.00	0.96	0.69	7.43	7.87	0.00	0.79	0.70
time (sec)	N/A	0.282	0.098	137.481	0.225	0.256	0.000	0.271	2.428

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	136	147	462	0	6874	0	236	354
N.S.	1	1.21	1.31	4.12	0.00	61.38	0.00	2.11	3.16
time (sec)	N/A	0.433	0.663	0.288	0.000	0.352	0.000	0.266	2.407

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	206	33	0	57	0	0	0
N.S.	1	1.00	3.17	0.51	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.276	0.514	0.986	0.000	0.078	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	92	171	0	101	0	0	0
N.S.	1	1.00	0.89	1.66	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.383	0.418	0.809	0.000	0.086	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	193	42	0	274	0	0	0
N.S.	1	1.00	1.87	0.41	0.00	2.66	0.00	0.00	0.00
time (sec)	N/A	0.388	0.588	0.912	0.000	0.086	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	81	97	0	29	0	0	0
N.S.	1	1.00	1.25	1.49	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.268	0.085	0.650	0.000	0.079	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	148	33	0	222	0	0	0
N.S.	1	1.00	1.32	0.29	0.00	1.98	0.00	0.00	0.00
time (sec)	N/A	0.411	0.372	0.587	0.000	0.087	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	133	37	0	679	0	0	0
N.S.	1	1.00	1.15	0.32	0.00	5.85	0.00	0.00	0.00
time (sec)	N/A	0.376	0.425	0.629	0.000	0.097	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	45	12	23	21	29	56	11
N.S.	1	1.00	1.96	0.52	1.00	0.91	1.26	2.43	0.48
time (sec)	N/A	0.164	0.007	0.343	0.190	0.247	0.090	0.254	0.061

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	18	88	43	44	17	17
N.S.	1	1.00	0.96	0.69	3.38	1.65	1.69	0.65	0.65
time (sec)	N/A	0.195	0.138	1.276	0.199	0.243	0.116	0.254	0.070

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	18	146	64	83	17	17
N.S.	1	1.00	0.96	0.69	5.62	2.46	3.19	0.65	0.65
time (sec)	N/A	0.193	0.208	6.512	0.206	0.237	0.163	0.257	0.074

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	27	18	34	36	17	17
N.S.	1	1.00	0.92	1.04	0.69	1.31	1.38	0.65	0.65
time (sec)	N/A	0.200	0.077	0.741	0.200	0.247	0.127	0.261	2.404

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	18	17	23	34	17	17
N.S.	1	1.00	1.00	0.75	0.71	0.96	1.42	0.71	0.71
time (sec)	N/A	0.198	0.174	0.667	0.195	0.254	0.296	0.267	0.073

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	18	17	49	66	17	17
N.S.	1	1.00	1.00	0.69	0.65	1.88	2.54	0.65	0.65
time (sec)	N/A	0.195	0.190	3.465	0.197	0.248	0.448	0.263	0.110

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	18	17	71	90	17	17
N.S.	1	1.00	1.00	0.69	0.65	2.73	3.46	0.65	0.65
time (sec)	N/A	0.195	0.200	19.502	0.205	0.259	0.613	0.269	2.293

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	23	17	24	0	17	15
N.S.	1	1.00	0.92	0.88	0.65	0.92	0.00	0.65	0.58
time (sec)	N/A	0.195	0.066	0.332	0.190	0.255	0.000	0.252	2.252

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	16	17	42	0	17	15
N.S.	1	1.00	0.92	0.62	0.65	1.62	0.00	0.65	0.58
time (sec)	N/A	0.201	0.079	0.270	0.191	0.255	0.000	0.252	2.226

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	47	16	24	22	29	56	15
N.S.	1	1.00	1.96	0.67	1.00	0.92	1.21	2.33	0.62
time (sec)	N/A	0.160	0.041	0.369	0.190	0.255	0.102	0.271	0.050

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	89	43	44	17	17
N.S.	1	1.00	1.00	0.67	3.30	1.59	1.63	0.63	0.63
time (sec)	N/A	0.195	0.088	1.310	0.190	0.240	0.105	0.257	2.209

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	147	62	83	17	17
N.S.	1	1.00	1.00	0.67	5.44	2.30	3.07	0.63	0.63
time (sec)	N/A	0.192	0.120	6.793	0.213	0.244	0.144	0.261	2.306

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	29	20	39	37	23	21
N.S.	1	1.00	0.96	1.04	0.71	1.39	1.32	0.82	0.75
time (sec)	N/A	0.195	0.057	0.739	0.202	0.236	0.117	0.255	2.365

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	14	13	20	32	13	13
N.S.	1	1.00	0.92	0.58	0.54	0.83	1.33	0.54	0.54
time (sec)	N/A	0.190	0.007	0.639	0.192	0.247	0.207	0.254	2.252

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	17	41	65	17	17
N.S.	1	1.00	1.00	0.67	0.63	1.52	2.41	0.63	0.63
time (sec)	N/A	0.190	0.114	3.433	0.195	0.251	0.278	0.257	0.101

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	17	59	88	17	17
N.S.	1	1.00	1.00	0.67	0.63	2.19	3.26	0.63	0.63
time (sec)	N/A	0.195	0.137	19.299	0.202	0.250	0.618	0.264	2.298

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	19	17	24	0	17	18
N.S.	1	1.00	0.96	0.70	0.63	0.89	0.00	0.63	0.67
time (sec)	N/A	0.191	0.040	0.450	0.198	0.236	0.000	0.259	2.602

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	19	17	40	0	17	27
N.S.	1	1.00	0.96	0.70	0.63	1.48	0.00	0.63	1.00
time (sec)	N/A	0.199	0.035	0.287	0.201	0.247	0.000	0.261	0.190

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	184	355	160	279	2040	0	240	495
N.S.	1	1.48	2.86	1.29	2.25	16.45	0.00	1.94	3.99
time (sec)	N/A	0.457	1.301	198.102	0.293	0.266	0.000	0.262	2.909

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	106	79	84	210	207	0	110	145
N.S.	1	1.06	0.79	0.84	2.10	2.07	0.00	1.10	1.45
time (sec)	N/A	0.594	0.241	52.869	0.223	0.252	0.000	0.267	2.360

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	92	194	64	120	502	0	117	233
N.S.	1	1.59	3.34	1.10	2.07	8.66	0.00	2.02	4.02
time (sec)	N/A	0.352	1.299	13.145	0.278	0.269	0.000	0.258	2.505

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	26	43	42	0	31	33
N.S.	1	1.00	0.90	0.90	1.48	1.45	0.00	1.07	1.14
time (sec)	N/A	0.281	0.035	2.971	0.195	0.259	0.000	0.257	2.326

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	30	19	21	40
N.S.	1	1.00	1.00	1.09	1.00	2.73	1.73	1.91	3.64
time (sec)	N/A	0.148	0.006	0.359	0.198	0.251	0.196	0.263	0.073

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	27	28	27	32	22	25
N.S.	1	1.00	1.00	2.45	2.55	2.45	2.91	2.00	2.27
time (sec)	N/A	0.243	0.003	0.431	0.190	0.261	0.154	0.262	0.085

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	80	502	101	100	362	0	97	132
N.S.	1	1.29	8.10	1.63	1.61	5.84	0.00	1.56	2.13
time (sec)	N/A	0.452	1.588	2.048	0.283	0.257	0.000	0.272	2.439

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	45	42	78	117	543	651	75	0
N.S.	1	0.94	0.88	1.62	2.44	11.31	13.56	1.56	0.00
time (sec)	N/A	0.313	0.100	11.794	0.203	0.263	1.299	0.264	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	B	B	B	F	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	146	178	3430	357	375	2978	0	267	0
N.S.	1	1.22	23.49	2.45	2.57	20.40	0.00	1.83	0.00
time (sec)	N/A	0.884	7.169	90.760	0.321	0.286	0.000	0.283	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	83	83	176	297	2640	2162	152	0
N.S.	1	0.87	0.87	1.85	3.13	27.79	22.76	1.60	0.00
time (sec)	N/A	0.354	0.167	289.964	0.251	0.283	6.197	0.283	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	62	78	235	94	0	34	90
N.S.	1	1.00	1.55	1.95	5.88	2.35	0.00	0.85	2.25
time (sec)	N/A	0.285	0.041	0.488	0.277	0.255	0.000	0.257	2.438

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	74	60	181	52	0	22	65
N.S.	1	1.00	1.95	1.58	4.76	1.37	0.00	0.58	1.71
time (sec)	N/A	0.392	0.119	0.733	0.195	0.244	0.000	0.255	2.361

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	39	26	73	50	0	21	39
N.S.	1	1.00	1.50	1.00	2.81	1.92	0.00	0.81	1.50
time (sec)	N/A	0.273	0.028	73.087	0.285	0.244	0.000	0.263	2.317

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	16	15	25	17	0	12	14
N.S.	1	1.00	0.80	0.75	1.25	0.85	0.00	0.60	0.70
time (sec)	N/A	0.305	0.012	4.597	0.220	0.244	0.000	0.255	2.614

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	9	11	17	18	14
N.S.	1	1.00	1.00	1.00	0.82	1.00	1.55	1.64	1.27
time (sec)	N/A	0.148	0.005	1.164	0.205	0.255	0.229	0.253	0.057

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	17	15	15	11	22	11	14
N.S.	1	1.00	1.31	1.15	1.15	0.85	1.69	0.85	1.08
time (sec)	N/A	0.224	0.018	1.461	0.196	0.249	0.114	0.258	2.475

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	31	15	14	17	0	12	14
N.S.	1	1.00	1.55	0.75	0.70	0.85	0.00	0.60	0.70
time (sec)	N/A	0.249	0.024	7.432	0.200	0.252	0.000	0.257	0.098

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	40	26	33	50	432	21	41
N.S.	1	1.00	1.43	0.93	1.18	1.79	15.43	0.75	1.46
time (sec)	N/A	0.267	0.033	166.629	0.201	0.260	0.982	0.265	0.203

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	75	41	40	52	0	22	67
N.S.	1	1.00	1.97	1.08	1.05	1.37	0.00	0.58	1.76
time (sec)	N/A	0.331	0.066	0.408	0.198	0.252	0.000	0.253	0.192

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	38	45	68	58	94	1445	34	94
N.S.	1	0.90	1.07	1.62	1.38	2.24	34.40	0.81	2.24
time (sec)	N/A	0.277	0.084	0.396	0.221	0.269	4.262	0.252	2.422

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	40	62	78	235	94	0	34	94
N.S.	1	0.95	1.48	1.86	5.60	2.24	0.00	0.81	2.24
time (sec)	N/A	0.280	0.037	0.323	0.280	0.259	0.000	0.255	2.336

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	75	26	181	52	0	22	67
N.S.	1	1.00	1.97	0.68	4.76	1.37	0.00	0.58	1.76
time (sec)	N/A	0.381	0.066	278.081	0.202	0.246	0.000	0.255	2.290

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	26	39	26	73	50	0	21	41
N.S.	1	0.93	1.39	0.93	2.61	1.79	0.00	0.75	1.46
time (sec)	N/A	0.269	0.027	15.784	0.276	0.257	0.000	0.254	0.166

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	16	15	25	17	0	12	14
N.S.	1	1.00	0.80	0.75	1.25	0.85	0.00	0.60	0.70
time (sec)	N/A	0.311	0.005	6.089	0.189	0.249	0.000	0.252	0.120

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	9	11	17	18	14
N.S.	1	1.00	1.00	1.00	0.82	1.00	1.55	1.64	1.27
time (sec)	N/A	0.146	0.004	1.622	0.212	0.249	0.212	0.260	2.197

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	13	17	15	15	11	22	11	14
N.S.	1	1.18	1.55	1.36	1.36	1.00	2.00	1.00	1.27
time (sec)	N/A	0.229	0.018	1.918	0.209	0.258	0.131	0.258	0.137

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	31	15	14	17	0	12	14
N.S.	1	1.00	1.55	0.75	0.70	0.85	0.00	0.60	0.70
time (sec)	N/A	0.249	0.025	9.132	0.200	0.248	0.000	0.245	2.241

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	28	27	26	33	50	432	21	39
N.S.	1	1.08	1.04	1.00	1.27	1.92	16.62	0.81	1.50
time (sec)	N/A	0.268	0.033	44.040	0.212	0.257	0.972	0.261	0.190

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	74	41	40	52	0	22	65
N.S.	1	1.00	1.95	1.08	1.05	1.37	0.00	0.58	1.71
time (sec)	N/A	0.327	0.052	0.647	0.205	0.246	0.000	0.258	2.370

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	38	45	68	58	94	1445	34	90
N.S.	1	0.95	1.12	1.70	1.45	2.35	36.12	0.85	2.25
time (sec)	N/A	0.273	0.084	0.511	0.206	0.256	4.418	0.262	2.777

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	173	143	159	330	2716	0	234	392
N.S.	1	1.40	1.15	1.28	2.66	21.90	0.00	1.89	3.16
time (sec)	N/A	0.513	0.845	55.342	0.197	0.276	0.000	0.268	0.211

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	108	95	84	214	209	0	112	146
N.S.	1	1.07	0.94	0.83	2.12	2.07	0.00	1.11	1.45
time (sec)	N/A	0.611	0.290	20.292	0.190	0.259	0.000	0.260	2.417

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	97	78	66	152	674	0	115	169
N.S.	1	1.64	1.32	1.12	2.58	11.42	0.00	1.95	2.86
time (sec)	N/A	0.401	0.478	5.105	0.199	0.279	0.000	0.268	0.110

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	27	45	37	0	29	33
N.S.	1	1.00	0.85	1.00	1.67	1.37	0.00	1.07	1.22
time (sec)	N/A	0.298	0.165	1.339	0.188	0.249	0.000	0.266	2.311

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	30	14	13	29	22	33	35
N.S.	1	1.00	2.50	1.17	1.08	2.42	1.83	2.75	2.92
time (sec)	N/A	0.149	0.032	0.366	0.186	0.246	0.245	0.256	0.070

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	27	26	27	0	19	23
N.S.	1	1.00	1.00	2.45	2.36	2.45	0.00	1.73	2.09
time (sec)	N/A	0.259	0.003	0.445	0.208	0.250	0.000	0.245	2.198

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	72	61	96	0	682	0	68	139
N.S.	1	1.07	0.91	1.43	0.00	10.18	0.00	1.01	2.07
time (sec)	N/A	0.457	0.320	2.025	0.000	0.270	0.000	0.262	2.483

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	47	77	75	111	521	0	66	0
N.S.	1	0.94	1.54	1.50	2.22	10.42	0.00	1.32	0.00
time (sec)	N/A	0.338	0.154	10.248	0.203	0.253	0.000	0.264	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	179	150	206	0	5830	0	242	0
N.S.	1	1.13	0.94	1.30	0.00	36.67	0.00	1.52	0.00
time (sec)	N/A	0.896	0.371	40.024	0.000	0.353	0.000	0.269	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	86	138	174	285	2564	0	135	0
N.S.	1	0.88	1.41	1.78	2.91	26.16	0.00	1.38	0.00
time (sec)	N/A	0.381	0.319	126.517	0.222	0.282	0.000	0.267	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	32	32	236	270	0	33	81
N.S.	1	1.00	1.14	1.14	8.43	9.64	0.00	1.18	2.89
time (sec)	N/A	0.307	0.143	5.023	0.201	0.261	0.000	0.249	2.214

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	183	68	0	22	59
N.S.	1	1.00	1.00	0.77	6.10	2.27	0.00	0.73	1.97
time (sec)	N/A	0.414	0.056	1.820	0.189	0.251	0.000	0.261	2.194

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	20	22	66	91	0	22	33
N.S.	1	1.00	1.11	1.22	3.67	5.06	0.00	1.22	1.83
time (sec)	N/A	0.285	0.097	0.665	0.190	0.244	0.000	0.247	0.049

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	10	11	25	22	0	10	10
N.S.	1	1.00	0.71	0.79	1.79	1.57	0.00	0.71	0.71
time (sec)	N/A	0.309	0.037	0.245	0.185	0.253	0.000	0.260	2.305

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	23	10	9	13	19	25	11
N.S.	1	1.00	2.56	1.11	1.00	1.44	2.11	2.78	1.22
time (sec)	N/A	0.140	0.003	0.160	0.189	0.242	0.256	0.260	0.038

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	9	12	11	13	0	11	11
N.S.	1	1.00	1.80	2.40	2.20	2.60	0.00	2.20	2.20
time (sec)	N/A	0.224	0.026	0.282	0.208	0.255	0.000	0.252	0.045

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	11	12	20	0	10	10
N.S.	1	1.00	0.83	0.92	1.00	1.67	0.00	0.83	0.83
time (sec)	N/A	0.248	0.028	0.496	0.187	0.245	0.000	0.254	0.183

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	22	31	89	0	21	33
N.S.	1	1.00	1.29	1.57	2.21	6.36	0.00	1.50	2.36
time (sec)	N/A	0.281	0.032	1.332	0.206	0.244	0.000	0.253	2.179

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	2	38	68	0	22	57
N.S.	1	1.00	1.15	0.08	1.46	2.62	0.00	0.85	2.19
time (sec)	N/A	0.329	0.021	3.747	0.199	0.256	0.000	0.263	2.183

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	32	2	52	266	0	30	79
N.S.	1	1.00	1.45	0.09	2.36	12.09	0.00	1.36	3.59
time (sec)	N/A	0.301	0.028	10.657	0.206	0.253	0.000	0.269	2.161

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	30	28	238	265	0	28	77
N.S.	1	1.00	1.25	1.17	9.92	11.04	0.00	1.17	3.21
time (sec)	N/A	0.311	0.083	0.481	0.206	0.247	0.000	0.269	2.208

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	23	183	68	0	22	57
N.S.	1	1.00	1.15	0.88	7.04	2.62	0.00	0.85	2.19
time (sec)	N/A	0.412	0.008	0.282	0.192	0.252	0.000	0.248	2.191

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	20	68	88	0	19	31
N.S.	1	1.00	1.12	1.25	4.25	5.50	0.00	1.19	1.94
time (sec)	N/A	0.289	0.058	0.269	0.194	0.258	0.000	0.259	2.162

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	11	25	20	0	10	10
N.S.	1	1.00	1.50	0.92	2.08	1.67	0.00	0.83	0.83
time (sec)	N/A	0.333	0.002	0.124	0.200	0.248	0.000	0.267	0.063

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	27	12	11	11	19	25	9
N.S.	1	1.00	2.45	1.09	1.00	1.00	1.73	2.27	0.82
time (sec)	N/A	0.156	0.008	0.116	0.175	0.254	0.262	0.257	2.176

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	13	11	0	10	9
N.S.	1	1.00	1.00	1.11	1.44	1.22	0.00	1.11	1.00
time (sec)	N/A	0.247	0.035	0.266	0.192	0.249	0.000	0.253	0.043

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	24	11	12	22	0	10	10
N.S.	1	1.00	1.71	0.79	0.86	1.57	0.00	0.71	0.71
time (sec)	N/A	0.261	0.001	0.446	0.195	0.256	0.000	0.249	2.342

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	27	20	35	90	0	20	31
N.S.	1	1.00	1.35	1.00	1.75	4.50	0.00	1.00	1.55
time (sec)	N/A	0.296	0.048	1.336	0.194	0.249	0.000	0.243	0.051

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	23	38	68	0	22	59
N.S.	1	1.00	0.93	0.77	1.27	2.27	0.00	0.73	1.97
time (sec)	N/A	0.339	0.008	3.787	0.204	0.265	0.000	0.260	2.185

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	39	2	58	269	0	31	79
N.S.	1	1.00	1.30	0.07	1.93	8.97	0.00	1.03	2.63
time (sec)	N/A	0.299	0.052	10.357	0.197	0.254	0.000	0.255	2.155

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	19	9	8	53	8	24	27
N.S.	1	1.00	2.38	1.12	1.00	6.62	1.00	3.00	3.38
time (sec)	N/A	0.142	0.005	0.136	0.174	0.246	0.168	0.255	0.045

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	29	18	15	26	32	0	39	26
N.S.	1	1.32	0.82	0.68	1.18	1.45	0.00	1.77	1.18
time (sec)	N/A	0.208	0.004	0.400	0.186	0.253	0.000	0.256	2.226

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	39	67	28	67	616	0	62	71
N.S.	1	1.15	1.97	0.82	1.97	18.12	0.00	1.82	2.09
time (sec)	N/A	0.283	0.321	1.198	0.184	0.258	0.000	0.250	0.067

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	35	42	54	55	0	0	13
N.S.	1	1.00	2.69	3.23	4.15	4.23	0.00	0.00	1.00
time (sec)	N/A	0.423	0.038	0.931	0.280	0.246	0.000	0.000	2.233

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	74	21	0	109	97	0	0	0
N.S.	1	2.39	0.68	0.00	3.52	3.13	0.00	0.00	0.00
time (sec)	N/A	0.493	0.020	0.000	0.291	0.246	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	99	44	0	163	259	0	0	0
N.S.	1	1.98	0.88	0.00	3.26	5.18	0.00	0.00	0.00
time (sec)	N/A	0.598	0.223	0.000	0.287	0.251	0.000	0.000	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	42	10	16	16
N.S.	1	1.00	1.00	1.12	1.00	5.25	1.25	2.00	2.00
time (sec)	N/A	0.139	0.007	0.569	0.181	0.269	0.197	0.250	2.201

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	31	16	13	26	30	0	37	26
N.S.	1	1.41	0.73	0.59	1.18	1.36	0.00	1.68	1.18
time (sec)	N/A	0.206	0.029	0.352	0.194	0.254	0.000	0.253	2.229

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	37	37	29	56	486	0	66	57
N.S.	1	1.09	1.09	0.85	1.65	14.29	0.00	1.94	1.68
time (sec)	N/A	0.278	0.025	3.722	0.268	0.260	0.000	0.250	0.064

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	43	39	57	0	0	15
N.S.	1	1.00	1.00	3.07	2.79	4.07	0.00	0.00	1.07
time (sec)	N/A	0.314	0.080	0.605	0.289	0.254	0.000	0.000	2.280

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	33	84	24	0	77	99	0	0	0
N.S.	1	2.55	0.73	0.00	2.33	3.00	0.00	0.00	0.00
time (sec)	N/A	0.418	0.105	0.000	0.294	0.251	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	115	30	0	115	257	0	0	0
N.S.	1	2.17	0.57	0.00	2.17	4.85	0.00	0.00	0.00
time (sec)	N/A	0.527	0.148	0.000	0.292	0.268	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	34	35	17	39	96	0	43	39
N.S.	1	1.89	1.94	0.94	2.17	5.33	0.00	2.39	2.17
time (sec)	N/A	0.402	0.034	0.297	0.177	0.248	0.000	0.253	0.047

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	34	50	17	40	96	0	43	41
N.S.	1	1.70	2.50	0.85	2.00	4.80	0.00	2.15	2.05
time (sec)	N/A	0.406	0.034	0.300	0.190	0.241	0.000	0.252	0.042

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	48	29	39	40	43	143	43	30
N.S.	1	1.23	0.74	1.00	1.03	1.10	3.67	1.10	0.77
time (sec)	N/A	0.323	0.126	0.351	0.189	0.265	0.265	0.257	0.094

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	82	89	93	0	435	685	61	157
N.S.	1	1.11	1.20	1.26	0.00	5.88	9.26	0.82	2.12
time (sec)	N/A	0.392	0.270	0.458	0.000	0.262	127.105	0.264	2.470

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	116	75	106	87	337	0	114	86
N.S.	1	1.15	0.74	1.05	0.86	3.34	0.00	1.13	0.85
time (sec)	N/A	0.548	0.233	1.254	0.186	0.251	0.000	0.260	2.423

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	38	41	42	150	43	29
N.S.	1	1.00	0.74	0.97	1.05	1.08	3.85	1.10	0.74
time (sec)	N/A	0.309	0.021	0.246	0.182	0.263	0.322	0.265	0.053

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	82	80	93	0	435	774	61	157
N.S.	1	1.11	1.08	1.26	0.00	5.88	10.46	0.82	2.12
time (sec)	N/A	0.376	0.167	0.384	0.000	0.274	123.841	0.266	2.432

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	103	75	104	86	331	0	111	84
N.S.	1	1.02	0.74	1.03	0.85	3.28	0.00	1.10	0.83
time (sec)	N/A	0.524	0.100	0.869	0.194	0.265	0.000	0.266	2.403

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	60	60	54	0	200	0	48	164
N.S.	1	1.20	1.20	1.08	0.00	4.00	0.00	0.96	3.28
time (sec)	N/A	0.297	0.134	0.971	0.000	0.276	0.000	0.253	4.320

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	60	68	53	0	239	0	60	177
N.S.	1	1.18	1.33	1.04	0.00	4.69	0.00	1.18	3.47
time (sec)	N/A	0.303	0.103	1.109	0.000	0.274	0.000	0.274	2.610

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	77	125	99	0	594	0	72	183
N.S.	1	1.17	1.89	1.50	0.00	9.00	0.00	1.09	2.77
time (sec)	N/A	0.326	0.282	1.663	0.000	0.257	0.000	0.262	2.575

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	83	61	101	104	348	983	113	108
N.S.	1	1.22	0.90	1.49	1.53	5.12	14.46	1.66	1.59
time (sec)	N/A	0.537	0.371	0.400	0.214	0.261	0.823	0.269	2.743

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	327	205	121	0	1633	0	174	255
N.S.	1	1.68	1.05	0.62	0.00	8.37	0.00	0.89	1.31
time (sec)	N/A	1.339	0.447	1.045	0.000	0.287	0.000	0.271	2.569

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	73	124	98	0	596	0	72	183
N.S.	1	1.14	1.94	1.53	0.00	9.31	0.00	1.12	2.86
time (sec)	N/A	0.302	0.102	1.089	0.000	0.258	0.000	0.263	2.420

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	80	66	104	104	348	952	114	104
N.S.	1	1.19	0.99	1.55	1.55	5.19	14.21	1.70	1.55
time (sec)	N/A	0.513	0.209	0.400	0.205	0.260	0.760	0.249	0.453

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	200	204	124	0	1645	0	174	255
N.S.	1	1.50	1.53	0.93	0.00	12.37	0.00	1.31	1.92
time (sec)	N/A	0.695	0.326	0.679	0.000	0.287	0.000	0.257	2.597

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	54	31	167	216	0	50	42
N.S.	1	1.00	2.84	1.63	8.79	11.37	0.00	2.63	2.21
time (sec)	N/A	0.207	0.257	12.404	0.204	0.239	0.000	0.264	2.419

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	140	117	160	289	1268	3813	251	159
N.S.	1	1.35	1.12	1.54	2.78	12.19	36.66	2.41	1.53
time (sec)	N/A	0.743	0.688	0.552	0.219	0.288	1.861	0.273	2.555

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	26	40	41	167	216	0	48	42
N.S.	1	1.37	2.11	2.16	8.79	11.37	0.00	2.53	2.21
time (sec)	N/A	0.206	0.069	5.859	0.198	0.250	0.000	0.254	2.262

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	130	119	158	292	1269	3840	251	159
N.S.	1	1.25	1.14	1.52	2.81	12.20	36.92	2.41	1.53
time (sec)	N/A	0.717	0.631	0.542	0.213	0.263	1.771	0.266	2.347

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	88	79	92	0	427	678	60	157
N.S.	1	1.22	1.10	1.28	0.00	5.93	9.42	0.83	2.18
time (sec)	N/A	0.436	0.159	0.247	0.000	0.282	124.528	0.277	2.383

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	110	73	98	83	334	0	101	81
N.S.	1	1.08	0.72	0.96	0.81	3.27	0.00	0.99	0.79
time (sec)	N/A	0.629	0.144	0.587	0.197	0.254	0.000	0.259	2.451

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	153	180	166	0	1861	0	163	261
N.S.	1	1.12	1.31	1.21	0.00	13.58	0.00	1.19	1.91
time (sec)	N/A	0.747	0.781	3.858	0.000	0.291	0.000	0.250	2.530

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	113	73	99	84	334	0	102	81
N.S.	1	1.11	0.72	0.97	0.82	3.27	0.00	1.00	0.79
time (sec)	N/A	0.620	0.372	0.617	0.194	0.259	0.000	0.257	2.413

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	142	179	168	0	1847	0	159	260
N.S.	1	1.16	1.47	1.38	0.00	15.14	0.00	1.30	2.13
time (sec)	N/A	0.825	0.792	2.379	0.000	0.279	0.000	0.266	2.473

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	198	128	160	153	1158	0	199	127
N.S.	1	1.02	0.66	0.82	0.79	5.97	0.00	1.03	0.65
time (sec)	N/A	1.236	0.433	10.944	0.205	0.267	0.000	0.255	2.699

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	156	167	174	0	1829	0	163	259
N.S.	1	1.14	1.22	1.27	0.00	13.35	0.00	1.19	1.89
time (sec)	N/A	0.747	0.865	1.922	0.000	0.297	0.000	0.257	2.608

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	190	126	161	150	1162	0	202	129
N.S.	1	0.98	0.65	0.83	0.77	5.99	0.00	1.04	0.66
time (sec)	N/A	1.224	0.379	7.100	0.200	0.281	0.000	0.281	2.665

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	0	325	266	0	4935	0	325	371
N.S.	1	0.00	1.53	1.25	0.00	23.28	0.00	1.53	1.75
time (sec)	N/A	0.000	1.631	31.595	0.000	0.324	0.000	0.276	2.810

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	146	60	93	107	376	962	128	98
N.S.	1	1.57	0.65	1.00	1.15	4.04	10.34	1.38	1.05
time (sec)	N/A	0.819	0.137	0.279	0.210	0.266	0.835	0.268	2.766

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	287	222	128	0	1819	0	179	397
N.S.	1	1.74	1.35	0.78	0.00	11.02	0.00	1.08	2.41
time (sec)	N/A	1.538	0.798	0.675	0.000	0.288	0.000	0.260	2.733

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	B	F(-1)	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	0	176	177	241	1655	0	238	127
N.S.	1	0.00	0.82	0.82	1.12	7.70	0.00	1.11	0.59
time (sec)	N/A	0.000	0.804	2.153	0.209	0.285	0.000	0.285	2.705

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	290	264	130	0	1805	0	179	397
N.S.	1	1.78	1.62	0.80	0.00	11.07	0.00	1.10	2.44
time (sec)	N/A	1.530	0.662	0.687	0.000	0.289	0.000	0.256	2.994

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	B	F(-1)	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	0	174	190	244	1726	0	232	132
N.S.	1	0.00	0.85	0.93	1.19	8.42	0.00	1.13	0.64
time (sec)	N/A	0.000	1.348	1.681	0.207	0.286	0.000	0.265	2.670

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	0	474	208	0	5061	0	310	592
N.S.	1	0.00	1.82	0.80	0.00	19.39	0.00	1.19	2.27
time (sec)	N/A	0.000	2.783	6.113	0.000	0.332	0.000	0.269	2.882

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	B	F(-1)	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	0	183	178	240	1661	0	240	127
N.S.	1	0.00	0.85	0.83	1.12	7.73	0.00	1.12	0.59
time (sec)	N/A	0.000	0.866	1.681	0.198	0.284	0.000	0.257	2.590

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	0	481	198	0	5031	0	310	590
N.S.	1	0.00	1.86	0.76	0.00	19.42	0.00	1.20	2.28
time (sec)	N/A	0.000	1.465	4.435	0.000	0.361	0.000	0.278	2.742

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	B	F(-1)	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	0	366	276	384	4001	0	384	173
N.S.	1	0.00	1.17	0.88	1.22	12.74	0.00	1.22	0.55
time (sec)	N/A	0.000	1.031	17.210	0.211	0.315	0.000	0.267	2.759

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	89	78	126	0	233	367	80	178
N.S.	1	1.11	0.98	1.58	0.00	2.91	4.59	1.00	2.22
time (sec)	N/A	0.328	0.608	0.844	0.000	0.281	26.639	0.263	4.745

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	90	155	115	0	679	0	83	168
N.S.	1	1.10	1.89	1.40	0.00	8.28	0.00	1.01	2.05
time (sec)	N/A	0.350	0.484	5.330	0.000	0.272	0.000	0.261	2.637

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	145	134	187	0	1855	0	152	217
N.S.	1	1.18	1.09	1.52	0.00	15.08	0.00	1.24	1.76
time (sec)	N/A	0.526	1.026	31.777	0.000	0.313	0.000	0.267	2.553

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	89	78	126	0	234	697	80	177
N.S.	1	1.11	0.98	1.58	0.00	2.92	8.71	1.00	2.21
time (sec)	N/A	0.331	0.150	0.611	0.000	0.267	30.011	0.260	3.893

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	87	151	116	0	680	0	83	168
N.S.	1	1.12	1.94	1.49	0.00	8.72	0.00	1.06	2.15
time (sec)	N/A	0.340	0.221	2.604	0.000	0.274	0.000	0.256	2.608

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	143	134	211	0	1855	0	152	216
N.S.	1	1.19	1.12	1.76	0.00	15.46	0.00	1.27	1.80
time (sec)	N/A	0.505	0.759	13.826	0.000	0.299	0.000	0.258	2.395

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	7	6	16	8	6	6
N.S.	1	1.00	1.55	0.64	0.55	1.45	0.73	0.55	0.55
time (sec)	N/A	0.211	0.032	0.346	0.194	0.243	0.215	0.269	0.068

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	7	6	19	10	6	6
N.S.	1	1.00	1.55	0.64	0.55	1.73	0.91	0.55	0.55
time (sec)	N/A	0.210	0.024	0.306	0.189	0.256	0.177	0.261	2.236

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	15	17	10	13	12	13	16
N.S.	1	1.00	1.07	1.21	0.71	0.93	0.86	0.93	1.14
time (sec)	N/A	0.196	0.051	0.694	0.190	0.253	0.075	0.254	0.095

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	56	87	60	326	54	53
N.S.	1	1.00	0.81	1.06	1.64	1.13	6.15	1.02	1.00
time (sec)	N/A	0.243	0.201	0.444	0.204	0.256	0.339	0.257	2.365

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	87	87	118	0	749	0	88	199
N.S.	1	1.12	1.12	1.51	0.00	9.60	0.00	1.13	2.55
time (sec)	N/A	0.343	0.305	1.653	0.000	0.285	0.000	0.253	2.733

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	70	63	337	232	0	70	67
N.S.	1	1.00	0.99	0.89	4.75	3.27	0.00	0.99	0.94
time (sec)	N/A	0.323	0.255	14.047	0.218	0.253	0.000	0.263	2.495

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	101	90	175	0	264	643	89	302
N.S.	1	1.10	0.98	1.90	0.00	2.87	6.99	0.97	3.28
time (sec)	N/A	0.352	0.540	0.641	0.000	0.273	30.671	0.252	4.825

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	97	106	133	0	799	0	95	210
N.S.	1	1.10	1.20	1.51	0.00	9.08	0.00	1.08	2.39
time (sec)	N/A	0.362	0.482	2.408	0.000	0.287	0.000	0.253	2.575

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	158	146	228	0	1931	0	183	224
N.S.	1	1.17	1.08	1.69	0.00	14.30	0.00	1.36	1.66
time (sec)	N/A	0.551	0.920	12.656	0.000	0.298	0.000	0.258	2.497

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	128	116	97	137	160	196	219	131
N.S.	1	1.08	0.97	0.82	1.15	1.34	1.65	1.84	1.10
time (sec)	N/A	0.412	0.131	4.824	0.192	0.258	0.145	0.250	0.164

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	57	54	54	63	59	100	83	55
N.S.	1	0.97	0.92	0.92	1.07	1.00	1.69	1.41	0.93
time (sec)	N/A	0.242	0.067	1.001	0.188	0.245	0.095	0.247	2.353

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	26	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	2.17	1.00
time (sec)	N/A	0.147	0.038	0.233	0.196	0.245	0.067	0.266	0.055

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	56	54	53	0	248	0	46	78
N.S.	1	1.10	1.06	1.04	0.00	4.86	0.00	0.90	1.53
time (sec)	N/A	0.235	0.060	0.572	0.000	0.273	0.000	0.258	0.223

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	95	105	191	0	1268	0	111	0
N.S.	1	1.06	1.17	2.12	0.00	14.09	0.00	1.23	0.00
time (sec)	N/A	0.356	0.193	3.235	0.000	0.274	0.000	0.262	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	168	183	577	0	7379	0	304	0
N.S.	1	1.15	1.25	3.95	0.00	50.54	0.00	2.08	0.00
time (sec)	N/A	0.551	0.356	21.914	0.000	0.332	0.000	0.273	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	261	488	1588	0	23093	0	717	0
N.S.	1	1.19	2.22	7.22	0.00	104.97	0.00	3.26	0.00
time (sec)	N/A	0.902	0.628	118.960	0.000	0.494	0.000	0.297	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	115	112	96	137	144	189	186	131
N.S.	1	1.10	1.07	0.91	1.30	1.37	1.80	1.77	1.25
time (sec)	N/A	0.404	0.127	4.421	0.187	0.255	0.122	0.258	0.161

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	55	55	55	63	57	100	81	51
N.S.	1	0.96	0.96	0.96	1.11	1.00	1.75	1.42	0.89
time (sec)	N/A	0.242	0.051	0.971	0.193	0.251	0.082	0.249	2.308

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	26	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	2.17	1.00
time (sec)	N/A	0.153	0.003	0.223	0.182	0.245	0.053	0.263	2.212

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	35	14	36	32	17	39	46
N.S.	1	1.00	2.33	0.93	2.40	2.13	1.13	2.60	3.07
time (sec)	N/A	0.190	0.030	0.521	0.192	0.250	0.255	0.259	0.174

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	87	49	86	236	0	84	0
N.S.	1	1.00	2.02	1.14	2.00	5.49	0.00	1.95	0.00
time (sec)	N/A	0.287	0.232	2.254	0.200	0.261	0.000	0.261	0.000

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	92	148	103	248	1504	0	205	0
N.S.	1	1.03	1.66	1.16	2.79	16.90	0.00	2.30	0.00
time (sec)	N/A	0.448	0.370	11.128	0.215	0.266	0.000	0.258	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	151	300	178	487	4015	0	377	0
N.S.	1	1.08	2.14	1.27	3.48	28.68	0.00	2.69	0.00
time (sec)	N/A	0.692	0.413	44.907	0.228	0.303	0.000	0.270	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	187	208	321	277	1293	626	390	361
N.S.	1	0.99	1.11	1.71	1.47	6.88	3.33	2.07	1.92
time (sec)	N/A	0.581	0.325	1.935	0.201	0.273	0.306	0.276	0.454

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	132	134	161	161	664	298	194	144
N.S.	1	0.97	0.99	1.18	1.18	4.88	2.19	1.43	1.06
time (sec)	N/A	0.411	0.197	0.634	0.198	0.265	0.210	0.255	2.278

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	77	72	80	79	238	122	96	70
N.S.	1	0.86	0.80	0.89	0.88	2.64	1.36	1.07	0.78
time (sec)	N/A	0.273	0.100	0.443	0.182	0.261	0.119	0.251	2.252

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	61	20	36	22
N.S.	1	1.00	1.00	0.96	0.92	2.54	0.83	1.50	0.92
time (sec)	N/A	0.156	0.006	0.199	0.188	0.252	0.069	0.265	0.068

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	36	25	0	88	0	0	0
N.S.	1	1.00	1.06	0.74	0.00	2.59	0.00	0.00	0.00
time (sec)	N/A	0.225	0.066	0.401	0.000	0.268	0.000	0.000	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	68	47	0	660	0	0	0
N.S.	1	1.00	0.68	0.47	0.00	6.60	0.00	0.00	0.00
time (sec)	N/A	0.381	0.116	0.559	0.000	0.288	0.000	0.000	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	168	184	92	0	3035	0	0	0
N.S.	1	1.15	1.26	0.63	0.00	20.79	0.00	0.00	0.00
time (sec)	N/A	0.541	0.285	0.684	0.000	0.451	0.000	0.000	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	236	425	184	0	6590	0	0	0
N.S.	1	1.19	2.15	0.93	0.00	33.28	0.00	0.00	0.00
time (sec)	N/A	0.743	0.619	0.883	0.000	1.605	0.000	0.000	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	294	306	3775	899	0	928	0	0	0
N.S.	1	1.04	12.84	3.06	0.00	3.16	0.00	0.00	0.00
time (sec)	N/A	1.491	6.275	1.742	0.000	0.101	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	249	250	2292	321	0	463	0	0	0
N.S.	1	1.00	9.20	1.29	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	1.045	6.116	0.794	0.000	0.088	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	1401	317	0	314	0	0	0
N.S.	1	1.00	13.74	3.11	0.00	3.08	0.00	0.00	0.00
time (sec)	N/A	0.379	6.090	1.824	0.000	0.088	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	237	251	0	108	0	0	0
N.S.	1	1.00	2.32	2.46	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.379	0.407	0.742	0.000	0.075	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	156	156	806	888	0	798	0	0	0
N.S.	1	1.00	5.17	5.69	0.00	5.12	0.00	0.00	0.00
time (sec)	N/A	0.523	6.019	1.023	0.000	0.091	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	322	335	2492	2160	0	3730	0	0	0
N.S.	1	1.04	7.74	6.71	0.00	11.58	0.00	0.00	0.00
time (sec)	N/A	1.378	6.234	2.506	0.000	0.175	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	411	443	4093	57909	0	13897	0	0	0
N.S.	1	1.08	9.96	140.90	0.00	33.81	0.00	0.00	0.00
time (sec)	N/A	2.015	6.424	8.225	0.000	0.742	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	140	149	4500	289	1783	784	0	315	0
N.S.	1	1.06	32.14	2.06	12.74	5.60	0.00	2.25	0.00
time (sec)	N/A	0.523	56.315	0.604	2.151	0.323	0.000	0.304	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	4392	189	640	329	0	183	0
N.S.	1	1.00	47.74	2.05	6.96	3.58	0.00	1.99	0.00
time (sec)	N/A	0.367	35.933	0.210	0.527	0.300	0.000	0.278	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	455	125	153	143	0	81	0
N.S.	1	1.00	12.30	3.38	4.14	3.86	0.00	2.19	0.00
time (sec)	N/A	0.229	31.802	0.215	0.363	0.276	0.000	0.269	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	211	129	0	681	0	538	0
N.S.	1	1.00	2.13	1.30	0.00	6.88	0.00	5.43	0.00
time (sec)	N/A	0.375	49.812	0.387	0.000	0.309	0.000	0.485	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	0	417	0	1801	0	0	0
N.S.	1	1.00	0.00	2.69	0.00	11.62	0.00	0.00	0.00
time (sec)	N/A	0.534	0.000	0.238	0.000	0.434	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	205	225	0	817	0	5297	0	0	0
N.S.	1	1.10	0.00	3.99	0.00	25.84	0.00	0.00	0.00
time (sec)	N/A	0.742	0.000	0.268	0.000	1.553	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	146	155	4368	275	1789	784	0	315	0
N.S.	1	1.06	29.92	1.88	12.25	5.37	0.00	2.16	0.00
time (sec)	N/A	0.530	56.146	0.602	2.284	0.329	0.000	0.318	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	4260	189	644	329	0	184	0
N.S.	1	1.00	44.38	1.97	6.71	3.43	0.00	1.92	0.00
time (sec)	N/A	0.373	31.280	0.221	0.470	0.291	0.000	0.280	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	4196	131	156	143	0	81	0
N.S.	1	1.00	107.59	3.36	4.00	3.67	0.00	2.08	0.00
time (sec)	N/A	0.229	35.969	0.227	0.337	0.279	0.000	0.295	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	52609	129	0	680	0	546	0
N.S.	1	1.00	515.77	1.26	0.00	6.67	0.00	5.35	0.00
time (sec)	N/A	0.371	54.832	0.405	0.000	0.324	0.000	0.524	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	0	415	0	2137	0	0	0
N.S.	1	1.00	0.00	2.61	0.00	13.44	0.00	0.00	0.00
time (sec)	N/A	0.549	0.000	0.234	0.000	0.465	0.000	0.000	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	211	231	0	817	0	5675	0	0	0
N.S.	1	1.09	0.00	3.87	0.00	26.90	0.00	0.00	0.00
time (sec)	N/A	0.764	0.000	0.287	0.000	1.632	0.000	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	113	86	177	0	429	0	106	472
N.S.	1	1.06	0.80	1.65	0.00	4.01	0.00	0.99	4.41
time (sec)	N/A	0.412	0.211	0.431	0.000	0.265	0.000	0.265	7.253

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	120	86	178	0	438	0	106	324
N.S.	1	1.06	0.76	1.58	0.00	3.88	0.00	0.94	2.87
time (sec)	N/A	0.431	0.423	0.280	0.000	0.284	0.000	0.270	0.742

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	120	86	181	0	455	0	106	325
N.S.	1	1.15	0.83	1.74	0.00	4.38	0.00	1.02	3.12
time (sec)	N/A	0.371	0.386	0.296	0.000	0.275	0.000	0.259	2.951

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	28	18	11	10	19	34	10	10
N.S.	1	1.56	1.00	0.61	0.56	1.06	1.89	0.56	0.56
time (sec)	N/A	0.193	0.114	0.108	0.206	0.247	0.319	0.267	0.048

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	60	54	53	0	234	0	46	78
N.S.	1	1.11	1.00	0.98	0.00	4.33	0.00	0.85	1.44
time (sec)	N/A	0.292	0.059	0.265	0.000	0.273	0.000	0.259	0.196

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	144	96	167	0	486	0	126	1069
N.S.	1	0.99	0.66	1.14	0.00	3.33	0.00	0.86	7.32
time (sec)	N/A	0.653	0.318	2.089	0.000	0.919	0.000	0.252	30.903

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	25	28	12	11	17	0	13	11
N.S.	1	1.32	1.47	0.63	0.58	0.89	0.00	0.68	0.58
time (sec)	N/A	0.261	0.224	0.127	0.199	0.252	0.000	0.254	0.063

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	62	54	53	0	244	0	46	78
N.S.	1	1.24	1.08	1.06	0.00	4.88	0.00	0.92	1.56
time (sec)	N/A	0.299	0.069	0.244	0.000	0.265	0.000	0.254	0.219

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	108	124	0	546	0	122	1069
N.S.	1	1.00	0.92	1.05	0.00	4.63	0.00	1.03	9.06
time (sec)	N/A	0.628	1.565	0.429	0.000	0.900	0.000	0.274	8.181

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	115	104	200	0	505	0	122	377
N.S.	1	0.96	0.87	1.67	0.00	4.21	0.00	1.02	3.14
time (sec)	N/A	0.392	1.008	0.371	0.000	0.279	0.000	0.257	0.783

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	113	130	224	0	2211	0	179	0
N.S.	1	1.05	1.20	2.07	0.00	20.47	0.00	1.66	0.00
time (sec)	N/A	0.402	0.540	1.709	0.000	0.305	0.000	0.274	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	219	373	836	0	12285	0	625	0
N.S.	1	1.11	1.88	4.22	0.00	62.05	0.00	3.16	0.00
time (sec)	N/A	0.696	0.997	11.623	0.000	0.594	0.000	0.277	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	116	104	197	0	508	0	122	375
N.S.	1	0.96	0.86	1.63	0.00	4.20	0.00	1.01	3.10
time (sec)	N/A	0.388	0.211	0.345	0.000	0.276	0.000	0.262	0.730

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	113	125	225	0	2228	0	177	0
N.S.	1	1.05	1.16	2.08	0.00	20.63	0.00	1.64	0.00
time (sec)	N/A	0.397	0.237	1.701	0.000	0.314	0.000	0.257	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	216	336	857	0	12366	0	625	0
N.S.	1	1.11	1.73	4.42	0.00	63.74	0.00	3.22	0.00
time (sec)	N/A	0.706	0.504	11.589	0.000	0.603	0.000	0.278	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	130	107	226	0	583	0	125	376
N.S.	1	1.04	0.86	1.81	0.00	4.66	0.00	1.00	3.01
time (sec)	N/A	0.398	0.838	0.375	0.000	0.285	0.000	0.256	0.765

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	113	123	225	0	2119	0	179	0
N.S.	1	1.05	1.14	2.08	0.00	19.62	0.00	1.66	0.00
time (sec)	N/A	0.406	0.494	1.722	0.000	0.306	0.000	0.263	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	215	319	800	0	10107	0	577	0
N.S.	1	1.11	1.64	4.12	0.00	52.10	0.00	2.97	0.00
time (sec)	N/A	0.673	1.039	11.484	0.000	0.589	0.000	0.273	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	142	119	237	0	605	0	136	454
N.S.	1	1.04	0.87	1.73	0.00	4.42	0.00	0.99	3.31
time (sec)	N/A	0.425	0.941	0.413	0.000	0.296	0.000	0.279	3.270

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	126	143	249	0	2541	0	207	0
N.S.	1	1.04	1.18	2.06	0.00	21.00	0.00	1.71	0.00
time (sec)	N/A	0.416	0.713	1.736	0.000	0.309	0.000	0.280	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	255	465	1084	0	13813	0	819	0
N.S.	1	1.09	2.00	4.65	0.00	59.28	0.00	3.52	0.00
time (sec)	N/A	0.776	1.317	11.799	0.000	0.728	0.000	0.296	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	34	36	0	55	0	35	0
N.S.	1	1.00	1.55	1.64	0.00	2.50	0.00	1.59	0.00
time (sec)	N/A	0.250	0.148	1.692	0.000	0.264	0.000	0.274	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	86	72	58	107	753	58	57
N.S.	1	1.00	1.21	1.01	0.82	1.51	10.61	0.82	0.80
time (sec)	N/A	0.246	0.349	0.245	0.195	0.265	2.205	0.267	2.327

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	84	72	57	110	806	58	57
N.S.	1	1.00	1.09	0.94	0.74	1.43	10.47	0.75	0.74
time (sec)	N/A	0.238	0.150	0.237	0.194	0.258	2.263	0.276	2.416

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	103	121	99	134	1321	79	75
N.S.	1	1.00	1.20	1.41	1.15	1.56	15.36	0.92	0.87
time (sec)	N/A	0.272	0.376	0.253	0.199	0.264	2.604	0.267	2.394

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	86	62	65	59	852	49	48
N.S.	1	1.00	1.12	0.81	0.84	0.77	11.06	0.64	0.62
time (sec)	N/A	0.244	0.374	0.247	0.198	0.266	2.283	0.268	0.128

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	86	62	62	56	904	48	47
N.S.	1	1.00	1.10	0.79	0.79	0.72	11.59	0.62	0.60
time (sec)	N/A	0.238	0.138	0.262	0.211	0.260	2.354	0.267	2.295

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	102	108	105	70	1420	69	64
N.S.	1	1.00	1.26	1.33	1.30	0.86	17.53	0.85	0.79
time (sec)	N/A	0.261	0.366	0.239	0.214	0.261	2.719	0.278	2.360

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	9	24	35	19	172	5	5
N.S.	1	1.00	3.00	8.00	11.67	6.33	57.33	1.67	1.67
time (sec)	N/A	0.182	0.004	1.046	0.284	0.256	3.771	0.270	0.050

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	8	11	8	40	48	10	10
N.S.	1	1.00	0.73	1.00	0.73	3.64	4.36	0.91	0.91
time (sec)	N/A	0.193	0.004	30.154	0.201	0.254	0.681	0.265	2.255

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	30	22	2	64	304	3602	46	28
N.S.	1	1.15	0.85	0.08	2.46	11.69	138.54	1.77	1.08
time (sec)	N/A	0.207	0.006	29.067	0.297	0.266	131.065	0.279	2.287

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	10	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	10.00	1.00	1.00
time (sec)	N/A	0.173	0.000	0.606	0.200	0.248	0.224	0.263	2.365

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	22	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	22.00	1.00	1.00
time (sec)	N/A	0.176	0.000	11.536	0.190	0.253	0.431	0.262	0.039

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	34	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	34.00	1.00	1.00
time (sec)	N/A	0.176	0.001	98.956	0.197	0.238	0.742	0.275	0.020

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.171	0.001	1.068	0.207	0.246	0.000	0.265	2.257

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.172	0.000	38.338	0.189	0.249	0.000	0.264	2.194

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.172	0.001	50.523	0.201	0.276	0.000	0.268	2.219

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	21	529	39	64	70	0	41	56
N.S.	1	1.11	27.84	2.05	3.37	3.68	0.00	2.16	2.95
time (sec)	N/A	0.206	0.667	0.464	0.281	0.278	0.000	0.272	0.185

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	33	549	2	88	266	0	63	78
N.S.	1	1.06	17.71	0.06	2.84	8.58	0.00	2.03	2.52
time (sec)	N/A	0.224	6.246	16.123	0.290	0.287	0.000	0.274	2.242

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	54	64	765	2	114	717	0	77	114
N.S.	1	1.19	14.17	0.04	2.11	13.28	0.00	1.43	2.11
time (sec)	N/A	0.244	4.747	210.079	0.279	0.281	0.000	0.276	0.071

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	20	18	37	36	67	0	36	54
N.S.	1	1.11	1.00	2.06	2.00	3.72	0.00	2.00	3.00
time (sec)	N/A	0.207	0.085	0.834	0.279	0.261	0.000	0.270	0.158

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	34	64	61	60	262	0	60	77
N.S.	1	1.06	2.00	1.91	1.88	8.19	0.00	1.88	2.41
time (sec)	N/A	0.222	0.117	26.167	0.298	0.275	0.000	0.271	0.060

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	66	66	2	84	715	0	72	112
N.S.	1	1.22	1.22	0.04	1.56	13.24	0.00	1.33	2.07
time (sec)	N/A	0.264	0.188	37.526	0.316	0.285	0.000	0.273	2.232

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.172	0.001	0.372	0.198	0.247	0.000	0.267	0.072

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.172	0.000	10.525	0.199	0.240	0.000	0.262	0.027

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.175	0.000	116.966	0.206	0.272	0.000	0.289	2.299

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	275	217	74	0	3313	0	1	0
N.S.	1	1.01	0.80	0.27	0.00	12.23	0.00	0.00	0.00
time (sec)	N/A	0.641	1.273	0.234	0.000	0.375	0.000	62.360	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	287	244	70	0	3309	0	0	0
N.S.	1	1.02	0.87	0.25	0.00	11.82	0.00	0.00	0.00
time (sec)	N/A	0.916	2.104	0.201	0.000	0.363	0.000	0.000	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	283	108	0	4943	0	5	0
N.S.	1	1.00	0.92	0.35	0.00	16.00	0.00	0.02	0.00
time (sec)	N/A	1.027	2.600	0.899	0.000	0.600	0.000	0.762	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	367	326	144	0	6680	0	24	0
N.S.	1	1.01	0.90	0.40	0.00	18.40	0.00	0.07	0.00
time (sec)	N/A	2.993	0.619	1.297	0.000	1.109	0.000	0.815	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	21	14	28	225	57	0	28	48
N.S.	1	1.75	1.17	2.33	18.75	4.75	0.00	2.33	4.00
time (sec)	N/A	0.317	0.173	0.187	0.283	0.245	0.000	0.288	2.656

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	306	258	79	0	6841	0	1	0
N.S.	1	1.02	0.86	0.26	0.00	22.80	0.00	0.00	0.00
time (sec)	N/A	0.661	1.734	3.666	0.000	2.974	0.000	1.554	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	198	208	0	3485	0	1	0
N.S.	1	1.00	0.89	0.93	0.00	15.63	0.00	0.00	0.00
time (sec)	N/A	0.607	0.786	0.536	0.000	0.380	0.000	61.080	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	227	204	0	3505	0	0	0
N.S.	1	1.00	0.99	0.89	0.00	15.24	0.00	0.00	0.00
time (sec)	N/A	0.641	0.817	0.523	0.000	0.382	0.000	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	264	274	0	5079	0	5	0
N.S.	1	1.00	1.04	1.07	0.00	19.92	0.00	0.02	0.00
time (sec)	N/A	1.044	0.750	0.908	0.000	0.584	0.000	0.728	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	309	354	0	6794	0	24	0
N.S.	1	1.00	1.03	1.18	0.00	22.72	0.00	0.08	0.00
time (sec)	N/A	3.530	0.433	1.499	0.000	1.083	0.000	0.795	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	20	11	12	0	54	0	26	51
N.S.	1	1.82	1.00	1.09	0.00	4.91	0.00	2.36	4.64
time (sec)	N/A	0.312	0.015	0.165	0.000	0.253	0.000	0.260	2.675

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	241	254	0	6997	0	1	0
N.S.	1	1.00	0.98	1.03	0.00	28.44	0.00	0.00	0.00
time (sec)	N/A	0.560	0.768	3.684	0.000	3.063	0.000	1.564	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	41	34	92	74	367	202	46	209
N.S.	1	1.05	0.87	2.36	1.90	9.41	5.18	1.18	5.36
time (sec)	N/A	0.305	0.315	0.178	0.295	0.298	0.589	0.270	2.913

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	40	33	92	74	363	224	45	208
N.S.	1	1.05	0.87	2.42	1.95	9.55	5.89	1.18	5.47
time (sec)	N/A	0.274	0.060	0.155	0.287	0.286	0.581	0.254	2.673

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	40	40	47	73	95	136	33	25
N.S.	1	1.05	1.05	1.24	1.92	2.50	3.58	0.87	0.66
time (sec)	N/A	0.331	5.035	0.167	0.292	0.279	0.502	0.255	2.484

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	40	40	47	73	95	136	33	25
N.S.	1	1.05	1.05	1.24	1.92	2.50	3.58	0.87	0.66
time (sec)	N/A	0.289	5.034	0.153	0.285	0.271	0.493	0.260	0.095

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	46	47	136	36	91	0	0	0
N.S.	1	0.78	0.80	2.31	0.61	1.54	0.00	0.00	0.00
time (sec)	N/A	0.687	0.119	0.141	0.295	0.262	0.000	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	68	71	209	60	188	0	0	0
N.S.	1	0.65	0.68	2.01	0.58	1.81	0.00	0.00	0.00
time (sec)	N/A	0.826	0.106	0.094	0.307	0.275	0.000	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	98	93	281	80	272	0	0	0
N.S.	1	0.65	0.62	1.87	0.53	1.81	0.00	0.00	0.00
time (sec)	N/A	0.943	0.109	0.092	0.305	0.263	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	59	43	175	43	152	0	0	0
N.S.	1	0.81	0.59	2.40	0.59	2.08	0.00	0.00	0.00
time (sec)	N/A	0.663	0.033	0.147	0.316	0.290	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	73	57	253	67	294	0	0	0
N.S.	1	0.74	0.58	2.58	0.68	3.00	0.00	0.00	0.00
time (sec)	N/A	0.811	0.037	0.100	0.310	0.276	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	92	68	329	87	427	0	0	0
N.S.	1	0.71	0.53	2.55	0.67	3.31	0.00	0.00	0.00
time (sec)	N/A	0.891	0.043	0.110	0.301	0.278	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	44	66	150	60	351	0	0	0
N.S.	1	0.50	0.75	1.70	0.68	3.99	0.00	0.00	0.00
time (sec)	N/A	0.540	0.074	0.137	0.283	0.294	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	114	128	0	0	786	0	0	0
N.S.	1	0.61	0.68	0.00	0.00	4.20	0.00	0.00	0.00
time (sec)	N/A	0.728	0.181	0.000	0.000	0.280	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	170	180	0	0	1202	0	0	0
N.S.	1	0.59	0.63	0.00	0.00	4.19	0.00	0.00	0.00
time (sec)	N/A	0.823	0.263	0.000	0.000	0.294	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	69	71	252	92	1757	0	0	0
N.S.	1	0.52	0.54	1.91	0.70	13.31	0.00	0.00	0.00
time (sec)	N/A	0.587	0.245	0.127	0.293	0.273	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	98	113	441	154	3431	0	0	0
N.S.	1	0.48	0.55	2.16	0.75	16.82	0.00	0.00	0.00
time (sec)	N/A	0.868	0.366	0.130	0.295	0.322	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	170	149	602	207	4629	0	0	0
N.S.	1	0.52	0.46	1.85	0.63	14.20	0.00	0.00	0.00
time (sec)	N/A	0.912	0.615	0.124	0.289	0.355	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	162	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	0.635	0.000	0.000	0.000	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	114	77	99	126	164	190	138	79
N.S.	1	1.05	0.71	0.91	1.16	1.50	1.74	1.27	0.72
time (sec)	N/A	0.402	0.654	88.697	0.197	0.242	0.363	0.267	2.633

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	50	66	63	80	129	81	44
N.S.	1	1.00	0.79	1.05	1.00	1.27	2.05	1.29	0.70
time (sec)	N/A	0.257	0.390	7.153	0.191	0.252	0.174	0.271	2.485

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	38	19	18	31	24	34	18
N.S.	1	1.00	1.90	0.95	0.90	1.55	1.20	1.70	0.90
time (sec)	N/A	0.162	0.043	0.241	0.191	0.248	0.088	0.269	0.060

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	39	48	133	73	299	0	79	343
N.S.	1	0.89	1.09	3.02	1.66	6.80	0.00	1.80	7.80
time (sec)	N/A	0.277	0.240	1.060	0.287	0.258	0.000	0.373	2.946

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	87	90	213	150	765	0	140	229
N.S.	1	0.98	1.01	2.39	1.69	8.60	0.00	1.57	2.57
time (sec)	N/A	0.409	0.497	15.759	0.282	0.257	0.000	0.421	2.793

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	154	121	481	327	2439	0	256	0
N.S.	1	1.08	0.85	3.36	2.29	17.06	0.00	1.79	0.00
time (sec)	N/A	0.620	0.706	211.638	0.299	0.293	0.000	0.596	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	297	239	1260	0	0	0	0	0
N.S.	1	0.99	0.79	4.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.306	1.473	6.296	0.000	0.000	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	244	202	935	0	0	0	0	0
N.S.	1	0.98	0.81	3.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.952	0.963	0.645	0.000	0.000	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	94	352	0	0	0	0	0
N.S.	1	1.00	0.98	3.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.391	0.291	0.464	0.000	0.000	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	90	181	0	143	0	0	0
N.S.	1	1.00	0.94	1.89	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.387	0.310	0.247	0.000	0.085	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	119	630	0	1283	0	0	0
N.S.	1	1.00	0.75	3.99	0.00	8.12	0.00	0.00	0.00
time (sec)	N/A	0.537	0.671	0.411	0.000	0.107	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	237	641	0	4231	0	0	0
N.S.	1	1.00	0.73	1.97	0.00	13.02	0.00	0.00	0.00
time (sec)	N/A	1.283	1.704	0.753	0.000	0.190	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	349	279	687	0	1488	0	0	0
N.S.	1	0.90	0.72	1.78	0.00	3.85	0.00	0.00	0.00
time (sec)	N/A	1.399	0.133	1.554	0.000	0.287	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	271	210	530	0	1122	0	0	0
N.S.	1	0.96	0.75	1.89	0.00	3.99	0.00	0.00	0.00
time (sec)	N/A	1.063	0.063	0.939	0.000	0.282	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	191	143	376	0	754	0	0	0
N.S.	1	1.03	0.77	2.02	0.00	4.05	0.00	0.00	0.00
time (sec)	N/A	0.704	0.048	0.913	0.000	0.277	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.07	1.00	1.14	1.14
time (sec)	N/A	0.267	0.964	0.097	0.265	0.264	66.010	0.256	2.292

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	0.055	0.000	0.000	0.000	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	53	50	55	53	105	124	54	45
N.S.	1	0.80	0.76	0.83	0.80	1.59	1.88	0.82	0.68
time (sec)	N/A	0.196	0.029	0.347	0.182	0.252	0.876	0.261	0.278

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	33	32	33	37	92	139	32	32
N.S.	1	0.82	0.80	0.82	0.92	2.30	3.48	0.80	0.80
time (sec)	N/A	0.197	0.015	0.154	0.188	0.270	0.439	0.283	2.370

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	31	25	27	26	54	54	26	25
N.S.	1	0.97	0.78	0.84	0.81	1.69	1.69	0.81	0.78
time (sec)	N/A	0.179	0.011	0.096	0.186	0.263	0.210	0.259	0.071

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	23	24	40	45	53	0	34	39
N.S.	1	0.88	0.92	1.54	1.73	2.04	0.00	1.31	1.50
time (sec)	N/A	0.173	0.015	0.105	0.186	0.262	0.000	0.257	0.082

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	37	37	43	62	104	0	44	37
N.S.	1	0.88	0.88	1.02	1.48	2.48	0.00	1.05	0.88
time (sec)	N/A	0.192	0.034	0.168	0.190	0.273	0.000	0.271	2.338

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	80	61	67	88	388	0	64	90
N.S.	1	1.10	0.84	0.92	1.21	5.32	0.00	0.88	1.23
time (sec)	N/A	0.194	0.041	0.365	0.182	0.252	0.000	0.272	2.325

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	134	108	85	0	316	976	84	127
N.S.	1	0.96	0.78	0.61	0.00	2.27	7.02	0.60	0.91
time (sec)	N/A	0.343	0.377	0.408	0.000	0.247	2.350	0.281	2.920

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	58	57	0	149	428	56	105
N.S.	1	1.00	0.66	0.65	0.00	1.69	4.86	0.64	1.19
time (sec)	N/A	0.264	0.116	0.215	0.000	0.257	0.845	0.263	0.254

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	37	0	67	201	40	54
N.S.	1	1.00	0.70	0.69	0.00	1.24	3.72	0.74	1.00
time (sec)	N/A	0.199	0.052	0.122	0.000	0.257	0.383	0.259	0.108

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	49	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.258	0.000	0.000	0.000	0.000	0.000	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	137	0	0	0	0	0	0
N.S.	1	1.00	2.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	1.232	0.000	0.000	0.000	0.000	0.000	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	107	82	0	0	0	0	0	0
N.S.	1	1.07	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	1.234	0.000	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.345	0.054	0.000	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	134	106	85	0	313	1085	84	125
N.S.	1	0.96	0.76	0.61	0.00	2.25	7.81	0.60	0.90
time (sec)	N/A	0.348	0.366	0.575	0.000	0.272	2.294	0.266	2.913

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	56	57	0	146	432	56	68
N.S.	1	1.00	0.64	0.65	0.00	1.66	4.91	0.64	0.77
time (sec)	N/A	0.260	0.114	0.233	0.000	0.252	0.841	0.267	2.543

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	37	0	66	167	40	53
N.S.	1	1.00	0.70	0.69	0.00	1.22	3.09	0.74	0.98
time (sec)	N/A	0.201	0.053	0.129	0.000	0.261	0.381	0.279	0.080

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	51	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	109	80	0	0	0	0	0	0
N.S.	1	1.06	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	0.104	0.000	0.000	0.000	0.000	0.000	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	108	89	0	0	0	0	0	0
N.S.	1	1.20	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	110	90	0	0	0	0	0	0
N.S.	1	1.21	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	0.077	0.000	0.000	0.000	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	240	196	195	189	441	2377	1546	252
N.S.	1	0.94	0.77	0.77	0.74	1.74	9.36	6.09	0.99
time (sec)	N/A	0.655	7.343	0.863	0.206	0.261	35.866	0.333	3.538

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	105	93	91	88	135	510	885	88
N.S.	1	0.99	0.88	0.86	0.83	1.27	4.81	8.35	0.83
time (sec)	N/A	0.417	0.720	0.238	0.199	0.263	1.190	0.294	2.669

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	104	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	1.154	0.000	0.000	0.000	0.000	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	209	250	0	0	0	0	0	0
N.S.	1	1.07	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.455	1.299	0.000	0.000	0.000	0.000	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	237	230	192	187	2340	2346	1548	288
N.S.	1	0.94	0.92	0.76	0.75	9.32	9.35	6.17	1.15
time (sec)	N/A	0.594	0.579	0.480	0.218	0.339	2.141	0.331	2.704

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	100	88	88	87	430	488	886	134
N.S.	1	0.99	0.87	0.87	0.86	4.26	4.83	8.77	1.33
time (sec)	N/A	0.400	0.222	0.179	0.206	0.263	0.649	0.296	2.484

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	164	127	0	0	0	0	0	0
N.S.	1	1.09	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.427	0.246	0.000	0.000	0.000	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	62	51	44	56	111	139	52	50
N.S.	1	0.90	0.74	0.64	0.81	1.61	2.01	0.75	0.72
time (sec)	N/A	0.212	0.051	284.720	0.196	0.249	2.172	0.271	0.541

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	47	45	47	50	95	177	57	43
N.S.	1	0.82	0.79	0.82	0.88	1.67	3.11	1.00	0.75
time (sec)	N/A	0.217	0.052	19.050	0.188	0.250	0.934	0.260	2.540

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	32	28	26	29	53	76	26	26
N.S.	1	0.91	0.80	0.74	0.83	1.51	2.17	0.74	0.74
time (sec)	N/A	0.198	0.005	1.799	0.181	0.243	0.391	0.258	2.364

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	22	22	27	38	49	0	32	38
N.S.	1	0.88	0.88	1.08	1.52	1.96	0.00	1.28	1.52
time (sec)	N/A	0.170	0.007	0.063	0.189	0.270	0.000	0.261	0.065

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	38	34	25	45	103	0	46	36
N.S.	1	0.93	0.83	0.61	1.10	2.51	0.00	1.12	0.88
time (sec)	N/A	0.203	0.039	0.118	0.185	0.261	0.000	0.260	2.365

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	77	59	55	78	387	0	62	102
N.S.	1	1.10	0.84	0.79	1.11	5.53	0.00	0.89	1.46
time (sec)	N/A	0.214	0.063	0.441	0.200	0.253	0.000	0.262	0.148

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	73	67	89	77	167	294	81	65
N.S.	1	0.80	0.74	0.98	0.85	1.84	3.23	0.89	0.71
time (sec)	N/A	0.240	0.071	0.032	0.183	0.261	5.104	0.272	0.630

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	45	40	41	38	90	144	36	36
N.S.	1	0.92	0.82	0.84	0.78	1.84	2.94	0.73	0.73
time (sec)	N/A	0.222	0.024	96.043	0.199	0.258	2.201	0.254	0.520

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	45	43	47	50	96	175	57	42
N.S.	1	0.79	0.75	0.82	0.88	1.68	3.07	1.00	0.74
time (sec)	N/A	0.215	0.050	10.561	0.191	0.253	0.881	0.276	0.287

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	48	37	41	50	72	0	39	35
N.S.	1	1.14	0.88	0.98	1.19	1.71	0.00	0.93	0.83
time (sec)	N/A	0.211	0.026	0.372	0.203	0.271	0.000	0.263	2.359

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	47	179	48	62	198	0	56	62
N.S.	1	0.89	3.38	0.91	1.17	3.74	0.00	1.06	1.17
time (sec)	N/A	0.198	1.167	0.081	0.202	0.268	0.000	0.251	2.389

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	59	46	35	69	262	0	59	65
N.S.	1	0.95	0.74	0.56	1.11	4.23	0.00	0.95	1.05
time (sec)	N/A	0.229	0.070	0.147	0.197	0.254	0.000	0.263	2.461

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	60	51	88	54	154	202	52	50
N.S.	1	0.87	0.74	1.28	0.78	2.23	2.93	0.75	0.72
time (sec)	N/A	0.227	0.032	0.047	0.190	0.254	11.958	0.272	2.708

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	75	67	84	77	165	325	81	65
N.S.	1	0.82	0.74	0.92	0.85	1.81	3.57	0.89	0.71
time (sec)	N/A	0.243	0.075	0.028	0.197	0.250	5.273	0.257	0.622

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	62	51	52	56	111	139	52	50
N.S.	1	0.90	0.74	0.75	0.81	1.61	2.01	0.75	0.72
time (sec)	N/A	0.216	0.030	48.589	0.201	0.253	2.211	0.247	0.526

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	51	68	50	65	170	0	57	66
N.S.	1	0.86	1.15	0.85	1.10	2.88	0.00	0.97	1.12
time (sec)	N/A	0.217	0.057	5.153	0.190	0.253	0.000	0.263	2.372

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	64	52	56	68	213	0	63	53
N.S.	1	1.02	0.83	0.89	1.08	3.38	0.00	1.00	0.84
time (sec)	N/A	0.226	0.083	0.802	0.204	0.252	0.000	0.252	2.474

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	72	286	77	88	459	0	72	97
N.S.	1	0.89	3.53	0.95	1.09	5.67	0.00	0.89	1.20
time (sec)	N/A	0.215	1.055	0.139	0.192	0.265	0.000	0.267	0.081

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	47	52	152	235	46	42
N.S.	1	1.00	0.75	0.82	0.91	2.67	4.12	0.81	0.74
time (sec)	N/A	0.221	0.078	290.280	0.208	0.260	2.192	0.248	0.575

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	57	51	55	53	105	128	54	47
N.S.	1	0.86	0.77	0.83	0.80	1.59	1.94	0.82	0.71
time (sec)	N/A	0.218	0.068	21.420	0.189	0.268	0.873	0.262	0.251

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	32	25	19	24	91	117	18	18
N.S.	1	1.39	1.09	0.83	1.04	3.96	5.09	0.78	0.78
time (sec)	N/A	0.198	0.014	1.926	0.189	0.251	0.430	0.264	2.431

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	38	57	64	0	30	30
N.S.	1	1.00	0.92	1.03	1.54	1.73	0.00	0.81	0.81
time (sec)	N/A	0.197	0.028	0.104	0.190	0.263	0.000	0.258	0.064

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	47	66	65	76	200	0	55	63
N.S.	1	0.87	1.22	1.20	1.41	3.70	0.00	1.02	1.17
time (sec)	N/A	0.200	0.092	0.162	0.199	0.267	0.000	0.262	2.382

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	55	48	56	86	262	0	48	66
N.S.	1	0.87	0.76	0.89	1.37	4.16	0.00	0.76	1.05
time (sec)	N/A	0.228	0.061	0.430	0.197	0.265	0.000	0.260	2.420

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	85	73	108	78	175	197	82	69
N.S.	1	0.85	0.73	1.08	0.78	1.75	1.97	0.82	0.69
time (sec)	N/A	0.238	0.076	0.024	0.199	0.271	5.636	0.265	0.609

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	48	38	44	42	108	128	43	39
N.S.	1	0.92	0.73	0.85	0.81	2.08	2.46	0.83	0.75
time (sec)	N/A	0.227	0.028	107.880	0.197	0.264	2.350	0.266	0.542

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	55	54	55	53	105	128	54	47
N.S.	1	0.83	0.82	0.83	0.80	1.59	1.94	0.82	0.71
time (sec)	N/A	0.213	0.032	10.725	0.195	0.260	1.011	0.275	2.527

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	41	58	54	61	98	0	46	53
N.S.	1	0.91	1.29	1.20	1.36	2.18	0.00	1.02	1.18
time (sec)	N/A	0.201	0.097	0.515	0.191	0.254	0.000	0.261	0.089

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	53	49	57	86	195	0	56	51
N.S.	1	0.90	0.83	0.97	1.46	3.31	0.00	0.95	0.86
time (sec)	N/A	0.206	0.050	0.218	0.193	0.266	0.000	0.289	2.375

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	84	247	78	96	459	0	72	98
N.S.	1	0.99	2.91	0.92	1.13	5.40	0.00	0.85	1.15
time (sec)	N/A	0.224	3.665	0.292	0.197	0.270	0.000	0.281	2.492

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	59	45	61	52	186	382	46	46
N.S.	1	1.04	0.79	1.07	0.91	3.26	6.70	0.81	0.81
time (sec)	N/A	0.231	0.040	0.025	0.202	0.260	12.490	0.255	2.549

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	85	73	108	76	176	197	82	69
N.S.	1	0.85	0.73	1.08	0.76	1.76	1.97	0.82	0.69
time (sec)	N/A	0.237	0.072	0.043	0.196	0.271	5.490	0.265	0.602

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	59	43	47	52	152	233	46	42
N.S.	1	1.04	0.75	0.82	0.91	2.67	4.09	0.81	0.74
time (sec)	N/A	0.221	0.044	48.701	0.192	0.256	2.334	0.262	0.576

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	50	48	55	70	127	0	50	49
N.S.	1	0.85	0.81	0.93	1.19	2.15	0.00	0.85	0.83
time (sec)	N/A	0.228	0.040	5.520	0.196	0.277	0.000	0.282	0.102

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	66	220	79	87	272	0	69	77
N.S.	1	0.90	3.01	1.08	1.19	3.73	0.00	0.95	1.05
time (sec)	N/A	0.224	1.011	1.105	0.200	0.280	0.000	0.264	2.517

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	73	61	70	106	398	0	70	80
N.S.	1	0.91	0.76	0.88	1.32	4.98	0.00	0.88	1.00
time (sec)	N/A	0.224	0.102	0.334	0.199	0.272	0.000	0.276	0.078

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	127	42	40	90	373	0	90	91
N.S.	1	1.12	0.37	0.35	0.80	3.30	0.00	0.80	0.81
time (sec)	N/A	0.335	0.029	0.635	0.283	0.263	0.000	0.261	0.279

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	148	120	44	101	882	0	95	122
N.S.	1	1.15	0.93	0.34	0.78	6.84	0.00	0.74	0.95
time (sec)	N/A	0.360	0.075	5.142	0.278	0.278	0.000	0.263	2.653

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	150	58	48	105	920	0	99	112
N.S.	1	1.15	0.45	0.37	0.81	7.08	0.00	0.76	0.86
time (sec)	N/A	0.377	0.053	1.463	0.284	0.269	0.000	0.269	2.674

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	171	64	50	115	1616	0	103	154
N.S.	1	1.15	0.43	0.34	0.77	10.85	0.00	0.69	1.03
time (sec)	N/A	0.405	0.065	13.555	0.276	0.286	0.000	0.266	2.831

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	44	31	24	34	202	0	35	38
N.S.	1	1.29	0.91	0.71	1.00	5.94	0.00	1.03	1.12
time (sec)	N/A	0.198	0.039	0.176	0.272	0.270	0.000	0.261	0.216

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	67	54	54	47	522	0	42	80
N.S.	1	1.26	1.02	1.02	0.89	9.85	0.00	0.79	1.51
time (sec)	N/A	0.220	0.064	0.527	0.266	0.272	0.000	0.261	2.713

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	69	161	56	47	557	0	42	62
N.S.	1	1.25	2.93	1.02	0.85	10.13	0.00	0.76	1.13
time (sec)	N/A	0.236	2.620	0.238	0.283	0.271	0.000	0.267	2.700

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	92	310	60	59	992	0	48	114
N.S.	1	1.23	4.13	0.80	0.79	13.23	0.00	0.64	1.52
time (sec)	N/A	0.263	4.683	0.470	0.266	0.279	0.000	0.259	2.924

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	86	202	0	505	1295	93	228
N.S.	1	1.00	0.63	1.47	0.00	3.69	9.45	0.68	1.66
time (sec)	N/A	0.323	0.776	0.040	0.000	0.270	7.773	0.261	1.196

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	80	178	0	379	972	86	126
N.S.	1	1.00	0.63	1.40	0.00	2.98	7.65	0.68	0.99
time (sec)	N/A	0.317	0.756	24.584	0.000	0.264	2.742	0.266	2.997

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	69	47	61	0	142	304	47	58
N.S.	1	1.05	0.71	0.92	0.00	2.15	4.61	0.71	0.88
time (sec)	N/A	0.267	0.062	2.071	0.000	0.276	0.970	0.255	2.561

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	37	0	97	184	40	54
N.S.	1	1.00	0.70	0.69	0.00	1.80	3.41	0.74	1.00
time (sec)	N/A	0.205	0.019	0.120	0.000	0.265	0.457	0.260	2.516

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	120	0	0	0	0	0	0
N.S.	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	0.519	0.000	0.000	0.000	0.000	0.000	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	105	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.441	0.345	0.000	0.000	0.000	0.000	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	159	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.466	1.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	117	278	0	919	2693	132	395
N.S.	1	1.00	0.60	1.43	0.00	4.71	13.81	0.68	2.03
time (sec)	N/A	0.388	0.784	0.027	0.000	0.277	26.295	0.271	3.311

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	97	0	303	819	58	96
N.S.	1	1.00	0.70	1.17	0.00	3.65	9.87	0.70	1.16
time (sec)	N/A	0.281	0.271	112.680	0.000	0.262	6.137	0.265	3.389

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	80	178	0	381	972	86	126
N.S.	1	1.00	0.63	1.40	0.00	3.00	7.65	0.68	0.99
time (sec)	N/A	0.302	0.390	11.292	0.000	0.263	2.598	0.255	2.904

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	55	60	0	176	435	58	68
N.S.	1	1.00	0.58	0.63	0.00	1.85	4.58	0.61	0.72
time (sec)	N/A	0.265	0.120	0.217	0.000	0.264	1.052	0.268	0.267

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	115	85	0	0	0	0	0	0
N.S.	1	1.12	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.419	0.220	0.000	0.000	0.000	0.000	0.000	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	148	0	0	0	0	0	0
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	0.721	0.000	0.000	0.000	0.000	0.000	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	111	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	0.994	0.000	0.000	0.000	0.000	0.000	0.000

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	113	202	0	676	1916	93	182
N.S.	1	1.00	0.82	1.47	0.00	4.93	13.99	0.68	1.33
time (sec)	N/A	0.331	0.719	0.030	0.000	0.269	68.042	0.277	1.003

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	118	278	0	917	2761	132	393
N.S.	1	1.00	0.61	1.43	0.00	4.70	14.16	0.68	2.02
time (sec)	N/A	0.374	0.808	0.025	0.000	0.258	25.243	0.275	3.204

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	86	202	0	501	1292	93	163
N.S.	1	1.00	0.63	1.47	0.00	3.66	9.43	0.68	1.19
time (sec)	N/A	0.314	0.596	63.865	0.000	0.266	7.786	0.264	3.467

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	141	106	100	0	381	1046	86	125
N.S.	1	0.98	0.74	0.69	0.00	2.65	7.26	0.60	0.87
time (sec)	N/A	0.352	0.348	0.532	0.000	0.256	2.951	0.262	2.924

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	152	172	0	0	0	0	0	0
N.S.	1	1.22	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.480	0.844	0.000	0.000	0.000	0.000	0.000	0.000

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	172	145	0	0	0	0	0	0
N.S.	1	1.08	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	0.966	0.000	0.000	0.000	0.000	0.000	0.000

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	176	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.375	1.818	0.000	0.000	0.000	0.000	0.000	0.000

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	51	0	0	0	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.733	4.796	0.000	0.000	0.000	0.000	0.000	0.000

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	26	36	30	16
N.S.	1	1.00	1.00	1.00	0.94	1.53	2.12	1.76	0.94
time (sec)	N/A	0.202	0.034	2.457	0.181	0.257	0.139	0.281	0.101

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	22	22	35	49	38	38
N.S.	1	1.00	1.00	1.00	1.00	1.59	2.23	1.73	1.73
time (sec)	N/A	0.203	0.100	0.174	0.190	0.252	1.399	0.273	2.488

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	22	35	48	38	38
N.S.	1	1.00	0.96	0.96	0.96	1.52	2.09	1.65	1.65
time (sec)	N/A	0.206	0.021	0.105	0.195	0.258	0.311	0.293	2.413

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	13	13	13	0	0	13	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.200	0.053	0.000	0.000	0.243	0.000	0.000	0.000

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	18	0	0	19	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.206	0.083	0.000	0.000	0.257	0.000	0.000	0.000

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	18	0	0	19	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.206	0.037	0.000	0.000	0.253	0.000	0.000	0.000

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	26	36	30	16
N.S.	1	1.00	1.00	1.00	0.94	1.53	2.12	1.76	0.94
time (sec)	N/A	0.191	0.013	1.490	0.186	0.249	0.169	0.268	0.073

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	22	22	35	49	38	38
N.S.	1	1.00	1.00	1.00	1.00	1.59	2.23	1.73	1.73
time (sec)	N/A	0.200	0.097	0.224	0.194	0.254	1.874	0.278	2.423

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	22	35	48	38	38
N.S.	1	1.00	0.96	0.96	0.96	1.52	2.09	1.65	1.65
time (sec)	N/A	0.194	0.023	0.158	0.192	0.243	0.380	0.264	2.369

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	13	13	13	0	0	13	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.071	0.000	0.000	0.258	0.000	0.000	0.000

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	18	0	0	19	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.208	0.095	0.000	0.000	0.259	0.000	0.000	0.000

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	18	0	0	19	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.201	0.035	0.000	0.000	0.247	0.000	0.000	0.000

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	42	0	45	50
N.S.	1	1.00	1.00	1.09	1.00	3.82	0.00	4.09	4.55
time (sec)	N/A	0.203	0.009	1.499	0.198	0.260	0.000	0.251	0.188

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	9	24	35	19	0	5	5
N.S.	1	1.00	3.00	8.00	11.67	6.33	0.00	1.67	1.67
time (sec)	N/A	0.196	0.001	1.429	0.341	0.249	0.000	0.258	0.076

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	26	11	21	0	11	11
N.S.	1	1.00	0.82	2.36	1.00	1.91	0.00	1.00	1.00
time (sec)	N/A	0.197	0.061	1.617	0.307	0.246	0.000	0.245	2.355

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	69	0	39	54
N.S.	1	1.00	1.00	1.05	1.00	3.63	0.00	2.05	2.84
time (sec)	N/A	0.208	0.142	30.523	0.192	0.275	0.000	0.258	2.532

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	6	11	13	12	14	29	12	12
N.S.	1	1.50	2.75	3.25	3.00	3.50	7.25	3.00	3.00
time (sec)	N/A	0.224	0.004	1.013	0.190	0.242	0.318	0.251	2.453

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	11	13	12	14	29	12	12
N.S.	1	1.00	2.75	3.25	3.00	3.50	7.25	3.00	3.00
time (sec)	N/A	0.244	0.001	0.901	0.227	0.249	0.311	0.251	2.391

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	27	26	0	23	0	9	9
N.S.	1	1.00	5.40	5.20	0.00	4.60	0.00	1.80	1.80
time (sec)	N/A	0.229	0.041	2.361	0.000	0.239	0.000	0.254	2.384

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	11	24	29	77	0	29	23
N.S.	1	1.00	0.73	1.60	1.93	5.13	0.00	1.93	1.53
time (sec)	N/A	0.246	0.011	1.735	0.205	0.246	0.000	0.262	0.094

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	11	21	32	53	0	26	20
N.S.	1	1.00	0.73	1.40	2.13	3.53	0.00	1.73	1.33
time (sec)	N/A	0.245	0.078	1.949	0.228	0.275	0.000	0.286	2.410

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	115	74	31	0	309	0	1	169
N.S.	1	1.13	0.73	0.30	0.00	3.03	0.00	0.01	1.66
time (sec)	N/A	0.359	0.232	2.774	0.000	0.272	0.000	0.265	4.110

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	20	22	40	0	32	0	19	19
N.S.	1	0.91	1.00	1.82	0.00	1.45	0.00	0.86	0.86
time (sec)	N/A	0.242	0.099	2.085	0.000	0.254	0.000	0.283	0.109

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	75	66	172	0	113	297
N.S.	1	1.00	0.93	2.68	2.36	6.14	0.00	4.04	10.61
time (sec)	N/A	0.299	0.204	1.825	0.282	0.254	0.000	0.285	2.759

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	60	151	688	0	264	107
N.S.	1	1.00	0.98	1.13	2.85	12.98	0.00	4.98	2.02
time (sec)	N/A	0.372	0.281	4.509	0.290	0.266	0.000	0.286	2.785

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	122	116	276	1975	0	543	1347
N.S.	1	1.00	1.56	1.49	3.54	25.32	0.00	6.96	17.27
time (sec)	N/A	0.379	0.485	11.445	0.295	0.286	0.000	0.296	3.338

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	73	0	32	32
N.S.	1	1.00	1.00	0.92	0.83	6.08	0.00	2.67	2.67
time (sec)	N/A	0.267	0.104	11.843	0.189	0.238	0.000	0.287	2.539

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	45	67	26	25	778	0	66	820
N.S.	1	1.36	2.03	0.79	0.76	23.58	0.00	2.00	24.85
time (sec)	N/A	0.326	0.014	0.035	0.190	0.245	0.000	0.296	2.408

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	24	122	50	0	28	47
N.S.	1	1.00	1.04	0.92	4.69	1.92	0.00	1.08	1.81
time (sec)	N/A	0.292	3.093	3.658	0.284	0.251	0.000	0.272	0.528

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	12	14	0	12	12
N.S.	1	1.00	1.00	1.25	3.00	3.50	0.00	3.00	3.00
time (sec)	N/A	0.183	0.001	0.142	0.188	0.243	0.000	0.271	2.368

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	40	20	36	0	67	0	35	50
N.S.	1	2.00	1.00	1.80	0.00	3.35	0.00	1.75	2.50
time (sec)	N/A	0.322	0.090	0.333	0.000	0.248	0.000	0.267	0.179

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	9	9	43	0	0	112	0	44	0
N.S.	1	1.00	4.78	0.00	0.00	12.44	0.00	4.89	0.00
time (sec)	N/A	0.217	0.040	0.000	0.000	0.243	0.000	0.264	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	47	8	0	118	0	44	0
N.S.	1	1.00	5.22	0.89	0.00	13.11	0.00	4.89	0.00
time (sec)	N/A	0.216	0.045	0.154	0.000	0.246	0.000	0.266	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	46	13	0	154	0	85	0
N.S.	1	1.00	3.29	0.93	0.00	11.00	0.00	6.07	0.00
time (sec)	N/A	0.215	0.034	0.156	0.000	0.249	0.000	0.283	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	51	32	0	219	0	120	0
N.S.	1	1.00	2.68	1.68	0.00	11.53	0.00	6.32	0.00
time (sec)	N/A	0.221	0.309	0.161	0.000	0.250	0.000	0.285	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	55	19	0	334	0	145	0
N.S.	1	1.00	2.29	0.79	0.00	13.92	0.00	6.04	0.00
time (sec)	N/A	0.213	0.072	0.147	0.000	0.249	0.000	0.279	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	25	14	34	340	19	41	375
N.S.	1	1.00	1.47	0.82	2.00	20.00	1.12	2.41	22.06
time (sec)	N/A	0.296	0.012	0.023	0.199	0.248	0.920	0.266	2.481

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	28	61	104	73	0	0	108
N.S.	1	0.95	0.65	1.42	2.42	1.70	0.00	0.00	2.51
time (sec)	N/A	0.231	0.087	0.801	0.258	0.249	0.000	0.000	2.524

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	28	61	104	73	0	0	108
N.S.	1	0.95	0.65	1.42	2.42	1.70	0.00	0.00	2.51
time (sec)	N/A	0.230	0.010	0.654	0.270	0.257	0.000	0.000	0.002

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	62	36	65	117	91	0	255	127
N.S.	1	0.97	0.56	1.02	1.83	1.42	0.00	3.98	1.98
time (sec)	N/A	0.233	0.095	0.802	0.264	0.247	0.000	0.326	2.571

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	62	36	65	117	91	0	255	127
N.S.	1	0.97	0.56	1.02	1.83	1.42	0.00	3.98	1.98
time (sec)	N/A	0.232	0.010	0.644	0.263	0.249	0.000	0.319	0.003

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	28	59	103	73	0	0	107
N.S.	1	0.95	0.65	1.37	2.40	1.70	0.00	0.00	2.49
time (sec)	N/A	0.233	0.076	0.770	0.258	0.252	0.000	0.000	2.449

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	28	59	103	73	0	0	107
N.S.	1	0.95	0.65	1.37	2.40	1.70	0.00	0.00	2.49
time (sec)	N/A	0.231	0.010	0.649	0.256	0.269	0.000	0.000	0.002

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	62	36	63	117	91	0	254	127
N.S.	1	0.97	0.56	0.98	1.83	1.42	0.00	3.97	1.98
time (sec)	N/A	0.232	0.087	0.816	0.264	0.261	0.000	0.303	2.430

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	62	36	63	117	91	0	254	127
N.S.	1	0.97	0.56	0.98	1.83	1.42	0.00	3.97	1.98
time (sec)	N/A	0.233	0.010	0.652	0.260	0.264	0.000	0.295	0.002

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	95	12	0	20	21
N.S.	1	1.00	1.00	0.89	10.56	1.33	0.00	2.22	2.33
time (sec)	N/A	0.182	0.006	1.721	0.961	0.258	0.000	0.275	2.409

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	12	0	20	21
N.S.	1	1.00	1.00	0.89	0.78	1.33	0.00	2.22	2.33
time (sec)	N/A	0.180	0.009	0.139	0.190	0.247	0.000	0.263	2.364

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.222	0.068	0.025	0.294	0.240	0.627	0.261	2.391

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.220	0.047	0.022	0.298	0.254	0.538	0.259	2.344

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.225	0.091	0.031	0.289	0.253	0.817	0.262	2.440

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.228	0.090	0.030	0.293	0.257	3.170	0.271	2.425

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	23	87	12	24	26
N.S.	1	1.00	1.00	0.92	1.77	6.69	0.92	1.85	2.00
time (sec)	N/A	0.228	0.010	1.414	0.191	0.248	0.150	0.272	2.491

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	12	43	0	46	51
N.S.	1	1.00	1.67	1.08	1.00	3.58	0.00	3.83	4.25
time (sec)	N/A	0.221	0.026	0.301	0.193	0.273	0.000	0.265	0.200

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	70	0	40	55
N.S.	1	1.00	1.00	1.05	1.00	3.50	0.00	2.00	2.75
time (sec)	N/A	0.224	0.293	19.686	0.193	0.272	0.000	0.262	2.515

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	12	14	0	12	12
N.S.	1	1.00	1.00	1.25	3.00	3.50	0.00	3.00	3.00
time (sec)	N/A	0.177	0.003	0.125	0.211	0.270	0.000	0.256	0.052

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	6	19	13	12	14	0	12	12
N.S.	1	1.50	4.75	3.25	3.00	3.50	0.00	3.00	3.00
time (sec)	N/A	0.257	0.005	0.175	0.195	0.254	0.000	0.260	2.411

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	56	75	77	174	0	113	297
N.S.	1	1.00	2.00	2.68	2.75	6.21	0.00	4.04	10.61
time (sec)	N/A	0.293	2.932	0.523	0.213	0.255	0.000	0.254	2.722

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	62	62	177	694	0	265	107
N.S.	1	1.00	1.17	1.17	3.34	13.09	0.00	5.00	2.02
time (sec)	N/A	0.358	3.515	0.832	0.204	0.286	0.000	0.276	2.755

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	136	118	316	1980	0	544	1346
N.S.	1	1.00	1.74	1.51	4.05	25.38	0.00	6.97	17.26
time (sec)	N/A	0.370	5.200	1.854	0.219	0.306	0.000	0.273	3.166

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	40	136	40	39	386	44	224	39
N.S.	1	1.11	3.78	1.11	1.08	10.72	1.22	6.22	1.08
time (sec)	N/A	0.278	0.278	0.035	0.200	0.282	0.950	0.262	2.445

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	40	114	40	39	386	44	224	39
N.S.	1	1.11	3.17	1.11	1.08	10.72	1.22	6.22	1.08
time (sec)	N/A	0.270	0.430	0.036	0.190	0.248	1.020	0.264	0.212

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	154	46	0	15
N.S.	1	1.00	1.00	0.84	0.79	8.11	2.42	0.00	0.79
time (sec)	N/A	0.245	0.009	0.122	0.191	0.260	0.258	0.000	2.573

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	41	22	0	53	0	0	21
N.S.	1	1.00	1.52	0.81	0.00	1.96	0.00	0.00	0.78
time (sec)	N/A	0.306	0.029	0.119	0.000	0.264	0.000	0.000	2.584

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	17	37	8	17	8
N.S.	1	1.00	1.00	0.88	2.12	4.62	1.00	2.12	1.00
time (sec)	N/A	0.356	0.017	0.049	0.188	0.281	1.735	0.268	2.325

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	7	6	17	7	17	6
N.S.	1	1.00	1.50	0.88	0.75	2.12	0.88	2.12	0.75
time (sec)	N/A	0.170	0.006	0.042	0.187	0.258	0.114	0.254	2.343

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	15	37	8	15	8
N.S.	1	1.00	1.00	0.88	1.88	4.62	1.00	1.88	1.00
time (sec)	N/A	0.354	0.017	0.034	0.200	0.255	0.128	0.258	2.300

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	90	59	86	85	251	0	92	273
N.S.	1	1.73	1.13	1.65	1.63	4.83	0.00	1.77	5.25
time (sec)	N/A	0.453	0.121	1.449	0.279	0.273	0.000	0.261	0.443

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	74	59	78	85	251	0	92	137
N.S.	1	1.42	1.13	1.50	1.63	4.83	0.00	1.77	2.63
time (sec)	N/A	0.350	0.077	1.316	0.298	0.269	0.000	0.269	2.528

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	66	48	61	0	303	0	47	153
N.S.	1	1.27	0.92	1.17	0.00	5.83	0.00	0.90	2.94
time (sec)	N/A	0.342	0.115	1.344	0.000	0.265	0.000	0.254	2.732

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	54	50	63	0	297	0	49	120
N.S.	1	1.04	0.96	1.21	0.00	5.71	0.00	0.94	2.31
time (sec)	N/A	0.287	0.084	1.322	0.000	0.278	0.000	0.255	0.264

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	31	39	18	335	0	36	0
N.S.	1	1.00	1.03	1.30	0.60	11.17	0.00	1.20	0.00
time (sec)	N/A	0.230	0.027	0.155	0.297	0.273	0.000	0.264	0.000

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	40	174	0	0	0
N.S.	1	1.00	1.00	1.00	1.29	5.61	0.00	0.00	0.00
time (sec)	N/A	0.231	0.046	0.148	0.321	0.274	0.000	0.000	0.000

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	60	68	0	124	0	0	0
N.S.	1	1.00	1.40	1.58	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.252	0.032	0.418	0.000	0.256	0.000	0.000	0.000

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	26	33	91	0	47	25
N.S.	1	1.00	1.00	2.17	2.75	7.58	0.00	3.92	2.08
time (sec)	N/A	0.206	0.018	1.472	0.306	0.249	0.000	0.258	2.394

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	68	75	0	408	0	0	0
N.S.	1	1.00	1.28	1.42	0.00	7.70	0.00	0.00	0.00
time (sec)	N/A	0.304	0.095	3.754	0.000	0.269	0.000	0.000	0.000

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	51
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	2.55
time (sec)	N/A	0.324	2.009	0.000	0.000	0.000	0.000	0.000	2.479

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	32	22	11	22	29	31	10	10
N.S.	1	1.45	1.00	0.50	1.00	1.32	1.41	0.45	0.45
time (sec)	N/A	0.218	0.002	0.207	0.182	0.261	0.072	0.243	0.059

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	37	28	29	95	381	27	27
N.S.	1	1.00	0.54	0.41	0.42	1.38	5.52	0.39	0.39
time (sec)	N/A	0.406	0.050	0.408	0.185	0.255	0.664	0.272	0.057

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	149	120	194	213	396	241	386	213
N.S.	1	1.16	0.93	1.50	1.65	3.07	1.87	2.99	1.65
time (sec)	N/A	0.611	0.298	0.223	0.202	0.256	3.973	0.267	0.251

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	36	33	28	27	95	0	27	27
N.S.	1	0.92	0.85	0.72	0.69	2.44	0.00	0.69	0.69
time (sec)	N/A	0.205	0.020	0.158	0.183	0.261	0.000	0.256	0.057

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	30	18	0	431	0	46	148
N.S.	1	1.00	1.20	0.72	0.00	17.24	0.00	1.84	5.92
time (sec)	N/A	0.309	0.208	0.116	0.000	0.267	0.000	0.276	2.451

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	26	389	271	0	0	207
N.S.	1	1.00	0.92	0.70	10.51	7.32	0.00	0.00	5.59
time (sec)	N/A	0.314	0.044	0.128	0.281	0.285	0.000	0.000	3.082

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	21	13	0	0	4
N.S.	1	1.00	1.00	1.25	5.25	3.25	0.00	0.00	1.00
time (sec)	N/A	0.289	0.075	12.679	0.411	0.254	0.000	0.000	2.577

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	47	33	27	28	70	65	274	0
N.S.	1	1.74	1.22	1.00	1.04	2.59	2.41	10.15	0.00
time (sec)	N/A	0.335	0.065	5.315	0.198	0.258	0.430	0.294	0.000

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	54	37	37	36	76	63	282	0
N.S.	1	1.69	1.16	1.16	1.12	2.38	1.97	8.81	0.00
time (sec)	N/A	0.346	0.092	5.187	0.210	0.257	0.454	0.270	0.000

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	49	25	74	0	192	97	34	77
N.S.	1	0.96	0.49	1.45	0.00	3.76	1.90	0.67	1.51
time (sec)	N/A	0.359	0.026	0.280	0.000	0.277	2.222	0.730	0.220

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	45	115	68	93	127	202	37	48
N.S.	1	0.96	2.45	1.45	1.98	2.70	4.30	0.79	1.02
time (sec)	N/A	0.467	6.936	0.351	0.292	0.277	5.099	0.330	2.386

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	30	49	38	53	44	25
N.S.	1	1.00	1.55	2.73	4.45	3.45	4.82	4.00	2.27
time (sec)	N/A	0.304	0.006	0.291	0.294	0.245	0.445	0.280	2.272

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	65	15	14	40	37	14	14
N.S.	1	1.00	2.95	0.68	0.64	1.82	1.68	0.64	0.64
time (sec)	N/A	0.242	0.050	0.141	0.195	0.257	0.250	0.266	0.093

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	65	15	14	40	0	14	14
N.S.	1	1.00	4.64	1.07	1.00	2.86	0.00	1.00	1.00
time (sec)	N/A	0.346	0.037	0.206	0.204	0.273	0.000	0.289	0.083

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	17	30	50	37	0	43	26
N.S.	1	1.00	1.42	2.50	4.17	3.08	0.00	3.58	2.17
time (sec)	N/A	0.392	0.005	0.247	0.292	0.262	0.000	0.285	2.331

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	45	52	68	93	127	0	37	48
N.S.	1	0.96	1.11	1.45	1.98	2.70	0.00	0.79	1.02
time (sec)	N/A	0.536	6.127	0.431	0.292	0.252	0.000	0.309	2.379

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	49	26	74	0	192	0	34	77
N.S.	1	0.96	0.51	1.45	0.00	3.76	0.00	0.67	1.51
time (sec)	N/A	1.193	0.018	0.299	0.000	0.250	0.000	0.428	2.384

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [421] had the largest ratio of [2.89999999999999991]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	14	0.286
2	A	4	3	1.00	21	0.143
3	A	4	3	1.00	23	0.130
4	A	5	4	1.00	21	0.190
5	A	5	4	1.00	23	0.174
6	A	5	4	1.00	23	0.174
7	A	5	4	1.00	23	0.174
8	A	5	4	1.00	13	0.308
9	A	4	3	1.00	15	0.200
10	A	5	4	0.95	17	0.235
11	A	5	4	0.92	17	0.235
12	A	5	4	1.00	15	0.267
13	A	6	5	0.98	17	0.294
14	A	6	5	0.92	17	0.294
15	A	6	6	1.11	17	0.353
16	A	9	9	1.14	17	0.529
17	A	12	12	1.16	17	0.706
18	A	8	8	1.15	17	0.471
19	A	11	11	1.17	17	0.647
20	A	14	14	1.18	17	0.824
21	A	10	10	1.17	17	0.588
22	A	13	13	1.18	17	0.765

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	16	16	1.19	17	0.941
24	C	5	4	1.36	13	0.308
25	A	7	6	0.87	15	0.400
26	C	6	5	1.04	15	0.333
27	A	7	6	0.84	15	0.400
28	C	7	6	1.02	15	0.400
29	C	7	6	1.21	15	0.400
30	C	6	5	1.22	17	0.294
31	C	7	6	1.22	17	0.353
32	C	6	5	1.11	17	0.294
33	C	9	8	1.30	17	0.471
34	C	6	5	1.04	15	0.333
35	A	7	6	1.06	17	0.353
36	C	7	6	0.95	17	0.353
37	A	7	6	0.97	17	0.353
38	C	7	6	0.91	17	0.353
39	C	6	5	1.16	15	0.333
40	C	5	4	1.14	17	0.235
41	C	6	5	1.12	17	0.294
42	C	5	4	1.06	17	0.235
43	C	8	7	1.18	17	0.412
44	C	7	6	1.05	15	0.400
45	A	9	8	1.17	17	0.471
46	C	7	6	0.91	17	0.353
47	A	9	8	1.06	17	0.471
48	C	7	6	0.94	17	0.353
49	A	10	9	1.05	21	0.429
50	A	9	8	1.06	21	0.381
51	A	8	7	1.06	21	0.333
52	A	7	6	1.07	21	0.286
53	A	6	5	1.07	21	0.238
54	A	9	8	1.06	21	0.381
55	A	8	7	1.06	21	0.333
56	A	11	10	1.05	21	0.476

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	13	12	0.88	21	0.571
58	A	12	11	0.88	21	0.524
59	A	13	12	1.00	21	0.571
60	A	12	11	0.99	21	0.524
61	A	11	10	0.85	21	0.476
62	A	10	9	0.85	21	0.429
63	A	11	10	1.00	21	0.476
64	A	14	13	0.99	21	0.619
65	A	13	12	0.88	21	0.571
66	A	12	11	0.88	21	0.524
67	A	2	2	1.00	13	0.154
68	A	2	2	1.00	13	0.154
69	A	4	3	1.00	9	0.333
70	A	7	6	0.91	13	0.462
71	A	6	5	1.05	15	0.333
72	A	6	5	1.00	15	0.333
73	A	6	5	0.84	15	0.333
74	A	6	5	0.89	15	0.333
75	A	6	5	1.28	17	0.294
76	A	7	6	0.81	17	0.353
77	A	5	4	0.84	15	0.267
78	A	6	5	0.87	17	0.294
79	A	8	7	0.94	17	0.412
80	A	7	6	0.98	15	0.400
81	A	5	4	1.00	13	0.308
82	A	5	4	1.00	15	0.267
83	A	5	4	1.00	17	0.235
84	A	5	4	1.00	17	0.235
85	A	5	4	1.00	17	0.235
86	A	4	3	1.00	17	0.176
87	A	5	4	0.93	15	0.267
88	A	7	6	0.94	17	0.353
89	A	7	6	0.94	19	0.316
90	C	6	5	1.16	17	0.294

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	C	5	4	1.14	19	0.211
92	C	5	4	1.15	17	0.235
93	A	5	5	1.00	15	0.333
94	A	8	8	1.09	15	0.533
95	A	7	7	1.09	17	0.412
96	A	6	5	1.00	7	0.714
97	A	6	5	1.00	9	0.556
98	A	10	10	1.26	9	1.111
99	A	9	9	1.28	9	1.000
100	C	6	5	1.36	9	0.556
101	A	6	5	0.91	13	0.385
102	C	6	5	1.27	15	0.333
103	A	7	6	1.04	15	0.400
104	C	5	4	1.14	15	0.267
105	C	6	5	1.04	15	0.333
106	C	7	6	1.55	17	0.353
107	C	7	6	0.95	17	0.353
108	A	6	5	0.87	15	0.333
109	C	6	5	1.11	17	0.294
110	A	7	6	0.95	17	0.353
111	C	7	6	1.05	15	0.400
112	A	4	3	1.00	13	0.231
113	A	5	4	1.00	15	0.267
114	A	4	3	1.00	17	0.176
115	A	4	3	1.00	17	0.176
116	A	5	4	1.00	17	0.235
117	A	5	4	1.00	17	0.235
118	C	5	4	1.19	15	0.267
119	C	6	5	1.16	17	0.294
120	A	6	5	0.95	19	0.263
121	C	7	7	1.24	15	0.467
122	C	11	11	1.27	17	0.647
123	C	11	11	1.27	15	0.733
124	C	6	5	1.47	9	0.556

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	7	6	1.00	9	0.667
126	C	5	4	1.35	9	0.444
127	C	15	15	1.58	9	1.667
128	C	6	5	1.40	9	0.556
129	C	5	4	1.34	11	0.364
130	C	6	5	1.36	9	0.556
131	A	2	2	1.00	13	0.154
132	A	2	2	1.00	14	0.143
133	A	2	2	1.00	13	0.154
134	A	2	2	1.00	14	0.143
135	A	5	5	1.24	13	0.385
136	A	5	5	1.17	14	0.357
137	C	5	5	1.43	13	0.385
138	C	5	5	1.39	14	0.357
139	A	4	4	1.00	13	0.308
140	A	4	4	1.00	14	0.286
141	C	4	4	1.22	13	0.308
142	C	4	4	1.24	14	0.286
143	A	4	4	1.00	13	0.308
144	A	11	10	1.00	15	0.667
145	A	16	15	1.19	15	1.000
146	A	5	5	1.00	13	0.385
147	A	10	9	1.00	15	0.600
148	C	18	17	1.29	15	1.133
149	A	5	5	1.00	13	0.385
150	A	7	6	1.00	15	0.400
151	A	8	7	1.00	15	0.467
152	C	5	5	1.15	13	0.385
153	A	7	6	1.00	15	0.400
154	A	9	8	1.00	15	0.533
155	A	5	5	1.00	13	0.385
156	A	10	9	1.00	15	0.600
157	A	17	16	1.19	15	1.067
158	A	5	5	1.00	13	0.385

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	10	9	1.00	15	0.600
160	C	18	17	1.29	15	1.133
161	A	5	5	1.00	13	0.385
162	A	7	6	1.00	15	0.400
163	A	8	7	1.00	15	0.467
164	C	5	5	1.15	13	0.385
165	A	7	6	1.00	15	0.400
166	A	9	8	1.00	15	0.533
167	A	2	2	1.00	13	0.154
168	A	2	2	1.00	15	0.133
169	A	2	2	1.00	15	0.133
170	A	2	2	1.00	17	0.118
171	A	2	2	1.00	17	0.118
172	A	2	2	1.00	17	0.118
173	A	2	2	1.00	13	0.154
174	A	2	2	1.00	15	0.133
175	A	2	2	1.00	15	0.133
176	A	2	2	1.00	17	0.118
177	A	2	2	1.00	17	0.118
178	A	2	2	1.00	17	0.118
179	A	2	2	1.00	13	0.154
180	A	2	2	1.00	15	0.133
181	A	2	2	1.00	15	0.133
182	A	2	2	1.00	15	0.133
183	A	2	2	1.00	17	0.118
184	A	2	2	1.00	17	0.118
185	A	2	2	1.00	15	0.133
186	A	2	2	1.00	17	0.118
187	A	2	2	1.00	17	0.118
188	A	2	2	1.00	13	0.154
189	A	2	2	1.00	13	0.154
190	A	2	2	1.00	13	0.154
191	A	2	2	1.00	13	0.154
192	A	3	3	1.88	7	0.429

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	3	3	1.00	7	0.429
194	A	3	3	1.00	7	0.429
195	A	2	2	1.00	7	0.286
196	C	3	3	1.53	7	0.429
197	C	3	3	1.47	7	0.429
198	C	3	3	1.47	7	0.429
199	A	2	2	1.00	7	0.286
200	A	7	6	1.42	7	0.857
201	A	9	8	1.00	7	1.143
202	A	9	8	1.16	7	1.143
203	A	7	6	1.00	7	0.857
204	A	7	6	1.07	7	0.857
205	A	2	2	1.00	7	0.286
206	A	6	5	1.40	7	0.714
207	A	5	4	1.00	7	0.571
208	A	6	5	1.07	7	0.714
209	A	5	4	1.00	7	0.571
210	A	6	5	1.11	7	0.714
211	A	5	4	1.00	7	0.571
212	A	9	8	1.29	7	1.143
213	A	6	5	1.00	7	0.714
214	A	7	6	1.29	7	0.857
215	A	6	5	1.00	7	0.714
216	A	4	4	1.00	7	0.571
217	A	4	3	1.00	7	0.429
218	A	5	4	1.00	7	0.571
219	A	5	4	1.16	7	0.571
220	A	6	5	1.06	7	0.714
221	C	3	3	2.88	7	0.429
222	C	3	3	1.47	7	0.429
223	C	3	3	1.47	7	0.429
224	A	2	2	1.00	7	0.286
225	A	2	2	1.00	7	0.286
226	A	2	2	1.00	7	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
227	A	2	2	1.00	7	0.286
228	A	2	2	1.00	7	0.286
229	A	7	6	1.42	7	0.857
230	A	6	5	1.00	7	0.714
231	A	9	8	1.16	7	1.143
232	A	6	5	1.00	7	0.714
233	A	7	6	1.07	7	0.857
234	A	7	6	1.40	7	0.857
235	A	9	8	1.22	7	1.143
236	A	7	6	1.07	7	0.857
237	A	6	5	1.00	7	0.714
238	A	7	6	1.11	7	0.857
239	A	2	2	1.00	7	0.286
240	A	4	3	1.00	7	0.429
241	A	4	3	1.00	7	0.429
242	A	5	4	1.00	7	0.571
243	A	5	4	1.19	7	0.571
244	A	5	4	1.00	7	0.571
245	A	7	7	1.00	7	1.000
246	A	9	8	1.29	7	1.143
247	A	6	5	1.00	7	0.714
248	A	8	7	1.29	7	1.000
249	A	7	6	1.03	7	0.857
250	C	6	6	1.17	16	0.375
251	A	10	10	1.11	16	0.625
252	A	5	5	1.03	16	0.312
253	A	5	5	1.11	14	0.357
254	A	5	4	1.00	13	0.308
255	C	10	10	1.22	16	0.625
256	C	12	12	1.26	16	0.750
257	C	16	16	1.28	16	1.000
258	C	18	18	1.14	16	1.125
259	A	2	2	1.00	18	0.111
260	C	14	13	1.30	18	0.722

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	9	9	1.10	18	0.500
262	C	5	4	1.13	16	0.250
263	A	5	4	1.00	15	0.267
264	A	2	2	1.00	18	0.111
265	A	2	2	1.00	18	0.111
266	A	2	2	1.00	18	0.111
267	A	2	2	1.00	18	0.111
268	A	2	2	1.00	18	0.111
269	A	13	13	1.34	18	0.722
270	A	6	6	1.10	18	0.333
271	A	6	6	1.15	16	0.375
272	A	5	4	1.00	15	0.267
273	A	2	2	1.00	18	0.111
274	A	2	2	1.00	18	0.111
275	A	2	2	1.00	18	0.111
276	A	2	2	1.00	18	0.111
277	A	5	5	1.00	8	0.625
278	C	7	7	1.62	8	0.875
279	C	11	11	1.48	8	1.375
280	A	2	2	1.00	18	0.111
281	C	17	16	1.28	18	0.889
282	C	10	10	1.23	18	0.556
283	A	6	5	0.96	16	0.312
284	A	5	4	1.00	15	0.267
285	A	2	2	1.00	18	0.111
286	A	2	2	1.00	18	0.111
287	A	2	2	1.00	18	0.111
288	A	2	2	1.00	18	0.111
289	A	2	2	1.00	20	0.100
290	A	2	2	1.00	20	0.100
291	A	2	2	1.00	20	0.100
292	A	2	2	1.00	18	0.111
293	A	6	6	1.11	17	0.353
294	A	2	2	1.00	20	0.100

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
295	A	2	2	1.00	20	0.100
296	A	2	2	1.00	20	0.100
297	A	2	2	1.00	20	0.100
298	A	2	2	1.00	20	0.100
299	A	2	2	1.00	20	0.100
300	A	2	2	1.00	20	0.100
301	A	2	2	1.00	18	0.111
302	C	6	5	1.16	17	0.294
303	A	2	2	1.00	20	0.100
304	A	2	2	1.00	20	0.100
305	A	2	2	1.00	20	0.100
306	A	2	2	1.00	20	0.100
307	A	2	2	1.00	18	0.111
308	A	19	19	1.34	18	1.056
309	A	8	8	1.10	18	0.444
310	A	8	8	1.15	16	0.500
311	A	5	4	1.00	15	0.267
312	A	2	2	1.00	18	0.111
313	A	2	2	1.00	18	0.111
314	A	2	2	1.00	18	0.111
315	A	2	2	1.00	18	0.111
316	A	2	2	1.00	20	0.100
317	A	2	2	1.00	20	0.100
318	A	2	2	1.00	20	0.100
319	A	2	2	1.00	18	0.111
320	A	6	5	0.97	17	0.294
321	A	2	2	1.00	20	0.100
322	A	2	2	1.00	20	0.100
323	A	2	2	1.00	20	0.100
324	A	2	2	1.00	20	0.100
325	A	2	2	1.00	20	0.100
326	A	2	2	1.00	20	0.100
327	A	2	2	1.00	20	0.100
328	A	2	2	1.00	18	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	6	5	0.94	17	0.294
330	A	2	2	1.00	20	0.100
331	A	2	2	1.00	20	0.100
332	A	2	2	1.00	20	0.100
333	A	2	2	1.00	20	0.100
334	N/A	3	0	1.00	10	0.000
335	C	9	8	1.30	10	0.800
336	C	8	7	1.35	10	0.700
337	C	7	6	1.31	8	0.750
338	A	3	3	1.00	6	0.500
339	N/A	3	0	1.00	10	0.000
340	N/A	3	0	1.00	10	0.000
341	N/A	1	0	1.00	16	0.000
342	A	7	6	1.11	16	0.375
343	A	6	5	1.09	16	0.312
344	A	3	3	1.00	14	0.214
345	A	5	4	1.00	13	0.308
346	N/A	1	0	1.00	16	0.000
347	N/A	1	0	1.00	16	0.000
348	N/A	1	0	1.00	18	0.000
349	C	11	10	1.23	18	0.556
350	A	7	7	1.12	18	0.389
351	A	5	4	1.00	16	0.250
352	A	5	4	1.00	15	0.267
353	N/A	1	0	1.00	18	0.000
354	N/A	1	0	1.00	18	0.000
355	N/A	6	0	1.00	16	0.000
356	A	19	18	1.16	16	1.125
357	A	14	13	1.13	16	0.812
358	A	11	10	1.00	14	0.714
359	A	7	6	0.91	13	0.462
360	N/A	9	0	1.00	16	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
361	N/A	13	0	1.00	16	0.000
362	N/A	3	0	1.00	12	0.000
363	C	13	12	1.27	12	1.000
364	C	12	11	1.29	12	0.917
365	A	8	8	1.00	10	0.800
366	A	4	4	1.00	8	0.500
367	N/A	3	0	1.00	12	0.000
368	N/A	3	0	1.00	12	0.000
369	N/A	3	0	1.00	18	0.000
370	A	14	13	1.75	18	0.722
371	A	13	12	1.81	18	0.667
372	A	11	10	1.62	16	0.625
373	A	5	5	1.00	15	0.333
374	N/A	3	0	1.00	18	0.000
375	N/A	3	0	1.00	18	0.000
376	N/A	10	0	1.00	18	0.000
377	C	20	19	1.20	18	1.056
378	C	14	13	1.18	18	0.722
379	C	13	12	1.21	16	0.750
380	A	6	5	0.89	15	0.333
381	N/A	14	0	1.00	18	0.000
382	N/A	16	0	1.00	18	0.000
383	N/A	6	0	1.00	18	0.000
384	A	19	18	1.24	18	1.000
385	A	16	15	1.20	18	0.833
386	C	9	9	1.22	16	0.562
387	A	6	5	1.05	15	0.333
388	N/A	10	0	1.00	18	0.000
389	N/A	12	0	1.00	18	0.000
390	N/A	3	0	1.00	12	0.000
391	C	24	23	1.26	12	1.917
392	C	20	19	1.26	12	1.583

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
393	C	15	14	1.22	10	1.400
394	C	7	7	1.33	8	0.875
395	N/A	3	0	1.00	12	0.000
396	N/A	3	0	1.00	12	0.000
397	N/A	3	0	1.00	10	0.000
398	C	9	8	1.63	10	0.800
399	C	8	7	1.68	10	0.700
400	C	7	6	1.58	8	0.750
401	C	3	3	1.36	6	0.500
402	N/A	3	0	1.00	10	0.000
403	N/A	3	0	1.00	10	0.000
404	N/A	6	0	1.00	16	0.000
405	C	18	17	1.36	16	1.062
406	C	15	14	1.31	16	0.875
407	C	10	9	1.33	14	0.643
408	A	6	5	0.91	13	0.385
409	N/A	10	0	1.00	16	0.000
410	N/A	12	0	1.00	16	0.000
411	N/A	10	0	1.00	18	0.000
412	C	20	19	1.37	18	1.056
413	C	14	13	1.37	18	0.722
414	C	13	12	1.36	16	0.750
415	C	6	5	1.04	15	0.333
416	N/A	14	0	1.00	18	0.000
417	N/A	16	0	1.00	18	0.000
418	A	11	11	1.00	10	1.100
419	C	19	18	1.34	12	1.500
420	C	20	19	1.23	12	1.583
421	C	30	29	2.24	10	2.900
422	F	0	0	N/A	0.000	N/A
423	F	0	0	N/A	0.000	N/A
424	N/A	1	0	1.00	16	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
425	C	8	7	1.26	16	0.438
426	C	7	6	1.24	16	0.375
427	A	4	4	1.00	14	0.286
428	A	4	3	1.00	13	0.231
429	N/A	1	0	1.00	16	0.000
430	N/A	1	0	1.00	16	0.000
431	N/A	3	0	1.00	12	0.000
432	C	13	12	1.51	12	1.000
433	C	12	11	1.48	12	0.917
434	C	8	8	1.13	10	0.800
435	A	4	4	1.00	8	0.500
436	N/A	3	0	1.00	12	0.000
437	N/A	3	0	1.00	12	0.000
438	N/A	6	0	1.00	18	0.000
439	C	21	20	1.29	18	1.111
440	C	16	15	1.27	18	0.833
441	A	11	11	1.00	16	0.688
442	C	6	5	1.27	15	0.333
443	N/A	9	0	1.00	18	0.000
444	N/A	13	0	1.00	18	0.000
445	N/A	1	0	1.00	18	0.000
446	C	12	11	1.37	18	0.611
447	C	8	8	1.24	18	0.444
448	A	6	5	1.00	16	0.312
449	A	5	4	1.00	15	0.267
450	N/A	1	0	1.00	18	0.000
451	N/A	1	0	1.00	18	0.000
452	N/A	4	0	1.00	18	0.000
453	C	17	16	1.98	18	0.889
454	C	16	15	2.08	18	0.833
455	C	14	13	1.87	16	0.812
456	C	7	7	1.24	15	0.467

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
457	N/A	4	0	1.00	18	0.000
458	N/A	4	0	1.00	18	0.000
459	N/A	3	0	1.00	12	0.000
460	C	24	23	1.49	12	1.917
461	C	20	19	1.47	12	1.583
462	C	15	14	1.37	10	1.400
463	C	7	7	1.48	8	0.875
464	N/A	3	0	1.00	12	0.000
465	N/A	3	0	1.00	12	0.000
466	N/A	1	0	1.00	16	0.000
467	C	9	8	1.24	16	0.500
468	C	8	7	1.28	16	0.438
469	C	7	6	1.17	14	0.429
470	C	5	4	1.36	13	0.308
471	N/A	4	0	1.00	16	0.000
472	N/A	4	0	1.00	16	0.000
473	N/A	1	0	1.00	18	0.000
474	A	4	4	1.16	18	0.222
475	A	4	4	1.23	18	0.222
476	A	2	2	1.00	16	0.125
477	A	7	6	0.87	15	0.400
478	N/A	1	0	1.00	18	0.000
479	N/A	1	0	1.00	18	0.000
480	N/A	1	0	1.00	18	0.000
481	A	4	4	1.08	18	0.222
482	A	4	4	1.00	18	0.222
483	A	2	2	1.00	16	0.125
484	C	6	5	1.04	15	0.333
485	N/A	1	0	1.00	18	0.000
486	N/A	1	0	1.00	18	0.000
487	N/A	1	0	1.00	18	0.000
488	A	4	4	1.16	18	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
489	A	4	4	1.22	18	0.222
490	A	2	2	1.00	16	0.125
491	C	7	6	1.21	15	0.400
492	N/A	1	0	1.00	18	0.000
493	N/A	1	0	1.00	18	0.000
494	N/A	1	0	1.00	20	0.000
495	C	13	12	1.59	20	0.600
496	C	12	11	1.50	20	0.550
497	C	8	8	1.37	18	0.444
498	C	6	5	1.22	17	0.294
499	N/A	4	0	1.00	20	0.000
500	N/A	4	0	1.00	20	0.000
501	N/A	1	0	1.00	20	0.000
502	A	4	4	1.00	20	0.200
503	A	2	2	1.00	18	0.111
504	C	7	6	1.22	17	0.353
505	N/A	1	0	1.00	20	0.000
506	N/A	1	0	1.00	20	0.000
507	N/A	1	0	1.00	18	0.000
508	A	4	4	1.09	18	0.222
509	A	4	4	1.00	18	0.222
510	A	2	2	1.00	16	0.125
511	C	6	5	1.04	15	0.333
512	N/A	1	0	1.00	18	0.000
513	N/A	1	0	1.00	18	0.000
514	N/A	1	0	1.00	20	0.000
515	A	4	4	1.09	20	0.200
516	A	4	4	1.00	20	0.200
517	A	2	2	1.00	18	0.111
518	A	7	6	1.06	17	0.353
519	N/A	1	0	1.00	20	0.000
520	N/A	1	0	1.00	20	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
521	N/A	1	0	1.00	20	0.000
522	C	15	14	1.30	20	0.700
523	C	13	12	1.32	20	0.600
524	C	11	10	1.31	18	0.556
525	C	7	6	0.95	17	0.353
526	N/A	4	0	1.00	20	0.000
527	N/A	4	0	1.00	20	0.000
528	A	7	7	1.15	18	0.389
529	A	5	5	1.12	18	0.278
530	A	5	5	1.12	18	0.278
531	A	3	3	1.00	18	0.167
532	A	3	3	1.00	18	0.167
533	A	5	5	1.06	18	0.278
534	A	5	5	1.12	18	0.278
535	A	7	7	1.10	18	0.389
536	A	9	9	1.08	18	0.500
537	A	7	7	1.10	18	0.389
538	A	7	7	1.05	18	0.389
539	A	5	5	1.00	18	0.278
540	A	5	5	1.00	18	0.278
541	A	7	7	1.10	18	0.389
542	A	7	7	1.10	18	0.389
543	A	9	9	1.12	18	0.500
544	A	9	9	1.11	18	0.500
545	A	7	7	1.08	18	0.389
546	A	7	7	1.08	18	0.389
547	A	5	5	1.00	18	0.278
548	A	5	5	1.00	18	0.278
549	A	7	7	1.04	18	0.389
550	A	7	7	1.08	18	0.389
551	A	9	9	1.07	18	0.500
552	A	9	9	1.07	18	0.500
553	A	7	7	1.08	18	0.389

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
554	A	7	7	1.04	18	0.389
555	A	5	5	1.00	18	0.278
556	A	5	5	1.00	18	0.278
557	A	7	7	1.08	18	0.389
558	A	7	7	1.08	18	0.389
559	A	9	9	1.11	18	0.500
560	A	6	6	1.00	9	0.667
561	C	8	8	2.71	9	0.889
562	C	10	10	2.28	9	1.111
563	A	6	6	1.00	9	0.667
564	C	8	8	2.39	9	0.889
565	C	10	10	1.98	9	1.111
566	A	3	3	1.00	14	0.214
567	A	3	3	1.00	15	0.200
568	A	3	3	1.00	14	0.214
569	A	3	3	1.00	15	0.200
570	A	5	4	1.00	17	0.235
571	A	6	5	1.00	16	0.312
572	A	9	8	1.00	15	0.533
573	C	15	14	1.22	15	0.933
574	A	6	5	1.00	17	0.294
575	A	8	7	1.00	17	0.412
576	A	10	9	1.00	17	0.529
577	A	6	5	1.00	17	0.294
578	A	12	11	1.07	17	0.647
579	C	18	17	1.26	17	1.000
580	A	1	1	1.00	9	0.111
581	A	3	3	1.00	11	0.273
582	C	4	3	1.46	11	0.273
583	A	5	5	1.04	11	0.455
584	C	5	4	1.36	11	0.364
585	C	4	3	1.24	11	0.273
586	A	2	2	1.00	11	0.182
587	C	6	5	1.10	11	0.455

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
588	A	4	4	1.00	11	0.364
589	C	8	7	1.21	11	0.636
590	A	4	4	1.00	13	0.308
591	A	6	6	1.00	13	0.462
592	A	6	6	1.00	13	0.462
593	A	4	4	1.00	13	0.308
594	A	6	6	1.00	13	0.462
595	A	6	6	1.00	13	0.462
596	A	1	1	1.00	17	0.059
597	A	2	2	1.00	19	0.105
598	A	2	2	1.00	19	0.105
599	A	2	2	1.00	19	0.105
600	A	2	2	1.00	19	0.105
601	A	2	2	1.00	19	0.105
602	A	2	2	1.00	19	0.105
603	A	2	2	1.00	21	0.095
604	A	2	2	1.00	21	0.095
605	A	1	1	1.00	18	0.056
606	A	2	2	1.00	20	0.100
607	A	2	2	1.00	20	0.100
608	A	2	2	1.00	20	0.100
609	A	2	2	1.00	20	0.100
610	A	2	2	1.00	20	0.100
611	A	2	2	1.00	20	0.100
612	A	2	2	1.00	22	0.091
613	A	2	2	1.00	22	0.091
614	A	10	9	1.48	11	0.818
615	A	10	10	1.06	11	0.909
616	A	8	7	1.59	11	0.636
617	A	5	5	1.00	11	0.455
618	A	1	1	1.00	9	0.111
619	A	6	5	1.00	11	0.455
620	C	13	12	1.29	11	1.091
621	A	8	7	0.94	11	0.636

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
622	C	19	18	1.22	11	1.636
623	A	7	6	0.87	11	0.545
624	A	7	6	1.00	11	0.545
625	A	9	9	1.00	11	0.818
626	A	7	6	1.00	11	0.545
627	A	7	7	1.00	11	0.636
628	A	1	1	1.00	9	0.111
629	A	6	5	1.00	11	0.455
630	A	5	5	1.00	11	0.455
631	A	7	6	1.00	11	0.545
632	A	7	7	1.00	11	0.636
633	A	7	6	0.90	11	0.545
634	A	7	6	0.95	11	0.545
635	A	9	9	1.00	11	0.818
636	A	7	6	0.93	11	0.545
637	A	7	7	1.00	11	0.636
638	A	1	1	1.00	9	0.111
639	A	6	5	1.18	11	0.455
640	A	5	5	1.00	11	0.455
641	A	7	6	1.08	11	0.545
642	A	7	7	1.00	11	0.636
643	A	7	6	0.95	11	0.545
644	A	10	9	1.40	11	0.818
645	A	11	11	1.07	11	1.000
646	A	10	9	1.64	11	0.818
647	A	8	8	1.00	11	0.727
648	A	1	1	1.00	9	0.111
649	A	9	8	1.00	11	0.727
650	A	14	13	1.07	11	1.182
651	A	10	9	0.94	11	0.818
652	A	19	18	1.13	11	1.636
653	A	10	9	0.88	11	0.818
654	A	10	9	1.00	7	1.286
655	A	11	11	1.00	7	1.571

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	10	9	1.00	7	1.286
657	A	12	12	1.00	7	1.714
658	A	1	1	1.00	5	0.200
659	A	9	8	1.00	7	1.143
660	A	8	8	1.00	7	1.143
661	A	10	9	1.00	7	1.286
662	A	8	8	1.00	7	1.143
663	A	10	9	1.00	7	1.286
664	A	10	9	1.00	9	1.000
665	A	11	11	1.00	9	1.222
666	A	10	9	1.00	9	1.000
667	A	12	12	1.00	9	1.333
668	A	1	1	1.00	7	0.143
669	A	9	8	1.00	9	0.889
670	A	8	8	1.00	9	0.889
671	A	10	9	1.00	9	1.000
672	A	8	8	1.00	9	0.889
673	A	10	9	1.00	9	1.000
674	A	1	1	1.00	5	0.200
675	A	6	5	1.32	7	0.714
676	A	9	8	1.15	7	1.143
677	A	8	8	1.00	9	0.889
678	C	10	10	2.39	9	1.111
679	C	12	12	1.98	9	1.333
680	A	1	1	1.00	7	0.143
681	A	6	5	1.41	9	0.556
682	A	11	10	1.09	9	1.111
683	A	6	6	1.00	11	0.545
684	C	8	8	2.55	11	0.727
685	C	10	10	2.17	11	0.909
686	C	18	17	1.89	7	2.429
687	C	18	17	1.70	9	1.889
688	C	6	6	1.23	14	0.429
689	C	10	9	1.11	16	0.562

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
690	C	14	14	1.15	16	0.875
691	A	5	5	1.00	14	0.357
692	C	7	6	1.11	16	0.375
693	A	9	9	1.02	16	0.562
694	C	4	4	1.20	14	0.286
695	C	4	4	1.18	14	0.286
696	C	7	6	1.17	14	0.429
697	A	10	10	1.22	16	0.625
698	C	4	4	1.68	16	0.250
699	C	6	5	1.14	14	0.357
700	A	9	9	1.19	16	0.562
701	A	5	4	1.50	16	0.250
702	A	5	4	1.00	14	0.286
703	C	13	13	1.35	16	0.812
704	C	5	4	1.37	14	0.286
705	A	11	11	1.25	16	0.688
706	C	11	10	1.22	16	0.625
707	C	17	16	1.08	18	0.889
708	C	20	19	1.12	18	1.056
709	C	15	14	1.11	18	0.778
710	C	21	20	1.16	20	1.000
711	C	29	28	1.02	20	1.400
712	C	16	15	1.14	18	0.833
713	C	27	26	0.98	20	1.300
714	F	0	0	N/A	0.000	N/A
715	C	14	14	1.57	16	0.875
716	C	28	27	1.74	18	1.500
717	F	0	0	N/A	0.000	N/A
718	C	24	23	1.78	18	1.278
719	F	0	0	N/A	0.000	N/A
720	F	0	0	N/A	0.000	N/A
721	F	0	0	N/A	0.000	N/A
722	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
723	F	0	0	N/A	0.000	N/A
724	C	6	5	1.11	18	0.278
725	C	6	5	1.10	18	0.278
726	C	9	8	1.18	18	0.444
727	C	6	5	1.11	18	0.278
728	C	6	5	1.12	18	0.278
729	C	8	7	1.19	18	0.389
730	A	2	2	1.00	15	0.133
731	A	2	2	1.00	15	0.133
732	A	2	2	1.00	21	0.095
733	A	2	2	1.00	21	0.095
734	C	6	5	1.12	21	0.238
735	A	5	5	1.00	21	0.238
736	C	6	5	1.10	22	0.227
737	C	6	5	1.10	22	0.227
738	C	8	7	1.17	22	0.318
739	A	5	5	1.08	12	0.417
740	A	3	3	0.97	12	0.250
741	A	1	1	1.00	10	0.100
742	A	5	4	1.10	12	0.333
743	A	9	8	1.06	12	0.667
744	A	10	9	1.15	12	0.750
745	A	13	12	1.19	12	1.000
746	A	5	5	1.10	12	0.417
747	A	3	3	0.96	12	0.250
748	A	1	1	1.00	10	0.100
749	A	4	3	1.00	12	0.250
750	A	8	7	1.00	12	0.583
751	A	9	8	1.03	12	0.667
752	A	12	11	1.08	12	0.917
753	A	7	7	0.99	24	0.292
754	A	5	5	0.97	24	0.208
755	A	3	3	0.86	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
756	A	1	1	1.00	22	0.045
757	A	2	2	1.00	24	0.083
758	A	4	4	1.00	24	0.167
759	A	6	6	1.15	24	0.250
760	A	8	8	1.19	24	0.333
761	A	15	15	1.04	14	1.071
762	A	12	12	1.00	14	0.857
763	A	4	4	1.00	14	0.286
764	A	4	4	1.00	14	0.286
765	A	6	6	1.00	14	0.429
766	A	15	15	1.04	14	1.071
767	A	18	18	1.08	14	1.286
768	A	6	6	1.06	26	0.231
769	A	4	4	1.00	26	0.154
770	A	2	2	1.00	26	0.077
771	A	6	5	1.00	26	0.192
772	A	8	7	1.00	26	0.269
773	A	10	9	1.10	26	0.346
774	A	6	6	1.06	28	0.214
775	A	4	4	1.00	28	0.143
776	A	2	2	1.00	28	0.071
777	A	6	5	1.00	28	0.179
778	A	8	7	1.00	28	0.250
779	A	10	9	1.09	28	0.321
780	A	9	8	1.06	12	0.667
781	A	12	11	1.06	12	0.917
782	C	8	7	1.15	15	0.467
783	C	3	3	1.56	11	0.273
784	A	7	6	1.11	15	0.400
785	A	15	14	0.99	17	0.824
786	C	8	7	1.32	15	0.467
787	C	8	7	1.24	15	0.467
788	A	12	11	1.00	17	0.647
789	A	7	6	0.96	19	0.316

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
790	A	7	6	1.05	19	0.316
791	A	10	9	1.11	19	0.474
792	A	7	6	0.96	19	0.316
793	A	7	6	1.05	19	0.316
794	A	10	9	1.11	19	0.474
795	A	7	6	1.04	22	0.273
796	A	7	6	1.05	22	0.273
797	A	9	8	1.11	22	0.364
798	A	7	6	1.04	23	0.261
799	A	7	6	1.04	23	0.261
800	A	10	9	1.09	23	0.391
801	A	2	2	1.00	32	0.062
802	A	2	2	1.00	19	0.105
803	A	2	2	1.00	19	0.105
804	A	2	2	1.00	23	0.087
805	A	2	2	1.00	20	0.100
806	A	2	2	1.00	20	0.100
807	A	2	2	1.00	24	0.083
808	A	4	3	1.00	11	0.273
809	A	4	3	1.00	11	0.273
810	A	6	5	1.15	11	0.455
811	A	3	3	1.00	13	0.231
812	A	3	3	1.00	13	0.231
813	A	3	3	1.00	13	0.231
814	A	3	3	1.00	11	0.273
815	A	3	3	1.00	11	0.273
816	A	3	3	1.00	11	0.273
817	A	5	4	1.11	13	0.308
818	A	7	6	1.06	13	0.462
819	A	8	7	1.19	13	0.538
820	A	5	4	1.11	11	0.364
821	A	7	6	1.06	11	0.545
822	A	9	8	1.22	11	0.727
823	A	3	3	1.00	13	0.231

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
824	A	3	3	1.00	13	0.231
825	A	3	3	1.00	13	0.231
826	A	7	6	1.01	14	0.429
827	A	4	4	1.02	17	0.235
828	A	4	4	1.00	19	0.211
829	A	4	4	1.01	19	0.211
830	A	6	6	1.75	27	0.222
831	A	7	6	1.02	21	0.286
832	A	6	5	1.00	14	0.357
833	A	3	3	1.00	17	0.176
834	A	3	3	1.00	19	0.158
835	A	3	3	1.00	19	0.158
836	A	6	6	1.82	27	0.222
837	A	6	5	1.00	21	0.238
838	A	8	7	1.05	20	0.350
839	A	6	5	1.05	20	0.250
840	A	7	6	1.05	16	0.375
841	A	5	4	1.05	16	0.250
842	C	7	6	0.78	16	0.375
843	C	8	7	0.65	18	0.389
844	C	9	8	0.65	18	0.444
845	C	9	8	0.81	16	0.500
846	C	10	9	0.74	18	0.500
847	C	11	10	0.71	18	0.556
848	A	3	3	0.50	16	0.188
849	A	5	5	0.61	18	0.278
850	A	5	5	0.59	18	0.278
851	A	3	3	0.52	16	0.188
852	A	5	5	0.48	18	0.278
853	A	5	5	0.52	18	0.278
854	A	7	6	1.00	18	0.333
855	A	7	7	1.05	18	0.389
856	A	4	4	1.00	18	0.222
857	A	1	1	1.00	16	0.062

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
858	A	7	6	0.89	18	0.333
859	A	10	9	0.98	18	0.500
860	A	14	13	1.08	18	0.722
861	A	17	17	0.99	20	0.850
862	A	14	14	0.98	20	0.700
863	A	6	6	1.00	20	0.300
864	A	6	6	1.00	20	0.300
865	A	9	9	1.00	20	0.450
866	A	18	18	1.00	20	0.900
867	A	12	11	0.90	14	0.786
868	A	11	10	0.96	14	0.714
869	A	10	9	1.03	12	0.750
870	N/A	3	0	1.00	14	0.000
871	A	2	2	1.00	18	0.111
872	A	5	4	0.80	18	0.222
873	A	6	5	0.82	18	0.278
874	A	4	3	0.97	16	0.188
875	A	5	4	0.88	16	0.250
876	A	6	5	0.88	18	0.278
877	A	6	5	1.10	18	0.278
878	A	2	2	0.96	16	0.125
879	A	2	2	1.00	16	0.125
880	A	1	1	1.00	14	0.071
881	A	1	1	1.00	14	0.071
882	A	1	1	1.00	16	0.062
883	A	2	2	1.07	16	0.125
884	A	2	2	1.00	18	0.111
885	A	2	2	0.96	16	0.125
886	A	2	2	1.00	16	0.125
887	A	1	1	1.00	14	0.071
888	A	1	1	1.00	14	0.071
889	A	1	1	1.00	16	0.062
890	A	2	2	1.06	16	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
891	A	2	2	1.20	18	0.111
892	A	2	2	1.21	18	0.111
893	A	4	4	0.94	25	0.160
894	A	4	4	0.99	23	0.174
895	A	2	2	1.00	25	0.080
896	A	3	3	1.07	25	0.120
897	A	4	4	0.94	22	0.182
898	A	4	4	0.99	20	0.200
899	A	2	2	1.00	22	0.091
900	A	3	3	1.09	22	0.136
901	A	5	4	0.90	22	0.182
902	A	6	5	0.82	22	0.227
903	A	5	4	0.91	20	0.200
904	A	5	4	0.88	14	0.286
905	A	6	5	0.93	20	0.250
906	A	7	6	1.10	22	0.273
907	A	6	5	0.80	24	0.208
908	A	5	4	0.92	24	0.167
909	A	6	5	0.79	22	0.227
910	A	6	5	1.14	20	0.250
911	A	4	3	0.89	16	0.188
912	A	6	5	0.95	22	0.227
913	A	5	4	0.87	24	0.167
914	A	6	5	0.82	24	0.208
915	A	5	4	0.90	22	0.182
916	A	5	4	0.86	22	0.182
917	A	6	5	1.02	22	0.227
918	A	5	4	0.89	16	0.250
919	A	6	5	1.00	24	0.208
920	A	5	4	0.86	24	0.167
921	A	5	4	1.39	22	0.182
922	A	6	5	1.00	16	0.312
923	A	7	6	0.87	22	0.273
924	A	6	5	0.87	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
925	A	5	4	0.85	26	0.154
926	A	5	4	0.92	26	0.154
927	A	5	4	0.83	24	0.167
928	A	5	4	0.91	22	0.182
929	A	5	4	0.90	18	0.222
930	A	8	7	0.99	24	0.292
931	A	6	5	1.04	26	0.192
932	A	5	4	0.85	26	0.154
933	A	6	5	1.04	24	0.208
934	A	6	5	0.85	24	0.208
935	A	5	4	0.90	24	0.167
936	A	6	5	0.91	18	0.278
937	A	12	11	1.12	12	0.917
938	A	13	12	1.15	14	0.857
939	A	14	13	1.15	14	0.929
940	A	15	14	1.15	16	0.875
941	A	7	6	1.29	12	0.500
942	A	8	7	1.26	14	0.500
943	A	9	8	1.25	14	0.571
944	A	10	9	1.23	16	0.562
945	A	2	2	1.00	22	0.091
946	A	2	2	1.00	22	0.091
947	A	3	3	1.05	20	0.150
948	A	1	1	1.00	14	0.071
949	A	2	2	1.00	14	0.143
950	A	2	2	1.00	20	0.100
951	A	2	2	1.00	22	0.091
952	A	2	2	1.00	24	0.083
953	A	2	2	1.00	24	0.083
954	A	2	2	1.00	22	0.091
955	A	2	2	1.00	16	0.125
956	A	2	2	1.12	20	0.100
957	A	2	2	1.00	16	0.125
958	A	2	2	1.00	22	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
959	A	2	2	1.00	24	0.083
960	A	2	2	1.00	24	0.083
961	A	2	2	1.00	22	0.091
962	A	2	2	0.98	16	0.125
963	A	2	2	1.22	22	0.091
964	A	2	2	1.08	22	0.091
965	A	2	2	1.00	16	0.125
966	A	1	1	1.00	56	0.018
967	A	3	2	1.00	17	0.118
968	A	3	2	1.00	22	0.091
969	A	3	2	1.00	22	0.091
970	A	3	2	1.00	17	0.118
971	A	3	2	1.00	22	0.091
972	A	3	2	1.00	22	0.091
973	A	3	2	1.00	17	0.118
974	A	3	2	1.00	22	0.091
975	A	3	2	1.00	22	0.091
976	A	3	2	1.00	17	0.118
977	A	3	2	1.00	22	0.091
978	A	3	2	1.00	22	0.091
979	A	4	3	1.00	13	0.231
980	A	4	3	1.00	13	0.231
981	A	4	3	1.00	13	0.231
982	A	4	3	1.00	13	0.231
983	A	4	3	1.50	17	0.176
984	A	3	3	1.00	23	0.130
985	A	5	4	1.00	17	0.235
986	A	5	4	1.00	16	0.250
987	A	6	5	1.00	18	0.278
988	A	11	10	1.13	15	0.667
989	A	5	4	0.91	19	0.211
990	A	5	4	1.00	19	0.211
991	A	5	4	1.00	21	0.190
992	A	5	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
993	A	6	5	1.00	17	0.294
994	C	10	9	1.36	19	0.474
995	A	7	6	1.00	19	0.316
996	A	3	3	1.00	11	0.273
997	A	5	4	2.00	17	0.235
998	A	4	3	1.00	17	0.176
999	A	4	3	1.00	17	0.176
1000	A	5	4	1.00	15	0.267
1001	A	5	4	1.00	15	0.267
1002	A	5	4	1.00	15	0.267
1003	A	10	9	1.00	15	0.600
1004	A	5	4	0.95	20	0.200
1005	A	5	4	0.95	19	0.211
1006	A	5	4	0.97	24	0.167
1007	A	5	4	0.97	21	0.190
1008	A	5	4	0.95	20	0.200
1009	A	5	4	0.95	19	0.211
1010	A	5	4	0.97	24	0.167
1011	A	5	4	0.97	21	0.190
1012	A	1	1	1.00	8	0.125
1013	A	1	1	1.00	8	0.125
1014	N/A	3	0	1.00	18	0.000
1015	N/A	3	0	1.00	18	0.000
1016	N/A	3	0	1.00	20	0.000
1017	N/A	3	0	1.00	20	0.000
1018	A	7	6	1.00	13	0.462
1019	A	5	4	1.00	13	0.308
1020	A	5	4	1.00	13	0.308
1021	A	5	5	1.00	11	0.455
1022	A	8	7	1.50	19	0.368
1023	A	6	5	1.00	19	0.263
1024	A	6	5	1.00	21	0.238
1025	A	6	5	1.00	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1026	A	7	6	1.11	17	0.353
1027	A	8	7	1.11	17	0.412
1028	A	6	5	1.00	17	0.294
1029	A	4	3	1.00	16	0.188
1030	A	5	4	1.00	18	0.222
1031	A	1	1	1.00	18	0.056
1032	A	6	5	1.00	18	0.278
1033	A	13	12	1.73	15	0.800
1034	A	5	4	1.42	15	0.267
1035	A	7	6	1.27	15	0.400
1036	A	5	4	1.04	15	0.267
1037	A	6	5	1.00	21	0.238
1038	A	6	5	1.00	21	0.238
1039	A	2	2	1.00	8	0.250
1040	A	2	2	1.00	10	0.200
1041	A	2	2	1.00	10	0.200
1042	A	2	2	1.00	18	0.111
1043	C	4	4	1.45	8	0.500
1044	A	8	7	1.00	15	0.467
1045	A	12	11	1.16	22	0.500
1046	A	5	4	0.92	12	0.333
1047	A	10	9	1.00	15	0.600
1048	A	8	7	1.00	15	0.467
1049	A	2	2	1.00	13	0.154
1050	A	4	4	1.74	23	0.174
1051	A	4	4	1.69	25	0.160
1052	A	6	5	0.96	39	0.128
1053	A	5	4	0.96	39	0.103
1054	A	6	5	1.00	39	0.128
1055	A	2	2	1.00	31	0.065
1056	A	4	3	1.00	31	0.097
1057	A	4	3	1.00	39	0.077
1058	A	6	5	0.96	39	0.128
1059	A	7	6	0.96	39	0.154

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{2}{-1+3\cosh(4+6x)}\,dx$

3.1.1 Optimal result

Integrand size = 14, antiderivative size = 22

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx=\frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

output
```
1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

3.1.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx=\frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[2/(-1 + 3*Cosh[4 + 6*x]),x]
```

output
```
ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

3.1.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {27, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2}{3\cosh(6x+4)-1}dx$$

$$\Big\downarrow \; 27$$

$$2\int \frac{1}{3\cosh(6x+4)-1}dx$$

$$\Big\downarrow \; 3042$$

$$2\int \frac{1}{3\sin\left(6ix+\frac{\pi}{2}+4i\right)-1}dx$$

$$\Big\downarrow \; 3138$$

$$-\frac{2}{3}i\int \frac{1}{4\tanh^2(3x+2)+2}d(i\tanh(3x+2))$$

$$\Big\downarrow \; 219$$

$$\frac{\arctan\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[2/(-1 + 3*Cosh[4 + 6*x]),x]
```

output
```
ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

3.1.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.1.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\arctan\left(\sqrt{2}\,\tanh(2+3x)\right)\sqrt{2}}{6}$	17
default	$\frac{\arctan\left(\sqrt{2}\,\tanh(2+3x)\right)\sqrt{2}}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	44

input
```
int(2/(-1+3*cosh(4+6*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

3.1.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}\cosh\left(6x+4\right)+\frac{3}{4}\sqrt{2}\sinh\left(6x+4\right)-\frac{1}{4}\sqrt{2}\right)$$

input
```
integrate(2/(-1+3*cosh(4+6*x)),x, algorithm="fricas")
```

output
```
1/6*sqrt(2)*arctan(3/4*sqrt(2)*cosh(6*x + 4) + 3/4*sqrt(2)*sinh(6*x + 4) -
 1/4*sqrt(2))
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tanh\left(3x+2\right)\right)}{6}$$

input
```
integrate(2/(-1+3*cosh(4+6*x)),x)
```

output
```
sqrt(2)*atan(sqrt(2)*tanh(3*x + 2))/6
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}(3e^{(-6x-4)}-1)\right)$$

input
```
integrate(2/(-1+3*cosh(4+6*x)),x, algorithm="maxima")
```

output
```
-1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}(3e^{(6x+4)}-1)\right)$$

input
```
integrate(2/(-1+3*cosh(4+6*x)),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) - 1))
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(3\,\mathrm{e}^{6\,x+4}-1)}{4}\right)}{6}$$

input
```
int(2/(3*cosh(6*x + 4) - 1),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) - 1))/4))/6
```

3.2 $\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx$

3.2.1 Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = \frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

output
```
1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = \frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[(Cosh[2 + 3*x]^2 + 2*Sinh[2 + 3*x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

3.2.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2\sinh^2(3x+2)+\cosh^2(3x+2)}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\cos(3ix+2i)^2-2\sin(3ix+2i)^2}dx$$
$$\Big\downarrow \; 4889$$
$$\frac{1}{3}\int \frac{1}{2\tanh^2(3x+2)+1}d\tanh(3x+2)$$
$$\Big\downarrow \; 216$$
$$\frac{\arctan\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[(Cosh[2 + 3*x]^2 + 2*Sinh[2 + 3*x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

3.2.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.2.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

method	result	size
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	44
derivativedivides	$-\frac{\left(3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{9\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{\left(-3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{9\left(2\sqrt{3}-2\sqrt{2}\right)}$	92
default	$-\frac{\left(3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{9\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{\left(-3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{9\left(2\sqrt{3}-2\sqrt{2}\right)}$	92

input
```
int(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/12*I*2^(1/2)*ln(exp(4+6*x)-1/3+2/3*I*2^(1/2))-1/12*I*2^(1/2)*ln(exp(4+6*
x)-1/3-2/3*I*2^(1/2))
```

3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(16) = 32.

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(3x+2)+2\sqrt{2}\sinh(3x+2)}{2(\cosh(3x+2)-\sinh(3x+2))}\right)$$

input
```
integrate(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*cosh(3*x + 2) + 2*sqrt(2)*sinh(3*x + 2))
/(cosh(3*x + 2) - sinh(3*x + 2)))
```

3.2.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(19) = 38$.

Time = 3.54 (sec) , antiderivative size = 185, normalized size of antiderivative = 8.41

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = \frac{2093258\sqrt{5-2\sqrt{6}}\operatorname{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{5-2\sqrt{6}}}\right)}{1152360\sqrt{6}+2822694} + \frac{854569\sqrt{6}\sqrt{5-2\sqrt{6}}\operatorname{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{5-2\sqrt{6}}}\right)}{1152360\sqrt{6}+2822694} - \frac{86329\sqrt{6}\sqrt{2\sqrt{6}+5}\operatorname{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{6}+5}}\right)}{1152360\sqrt{6}+2822694} - \frac{211462\sqrt{2\sqrt{6}+5}\operatorname{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{6}+5}}\right)}{1152360\sqrt{6}+2822694}$$

input
```
integrate(1/(cosh(2+3*x)**2+2*sinh(2+3*x)**2),x)
```

output
```
2093258*sqrt(5 - 2*sqrt(6))*atan(tanh(3*x/2 + 1)/sqrt(5 - 2*sqrt(6)))/(115
2360*sqrt(6) + 2822694) + 854569*sqrt(6)*sqrt(5 - 2*sqrt(6))*atan(tanh(3*x
/2 + 1)/sqrt(5 - 2*sqrt(6)))/(1152360*sqrt(6) + 2822694) - 86329*sqrt(6)*s
qrt(2*sqrt(6) + 5)*atan(tanh(3*x/2 + 1)/sqrt(2*sqrt(6) + 5))/(1152360*sqrt
(6) + 2822694) - 211462*sqrt(2*sqrt(6) + 5)*atan(tanh(3*x/2 + 1)/sqrt(2*sq
rt(6) + 5))/(1152360*sqrt(6) + 2822694)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3\,e^{(-6\,x-4)}-1\right)\right)$$

input
```
integrate(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3\,e^{(6\,x+4)}-1\right)\right)$$

input
```
integrate(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) - 1))
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(3\,\mathrm{e}^{6\,x+4}-1\right)}{4}\right)}{6}$$

input
```
int(1/(2*sinh(3*x + 2)^2 + cosh(3*x + 2)^2),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) - 1))/4))/6
```

3.3 $\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx$

3.3.1 Optimal result

Integrand size = 23, antiderivative size = 22

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx=\frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

output
```
1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx=\frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[Sech[2 + 3*x]^2/(1 + 2*Tanh[2 + 3*x]^2),x]
```

output
```
ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

3.3.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4158, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(3x+2)}{2\tanh^2(3x+2)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(3ix+2i)^2}{1-2\tan(3ix+2i)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{1}{3}\int \frac{1}{2\tanh^2(3x+2)+1}d\tanh(3x+2)$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[Sech[2 + 3*x]^2/(1 + 2*Tanh[2 + 3*x]^2),x]
```

output
```
ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

3.3.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.3.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

method	result	size
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	44
derivativedivides	$-\frac{\left(3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{9\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{\left(-3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{9\left(2\sqrt{3}-2\sqrt{2}\right)}$	92
default	$-\frac{\left(3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{9\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{\left(-3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{9\left(2\sqrt{3}-2\sqrt{2}\right)}$	92

input
```
int(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/12*I*2^(1/2)*ln(exp(4+6*x)-1/3+2/3*I*2^(1/2))-1/12*I*2^(1/2)*ln(exp(4+6*
x)-1/3-2/3*I*2^(1/2))
```

3.3.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \frac{\mathrm{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx=-\frac{1}{6}\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh\left(3\,x+2\right)+2\,\sqrt{2}\sinh\left(3\,x+2\right)}{2\left(\cosh\left(3\,x+2\right)-\sinh\left(3\,x+2\right)\right)}\right)$$

input
```
integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*cosh(3*x + 2) + 2*sqrt(2)*sinh(3*x + 2))
/(cosh(3*x + 2) - sinh(3*x + 2)))
```

3.3.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx = \int \frac{\operatorname{sech}^2(3x+2)}{2\tanh^2(3x+2)+1}\,dx$$

input
```
integrate(sech(2+3*x)**2/(1+2*tanh(2+3*x)**2),x)
```

output
```
Integral(sech(3*x + 2)**2/(2*tanh(3*x + 2)**2 + 1), x)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}-1\right)\right)$$

input
```
integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(6x+4)}-1\right)\right)$$

input
```
integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) - 1))
```

3.3.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx=\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\left(3\,e^{6\,x+4}-1\right)}{4}\right)}{6}$$

input
```
int(1/(cosh(3*x + 2)^2*(2*tanh(3*x + 2)^2 + 1)),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) - 1))/4))/6
```

3.4 $\int \frac{\text{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx$

3.4.1 Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{\text{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx=\frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

output
```
1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\text{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx=\frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[Csch[2 + 3*x]^2/(2 + Coth[2 + 3*x]^2),x]
```

output
```
ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

3.4.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 25, 4158, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(3x+2)}{\coth^2(3x+2)+2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sec\left(3ix+\frac{\pi}{2}+2i\right)^2}{2-\tan\left(3ix+\frac{\pi}{2}+2i\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sec\left(3ix+\frac{1}{2}(4i+\pi)\right)^2}{2-\tan\left(3ix+\frac{1}{2}(4i+\pi)\right)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{1}{3}i\int \frac{1}{\coth^2(3x+2)+2}d(i\coth(3x+2))$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arctan\left(\frac{\coth(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[Csch[2 + 3*x]^2/(2 + Coth[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTan[Coth[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.4.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

method	result	size
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	44
derivativedivides	$-\frac{\left(3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{9\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{\left(-3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{9\left(2\sqrt{3}-2\sqrt{2}\right)}$	92
default	$-\frac{\left(3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{9\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{\left(-3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{9\left(2\sqrt{3}-2\sqrt{2}\right)}$	92

input
```
int(csch(2+3*x)^2/(2+coth(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/12*I*2^(1/2)*ln(exp(4+6*x)-1/3+2/3*I*2^(1/2))-1/12*I*2^(1/2)*ln(exp(4+6*
x)-1/3-2/3*I*2^(1/2))
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh\left(3\,x+2\right)+2\sqrt{2}\sinh\left(3\,x+2\right)}{2\left(\cosh\left(3\,x+2\right)-\sinh\left(3\,x+2\right)\right)}\right)$$

input
```
integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*cosh(3*x + 2) + 2*sqrt(2)*sinh(3*x + 2))
/(cosh(3*x + 2) - sinh(3*x + 2)))
```

3.4.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx = \int \frac{\operatorname{csch}^2\left(3x+2\right)}{\coth^2\left(3x+2\right)+2}\,dx$$

input
```
integrate(csch(2+3*x)**2/(2+coth(2+3*x)**2),x)
```

output
```
Integral(csch(3*x + 2)**2/(coth(3*x + 2)**2 + 2), x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3\,e^{(-6\,x-4)}-1\right)\right)$$

input
```
integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3\,e^{(6\,x+4)}-1\right)\right)$$

input
```
integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) - 1))
```

3.4.9 Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx = \frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\left(3\,\mathrm{e}^{6\,x+4}-1\right)}{4}\right)}{6}$$

input
```
int(1/(sinh(3*x + 2)^2*(coth(3*x + 2)^2 + 2)),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) - 1))/4))/6
```

3.5 $\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx$

3.5.1 Optimal result

Integrand size = 23, antiderivative size = 22

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx=-\frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

output
```
-1/6*arctanh(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx=-\frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[Csch[2 + 3*x]^2/(2 - Coth[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tanh[2 + 3*x]]/Sqrt[2]
```

3.5.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 25, 4158, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(3x+2)}{2-\coth^2(3x+2)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sec\left(3ix+\frac{\pi}{2}+2i\right)^2}{2+\tan\left(3ix+\frac{\pi}{2}+2i\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sec\left(3ix+\frac{1}{2}(4i+\pi)\right)^2}{\tan\left(3ix+\frac{1}{2}(4i+\pi)\right)^2+2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{1}{3}i\int \frac{1}{2-\coth^2(3x+2)}d(i\coth(3x+2))$$

$$\Big\downarrow \text{216}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\coth(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[Csch[2 + 3*x]^2/(2 - Coth[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTanh[Coth[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.5.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(16) = 32$.

Time = 0.54 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

method	result	size
risch	$\frac{\sqrt{2}\, \ln\left(e^{4+6x}-3-2\sqrt{2}\right)}{12}-\frac{\sqrt{2}\, \ln\left(e^{4+6x}-3+2\sqrt{2}\right)}{12}$	40
derivativedivides	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(2\tanh\left(1+\frac{3x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{6}-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(2\tanh\left(1+\frac{3x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{6}$	44
default	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(2\tanh\left(1+\frac{3x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{6}-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(2\tanh\left(1+\frac{3x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{6}$	44

input
```
int(csch(2+3*x)^2/(2-coth(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(1/2)*ln(exp(4+6*x)-3-2*2^(1/2))-1/12*2^(1/2)*ln(exp(4+6*x)-3+2*2^(
1/2))
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.05

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx$$
$$= \frac{1}{12}\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh\left(3x+2\right)^2-4\left(3\sqrt{2}+4\right)\cosh\left(3x+2\right)\sinh\left(3x+2\right)+3\left(2\sqrt{2}+3\right)\sinh\left(\right.}{\cosh\left(3x+2\right)^2+\sinh\left(3x+2\right)^2-3}\right.$$

input
```
integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="fricas")
```

output
```
1/12*sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(3*x + 2)^2 - 4*(3*sqrt(2) + 4)*co
sh(3*x + 2)*sinh(3*x + 2) + 3*(2*sqrt(2) + 3)*sinh(3*x + 2)^2 - 2*sqrt(2)
- 3)/(cosh(3*x + 2)^2 + sinh(3*x + 2)^2 - 3))
```

3.5.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx = -\int \frac{\operatorname{csch}^2(3x+2)}{\coth^2(3x+2)-2}\,dx$$

input
```
integrate(csch(2+3*x)**2/(2-coth(2+3*x)**2),x)
```

output
```
-Integral(csch(3*x + 2)**2/(coth(3*x + 2)**2 - 2), x)
```

3.5.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.14

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx = -\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-3x-2)}+1}{\sqrt{2}+e^{(-3x-2)}-1}\right) + \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-3x-2)}-1}{\sqrt{2}+e^{(-3x-2)}+1}\right)$$

input
```
integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="maxima")
```

output
```
-1/12*sqrt(2)*log(-(sqrt(2) - e^(-3*x - 2) + 1)/(sqrt(2) + e^(-3*x - 2) -
1)) + 1/12*sqrt(2)*log(-(sqrt(2) - e^(-3*x - 2) - 1)/(sqrt(2) + e^(-3*x -
2) + 1))
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}e^4 - 6\,e^4 + 2\,e^{(6\,x+8)}\right|}{\left|4\sqrt{2}e^4 - 6\,e^4 + 2\,e^{(6\,x+8)}\right|}\right)$$

input
```
integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-4*sqrt(2)*e^4 - 6*e^4 + 2*e^(6*x + 8))/abs(4*sqrt(2)
*e^4 - 6*e^4 + 2*e^(6*x + 8)))
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx = \frac{\sqrt{2}\left(\ln\left(\sqrt{2}\left(3\,\mathrm{e}^{6\,x+4}-1\right)-4\,\mathrm{e}^{6\,x+4}\right)-\ln\left(-4\,\mathrm{e}^{6\,x+4}-\sqrt{2}\left(3\,\mathrm{e}^{6\,x+4}-1\right)\right)\right)}{12}$$

input
```
int(-1/(sinh(3*x + 2)^2*(coth(3*x + 2)^2 - 2)),x)
```

output
```
(2^(1/2)*(log(2^(1/2)*(3*exp(6*x + 4) - 1) - 4*exp(6*x + 4)) - log(- 4*exp
(6*x + 4) - 2^(1/2)*(3*exp(6*x + 4) - 1))))/12
```

3.6 $\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx$

3.6.1 Optimal result

Integrand size = 23, antiderivative size = 22

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx=\frac{\arctan\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

output
```
1/6*arctan(1/2*2^(1/2)*tanh(2+3*x))*2^(1/2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx=\frac{\arctan\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[Csch[2 + 3*x]^2/(1 + 2*Coth[2 + 3*x]^2),x]
```

output
```
ArcTan[Tanh[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

3.6.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 25, 4158, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(3x+2)}{2\coth^2(3x+2)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sec\left(3ix+\frac{\pi}{2}+2i\right)^2}{1-2\tan\left(3ix+\frac{\pi}{2}+2i\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sec\left(3ix+\frac{1}{2}(4i+\pi)\right)^2}{1-2\tan\left(3ix+\frac{1}{2}(4i+\pi)\right)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{1}{3}i\int \frac{1}{2\coth^2(3x+2)+1}d(i\coth(3x+2))$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arctan\left(\sqrt{2}\coth(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[Csch[2 + 3*x]^2/(1 + 2*Coth[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTan[Sqrt[2]*Coth[2 + 3*x]]/Sqrt[2]
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.6.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

method	result	size
risch	$\frac{i\sqrt{2}\,\ln\left({\mathrm e}^{4+6x}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{i\sqrt{2}\,\ln\left({\mathrm e}^{4+6x}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	44
derivativedivides	$-\frac{\sqrt{3}\left(3+\sqrt{3}\right)\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{\sqrt{6}+\sqrt{2}}\right)}{9\left(\sqrt{6}+\sqrt{2}\right)}-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{\sqrt{6}-\sqrt{2}}\right)}{9\left(\sqrt{6}-\sqrt{2}\right)}$	80
default	$-\frac{\sqrt{3}\left(3+\sqrt{3}\right)\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{\sqrt{6}+\sqrt{2}}\right)}{9\left(\sqrt{6}+\sqrt{2}\right)}-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{\sqrt{6}-\sqrt{2}}\right)}{9\left(\sqrt{6}-\sqrt{2}\right)}$	80

input
```
int(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/12*I*2^(1/2)*ln(exp(4+6*x)+1/3+2/3*I*2^(1/2))-1/12*I*2^(1/2)*ln(exp(4+6*
x)+1/3-2/3*I*2^(1/2))
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(-\frac{2\sqrt{2}\cosh(3x+2)+\sqrt{2}\sinh(3x+2)}{2(\cosh(3x+2)-\sinh(3x+2))}\right)$$

input
```
integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*arctan(-1/2*(2*sqrt(2)*cosh(3*x + 2) + sqrt(2)*sinh(3*x + 2))
/(cosh(3*x + 2) - sinh(3*x + 2)))
```

3.6.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx = \int \frac{\operatorname{csch}^2(3x+2)}{2\coth^2(3x+2)+1}\,dx$$

input
```
integrate(csch(2+3*x)**2/(1+2*coth(2+3*x)**2),x)
```

output
```
Integral(csch(3*x + 2)**2/(2*coth(3*x + 2)**2 + 1), x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}+1\right)\right)$$

input
```
integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) + 1))
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}(3\,e^{(6\,x+4)}+1)\right)$$

input
```
integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) + 1))
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(3\,\mathrm{e}^{6\,x+4}+1\right)}{4}\right)}{6}$$

input
```
int(1/(sinh(3*x + 2)^2*(2*coth(3*x + 2)^2 + 1)),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) + 1))/4))/6
```

3.7 $\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx$

3.7.1 Optimal result

Integrand size = 23, antiderivative size = 22

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

output
```
-1/6*arctanh(1/2*2^(1/2)*tanh(2+3*x))*2^(1/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[Csch[2 + 3*x]^2/(1 - 2*Coth[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTanh[Tanh[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.7.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 25, 4158, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(3x+2)}{1-2\coth^2(3x+2)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sec\left(3ix+\frac{\pi}{2}+2i\right)^2}{1+2\tan\left(3ix+\frac{\pi}{2}+2i\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sec\left(3ix+\frac{1}{2}(4i+\pi)\right)^2}{2\tan\left(3ix+\frac{1}{2}(4i+\pi)\right)^2+1}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{1}{3}i\int \frac{1}{1-2\coth^2(3x+2)}d(i\coth(3x+2))$$

$$\Big\downarrow \text{216}$$

$$-\frac{\operatorname{arctanh}\left(\sqrt{2}\coth(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[Csch[2 + 3*x]^2/(1 - 2*Coth[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Coth[2 + 3*x]]/Sqrt[2]
```

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.7.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(17) = 34$.

Time = 0.61 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

method	result
risch	$\frac{\sqrt{2}\,\ln\left(e^{4+6x}+3+2\sqrt{2}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{4+6x}+3-2\sqrt{2}\right)}{12}$
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(1+\frac{3x}{2}\right)^{2}+\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}+1}{\tanh\left(1+\frac{3x}{2}\right)^{2}-\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}-1\right)\right)}{24}+\frac{\sqrt{2}}{}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(1+\frac{3x}{2}\right)^{2}+\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}+1}{\tanh\left(1+\frac{3x}{2}\right)^{2}-\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}-1\right)\right)}{24}+\frac{\sqrt{2}}{}$

input
```
int(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(1/2)*ln(exp(4+6*x)+3+2*2^(1/2))-1/12*2^(1/2)*ln(exp(4+6*x)+3-2*2^(
1/2))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.05

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx$$
$$= \frac{1}{12}\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh\left(3x+2\right)^2 - 4\left(3\sqrt{2}+4\right)\cosh\left(3x+2\right)\sinh\left(3x+2\right) + 3\left(2\sqrt{2}+3\right)\sinh\left(\right.}{\cosh\left(3x+2\right)^2 + \sinh\left(3x+2\right)^2 + 3}\right.$$

input
```
integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="fricas")
```

output
```
1/12*sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(3*x + 2)^2 - 4*(3*sqrt(2) + 4)*co
sh(3*x + 2)*sinh(3*x + 2) + 3*(2*sqrt(2) + 3)*sinh(3*x + 2)^2 + 2*sqrt(2)
+ 3)/(cosh(3*x + 2)^2 + sinh(3*x + 2)^2 + 3))
```

3.7.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\int \frac{\operatorname{csch}^2(3x+2)}{2\coth^2(3x+2)-1}\,dx$$

input
```
integrate(csch(2+3*x)**2/(1-2*coth(2+3*x)**2),x)
```

output
```
-Integral(csch(3*x + 2)**2/(2*coth(3*x + 2)**2 - 1), x)
```

3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-6x-4)}-3}{2\sqrt{2}+e^{(-6x-4)}+3}\right)$$

input
```
integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(2*sqrt(2) - e^(-6*x - 4) - 3)/(2*sqrt(2) + e^(-6*x - 4)
 + 3))
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\frac{1}{12}\sqrt{2}\log\left(-\frac{2\sqrt{2}e^4-3e^4-e^{(6x+8)}}{2\sqrt{2}e^4+3e^4+e^{(6x+8)}}\right)$$

input
```
integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="giac")
```

output
```
-1/12*sqrt(2)*log(-(2*sqrt(2)*e^4 - 3*e^4 - e^(6*x + 8))/(2*sqrt(2)*e^4 +
3*e^4 + e^(6*x + 8)))
```

3.7.9 Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\frac{\sqrt{2}\left(\ln\left(4\,e^{6x+4}+\sqrt{2}\left(3\,e^{6x+4}+1\right)\right)-\ln\left(4\,e^{6x+4}-\sqrt{2}\left(3\,e^{6x+4}+1\right)\right)\right)}{12}$$

input
```
int(-1/(sinh(3*x + 2)^2*(2*coth(3*x + 2)^2 - 1)),x)
```

output
```
-(2^(1/2)*(log(4*exp(6*x + 4) + 2^(1/2)*(3*exp(6*x + 4) + 1)) - log(4*exp(
6*x + 4) - 2^(1/2)*(3*exp(6*x + 4) + 1))))/12
```

3.8 $\int \cosh(a+bx)\sinh(a+bx)\,dx$

3.8.1 Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\sinh^2(a+bx)}{2b}$$

output
```
1/2*sinh(b*x+a)^2/b
```

3.8.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{1}{2}\left(\frac{\cosh(2a)\cosh(2bx)}{2b} + \frac{\sinh(2a)\sinh(2bx)}{2b}\right)$$

input
```
Integrate[Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
((Cosh[2*a]*Cosh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b))/2
```

3.8.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 26, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)\cos(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \cos(ia+ibx)\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3044}$$

$$-\frac{\int i\sinh(a+bx)d(i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sinh^2(a+bx)}{2b}$$

input
```
Int[Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
Sinh[a + b*x]^2/(2*b)
```

3.8.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.8.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cosh(bx+a)^2}{2b}$	14
default	$\frac{\cosh(bx+a)^2}{2b}$	14
risch	$\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}$	30

input
```
int(cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*cosh(b*x+a)^2/b
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cosh(a+bx)\sinh(a+bx)\,dx=\frac{\cosh(bx+a)^2+\sinh(bx+a)^2}{4\,b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(cosh(b*x + a)^2 + sinh(b*x + a)^2)/b
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{\sinh^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sinh(a)\cosh(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a),x)
```

output
```
Piecewise((sinh(a + b*x)**2/(2*b), Ne(b, 0)), (x*sinh(a)*cosh(a), True))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^2}{2\,b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*cosh(b*x + a)^2/b
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)^2}{2\,b}$$

input
```
int(cosh(a + b*x)*sinh(a + b*x),x)
```

output
```
cosh(a + b*x)^2/(2*b)
```

3.9 $\int \cosh(a+bx)\sinh^n(a+bx)\,dx$

3.9.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx=\frac{\sinh^{1+n}(a+bx)}{b(1+n)}$$

output
```
sinh(b*x+a)^(1+n)/b/(1+n)
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx=\frac{\sinh^{1+n}(a+bx)}{b(1+n)}$$

input
```
Integrate[Cosh[a + b*x]*Sinh[a + b*x]^n,x]
```

output
```
Sinh[a + b*x]^(1 + n)/(b*(1 + n))
```

3.9.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(ia+ibx)(-i\sin(ia+ibx))^n dx$$

$$\Big\downarrow \; 3044$$

$$\frac{\int \sinh^n(a+bx)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \; 15$$

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}$$

input
```
Int[Cosh[a + b*x]*Sinh[a + b*x]^n,x]
```

output
```
Sinh[a + b*x]^(1 + n)/(b*(1 + n))
```

3.9.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.9.4 Maple [A] (verified)

Time = 6.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\sinh(bx+a)^{n+1}}{b(n+1)}$	20
default	$\frac{\sinh(bx+a)^{n+1}}{b(n+1)}$	20

input
```
int(cosh(b*x+a)*sinh(b*x+a)^n,x,method=_RETURNVERBOSE)
```

output
```
sinh(b*x+a)^(n+1)/b/(n+1)
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(19) = 38.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.58

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx = \frac{\cosh(n\log(\sinh(bx+a)))\sinh(bx+a)+\sinh(bx+a)\sinh(n\log(\sinh(bx+a)))}{(bn+b)\cosh(bx+a)^2-(bn+b)\sinh(bx+a)^2}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^n,x, algorithm="fricas")
```

output
```
(cosh(n*log(sinh(b*x + a)))*sinh(b*x + a) + sinh(b*x + a)*sinh(n*log(sinh(
b*x + a))))/((b*n + b)*cosh(b*x + a)^2 - (b*n + b)*sinh(b*x + a)^2)
```

3.9.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(14) = 28.

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx = \begin{cases} \frac{x\cosh(a)}{\sinh(a)} & \text{for } b=0 \wedge n=-1 \\ x\sinh^n(a)\cosh(a) & \text{for } b=0 \\ \frac{\log(\sinh(a+bx))}{b} & \text{for } n=-1 \\ \frac{\sinh(a+bx)\sinh^n(a+bx)}{bn+b} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)**n,x)
```

output
```
Piecewise((x*cosh(a)/sinh(a), Eq(b, 0) & Eq(n, -1)), (x*sinh(a)**n*cosh(a)
, Eq(b, 0)), (log(sinh(a + b*x))/b, Eq(n, -1)), (sinh(a + b*x)*sinh(a + b*
x)**n/(b*n + b), True))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx = \frac{\sinh(bx+a)^{n+1}}{b(n+1)}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^n,x, algorithm="maxima")
```

output
```
sinh(b*x + a)^(n + 1)/(b*(n + 1))
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx = \frac{\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)^{n+1}}{b(n+1)}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^n,x, algorithm="giac")
```

output
```
(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a))^(n + 1)/(b*(n + 1))
```

3.9.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx = \frac{\sinh(a+b\,x)^{n+1}}{b\,(n+1)}$$

input
```
int(cosh(a + b*x)*sinh(a + b*x)^n,x)
```

output
```
sinh(a + b*x)^(n + 1)/(b*(n + 1))
```

3.10 $\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx$

3.10.1 Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx = \frac{\sinh^{1+n}(a+bx)}{b(1+n)} + \frac{\sinh^{3+n}(a+bx)}{b(3+n)}$$

output
```
sinh(b*x+a)^(1+n)/b/(1+n)+sinh(b*x+a)^(3+n)/b/(3+n)
```

3.10.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx = \frac{\sinh^{1+n}(a+bx)}{b(1+n)} + \frac{\sinh^{3+n}(a+bx)}{b(3+n)}$$

input
```
Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^n,x]
```

output
```
Sinh[a + b*x]^(1 + n)/(b*(1 + n)) + Sinh[a + b*x]^(3 + n)/(b*(3 + n))
```

3.10.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cos(ia+ibx)^3(-i\sin(ia+ibx))^n dx$$

$$\Big\downarrow \ 3044$$

$$\frac{\int \sinh^n(a+bx)\left(\sinh^2(a+bx)+1\right)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \ 244$$

$$\frac{\int \left(\sinh^n(a+bx)+\sinh^{n+2}(a+bx)\right)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\sinh^{n+1}(a+bx)}{n+1}+\frac{\sinh^{n+3}(a+bx)}{n+3}}{b}$$

input
```
Int[Cosh[a + b*x]^3*Sinh[a + b*x]^n,x]
```

output
```
(Sinh[a + b*x]^(1 + n)/(1 + n) + Sinh[a + b*x]^(3 + n)/(3 + n))/b
```

3.10.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.10.4 Maple [A] (verified)

Time = 197.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{\sinh(bx+a)^3 e^{n\ln(\sinh(bx+a))}}{b(3+n)}+\frac{\sinh(bx+a)e^{n\ln(\sinh(bx+a))}}{b(n+1)}$	54
default	$\frac{\sinh(bx+a)^3 e^{n\ln(\sinh(bx+a))}}{b(3+n)}+\frac{\sinh(bx+a)e^{n\ln(\sinh(bx+a))}}{b(n+1)}$	54

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^n,x,method=_RETURNVERBOSE)
```

output
```
1/b/(3+n)*sinh(b*x+a)^3*exp(n*ln(sinh(b*x+a)))+1/b/(n+1)*sinh(b*x+a)*exp(n
*ln(sinh(b*x+a)))
```

3.10.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.49

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx$$
$$=\frac{\left((n+1)\sinh\left(bx+a\right)^3+\left(3\left(n+1\right)\cosh\left(bx+a\right)^2+n+9\right)\sinh\left(bx+a\right)\right)\cosh\left(n\log\left(\sinh\left(bx+a\right)\right)\right)+(}{4\left(\left(bn^2+4\,bn+3\,b\right)\cosh\left(bx+a\right)^4-2\left(bn^2+4\,bn+3\,b\right)\cosh(}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^n,x, algorithm="fricas")
```

output
```
1/4*(((n + 1)*sinh(b*x + a)^3 + (3*(n + 1)*cosh(b*x + a)^2 + n + 9)*sinh(b
*x + a))*cosh(n*log(sinh(b*x + a))) + ((n + 1)*sinh(b*x + a)^3 + (3*(n + 1
)*cosh(b*x + a)^2 + n + 9)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a))))/((b*
n^2 + 4*b*n + 3*b)*cosh(b*x + a)^4 - 2*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a)
^2*sinh(b*x + a)^2 + (b*n^2 + 4*b*n + 3*b)*sinh(b*x + a)^4)
```

3.10.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(29) = 58$.

Time = 1.21 (sec) , antiderivative size = 638, normalized size of antiderivative = 16.36

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx$$

$$= \begin{cases} x\sinh^n(a)\cosh^3(a) \\ \frac{\log(\sinh(a+bx))}{b} - \frac{\cosh^2(a+bx)}{2b\sinh^2(a+bx)} \\ \frac{bx\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} - \frac{2bx\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} + \frac{bx}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} - \frac{2\log\left(\tan\cdots\right.}{b\tanh^4\left(\cdots\right.} \\ \frac{n\sinh(a+bx)\sinh^n(a+bx)\cosh^2(a+bx)}{bn^2+4bn+3b} - \frac{2\sinh^3(a+bx)\sinh^n(a+bx)}{bn^2+4bn+3b} + \frac{3\sinh(a+bx)\sinh^n(a+bx)\cosh^2(a+bx)}{bn^2+4bn+3b} \end{cases}$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)**n,x)
```

output
```
Piecewise((x*sinh(a)**n*cosh(a)**3, Eq(b, 0)), (log(sinh(a + b*x))/b - cos
h(a + b*x)**2/(2*b*sinh(a + b*x)**2), Eq(n, -3)), (b*x*tanh(a/2 + b*x/2)**
4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*b*x*tanh(a/2
 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + b*x
/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2
+ b*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2
+ b*x/2)**2 + b) + 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*ta
nh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/
2) + 1)/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + log(tanh
(a/2 + b*x/2))*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2
 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**2/(b*tanh(
a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*x/2))/(
b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*tanh(a/2 + b*x/
2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b), Eq(n, -1)),
 (n*sinh(a + b*x)*sinh(a + b*x)**n*cosh(a + b*x)**2/(b*n**2 + 4*b*n + 3*b)
 - 2*sinh(a + b*x)**3*sinh(a + b*x)**n/(b*n**2 + 4*b*n + 3*b) + 3*sinh(a +
 b*x)*sinh(a + b*x)**n*cosh(a + b*x)**2/(b*n**2 + 4*b*n + 3*b), True))
```

3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(39) = 78.

Time = 0.31 (sec) , antiderivative size = 373, normalized size of antiderivative = 9.56

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx = \frac{ne^{((bx+a)n+3\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+3\,a)}}{8\,(2^n n^2+2^{n+2}n+3\cdot 2^n)b} + \frac{(n+9)e^{((bx+a)n+bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+a)}}{8\,(2^n n^2+2^{n+2}n+3\cdot 2^n)b} - \frac{(n+9)e^{((bx+a)n-bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-a)}}{8\,(2^n n^2+2^{n+2}n+3\cdot 2^n)b} - \frac{(n+1)e^{((bx+a)n-3\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-3\,a)}}{8\,(2^n n^2+2^{n+2}n+3\cdot 2^n)b} + \frac{e^{((bx+a)n+3\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+3\,a)}}{8\,(2^n n^2+2^{n+2}n+3\cdot 2^n)b}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^n,x, algorithm="maxima")
```

output
```
1/8*n*e^((b*x + a)*n + 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x -
a) + 1) + 3*a)/((2^n*n^2 + 2^(n + 2)*n + 3*2^n)*b) + 1/8*(n + 9)*e^((b*x +
 a)*n + b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) + a)/((2^
n*n^2 + 2^(n + 2)*n + 3*2^n)*b) - 1/8*(n + 9)*e^((b*x + a)*n - b*x + n*log
(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) - a)/((2^n*n^2 + 2^(n + 2)*n
 + 3*2^n)*b) - 1/8*(n + 1)*e^((b*x + a)*n - 3*b*x + n*log(e^(-b*x - a) + 1
) + n*log(-e^(-b*x - a) + 1) - 3*a)/((2^n*n^2 + 2^(n + 2)*n + 3*2^n)*b) +
1/8*e^((b*x + a)*n + 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a)
 + 1) + 3*a)/((2^n*n^2 + 2^(n + 2)*n + 3*2^n)*b)
```

3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(39) = 78.

Time = 0.33 (sec) , antiderivative size = 327, normalized size of antiderivative = 8.38

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx = \frac{ne^{\left(7\,bx+n\log\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)+7\,a\right)} + ne^{\left(5\,bx+n\log\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)+5\,a\right)} - ne^{\left(3\,bx+n\log\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}-1\right)e^{(-\right.}\right.}}{}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^n,x, algorithm="giac")
```

output
```
1/8*(n*e^(7*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 7*a) + n
*e^(5*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 5*a) - n*e^(3*
b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 3*a) - n*e^(b*x + n*
log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + a) + e^(7*b*x + n*log(1/2*(e
^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 7*a) + 9*e^(5*b*x + n*log(1/2*(e^(2*b*
x + 2*a) - 1)*e^(-b*x - a)) + 5*a) - 9*e^(3*b*x + n*log(1/2*(e^(2*b*x + 2*
a) - 1)*e^(-b*x - a)) + 3*a) - e^(b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^
(-b*x - a)) + a))/(b*n^2*e^(4*b*x + 4*a) + 4*b*n*e^(4*b*x + 4*a) + 3*b*e^(
4*b*x + 4*a))
```

3.10.9 Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.46

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx = -\left(\frac{1}{2}\right)^n e^{-3a-3bx}\left(e^{a+bx}-e^{-a-bx}\right)^n\left(\frac{\frac{n}{8}+\frac{1}{8}}{b\,(n^2+4\,n+3)} + \frac{e^{2a+2bx}\,(n+9)}{8\,b\,(n^2+4\,n+3)} - \frac{e^{6a+6bx}\,(n+1)}{8\,b\,(n^2+4\,n+3)} - \frac{e^{4a+4bx}\,(n+9)}{8\,b\,(n^2+4\,n+3)}\right)$$

input
```
int(cosh(a + b*x)^3*sinh(a + b*x)^n,x)
```

output
```
-(1/2)^n*exp(- 3*a - 3*b*x)*(exp(a + b*x) - exp(- a - b*x))^n*((n/8 + 1/8)
/(b*(4*n + n^2 + 3)) + (exp(2*a + 2*b*x)*(n + 9))/(8*b*(4*n + n^2 + 3)) -
(exp(6*a + 6*b*x)*(n + 1))/(8*b*(4*n + n^2 + 3)) - (exp(4*a + 4*b*x)*(n +
9))/(8*b*(4*n + n^2 + 3)))
```

3.11 $\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx$

3.11.1 Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx=\frac{\sinh^{1+n}(a+bx)}{b(1+n)}+\frac{2\sinh^{3+n}(a+bx)}{b(3+n)}+\frac{\sinh^{5+n}(a+bx)}{b(5+n)}$$

output
```
sinh(b*x+a)^(1+n)/b/(1+n)+2*sinh(b*x+a)^(3+n)/b/(3+n)+sinh(b*x+a)^(5+n)/b/
(5+n)
```

3.11.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx=\frac{\sinh^{1+n}(a+bx)\left(\frac{1}{1+n}+\frac{2\sinh^2(a+bx)}{3+n}+\frac{\sinh^4(a+bx)}{5+n}\right)}{b}$$

input
```
Integrate[Cosh[a + b*x]^5*Sinh[a + b*x]^n,x]
```

output
```
(Sinh[a + b*x]^(1 + n)*((1 + n)^(-1) + (2*Sinh[a + b*x]^2)/(3 + n) + Sinh[
a + b*x]^4/(5 + n)))/b
```

3.11.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ia+ibx)^5(-i\sin(ia+ibx))^n dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sinh^n(a+bx)\left(\sinh^2(a+bx)+1\right)^2 d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\sinh^n(a+bx)+2\sinh^{n+2}(a+bx)+\sinh^{n+4}(a+bx)\right) d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sinh^{n+1}(a+bx)}{n+1}+\frac{2\sinh^{n+3}(a+bx)}{n+3}+\frac{\sinh^{n+5}(a+bx)}{n+5}}{b}$$

input
```
Int[Cosh[a + b*x]^5*Sinh[a + b*x]^n,x]
```

output
```
(Sinh[a + b*x]^(1 + n)/(1 + n) + (2*Sinh[a + b*x]^(3 + n))/(3 + n) + Sinh[
a + b*x]^(5 + n)/(5 + n))/b
```

3.11.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.11.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\frac{\sinh\left(bx+a\right)^{5} \mathrm{e}^{n\ln(\sinh(bx+a))}}{b\left(5+n\right)}+\frac{\sinh\left(bx+a\right) \mathrm{e}^{n\ln(\sinh(bx+a))}}{b\left(n+1\right)}+\frac{2\sinh\left(bx+a\right)^{3} \mathrm{e}^{n\ln(\sinh(bx+a))}}{b\left(3+n\right)}$$

input
```
int(cosh(b*x+a)^5*sinh(b*x+a)^n,x)
```

output
```
1/b/(5+n)*sinh(b*x+a)^5*exp(n*ln(sinh(b*x+a)))+1/b/(n+1)*sinh(b*x+a)*exp(n
*ln(sinh(b*x+a)))+2/b/(3+n)*sinh(b*x+a)^3*exp(n*ln(sinh(b*x+a)))
```

3.11.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(59) = 118.

Time = 0.27 (sec) , antiderivative size = 379, normalized size of antiderivative = 6.42

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx$$
$$=\frac{\left(\left(n^2+4n+3\right)\sinh\left(bx+a\right)^5+\left(10\left(n^2+4n+3\right)\cosh\left(bx+a\right)^2+3n^2+28n+25\right)\sinh\left(bx+a\right)^3+\left(5\right.}{}$$

input
```
integrate(cosh(b*x+a)^5*sinh(b*x+a)^n,x, algorithm="fricas")
```

output
```
1/16*(((n^2 + 4*n + 3)*sinh(b*x + a)^5 + (10*(n^2 + 4*n + 3)*cosh(b*x + a)
^2 + 3*n^2 + 28*n + 25)*sinh(b*x + a)^3 + (5*(n^2 + 4*n + 3)*cosh(b*x + a)
^4 + 3*(3*n^2 + 28*n + 25)*cosh(b*x + a)^2 + 2*n^2 + 24*n + 150)*sinh(b*x
+ a))*cosh(n*log(sinh(b*x + a))) + ((n^2 + 4*n + 3)*sinh(b*x + a)^5 + (10*
(n^2 + 4*n + 3)*cosh(b*x + a)^2 + 3*n^2 + 28*n + 25)*sinh(b*x + a)^3 + (5*
(n^2 + 4*n + 3)*cosh(b*x + a)^4 + 3*(3*n^2 + 28*n + 25)*cosh(b*x + a)^2 +
2*n^2 + 24*n + 150)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a))))/((b*n^3 + 9
*b*n^2 + 23*b*n + 15*b)*cosh(b*x + a)^6 - 3*(b*n^3 + 9*b*n^2 + 23*b*n + 15
*b)*cosh(b*x + a)^4*sinh(b*x + a)^2 + 3*(b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*
cosh(b*x + a)^2*sinh(b*x + a)^4 - (b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*sinh(b
*x + a)^6)
```

3.11.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2574 vs. $2(46) = 92$.

Time = 4.32 (sec) , antiderivative size = 2574, normalized size of antiderivative = 43.63

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)**5*sinh(b*x+a)**n,x)
```

output
```
Piecewise((x*sinh(a)**n*cosh(a)**5, Eq(b, 0)), (log(sinh(a + b*x))/b - cos
h(a + b*x)**2/(2*b*sinh(a + b*x)**2) - cosh(a + b*x)**4/(4*b*sinh(a + b*x)
**4), Eq(n, -5)), (16*b*x*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/2)**6 -
 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*b*x*tanh(a/2 +
 b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tan
h(a/2 + b*x/2)**2) + 16*b*x*tanh(a/2 + b*x/2)**2/(8*b*tanh(a/2 + b*x/2)**6
 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*log(tanh(a/2
 + b*x/2) + 1)*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(
a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 64*log(tanh(a/2 + b*x/2) + 1
)*tanh(a/2 + b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)*
*4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 +
b*x/2)**2/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh
(a/2 + b*x/2)**2) + 16*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**6/(8*b*ta
nh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2)
 - 32*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**
6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 16*log(tanh(a/
2 + b*x/2))*tanh(a/2 + b*x/2)**2/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2
 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - tanh(a/2 + b*x/2)**8/(8*b*tanh(
a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) +
18*tanh(a/2 + b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x...
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(59) = 118$.

Time = 0.30 (sec) , antiderivative size = 686, normalized size of antiderivative = 11.63

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx = \frac{n^2 e^{\left((bx+a)n+5\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+5\,a\right)}}{32\,(2^n n^3+9\cdot 2^n n^2+23\cdot 2^n n+15\cdot 2^n)b} + \frac{n e^{\left((bx+a)n+5\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+5\,a\right)}}{8\,(2^n n^3+9\cdot 2^n n^2+23\cdot 2^n n+15\cdot 2^n)b} + \frac{(3\,n^2+28\,n+25)e^{\left((bx+a)n+3\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+3\,a\right)}}{32\,(2^n n^3+9\cdot 2^n n^2+23\cdot 2^n n+15\cdot 2^n)b} + \frac{(n^2+12\,n+75)e^{\left((bx+a)n+bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+a\right)}}{16\,(2^n n^3+9\cdot 2^n n^2+23\cdot 2^n n+15\cdot 2^n)b} - \frac{(n^2+12\,n+75)e^{\left((bx+a)n-bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-a\right)}}{16\,(2^n n^3+9\cdot 2^n n^2+23\cdot 2^n n+15\cdot 2^n)b} - \frac{(3\,n^2+28\,n+25)e^{\left((bx+a)n-3\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-3\,a\right)}}{32\,(2^n n^3+9\cdot 2^n n^2+23\cdot 2^n n+15\cdot 2^n)b} - \frac{(n^2+4\,n+3)e^{\left((bx+a)n-5\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-5\,a\right)}}{32\,(2^n n^3+9\cdot 2^n n^2+23\cdot 2^n n+15\cdot 2^n)b} + \frac{3\,e^{\left((bx+a)n+5\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+5\,a\right)}}{32\,(2^n n^3+9\cdot 2^n n^2+23\cdot 2^n n+15\cdot 2^n)b}$$

input
```
integrate(cosh(b*x+a)^5*sinh(b*x+a)^n,x, algorithm="maxima")
```

output
```
1/32*n^2*e^((b*x + a)*n + 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x
 - a) + 1) + 5*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 1/8*n*e^
((b*x + a)*n + 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1)
+ 5*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 1/32*(3*n^2 + 28*n
+ 25)*e^((b*x + a)*n + 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x -
a) + 1) + 3*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 1/16*(n^2 +
 12*n + 75)*e^((b*x + a)*n + b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*
x - a) + 1) + a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) - 1/16*(n^2
 + 12*n + 75)*e^((b*x + a)*n - b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-
b*x - a) + 1) - a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) - 1/32*(3
*n^2 + 28*n + 25)*e^((b*x + a)*n - 3*b*x + n*log(e^(-b*x - a) + 1) + n*log
(-e^(-b*x - a) + 1) - 3*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) -
 1/32*(n^2 + 4*n + 3)*e^((b*x + a)*n - 5*b*x + n*log(e^(-b*x - a) + 1) + n
*log(-e^(-b*x - a) + 1) - 5*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*
b) + 3/32*e^((b*x + a)*n + 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*
x - a) + 1) + 5*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b)
```

3.11. $\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx$

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(59) = 118$.

Time = 0.38 (sec) , antiderivative size = 722, normalized size of antiderivative = 12.24

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^5*sinh(b*x+a)^n,x, algorithm="giac")
```

output
```
1/32*(n^2*e^(11*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 11*a
) + 3*n^2*e^(9*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 9*a)
+ 2*n^2*e^(7*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 7*a) -
2*n^2*e^(5*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 5*a) - 3*
n^2*e^(3*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 3*a) - n^2*
e^(b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + a) + 4*n*e^(11*b*
x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 11*a) + 28*n*e^(9*b*x
+ n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 9*a) + 24*n*e^(7*b*x + n
*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 7*a) - 24*n*e^(5*b*x + n*lo
g(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 5*a) - 28*n*e^(3*b*x + n*log(1
/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 3*a) - 4*n*e^(b*x + n*log(1/2*(e^
(2*b*x + 2*a) - 1)*e^(-b*x - a)) + a) + 3*e^(11*b*x + n*log(1/2*(e^(2*b*x
+ 2*a) - 1)*e^(-b*x - a)) + 11*a) + 25*e^(9*b*x + n*log(1/2*(e^(2*b*x + 2*
a) - 1)*e^(-b*x - a)) + 9*a) + 150*e^(7*b*x + n*log(1/2*(e^(2*b*x + 2*a) -
 1)*e^(-b*x - a)) + 7*a) - 150*e^(5*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*
e^(-b*x - a)) + 5*a) - 25*e^(3*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b
*x - a)) + 3*a) - 3*e^(b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a))
 + a))/(b*n^3*e^(6*b*x + 6*a) + 9*b*n^2*e^(6*b*x + 6*a) + 23*b*n*e^(6*b*x
+ 6*a) + 15*b*e^(6*b*x + 6*a))
```

3.11.9 Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 255, normalized size of antiderivative = 4.32

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx$$
$$= -e^{-5a-5bx}\left(\frac{e^{a+bx}}{2}-\frac{e^{-a-bx}}{2}\right)^n\left(\frac{n^2+4n+3}{32b\,(n^3+9n^2+23n+15)} - \frac{e^{10a+10bx}\,(n^2+4n+3)}{32b\,(n^3+9n^2+23n+15)} + \frac{e^{2a+2bx}\,(3n^2+28n+25)}{32b\,(n^3+9n^2+23n+15)} - \frac{e^{8a+8bx}\,(3n^2+28n+25)}{32b\,(n^3+9n^2+23n+15)} + \frac{e^{4a+4bx}\,(2n^2+24n+150)}{32b\,(n^3+9n^2+23n+15)} - \frac{e^{6a+6bx}\,(2n^2+24n+150)}{32b\,(n^3+9n^2+23n+15)}\right)$$

input
```
int(cosh(a + b*x)^5*sinh(a + b*x)^n,x)
```

output
```
-exp(- 5*a - 5*b*x)*(exp(a + b*x)/2 - exp(- a - b*x)/2)^n*((4*n + n^2 + 3)
/(32*b*(23*n + 9*n^2 + n^3 + 15)) - (exp(10*a + 10*b*x)*(4*n + n^2 + 3))/(
32*b*(23*n + 9*n^2 + n^3 + 15)) + (exp(2*a + 2*b*x)*(28*n + 3*n^2 + 25))/(
32*b*(23*n + 9*n^2 + n^3 + 15)) - (exp(8*a + 8*b*x)*(28*n + 3*n^2 + 25))/(
32*b*(23*n + 9*n^2 + n^3 + 15)) + (exp(4*a + 4*b*x)*(24*n + 2*n^2 + 150))/
(32*b*(23*n + 9*n^2 + n^3 + 15)) - (exp(6*a + 6*b*x)*(24*n + 2*n^2 + 150))
/(32*b*(23*n + 9*n^2 + n^3 + 15)))
```

3.12 $\int \cosh^m(a+bx)\sinh(a+bx)\,dx$

3.12.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx=\frac{\cosh^{1+m}(a+bx)}{b(1+m)}$$

output
```
cosh(b*x+a)^(1+m)/b/(1+m)
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx=\frac{\cosh^{1+m}(a+bx)}{b(1+m)}$$

input
```
Integrate[Cosh[a + b*x]^m*Sinh[a + b*x],x]
```

output
```
Cosh[a + b*x]^(1 + m)/(b*(1 + m))
```

3.12.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 26, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\cosh^m(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)\cos(ia+ibx)^m dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \cos(ia+ibx)^m\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3045}$$

$$\frac{\int \cosh^m(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

input
```
Int[Cosh[a + b*x]^m*Sinh[a + b*x],x]
```

output
```
Cosh[a + b*x]^(1 + m)/(b*(1 + m))
```

3.12.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.12.4 Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\cosh(bx+a)^{1+m}}{b(1+m)}$
default	$\frac{\cosh(bx+a)^{1+m}}{b(1+m)}$
risch	$\frac{\left(e^{bx+a}\right)^{-m}\left(1+e^{2bx+2a}\right)^{m}\left(\frac{1}{2}\right)^{m}\left(e^{2bx+2a}e^{-\frac{i\operatorname{csgn}\left(i\left(1+e^{2bx+2a}\right)e^{-bx-a}\right)^{3}\pi m}{2}}e^{\frac{i\operatorname{csgn}\left(i\left(1+e^{2bx+2a}\right)e^{-bx-a}\right)^{2}\operatorname{csgn}\left(ie^{-bx}\right.}{2}}\right.}{}$

input
```
int(cosh(b*x+a)^m*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
cosh(b*x+a)^(1+m)/b/(1+m)
```

3.12.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(19) = 38.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.58

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)\cosh(m\log(\cosh(bx+a)))+\cosh(bx+a)\sinh(m\log(\cosh(bx+a)))}{(bm+b)\cosh(bx+a)^2-(bm+b)\sinh(bx+a)^2}$$

input
```
integrate(cosh(b*x+a)^m*sinh(b*x+a),x, algorithm="fricas")
```

output
```
(cosh(b*x + a)*cosh(m*log(cosh(b*x + a))) + cosh(b*x + a)*sinh(m*log(cosh(
b*x + a))))/((b*m + b)*cosh(b*x + a)^2 - (b*m + b)*sinh(b*x + a)^2)
```

3.12.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{x\sinh(a)}{\cosh(a)} & \text{for } b = 0 \wedge m = -1 \\ x\sinh(a)\cosh^m(a) & \text{for } b = 0 \\ \frac{\log(\cosh(a+bx))}{b} & \text{for } m = -1 \\ \frac{\cosh(a+bx)\cosh^m(a+bx)}{bm+b} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**m*sinh(b*x+a),x)
```

output
```
Piecewise((x*sinh(a)/cosh(a), Eq(b, 0) & Eq(m, -1)), (x*sinh(a)*cosh(a)**m
, Eq(b, 0)), (log(cosh(a + b*x))/b, Eq(m, -1)), (cosh(a + b*x)*cosh(a + b*
x)**m/(b*m + b), True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^{m+1}}{b(m+1)}$$

input
```
integrate(cosh(b*x+a)^m*sinh(b*x+a),x, algorithm="maxima")
```

output
```
cosh(b*x + a)^(m + 1)/(b*(m + 1))
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx = \frac{\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}\right)^{m+1}}{b(m+1)}$$

input
```
integrate(cosh(b*x+a)^m*sinh(b*x+a),x, algorithm="giac")
```

output
```
(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a))^(m + 1)/(b*(m + 1))
```

3.12.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)^{m+1}}{b\,(m+1)}$$

input
```
int(cosh(a + b*x)^m*sinh(a + b*x),x)
```

output
```
cosh(a + b*x)^(m + 1)/(b*(m + 1))
```

3.13 $\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx$

3.13.1 Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx=-\frac{\cosh^{1+m}(a+bx)}{b(1+m)}+\frac{\cosh^{3+m}(a+bx)}{b(3+m)}$$

output
```
-cosh(b*x+a)^(1+m)/b/(1+m)+cosh(b*x+a)^(3+m)/b/(3+m)
```

3.13.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx=\frac{\cosh^{1+m}(a+bx)(-5-m+(1+m)\cosh(2(a+bx)))}{2b(1+m)(3+m)}$$

input
```
Integrate[Cosh[a + b*x]^m*Sinh[a + b*x]^3,x]
```

output
```
(Cosh[a + b*x]^(1 + m)*(-5 - m + (1 + m)*Cosh[2*(a + b*x)]))/(2*b*(1 + m)*
(3 + m))
```

3.13.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 26, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\cosh^m(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ia+ibx)^3\cos(ia+ibx)^m dx$$

$$\Big\downarrow \text{26}$$

$$i\int \cos(ia+ibx)^m\sin(ia+ibx)^3 dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \cosh^m(a+bx)\left(1-\cosh^2(a+bx)\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\cosh^m(a+bx)-\cosh^{m+2}(a+bx)\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\cosh^{m+1}(a+bx)}{m+1}-\frac{\cosh^{m+3}(a+bx)}{m+3}}{b}$$

input
```
Int[Cosh[a + b*x]^m*Sinh[a + b*x]^3,x]
```

output
```
-((Cosh[a + b*x]^(1 + m)/(1 + m) - Cosh[a + b*x]^(3 + m)/(3 + m))/b)
```

3.13.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.13.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\frac{\cosh\left(bx+a\right)^{3} \mathrm{e}^{m\ln(\cosh(bx+a))}}{b\left(3+m\right)}-\frac{\cosh\left(bx+a\right) \mathrm{e}^{m\ln(\cosh(bx+a))}}{b\left(1+m\right)}$$

input
```
int(cosh(b*x+a)^m*sinh(b*x+a)^3,x)
```

output
```
1/b/(3+m)*cosh(b*x+a)^3*exp(m*ln(cosh(b*x+a)))-1/b/(1+m)*cosh(b*x+a)*exp(m
*ln(cosh(b*x+a)))
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.72

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{\left((m+1)\cosh(bx+a)^3 + 3(m+1)\cosh(bx+a)\sinh(bx+a)^2 - (m+9)\cosh(bx+a)\right)\cosh(m\log(\mathrm{co}}{4\left((bm^2+4bm+3b)\cosh(bx+a)^4 - 2(bm^2+\right.}$$

input
```
integrate(cosh(b*x+a)^m*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(((m + 1)*cosh(b*x + a)^3 + 3*(m + 1)*cosh(b*x + a)*sinh(b*x + a)^2 -
(m + 9)*cosh(b*x + a))*cosh(m*log(cosh(b*x + a))) + ((m + 1)*cosh(b*x + a)
^3 + 3*(m + 1)*cosh(b*x + a)*sinh(b*x + a)^2 - (m + 9)*cosh(b*x + a))*sinh
(m*log(cosh(b*x + a))))/((b*m^2 + 4*b*m + 3*b)*cosh(b*x + a)^4 - 2*(b*m^2
+ 4*b*m + 3*b)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (b*m^2 + 4*b*m + 3*b)*sin
h(b*x + a)^4)
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(29) = 58$.

Time = 1.22 (sec) , antiderivative size = 648, normalized size of antiderivative = 16.20

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} x\sinh^3(a)\cosh^m(a) \\ \frac{\log(\cosh(a+bx))}{b} - \frac{\sinh^2(a+bx)}{2b\cosh^2(a+bx)} \\ -\frac{bx\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} + \frac{2bx\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} - \frac{bx}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} + \frac{2\log(\mathrm{t}}{b\tanh} \\ \frac{m\sinh^2(a+bx)\cosh(a+bx)\cosh^m(a+bx)}{bm^2+4bm+3b} + \frac{3\sinh^2(a+bx)\cosh(a+bx)\cosh^m(a+bx)}{bm^2+4bm+3b} - \frac{2\cosh^3(a+bx)\cosh^m(a+bx)}{bm^2+4bm+3b} \end{cases}$$

input
```
integrate(cosh(b*x+a)**m*sinh(b*x+a)**3,x)
```

output
```
Piecewise((x*sinh(a)**3*cosh(a)**m, Eq(b, 0)), (log(cosh(a + b*x))/b - sin
h(a + b*x)**2/(2*b*cosh(a + b*x)**2), Eq(m, -3)), (-b*x*tanh(a/2 + b*x/2)*
*4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*b*x*tanh(a/
2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - b*
x/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*log(tanh(a/2
 + b*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2
 + b*x/2)**2 + b) - 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*t
anh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*log(tanh(a/2 + b*x
/2) + 1)/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - log(tan
h(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*
tanh(a/2 + b*x/2)**2 + b) + 2*log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x
/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - log(tanh(
a/2 + b*x/2)**2 + 1)/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 +
b) + 2*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2
)**2 + b), Eq(m, -1)), (m*sinh(a + b*x)**2*cosh(a + b*x)*cosh(a + b*x)**m/
(b*m**2 + 4*b*m + 3*b) + 3*sinh(a + b*x)**2*cosh(a + b*x)*cosh(a + b*x)**m
/(b*m**2 + 4*b*m + 3*b) - 2*cosh(a + b*x)**3*cosh(a + b*x)**m/(b*m**2 + 4*
b*m + 3*b), True))
```

3.13.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(40) = 80$.

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 7.32

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx = \frac{me^{\left((bx+a)m+3\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+3\,a\right)}}{8\,(2^m m^2+2^{m+2}m+3\cdot 2^m)b} - \frac{(m+9)e^{\left((bx+a)m+bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+a\right)}}{8\,(2^m m^2+2^{m+2}m+3\cdot 2^m)b} - \frac{(m+9)e^{\left((bx+a)m-bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)-a\right)}}{8\,(2^m m^2+2^{m+2}m+3\cdot 2^m)b} + \frac{(m+1)e^{\left((bx+a)m-3\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)-3\,a\right)}}{8\,(2^m m^2+2^{m+2}m+3\cdot 2^m)b} + \frac{e^{\left((bx+a)m+3\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+3\,a\right)}}{8\,(2^m m^2+2^{m+2}m+3\cdot 2^m)b}$$

input
```
integrate(cosh(b*x+a)^m*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*m*e^((b*x + a)*m + 3*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 3*a)/((2^m*m^
2 + 2^(m + 2)*m + 3*2^m)*b) - 1/8*(m + 9)*e^((b*x + a)*m + b*x + m*log(e^(
-2*b*x - 2*a) + 1) + a)/((2^m*m^2 + 2^(m + 2)*m + 3*2^m)*b) - 1/8*(m + 9)*
e^((b*x + a)*m - b*x + m*log(e^(-2*b*x - 2*a) + 1) - a)/((2^m*m^2 + 2^(m +
 2)*m + 3*2^m)*b) + 1/8*(m + 1)*e^((b*x + a)*m - 3*b*x + m*log(e^(-2*b*x -
 2*a) + 1) - 3*a)/((2^m*m^2 + 2^(m + 2)*m + 3*2^m)*b) + 1/8*e^((b*x + a)*m
 + 3*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 3*a)/((2^m*m^2 + 2^(m + 2)*m + 3*
2^m)*b)
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(40) = 80$.

Time = 0.32 (sec) , antiderivative size = 325, normalized size of antiderivative = 8.12

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{me^{\left(7\,bx+m\log\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}\right)+7\,a\right)} - me^{\left(5\,bx+m\log\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}\right)+5\,a\right)} - me^{\left(3\,bx+m\log\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}+1\right.\right.\right.}}{}$$

input
```
integrate(cosh(b*x+a)^m*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*(m*e^(7*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 7*a) - m
*e^(5*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 5*a) - m*e^(3*
b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 3*a) + m*e^(b*x + m*
log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + a) + e^(7*b*x + m*log(1/2*(e
^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 7*a) - 9*e^(5*b*x + m*log(1/2*(e^(2*b*
x + 2*a) + 1)*e^(-b*x - a)) + 5*a) - 9*e^(3*b*x + m*log(1/2*(e^(2*b*x + 2*
a) + 1)*e^(-b*x - a)) + 3*a) + e^(b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^
(-b*x - a)) + a))/(b*m^2*e^(4*b*x + 4*a) + 4*b*m*e^(4*b*x + 4*a) + 3*b*e^(
4*b*x + 4*a))
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.30

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx = \left(\frac{1}{2}\right)^m e^{-3a-3bx}\left(e^{a+bx}+e^{-a-bx}\right)^m\left(\frac{\frac{m}{8}+\frac{1}{8}}{b\,(m^2+4m+3)} - \frac{e^{2a+2bx}\,(m+9)}{8b\,(m^2+4m+3)} + \frac{e^{6a+6bx}\,(m+1)}{8b\,(m^2+4m+3)} - \frac{e^{4a+4bx}\,(m+9)}{8b\,(m^2+4m+3)}\right)$$

input
```
int(cosh(a + b*x)^m*sinh(a + b*x)^3,x)
```

output
```
(1/2)^m*exp(- 3*a - 3*b*x)*(exp(a + b*x) + exp(- a - b*x))^m*((m/8 + 1/8)/
(b*(4*m + m^2 + 3)) - (exp(2*a + 2*b*x)*(m + 9))/(8*b*(4*m + m^2 + 3)) + (
exp(6*a + 6*b*x)*(m + 1))/(8*b*(4*m + m^2 + 3)) - (exp(4*a + 4*b*x)*(m + 9
))/(8*b*(4*m + m^2 + 3)))
```

3.14 $\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx$

3.14.1 Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx=\frac{\cosh^{1+m}(a+bx)}{b(1+m)}-\frac{2\cosh^{3+m}(a+bx)}{b(3+m)}+\frac{\cosh^{5+m}(a+bx)}{b(5+m)}$$

output
```
cosh(b*x+a)^(1+m)/b/(1+m)-2*cosh(b*x+a)^(3+m)/b/(3+m)+cosh(b*x+a)^(5+m)/b/
(5+m)
```

3.14.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx$$
$$=\frac{\cosh^{1+m}(a+bx)\left(89+28m+3m^2-4(7+8m+m^2)\cosh(2(a+bx))+(3+4m+m^2)\cosh(4(a+bx))\right)}{8b(1+m)(3+m)(5+m)}$$

input
```
Integrate[Cosh[a + b*x]^m*Sinh[a + b*x]^5,x]
```

output
```
(Cosh[a + b*x]^(1 + m)*(89 + 28*m + 3*m^2 - 4*(7 + 8*m + m^2)*Cosh[2*(a +
b*x)] + (3 + 4*m + m^2)*Cosh[4*(a + b*x)]))/(8*b*(1 + m)*(3 + m)*(5 + m))
```

3.14.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 26, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^5(a+bx)\cosh^m(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -i\sin(ia+ibx)^5\cos(ia+ibx)^m dx$$

$$\Big\downarrow \ 26$$

$$-i\int \cos(ia+ibx)^m\sin(ia+ibx)^5 dx$$

$$\Big\downarrow \ 3045$$

$$\frac{\int \cosh^m(a+bx)\left(1-\cosh^2(a+bx)\right)^2 d\cosh(a+bx)}{b}$$

$$\Big\downarrow \ 244$$

$$\frac{\int \left(\cosh^m(a+bx)-2\cosh^{m+2}(a+bx)+\cosh^{m+4}(a+bx)\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\cosh^{m+1}(a+bx)}{m+1}-\frac{2\cosh^{m+3}(a+bx)}{m+3}+\frac{\cosh^{m+5}(a+bx)}{m+5}}{b}$$

input
```
Int[Cosh[a + b*x]^m*Sinh[a + b*x]^5,x]
```

output
```
(Cosh[a + b*x]^(1 + m)/(1 + m) - (2*Cosh[a + b*x]^(3 + m))/(3 + m) + Cosh[
a + b*x]^(5 + m)/(5 + m))/b
```

3.14.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.14.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\frac{\cosh(bx+a)\,e^{m\ln(\cosh(bx+a))}}{b(1+m)}+\frac{\cosh(bx+a)^5\,e^{m\ln(\cosh(bx+a))}}{b(5+m)}-\frac{2\cosh(bx+a)^3\,e^{m\ln(\cosh(bx+a))}}{b(3+m)}$$

input
```
int(cosh(b*x+a)^m*sinh(b*x+a)^5,x)
```

output
```
1/b/(1+m)*cosh(b*x+a)*exp(m*ln(cosh(b*x+a)))+1/b/(5+m)*cosh(b*x+a)^5*exp(m
*ln(cosh(b*x+a)))-2/b/(3+m)*cosh(b*x+a)^3*exp(m*ln(cosh(b*x+a)))
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(59) = 118.

Time = 0.26 (sec) , antiderivative size = 407, normalized size of antiderivative = 6.90

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx$$
$$= \frac{\left((m^2+4\,m+3)\cosh\left(bx+a\right)^5+5\,(m^2+4\,m+3)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-(3\,m^2+28\,m+25)\cos\right.}{}$$

input
```
integrate(cosh(b*x+a)^m*sinh(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/16*(((m^2 + 4*m + 3)*cosh(b*x + a)^5 + 5*(m^2 + 4*m + 3)*cosh(b*x + a)*s
inh(b*x + a)^4 - (3*m^2 + 28*m + 25)*cosh(b*x + a)^3 + (10*(m^2 + 4*m + 3)
*cosh(b*x + a)^3 - 3*(3*m^2 + 28*m + 25)*cosh(b*x + a))*sinh(b*x + a)^2 +
2*(m^2 + 12*m + 75)*cosh(b*x + a))*cosh(m*log(cosh(b*x + a))) + ((m^2 + 4*
m + 3)*cosh(b*x + a)^5 + 5*(m^2 + 4*m + 3)*cosh(b*x + a)*sinh(b*x + a)^4 -
 (3*m^2 + 28*m + 25)*cosh(b*x + a)^3 + (10*(m^2 + 4*m + 3)*cosh(b*x + a)^3
 - 3*(3*m^2 + 28*m + 25)*cosh(b*x + a))*sinh(b*x + a)^2 + 2*(m^2 + 12*m +
75)*cosh(b*x + a))*sinh(m*log(cosh(b*x + a))))/((b*m^3 + 9*b*m^2 + 23*b*m
+ 15*b)*cosh(b*x + a)^6 - 3*(b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*cosh(b*x + a
)^4*sinh(b*x + a)^2 + 3*(b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*cosh(b*x + a)^2*
sinh(b*x + a)^4 - (b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*sinh(b*x + a)^6)
```

3.14.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2351 vs. 2(46) = 92.

Time = 4.54 (sec) , antiderivative size = 2351, normalized size of antiderivative = 39.85

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)**m*sinh(b*x+a)**5,x)
```

output
```
Piecewise((x*sinh(a)**5*cosh(a)**m, Eq(b, 0)), (log(cosh(a + b*x))/b - sin
h(a + b*x)**4/(4*b*cosh(a + b*x)**4) - sinh(a + b*x)**2/(2*b*cosh(a + b*x)
**2), Eq(m, -5)), (-2*b*x*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 2
*b*tanh(a/2 + b*x/2)**4 + b) + 4*b*x*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*
x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 2*b*x/(b*tanh(a/2 + b*x/2)**8 -
2*b*tanh(a/2 + b*x/2)**4 + b) + 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*
x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 8*log(ta
nh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 2*b*ta
nh(a/2 + b*x/2)**4 + b) + 4*log(tanh(a/2 + b*x/2) + 1)/(b*tanh(a/2 + b*x/2
)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 2*log(tanh(a/2 + b*x/2)**2 + 1)*tan
h(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b)
+ 4*log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2
)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 2*log(tanh(a/2 + b*x/2)**2 + 1)/(b*
tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*tanh(a/2 + b*x/2)
**6/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*tanh(a/2 +
 b*x/2)**2/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b), Eq(m,
-3)), (b*x*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b
*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*b*
x*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6
+ 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 6*b*x*tanh...
```

3.14.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(59) = 118$.

Time = 0.31 (sec) , antiderivative size = 558, normalized size of antiderivative = 9.46

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx$$
$$= \frac{m^2 e^{\left((bx+a)m+5\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{32\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} + \frac{m e^{\left((bx+a)m+5\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{8\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} - \frac{(3\,m^2+28\,m+25)e^{\left((bx+a)m+3\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+3\,a\right)}}{32\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} + \frac{(m^2+12\,m+75)e^{\left((bx+a)m+bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+a\right)}}{16\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} + \frac{(m^2+12\,m+75)e^{\left((bx+a)m-bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)-a\right)}}{16\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} - \frac{(3\,m^2+28\,m+25)e^{\left((bx+a)m-3\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)-3\,a\right)}}{32\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} + \frac{(m^2+4\,m+3)e^{\left((bx+a)m-5\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)-5\,a\right)}}{32\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} + \frac{3\,e^{\left((bx+a)m+5\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{32\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b}$$

input
```
integrate(cosh(b*x+a)^m*sinh(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/32*m^2*e^((b*x + a)*m + 5*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^m
*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 1/8*m*e^((b*x + a)*m + 5*b*x +
m*log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2
^m)*b) - 1/32*(3*m^2 + 28*m + 25)*e^((b*x + a)*m + 3*b*x + m*log(e^(-2*b*x
 - 2*a) + 1) + 3*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 1/16*(
m^2 + 12*m + 75)*e^((b*x + a)*m + b*x + m*log(e^(-2*b*x - 2*a) + 1) + a)/(
(2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 1/16*(m^2 + 12*m + 75)*e^((
b*x + a)*m - b*x + m*log(e^(-2*b*x - 2*a) + 1) - a)/((2^m*m^3 + 9*2^m*m^2
+ 23*2^m*m + 15*2^m)*b) - 1/32*(3*m^2 + 28*m + 25)*e^((b*x + a)*m - 3*b*x
+ m*log(e^(-2*b*x - 2*a) + 1) - 3*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15
*2^m)*b) + 1/32*(m^2 + 4*m + 3)*e^((b*x + a)*m - 5*b*x + m*log(e^(-2*b*x -
 2*a) + 1) - 5*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 3/32*e^(
(b*x + a)*m + 5*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^m*m^3 + 9*2^m
*m^2 + 23*2^m*m + 15*2^m)*b)
```

3.14. $\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx$

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(59) = 118$.

Time = 0.36 (sec) , antiderivative size = 721, normalized size of antiderivative = 12.22

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^m*sinh(b*x+a)^5,x, algorithm="giac")
```

output
```
1/32*(m^2*e^(11*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 11*a
) - 3*m^2*e^(9*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 9*a)
+ 2*m^2*e^(7*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 7*a) +
2*m^2*e^(5*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 5*a) - 3*
m^2*e^(3*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 3*a) + m^2*
e^(b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + a) + 4*m*e^(11*b*
x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 11*a) - 28*m*e^(9*b*x
+ m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 9*a) + 24*m*e^(7*b*x + m
*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 7*a) + 24*m*e^(5*b*x + m*lo
g(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 5*a) - 28*m*e^(3*b*x + m*log(1
/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 3*a) + 4*m*e^(b*x + m*log(1/2*(e^
(2*b*x + 2*a) + 1)*e^(-b*x - a)) + a) + 3*e^(11*b*x + m*log(1/2*(e^(2*b*x
+ 2*a) + 1)*e^(-b*x - a)) + 11*a) - 25*e^(9*b*x + m*log(1/2*(e^(2*b*x + 2*
a) + 1)*e^(-b*x - a)) + 9*a) + 150*e^(7*b*x + m*log(1/2*(e^(2*b*x + 2*a) +
 1)*e^(-b*x - a)) + 7*a) + 150*e^(5*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*
e^(-b*x - a)) + 5*a) - 25*e^(3*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b
*x - a)) + 3*a) + 3*e^(b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a))
 + a))/(b*m^3*e^(6*b*x + 6*a) + 9*b*m^2*e^(6*b*x + 6*a) + 23*b*m*e^(6*b*x
+ 6*a) + 15*b*e^(6*b*x + 6*a))
```

3.14.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 254, normalized size of antiderivative = 4.31

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx$$
$$= e^{-5a-5bx}\left(\frac{e^{a+bx}}{2}+\frac{e^{-a-bx}}{2}\right)^m\left(\frac{m^2+4m+3}{32b\,(m^3+9m^2+23m+15)}+\frac{e^{10a+10bx}\,(m^2+4m+3)}{32b\,(m^3+9m^2+23m+15)}-\frac{e^{2a+2bx}\,(3m^2+28m+25)}{32b\,(m^3+9m^2+23m+15)}-\frac{e^{8a+8bx}\,(3m^2+28m+25)}{32b\,(m^3+9m^2+23m+15)}+\frac{e^{4a+4bx}\,(2m^2+24m+150)}{32b\,(m^3+9m^2+23m+15)}+\frac{e^{6a+6bx}\,(2m^2+24m+150)}{32b\,(m^3+9m^2+23m+15)}\right)$$

input
```
int(cosh(a + b*x)^m*sinh(a + b*x)^5,x)
```

output
```
exp(- 5*a - 5*b*x)*(exp(a + b*x)/2 + exp(- a - b*x)/2)^m*((4*m + m^2 + 3)/
(32*b*(23*m + 9*m^2 + m^3 + 15)) + (exp(10*a + 10*b*x)*(4*m + m^2 + 3))/(3
2*b*(23*m + 9*m^2 + m^3 + 15)) - (exp(2*a + 2*b*x)*(28*m + 3*m^2 + 25))/(3
2*b*(23*m + 9*m^2 + m^3 + 15)) - (exp(8*a + 8*b*x)*(28*m + 3*m^2 + 25))/(3
2*b*(23*m + 9*m^2 + m^3 + 15)) + (exp(4*a + 4*b*x)*(24*m + 2*m^2 + 150))/(
32*b*(23*m + 9*m^2 + m^3 + 15)) + (exp(6*a + 6*b*x)*(24*m + 2*m^2 + 150))/
(32*b*(23*m + 9*m^2 + m^3 + 15)))
```

3.15 $\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

3.15.1 Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx=-\frac{x}{8}-\frac{\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}$$

output
```
-1/8*x-1/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)^3*sinh(b*x+a)/b
```

3.15.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx=\frac{-4(a+bx)+\sinh(4(a+bx))}{32b}$$

input
```
Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(-4*(a + b*x) + Sinh[4*(a + b*x)])/(32*b)
```

3.15.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 25, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^2\left(-\cos(ia+ibx)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(ia+ibx)^2\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{1}{4}\int \cosh^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{1}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}\left(-\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{1}{4}\left(-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}\right)$$

input
```
Int[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (-1/2*x - (Cosh[a + b*x]*Sinh[a +
b*x])/(2*b))/4
```

3.15.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.15.4 Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{x}{8}+\frac{{\mathrm e}^{4 b x +4 a}}{64 b}-\frac{{\mathrm e}^{-4 b x -4 a}}{64 b}$	33
derivativedivides	$\frac{\frac{\cosh \left(b x +a \right)^{3} \sinh \left(b x +a \right)}{4}-\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{8}-\frac{b x}{8}-\frac{a}{8}}{b}$	43
default	$\frac{\frac{\cosh \left(b x +a \right)^{3} \sinh \left(b x +a \right)}{4}-\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{8}-\frac{b x}{8}-\frac{a}{8}}{b}$	43

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*x+1/64*exp(4*b*x+4*a)/b-1/64*exp(-4*b*x-4*a)/b
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\cosh(bx+a)^3\sinh(bx+a)+\cosh(bx+a)\sinh(bx+a)^3-bx}{8\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*x + a)^3*sinh(b*x + a) + cosh(b*x + a)*sinh(b*x + a)^3 - b*x)/
b
```

3.15.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(37) = 74$.

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{x\sinh^4(a+bx)}{8}+\frac{x\sinh^2(a+bx)\cosh^2(a+bx)}{4}-\frac{x\cosh^4(a+bx)}{8}+\frac{\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for } b \neq 0 \\ x\sinh^2(a)\cosh^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-x*sinh(a + b*x)**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 -
 x*cosh(a + b*x)**4/8 + sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*
x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**2, True))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{bx+a}{8\,b} + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(b*x + a)/b + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(4\,a+4\,b\,x)}{32\,b} - \frac{x}{8}$$

input
```
int(cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

output
```
sinh(4*a + 4*b*x)/(32*b) - x/8
```

3.16 $\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$

3.16.1 Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{x}{16}+\frac{\cosh(a+bx)\sinh(a+bx)}{16b}-\frac{\cosh^3(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh^3(a+bx)}{6b}$$

output
```
1/16*x+1/16*cosh(b*x+a)*sinh(b*x+a)/b-1/8*cosh(b*x+a)^3*sinh(b*x+a)/b+1/6*
cosh(b*x+a)^3*sinh(b*x+a)^3/b
```

3.16.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{12bx-3\sinh(2(a+bx))-3\sinh(4(a+bx))+\sinh(6(a+bx))}{192b}$$

input
```
Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^4,x]
```

output
```
(12*b*x - 3*Sinh[2*(a + b*x)] - 3*Sinh[4*(a + b*x)] + Sinh[6*(a + b*x)])/(
192*b)
```

3.16.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3042, 3048, 25, 3042, 25, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^4\cos(ia+ibx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{2}\int -\cosh^2(a+bx)\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}-\frac{1}{2}\int \cosh^2(a+bx)\sinh^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}-\frac{1}{2}\int -\cos(ia+ibx)^2\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}+\frac{1}{2}\int \cos(ia+ibx)^2\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \cosh^2(a+bx)dx-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}+\frac{1}{2}\left(-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{1}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}$$

$$\Big\downarrow 24$$

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}+\frac{1}{2}\left(\frac{1}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)$$

input
```
Int[Cosh[a + b*x]^2*Sinh[a + b*x]^4,x]
```

output
```
(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/(6*b) + (-1/4*(Cosh[a + b*x]^3*Sinh[a +
b*x])/b + (x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b))/4)/2
```

3.16.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.16.4 Maple [A] (verified)

Time = 50.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)^3}{6}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{8}+\frac{\cosh(bx+a)\sinh(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}}{b}$	61
default	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)^3}{6}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{8}+\frac{\cosh(bx+a)\sinh(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}}{b}$	61
risch	$\frac{x}{16}+\frac{e^{6bx+6a}}{384b}-\frac{e^{4bx+4a}}{128b}-\frac{e^{2bx+2a}}{128b}+\frac{e^{-2bx-2a}}{128b}+\frac{e^{-4bx-4a}}{128b}-\frac{e^{-6bx-6a}}{384b}$	89

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/6*cosh(b*x+a)^3*sinh(b*x+a)^3-1/8*cosh(b*x+a)^3*sinh(b*x+a)+1/16*co
sh(b*x+a)*sinh(b*x+a)+1/16*b*x+1/16*a)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$$
$$=\frac{3\cosh\left(bx+a\right)\sinh\left(bx+a\right)^5+2\left(5\cosh\left(bx+a\right)^3-3\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^3+6\,bx+3\left(\cosh\left(ba\right.\right.}{96\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 - 3*cosh(b*x
+ a))*sinh(b*x + a)^3 + 6*b*x + 3*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a))/b
```

3.16.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(58) = 116$.

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.97

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \begin{cases} -\frac{x\sinh^6(a+bx)}{16} + \frac{3x\sinh^4(a+bx)\cosh^2(a+bx)}{16} - \frac{3x\sinh^2(a+bx)\cosh^4(a+bx)}{16} + \frac{x\cosh^6(a+bx)}{16} + \frac{\sinh^5(a+bx)\cosh(a+bx)}{16b} + \text{si} \\ x\sinh^4(a)\cosh^2(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**4,x)
```

output
```
Piecewise((-x*sinh(a + b*x)**6/16 + 3*x*sinh(a + b*x)**4*cosh(a + b*x)**2/
16 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 + x*cosh(a + b*x)**6/16 + si
nh(a + b*x)**5*cosh(a + b*x)/(16*b) + sinh(a + b*x)**3*cosh(a + b*x)**3/(6
*b) - sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)**4*cosh
(a)**2, True))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = -\frac{\left(3\,e^{(-2\,bx-2\,a)} + 3\,e^{(-4\,bx-4\,a)} - 1\right)e^{(6\,bx+6\,a)}}{384\,b} + \frac{bx+a}{16\,b} + \frac{3\,e^{(-2\,bx-2\,a)} + 3\,e^{(-4\,bx-4\,a)} - e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/384*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b + 1
/16*(b*x + a)/b + 1/384*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - e^(-6*b
*x - 6*a))/b
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{1}{16}\,x + \frac{e^{(6\,bx+6\,a)}}{384\,b} - \frac{e^{(4\,bx+4\,a)}}{128\,b} - \frac{e^{(2\,bx+2\,a)}}{128\,b} + \frac{e^{(-2\,bx-2\,a)}}{128\,b} + \frac{e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="giac")
```

output
```
1/16*x + 1/384*e^(6*b*x + 6*a)/b - 1/128*e^(4*b*x + 4*a)/b - 1/128*e^(2*b*
x + 2*a)/b + 1/128*e^(-2*b*x - 2*a)/b + 1/128*e^(-4*b*x - 4*a)/b - 1/384*e
^(-6*b*x - 6*a)/b
```

3.16.9 Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{x}{16} - \frac{\frac{\sinh(2\,a+2\,b\,x)}{64} + \frac{\sinh(4\,a+4\,b\,x)}{64} - \frac{\sinh(6\,a+6\,b\,x)}{192}}{b}$$

input
```
int(cosh(a + b*x)^2*sinh(a + b*x)^4,x)
```

output
```
x/16 - (sinh(2*a + 2*b*x)/64 + sinh(4*a + 4*b*x)/64 - sinh(6*a + 6*b*x)/19
2)/b
```

3.17 $\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$

3.17.1 Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = -\frac{5x}{128}-\frac{5\cosh(a+bx)\sinh(a+bx)}{128b}+\frac{5\cosh^3(a+bx)\sinh(a+bx)}{64b}-\frac{5\cosh^3(a+bx)\sinh^3(a+bx)}{48b}+\frac{\cosh^3(a+bx)\sinh^5(a+bx)}{8b}$$

output
```
-5/128*x-5/128*cosh(b*x+a)*sinh(b*x+a)/b+5/64*cosh(b*x+a)^3*sinh(b*x+a)/b-
5/48*cosh(b*x+a)^3*sinh(b*x+a)^3/b+1/8*cosh(b*x+a)^3*sinh(b*x+a)^5/b
```

3.17.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = \frac{-120bx+48\sinh(2(a+bx))+24\sinh(4(a+bx))-16\sinh(6(a+bx))+3\sinh(8(a+bx))}{3072b}$$

input
```
Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^6,x]
```

output
```
(-120*b*x + 48*Sinh[2*(a + b*x)] + 24*Sinh[4*(a + b*x)] - 16*Sinh[6*(a + b
*x)] + 3*Sinh[8*(a + b*x)])/(3072*b)
```

3.17.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {3042, 25, 3048, 3042, 3048, 25, 3042, 25, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^6(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^6\left(-\cos(ia+ibx)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(ia+ibx)^2\sin(ia+ibx)^6dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\int \cosh^2(a+bx)\sinh^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\int \cos(ia+ibx)^2\sin(ia+ibx)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{1}{2}\int -\cosh^2(a+bx)\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}-\frac{1}{2}\int \cosh^2(a+bx)\sinh^2(a+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}-\frac{1}{2}\int -\cos(ia+ibx)^2\sin(ia+ibx)^2dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}+\frac{1}{2}\int \cos(ia+ibx)^2\sin(ia+ibx)^2dx\right)$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\int \cosh^2(a+bx)dx-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}+\frac{1}{2}\left(-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{1}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}+\frac{1}{2}\left(\frac{1}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)\right)$$

input
```
Int[Cosh[a + b*x]^2*Sinh[a + b*x]^6,x]
```

output
```
(Cosh[a + b*x]^3*Sinh[a + b*x]^5)/(8*b) - (5*((Cosh[a + b*x]^3*Sinh[a + b*
x]^3)/(6*b) + (-1/4*(Cosh[a + b*x]^3*Sinh[a + b*x])/b + (x/2 + (Cosh[a + b
*x]*Sinh[a + b*x])/(2*b))/4)/2))/8
```

3.17. $\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$

3.17.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.17.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$\frac{\frac{\sinh(bx+a)^5\cosh(bx+a)^3}{8}-\frac{5\cosh(bx+a)^3\sinh(bx+a)^3}{48}+\frac{5\cosh(bx+a)^3\sinh(bx+a)}{64}-\frac{5\cosh(bx+a)\sinh(bx+a)}{128}-\frac{5bx}{128}-\frac{5a}{128}}{b}$$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^6,x)
```

output
```
1/b*(1/8*sinh(b*x+a)^5*cosh(b*x+a)^3-5/48*cosh(b*x+a)^3*sinh(b*x+a)^3+5/64
*cosh(b*x+a)^3*sinh(b*x+a)-5/128*cosh(b*x+a)*sinh(b*x+a)-5/128*b*x-5/128*a
)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.50

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$$
$$= \frac{3\cosh(bx+a)\sinh(bx+a)^7 + 3\left(7\cosh(bx+a)^3 - 4\cosh(bx+a)\right)\sinh(bx+a)^5 + \left(21\cosh(bx+a)\right.}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/384*(3*cosh(b*x + a)*sinh(b*x + a)^7 + 3*(7*cosh(b*x + a)^3 - 4*cosh(b*x
 + a))*sinh(b*x + a)^5 + (21*cosh(b*x + a)^5 - 40*cosh(b*x + a)^3 + 12*cos
h(b*x + a))*sinh(b*x + a)^3 - 15*b*x + 3*(cosh(b*x + a)^7 - 4*cosh(b*x + a
)^5 + 4*cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a))/b
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(85) = 170$.

Time = 0.68 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.05

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$$
$$= \begin{cases} -\frac{5x\sinh^8(a+bx)}{128} + \frac{5x\sinh^6(a+bx)\cosh^2(a+bx)}{32} - \frac{15x\sinh^4(a+bx)\cosh^4(a+bx)}{64} + \frac{5x\sinh^2(a+bx)\cosh^6(a+bx)}{32} - \frac{5x\cosh^8(a+bx)}{128} \\ x\sinh^6(a)\cosh^2(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**6,x)
```

output
```
Piecewise((-5*x*sinh(a + b*x)**8/128 + 5*x*sinh(a + b*x)**6*cosh(a + b*x)*
*2/32 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 + 5*x*sinh(a + b*x)**2*c
osh(a + b*x)**6/32 - 5*x*cosh(a + b*x)**8/128 + 5*sinh(a + b*x)**7*cosh(a
+ b*x)/(128*b) + 73*sinh(a + b*x)**5*cosh(a + b*x)**3/(384*b) - 55*sinh(a
+ b*x)**3*cosh(a + b*x)**5/(384*b) + 5*sinh(a + b*x)*cosh(a + b*x)**7/(128
*b), Ne(b, 0)), (x*sinh(a)**6*cosh(a)**2, True))
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$$
$$= -\frac{\left(16\,e^{(-2\,bx-2\,a)}-24\,e^{(-4\,bx-4\,a)}-48\,e^{(-6\,bx-6\,a)}-3\right)e^{(8\,bx+8\,a)}}{6144\,b}-\frac{5\,(bx+a)}{128\,b}$$
$$-\frac{48\,e^{(-2\,bx-2\,a)}+24\,e^{(-4\,bx-4\,a)}-16\,e^{(-6\,bx-6\,a)}+3\,e^{(-8\,bx-8\,a)}}{6144\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/6144*(16*e^(-2*b*x - 2*a) - 24*e^(-4*b*x - 4*a) - 48*e^(-6*b*x - 6*a) -
 3)*e^(8*b*x + 8*a)/b - 5/128*(b*x + a)/b - 1/6144*(48*e^(-2*b*x - 2*a) +
24*e^(-4*b*x - 4*a) - 16*e^(-6*b*x - 6*a) + 3*e^(-8*b*x - 8*a))/b
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.26

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = -\frac{5}{128}\,x+\frac{e^{(8\,bx+8\,a)}}{2048\,b}-\frac{e^{(6\,bx+6\,a)}}{384\,b}+\frac{e^{(4\,bx+4\,a)}}{256\,b}+\frac{e^{(2\,bx+2\,a)}}{128\,b}$$
$$-\frac{e^{(-2\,bx-2\,a)}}{128\,b}-\frac{e^{(-4\,bx-4\,a)}}{256\,b}+\frac{e^{(-6\,bx-6\,a)}}{384\,b}-\frac{e^{(-8\,bx-8\,a)}}{2048\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="giac")
```

output
```
-5/128*x + 1/2048*e^(8*b*x + 8*a)/b - 1/384*e^(6*b*x + 6*a)/b + 1/256*e^(4
*b*x + 4*a)/b + 1/128*e^(2*b*x + 2*a)/b - 1/128*e^(-2*b*x - 2*a)/b - 1/256
*e^(-4*b*x - 4*a)/b + 1/384*e^(-6*b*x - 6*a)/b - 1/2048*e^(-8*b*x - 8*a)/b
```

3.17.9 Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = \frac{\frac{\sinh(2\,a+2\,b\,x)}{64}+\frac{\sinh(4\,a+4\,b\,x)}{128}-\frac{\sinh(6\,a+6\,b\,x)}{192}+\frac{\sinh(8\,a+8\,b\,x)}{1024}}{b} - \frac{5\,x}{128}$$

input
```
int(cosh(a + b*x)^2*sinh(a + b*x)^6,x)
```

output
```
(sinh(2*a + 2*b*x)/64 + sinh(4*a + 4*b*x)/128 - sinh(6*a + 6*b*x)/192 + si
nh(8*a + 8*b*x)/1024)/b - (5*x)/128
```

3.18 $\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$

3.18.1 Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = -\frac{x}{16}-\frac{\cosh(a+bx)\sinh(a+bx)}{16b}-\frac{\cosh^3(a+bx)\sinh(a+bx)}{24b}+\frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}$$

output
```
-1/16*x-1/16*cosh(b*x+a)*sinh(b*x+a)/b-1/24*cosh(b*x+a)^3*sinh(b*x+a)/b+1/
6*cosh(b*x+a)^5*sinh(b*x+a)/b
```

3.18.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{-12bx-3\sinh(2(a+bx))+3\sinh(4(a+bx))+\sinh(6(a+bx))}{192b}$$

input
```
Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^2,x]
```

output
```
(-12*b*x - 3*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)] + Sinh[6*(a + b*x)])/
(192*b)
```

3.18.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 25, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\cosh^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^2\left(-\cos(ia+ibx)^4\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(ia+ibx)^4\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{1}{6}\int \cosh^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{1}{6}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}\left(-\frac{3}{4}\int \cosh^2(a+bx)dx-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{1}{6}\left(-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{3}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}\left(-\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+$$
$$\frac{1}{6}\left(-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)\right)$$

input
```
Int[Cosh[a + b*x]^4*Sinh[a + b*x]^2,x]
```

output
```
(Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b) + (-1/4*(Cosh[a + b*x]^3*Sinh[a + b*
x])/b - (3*(x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4)/6
```

3.18.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.18.4 Maple [A] (verified)

Time = 22.63 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^5}{6}-\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{6}-\frac{bx}{16}-\frac{a}{16}}{b}$	56
default	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^5}{6}-\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{6}-\frac{bx}{16}-\frac{a}{16}}{b}$	56
risch	$-\frac{x}{16}+\frac{e^{6bx+6a}}{384b}+\frac{e^{4bx+4a}}{128b}-\frac{e^{2bx+2a}}{128b}+\frac{e^{-2bx-2a}}{128b}-\frac{e^{-4bx-4a}}{128b}-\frac{e^{-6bx-6a}}{384b}$	89

input
```
int(cosh(b*x+a)^4*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/6*sinh(b*x+a)*cosh(b*x+a)^5-1/6*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))
*sinh(b*x+a)-1/16*b*x-1/16*a)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$$
$$= \frac{3\,\cosh(bx+a)\sinh(bx+a)^5+2\left(5\,\cosh(bx+a)^3+3\,\cosh(bx+a)\right)\sinh(bx+a)^3-6\,bx+3\,(\cosh(ba}{96\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 + 3*cosh(b*x
+ a))*sinh(b*x + a)^3 - 6*b*x + 3*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a))/b
```

3.18.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(56) = 112.

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.03

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$$
$$= \begin{cases} \frac{x\sinh^6(a+bx)}{16} - \frac{3x\sinh^4(a+bx)\cosh^2(a+bx)}{16} + \frac{3x\sinh^2(a+bx)\cosh^4(a+bx)}{16} - \frac{x\cosh^6(a+bx)}{16} - \frac{\sinh^5(a+bx)\cosh(a+bx)}{16b} + \frac{\sinh \\ x\sinh^2(a)\cosh^4(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**4*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x*sinh(a + b*x)**6/16 - 3*x*sinh(a + b*x)**4*cosh(a + b*x)**2/1
6 + 3*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 - x*cosh(a + b*x)**6/16 - sin
h(a + b*x)**5*cosh(a + b*x)/(16*b) + sinh(a + b*x)**3*cosh(a + b*x)**3/(6*
b) + sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)**2*cosh(
a)**4, True))
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + 1\right)e^{(6\,bx+6\,a)}}{384\,b} - \frac{bx+a}{16\,b} + \frac{3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} - e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/384*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + 1)*e^(6*b*x + 6*a)/b - 1/
16*(b*x + a)/b + 1/384*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) - e^(-6*b*
x - 6*a))/b
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{16}x + \frac{e^{(6\,bx+6\,a)}}{384\,b} + \frac{e^{(4\,bx+4\,a)}}{128\,b} - \frac{e^{(2\,bx+2\,a)}}{128\,b} + \frac{e^{(-2\,bx-2\,a)}}{128\,b} - \frac{e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/16*x + 1/384*e^(6*b*x + 6*a)/b + 1/128*e^(4*b*x + 4*a)/b - 1/128*e^(2*b
*x + 2*a)/b + 1/128*e^(-2*b*x - 2*a)/b - 1/128*e^(-4*b*x - 4*a)/b - 1/384*
e^(-6*b*x - 6*a)/b
```

3.18.9 Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{\sinh(4\,a+4\,b\,x)}{64} - \frac{\sinh(2\,a+2\,b\,x)}{64} + \frac{\sinh(6\,a+6\,b\,x)}{192}}{b} - \frac{x}{16}$$

input
```
int(cosh(a + b*x)^4*sinh(a + b*x)^2,x)
```

output
```
(sinh(4*a + 4*b*x)/64 - sinh(2*a + 2*b*x)/64 + sinh(6*a + 6*b*x)/192)/b -
x/16
```

3.19 $\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$

3.19.1 Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{3x}{128}+\frac{3\cosh(a+bx)\sinh(a+bx)}{128b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{64b}-\frac{\cosh^5(a+bx)\sinh(a+bx)}{16b}+\frac{\cosh^5(a+bx)\sinh^3(a+bx)}{8b}$$

output
```
3/128*x+3/128*cosh(b*x+a)*sinh(b*x+a)/b+1/64*cosh(b*x+a)^3*sinh(b*x+a)/b-1
/16*cosh(b*x+a)^5*sinh(b*x+a)/b+1/8*cosh(b*x+a)^5*sinh(b*x+a)^3/b
```

3.19.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.37

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{24(a+bx)-8\sinh(4(a+bx))+\sinh(8(a+bx))}{1024b}$$

input
```
Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^4,x]
```

output
```
(24*(a + b*x) - 8*Sinh[4*(a + b*x)] + Sinh[8*(a + b*x)])/(1024*b)
```

3.19.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 3048, 25, 3042, 25, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(a+bx)\cosh^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^4\cos(ia+ibx)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{8}\int -\cosh^4(a+bx)\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{3}{8}\int \cosh^4(a+bx)\sinh^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{3}{8}\int -\cos(ia+ibx)^4\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}+\frac{3}{8}\int \cos(ia+ibx)^4\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{8}\left(\frac{1}{6}\int \cosh^4(a+bx)dx-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)+\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}+\frac{3}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{1}{6}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)+\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}+\frac{3}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{1}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)+\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}+\frac{3}{8}\left(\frac{1}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)$$

input
```
Int[Cosh[a + b*x]^4*Sinh[a + b*x]^4,x]
```

output
```
(Cosh[a + b*x]^5*Sinh[a + b*x]^3)/(8*b) + (3*(-1/6*(Cosh[a + b*x]^5*Sinh[a
 + b*x])/b + ((Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (3*(x/2 + (Cosh[a +
b*x]*Sinh[a + b*x])/(2*b)))/4)/6))/8
```

3.19.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.19.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\frac{\frac{\sinh(bx+a)^3\cosh(bx+a)^5}{8}-\frac{\sinh(bx+a)\cosh(bx+a)^5}{16}+\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{16}+\frac{3bx}{128}+\frac{3a}{128}}{b}$$

input
```
int(cosh(b*x+a)^4*sinh(b*x+a)^4,x)
```

output
```
1/b*(1/8*sinh(b*x+a)^3*cosh(b*x+a)^5-1/16*sinh(b*x+a)*cosh(b*x+a)^5+1/16*(
1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)+3/128*b*x+3/128*a)
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$$
$$=\frac{7\cosh(bx+a)^3\sinh(bx+a)^5+\cosh(bx+a)\sinh(bx+a)^7+\left(7\cosh(bx+a)^5-4\cosh(bx+a)\right)\sinh}{128\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/128*(7*cosh(b*x + a)^3*sinh(b*x + a)^5 + cosh(b*x + a)*sinh(b*x + a)^7 +
 (7*cosh(b*x + a)^5 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x
 + a)^7 - 4*cosh(b*x + a)^3)*sinh(b*x + a))/b
```

3.19.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(80) = 160.

Time = 0.69 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.10

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$$
$$=\begin{cases} \frac{3x\sinh^8(a+bx)}{128}-\frac{3x\sinh^6(a+bx)\cosh^2(a+bx)}{32}+\frac{9x\sinh^4(a+bx)\cosh^4(a+bx)}{64}-\frac{3x\sinh^2(a+bx)\cosh^6(a+bx)}{32}+\frac{3x\cosh^8(a+bx)}{128}- \\ x\sinh^4(a)\cosh^4(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**4*sinh(b*x+a)**4,x)
```

output
```
Piecewise((3*x*sinh(a + b*x)**8/128 - 3*x*sinh(a + b*x)**6*cosh(a + b*x)**
2/32 + 9*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 - 3*x*sinh(a + b*x)**2*cos
h(a + b*x)**6/32 + 3*x*cosh(a + b*x)**8/128 - 3*sinh(a + b*x)**7*cosh(a +
b*x)/(128*b) + 11*sinh(a + b*x)**5*cosh(a + b*x)**3/(128*b) + 11*sinh(a +
b*x)**3*cosh(a + b*x)**5/(128*b) - 3*sinh(a + b*x)*cosh(a + b*x)**7/(128*b
), Ne(b, 0)), (x*sinh(a)**4*cosh(a)**4, True))
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = -\frac{\left(8\,e^{(-4\,bx-4\,a)}-1\right)e^{(8\,bx+8\,a)}}{2048\,b} + \frac{3\,(bx+a)}{128\,b} + \frac{8\,e^{(-4\,bx-4\,a)}-e^{(-8\,bx-8\,a)}}{2048\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/2048*(8*e^(-4*b*x - 4*a) - 1)*e^(8*b*x + 8*a)/b + 3/128*(b*x + a)/b + 1
/2048*(8*e^(-4*b*x - 4*a) - e^(-8*b*x - 8*a))/b
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{3}{128}\,x + \frac{e^{(8\,bx+8\,a)}}{2048\,b} - \frac{e^{(4\,bx+4\,a)}}{256\,b} + \frac{e^{(-4\,bx-4\,a)}}{256\,b} - \frac{e^{(-8\,bx-8\,a)}}{2048\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="giac")
```

output
```
3/128*x + 1/2048*e^(8*b*x + 8*a)/b - 1/256*e^(4*b*x + 4*a)/b + 1/256*e^(-4
*b*x - 4*a)/b - 1/2048*e^(-8*b*x - 8*a)/b
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.36

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{3\,x}{128} - \frac{\frac{\sinh(4\,a+4\,b\,x)}{128} - \frac{\sinh(8\,a+8\,b\,x)}{1024}}{b}$$

input
```
int(cosh(a + b*x)^4*sinh(a + b*x)^4,x)
```

output
```
(3*x)/128 - (sinh(4*a + 4*b*x)/128 - sinh(8*a + 8*b*x)/1024)/b
```

3.20 $\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$

3.20.1 Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = -\frac{3x}{256}-\frac{3\cosh(a+bx)\sinh(a+bx)}{256b}-\frac{\cosh^3(a+bx)\sinh(a+bx)}{128b}+\frac{\cosh^5(a+bx)\sinh(a+bx)}{32b}-\frac{\cosh^5(a+bx)\sinh^3(a+bx)}{16b}+\frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b}$$

output
```
-3/256*x-3/256*cosh(b*x+a)*sinh(b*x+a)/b-1/128*cosh(b*x+a)^3*sinh(b*x+a)/b
+1/32*cosh(b*x+a)^5*sinh(b*x+a)/b-1/16*cosh(b*x+a)^5*sinh(b*x+a)^3/b+1/10*
cosh(b*x+a)^5*sinh(b*x+a)^5/b
```

3.20.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$$
$$= \frac{-120bx+20\sinh(2(a+bx))+40\sinh(4(a+bx))-10\sinh(6(a+bx))-5\sinh(8(a+bx))+2\sinh(10(a}{10240b}$$

input
```
Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^6,x]
```

output
```
(-120*b*x + 20*Sinh[2*(a + b*x)] + 40*Sinh[4*(a + b*x)] - 10*Sinh[6*(a + b
*x)] - 5*Sinh[8*(a + b*x)] + 2*Sinh[10*(a + b*x)])/(10240*b)
```

3.20.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {3042, 25, 3048, 3042, 3048, 25, 3042, 25, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^6(a+bx)\cosh^4(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin(ia+ibx)^6\left(-\cos(ia+ibx)^4\right)dx$$
$$\Big\downarrow \text{ 25}$$
$$-\int \cos(ia+ibx)^4\sin(ia+ibx)^6dx$$
$$\Big\downarrow \text{ 3048}$$
$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}-\frac{1}{2}\int \cosh^4(a+bx)\sinh^4(a+bx)dx$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}-\frac{1}{2}\int \cos(ia+ibx)^4\sin(ia+ibx)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{2}\left(-\frac{3}{8}\int -\cosh^4(a+bx)\sinh^2(a+bx)dx-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}\right)+\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{3}{8}\int \cosh^4(a+bx)\sinh^2(a+bx)dx-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}\right)+\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}+\frac{1}{2}\left(-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}+\frac{3}{8}\int -\cos(ia+ibx)^4\sin(ia+ibx)^2dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}+\frac{1}{2}\left(-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{3}{8}\int \cos(ia+ibx)^4\sin(ia+ibx)^2dx\right)$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{1}{6}\int \cosh^4(a+bx)dx-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}\right)+\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}+\frac{1}{2}\left(-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{3}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{1}{6}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)-\frac{\sinh^3(a+bx)}{8}\right.$$
$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

3.20. $\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}+$$
$$\frac{1}{2}\left(-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{3}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{1}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\right.\right.\right.$$

$$\Big\downarrow 3115$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right.\right.$$
$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}$$

$$\Big\downarrow 24$$

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}+$$
$$\frac{1}{2}\left(-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{3}{8}\left(\frac{1}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)\right)-\right.\right.$$

input
```
Int[Cosh[a + b*x]^4*Sinh[a + b*x]^6,x]
```

output
```
(Cosh[a + b*x]^5*Sinh[a + b*x]^5)/(10*b) + (-1/8*(Cosh[a + b*x]^5*Sinh[a +
 b*x]^3)/b - (3*(-1/6*(Cosh[a + b*x]^5*Sinh[a + b*x])/b + ((Cosh[a + b*x]^
3*Sinh[a + b*x])/(4*b) + (3*(x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4
)/6))/8)/2
```

3.20.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.20.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\frac{\frac{\sinh(bx+a)^5\cosh(bx+a)^5}{10} - \frac{\sinh(bx+a)^3\cosh(bx+a)^5}{16} + \frac{\sinh(bx+a)\cosh(bx+a)^5}{32} - \frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{32} - \frac{3bx}{256} - \frac{3a}{256}}{b}$$

input
```
int(cosh(b*x+a)^4*sinh(b*x+a)^6,x)
```

output
```
1/b*(1/10*sinh(b*x+a)^5*cosh(b*x+a)^5-1/16*sinh(b*x+a)^3*cosh(b*x+a)^5+1/3
2*sinh(b*x+a)*cosh(b*x+a)^5-1/32*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(
b*x+a)-3/256*b*x-3/256*a)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.74

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$$
$$= \frac{5\cosh(bx+a)\sinh(bx+a)^9 + 10\left(6\cosh(bx+a)^3 - \cosh(bx+a)\right)\sinh(bx+a)^7 + \left(126\cosh(bx+a\right.}{}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/2560*(5*cosh(b*x + a)*sinh(b*x + a)^9 + 10*(6*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a)^7 + (126*cosh(b*x + a)^5 - 70*cosh(b*x + a)^3 - 15*co
sh(b*x + a))*sinh(b*x + a)^5 + 10*(6*cosh(b*x + a)^7 - 7*cosh(b*x + a)^5 -
 5*cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^3 - 30*b*x + 5*(cosh(b
*x + a)^9 - 2*cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 8*cosh(b*x + a)^3 + 2*
cosh(b*x + a))*sinh(b*x + a))/b
```

3.20.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(100) = 200$.

Time = 1.30 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.04

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$$
$$= \begin{cases} \frac{3x\sinh^{10}(a+bx)}{256} - \frac{15x\sinh^8(a+bx)\cosh^2(a+bx)}{256} + \frac{15x\sinh^6(a+bx)\cosh^4(a+bx)}{128} - \frac{15x\sinh^4(a+bx)\cosh^6(a+bx)}{128} + \frac{15x\sinh^2(a+ \\ x\sinh^6(a)\cosh^4(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**4*sinh(b*x+a)**6,x)
```

output
```
Piecewise((3*x*sinh(a + b*x)**10/256 - 15*x*sinh(a + b*x)**8*cosh(a + b*x)
**2/256 + 15*x*sinh(a + b*x)**6*cosh(a + b*x)**4/128 - 15*x*sinh(a + b*x)*
*4*cosh(a + b*x)**6/128 + 15*x*sinh(a + b*x)**2*cosh(a + b*x)**8/256 - 3*x
*cosh(a + b*x)**10/256 - 3*sinh(a + b*x)**9*cosh(a + b*x)/(256*b) + 7*sinh
(a + b*x)**7*cosh(a + b*x)**3/(128*b) + sinh(a + b*x)**5*cosh(a + b*x)**5/
(10*b) - 7*sinh(a + b*x)**3*cosh(a + b*x)**7/(128*b) + 3*sinh(a + b*x)*cos
h(a + b*x)**9/(256*b), Ne(b, 0)), (x*sinh(a)**6*cosh(a)**4, True))
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$$
$$= -\frac{\left(5\,e^{(-2\,bx-2\,a)} + 10\,e^{(-4\,bx-4\,a)} - 40\,e^{(-6\,bx-6\,a)} - 20\,e^{(-8\,bx-8\,a)} - 2\right)e^{(10\,bx+10\,a)}}{20480\,b} - \frac{3\,(bx+a)}{256\,b} - \frac{20\,e^{(-2\,bx-2\,a)} + 40\,e^{(-4\,bx-4\,a)} - 10\,e^{(-6\,bx-6\,a)} - 5\,e^{(-8\,bx-8\,a)} + 2\,e^{(-10\,bx-10\,a)}}{20480\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/20480*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) - 40*e^(-6*b*x - 6*a) -
 20*e^(-8*b*x - 8*a) - 2)*e^(10*b*x + 10*a)/b - 3/256*(b*x + a)/b - 1/2048
0*(20*e^(-2*b*x - 2*a) + 40*e^(-4*b*x - 4*a) - 10*e^(-6*b*x - 6*a) - 5*e^(
-8*b*x - 8*a) + 2*e^(-10*b*x - 10*a))/b
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = -\frac{3}{256}x + \frac{e^{(10\,bx+10\,a)}}{10240\,b} - \frac{e^{(8\,bx+8\,a)}}{4096\,b} - \frac{e^{(6\,bx+6\,a)}}{2048\,b} + \frac{e^{(4\,bx+4\,a)}}{512\,b} + \frac{e^{(2\,bx+2\,a)}}{1024\,b} - \frac{e^{(-2\,bx-2\,a)}}{1024\,b} - \frac{e^{(-4\,bx-4\,a)}}{512\,b} + \frac{e^{(-6\,bx-6\,a)}}{2048\,b} + \frac{e^{(-8\,bx-8\,a)}}{4096\,b} - \frac{e^{(-10\,bx-10\,a)}}{10240\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="giac")
```

output
```
-3/256*x + 1/10240*e^(10*b*x + 10*a)/b - 1/4096*e^(8*b*x + 8*a)/b - 1/2048
*e^(6*b*x + 6*a)/b + 1/512*e^(4*b*x + 4*a)/b + 1/1024*e^(2*b*x + 2*a)/b -
1/1024*e^(-2*b*x - 2*a)/b - 1/512*e^(-4*b*x - 4*a)/b + 1/2048*e^(-6*b*x -
6*a)/b + 1/4096*e^(-8*b*x - 8*a)/b - 1/10240*e^(-10*b*x - 10*a)/b
```

3.20.9 Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = \frac{20\sinh(2\,a+2\,b\,x)+40\sinh(4\,a+4\,b\,x)-10\sinh(6\,a+6\,b\,x)-5\sinh(8\,a+8\,b\,x)+2\sinh(10\,a+10\,b\,x}{10240\,b}$$

input
```
int(cosh(a + b*x)^4*sinh(a + b*x)^6,x)
```

output
```
(20*sinh(2*a + 2*b*x) + 40*sinh(4*a + 4*b*x) - 10*sinh(6*a + 6*b*x) - 5*si
nh(8*a + 8*b*x) + 2*sinh(10*a + 10*b*x) - 120*b*x)/(10240*b)
```

3.21 $\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$

3.21.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = -\frac{5x}{128}-\frac{5\cosh(a+bx)\sinh(a+bx)}{128b}-\frac{5\cosh^3(a+bx)\sinh(a+bx)}{192b}-\frac{\cosh^5(a+bx)\sinh(a+bx)}{48b}+\frac{\cosh^7(a+bx)\sinh(a+bx)}{8b}$$

output
```
-5/128*x-5/128*cosh(b*x+a)*sinh(b*x+a)/b-5/192*cosh(b*x+a)^3*sinh(b*x+a)/b
-1/48*cosh(b*x+a)^5*sinh(b*x+a)/b+1/8*cosh(b*x+a)^7*sinh(b*x+a)/b
```

3.21.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = \frac{-120bx-48\sinh(2(a+bx))+24\sinh(4(a+bx))+16\sinh(6(a+bx))+3\sinh(8(a+bx))}{3072b}$$

input
```
Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^2,x]
```

output
```
(-120*b*x - 48*Sinh[2*(a + b*x)] + 24*Sinh[4*(a + b*x)] + 16*Sinh[6*(a + b
*x)] + 3*Sinh[8*(a + b*x)])/(3072*b)
```

3.21.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {3042, 25, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\cosh^6(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(ia+ibx)^2\left(-\cos(ia+ibx)^6\right)dx$$

$$\Big\downarrow \; 25$$

$$-\int \cos(ia+ibx)^6\sin(ia+ibx)^2dx$$

$$\Big\downarrow \; 3048$$

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}-\frac{1}{8}\int \cosh^6(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}-\frac{1}{8}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{8}\left(-\frac{5}{6}\int \cosh^4(a+bx)dx-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)+\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{5}{6}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx\right)$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{8}\left(-\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)+\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{8}\left(-\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)+\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)\right)\right)$$

input
```
Int[Cosh[a + b*x]^6*Sinh[a + b*x]^2,x]
```

output
```
(Cosh[a + b*x]^7*Sinh[a + b*x])/(8*b) + (-1/6*(Cosh[a + b*x]^5*Sinh[a + b*
x])/b - (5*((Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (3*(x/2 + (Cosh[a + b*
x]*Sinh[a + b*x])/(2*b)))/4))/6)/8
```

3.21.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.21.4 Maple [A] (verified)

Time = 127.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

method	result	s
derivativedivides	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^7}{8}-\frac{\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)}{8}-\frac{5bx}{128}-\frac{5a}{128}}{b}$	6
default	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^7}{8}-\frac{\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)}{8}-\frac{5bx}{128}-\frac{5a}{128}}{b}$	6
risch	$-\frac{5x}{128}+\frac{e^{8bx+8a}}{2048b}+\frac{e^{6bx+6a}}{384b}+\frac{e^{4bx+4a}}{256b}-\frac{e^{2bx+2a}}{128b}+\frac{e^{-2bx-2a}}{128b}-\frac{e^{-4bx-4a}}{256b}-\frac{e^{-6bx-6a}}{384b}-\frac{e^{-8bx-8a}}{2048b}$	1

input
```
int(cosh(b*x+a)^6*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*sinh(b*x+a)*cosh(b*x+a)^7-1/8*(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)
^3+5/16*cosh(b*x+a))*sinh(b*x+a)-5/128*b*x-5/128*a)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.57

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)\sinh(bx+a)^7+3\left(7\cosh(bx+a)^3+4\cosh(bx+a)\right)\sinh(bx+a)^5+\left(21\cosh(bx+a)\right.}{}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/384*(3*cosh(b*x + a)*sinh(b*x + a)^7 + 3*(7*cosh(b*x + a)^3 + 4*cosh(b*x
 + a))*sinh(b*x + a)^5 + (21*cosh(b*x + a)^5 + 40*cosh(b*x + a)^3 + 12*cos
h(b*x + a))*sinh(b*x + a)^3 - 15*b*x + 3*(cosh(b*x + a)^7 + 4*cosh(b*x + a
)^5 + 4*cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a))/b
```

3.21.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(80) = 160$.

Time = 0.67 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.15

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$$
$$= \begin{cases} -\frac{5x\sinh^8(a+bx)}{128} + \frac{5x\sinh^6(a+bx)\cosh^2(a+bx)}{32} - \frac{15x\sinh^4(a+bx)\cosh^4(a+bx)}{64} + \frac{5x\sinh^2(a+bx)\cosh^6(a+bx)}{32} - \frac{5x\cosh^8(a+bx)}{128} \\ x\sinh^2(a)\cosh^6(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**6*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-5*x*sinh(a + b*x)**8/128 + 5*x*sinh(a + b*x)**6*cosh(a + b*x)*
*2/32 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 + 5*x*sinh(a + b*x)**2*c
osh(a + b*x)**6/32 - 5*x*cosh(a + b*x)**8/128 + 5*sinh(a + b*x)**7*cosh(a
+ b*x)/(128*b) - 55*sinh(a + b*x)**5*cosh(a + b*x)**3/(384*b) + 73*sinh(a
+ b*x)**3*cosh(a + b*x)**5/(384*b) + 5*sinh(a + b*x)*cosh(a + b*x)**7/(128
*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**6, True))
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$$
$$= \frac{\left(16\,e^{(-2\,bx-2\,a)} + 24\,e^{(-4\,bx-4\,a)} - 48\,e^{(-6\,bx-6\,a)} + 3\right)e^{(8\,bx+8\,a)}}{6144\,b} - \frac{5\,(bx+a)}{128\,b}$$
$$+ \frac{48\,e^{(-2\,bx-2\,a)} - 24\,e^{(-4\,bx-4\,a)} - 16\,e^{(-6\,bx-6\,a)} - 3\,e^{(-8\,bx-8\,a)}}{6144\,b}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/6144*(16*e^(-2*b*x - 2*a) + 24*e^(-4*b*x - 4*a) - 48*e^(-6*b*x - 6*a) +
3)*e^(8*b*x + 8*a)/b - 5/128*(b*x + a)/b + 1/6144*(48*e^(-2*b*x - 2*a) - 2
4*e^(-4*b*x - 4*a) - 16*e^(-6*b*x - 6*a) - 3*e^(-8*b*x - 8*a))/b
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = -\frac{5}{128}\,x + \frac{e^{(8\,bx+8\,a)}}{2048\,b} + \frac{e^{(6\,bx+6\,a)}}{384\,b} + \frac{e^{(4\,bx+4\,a)}}{256\,b} - \frac{e^{(2\,bx+2\,a)}}{128\,b} + \frac{e^{(-2\,bx-2\,a)}}{128\,b} - \frac{e^{(-4\,bx-4\,a)}}{256\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b} - \frac{e^{(-8\,bx-8\,a)}}{2048\,b}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-5/128*x + 1/2048*e^(8*b*x + 8*a)/b + 1/384*e^(6*b*x + 6*a)/b + 1/256*e^(4
*b*x + 4*a)/b - 1/128*e^(2*b*x + 2*a)/b + 1/128*e^(-2*b*x - 2*a)/b - 1/256
*e^(-4*b*x - 4*a)/b - 1/384*e^(-6*b*x - 6*a)/b - 1/2048*e^(-8*b*x - 8*a)/b
```

3.21.9 Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{\sinh(4\,a+4\,b\,x)}{128} - \frac{\sinh(2\,a+2\,b\,x)}{64} + \frac{\sinh(6\,a+6\,b\,x)}{192} + \frac{\sinh(8\,a+8\,b\,x)}{1024}}{b} - \frac{5\,x}{128}$$

input
```
int(cosh(a + b*x)^6*sinh(a + b*x)^2,x)
```

output
```
(sinh(4*a + 4*b*x)/128 - sinh(2*a + 2*b*x)/64 + sinh(6*a + 6*b*x)/192 + si
nh(8*a + 8*b*x)/1024)/b - (5*x)/128
```

3.22 $\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$

3.22.1 Optimal result

Integrand size = 17, antiderivative size = 111

$$\begin{aligned}\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = {} & \frac{3x}{256}+\frac{3\cosh(a+bx)\sinh(a+bx)}{256b} \\ & +\frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} \\ & +\frac{\cosh^5(a+bx)\sinh(a+bx)}{160b} \\ & -\frac{3\cosh^7(a+bx)\sinh(a+bx)}{80b} \\ & +\frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b}\end{aligned}$$

output
```
3/256*x+3/256*cosh(b*x+a)*sinh(b*x+a)/b+1/128*cosh(b*x+a)^3*sinh(b*x+a)/b+
1/160*cosh(b*x+a)^5*sinh(b*x+a)/b-3/80*cosh(b*x+a)^7*sinh(b*x+a)/b+1/10*co
sh(b*x+a)^7*sinh(b*x+a)^3/b
```

3.22.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{120bx+20\sinh(2(a+bx))-40\sinh(4(a+bx))-10\sinh(6(a+bx))+5\sinh(8(a+bx))+2\sinh(10(a+bx))}{10240b}$$

input
```
Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^4,x]
```

output
```
(120*b*x + 20*Sinh[2*(a + b*x)] - 40*Sinh[4*(a + b*x)] - 10*Sinh[6*(a + b*
x)] + 5*Sinh[8*(a + b*x)] + 2*Sinh[10*(a + b*x)])/(10240*b)
```

3.22.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {3042, 3048, 25, 3042, 25, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(a+bx)\cosh^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^4\cos(ia+ibx)^6dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{10}\int -\cosh^6(a+bx)\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}-\frac{3}{10}\int \cosh^6(a+bx)\sinh^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}-\frac{3}{10}\int -\cos(ia+ibx)^6\sin(ia+ibx)^2dx$$

$$\Big\downarrow \; 25$$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+\frac{3}{10}\int \cos(ia+ibx)^6\sin(ia+ibx)^2dx$$

$$\Big\downarrow \; 3048$$

$$\frac{3}{10}\left(\frac{1}{8}\int \cosh^6(a+bx)dx-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}\right)+\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+\frac{3}{10}\left(-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^6dx\right)$$

$$\Big\downarrow \; 3115$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \cosh^4(a+bx)dx+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}\right)+\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+\frac{3}{10}\left(-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\left(\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx\right)\right)$$

$$\Big\downarrow \; 3115$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)-\frac{\sinh(a+bx)\,\mathrm{c}}{8b}\right.\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+\frac{3}{10}\left(-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\left(\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\left(i\right.\right.\right.\right.$$

$$\Big\downarrow \; 3115$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right.\right.\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}$$

$$\Big\downarrow \; 24$$

3.22. $\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+$$
$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)\right)\right)-\right.$$

input
```
Int[Cosh[a + b*x]^6*Sinh[a + b*x]^4,x]
```

output
```
(Cosh[a + b*x]^7*Sinh[a + b*x]^3)/(10*b) + (3*(-1/8*(Cosh[a + b*x]^7*Sinh[
a + b*x])/b + ((Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b) + (5*((Cosh[a + b*x]^
3*Sinh[a + b*x])/(4*b) + (3*(x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4
))/6)/8))/10
```

3.22.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.22.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\frac{\frac{\sinh(bx+a)^3\cosh(bx+a)^7}{10}-\frac{3\sinh(bx+a)\cosh(bx+a)^7}{80}+\frac{3\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)}{80}+\frac{3bx}{256}+\frac{3a}{256}}{b}$$

input
```
int(cosh(b*x+a)^6*sinh(b*x+a)^4,x)
```

output
```
1/b*(1/10*sinh(b*x+a)^3*cosh(b*x+a)^7-3/80*sinh(b*x+a)*cosh(b*x+a)^7+3/80*
(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^3+5/16*cosh(b*x+a))*sinh(b*x+a)+3/256*
b*x+3/256*a)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.76

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$$
$$=\frac{5\cosh(bx+a)\sinh(bx+a)^9+10\left(6\cosh(bx+a)^3+\cosh(bx+a)\right)\sinh(bx+a)^7+\left(126\cosh(bx+a\right.}{}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/2560*(5*cosh(b*x + a)*sinh(b*x + a)^9 + 10*(6*cosh(b*x + a)^3 + cosh(b*x
 + a))*sinh(b*x + a)^7 + (126*cosh(b*x + a)^5 + 70*cosh(b*x + a)^3 - 15*co
sh(b*x + a))*sinh(b*x + a)^5 + 10*(6*cosh(b*x + a)^7 + 7*cosh(b*x + a)^5 -
 5*cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 30*b*x + 5*(cosh(b
*x + a)^9 + 2*cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 - 8*cosh(b*x + a)^3 + 2*
cosh(b*x + a))*sinh(b*x + a))/b
```

3.22.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(100) = 200$.

Time = 1.31 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.08

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$$
$$= \begin{cases} -\frac{3x\sinh^{10}(a+bx)}{256} + \frac{15x\sinh^8(a+bx)\cosh^2(a+bx)}{256} - \frac{15x\sinh^6(a+bx)\cosh^4(a+bx)}{128} + \frac{15x\sinh^4(a+bx)\cosh^6(a+bx)}{128} - \frac{15x\sinh^2(a\\ x\sinh^4(a)\cosh^6(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**6*sinh(b*x+a)**4,x)
```

output
```
Piecewise((-3*x*sinh(a + b*x)**10/256 + 15*x*sinh(a + b*x)**8*cosh(a + b*x
)**2/256 - 15*x*sinh(a + b*x)**6*cosh(a + b*x)**4/128 + 15*x*sinh(a + b*x)
**4*cosh(a + b*x)**6/128 - 15*x*sinh(a + b*x)**2*cosh(a + b*x)**8/256 + 3*
x*cosh(a + b*x)**10/256 + 3*sinh(a + b*x)**9*cosh(a + b*x)/(256*b) - 7*sin
h(a + b*x)**7*cosh(a + b*x)**3/(128*b) + sinh(a + b*x)**5*cosh(a + b*x)**5
/(10*b) + 7*sinh(a + b*x)**3*cosh(a + b*x)**7/(128*b) - 3*sinh(a + b*x)*co
sh(a + b*x)**9/(256*b), Ne(b, 0)), (x*sinh(a)**4*cosh(a)**6, True))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$$
$$= \frac{\left(5\,e^{(-2\,bx-2\,a)} - 10\,e^{(-4\,bx-4\,a)} - 40\,e^{(-6\,bx-6\,a)} + 20\,e^{(-8\,bx-8\,a)} + 2\right)e^{(10\,bx+10\,a)}}{20480\,b} + \frac{3\,(bx+a)}{256\,b}$$
$$- \frac{20\,e^{(-2\,bx-2\,a)} - 40\,e^{(-4\,bx-4\,a)} - 10\,e^{(-6\,bx-6\,a)} + 5\,e^{(-8\,bx-8\,a)} + 2\,e^{(-10\,bx-10\,a)}}{20480\,b}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/20480*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) - 40*e^(-6*b*x - 6*a) +
20*e^(-8*b*x - 8*a) + 2)*e^(10*b*x + 10*a)/b + 3/256*(b*x + a)/b - 1/20480
*(20*e^(-2*b*x - 2*a) - 40*e^(-4*b*x - 4*a) - 10*e^(-6*b*x - 6*a) + 5*e^(-
8*b*x - 8*a) + 2*e^(-10*b*x - 10*a))/b
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.30

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{3}{256}\,x + \frac{e^{(10\,bx+10\,a)}}{10240\,b} + \frac{e^{(8\,bx+8\,a)}}{4096\,b} - \frac{e^{(6\,bx+6\,a)}}{2048\,b} - \frac{e^{(4\,bx+4\,a)}}{512\,b} + \frac{e^{(2\,bx+2\,a)}}{1024\,b} - \frac{e^{(-2\,bx-2\,a)}}{1024\,b} + \frac{e^{(-4\,bx-4\,a)}}{512\,b} + \frac{e^{(-6\,bx-6\,a)}}{2048\,b} - \frac{e^{(-8\,bx-8\,a)}}{4096\,b} - \frac{e^{(-10\,bx-10\,a)}}{10240\,b}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="giac")
```

output
```
3/256*x + 1/10240*e^(10*b*x + 10*a)/b + 1/4096*e^(8*b*x + 8*a)/b - 1/2048*
e^(6*b*x + 6*a)/b - 1/512*e^(4*b*x + 4*a)/b + 1/1024*e^(2*b*x + 2*a)/b - 1
/1024*e^(-2*b*x - 2*a)/b + 1/512*e^(-4*b*x - 4*a)/b + 1/2048*e^(-6*b*x - 6
*a)/b - 1/4096*e^(-8*b*x - 8*a)/b - 1/10240*e^(-10*b*x - 10*a)/b
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.59

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{20\sinh(2\,a+2\,b\,x) - 40\sinh(4\,a+4\,b\,x) - 10\sinh(6\,a+6\,b\,x) + 5\sinh(8\,a+8\,b\,x) + 2\sinh(10\,a+10\,b\,x}{10240\,b}$$

input
```
int(cosh(a + b*x)^6*sinh(a + b*x)^4,x)
```

output
```
(20*sinh(2*a + 2*b*x) - 40*sinh(4*a + 4*b*x) - 10*sinh(6*a + 6*b*x) + 5*si
nh(8*a + 8*b*x) + 2*sinh(10*a + 10*b*x) + 120*b*x)/(10240*b)
```

3.23 $\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$

3.23.1 Optimal result

Integrand size = 17, antiderivative size = 134

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = -\frac{5x}{1024} - \frac{5\cosh(a+bx)\sinh(a+bx)}{1024b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{1536b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{384b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b}$$

output
```
-5/1024*x-5/1024*cosh(b*x+a)*sinh(b*x+a)/b-5/1536*cosh(b*x+a)^3*sinh(b*x+a
)/b-1/384*cosh(b*x+a)^5*sinh(b*x+a)/b+1/64*cosh(b*x+a)^7*sinh(b*x+a)/b-1/2
4*cosh(b*x+a)^7*sinh(b*x+a)^3/b+1/12*cosh(b*x+a)^7*sinh(b*x+a)^5/b
```

3.23.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = \frac{-120a-120bx+45\sinh(4(a+bx))-9\sinh(8(a+bx))+\sinh(12(a+bx))}{24576b}$$

input
```
Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^6,x]
```

output
```
(-120*a - 120*b*x + 45*Sinh[4*(a + b*x)] - 9*Sinh[8*(a + b*x)] + Sinh[12*(
a + b*x)])/(24576*b)
```

3.23.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.941$, Rules used = {3042, 25, 3048, 3042, 3048, 25, 3042, 25, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^6(a+bx)\cosh^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^6\left(-\cos(ia+ibx)^6\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(ia+ibx)^6\sin(ia+ibx)^6dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\int \cosh^6(a+bx)\sinh^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\int \cos(ia+ibx)^6\sin(ia+ibx)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{3}{10}\int -\cosh^6(a+bx)\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}-\frac{3}{10}\int \cosh^6(a+bx)\sinh^2(a+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}-\frac{3}{10}\int -\cos(ia+ibx)^6\sin(ia+ibx)^2dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+\frac{3}{10}\int \cos(ia+ibx)^6\sin(ia+ibx)^2dx\right)$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\int \cosh^6(a+bx)dx-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}\right)+\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+\frac{3}{10}\left(-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^6dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \cosh^4(a+bx)dx+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}\right)+\frac{\sinh^3(a+bx)}{1}\right.$$

$$\Big\downarrow \text{3042}$$

3.23. $\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-$$
$$\frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+\frac{3}{10}\left(-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\left(\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\int \sin\right.\right.\right.$$

$\big\downarrow$ 3115

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-$$
$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)-\frac{\sinh(a+\right.\right.$$

$\big\downarrow$ 3042

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-$$
$$\frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+\frac{3}{10}\left(-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\left(\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sinh}{}\right.\right.\right.\right.$$

$\big\downarrow$ 3115

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-$$
$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+}{6b}\right.\right.\right.$$

$\big\downarrow$ 24

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-$$
$$\frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+\frac{3}{10}\left(\frac{1}{8}\left(\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin}{}\right.\right.\right.\right.\right.$$

input
```
Int[Cosh[a + b*x]^6*Sinh[a + b*x]^6,x]
```

output
```
(Cosh[a + b*x]^7*Sinh[a + b*x]^5)/(12*b) - (5*((Cosh[a + b*x]^7*Sinh[a + b
*x]^3)/(10*b) + (3*(-1/8*(Cosh[a + b*x]^7*Sinh[a + b*x])/b + ((Cosh[a + b*
x]^5*Sinh[a + b*x])/(6*b) + (5*((Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (3
*(x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4))/6)/8))/10))/12
```

3.23. $\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$

3.23.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.23.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.76

$$\frac{\frac{\sinh(bx+a)^5\cosh(bx+a)^7}{12}-\frac{\sinh(bx+a)^3\cosh(bx+a)^7}{24}+\frac{\sinh(bx+a)\cosh(bx+a)^7}{64}-\frac{\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)}{64}}{b}$$

input
```
int(cosh(b*x+a)^6*sinh(b*x+a)^6,x)
```

output
```
1/b*(1/12*sinh(b*x+a)^5*cosh(b*x+a)^7-1/24*sinh(b*x+a)^3*cosh(b*x+a)^7+1/6
4*sinh(b*x+a)*cosh(b*x+a)^7-1/64*(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^3+5/1
6*cosh(b*x+a))*sinh(b*x+a)-5/1024*b*x-5/1024*a)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.34

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$$
$$= \frac{55\cosh(bx+a)^3\sinh(bx+a)^9 + 3\cosh(bx+a)\sinh(bx+a)^{11} + 18\left(11\cosh(bx+a)^5 - \cosh(bx+a)\right.}{}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/6144*(55*cosh(b*x + a)^3*sinh(b*x + a)^9 + 3*cosh(b*x + a)*sinh(b*x + a)
^11 + 18*(11*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^7 + 18*(11*cos
h(b*x + a)^7 - 7*cosh(b*x + a)^3)*sinh(b*x + a)^5 + (55*cosh(b*x + a)^9 -
126*cosh(b*x + a)^5 + 45*cosh(b*x + a))*sinh(b*x + a)^3 - 30*b*x + 3*(cosh
(b*x + a)^11 - 6*cosh(b*x + a)^7 + 15*cosh(b*x + a)^3)*sinh(b*x + a))/b
```

3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(121) = 242$.

Time = 2.62 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.07

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$$
$$= \begin{cases} -\frac{5x\sinh^{12}(a+bx)}{1024} + \frac{15x\sinh^{10}(a+bx)\cosh^2(a+bx)}{512} - \frac{75x\sinh^8(a+bx)\cosh^4(a+bx)}{1024} + \frac{25x\sinh^6(a+bx)\cosh^6(a+bx)}{256} - \frac{75x\sinh^4(}{} \\ x\sinh^6(a)\cosh^6(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**6*sinh(b*x+a)**6,x)
```

output
```
Piecewise((-5*x*sinh(a + b*x)**12/1024 + 15*x*sinh(a + b*x)**10*cosh(a + b
*x)**2/512 - 75*x*sinh(a + b*x)**8*cosh(a + b*x)**4/1024 + 25*x*sinh(a + b
*x)**6*cosh(a + b*x)**6/256 - 75*x*sinh(a + b*x)**4*cosh(a + b*x)**8/1024
+ 15*x*sinh(a + b*x)**2*cosh(a + b*x)**10/512 - 5*x*cosh(a + b*x)**12/1024
 + 5*sinh(a + b*x)**11*cosh(a + b*x)/(1024*b) - 85*sinh(a + b*x)**9*cosh(a
 + b*x)**3/(3072*b) + 33*sinh(a + b*x)**7*cosh(a + b*x)**5/(512*b) + 33*si
nh(a + b*x)**5*cosh(a + b*x)**7/(512*b) - 85*sinh(a + b*x)**3*cosh(a + b*x
)**9/(3072*b) + 5*sinh(a + b*x)*cosh(a + b*x)**11/(1024*b), Ne(b, 0)), (x*
sinh(a)**6*cosh(a)**6, True))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.64

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = -\frac{\left(9\,e^{(-4\,bx-4\,a)}-45\,e^{(-8\,bx-8\,a)}-1\right)e^{(12\,bx+12\,a)}}{49152\,b} - \frac{5\,(bx+a)}{1024\,b} - \frac{45\,e^{(-4\,bx-4\,a)}-9\,e^{(-8\,bx-8\,a)}+e^{(-12\,bx-12\,a)}}{49152\,b}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/49152*(9*e^(-4*b*x - 4*a) - 45*e^(-8*b*x - 8*a) - 1)*e^(12*b*x + 12*a)/
b - 5/1024*(b*x + a)/b - 1/49152*(45*e^(-4*b*x - 4*a) - 9*e^(-8*b*x - 8*a)
 + e^(-12*b*x - 12*a))/b
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = -\frac{5}{1024}\,x + \frac{e^{(12\,bx+12\,a)}}{49152\,b} - \frac{3\,e^{(8\,bx+8\,a)}}{16384\,b} + \frac{15\,e^{(4\,bx+4\,a)}}{16384\,b} - \frac{15\,e^{(-4\,bx-4\,a)}}{16384\,b} + \frac{3\,e^{(-8\,bx-8\,a)}}{16384\,b} - \frac{e^{(-12\,bx-12\,a)}}{49152\,b}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="giac")
```

output
```
-5/1024*x + 1/49152*e^(12*b*x + 12*a)/b - 3/16384*e^(8*b*x + 8*a)/b + 15/1
6384*e^(4*b*x + 4*a)/b - 15/16384*e^(-4*b*x - 4*a)/b + 3/16384*e^(-8*b*x -
 8*a)/b - 1/49152*e^(-12*b*x - 12*a)/b
```

3.23.9 Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.31

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = \frac{\frac{15\sinh(4\,a+4\,b\,x)}{8192}-\frac{3\sinh(8\,a+8\,b\,x)}{8192}+\frac{\sinh(12\,a+12\,b\,x)}{24576}}{b} - \frac{5\,x}{1024}$$

input
```
int(cosh(a + b*x)^6*sinh(a + b*x)^6,x)
```

output
```
((15*sinh(4*a + 4*b*x))/8192 - (3*sinh(8*a + 8*b*x))/8192 + sinh(12*a + 12
*b*x)/24576)/b - (5*x)/1024
```

3.23. $\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$

3.24 $\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$

3.24.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}$$

output
```
ln(tanh(b*x+a))/b
```

3.24.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx=2\left(-\frac{\log(\cosh(a+bx))}{2b}+\frac{\log(\sinh(a+bx))}{2b}\right)$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
2*(-1/2*Log[Cosh[a + b*x]]/b + Log[Sinh[a + b*x]]/(2*b))
```

3.24.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 26, 3100, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)dx$$

$$\Big\downarrow \text{ 3100}$$

$$\frac{\int -i\coth(a+bx)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\log(i\tanh(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
Log[I*Tanh[a + b*x]]/b
```

3.24.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.24.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(\tanh(bx+a))}{b}$	12
default	$\frac{\ln(\tanh(bx+a))}{b}$	12
risch	$\frac{\ln(e^{2bx+2a}-1)}{b} - \frac{\ln(1+e^{2bx+2a})}{b}$	35

input
```
int(csch(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(tanh(b*x+a))/b
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(11) = 22.

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 5.45

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right) - \log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

output
```
-(log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - log(2*sinh(b*x +
a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

3.24.6 Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x), x)
```

3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 4.55

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

output
```
log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
 1)/b
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.73

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(e^{(2\,bx+2\,a)}+1\right) - \log\left(e^{(bx+a)}+1\right) - \log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

output
```
-(log(e^(2*b*x + 2*a) + 1) - log(e^(b*x + a) + 1) - log(abs(e^(b*x + a) -
1)))/b
```

3.24.9 Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)),x)
```

output
```
-(2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

3.25 $\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

3.25.1 Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

output
```
-arctanh(cosh(b*x+a))/b+sech(b*x+a)/b
```

3.25.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx=-\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
-(Log[Cosh[(a + b*x)/2]]/b) + Log[Sinh[(a + b*x)/2]]/b + Sech[a + b*x]/b
```

3.25.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3102, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 3102}$$

$$\frac{\int -\frac{\operatorname{sech}^2(a+bx)}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\operatorname{sech}^2(a+bx)}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\operatorname{sech}(a+bx)-\int \frac{1}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\operatorname{sech}(a+bx)-\operatorname{arctanh}(\operatorname{sech}(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
(-ArcTanh[Sech[a + b*x]] + Sech[a + b*x])/b
```

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.25.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{1}{\cosh(bx+a)}-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	23
default	$\frac{\frac{1}{\cosh(bx+a)}-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	23
risch	$\frac{2\,\mathrm{e}^{bx+a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}$	53

input
```
int(csch(b*x+a)*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

3.25.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(23) = 46.

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 6.74

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\left(\cosh\left(bx+a\right)^2+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2+1\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right.}{b\cosh\left(bx\right.}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*
log(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a
)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(cosh(b*x + a) + sinh(b*x + a) -
 1) - 2*cosh(b*x + a) - 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x
 + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

3.25.6 Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**2, x)
```

3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(23) = 46$.

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\,e^{(-bx-a)}}{b(e^{(-2\,bx-2\,a)}+1)}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
-log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-b*x - a)/(b*(e^
(-2*b*x - 2*a) + 1))
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(23) = 46$.

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}}-\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{2\,b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(4/(e^(b*x + a) + e^(-b*x - a)) - log(e^(b*x + a) + e^(-b*x - a) + 2)
+ log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)),x)
```

output
```
(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(b*x)*exp(a)*(-b
^2)^(1/2))/b))/(-b^2)^(1/2)
```

3.25. $\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

3.26 $\int \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$

3.26.1 Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

output
```
ln(tanh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

3.26.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=-\frac{2\log(\cosh(a+bx))-2\log(\sinh(a+bx))-\text{sech}^2(a+bx)}{2b}$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
-1/2*(2*Log[Cosh[a + b*x]] - 2*Log[Sinh[a + b*x]] - Sech[a + b*x]^2)/b
```

3.26.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int -i\coth(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int (i\tanh(a+bx)-i\coth(a+bx))d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\tanh^2(a+bx)+\log(i\tanh(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
(Log[I*Tanh[a + b*x]] - Tanh[a + b*x]^2/2)/b
```

3.26.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.26.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{1}{2\cosh(bx+a)^{2}}+\ln(\tanh(bx+a))}{b}$	23
default	$\frac{\frac{1}{2\cosh(bx+a)^{2}}+\ln(\tanh(bx+a))}{b}$	23
risch	$\frac{2\,\mathrm{e}^{2bx+2a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)^{2}}-\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	62

input
```
int(csch(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2/cosh(b*x+a)^2+ln(tanh(b*x+a)))
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 371, normalized size of antiderivative = 13.74

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{2\cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2\right.\right.}{}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
(2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x +
 a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x
+ a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*
x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*
x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(
b*x + a)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(
b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*
b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x
 + a))*sinh(b*x + a) + b)
```

3.26.6 Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**3, x)
```

3.26.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1\right)}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

3.26.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\frac{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+6}{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2} - \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right) + \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{2\,b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
 - 2*a) + 2) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x
+ 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

3.26.9 Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.89

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{2}{b\,\left(e^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

output
```
2/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/
b))/(-b^2)^(1/2) - 2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
```

3.27 $\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx$

3.27.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\operatorname{sech}(a+bx)}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}$$

output
```
-arctanh(cosh(b*x+a))/b+sech(b*x+a)/b+1/3*sech(b*x+a)^3/b
```

3.27.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx=-\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\operatorname{sech}(a+bx)}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x]^4,x]
```

output
```
-(Log[Cosh[(a + b*x)/2]]/b) + Log[Sinh[(a + b*x)/2]]/b + Sech[a + b*x]/b +
 Sech[a + b*x]^3/(3*b)
```

3.27.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3102, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\operatorname{sech}^4(a+bx)}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\operatorname{sech}^4(a+bx)}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\int \left(-\operatorname{sech}^2(a+bx)+\frac{1}{1-\operatorname{sech}^2(a+bx)}-1\right)d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\operatorname{arctanh}(\operatorname{sech}(a+bx))+\frac{1}{3}\operatorname{sech}^3(a+bx)+\operatorname{sech}(a+bx)}{b}$$

input
```
Int[Csch[a + b*x]*Sech[a + b*x]^4,x]
```

output
```
(-ArcTanh[Sech[a + b*x]] + Sech[a + b*x] + Sech[a + b*x]^3/3)/b
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.27.4 Maple [A] (verified)

Time = 7.68 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{1}{3\cosh(bx+a)^3}+\frac{1}{\cosh(bx+a)}-2\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	33
default	$\frac{\frac{1}{3\cosh(bx+a)^3}+\frac{1}{\cosh(bx+a)}-2\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	33
risch	$\frac{2\,e^{bx+a}\left(3\,e^{4bx+4a}+10\,e^{2bx+2a}+3\right)}{3b\left(1+e^{2bx+2a}\right)^3}-\frac{\ln\left(e^{bx+a}+1\right)}{b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}$	77

input
```
int(csch(b*x+a)*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3/cosh(b*x+a)^3+1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 697, normalized size of antiderivative = 18.34

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{6\cosh(bx+a)^5 + 30\cosh(bx+a)\sinh(bx+a)^4 + 6\sinh(bx+a)^5 + 20\left(3\cosh(bx+a)^2+1\right)\sinh(bx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/3*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a
)^5 + 20*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 20*cosh(b*x + a)^3 + 60
*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6 +
6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 +
 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*
x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1
) + 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
 + 3*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*co
sh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 +
6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x +
 a)^5 + 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x
 + a) + sinh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 + 10*cosh(b*x + a)^2 + 1
)*sinh(b*x + a) + 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*
sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*
x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
)*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*cos
h(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 + 2*b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

3.27.6 Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx = \int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**4,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**4, x)
```

3.27.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(36) = 72$.

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.84

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\left(3\,e^{(-bx-a)}+10\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}\right)}{3\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
-log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2/3*(3*e^(-b*x - a) +
 10*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-
4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1))
```

3.27.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(36) = 72$.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.32

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+4\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3} - 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{6\,b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="giac")
```

output
```
1/6*(4*(3*(e^(b*x + a) + e^(-b*x - a))^2 + 4)/(e^(b*x + a) + e^(-b*x - a))
^3 - 3*log(e^(b*x + a) + e^(-b*x - a) + 2) + 3*log(e^(b*x + a) + e^(-b*x -
 a) - 2))/b
```

3.27.9 Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.50

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{8\,e^{a+b\,x}}{3\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} + \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^4*sinh(a + b*x)),x)
```

output
```
(8*exp(a + b*x))/(3*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (2*at
an((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (8*exp(a + b*x))/(3*b
*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + (2*ex
p(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.28 $\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx$

3.28.1 Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{b}+\frac{\tanh^4(a+bx)}{4b}$$

output
```
ln(tanh(b*x+a))/b-tanh(b*x+a)^2/b+1/4*tanh(b*x+a)^4/b
```

3.28.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx$$
$$=-\frac{4\log(\cosh(a+bx))-4\log(\sinh(a+bx))-2\operatorname{sech}^2(a+bx)-\operatorname{sech}^4(a+bx)}{4b}$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x]^5,x]
```

output
```
-1/4*(4*Log[Cosh[a + b*x]] - 4*Log[Sinh[a + b*x]] - 2*Sech[a + b*x]^2 - Se
ch[a + b*x]^4)/b
```

3.28.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(a+bx)\text{sech}^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int -i\coth(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int -i\coth(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(-\tanh^2(a+bx)-i\coth(a+bx)+2\right)d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\tanh^2(a+bx)+2i\tanh(a+bx)+\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input

```
Int[Csch[a + b*x]*Sech[a + b*x]^5,x]
```

output

```
(Log[-Tanh[a + b*x]^2] + (2*I)*Tanh[a + b*x] - Tanh[a + b*x]^2/2)/(2*b)
```

3.28.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.28.4 Maple [A] (verified)

Time = 21.98 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{1}{4\cosh(bx+a)^4}+\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	33
default	$\frac{\frac{1}{4\cosh(bx+a)^4}+\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	33
risch	$\frac{2\,\mathrm{e}^{2bx+2a}\left(\mathrm{e}^{4bx+4a}+4\,\mathrm{e}^{2bx+2a}+1\right)}{b\left(1+\mathrm{e}^{2bx+2a}\right)^4}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}-\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}$	84

input
```
int(csch(b*x+a)*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4/cosh(b*x+a)^4+1/2/cosh(b*x+a)^2+ln(tanh(b*x+a)))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. 2(38) = 76.

Time = 0.26 (sec) , antiderivative size = 1073, normalized size of antiderivative = 26.82

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
(2*cosh(b*x + a)^6 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6
+ 2*(15*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^4 + 8*cosh(b*x + a)^4 + 8*(5*co
sh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^3 + 2*(15*cosh(b*x + a)^4 +
 24*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b*x +
 a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x
+ a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 3
*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x + a)
^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 10*co
sh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 +
15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x +
 a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh(
b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x
 + a))) + (cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x +
a)^8 + 4*(7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(
7*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)
^4 + 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*co
sh(b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*
(7*cosh(b*x + a)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x
+ a)^2 + 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*co
sh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(...
```

3.28.6 Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx=\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**5,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**5, x)
```

3.28.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(38) = 76$.

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.28

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx =\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{\log\left(e^{(-2bx-2a)}+1\right)}{b}+\frac{2\left(e^{(-2bx-2a)}+4e^{(-4bx-4a)}+e^{(-6bx-6a)}\right)}{b(4e^{(-2bx-2a)}+6e^{(-4bx-4a)}+4e^{(-6bx-6a)}+e^{(-8bx-8a)}+1)}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
 1)/b + 2*(e^(-2*b*x - 2*a) + 4*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/(b*(4
*e^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x -
8*a) + 1))
```

3.28.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(38) = 76$.

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.05

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx =\frac{\frac{3\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2+20e^{(2bx+2a)}+20e^{(-2bx-2a)}+44}{\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)^2}-2\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)+2\log\left(e^{(2bx+2a)}+\cdots\right.}{4b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="giac")
```

output
```
1/4*((3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 + 20*e^(2*b*x + 2*a) + 20*e
^(-2*b*x - 2*a) + 44)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)^2 - 2*log(e
^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + 2*log(e^(2*b*x + 2*a) + e^(-2*b*x
 - 2*a) - 2))/b
```

3.28.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 4.22

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{2}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)} - \frac{8}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)} + \frac{4}{b\,\left(4\,\mathrm{e}^{2\,a+2\,b\,x}+6\,\mathrm{e}^{4\,a+4\,b\,x}+4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^5*sinh(a + b*x)),x)
```

output
```
2/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/
b))/(-b^2)^(1/2) + 2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - 8/(
b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + 4/(b
*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*x) + exp(8*a +
 8*b*x) + 1))
```

3.29 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$

3.29.1 Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

output
```
-arctan(sinh(b*x+a))/b-csch(b*x+a)/b
```

3.29.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx\\=-\frac{\operatorname{csch}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\sinh^2(a+bx)\right)}{b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
-((Csch[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[a + b*x]^2])/b)
```

3.29.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 25, 3101, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)dx$$

$$\Big\downarrow \text{ 3101}$$

$$-\frac{i\int \frac{\operatorname{csch}^2(a+bx)}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\int -\frac{\operatorname{csch}^2(a+bx)}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{i\left(-\int \frac{1}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))-i\operatorname{csch}(a+bx)\right)}{b}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{i(i\arctan(\operatorname{csch}(a+bx))-i\operatorname{csch}(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
((-I)*(I*ArcTan[Csch[a + b*x]] - I*Csch[a + b*x]))/b
```

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.29.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)}-2\arctan\left(e^{bx+a}\right)}{b}$	25
default	$\frac{-\frac{1}{\sinh(bx+a)}-2\arctan\left(e^{bx+a}\right)}{b}$	25
risch	$-\frac{2\,e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}+\frac{i\ln\left(e^{bx+a}-i\right)}{b}-\frac{i\ln\left(e^{bx+a}+i\right)}{b}$	58

input
```
int(csch(b*x+a)^2*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)-2*arctan(exp(b*x+a)))
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(24) = 48$.

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.29

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx =$$
$$-\frac{2\left(\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2-1\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(b\right.\right.\right.}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

output
```
-2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1
)*arctan(cosh(b*x + a) + sinh(b*x + a)) + cosh(b*x + a) + sinh(b*x + a))/(
b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 -
b)
```

3.29.6 Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^2\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x), x)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{2\,e^{(-bx-a)}}{b(e^{(-2\,bx-2\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

output
```
2*arctan(e^(-b*x - a))/b + 2*e^(-b*x - a)/(b*(e^(-2*b*x - 2*a) - 1))
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\pi + \frac{4}{e^{(bx+a)}-e^{(-bx-a)}} + 2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{2b}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(pi + 4/(e^(b*x + a) - e^(-b*x - a)) + 2*arctan(1/2*(e^(2*b*x + 2*a)
- 1)*e^(-b*x - a)))/b
```

3.29.9 Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

output
```
- (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))
/(b*(exp(2*a + 2*b*x) - 1))
```

3.30 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$

3.30.1 Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\coth(a+bx)}{b} - \frac{\tanh(a+bx)}{b}$$

output
```
-coth(b*x+a)/b-tanh(b*x+a)/b
```

3.30.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{2\coth(2(a+bx))}{b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
(-2*Coth[2*(a + b*x)])/b
```

3.30.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 25, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 3100}$$

$$\frac{i\int -\coth^2(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{i\int \left(1-\coth^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i(i\tanh(a+bx)+i\coth(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
(I*(I*Coth[a + b*x] + I*Tanh[a + b*x]))/b
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.30.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)}-2\tanh(bx+a)}{b}$	32
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)}-2\tanh(bx+a)}{b}$	32
risch	$-\frac{4}{b\left(e^{2bx+2a}-1\right)\left(1+e^{2bx+2a}\right)}$	32

input
```
int(csch(b*x+a)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)/cosh(b*x+a)-2*tanh(b*x+a))
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.52

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4}{b\cosh(bx+a)^4 + 4\,b\cosh(bx+a)^3\sinh(bx+a) + 6\,b\cosh(bx+a)^2\sinh(bx+a)^2 + 4\,b\cosh(bx+a)$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-4/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x +
 a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a
)^4 - b)
```

3.30.6 Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**2, x)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{4}{b(e^{(-4\,bx-4\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
4/(b*(e^(-4*b*x - 4*a) - 1))
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4}{b(e^{(4\,bx+4\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
-4/(b*(e^(4*b*x + 4*a) - 1))
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

output
```
-4/(b*(exp(4*a + 4*b*x) - 1))
```

3.31 $\int \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx$

3.31.1 Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{3\arctan(\sinh(a+bx))}{2b} - \frac{3\text{csch}(a+bx)}{2b} + \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}$$

output
```
-3/2*arctan(sinh(b*x+a))/b-3/2*csch(b*x+a)/b+1/2*csch(b*x+a)*sech(b*x+a)^2
/b
```

3.31.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{\text{csch}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},-\sinh^2(a+bx)\right)}{b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

output
```
-((Csch[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, -Sinh[a + b*x]^2])/b)
```

3.31.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 25, 3101, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{ 3101}$$

$$-\frac{i\int \frac{\text{csch}^4(a+bx)}{\left(\text{csch}^2(a+bx)+1\right)^2}d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{ 252}$$

$$-\frac{i\left(\frac{i\text{csch}^3(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}-\frac{3}{2}\int -\frac{\text{csch}^2(a+bx)}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))\right)}{b}$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{i\left(\frac{i\text{csch}^3(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}-\frac{3}{2}\left(\int \frac{1}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))+i\text{csch}(a+bx)\right)\right)}{b}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{i\left(\frac{i\text{csch}^3(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}-\frac{3}{2}(i\text{csch}(a+bx)-i\arctan(\text{csch}(a+bx)))\right)}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

output
```
((-I)*((-3*((-I)*ArcTan[Csch[a + b*x]] + I*Csch[a + b*x]))/2 + ((I/2)*Csch
[a + b*x]^3)/(1 + Csch[a + b*x]^2)))/b
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.31.4 Maple [A] (verified)

Time = 6.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	47
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	47
risch	$-\frac{\mathrm{e}^{bx+a}\left(3\,\mathrm{e}^{4bx+4a}+2\,\mathrm{e}^{2bx+2a}+3\right)}{b\left(1+\mathrm{e}^{2bx+2a}\right)^2\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{3i\ln\left(\mathrm{e}^{bx+a}-i\right)}{2b}-\frac{3i\ln\left(\mathrm{e}^{bx+a}+i\right)}{2b}$	95

input
```
int(csch(b*x+a)^2*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^2-3/2*sech(b*x+a)*tanh(b*x+a)-3*arctan(exp
(b*x+a)))
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 511, normalized size of antiderivative = 10.43

$$\int \mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx =$$
$$-\frac{3\cosh(bx+a)^5+15\cosh(bx+a)\sinh(bx+a)^4+3\sinh(bx+a)^5+2\left(15\cosh(bx+a)^2+1\right)\sinh(\cdots}{\cdots}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5
 + 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*(5*c
osh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*
sinh(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*
sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b
*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(5*c
osh(b*x + a)^4 + 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b*x + a))/(
b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6
+ b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*
cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x + a)^2 + (
15*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(3*b*c
osh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) - b)
```

3.31.6 Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**3, x)
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(43) = 86$.

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.84

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{3\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{3\,e^{(-bx-a)} + 2\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)}}{b(e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - e^{(-6\,bx-6\,a)} + 1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
3*arctan(e^(-b*x - a))/b - (3*e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + 3*e^(-5*
b*x - 5*a))/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - e^(-6*b*x - 6*a) + 1
))
```

3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.08

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\,\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+8\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+4\,e^{(bx+a)}-4\,e^{(-bx-a)}} + 6\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{4\,b}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 8)/((e^(b*x + a) - e^(-
b*x - a))^3 + 4*e^(b*x + a) - 4*e^(-b*x - a)) + 6*arctan(1/2*(e^(2*b*x + 2
*a) - 1)*e^(-b*x - a)))/b
```

3.31.9 Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.18

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)} - \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

output
```
(2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (3*atan
((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(exp(
2*a + 2*b*x) - 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

3.32 $\int \mathrm{csch}^2(a+bx)\mathrm{sech}^4(a+bx)\,dx$

3.32.1 Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \mathrm{csch}^2(a+bx)\mathrm{sech}^4(a+bx)\,dx=-\frac{\coth(a+bx)}{b}-\frac{2\tanh(a+bx)}{b}+\frac{\tanh^3(a+bx)}{3b}$$

output
```
-coth(b*x+a)/b-2*tanh(b*x+a)/b+1/3*tanh(b*x+a)^3/b
```

3.32.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \mathrm{csch}^2(a+bx)\mathrm{sech}^4(a+bx)\,dx=-\frac{\coth(a+bx)}{b}-\frac{5\tanh(a+bx)}{3b}-\frac{\mathrm{sech}^2(a+bx)\tanh(a+bx)}{3b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x]^4,x]
```

output
```
-(Coth[a + b*x]/b) - (5*Tanh[a + b*x])/(3*b) - (Sech[a + b*x]^2*Tanh[a + b
*x])/(3*b)
```

3.32.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 25, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(a+bx)\text{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{25}$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{i\int -\coth^2(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{i\int \left(-\coth^2(a+bx)-\tanh^2(a+bx)+2\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{1}{3}i\tanh^3(a+bx)+2i\tanh(a+bx)+i\coth(a+bx)\right)}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x]^4,x]
```

output
```
(I*(I*Coth[a + b*x] + (2*I)*Tanh[a + b*x] - (I/3)*Tanh[a + b*x]^3))/b
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.32.4 Maple [A] (verified)

Time = 16.52 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^3}-4\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	44
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^3}-4\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	44
risch	$-\frac{16\left(2\,e^{2bx+2a}+1\right)}{3b\left(e^{2bx+2a}-1\right)\left(1+e^{2bx+2a}\right)^3}$	45

input
```
int(csch(b*x+a)^2*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^3-4*(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a))
```

3.32.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(36) = 72.

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 6.05

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{7}+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{6}+b\sinh\left(bx+a\right)^{7}+2\,b\cosh\left(bx+a\right)^{5}+\left(21\,b\cosh\right.\right.}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
-16/3*(3*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x
+ a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^5 + (21*b*cos
h(b*x + a)^2 + 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 2*b*cosh(b*
x + a))*sinh(b*x + a)^4 + 5*(7*b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^2)*si
nh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3)*sinh(b*x + a
)^2 - 3*b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 + 10*b*cosh(b*x + a)^4 - b)
*sinh(b*x + a))
```

3.32.6 Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = \int \operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^4\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**4,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**4, x)
```

3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(36) = 72.

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.47

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{32\,e^{(-2\,bx-2\,a)}}{3\,b(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}+1)} - \frac{16}{3\,b(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}+1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
-32/3*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) - e^(-8
*b*x - 8*a) + 1)) - 16/3/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) - e^(
-8*b*x - 8*a) + 1))
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{2\left(\frac{3}{e^{(2\,bx+2\,a)}-1} - \frac{3\,e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}+5}{\left(e^{(2\,bx+2\,a)}+1\right)^3}\right)}{3\,b}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="giac")
```

output
```
-2/3*(3/(e^(2*b*x + 2*a) - 1) - (3*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a) +
5)/(e^(2*b*x + 2*a) + 1)^3)/b
```

3.32.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{\frac{2}{3\,b}+\frac{4\,e^{2\,a+2\,b\,x}}{b}+\frac{2\,e^{4\,a+4\,b\,x}}{3\,b}}{3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1} + \frac{\frac{2}{b}+\frac{2\,e^{2\,a+2\,b\,x}}{3\,b}}{2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} + \frac{2}{3\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^4*sinh(a + b*x)^2),x)
```

output
```
(2/(3*b) + (4*exp(2*a + 2*b*x))/b + (2*exp(4*a + 4*b*x))/(3*b))/(3*exp(2*a
 + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1) + (2/b + (2*exp(2*a
 + 2*b*x))/(3*b))/(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1) - 2/(b*(exp(
2*a + 2*b*x) - 1)) + 2/(3*b*(exp(2*a + 2*b*x) + 1))
```

3.32. $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx$

3.33 $\int \text{csch}^2(a+bx)\text{sech}^5(a+bx)\,dx$

3.33.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \text{csch}^2(a+bx)\text{sech}^5(a+bx)\,dx = -\frac{15\arctan(\sinh(a+bx))}{8b} - \frac{15\text{csch}(a+bx)}{8b} + \frac{5\text{csch}(a+bx)\text{sech}^2(a+bx)}{8b} + \frac{\text{csch}(a+bx)\text{sech}^4(a+bx)}{4b}$$

output
```
-15/8*arctan(sinh(b*x+a))/b-15/8*csch(b*x+a)/b+5/8*csch(b*x+a)*sech(b*x+a)
^2/b+1/4*csch(b*x+a)*sech(b*x+a)^4/b
```

3.33.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

$$\int \text{csch}^2(a+bx)\text{sech}^5(a+bx)\,dx = -\frac{\text{csch}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},-\sinh^2(a+bx)\right)}{b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x]^5,x]
```

output
```
-((Csch[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, -Sinh[a + b*x]^2])/b)
```

3.33.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 25, 3101, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(a+bx)\text{sech}^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{25}$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{i\int \frac{\text{csch}^6(a+bx)}{\left(\text{csch}^2(a+bx)+1\right)^3}d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int -\frac{\text{csch}^6(a+bx)}{\left(\text{csch}^2(a+bx)+1\right)^3}d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{i\left(\frac{5}{4}\int \frac{\text{csch}^4(a+bx)}{\left(\text{csch}^2(a+bx)+1\right)^2}d(-i\text{csch}(a+bx))+\frac{i\text{csch}^5(a+bx)}{4\left(\text{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{i\left(\frac{5}{4}\left(\frac{i\text{csch}^3(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}-\frac{3}{2}\int -\frac{\text{csch}^2(a+bx)}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))\right)+\frac{i\text{csch}^5(a+bx)}{4\left(\text{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{i\left(\frac{5}{4}\left(\frac{i\text{csch}^3(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}-\frac{3}{2}\left(\int \frac{1}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))+i\text{csch}(a+bx)\right)\right)+\frac{i\text{csch}^5(a+bx)}{4\left(\text{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

$$\Big\downarrow \; 219$$

$$-\frac{i\left(\frac{5}{4}\left(\frac{i\text{csch}^3(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}-\frac{3}{2}(i\text{csch}(a+bx)-i\arctan(\text{csch}(a+bx)))\right)+\frac{i\text{csch}^5(a+bx)}{4\left(\text{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x]^5,x]
```

output
```
((-I)*(((I/4)*Csch[a + b*x]^5)/(1 + Csch[a + b*x]^2)^2 + (5*((-3*((-I)*Arc
Tan[Csch[a + b*x]] + I*Csch[a + b*x]))/2 + ((I/2)*Csch[a + b*x]^3)/(1 + Cs
ch[a + b*x]^2)))/4))/b
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.33.4 Maple [A] (verified)

Time = 39.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^4}-5\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)-\frac{15\arctan\left(e^{bx+a}\right)}{4}}{b}$	60
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^4}-5\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)-\frac{15\arctan\left(e^{bx+a}\right)}{4}}{b}$	60
risch	$-\frac{e^{bx+a}\left(15\,e^{8bx+8a}+40\,e^{6bx+6a}+18\,e^{4bx+4a}+40\,e^{2bx+2a}+15\right)}{4b\left(1+e^{2bx+2a}\right)^4\left(e^{2bx+2a}-1\right)}+\frac{15i\ln\left(e^{bx+a}-i\right)}{8b}-\frac{15i\ln\left(e^{bx+a}+i\right)}{8b}$	117

input
```
int(csch(b*x+a)^2*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^4-5*(1/4*sech(b*x+a)^3+3/8*sech(b*x+a))*ta
nh(b*x+a)-15/4*arctan(exp(b*x+a)))
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. 2(62) = 124.

Time = 0.26 (sec) , antiderivative size = 1183, normalized size of antiderivative = 16.90

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/4*(15*cosh(b*x + a)^9 + 135*cosh(b*x + a)*sinh(b*x + a)^8 + 15*sinh(b*x
 + a)^9 + 20*(27*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^7 + 40*cosh(b*x + a)^7
 + 140*(9*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^6 + 6*(315*cosh
(b*x + a)^4 + 140*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^5 + 18*cosh(b*x + a)^
5 + 10*(189*cosh(b*x + a)^5 + 140*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(
b*x + a)^4 + 20*(63*cosh(b*x + a)^6 + 70*cosh(b*x + a)^4 + 9*cosh(b*x + a)
^2 + 2)*sinh(b*x + a)^3 + 40*cosh(b*x + a)^3 + 60*(9*cosh(b*x + a)^7 + 14*
cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^2 + 1
5*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10
+ 3*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + 3*cosh(b*x + a)^8 + 24*(5*c
osh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 +
 42*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(b*x + a)^6 + 12*(21*cosh
(b*x + a)^5 + 14*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105
*cosh(b*x + a)^6 + 105*cosh(b*x + a)^4 + 15*cosh(b*x + a)^2 - 1)*sinh(b*x
+ a)^4 - 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 21*cosh(b*x + a)^5 +
5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + 3*(15*cosh(b*x + a)^8
 + 28*cosh(b*x + a)^6 + 10*cosh(b*x + a)^4 - 4*cosh(b*x + a)^2 - 1)*sinh(b
*x + a)^2 - 3*cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 + 12*cosh(b*x + a)^7
+ 6*cosh(b*x + a)^5 - 4*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a) -
 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 5*(27*cosh(b*x + a)^8 + 56*...
```

3.33.6 Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = \int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**5,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**5, x)
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(62) = 124.

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.94

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx$$
$$= \frac{15\arctan\left(e^{(-bx-a)}\right)}{4b} - \frac{15\,e^{(-bx-a)}+40\,e^{(-3\,bx-3\,a)}+18\,e^{(-5\,bx-5\,a)}+40\,e^{(-7\,bx-7\,a)}+15\,e^{(-9\,bx-9\,a)}}{4\,b(3\,e^{(-2\,bx-2\,a)}+2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}-3\,e^{(-8\,bx-8\,a)}-e^{(-10\,bx-10\,a)}+1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
15/4*arctan(e^(-b*x - a))/b - 1/4*(15*e^(-b*x - a) + 40*e^(-3*b*x - 3*a) +
 18*e^(-5*b*x - 5*a) + 40*e^(-7*b*x - 7*a) + 15*e^(-9*b*x - 9*a))/(b*(3*e^
(-2*b*x - 2*a) + 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a) - 3*e^(-8*b*x - 8
*a) - e^(-10*b*x - 10*a) + 1))
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.77

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx =$$
$$-\frac{15\,\pi+\frac{4\left(7\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+36\,e^{(bx+a)}-36\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2}+\frac{32}{e^{(bx+a)}-e^{(-bx-a)}}+30\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{16\,b}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/16*(15*pi + 4*(7*(e^(b*x + a) - e^(-b*x - a))^3 + 36*e^(b*x + a) - 36*e
^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 32/(e^(b*x + a) - e^
(-b*x - a)) + 30*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{3\,e^{a+b\,x}}{2\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}-\frac{15\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^a\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}}+\frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)}-\frac{4\,e^{a+b\,x}}{b\,\left(4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}-\frac{7\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^5*sinh(a + b*x)^2),x)
```

output
```
(3*exp(a + b*x))/(2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (15*a
tan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) + (6*exp(a + b*x))/(
b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) - (4*e
xp(a + b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b
*x) + exp(8*a + 8*b*x) + 1)) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
 - (7*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

3.34 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$

3.34.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\coth^2(a+bx)}{2b} - \frac{\log(\tanh(a+bx))}{b}$$

output
```
-1/2*coth(b*x+a)^2/b-ln(tanh(b*x+a))/b
```

3.34.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\operatorname{csch}^2(a+bx) - 2\log(\cosh(a+bx)) + 2\log(\sinh(a+bx))}{2b}$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
-1/2*(Csch[a + b*x]^2 - 2*Log[Cosh[a + b*x]] + 2*Log[Sinh[a + b*x]])/b
```

3.34.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)dx$$

$$\Big\downarrow \text{3100}$$

$$-\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(i\coth^3(a+bx)-i\coth(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\coth^2(a+bx)+\log(i\tanh(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
-((Coth[a + b*x]^2/2 + Log[I*Tanh[a + b*x]])/b)
```

3.34.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.34.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2}-\ln(\tanh(bx+a))}{b}$	25
default	$\frac{-\frac{1}{2\sinh(bx+a)^2}-\ln(\tanh(bx+a))}{b}$	25
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)^2}-\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}+\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}$	62

input
```
int(csch(b*x+a)^3*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2-ln(tanh(b*x+a)))
```

3.34.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 379, normalized size of antiderivative = 13.54

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx =$$

$$-\frac{2\cosh(bx+a)^2-\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a\right.\right.}{}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

output
```
-(2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
 sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh
(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x
 + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b
*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b
*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh
(b*x + a)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh
(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3
*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*
x + a))*sinh(b*x + a) + b)
```

3.34.6 Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x), x)
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.25

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

output
```
-log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + log(e^(-2*b*x - 2*a)
+ 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1)
)
```

3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.32

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\frac{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-6}{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2} + \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right) - \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{2\,b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

output
```
1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
 - 2*a) - 2) + log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - log(e^(2*b*x
+ 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.79

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{2}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

output
```
(2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 2/(b*(exp(2*
a + 2*b*x) - 1)) - 2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

3.35 $\int \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$

3.35.1 Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \frac{3\text{arctanh}(\cosh(a+bx))}{2b} - \frac{3\text{sech}(a+bx)}{2b} - \frac{\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

output
```
3/2*arctanh(cosh(b*x+a))/b-3/2*sech(b*x+a)/b-1/2*csch(b*x+a)^2*sech(b*x+a)
/b
```

3.35.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\text{sech}(a+bx)}{b}$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
-1/8*Csch[(a + b*x)/2]^2/b + (3*Log[Cosh[(a + b*x)/2]])/(2*b) - (3*Log[Sin
h[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b) - Sech[a + b*x]/b
```

3.35.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 3102, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 3102}$$

$$-\frac{\int \frac{\operatorname{sech}^4(a+bx)}{\left(1-\operatorname{sech}^2(a+bx)\right)^2}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{ 252}$$

$$-\frac{\frac{\operatorname{sech}^3(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{3}{2}\int \frac{\operatorname{sech}^2(a+bx)}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{\frac{\operatorname{sech}^3(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{3}{2}\left(\int \frac{1}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)-\operatorname{sech}(a+bx)\right)}{b}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{\operatorname{sech}^3(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{3}{2}(\operatorname{arctanh}(\operatorname{sech}(a+bx))-\operatorname{sech}(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
-(((-3*(ArcTanh[Sech[a + b*x]] - Sech[a + b*x]))/2 + Sech[a + b*x]^3/(2*(1
 - Sech[a + b*x]^2)))/b)
```

3.35.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.35.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)}-\frac{3}{2\cosh(bx+a)}+3\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	43
default	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)}-\frac{3}{2\cosh(bx+a)}+3\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	43
risch	$-\frac{\mathrm{e}^{bx+a}\left(3\,\mathrm{e}^{4bx+4a}-2\,\mathrm{e}^{2bx+2a}+3\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^2\left(1+\mathrm{e}^{2bx+2a}\right)}-\frac{3\ln\left(\mathrm{e}^{bx+a}-1\right)}{2b}+\frac{3\ln\left(\mathrm{e}^{bx+a}+1\right)}{2b}$	91

input
```
int(csch(b*x+a)^3*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)-3/2/cosh(b*x+a)+3*arctanh(exp(b*x+a)))
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(43) = 86$.

Time = 0.26 (sec) , antiderivative size = 709, normalized size of antiderivative = 14.47

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx =$$

$$-\frac{6\cosh(bx+a)^5+30\cosh(bx+a)\sinh(bx+a)^4+6\sinh(bx+a)^5+4\left(15\cosh(bx+a)^2-1\right)\sinh(}{}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x +
a)^5 + 4*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 + 12
*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6
+ 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x +
 a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b
*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) +
3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (
15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x
+ a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x
 + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + si
nh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 - 2*cosh(b*x + a)^2 + 1)*sinh(b*x
+ a) + 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x +
a)^5 + b*sinh(b*x + a)^6 - b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - b)*
sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^
3 - b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 - b)*s
inh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 - b*cosh(b*x
 + a))*sinh(b*x + a) + b)
```

3.35.6 Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**2, x)
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(43) = 86$.

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.16

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\log\left(e^{(-bx-a)}+1\right)}{2b} - \frac{3\log\left(e^{(-bx-a)}-1\right)}{2b} + \frac{3e^{(-bx-a)} - 2e^{(-3bx-3a)} + 3e^{(-5bx-5a)}}{b\left(e^{(-2bx-2a)} + e^{(-4bx-4a)} - e^{(-6bx-6a)} - 1\right)}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
3/2*log(e^(-b*x - a) + 1)/b - 3/2*log(e^(-b*x - a) - 1)/b + (3*e^(-b*x - a
) - 2*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(e^(-2*b*x - 2*a) + e^(-4*
b*x - 4*a) - e^(-6*b*x - 6*a) - 1))
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(43) = 86$.

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.24

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-8\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-4e^{(bx+a)}-4e^{(-bx-a)}} - 3\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + 3\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*(4*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 8)/((e^(b*x + a) + e^(-b*x - a
))^3 - 4*e^(b*x + a) - 4*e^(-b*x - a)) - 3*log(e^(b*x + a) + e^(-b*x - a)
+ 2) + 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

3.35.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.27

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

output
```
(3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x))
/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*
a + 2*b*x) - 1)) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.36 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

3.36.1 Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}+\frac{\tanh^2(a+bx)}{2b}$$

output
```
-1/2*coth(b*x+a)^2/b-2*ln(tanh(b*x+a))/b+1/2*tanh(b*x+a)^2/b
```

3.36.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=8\left(-\frac{\operatorname{csch}^2(a+bx)}{16b}+\frac{\log(\cosh(a+bx))}{4b}-\frac{\log(\sinh(a+bx))}{4b}-\frac{\operatorname{sech}^2(a+bx)}{16b}\right)$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
8*(-1/16*Csch[a + b*x]^2/b + Log[Cosh[a + b*x]]/(4*b) - Log[Sinh[a + b*x]]
/(4*b) - Sech[a + b*x]^2/(16*b))
```

3.36.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)^3dx$$

$$\Big\downarrow \ 26$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)^3dx$$

$$\Big\downarrow \ 3100$$

$$-\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \ 243$$

$$-\frac{\int -\coth^2(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \ 49$$

$$-\frac{\int \left(-\coth^2(a+bx)-2i\coth(a+bx)+1\right)d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\tanh^2(a+bx)+i\coth(a+bx)+2\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
-1/2*(I*Coth[a + b*x] + 2*Log[-Tanh[a + b*x]^2] - Tanh[a + b*x]^2)/b
```

3.36.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.36.4 Maple [A] (verified)

Time = 11.82 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^2}-\frac{1}{\cosh(bx+a)^2}-2\ln(\tanh(bx+a))}{b}$	43
default	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^2}-\frac{1}{\cosh(bx+a)^2}-2\ln(\tanh(bx+a))}{b}$	43
risch	$-\frac{4\,\mathrm{e}^{2bx+2a}\left(\mathrm{e}^{4bx+4a}+1\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^2\left(1+\mathrm{e}^{2bx+2a}\right)^2}+\frac{2\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}-\frac{2\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	87

input
```
int(csch(b*x+a)^3*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^2-1/cosh(b*x+a)^2-2*ln(tanh(b*x+a)))
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 774, normalized size of antiderivative = 18.00

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2*(2*cosh(b*x + a)^6 + 40*cosh(b*x + a)^3*sinh(b*x + a)^3 + 30*cosh(b*x +
 a)^2*sinh(b*x + a)^4 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)
^6 + 2*(15*cosh(b*x + a)^4 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cos
h(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sin
h(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*c
osh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x +
a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x
+ a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x +
 a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x
+ a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x
+ a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*
x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 -
 cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2
)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) +
1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(3*cosh(b*x +
a)^5 + cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^8 + 56*b*cosh(b*x +
a)^3*sinh(b*x + a)^5 + 28*b*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*cosh(b*x
 + a)*sinh(b*x + a)^7 + b*sinh(b*x + a)^8 - 2*b*cosh(b*x + a)^4 + 2*(35*b*
cosh(b*x + a)^4 - b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - b*cosh(b*x
 + a))*sinh(b*x + a)^3 + 4*(7*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*...
```

3.36.6 Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**3, x)
```

3.36.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(39) = 78.

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=-\frac{2\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{2\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\log\left(e^{(-2bx-2a)}+1\right)}{b}+\frac{4\left(e^{(-2bx-2a)}+e^{(-6bx-6a)}\right)}{b(2e^{(-4bx-4a)}-e^{(-8bx-8a)}-1)}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2*log(e^(-b*x - a) + 1)/b - 2*log(e^(-b*x - a) - 1)/b + 2*log(e^(-2*b*x -
 2*a) + 1)/b + 4*(e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/(b*(2*e^(-4*b*x - 4
*a) - e^(-8*b*x - 8*a) - 1))
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.23

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=-\frac{\frac{4\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)}{\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2-4}-\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)+\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}{b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
-(4*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/((e^(2*b*x + 2*a) + e^(-2*b*x - 2
*a))^2 - 4) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x +
 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

3.36.9 Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.23

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{4\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{4\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)} - \frac{8\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{8\,a+8\,b\,x}-2\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

output
```
(4*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (4*exp(2*a +
 2*b*x))/(b*(exp(4*a + 4*b*x) - 1)) - (8*exp(2*a + 2*b*x))/(b*(exp(8*a + 8
*b*x) - 2*exp(4*a + 4*b*x) + 1))
```

3.37 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx$

3.37.1 Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{5\operatorname{arctanh}(\cosh(a+bx))}{2b} - \frac{5\operatorname{sech}(a+bx)}{2b} - \frac{5\operatorname{sech}^3(a+bx)}{6b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b}$$

output
```
5/2*arctanh(cosh(b*x+a))/b-5/2*sech(b*x+a)/b-5/6*sech(b*x+a)^3/b-1/2*csch(
b*x+a)^2*sech(b*x+a)^3/b
```

3.37.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{5\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{5\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{2\operatorname{sech}(a+bx)}{b} - \frac{\operatorname{sech}^3(a+bx)}{3b}$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x]^4,x]
```

output
```
-1/8*Csch[(a + b*x)/2]^2/b + (5*Log[Cosh[(a + b*x)/2]])/(2*b) - (5*Log[Sin
h[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b) - (2*Sech[a + b*x])/b -
 Sech[a + b*x]^3/(3*b)
```

3.37.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 3102, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$-\frac{\int \frac{\operatorname{sech}^6(a+bx)}{\left(1-\operatorname{sech}^2(a+bx)\right)^2}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\operatorname{sech}^5(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{5}{2}\int \frac{\operatorname{sech}^4(a+bx)}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{\operatorname{sech}^5(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{5}{2}\int \left(-\operatorname{sech}^2(a+bx)+\frac{1}{1-\operatorname{sech}^2(a+bx)}-1\right)d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\operatorname{sech}^5(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{5}{2}\left(\operatorname{arctanh}(\operatorname{sech}(a+bx))-\frac{1}{3}\operatorname{sech}^3(a+bx)-\operatorname{sech}(a+bx)\right)}{b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x]^4,x]
```

output
```
-((Sech[a + b*x]^5/(2*(1 - Sech[a + b*x]^2)) - (5*(ArcTanh[Sech[a + b*x]]
- Sech[a + b*x] - Sech[a + b*x]^3/3))/2)/b)
```

3.37.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.37.4 Maple [A] (verified)

Time = 30.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^3}-\frac{5}{6\cosh(bx+a)^3}-\frac{5}{2\cosh(bx+a)}+5\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	53
default	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^3}-\frac{5}{6\cosh(bx+a)^3}-\frac{5}{2\cosh(bx+a)}+5\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	53
risch	$-\frac{\mathrm{e}^{bx+a}\left(15\,\mathrm{e}^{8bx+8a}+20\,\mathrm{e}^{6bx+6a}-22\,\mathrm{e}^{4bx+4a}+20\,\mathrm{e}^{2bx+2a}+15\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^2\left(1+\mathrm{e}^{2bx+2a}\right)^3}-\frac{5\ln\left(\mathrm{e}^{bx+a}-1\right)}{2b}+\frac{5\ln\left(\mathrm{e}^{bx+a}+1\right)}{2b}$	113

input
```
int(csch(b*x+a)^3*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^3-5/6/cosh(b*x+a)^3-5/2/cosh(b*x+a)+5*
arctanh(exp(b*x+a)))
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1573 vs. 2(58) = 116.

Time = 0.25 (sec) , antiderivative size = 1573, normalized size of antiderivative = 23.83

$$\int \mathrm{csch}^3(a+bx)\mathrm{sech}^4(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/6*(30*cosh(b*x + a)^9 + 270*cosh(b*x + a)*sinh(b*x + a)^8 + 30*sinh(b*x
 + a)^9 + 40*(27*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^7 + 40*cosh(b*x + a)^7
 + 280*(9*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^6 + 4*(945*cosh(b
*x + a)^4 + 210*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 44*cosh(b*x + a)^5
 + 20*(189*cosh(b*x + a)^5 + 70*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b
*x + a)^4 + 40*(63*cosh(b*x + a)^6 + 35*cosh(b*x + a)^4 - 11*cosh(b*x + a)
^2 + 1)*sinh(b*x + a)^3 + 40*cosh(b*x + a)^3 + 40*(27*cosh(b*x + a)^7 + 21
*cosh(b*x + a)^5 - 11*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 -
 15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^1
0 + (45*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + cosh(b*x + a)^8 + 8*(15*cos
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 + 1
4*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4*(63*cosh(b*
x + a)^5 + 14*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*
cosh(b*x + a)^6 + 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^4 - 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 7*cosh(b*x + a)^5 - 5*c
osh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^8 + 28
*cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 + 1)*sinh(b*x +
 a)^2 + cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 + 4*cosh(b*x + a)^7 - 6*cos
h(b*x + a)^5 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(c
osh(b*x + a) + sinh(b*x + a) + 1) + 15*(cosh(b*x + a)^10 + 10*cosh(b*x ...
```

3.37.6 Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**4,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**4, x)
```

3.37.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(58) = 116$.

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.26

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{5\,\log\left(e^{(-bx-a)}+1\right)}{2\,b} - \frac{5\,\log\left(e^{(-bx-a)}-1\right)}{2\,b} - \frac{15\,e^{(-bx-a)}+20\,e^{(-3\,bx-3\,a)}-22\,e^{(-5\,bx-5\,a)}+20\,e^{(-7\,bx-7\,a)}+15\,e^{(-9\,bx-9\,a)}}{3\,b(e^{(-2\,bx-2\,a)}-2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
5/2*log(e^(-b*x - a) + 1)/b - 5/2*log(e^(-b*x - a) - 1)/b - 1/3*(15*e^(-b*
x - a) + 20*e^(-3*b*x - 3*a) - 22*e^(-5*b*x - 5*a) + 20*e^(-7*b*x - 7*a) +
 15*e^(-9*b*x - 9*a))/(b*(e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) - 2*e^(-6*
b*x - 6*a) + e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1))
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(58) = 116$.

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.94

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{\frac{12\,\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} + \frac{16\,\left(3\,\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+2\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3} - 15\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + 15\,\log\left(e^{(bx+a)}+e^{(-bx}\right.}{12\,b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/12*(12*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4
) + 16*(3*(e^(b*x + a) + e^(-b*x - a))^2 + 2)/(e^(b*x + a) + e^(-b*x - a))
^3 - 15*log(e^(b*x + a) + e^(-b*x - a) + 2) + 15*log(e^(b*x + a) + e^(-b*x
 - a) - 2))/b
```

3.37.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.91

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{5\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} + \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{4\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input

```
int(1/(cosh(a + b*x)^4*sinh(a + b*x)^3),x)
```

output

```
(5*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x))
/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(
2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (8*exp(a + b*x))/(3*b*(3*exp
(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) - exp(a + b*x)
/(b*(exp(2*a + 2*b*x) - 1)) - (4*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.38 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$

3.38.1 Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{\coth^2(a+bx)}{2b}-\frac{3\log(\tanh(a+bx))}{b}+\frac{3\tanh^2(a+bx)}{2b}-\frac{\tanh^4(a+bx)}{4b}$$

output
```
-1/2*coth(b*x+a)^2/b-3*ln(tanh(b*x+a))/b+3/2*tanh(b*x+a)^2/b-1/4*tanh(b*x+
a)^4/b
```

3.38.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{2\operatorname{csch}^2(a+bx)-12\log(\cosh(a+bx))+12\log(\sinh(a+bx))+4\operatorname{sech}^2(a+bx)+\operatorname{sech}^4(a+bx)}{4b}$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x]^5,x]
```

output
```
-1/4*(2*Csch[a + b*x]^2 - 12*Log[Cosh[a + b*x]] + 12*Log[Sinh[a + b*x]] +
4*Sech[a + b*x]^2 + Sech[a + b*x]^4)/b
```

3.38.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^3(a+bx)\text{sech}^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{3100}$$

$$-\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)^3d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\int -\coth^2(a+bx)\left(1-\tanh^2(a+bx)\right)^3d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(-\coth^2(a+bx)-3i\coth(a+bx)-\tanh^2(a+bx)+3\right)d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{2}\tanh^2(a+bx)+3i\tanh(a+bx)+i\coth(a+bx)+3\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x]^5,x]
```

output
```
-1/2*(I*Coth[a + b*x] + 3*Log[-Tanh[a + b*x]^2] + (3*I)*Tanh[a + b*x] - Ta
nh[a + b*x]^2/2)/b
```

3.38.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.38.4 Maple [A] (verified)

Time = 97.62 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^4}-\frac{3}{4\cosh(bx+a)^4}-\frac{3}{2\cosh(bx+a)^2}-3\ln(\tanh(bx+a))}{b}$	53
default	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^4}-\frac{3}{4\cosh(bx+a)^4}-\frac{3}{2\cosh(bx+a)^2}-3\ln(\tanh(bx+a))}{b}$	53
risch	$-\frac{2\,e^{2bx+2a}\left(3\,e^{8bx+8a}+6\,e^{6bx+6a}-2\,e^{4bx+4a}+6\,e^{2bx+2a}+3\right)}{b\left(1+e^{2bx+2a}\right)^4\left(e^{2bx+2a}-1\right)^2}+\frac{3\ln\left(1+e^{2bx+2a}\right)}{b}-\frac{3\ln\left(e^{2bx+2a}-1\right)}{b}$	122

input
```
int(csch(b*x+a)^3*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^4-3/4/cosh(b*x+a)^4-3/2/cosh(b*x+a)^2-
3*ln(tanh(b*x+a)))
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2103 vs. $2(52) = 104$.

Time = 0.26 (sec) , antiderivative size = 2103, normalized size of antiderivative = 36.26

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
-(6*cosh(b*x + a)^10 + 60*cosh(b*x + a)*sinh(b*x + a)^9 + 6*sinh(b*x + a)^
10 + 6*(45*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^8 + 12*cosh(b*x + a)^8 + 48*
(15*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(315*cosh(b*x +
 a)^4 + 84*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 24*(
63*cosh(b*x + a)^5 + 28*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^5 +
 12*(105*cosh(b*x + a)^6 + 70*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 + 1)*sin
h(b*x + a)^4 + 12*cosh(b*x + a)^4 + 16*(45*cosh(b*x + a)^7 + 42*cosh(b*x +
 a)^5 - 5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(45*cosh(
b*x + a)^8 + 56*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^12 + 12*cosh(b
*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*cosh(b*x + a)^2 + 1)*s
inh(b*x + a)^10 + 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 + cosh(b*x +
 a))*sinh(b*x + a)^9 + (495*cosh(b*x + a)^4 + 90*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^8 - cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 + 30*cosh(b*x + a)^3
 - cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 + 105*cosh(b*x
+ a)^4 - 7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(9
9*cosh(b*x + a)^7 + 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 - 3*cosh(b*x +
a))*sinh(b*x + a)^5 + (495*cosh(b*x + a)^8 + 420*cosh(b*x + a)^6 - 70*cosh
(b*x + a)^4 - 60*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 +
4*(55*cosh(b*x + a)^9 + 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5 - 20*co...
```

3.38.6 Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx=\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**5,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**5, x)
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(52) = 104$.

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.12

$$\begin{aligned}&\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx\\&=-\frac{3\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{3\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{3\log\left(e^{(-2bx-2a)}+1\right)}{b}\\&\quad-\frac{2\left(3e^{(-2bx-2a)}+6e^{(-4bx-4a)}-2e^{(-6bx-6a)}+6e^{(-8bx-8a)}+3e^{(-10bx-10a)}\right)}{b(2e^{(-2bx-2a)}-e^{(-4bx-4a)}-4e^{(-6bx-6a)}-e^{(-8bx-8a)}+2e^{(-10bx-10a)}+e^{(-12bx-12a)}+1)}\end{aligned}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
-3*log(e^(-b*x - a) + 1)/b - 3*log(e^(-b*x - a) - 1)/b + 3*log(e^(-2*b*x -
 2*a) + 1)/b - 2*(3*e^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4*a) - 2*e^(-6*b*x -
6*a) + 6*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a))/(b*(2*e^(-2*b*x - 2*a) -
 e^(-4*b*x - 4*a) - 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) + 2*e^(-10*b*x -
 10*a) + e^(-12*b*x - 12*a) + 1))
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(52) = 104$.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.95

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$$
$$= \frac{\frac{2\left(3e^{(2bx+2a)}+3e^{(-2bx-2a)}-10\right)}{e^{(2bx+2a)}+e^{(-2bx-2a)}-2} - \frac{9\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2+52e^{(2bx+2a)}+52e^{(-2bx-2a)}+84}{\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)^2} + 6\log\left(e^{(2bx+2a)}+e^{(-2bx-2}\right.}{4b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="giac")
```

output
```
1/4*(2*(3*e^(2*b*x + 2*a) + 3*e^(-2*b*x - 2*a) - 10)/(e^(2*b*x + 2*a) + e^
(-2*b*x - 2*a) - 2) - (9*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 + 52*e^(2*
b*x + 2*a) + 52*e^(-2*b*x - 2*a) + 84)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a)
 + 2)^2 + 6*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - 6*log(e^(2*b*x +
 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.22

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{6\,\mathrm{atan}\left(\frac{e^{2a}\,e^{2bx}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{4}{b\,\left(e^{2a+2bx}+1\right)} - \frac{2}{b\,\left(e^{2a+2bx}-1\right)} - \frac{2}{b\,\left(e^{4a+4bx}-2\,e^{2a+2bx}+1\right)} + \frac{8}{b\,\left(3\,e^{2a+2bx}+3\,e^{4a+4bx}+e^{6a+6bx}+1\right)} - \frac{4}{b\,\left(4\,e^{2a+2bx}+6\,e^{4a+4bx}+4\,e^{6a+6bx}+e^{8a+8bx}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^5*sinh(a + b*x)^3),x)
```

output
```
(6*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 4/(b*(exp(2*
a + 2*b*x) + 1)) - 2/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(exp(4*a + 4*b*x) -
 2*exp(2*a + 2*b*x) + 1)) + 8/(b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x)
+ exp(6*a + 6*b*x) + 1)) - 4/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) +
 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

3.39 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx$

3.39.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}+\frac{\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

output
```
arctan(sinh(b*x+a))/b+csch(b*x+a)/b-1/3*csch(b*x+a)^3/b
```

3.39.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\operatorname{csch}^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\sinh^2(a+bx)\right)}{3b}$$

input
```
Integrate[Csch[a + b*x]^4*Sech[a + b*x],x]
```

output
```
-1/3*(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Sinh[a + b*x]^2])/
b
```

3.39.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3101, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc(ia+ibx)^4\sec(ia+ibx)dx$$

$$\Big\downarrow \text{ 3101}$$

$$\frac{i\int -\frac{\operatorname{csch}^4(a+bx)}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{i\int \frac{\operatorname{csch}^4(a+bx)}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{ 254}$$

$$-\frac{i\int \left(\operatorname{csch}^2(a+bx)+\frac{1}{\operatorname{csch}^2(a+bx)+1}-1\right)d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i\left(i\arctan(\operatorname{csch}(a+bx))+\frac{1}{3}i\operatorname{csch}^3(a+bx)-i\operatorname{csch}(a+bx)\right)}{b}$$

input
```
Int[Csch[a + b*x]^4*Sech[a + b*x],x]
```

output
```
(I*(I*ArcTan[Csch[a + b*x]] - I*Csch[a + b*x] + (I/3)*Csch[a + b*x]^3))/b
```

3.39.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.39.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^3}+\frac{1}{\sinh(bx+a)}+2\arctan\left(e^{bx+a}\right)}{b}$	33
default	$\frac{-\frac{1}{3\sinh(bx+a)^3}+\frac{1}{\sinh(bx+a)}+2\arctan\left(e^{bx+a}\right)}{b}$	33
risch	$\frac{2\,e^{bx+a}\left(3\,e^{4bx+4a}-10\,e^{2bx+2a}+3\right)}{3b\left(e^{2bx+2a}-1\right)^3}+\frac{i\ln\left(e^{bx+a}+i\right)}{b}-\frac{i\ln\left(e^{bx+a}-i\right)}{b}$	82

input
```
int(csch(b*x+a)^4*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sinh(b*x+a)^3+1/sinh(b*x+a)+2*arctan(exp(b*x+a)))
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. 2(35) = 70.

Time = 0.25 (sec) , antiderivative size = 515, normalized size of antiderivative = 13.92

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\left(3\cosh(bx+a)^5+15\cosh(bx+a)\sinh(bx+a)^4+3\sinh(bx+a)^5+10\left(3\cosh(bx+a)^2-1\right)\sinh(\cdots\right.}{\cdots}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="fricas")
```

output
```
2/3*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a
)^5 + 10*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 10*cosh(b*x + a)^3 + 30
*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 +
6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 -
 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*
x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a))
 + 3*(5*cosh(b*x + a)^4 - 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b
*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b
*x + a)^6 - 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a
)^4 + 4*(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*co
sh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x
 + a)^2 + 6*(b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*si
nh(b*x + a) - b)
```

3.39.6 Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**4*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)**4*sech(a + b*x), x)
```

3.39.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(35) = 70$.

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{2\left(3\,e^{(-bx-a)} - 10\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)}\right)}{3\,b\left(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} - 1\right)}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="maxima")
```

output
```
-2*arctan(e^(-b*x - a))/b - 2/3*(3*e^(-b*x - a) - 10*e^(-3*b*x - 3*a) + 3*
e^(-5*b*x - 5*a))/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) - 1))
```

3.39.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(35) = 70$.

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.16

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{3\,\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2-4\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3} + 6\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{6\,b}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="giac")
```

output
```
1/6*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^2 - 4)/(e^(b*x + a) - e^(-b*
x - a))^3 + 6*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.49

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^a\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} + \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)^4),x)
```

output
```
(2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (8*exp(a + b*x))/(
3*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(
3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) + (2*exp(
a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.40 $\int \mathbf{csch}^4(a+bx)\mathbf{sech}^2(a+bx)\,dx$

3.40.1 Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx=\frac{2\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

output
```
2*coth(b*x+a)/b-1/3*coth(b*x+a)^3/b+tanh(b*x+a)/b
```

3.40.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx=\frac{5\coth(a+bx)}{3b}-\frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

input
```
Integrate[Csch[a + b*x]^4*Sech[a + b*x]^2,x]
```

output
```
(5*Coth[a + b*x])/(3*b) - (Coth[a + b*x]*Csch[a + b*x]^2)/(3*b) + Tanh[a +
 b*x]/b
```

3.40.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(ia+ibx)^4\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{3100}$$

$$-\frac{i\int \coth^4(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{i\int \left(\coth^4(a+bx)-2\coth^2(a+bx)+1\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(i\tanh(a+bx)-\frac{1}{3}i\coth^3(a+bx)+2i\coth(a+bx)\right)}{b}$$

input
```
Int[Csch[a + b*x]^4*Sech[a + b*x]^2,x]
```

output
```
((-I)*((2*I)*Coth[a + b*x] - (I/3)*Coth[a + b*x]^3 + I*Tanh[a + b*x]))/b
```

3.40.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.40.4 Maple [A] (verified)

Time = 9.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

method	result	size
risch	$-\frac{16\left(2\,{\mathrm e}^{2 b x +2 a}-1\right)}{3 b \left({\mathrm e}^{2 b x +2 a}-1\right)^{3} \left(1+{\mathrm e}^{2 b x +2 a}\right)}$	45
derivativedivides	$\frac{-\frac{1}{3 \sinh \left(b x +a \right)^{3} \cosh \left(b x +a \right)}+\frac{4}{3 \sinh \left(b x +a \right) \cosh \left(b x +a \right)}+\frac{8 \tanh \left(b x +a \right)}{3}}{b}$	50
default	$\frac{-\frac{1}{3 \sinh \left(b x +a \right)^{3} \cosh \left(b x +a \right)}+\frac{4}{3 \sinh \left(b x +a \right) \cosh \left(b x +a \right)}+\frac{8 \tanh \left(b x +a \right)}{3}}{b}$	50

input
```
int(csch(b*x+a)^4*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-16/3*(2*exp(2*b*x+2*a)-1)/b/(exp(2*b*x+2*a)-1)^3/(1+exp(2*b*x+2*a))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(35) = 70.

Time = 0.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 6.19

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^7+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^6+b\sinh\left(bx+a\right)^7-2\,b\cosh\left(bx+a\right)^5+\left(21\,b\cosh\right.\right.}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-16/3*(cosh(b*x + a) + 3*sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x
+ a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 2*b*cosh(b*x + a)^5 + (21*b*cos
h(b*x + a)^2 - 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 - 2*b*cosh(b*
x + a))*sinh(b*x + a)^4 + 5*(7*b*cosh(b*x + a)^4 - 4*b*cosh(b*x + a)^2)*si
nh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 - 20*b*cosh(b*x + a)^3)*sinh(b*x + a
)^2 + b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 - 10*b*cosh(b*x + a)^4 + 3*b)
*sinh(b*x + a))
```

3.40.6 Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**4*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)**4*sech(a + b*x)**2, x)
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(35) = 70$.

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx=\frac{32\,e^{(-2\,bx-2\,a)}}{3\,b(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}-1)}-\frac{16}{3\,b(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
32/3*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) + e^(-8*
b*x - 8*a) - 1)) - 16/3/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) + e^(-
8*b*x - 8*a) - 1))
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{2\left(\frac{3}{e^{(2\,bx+2\,a)}+1} - \frac{3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}+5}{\left(e^{(2\,bx+2\,a)}-1\right)^3}\right)}{3\,b}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
-2/3*(3/(e^(2*b*x + 2*a) + 1) - (3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a) +
5)/(e^(2*b*x + 2*a) - 1)^3)/b
```

3.40.9 Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 4.14

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\frac{2}{3\,b}-\frac{4\,e^{2\,a+2\,b\,x}}{b}+\frac{2\,e^{4\,a+4\,b\,x}}{3\,b}}{3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1} - \frac{\frac{2}{b}-\frac{2\,e^{2\,a+2\,b\,x}}{3\,b}}{e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1} + \frac{2}{3\,b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)^4),x)
```

output
```
(2/(3*b) - (4*exp(2*a + 2*b*x))/b + (2*exp(4*a + 4*b*x))/(3*b))/(3*exp(2*a
 + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1) - (2/b - (2*exp(2*a
 + 2*b*x))/(3*b))/(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1) + 2/(3*b*(ex
p(2*a + 2*b*x) - 1)) - 2/(b*(exp(2*a + 2*b*x) + 1))
```

3.41 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx$

3.41.1 Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{5\arctan(\sinh(a+bx))}{2b}+\frac{5\operatorname{csch}(a+bx)}{2b}-\frac{5\operatorname{csch}^3(a+bx)}{6b}+\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

output
```
5/2*arctan(sinh(b*x+a))/b+5/2*csch(b*x+a)/b-5/6*csch(b*x+a)^3/b+1/2*csch(b
*x+a)^3*sech(b*x+a)^2/b
```

3.41.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.50

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\operatorname{csch}^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},-\sinh^2(a+bx)\right)}{3b}$$

input
```
Integrate[Csch[a + b*x]^4*Sech[a + b*x]^3,x]
```

output
```
-1/3*(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, -Sinh[a + b*x]^2])/
b
```

3.41.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3101, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(ia+ibx)^4\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$\frac{i\int -\frac{\operatorname{csch}^6(a+bx)}{\left(\operatorname{csch}^2(a+bx)+1\right)^2}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{i\left(-\frac{5}{2}\int \frac{\operatorname{csch}^4(a+bx)}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))-\frac{i\operatorname{csch}^5(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}\right)}{b}$$

$$\Big\downarrow \text{254}$$

$$\frac{i\left(-\frac{5}{2}\int \left(\operatorname{csch}^2(a+bx)+\frac{1}{\operatorname{csch}^2(a+bx)+1}-1\right)d(-i\operatorname{csch}(a+bx))-\frac{i\operatorname{csch}^5(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}\right)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{5}{2}\left(-i\arctan(\operatorname{csch}(a+bx))-\frac{1}{3}i\operatorname{csch}^3(a+bx)+i\operatorname{csch}(a+bx)\right)-\frac{i\operatorname{csch}^5(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}\right)}{b}$$

input
```
Int[Csch[a + b*x]^4*Sech[a + b*x]^3,x]
```

output
```
(I*(((-1/2*I)*Csch[a + b*x]^5)/(1 + Csch[a + b*x]^2) - (5*((-I)*ArcTan[Csc
h[a + b*x]] + I*Csch[a + b*x] - (I/3)*Csch[a + b*x]^3))/2))/b
```

3.41.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.41.4 Maple [A] (verified)

Time = 23.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^2}+\frac{5}{3\sinh(bx+a)\cosh(bx+a)^2}+\frac{5\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}+5\arctan(\mathrm{e}^{bx+a})}{b}$	65
default	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^2}+\frac{5}{3\sinh(bx+a)\cosh(bx+a)^2}+\frac{5\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}+5\arctan(\mathrm{e}^{bx+a})}{b}$	65
risch	$\frac{\mathrm{e}^{bx+a}(15\,\mathrm{e}^{8bx+8a}-20\,\mathrm{e}^{6bx+6a}-22\,\mathrm{e}^{4bx+4a}-20\,\mathrm{e}^{2bx+2a}+15)}{3b(1+\mathrm{e}^{2bx+2a})^2(\mathrm{e}^{2bx+2a}-1)^3}+\frac{5i\ln(\mathrm{e}^{bx+a}+i)}{2b}-\frac{5i\ln(\mathrm{e}^{bx+a}-i)}{2b}$	117

input
```
int(csch(b*x+a)^4*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sinh(b*x+a)^3/cosh(b*x+a)^2+5/3/sinh(b*x+a)/cosh(b*x+a)^2+5/2*se
ch(b*x+a)*tanh(b*x+a)+5*arctan(exp(b*x+a)))
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1176 vs. $2(58) = 116$.

Time = 0.25 (sec) , antiderivative size = 1176, normalized size of antiderivative = 17.82

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3*(15*cosh(b*x + a)^9 + 135*cosh(b*x + a)*sinh(b*x + a)^8 + 15*sinh(b*x
+ a)^9 + 20*(27*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^7 - 20*cosh(b*x + a)^7
+ 140*(9*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^6 + 2*(945*cosh(b*
x + a)^4 - 210*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 22*cosh(b*x + a)^5
+ 10*(189*cosh(b*x + a)^5 - 70*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*
x + a)^4 + 20*(63*cosh(b*x + a)^6 - 35*cosh(b*x + a)^4 - 11*cosh(b*x + a)^
2 - 1)*sinh(b*x + a)^3 - 20*cosh(b*x + a)^3 + 20*(27*cosh(b*x + a)^7 - 21*
cosh(b*x + a)^5 - 11*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 +
15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10
 + (45*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - cosh(b*x + a)^8 + 8*(15*cosh
(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 - 14
*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4*(63*cosh(b*x
 + a)^5 - 14*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*c
osh(b*x + a)^6 - 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 + 1)*sinh(b*x + a
)^4 + 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 - 7*cosh(b*x + a)^5 - 5*co
sh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^8 - 28*
cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2 + 1)*sinh(b*x +
a)^2 + cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 - 4*cosh(b*x + a)^7 - 6*cosh
(b*x + a)^5 + 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*arctan
(cosh(b*x + a) + sinh(b*x + a)) + 5*(27*cosh(b*x + a)^8 - 28*cosh(b*x +...
```

3.41.6 Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int \operatorname{csch}^4{(a+bx)}\operatorname{sech}^3{(a+bx)}\,dx$$

input
```
integrate(csch(b*x+a)**4*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)**4*sech(a + b*x)**3, x)
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(58) = 116$.

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.00

$$\begin{aligned}&\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx\\&=-\frac{5\arctan\left(e^{(-bx-a)}\right)}{b}\\&\quad-\frac{15\,e^{(-bx-a)}-20\,e^{(-3\,bx-3\,a)}-22\,e^{(-5\,bx-5\,a)}-20\,e^{(-7\,bx-7\,a)}+15\,e^{(-9\,bx-9\,a)}}{3\,b(e^{(-2\,bx-2\,a)}+2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1)}\end{aligned}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-5*arctan(e^(-b*x - a))/b - 1/3*(15*e^(-b*x - a) - 20*e^(-3*b*x - 3*a) - 2
2*e^(-5*b*x - 5*a) - 20*e^(-7*b*x - 7*a) + 15*e^(-9*b*x - 9*a))/(b*(e^(-2*
b*x - 2*a) + 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) +
e^(-10*b*x - 10*a) - 1))
```

3.41.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(58) = 116$.

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.88

$$\begin{aligned}&\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx\\&=\frac{15\,\pi+\frac{12\,\left(e^{(bx+a)}-e^{(-bx-a)}\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4}+\frac{16\,\left(3\,\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2-2\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3}+30\arctan\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{12\,b}\end{aligned}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
1/12*(15*pi + 12*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a)
)^2 + 4) + 16*(3*(e^(b*x + a) - e^(-b*x - a))^2 - 2)/(e^(b*x + a) - e^(-b*
x - a))^3 + 30*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

3.41.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.83

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{5\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} + \frac{4\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} + \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)^4),x)
```

output
```
(5*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (8*exp(a + b*x))/(
3*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (2*exp(a + b*x))/(b*(2*
exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(3*exp(2
*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) + (4*exp(a + b*x
))/(b*(exp(2*a + 2*b*x) - 1)) + exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

3.42 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx$

3.42.1 Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{3\coth(a+bx)}{b} - \frac{\coth^3(a+bx)}{3b} + \frac{3\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b}$$

output
```
3*coth(b*x+a)/b-1/3*coth(b*x+a)^3/b+3*tanh(b*x+a)/b-1/3*tanh(b*x+a)^3/b
```

3.42.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = 16\left(\frac{\coth(2(a+bx))}{3b} - \frac{\coth(2(a+bx))\operatorname{csch}^2(2(a+bx))}{6b}\right)$$

input
```
Integrate[Csch[a + b*x]^4*Sech[a + b*x]^4,x]
```

output
```
16*(Coth[2*(a + b*x)]/(3*b) - (Coth[2*(a + b*x)]*Csch[2*(a + b*x)]^2)/(6*b
))
```

3.42.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc(ia+ibx)^4\sec(ia+ibx)^4dx$$

$$\Big\downarrow \; 3100$$

$$-\frac{i\int \coth^4(a+bx)\left(1-\tanh^2(a+bx)\right)^3d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \; 244$$

$$-\frac{i\int \left(\coth^4(a+bx)-3\coth^2(a+bx)-\tanh^2(a+bx)+3\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{i\left(-\frac{1}{3}i\tanh^3(a+bx)+3i\tanh(a+bx)-\frac{1}{3}i\coth^3(a+bx)+3i\coth(a+bx)\right)}{b}$$

input
```
Int[Csch[a + b*x]^4*Sech[a + b*x]^4,x]
```

output
```
((-I)*((3*I)*Coth[a + b*x] - (I/3)*Coth[a + b*x]^3 + (3*I)*Tanh[a + b*x] -
 (I/3)*Tanh[a + b*x]^3))/b
```

3.42.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.42.4 Maple [A] (verified)

Time = 73.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{32\left(3\,\mathrm{e}^{4bx+4a}-1\right)}{3b\left(1+\mathrm{e}^{2bx+2a}\right)^{3}\left(\mathrm{e}^{2bx+2a}-1\right)^{3}}$	45
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^{3}\cosh(bx+a)^{3}}+\frac{2}{\sinh(bx+a)\cosh(bx+a)^{3}}+8\left(\frac{2}{3}+\frac{\mathrm{sech}(bx+a)^{2}}{3}\right)\tanh(bx+a)}{b}$	62
default	$\frac{-\frac{1}{3\sinh(bx+a)^{3}\cosh(bx+a)^{3}}+\frac{2}{\sinh(bx+a)\cosh(bx+a)^{3}}+8\left(\frac{2}{3}+\frac{\mathrm{sech}(bx+a)^{2}}{3}\right)\tanh(bx+a)}{b}$	62

input
```
int(csch(b*x+a)^4*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-32/3*(3*exp(4*b*x+4*a)-1)/b/(1+exp(2*b*x+2*a))^3/(exp(2*b*x+2*a)-1)^3
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(49) = 98$.

Time = 0.23 (sec) , antiderivative size = 330, normalized size of antiderivative = 6.23

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{10}+120\,b\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)^{7}+45\,b\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{8}+10\,b\cosh\right.}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
-64/3*(cosh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/
(b*cosh(b*x + a)^10 + 120*b*cosh(b*x + a)^3*sinh(b*x + a)^7 + 45*b*cosh(b*
x + a)^2*sinh(b*x + a)^8 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 + b*sinh(b*x
 + a)^10 - 3*b*cosh(b*x + a)^6 + 3*(70*b*cosh(b*x + a)^4 - b)*sinh(b*x + a
)^6 + 18*(14*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a)^5 + 15*(14
*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 60*(2*b*cosh(b
*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a)^3 + 2*b*cosh(b*x + a)^2 + (45
*b*cosh(b*x + a)^8 - 45*b*cosh(b*x + a)^4 + 2*b)*sinh(b*x + a)^2 + 2*(5*b*
cosh(b*x + a)^9 - 9*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a))*sinh(b*x + a))
```

3.42.6 Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = \int \operatorname{csch}^4\left(a+bx\right)\operatorname{sech}^4\left(a+bx\right)\,dx$$

input
```
integrate(csch(b*x+a)**4*sech(b*x+a)**4,x)
```

output
```
Integral(csch(a + b*x)**4*sech(a + b*x)**4, x)
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{32\,e^{(-4\,bx-4\,a)}}{b(3\,e^{(-4\,bx-4\,a)}-3\,e^{(-8\,bx-8\,a)}+e^{(-12\,bx-12\,a)}-1)} - \frac{32}{3\,b(3\,e^{(-4\,bx-4\,a)}-3\,e^{(-8\,bx-8\,a)}+e^{(-12\,bx-12\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
32*e^(-4*b*x - 4*a)/(b*(3*e^(-4*b*x - 4*a) - 3*e^(-8*b*x - 8*a) + e^(-12*b
*x - 12*a) - 1)) - 32/3/(b*(3*e^(-4*b*x - 4*a) - 3*e^(-8*b*x - 8*a) + e^(-
12*b*x - 12*a) - 1))
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{32\left(3\,e^{(4\,bx+4\,a)}-1\right)}{3\,b(e^{(4\,bx+4\,a)}-1)^3}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="giac")
```

output
```
-32/3*(3*e^(4*b*x + 4*a) - 1)/(b*(e^(4*b*x + 4*a) - 1)^3)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{32\left(3\,\mathrm{e}^{4\,a+4\,b\,x}-1\right)}{3\,b\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)^3}$$

input
```
int(1/(cosh(a + b*x)^4*sinh(a + b*x)^4),x)
```

output
```
-(32*(3*exp(4*a + 4*b*x) - 1))/(3*b*(exp(4*a + 4*b*x) - 1)^3)
```

3.43 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx$

3.43.1 Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{35\arctan(\sinh(a+bx))}{8b}+\frac{35\operatorname{csch}(a+bx)}{8b}-\frac{35\operatorname{csch}^3(a+bx)}{24b}+\frac{7\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{8b}+\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)}{4b}$$

output
```
35/8*arctan(sinh(b*x+a))/b+35/8*csch(b*x+a)/b-35/24*csch(b*x+a)^3/b+7/8*cs
ch(b*x+a)^3*sech(b*x+a)^2/b+1/4*csch(b*x+a)^3*sech(b*x+a)^4/b
```

3.43.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.37

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{\operatorname{csch}^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},3,-\frac{1}{2},-\sinh^2(a+bx)\right)}{3b}$$

input
```
Integrate[Csch[a + b*x]^4*Sech[a + b*x]^5,x]
```

output
```
-1/3*(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, -Sinh[a + b*x]^2])/
b
```

3.43.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 3101, 25, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc(ia+ibx)^4\sec(ia+ibx)^5dx$$

$$\Big\downarrow \ 3101$$

$$\frac{i\int -\frac{\operatorname{csch}^8(a+bx)}{\left(\operatorname{csch}^2(a+bx)+1\right)^3}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{i\int \frac{\operatorname{csch}^8(a+bx)}{\left(\operatorname{csch}^2(a+bx)+1\right)^3}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \ 252$$

$$\frac{i\left(\frac{7}{4}\int -\frac{\operatorname{csch}^6(a+bx)}{\left(\operatorname{csch}^2(a+bx)+1\right)^2}d(-i\operatorname{csch}(a+bx))-\frac{i\operatorname{csch}^7(a+bx)}{4\left(\operatorname{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

$$\Big\downarrow \ 252$$

$$\frac{i\left(\frac{7}{4}\left(-\frac{5}{2}\int \frac{\operatorname{csch}^4(a+bx)}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))-\frac{i\operatorname{csch}^5(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}\right)-\frac{i\operatorname{csch}^7(a+bx)}{4\left(\operatorname{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

$$\Big\downarrow \ 254$$

$$\frac{i\left(\frac{7}{4}\left(-\frac{5}{2}\int\left(\text{csch}^2(a+bx)+\frac{1}{\text{csch}^2(a+bx)+1}-1\right)d(-i\text{csch}(a+bx))-\frac{i\text{csch}^5(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}\right)-\frac{i\text{csch}^7(a+bx)}{4\left(\text{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(\frac{7}{4}\left(-\frac{5}{2}\left(-i\arctan(\text{csch}(a+bx))-\frac{1}{3}i\text{csch}^3(a+bx)+i\text{csch}(a+bx)\right)-\frac{i\text{csch}^5(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}\right)-\frac{i\text{csch}^7(a+bx)}{4\left(\text{csch}^2(a+bx)+1\right)^2}\right.}{b}$$

input
```
Int[Csch[a + b*x]^4*Sech[a + b*x]^5,x]
```

output
```
(I*(((-1/4*I)*Csch[a + b*x]^7)/(1 + Csch[a + b*x]^2)^2 + (7*(((-1/2*I)*Csc
h[a + b*x]^5)/(1 + Csch[a + b*x]^2) - (5*((-I)*ArcTan[Csch[a + b*x]] + I*C
sch[a + b*x] - (I/3)*Csch[a + b*x]^3))/2))/4))/b
```

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.43.4 Maple [A] (verified)

Time = 151.89 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^4}+\frac{7}{3\sinh(bx+a)\cosh(bx+a)^4}+\frac{35\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)}{3}+\frac{35\arctan\left(e^{bx+a}\right)}{4}}{b}$
default	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^4}+\frac{7}{3\sinh(bx+a)\cosh(bx+a)^4}+\frac{35\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)}{3}+\frac{35\arctan\left(e^{bx+a}\right)}{4}}{b}$
risch	$\frac{e^{bx+a}\left(105\,e^{12bx+12a}+70\,e^{10bx+10a}-329\,e^{8bx+8a}-204\,e^{6bx+6a}-329\,e^{4bx+4a}+70\,e^{2bx+2a}+105\right)}{12b\left(1+e^{2bx+2a}\right)^4\left(e^{2bx+2a}-1\right)^3}+\frac{35i\ln\left(e^{bx+a}+i\right)}{8b}-$

input
```
int(csch(b*x+a)^4*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sinh(b*x+a)^3/cosh(b*x+a)^4+7/3/sinh(b*x+a)/cosh(b*x+a)^4+35/3*(
1/4*sech(b*x+a)^3+3/8*sech(b*x+a))*tanh(b*x+a)+35/4*arctan(exp(b*x+a)))
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2092 vs. 2(79) = 158.

Time = 0.26 (sec) , antiderivative size = 2092, normalized size of antiderivative = 23.51

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/12*(105*cosh(b*x + a)^13 + 1365*cosh(b*x + a)*sinh(b*x + a)^12 + 105*sin
h(b*x + a)^13 + 70*(117*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^11 + 70*cosh(b*
x + a)^11 + 770*(39*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^10 + 7*
(10725*cosh(b*x + a)^4 + 550*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^9 - 329*c
osh(b*x + a)^9 + 21*(6435*cosh(b*x + a)^5 + 550*cosh(b*x + a)^3 - 141*cosh
(b*x + a))*sinh(b*x + a)^8 + 12*(15015*cosh(b*x + a)^6 + 1925*cosh(b*x + a
)^4 - 987*cosh(b*x + a)^2 - 17)*sinh(b*x + a)^7 - 204*cosh(b*x + a)^7 + 84
*(2145*cosh(b*x + a)^7 + 385*cosh(b*x + a)^5 - 329*cosh(b*x + a)^3 - 17*co
sh(b*x + a))*sinh(b*x + a)^6 + 7*(19305*cosh(b*x + a)^8 + 4620*cosh(b*x +
a)^6 - 5922*cosh(b*x + a)^4 - 612*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^5 -
329*cosh(b*x + a)^5 + 7*(10725*cosh(b*x + a)^9 + 3300*cosh(b*x + a)^7 - 59
22*cosh(b*x + a)^5 - 1020*cosh(b*x + a)^3 - 235*cosh(b*x + a))*sinh(b*x +
a)^4 + 14*(2145*cosh(b*x + a)^10 + 825*cosh(b*x + a)^8 - 1974*cosh(b*x + a
)^6 - 510*cosh(b*x + a)^4 - 235*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 70*
cosh(b*x + a)^3 + 14*(585*cosh(b*x + a)^11 + 275*cosh(b*x + a)^9 - 846*cos
h(b*x + a)^7 - 306*cosh(b*x + a)^5 - 235*cosh(b*x + a)^3 + 15*cosh(b*x + a
))*sinh(b*x + a)^2 + 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh(b*x + a
)^13 + sinh(b*x + a)^14 + (91*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^12 + cosh
(b*x + a)^12 + 4*(91*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^11 +
 (1001*cosh(b*x + a)^4 + 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^10 - 3*c...
```

3.43.6 Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx=\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**4*sech(b*x+a)**5,x)
```

output
```
Integral(csch(a + b*x)**4*sech(a + b*x)**5, x)
```

3.43.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(79) = 158$.

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{35\arctan\left(e^{(-bx-a)}\right)}{4\,b} + \frac{105\,e^{(-bx-a)} + 70\,e^{(-3\,bx-3\,a)} - 329\,e^{(-5\,bx-5\,a)} - 204\,e^{(-7\,bx-7\,a)} - 329\,e^{(-9\,bx-9\,a)} + 70\,e^{(-11\,bx-11\,a)} + 105\,e^{(-13\,bx-13\,a)}}{12\,b(e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} - 3\,e^{(-6\,bx-6\,a)} + 3\,e^{(-8\,bx-8\,a)} + 3\,e^{(-10\,bx-10\,a)} - e^{(-12\,bx-12\,a)} - e^{(-14\,bx-14\,a)} + 1)}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
-35/4*arctan(e^(-b*x - a))/b + 1/12*(105*e^(-b*x - a) + 70*e^(-3*b*x - 3*a
) - 329*e^(-5*b*x - 5*a) - 204*e^(-7*b*x - 7*a) - 329*e^(-9*b*x - 9*a) + 7
0*e^(-11*b*x - 11*a) + 105*e^(-13*b*x - 13*a))/(b*(e^(-2*b*x - 2*a) - 3*e^
(-4*b*x - 4*a) - 3*e^(-6*b*x - 6*a) + 3*e^(-8*b*x - 8*a) + 3*e^(-10*b*x -
10*a) - e^(-12*b*x - 12*a) - e^(-14*b*x - 14*a) + 1))
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{105\,\pi + \frac{12\left(11\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+52\,e^{(bx+a)}-52\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + \frac{32\left(9\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2-4\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3} + 210\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{48\,b}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="giac")
```

output
```
1/48*(105*pi + 12*(11*(e^(b*x + a) - e^(-b*x - a))^3 + 52*e^(b*x + a) - 52
*e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 32*(9*(e^(b*x + a)
 - e^(-b*x - a))^2 - 4)/(e^(b*x + a) - e^(-b*x - a))^3 + 210*arctan(1/2*(e
^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

3.43.9 Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.27

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{35\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{7\,e^{a+b\,x}}{2\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} + \frac{4\,e^{a+b\,x}}{b\,\left(4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)} + \frac{6\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} + \frac{11\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^5*sinh(a + b*x)^4),x)
```

output
```
(35*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) - (8*exp(a + b*
x))/(3*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (7*exp(a + b*x))/(
2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(
3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (6*exp(
a + b*x))/(b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) +
 1)) + (4*exp(a + b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*ex
p(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1)) + (6*exp(a + b*x))/(b*(exp(2*a + 2
*b*x) - 1)) + (11*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

3.44 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx$

3.44.1 Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx=\frac{\coth^2(a+bx)}{b}-\frac{\coth^4(a+bx)}{4b}+\frac{\log(\tanh(a+bx))}{b}$$

output
```
coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+ln(tanh(b*x+a))/b
```

3.44.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx =\frac{2\operatorname{csch}^2(a+bx)-\operatorname{csch}^4(a+bx)-4\log(\cosh(a+bx))+4\log(\sinh(a+bx))}{4b}$$

input
```
Integrate[Csch[a + b*x]^5*Sech[a + b*x],x]
```

output
```
(2*Csch[a + b*x]^2 - Csch[a + b*x]^4 - 4*Log[Cosh[a + b*x]] + 4*Log[Sinh[a
 + b*x]])/(4*b)
```

3.44.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)^5\sec(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)^5\sec(ia+ibx)dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int -i\coth^5(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)^2d\left(-\tanh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(i\coth^3(a+bx)-2\coth^2(a+bx)-i\coth(a+bx)\right)d\left(-\tanh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\coth^2(a+bx)+2i\coth(a+bx)+\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input
```
Int[Csch[a + b*x]^5*Sech[a + b*x],x]
```

output
```
((2*I)*Coth[a + b*x] + Coth[a + b*x]^2/2 + Log[-Tanh[a + b*x]^2])/(2*b)
```

3.44.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.44.4 Maple [A] (verified)

Time = 5.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4}+\frac{1}{2\sinh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	33
default	$\frac{-\frac{1}{4\sinh(bx+a)^4}+\frac{1}{2\sinh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	33
risch	$\frac{2\,e^{2bx+2a}\left(e^{4bx+4a}-4\,e^{2bx+2a}+1\right)}{b\left(e^{2bx+2a}-1\right)^4}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b}-\frac{\ln\left(1+e^{2bx+2a}\right)}{b}$	84

input
```
int(csch(b*x+a)^5*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sinh(b*x+a)^4+1/2/sinh(b*x+a)^2+ln(tanh(b*x+a)))
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 1082, normalized size of antiderivative = 27.74

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="fricas")
```

output
```
(2*cosh(b*x + a)^6 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6
+ 2*(15*cosh(b*x + a)^2 - 4)*sinh(b*x + a)^4 - 8*cosh(b*x + a)^4 + 8*(5*co
sh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 2*(15*cosh(b*x + a)^4 -
 24*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b*x +
 a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 - 3
*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x + a)
^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*co
sh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 -
15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x +
 a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(
b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x
 + a))) + (cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x +
a)^8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(
7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)
^4 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*co
sh(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*
(7*cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x
+ a)^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*co
sh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(...
```

3.44.6 Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx=\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**5*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)**5*sech(a + b*x), x)
```

3.44.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.41

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} - \frac{2\left(e^{(-2\,bx-2\,a)}-4\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}{b(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="maxima")
```

output
```
log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
 1)/b - 2*(e^(-2*b*x - 2*a) - 4*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/(b*(4
*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x -
8*a) - 1))
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.13

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\frac{3\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2-20\,e^{(2\,bx+2\,a)}-20\,e^{(-2\,bx-2\,a)}+44}{\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)^2}+2\,\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)-2\,\log\left(e^{(2\,bx+2\,a)}\right.}{4\,b}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="giac")
```

output
```
-1/4*((3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 20*e^(2*b*x + 2*a) - 20*
e^(-2*b*x - 2*a) + 44)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 2)^2 + 2*log(
e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - 2*log(e^(2*b*x + 2*a) + e^(-2*b*
x - 2*a) - 2))/b
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 4.33

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{2\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{8}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{4}{b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)^5),x)
```

output
```
2/(b*(exp(2*a + 2*b*x) - 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/
b))/(-b^2)^(1/2) - 2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - 8/(
b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - 4/(b
*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a +
 8*b*x) + 1))
```

3.45 $\int \mathbf{csch}^5(a+bx)\mathbf{sech}^2(a+bx)\,dx$

3.45.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{15\operatorname{arctanh}(\cosh(a+bx))}{8b}+\frac{15\operatorname{sech}(a+bx)}{8b}+\frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{8b}-\frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)}{4b}$$

output
```
-15/8*arctanh(cosh(b*x+a))/b+15/8*sech(b*x+a)/b+5/8*csch(b*x+a)^2*sech(b*x
+a)/b-1/4*csch(b*x+a)^4*sech(b*x+a)/b
```

3.45.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{7\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{15\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{15\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{7\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{\operatorname{sech}(a+bx)}{b}$$

input
```
Integrate[Csch[a + b*x]^5*Sech[a + b*x]^2,x]
```

output
```
(7*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) - (15*Log[Cosh
[(a + b*x)/2]])/(8*b) + (15*Log[Sinh[(a + b*x)/2]])/(8*b) + (7*Sech[(a + b
*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/(64*b) + Sech[a + b*x]/b
```

3.45.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 26, 3102, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)^5\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)^5\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\operatorname{sech}^6(a+bx)}{\left(1-\operatorname{sech}^2(a+bx)\right)^3}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\operatorname{sech}^6(a+bx)}{\left(1-\operatorname{sech}^2(a+bx)\right)^3}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\int \frac{\operatorname{sech}^4(a+bx)}{\left(1-\operatorname{sech}^2(a+bx)\right)^2}d\operatorname{sech}(a+bx)-\frac{\operatorname{sech}^5(a+bx)}{4\left(1-\operatorname{sech}^2(a+bx)\right)^2}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\left(\frac{\operatorname{sech}^3(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{3}{2}\int \frac{\operatorname{sech}^2(a+bx)}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)\right)-\frac{\operatorname{sech}^5(a+bx)}{4\left(1-\operatorname{sech}^2(a+bx)\right)^2}}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{5}{4}\left(\frac{\operatorname{sech}^3(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{3}{2}\left(\int \frac{1}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)-\operatorname{sech}(a+bx)\right)\right)-\frac{\operatorname{sech}^5(a+bx)}{4\left(1-\operatorname{sech}^2(a+bx)\right)^2}}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{5}{4}\left(\frac{\operatorname{sech}^3(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{3}{2}(\operatorname{arctanh}(\operatorname{sech}(a+bx))-\operatorname{sech}(a+bx))\right)-\frac{\operatorname{sech}^5(a+bx)}{4\left(1-\operatorname{sech}^2(a+bx)\right)^2}}{b}$$

input
```
Int[Csch[a + b*x]^5*Sech[a + b*x]^2,x]
```

output
```
(-1/4*Sech[a + b*x]^5/(1 - Sech[a + b*x]^2)^2 + (5*((-3*(ArcTanh[Sech[a +
b*x]] - Sech[a + b*x]))/2 + Sech[a + b*x]^3/(2*(1 - Sech[a + b*x]^2))))/4)
/b
```

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.45.4 Maple [A] (verified)

Time = 15.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)}+\frac{5}{8\sinh(bx+a)^2\cosh(bx+a)}+\frac{15}{8\cosh(bx+a)}-\frac{15\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$	61
default	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)}+\frac{5}{8\sinh(bx+a)^2\cosh(bx+a)}+\frac{15}{8\cosh(bx+a)}-\frac{15\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$	61
risch	$\frac{\mathrm{e}^{bx+a}\left(15\,\mathrm{e}^{8bx+8a}-40\,\mathrm{e}^{6bx+6a}+18\,\mathrm{e}^{4bx+4a}-40\,\mathrm{e}^{2bx+2a}+15\right)}{4b\left(\mathrm{e}^{2bx+2a}-1\right)^4\left(1+\mathrm{e}^{2bx+2a}\right)}-\frac{15\ln\left(\mathrm{e}^{bx+a}+1\right)}{8b}+\frac{15\ln\left(\mathrm{e}^{bx+a}-1\right)}{8b}$	113

input
```
int(csch(b*x+a)^5*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)+5/8/sinh(b*x+a)^2/cosh(b*x+a)+15/8/cos
h(b*x+a)-15/4*arctanh(exp(b*x+a)))
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1591 vs. $2(62) = 124$.

Time = 0.26 (sec) , antiderivative size = 1591, normalized size of antiderivative = 22.73

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(30*cosh(b*x + a)^9 + 270*cosh(b*x + a)*sinh(b*x + a)^8 + 30*sinh(b*x
+ a)^9 + 40*(27*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^7 - 80*cosh(b*x + a)^7
+ 280*(9*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^6 + 12*(315*cosh
(b*x + a)^4 - 140*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^5 + 36*cosh(b*x + a)^
5 + 20*(189*cosh(b*x + a)^5 - 140*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(
b*x + a)^4 + 40*(63*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 + 9*cosh(b*x + a)
^2 - 2)*sinh(b*x + a)^3 - 80*cosh(b*x + a)^3 + 120*(9*cosh(b*x + a)^7 - 14
*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^2 -
15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10
 + 3*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - 3*cosh(b*x + a)^8 + 24*(5*
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4
- 42*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(b*x + a)^6 + 12*(21*cos
h(b*x + a)^5 - 14*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^5 + 2*(10
5*cosh(b*x + a)^6 - 105*cosh(b*x + a)^4 + 15*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^4 + 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 - 21*cosh(b*x + a)^5 +
 5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + 3*(15*cosh(b*x + a)^
8 - 28*cosh(b*x + a)^6 + 10*cosh(b*x + a)^4 + 4*cosh(b*x + a)^2 - 1)*sinh(
b*x + a)^2 - 3*cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 - 12*cosh(b*x + a)^7
 + 6*cosh(b*x + a)^5 + 4*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)
+ 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 15*(cosh(b*x + a)^10 + 10...
```

3.45.6 Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**5*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)**5*sech(a + b*x)**2, x)
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(62) = 124$.

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.21

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx$$
$$= -\frac{15\,\log\left(e^{(-bx-a)}+1\right)}{8\,b} + \frac{15\,\log\left(e^{(-bx-a)}-1\right)}{8\,b} - \frac{15\,e^{(-bx-a)} - 40\,e^{(-3\,bx-3\,a)} + 18\,e^{(-5\,bx-5\,a)} - 40\,e^{(-7\,bx-7\,a)} + 15\,e^{(-9\,bx-9\,a)}}{4\,b(3\,e^{(-2\,bx-2\,a)} - 2\,e^{(-4\,bx-4\,a)} - 2\,e^{(-6\,bx-6\,a)} + 3\,e^{(-8\,bx-8\,a)} - e^{(-10\,bx-10\,a)} - 1)}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
-15/8*log(e^(-b*x - a) + 1)/b + 15/8*log(e^(-b*x - a) - 1)/b - 1/4*(15*e^(
-b*x - a) - 40*e^(-3*b*x - 3*a) + 18*e^(-5*b*x - 5*a) - 40*e^(-7*b*x - 7*a
) + 15*e^(-9*b*x - 9*a))/(b*(3*e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) - 2*e
^(-6*b*x - 6*a) + 3*e^(-8*b*x - 8*a) - e^(-10*b*x - 10*a) - 1))
```

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(62) = 124$.

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.86

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx$$
$$= \frac{\frac{4\left(7\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-36\,e^{(bx+a)}-36\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2} + \frac{32}{e^{(bx+a)}+e^{(-bx-a)}} - 15\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + 15\,\log\left(e^{(bx+\cdots\right.}{16\,b}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
1/16*(4*(7*(e^(b*x + a) + e^(-b*x - a))^3 - 36*e^(b*x + a) - 36*e^(-b*x -
a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 + 32/(e^(b*x + a) + e^(-b*x - a
)) - 15*log(e^(b*x + a) + e^(-b*x - a) + 2) + 15*log(e^(b*x + a) + e^(-b*x
 - a) - 2))/b
```

3.45.9 Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.06

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\,e^{a+b\,x}}{2\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{15\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}} - \frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{4\,e^{a+b\,x}}{b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)} + \frac{7\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}-1\right)} + \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)^5),x)
```

output
```
(3*exp(a + b*x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (15*a
tan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2)) - (6*exp(a + b*x))
/(b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4
*exp(a + b*x))/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6
*b*x) + exp(8*a + 8*b*x) + 1)) + (7*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) -
 1)) + (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.46 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$

3.46.1 Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx=\frac{3\coth^2(a+bx)}{2b}-\frac{\coth^4(a+bx)}{4b}+\frac{3\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

output
```
3/2*coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+3*ln(tanh(b*x+a))/b-1/2*tanh(b*x+a
)^2/b
```

3.46.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$=\frac{4\operatorname{csch}^2(a+bx)-\operatorname{csch}^4(a+bx)-12\log(\cosh(a+bx))+12\log(\sinh(a+bx))+2\operatorname{sech}^2(a+bx)}{4b}$$

input
```
Integrate[Csch[a + b*x]^5*Sech[a + b*x]^3,x]
```

output
```
(4*Csch[a + b*x]^2 - Csch[a + b*x]^4 - 12*Log[Cosh[a + b*x]] + 12*Log[Sinh
[a + b*x]] + 2*Sech[a + b*x]^2)/(4*b)
```

3.46.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int i\csc(ia+ibx)^5\sec(ia+ibx)^3dx$$

$$\Big\downarrow \; 26$$

$$i\int \csc(ia+ibx)^5\sec(ia+ibx)^3dx$$

$$\Big\downarrow \; 3100$$

$$\frac{\int -i\coth^5(a+bx)\left(1-\tanh^2(a+bx)\right)^3d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \; 243$$

$$\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)^3d\left(-\tanh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \; 49$$

$$\frac{\int \left(i\coth^3(a+bx)-3\coth^2(a+bx)-3i\coth(a+bx)+1\right)d\left(-\tanh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\tanh^2(a+bx)+\frac{1}{2}\coth^2(a+bx)+3i\coth(a+bx)+3\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input
```
Int[Csch[a + b*x]^5*Sech[a + b*x]^3,x]
```

output
```
((3*I)*Coth[a + b*x] + Coth[a + b*x]^2/2 + 3*Log[-Tanh[a + b*x]^2] - Tanh[
a + b*x]^2)/(2*b)
```

3.46.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.46.4 Maple [A] (verified)

Time = 36.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^2}+\frac{3}{4\sinh(bx+a)^2\cosh(bx+a)^2}+\frac{3}{2\cosh(bx+a)^2}+3\ln(\tanh(bx+a))}{b}$	61
default	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^2}+\frac{3}{4\sinh(bx+a)^2\cosh(bx+a)^2}+\frac{3}{2\cosh(bx+a)^2}+3\ln(\tanh(bx+a))}{b}$	61
risch	$\frac{2\,\mathrm{e}^{2bx+2a}\left(3\,\mathrm{e}^{8bx+8a}-6\,\mathrm{e}^{6bx+6a}-2\,\mathrm{e}^{4bx+4a}-6\,\mathrm{e}^{2bx+2a}+3\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^4\left(1+\mathrm{e}^{2bx+2a}\right)^2}+\frac{3\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}-\frac{3\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}$	122

input
```
int(csch(b*x+a)^5*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)^2+3/4/sinh(b*x+a)^2/cosh(b*x+a)^2+3/2/
cosh(b*x+a)^2+3*ln(tanh(b*x+a)))
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2114 vs. $2(52) = 104$.

Time = 0.27 (sec) , antiderivative size = 2114, normalized size of antiderivative = 36.45

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
(6*cosh(b*x + a)^10 + 60*cosh(b*x + a)*sinh(b*x + a)^9 + 6*sinh(b*x + a)^1
0 + 6*(45*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^8 - 12*cosh(b*x + a)^8 + 48*(
15*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(315*cosh(b*x +
a)^4 - 84*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 24*(6
3*cosh(b*x + a)^5 - 28*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^5 +
12*(105*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^4 - 12*cosh(b*x + a)^4 + 16*(45*cosh(b*x + a)^7 - 42*cosh(b*x +
a)^5 - 5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(45*cosh(b
*x + a)^8 - 56*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 +
 1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^12 + 12*cosh(b*
x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*cosh(b*x + a)^2 - 1)*si
nh(b*x + a)^10 - 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 - cosh(b*x +
a))*sinh(b*x + a)^9 + (495*cosh(b*x + a)^4 - 90*cosh(b*x + a)^2 - 1)*sinh(
b*x + a)^8 - cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 - 30*cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 - 105*cosh(b*x +
 a)^4 - 7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(99
*cosh(b*x + a)^7 - 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 + 3*cosh(b*x + a
))*sinh(b*x + a)^5 + (495*cosh(b*x + a)^8 - 420*cosh(b*x + a)^6 - 70*cosh(
b*x + a)^4 + 60*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4
*(55*cosh(b*x + a)^9 - 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5 + 20*cos...
```

3.46.6 Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**5*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)**5*sech(a + b*x)**3, x)
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(52) = 104.

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.09

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{3\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{3\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{3\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} - \frac{2\left(3\,e^{(-2\,bx-2\,a)} - 6\,e^{(-4\,bx-4\,a)} - 2\,e^{(-6\,bx-6\,a)} - 6\,e^{(-8\,bx-8\,a)} + 3\,e^{(-10\,bx-10\,a)}\right)}{b(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} - 4\,e^{(-6\,bx-6\,a)} + e^{(-8\,bx-8\,a)} + 2\,e^{(-10\,bx-10\,a)} - e^{(-12\,bx-12\,a)} - 1)}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
3*log(e^(-b*x - a) + 1)/b + 3*log(e^(-b*x - a) - 1)/b - 3*log(e^(-2*b*x -
2*a) + 1)/b - 2*(3*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6
*a) - 6*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a))/(b*(2*e^(-2*b*x - 2*a) +
e^(-4*b*x - 4*a) - 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 2*e^(-10*b*x -
10*a) - e^(-12*b*x - 12*a) - 1))
```

3.46.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(52) = 104$.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.95

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{\frac{2\left(3\,e^{(2\,bx+2\,a)}+3\,e^{(-2\,bx-2\,a)}+10\right)}{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2} - \frac{9\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2-52\,e^{(2\,bx+2\,a)}-52\,e^{(-2\,bx-2\,a)}+84}{\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)^2} - 6\,\log\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-}\right.}{4\,b}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(2*(3*e^(2*b*x + 2*a) + 3*e^(-2*b*x - 2*a) + 10)/(e^(2*b*x + 2*a) + e^
(-2*b*x - 2*a) + 2) - (9*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 52*e^(2*
b*x + 2*a) - 52*e^(-2*b*x - 2*a) + 84)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a)
 - 2)^2 - 6*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + 6*log(e^(2*b*x +
 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

3.46.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.22

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{4}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} + \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{6\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)} - \frac{8}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}-1\right)} - \frac{4}{b\,\left(6\,\mathrm{e}^{4\,a+4\,b\,x}-4\,\mathrm{e}^{2\,a+2\,b\,x}-4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)^5),x)
```

output
```
4/(b*(exp(2*a + 2*b*x) - 1)) + 2/(b*(exp(2*a + 2*b*x) + 1)) - (6*atan((exp
(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 2/(b*(2*exp(2*a + 2*b*x)
 + exp(4*a + 4*b*x) + 1)) - 8/(b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x)
+ exp(6*a + 6*b*x) - 1)) - 4/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) -
 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

3.47 $\int \mathrm{csch}^5(a+bx)\mathrm{sech}^4(a+bx)\,dx$

3.47.1 Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \mathrm{csch}^5(a+bx)\mathrm{sech}^4(a+bx)\,dx = -\frac{35\mathrm{arctanh}(\cosh(a+bx))}{8b}+\frac{35\mathrm{sech}(a+bx)}{8b}+\frac{35\mathrm{sech}^3(a+bx)}{24b}+\frac{7\mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)}{8b}-\frac{\mathrm{csch}^4(a+bx)\mathrm{sech}^3(a+bx)}{4b}$$

output
```
-35/8*arctanh(cosh(b*x+a))/b+35/8*sech(b*x+a)/b+35/24*sech(b*x+a)^3/b+7/8*
csch(b*x+a)^2*sech(b*x+a)^3/b-1/4*csch(b*x+a)^4*sech(b*x+a)^3/b
```

3.47.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.56

$$\int \mathrm{csch}^5(a+bx)\mathrm{sech}^4(a+bx)\,dx = \frac{11\mathrm{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\mathrm{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{35\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{35\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{11\mathrm{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\mathrm{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{3\mathrm{sech}(a+bx)}{b}+\frac{\mathrm{sech}^3(a+bx)}{3b}$$

input
```
Integrate[Csch[a + b*x]^5*Sech[a + b*x]^4,x]
```

output
```
(11*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) - (35*Log[Cos
h[(a + b*x)/2]])/(8*b) + (35*Log[Sinh[(a + b*x)/2]])/(8*b) + (11*Sech[(a +
 b*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/(64*b) + (3*Sech[a + b*x])/b + Se
ch[a + b*x]^3/(3*b)
```

3.47.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 26, 3102, 25, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^5(a+bx)\text{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)^5\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)^5\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\text{sech}^8(a+bx)}{\left(1-\text{sech}^2(a+bx)\right)^3}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\text{sech}^8(a+bx)}{\left(1-\text{sech}^2(a+bx)\right)^3}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{7}{4}\int \frac{\text{sech}^6(a+bx)}{\left(1-\text{sech}^2(a+bx)\right)^2}d\text{sech}(a+bx)-\frac{\text{sech}^7(a+bx)}{4\left(1-\text{sech}^2(a+bx)\right)^2}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{7}{4}\left(\frac{\text{sech}^5(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{5}{2}\int\frac{\text{sech}^4(a+bx)}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)\right)-\frac{\text{sech}^7(a+bx)}{4\left(1-\text{sech}^2(a+bx)\right)^2}}{b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{7}{4}\left(\frac{\text{sech}^5(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{5}{2}\int\left(-\text{sech}^2(a+bx)+\frac{1}{1-\text{sech}^2(a+bx)}-1\right)d\text{sech}(a+bx)\right)-\frac{\text{sech}^7(a+bx)}{4\left(1-\text{sech}^2(a+bx)\right)^2}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{7}{4}\left(\frac{\text{sech}^5(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{5}{2}\left(\text{arctanh}(\text{sech}(a+bx))-\frac{1}{3}\text{sech}^3(a+bx)-\text{sech}(a+bx)\right)\right)-\frac{\text{sech}^7(a+bx)}{4\left(1-\text{sech}^2(a+bx)\right)^2}}{b}$$

input
```
Int[Csch[a + b*x]^5*Sech[a + b*x]^4,x]
```

output
```
(-1/4*Sech[a + b*x]^7/(1 - Sech[a + b*x]^2)^2 + (7*(Sech[a + b*x]^5/(2*(1
- Sech[a + b*x]^2)) - (5*(ArcTanh[Sech[a + b*x]] - Sech[a + b*x] - Sech[a
+ b*x]^3/3))/2))/4)/b
```

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.47.4 Maple [A] (verified)

Time = 115.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^3}+\frac{7}{8\sinh(bx+a)^2\cosh(bx+a)^3}+\frac{35}{24\cosh(bx+a)^3}+\frac{35}{8\cosh(bx+a)}-\frac{35\,\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$
default	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^3}+\frac{7}{8\sinh(bx+a)^2\cosh(bx+a)^3}+\frac{35}{24\cosh(bx+a)^3}+\frac{35}{8\cosh(bx+a)}-\frac{35\,\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$
risch	$\frac{e^{bx+a}\left(105\,e^{12bx+12a}-70\,e^{10bx+10a}-329\,e^{8bx+8a}+204\,e^{6bx+6a}-329\,e^{4bx+4a}-70\,e^{2bx+2a}+105\right)}{12b\left(e^{2bx+2a}-1\right)^4\left(1+e^{2bx+2a}\right)^3}+\frac{35\ln\left(e^{bx+a}-1\right)}{8b}-$

input
```
int(csch(b*x+a)^5*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)^3+7/8/sinh(b*x+a)^2/cosh(b*x+a)^3+35/2
4/cosh(b*x+a)^3+35/8/cosh(b*x+a)-35/4*arctanh(exp(b*x+a)))
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2802 vs. $2(79) = 158$.

Time = 0.27 (sec) , antiderivative size = 2802, normalized size of antiderivative = 31.48

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/24*(210*cosh(b*x + a)^13 + 2730*cosh(b*x + a)*sinh(b*x + a)^12 + 210*sin
h(b*x + a)^13 + 140*(117*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^11 - 140*cosh(
b*x + a)^11 + 1540*(39*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^10 +
 14*(10725*cosh(b*x + a)^4 - 550*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^9 - 6
58*cosh(b*x + a)^9 + 42*(6435*cosh(b*x + a)^5 - 550*cosh(b*x + a)^3 - 141*
cosh(b*x + a))*sinh(b*x + a)^8 + 24*(15015*cosh(b*x + a)^6 - 1925*cosh(b*x
 + a)^4 - 987*cosh(b*x + a)^2 + 17)*sinh(b*x + a)^7 + 408*cosh(b*x + a)^7
+ 168*(2145*cosh(b*x + a)^7 - 385*cosh(b*x + a)^5 - 329*cosh(b*x + a)^3 +
17*cosh(b*x + a))*sinh(b*x + a)^6 + 14*(19305*cosh(b*x + a)^8 - 4620*cosh(
b*x + a)^6 - 5922*cosh(b*x + a)^4 + 612*cosh(b*x + a)^2 - 47)*sinh(b*x + a
)^5 - 658*cosh(b*x + a)^5 + 14*(10725*cosh(b*x + a)^9 - 3300*cosh(b*x + a)
^7 - 5922*cosh(b*x + a)^5 + 1020*cosh(b*x + a)^3 - 235*cosh(b*x + a))*sinh
(b*x + a)^4 + 28*(2145*cosh(b*x + a)^10 - 825*cosh(b*x + a)^8 - 1974*cosh(
b*x + a)^6 + 510*cosh(b*x + a)^4 - 235*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^
3 - 140*cosh(b*x + a)^3 + 28*(585*cosh(b*x + a)^11 - 275*cosh(b*x + a)^9 -
 846*cosh(b*x + a)^7 + 306*cosh(b*x + a)^5 - 235*cosh(b*x + a)^3 - 15*cosh
(b*x + a))*sinh(b*x + a)^2 - 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh
(b*x + a)^13 + sinh(b*x + a)^14 + (91*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^1
2 - cosh(b*x + a)^12 + 4*(91*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x +
 a)^11 + (1001*cosh(b*x + a)^4 - 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^...
```

3.47.6 Sympy [F]

$$\int \text{csch}^5(a+bx)\text{sech}^4(a+bx)\,dx = \int \text{csch}^5\,(a+bx)\,\text{sech}^4\,(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**5*sech(b*x+a)**4,x)
```

output
```
Integral(csch(a + b*x)**5*sech(a + b*x)**4, x)
```

3.47.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(79) = 158$.

Time = 0.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{35\log\left(e^{(-bx-a)}+1\right)}{8\,b} + \frac{35\log\left(e^{(-bx-a)}-1\right)}{8\,b} - \frac{105\,e^{(-bx-a)} - 70\,e^{(-3\,bx-3\,a)} - 329\,e^{(-5\,bx-5\,a)} + 204\,e^{(-7\,bx-7\,a)} - 329\,e^{(-9\,bx-9\,a)} - 70\,e^{(-11\,bx-11\,a)} + 105}{12\,b(e^{(-2\,bx-2\,a)} + 3\,e^{(-4\,bx-4\,a)} - 3\,e^{(-6\,bx-6\,a)} - 3\,e^{(-8\,bx-8\,a)} + 3\,e^{(-10\,bx-10\,a)} + e^{(-12\,bx-12\,a)} - e^{(-14\,bx-}}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
-35/8*log(e^(-b*x - a) + 1)/b + 35/8*log(e^(-b*x - a) - 1)/b - 1/12*(105*e
^(-b*x - a) - 70*e^(-3*b*x - 3*a) - 329*e^(-5*b*x - 5*a) + 204*e^(-7*b*x -
 7*a) - 329*e^(-9*b*x - 9*a) - 70*e^(-11*b*x - 11*a) + 105*e^(-13*b*x - 13
*a))/(b*(e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - 3*e^(-6*b*x - 6*a) - 3*e^
(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a) + e^(-12*b*x - 12*a) - e^(-14*b*x -
14*a) - 1))
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.71

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{\frac{12\left(11\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-52\,e^{(bx+a)}-52\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2} + \frac{32\left(9\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+4\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3} - 105\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) +}{48\,b}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="giac")
```

output
```
1/48*(12*(11*(e^(b*x + a) + e^(-b*x - a))^3 - 52*e^(b*x + a) - 52*e^(-b*x
- a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 + 32*(9*(e^(b*x + a) + e^(-b*
x - a))^2 + 4)/(e^(b*x + a) + e^(-b*x - a))^3 - 105*log(e^(b*x + a) + e^(-
b*x - a) + 2) + 105*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.31

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{7\,e^{a+b\,x}}{2\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{35\,\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}} + \frac{8\,e^{a+b\,x}}{3\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} - \frac{4\,e^{a+b\,x}}{b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)} + \frac{11\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}-1\right)} + \frac{6\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^4*sinh(a + b*x)^5),x)
```

output
```
(7*exp(a + b*x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (35*a
tan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2)) + (8*exp(a + b*x))
/(3*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (6*exp(a + b*x))/(b*(
3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (8*exp(
a + b*x))/(3*b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x)
 + 1)) - (4*exp(a + b*x))/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*
exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1)) + (11*exp(a + b*x))/(4*b*(exp(2*
a + 2*b*x) - 1)) + (6*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.48 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx$

3.48.1 Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{2\coth^2(a+bx)}{b} - \frac{\coth^4(a+bx)}{4b} + \frac{6\log(\tanh(a+bx))}{b} - \frac{2\tanh^2(a+bx)}{b} + \frac{\tanh^4(a+bx)}{4b}$$

output
```
2*coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+6*ln(tanh(b*x+a))/b-2*tanh(b*x+a)^2/
b+1/4*tanh(b*x+a)^4/b
```

3.48.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = 32\left(\frac{3\operatorname{csch}^2(a+bx)}{64b} - \frac{\operatorname{csch}^4(a+bx)}{128b} - \frac{3\log(\cosh(a+bx))}{16b} + \frac{3\log(\sinh(a+bx))}{16b} + \frac{3\operatorname{sech}^2(a+bx)}{64b} + \frac{\operatorname{sech}^4(a+bx)}{128b}\right)$$

input
```
Integrate[Csch[a + b*x]^5*Sech[a + b*x]^5,x]
```

output
```
32*((3*Csch[a + b*x]^2)/(64*b) - Csch[a + b*x]^4/(128*b) - (3*Log[Cosh[a +
 b*x]])/(16*b) + (3*Log[Sinh[a + b*x]])/(16*b) + (3*Sech[a + b*x]^2)/(64*b
) + Sech[a + b*x]^4/(128*b))
```

3.48.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^5(a+bx)\text{sech}^5(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int i\csc(ia+ibx)^5\sec(ia+ibx)^5dx$$

$$\Big\downarrow \ 26$$

$$i\int \csc(ia+ibx)^5\sec(ia+ibx)^5dx$$

$$\Big\downarrow \ 3100$$

$$\frac{\int -i\coth^5(a+bx)\left(1-\tanh^2(a+bx)\right)^4d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \ 243$$

$$\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)^4d\left(-\tanh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \ 49$$

$$\frac{\int \left(i\coth^3(a+bx)-4\coth^2(a+bx)-6i\coth(a+bx)-\tanh^2(a+bx)+4\right)d\left(-\tanh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{1}{2}\tanh^2(a+bx)+4i\tanh(a+bx)+\frac{1}{2}\coth^2(a+bx)+4i\coth(a+bx)+6\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input
```
Int[Csch[a + b*x]^5*Sech[a + b*x]^5,x]
```

output
```
((4*I)*Coth[a + b*x] + Coth[a + b*x]^2/2 + 6*Log[-Tanh[a + b*x]^2] + (4*I)
*Tanh[a + b*x] - Tanh[a + b*x]^2/2)/(2*b)
```

3.48.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.48.4 Maple [A] (verified)

Time = 242.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^4}+\frac{1}{\sinh(bx+a)^2\cosh(bx+a)^4}+\frac{3}{2\cosh(bx+a)^4}+\frac{3}{\cosh(bx+a)^2}+6\ln(\tanh(bx+a))}{b}$	70
default	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^4}+\frac{1}{\sinh(bx+a)^2\cosh(bx+a)^4}+\frac{3}{2\cosh(bx+a)^4}+\frac{3}{\cosh(bx+a)^2}+6\ln(\tanh(bx+a))}{b}$	70
risch	$\frac{4\,e^{2bx+2a}\left(3\,e^{12bx+12a}-11\,e^{8bx+8a}-11\,e^{4bx+4a}+3\right)}{b\left(1+e^{2bx+2a}\right)^4\left(e^{2bx+2a}-1\right)^4}+\frac{6\ln\left(e^{2bx+2a}-1\right)}{b}-\frac{6\ln\left(1+e^{2bx+2a}\right)}{b}$	111

input
```
int(csch(b*x+a)^5*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)^4+1/sinh(b*x+a)^2/cosh(b*x+a)^4+3/2/co
sh(b*x+a)^4+3/cosh(b*x+a)^2+6*ln(tanh(b*x+a)))
```

3.48.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. $2(65) = 130$.

Time = 0.25 (sec) , antiderivative size = 2231, normalized size of antiderivative = 32.33

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
2*(6*cosh(b*x + a)^14 + 2184*cosh(b*x + a)^3*sinh(b*x + a)^11 + 546*cosh(b
*x + a)^2*sinh(b*x + a)^12 + 84*cosh(b*x + a)*sinh(b*x + a)^13 + 6*sinh(b*
x + a)^14 + 22*(273*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^10 - 22*cosh(b*x +
a)^10 + 44*(273*cosh(b*x + a)^5 - 5*cosh(b*x + a))*sinh(b*x + a)^9 + 198*(
91*cosh(b*x + a)^6 - 5*cosh(b*x + a)^2)*sinh(b*x + a)^8 + 528*(39*cosh(b*x
 + a)^7 - 5*cosh(b*x + a)^3)*sinh(b*x + a)^7 + 22*(819*cosh(b*x + a)^8 - 2
10*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^6 - 22*cosh(b*x + a)^6 + 132*(91*cos
h(b*x + a)^9 - 42*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^5 + 66*(9
1*cosh(b*x + a)^10 - 70*cosh(b*x + a)^6 - 5*cosh(b*x + a)^2)*sinh(b*x + a)
^4 + 8*(273*cosh(b*x + a)^11 - 330*cosh(b*x + a)^7 - 55*cosh(b*x + a)^3)*s
inh(b*x + a)^3 + 6*(91*cosh(b*x + a)^12 - 165*cosh(b*x + a)^8 - 55*cosh(b*
x + a)^4 + 1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^16 +
560*cosh(b*x + a)^3*sinh(b*x + a)^13 + 120*cosh(b*x + a)^2*sinh(b*x + a)^1
4 + 16*cosh(b*x + a)*sinh(b*x + a)^15 + sinh(b*x + a)^16 + 4*(455*cosh(b*x
 + a)^4 - 1)*sinh(b*x + a)^12 - 4*cosh(b*x + a)^12 + 48*(91*cosh(b*x + a)^
5 - cosh(b*x + a))*sinh(b*x + a)^11 + 88*(91*cosh(b*x + a)^6 - 3*cosh(b*x
+ a)^2)*sinh(b*x + a)^10 + 880*(13*cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh
(b*x + a)^9 + 6*(2145*cosh(b*x + a)^8 - 330*cosh(b*x + a)^4 + 1)*sinh(b*x
+ a)^8 + 6*cosh(b*x + a)^8 + 16*(715*cosh(b*x + a)^9 - 198*cosh(b*x + a)^5
 + 3*cosh(b*x + a))*sinh(b*x + a)^7 + 56*(143*cosh(b*x + a)^10 - 66*cos...
```

3.48.6 Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**5*sech(b*x+a)**5,x)
```

output
```
Integral(csch(a + b*x)**5*sech(a + b*x)**5, x)
```

3.48.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(65) = 130.

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.17

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{6\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{6\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{6\log\left(e^{(-2bx-2a)}+1\right)}{b} - \frac{4\left(3e^{(-2bx-2a)}-11e^{(-6bx-6a)}-11e^{(-10bx-10a)}+3e^{(-14bx-14a)}\right)}{b(4e^{(-4bx-4a)}-6e^{(-8bx-8a)}+4e^{(-12bx-12a)}-e^{(-16bx-16a)}-1)}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
6*log(e^(-b*x - a) + 1)/b + 6*log(e^(-b*x - a) - 1)/b - 6*log(e^(-2*b*x -
2*a) + 1)/b - 4*(3*e^(-2*b*x - 2*a) - 11*e^(-6*b*x - 6*a) - 11*e^(-10*b*x
- 10*a) + 3*e^(-14*b*x - 14*a))/(b*(4*e^(-4*b*x - 4*a) - 6*e^(-8*b*x - 8*a
) + 4*e^(-12*b*x - 12*a) - e^(-16*b*x - 16*a) - 1))
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.80

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{\frac{4\left(3\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^3-20e^{(2bx+2a)}-20e^{(-2bx-2a)}\right)}{\left(\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2-4\right)^2}-3\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)+3\log\left(e^{(2bx+2a)}+\right.}{b}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="giac")
```

output
```
(4*(3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^3 - 20*e^(2*b*x + 2*a) - 20*e^(
-2*b*x - 2*a))/((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 4)^2 - 3*log(e^(2
*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + 3*log(e^(2*b*x + 2*a) + e^(-2*b*x -
2*a) - 2))/b
```

3.48.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.97

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{12\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}-\frac{12\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{8\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{8\,a+8\,b\,x}-2\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)} - \frac{32\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(3\,\mathrm{e}^{4\,a+4\,b\,x}-3\,\mathrm{e}^{8\,a+8\,b\,x}+\mathrm{e}^{12\,a+12\,b\,x}-1\right)} - \frac{64\,\mathrm{e}^{6\,a+6\,b\,x}}{b\,\left(6\,\mathrm{e}^{8\,a+8\,b\,x}-4\,\mathrm{e}^{4\,a+4\,b\,x}-4\,\mathrm{e}^{12\,a+12\,b\,x}+\mathrm{e}^{16\,a+16\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^5*sinh(a + b*x)^5),x)
```

output
```
(12*exp(2*a + 2*b*x))/(b*(exp(4*a + 4*b*x) - 1)) - (12*atan((exp(2*a)*exp(
2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (8*exp(2*a + 2*b*x))/(b*(exp(8*a +
 8*b*x) - 2*exp(4*a + 4*b*x) + 1)) - (32*exp(2*a + 2*b*x))/(b*(3*exp(4*a +
 4*b*x) - 3*exp(8*a + 8*b*x) + exp(12*a + 12*b*x) - 1)) - (64*exp(6*a + 6*
b*x))/(b*(6*exp(8*a + 8*b*x) - 4*exp(4*a + 4*b*x) - 4*exp(12*a + 12*b*x) +
 exp(16*a + 16*b*x) + 1))
```

3.49 $\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx$

3.49.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

output
```
-arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/s
inh(b*x+a)^(1/2))/b-2/5*sinh(b*x+a)^(5/2)/b/cosh(b*x+a)^(5/2)-2*sinh(b*x+a
)^(1/2)/b/cosh(b*x+a)^(1/2)
```

3.49.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\sqrt[4]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{9}{4},\frac{13}{4},-\sinh^2(a+bx)\right)\sinh^{\frac{9}{2}}(a+bx)}{9b\sqrt{\cosh(a+bx)}}$$

input
```
Integrate[Sinh[a + b*x]^(7/2)/Cosh[a + b*x]^(7/2),x]
```

output
```
(2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 9/4, 13/4, -Sinh[a + b*x
]^2]*Sinh[a + b*x]^(9/2))/(9*b*Sqrt[Cosh[a + b*x]])
```

3.49.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3046, 3042, 3046, 3042, 3055, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ia+ibx))^{7/2}}{\cos(ia+ibx)^{7/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}dx-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\int \frac{(-i\sin(ia+ibx))^{3/2}}{\cos(ia+ibx)^{3/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}dx-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\cos(ia+ibx)}}{\sqrt{-i\sin(ia+ibx)}}dx-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{3055}$$

$$\frac{2\int \frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}}{b}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\int \frac{1}{\coth(a+bx)+1}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

input
```
Int[Sinh[a + b*x]^(7/2)/Cosh[a + b*x]^(7/2),x]
```

output
```
(2*(-1/2*ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]] + ArcTanh[Sqrt[Co
sh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/2))/b - (2*Sqrt[Sinh[a + b*x]])/(b*Sqrt[
Cosh[a + b*x]]) - (2*Sinh[a + b*x]^(5/2))/(5*b*Cosh[a + b*x]^(5/2))
```

3.49.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

3.49.4 Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{\frac{7}{2}}}{\cosh\left(bx+a\right)^{\frac{7}{2}}}dx$$

input
```
int(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x)
```

output
```
int(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x)
```

3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. 2(88) = 176.

Time = 0.27 (sec) , antiderivative size = 997, normalized size of antiderivative = 9.41

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/10*(24*cosh(b*x + a)^6 + 144*cosh(b*x + a)*sinh(b*x + a)^5 + 24*sinh(b*
x + a)^6 + 72*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 72*cosh(b*x + a)^4
 + 96*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 72*(5*cosh(b
*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 - 10*(cosh(b*x + a)^6 +
 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b
*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*s
inh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^
3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*
x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b
*x + a)*sinh(b*x + a) - sinh(b*x + a)^2) + 72*cosh(b*x + a)^2 + 5*(cosh(b*
x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b
*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3
+ 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a
)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh
(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(-cosh(b*x + a)^2 + 2*(
cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2
*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2) + 16*(3*cosh(b*x + a)^5 +
15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5 + 2*(15*cosh(b*x + a)
^2 + 2)*sinh(b*x + a)^3 + 4*cosh(b*x + a)^3 + 6*(5*cosh(b*x + a)^3 + 2*...
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)**(7/2)/cosh(b*x+a)**(7/2),x)
```

output
```
Timed out
```

3.49.7 Maxima [F]

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{7}{2}}}{\cosh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(7/2)/cosh(b*x + a)^(7/2), x)
```

3.49.8 Giac [F]

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{7}{2}}}{\cosh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(7/2)/cosh(b*x + a)^(7/2), x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\mathrm{sinh}{\left(a+b\,x\right)}^{7/2}}{{\mathrm{cosh}\left(a+b\,x\right)}^{7/2}}\,dx$$

input
```
int(sinh(a + b*x)^(7/2)/cosh(a + b*x)^(7/2),x)
```

output
```
int(sinh(a + b*x)^(7/2)/cosh(a + b*x)^(7/2), x)
```

3.50 $\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

3.50.1 Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

output
```
-arctan(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2))/b+arctanh(sinh(b*x+a)^(1/2)/c
osh(b*x+a)^(1/2))/b-2/3*sinh(b*x+a)^(3/2)/b/cosh(b*x+a)^(3/2)
```

3.50.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\cosh^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{7}{4},\frac{11}{4},-\sinh^2(a+bx)\right)\sinh^{\frac{7}{2}}(a+bx)}{7b\cosh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Sinh[a + b*x]^(5/2)/Cosh[a + b*x]^(5/2),x]
```

output
```
(2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 7/4, 11/4, -Sinh[a + b*x
]^2]*Sinh[a + b*x]^(7/2))/(7*b*Cosh[a + b*x]^(3/2))
```

3.50.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3046, 3042, 3054, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ia+ibx))^{5/2}}{\cos(ia+ibx)^{5/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}dx-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}+\int \frac{\sqrt{-i\sin(ia+ibx)}}{\sqrt{\cos(ia+ibx)}}dx$$

$$\Big\downarrow \text{3054}$$

$$-\frac{2\int -\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{827}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{1}{\tanh(a+bx)+1}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\downarrow 219$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)\right)}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

input
```
Int[Sinh[a + b*x]^(5/2)/Cosh[a + b*x]^(5/2),x]
```

output
```
(-2*(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2 - ArcTanh[Sqrt[Sinh
[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2))/b - (2*Sinh[a + b*x]^(3/2))/(3*b*Cosh[
a + b*x]^(3/2))
```

3.50.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

3.50.4 Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{5}{2}}}{\cosh(bx+a)^{\frac{5}{2}}}dx$$

input
```
int(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x)
```

output
```
int(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x)
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. 2(67) = 134.

Time = 0.27 (sec) , antiderivative size = 591, normalized size of antiderivative = 7.30

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx =$$

$$-\frac{4\cosh(bx+a)^4+16\cosh(bx+a)\sinh(bx+a)^3+4\sinh(bx+a)^4+8\left(3\cosh(bx+a)^2+1\right)\sinh(b}{}$$

input
```
integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(4*cosh(b*x + a)^4 + 16*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x +
a)^4 + 8*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 6*(cosh(b*x + a)^4 + 4*
cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1
)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a)
)*sinh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x
 + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x
+ a) - sinh(b*x + a)^2) + 8*cosh(b*x + a)^2 + 3*(cosh(b*x + a)^4 + 4*cosh(
b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sin
h(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*s
qrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - s
inh(b*x + a)^2) + 8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + s
inh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*sq
rt(cosh(b*x + a))*sqrt(sinh(b*x + a)) + 16*(cosh(b*x + a)^3 + cosh(b*x + a
))*sinh(b*x + a) + 4)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)
^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + b)
*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) +
 b)
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)**(5/2)/cosh(b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.50.7 Maxima [F]

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{5}{2}}}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(5/2)/cosh(b*x + a)^(5/2), x)
```

3.50.8 Giac [F]

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{5}{2}}}{\cosh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(5/2)/cosh(b*x + a)^(5/2), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{5/2}}{\cosh\left(a+b\,x\right)^{5/2}}\,dx$$

input
```
int(sinh(a + b*x)^(5/2)/cosh(a + b*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)^(5/2)/cosh(a + b*x)^(5/2), x)
```

3.51 $\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$

3.51.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

output
```
-arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/s
inh(b*x+a)^(1/2))/b-2*sinh(b*x+a)^(1/2)/b/cosh(b*x+a)^(1/2)
```

3.51.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\sqrt[4]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{4},\frac{9}{4},-\sinh^2(a+bx)\right)\sinh^{\frac{5}{2}}(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

input
```
Integrate[Sinh[a + b*x]^(3/2)/Cosh[a + b*x]^(3/2),x]
```

output
```
(2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(5/2))/(5*b*Sqrt[Cosh[a + b*x]])
```

3.51.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3046, 3042, 3055, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ia+ibx))^{3/2}}{\cos(ia+ibx)^{3/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}dx-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}+\int \frac{\sqrt{\cos(ia+ibx)}}{\sqrt{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{3055}$$

$$\frac{2\int \frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\int \frac{1}{\coth(a+bx)+1}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow 219$$

$$\frac{2\left(\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

input
```
Int[Sinh[a + b*x]^(3/2)/Cosh[a + b*x]^(3/2),x]
```

output
```
(2*(-1/2*ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]] + ArcTanh[Sqrt[Co
sh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/2))/b - (2*Sqrt[Sinh[a + b*x]])/(b*Sqrt[
Cosh[a + b*x]])
```

3.51.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

3.51.4 Maple [F]

$$\int \frac{\sinh (bx+a)^{\frac{3}{2}}}{\cosh (bx+a)^{\frac{3}{2}}}dx$$

input
```
int(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x)
```

output
```
int(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x)
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(67) = 134.

Time = 0.26 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.92

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\left(\cosh (bx+a)^2+2\cosh (bx+a)\sinh (bx+a)+\sinh (bx+a)^2+1\right)\arctan\left(-\cosh (bx+a)^2+2(\cosh\right.}{}$$

input
```
integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
+ 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh
(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
+ a)^2) - 4*cosh(b*x + a)^2 - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
+ a) + sinh(b*x + a)^2 + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh
(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(
b*x + a) - sinh(b*x + a)^2) - 8*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(
b*x + a))*sqrt(sinh(b*x + a)) - 8*cosh(b*x + a)*sinh(b*x + a) - 4*sinh(b*x
 + a)^2 - 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh
(b*x + a)^2 + b)
```

3.51.6 Sympy [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)**(3/2)/cosh(b*x+a)**(3/2),x)
```

output
```
Integral(sinh(a + b*x)**(3/2)/cosh(a + b*x)**(3/2), x)
```

3.51.7 Maxima [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{3}{2}}}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(3/2)/cosh(b*x + a)^(3/2), x)
```

3.51.8 Giac [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{3}{2}}}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(3/2)/cosh(b*x + a)^(3/2), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{3/2}}{\cosh\left(a+b\,x\right)^{3/2}}\,dx$$

input
```
int(sinh(a + b*x)^(3/2)/cosh(a + b*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)^(3/2)/cosh(a + b*x)^(3/2), x)
```

3.52 $\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx$

3.52.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx=-\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

output
```
-arctan(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2))/b+arctanh(sinh(b*x+a)^(1/2)/c
osh(b*x+a)^(1/2))/b
```

3.52.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx=\frac{2\cosh^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},-\sinh^2(a+bx)\right)\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]],x]
```

output
```
(2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(3/2))/(3*b*Cosh[a + b*x]^(3/2))
```

3.52.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3054, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-i\sin(ia+ibx)}}{\sqrt{\cos(ia+ibx)}}dx$$

$$\Big\downarrow \text{3054}$$

$$-\frac{2\int -\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}$$

$$\Big\downarrow \text{827}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{1}{\tanh(a+bx)+1}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)\right)}{b}$$

input
```
Int[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]],x]
```

output
```
(-2*(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2 - ArcTanh[Sqrt[Sinh
[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2))/b
```

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

3.52.4 Maple [F]

$$\int \frac{\sqrt{\sinh \left(bx + a\right)}}{\sqrt{\cosh \left(bx + a\right)}}dx$$

input
```
int(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x)
```

output
```
int(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x)
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx =$$
$$-\frac{2\arctan\left(-\cosh(bx+a)^2+2\left(\cosh(bx+a)+\sinh(bx+a)\right)\sqrt{\cosh(bx+a)}\sqrt{\sinh(bx+a)}-2\cosh\right.}{}$$

input
```
integrate(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(c
osh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b
*x + a)^2) + log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt
(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh
(b*x + a)^2))/b
```

3.52.6 Sympy [F]

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx$$

input
```
integrate(sinh(b*x+a)**(1/2)/cosh(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(sinh(a + b*x))/sqrt(cosh(a + b*x)), x)
```

3.52.7 Maxima [F]

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh(bx+a)}}{\sqrt{\cosh(bx+a)}}\,dx$$

input
```
integrate(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sinh(b*x + a))/sqrt(cosh(b*x + a)), x)
```

3.52.8 Giac [F]

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh\left(bx+a\right)}}{\sqrt{\cosh\left(bx+a\right)}}\,dx$$

input
```
integrate(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sinh(b*x + a))/sqrt(cosh(b*x + a)), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh\left(a+b\,x\right)}}{\sqrt{\cosh\left(a+b\,x\right)}}\,dx$$

input
```
int(sinh(a + b*x)^(1/2)/cosh(a + b*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)^(1/2)/cosh(a + b*x)^(1/2), x)
```

3.53 $\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx$

3.53.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

output
```
-arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/s
inh(b*x+a)^(1/2))/b
```

3.53.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2\sqrt[4]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},-\sinh^2(a+bx)\right)\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

input
```
Integrate[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]],x]
```

output
```
(2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, -Sinh[a + b*x]
^2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b*x]])
```

3.53.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3055, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\sqrt{\cos(ia+ibx)}}{\sqrt{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \textbf{3055}$$

$$\frac{2\int \frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}}{b}$$

$$\Big\downarrow \textbf{827}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\int \frac{1}{\coth(a+bx)+1}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{2\left(\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}$$

input
```
Int[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]],x]
```

output
```
(2*(-1/2*ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]] + ArcTanh[Sqrt[Co
sh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/2))/b
```

3.53.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

3.53.4 Maple [F]

$$\int \frac{\sqrt{\cosh(bx+a)}}{\sqrt{\sinh(bx+a)}}dx$$

input
```
int(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x)
```

output
```
int(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x)
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(46) = 92$.

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.67

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2\arctan\left(-\cosh(bx+a)^2+2(\cosh(bx+a)+\sinh(bx+a))\sqrt{\cosh(bx+a)}\sqrt{\sinh(bx+a)}-2\cosh(b\right.}{}$$

input
```
integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(co
sh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*
x + a)^2) - log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(
cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(
b*x + a)^2))/b
```

3.53.6 Sympy [F]

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx$$

input
```
integrate(cosh(b*x+a)**(1/2)/sinh(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(cosh(a + b*x))/sqrt(sinh(a + b*x)), x)
```

3.53.7 Maxima [F]

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh(bx+a)}}{\sqrt{\sinh(bx+a)}}\,dx$$

input
```
integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cosh(b*x + a))/sqrt(sinh(b*x + a)), x)
```

3.53.8 Giac [F]

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh\left(bx+a\right)}}{\sqrt{\sinh\left(bx+a\right)}}\,dx$$

input
```
integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cosh(b*x + a))/sqrt(sinh(b*x + a)), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh\left(a+b\,x\right)}}{\sqrt{\sinh\left(a+b\,x\right)}}\,dx$$

input
```
int(cosh(a + b*x)^(1/2)/sinh(a + b*x)^(1/2),x)
```

output
```
int(cosh(a + b*x)^(1/2)/sinh(a + b*x)^(1/2), x)
```

3.54 $\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$

3.54.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

output
```
-arctan(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2))/b+arctanh(sinh(b*x+a)^(1/2)/c
osh(b*x+a)^(1/2))/b-2*cosh(b*x+a)^(1/2)/b/sinh(b*x+a)^(1/2)
```

3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\cosh^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},-\frac{1}{4},\frac{3}{4},-\sinh^2(a+bx)\right)}{b\cosh^{\frac{3}{2}}(a+bx)\sqrt{\sinh(a+bx)}}$$

input
```
Integrate[Cosh[a + b*x]^(3/2)/Sinh[a + b*x]^(3/2),x]
```

output
```
(-2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, -Sinh[a + b
*x]^2])/(b*Cosh[a + b*x]^(3/2)*Sqrt[Sinh[a + b*x]])
```

3.54.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3047, 3042, 3054, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ia+ibx)^{3/2}}{(-i\sin(ia+ibx))^{3/2}}dx$$

$$\Big\downarrow \text{3047}$$

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}dx-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}+\int \frac{\sqrt{-i\sin(ia+ibx)}}{\sqrt{\cos(ia+ibx)}}dx$$

$$\Big\downarrow \text{3054}$$

$$-\frac{2\int -\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{827}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{1}{\tanh(a+bx)+1}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow 219$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)\right)}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

input
```
Int[Cosh[a + b*x]^(3/2)/Sinh[a + b*x]^(3/2),x]
```

output
```
(-2*(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2 - ArcTanh[Sqrt[Sinh
[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2))/b - (2*Sqrt[Cosh[a + b*x]])/(b*Sqrt[Si
nh[a + b*x]])
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

3.54.4 Maple [F]

$$\int \frac{\cosh{(bx+a)}^{\frac{3}{2}}}{\sinh{(bx+a)}^{\frac{3}{2}}}dx$$

input
```
int(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x)
```

output
```
int(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x)
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(67) = 134$.

Time = 0.25 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.94

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\left(\cosh{(bx+a)}^2 + 2\cosh{(bx+a)}\sinh{(bx+a)} + \sinh{(bx+a)}^2 - 1\right)\arctan\left(-\cosh{(bx+a)}^2 + 2\,(\mathrm{co}\right.}{}$$

input
```
integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
 - 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cos
h(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
 + a)^2) + 4*cosh(b*x + a)^2 + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
 + a) + sinh(b*x + a)^2 - 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sin
h(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh
(b*x + a) - sinh(b*x + a)^2) + 8*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh
(b*x + a))*sqrt(sinh(b*x + a)) + 8*cosh(b*x + a)*sinh(b*x + a) + 4*sinh(b*
x + a)^2 - 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sin
h(b*x + a)^2 - b)
```

3.54.6 Sympy [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(cosh(b*x+a)**(3/2)/sinh(b*x+a)**(3/2),x)
```

output
```
Integral(cosh(a + b*x)**(3/2)/sinh(a + b*x)**(3/2), x)
```

3.54.7 Maxima [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{3}{2}}}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(3/2)/sinh(b*x + a)^(3/2), x)
```

3.54.8 Giac [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{3}{2}}}{\sinh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(3/2)/sinh(b*x + a)^(3/2), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{3/2}}{\sinh\left(a+b\,x\right)^{3/2}}\,dx$$

input
```
int(cosh(a + b*x)^(3/2)/sinh(a + b*x)^(3/2),x)
```

output
```
int(cosh(a + b*x)^(3/2)/sinh(a + b*x)^(3/2), x)
```

3.55 $\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)} dx$

3.55.1 Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)} dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

output
```
-arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/s
inh(b*x+a)^(1/2))/b-2/3*cosh(b*x+a)^(3/2)/b/sinh(b*x+a)^(3/2)
```

3.55.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)} dx = -\frac{2\sqrt[4]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{3}{4},\frac{1}{4},-\sinh^2(a+bx)\right)}{3b\sqrt{\cosh(a+bx)}\sinh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(5/2)/Sinh[a + b*x]^(5/2),x]
```

output
```
(-2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, -Sinh[a + b
*x]^2])/(3*b*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x]^(3/2))
```

3.55.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3047, 3042, 3055, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(ia+ibx)^{5/2}}{(-i\sin(ia+ibx))^{5/2}}dx$$

$$\Big\downarrow \ 3047$$

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}dx-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\int \frac{\sqrt{\cos(ia+ibx)}}{\sqrt{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \ 3055$$

$$\frac{2\int \frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \ 827$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\int \frac{1}{\coth(a+bx)+1}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \ 216$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \ 219$$

$$\frac{2\left(\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

input
```
Int[Cosh[a + b*x]^(5/2)/Sinh[a + b*x]^(5/2),x]
```

output
```
(2*(-1/2*ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]] + ArcTanh[Sqrt[Co
sh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/2))/b - (2*Cosh[a + b*x]^(3/2))/(3*b*Sin
h[a + b*x]^(3/2))
```

3.55.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

3.55.4 Maple [F]

$$\int \frac{\cosh (bx+a)^{\frac{5}{2}}}{\sinh (bx+a)^{\frac{5}{2}}}dx$$

input
```
int(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x)
```

output
```
int(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x)
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. 2(67) = 134.

Time = 0.27 (sec) , antiderivative size = 598, normalized size of antiderivative = 7.38

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{4\cosh (bx+a)^4 + 16\cosh (bx+a)\sinh (bx+a)^3 + 4\sinh (bx+a)^4 + 8\left(3\cosh (bx+a)^2 - 1\right)\sinh (b}{}$$

input
```
integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(4*cosh(b*x + a)^4 + 16*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x +
a)^4 + 8*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 6*(cosh(b*x + a)^4 + 4*
cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1
)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x
 + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x
+ a) - sinh(b*x + a)^2) - 8*cosh(b*x + a)^2 + 3*(cosh(b*x + a)^4 + 4*cosh(
b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sin
h(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sin
h(b*x + a) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*s
qrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - s
inh(b*x + a)^2) + 8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + s
inh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*sq
rt(cosh(b*x + a))*sqrt(sinh(b*x + a)) + 16*(cosh(b*x + a)^3 - cosh(b*x + a
))*sinh(b*x + a) + 4)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)
^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)
*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) +
 b)
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**(5/2)/sinh(b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.55.7 Maxima [F]

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{5}{2}}}{\sinh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(5/2)/sinh(b*x + a)^(5/2), x)
```

3.55.8 Giac [F]

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{5}{2}}}{\sinh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(5/2)/sinh(b*x + a)^(5/2), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{5/2}}{\sinh\left(a+b\,x\right)^{5/2}}\,dx$$

input
```
int(cosh(a + b*x)^(5/2)/sinh(a + b*x)^(5/2),x)
```

output
```
int(cosh(a + b*x)^(5/2)/sinh(a + b*x)^(5/2), x)
```

3.56 $\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

3.56.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx=-\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

output
```
-arctan(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2))/b+arctanh(sinh(b*x+a)^(1/2)/c
osh(b*x+a)^(1/2))/b-2/5*cosh(b*x+a)^(5/2)/b/sinh(b*x+a)^(5/2)-2*cosh(b*x+a
)^(1/2)/b/sinh(b*x+a)^(1/2)
```

3.56.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx=-\frac{2\cosh^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{5}{4},-\frac{1}{4},-\sinh^2(a+bx)\right)}{5b\cosh^{\frac{3}{2}}(a+bx)\sinh^{\frac{5}{2}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(7/2)/Sinh[a + b*x]^(7/2),x]
```

output
```
(-2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, -Sinh[a +
b*x]^2])/(5*b*Cosh[a + b*x]^(3/2)*Sinh[a + b*x]^(5/2))
```

3.56.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3047, 3042, 3047, 3042, 3054, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ia+ibx)^{7/2}}{(-i\sin(ia+ibx))^{7/2}}dx$$

$$\Big\downarrow \text{3047}$$

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}dx-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\int \frac{\cos(ia+ibx)^{3/2}}{(-i\sin(ia+ibx))^{3/2}}dx$$

$$\Big\downarrow \text{3047}$$

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}dx-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-i\sin(ia+ibx)}}{\sqrt{\cos(ia+ibx)}}dx-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3054}$$

$$-\frac{2\int -\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{827}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{1}{\tanh(a+bx)+1}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)\right)}{b}-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

input
```
Int[Cosh[a + b*x]^(7/2)/Sinh[a + b*x]^(7/2),x]
```

output
```
(-2*(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2 - ArcTanh[Sqrt[Sinh
[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2))/b - (2*Cosh[a + b*x]^(5/2))/(5*b*Sinh[
a + b*x]^(5/2)) - (2*Sqrt[Cosh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

3.56.4 Maple [F]

$$\int \frac{\cosh\left(bx+a\right)^{\frac{7}{2}}}{\sinh\left(bx+a\right)^{\frac{7}{2}}}dx$$

input
```
int(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x)
```

output
```
int(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x)
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. 2(88) = 176.

Time = 0.27 (sec) , antiderivative size = 1001, normalized size of antiderivative = 9.44

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/10*(24*cosh(b*x + a)^6 + 144*cosh(b*x + a)*sinh(b*x + a)^5 + 24*sinh(b*
x + a)^6 + 72*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 72*cosh(b*x + a)^4
 + 96*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 72*(5*cosh(b
*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 10*(cosh(b*x + a)^6 +
 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b
*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*s
inh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^
3 + cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*
x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b
*x + a)*sinh(b*x + a) - sinh(b*x + a)^2) + 72*cosh(b*x + a)^2 + 5*(cosh(b*
x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3
- 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a
)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh
(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*log(-cosh(b*x + a)^2 + 2*(
cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2
*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2) + 16*(3*cosh(b*x + a)^5 +
15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5 + 2*(15*cosh(b*x + a)
^2 - 2)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 + 6*(5*cosh(b*x + a)^3 - 2*...
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**(7/2)/sinh(b*x+a)**(7/2),x)
```

output
```
Timed out
```

3.56.7 Maxima [F]

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{7}{2}}}{\sinh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(7/2)/sinh(b*x + a)^(7/2), x)
```

3.56.8 Giac [F]

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{7}{2}}}{\sinh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(7/2)/sinh(b*x + a)^(7/2), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{{\mathrm{cosh}\left(a+b\,x\right)}^{7/2}}{{\mathrm{sinh}\left(a+b\,x\right)}^{7/2}}\,dx$$

input
```
int(cosh(a + b*x)^(7/2)/sinh(a + b*x)^(7/2),x)
```

output
```
int(cosh(a + b*x)^(7/2)/sinh(a + b*x)^(7/2), x)
```

3.57 $\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx$

3.57.1 Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

output
```
-1/2*ln(1-sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a)^(2/3
)/cosh(b*x+a)^(2/3)+sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3))/b-3/4*sinh(b*x+a)
^(4/3)/b/cosh(b*x+a)^(4/3)-1/2*arctan(1/3*(1+2*sinh(b*x+a)^(2/3)/cosh(b*x+
a)^(2/3))*3^(1/2))*3^(1/2)/b
```

3.57.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)} dx = \frac{3\cosh^2(a+bx)^{2/3}\text{Hypergeometric2F1}\left(\frac{5}{3},\frac{5}{3},\frac{8}{3},-\sinh^2(a+bx)\right)\sinh^{\frac{10}{3}}(a+bx)}{10b\cosh^{\frac{4}{3}}(a+bx)}$$

input
```
Integrate[Sinh[a + b*x]^(7/3)/Cosh[a + b*x]^(7/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[5/3, 5/3, 8/3, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(10/3))/(10*b*Cosh[a + b*x]^(4/3))
```

3.57.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.88, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3046, 3042, 3054, 25, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)} dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{(-i\sin(ia+ibx))^{7/3}}{\cos(ia+ibx)^{7/3}}dx$$

$$\Big\downarrow \textbf{3046}$$

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}dx - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} + \int \frac{\sqrt[3]{-i\sin(ia+ibx)}}{\sqrt[3]{\cos(ia+ibx)}}dx$$

$$\Big\downarrow \color{blue}{3054}$$

$$-\frac{3\int -\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{3\int \frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \color{blue}{807}$$

$$\frac{3\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)(1-\tanh(a+bx))}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \color{blue}{821}$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int \frac{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \color{blue}{16}$$

$$\frac{3\left(-\frac{1}{3}\int \frac{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \color{blue}{1142}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{3}{2}\int \frac{1}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \color{blue}{1083}$$

3.57. $\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}dx$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+3\int\frac{1}{-\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-4}d\left(\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\log\left(\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

input
```
Int[Sinh[a + b*x]^(7/3)/Cosh[a + b*x]^(7/3),x]
```

output
```
(3*(-1/3*Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)] + (-(Sqrt[3]*Arc
Tan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]) + Log[1 +
(2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3)]/2)/3))/(2*b) - (3*Sinh[a + b*
x]^(4/3))/(4*b*Cosh[a + b*x]^(4/3))
```

3.57.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

3.57. $\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

3.57. $\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx$

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

3.57.4 Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{7}{3}}}{\cosh(bx+a)^{\frac{7}{3}}}dx$$

input
```
int(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x)
```

output
```
int(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x)
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(124) = 248$.

Time = 0.28 (sec) , antiderivative size = 1042, normalized size of antiderivative = 6.72

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
-1/4*(2*(sqrt(3)*cosh(b*x + a)^4 + 4*sqrt(3)*cosh(b*x + a)*sinh(b*x + a)^3
 + sqrt(3)*sinh(b*x + a)^4 + 2*(3*sqrt(3)*cosh(b*x + a)^2 + sqrt(3))*sinh(
b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a)^3 + sqrt
(3)*cosh(b*x + a))*sinh(b*x + a) + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x +
 a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 +
4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh
(b*x + a)^(2/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 + 1)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x +
a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cos
h(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(
(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(
3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 2*(cosh(b*x +
 a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x +
a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)
^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x +
 a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x +
a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*
x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
+ a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4
*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x + ...
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)**(7/3)/cosh(b*x+a)**(7/3),x)
```

output
```
Timed out
```

3.57.7 Maxima [F]

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\sinh{(bx+a)}^{\frac{7}{3}}}{\cosh{(bx+a)}^{\frac{7}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(7/3)/cosh(b*x + a)^(7/3), x)
```

3.57.8 Giac [F]

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\sinh{(bx+a)}^{\frac{7}{3}}}{\cosh{(bx+a)}^{\frac{7}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(7/3)/cosh(b*x + a)^(7/3), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^{7/3}}{{\mathrm{cosh}\left(a+b\,x\right)}^{7/3}}\,dx$$

input
```
int(sinh(a + b*x)^(7/3)/cosh(a + b*x)^(7/3),x)
```

output
```
int(sinh(a + b*x)^(7/3)/cosh(a + b*x)^(7/3), x)
```

3.58 $\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx$

3.58.1 Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

output
```
-1/2*ln(1-cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b+1/4*ln(1+cosh(b*x+a)^(4/3
)/sinh(b*x+a)^(4/3)+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b-3/2*sinh(b*x+a)
^(2/3)/b/cosh(b*x+a)^(2/3)-1/2*arctan(1/3*(1+2*cosh(b*x+a)^(2/3)/sinh(b*x+
a)^(2/3))*3^(1/2))*3^(1/2)/b
```

3.58.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \frac{3\sqrt[3]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{4}{3},\frac{7}{3},-\sinh^2(a+bx)\right)\sinh^{\frac{8}{3}}(a+bx)}{8b\cosh^{\frac{2}{3}}(a+bx)}$$

input
```
Integrate[Sinh[a + b*x]^(5/3)/Cosh[a + b*x]^(5/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(8/3))/(8*b*Cosh[a + b*x]^(2/3))
```

3.58.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3046, 3042, 3055, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ia+ibx))^{5/3}}{\cos(ia+ibx)^{5/3}}dx$$

$$\Big\downarrow \text{3046}$$

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}dx-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}+\int \frac{\sqrt[3]{\cos(ia+ibx)}}{\sqrt[3]{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{3055}$$

$$\frac{3\int \frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{807}$$

$$\frac{3\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{(1-\coth(a+bx))\sinh^{\frac{2}{3}}(a+bx)}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{821}$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int \frac{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{16}$$

$$\frac{3\left(-\frac{1}{3}\int \frac{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{3}{2}\int \frac{1}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+3\int \frac{1}{-\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-4}d\left(\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{217}$$

3.58. $\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}dx$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\downarrow \textbf{1103}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\log\left(\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

input
```
Int[Sinh[a + b*x]^(5/3)/Cosh[a + b*x]^(5/3),x]
```

output
```
(3*(-1/3*Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)] + (-(Sqrt[3]*Arc
Tan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]) + Log[1 +
(2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3)]/2)/3))/(2*b) - (3*Sinh[a + b*
x]^(2/3))/(2*b*Cosh[a + b*x]^(2/3))
```

3.58.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

3.58. $\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)} \, dx$

3.58.4 Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{5}{3}}}{\cosh(bx+a)^{\frac{5}{3}}}dx$$

input
```
int(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x)
```

output
```
int(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x)
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(124) = 248$.

Time = 0.28 (sec) , antiderivative size = 751, normalized size of antiderivative = 4.85

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
-1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
 sqrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 +
2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(
3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a
)^(1/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh
(b*x + a)^2 - 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 + 1)*log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
 sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x +
 a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x +
a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b
*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh
(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 -
cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(
b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1
)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
+ 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x +
a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +...
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)**(5/3)/cosh(b*x+a)**(5/3),x)
```

output
```
Timed out
```

3.58.7 Maxima [F]

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sinh{(bx+a)}^{\frac{5}{3}}}{\cosh{(bx+a)}^{\frac{5}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(5/3)/cosh(b*x + a)^(5/3), x)
```

3.58.8 Giac [F]

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sinh{(bx+a)}^{\frac{5}{3}}}{\cosh{(bx+a)}^{\frac{5}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(5/3)/cosh(b*x + a)^(5/3), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{5/3}}{\cosh\left(a+b\,x\right)^{5/3}}\,dx$$

input
```
int(sinh(a + b*x)^(5/3)/cosh(a + b*x)^(5/3),x)
```

output
```
int(sinh(a + b*x)^(5/3)/cosh(a + b*x)^(5/3), x)
```

3.59 $\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$

3.59.1 Optimal result

Integrand size = 21, antiderivative size = 243

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} - \frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

output
```
arctanh(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b-1/4*ln(1+cosh(b*x+a)^(2/3)/
sinh(b*x+a)^(2/3)-cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b+1/4*ln(1+cosh(b*x
+a)^(2/3)/sinh(b*x+a)^(2/3)+cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b-3*sinh(
b*x+a)^(1/3)/b/cosh(b*x+a)^(1/3)+1/2*arctan(1/3*(1-2*cosh(b*x+a)^(1/3)/sin
h(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b-1/2*arctan(1/3*(1+2*cosh(b*x+a)^(1/3)/s
inh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b
```

3.59.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\sqrt[6]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{7}{6},\frac{13}{6},-\sinh^2(a+bx)\right)\sinh^{\frac{7}{3}}(a+bx)}{7b\sqrt[3]{\cosh(a+bx)}}$$

input
```
Integrate[Sinh[a + b*x]^(4/3)/Cosh[a + b*x]^(4/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(1/6)*Hypergeometric2F1[7/6, 7/6, 13/6, -Sinh[a + b*x
]^2]*Sinh[a + b*x]^(7/3))/(7*b*Cosh[a + b*x]^(1/3))
```

3.59.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3046, 3042, 3055, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ia+ibx))^{4/3}}{\cos(ia+ibx)^{4/3}}dx$$

$$\Big\downarrow \; 3046$$

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}dx-\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}+\int \frac{\cos(ia+ibx)^{2/3}}{(-i\sin(ia+ibx))^{2/3}}dx$$

$$\Big\downarrow \; 3055$$

$$\frac{3\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{(1-\coth^2(a+bx))\sinh^{\frac{4}{3}}(a+bx)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{b}-\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \; 825$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+\frac{1}{3}\int -\frac{\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{2\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+\frac{1}{3}\int -\frac{\cdots}{2\left(\frac{\cosh^{\frac{2}{3}}(\cdots}{\sinh^{\frac{2}{3}}(\cdots}\cdots\right)}\cdots\right)}{b}\cdots$$

$$\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{6}\int \frac{\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\cosh(\cdots}}{\sqrt[3]{\sinh(\cdots}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh\cdots}}{\sqrt[3]{\sinh\cdots}}\cdots}\cdots\right)}{b}\cdots$$

$$\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{3\left(-\frac{1}{6}\int \frac{\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+\cdots\right)}{b}\cdots$$

$$\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

3.59. $\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$

$\downarrow$ 1142

$$\frac{3\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{2}\int -\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right.\right.}{\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}}$$

$\downarrow$ 25

$$\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{3}{2}\int \frac{1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)\right.}{\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}}$$

$\downarrow$ 1083

$$\frac{3\left(\frac{1}{6}\left(3\int \frac{1}{-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-3}d\left(\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-1\right)+\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)+\frac{1}{6}\left(3\right.\right.}{\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}}$$

$\downarrow$ 217

$$\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{2}{\frac{\cosh^{\frac{2}{3}}(a}{\sinh^{\frac{2}{3}}(a}}\right.\right.}{\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}}$$

$\downarrow$ 1103

3.59. $\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$

$$\frac{3\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx}{\sinh^{\frac{2}{3}}(a+bx}\right.\right.\right.}{b}$$

$$\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

input
```
Int[Sinh[a + b*x]^(4/3)/Cosh[a + b*x]^(4/3),x]
```

output
```
(3*(ArcTanh[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/3 + (-(Sqrt[3]*ArcTan
[(-1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]]) - Log[1 + Co
sh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) - Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^
(1/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]
^(1/3))/Sqrt[3]]) + Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) + Cosh
[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/2)/6))/b - (3*Sinh[a + b*x]^(1/3))/(b
*Cosh[a + b*x]^(1/3))
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

3.59. $\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$

3.59.4 Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{4}{3}}}{\cosh(bx+a)^{\frac{4}{3}}}dx$$

input
```
int(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x)
```

output
```
int(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x)
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. $2(197) = 394$.

Time = 0.27 (sec) , antiderivative size = 1003, normalized size of antiderivative = 4.13

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
sqrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2
*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3
)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)
^(1/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(
b*x + a)^2 + 1)) + 2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*si
nh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(-1/3*(sqrt(3)*cosh
(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a
)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3
)*sinh(b*x + a)^(1/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 + 1)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(
b*x + a) + sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)
+ sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)
*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*s
inh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a
)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x
+ a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x
 + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - (cosh(b*x + a)^2 + 2*cosh(b...
```

3.59.6 Sympy [F]

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)**(4/3)/cosh(b*x+a)**(4/3),x)
```

output
```
Integral(sinh(a + b*x)**(4/3)/cosh(a + b*x)**(4/3), x)
```

3.59.7 Maxima [F]

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{4}{3}}}{\cosh(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(4/3)/cosh(b*x + a)^(4/3), x)
```

3.59.8 Giac [F]

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{4}{3}}}{\cosh(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(4/3)/cosh(b*x + a)^(4/3), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{4/3}}{\cosh\,(a+b\,x)^{4/3}}\,dx$$

input
```
int(sinh(a + b*x)^(4/3)/cosh(a + b*x)^(4/3),x)
```

output
```
int(sinh(a + b*x)^(4/3)/cosh(a + b*x)^(4/3), x)
```

3.60 $\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$

3.60.1 Optimal result

Integrand size = 21, antiderivative size = 218

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

output
```
arctanh(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))/b-1/4*ln(1-sinh(b*x+a)^(1/3)/
cosh(b*x+a)^(1/3)+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x
+a)^(1/3)/cosh(b*x+a)^(1/3)+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/2*arc
tan(1/3*(1-2*sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b-1/2*a
rctan(1/3*(1+2*sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b
```

3.60.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.27

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\cosh^2(a+bx)^{5/6}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{6},\frac{11}{6},-\sinh^2(a+bx)\right)\sinh^{\frac{5}{3}}(a+bx)}{5b\cosh^{\frac{5}{3}}(a+bx)}$$

input
```
Integrate[Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6, -Sinh[a + b*x
]^2]*Sinh[a + b*x]^(5/3))/(5*b*Cosh[a + b*x]^(5/3))
```

3.60.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3054, 25, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ia+ibx))^{2/3}}{\cos(ia+ibx)^{2/3}}dx$$

$$\Big\downarrow \text{3054}$$

$$-\frac{3\int -\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)(1-\tanh^2(a+bx))}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)(1-\tanh^2(a+bx))}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}$$

$\big\downarrow$ 825

$$-\frac{3\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\frac{1}{3}\int -\frac{\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{2\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\frac{1}{3}\int -\frac{}{2\left(\frac{\sin}{\cos}\right.}\right.}{b}$$

$\big\downarrow$ 27

$$-\frac{3\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{6}\int \frac{\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{s}}{\sqrt[3]{c}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+}\right.}{b}$$

$\big\downarrow$ 219

$$-\frac{3\left(\frac{1}{6}\int \frac{\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\right.}{b}$$

$\big\downarrow$ 1142

$$-\frac{3\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{2}\int -\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right.\right.}{}$$

$\big\downarrow$ 25

$$-\frac{3\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right.\right.}{}$$

$\big\downarrow$ 1083

3.60. $\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$

$$-\frac{3\left(\frac{1}{6}\left(-3\int \frac{1}{-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-3}d\left(\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-1\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)+\frac{1}{6}}{}$$

$\downarrow$ 217

$$-\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)+\frac{1}{6}\left(\sqrt{3}\arcta\right.}{}$$

$\downarrow$ 1103

$$-\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)\right)+\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{s}}{\sqrt[3]{c}}\right.\right.}{b}$$

input
```
Int[Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3),x]
```

output
```
(-3*(-1/3*ArcTanh[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3)] + (Sqrt[3]*ArcT
an[(-1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/Sqrt[3]] + Log[1 - S
inh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a + b*x]^(2/3)/Cosh[a + b*x]
^(2/3)]/2)/6 + (Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^
(1/3))/Sqrt[3]] - Log[1 + Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a
 + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/2)/6))/b
```

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.60. $\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

3.60.4 Maple [F]

$$\int \frac{\sinh (bx+a)^{\frac{2}{3}}}{\cosh (bx+a)^{\frac{2}{3}}}dx$$

input
```
int(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x)
```

output
```
int(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x)
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(176) = 352$.

Time = 0.28 (sec) , antiderivative size = 727, normalized size of antiderivative = 3.33

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sq
rt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) - sqrt(3))/(c
osh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 2
*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*si
nh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)
*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) - sqrt(3))/(cosh(b
*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + log((c
osh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sin
h(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*s
inh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) -
 log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2
/3)*sinh(b*x + a)^(1/3) - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^
(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^
2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
- 1)) + 2*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*
x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*
x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 - 1)) - 2*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + ...
```

3.60.6 Sympy [F]

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)**(2/3)/cosh(b*x+a)**(2/3),x)
```

output
```
Integral(sinh(a + b*x)**(2/3)/cosh(a + b*x)**(2/3), x)
```

3.60.7 Maxima [F]

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{2}{3}}}{\cosh(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(2/3)/cosh(b*x + a)^(2/3), x)
```

3.60.8 Giac [F]

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{2}{3}}}{\cosh(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(2/3)/cosh(b*x + a)^(2/3), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{2/3}}{\cosh(a+b\,x)^{2/3}}\,dx$$

input
```
int(sinh(a + b*x)^(2/3)/cosh(a + b*x)^(2/3),x)
```

output
```
int(sinh(a + b*x)^(2/3)/cosh(a + b*x)^(2/3), x)
```

3.61 $\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b}$$

output
```
-1/2*ln(1-sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a)^(2/3
)/cosh(b*x+a)^(2/3)+sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3))/b-1/2*arctan(1/3*
(1+2*sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))*3^(1/2))*3^(1/2)/b
```

3.61.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = \frac{3\cosh^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{2}{3},\frac{5}{3},-\sinh^2(a+bx)\right)\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

input
```
Integrate[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(4/3))/(4*b*Cosh[a + b*x]^(4/3))
```

3.61.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3054, 25, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{-i\sin(ia+ibx)}}{\sqrt[3]{\cos(ia+ibx)}}dx$$

$$\Big\downarrow \text{3054}$$

$$-\frac{3\int -\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\int \frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}$$

$$\Big\downarrow \text{807}$$

$$\frac{3\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)(1-\tanh(a+bx))}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{2b}$$

$$\Big\downarrow \text{821}$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int \frac{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$\downarrow 16$$

$$\frac{3\left(-\frac{1}{3}\int \frac{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\downarrow 1142$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{3}{2}\int \frac{1}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\downarrow 1083$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+3\int \frac{1}{-\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-4}d\left(\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\downarrow 217$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\downarrow 1103$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\log\left(\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

input
```
Int[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3),x]
```

output
```
(3*(-1/3*Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)] + (-(Sqrt[3]*Arc
Tan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]) + Log[1 +
(2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3)]/2)/3))/(2*b)
```

3.61. $\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$

3.61.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.61. $\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

3.61.4 Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{1}{3}}}{\cosh(bx+a)^{\frac{1}{3}}}dx$$

input
```
int(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x)
```

output
```
int(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x)
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(103) = 206$.

Time = 0.28 (sec) , antiderivative size = 572, normalized size of antiderivative = 4.47

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\cosh(bx+a)^2+2\sqrt{3}\cosh(bx+a)\sinh(bx+a)+\sqrt{3}\sinh(bx+a)^2+4\left(\sqrt{3}\cosh(bx+a)+\sqrt{3}\sinh(bx+a)\right)\cosh(bx+a)^{\frac{1}{3}}\sinh}{3\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1\right)}\right.}{}$$

input
```
integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
 a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + s
qrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + sqrt(3))/(
cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) -
log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 2*(cosh(b
*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*
x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x
+ a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b
*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*
x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(
b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2
 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*log(-(cosh(
b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*
x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(
b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)))/b
```

3.61.6 Sympy [F]

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = \int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$$

input
```
integrate(sinh(b*x+a)**(1/3)/cosh(b*x+a)**(1/3),x)
```

output
```
Integral(sinh(a + b*x)**(1/3)/cosh(a + b*x)**(1/3), x)
```

3.61.7 Maxima [F]

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = \int \frac{\sinh(bx+a)^{\frac{1}{3}}}{\cosh(bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(1/3)/cosh(b*x + a)^(1/3), x)
```

3.61.8 Giac [F]

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{1}{3}}}{\cosh\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(1/3)/cosh(b*x + a)^(1/3), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = \int \frac{\sinh(a+b\,x)^{1/3}}{\cosh\left(a+b\,x\right)^{1/3}}\,dx$$

input
```
int(sinh(a + b*x)^(1/3)/cosh(a + b*x)^(1/3),x)
```

output
```
int(sinh(a + b*x)^(1/3)/cosh(a + b*x)^(1/3), x)
```

3.62 $\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$

3.62.1 Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

output
```
-1/2*ln(1-cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b+1/4*ln(1+cosh(b*x+a)^(4/3
)/sinh(b*x+a)^(4/3)+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b-1/2*arctan(1/3*
(1+2*cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))*3^(1/2))*3^(1/2)/b
```

3.62.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \frac{3\sqrt[3]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{3},\frac{4}{3},-\sinh^2(a+bx)\right)\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(2/3))/(2*b*Cosh[a + b*x]^(2/3))
```

3.62.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3055, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{\cos(ia+ibx)}}{\sqrt[3]{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{3055}$$

$$\frac{3\int \frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{b}$$

$$\Big\downarrow \text{807}$$

$$\frac{3\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{(1-\coth(a+bx))\sinh^{\frac{2}{3}}(a+bx)}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{2b}$$

$$\Big\downarrow \text{821}$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int \frac{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$\Big\downarrow \text{16}$$

$$\frac{3\left(-\frac{1}{3}\int \frac{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{3}{2}\int \frac{1}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+3\int \frac{1}{-\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-4}d\left(\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\Big\downarrow \text{217}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\log\left(\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

input
```
Int[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3),x]
```

output
```
(3*(-1/3*Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)] + (-(Sqrt[3]*Arc
Tan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]) + Log[1 +
(2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3)]/2)/3))/(2*b)
```

3.62. $\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$

3.62.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.62. $\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

3.62.4 Maple [F]

$$\int \frac{\cosh \left(bx+a\right)^{\frac{1}{3}}}{\sinh \left(bx+a\right)^{\frac{1}{3}}}dx$$

input
```
int(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x)
```

output
```
int(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x)
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(103) = 206$.

Time = 0.27 (sec) , antiderivative size = 578, normalized size of antiderivative = 4.52

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\cosh(bx+a)^2+2\sqrt{3}\cosh(bx+a)\sinh(bx+a)+\sqrt{3}\sinh(bx+a)^2+4\left(\sqrt{3}\cosh(bx+a)+\sqrt{3}\sinh(bx+a)\right)\cosh(bx+a)^{\frac{2}{3}}\sinh}{3\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)}\right.}{}$$

input
```
integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
 a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + s
qrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) - sqrt(3))/(
cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) -
log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 2*(cosh(b
*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*
x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x
+ a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b
*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*
x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sin
h(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(
b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*log(-(cosh(
b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*
x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(
b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/b
```

3.62.6 Sympy [F]

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \int \frac{\sqrt[3]{\cosh{\left(a+bx \right)}}}{\sqrt[3]{\sinh{\left(a+bx \right)}}}\,dx$$

input
```
integrate(cosh(b*x+a)**(1/3)/sinh(b*x+a)**(1/3),x)
```

output
```
Integral(cosh(a + b*x)**(1/3)/sinh(a + b*x)**(1/3), x)
```

3.62.7 Maxima [F]

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{1}{3}}}{\sinh\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(1/3)/sinh(b*x + a)^(1/3), x)
```

3.62.8 Giac [F]

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx=\int \frac{\cosh\left(bx+a\right)^{\frac{1}{3}}}{\sinh\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(1/3)/sinh(b*x + a)^(1/3), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx=\int \frac{{\cosh(a+b\,x)}^{1/3}}{{\sinh\left(a+b\,x\right)}^{1/3}}\,dx$$

input
```
int(cosh(a + b*x)^(1/3)/sinh(a + b*x)^(1/3),x)
```

output
```
int(cosh(a + b*x)^(1/3)/sinh(a + b*x)^(1/3), x)
```

3.63 $\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$

3.63.1 Optimal result

Integrand size = 21, antiderivative size = 218

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b}$$

output
```
arctanh(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b-1/4*ln(1+cosh(b*x+a)^(2/3)/
sinh(b*x+a)^(2/3)-cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b+1/4*ln(1+cosh(b*x
+a)^(2/3)/sinh(b*x+a)^(2/3)+cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b+1/2*arc
tan(1/3*(1-2*cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b-1/2*a
rctan(1/3*(1+2*cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b
```

3.63.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.26

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\sqrt[6]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{6},\frac{7}{6},-\sinh^2(a+bx)\right)\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

input
```
Integrate[Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/6, 7/6, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(1/3))/(b*Cosh[a + b*x]^(1/3))
```

3.63.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3055, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ia+ibx)^{2/3}}{(-i\sin(ia+ibx))^{2/3}}dx$$

$$\Big\downarrow \text{3055}$$

$$\frac{3\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{(1-\coth^2(a+bx))\sinh^{\frac{4}{3}}(a+bx)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{b}$$

$$\Big\downarrow \text{825}$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+\frac{1}{3}\int -\frac{\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{2\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+\frac{1}{3}\int -\ldots\right.}{b}$$

$\downarrow$ 27

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{6}\int \frac{\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\cosh(\ldots}}{\sqrt[3]{\sinh(\ldots}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\ldots}}{\sqrt[3]{\ldots}}}\ldots\right.}{b}$$

$\downarrow$ 219

$$\frac{3\left(-\frac{1}{6}\int \frac{\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+\ldots\right.}{b}$$

$\downarrow$ 1142

$$\frac{3\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{2}\int -\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\ldots\right.\right.}{\ldots}$$

$\downarrow$ 25

$$\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{3}{2}\int \frac{1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)\ldots\right.}{\ldots}$$

$\downarrow$ 1083

3.63. $\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$

$$3\left(\frac{1}{6}\left(3\int \frac{1}{-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-3}d\left(\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-1\right)+\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)+\frac{1}{6}\left(3\right.\right.$$

$\downarrow$ 217

$$3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{}{\frac{\cosh^{\frac{2}{3}}(a}{\sinh^{\frac{2}{3}}(a}}\right.\right.$$

$\downarrow$ 1103

$$3\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx}{\sinh^{\frac{2}{3}}(a+bx}\right.\right.\right.$$
$$b$$

input
```
Int[Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3),x]
```

output
```
(3*(ArcTanh[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/3 + (-(Sqrt[3]*ArcTan
[(-1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]]) - Log[1 + Co
sh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) - Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^
(1/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]
^(1/3))/Sqrt[3]]) + Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) + Cosh
[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/2)/6))/b
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.63. $\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.63. $\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

3.63.4 Maple [F]

$$\int \frac{\cosh \left(bx + a\right)^{\frac{2}{3}}}{\sinh \left(bx + a\right)^{\frac{2}{3}}}dx$$

input
```
int(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x)
```

output
```
int(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x)
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 723 vs. $2(176) = 352$.

Time = 0.28 (sec) , antiderivative size = 723, normalized size of antiderivative = 3.32

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sq
rt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + sqrt(3))/(c
osh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2
*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*si
nh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)
*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + sqrt(3))/(cosh(b
*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + log((c
osh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sin
h(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*s
inh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) +
 2*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^
(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^
2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
+ 1)) - log((cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x
+ a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*
x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*
x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 + 1)) - 2*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + ...
```

3.63.6 Sympy [F]

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$$

input
```
integrate(cosh(b*x+a)**(2/3)/sinh(b*x+a)**(2/3),x)
```

output
```
Integral(cosh(a + b*x)**(2/3)/sinh(a + b*x)**(2/3), x)
```

3.63.7 Maxima [F]

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx=\int \frac{\cosh\left(bx+a\right)^{\frac{2}{3}}}{\sinh\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(2/3)/sinh(b*x + a)^(2/3), x)
```

3.63.8 Giac [F]

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx=\int \frac{\cosh\left(bx+a\right)^{\frac{2}{3}}}{\sinh\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(2/3)/sinh(b*x + a)^(2/3), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx=\int \frac{\cosh(a+b\,x)^{2/3}}{\sinh\left(a+b\,x\right)^{2/3}}\,dx$$

input
```
int(cosh(a + b*x)^(2/3)/sinh(a + b*x)^(2/3),x)
```

output
```
int(cosh(a + b*x)^(2/3)/sinh(a + b*x)^(2/3), x)
```

3.64 $\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx$

3.64.1 Optimal result

Integrand size = 21, antiderivative size = 243

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

output
```
arctanh(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))/b-1/4*ln(1-sinh(b*x+a)^(1/3)/
cosh(b*x+a)^(1/3)+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x
+a)^(1/3)/cosh(b*x+a)^(1/3)+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b-3*cosh(
b*x+a)^(1/3)/b/sinh(b*x+a)^(1/3)+1/2*arctan(1/3*(1-2*sinh(b*x+a)^(1/3)/cos
h(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b-1/2*arctan(1/3*(1+2*sinh(b*x+a)^(1/3)/c
osh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b
```

3.64.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\cosh^2(a+bx)^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},-\frac{1}{6},\frac{5}{6},-\sinh^2(a+bx)\right)}{b\cosh^{\frac{5}{3}}(a+bx)\sqrt[3]{\sinh(a+bx)}}$$

input
```
Integrate[Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3),x]
```

output
```
(-3*(Cosh[a + b*x]^2)^(5/6)*Hypergeometric2F1[-1/6, -1/6, 5/6, -Sinh[a + b
*x]^2])/(b*Cosh[a + b*x]^(5/3)*Sinh[a + b*x]^(1/3))
```

3.64.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3047, 3042, 3054, 25, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ia+ibx)^{4/3}}{(-i\sin(ia+ibx))^{4/3}}dx$$

$$\Big\downarrow \text{3047}$$

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}dx - \frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}} + \int \frac{(-i\sin(ia+ibx))^{2/3}}{\cos(ia+ibx)^{2/3}}dx$$

$$\Big\downarrow \; 3054$$

$$-\frac{3\int -\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)(1-\tanh^2(a+bx))}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b} - \frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{3\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)(1-\tanh^2(a+bx))}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b} - \frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \; 825$$

$$-\frac{3\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}} - \frac{1}{3}\int -\frac{\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{2\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}} - \frac{1}{3}\int - \cdots \right.}{b}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{3\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}} + \frac{1}{6}\int \frac{\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}} + \frac{1}{6}\int \frac{1-\cdots}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\cdots} \right.}{b}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \; 219$$

3.64. $\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}dx$

$$-\frac{3\left(\frac{1}{6}\int \frac{\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\cdots}{b}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$\downarrow$ 1142

$$-\frac{3\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{2}\int -\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\cdots\right.\right.}{\cdots}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$\downarrow$ 25

$$-\frac{3\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\cdots\right.\right.}{\cdots}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$\downarrow$ 1083

$$-\frac{3\left(\frac{1}{6}\left(-3\int \frac{1}{-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-3}d\left(\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-1\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)+\frac{1}{6}\cdots\right.}{\cdots}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$\downarrow$ 217

3.64. $\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx$

$$-\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-1}{\sqrt{3}}\right)-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)+\frac{1}{6}\left(\sqrt{3}\arctan\right.\right.}{}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$\Big\downarrow$ 1103

$$-\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)\right)+\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{s}}{\sqrt[3]{c}}\right.\right.\right.}{b}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

input
```
Int[Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3),x]
```

output
```
(-3*(-1/3*ArcTanh[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3)] + (Sqrt[3]*ArcT
an[(-1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/Sqrt[3]] + Log[1 - S
inh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a + b*x]^(2/3)/Cosh[a + b*x]
^(2/3)]/2)/6 + (Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^
(1/3))/Sqrt[3]] - Log[1 + Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a
 + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/2)/6))/b - (3*Cosh[a + b*x]^(1/3))/(b*S
inh[a + b*x]^(1/3))
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.64. $\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx$

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

3.64.4 Maple [F]

$$\int \frac{\cosh \left(bx+a\right)^{\frac{4}{3}}}{\sinh \left(bx+a\right)^{\frac{4}{3}}}dx$$

input
```
int(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x)
```

output
```
int(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x)
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. 2(197) = 394.

Time = 0.28 (sec) , antiderivative size = 1013, normalized size of antiderivative = 4.17

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
sqrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2
*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3
)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)
^(2/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(
b*x + a)^2 - 1)) + 2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*si
nh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(-1/3*(sqrt(3)*cosh
(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a
)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3
)*sinh(b*x + a)^(2/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 - 1)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(
b*x + a) + sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)
+ sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)
*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*s
inh(b*x + a) + sinh(b*x + a)^2 - 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x +
a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) - 2*(cosh(b*x
+ a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x
 + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*...
```

3.64.6 Sympy [F]

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx$$

input
```
integrate(cosh(b*x+a)**(4/3)/sinh(b*x+a)**(4/3),x)
```

output
```
Integral(cosh(a + b*x)**(4/3)/sinh(a + b*x)**(4/3), x)
```

3.64.7 Maxima [F]

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{4}{3}}}{\sinh(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(4/3)/sinh(b*x + a)^(4/3), x)
```

3.64.8 Giac [F]

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{4}{3}}}{\sinh(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(4/3)/sinh(b*x + a)^(4/3), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{4/3}}{\sinh(a+b\,x)^{4/3}}\,dx$$

input
```
int(cosh(a + b*x)^(4/3)/sinh(a + b*x)^(4/3),x)
```

output
```
int(cosh(a + b*x)^(4/3)/sinh(a + b*x)^(4/3), x)
```

3.65 $\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx$

3.65.1 Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

output
```
-1/2*ln(1-sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a)^(2/3
)/cosh(b*x+a)^(2/3)+sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3))/b-3/2*cosh(b*x+a)
^(2/3)/b/sinh(b*x+a)^(2/3)-1/2*arctan(1/3*(1+2*sinh(b*x+a)^(2/3)/cosh(b*x+
a)^(2/3))*3^(1/2))*3^(1/2)/b
```

3.65.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\cosh^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{3},\frac{2}{3},-\sinh^2(a+bx)\right)}{2b\cosh^{\frac{4}{3}}(a+bx)\sinh^{\frac{2}{3}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(5/3)/Sinh[a + b*x]^(5/3),x]
```

output
```
(-3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, -Sinh[a + b
*x]^2])/(2*b*Cosh[a + b*x]^(4/3)*Sinh[a + b*x]^(2/3))
```

3.65.3 Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.88, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3047, 3042, 3054, 25, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ia+ibx)^{5/3}}{(-i\sin(ia+ibx))^{5/3}}dx$$

$$\Big\downarrow \text{3047}$$

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}dx - \frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} + \int \frac{\sqrt[3]{-i\sin(ia+ibx)}}{\sqrt[3]{\cos(ia+ibx)}}dx$$

$$\Big\downarrow \text{3054}$$

$$-\frac{3\int -\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\int \frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{807}$$

$$\frac{3\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)(1-\tanh(a+bx))}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{821}$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int \frac{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{16}$$

$$\frac{3\left(-\frac{1}{3}\int \frac{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{3}{2}\int \frac{1}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+3\int \frac{1}{-\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-4}d\left(\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

3.65. $\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}dx$

$$\Big\downarrow \text{217}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\log\left(\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

input
```
Int[Cosh[a + b*x]^(5/3)/Sinh[a + b*x]^(5/3),x]
```

output
```
(3*(-1/3*Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)] + (-(Sqrt[3]*Arc
Tan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]) + Log[1 +
(2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3)]/2)/3))/(2*b) - (3*Cosh[a + b*
x]^(2/3))/(2*b*Sinh[a + b*x]^(2/3))
```

3.65.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

3.65. $\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx$

3.65.4 Maple [F]

$$\int \frac{\cosh(bx+a)^{\frac{5}{3}}}{\sinh(bx+a)^{\frac{5}{3}}}dx$$

input
```
int(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x)
```

output
```
int(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x)
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(124) = 248$.

Time = 0.27 (sec) , antiderivative size = 749, normalized size of antiderivative = 4.83

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
-1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
 sqrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 +
2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(
3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a
)^(2/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh
(b*x + a)^2 + 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 - 1)*log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
 sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x
+ a)^2 + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x +
 a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x +
a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b
*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh
(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 +
cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(
b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 +
 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1
)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
- 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x +
a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +...
```

3.65.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**(5/3)/sinh(b*x+a)**(5/3),x)
```

output
```
Timed out
```

3.65.7 Maxima [F]

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{5}{3}}}{\sinh(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(5/3)/sinh(b*x + a)^(5/3), x)
```

3.65.8 Giac [F]

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{5}{3}}}{\sinh(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(5/3)/sinh(b*x + a)^(5/3), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{5/3}}{\sinh\left(a+b\,x\right)^{5/3}}\,dx$$

input
```
int(cosh(a + b*x)^(5/3)/sinh(a + b*x)^(5/3),x)
```

output
```
int(cosh(a + b*x)^(5/3)/sinh(a + b*x)^(5/3), x)
```

3.66 $\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx$

3.66.1 Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

output
```
-1/2*ln(1-cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b+1/4*ln(1+cosh(b*x+a)^(4/3
)/sinh(b*x+a)^(4/3)+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b-3/4*cosh(b*x+a)
^(4/3)/b/sinh(b*x+a)^(4/3)-1/2*arctan(1/3*(1+2*cosh(b*x+a)^(2/3)/sinh(b*x+
a)^(2/3))*3^(1/2))*3^(1/2)/b
```

3.66.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{3},\frac{1}{3},-\sinh^2(a+bx)\right)}{4b\cosh^{\frac{2}{3}}(a+bx)\sinh^{\frac{4}{3}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(7/3)/Sinh[a + b*x]^(7/3),x]
```

output
```
(-3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, -Sinh[a + b
*x]^2])/(4*b*Cosh[a + b*x]^(2/3)*Sinh[a + b*x]^(4/3))
```

3.66.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3047, 3042, 3055, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ia+ibx)^{7/3}}{(-i\sin(ia+ibx))^{7/3}}dx$$

$$\Big\downarrow \text{ 3047}$$

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}dx-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}+\int \frac{\sqrt[3]{\cos(ia+ibx)}}{\sqrt[3]{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{ 3055}$$

$$\frac{3\int \frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{807}$$

$$\frac{3\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{(1-\coth(a+bx))\sinh^{\frac{2}{3}}(a+bx)}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{821}$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int \frac{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{16}$$

$$\frac{3\left(-\frac{1}{3}\int \frac{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{3}{2}\int \frac{1}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+3\int \frac{1}{-\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-4}d\left(\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{217}$$

3.66. $\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}dx$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\log\left(\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

input
```
Int[Cosh[a + b*x]^(7/3)/Sinh[a + b*x]^(7/3),x]
```

output
```
(3*(-1/3*Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)] + (-(Sqrt[3]*Arc
Tan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]) + Log[1 +
(2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3)]/2)/3))/(2*b) - (3*Cosh[a + b*
x]^(4/3))/(4*b*Sinh[a + b*x]^(4/3))
```

3.66.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

3.66. $\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx$

3.66.4 Maple [F]

$$\int \frac{\cosh(bx+a)^{\frac{7}{3}}}{\sinh(bx+a)^{\frac{7}{3}}}dx$$

input
```
int(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x)
```

output
```
int(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x)
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(124) = 248$.

Time = 0.27 (sec) , antiderivative size = 1056, normalized size of antiderivative = 6.81

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
-1/4*(2*(sqrt(3)*cosh(b*x + a)^4 + 4*sqrt(3)*cosh(b*x + a)*sinh(b*x + a)^3
 + sqrt(3)*sinh(b*x + a)^4 + 2*(3*sqrt(3)*cosh(b*x + a)^2 - sqrt(3))*sinh(
b*x + a)^2 - 2*sqrt(3)*cosh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a)^3 - sqrt
(3)*cosh(b*x + a))*sinh(b*x + a) + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x +
 a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 +
4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh
(b*x + a)^(1/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 - 1)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x +
a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cos
h(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(
(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(
3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 2*(cosh(b*x +
 a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x +
a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)
^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x +
 a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x +
a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*
x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
+ a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4
*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x + ...
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**(7/3)/sinh(b*x+a)**(7/3),x)
```

output
```
Timed out
```

3.66.7 Maxima [F]

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{7}{3}}}{\sinh(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(7/3)/sinh(b*x + a)^(7/3), x)
```

3.66.8 Giac [F]

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{7}{3}}}{\sinh(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(7/3)/sinh(b*x + a)^(7/3), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx=\int \frac{\cosh(a+b\,x)^{7/3}}{\sinh\left(a+b\,x\right)^{7/3}}\,dx$$

input
```
int(cosh(a + b*x)^(7/3)/sinh(a + b*x)^(7/3),x)
```

output
```
int(cosh(a + b*x)^(7/3)/sinh(a + b*x)^(7/3), x)
```

3.67 $\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx$

3.67.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = -\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

output
```
-3/5*cosh(x)^(5/3)/sinh(x)^(5/3)
```

3.67.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = -\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

input
```
Integrate[Cosh[x]^(2/3)/Sinh[x]^(8/3),x]
```

output
```
(-3*Cosh[x]^(5/3))/(5*Sinh[x]^(5/3))
```

3.67.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)} dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(ix)^{2/3}}{(-i\sin(ix))^{8/3}}dx$$
$$\Big\downarrow \text{ 3043}$$
$$-\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

input
```
Int[Cosh[x]^(2/3)/Sinh[x]^(8/3),x]
```

output
```
(-3*Cosh[x]^(5/3))/(5*Sinh[x]^(5/3))
```

3.67.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

3.67.4 Maple [F]

$$\int \frac{\cosh(x)^{\frac{2}{3}}}{\sinh(x)^{\frac{8}{3}}}dx$$

input
```
int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

output
```
int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(10) = 20.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 5.81

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = -\frac{6 \left(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 + 1\right)\sinh(x) + \cosh(x)\right) \cosh}{5 \left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 1\right)\sinh(x)^2 - 2\cosh(x)^2 + 4 (\cosh (x\right.}$$

input
```
integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="fricas")
```

output
```
-6/5*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh
(x) + cosh(x))*cosh(x)^(2/3)*sinh(x)^(1/3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^
3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3
 - cosh(x))*sinh(x) + 1)
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**(2/3)/sinh(x)**(8/3),x)
```

output
```
Timed out
```

3.67.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.81

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = \frac{3\left(e^{(-2x)}+1\right)^{\frac{2}{3}}e^{(-4x)}}{5\left(e^{(-x)}+1\right)^{\frac{8}{3}}\left(-e^{(-x)}+1\right)^{\frac{8}{3}}} - \frac{3\left(e^{(-2x)}+1\right)^{\frac{2}{3}}}{5\left(e^{(-x)}+1\right)^{\frac{8}{3}}\left(-e^{(-x)}+1\right)^{\frac{8}{3}}}$$

input
```
integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="maxima")
```

output
```
3/5*(e^(-2*x) + 1)^(2/3)*e^(-4*x)/((e^(-x) + 1)^(8/3)*(-e^(-x) + 1)^(8/3))
 - 3/5*(e^(-2*x) + 1)^(2/3)/((e^(-x) + 1)^(8/3)*(-e^(-x) + 1)^(8/3))
```

3.67.8 Giac [F]

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = \int \frac{\cosh(x)^{\frac{2}{3}}}{\sinh(x)^{\frac{8}{3}}}\,dx$$

input
```
integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="giac")
```

output
```
integrate(cosh(x)^(2/3)/sinh(x)^(8/3), x)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.38

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = -\frac{3\coth(x)^{5/3}}{5}$$

input
```
int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

output
```
-(3*coth(x)^(5/3))/5
```

3.68 $\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx$

3.68.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

output
```
3/5*sinh(x)^(5/3)/cosh(x)^(5/3)
```

3.68.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

input
```
Integrate[Sinh[x]^(2/3)/Cosh[x]^(8/3),x]
```

output
```
(3*Sinh[x]^(5/3))/(5*Cosh[x]^(5/3))
```

3.68.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(-i\sin(ix))^{2/3}}{\cos(ix)^{8/3}}dx$$
$$\Big\downarrow \text{3043}$$
$$\frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

input
```
Int[Sinh[x]^(2/3)/Cosh[x]^(8/3),x]
```

output
```
(3*Sinh[x]^(5/3))/(5*Cosh[x]^(5/3))
```

3.68.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

3.68.4 Maple [F]

$$\int \frac{\sinh(x)^{\frac{2}{3}}}{\cosh(x)^{\frac{8}{3}}}dx$$

input
```
int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

output
```
int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 5.81

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \frac{6\left(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 - 1\right)\sinh(x) - \cosh(x)\right)\cosh(}{5\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 + 1\right)\sinh(x)^2 + 2\cosh(x)^2 + 4\left(\cosh(x)^3\right.\right.}$$

input
```
integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="fricas")
```

output
```
6/5*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(
x) - cosh(x))*cosh(x)^(1/3)*sinh(x)^(2/3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3
+ cosh(x))*sinh(x) + 1)
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**(2/3)/cosh(x)**(8/3),x)
```

output
```
Timed out
```

3.68.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.81

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = -\frac{3\left(e^{(-x)}+1\right)^{\frac{2}{3}}\left(-e^{(-x)}+1\right)^{\frac{2}{3}}e^{(-4x)}}{5\left(e^{(-2x)}+1\right)^{\frac{8}{3}}} + \frac{3\left(e^{(-x)}+1\right)^{\frac{2}{3}}\left(-e^{(-x)}+1\right)^{\frac{2}{3}}}{5\left(e^{(-2x)}+1\right)^{\frac{8}{3}}}$$

input
```
integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="maxima")
```

output
```
-3/5*(e^(-x) + 1)^(2/3)*(-e^(-x) + 1)^(2/3)*e^(-4*x)/(e^(-2*x) + 1)^(8/3)
+ 3/5*(e^(-x) + 1)^(2/3)*(-e^(-x) + 1)^(2/3)/(e^(-2*x) + 1)^(8/3)
```

3.68.8 Giac [F]

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \int \frac{\sinh(x)^{\frac{2}{3}}}{\cosh(x)^{\frac{8}{3}}}\,dx$$

input
```
integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="giac")
```

output
```
integrate(sinh(x)^(2/3)/cosh(x)^(8/3), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \int \frac{\sinh(x)^{2/3}}{\cosh(x)^{8/3}}\,dx$$

input
```
int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

output
```
int(sinh(x)^(2/3)/cosh(x)^(8/3), x)
```

3.69 $\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx$

3.69.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3}{4}\text{csch}^{\frac{4}{3}}(x)$$

output
```
-3/4*csch(x)^(4/3)
```

3.69.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3}{4}\text{csch}^{\frac{4}{3}}(x)$$

input
```
Integrate[Cosh[x]*Csch[x]^(7/3),x]
```

output
```
(-3*Csch[x]^(4/3))/4
```

3.69.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3101, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(i\csc(ix))^{7/3}}{\sec(ix)}dx$$

$$\Big\downarrow \; 3101$$

$$-\int \sqrt[3]{\text{csch}(x)}d\text{csch}(x)$$

$$\Big\downarrow \; 15$$

$$-\frac{3}{4}\text{csch}^{\frac{4}{3}}(x)$$

input
```
Int[Cosh[x]*Csch[x]^(7/3),x]
```

output
```
(-3*Csch[x]^(4/3))/4
```

3.69.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.69.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{3 \operatorname{csch}(x)^{\frac{4}{3}}}{4}$	7
default	$-\frac{3 \operatorname{csch}(x)^{\frac{4}{3}}}{4}$	7

input
```
int(cosh(x)*csch(x)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4*csch(x)^(4/3)
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 5.40

$$\int \cosh(x)\operatorname{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3 \cdot 2^{\frac{1}{3}} \left(\frac{\cosh(x)+\sinh(x)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}\right)^{\frac{1}{3}} (\cosh(x)+\sinh(x))}{2\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)}$$

input
```
integrate(cosh(x)*csch(x)^(7/3),x, algorithm="fricas")
```

output
```
-3/2*2^(1/3)*((cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1))^(1/3)*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x
)^2 - 1)
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int \cosh(x)\operatorname{csch}^{\frac{7}{3}}(x)\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)*csch(x)**(7/3),x)
```

output
```
Timed out
```

3.69.7 Maxima [F]

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = \int \cosh\left(x\right)\text{csch}\left(x\right)^{\frac{7}{3}}\,dx$$

input
```
integrate(cosh(x)*csch(x)^(7/3),x, algorithm="maxima")
```

output
```
integrate(cosh(x)*csch(x)^(7/3), x)
```

3.69.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3\cdot 2^{\frac{1}{3}}e^{\left(\frac{4}{3}\,x\right)}}{2\,\left(e^{\left(2\,x\right)}-1\right)^{\frac{4}{3}}}$$

input
```
integrate(cosh(x)*csch(x)^(7/3),x, algorithm="giac")
```

output
```
-3/2*2^(1/3)*e^(4/3*x)/(e^(2*x) - 1)^(4/3)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.10

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3\,\mathrm{e}^{x}\,{\left(-\frac{1}{\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}}\right)}^{1/3}}{2\,\left(\mathrm{e}^{2\,x}-1\right)}$$

input
```
int(cosh(x)*(1/sinh(x))^(7/3),x)
```

output
```
-(3*exp(x)*(-1/(exp(-x)/2 - exp(x)/2))^(1/3))/(2*(exp(2*x) - 1))
```

3.70 $\int \sinh(a+bx)\tanh(a+bx)\,dx$

3.70.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \sinh(a+bx)\tanh(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}+\frac{\sinh(a+bx)}{b}$$

output
```
-arctan(sinh(b*x+a))/b+sinh(b*x+a)/b
```

3.70.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}+\frac{\sinh(a+bx)}{b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
-(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

3.70.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 25, 3072, 25, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\sin(ia+ibx)\tan(ia+ibx)dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \sin(ia+ibx)\tan(ia+ibx)dx$$

$$\Big\downarrow \text{ 3072}$$

$$-\frac{\int -\frac{\sinh^2(a+bx)}{\sinh^2(a+bx)+1}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\sinh^2(a+bx)}{\sinh^2(a+bx)+1}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{\int \frac{1}{\sinh^2(a+bx)+1}d\sinh(a+bx)-\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{\arctan(\sinh(a+bx))-\sinh(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
-((ArcTan[Sinh[a + b*x]] - Sinh[a + b*x])/b)
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.70.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\sinh(bx+a)-2\arctan\left(e^{bx+a}\right)}{b}$	21
default	$\frac{\sinh(bx+a)-2\arctan\left(e^{bx+a}\right)}{b}$	21
risch	$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{i\ln\left(e^{bx+a}-i\right)}{b}-\frac{i\ln\left(e^{bx+a}+i\right)}{b}$	59

input
```
int(sinh(b*x+a)*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x+a)-2*arctan(exp(b*x+a)))
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.74

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{4\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-\cosh\left(bx+a\right)^{2}-2\,\cosh\left(bx+\right.}{2\left(b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)\right)}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(4*(cosh(b*x + a) + sinh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x +
a)) - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2 +
1)/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

3.70.6 Sympy [F]

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \int \sinh\left(a+bx\right)\tanh\left(a+bx\right)dx$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*tanh(a + b*x), x)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{2\,\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{2\,b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="maxima")
```

output
```
2*arctan(e^(-b*x - a))/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{4\arctan\left(e^{(bx+a)}\right) - e^{(bx+a)} + e^{(-bx-a)}}{2\,b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(4*arctan(e^(b*x + a)) - e^(b*x + a) + e^(-b*x - a))/b
```

3.70.9 Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.13

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{e^{-a-b\,x}}{2\,b}$$

input
```
int(sinh(a + b*x)*tanh(a + b*x),x)
```

output
```
exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
 - exp(- a - b*x)/(2*b)
```

3.71 $\int \sinh(a+bx)\tanh^2(a+bx)\,dx$

3.71.1 Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

output
```
cosh(b*x+a)/b+sech(b*x+a)/b
```

3.71.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
Cosh[a + b*x]/b + Sech[a + b*x]/b
```

3.71.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \left(1-\cosh^2(a+bx)\right)\operatorname{sech}^2(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\operatorname{sech}^2(a+bx)-1\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\cosh(a+bx)-\operatorname{sech}(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
-((-Cosh[a + b*x] - Sech[a + b*x])/b)
```

3.71.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.71.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$	33
default	$\frac{\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$	33
risch	$\frac{e^{3bx+3a}+6\,e^{bx+a}+e^{-bx-a}}{2b\left(1+e^{2bx+2a}\right)}$	46

input
```
int(sinh(b*x+a)*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x+a)^2/cosh(b*x+a)+2/cosh(b*x+a))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh{(bx+a)}^2+\sinh{(bx+a)}^2+3}{2\,b\cosh{(bx+a)}}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 + 3)/(b*cosh(b*x + a))
```

3.71.6 Sympy [F]

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \int \sinh{(a+bx)}\tanh^2{(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)**2,x)
```

output
```
Integral(sinh(a + b*x)*tanh(a + b*x)**2, x)
```

3.71.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{(-bx-a)}}{2\,b}+\frac{5\,e^{(-2\,bx-2\,a)}+1}{2\,b(e^{(-bx-a)}+e^{(-3\,bx-3\,a)})}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*e^(-b*x - a)/b + 1/2*(5*e^(-2*b*x - 2*a) + 1)/(b*(e^(-b*x - a) + e^(-3
*b*x - 3*a)))
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}} + e^{(bx+a)} + e^{(-bx-a)}}{2\,b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(4/(e^(b*x + a) + e^(-b*x - a)) + e^(b*x + a) + e^(-b*x - a))/b
```

3.71.9 Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{-a-b\,x}\,\left(6\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}{2\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(sinh(a + b*x)*tanh(a + b*x)^2,x)
```

output
```
(exp(- a - b*x)*(6*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))/(2*b*(exp(2*a
 + 2*b*x) + 1))
```

3.72 $\int \sinh(a+bx)\tanh^3(a+bx)\,dx$

3.72.1 Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx=-\frac{3\arctan(\sinh(a+bx))}{2b}+\frac{3\sinh(a+bx)}{2b}-\frac{\sinh(a+bx)\tanh^2(a+bx)}{2b}$$

output
```
-3/2*arctan(sinh(b*x+a))/b+3/2*sinh(b*x+a)/b-1/2*sinh(b*x+a)*tanh(b*x+a)^2
/b
```

3.72.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx=-\frac{3\arctan(\sinh(a+bx))}{2b}+\frac{3\text{sech}(a+bx)\tanh(a+bx)}{2b}+\frac{\sinh(a+bx)\tanh^2(a+bx)}{b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[a + b*x]^3,x]
```

output
```
(-3*ArcTan[Sinh[a + b*x]])/(2*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(2*b) +
 (Sinh[a + b*x]*Tanh[a + b*x]^2)/b
```

3.72.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3072, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{\sinh^4(a+bx)}{(\sinh^2(a+bx)+1)^2}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{3}{2}\int \frac{\sinh^2(a+bx)}{\sinh^2(a+bx)+1}d\sinh(a+bx)-\frac{\sinh^3(a+bx)}{2(\sinh^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{3}{2}\left(\sinh(a+bx)-\int \frac{1}{\sinh^2(a+bx)+1}d\sinh(a+bx)\right)-\frac{\sinh^3(a+bx)}{2(\sinh^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{3}{2}(\sinh(a+bx)-\arctan(\sinh(a+bx)))-\frac{\sinh^3(a+bx)}{2(\sinh^2(a+bx)+1)}}{b}$$

input
```
Int[Sinh[a + b*x]*Tanh[a + b*x]^3,x]
```

output
```
((3*(-ArcTan[Sinh[a + b*x]] + Sinh[a + b*x]))/2 - Sinh[a + b*x]^3/(2*(1 +
Sinh[a + b*x]^2)))/b
```

3.72.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.72.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^3}{\cosh(bx+a)^2}+\frac{3\sinh(bx+a)}{\cosh(bx+a)^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	62
default	$\frac{\frac{\sinh(bx+a)^3}{\cosh(bx+a)^2}+\frac{3\sinh(bx+a)}{\cosh(bx+a)^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	62
risch	$\frac{\mathrm{e}^{bx+a}}{2b}-\frac{\mathrm{e}^{-bx-a}}{2b}+\frac{\mathrm{e}^{bx+a}\left(\mathrm{e}^{2bx+2a}-1\right)}{b\left(1+\mathrm{e}^{2bx+2a}\right)^2}+\frac{3i\ln\left(\mathrm{e}^{bx+a}-i\right)}{2b}-\frac{3i\ln\left(\mathrm{e}^{bx+a}+i\right)}{2b}$	93

input
```
int(sinh(b*x+a)*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x+a)^3/cosh(b*x+a)^2+3/cosh(b*x+a)^2*sinh(b*x+a)-3/2*sech(b*x+
a)*tanh(b*x+a)-3*arctan(exp(b*x+a)))
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(43) = 86$.

Time = 0.25 (sec) , antiderivative size = 463, normalized size of antiderivative = 9.45

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 3\left(5\cosh(bx+a)^2+1\right)\sinh(bx+a)^4}{}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 3*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh
(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*
cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 6*(cosh(b*x + a)^5 + 5*cosh(b*x + a
)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^2 + 1)*sinh(b*x +
 a)^3 + 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b
*x + a)^2 + (5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + co
sh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a)) - 3*cosh(b*x + a)^2 + 6
*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)/
(b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5
 + 2*b*cosh(b*x + a)^3 + 2*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^3 + 2*(
5*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + b*cosh(b*x + a)
 + (5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a))
```

3.72.6 Sympy [F]

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = \int \sinh{(a+bx)}\tanh^3{(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)**3,x)
```

output
```
Integral(sinh(a + b*x)*tanh(a + b*x)**3, x)
```

3.72.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(43) = 86$.

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.86

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = \frac{3\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{e^{(-bx-a)}}{2\,b} + \frac{4\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} + 1}{2\,b\left(e^{(-bx-a)} + 2\,e^{(-3\,bx-3\,a)} + e^{(-5\,bx-5\,a)}\right)}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
3*arctan(e^(-b*x - a))/b - 1/2*e^(-b*x - a)/b + 1/2*(4*e^(-2*b*x - 2*a) -
e^(-4*b*x - 4*a) + 1)/(b*(e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + e^(-5*b*x -
5*a)))
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.98

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = -\frac{3\,\pi - \frac{4\left(e^{(bx+a)}-e^{(-bx-a)}\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4} + 6\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right) - 2\,e^{(bx+a)} + 2\,e^{(-bx-a)}}{4\,b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(3*pi - 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^
2 + 4) + 6*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) - 2*e^(b*x + a)
+ 2*e^(-b*x - a))/b
```

3.72.9 Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.18

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{e^{-a-b\,x}}{2\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} + \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(sinh(a + b*x)*tanh(a + b*x)^3,x)
```

output
```
exp(a + b*x)/(2*b) - (3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
 - exp(- a - b*x)/(2*b) - (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*
a + 4*b*x) + 1)) + exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

3.73 $\int \sinh(a+bx)\tanh^4(a+bx)\,dx$

3.73.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{2\text{sech}(a+bx)}{b}-\frac{\text{sech}^3(a+bx)}{3b}$$

output
```
cosh(b*x+a)/b+2*sech(b*x+a)/b-1/3*sech(b*x+a)^3/b
```

3.73.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{2\text{sech}(a+bx)}{b}-\frac{\text{sech}^3(a+bx)}{3b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[a + b*x]^4,x]
```

output
```
Cosh[a + b*x]/b + (2*Sech[a + b*x])/b - Sech[a + b*x]^3/(3*b)
```

3.73.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sin(ia+ibx)\tan(ia+ibx)^4dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \sin(ia+ibx)\tan(ia+ibx)^4dx$$

$$\Big\downarrow \text{ 3070}$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)^2\text{sech}^4(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left(\text{sech}^4(a+bx)-2\text{sech}^2(a+bx)+1\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\cosh(a+bx)-\frac{1}{3}\text{sech}^3(a+bx)+2\text{sech}(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]*Tanh[a + b*x]^4,x]
```

output
```
(Cosh[a + b*x] + 2*Sech[a + b*x] - Sech[a + b*x]^3/3)/b
```

3.73.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.73.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^4}{\cosh(bx+a)^3}+\frac{4\sinh(bx+a)^2}{\cosh(bx+a)^3}+\frac{8}{3\cosh(bx+a)^3}}{b}$	51
default	$\frac{\frac{\sinh(bx+a)^4}{\cosh(bx+a)^3}+\frac{4\sinh(bx+a)^2}{\cosh(bx+a)^3}+\frac{8}{3\cosh(bx+a)^3}}{b}$	51
risch	$\frac{3\,\mathrm{e}^{7bx+7a}+36\,\mathrm{e}^{5bx+5a}+50\,\mathrm{e}^{3bx+3a}+36\,\mathrm{e}^{bx+a}+3\,\mathrm{e}^{-bx-a}}{6b\left(1+\mathrm{e}^{2bx+2a}\right)^3}$	72

input
```
int(sinh(b*x+a)*tanh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x+a)^4/cosh(b*x+a)^3+4*sinh(b*x+a)^2/cosh(b*x+a)^3+8/3/cosh(b*
x+a)^3)
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(35) = 70.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.51

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)^4+3\sinh(bx+a)^4+18\left(\cosh(bx+a)^2+2\right)\sinh(bx+a)^2+36\cosh(bx+a)^2+25}{6\left(b\cosh(bx+a)^3+3\,b\cosh(bx+a)\sinh(bx+a)^2+3\,b\cosh(bx+a)\right)}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/6*(3*cosh(b*x + a)^4 + 3*sinh(b*x + a)^4 + 18*(cosh(b*x + a)^2 + 2)*sinh
(b*x + a)^2 + 36*cosh(b*x + a)^2 + 25)/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x +
 a)*sinh(b*x + a)^2 + 3*b*cosh(b*x + a))
```

3.73.6 Sympy [F]

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx=\int \sinh(a+bx)\tanh^4(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)**4,x)
```

output
```
Integral(sinh(a + b*x)*tanh(a + b*x)**4, x)
```

3.73.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(35) = 70.

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.65

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx=\frac{e^{(-bx-a)}}{2\,b}+\frac{33\,e^{(-2\,bx-2\,a)}+41\,e^{(-4\,bx-4\,a)}+27\,e^{(-6\,bx-6\,a)}+3}{6\,b(e^{(-bx-a)}+3\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}+e^{(-7\,bx-7\,a)})}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/2*e^(-b*x - a)/b + 1/6*(33*e^(-2*b*x - 2*a) + 41*e^(-4*b*x - 4*a) + 27*e
^(-6*b*x - 6*a) + 3)/(b*(e^(-b*x - a) + 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x -
 5*a) + e^(-7*b*x - 7*a)))
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx = \frac{\frac{8\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-2\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3}+3\,e^{(bx+a)}+3\,e^{(-bx-a)}}{6\,b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="giac")
```

output
```
1/6*(8*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 2)/(e^(b*x + a) + e^(-b*x - a))
^3 + 3*e^(b*x + a) + 3*e^(-b*x - a))/b
```

3.73.9 Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.54

$$\begin{aligned}\int \sinh(a+bx)\tanh^4(a+bx)\,dx &= \frac{e^{a+b\,x}}{2\,b}+\frac{e^{-a-b\,x}}{2\,b}-\frac{8\,e^{a+b\,x}}{3\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}\\ &\quad+\frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)}\\ &\quad+\frac{4\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}\end{aligned}$$

input
```
int(sinh(a + b*x)*tanh(a + b*x)^4,x)
```

output
```
exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) - (8*exp(a + b*x))/(3*b*(2*exp(2
*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (8*exp(a + b*x))/(3*b*(3*exp(2*a +
2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + (4*exp(a + b*x))/(b
*(exp(2*a + 2*b*x) + 1))
```

3.74 $\int \sinh^2(a+bx)\tanh(a+bx)\,dx$

3.74.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx=\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

output
```
1/2*cosh(b*x+a)^2/b-ln(cosh(b*x+a))/b
```

3.74.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx=-\frac{-\frac{1}{2}\cosh^2(a+bx)+\log(\cosh(a+bx))}{b}$$

input
```
Integrate[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-((-1/2*Cosh[a + b*x]^2 + Log[Cosh[a + b*x]])/b)
```

3.74.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ia+ibx)^2\tan(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin(ia+ibx)^2\tan(ia+ibx)dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \left(1-\cosh^2(a+bx)\right)\operatorname{sech}(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int (\operatorname{sech}(a+bx)-\cosh(a+bx))d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(\cosh(a+bx))-\frac{1}{2}\cosh^2(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-((-1/2*Cosh[a + b*x]^2 + Log[Cosh[a + b*x]])/b)
```

3.74.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.74.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^2}{2}-\ln(\cosh(bx+a))}{b}$	25
default	$\frac{\frac{\sinh(bx+a)^2}{2}-\ln(\cosh(bx+a))}{b}$	25
risch	$x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}+\frac{2a}{b}-\frac{\ln(1+e^{2bx+2a})}{b}$	54

input
```
int(sinh(b*x+a)^2*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sinh(b*x+a)^2-ln(cosh(b*x+a)))
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 7.04

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{8\,bx\cosh\left(bx+a\right)^2 + \cosh\left(bx+a\right)^4 + 4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3 + \sinh\left(bx+a\right)^4 + 2\left(4\,bx + 3\,\cosh\left(\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(8*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x +
a)^3 + sinh(b*x + a)^4 + 2*(4*b*x + 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8
*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2
*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) +
 cosh(b*x + a)^3)*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.74.6 Sympy [F]

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = \int \sinh^2\left(a+bx\right)\tanh\left(a+bx\right)dx$$

input
```
integrate(sinh(b*x+a)**2*tanh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)**2*tanh(a + b*x), x)
```

3.74.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{bx+a}{b} + \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="maxima")
```

output
```
-(b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b - log(e^(-2*
b*x - 2*a) + 1)/b
```

3.74.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{8\,bx - \left(4\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 8\,a + e^{(2\,bx+2\,a)} - 8\,\log\left(e^{(2\,bx+2\,a)} + 1\right)}{8\,b}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="giac")
```

output
```
1/8*(8*b*x - (4*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 8*a + e^(2*b*x + 2
*a) - 8*log(e^(2*b*x + 2*a) + 1))/b
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = x - \frac{\ln\left(e^{2\,a}\,e^{2\,b\,x} + 1\right)}{b} + \frac{e^{-2\,a-2\,b\,x}}{8\,b} + \frac{e^{2\,a+2\,b\,x}}{8\,b}$$

input
```
int(sinh(a + b*x)^2*tanh(a + b*x),x)
```

output
```
x - log(exp(2*a)*exp(2*b*x) + 1)/b + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a +
2*b*x)/(8*b)
```

3.75 $\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx$

3.75.1 Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx=-\frac{3x}{2}+\frac{3\tanh(a+bx)}{2b}+\frac{\sinh^2(a+bx)\tanh(a+bx)}{2b}$$

output
```
-3/2*x+3/2*tanh(b*x+a)/b+1/2*sinh(b*x+a)^2*tanh(b*x+a)/b
```

3.75.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx=\frac{-6(a+bx)+\sinh(2(a+bx))+4\tanh(a+bx)}{4b}$$

input
```
Integrate[Sinh[a + b*x]^2*Tanh[a + b*x]^2,x]
```

output
```
(-6*(a + b*x) + Sinh[2*(a + b*x)] + 4*Tanh[a + b*x])/(4*b)
```

3.75.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3071, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^2\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{\int \frac{\tanh^4(a+bx)}{(1-\tanh^2(a+bx))^2}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\tanh^3(a+bx)}{2(1-\tanh^2(a+bx))}-\frac{3}{2}\int \frac{\tanh^2(a+bx)}{1-\tanh^2(a+bx)}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\tanh^3(a+bx)}{2(1-\tanh^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\tanh^2(a+bx)}d\tanh(a+bx)-\tanh(a+bx)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\tanh^3(a+bx)}{2(1-\tanh^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\tanh(a+bx))-\tanh(a+bx))}{b}$$

input
```
Int[Sinh[a + b*x]^2*Tanh[a + b*x]^2,x]
```

output
```
((-3*(ArcTanh[Tanh[a + b*x]] - Tanh[a + b*x]))/2 + Tanh[a + b*x]^3/(2*(1 -
 Tanh[a + b*x]^2)))/b
```

3.75.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.75.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^3}{2\cosh(bx+a)}-\frac{3bx}{2}-\frac{3a}{2}+\frac{3\tanh(bx+a)}{2}}{b}$	39
default	$\frac{\frac{\sinh(bx+a)^3}{2\cosh(bx+a)}-\frac{3bx}{2}-\frac{3a}{2}+\frac{3\tanh(bx+a)}{2}}{b}$	39
risch	$-\frac{3x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}-\frac{2}{b(1+e^{2bx+2a})}$	51

input
```
int(sinh(b*x+a)^2*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sinh(b*x+a)^3/cosh(b*x+a)-3/2*b*x-3/2*a+3/2*tanh(b*x+a))
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \frac{\sinh(bx+a)^3 - 4(3bx+2)\cosh(bx+a) + 3\left(\cosh(bx+a)^2+3\right)\sinh(bx+a)}{8b\cosh(bx+a)}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(sinh(b*x + a)^3 - 4*(3*b*x + 2)*cosh(b*x + a) + 3*(cosh(b*x + a)^2 +
3)*sinh(b*x + a))/(b*cosh(b*x + a))
```

3.75.6 Sympy [F]

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \int \sinh^2(a+bx)\tanh^2(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)**2*tanh(b*x+a)**2,x)
```

output
```
Integral(sinh(a + b*x)**2*tanh(a + b*x)**2, x)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.60

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = -\frac{3\,(bx+a)}{2\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b} + \frac{17\,e^{(-2\,bx-2\,a)}+1}{8\,b(e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)})}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-3/2*(b*x + a)/b - 1/8*e^(-2*b*x - 2*a)/b + 1/8*(17*e^(-2*b*x - 2*a) + 1)/
(b*(e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a)))
```

3.75.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(34) = 68.

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = -\frac{12\,bx+12\,a-\frac{3\,e^{(4\,bx+4\,a)}-14\,e^{(2\,bx+2\,a)}-1}{e^{(4\,bx+4\,a)}+e^{(2\,bx+2\,a)}}-e^{(2\,bx+2\,a)}}{8\,b}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8*(12*b*x + 12*a - (3*e^(4*b*x + 4*a) - 14*e^(2*b*x + 2*a) - 1)/(e^(4*b
*x + 4*a) + e^(2*b*x + 2*a)) - e^(2*b*x + 2*a))/b
```

3.75.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \frac{\mathrm{e}^{2\,a+2\,b\,x}}{8\,b} - \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{\mathrm{e}^{-2\,a-2\,b\,x}}{8\,b} - \frac{3\,x}{2}$$

input
```
int(sinh(a + b*x)^2*tanh(a + b*x)^2,x)
```

output
```
exp(2*a + 2*b*x)/(8*b) - 2/(b*(exp(2*a + 2*b*x) + 1)) - exp(- 2*a - 2*b*x)
/(8*b) - (3*x)/2
```

3.76 $\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx$

3.76.1 Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = \frac{\cosh^2(a+bx)}{2b} - \frac{2\log(\cosh(a+bx))}{b} - \frac{\operatorname{sech}^2(a+bx)}{2b}$$

output
```
1/2*cosh(b*x+a)^2/b-2*ln(cosh(b*x+a))/b-1/2*sech(b*x+a)^2/b
```

3.76.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = -\frac{4\log(\cosh(a+bx)) + \operatorname{sech}^2(a+bx) - \sinh^2(a+bx)}{2b}$$

input
```
Integrate[Sinh[a + b*x]^2*Tanh[a + b*x]^3,x]
```

output
```
-1/2*(4*Log[Cosh[a + b*x]] + Sech[a + b*x]^2 - Sinh[a + b*x]^2)/b
```

3.76.3 Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)^2\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ia+ibx)^2\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)^2\text{sech}^3(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)^2\text{sech}^2(a+bx)d\cosh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\text{sech}^2(a+bx)-2\text{sech}(a+bx)+1\right)d\cosh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh^2(a+bx)-\text{sech}(a+bx)-2\log\left(\cosh^2(a+bx)\right)}{2b}$$

input
```
Int[Sinh[a + b*x]^2*Tanh[a + b*x]^3,x]
```

output
```
(Cosh[a + b*x]^2 - 2*Log[Cosh[a + b*x]^2] - Sech[a + b*x])/(2*b)
```

3.76.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.76.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^4}{2\cosh(bx+a)^2}-2\ln(\cosh(bx+a))+\tanh(bx+a)^2}{b}$	41
default	$\frac{\frac{\sinh(bx+a)^4}{2\cosh(bx+a)^2}-2\ln(\cosh(bx+a))+\tanh(bx+a)^2}{b}$	41
risch	$2x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}+\frac{4a}{b}-\frac{2\,e^{2bx+2a}}{b\left(1+e^{2bx+2a}\right)^2}-\frac{2\ln\left(1+e^{2bx+2a}\right)}{b}$	83

input
```
int(sinh(b*x+a)^2*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sinh(b*x+a)^4/cosh(b*x+a)^2-2*ln(cosh(b*x+a))+tanh(b*x+a)^2)
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. 2(39) = 78.

Time = 0.26 (sec) , antiderivative size = 742, normalized size of antiderivative = 17.26

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx$$

$$= \frac{\cosh(bx+a)^8 + 8\cosh(bx+a)\sinh(bx+a)^7 + \sinh(bx+a)^8 + 2(8bx+1)\cosh(bx+a)^6 + 2(8bx+}{}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 +
 2*(8*b*x + 1)*cosh(b*x + a)^6 + 2*(8*b*x + 14*cosh(b*x + a)^2 + 1)*sinh(b
*x + a)^6 + 4*(14*cosh(b*x + a)^3 + 3*(8*b*x + 1)*cosh(b*x + a))*sinh(b*x
+ a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^4 + 2*(35*cosh(b*x + a)^4 + 15*(8*b*
x + 1)*cosh(b*x + a)^2 + 16*b*x - 7)*sinh(b*x + a)^4 + 8*(7*cosh(b*x + a)^
5 + 5*(8*b*x + 1)*cosh(b*x + a)^3 + (16*b*x - 7)*cosh(b*x + a))*sinh(b*x +
 a)^3 + 2*(8*b*x + 1)*cosh(b*x + a)^2 + 2*(14*cosh(b*x + a)^6 + 15*(8*b*x
+ 1)*cosh(b*x + a)^4 + 6*(16*b*x - 7)*cosh(b*x + a)^2 + 8*b*x + 1)*sinh(b*
x + a)^2 - 16*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*
x + a)^6 + (15*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^4 + 2*cosh(b*x + a)^4 +
4*(5*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a
)^4 + 12*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x + a)^2 + 2*(3*cos
h(b*x + a)^5 + 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))*log(2*cos
h(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(2*cosh(b*x + a)^7 + 3*(8*
b*x + 1)*cosh(b*x + a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^3 + (8*b*x + 1)*co
sh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sin
h(b*x + a)^5 + b*sinh(b*x + a)^6 + 2*b*cosh(b*x + a)^4 + (15*b*cosh(b*x +
a)^2 + 2*b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 2*b*cosh(b*x + a))*
sinh(b*x + a)^3 + b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 + 12*b*cosh(b*
x + a)^2 + b)*sinh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 + 4*b*cosh(b*x +...
```

3.76.6 Sympy [F]

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = \int \sinh^2{(a+bx)}\tanh^3{(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)**2*tanh(b*x+a)**3,x)
```

output
```
Integral(sinh(a + b*x)**2*tanh(a + b*x)**3, x)
```

3.76.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(39) = 78$.

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\,(bx+a)}{b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{2\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)} - 15\,e^{(-4\,bx-4\,a)} + 1}{8\,b(e^{(-2\,bx-2\,a)} + 2\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)})}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2*(b*x + a)/b + 1/8*e^(-2*b*x - 2*a)/b - 2*log(e^(-2*b*x - 2*a) + 1)/b +
1/8*(2*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) + 1)/(b*(e^(-2*b*x - 2*a) +
2*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

3.76.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(39) = 78$.

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = \frac{16\,bx - \left(8\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 16\,a + \frac{8\,\left(3\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+3\right)}{\left(e^{(2\,bx+2\,a)}+1\right)^2} + e^{(2\,bx+2\,a)} - 16\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{8\,b}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*(16*b*x - (8*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 16*a + 8*(3*e^(4*
b*x + 4*a) + 4*e^(2*b*x + 2*a) + 3)/(e^(2*b*x + 2*a) + 1)^2 + e^(2*b*x + 2
*a) - 16*log(e^(2*b*x + 2*a) + 1))/b
```

3.76.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.26

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = 2\,x - \frac{2\,\ln\left(e^{2\,a}\,e^{2\,b\,x}+1\right)}{b} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}+1\right)} + \frac{2}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} + \frac{e^{-2\,a-2\,b\,x}}{8\,b} + \frac{e^{2\,a+2\,b\,x}}{8\,b}$$

input
```
int(sinh(a + b*x)^2*tanh(a + b*x)^3,x)
```

output
```
2*x - (2*log(exp(2*a)*exp(2*b*x) + 1))/b - 2/(b*(exp(2*a + 2*b*x) + 1)) +
2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + exp(- 2*a - 2*b*x)/(8*
b) + exp(2*a + 2*b*x)/(8*b)
```

3.77 $\int \sinh^3(a+bx)\tanh(a+bx)\,dx$

3.77.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}-\frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

output
```
arctan(sinh(b*x+a))/b-sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b
```

3.77.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}-\frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

input
```
Integrate[Sinh[a + b*x]^3*Tanh[a + b*x],x]
```

output
```
ArcTan[Sinh[a + b*x]]/b - Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

3.77.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3072, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^3\tan(ia+ibx)dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{\sinh^4(a+bx)}{\sinh^2(a+bx)+1}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\int \left(\sinh^2(a+bx)+\frac{1}{\sinh^2(a+bx)+1}-1\right)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan(\sinh(a+bx))+\frac{1}{3}\sinh^3(a+bx)-\sinh(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]^3*Tanh[a + b*x],x]
```

output
```
(ArcTan[Sinh[a + b*x]] - Sinh[a + b*x] + Sinh[a + b*x]^3/3)/b
```

3.77.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.77.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^3}{3}-\sinh(bx+a)+2\arctan(\mathrm{e}^{bx+a})}{b}$	33
default	$\frac{\frac{\sinh(bx+a)^3}{3}-\sinh(bx+a)+2\arctan(\mathrm{e}^{bx+a})}{b}$	33
risch	$\frac{\mathrm{e}^{3bx+3a}}{24b}-\frac{5\,\mathrm{e}^{bx+a}}{8b}+\frac{5\,\mathrm{e}^{-bx-a}}{8b}-\frac{\mathrm{e}^{-3bx-3a}}{24b}+\frac{i\ln(\mathrm{e}^{bx+a}+i)}{b}-\frac{i\ln(\mathrm{e}^{bx+a}-i)}{b}$	87

input
```
int(sinh(b*x+a)^3*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*sinh(b*x+a)^3-sinh(b*x+a)+2*arctan(exp(b*x+a)))
```

3.77.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 290, normalized size of antiderivative = 7.63

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^6+6\cosh(bx+a)\sinh(bx+a)^5+\sinh(bx+a)^6+15\left(\cosh(bx+a)^2-1\right)\sinh(bx+a)^4 \cdots}{\cdots}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="fricas")
```

output
```
1/24*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
+ 15*(cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 15*cosh(b*x + a)^4 + 20*(cosh
(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 15*(cosh(b*x + a)^4 - 6*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 48*(cosh(b*x + a)^3 + 3*cosh(b*x + a
)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3)*arc
tan(cosh(b*x + a) + sinh(b*x + a)) + 15*cosh(b*x + a)^2 + 6*(cosh(b*x + a)
^5 - 10*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b*x
+ a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x +
a)^2 + b*sinh(b*x + a)^3)
```

3.77.6 Sympy [F]

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx=\int \sinh^3{(a+bx)}\tanh{(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)**3*tanh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)**3*tanh(a + b*x), x)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx=-\frac{\left(15\,e^{(-2\,bx-2\,a)}-1\right)e^{(3\,bx+3\,a)}}{24\,b}+\frac{15\,e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{24\,b}-\frac{2\,\arctan\left(e^{(-bx-a)}\right)}{b}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="maxima")
```

output
```
-1/24*(15*e^(-2*b*x - 2*a) - 1)*e^(3*b*x + 3*a)/b + 1/24*(15*e^(-b*x - a)
- e^(-3*b*x - 3*a))/b - 2*arctan(e^(-b*x - a))/b
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \frac{\left(15\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}+48\,\arctan\left(e^{(bx+a)}\right)+e^{(3\,bx+3\,a)}-15\,e^{(bx+a)}}{24\,b}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="giac")
```

output
```
1/24*((15*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + 48*arctan(e^(b*x + a)) +
 e^(3*b*x + 3*a) - 15*e^(b*x + a))/b
```

3.77.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{5\,\mathrm{e}^{a+b\,x}}{8\,b}+\frac{5\,\mathrm{e}^{-a-b\,x}}{8\,b}-\frac{\mathrm{e}^{-3\,a-3\,b\,x}}{24\,b}+\frac{\mathrm{e}^{3\,a+3\,b\,x}}{24\,b}$$

input
```
int(sinh(a + b*x)^3*tanh(a + b*x),x)
```

output
```
(2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (5*exp(a + b*x))/(
8*b) + (5*exp(- a - b*x))/(8*b) - exp(- 3*a - 3*b*x)/(24*b) + exp(3*a + 3*
b*x)/(24*b)
```

3.78 $\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx$

3.78.1 Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = -\frac{2\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}-\frac{\operatorname{sech}(a+bx)}{b}$$

output
```
-2*cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b-sech(b*x+a)/b
```

3.78.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = -\frac{7\cosh(a+bx)}{4b}+\frac{\cosh(3(a+bx))}{12b}-\frac{\operatorname{sech}(a+bx)}{b}$$

input
```
Integrate[Sinh[a + b*x]^3*Tanh[a + b*x]^2,x]
```

output
```
(-7*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Sech[a + b*x]/b
```

3.78.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)^3\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ia+ibx)^3\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)^2\text{sech}^2(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cosh^2(a+bx)+\text{sech}^2(a+bx)-2\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\cosh^3(a+bx)-2\cosh(a+bx)-\text{sech}(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]^3*Tanh[a + b*x]^2,x]
```

output
```
(-2*Cosh[a + b*x] + Cosh[a + b*x]^3/3 - Sech[a + b*x])/b
```

3.78.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.78.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^4}{3\cosh(bx+a)}-\frac{4\sinh(bx+a)^2}{3\cosh(bx+a)}-\frac{8}{3\cosh(bx+a)}}{b}$	52
default	$\frac{\frac{\sinh(bx+a)^4}{3\cosh(bx+a)}-\frac{4\sinh(bx+a)^2}{3\cosh(bx+a)}-\frac{8}{3\cosh(bx+a)}}{b}$	52
risch	$\frac{e^{5bx+5a}-20\,e^{3bx+3a}-90\,e^{bx+a}-20\,e^{-bx-a}+e^{-3bx-3a}}{24b\left(1+e^{2bx+2a}\right)}$	68

input
```
int(sinh(b*x+a)^3*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*sinh(b*x+a)^4/cosh(b*x+a)-4/3*sinh(b*x+a)^2/cosh(b*x+a)-8/3/cosh(
b*x+a))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh(bx+a)^4+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2-10\right)\sinh(bx+a)^2-20\cosh(bx+a)^2-45}{24\,b\cosh(bx+a)}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/24*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 10)*sinh(
b*x + a)^2 - 20*cosh(b*x + a)^2 - 45)/(b*cosh(b*x + a))
```

3.78.6 Sympy [F]

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \int \sinh^3(a+bx)\tanh^2(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)**3*tanh(b*x+a)**2,x)
```

output
```
Integral(sinh(a + b*x)**3*tanh(a + b*x)**2, x)
```

3.78.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(36) = 72.

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.08

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = -\frac{21\,e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{24\,b} - \frac{20\,e^{(-2\,bx-2\,a)}+69\,e^{(-4\,bx-4\,a)}-1}{24\,b\left(e^{(-3\,bx-3\,a)}+e^{(-5\,bx-5\,a)}\right)}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/24*(21*e^(-b*x - a) - e^(-3*b*x - 3*a))/b - 1/24*(20*e^(-2*b*x - 2*a) +
 69*e^(-4*b*x - 4*a) - 1)/(b*(e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a)))
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx$$
$$= \frac{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3 - \frac{48}{e^{(bx+a)}+e^{(-bx-a)}} - 24\,e^{(bx+a)} - 24\,e^{(-bx-a)}}{24\,b}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/24*((e^(b*x + a) + e^(-b*x - a))^3 - 48/(e^(b*x + a) + e^(-b*x - a)) - 2
4*e^(b*x + a) - 24*e^(-b*x - a))/b
```

3.78.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{-3\,a-3\,b\,x}}{24\,b} - \frac{7\,e^{-a-b\,x}}{8\,b} - \frac{7\,e^{a+b\,x}}{8\,b} + \frac{e^{3\,a+3\,b\,x}}{24\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(sinh(a + b*x)^3*tanh(a + b*x)^2,x)
```

output
```
exp(- 3*a - 3*b*x)/(24*b) - (7*exp(- a - b*x))/(8*b) - (7*exp(a + b*x))/(8
*b) + exp(3*a + 3*b*x)/(24*b) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1)
)
```

3.79 $\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$

3.79.1 Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{5\arctan(\sinh(a+bx))}{2b} - \frac{5\sinh(a+bx)}{2b} + \frac{5\sinh^3(a+bx)}{6b} - \frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b}$$

output
```
5/2*arctan(sinh(b*x+a))/b-5/2*sinh(b*x+a)/b+5/6*sinh(b*x+a)^3/b-1/2*sinh(b
*x+a)^3*tanh(b*x+a)^2/b
```

3.79.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{5\arctan(\sinh(a+bx))}{2b} - \frac{5\operatorname{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{5\sinh(a+bx)\tanh^2(a+bx)}{3b} + \frac{\sinh^3(a+bx)\tanh^2(a+bx)}{3b}$$

input
```
Integrate[Sinh[a + b*x]^3*Tanh[a + b*x]^3,x]
```

output
```
(5*ArcTan[Sinh[a + b*x]])/(2*b) - (5*Sech[a + b*x]*Tanh[a + b*x])/(2*b) -
(5*Sinh[a + b*x]*Tanh[a + b*x]^2)/(3*b) + (Sinh[a + b*x]^3*Tanh[a + b*x]^2
)/(3*b)
```

3.79.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 25, 3072, 25, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\sin(ia+ibx)^3\tan(ia+ibx)^3dx$$

$$\Big\downarrow \; 25$$

$$-\int \sin(ia+ibx)^3\tan(ia+ibx)^3dx$$

$$\Big\downarrow \; 3072$$

$$-\frac{\int -\frac{\sinh^6(a+bx)}{\left(\sinh^2(a+bx)+1\right)^2}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sinh^6(a+bx)}{\left(\sinh^2(a+bx)+1\right)^2}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \; 252$$

$$-\frac{\frac{\sinh^5(a+bx)}{2\left(\sinh^2(a+bx)+1\right)}-\frac{5}{2}\int \frac{\sinh^4(a+bx)}{\sinh^2(a+bx)+1}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \; 254$$

$$-\frac{\frac{\sinh^5(a+bx)}{2\left(\sinh^2(a+bx)+1\right)}-\frac{5}{2}\int \left(\sinh^2(a+bx)+\frac{1}{\sinh^2(a+bx)+1}-1\right)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{\sinh^5(a+bx)}{2(\sinh^2(a+bx)+1)}-\frac{5}{2}\left(\arctan(\sinh(a+bx))+\frac{1}{3}\sinh^3(a+bx)-\sinh(a+bx)\right)}{b}$$

input
```
Int[Sinh[a + b*x]^3*Tanh[a + b*x]^3,x]
```

output
```
-((Sinh[a + b*x]^5/(2*(1 + Sinh[a + b*x]^2)) - (5*(ArcTan[Sinh[a + b*x]] -
 Sinh[a + b*x] + Sinh[a + b*x]^3/3))/2)/b)
```

3.79.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.79.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

method	result	siz
derivativedivides	$\frac{\frac{\sinh(bx+a)^5}{3\cosh(bx+a)^2}-\frac{5\sinh(bx+a)^3}{3\cosh(bx+a)^2}-\frac{5\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{5\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}+5\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	81
default	$\frac{\frac{\sinh(bx+a)^5}{3\cosh(bx+a)^2}-\frac{5\sinh(bx+a)^3}{3\cosh(bx+a)^2}-\frac{5\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{5\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}+5\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	81
risch	$\frac{\mathrm{e}^{3bx+3a}}{24b}-\frac{9\,\mathrm{e}^{bx+a}}{8b}+\frac{9\,\mathrm{e}^{-bx-a}}{8b}-\frac{\mathrm{e}^{-3bx-3a}}{24b}-\frac{\mathrm{e}^{bx+a}\left(\mathrm{e}^{2bx+2a}-1\right)}{b\left(1+\mathrm{e}^{2bx+2a}\right)^2}+\frac{5i\ln\left(\mathrm{e}^{bx+a}+i\right)}{2b}-\frac{5i\ln\left(\mathrm{e}^{bx+a}-i\right)}{2b}$	12

input
```
int(sinh(b*x+a)^3*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*sinh(b*x+a)^5/cosh(b*x+a)^2-5/3*sinh(b*x+a)^3/cosh(b*x+a)^2-5/cos
h(b*x+a)^2*sinh(b*x+a)+5/2*sech(b*x+a)*tanh(b*x+a)+5*arctan(exp(b*x+a)))
```

3.79.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. 2(58) = 116.

Time = 0.26 (sec) , antiderivative size = 851, normalized size of antiderivative = 12.89

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/24*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^
10 + 5*(9*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^8 - 25*cosh(b*x + a)^8 + 40*(
3*cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^7 + 10*(21*cosh(b*x + a
)^4 - 70*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^6 - 50*cosh(b*x + a)^6 + 4*(63
*cosh(b*x + a)^5 - 350*cosh(b*x + a)^3 - 75*cosh(b*x + a))*sinh(b*x + a)^5
 + 10*(21*cosh(b*x + a)^6 - 175*cosh(b*x + a)^4 - 75*cosh(b*x + a)^2 + 5)*
sinh(b*x + a)^4 + 50*cosh(b*x + a)^4 + 40*(3*cosh(b*x + a)^7 - 35*cosh(b*x
 + a)^5 - 25*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^3 + 5*(9*cos
h(b*x + a)^8 - 140*cosh(b*x + a)^6 - 150*cosh(b*x + a)^4 + 60*cosh(b*x + a
)^2 + 5)*sinh(b*x + a)^2 + 120*(cosh(b*x + a)^7 + 7*cosh(b*x + a)*sinh(b*x
 + a)^6 + sinh(b*x + a)^7 + (21*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^5 + 2*c
osh(b*x + a)^5 + 5*(7*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^4 +
 (35*cosh(b*x + a)^4 + 20*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + cosh(b*x
+ a)^3 + (21*cosh(b*x + a)^5 + 20*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(
b*x + a)^2 + (7*cosh(b*x + a)^6 + 10*cosh(b*x + a)^4 + 3*cosh(b*x + a)^2)*
sinh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a)) + 25*cosh(b*x + a)^2
+ 10*(cosh(b*x + a)^9 - 20*cosh(b*x + a)^7 - 30*cosh(b*x + a)^5 + 20*cosh(
b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^7 + 7*b*
cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^5 +
(21*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 +...
```

3.79.6 Sympy [F]

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)**3*tanh(b*x+a)**3,x)
```

output
```
Integral(sinh(a + b*x)**3*tanh(a + b*x)**3, x)
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.76

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{27\,e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{24\,b} - \frac{5\,\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{25\,e^{(-2\,bx-2\,a)} + 77\,e^{(-4\,bx-4\,a)} + 3\,e^{(-6\,bx-6\,a)} - 1}{24\,b(e^{(-3\,bx-3\,a)} + 2\,e^{(-5\,bx-5\,a)} + e^{(-7\,bx-7\,a)})}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*(27*e^(-b*x - a) - e^(-3*b*x - 3*a))/b - 5*arctan(e^(-b*x - a))/b - 1
/24*(25*e^(-2*b*x - 2*a) + 77*e^(-4*b*x - 4*a) + 3*e^(-6*b*x - 6*a) - 1)/(
b*(e^(-3*b*x - 3*a) + 2*e^(-5*b*x - 5*a) + e^(-7*b*x - 7*a)))
```

3.79.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(58) = 116.

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.77

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$$
$$= \frac{30\,\pi + \left(e^{(bx+a)} - e^{(-bx-a)}\right)^3 - \frac{24\,\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4} + 60\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right) - 24\,e^{(bx+a)} + 2}{24\,b}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/24*(30*pi + (e^(b*x + a) - e^(-b*x - a))^3 - 24*(e^(b*x + a) - e^(-b*x -
 a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4) + 60*arctan(1/2*(e^(2*b*x + 2*a)
 - 1)*e^(-b*x - a)) - 24*e^(b*x + a) + 24*e^(-b*x - a))/b
```

3.79.9 Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.06

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{5\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{9\,e^{a+b\,x}}{8\,b} + \frac{9\,e^{-a-b\,x}}{8\,b} - \frac{e^{-3\,a-3\,b\,x}}{24\,b} + \frac{e^{3\,a+3\,b\,x}}{24\,b} + \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(sinh(a + b*x)^3*tanh(a + b*x)^3,x)
```

output
```
(5*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (9*exp(a + b*x))/(
8*b) + (9*exp(- a - b*x))/(8*b) - exp(- 3*a - 3*b*x)/(24*b) + exp(3*a + 3*
b*x)/(24*b) + (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) +
 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

3.80 $\int \sinh^4(a+bx)\tanh(a+bx)\,dx$

3.80.1 Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx=-\frac{\cosh^2(a+bx)}{b}+\frac{\cosh^4(a+bx)}{4b}+\frac{\log(\cosh(a+bx))}{b}$$

output
```
-cosh(b*x+a)^2/b+1/4*cosh(b*x+a)^4/b+ln(cosh(b*x+a))/b
```

3.80.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx=\frac{-\cosh^2(a+bx)+\frac{1}{4}\cosh^4(a+bx)+\log(\cosh(a+bx))}{b}$$

input
```
Integrate[Sinh[a + b*x]^4*Tanh[a + b*x],x]
```

output
```
(-Cosh[a + b*x]^2 + Cosh[a + b*x]^4/4 + Log[Cosh[a + b*x]])/b
```

3.80.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)^4\tan(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ia+ibx)^4\tan(ia+ibx)dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)^2\operatorname{sech}(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)^2\operatorname{sech}(a+bx)d\cosh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\cosh^2(a+bx)+\operatorname{sech}(a+bx)-2\right)d\cosh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\cosh^4(a+bx)-2\cosh^2(a+bx)+\log\left(\cosh^2(a+bx)\right)}{2b}$$

input
```
Int[Sinh[a + b*x]^4*Tanh[a + b*x],x]
```

output
```
(-2*Cosh[a + b*x]^2 + Cosh[a + b*x]^4/2 + Log[Cosh[a + b*x]^2])/(2*b)
```

3.80.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.80.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^4}{4}-\frac{\sinh(bx+a)^2}{2}+\ln(\cosh(bx+a))}{b}$	33
default	$\frac{\frac{\sinh(bx+a)^4}{4}-\frac{\sinh(bx+a)^2}{2}+\ln(\cosh(bx+a))}{b}$	33
risch	$-x+\frac{e^{4bx+4a}}{64b}-\frac{3\,e^{2bx+2a}}{16b}-\frac{3\,e^{-2bx-2a}}{16b}+\frac{e^{-4bx-4a}}{64b}-\frac{2a}{b}+\frac{\ln\left(1+e^{2bx+2a}\right)}{b}$	83

input
```
int(sinh(b*x+a)^4*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*sinh(b*x+a)^4-1/2*sinh(b*x+a)^2+ln(cosh(b*x+a)))
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(38) = 76$.

Time = 0.27 (sec) , antiderivative size = 457, normalized size of antiderivative = 11.42

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^8 + 8\cosh(bx+a)\sinh(bx+a)^7 + \sinh(bx+a)^8 + 4\left(7\cosh(bx+a)^2 - 3\right)\sinh(bx+a)^6}{}$$

input
```
integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="fricas")
```

output
```
1/64*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8
+ 4*(7*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^6 - 64*b*x*cosh(b*x + a)^4 - 12*
cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 - 9*cosh(b*x + a))*sinh(b*x + a)^5
+ 2*(35*cosh(b*x + a)^4 - 32*b*x - 90*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8
*(7*cosh(b*x + a)^5 - 32*b*x*cosh(b*x + a) - 30*cosh(b*x + a)^3)*sinh(b*x
+ a)^3 + 4*(7*cosh(b*x + a)^6 - 96*b*x*cosh(b*x + a)^2 - 45*cosh(b*x + a)^
4 - 3)*sinh(b*x + a)^2 - 12*cosh(b*x + a)^2 + 64*(cosh(b*x + a)^4 + 4*cosh
(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x
 + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4)*log(2*cosh(b*x + a)/(cosh(b*x + a
) - sinh(b*x + a))) + 8*(cosh(b*x + a)^7 - 32*b*x*cosh(b*x + a)^3 - 9*cosh
(b*x + a)^5 - 3*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^4 + 4*b
*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b
*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4)
```

3.80.6 Sympy [F]

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = \int \sinh^4(a+bx)\tanh(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)**4*tanh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)**4*tanh(a + b*x), x)
```

3.80.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(38) = 76$.

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.02

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = -\frac{\left(12\,e^{(-2\,bx-2\,a)}-1\right)e^{(4\,bx+4\,a)}}{64\,b} + \frac{bx+a}{b} - \frac{12\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}}{64\,b} + \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

input
```
integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="maxima")
```

output
```
-1/64*(12*e^(-2*b*x - 2*a) - 1)*e^(4*b*x + 4*a)/b + (b*x + a)/b - 1/64*(12
*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a))/b + log(e^(-2*b*x - 2*a) + 1)/b
```

3.80.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(38) = 76$.

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.10

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = -\frac{64\,bx - \left(48\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}+1\right)e^{(-4\,bx-4\,a)} + 64\,a - e^{(4\,bx+4\,a)} + 12\,e^{(2\,bx+2\,a)} - 64\,\log\left(e^{(2\,bx+2\,a)}\right.}{64\,b}$$

input
```
integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="giac")
```

output
```
-1/64*(64*b*x - (48*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a) + 1)*e^(-4*b*x -
4*a) + 64*a - e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a) - 64*log(e^(2*b*x + 2*a
) + 1))/b
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.92

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = \frac{\ln\left(e^{2a}\,e^{2bx}+1\right)}{b} - x - \frac{3\,e^{-2a-2bx}}{16\,b} - \frac{3\,e^{2a+2bx}}{16\,b} + \frac{e^{-4a-4bx}}{64\,b} + \frac{e^{4a+4bx}}{64\,b}$$

input
```
int(sinh(a + b*x)^4*tanh(a + b*x),x)
```

output
```
log(exp(2*a)*exp(2*b*x) + 1)/b - x - (3*exp(- 2*a - 2*b*x))/(16*b) - (3*ex
p(2*a + 2*b*x))/(16*b) + exp(- 4*a - 4*b*x)/(64*b) + exp(4*a + 4*b*x)/(64*
b)
```

3.81 $\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$

3.81.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}(a+bx)}{b}$$

output
```
-sech(b*x+a)/b
```

3.81.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}(a+bx)}{b}$$

input
```
Integrate[Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
-(Sech[a + b*x]/b)
```

3.81.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 26, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(a+bx)\text{sech}(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -i\tan(ia+ibx)\sec(ia+ibx)dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \sec(ia+ibx)\tan(ia+ibx)dx$$
$$\Big\downarrow \text{ 3086}$$
$$-\frac{\int 1d\text{sech}(a+bx)}{b}$$
$$\Big\downarrow \text{ 24}$$
$$-\frac{\text{sech}(a+bx)}{b}$$

input
```
Int[Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
-(Sech[a + b*x]/b)
```

3.81.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.81.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\operatorname{sech}(bx+a)}{b}$	12
default	$-\frac{\operatorname{sech}(bx+a)}{b}$	12
risch	$-\frac{2\,\mathrm{e}^{bx+a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)}$	25

input
```
int(sech(b*x+a)*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-sech(b*x+a)/b
```

3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(11) = 22.

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.91

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$$
$$= -\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2+b}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

3.81.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \begin{cases} -\frac{\operatorname{sech}(a+bx)}{b} & \text{for } b \neq 0 \\ x\tanh(a)\operatorname{sech}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a),x)
```

output
```
Piecewise((-sech(a + b*x)/b, Ne(b, 0)), (x*tanh(a)*sech(a), True))
```

3.81.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)} + e^{(-bx-a)})}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="maxima")
```

output
```
-2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

3.81.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)} + e^{(-bx-a)})}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="giac")
```

output
```
-2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

3.81.9 Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(tanh(a + b*x)/cosh(a + b*x),x)
```

output
```
-(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.82 $\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx$

3.82.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

output
```
-1/2*sech(b*x+a)^2/b
```

3.82.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

input
```
Integrate[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-1/2*Sech[a + b*x]^2/b
```

3.82.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 26, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -i\tan(ia+ibx)\sec(ia+ibx)^2dx$$

$$\Big\downarrow \; 26$$

$$-i\int \sec(ia+ibx)^2\tan(ia+ibx)dx$$

$$\Big\downarrow \; 3086$$

$$-\frac{\int \text{sech}(a+bx)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \; 15$$

$$-\frac{\text{sech}^2(a+bx)}{2b}$$

input
```
Int[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-1/2*Sech[a + b*x]^2/b
```

3.82.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.82.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tanh(bx+a)^2}{2b}$	14
default	$\frac{\tanh(bx+a)^2}{2b}$	14
risch	$-\frac{2\,e^{2bx+2a}}{b\left(1+e^{2bx+2a}\right)^2}$	28

input
```
int(sech(b*x+a)^2*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*tanh(b*x+a)^2/b
```

3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 5.60

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+b\sinh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)+\left(3\,b\cosh\left(bx+\right.\right.}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*
sinh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a))
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \begin{cases} -\frac{\operatorname{sech}^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\tanh(a)\operatorname{sech}^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sech(b*x+a)**2*tanh(b*x+a),x)
```

output
```
Piecewise((-sech(a + b*x)**2/(2*b), Ne(b, 0)), (x*tanh(a)*sech(a)**2, True
))
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \frac{\tanh(bx+a)^2}{2\,b}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*tanh(b*x + a)^2/b
```

3.82.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2\,e^{(2\,bx+2\,a)}}{b(e^{(2\,bx+2\,a)}+1)^2}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="giac")
```

output
```
-2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{1}{2\,b\cosh\left(a+b\,x\right)^2}$$

input
```
int(tanh(a + b*x)/cosh(a + b*x)^2,x)
```

output
```
-1/(2*b*cosh(a + b*x)^2)
```

3.83 $\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx$

3.83.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = -\frac{\operatorname{sech}^n(a+bx)}{bn}$$

output
```
-sech(b*x+a)^n/b/n
```

3.83.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = -\frac{\operatorname{sech}^n(a+bx)}{bn}$$

input
```
Integrate[Sech[a + b*x]^(1 + n)*Sinh[a + b*x],x]
```

output
```
-(Sech[a + b*x]^n/(b*n))
```

3.83.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 26, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\text{sech}^{n+1}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sec(ia+ibx)^{n+1}}{\csc(ia+ibx)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sec(ia+ibx)^{n+1}}{\csc(ia+ibx)}dx$$

$$\Big\downarrow \text{3102}$$

$$-\frac{\int \text{sech}^{n-1}(a+bx)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\text{sech}^n(a+bx)}{bn}$$

input
```
Int[Sech[a + b*x]^(1 + n)*Sinh[a + b*x],x]
```

output
```
-(Sech[a + b*x]^n/(b*n))
```

3.83.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.83.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{\operatorname{sech}(bx+a)^n}{bn}$
default	$-\frac{\operatorname{sech}(bx+a)^n}{bn}$
risch	$-\frac{2^n \left(\mathrm{e}^{bx+a}\right)^n \left(1+\mathrm{e}^{2bx+2a}\right)^{-n} \mathrm{e}^{-\frac{i \operatorname{csgn}\left(\frac{i \mathrm{e}^{bx+a}}{1+\mathrm{e}^{2bx+2a}}\right) \pi n \left(-\operatorname{csgn}\left(\frac{i \mathrm{e}^{bx+a}}{1+\mathrm{e}^{2bx+2a}}\right)+\operatorname{csgn}\left(\frac{i}{1+\mathrm{e}^{2bx+2a}}\right)\right) \left(-\operatorname{csgn}\left(\frac{i \mathrm{e}^{bx+a}}{1+\mathrm{e}^{2bx+2a}}\right)+\right.}{2}}}{bn}$

input
```
int(sech(b*x+a)^n*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-sech(b*x+a)^n/b/n
```

3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 7.19

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = -\frac{\cosh\left(n\log\left(\frac{2\left(\cosh(bx+a)+\sinh(bx+a)\right)}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1}\right)\right)+\sinh\left(n\log\left(\frac{2\left(\cosh(bx+a)+\sinh(bx+\right.}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a}\right.\right.}{bn}$$

input
```
integrate(sech(b*x+a)^n*tanh(b*x+a),x, algorithm="fricas")
```

output
```
-(cosh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b
*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))) + sinh(n*log(2*(cosh(b*x +
a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 + 1))))/(b*n)
```

3.83.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = \begin{cases} x\tanh(a) & \text{for } b=0 \wedge n=0 \\ x\tanh(a)\operatorname{sech}^n(a) & \text{for } b=0 \\ x-\frac{\log(\tanh(a+bx)+1)}{b} & \text{for } n=0 \\ -\frac{\operatorname{sech}^n(a+bx)}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(sech(b*x+a)**n*tanh(b*x+a),x)
```

output
```
Piecewise((x*tanh(a), Eq(b, 0) & Eq(n, 0)), (x*tanh(a)*sech(a)**n, Eq(b, 0
)), (x - log(tanh(a + b*x) + 1)/b, Eq(n, 0)), (-sech(a + b*x)**n/(b*n), Tr
ue))
```

3.83.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = -\frac{2^n e^{\left(-(bx+a)n-n\log\left(e^{(-2bx-2a)}+1\right)\right)}}{bn}$$

input
```
integrate(sech(b*x+a)^n*tanh(b*x+a),x, algorithm="maxima")
```

output
```
-2^n*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/(b*n)
```

3.83.8 Giac [F]

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = \int \operatorname{sech}\left(bx+a\right)^{n}\tanh\left(bx+a\right)\,dx$$

input
```
integrate(sech(b*x+a)^n*tanh(b*x+a),x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^n*tanh(b*x + a), x)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = -\frac{\left(\frac{2\,\mathrm{e}^{a+b\,x}}{\mathrm{e}^{2\,a+2\,b\,x}+1}\right)^n}{b\,n}$$

input
```
int(tanh(a + b*x)*(1/cosh(a + b*x))^n,x)
```

output
```
-((2*exp(a + b*x))/(exp(2*a + 2*b*x) + 1))^n/(b*n)
```

3.84 $\int \text{sech}^2(a+bx)\tanh^2(a+bx)\,dx$

3.84.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \text{sech}^2(a+bx)\tanh^2(a+bx)\,dx=\frac{\tanh^3(a+bx)}{3b}$$

output
```
1/3*tanh(b*x+a)^3/b
```

3.84.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \text{sech}^2(a+bx)\tanh^2(a+bx)\,dx=\frac{\tanh^3(a+bx)}{3b}$$

input
```
Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^2,x]
```

output
```
Tanh[a + b*x]^3/(3*b)
```

3.84.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 25, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ia+ibx)^2\left(-\sec(ia+ibx)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ia+ibx)^2\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{i\int -\tanh^2(a+bx)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\tanh^3(a+bx)}{3b}$$

input
```
Int[Sech[a + b*x]^2*Tanh[a + b*x]^2,x]
```

output
```
Tanh[a + b*x]^3/(3*b)
```

3.84.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.84.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tanh(bx+a)^3}{3b}$	14
default	$\frac{\tanh(bx+a)^3}{3b}$	14
risch	$-\frac{2(3\,e^{4bx+4a}+1)}{3b(1+e^{2bx+2a})^3}$	32

input
```
int(sech(b*x+a)^2*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*tanh(b*x+a)^3/b
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(13) = 26.

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 9.20

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx =$$

$$-\frac{8\left(\cosh\left(bx+a\right)^2+\cosh\left(bx+a\right)\sinh}{3\left(b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+b\sinh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)^2+2\left(3\,b\cosh\right.\right.}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-8/3*(cosh(b*x + a)^2 + cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*
cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 +
4*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

3.84.6 Sympy [F]

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = \int \tanh^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**2*tanh(b*x+a)**2,x)
```

output
```
Integral(tanh(a + b*x)**2*sech(a + b*x)**2, x)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = \frac{\tanh(bx+a)^3}{3\,b}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*tanh(b*x + a)^3/b
```

3.84.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = -\frac{2\left(3\,e^{(4\,bx+4\,a)}+1\right)}{3\,b(e^{(2\,bx+2\,a)}+1)^3}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
-2/3*(3*e^(4*b*x + 4*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^3)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx=-\frac{2\left(3\,e^{4\,a+4\,b\,x}+1\right)}{3\,b\,\left(e^{2\,a+2\,b\,x}+1\right)^3}$$

input
```
int(tanh(a + b*x)^2/cosh(a + b*x)^2,x)
```

output
```
-(2*(3*exp(4*a + 4*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) + 1)^3)
```

3.85 $\int \mathbf{sech}^2(a+bx)\tanh^3(a+bx)\,dx$

3.85.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx=\frac{\tanh^4(a+bx)}{4b}$$

output
```
1/4*tanh(b*x+a)^4/b
```

3.85.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx=\frac{\tanh^4(a+bx)}{4b}$$

input
```
Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^3,x]
```

output
```
Tanh[a + b*x]^4/(4*b)
```

3.85.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 26, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\tan(ia+ibx)^3\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \sec(ia+ibx)^2\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{\int -i\tanh^3(a+bx)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\tanh^4(a+bx)}{4b}$$

input
```
Int[Sech[a + b*x]^2*Tanh[a + b*x]^3,x]
```

output
```
Tanh[a + b*x]^4/(4*b)
```

3.85.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.85.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tanh(bx+a)^4}{4b}$	14
default	$\frac{\tanh(bx+a)^4}{4b}$	14
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}\left(\mathrm{e}^{4bx+4a}+1\right)}{b\left(1+\mathrm{e}^{2bx+2a}\right)^4}$	39

input
```
int(sech(b*x+a)^2*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*tanh(b*x+a)^4/b
```

3.85.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(13) = 26$.

Time = 0.24 (sec) , antiderivative size = 208, normalized size of antiderivative = 13.87

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx =$$

$$-\frac{2\left(\cosh\left(bx+a\right)^3+3\,\cosh\left(bx\right.\right.}{b\cosh\left(bx+a\right)^5+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+b\sinh\left(bx+a\right)^5+5\,b\cosh\left(bx+a\right)^3+\left(10\,b\cosh\left(bx\right.\right.}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 +
(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a)^5
+ 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 5*b*cosh(b*x + a
)^3 + (10*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^3 + 5*(2*b*cosh(b*x + a)^
3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + 10*b*cosh(b*x + a) + (5*b*cosh(b*
x + a)^4 + 9*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

3.85.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = \begin{cases} -\frac{\tanh^2(a+bx)\operatorname{sech}^2(a+bx)}{4b} - \frac{\operatorname{sech}^2(a+bx)}{4b} & \text{for } b \neq 0 \\ x\tanh^3(a)\operatorname{sech}^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sech(b*x+a)**2*tanh(b*x+a)**3,x)
```

output
```
Piecewise((-tanh(a + b*x)**2*sech(a + b*x)**2/(4*b) - sech(a + b*x)**2/(4*
b), Ne(b, 0)), (x*tanh(a)**3*sech(a)**2, True))
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = \frac{\tanh(bx+a)^4}{4b}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*tanh(b*x + a)^4/b
```

3.85.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\left(e^{(6\,bx+6\,a)}+e^{(2\,bx+2\,a)}\right)}{b\left(e^{(2\,bx+2\,a)}+1\right)^4}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
-2*(e^(6*b*x + 6*a) + e^(2*b*x + 2*a))/(b*(e^(2*b*x + 2*a) + 1)^4)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 15.33

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = \frac{\frac{1}{2\,b}-\frac{3\,e^{2\,a+2\,b\,x}}{2\,b}+\frac{3\,e^{4\,a+4\,b\,x}}{2\,b}-\frac{e^{6\,a+6\,b\,x}}{2\,b}}{4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1} - \frac{\frac{1}{2\,b}-\frac{e^{2\,a+2\,b\,x}}{b}+\frac{e^{4\,a+4\,b\,x}}{2\,b}}{3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1} + \frac{\frac{1}{2\,b}-\frac{e^{2\,a+2\,b\,x}}{2\,b}}{2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1} - \frac{1}{2\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(tanh(a + b*x)^3/cosh(a + b*x)^2,x)
```

output
```
(1/(2*b) - (3*exp(2*a + 2*b*x))/(2*b) + (3*exp(4*a + 4*b*x))/(2*b) - exp(6
*a + 6*b*x)/(2*b))/(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a +
6*b*x) + exp(8*a + 8*b*x) + 1) - (1/(2*b) - exp(2*a + 2*b*x)/b + exp(4*a +
 4*b*x)/(2*b))/(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x)
 + 1) + (1/(2*b) - exp(2*a + 2*b*x)/(2*b))/(2*exp(2*a + 2*b*x) + exp(4*a +
 4*b*x) + 1) - 1/(2*b*(exp(2*a + 2*b*x) + 1))
```

3.86 $\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx$

3.86.1 Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx=\frac{\tanh^{1+n}(a+bx)}{b(1+n)}$$

output
```
tanh(b*x+a)^(1+n)/b/(1+n)
```

3.86.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx=\frac{\tanh^{1+n}(a+bx)}{b(1+n)}$$

input
```
Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^n,x]
```

output
```
Tanh[a + b*x]^(1 + n)/(b*(1 + n))
```

3.86.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(ia+ibx)^2(-i\tan(ia+ibx))^n dx$$

$$\Big\downarrow \text{3087}$$

$$-\frac{i\int \tanh^n(a+bx)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{\tanh^{n+1}(a+bx)}{b(n+1)}$$

input
```
Int[Sech[a + b*x]^2*Tanh[a + b*x]^n,x]
```

output
```
Tanh[a + b*x]^(1 + n)/(b*(1 + n))
```

3.86.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.86.4 Maple [A] (verified)

Time = 6.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\tanh(bx+a)^{n+1}}{b(n+1)}$
default	$\frac{\tanh(bx+a)^{n+1}}{b(n+1)}$
risch	$\frac{(e^{2bx+2a}-1)(e^{bx+a}-1)^n(e^{bx+a}+1)^n(1+e^{2bx+2a})^{-n}e^{-\frac{i\pi n\left(-\operatorname{csgn}\left(\frac{i}{1+e^{2bx+2a}}\right)\operatorname{csgn}\left(\frac{i\left(e^{bx+a}+1\right)}{1+e^{2bx+2a}}\right)^2+\operatorname{csgn}\left(\frac{i}{1+e^{2bx+2a}}\right.}{}}}{}$

input
```
int(sech(b*x+a)^2*tanh(b*x+a)^n,x,method=_RETURNVERBOSE)
```

output
```
tanh(b*x+a)^(n+1)/b/(n+1)
```

3.86.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.63

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx$$
$$=\frac{\cosh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)\sinh(bx+a)+\sinh(bx+a)\sinh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)}{(bn+b)\cosh(bx+a)}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="fricas")
```

output
```
(cosh(n*log(sinh(b*x + a)/cosh(b*x + a)))*sinh(b*x + a) + sinh(b*x + a)*si
nh(n*log(sinh(b*x + a)/cosh(b*x + a))))/((b*n + b)*cosh(b*x + a))
```

3.86.6 Sympy [F]

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \int \tanh^n(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**2*tanh(b*x+a)**n,x)
```

output
```
Integral(tanh(a + b*x)**n*sech(a + b*x)**2, x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \frac{\tanh(bx+a)^{n+1}}{b(n+1)}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="maxima")
```

output
```
tanh(b*x + a)^(n + 1)/(b*(n + 1))
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \frac{\left(\frac{e^{(2\,bx+2\,a)}-1}{e^{(2\,bx+2\,a)}+1}\right)^{n+1}}{b(n+1)}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="giac")
```

output
```
((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1))^(n + 1)/(b*(n + 1))
```

3.86.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \frac{\tanh(a+b\,x)\,\left(\frac{e^{2\,a+2\,b\,x}-1}{e^{2\,a+2\,b\,x}+1}\right)^n}{b\,(n+1)}$$

input
```
int(tanh(a + b*x)^n/cosh(a + b*x)^2,x)
```

output
```
(tanh(a + b*x)*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^n)/(b*(n +
1))
```

3.87 $\int \text{sech}(a+bx)\tanh^3(a+bx)\,dx$

3.87.1 Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \text{sech}(a+bx)\tanh^3(a+bx)\,dx = -\frac{\text{sech}(a+bx)}{b}+\frac{\text{sech}^3(a+bx)}{3b}$$

output
```
-sech(b*x+a)/b+1/3*sech(b*x+a)^3/b
```

3.87.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \text{sech}(a+bx)\tanh^3(a+bx)\,dx = -\frac{\text{sech}(a+bx)}{b}+\frac{\text{sech}^3(a+bx)}{3b}$$

input
```
Integrate[Sech[a + b*x]*Tanh[a + b*x]^3,x]
```

output
```
-(Sech[a + b*x]/b) + Sech[a + b*x]^3/(3*b)
```

3.87.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 26, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ia+ibx)^3\sec(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec(ia+ibx)\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \left(\text{sech}^2(a+bx)-1\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\text{sech}^3(a+bx)-\text{sech}(a+bx)}{b}$$

input
```
Int[Sech[a + b*x]*Tanh[a + b*x]^3,x]
```

output
```
(-Sech[a + b*x] + Sech[a + b*x]^3/3)/b
```

3.87.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.87.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\operatorname{sech}(bx+a)^3}{3}-\operatorname{sech}(bx+a)}{b}$	24
default	$\frac{\frac{\operatorname{sech}(bx+a)^3}{3}-\operatorname{sech}(bx+a)}{b}$	24
risch	$-\frac{2\,\mathrm{e}^{bx+a}\left(3\,\mathrm{e}^{4bx+4a}+2\,\mathrm{e}^{2bx+2a}+3\right)}{3b\left(1+\mathrm{e}^{2bx+2a}\right)^3}$	49

input
```
int(sech(b*x+a)*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*sech(b*x+a)^3-sech(b*x+a))
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(25) = 50.

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 6.37

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx =$$
$$-\frac{2\left(3\,\cosh\left(bx+a\right)^3+9\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+3\,\sinh\left(bx+a\right)^3}{3\left(b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+b\sinh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)^2+2\left(3\,b\cosh\right.\right.}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 + 3*sinh(b*x + a
)^3 + (9*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + 5*cosh(b*x + a))/(b*cosh(b*x
 + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 4*b*cosh
(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a)^2 + 4*(b*cosh(b*
x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

3.87.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = \begin{cases} -\frac{\tanh^2(a+bx)\operatorname{sech}(a+bx)}{3b} - \frac{2\operatorname{sech}(a+bx)}{3b} & \text{for } b \neq 0 \\ x\tanh^3(a)\operatorname{sech}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)**3,x)
```

output
```
Piecewise((-tanh(a + b*x)**2*sech(a + b*x)/(3*b) - 2*sech(a + b*x)/(3*b),
Ne(b, 0)), (x*tanh(a)**3*sech(a), True))
```

3.87.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(25) = 50$.

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 5.48

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = -\frac{2e^{(-bx-a)}}{b(3e^{(-2bx-2a)} + 3e^{(-4bx-4a)} + e^{(-6bx-6a)} + 1)} - \frac{4e^{(-3bx-3a)}}{3b(3e^{(-2bx-2a)} + 3e^{(-4bx-4a)} + e^{(-6bx-6a)} + 1)} - \frac{2e^{(-5bx-5a)}}{b(3e^{(-2bx-2a)} + 3e^{(-4bx-4a)} + e^{(-6bx-6a)} + 1)}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2*e^(-b*x - a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x -
6*a) + 1)) - 4/3*e^(-3*b*x - 3*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4
*a) + e^(-6*b*x - 6*a) + 1)) - 2*e^(-5*b*x - 5*a)/(b*(3*e^(-2*b*x - 2*a) +
 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1))
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)}{3\,b\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
-2/3*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 4)/(b*(e^(b*x + a) + e^(-b*x - a)
)^3)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\,e^{a+b\,x}\,\left(2\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+3\right)}{3\,b\,\left(e^{2\,a+2\,b\,x}+1\right)^3}$$

input
```
int(tanh(a + b*x)^3/cosh(a + b*x),x)
```

output
```
-(2*exp(a + b*x)*(2*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + 3))/(3*b*(exp(
2*a + 2*b*x) + 1)^3)
```

3.88 $\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx$

3.88.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx=-\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{\operatorname{sech}^5(a+bx)}{5b}$$

output
```
-1/3*sech(b*x+a)^3/b+1/5*sech(b*x+a)^5/b
```

3.88.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx=-\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{\operatorname{sech}^5(a+bx)}{5b}$$

input
```
Integrate[Sech[a + b*x]^3*Tanh[a + b*x]^3,x]
```

output
```
-1/3*Sech[a + b*x]^3/b + Sech[a + b*x]^5/(5*b)
```

3.88.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ia+ibx)^3\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec(ia+ibx)^3\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int -\text{sech}^2(a+bx)\left(1-\text{sech}^2(a+bx)\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \text{sech}^2(a+bx)\left(1-\text{sech}^2(a+bx)\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\text{sech}^2(a+bx)-\text{sech}^4(a+bx)\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}\text{sech}^5(a+bx)-\frac{1}{3}\text{sech}^3(a+bx)}{b}$$

input
```
Int[Sech[a + b*x]^3*Tanh[a + b*x]^3,x]
```

output
```
(-1/3*Sech[a + b*x]^3 + Sech[a + b*x]^5/5)/b
```

3.88.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.88.4 Maple [A] (verified)

Time = 3.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\operatorname{sech}(bx+a)^5}{5}-\frac{\operatorname{sech}(bx+a)^3}{3}}{b}$	26
default	$\frac{\frac{\operatorname{sech}(bx+a)^5}{5}-\frac{\operatorname{sech}(bx+a)^3}{3}}{b}$	26
risch	$-\frac{8\,{\rm e}^{3bx+3a}\left(5\,{\rm e}^{4bx+4a}-2\,{\rm e}^{2bx+2a}+5\right)}{15b\left(1+{\rm e}^{2bx+2a}\right)^5}$	52

input
```
int(sech(b*x+a)^3*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5*sech(b*x+a)^5-1/3*sech(b*x+a)^3)
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 345, normalized size of antiderivative = 11.13

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^7+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^6+b\sinh\left(bx+a\right)^7+5\,b\cosh\left(bx+a\right)^5+\left(21\,b\cos\right.\right.}$$

input
```
integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-8/15*(5*cosh(b*x + a)^4 + 20*cosh(b*x + a)*sinh(b*x + a)^3 + 5*sinh(b*x +
 a)^4 + 2*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4
*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 5)/(b*cosh(b*x + a)^7
 + 7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 5*b*cosh(b*x +
a)^5 + (21*b*cosh(b*x + a)^2 + 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)
^3 + 5*b*cosh(b*x + a))*sinh(b*x + a)^4 + 11*b*cosh(b*x + a)^3 + (35*b*cos
h(b*x + a)^4 + 50*b*cosh(b*x + a)^2 + 9*b)*sinh(b*x + a)^3 + (21*b*cosh(b*
x + a)^5 + 50*b*cosh(b*x + a)^3 + 33*b*cosh(b*x + a))*sinh(b*x + a)^2 + 15
*b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 + 25*b*cosh(b*x + a)^4 + 27*b*cosh
(b*x + a)^2 + 5*b)*sinh(b*x + a))
```

3.88.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx = \begin{cases} -\frac{\tanh^2(a+bx)\operatorname{sech}^3(a+bx)}{5b} - \frac{2\operatorname{sech}^3(a+bx)}{15b} & \text{for } b \neq 0 \\ x\tanh^3(a)\operatorname{sech}^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sech(b*x+a)**3*tanh(b*x+a)**3,x)
```

output
```
Piecewise((-tanh(a + b*x)**2*sech(a + b*x)**3/(5*b) - 2*sech(a + b*x)**3/(
15*b), Ne(b, 0)), (x*tanh(a)**3*sech(a)**3, True))
```

3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(27) = 54$.

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 6.90

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx$$
$$= -\frac{8\,e^{(-3\,bx-3\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$
$$+\frac{16\,e^{(-5\,bx-5\,a)}}{15\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$
$$-\frac{8\,e^{(-7\,bx-7\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$

input
```
integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-8/3*e^(-3*b*x - 3*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^
(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 16/15*e^(
-5*b*x - 5*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x
- 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) - 8/3*e^(-7*b*x - 7
*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5
*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1))
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx = -\frac{8\left(5\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-12\right)}{15\,b\left(e^{(bx+a)}+e^{(-bx-a)}\right)^5}$$

input
```
integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
-8/15*(5*(e^(b*x + a) + e^(-b*x - a))^2 - 12)/(b*(e^(b*x + a) + e^(-b*x -
a))^5)
```

3.88.9 Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 251, normalized size of antiderivative = 8.10

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx$$
$$= \frac{\frac{4\,e^{a+b\,x}}{5\,b}-\frac{12\,e^{3\,a+3\,b\,x}}{5\,b}+\frac{12\,e^{5\,a+5\,b\,x}}{5\,b}-\frac{4\,e^{7\,a+7\,b\,x}}{5\,b}}{5\,e^{2\,a+2\,b\,x}+10\,e^{4\,a+4\,b\,x}+10\,e^{6\,a+6\,b\,x}+5\,e^{8\,a+8\,b\,x}+e^{10\,a+10\,b\,x}+1} - \frac{28\,e^{a+b\,x}}{15\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}+\frac{64\,e^{a+b\,x}}{15\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} - \frac{16\,e^{a+b\,x}}{5\,b\,\left(4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}$$

input
```
int(tanh(a + b*x)^3/cosh(a + b*x)^3,x)
```

output
```
((4*exp(a + b*x))/(5*b) - (12*exp(3*a + 3*b*x))/(5*b) + (12*exp(5*a + 5*b*
x))/(5*b) - (4*exp(7*a + 7*b*x))/(5*b))/(5*exp(2*a + 2*b*x) + 10*exp(4*a +
 4*b*x) + 10*exp(6*a + 6*b*x) + 5*exp(8*a + 8*b*x) + exp(10*a + 10*b*x) +
1) - (28*exp(a + b*x))/(15*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
+ (64*exp(a + b*x))/(15*b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6
*a + 6*b*x) + 1)) - (16*exp(a + b*x))/(5*b*(4*exp(2*a + 2*b*x) + 6*exp(4*a
 + 4*b*x) + 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

3.89 $\int \text{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx$

3.89.1 Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \text{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx = -\frac{\text{sech}^n(a+bx)}{bn}+\frac{\text{sech}^{2+n}(a+bx)}{b(2+n)}$$

output
```
-sech(b*x+a)^n/b/n+sech(b*x+a)^(2+n)/b/(2+n)
```

3.89.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \text{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx = \frac{\text{sech}^n(a+bx)\left(-\frac{1}{n}+\frac{\text{sech}^2(a+bx)}{2+n}\right)}{b}$$

input
```
Integrate[Sech[a + b*x]^(3 + n)*Sinh[a + b*x]^3,x]
```

output
```
(Sech[a + b*x]^n*(-n^(-1) + Sech[a + b*x]^2/(2 + n)))/b
```

3.89.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 26, 3102, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\text{sech}^{n+3}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sec(ia+ibx)^{n+3}}{\csc(ia+ibx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sec(ia+ibx)^{n+3}}{\csc(ia+ibx)^3}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\text{sech}^{n-1}(a+bx)\left(1-\text{sech}^2(a+bx)\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \text{sech}^{n-1}(a+bx)\left(1-\text{sech}^2(a+bx)\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\text{sech}^{n-1}(a+bx)-\text{sech}^{n+1}(a+bx)\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\text{sech}^{n+2}(a+bx)}{n+2}-\frac{\text{sech}^n(a+bx)}{n}}{b}$$

input
```
Int[Sech[a + b*x]^(3 + n)*Sinh[a + b*x]^3,x]
```

output
```
(-(Sech[a + b*x]^n/n) + Sech[a + b*x]^(2 + n)/(2 + n))/b
```

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.89.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.18 (sec) , antiderivative size = 209, normalized size of antiderivative = 5.81

method	result
risch	$-\frac{\left(n\,{\mathrm e}^{4 b x +4 a}+2\,{\mathrm e}^{4 b x +4 a}-2 n\,{\mathrm e}^{2 b x +2 a}+4\,{\mathrm e}^{2 b x +2 a}+n +2\right) 2^{n} \left({\mathrm e}^{b x +a}\right)^{n} \left(1+{\mathrm e}^{2 b x +2 a}\right)^{-n} {\mathrm e}^{-\frac{i \,\mathrm{csgn}\left(\frac{i {\mathrm e}^{b x +a}}{1+{\mathrm e}^{2 b x +2 a}}\right) \pi n \left(-\mathrm{csgn}\left(\frac{i {\mathrm e}^{b x +a}}{1+{\mathrm e}^{2 b x +2 a}}\right)+\cdots\right.}{\cdots}}}{b n \left(n +2\right) \left(1+{\mathrm e}^{2 b x +2 a}\right)^{2}}$

input
```
int(sech(b*x+a)^n*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-(n*exp(4*b*x+4*a)+2*exp(4*b*x+4*a)-2*n*exp(2*b*x+2*a)+4*exp(2*b*x+2*a)+n+
2)/b/n/(n+2)/(1+exp(2*b*x+2*a))^2*2^n*exp(b*x+a)^n*(1+exp(2*b*x+2*a))^(-n)
*exp(-1/2*I*csgn(I*exp(b*x+a)/(1+exp(2*b*x+2*a)))*Pi*n*(-csgn(I*exp(b*x+a)
/(1+exp(2*b*x+2*a)))+csgn(I/(1+exp(2*b*x+2*a))))*(-csgn(I*exp(b*x+a)/(1+ex
p(2*b*x+2*a)))+csgn(I*exp(b*x+a))))
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(36) = 72$.

Time = 0.24 (sec) , antiderivative size = 219, normalized size of antiderivative = 6.08

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx =$$

$$-\frac{\left((n+2)\cosh(bx+a)^2+(n+2)\sinh(bx+a)^2-n+2\right)\cosh\left(n\log\left(\frac{2(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\mathrm{si}}\right.\right.}{bn^2+(bn^2+2bn)\,\mathrm{c}}$$

input
```
integrate(sech(b*x+a)^n*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*x + a)^2 - n + 2)*cosh(n*log(2
*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 + 1))) + ((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh
(b*x + a)^2 - n + 2)*sinh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*
x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))))/(b*n^2
+ (b*n^2 + 2*b*n)*cosh(b*x + a)^2 + (b*n^2 + 2*b*n)*sinh(b*x + a)^2 + 2*b*
n)
```

3.89.6 Sympy [F]

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx = \begin{cases} x\tanh^3(a)\operatorname{sech}^n(a) & \text{for } b=0 \\ \int \frac{\tanh^3(a+bx)}{\operatorname{sech}^2(a+bx)}\,dx & \text{for } n=-2 \\ x-\frac{\log(\tanh(a+bx)+1)}{b}-\frac{\tanh^2(a+bx)}{2b} & \text{for } n=0 \\ -\frac{n\tanh^2(a+bx)\operatorname{sech}^n(a+bx)}{bn^2+2bn}-\frac{2\operatorname{sech}^n(a+bx)}{bn^2+2bn} & \text{otherwise} \end{cases}$$

input
```
integrate(sech(b*x+a)**n*tanh(b*x+a)**3,x)
```

output
```
Piecewise((x*tanh(a)**3*sech(a)**n, Eq(b, 0)), (Integral(tanh(a + b*x)**3/
sech(a + b*x)**2, x), Eq(n, -2)), (x - log(tanh(a + b*x) + 1)/b - tanh(a +
 b*x)**2/(2*b), Eq(n, 0)), (-n*tanh(a + b*x)**2*sech(a + b*x)**n/(b*n**2 +
 2*b*n) - 2*sech(a + b*x)**n/(b*n**2 + 2*b*n), True))
```

3.89.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(36) = 72$.

Time = 0.34 (sec) , antiderivative size = 345, normalized size of antiderivative = 9.58

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx$$
$$= -\frac{2^n ne^{\left(-(bx+a)n-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{\left(n^2+2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n\right)b} + \frac{\left(2^{n+1}n-2^{n+2}\right)e^{\left(-(bx+a)n-2\,bx-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-2\,a\right)}}{\left(n^2+2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n\right)b} - \frac{\left(2^n n+2^{n+1}\right)e^{\left(-(bx+a)n-4\,bx-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-4\,a\right)}}{\left(n^2+2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n\right)b} - \frac{2^{n+1}e^{\left(-(bx+a)n-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{\left(n^2+2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n\right)b}$$

input
```
integrate(sech(b*x+a)^n*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2^n*n*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/((n^2 + 2*(n^2 + 2*n
)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b) + (2^(n + 1)*n
 - 2^(n + 2))*e^(-(b*x + a)*n - 2*b*x - n*log(e^(-2*b*x - 2*a) + 1) - 2*a)
/((n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2
*n)*b) - (2^n*n + 2^(n + 1))*e^(-(b*x + a)*n - 4*b*x - n*log(e^(-2*b*x - 2
*a) + 1) - 4*a)/((n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4
*b*x - 4*a) + 2*n)*b) - 2^(n + 1)*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a)
 + 1))/((n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*
a) + 2*n)*b)
```

3.89.8 Giac [F]

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx = \int \operatorname{sech}(bx+a)^n \tanh(bx+a)^3\,dx$$

input
```
integrate(sech(b*x+a)^n*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^n*tanh(b*x + a)^3, x)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.81

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(\frac{1}{\frac{e^{a+b\,x}}{2}+\frac{e^{-a-b\,x}}{2}}\right)^n\left(\frac{1}{b\,n}+\frac{e^{4\,a+4\,b\,x}}{b\,n}-\frac{e^{2\,a+2\,b\,x}\,(2\,n-4)}{b\,n\,(n+2)}\right)}{2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1}$$

input
```
int(tanh(a + b*x)^3*(1/cosh(a + b*x))^n,x)
```

output
```
-((1/(exp(a + b*x)/2 + exp(- a - b*x)/2))^n*(1/(b*n) + exp(4*a + 4*b*x)/(b
*n) - (exp(2*a + 2*b*x)*(2*n - 4))/(b*n*(n + 2))))/(2*exp(2*a + 2*b*x) + e
xp(4*a + 4*b*x) + 1)
```

3.90 $\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx$

3.90.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx=\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh^5(a+bx)}{5b}$$

output
```
1/3*tanh(b*x+a)^3/b-1/5*tanh(b*x+a)^5/b
```

3.90.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx=\frac{2\tanh(a+bx)}{15b}+\frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{15b}-\frac{\operatorname{sech}^4(a+bx)\tanh(a+bx)}{5b}$$

input
```
Integrate[Sech[a + b*x]^4*Tanh[a + b*x]^2,x]
```

output
```
(2*Tanh[a + b*x])/(15*b) + (Sech[a + b*x]^2*Tanh[a + b*x])/(15*b) - (Sech[
a + b*x]^4*Tanh[a + b*x])/(5*b)
```

3.90.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 25, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(a+bx)\text{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ia+ibx)^2\left(-\sec(ia+ibx)^4\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ia+ibx)^4\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{i\int -\tanh^2(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{i\int \left(\tanh^4(a+bx)-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(\frac{1}{5}i\tanh^5(a+bx)-\frac{1}{3}i\tanh^3(a+bx)\right)}{b}$$

input
```
Int[Sech[a + b*x]^4*Tanh[a + b*x]^2,x]
```

output
```
(I*((-1/3*I)*Tanh[a + b*x]^3 + (I/5)*Tanh[a + b*x]^5))/b
```

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.90.4 Maple [A] (verified)

Time = 5.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{\tanh(bx+a)^5}{5}+\frac{\tanh(bx+a)^3}{3}}{b}$	26
default	$\frac{-\frac{\tanh(bx+a)^5}{5}+\frac{\tanh(bx+a)^3}{3}}{b}$	26
risch	$-\frac{4\left(15\,\mathrm{e}^{6bx+6a}-5\,\mathrm{e}^{4bx+4a}+5\,\mathrm{e}^{2bx+2a}+1\right)}{15b\left(1+\mathrm{e}^{2bx+2a}\right)^5}$	54

input
```
int(sech(b*x+a)^4*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/5*tanh(b*x+a)^5+1/3*tanh(b*x+a)^3)
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(27) = 54$.

Time = 0.24 (sec) , antiderivative size = 304, normalized size of antiderivative = 9.81

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^{7}+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{6}+b\sinh\left(bx+a\right)^{7}+5\,b\cosh\left(bx+a\right)^{5}+\left(21\,b\cos\right.\right.}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-8/15*(8*cosh(b*x + a)^3 + 24*cosh(b*x + a)*sinh(b*x + a)^2 + 7*sinh(b*x +
 a)^3 + (21*cosh(b*x + a)^2 - 5)*sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*c
osh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 5*b*cosh(b*x + a)^5 + (
21*b*cosh(b*x + a)^2 + 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 5*b
*cosh(b*x + a))*sinh(b*x + a)^4 + 11*b*cosh(b*x + a)^3 + (35*b*cosh(b*x +
a)^4 + 50*b*cosh(b*x + a)^2 + 9*b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^5
 + 50*b*cosh(b*x + a)^3 + 33*b*cosh(b*x + a))*sinh(b*x + a)^2 + 15*b*cosh(
b*x + a) + (7*b*cosh(b*x + a)^6 + 25*b*cosh(b*x + a)^4 + 27*b*cosh(b*x + a
)^2 + 5*b)*sinh(b*x + a))
```

3.90.6 Sympy [F]

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx = \int \tanh^2\left(a+bx\right)\operatorname{sech}^4\left(a+bx\right)dx$$

input
```
integrate(sech(b*x+a)**4*tanh(b*x+a)**2,x)
```

output
```
Integral(tanh(a + b*x)**2*sech(a + b*x)**4, x)
```

3.90.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(27) = 54$.

Time = 0.20 (sec) , antiderivative size = 276, normalized size of antiderivative = 8.90

$$\begin{aligned}\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx &= \frac{4\,e^{(-2\,bx-2\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)} \\ &\quad - \frac{4\,e^{(-4\,bx-4\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)} \\ &\quad + \frac{4\,e^{(-6\,bx-6\,a)}}{b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)} \\ &\quad + \frac{4}{15\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}\end{aligned}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
4/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(
-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) - 4/3*e^(-4*
b*x - 4*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6
*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 4*e^(-6*b*x - 6*a)/(
b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-
8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 4/15/(b*(5*e^(-2*b*x - 2*a) + 10
*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x
- 10*a) + 1))
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx = -\frac{4\left(15\,e^{(6\,bx+6\,a)}-5\,e^{(4\,bx+4\,a)}+5\,e^{(2\,bx+2\,a)}+1\right)}{15\,b(e^{(2\,bx+2\,a)}+1)^5}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
-4/15*(15*e^(6*b*x + 6*a) - 5*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) + 1)/(b*
(e^(2*b*x + 2*a) + 1)^5)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 270, normalized size of antiderivative = 8.71

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx$$
$$= \frac{\frac{8}{15\,b}-\frac{4\,e^{2\,a+2\,b\,x}}{5\,b}}{3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1} - \frac{\frac{2}{5\,b}-\frac{8\,e^{2\,a+2\,b\,x}}{5\,b}+\frac{6\,e^{4\,a+4\,b\,x}}{5\,b}}{4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1} - \frac{\frac{8\,e^{2\,a+2\,b\,x}}{5\,b}-\frac{16\,e^{4\,a+4\,b\,x}}{5\,b}+\frac{8\,e^{6\,a+6\,b\,x}}{5\,b}}{5\,e^{2\,a+2\,b\,x}+10\,e^{4\,a+4\,b\,x}+10\,e^{6\,a+6\,b\,x}+5\,e^{8\,a+8\,b\,x}+e^{10\,a+10\,b\,x}+1} - \frac{2}{5\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}$$

input
```
int(tanh(a + b*x)^2/cosh(a + b*x)^4,x)
```

output
```
(8/(15*b) - (4*exp(2*a + 2*b*x))/(5*b))/(3*exp(2*a + 2*b*x) + 3*exp(4*a +
4*b*x) + exp(6*a + 6*b*x) + 1) - (2/(5*b) - (8*exp(2*a + 2*b*x))/(5*b) + (
6*exp(4*a + 4*b*x))/(5*b))/(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*ex
p(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1) - ((8*exp(2*a + 2*b*x))/(5*b) - (16
*exp(4*a + 4*b*x))/(5*b) + (8*exp(6*a + 6*b*x))/(5*b))/(5*exp(2*a + 2*b*x)
 + 10*exp(4*a + 4*b*x) + 10*exp(6*a + 6*b*x) + 5*exp(8*a + 8*b*x) + exp(10
*a + 10*b*x) + 1) - 2/(5*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
```

3.91 $\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx$

3.91.1 Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx=\frac{2\tanh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\tanh^{\frac{7}{2}}(a+bx)}{7b}$$

output
```
2/3*tanh(b*x+a)^(3/2)/b-2/7*tanh(b*x+a)^(7/2)/b
```

3.91.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx=\frac{2\left(4+3\operatorname{sech}^2(a+bx)\right)\tanh^{\frac{3}{2}}(a+bx)}{21b}$$

input
```
Integrate[Sech[a + b*x]^4*Sqrt[Tanh[a + b*x]],x]
```

output
```
(2*(4 + 3*Sech[a + b*x]^2)*Tanh[a + b*x]^(3/2))/(21*b)
```

3.91.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tanh(a+bx)}\operatorname{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-i\tan(ia+ibx)}\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{3087}$$

$$-\frac{i\int \sqrt{\tanh(a+bx)}\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{i\int \left(\sqrt{\tanh(a+bx)}-\tanh^{\frac{5}{2}}(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{2}{3}i\tanh^{\frac{3}{2}}(a+bx)-\frac{2}{7}i\tanh^{\frac{7}{2}}(a+bx)\right)}{b}$$

input
```
Int[Sech[a + b*x]^4*Sqrt[Tanh[a + b*x]],x]
```

output
```
((-I)*(((2*I)/3)*Tanh[a + b*x]^(3/2) - ((2*I)/7)*Tanh[a + b*x]^(7/2)))/b
```

3.91.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.91.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{-\frac{2\tanh(bx+a)^{\frac{7}{2}}}{7}+\frac{2\tanh(bx+a)^{\frac{3}{2}}}{3}}{b}$	26
default	$\frac{-\frac{2\tanh(bx+a)^{\frac{7}{2}}}{7}+\frac{2\tanh(bx+a)^{\frac{3}{2}}}{3}}{b}$	26

input
```
int(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-2/7*tanh(b*x+a)^(7/2)+2/3*tanh(b*x+a)^(3/2))
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 551, normalized size of antiderivative = 15.74

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx$$
$$= \frac{8\left(\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 3\left(5\cosh(bx+a)^2+1\right)\sinh(bx+a\right.}{}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
8/21*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
+ 3*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cos
h(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x +
a)^5 + 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + (cosh(b*x + a)^6
 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2
 + 4)*sinh(b*x + a)^4 + 4*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 4*cosh(
b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 24*cosh(b*x + a)^2 - 4)*
sinh(b*x + a)^2 - 4*cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 8*cosh(b*x +
a)^3 - 4*cosh(b*x + a))*sinh(b*x + a) - 1)*sqrt(sinh(b*x + a)/cosh(b*x + a
)) + 1)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*
x + a)^6 + 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)
^4 + 4*(5*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cos
h(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x
+ a)^2 + 6*(b*cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sin
h(b*x + a) + b)
```

3.91.6 Sympy [F]

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = \int \sqrt{\tanh(a+bx)}\operatorname{sech}^4(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**4*tanh(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(tanh(a + b*x))*sech(a + b*x)**4, x)
```

3.91.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(27) = 54$.

Time = 0.31 (sec) , antiderivative size = 352, normalized size of antiderivative = 10.06

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx$$
$$= \frac{32\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}e^{(-2\,bx-2\,a)}}{21\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)\sqrt{e^{(-2\,bx-2\,a)}+1}}$$
$$- \frac{32\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}e^{(-4\,bx-4\,a)}}{21\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)\sqrt{e^{(-2\,bx-2\,a)}+1}}$$
$$- \frac{8\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}e^{(-6\,bx-6\,a)}}{21\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)\sqrt{e^{(-2\,bx-2\,a)}+1}}$$
$$+ \frac{8\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}}{21\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)\sqrt{e^{(-2\,bx-2\,a)}+1}}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
32/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b*x - a) + 1)*e^(-2*b*x - 2*a)/(b*(
3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2
*b*x - 2*a) + 1)) - 32/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b*x - a) + 1)*e
^(-4*b*x - 4*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x -
6*a) + 1)*sqrt(e^(-2*b*x - 2*a) + 1)) - 8/21*sqrt(e^(-b*x - a) + 1)*sqrt(-
e^(-b*x - a) + 1)*e^(-6*b*x - 6*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x -
4*a) + e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b*x - 2*a) + 1)) + 8/21*sqrt(e^(-b
*x - a) + 1)*sqrt(-e^(-b*x - a) + 1)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x
- 4*a) + e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b*x - 2*a) + 1))
```

3.91.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(27) = 54$.

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.23

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx$$
$$= \frac{16\left(21\left(\sqrt{e^{(4\,bx+4\,a)}-1}-e^{(2\,bx+2\,a)}\right)^5-7\left(\sqrt{e^{(4\,bx+4\,a)}-1}-e^{(2\,bx+2\,a)}\right)^4+28\left(\sqrt{e^{(4\,bx+4\,a)}-1}-e^{(2\,bx+2\,a}\right.\right.}{21\,b\left(\sqrt{e^{(4\,bx+4\,a)}-1}-e^{(2\,bx+2\,a)}-1\right)^7}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
16/21*(21*(sqrt(e^(4*b*x + 4*a) - 1) - e^(2*b*x + 2*a))^5 - 7*(sqrt(e^(4*b
*x + 4*a) - 1) - e^(2*b*x + 2*a))^4 + 28*(sqrt(e^(4*b*x + 4*a) - 1) - e^(2
*b*x + 2*a))^3 + 7*sqrt(e^(4*b*x + 4*a) - 1) - 7*e^(2*b*x + 2*a) - 1)/(b*(
sqrt(e^(4*b*x + 4*a) - 1) - e^(2*b*x + 2*a) - 1)^7)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 4.80

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = \frac{8\sqrt{\frac{e^{2a+2bx}-1}{e^{2a+2bx}+1}}}{21b} + \frac{8\sqrt{\frac{e^{2a+2bx}-1}{e^{2a+2bx}+1}}}{21b\left(e^{2a+2bx}+1\right)} - \frac{24\sqrt{\frac{e^{2a+2bx}-1}{e^{2a+2bx}+1}}}{7b\left(e^{2a+2bx}+1\right)^2} + \frac{16\sqrt{\frac{e^{2a+2bx}-1}{e^{2a+2bx}+1}}}{7b\left(e^{2a+2bx}+1\right)^3}$$

input
```
int(tanh(a + b*x)^(1/2)/cosh(a + b*x)^4,x)
```

output
```
(8*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^(1/2))/(21*b) + (8*((ex
p(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^(1/2))/(21*b*(exp(2*a + 2*b*x)
 + 1)) - (24*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^(1/2))/(7*b*(
exp(2*a + 2*b*x) + 1)^2) + (16*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) +
 1))^(1/2))/(7*b*(exp(2*a + 2*b*x) + 1)^3)
```

3.92 $\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx$

3.92.1 Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx=\frac{\tanh^{1+n}(a+bx)}{b(1+n)}-\frac{\tanh^{3+n}(a+bx)}{b(3+n)}$$

output
```
tanh(b*x+a)^(1+n)/b/(1+n)-tanh(b*x+a)^(3+n)/b/(3+n)
```

3.92.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.82

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx =\frac{\tanh^{-1+n}(a+bx)\left((2+n+\cosh(2(a+bx)))\operatorname{sech}^2(a+bx)\tanh^2(a+bx)-2\tanh^2(a+bx)^{\frac{1-n}{2}}\right)}{b(1+n)(3+n)}$$

input
```
Integrate[Sech[a + b*x]^4*Tanh[a + b*x]^n,x]
```

output
```
(Tanh[a + b*x]^(-1 + n)*((2 + n + Cosh[2*(a + b*x)])*Sech[a + b*x]^2*Tanh[
a + b*x]^2 - 2*(Tanh[a + b*x]^2)^((1 - n)/2)))/(b*(1 + n)*(3 + n))
```

3.92.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(ia+ibx)^4(-i\tan(ia+ibx))^n dx$$

$$\Big\downarrow \text{3087}$$

$$-\frac{i\int \tanh^n(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{i\int \left(\tanh^n(a+bx)-\tanh^{n+2}(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{i\tanh^{n+1}(a+bx)}{n+1}-\frac{i\tanh^{n+3}(a+bx)}{n+3}\right)}{b}$$

input
```
Int[Sech[a + b*x]^4*Tanh[a + b*x]^n,x]
```

output
```
((-I)*((I*Tanh[a + b*x]^(1 + n))/(1 + n) - (I*Tanh[a + b*x]^(3 + n))/(3 +
n)))/b
```

3.92.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.92.4 Maple [A] (verified)

Time = 117.68 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{\tanh(bx+a)\,\mathrm{e}^{n\ln(\tanh(bx+a))}}{b(n+1)}-\frac{\tanh(bx+a)^{3}\,\mathrm{e}^{n\ln(\tanh(bx+a))}}{b(3+n)}$
default	$\frac{\tanh(bx+a)\,\mathrm{e}^{n\ln(\tanh(bx+a))}}{b(n+1)}-\frac{\tanh(bx+a)^{3}\,\mathrm{e}^{n\ln(\tanh(bx+a))}}{b(3+n)}$
risch	$2\left(\mathrm{e}^{6bx+6a}+2n\,\mathrm{e}^{4bx+4a}+3\,\mathrm{e}^{4bx+4a}-2n\,\mathrm{e}^{2bx+2a}-3\,\mathrm{e}^{2bx+2a}-1\right)\left(\mathrm{e}^{bx+a}-1\right)^{n}\left(\mathrm{e}^{bx+a}+1\right)^{n}\left(1+\mathrm{e}^{2bx+2a}\right)^{-n}\mathrm{e}^{-\frac{i\pi n\left(-\mathrm{csgn}\left(\right.\right.}{}}$

input
```
int(sech(b*x+a)^4*tanh(b*x+a)^n,x,method=_RETURNVERBOSE)
```

output
```
1/b/(n+1)*tanh(b*x+a)*exp(n*ln(tanh(b*x+a)))-1/b/(3+n)*tanh(b*x+a)^3*exp(n
*ln(tanh(b*x+a)))
```

3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 4.50

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx$$
$$= \frac{2\left(\left(\sinh(bx+a)^3 + \left(3\cosh(bx+a)^2 + 2n + 3\right)\sinh(bx+a)\right)\cosh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right) + \left(\sinh(bx+a\right.}{(bn^2+4bn+3b)\cosh(bx+a)^3 + 3(bn^2+4bn+3b)\cosh(bx+a)\sinh(b}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="fricas")
```

output
```
2*((sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 2*n + 3)*sinh(b*x + a))*cosh(n*
log(sinh(b*x + a)/cosh(b*x + a))) + (sinh(b*x + a)^3 + (3*cosh(b*x + a)^2
+ 2*n + 3)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a)/cosh(b*x + a))))/((b*n^
2 + 4*b*n + 3*b)*cosh(b*x + a)^3 + 3*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a)*s
inh(b*x + a)^2 + 3*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a))
```

3.92.6 Sympy [F]

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx = \int \tanh^n(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**4*tanh(b*x+a)**n,x)
```

output
```
Integral(tanh(a + b*x)**n*sech(a + b*x)**4, x)
```

3.92.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(40) = 80$.

Time = 0.30 (sec) , antiderivative size = 504, normalized size of antiderivative = 12.60

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx = \frac{2\,(2\,n+3)e^{\left(-2\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-2\,a\right)}}{\left(n^2+3\,(n^2+4\,n+3)e^{(-2\,bx-2\,a)}+3\,(n^2+4\,n+3)e^{(-4\,bx-4\,a)}+(n^2+4\,n+3)e^{(-6\,bx-6\,a)}+4\,n+3\right)b} - \frac{2\,(2\,n+3)e^{\left(-4\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-4\,a\right)}}{\left(n^2+3\,(n^2+4\,n+3)e^{(-2\,bx-2\,a)}+3\,(n^2+4\,n+3)e^{(-4\,bx-4\,a)}+(n^2+4\,n+3)e^{(-6\,bx-6\,a)}+4\,n+3\right)b} - \frac{2\,e^{\left(-6\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-6\,a\right)}}{\left(n^2+3\,(n^2+4\,n+3)e^{(-2\,bx-2\,a)}+3\,(n^2+4\,n+3)e^{(-4\,bx-4\,a)}+(n^2+4\,n+3)e^{(-6\,bx-6\,a)}+4\,n+3\right)b} + \frac{2\,e^{\left(n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{\left(n^2+3\,(n^2+4\,n+3)e^{(-2\,bx-2\,a)}+3\,(n^2+4\,n+3)e^{(-4\,bx-4\,a)}+(n^2+4\,n+3)e^{(-6\,bx-6\,a)}+4\,n+3\right)b}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="maxima")
```

output
```
2*(2*n + 3)*e^(-2*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1)
 - n*log(e^(-2*b*x - 2*a) + 1) - 2*a)/((n^2 + 3*(n^2 + 4*n + 3)*e^(-2*b*x
- 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 + 4*n + 3)*e^(-6*b*x -
6*a) + 4*n + 3)*b) - 2*(2*n + 3)*e^(-4*b*x + n*log(e^(-b*x - a) + 1) + n*l
og(-e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1) - 4*a)/((n^2 + 3*(n^2
+ 4*n + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 +
4*n + 3)*e^(-6*b*x - 6*a) + 4*n + 3)*b) - 2*e^(-6*b*x + n*log(e^(-b*x - a)
 + 1) + n*log(-e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1) - 6*a)/((n^
2 + 3*(n^2 + 4*n + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a
) + (n^2 + 4*n + 3)*e^(-6*b*x - 6*a) + 4*n + 3)*b) + 2*e^(n*log(e^(-b*x -
a) + 1) + n*log(-e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1))/((n^2 +
3*(n^2 + 4*n + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) +
(n^2 + 4*n + 3)*e^(-6*b*x - 6*a) + 4*n + 3)*b)
```

3.92.8 Giac [F]

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx = \int \tanh\left(bx+a\right)^n \operatorname{sech}\left(bx+a\right)^4\,dx$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(tanh(b*x + a)^n*sech(b*x + a)^4, x)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.88

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx$$
$$= \frac{e^{-3a-3bx}\left(\frac{4e^{3a+3bx}\sinh(3a+3bx)}{b(n^2+4n+3)}+\frac{2e^{3a+3bx}\sinh(a+bx)(4n+6)}{b(n^2+4n+3)}\right)\left(\frac{e^{2a+2bx}-1}{e^{2a+2bx}+1}\right)^n}{8\cosh(a+bx)^3}$$

input
```
int(tanh(a + b*x)^n/cosh(a + b*x)^4,x)
```

output
```
(exp(- 3*a - 3*b*x)*((4*exp(3*a + 3*b*x)*sinh(3*a + 3*b*x))/(b*(4*n + n^2
+ 3)) + (2*exp(3*a + 3*b*x)*sinh(a + b*x)*(4*n + 6))/(b*(4*n + n^2 + 3)))*
((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^n)/(8*cosh(a + b*x)^3)
```

3.93 $\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

3.93.1 Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}-\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

output
```
1/2*arctan(sinh(b*x+a))/b-1/2*sech(b*x+a)*tanh(b*x+a)/b
```

3.93.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}-\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

input
```
Integrate[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

3.93.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ia+ibx)^2(-\sec(ia+ibx))dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{2}\int \text{sech}(a+bx)dx-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}+\frac{1}{2}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\arctan(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

input
```
Int[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.93.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{-\frac{\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(e^{bx+a}\right)}{b}$	43
default	$\frac{-\frac{\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(e^{bx+a}\right)}{b}$	43
risch	$-\frac{e^{bx+a}\left(e^{2bx+2a}-1\right)}{b\left(1+e^{2bx+2a}\right)^2}+\frac{i\ln\left(e^{bx+a}+i\right)}{2b}-\frac{i\ln\left(e^{bx+a}-i\right)}{2b}$	69

input
```
int(sech(b*x+a)*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/cosh(b*x+a)^2*sinh(b*x+a)+1/2*sech(b*x+a)*tanh(b*x+a)+arctan(exp(b
*x+a)))
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 269, normalized size of antiderivative = 7.91

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = -\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 - (\cosh(bx+a)^4 + 4\cosh(bx+a)\sin}{b\cosh(bx+a)^4 + 4b\cosh}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (c
osh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*
cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a
)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x +
a)) + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x +
 a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b
*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

3.93.6 Sympy [F]

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int \tanh^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)**2,x)
```

output
```
Integral(tanh(a + b*x)**2*sech(a + b*x), x)
```

3.93.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = -\frac{\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1)}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-arctan(e^(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*
x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

3.93.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\pi - \frac{4\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4} + 2\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{4\,b}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*(pi - 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
 4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^a\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} + \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x} + e^{4\,a+4\,b\,x} + 1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x} + 1\right)}$$

input
```
int(tanh(a + b*x)^2/cosh(a + b*x),x)
```

output
```
atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) + (2*exp(a + b*x))/(b*(2
*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*
b*x) + 1))
```

3.93. $\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

3.94 $\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx$

3.94.1 Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx=\frac{3\arctan(\sinh(a+bx))}{8b}-\frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{4b}$$

output
```
3/8*arctan(sinh(b*x+a))/b-3/8*sech(b*x+a)*tanh(b*x+a)/b-1/4*sech(b*x+a)*ta
nh(b*x+a)^3/b
```

3.94.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx=\frac{3\arctan(\sinh(a+bx))}{8b}+\frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{3\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}-\frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{b}$$

input
```
Integrate[Sech[a + b*x]*Tanh[a + b*x]^4,x]
```

output
```
(3*ArcTan[Sinh[a + b*x]])/(8*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) -
(3*Sech[a + b*x]^3*Tanh[a + b*x])/(4*b) - (Sech[a + b*x]*Tanh[a + b*x]^3)/
b
```

3.94.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 3091, 25, 3042, 25, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ia+ibx)^4\sec(ia+ibx)dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{3}{4}\int -\text{sech}(a+bx)\tanh^2(a+bx)dx-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{4}\int \text{sech}(a+bx)\tanh^2(a+bx)dx-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}+\frac{3}{4}\int -\sec(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}-\frac{3}{4}\int \sec(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{3}{4}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}-\frac{1}{2}\int \text{sech}(a+bx)dx\right)-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}-\frac{3}{4}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}-\frac{1}{2}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{4257}$$

$$-\frac{3}{4}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}-\frac{\arctan(\sinh(a+bx))}{2b}\right)-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}$$

input
```
Int[Sech[a + b*x]*Tanh[a + b*x]^4,x]
```

output
```
-1/4*(Sech[a + b*x]*Tanh[a + b*x]^3)/b - (3*(-1/2*ArcTan[Sinh[a + b*x]]/b
+ (Sech[a + b*x]*Tanh[a + b*x])/(2*b)))/4
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.94.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

method	result	size
derivativedivides	$\frac{-\frac{\sinh(bx+a)^3}{\cosh(bx+a)^4}-\frac{\sinh(bx+a)}{\cosh(bx+a)^4}+\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)+\frac{3\arctan\left(e^{bx+a}\right)}{4}}{b}$	75
default	$\frac{-\frac{\sinh(bx+a)^3}{\cosh(bx+a)^4}-\frac{\sinh(bx+a)}{\cosh(bx+a)^4}+\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)+\frac{3\arctan\left(e^{bx+a}\right)}{4}}{b}$	75
risch	$-\frac{e^{bx+a}\left(5\,e^{6bx+6a}-3\,e^{4bx+4a}+3\,e^{2bx+2a}-5\right)}{4b\left(1+e^{2bx+2a}\right)^4}+\frac{3i\ln\left(e^{bx+a}+i\right)}{8b}-\frac{3i\ln\left(e^{bx+a}-i\right)}{8b}$	93

input
```
int(sech(b*x+a)*tanh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-sinh(b*x+a)^3/cosh(b*x+a)^4-1/cosh(b*x+a)^4*sinh(b*x+a)+(1/4*sech(b*
x+a)^3+3/8*sech(b*x+a))*tanh(b*x+a)+3/4*arctan(exp(b*x+a)))
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(49) = 98$.

Time = 0.25 (sec) , antiderivative size = 814, normalized size of antiderivative = 14.80

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/4*(5*cosh(b*x + a)^7 + 35*cosh(b*x + a)*sinh(b*x + a)^6 + 5*sinh(b*x +
a)^7 + 3*(35*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^5 - 3*cosh(b*x + a)^5 + 5*
(35*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^4 + (175*cosh(b*x + a
)^4 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^3 + 3*cosh(b*x + a)^3 + 3*(35*
cosh(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 -
3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4
*(7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b
*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*
cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x +
 a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(
b*x + a)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 +
 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x +
 a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x
+ a)) + (35*cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 5)*
sinh(b*x + a) - 5*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*si
nh(b*x + a)^7 + b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x
+ a)^2 + b)*sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*
sinh(b*x + a)^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cos
h(b*x + a)^2 + 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b
*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + ...
```

3.94.6 Sympy [F]

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = \int \tanh^4(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)**4,x)
```

output
```
Integral(tanh(a + b*x)**4*sech(a + b*x), x)
```

3.94.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx$$
$$= -\frac{3\arctan\left(e^{(-bx-a)}\right)}{4b} - \frac{5e^{(-bx-a)} - 3e^{(-3bx-3a)} + 3e^{(-5bx-5a)} - 5e^{(-7bx-7a)}}{4b(4e^{(-2bx-2a)} + 6e^{(-4bx-4a)} + 4e^{(-6bx-6a)} + e^{(-8bx-8a)} + 1)}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="maxima")
```

output
```
-3/4*arctan(e^(-b*x - a))/b - 1/4*(5*e^(-b*x - a) - 3*e^(-3*b*x - 3*a) + 3
*e^(-5*b*x - 5*a) - 5*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b
*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(49) = 98$.

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.85

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx$$
$$= \frac{3\pi - \frac{4\left(5\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+12e^{(bx+a)}-12e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + 6\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{16b}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="giac")
```

output
```
1/16*(3*pi - 4*(5*(e^(b*x + a) - e^(-b*x - a))^3 + 12*e^(b*x + a) - 12*e^(
-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 6*arctan(1/2*(e^(2*b*x
 + 2*a) - 1)*e^(-b*x - a)))/b
```

3.94.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.38

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}}+\frac{9\,e^{a+b\,x}}{2\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}-\frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)}+\frac{4\,e^{a+b\,x}}{b\,\left(4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}-\frac{5\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(tanh(a + b*x)^4/cosh(a + b*x),x)
```

output
```
(3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) + (9*exp(a + b*x
))/(2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (6*exp(a + b*x))/(b
*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + (4*ex
p(a + b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*
x) + exp(8*a + 8*b*x) + 1)) - (5*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1)
)
```

3.95 $\int \text{sech}^3(a+bx)\tanh^2(a+bx)\,dx$

3.95.1 Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \text{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\arctan(\sinh(a+bx))}{8b} + \frac{\text{sech}(a+bx)\tanh(a+bx)}{8b} - \frac{\text{sech}^3(a+bx)\tanh(a+bx)}{4b}$$

output
```
1/8*arctan(sinh(b*x+a))/b+1/8*sech(b*x+a)*tanh(b*x+a)/b-1/4*sech(b*x+a)^3*
tanh(b*x+a)/b
```

3.95.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \text{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\arctan(\sinh(a+bx))}{8b} + \frac{\text{sech}(a+bx)\tanh(a+bx)}{8b} - \frac{\text{sech}^3(a+bx)\tanh(a+bx)}{4b}$$

input
```
Integrate[Sech[a + b*x]^3*Tanh[a + b*x]^2,x]
```

output
```
ArcTan[Sinh[a + b*x]]/(8*b) + (Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (Sech[
a + b*x]^3*Tanh[a + b*x])/(4*b)
```

3.95.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 25, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ia+ibx)^2\left(-\sec(ia+ibx)^3\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ia+ibx)^3\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{4}\int \text{sech}^3(a+bx)dx-\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}+\frac{1}{4}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \text{sech}(a+bx)dx+\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}\right)-\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}+\frac{1}{4}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}+\frac{1}{2}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(\frac{\arctan(\sinh(a+bx))}{2b}+\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}\right)-\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}$$

input
```
Int[Sech[a + b*x]^3*Tanh[a + b*x]^2,x]
```

output
```
-1/4*(Sech[a + b*x]^3*Tanh[a + b*x])/b + (ArcTan[Sinh[a + b*x]]/(2*b) + (S
ech[a + b*x]*Tanh[a + b*x])/(2*b))/4
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.95.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{-\frac{\sinh(bx+a)}{3\cosh(bx+a)^4}+\frac{\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)}{3}+\frac{\arctan\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$	58
default	$\frac{-\frac{\sinh(bx+a)}{3\cosh(bx+a)^4}+\frac{\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)}{3}+\frac{\arctan\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$	58
risch	$\frac{\mathrm{e}^{bx+a}\left(\mathrm{e}^{6bx+6a}-7\,\mathrm{e}^{4bx+4a}+7\,\mathrm{e}^{2bx+2a}-1\right)}{4b\left(1+\mathrm{e}^{2bx+2a}\right)^4}+\frac{i\ln\left(\mathrm{e}^{bx+a}+i\right)}{8b}-\frac{i\ln\left(\mathrm{e}^{bx+a}-i\right)}{8b}$	91

input
```
int(sech(b*x+a)^3*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/cosh(b*x+a)^4*sinh(b*x+a)+1/3*(1/4*sech(b*x+a)^3+3/8*sech(b*x+a)
)*tanh(b*x+a)+1/4*arctan(exp(b*x+a)))
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(49) = 98$.

Time = 0.28 (sec) , antiderivative size = 808, normalized size of antiderivative = 14.69

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(cosh(b*x + a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 + sinh(b*x + a)^7 +
 7*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^5 - 7*cosh(b*x + a)^5 + 35*(cosh(
b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^4 + 7*(5*cosh(b*x + a)^4 - 10*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 7*cosh(b*x + a)^3 + 7*(3*cosh(b*x + a
)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 + (cosh(b*x +
a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x +
 a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 3*
cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x + a)^
2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 10*cos
h(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 + 1
5*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x +
a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh(b
*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (7*cos
h(b*x + a)^6 - 35*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
- cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 +
b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh
(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5
+ 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3
*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*
cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x ...
```

3.95.6 Sympy [F]

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \int \tanh^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**3*tanh(b*x+a)**2,x)
```

output
```
Integral(tanh(a + b*x)**2*sech(a + b*x)**3, x)
```

3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(49) = 98.

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = -\frac{\arctan\left(e^{(-bx-a)}\right)}{4\,b} + \frac{e^{(-bx-a)} - 7\,e^{(-3\,bx-3\,a)} + 7\,e^{(-5\,bx-5\,a)} - e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)} + 6\,e^{(-4\,bx-4\,a)} + 4\,e^{(-6\,bx-6\,a)} + e^{(-8\,bx-8\,a)} + 1)}$$

input
```
integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*arctan(e^(-b*x - a))/b + 1/4*(e^(-b*x - a) - 7*e^(-3*b*x - 3*a) + 7*e
^(-5*b*x - 5*a) - e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b*x -
 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.78

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\pi + \frac{4\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3 - 4\,e^{(bx+a)} + 4\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{16\,b}$$

input
```
integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/16*(pi + 4*((e^(b*x + a) - e^(-b*x - a))^3 - 4*e^(b*x + a) + 4*e^(-b*x -
 a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 2*arctan(1/2*(e^(2*b*x + 2*a
) - 1)*e^(-b*x - a)))/b
```

3.95.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.91

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}} - \frac{\frac{e^{a+b\,x}}{b}-\frac{2\,e^{3\,a+3\,b\,x}}{b}+\frac{e^{5\,a+5\,b\,x}}{b}}{4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1} - \frac{3\,e^{a+b\,x}}{2\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} + \frac{2\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} + \frac{e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(tanh(a + b*x)^2/cosh(a + b*x)^3,x)
```

output
```
atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(4*(b^2)^(1/2)) - (exp(a + b*x)/b -
(2*exp(3*a + 3*b*x))/b + exp(5*a + 5*b*x)/b)/(4*exp(2*a + 2*b*x) + 6*exp(4
*a + 4*b*x) + 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1) - (3*exp(a + b*x)
)/(2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (2*exp(a + b*x))/(b*
(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + exp(a
+ b*x)/(4*b*(exp(2*a + 2*b*x) + 1))
```

3.96 $\int \operatorname{sech}(x) \tanh^5(x) \, dx$

3.96.1 Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \operatorname{sech}(x) \tanh^5(x) \, dx = -\operatorname{sech}(x) + \frac{2\operatorname{sech}^3(x)}{3} - \frac{\operatorname{sech}^5(x)}{5}$$

output
```
-sech(x)+2/3*sech(x)^3-1/5*sech(x)^5
```

3.96.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(x) \tanh^5(x) \, dx = -\operatorname{sech}(x) + \frac{2\operatorname{sech}^3(x)}{3} - \frac{\operatorname{sech}^5(x)}{5}$$

input
```
Integrate[Sech[x]*Tanh[x]^5,x]
```

output
```
-Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5
```

3.96.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 26, 3086, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^5(x)\text{sech}(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)^5\sec(ix)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec(ix)\tan(ix)^5dx$$

$$\Big\downarrow \text{3086}$$

$$-\int \left(\text{sech}^2(x)-1\right)^2d\text{sech}(x)$$

$$\Big\downarrow \text{210}$$

$$-\int \left(\text{sech}^4(x)-2\text{sech}^2(x)+1\right)d\text{sech}(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\text{sech}^5(x)+\frac{2\text{sech}^3(x)}{3}-\text{sech}(x)$$

input
```
Int[Sech[x]*Tanh[x]^5,x]
```

output
```
-Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5
```

3.96.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.96.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\operatorname{sech}(x)+\frac{2\operatorname{sech}(x)^{3}}{3}-\frac{\operatorname{sech}(x)^{5}}{5}$	18
default	$-\operatorname{sech}(x)+\frac{2\operatorname{sech}(x)^{3}}{3}-\frac{\operatorname{sech}(x)^{5}}{5}$	18
risch	$-\frac{2\,e^{x}\left(15\,e^{8x}+20\,e^{6x}+58\,e^{4x}+20\,e^{2x}+15\right)}{15\left(1+e^{2x}\right)^{5}}$	39

input
```
int(sech(x)*tanh(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-sech(x)+2/3*sech(x)^3-1/5*sech(x)^5
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(17) = 34$.

Time = 0.24 (sec) , antiderivative size = 185, normalized size of antiderivative = 8.81

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx =$$
$$-\frac{2\left(15\cosh(x)^5+75\cosh(x)\sinh(x)^4+15\sinh(x)^5+5\left(30\cosh(x)^2+1\right)\sin\cdots\right.}{15\left(\cosh(x)^6+6\cosh(x)\sinh(x)^5+\sinh(x)^6+3\left(5\cosh(x)^2+2\right)\sinh(x)^4+6\cosh(x)^4+4\left(5\cos\cdots\right.\right.}$$

input
```
integrate(sech(x)*tanh(x)^5,x, algorithm="fricas")
```

output
```
-2/15*(15*cosh(x)^5 + 75*cosh(x)*sinh(x)^4 + 15*sinh(x)^5 + 5*(30*cosh(x)^
2 + 1)*sinh(x)^3 + 35*cosh(x)^3 + 15*(10*cosh(x)^3 + 7*cosh(x))*sinh(x)^2
+ (75*cosh(x)^4 + 15*cosh(x)^2 + 38)*sinh(x) + 78*cosh(x))/(cosh(x)^6 + 6*
cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 2)*sinh(x)^4 + 6*cosh(x)^
4 + 4*(5*cosh(x)^3 + 4*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 12*cosh(x)^2
+ 5)*sinh(x)^2 + 15*cosh(x)^2 + 2*(3*cosh(x)^5 + 8*cosh(x)^3 + 5*cosh(x))*
sinh(x) + 10)
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx = -\frac{\tanh^4(x)\operatorname{sech}(x)}{5} - \frac{4\tanh^2(x)\operatorname{sech}(x)}{15} - \frac{8\operatorname{sech}(x)}{15}$$

input
```
integrate(sech(x)*tanh(x)**5,x)
```

output
```
-tanh(x)**4*sech(x)/5 - 4*tanh(x)**2*sech(x)/15 - 8*sech(x)/15
```

3.96.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(17) = 34$.

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 9.10

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx = -\frac{2\,e^{(-x)}}{5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1} - \frac{8\,e^{(-3\,x)}}{3\,(5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1)} - \frac{116\,e^{(-5\,x)}}{15\,(5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1)} - \frac{8\,e^{(-7\,x)}}{3\,(5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1)} - \frac{2\,e^{(-9\,x)}}{5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1}$$

input
```
integrate(sech(x)*tanh(x)^5,x, algorithm="maxima")
```

output
```
-2*e^(-x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x)
 + 1) - 8/3*e^(-3*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x)
+ e^(-10*x) + 1) - 116/15*e^(-5*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x)
 + 5*e^(-8*x) + e^(-10*x) + 1) - 8/3*e^(-7*x)/(5*e^(-2*x) + 10*e^(-4*x) +
10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1) - 2*e^(-9*x)/(5*e^(-2*x) + 10*e^
(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1)
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx = -\frac{2\left(15\,(e^{(-x)} + e^{x})^4 - 40\,(e^{(-x)} + e^{x})^2 + 48\right)}{15\,(e^{(-x)} + e^{x})^5}$$

input
```
integrate(sech(x)*tanh(x)^5,x, algorithm="giac")
```

output
```
-2/15*(15*(e^(-x) + e^x)^4 - 40*(e^(-x) + e^x)^2 + 48)/(e^(-x) + e^x)^5
```

3.96.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 6.14

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx = \frac{64\,e^x}{5\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)} - \frac{2\,e^x}{e^{2\,x}+1} - \frac{176\,e^x}{15\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)} - \frac{32\,e^x}{5\,\left(5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+e^{10\,x}+1\right)} + \frac{16\,e^x}{3\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}$$

input
```
int(tanh(x)^5/cosh(x),x)
```

output
```
(64*exp(x))/(5*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) - (2
*exp(x))/(exp(2*x) + 1) - (176*exp(x))/(15*(3*exp(2*x) + 3*exp(4*x) + exp(
6*x) + 1)) - (32*exp(x))/(5*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*ex
p(8*x) + exp(10*x) + 1)) + (16*exp(x))/(3*(2*exp(2*x) + exp(4*x) + 1))
```

3.97 $\int \operatorname{sech}^7(x)\tanh^5(x)\,dx$

3.97.1 Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = -\frac{1}{7}\operatorname{sech}^7(x) + \frac{2\operatorname{sech}^9(x)}{9} - \frac{\operatorname{sech}^{11}(x)}{11}$$

output
```
-1/7*sech(x)^7+2/9*sech(x)^9-1/11*sech(x)^11
```

3.97.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = -\frac{1}{7}\operatorname{sech}^7(x) + \frac{2\operatorname{sech}^9(x)}{9} - \frac{\operatorname{sech}^{11}(x)}{11}$$

input
```
Integrate[Sech[x]^7*Tanh[x]^5,x]
```

output
```
-1/7*Sech[x]^7 + (2*Sech[x]^9)/9 - Sech[x]^11/11
```

3.97.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 26, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^5(x)\text{sech}^7(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)^5\sec(ix)^7dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec(ix)^7\tan(ix)^5dx$$

$$\Big\downarrow \text{3086}$$

$$-\int \text{sech}^6(x)\left(1-\text{sech}^2(x)\right)^2d\text{sech}(x)$$

$$\Big\downarrow \text{244}$$

$$-\int \left(\text{sech}^{10}(x)-2\text{sech}^8(x)+\text{sech}^6(x)\right)d\text{sech}(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{11}\text{sech}^{11}(x)+\frac{2\text{sech}^9(x)}{9}-\frac{\text{sech}^7(x)}{7}$$

input
```
Int[Sech[x]^7*Tanh[x]^5,x]
```

output
```
-1/7*Sech[x]^7 + (2*Sech[x]^9)/9 - Sech[x]^11/11
```

3.97.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.97.4 Maple [F(-1)]

Timed out.

hanged

input
```
int(sech(x)^7*tanh(x)^5,x)
```

output
```
int(sech(x)^7*tanh(x)^5,x)
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(19) = 38$.

Time = 0.25 (sec) , antiderivative size = 634, normalized size of antiderivative = 25.36

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^7*tanh(x)^5,x, algorithm="fricas")
```

output
```
-128/693*(99*cosh(x)^8 + 792*cosh(x)*sinh(x)^7 + 99*sinh(x)^8 + 44*(63*cos
h(x)^2 - 5)*sinh(x)^6 - 220*cosh(x)^6 + 264*(21*cosh(x)^3 - 5*cosh(x))*sin
h(x)^5 + 10*(693*cosh(x)^4 - 330*cosh(x)^2 + 37)*sinh(x)^4 + 370*cosh(x)^4
 + 8*(693*cosh(x)^5 - 550*cosh(x)^3 + 185*cosh(x))*sinh(x)^3 + 4*(693*cosh
(x)^6 - 825*cosh(x)^4 + 555*cosh(x)^2 - 55)*sinh(x)^2 - 220*cosh(x)^2 + 8*
(99*cosh(x)^7 - 165*cosh(x)^5 + 185*cosh(x)^3 - 55*cosh(x))*sinh(x) + 99)/
(cosh(x)^15 + 15*cosh(x)*sinh(x)^14 + sinh(x)^15 + (105*cosh(x)^2 + 11)*si
nh(x)^13 + 11*cosh(x)^13 + 13*(35*cosh(x)^3 + 11*cosh(x))*sinh(x)^12 + (13
65*cosh(x)^4 + 858*cosh(x)^2 + 55)*sinh(x)^11 + 55*cosh(x)^11 + 11*(273*co
sh(x)^5 + 286*cosh(x)^3 + 55*cosh(x))*sinh(x)^10 + 55*(91*cosh(x)^6 + 143*
cosh(x)^4 + 55*cosh(x)^2 + 3)*sinh(x)^9 + 165*cosh(x)^9 + 33*(195*cosh(x)^
7 + 429*cosh(x)^5 + 275*cosh(x)^3 + 45*cosh(x))*sinh(x)^8 + (6435*cosh(x)^
8 + 18876*cosh(x)^6 + 18150*cosh(x)^4 + 5940*cosh(x)^2 + 329)*sinh(x)^7 +
331*cosh(x)^7 + (5005*cosh(x)^9 + 18876*cosh(x)^7 + 25410*cosh(x)^5 + 1386
0*cosh(x)^3 + 2317*cosh(x))*sinh(x)^6 + (3003*cosh(x)^10 + 14157*cosh(x)^8
 + 25410*cosh(x)^6 + 20790*cosh(x)^4 + 6909*cosh(x)^2 + 451)*sinh(x)^5 + 4
73*cosh(x)^5 + 5*(273*cosh(x)^11 + 1573*cosh(x)^9 + 3630*cosh(x)^7 + 4158*
cosh(x)^5 + 2317*cosh(x)^3 + 473*cosh(x))*sinh(x)^4 + (455*cosh(x)^12 + 31
46*cosh(x)^10 + 9075*cosh(x)^8 + 13860*cosh(x)^6 + 11515*cosh(x)^4 + 4510*
cosh(x)^2 + 407)*sinh(x)^3 + 517*cosh(x)^3 + (105*cosh(x)^13 + 858*cosh...
```

3.97.6 Sympy [A] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = -\frac{\tanh^4(x)\operatorname{sech}^7(x)}{11} - \frac{4\tanh^2(x)\operatorname{sech}^7(x)}{99} - \frac{8\operatorname{sech}^7(x)}{693}$$

input
```
integrate(sech(x)**7*tanh(x)**5,x)
```

output
```
-tanh(x)**4*sech(x)**7/11 - 4*tanh(x)**2*sech(x)**7/99 - 8*sech(x)**7/693
```

3.97.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(19) = 38$.

Time = 0.20 (sec) , antiderivative size = 371, normalized size of antiderivative = 14.84

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx =$$

$$-\frac{128\,e^{(-7\,x)}}{7\,(11\,e^{(-2\,x)}+55\,e^{(-4\,x)}+165\,e^{(-6\,x)}+330\,e^{(-8\,x)}+462\,e^{(-10\,x)}+462\,e^{(-12\,x)}+330\,e^{(-14\,x)}+165\,e^{(-16\,x)}}$$
$$+\frac{2560\,e^{(-9\,x)}}{63\,(11\,e^{(-2\,x)}+55\,e^{(-4\,x)}+165\,e^{(-6\,x)}+330\,e^{(-8\,x)}+462\,e^{(-10\,x)}+462\,e^{(-12\,x)}+330\,e^{(-14\,x)}+165\,e^{(-16}}$$
$$-\frac{47360\,e^{(-11\,x)}}{693\,(11\,e^{(-2\,x)}+55\,e^{(-4\,x)}+165\,e^{(-6\,x)}+330\,e^{(-8\,x)}+462\,e^{(-10\,x)}+462\,e^{(-12\,x)}+330\,e^{(-14\,x)}+165\,e^{(-1}}$$
$$+\frac{2560\,e^{(-13\,x)}}{63\,(11\,e^{(-2\,x)}+55\,e^{(-4\,x)}+165\,e^{(-6\,x)}+330\,e^{(-8\,x)}+462\,e^{(-10\,x)}+462\,e^{(-12\,x)}+330\,e^{(-14\,x)}+165\,e^{(-16}}$$
$$-\frac{128\,e^{(-15\,x)}}{7\,(11\,e^{(-2\,x)}+55\,e^{(-4\,x)}+165\,e^{(-6\,x)}+330\,e^{(-8\,x)}+462\,e^{(-10\,x)}+462\,e^{(-12\,x)}+330\,e^{(-14\,x)}+165\,e^{(-16\,x)}}$$

input
```
integrate(sech(x)^7*tanh(x)^5,x, algorithm="maxima")
```

output
```
-128/7*e^(-7*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) +
 462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18
*x) + 11*e^(-20*x) + e^(-22*x) + 1) + 2560/63*e^(-9*x)/(11*e^(-2*x) + 55*e
^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) + 462*e^(-10*x) + 462*e^(-12*x) + 33
0*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*x) + 11*e^(-20*x) + e^(-22*x) + 1)
 - 47360/693*e^(-11*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(
-8*x) + 462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55
*e^(-18*x) + 11*e^(-20*x) + e^(-22*x) + 1) + 2560/63*e^(-13*x)/(11*e^(-2*x
) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) + 462*e^(-10*x) + 462*e^(-12
*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*x) + 11*e^(-20*x) + e^(-22
*x) + 1) - 128/7*e^(-15*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330
*e^(-8*x) + 462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x)
+ 55*e^(-18*x) + 11*e^(-20*x) + e^(-22*x) + 1)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = -\frac{128\left(99\left(e^{(-x)}+e^x\right)^4-616\left(e^{(-x)}+e^x\right)^2+1008\right)}{693\left(e^{(-x)}+e^x\right)^{11}}$$

input
```
integrate(sech(x)^7*tanh(x)^5,x, algorithm="giac")
```

output
```
-128/693*(99*(e^(-x) + e^x)^4 - 616*(e^(-x) + e^x)^2 + 1008)/(e^(-x) + e^x
)^11
```

3.97.9 Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 520, normalized size of antiderivative = 20.80

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx$$
$$= \frac{\frac{64\,e^{5x}}{11}-\frac{320\,e^{7x}}{11}+\frac{640\,e^{9x}}{11}-\frac{640\,e^{11x}}{11}+\frac{320\,e^{13x}}{11}-\frac{64\,e^{15x}}{11}}{11\,e^{2x}+55\,e^{4x}+165\,e^{6x}+330\,e^{8x}+462\,e^{10x}+462\,e^{12x}+330\,e^{14x}+165\,e^{16x}+55\,e^{18x}+11\,e^{20x}+e^{22x}}$$
$$-\frac{38464\,e^{x}}{693\,\left(6\,e^{2x}+15\,e^{4x}+20\,e^{6x}+15\,e^{8x}+6\,e^{10x}+e^{12x}+1\right)}$$
$$-\frac{640\,e^{x}}{33\,\left(8\,e^{2x}+28\,e^{4x}+56\,e^{6x}+70\,e^{8x}+56\,e^{10x}+28\,e^{12x}+8\,e^{14x}+e^{16x}+1\right)}$$
$$-\frac{104\,e^{x}}{21\,\left(4\,e^{2x}+6\,e^{4x}+4\,e^{6x}+e^{8x}+1\right)}+\frac{1664\,e^{x}}{63\,\left(5\,e^{2x}+10\,e^{4x}+10\,e^{6x}+5\,e^{8x}+e^{10x}+1\right)}$$
$$+\frac{4096\,e^{x}}{77\,\left(7\,e^{2x}+21\,e^{4x}+35\,e^{6x}+35\,e^{8x}+21\,e^{10x}+7\,e^{12x}+e^{14x}+1\right)}$$
$$+\frac{\frac{16\,e^{3x}}{11}-\frac{112\,e^{5x}}{11}+\frac{288\,e^{7x}}{11}-32\,e^{9x}+\frac{208\,e^{11x}}{11}-\frac{48\,e^{13x}}{11}}{10\,e^{2x}+45\,e^{4x}+120\,e^{6x}+210\,e^{8x}+252\,e^{10x}+210\,e^{12x}+120\,e^{14x}+45\,e^{16x}+10\,e^{18x}+e^{20x}+1}$$
$$-\frac{\frac{280\,e^{3x}}{99}-\frac{112\,e^{5x}}{11}+16\,e^{7x}-\frac{104\,e^{9x}}{9}+\frac{104\,e^{11x}}{33}-\frac{8\,e^{x}}{33}}{9\,e^{2x}+36\,e^{4x}+84\,e^{6x}+126\,e^{8x}+126\,e^{10x}+84\,e^{12x}+36\,e^{14x}+9\,e^{16x}+e^{18x}+1}$$

input
```
int(tanh(x)^5/cosh(x)^7,x)
```

output
```
((64*exp(5*x))/11 - (320*exp(7*x))/11 + (640*exp(9*x))/11 - (640*exp(11*x)
)/11 + (320*exp(13*x))/11 - (64*exp(15*x))/11)/(11*exp(2*x) + 55*exp(4*x)
+ 165*exp(6*x) + 330*exp(8*x) + 462*exp(10*x) + 462*exp(12*x) + 330*exp(14
*x) + 165*exp(16*x) + 55*exp(18*x) + 11*exp(20*x) + exp(22*x) + 1) - (3846
4*exp(x))/(693*(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8*x) + 6*e
xp(10*x) + exp(12*x) + 1)) - (640*exp(x))/(33*(8*exp(2*x) + 28*exp(4*x) +
56*exp(6*x) + 70*exp(8*x) + 56*exp(10*x) + 28*exp(12*x) + 8*exp(14*x) + ex
p(16*x) + 1)) - (104*exp(x))/(21*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + e
xp(8*x) + 1)) + (1664*exp(x))/(63*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x)
+ 5*exp(8*x) + exp(10*x) + 1)) + (4096*exp(x))/(77*(7*exp(2*x) + 21*exp(4*
x) + 35*exp(6*x) + 35*exp(8*x) + 21*exp(10*x) + 7*exp(12*x) + exp(14*x) +
1)) + ((16*exp(3*x))/11 - (112*exp(5*x))/11 + (288*exp(7*x))/11 - 32*exp(9
*x) + (208*exp(11*x))/11 - (48*exp(13*x))/11)/(10*exp(2*x) + 45*exp(4*x) +
 120*exp(6*x) + 210*exp(8*x) + 252*exp(10*x) + 210*exp(12*x) + 120*exp(14*
x) + 45*exp(16*x) + 10*exp(18*x) + exp(20*x) + 1) - ((280*exp(3*x))/99 - (
112*exp(5*x))/11 + 16*exp(7*x) - (104*exp(9*x))/9 + (104*exp(11*x))/33 - (
8*exp(x))/33)/(9*exp(2*x) + 36*exp(4*x) + 84*exp(6*x) + 126*exp(8*x) + 126
*exp(10*x) + 84*exp(12*x) + 36*exp(14*x) + 9*exp(16*x) + exp(18*x) + 1)
```

3.97. $\int \operatorname{sech}^7(x)\tanh^5(x)\,dx$

3.98 $\int \mathrm{sech}^3(x)\tanh^4(x)\,dx$

3.98.1 Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \mathrm{sech}^3(x)\tanh^4(x)\,dx=\frac{1}{16}\arctan(\sinh(x))+\frac{1}{16}\mathrm{sech}(x)\tanh(x)-\frac{1}{8}\mathrm{sech}^3(x)\tanh(x)-\frac{1}{6}\mathrm{sech}^3(x)\tanh^3(x)$$

output
```
1/16*arctan(sinh(x))+1/16*sech(x)*tanh(x)-1/8*sech(x)^3*tanh(x)-1/6*sech(x
)^3*tanh(x)^3
```

3.98.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \mathrm{sech}^3(x)\tanh^4(x)\,dx=\frac{1}{16}\arctan(\sinh(x))+\frac{1}{16}\mathrm{sech}(x)\tanh(x)+\frac{1}{24}\mathrm{sech}^3(x)\tanh(x)-\frac{1}{6}\mathrm{sech}^5(x)\tanh(x)-\frac{1}{3}\mathrm{sech}^3(x)\tanh^3(x)$$

input
```
Integrate[Sech[x]^3*Tanh[x]^4,x]
```

output
```
ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech
[x]^5*Tanh[x])/6 - (Sech[x]^3*Tanh[x]^3)/3
```

3.98.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {3042, 3091, 25, 3042, 25, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(x)\text{sech}^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ix)^4\sec(ix)^3dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{1}{2}\int -\text{sech}^3(x)\tanh^2(x)dx-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \text{sech}^3(x)\tanh^2(x)dx-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)+\frac{1}{2}\int -\sec(ix)^3\tan(ix)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)-\frac{1}{2}\int \sec(ix)^3\tan(ix)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \text{sech}^3(x)dx-\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)+\frac{1}{2}\left(-\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{1}{4}\int \csc\left(ix+\frac{\pi}{2}\right)^3dx\right)$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int \text{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\text{sech}(x)\right)-\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)+\frac{1}{2}\left(-\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{1}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x)+\frac{1}{2}\int\csc\left(ix+\frac{\pi}{2}\right)dx\right)\right)$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\text{sech}(x)\right)-\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)$$

input
```
Int[Sech[x]^3*Tanh[x]^4,x]
```

output
```
-1/6*(Sech[x]^3*Tanh[x]^3) + (-1/4*(Sech[x]^3*Tanh[x]) + (ArcTan[Sinh[x]]/
2 + (Sech[x]*Tanh[x])/2)/4)/2
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.98.4 Maple [A] (verified)

Time = 16.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{\sinh(x)^3}{3\cosh(x)^6}-\frac{\sinh(x)}{5\cosh(x)^6}+\frac{\left(\frac{\operatorname{sech}(x)^5}{6}+\frac{5\operatorname{sech}(x)^3}{24}+\frac{5\operatorname{sech}(x)}{16}\right)\tanh(x)}{5}+\frac{\arctan(e^x)}{8}$	46
risch	$\frac{e^x(3e^{10x}-47e^{8x}+78e^{6x}-78e^{4x}+47e^{2x}-3)}{24(1+e^{2x})^6}+\frac{i\ln(e^x+i)}{16}-\frac{i\ln(e^x-i)}{16}$	64

input
```
int(sech(x)^3*tanh(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*sinh(x)^3/cosh(x)^6-1/5*sinh(x)/cosh(x)^6+1/5*(1/6*sech(x)^5+5/24*sec
h(x)^3+5/16*sech(x))*tanh(x)+1/8*arctan(exp(x))
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(30) = 60.

Time = 0.25 (sec) , antiderivative size = 925, normalized size of antiderivative = 24.34

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^3*tanh(x)^4,x, algorithm="fricas")
```

output
```
1/24*(3*cosh(x)^11 + 33*cosh(x)*sinh(x)^10 + 3*sinh(x)^11 + (165*cosh(x)^2
 - 47)*sinh(x)^9 - 47*cosh(x)^9 + 9*(55*cosh(x)^3 - 47*cosh(x))*sinh(x)^8
+ 6*(165*cosh(x)^4 - 282*cosh(x)^2 + 13)*sinh(x)^7 + 78*cosh(x)^7 + 42*(33
*cosh(x)^5 - 94*cosh(x)^3 + 13*cosh(x))*sinh(x)^6 + 6*(231*cosh(x)^6 - 987
*cosh(x)^4 + 273*cosh(x)^2 - 13)*sinh(x)^5 - 78*cosh(x)^5 + 6*(165*cosh(x)
^7 - 987*cosh(x)^5 + 455*cosh(x)^3 - 65*cosh(x))*sinh(x)^4 + (495*cosh(x)^
8 - 3948*cosh(x)^6 + 2730*cosh(x)^4 - 780*cosh(x)^2 + 47)*sinh(x)^3 + 47*c
osh(x)^3 + 3*(55*cosh(x)^9 - 564*cosh(x)^7 + 546*cosh(x)^5 - 260*cosh(x)^3
 + 47*cosh(x))*sinh(x)^2 + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)
^12 + 6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3 +
3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 + 15
*cosh(x)^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(2
31*cosh(x)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*sinh(x)^6 + 20*cosh(x)^6
 + 24*(33*cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 + 5*cosh(x))*sinh(x)^5 +
 15*(33*cosh(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x
)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 + 42*cosh(x)^5 + 20*c
osh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45*cosh(x)^8 + 70*cos
h(x)^6 + 50*cosh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 12*(co
sh(x)^11 + 5*cosh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*cosh(x)^3 + cosh(
x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(11*cosh(x)^10 - 141*cos...
```

3.98.6 Sympy [F]

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \int \tanh^4(x)\operatorname{sech}^3(x)\,dx$$

input
```
integrate(sech(x)**3*tanh(x)**4,x)
```

output
```
Integral(tanh(x)**4*sech(x)**3, x)
```

3.98.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{3\,e^{(-x)} - 47\,e^{(-3\,x)} + 78\,e^{(-5\,x)} - 78\,e^{(-7\,x)} + 47\,e^{(-9\,x)} - 3\,e^{(-11\,x)}}{24\,(6\,e^{(-2\,x)} + 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} + 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} + e^{(-12\,x)} + 1)} - \frac{1}{8}\arctan\left(e^{(-x)}\right)$$

input
```
integrate(sech(x)^3*tanh(x)^4,x, algorithm="maxima")
```

output
```
1/24*(3*e^(-x) - 47*e^(-3*x) + 78*e^(-5*x) - 78*e^(-7*x) + 47*e^(-9*x) - 3
*e^(-11*x))/(6*e^(-2*x) + 15*e^(-4*x) + 20*e^(-6*x) + 15*e^(-8*x) + 6*e^(-
10*x) + e^(-12*x) + 1) - 1/8*arctan(e^(-x))
```

3.98.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{1}{32}\,\pi - \frac{3\left(e^{(-x)} - e^{x}\right)^5 - 32\left(e^{(-x)} - e^{x}\right)^3 - 48\,e^{(-x)} + 48\,e^{x}}{24\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)^3} + \frac{1}{16}\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

input
```
integrate(sech(x)^3*tanh(x)^4,x, algorithm="giac")
```

output
```
1/32*pi - 1/24*(3*(e^(-x) - e^x)^5 - 32*(e^(-x) - e^x)^3 - 48*e^(-x) + 48*
e^x)/((e^(-x) - e^x)^2 + 4)^3 + 1/16*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 5.26

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{\operatorname{atan}(e^x)}{8} - \frac{10\,e^x}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1} + \frac{e^x}{8\,\left(e^{2\,x}+1\right)} + \frac{7\,e^x}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1} - \frac{4\,e^{5\,x}-\frac{8\,e^{3\,x}}{3}-\frac{8\,e^{7\,x}}{3}+\frac{2\,e^{9\,x}}{3}+\frac{2\,e^x}{3}}{6\,e^{2\,x}+15\,e^{4\,x}+20\,e^{6\,x}+15\,e^{8\,x}+6\,e^{10\,x}+e^{12\,x}+1} + \frac{16\,e^x}{3\,\left(5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+e^{10\,x}+1\right)} - \frac{23\,e^x}{12\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}$$

input
```
int(tanh(x)^4/cosh(x)^3,x)
```

output
```
atan(exp(x))/8 - (10*exp(x))/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8
*x) + 1) + exp(x)/(8*(exp(2*x) + 1)) + (7*exp(x))/(3*exp(2*x) + 3*exp(4*x)
 + exp(6*x) + 1) - (4*exp(5*x) - (8*exp(3*x))/3 - (8*exp(7*x))/3 + (2*exp(
9*x))/3 + (2*exp(x))/3)/(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8
*x) + 6*exp(10*x) + exp(12*x) + 1) + (16*exp(x))/(3*(5*exp(2*x) + 10*exp(4
*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1)) - (23*exp(x))/(12*(2*exp(
2*x) + exp(4*x) + 1))
```

3.99 $\int \text{sech}^5(x) \tanh^2(x) \, dx$

3.99.1 Optimal result

Integrand size = 9, antiderivative size = 36

$$\int \text{sech}^5(x) \tanh^2(x) \, dx = \frac{1}{16} \arctan(\sinh(x)) + \frac{1}{16} \text{sech}(x) \tanh(x) + \frac{1}{24} \text{sech}^3(x) \tanh(x) - \frac{1}{6} \text{sech}^5(x) \tanh(x)$$

output
```
1/16*arctan(sinh(x))+1/16*sech(x)*tanh(x)+1/24*sech(x)^3*tanh(x)-1/6*sech(
x)^5*tanh(x)
```

3.99.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \text{sech}^5(x) \tanh^2(x) \, dx = \frac{1}{16} \arctan(\sinh(x)) + \frac{1}{16} \text{sech}(x) \tanh(x) + \frac{1}{24} \text{sech}^3(x) \tanh(x) - \frac{1}{6} \text{sech}^5(x) \tanh(x)$$

input
```
Integrate[Sech[x]^5*Tanh[x]^2,x]
```

output
```
ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech
[x]^5*Tanh[x])/6
```

3.99.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 25, 3091, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(x)\text{sech}^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ix)^2\left(-\sec(ix)^5\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ix)^5\tan(ix)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{6}\int \text{sech}^5(x)dx-\frac{1}{6}\tanh(x)\text{sech}^5(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\tanh(x)\text{sech}^5(x)+\frac{1}{6}\int \csc\left(ix+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{6}\left(\frac{3}{4}\int \text{sech}^3(x)dx+\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)-\frac{1}{6}\tanh(x)\text{sech}^5(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\tanh(x)\text{sech}^5(x)+\frac{1}{6}\left(\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{4}\int \csc\left(ix+\frac{\pi}{2}\right)^3dx\right)$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int \text{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\text{sech}(x)\right)+\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)-\frac{1}{6}\tanh(x)\text{sech}^5(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\tanh(x)\text{sech}^5(x)+\frac{1}{6}\left(\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x)+\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx\right)\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\text{sech}(x)\right)+\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)-\frac{1}{6}\tanh(x)\text{sech}^5(x)$$

input
```
Int[Sech[x]^5*Tanh[x]^2,x]
```

output
```
-1/6*(Sech[x]^5*Tanh[x]) + ((Sech[x]^3*Tanh[x])/4 + (3*(ArcTan[Sinh[x]]/2
+ (Sech[x]*Tanh[x])/2))/4)/6
```

3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.99.4 Maple [A] (verified)

Time = 169.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\sinh(x)}{5\cosh(x)^6}+\frac{\left(\frac{\operatorname{sech}(x)^5}{6}+\frac{5\operatorname{sech}(x)^3}{24}+\frac{5\operatorname{sech}(x)}{16}\right)\tanh(x)}{5}+\frac{\arctan(e^x)}{8}$	36
risch	$\frac{e^x(3e^{10x}+17e^{8x}-114e^{6x}+114e^{4x}-17e^{2x}-3)}{24(1+e^{2x})^6}+\frac{i\ln(e^x+i)}{16}-\frac{i\ln(e^x-i)}{16}$	64

input
```
int(sech(x)^5*tanh(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*sinh(x)/cosh(x)^6+1/5*(1/6*sech(x)^5+5/24*sech(x)^3+5/16*sech(x))*tan
h(x)+1/8*arctan(exp(x))
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(28) = 56.

Time = 0.25 (sec) , antiderivative size = 925, normalized size of antiderivative = 25.69

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^5*tanh(x)^2,x, algorithm="fricas")
```

output
```
1/24*(3*cosh(x)^11 + 33*cosh(x)*sinh(x)^10 + 3*sinh(x)^11 + (165*cosh(x)^2
 + 17)*sinh(x)^9 + 17*cosh(x)^9 + 9*(55*cosh(x)^3 + 17*cosh(x))*sinh(x)^8
+ 6*(165*cosh(x)^4 + 102*cosh(x)^2 - 19)*sinh(x)^7 - 114*cosh(x)^7 + 42*(3
3*cosh(x)^5 + 34*cosh(x)^3 - 19*cosh(x))*sinh(x)^6 + 6*(231*cosh(x)^6 + 35
7*cosh(x)^4 - 399*cosh(x)^2 + 19)*sinh(x)^5 + 114*cosh(x)^5 + 6*(165*cosh(
x)^7 + 357*cosh(x)^5 - 665*cosh(x)^3 + 95*cosh(x))*sinh(x)^4 + (495*cosh(x
)^8 + 1428*cosh(x)^6 - 3990*cosh(x)^4 + 1140*cosh(x)^2 - 17)*sinh(x)^3 - 1
7*cosh(x)^3 + 3*(55*cosh(x)^9 + 204*cosh(x)^7 - 798*cosh(x)^5 + 380*cosh(x
)^3 - 17*cosh(x))*sinh(x)^2 + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh
(x)^12 + 6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3
 + 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 +
 15*cosh(x)^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4
*(231*cosh(x)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*sinh(x)^6 + 20*cosh(x
)^6 + 24*(33*cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 + 5*cosh(x))*sinh(x)^
5 + 15*(33*cosh(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*cosh(x)^2 + 1)*sin
h(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 + 42*cosh(x)^5 + 2
0*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45*cosh(x)^8 + 70*
cosh(x)^6 + 50*cosh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 12*
(cosh(x)^11 + 5*cosh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*cosh(x)^3 + co
sh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(11*cosh(x)^10 + 51*c...
```

3.99.6 Sympy [F]

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \int \tanh^2(x)\operatorname{sech}^5(x)\,dx$$

input
```
integrate(sech(x)**5*tanh(x)**2,x)
```

output
```
Integral(tanh(x)**2*sech(x)**5, x)
```

3.99.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(28) = 56.

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \frac{3\,e^{(-x)} + 17\,e^{(-3\,x)} - 114\,e^{(-5\,x)} + 114\,e^{(-7\,x)} - 17\,e^{(-9\,x)} - 3\,e^{(-11\,x)}}{24\,(6\,e^{(-2\,x)} + 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} + 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} + e^{(-12\,x)} + 1)} - \frac{1}{8}\arctan\left(e^{(-x)}\right)$$

input
```
integrate(sech(x)^5*tanh(x)^2,x, algorithm="maxima")
```

output
```
1/24*(3*e^(-x) + 17*e^(-3*x) - 114*e^(-5*x) + 114*e^(-7*x) - 17*e^(-9*x) -
 3*e^(-11*x))/(6*e^(-2*x) + 15*e^(-4*x) + 20*e^(-6*x) + 15*e^(-8*x) + 6*e^
(-10*x) + e^(-12*x) + 1) - 1/8*arctan(e^(-x))
```

3.99.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.03

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \frac{1}{32}\,\pi - \frac{3\,\left(e^{(-x)} - e^{x}\right)^5 + 32\,\left(e^{(-x)} - e^{x}\right)^3 - 48\,e^{(-x)} + 48\,e^{x}}{24\,\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)^3} + \frac{1}{16}\arctan\left(\frac{1}{2}\,\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

input
```
integrate(sech(x)^5*tanh(x)^2,x, algorithm="giac")
```

output
```
1/32*pi - 1/24*(3*(e^(-x) - e^x)^5 + 32*(e^(-x) - e^x)^3 - 48*e^(-x) + 48*
e^x)/((e^(-x) - e^x)^2 + 4)^3 + 1/16*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

3.99.9 Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 5.72

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \frac{\operatorname{atan}(e^x)}{8} + \frac{34\,e^x}{15\,(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1)} + \frac{e^x}{8\,(e^{2\,x}+1)} - \frac{9\,e^x}{5\,(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1)} - \frac{\frac{8\,e^{3\,x}}{3}-\frac{16\,e^{5\,x}}{3}+\frac{8\,e^{7\,x}}{3}}{6\,e^{2\,x}+15\,e^{4\,x}+20\,e^{6\,x}+15\,e^{8\,x}+6\,e^{10\,x}+e^{12\,x}+1} - \frac{\frac{28\,e^{5\,x}}{15}-\frac{8\,e^{3\,x}}{3}+\frac{4\,e^x}{5}}{5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+e^{10\,x}+1} + \frac{e^x}{12\,(2\,e^{2\,x}+e^{4\,x}+1)}$$

input
```
int(tanh(x)^2/cosh(x)^5,x)
```

output
```
atan(exp(x))/8 + (34*exp(x))/(15*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + e
xp(8*x) + 1)) + exp(x)/(8*(exp(2*x) + 1)) - (9*exp(x))/(5*(3*exp(2*x) + 3*
exp(4*x) + exp(6*x) + 1)) - ((8*exp(3*x))/3 - (16*exp(5*x))/3 + (8*exp(7*x
))/3)/(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x)
+ exp(12*x) + 1) - ((28*exp(5*x))/15 - (8*exp(3*x))/3 + (4*exp(x))/5)/(5*e
xp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1) + exp(x)
/(12*(2*exp(2*x) + exp(4*x) + 1))
```

3.100 $\int \operatorname{sech}^8(x) \tanh^6(x) \, dx$

3.100.1 Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \operatorname{sech}^8(x) \tanh^6(x) \, dx = \frac{\tanh^7(x)}{7} - \frac{\tanh^9(x)}{3} + \frac{3\tanh^{11}(x)}{11} - \frac{\tanh^{13}(x)}{13}$$

output
```
1/7*tanh(x)^7-1/3*tanh(x)^9+3/11*tanh(x)^11-1/13*tanh(x)^13
```

3.100.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(33) = 66.

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\begin{aligned}\int \operatorname{sech}^8(x) \tanh^6(x) \, dx &= \frac{16\tanh(x)}{3003} + \frac{8\operatorname{sech}^2(x)\tanh(x)}{3003} + \frac{2\operatorname{sech}^4(x)\tanh(x)}{1001} \\ &\quad + \frac{5\operatorname{sech}^6(x)\tanh(x)}{3003} - \frac{53}{429}\operatorname{sech}^8(x)\tanh(x) \\ &\quad + \frac{27}{143}\operatorname{sech}^{10}(x)\tanh(x) - \frac{1}{13}\operatorname{sech}^{12}(x)\tanh(x)\end{aligned}$$

input
```
Integrate[Sech[x]^8*Tanh[x]^6,x]
```

output
```
(16*Tanh[x])/3003 + (8*Sech[x]^2*Tanh[x])/3003 + (2*Sech[x]^4*Tanh[x])/100
1 + (5*Sech[x]^6*Tanh[x])/3003 - (53*Sech[x]^8*Tanh[x])/429 + (27*Sech[x]^
10*Tanh[x])/143 - (Sech[x]^12*Tanh[x])/13
```

3.100.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 25, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^6(x)\text{sech}^8(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ix)^6\left(-\sec(ix)^8\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ix)^8\tan(ix)^6dx$$

$$\Big\downarrow \text{3087}$$

$$i\int -\tanh^6(x)\left(1-\tanh^2(x)\right)^3d(i\tanh(x))$$

$$\Big\downarrow \text{244}$$

$$i\int \left(\tanh^{12}(x)-3\tanh^{10}(x)+3\tanh^8(x)-\tanh^6(x)\right)d(i\tanh(x))$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{1}{13}i\tanh^{13}(x)-\frac{3}{11}i\tanh^{11}(x)+\frac{1}{3}i\tanh^9(x)-\frac{1}{7}i\tanh^7(x)\right)$$

input
```
Int[Sech[x]^8*Tanh[x]^6,x]
```

output
```
I*((-1/7*I)*Tanh[x]^7 + (I/3)*Tanh[x]^9 - ((3*I)/11)*Tanh[x]^11 + (I/13)*T
anh[x]^13)
```

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.100.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\frac{\tanh(x)^{7}}{7}-\frac{\tanh(x)^{9}}{3}+\frac{3\tanh(x)^{11}}{11}-\frac{\tanh(x)^{13}}{13}$$

input
```
int(sech(x)^8*tanh(x)^6,x)
```

output
```
1/7*tanh(x)^7-1/3*tanh(x)^9+3/11*tanh(x)^11-1/13*tanh(x)^13
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. 2(25) = 50.

Time = 0.23 (sec) , antiderivative size = 778, normalized size of antiderivative = 23.58

$$\int \operatorname{sech}^8(x) \tanh^6(x)\, dx = \text{Too large to display}$$

input
```
integrate(sech(x)^8*tanh(x)^6,x, algorithm="fricas")
```

output
```
-64/3003*(1502*cosh(x)^9 + 13518*cosh(x)*sinh(x)^8 + 1501*sinh(x)^9 + (540
36*cosh(x)^2 - 4511)*sinh(x)^7 - 4498*cosh(x)^7 + 14*(9012*cosh(x)^3 - 224
9*cosh(x))*sinh(x)^6 + 3*(63042*cosh(x)^4 - 31577*cosh(x)^2 + 2990)*sinh(x
)^5 + 9048*cosh(x)^5 + 2*(94626*cosh(x)^5 - 78715*cosh(x)^3 + 22620*cosh(x
))*sinh(x)^4 + (126084*cosh(x)^6 - 157885*cosh(x)^4 + 89700*cosh(x)^2 - 82
94)*sinh(x)^3 - 8008*cosh(x)^3 + 6*(9012*cosh(x)^7 - 15743*cosh(x)^5 + 150
80*cosh(x)^3 - 4004*cosh(x))*sinh(x)^2 + (13509*cosh(x)^8 - 31577*cosh(x)^
6 + 44850*cosh(x)^4 - 24882*cosh(x)^2 + 6292)*sinh(x) + 4004*cosh(x))/(cos
h(x)^17 + 17*cosh(x)*sinh(x)^16 + sinh(x)^17 + (136*cosh(x)^2 + 13)*sinh(x
)^15 + 13*cosh(x)^15 + 5*(136*cosh(x)^3 + 39*cosh(x))*sinh(x)^14 + (2380*c
osh(x)^4 + 1365*cosh(x)^2 + 78)*sinh(x)^13 + 78*cosh(x)^13 + 13*(476*cosh(
x)^5 + 455*cosh(x)^3 + 78*cosh(x))*sinh(x)^12 + 13*(952*cosh(x)^6 + 1365*c
osh(x)^4 + 468*cosh(x)^2 + 22)*sinh(x)^11 + 286*cosh(x)^11 + 143*(136*cosh
(x)^7 + 273*cosh(x)^5 + 156*cosh(x)^3 + 22*cosh(x))*sinh(x)^10 + (24310*co
sh(x)^8 + 65065*cosh(x)^6 + 55770*cosh(x)^4 + 15730*cosh(x)^2 + 714)*sinh(
x)^9 + 716*cosh(x)^9 + (24310*cosh(x)^9 + 83655*cosh(x)^7 + 100386*cosh(x)
^5 + 47190*cosh(x)^3 + 6444*cosh(x))*sinh(x)^8 + (19448*cosh(x)^10 + 83655
*cosh(x)^8 + 133848*cosh(x)^6 + 94380*cosh(x)^4 + 25704*cosh(x)^2 + 1274)*
sinh(x)^7 + 1300*cosh(x)^7 + (12376*cosh(x)^11 + 65065*cosh(x)^9 + 133848*
cosh(x)^7 + 132132*cosh(x)^5 + 60144*cosh(x)^3 + 9100*cosh(x))*sinh(x)^...
```

3.100.6 Sympy [F]

$$\int \operatorname{sech}^8(x) \tanh^6(x)\, dx = \int \tanh^6(x) \operatorname{sech}^8(x)\, dx$$

input
```
integrate(sech(x)**8*tanh(x)**6,x)
```

output
```
Integral(tanh(x)**6*sech(x)**8, x)
```

3.100.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(25) = 50$.

Time = 0.21 (sec) , antiderivative size = 857, normalized size of antiderivative = 25.97

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^8*tanh(x)^6,x, algorithm="maxima")
```

output
```
32/231*e^(-2*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) +
 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e
^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) +
64/77*e^(-4*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) +
1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^
(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 6
4/21*e^(-6*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1
287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(
-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 51
2/21*e^(-8*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1
287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(
-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 76
8/7*e^(-10*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1
287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(
-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 12
16/7*e^(-12*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) +
1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^
(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 1
92*e^(-14*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 12
87*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e...
```

3.100.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx = -\frac{32\left(3003\,e^{(18\,x)} - 9009\,e^{(16\,x)} + 18018\,e^{(14\,x)} - 16302\,e^{(12\,x)} + 10296\,e^{(10\,x)} - 2288\,e^{(8\,x)} + 286\,e^{(6\,x)} + 78\,e\right.}{3003\,\left(e^{(2\,x)} + 1\right)^{13}}$$

input
```
integrate(sech(x)^8*tanh(x)^6,x, algorithm="giac")
```

output
```
-32/3003*(3003*e^(18*x) - 9009*e^(16*x) + 18018*e^(14*x) - 16302*e^(12*x)
+ 10296*e^(10*x) - 2288*e^(8*x) + 286*e^(6*x) + 78*e^(4*x) + 13*e^(2*x) +
1)/(e^(2*x) + 1)^13
```

3.100.9 Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 820, normalized size of antiderivative = 24.85

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx = \text{Too large to display}$$

input
```
int(tanh(x)^6/cosh(x)^8,x)
```

output
```
- ((64*exp(4*x))/143 - (256*exp(2*x))/429 + 80/429)/(6*exp(2*x) + 15*exp(4
*x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + exp(12*x) + 1) - ((64*exp(
2*x))/143 - (768*exp(4*x))/143 + (3200*exp(6*x))/143 - (6400*exp(8*x))/143
 + (6720*exp(10*x))/143 - (3584*exp(12*x))/143 + (768*exp(14*x))/143)/(11*
exp(2*x) + 55*exp(4*x) + 165*exp(6*x) + 330*exp(8*x) + 462*exp(10*x) + 462
*exp(12*x) + 330*exp(14*x) + 165*exp(16*x) + 55*exp(18*x) + 11*exp(20*x) +
 exp(22*x) + 1) - ((160*exp(2*x))/143 - (256*exp(4*x))/143 + (128*exp(6*x)
)/143 - 640/3003)/(7*exp(2*x) + 21*exp(4*x) + 35*exp(6*x) + 35*exp(8*x) +
21*exp(10*x) + 7*exp(12*x) + exp(14*x) + 1) - ((128*exp(6*x))/13 - (768*ex
p(8*x))/13 + (1920*exp(10*x))/13 - (2560*exp(12*x))/13 + (1920*exp(14*x))/
13 - (768*exp(16*x))/13 + (128*exp(18*x))/13)/(13*exp(2*x) + 78*exp(4*x) +
 286*exp(6*x) + 715*exp(8*x) + 1287*exp(10*x) + 1716*exp(12*x) + 1716*exp(
14*x) + 1287*exp(16*x) + 715*exp(18*x) + 286*exp(20*x) + 78*exp(22*x) + 13
*exp(24*x) + exp(26*x) + 1) - ((560*exp(4*x))/143 - (640*exp(2*x))/429 - (
1792*exp(6*x))/429 + (224*exp(8*x))/143 + 80/429)/(8*exp(2*x) + 28*exp(4*x
) + 56*exp(6*x) + 70*exp(8*x) + 56*exp(10*x) + 28*exp(12*x) + 8*exp(14*x)
+ exp(16*x) + 1) - ((640*exp(2*x))/429 - (2560*exp(4*x))/429 + (4480*exp(6
*x))/429 - (3584*exp(8*x))/429 + (1792*exp(10*x))/715 - 256/2145)/(9*exp(2
*x) + 36*exp(4*x) + 84*exp(6*x) + 126*exp(8*x) + 126*exp(10*x) + 84*exp(12
*x) + 36*exp(14*x) + 9*exp(16*x) + exp(18*x) + 1) - ((32*exp(4*x))/13 -...
```

3.101 $\int \cosh(a+bx)\coth(a+bx)\,dx$

3.101.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \cosh(a+bx)\coth(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\cosh(a+bx)}{b}$$

output
```
-arctanh(cosh(b*x+a))/b+cosh(b*x+a)/b
```

3.101.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \cosh(a+bx)\coth(a+bx)\,dx=\frac{\cosh(a+bx)}{b}-\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
Cosh[a + b*x]/b - Log[Cosh[(a + b*x)/2]]/b + Log[Sinh[(a + b*x)/2]]/b
```

3.101.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin\left(ia+ibx+\frac{\pi}{2}\right)\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{\int \frac{\cosh^2(a+bx)}{1-\cosh^2(a+bx)}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\int \frac{1}{1-\cosh^2(a+bx)}d\cosh(a+bx)-\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}(\cosh(a+bx))-\cosh(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
-((ArcTanh[Cosh[a + b*x]] - Cosh[a + b*x])/b)
```

3.101.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.101.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\cosh(bx+a)-2\,\operatorname{arctanh}(e^{bx+a})}{b}$	21
default	$\frac{\cosh(bx+a)-2\,\operatorname{arctanh}(e^{bx+a})}{b}$	21
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{\ln(e^{bx+a}-1)}{b}-\frac{\ln(e^{bx+a}+1)}{b}$	54

input
```
int(cosh(b*x+a)*coth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.91

$$\int \cosh(a+bx)\coth(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^2 - 2(\cosh(bx+a)+\sinh(bx+a))\log(\cosh(bx+a)+\sinh(bx+a)+1) + 2(\cosh(bx+}{2(b\cosh(bx+}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
 + sinh(b*x + a) + 1) + 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a
) + sinh(b*x + a) - 1) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 +
 1)/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

3.101.6 Sympy [F]

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \int \cosh(a+bx)\coth(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a),x)
```

output
```
Integral(cosh(a + b*x)*coth(a + b*x), x)
```

3.101.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(23) = 46$.

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.57

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{(bx+a)}}{2\,b} + \frac{e^{(-bx-a)}}{2\,b} - \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="maxima")
```

output
```
1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b - log(e^(-b*x - a) + 1)/b + log(e^(
-b*x - a) - 1)/b
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \cosh(a+bx)\coth(a+bx)\,dx \\ = \frac{e^{(bx+a)} + e^{(-bx-a)} - 2\,\log\left(e^{(bx+a)}+1\right) + 2\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="giac")
```

output
```
1/2*(e^(b*x + a) + e^(-b*x - a) - 2*log(e^(b*x + a) + 1) + 2*log(abs(e^(b*
x + a) - 1)))/b
```

3.101.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{\mathrm{e}^{a+b\,x}}{2\,b} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{\mathrm{e}^{-a-b\,x}}{2\,b}$$

input
```
int(cosh(a + b*x)*coth(a + b*x),x)
```

output
```
exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/
2) + exp(- a - b*x)/(2*b)
```

3.102 $\int \cosh(a+bx)\coth^2(a+bx)\,dx$

3.102.1 Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}+\frac{\sinh(a+bx)}{b}$$

output
```
-csch(b*x+a)/b+sinh(b*x+a)/b
```

3.102.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}+\frac{\sinh(a+bx)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
-(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

3.102.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\sin\left(ia+ibx+\frac{\pi}{2}\right)\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \ 25$$

$$-\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx$$

$$\Big\downarrow \ 3070$$

$$-\frac{i\int -\text{csch}^2(a+bx)\left(\sinh^2(a+bx)+1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \ 244$$

$$-\frac{i\int \left(-\text{csch}^2(a+bx)-1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{i(i\sinh(a+bx)-i\text{csch}(a+bx))}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
((-I)*((-I)*Csch[a + b*x] + I*Sinh[a + b*x]))/b
```

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.102.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	33
default	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	33
risch	$\frac{e^{3bx+3a}-6\,e^{bx+a}+e^{-bx-a}}{2b\left(e^{2bx+2a}-1\right)}$	46

input
```
int(cosh(b*x+a)*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/sinh(b*x+a)*cosh(b*x+a)^2-2/sinh(b*x+a))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{\cosh(bx+a)^2+\sinh(bx+a)^2-3}{2\,b\sinh(bx+a)}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 - 3)/(b*sinh(b*x + a))
```

3.102.6 Sympy [F]

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \int \cosh(a+bx)\coth^2(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)**2,x)
```

output
```
Integral(cosh(a + b*x)*coth(a + b*x)**2, x)
```

3.102.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(22) = 44.

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{e^{(-bx-a)}}{2\,b} - \frac{5\,e^{(-2\,bx-2\,a)}-1}{2\,b(e^{(-bx-a)}-e^{(-3\,bx-3\,a)})}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*e^(-b*x - a)/b - 1/2*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-b*x - a) - e^(-
3*b*x - 3*a)))
```

3.102.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{\frac{4}{e^{(bx+a)}-e^{(-bx-a)}} - e^{(bx+a)} + e^{(-bx-a)}}{2\,b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(4/(e^(b*x + a) - e^(-b*x - a)) - e^(b*x + a) + e^(-b*x - a))/b
```

3.102.9 Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{e^{-a-b\,x}\,\left(e^{4\,a+4\,b\,x}-6\,e^{2\,a+2\,b\,x}+1\right)}{2\,b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(cosh(a + b*x)*coth(a + b*x)^2,x)
```

output
```
(exp(- a - b*x)*(exp(4*a + 4*b*x) - 6*exp(2*a + 2*b*x) + 1))/(2*b*(exp(2*a
 + 2*b*x) - 1))
```

3.103 $\int \cosh(a+bx)\coth^3(a+bx)\,dx$

3.103.1 Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = -\frac{3\text{arctanh}(\cosh(a+bx))}{2b}+\frac{3\cosh(a+bx)}{2b}-\frac{\cosh(a+bx)\coth^2(a+bx)}{2b}$$

output
```
-3/2*arctanh(cosh(b*x+a))/b+3/2*cosh(b*x+a)/b-1/2*cosh(b*x+a)*coth(b*x+a)^
2/b
```

3.103.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \frac{\cosh(a+bx)}{b}-\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

input
```
Integrate[Cosh[a + b*x]*Coth[a + b*x]^3,x]
```

output
```
Cosh[a + b*x]/b - Csch[(a + b*x)/2]^2/(8*b) - (3*Log[Cosh[(a + b*x)/2]])/(
2*b) + (3*Log[Sinh[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b)
```

3.103.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3072, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin\left(ia+ibx+\frac{\pi}{2}\right)\tan\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{\cosh^4(a+bx)}{(1-\cosh^2(a+bx))^2}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\cosh^3(a+bx)}{2(1-\cosh^2(a+bx))}-\frac{3}{2}\int \frac{\cosh^2(a+bx)}{1-\cosh^2(a+bx)}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\cosh^3(a+bx)}{2(1-\cosh^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\cosh^2(a+bx)}d\cosh(a+bx)-\cosh(a+bx)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\cosh^3(a+bx)}{2(1-\cosh^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\cosh(a+bx))-\cosh(a+bx))}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[a + b*x]^3,x]
```

output
```
((-3*(ArcTanh[Cosh[a + b*x]] - Cosh[a + b*x]))/2 + Cosh[a + b*x]^3/(2*(1 -
 Cosh[a + b*x]^2)))/b
```

3.103.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.103.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^3}{\sinh(bx+a)^2}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{3\coth(bx+a)\operatorname{csch}(bx+a)}{2}-3\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	62
default	$\frac{\frac{\cosh(bx+a)^3}{\sinh(bx+a)^2}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{3\coth(bx+a)\operatorname{csch}(bx+a)}{2}-3\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	62
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}-\frac{e^{bx+a}\left(1+e^{2bx+2a}\right)}{b\left(e^{2bx+2a}-1\right)^2}+\frac{3\ln\left(e^{bx+a}-1\right)}{2b}-\frac{3\ln\left(e^{bx+a}+1\right)}{2b}$	90

input
```
int(cosh(b*x+a)*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/sinh(b*x+a)^2*cosh(b*x+a)^3-3*cosh(b*x+a)/sinh(b*x+a)^2+3/2*coth(b*
x+a)*csch(b*x+a)-3*arctanh(exp(b*x+a)))
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(43) = 86.

Time = 0.29 (sec) , antiderivative size = 612, normalized size of antiderivative = 12.49

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx$$
$$= \frac{\cosh\left(bx+a\right)^6+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^5+\sinh\left(bx+a\right)^6+3\left(5\,\cosh\left(bx+a\right)^2-1\right)\sinh\left(bx+a\right)^4}{}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 3*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh
(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*
cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 3*cosh(b*x + a)^2 - 3*(cosh(b*x + a
)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(b*x +
a)^2 - 1)*sinh(b*x + a)^3 - 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3 - 3*c
osh(b*x + a))*sinh(b*x + a)^2 + (5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1
)*sinh(b*x + a) + cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) +
3*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2
*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 2*cosh(b*x + a)^3 + 2*(5*cosh(b
*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 + (5*cosh(b*x + a)^4 - 6*cosh
(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*log(cosh(b*x + a) + sinh(b
*x + a) - 1) + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sin
h(b*x + a) + 1)/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b
*sinh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 + 2*(5*b*cosh(b*x + a)^2 - b)*sinh(
b*x + a)^3 + 2*(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^2 +
 b*cosh(b*x + a) + (5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*
x + a))
```

3.103.6 Sympy [F]

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \int \cosh{(a+bx)}\coth^3{(a+bx)}\,dx$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)**3,x)
```

output
```
Integral(cosh(a + b*x)*coth(a + b*x)**3, x)
```

3.103.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(43) = 86$.

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.20

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \frac{e^{(-bx-a)}}{2\,b} - \frac{3\,\log\left(e^{(-bx-a)}+1\right)}{2\,b} + \frac{3\,\log\left(e^{(-bx-a)}-1\right)}{2\,b} - \frac{4\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} - 1}{2\,b(e^{(-bx-a)} - 2\,e^{(-3\,bx-3\,a)} + e^{(-5\,bx-5\,a)})}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*e^(-b*x - a)/b - 3/2*log(e^(-b*x - a) + 1)/b + 3/2*log(e^(-b*x - a) -
1)/b - 1/2*(4*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) - 1)/(b*(e^(-b*x - a) -
2*e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a)))
```

3.103.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(43) = 86.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.14

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = -\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} - 2\,e^{(bx+a)} - 2\,e^{(-bx-a)} + 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) - 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)} - \right.}{4\,b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)
- 2*e^(b*x + a) - 2*e^(-b*x - a) + 3*log(e^(b*x + a) + e^(-b*x - a) + 2) -
 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

3.103.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{e^{-a-b\,x}}{2\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(cosh(a + b*x)*coth(a + b*x)^3,x)
```

output
```
exp(a + b*x)/(2*b) - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/
2) + exp(- a - b*x)/(2*b) - (2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(
2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) - 1))
```

3.104 $\int \cosh(a+bx)\coth^4(a+bx)\,dx$

3.104.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx=-\frac{2\text{csch}(a+bx)}{b}-\frac{\text{csch}^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

output
```
-2*csch(b*x+a)/b-1/3*csch(b*x+a)^3/b+sinh(b*x+a)/b
```

3.104.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx=-\frac{2\text{csch}(a+bx)}{b}-\frac{\text{csch}^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Coth[a + b*x]^4,x]
```

output
```
(-2*Csch[a + b*x])/b - Csch[a + b*x]^3/(3*b) + Sinh[a + b*x]/b
```

3.104.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)\tan\left(ia+ibx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{i\int \text{csch}^4(a+bx)\left(\sinh^2(a+bx)+1\right)^2d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{i\int \left(\text{csch}^4(a+bx)+2\text{csch}^2(a+bx)+1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-i\sinh(a+bx)+\frac{1}{3}i\text{csch}^3(a+bx)+2i\text{csch}(a+bx)\right)}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[a + b*x]^4,x]
```

output
```
(I*((2*I)*Csch[a + b*x] + (I/3)*Csch[a + b*x]^3 - I*Sinh[a + b*x]))/b
```

3.104.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.104.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{\sinh(bx+a)^3}-\frac{4\cosh(bx+a)^2}{\sinh(bx+a)^3}+\frac{8}{3\sinh(bx+a)^3}}{b}$	51
default	$\frac{\frac{\cosh(bx+a)^4}{\sinh(bx+a)^3}-\frac{4\cosh(bx+a)^2}{\sinh(bx+a)^3}+\frac{8}{3\sinh(bx+a)^3}}{b}$	51
risch	$-\frac{-3\,\mathrm{e}^{7bx+7a}+36\,\mathrm{e}^{5bx+5a}-50\,\mathrm{e}^{3bx+3a}+36\,\mathrm{e}^{bx+a}-3\,\mathrm{e}^{-bx-a}}{6b\left(\mathrm{e}^{2bx+2a}-1\right)^3}$	72

input
```
int(cosh(b*x+a)*coth(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(cosh(b*x+a)^4/sinh(b*x+a)^3-4*cosh(b*x+a)^2/sinh(b*x+a)^3+8/3/sinh(b*
x+a)^3)
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(35) = 70$.

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.41

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)^4+3\sinh(bx+a)^4+18\left(\cosh(bx+a)^2-2\right)\sinh(bx+a)^2-36\cosh(bx+a)^2+25}{6\left(b\sinh(bx+a)^3+3\left(b\cosh(bx+a)^2-b\right)\sinh(bx+a)\right)}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/6*(3*cosh(b*x + a)^4 + 3*sinh(b*x + a)^4 + 18*(cosh(b*x + a)^2 - 2)*sinh
(b*x + a)^2 - 36*cosh(b*x + a)^2 + 25)/(b*sinh(b*x + a)^3 + 3*(b*cosh(b*x
+ a)^2 - b)*sinh(b*x + a))
```

3.104.6 Sympy [F]

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = \int \cosh\left(a+bx\right)\coth^4\left(a+bx\right)dx$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)**4,x)
```

output
```
Integral(cosh(a + b*x)*coth(a + b*x)**4, x)
```

3.104.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(35) = 70$.

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.70

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = -\frac{e^{(-bx-a)}}{2\,b} - \frac{33\,e^{(-2\,bx-2\,a)} - 41\,e^{(-4\,bx-4\,a)} + 27\,e^{(-6\,bx-6\,a)} - 3}{6\,b(e^{(-bx-a)} - 3\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)} - e^{(-7\,bx-7\,a)})}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/2*e^(-b*x - a)/b - 1/6*(33*e^(-2*b*x - 2*a) - 41*e^(-4*b*x - 4*a) + 27*
e^(-6*b*x - 6*a) - 3)/(b*(e^(-b*x - a) - 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x
- 5*a) - e^(-7*b*x - 7*a)))
```

3.104.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(35) = 70$.

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.92

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = -\frac{\frac{8\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+2\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3} - 3\,e^{(bx+a)} + 3\,e^{(-bx-a)}}{6\,b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/6*(8*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 2)/(e^(b*x + a) - e^(-b*x - a)
)^3 - 3*e^(b*x + a) + 3*e^(-b*x - a))/b
```

3.104.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.54

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{e^{-a-b\,x}}{2\,b} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{4\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(cosh(a + b*x)*coth(a + b*x)^4,x)
```

output
```
exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (8*exp(a + b*x))/(3*b*(exp(4*a
 + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(3*exp(2*a +
2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4*exp(a + b*x))/(b
*(exp(2*a + 2*b*x) - 1))
```

3.105 $\int \cosh^2(a+bx)\coth(a+bx)\,dx$

3.105.1 Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{2b}$$

output
```
ln(sinh(b*x+a))/b+1/2*sinh(b*x+a)^2/b
```

3.105.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx=\frac{2\log(\sinh(a+bx))+\sinh^2(a+bx)}{2b}$$

input
```
Integrate[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(2*Log[Sinh[a + b*x]] + Sinh[a + b*x]^2)/(2*b)
```

3.105.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -i\sin\left(ia+ibx+\frac{\pi}{2}\right)^2\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$
$$\Big\downarrow \text{3070}$$
$$\frac{\int i\operatorname{csch}(a+bx)\left(\sinh^2(a+bx)+1\right)d(-i\sinh(a+bx))}{b}$$
$$\Big\downarrow \text{244}$$
$$\frac{\int (i\operatorname{csch}(a+bx)+i\sinh(a+bx))d(-i\sinh(a+bx))}{b}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{1}{2}\sinh^2(a+bx)+\log(-i\sinh(a+bx))}{b}$$

input
```
Int[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(Log[(-I)*Sinh[a + b*x]] + Sinh[a + b*x]^2/2)/b
```

3.105.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.105.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	23
default	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	23
risch	$-x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}-\frac{2a}{b}+\frac{\ln(e^{2bx+2a}-1)}{b}$	55

input
```
int(cosh(b*x+a)^2*coth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*cosh(b*x+a)^2+ln(sinh(b*x+a)))
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(25) = 50.

Time = 0.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 7.52

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx =$$

$$-\frac{8\,bx\cosh\left(bx+a\right)^2-\cosh\left(bx+a\right)^4-4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\sinh\left(bx+a\right)^4+2\left(4\,bx-3\,\cos\right.}{}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(8*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 - 4*cosh(b*x + a)*sinh(b*x +
 a)^3 - sinh(b*x + a)^4 + 2*(4*b*x - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 -
8*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(
2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a)
- cosh(b*x + a)^3)*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x +
a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.105.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(20) = 40.

Time = 0.83 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.89

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx$$

$$= \begin{cases} \tilde{\infty}x \\ x\cosh^2\left(a\right)\coth\left(a\right) \\ \tilde{\infty}x \\ -\frac{x\sinh^2\left(a+bx\right)\coth\left(a+bx\right)}{2}+\frac{x\cosh^2\left(a+bx\right)\coth\left(a+bx\right)}{2}-\frac{x\cosh\left(a+bx\right)}{2\sinh\left(a+bx\right)}+\frac{\log\left(\sinh\left(a+bx\right)\right)}{b}+\frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)\coth\left(a+b\right.}{2b} \end{cases}$$

input
```
integrate(cosh(b*x+a)**2*coth(b*x+a),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x*cosh(a)**2*coth(a), Eq(b, 0)),
(zoo*x, Eq(a, -b*x)), (-x*sinh(a + b*x)**2*coth(a + b*x)/2 + x*cosh(a + b*
x)**2*coth(a + b*x)/2 - x*cosh(a + b*x)/(2*sinh(a + b*x)) + log(sinh(a + b
*x))/b + sinh(a + b*x)*cosh(a + b*x)*coth(a + b*x)/(2*b), True))
```

3.105.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(25) = 50$.

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{bx+a}{b} + \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="maxima")
```

output
```
(b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b + log(e^(-b*x
 - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

3.105.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{8\,bx - \left(4\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)} + 8\,a - e^{(2\,bx+2\,a)} - 8\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{8\,b}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(8*b*x - (4*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 8*a - e^(2*b*x +
2*a) - 8*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.105.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\ln\left(e^{2a}\,e^{2bx}-1\right)}{b} - x + \frac{e^{-2a-2bx}}{8b} + \frac{e^{2a+2bx}}{8b}$$

input
```
int(cosh(a + b*x)^2*coth(a + b*x),x)
```

output
```
log(exp(2*a)*exp(2*b*x) - 1)/b - x + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a +
2*b*x)/(8*b)
```

3.106 $\int \cosh^2(a+bx)\coth^2(a+bx)\,dx$

3.106.1 Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx=\frac{3x}{2}-\frac{3\coth(a+bx)}{2b}+\frac{\cosh^2(a+bx)\coth(a+bx)}{2b}$$

output
```
3/2*x-3/2*coth(b*x+a)/b+1/2*cosh(b*x+a)^2*coth(b*x+a)/b
```

3.106.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx=\frac{6(a+bx)-4\coth(a+bx)+\sinh(2(a+bx))}{4b}$$

input
```
Integrate[Cosh[a + b*x]^2*Coth[a + b*x]^2,x]
```

output
```
(6*(a + b*x) - 4*Coth[a + b*x] + Sinh[2*(a + b*x)])/(4*b)
```

3.106.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 25, 3071, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(ia+ibx+\frac{\pi}{2}\right)^2\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{i\int \frac{\coth^4(a+bx)}{(1-\coth^2(a+bx))^2}d(i\coth(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{i\left(\frac{3}{2}\int -\frac{\coth^2(a+bx)}{1-\coth^2(a+bx)}d(i\coth(a+bx))+\frac{i\coth^3(a+bx)}{2(1-\coth^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{i\left(\frac{3}{2}\left(i\coth(a+bx)-\int \frac{1}{1-\coth^2(a+bx)}d(i\coth(a+bx))\right)+\frac{i\coth^3(a+bx)}{2(1-\coth^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{i\left(\frac{3}{2}(i\coth(a+bx)-i\text{arctanh}(\coth(a+bx)))+\frac{i\coth^3(a+bx)}{2(1-\coth^2(a+bx))}\right)}{b}$$

input
```
Int[Cosh[a + b*x]^2*Coth[a + b*x]^2,x]
```

output
```
(I*((3*((-I)*ArcTanh[Coth[a + b*x]] + I*Coth[a + b*x]))/2 + ((I/2)*Coth[a
+ b*x]^3)/(1 - Coth[a + b*x]^2)))/b
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.106.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^3}{2\sinh(bx+a)}+\frac{3bx}{2}+\frac{3a}{2}-\frac{3\coth(bx+a)}{2}}{b}$	39
default	$\frac{\frac{\cosh(bx+a)^3}{2\sinh(bx+a)}+\frac{3bx}{2}+\frac{3a}{2}-\frac{3\coth(bx+a)}{2}}{b}$	39
risch	$\frac{3x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}-\frac{2}{b(e^{2bx+2a}-1)}$	51

input
```
int(cosh(b*x+a)^2*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2/sinh(b*x+a)*cosh(b*x+a)^3+3/2*b*x+3/2*a-3/2*coth(b*x+a))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^3+3\cosh(bx+a)\sinh(bx+a)^2+4\,(3\,bx+2)\sinh(bx+a)-9\cosh(bx+a)}{8\,b\sinh(bx+a)}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + 4*(3*b*x + 2)*sin
h(b*x + a) - 9*cosh(b*x + a))/(b*sinh(b*x + a))
```

3.106.6 Sympy [F]

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx=\int \cosh^2(a+bx)\coth^2(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**2*coth(b*x+a)**2,x)
```

output
```
Integral(cosh(a + b*x)**2*coth(a + b*x)**2, x)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = \frac{3\,(bx+a)}{2\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{17\,e^{(-2\,bx-2\,a)}-1}{8\,b(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)})}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="maxima")
```

output
```
3/2*(b*x + a)/b - 1/8*e^(-2*b*x - 2*a)/b - 1/8*(17*e^(-2*b*x - 2*a) - 1)/(
b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a)))
```

3.106.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = \frac{12\,bx+12\,a-\frac{3\,e^{(4\,bx+4\,a)}+14\,e^{(2\,bx+2\,a)}-1}{e^{(4\,bx+4\,a)}-e^{(2\,bx+2\,a)}}+e^{(2\,bx+2\,a)}}{8\,b}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="giac")
```

output
```
1/8*(12*b*x + 12*a - (3*e^(4*b*x + 4*a) + 14*e^(2*b*x + 2*a) - 1)/(e^(4*b*
x + 4*a) - e^(2*b*x + 2*a)) + e^(2*b*x + 2*a))/b
```

3.106.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = \frac{3\,x}{2} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{e^{-2\,a-2\,b\,x}}{8\,b} + \frac{e^{2\,a+2\,b\,x}}{8\,b}$$

input
```
int(cosh(a + b*x)^2*coth(a + b*x)^2,x)
```

output
```
(3*x)/2 - 2/(b*(exp(2*a + 2*b*x) - 1)) - exp(- 2*a - 2*b*x)/(8*b) + exp(2*
a + 2*b*x)/(8*b)
```

3.107 $\int \cosh^2(a+bx)\coth^3(a+bx)\,dx$

3.107.1 Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx=-\frac{\operatorname{csch}^2(a+bx)}{2b}+\frac{2\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{2b}$$

output
```
-1/2*csch(b*x+a)^2/b+2*ln(sinh(b*x+a))/b+1/2*sinh(b*x+a)^2/b
```

3.107.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx=-\frac{\operatorname{csch}^2(a+bx)-4\log(\sinh(a+bx))-\sinh^2(a+bx)}{2b}$$

input
```
Integrate[Cosh[a + b*x]^2*Coth[a + b*x]^3,x]
```

output
```
-1/2*(Csch[a + b*x]^2 - 4*Log[Sinh[a + b*x]] - Sinh[a + b*x]^2)/b
```

3.107.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin\left(ia+ibx+\frac{\pi}{2}\right)^2\tan\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int -i\text{csch}^3(a+bx)\left(\sinh^2(a+bx)+1\right)^2d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\int -\text{csch}^2(a+bx)(i\sinh(a+bx)+1)^2d(-\sinh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(-\text{csch}^2(a+bx)-2i\text{csch}(a+bx)+1\right)d(-\sinh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\sinh^2(a+bx)-i\text{csch}(a+bx)-2\log\left(-\sinh^2(a+bx)\right)}{2b}$$

input
```
Int[Cosh[a + b*x]^2*Coth[a + b*x]^3,x]
```

output
```
-1/2*((-I)*Csch[a + b*x] - 2*Log[-Sinh[a + b*x]^2] - Sinh[a + b*x]^2)/b
```

3.107.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.107.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{2\sinh(bx+a)^2}+2\ln(\sinh(bx+a))-\coth(bx+a)^2}{b}$	43
default	$\frac{\frac{\cosh(bx+a)^4}{2\sinh(bx+a)^2}+2\ln(\sinh(bx+a))-\coth(bx+a)^2}{b}$	43
risch	$-2x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}-\frac{4a}{b}-\frac{2\,e^{2bx+2a}}{b\left(e^{2bx+2a}-1\right)^2}+\frac{2\ln\left(e^{2bx+2a}-1\right)}{b}$	83

input
```
int(cosh(b*x+a)^2*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*cosh(b*x+a)^4/sinh(b*x+a)^2+2*ln(sinh(b*x+a))-coth(b*x+a)^2)
```

3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 743, normalized size of antiderivative = 17.28

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx$$

$$= \frac{\cosh(bx+a)^8 + 8\cosh(bx+a)\sinh(bx+a)^7 + \sinh(bx+a)^8 - 2(8bx+1)\cosh(bx+a)^6 - 2(8bx-}{}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 -
 2*(8*b*x + 1)*cosh(b*x + a)^6 - 2*(8*b*x - 14*cosh(b*x + a)^2 + 1)*sinh(b
*x + a)^6 + 4*(14*cosh(b*x + a)^3 - 3*(8*b*x + 1)*cosh(b*x + a))*sinh(b*x
+ a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^4 + 2*(35*cosh(b*x + a)^4 - 15*(8*b*
x + 1)*cosh(b*x + a)^2 + 16*b*x - 7)*sinh(b*x + a)^4 + 8*(7*cosh(b*x + a)^
5 - 5*(8*b*x + 1)*cosh(b*x + a)^3 + (16*b*x - 7)*cosh(b*x + a))*sinh(b*x +
 a)^3 - 2*(8*b*x + 1)*cosh(b*x + a)^2 + 2*(14*cosh(b*x + a)^6 - 15*(8*b*x
+ 1)*cosh(b*x + a)^4 + 6*(16*b*x - 7)*cosh(b*x + a)^2 - 8*b*x - 1)*sinh(b*
x + a)^2 + 16*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*
x + a)^6 + (15*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 +
4*(5*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a
)^4 - 12*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x + a)^2 + 2*(3*cos
h(b*x + a)^5 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))*log(2*sin
h(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(2*cosh(b*x + a)^7 - 3*(8*
b*x + 1)*cosh(b*x + a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^3 - (8*b*x + 1)*co
sh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sin
h(b*x + a)^5 + b*sinh(b*x + a)^6 - 2*b*cosh(b*x + a)^4 + (15*b*cosh(b*x +
a)^2 - 2*b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 2*b*cosh(b*x + a))*
sinh(b*x + a)^3 + b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 - 12*b*cosh(b*
x + a)^2 + b)*sinh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 - 4*b*cosh(b*x +...
```

3.107.6 Sympy [F]

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = \int \cosh^2(a+bx)\coth^3(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**2*coth(b*x+a)**3,x)
```

output
```
Integral(cosh(a + b*x)**2*coth(a + b*x)**3, x)
```

3.107.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(39) = 78$.

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.79

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = \frac{2(bx+a)}{b} + \frac{e^{(-2bx-2a)}}{8b} + \frac{2\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{2\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{2e^{(-2bx-2a)}+15e^{(-4bx-4a)}-1}{8b\left(e^{(-2bx-2a)}-2e^{(-4bx-4a)}+e^{(-6bx-6a)}\right)}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="maxima")
```

output
```
2*(b*x + a)/b + 1/8*e^(-2*b*x - 2*a)/b + 2*log(e^(-b*x - a) + 1)/b + 2*log
(e^(-b*x - a) - 1)/b - 1/8*(2*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) - 1)/
(b*(e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

3.107.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(39) = 78$.

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx =$$
$$-\frac{16bx - \left(8e^{(2bx+2a)}+1\right)e^{(-2bx-2a)} + 16a + \frac{8\left(3e^{(4bx+4a)}-4e^{(2bx+2a)}+3\right)}{\left(e^{(2bx+2a)}-1\right)^2} - e^{(2bx+2a)} - 16\log\left(\left|e^{(2bx+2a)} - \right.\right.}{8b}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/8*(16*b*x - (8*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 16*a + 8*(3*e^(4
*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 3)/(e^(2*b*x + 2*a) - 1)^2 - e^(2*b*x +
2*a) - 16*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.107.9 Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.26

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = \frac{2\ln\left(e^{2a}\,e^{2bx}-1\right)}{b} - 2x - \frac{2}{b\left(e^{2a+2bx}-1\right)} - \frac{2}{b\left(e^{4a+4bx}-2e^{2a+2bx}+1\right)} + \frac{e^{-2a-2bx}}{8b} + \frac{e^{2a+2bx}}{8b}$$

input
```
int(cosh(a + b*x)^2*coth(a + b*x)^3,x)
```

output
```
(2*log(exp(2*a)*exp(2*b*x) - 1))/b - 2*x - 2/(b*(exp(2*a + 2*b*x) - 1)) -
2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) + exp(- 2*a - 2*b*x)/(8*
b) + exp(2*a + 2*b*x)/(8*b)
```

3.108 $\int \cosh^3(a+bx)\coth(a+bx)\,dx$

3.108.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}$$

output
```
-arctanh(cosh(b*x+a))/b+cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b
```

3.108.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx=\frac{5\cosh(a+bx)}{4b}+\frac{\cosh(3(a+bx))}{12b}-\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

input
```
Integrate[Cosh[a + b*x]^3*Coth[a + b*x],x]
```

output
```
(5*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Log[Cosh[(a + b*x)/2]
]/b + Log[Sinh[(a + b*x)/2]]/b
```

3.108.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3072, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin\left(ia+ibx+\frac{\pi}{2}\right)^3\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{\int \frac{\cosh^4(a+bx)}{1-\cosh^2(a+bx)}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\int \left(-\cosh^2(a+bx)+\frac{1}{1-\cosh^2(a+bx)}-1\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}(\cosh(a+bx))-\frac{1}{3}\cosh^3(a+bx)-\cosh(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x]^3*Coth[a + b*x],x]
```

output
```
-((ArcTanh[Cosh[a + b*x]] - Cosh[a + b*x] - Cosh[a + b*x]^3/3)/b)
```

3.108.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.108.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^3}{3}+\cosh(bx+a)-2\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	31
default	$\frac{\frac{\cosh(bx+a)^3}{3}+\cosh(bx+a)-2\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	31
risch	$\frac{e^{3bx+3a}}{24b}+\frac{5\,e^{bx+a}}{8b}+\frac{5\,e^{-bx-a}}{8b}+\frac{e^{-3bx-3a}}{24b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}-\frac{\ln\left(e^{bx+a}+1\right)}{b}$	82

input
```
int(cosh(b*x+a)^3*coth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*cosh(b*x+a)^3+cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 357, normalized size of antiderivative = 9.39

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 15\left(\cosh(bx+a)^2+1\right)\sinh(bx+a)^4}{}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="fricas")
```

output
```
1/24*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
+ 15*(cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 15*cosh(b*x + a)^4 + 20*(cosh
(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 15*(cosh(b*x + a)^4 + 6*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 15*cosh(b*x + a)^2 - 24*(cosh(b*x +
a)^3 + 3*cosh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 +
 sinh(b*x + a)^3)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 24*(cosh(b*x +
a)^3 + 3*cosh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 +
 sinh(b*x + a)^3)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 6*(cosh(b*x + a
)^5 + 10*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x
 + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x +
 a)^2 + b*sinh(b*x + a)^3)
```

3.108.6 Sympy [F]

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \int \cosh^3(a+bx)\coth(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**3*coth(b*x+a),x)
```

output
```
Integral(cosh(a + b*x)**3*coth(a + b*x), x)
```

3.108.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(36) = 72.

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{\left(15\,e^{(-2\,bx-2\,a)}+1\right)e^{(3\,bx+3\,a)}}{24\,b} + \frac{15\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{24\,b} - \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="maxima")
```

output
```
1/24*(15*e^(-2*b*x - 2*a) + 1)*e^(3*b*x + 3*a)/b + 1/24*(15*e^(-b*x - a) +
 e^(-3*b*x - 3*a))/b - log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{\left(15\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}+e^{(3\,bx+3\,a)}+15\,e^{(bx+a)}-24\,\log\left(e^{(bx+a)}+1\right)+24\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{24\,b}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="giac")
```

output
```
1/24*((15*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) + e^(3*b*x + 3*a) + 15*e^(
b*x + a) - 24*log(e^(b*x + a) + 1) + 24*log(abs(e^(b*x + a) - 1)))/b
```

3.108.9 Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.13

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{5\,\mathrm{e}^{a+b\,x}}{8\,b} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{5\,\mathrm{e}^{-a-b\,x}}{8\,b} + \frac{\mathrm{e}^{-3\,a-3\,b\,x}}{24\,b} + \frac{\mathrm{e}^{3\,a+3\,b\,x}}{24\,b}$$

input
```
int(cosh(a + b*x)^3*coth(a + b*x),x)
```

output
```
(5*exp(a + b*x))/(8*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)
^(1/2) + (5*exp(- a - b*x))/(8*b) + exp(- 3*a - 3*b*x)/(24*b) + exp(3*a +
3*b*x)/(24*b)
```

3.108. $\int \cosh^3(a+bx)\coth(a+bx)\,dx$

3.109 $\int \cosh^3(a+bx)\coth^2(a+bx)\,dx$

3.109.1 Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}+\frac{2\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

output
```
-csch(b*x+a)/b+2*sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b
```

3.109.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}+\frac{2\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

input
```
Integrate[Cosh[a + b*x]^3*Coth[a + b*x]^2,x]
```

output
```
-(Csch[a + b*x]/b) + (2*Sinh[a + b*x])/b + Sinh[a + b*x]^3/(3*b)
```

3.109.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 25, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(ia+ibx+\frac{\pi}{2}\right)^3\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{i\int -\text{csch}^2(a+bx)\left(\sinh^2(a+bx)+1\right)^2d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{i\int \left(-\text{csch}^2(a+bx)-\sinh^2(a+bx)-2\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{3}i\sinh^3(a+bx)+2i\sinh(a+bx)-i\text{csch}(a+bx)\right)}{b}$$

input
```
Int[Cosh[a + b*x]^3*Coth[a + b*x]^2,x]
```

output
```
((-I)*((-I)*Csch[a + b*x] + (2*I)*Sinh[a + b*x] + (I/3)*Sinh[a + b*x]^3))/
b
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.109.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{3\sinh(bx+a)}+\frac{4\cosh(bx+a)^2}{3\sinh(bx+a)}-\frac{8}{3\sinh(bx+a)}}{b}$	52
default	$\frac{\frac{\cosh(bx+a)^4}{3\sinh(bx+a)}+\frac{4\cosh(bx+a)^2}{3\sinh(bx+a)}-\frac{8}{3\sinh(bx+a)}}{b}$	52
risch	$\frac{e^{5bx+5a}+20\,e^{3bx+3a}-90\,e^{bx+a}+20\,e^{-bx-a}+e^{-3bx-3a}}{24b\left(e^{2bx+2a}-1\right)}$	68

input
```
int(cosh(b*x+a)^3*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3/sinh(b*x+a)*cosh(b*x+a)^4+4/3/sinh(b*x+a)*cosh(b*x+a)^2-8/3/sinh(
b*x+a))
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \frac{\cosh(bx+a)^4+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2+10\right)\sinh(bx+a)^2+20\cosh(bx+a)^2-45}{24\,b\sinh(bx+a)}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/24*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 10)*sinh(
b*x + a)^2 + 20*cosh(b*x + a)^2 - 45)/(b*sinh(b*x + a))
```

3.109.6 Sympy [F]

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \int \cosh^3(a+bx)\coth^2(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**3*coth(b*x+a)**2,x)
```

output
```
Integral(cosh(a + b*x)**3*coth(a + b*x)**2, x)
```

3.109.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(36) = 72.

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.08

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = -\frac{21\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{24\,b} + \frac{20\,e^{(-2\,bx-2\,a)}-69\,e^{(-4\,bx-4\,a)}+1}{24\,b\left(e^{(-3\,bx-3\,a)}-e^{(-5\,bx-5\,a)}\right)}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/24*(21*e^(-b*x - a) + e^(-3*b*x - 3*a))/b + 1/24*(20*e^(-2*b*x - 2*a) -
 69*e^(-4*b*x - 4*a) + 1)/(b*(e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a)))
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \frac{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^3 - \frac{48}{e^{(bx+a)} - e^{(-bx-a)}} + 24\,e^{(bx+a)} - 24\,e^{(-bx-a)}}{24\,b}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="giac")
```

output
```
1/24*((e^(b*x + a) - e^(-b*x - a))^3 - 48/(e^(b*x + a) - e^(-b*x - a)) + 2
4*e^(b*x + a) - 24*e^(-b*x - a))/b
```

3.109.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \frac{7\,e^{a+b\,x}}{8\,b} - \frac{7\,e^{-a-b\,x}}{8\,b} - \frac{e^{-3\,a-3\,b\,x}}{24\,b} + \frac{e^{3\,a+3\,b\,x}}{24\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(cosh(a + b*x)^3*coth(a + b*x)^2,x)
```

output
```
(7*exp(a + b*x))/(8*b) - (7*exp(- a - b*x))/(8*b) - exp(- 3*a - 3*b*x)/(24
*b) + exp(3*a + 3*b*x)/(24*b) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1)
)
```

3.110 $\int \cosh^3(a+bx)\coth^3(a+bx)\,dx$

3.110.1 Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx=-\frac{5\text{arctanh}(\cosh(a+bx))}{2b}+\frac{5\cosh(a+bx)}{2b}+\frac{5\cosh^3(a+bx)}{6b}-\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b}$$

output
```
-5/2*arctanh(cosh(b*x+a))/b+5/2*cosh(b*x+a)/b+5/6*cosh(b*x+a)^3/b-1/2*cosh
(b*x+a)^3*coth(b*x+a)^2/b
```

3.110.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.56

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx=\frac{9\cosh(a+bx)}{4b}+\frac{\cosh(3(a+bx))}{12b}-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{5\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{5\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

input
```
Integrate[Cosh[a + b*x]^3*Coth[a + b*x]^3,x]
```

output
```
(9*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Csch[(a + b*x)/2]^2/(
8*b) - (5*Log[Cosh[(a + b*x)/2]])/(2*b) + (5*Log[Sinh[(a + b*x)/2]])/(2*b)
 - Sech[(a + b*x)/2]^2/(8*b)
```

3.110.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 3072, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin\left(ia+ibx+\frac{\pi}{2}\right)^3\tan\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{\cosh^6(a+bx)}{(1-\cosh^2(a+bx))^2}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\cosh^5(a+bx)}{2(1-\cosh^2(a+bx))}-\frac{5}{2}\int \frac{\cosh^4(a+bx)}{1-\cosh^2(a+bx)}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{\cosh^5(a+bx)}{2(1-\cosh^2(a+bx))}-\frac{5}{2}\int \left(-\cosh^2(a+bx)+\frac{1}{1-\cosh^2(a+bx)}-1\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\cosh^5(a+bx)}{2(1-\cosh^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\cosh(a+bx))-\frac{1}{3}\cosh^3(a+bx)-\cosh(a+bx)\right)}{b}$$

input
```
Int[Cosh[a + b*x]^3*Coth[a + b*x]^3,x]
```

output
```
(Cosh[a + b*x]^5/(2*(1 - Cosh[a + b*x]^2)) - (5*(ArcTanh[Cosh[a + b*x]] -
Cosh[a + b*x] - Cosh[a + b*x]^3/3))/2)/b
```

3.110.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.110.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^5}{3\sinh(bx+a)^2}+\frac{5\cosh(bx+a)^3}{3\sinh(bx+a)^2}-\frac{5\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{5\coth(bx+a)\operatorname{csch}(bx+a)}{2}-5\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	81
default	$\frac{\frac{\cosh(bx+a)^5}{3\sinh(bx+a)^2}+\frac{5\cosh(bx+a)^3}{3\sinh(bx+a)^2}-\frac{5\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{5\coth(bx+a)\operatorname{csch}(bx+a)}{2}-5\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	81
risch	$\frac{e^{3bx+3a}}{24b}+\frac{9\,e^{bx+a}}{8b}+\frac{9\,e^{-bx-a}}{8b}+\frac{e^{-3bx-3a}}{24b}-\frac{e^{bx+a}\left(1+e^{2bx+2a}\right)}{b\left(e^{2bx+2a}-1\right)^2}-\frac{5\ln\left(e^{bx+a}+1\right)}{2b}+\frac{5\ln\left(e^{bx+a}-1\right)}{2b}$	118

input
```
int(cosh(b*x+a)^3*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3/sinh(b*x+a)^2*cosh(b*x+a)^5+5/3/sinh(b*x+a)^2*cosh(b*x+a)^3-5*cos
h(b*x+a)/sinh(b*x+a)^2+5/2*coth(b*x+a)*csch(b*x+a)-5*arctanh(exp(b*x+a)))
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. 2(58) = 116.

Time = 0.28 (sec) , antiderivative size = 1077, normalized size of antiderivative = 16.32

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/24*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^
10 + 5*(9*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^8 + 25*cosh(b*x + a)^8 + 40*(
3*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^7 + 10*(21*cosh(b*x + a
)^4 + 70*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^6 - 50*cosh(b*x + a)^6 + 4*(63
*cosh(b*x + a)^5 + 350*cosh(b*x + a)^3 - 75*cosh(b*x + a))*sinh(b*x + a)^5
 + 10*(21*cosh(b*x + a)^6 + 175*cosh(b*x + a)^4 - 75*cosh(b*x + a)^2 - 5)*
sinh(b*x + a)^4 - 50*cosh(b*x + a)^4 + 40*(3*cosh(b*x + a)^7 + 35*cosh(b*x
 + a)^5 - 25*cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^3 + 5*(9*cos
h(b*x + a)^8 + 140*cosh(b*x + a)^6 - 150*cosh(b*x + a)^4 - 60*cosh(b*x + a
)^2 + 5)*sinh(b*x + a)^2 + 25*cosh(b*x + a)^2 - 60*(cosh(b*x + a)^7 + 7*co
sh(b*x + a)*sinh(b*x + a)^6 + sinh(b*x + a)^7 + (21*cosh(b*x + a)^2 - 2)*s
inh(b*x + a)^5 - 2*cosh(b*x + a)^5 + 5*(7*cosh(b*x + a)^3 - 2*cosh(b*x + a
))*sinh(b*x + a)^4 + (35*cosh(b*x + a)^4 - 20*cosh(b*x + a)^2 + 1)*sinh(b*
x + a)^3 + cosh(b*x + a)^3 + (21*cosh(b*x + a)^5 - 20*cosh(b*x + a)^3 + 3*
cosh(b*x + a))*sinh(b*x + a)^2 + (7*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 +
 3*cosh(b*x + a)^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1)
+ 60*(cosh(b*x + a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 + sinh(b*x + a)^7
+ (21*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^5 - 2*cosh(b*x + a)^5 + 5*(7*cosh
(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^4 + (35*cosh(b*x + a)^4 - 20*
cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + cosh(b*x + a)^3 + (21*cosh(b*x +...
```

3.110.6 Sympy [F]

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \int \cosh^3{(a+bx)}\coth^3{(a+bx)}\,dx$$

input
```
integrate(cosh(b*x+a)**3*coth(b*x+a)**3,x)
```

output
```
Integral(cosh(a + b*x)**3*coth(a + b*x)**3, x)
```

3.110.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(58) = 116$.

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.02

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{27\,e^{(-bx-a)} + e^{(-3\,bx-3\,a)}}{24\,b} - \frac{5\,\log\left(e^{(-bx-a)}+1\right)}{2\,b} + \frac{5\,\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{25\,e^{(-2\,bx-2\,a)} - 77\,e^{(-4\,bx-4\,a)} + 3\,e^{(-6\,bx-6\,a)} + 1}{24\,b\left(e^{(-3\,bx-3\,a)} - 2\,e^{(-5\,bx-5\,a)} + e^{(-7\,bx-7\,a)}\right)}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*(27*e^(-b*x - a) + e^(-3*b*x - 3*a))/b - 5/2*log(e^(-b*x - a) + 1)/b
+ 5/2*log(e^(-b*x - a) - 1)/b + 1/24*(25*e^(-2*b*x - 2*a) - 77*e^(-4*b*x -
 4*a) + 3*e^(-6*b*x - 6*a) + 1)/(b*(e^(-3*b*x - 3*a) - 2*e^(-5*b*x - 5*a)
+ e^(-7*b*x - 7*a)))
```

3.110.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(58) = 116$.

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.86

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{\left(e^{(bx+a)} + e^{(-bx-a)}\right)^3 - \frac{24\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} + 24\,e^{(bx+a)} + 24\,e^{(-bx-a)} - 30\,\log\left(e^{(bx+a)} + e^{(-bx-a)} + 2\right) + 30}{24\,b}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="giac")
```

output
```
1/24*((e^(b*x + a) + e^(-b*x - a))^3 - 24*(e^(b*x + a) + e^(-b*x - a))/((e
^(b*x + a) + e^(-b*x - a))^2 - 4) + 24*e^(b*x + a) + 24*e^(-b*x - a) - 30*
log(e^(b*x + a) + e^(-b*x - a) + 2) + 30*log(e^(b*x + a) + e^(-b*x - a) -
2))/b
```

3.110.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.12

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{9\,e^{a+b\,x}}{8\,b} - \frac{5\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{9\,e^{-a-b\,x}}{8\,b} + \frac{e^{-3\,a-3\,b\,x}}{24\,b} + \frac{e^{3\,a+3\,b\,x}}{24\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(cosh(a + b*x)^3*coth(a + b*x)^3,x)
```

output
```
(9*exp(a + b*x))/(8*b) - (5*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)
^(1/2) + (9*exp(- a - b*x))/(8*b) + exp(- 3*a - 3*b*x)/(24*b) + exp(3*a +
3*b*x)/(24*b) - (2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x)
 + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) - 1))
```

3.111 $\int \cosh^4(a+bx)\coth(a+bx)\,dx$

3.111.1 Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{b}+\frac{\sinh^4(a+bx)}{4b}$$

output
```
ln(sinh(b*x+a))/b+sinh(b*x+a)^2/b+1/4*sinh(b*x+a)^4/b
```

3.111.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx=\frac{4\log(\sinh(a+bx))+4\sinh^2(a+bx)+\sinh^4(a+bx)}{4b}$$

input
```
Integrate[Cosh[a + b*x]^4*Coth[a + b*x],x]
```

output
```
(4*Log[Sinh[a + b*x]] + 4*Sinh[a + b*x]^2 + Sinh[a + b*x]^4)/(4*b)
```

3.111.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin\left(ia+ibx+\frac{\pi}{2}\right)^4\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^4\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int i\,\text{csch}(a+bx)\left(\sinh^2(a+bx)+1\right)^2d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int i\,\text{csch}(a+bx)(i\sinh(a+bx)+1)^2d(-\sinh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(-\sinh^2(a+bx)+i\,\text{csch}(a+bx)-2\right)d(-\sinh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\sinh^2(a+bx)+2i\sinh(a+bx)+\log\left(-\sinh^2(a+bx)\right)}{2b}$$

input
```
Int[Cosh[a + b*x]^4*Coth[a + b*x],x]
```

output
```
(Log[-Sinh[a + b*x]^2] + (2*I)*Sinh[a + b*x] - Sinh[a + b*x]^2/2)/(2*b)
```

3.111.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.111.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{4}+\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	33
default	$\frac{\frac{\cosh(bx+a)^4}{4}+\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	33
risch	$-x+\frac{e^{4bx+4a}}{64b}+\frac{3\,e^{2bx+2a}}{16b}+\frac{3\,e^{-2bx-2a}}{16b}+\frac{e^{-4bx-4a}}{64b}-\frac{2a}{b}+\frac{\ln(e^{2bx+2a}-1)}{b}$	83

input
```
int(cosh(b*x+a)^4*coth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*cosh(b*x+a)^4+1/2*cosh(b*x+a)^2+ln(sinh(b*x+a)))
```

3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(37) = 74.

Time = 0.26 (sec) , antiderivative size = 457, normalized size of antiderivative = 11.72

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^8 + 8\cosh(bx+a)\sinh(bx+a)^7 + \sinh(bx+a)^8 + 4\left(7\cosh(bx+a)^2+3\right)\sinh(bx+a)^6}{}$$

input
```
integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="fricas")
```

output
```
1/64*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8
+ 4*(7*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^6 - 64*b*x*cosh(b*x + a)^4 + 12*
cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x + a)^5
+ 2*(35*cosh(b*x + a)^4 - 32*b*x + 90*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8
*(7*cosh(b*x + a)^5 - 32*b*x*cosh(b*x + a) + 30*cosh(b*x + a)^3)*sinh(b*x
+ a)^3 + 4*(7*cosh(b*x + a)^6 - 96*b*x*cosh(b*x + a)^2 + 45*cosh(b*x + a)^
4 + 3)*sinh(b*x + a)^2 + 12*cosh(b*x + a)^2 + 64*(cosh(b*x + a)^4 + 4*cosh
(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x
 + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4)*log(2*sinh(b*x + a)/(cosh(b*x + a
) - sinh(b*x + a))) + 8*(cosh(b*x + a)^7 - 32*b*x*cosh(b*x + a)^3 + 9*cosh
(b*x + a)^5 + 3*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^4 + 4*b
*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b
*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4)
```

3.111.6 Sympy [F]

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = \int \cosh^4(a+bx)\coth(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**4*coth(b*x+a),x)
```

output
```
Integral(cosh(a + b*x)**4*coth(a + b*x), x)
```

3.111.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(37) = 74$.

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = \frac{\left(12\,e^{(-2\,bx-2\,a)}+1\right)e^{(4\,bx+4\,a)}}{64\,b} + \frac{bx+a}{b} + \frac{12\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}}{64\,b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="maxima")
```

output
```
1/64*(12*e^(-2*b*x - 2*a) + 1)*e^(4*b*x + 4*a)/b + (b*x + a)/b + 1/64*(12*
e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/b + log(e^(-b*x - a) + 1)/b + log(e^(
-b*x - a) - 1)/b
```

3.111.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.18

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = -\frac{64\,bx-\left(48\,e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}+1\right)e^{(-4\,bx-4\,a)}+64\,a-e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}-64\,\log\left(\left|e^{(2\,bx+2\,a}\right.\right.}{64\,b}$$

input
```
integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="giac")
```

output
```
-1/64*(64*b*x - (48*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a) + 1)*e^(-4*b*x -
4*a) + 64*a - e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a) - 64*log(abs(e^(2*b*x +
 2*a) - 1)))/b
```

3.111.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.97

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = \frac{\ln\left(e^{2a}\,e^{2bx}-1\right)}{b} - x + \frac{3\,e^{-2a-2bx}}{16\,b} + \frac{3\,e^{2a+2bx}}{16\,b} + \frac{e^{-4a-4bx}}{64\,b} + \frac{e^{4a+4bx}}{64\,b}$$

input

```
int(cosh(a + b*x)^4*coth(a + b*x),x)
```

output

```
log(exp(2*a)*exp(2*b*x) - 1)/b - x + (3*exp(- 2*a - 2*b*x))/(16*b) + (3*ex
p(2*a + 2*b*x))/(16*b) + exp(- 4*a - 4*b*x)/(64*b) + exp(4*a + 4*b*x)/(64*
b)
```

3.112 $\int \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

3.112.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx=-\frac{\text{csch}(a+bx)}{b}$$

output
```
-csch(b*x+a)/b
```

3.112.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx=-\frac{\text{csch}(a+bx)}{b}$$

input
```
Integrate[Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
-(Csch[a + b*x]/b)
```

3.112.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan\left(ia+ibx-\frac{\pi}{2}\right)\sec\left(ia+ibx-\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{3086}$$
$$-\frac{i\int 1d(-i\text{csch}(a+bx))}{b}$$
$$\Big\downarrow \text{24}$$
$$-\frac{\text{csch}(a+bx)}{b}$$

input
```
Int[Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
-(Csch[a + b*x]/b)
```

3.112.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.112.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(bx+a)}{b}$	12
default	$-\frac{\operatorname{csch}(bx+a)}{b}$	12
risch	$-\frac{2\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}$	25

input
```
int(coth(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-csch(b*x+a)/b
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(11) = 22$.

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 5.09

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}-b}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

3.112.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \begin{cases} -\frac{\operatorname{csch}\left(a+bx\right)}{b} & \text{for } b\neq 0 \\ x\coth\left(a\right)\operatorname{csch}\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a),x)
```

output
```
Piecewise((-csch(a + b*x)/b, Ne(b, 0)), (x*coth(a)*csch(a), True))
```

3.112.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2}{b\left(e^{(bx+a)} - e^{(-bx-a)}\right)}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

output
```
-2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

3.112.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2}{b\left(e^{(bx+a)} - e^{(-bx-a)}\right)}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

output
```
-2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

3.112.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(coth(a + b*x)/sinh(a + b*x),x)
```

output
```
-(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.113 $\int \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

3.113.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\text{csch}^2(a+bx)}{2b}$$

output
```
-1/2*csch(b*x+a)^2/b
```

3.113.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\text{csch}^2(a+bx)}{2b}$$

input
```
Integrate[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
-1/2*Csch[a + b*x]^2/b
```

3.113.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 26, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan\left(ia+ibx-\frac{\pi}{2}\right)\sec\left(ia+ibx-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int -i\text{csch}(a+bx)d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\text{csch}^2(a+bx)}{2b}$$

input
```
Int[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
-1/2*Csch[a + b*x]^2/b
```

3.113.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.113.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\coth(bx+a)^2}{2b}$	14
default	$-\frac{\coth(bx+a)^2}{2b}$	14
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)^2}$	28

input
```
int(coth(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*coth(b*x+a)^2/b
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 5.73

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx =$$

$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+b\sinh\left(bx+a\right)^3-b\cosh\left(bx+a\right)+3\,\left(b\cosh\left(bx+\right.\right.}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*
sinh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)
^2 - b)*sinh(b*x + a))
```

3.113.6 Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \coth(a+bx)\mathrm{csch}^2(a+bx)\,dx = \begin{cases} -\frac{\mathrm{csch}^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\coth(a)\,\mathrm{csch}^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a)**2,x)
```

output
```
Piecewise((-csch(a + b*x)**2/(2*b), Ne(b, 0)), (x*coth(a)*csch(a)**2, True
))
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth(a+bx)\mathrm{csch}^2(a+bx)\,dx = -\frac{\coth(bx+a)^2}{2\,b}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*coth(b*x + a)^2/b
```

3.113.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \coth(a+bx)\mathrm{csch}^2(a+bx)\,dx = -\frac{2\,e^{(2\,bx+2\,a)}}{b(e^{(2\,bx+2\,a)}-1)^2}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{1}{2\,b\sinh\left(a+b\,x\right)^2}$$

input
```
int(coth(a + b*x)/sinh(a + b*x)^2,x)
```

output
```
-1/(2*b*sinh(a + b*x)^2)
```

3.114 $\int \cosh(a+bx)\mathbf{csch}^{1+n}(a+bx)\,dx$

3.114.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \cosh(a+bx)\operatorname{csch}^{1+n}(a+bx)\,dx=-\frac{\operatorname{csch}^n(a+bx)}{bn}$$

output
```
-csch(b*x+a)^n/b/n
```

3.114.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\operatorname{csch}^{1+n}(a+bx)\,dx=-\frac{\operatorname{csch}^n(a+bx)}{bn}$$

input
```
Integrate[Cosh[a + b*x]*Csch[a + b*x]^(1 + n),x]
```

output
```
-(Csch[a + b*x]^n/(b*n))
```

3.114.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3101, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\text{csch}^{n+1}(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(i\csc(ia+ibx))^{n+1}}{\sec(ia+ibx)}dx$$

$$\Big\downarrow \ 3101$$

$$-\frac{\int \text{csch}^{n-1}(a+bx)d\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \ 15$$

$$-\frac{\text{csch}^n(a+bx)}{bn}$$

input
```
Int[Cosh[a + b*x]*Csch[a + b*x]^(1 + n),x]
```

output
```
-(Csch[a + b*x]^n/(b*n))
```

3.114.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.114.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{\operatorname{csch}(bx+a)^n}{bn}$
default	$-\frac{\operatorname{csch}(bx+a)^n}{bn}$
risch	$-\frac{2^n (e^{bx+a})^n (e^{bx+a}-1)^{-n} (e^{bx+a}+1)^{-n} e^{-\frac{i\pi n \left(\operatorname{csgn}\left(\frac{i}{e^{bx+a}-1}\right) \operatorname{csgn}\left(\frac{i}{e^{bx+a}+1}\right) \operatorname{csgn}\left(\frac{i}{(e^{bx+a}-1)(e^{bx+a}+1)}\right) -\operatorname{csgn}\left(\right.\right.}{}}}{}$

input
```
int(coth(b*x+a)*csch(b*x+a)^n,x,method=_RETURNVERBOSE)
```

output
```
-csch(b*x+a)^n/b/n
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 7.19

$$\int \cosh(a+bx)\operatorname{csch}^{1+n}(a+bx)\,dx =$$
$$-\frac{\cosh\left(n\log\left(\frac{2(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1}\right)\right)+\sinh\left(n\log\left(\frac{2(\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a}\right.\right.}{bn}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a)^n,x, algorithm="fricas")
```

output
```
-(cosh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b
*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))) + sinh(n*log(2*(cosh(b*x +
a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 - 1))))/(b*n)
```

3.114.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(12) = 24.

Time = 2.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \cosh(a+bx)\operatorname{csch}^{1+n}(a+bx)\,dx = \begin{cases} x\coth(a)\operatorname{csch}^n(a) & \text{for } b=0 \\ \begin{cases} x\coth(a) & \text{for } b=0 \\ \frac{\log(\sinh(a+bx))}{b} & \text{otherwise} \end{cases} & \text{for } n=0 \\ -\frac{\operatorname{csch}^n(a+bx)}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a)**n,x)
```

output
```
Piecewise((x*coth(a)*csch(a)**n, Eq(b, 0)), (Piecewise((x*coth(a), Eq(b, 0
)), (log(sinh(a + b*x))/b, True)), Eq(n, 0)), (-csch(a + b*x)**n/(b*n), Tr
ue))
```

3.114.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(16) = 32.

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int \cosh(a+bx)\operatorname{csch}^{1+n}(a+bx)\,dx = -\frac{2^n e^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{bn}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a)^n,x, algorithm="maxima")
```

output
```
-2^n*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1))
/(b*n)
```

3.114.8 Giac [F]

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx=\int \text{csch}\,(bx+a)^n\coth\,(bx+a)\;dx$$

input
```
integrate(coth(b*x+a)*csch(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^n*coth(b*x + a), x)
```

3.114.9 Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx=-\frac{\left(\frac{2\,\mathrm{e}^{a+b\,x}}{\mathrm{e}^{2\,a+2\,b\,x}-1}\right)^n}{b\,n}$$

input
```
int(coth(a + b*x)*(1/sinh(a + b*x))^n,x)
```

output
```
-((2*exp(a + b*x))/(exp(2*a + 2*b*x) - 1))^n/(b*n)
```

3.115 $\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx$

3.115.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\coth^3(a+bx)}{3b}$$

output
```
-1/3*coth(b*x+a)^3/b
```

3.115.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\coth^3(a+bx)}{3b}$$

input
```
Integrate[Coth[a + b*x]^2*Csch[a + b*x]^2,x]
```

output
```
-1/3*Coth[a + b*x]^3/b
```

3.115.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(ia+ibx-\frac{\pi}{2}\right)^2\sec\left(ia+ibx-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3087}$$

$$-\frac{i\int -\coth^2(a+bx)d(i\coth(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\coth^3(a+bx)}{3b}$$

input
```
Int[Coth[a + b*x]^2*Csch[a + b*x]^2,x]
```

output
```
-1/3*Coth[a + b*x]^3/b
```

3.115.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.115.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\coth(bx+a)^3}{3b}$	14
default	$-\frac{\coth(bx+a)^3}{3b}$	14
risch	$-\frac{2(3\,e^{4bx+4a}+1)}{3b(e^{2bx+2a}-1)^3}$	32

input
```
int(csch(b*x+a)^2*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*coth(b*x+a)^3/b
```

3.115.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 9.27

$$\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx =$$

$$-\frac{8\left(\cosh\left(bx+a\right)^2+\cosh\left(bx+a\right)\sinh}{3\left(b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+b\sinh\left(bx+a\right)^4-4\,b\cosh\left(bx+a\right)^2+2\left(3\,b\cosh\right.\right.}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-8/3*(cosh(b*x + a)^2 + cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*
cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 -
4*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

3.115.6 Sympy [F]

$$\int \coth^2(a+bx)\mathrm{csch}^2(a+bx)\,dx = \int \coth^2(a+bx)\,\mathrm{csch}^2(a+bx)\,dx$$

input
```
integrate(coth(b*x+a)**2*csch(b*x+a)**2,x)
```

output
```
Integral(coth(a + b*x)**2*csch(a + b*x)**2, x)
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth^2(a+bx)\mathrm{csch}^2(a+bx)\,dx = -\frac{\coth(bx+a)^3}{3\,b}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3*coth(b*x + a)^3/b
```

3.115.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \coth^2(a+bx)\mathrm{csch}^2(a+bx)\,dx = -\frac{2\left(3\,e^{(4\,bx+4\,a)}+1\right)}{3\,b(e^{(2\,bx+2\,a)}-1)^3}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-2/3*(3*e^(4*b*x + 4*a) + 1)/(b*(e^(2*b*x + 2*a) - 1)^3)
```

3.115.9 Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{2\left(3\,e^{4\,a+4\,b\,x}+1\right)}{3\,b\left(e^{2\,a+2\,b\,x}-1\right)^3}$$

input
```
int(coth(a + b*x)^2/sinh(a + b*x)^2,x)
```

output
```
-(2*(3*exp(4*a + 4*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) - 1)^3)
```

3.116 $\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx$

3.116.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth^4(a+bx)}{4b}$$

output
```
-1/4*coth(b*x+a)^4/b
```

3.116.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth^4(a+bx)}{4b}$$

input
```
Integrate[Coth[a + b*x]^3*Csch[a + b*x]^2,x]
```

output
```
-1/4*Coth[a + b*x]^4/b
```

3.116.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 26, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan\left(ia+ibx-\frac{\pi}{2}\right)^3\sec\left(ia+ibx-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{3087}$$

$$-\frac{\int -i\coth^3(a+bx)d(i\coth(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\coth^4(a+bx)}{4b}$$

input
```
Int[Coth[a + b*x]^3*Csch[a + b*x]^2,x]
```

output
```
-1/4*Coth[a + b*x]^4/b
```

3.116.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.116.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\coth(bx+a)^4}{4b}$	14
default	$-\frac{\coth(bx+a)^4}{4b}$	14
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}\left(\mathrm{e}^{4bx+4a}+1\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^4}$	39

input
```
int(coth(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*coth(b*x+a)^4/b
```

3.116.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 13.87

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx =$$

$$-\frac{2\left(\cosh\left(bx+a\right)^3+3\,\cosh\left(bx\right.\right.}{b\cosh\left(bx+a\right)^5+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+b\sinh\left(bx+a\right)^5-3\,b\cosh\left(bx+a\right)^3+5\left(2\,b\cosh\left(b\right.\right.}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 +
(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a)^5
+ 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 - 3*b*cosh(b*x + a
)^3 + 5*(2*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3
- 9*b*cosh(b*x + a))*sinh(b*x + a)^2 + 2*b*cosh(b*x + a) + 5*(b*cosh(b*x +
 a)^4 - 3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

3.116.6 Sympy [F]

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = \int \coth^3(a+bx)\,\text{csch}^2(a+bx)\,dx$$

input
```
integrate(coth(b*x+a)**3*csch(b*x+a)**2,x)
```

output
```
Integral(coth(a + b*x)**3*csch(a + b*x)**2, x)
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth(bx+a)^4}{4\,b}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*coth(b*x + a)^4/b
```

3.116.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2\left(e^{(6\,bx+6\,a)}+e^{(2\,bx+2\,a)}\right)}{b(e^{(2\,bx+2\,a)}-1)^4}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-2*(e^(6*b*x + 6*a) + e^(2*b*x + 2*a))/(b*(e^(2*b*x + 2*a) - 1)^4)
```

3.116.9 Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 15.40

$$\int \coth^3(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\frac{1}{2b}+\frac{3e^{2a+2bx}}{2b}+\frac{3e^{4a+4bx}}{2b}+\frac{e^{6a+6bx}}{2b}}{6e^{4a+4bx}-4e^{2a+2bx}-4e^{6a+6bx}+e^{8a+8bx}+1} - \frac{\frac{1}{2b}+\frac{e^{2a+2bx}}{b}+\frac{e^{4a+4bx}}{2b}}{3e^{2a+2bx}-3e^{4a+4bx}+e^{6a+6bx}-1} - \frac{\frac{1}{2b}+\frac{e^{2a+2bx}}{2b}}{e^{4a+4bx}-2e^{2a+2bx}+1} - \frac{1}{2b\,(e^{2a+2bx}-1)}$$

input
```
int(coth(a + b*x)^3/sinh(a + b*x)^2,x)
```

output
```
- (1/(2*b) + (3*exp(2*a + 2*b*x))/(2*b) + (3*exp(4*a + 4*b*x))/(2*b) + exp
(6*a + 6*b*x)/(2*b))/(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a
+ 6*b*x) + exp(8*a + 8*b*x) + 1) - (1/(2*b) + exp(2*a + 2*b*x)/b + exp(4*a
 + 4*b*x)/(2*b))/(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*
x) - 1) - (1/(2*b) + exp(2*a + 2*b*x)/(2*b))/(exp(4*a + 4*b*x) - 2*exp(2*a
 + 2*b*x) + 1) - 1/(2*b*(exp(2*a + 2*b*x) - 1))
```

3.117 $\int \coth^n(a+bx)\mathbf{csch}^2(a+bx)\,dx$

3.117.1 Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \coth^n(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth^{1+n}(a+bx)}{b(1+n)}$$

output
```
-coth(b*x+a)^(1+n)/b/(1+n)
```

3.117.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \coth^n(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth^{1+n}(a+bx)}{b(1+n)}$$

input
```
Integrate[Coth[a + b*x]^n*Csch[a + b*x]^2,x]
```

output
```
-(Coth[a + b*x]^(1 + n)/(b*(1 + n)))
```

3.117.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 25, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(a+bx)\coth^n(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sec\left(ia+ibx-\frac{\pi}{2}\right)^2\left(-i\tan\left(ia+ibx-\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2\left(-i\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)\right)^n dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{i\int \coth^n(a+bx)d(i\coth(a+bx))}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{\coth^{n+1}(a+bx)}{b(n+1)}$$

input
```
Int[Coth[a + b*x]^n*Csch[a + b*x]^2,x]
```

output
```
-(Coth[a + b*x]^(1 + n)/(b*(1 + n)))
```

3.117.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.117.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{\coth(bx+a)^{n+1}}{b(n+1)}$
default	$-\frac{\coth(bx+a)^{n+1}}{b(n+1)}$
risch	$-\frac{\left(1+\mathrm{e}^{2bx+2a}\right)\left(\mathrm{e}^{bx+a}-1\right)^{-n}\left(\mathrm{e}^{bx+a}+1\right)^{-n}\left(1+\mathrm{e}^{2bx+2a}\right)^{n}\mathrm{e}^{-\frac{i\pi n\left(\mathrm{csgn}\left(\frac{i\left(1+\mathrm{e}^{2bx+2a}\right)}{\left(\mathrm{e}^{bx+a}-1\right)\left(\mathrm{e}^{bx+a}+1\right)}\right)^{3}-\mathrm{csgn}\left(\frac{i\left(1+\mathrm{e}^{2bx+2}\right.}{\left(\mathrm{e}^{bx+a}-1\right)\left(\mathrm{e}^{bx}\right.}\right.\right.}{}}}{}$

input
```
int(coth(b*x+a)^n*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-coth(b*x+a)^(n+1)/b/(n+1)
```

3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx$$
$$= -\frac{\cosh(bx+a)\cosh\left(n\log\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)\right)+\cosh(bx+a)\sinh\left(n\log\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)\right)}{(bn+b)\sinh(bx+a)}$$

input
```
integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(cosh(b*x + a)*cosh(n*log(cosh(b*x + a)/sinh(b*x + a))) + cosh(b*x + a)*s
inh(n*log(cosh(b*x + a)/sinh(b*x + a))))/((b*n + b)*sinh(b*x + a))
```

3.117.6 Sympy [F]

$$\int \coth^n(a+bx)\text{csch}^2(a+bx)\,dx = \int \coth^n(a+bx)\,\text{csch}^2(a+bx)\,dx$$

input
```
integrate(coth(b*x+a)**n*csch(b*x+a)**2,x)
```

output
```
Integral(coth(a + b*x)**n*csch(a + b*x)**2, x)
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \coth^n(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth(bx+a)^{n+1}}{b(n+1)}$$

input
```
integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-coth(b*x + a)^(n + 1)/(b*(n + 1))
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \coth^n(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\left(\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}\right)^{n+1}}{b(n+1)}$$

input
```
integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1))^(n + 1)/(b*(n + 1))
```

3.117.9 Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\coth(a+b\,x)\,\left(\frac{e^{2\,a+2\,b\,x}+1}{e^{2\,a+2\,b\,x}-1}\right)^n}{b\,(n+1)}$$

input
```
int(coth(a + b*x)^n/sinh(a + b*x)^2,x)
```

output
```
-(coth(a + b*x)*((exp(2*a + 2*b*x) + 1)/(exp(2*a + 2*b*x) - 1))^n)/(b*(n +
 1))
```

3.118 $\int \coth^3(a+bx)\mathbf{csch}(a+bx)\,dx$

3.118.1 Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

output
```
-csch(b*x+a)/b-1/3*csch(b*x+a)^3/b
```

3.118.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

input
```
Integrate[Coth[a + b*x]^3*Csch[a + b*x],x]
```

output
```
-(Csch[a + b*x]/b) - Csch[a + b*x]^3/(3*b)
```

3.118.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 25, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(ia+ibx-\frac{\pi}{2}\right)^3\left(-\sec\left(ia+ibx-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{i\int\left(-\text{csch}^2(a+bx)-1\right)d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(\frac{1}{3}i\text{csch}^3(a+bx)+i\text{csch}(a+bx)\right)}{b}$$

input
```
Int[Coth[a + b*x]^3*Csch[a + b*x],x]
```

output
```
(I*(I*Csch[a + b*x] + (I/3)*Csch[a + b*x]^3))/b
```

3.118.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.118.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{csch}(bx+a)^3}{3}-\operatorname{csch}(bx+a)}{b}$	24
default	$\frac{-\frac{\operatorname{csch}(bx+a)^3}{3}-\operatorname{csch}(bx+a)}{b}$	24
risch	$-\frac{2\,\mathrm{e}^{bx+a}\left(3\,\mathrm{e}^{4bx+4a}-2\,\mathrm{e}^{2bx+2a}+3\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^3}$	49

input
```
int(coth(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3*csch(b*x+a)^3-csch(b*x+a))
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.33

$$\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx =$$

$$-\frac{2\left(3\cosh\left(bx+a\right)^3+9\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+3\sinh\left(bx+a\right)\right.}{3\left(b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+b\sinh\left(bx+a\right)^4-4\,b\cosh\left(bx+a\right)^2+2\left(3\,b\cosh\right.\right.}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 + 3*sinh(b*x + a
)^3 + (9*cosh(b*x + a)^2 - 5)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x +
 a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 4*b*cosh(b
*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x
+ a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

3.118.6 Sympy [F]

$$\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx = \int \coth^3\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

input
```
integrate(coth(b*x+a)**3*csch(b*x+a),x)
```

output
```
Integral(coth(a + b*x)**3*csch(a + b*x), x)
```

3.118.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 5.48

$$\begin{aligned}\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx &= \frac{2\,e^{(-bx-a)}}{b(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1)} \\ &\quad-\frac{4\,e^{(-3\,bx-3\,a)}}{3\,b(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1)} \\ &\quad+\frac{2\,e^{(-5\,bx-5\,a)}}{b(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1)}\end{aligned}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

output
```
2*e^(-b*x - a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6
*a) - 1)) - 4/3*e^(-3*b*x - 3*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*
a) + e^(-6*b*x - 6*a) - 1)) + 2*e^(-5*b*x - 5*a)/(b*(3*e^(-2*b*x - 2*a) -
3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) - 1))
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \coth^3(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)}{3\,b\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

output
```
-2/3*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 4)/(b*(e^(b*x + a) - e^(-b*x - a)
)^3)
```

3.118.9 Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \coth^3(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\,e^{a+b\,x}\,\left(3\,e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+3\right)}{3\,b\,\left(e^{2\,a+2\,b\,x}-1\right)^3}$$

input
```
int(coth(a + b*x)^3/sinh(a + b*x),x)
```

output
```
-(2*exp(a + b*x)*(3*exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 3))/(3*b*(exp(
2*a + 2*b*x) - 1)^3)
```

3.119 $\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx$

3.119.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx=-\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{\operatorname{csch}^5(a+bx)}{5b}$$

output
```
-1/3*csch(b*x+a)^3/b-1/5*csch(b*x+a)^5/b
```

3.119.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx=-\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{\operatorname{csch}^5(a+bx)}{5b}$$

input
```
Integrate[Coth[a + b*x]^3*Csch[a + b*x]^3,x]
```

output
```
-1/3*Csch[a + b*x]^3/b - Csch[a + b*x]^5/(5*b)
```

3.119.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(a+bx)\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(ia+ibx-\frac{\pi}{2}\right)^3\sec\left(ia+ibx-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{i\int \text{csch}^2(a+bx)\left(\text{csch}^2(a+bx)+1\right)d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int -\text{csch}^2(a+bx)\left(\text{csch}^2(a+bx)+1\right)d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{i\int \left(-\text{csch}^4(a+bx)-\text{csch}^2(a+bx)\right)d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{1}{5}i\text{csch}^5(a+bx)-\frac{1}{3}i\text{csch}^3(a+bx)\right)}{b}$$

input
```
Int[Coth[a + b*x]^3*Csch[a + b*x]^3,x]
```

output
```
((-I)*((-1/3*I)*Csch[a + b*x]^3 - (I/5)*Csch[a + b*x]^5))/b
```

3.119.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.119.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$-\frac{\frac{\operatorname{csch}(bx+a)^{5}}{5}+\frac{\operatorname{csch}(bx+a)^{3}}{3}}{b}$	27
default	$-\frac{\frac{\operatorname{csch}(bx+a)^{5}}{5}+\frac{\operatorname{csch}(bx+a)^{3}}{3}}{b}$	27
risch	$-\frac{8\,{\mathrm e}^{3bx+3a}\left(5\,{\mathrm e}^{4bx+4a}+2\,{\mathrm e}^{2bx+2a}+5\right)}{15b\left({\mathrm e}^{2bx+2a}-1\right)^{5}}$	52

input
```
int(coth(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/5*csch(b*x+a)^5+1/3*csch(b*x+a)^3)
```

3.119.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(27) = 54$.

Time = 0.24 (sec) , antiderivative size = 343, normalized size of antiderivative = 11.06

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^{7}+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{6}+b\sinh\left(bx+a\right)^{7}-5\,b\cosh\left(bx+a\right)^{5}+\left(21\,b\cos\right.\right.}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-8/15*(5*cosh(b*x + a)^4 + 20*cosh(b*x + a)*sinh(b*x + a)^3 + 5*sinh(b*x +
 a)^4 + 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4
*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 5)/(b*cosh(b*x + a)^7
 + 7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 5*b*cosh(b*x +
a)^5 + (21*b*cosh(b*x + a)^2 - 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)
^3 - 5*b*cosh(b*x + a))*sinh(b*x + a)^4 + 9*b*cosh(b*x + a)^3 + (35*b*cosh
(b*x + a)^4 - 50*b*cosh(b*x + a)^2 + 11*b)*sinh(b*x + a)^3 + (21*b*cosh(b*
x + a)^5 - 50*b*cosh(b*x + a)^3 + 27*b*cosh(b*x + a))*sinh(b*x + a)^2 - 5*
b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 - 25*b*cosh(b*x + a)^4 + 33*b*cosh(
b*x + a)^2 - 15*b)*sinh(b*x + a))
```

3.119.6 Sympy [F]

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx = \int \coth^{3}\left(a+bx\right)\operatorname{csch}^{3}\left(a+bx\right)\,dx$$

input
```
integrate(coth(b*x+a)**3*csch(b*x+a)**3,x)
```

output
```
Integral(coth(a + b*x)**3*csch(a + b*x)**3, x)
```

3.119.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(27) = 54$.

Time = 0.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 6.90

$$\int \coth^3(a+bx)\text{csch}^3(a+bx)\,dx = \frac{8\,e^{(-3\,bx-3\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1)} + \frac{16\,e^{(-5\,bx-5\,a)}}{15\,b(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1)} + \frac{8\,e^{(-7\,bx-7\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1)}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
8/3*e^(-3*b*x - 3*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(
-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 16/15*e^(-
5*b*x - 5*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x -
 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 8/3*e^(-7*b*x - 7*
a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) - 5*
e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1))
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \coth^3(a+bx)\text{csch}^3(a+bx)\,dx = -\frac{8\left(5\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+12\right)}{15\,b(e^{(bx+a)}-e^{(-bx-a)})^5}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-8/15*(5*(e^(b*x + a) - e^(-b*x - a))^2 + 12)/(b*(e^(b*x + a) - e^(-b*x -
a))^5)
```

3.119.9 Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 252, normalized size of antiderivative = 8.13

$$\int \coth^3(a+bx)\text{csch}^3(a+bx)\,dx$$
$$= -\frac{\frac{4\,e^{a+b\,x}}{5\,b}+\frac{12\,e^{3\,a+3\,b\,x}}{5\,b}+\frac{12\,e^{5\,a+5\,b\,x}}{5\,b}+\frac{4\,e^{7\,a+7\,b\,x}}{5\,b}}{5\,e^{2\,a+2\,b\,x}-10\,e^{4\,a+4\,b\,x}+10\,e^{6\,a+6\,b\,x}-5\,e^{8\,a+8\,b\,x}+e^{10\,a+10\,b\,x}-1}$$
$$-\frac{28\,e^{a+b\,x}}{15\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{64\,e^{a+b\,x}}{15\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)}$$
$$-\frac{16\,e^{a+b\,x}}{5\,b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}$$

input
```
int(coth(a + b*x)^3/sinh(a + b*x)^3,x)
```

output
```
- ((4*exp(a + b*x))/(5*b) + (12*exp(3*a + 3*b*x))/(5*b) + (12*exp(5*a + 5*
b*x))/(5*b) + (4*exp(7*a + 7*b*x))/(5*b))/(5*exp(2*a + 2*b*x) - 10*exp(4*a
 + 4*b*x) + 10*exp(6*a + 6*b*x) - 5*exp(8*a + 8*b*x) + exp(10*a + 10*b*x)
- 1) - (28*exp(a + b*x))/(15*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)
) - (64*exp(a + b*x))/(15*b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp
(6*a + 6*b*x) - 1)) - (16*exp(a + b*x))/(5*b*(6*exp(4*a + 4*b*x) - 4*exp(2
*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

3.120 $\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx$

3.120.1 Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx=-\frac{\operatorname{csch}^n(a+bx)}{bn}-\frac{\operatorname{csch}^{2+n}(a+bx)}{b(2+n)}$$

output
```
-csch(b*x+a)^n/b/n-csch(b*x+a)^(2+n)/b/(2+n)
```

3.120.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx=-\frac{\operatorname{csch}^n(a+bx)\left(2+n+n\operatorname{csch}^2(a+bx)\right)}{bn(2+n)}$$

input
```
Integrate[Cosh[a + b*x]^3*Csch[a + b*x]^(3 + n),x]
```

output
```
-((Csch[a + b*x]^n*(2 + n + n*Csch[a + b*x]^2))/(b*n*(2 + n)))
```

3.120.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3101, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\text{csch}^{n+3}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\csc(ia+ibx))^{n+3}}{\sec(ia+ibx)^3}dx$$

$$\Big\downarrow \text{3101}$$

$$\frac{\int -\text{csch}^{n-1}(a+bx)\left(\text{csch}^2(a+bx)+1\right)d\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \text{csch}^{n-1}(a+bx)\left(\text{csch}^2(a+bx)+1\right)d\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\text{csch}^{n-1}(a+bx)+\text{csch}^{n+1}(a+bx)\right)d\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\text{csch}^{n+2}(a+bx)}{n+2}-\frac{\text{csch}^{n}(a+bx)}{n}}{b}$$

input
```
Int[Cosh[a + b*x]^3*Csch[a + b*x]^(3 + n),x]
```

output
```
(-(Csch[a + b*x]^n/n) - Csch[a + b*x]^(2 + n)/(2 + n))/b
```

3.120.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.120.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.86 (sec) , antiderivative size = 479, normalized size of antiderivative = 12.95

method	result
risch	$-\frac{\left(n\,{\mathrm e}^{4 b x +4 a}+2\,{\mathrm e}^{4 b x +4 a}+2 n\,{\mathrm e}^{2 b x +2 a}-4\,{\mathrm e}^{2 b x +2 a}+n +2\right) 2^{n} \left({\mathrm e}^{b x +a}-1\right)^{-n} \left({\mathrm e}^{b x +a}\right)^{n} \left({\mathrm e}^{b x +a}+1\right)^{-n} {\mathrm e}^{-\frac{i \pi n \left(\operatorname{csgn}\left(\frac{i}{{\mathrm e}^{b x +a}-1}\right) \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{b x +a}}\right.\right.}{}}}{}$

input
```
int(coth(b*x+a)^3*csch(b*x+a)^n,x,method=_RETURNVERBOSE)
```

output
```
-(n*exp(4*b*x+4*a)+2*exp(4*b*x+4*a)+2*n*exp(2*b*x+2*a)-4*exp(2*b*x+2*a)+n+
2)/b/n/(n+2)/(exp(2*b*x+2*a)-1)^2*2^n*(exp(b*x+a)-1)^(-n)*exp(b*x+a)^n*(ex
p(b*x+a)+1)^(-n)*exp(-1/2*I*Pi*n*(csgn(I/(exp(b*x+a)-1))*csgn(I/(exp(b*x+a
)+1))*csgn(I/(exp(b*x+a)-1)/(exp(b*x+a)+1))-csgn(I/(exp(b*x+a)-1))*csgn(I/
(exp(b*x+a)-1)/(exp(b*x+a)+1))^2+csgn(I*exp(b*x+a)/(exp(b*x+a)+1)/(exp(b*x
+a)-1))^3-csgn(I*exp(b*x+a)/(exp(b*x+a)+1)/(exp(b*x+a)-1))^2*csgn(I*exp(b*
x+a))-csgn(I*exp(b*x+a)/(exp(b*x+a)+1)/(exp(b*x+a)-1))^2*csgn(I/(exp(b*x+a
)-1)/(exp(b*x+a)+1))+csgn(I*exp(b*x+a)/(exp(b*x+a)+1)/(exp(b*x+a)-1))*csgn
(I*exp(b*x+a))*csgn(I/(exp(b*x+a)-1)/(exp(b*x+a)+1))-csgn(I/(exp(b*x+a)+1)
)*csgn(I/(exp(b*x+a)-1)/(exp(b*x+a)+1))^2+csgn(I/(exp(b*x+a)-1)/(exp(b*x+a
)+1))^3))
```

3.120.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 5.84

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx$$
$$= \frac{\left((n+2)\cosh{(bx+a)}^2+(n+2)\sinh{(bx+a)}^2+n-2\right)\cosh\left(n\log\left(\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\,\cosh(bx+a)\sinh(bx+a)+\sinh(\cdots}\right.\right.}{bn^2-(bn^2+2\,bn)\cosh\cdots}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^n,x, algorithm="fricas")
```

output
```
(((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*x + a)^2 + n - 2)*cosh(n*log(2*
(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*
x + a) + sinh(b*x + a)^2 - 1))) + ((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(
b*x + a)^2 + n - 2)*sinh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))))/(b*n^2 -
 (b*n^2 + 2*b*n)*cosh(b*x + a)^2 - (b*n^2 + 2*b*n)*sinh(b*x + a)^2 + 2*b*n
)
```

3.120.6 Sympy [F]

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx = \int \coth^3(a+bx)\operatorname{csch}^n(a+bx)\,dx$$

input
```
integrate(coth(b*x+a)**3*csch(b*x+a)**n,x)
```

output
```
Integral(coth(a + b*x)**3*csch(a + b*x)**n, x)
```

3.120.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(37) = 74$.

Time = 0.34 (sec) , antiderivative size = 414, normalized size of antiderivative = 11.19

$$\begin{aligned}
&\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx \\
&= -\frac{2^n n e^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{\left(n^2-2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n\right)b} \\
&\quad - \frac{(2^{n+1}n-2^{n+2})e^{\left(-(bx+a)n-2\,bx-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)-2\,a\right)}}{\left(n^2-2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n\right)b} \\
&\quad - \frac{(2^n n+2^{n+1})e^{\left(-(bx+a)n-4\,bx-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)-4\,a\right)}}{\left(n^2-2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n\right)b} \\
&\quad - \frac{2^{n+1}e^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{\left(n^2-2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n\right)b}
\end{aligned}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^n,x, algorithm="maxima")
```

output
```
-2^n*n*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1
))/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) +
 2*n)*b) - (2^(n + 1)*n - 2^(n + 2))*e^(-(b*x + a)*n - 2*b*x - n*log(e^(-b
*x - a) + 1) - n*log(-e^(-b*x - a) + 1) - 2*a)/((n^2 - 2*(n^2 + 2*n)*e^(-2
*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b) - (2^n*n + 2^(n + 1))
*e^(-(b*x + a)*n - 4*b*x - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) +
 1) - 4*a)/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x
- 4*a) + 2*n)*b) - 2^(n + 1)*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n
*log(-e^(-b*x - a) + 1))/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2
*n)*e^(-4*b*x - 4*a) + 2*n)*b)
```

3.120.8 Giac [F]

$$\int \cosh^3(a+bx)\text{csch}^{3+n}(a+bx)\,dx = \int \text{csch}\,(bx+a)^n \coth\,(bx+a)^3\,dx$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^n*coth(b*x + a)^3, x)
```

3.120.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.70

$$\int \cosh^3(a+bx)\text{csch}^{3+n}(a+bx)\,dx = -\frac{\left(\frac{1}{\frac{e^{a+b\,x}}{2}-\frac{e^{-a-b\,x}}{2}}\right)^n\left(\frac{1}{b\,n}+\frac{e^{4\,a+4\,b\,x}}{b\,n}+\frac{e^{2\,a+2\,b\,x}\,(2\,n-4)}{b\,n\,(n+2)}\right)}{e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1}$$

input
```
int(coth(a + b*x)^3*(1/sinh(a + b*x))^n,x)
```

output
```
-((1/(exp(a + b*x)/2 - exp(- a - b*x)/2))^n*(1/(b*n) + exp(4*a + 4*b*x)/(b
*n) + (exp(2*a + 2*b*x)*(2*n - 4))/(b*n*(n + 2))))/(exp(4*a + 4*b*x) - 2*e
xp(2*a + 2*b*x) + 1)
```

3.121 $\int \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

3.121.1 Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx=-\frac{\text{arctanh}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

output
```
-1/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b
```

3.121.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. $2(34)=68$.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx=-\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

input
```
Integrate[Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
-1/8*Csch[(a + b*x)/2]^2/b - Log[Cosh[(a + b*x)/2]]/(2*b) + Log[Sinh[(a +
b*x)/2]]/(2*b) - Sech[(a + b*x)/2]^2/(8*b)
```

3.121.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan\left(ia+ibx-\frac{\pi}{2}\right)^2\sec\left(ia+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3091}$$

$$-i\left(-\frac{1}{2}\int -i\text{csch}(a+bx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}i\int \text{csch}(a+bx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{2}i\int i\csc(ia+ibx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{1}{2}\int \csc(ia+ibx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(-\frac{i\text{arctanh}(\cosh(a+bx))}{2b}-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

input
```
Int[Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
(-I)*(((-1/2*I)*ArcTanh[Cosh[a + b*x]])/b - ((I/2)*Coth[a + b*x]*Csch[a +
b*x])/b)
```

3.121.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.121.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\operatorname{csch}(bx+a)}{2}-\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	45
default	$\frac{-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\operatorname{csch}(bx+a)}{2}-\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	45
risch	$-\frac{\mathrm{e}^{bx+a}\left(1+\mathrm{e}^{2bx+2a}\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^2}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{2b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{2b}$	65

input
```
int(csch(b*x+a)*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-cosh(b*x+a)/sinh(b*x+a)^2+1/2*coth(b*x+a)*csch(b*x+a)-arctanh(exp(b*
x+a)))
```

3.121.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 387, normalized size of antiderivative = 11.38

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx =$$

$$-\frac{2\cosh(bx+a)^3 + 6\cosh(bx+a)\sinh(bx+a)^2 + 2\sinh(bx+a)^3 + \left(\cosh(bx+a)^4 + 4\cosh(bx+a\right.}{}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a
)^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh
(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b
*x + a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b
*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
+ 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a)
 + sinh(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b
*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b
*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a
)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

3.121.6 Sympy [F]

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int \coth^2(a+bx)\,\text{csch}\,(a+bx)\,dx$$

input
```
integrate(coth(b*x+a)**2*csch(b*x+a),x)
```

output
```
Integral(coth(a + b*x)**2*csch(a + b*x), x)
```

3.121.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.47

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1)}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*log(e^(-b*x - a) + 1)/b + 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a)
 + e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.47

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} + \log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) - \log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

output
```
-1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)
+ log(e^(b*x + a) + e^(-b*x - a) + 2) - log(e^(b*x + a) + e^(-b*x - a) - 2
))/b
```

3.121.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.56

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(coth(a + b*x)^2/sinh(a + b*x),x)
```

output
```
- atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(
b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a
+ 2*b*x) - 1))
```

3.122 $\int \coth^2(a+bx)\mathbf{csch}^3(a+bx)\,dx$

3.122.1 Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx=\frac{\operatorname{arctanh}(\cosh(a+bx))}{8b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}$$

output
```
1/8*arctanh(cosh(b*x+a))/b-1/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)*c
sch(b*x+a)^3/b
```

3.122.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(55) = 110.

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx=-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}-\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}$$

input
```
Integrate[Coth[a + b*x]^2*Csch[a + b*x]^3,x]
```

output
```
-1/32*Csch[(a + b*x)/2]^2/b - Csch[(a + b*x)/2]^4/(64*b) + Log[Cosh[(a + b
*x)/2]]/(8*b) - Log[Sinh[(a + b*x)/2]]/(8*b) - Sech[(a + b*x)/2]^2/(32*b)
+ Sech[(a + b*x)/2]^4/(64*b)
```

3.122.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {3042, 26, 3091, 26, 3042, 26, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(a+bx)\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan\left(ia+ibx-\frac{\pi}{2}\right)^2\sec\left(ia+ibx-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)^3\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3091}$$

$$i\left(\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}-\frac{1}{4}\int i\text{csch}^3(a+bx)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}-\frac{1}{4}i\int \text{csch}^3(a+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}-\frac{1}{4}i\int -i\csc(ia+ibx)^3dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}-\frac{1}{4}\int \csc(ia+ibx)^3dx\right)$$

$$\Big\downarrow \text{4255}$$

$$i\left(\frac{1}{4}\left(\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}-\frac{1}{2}\int -i\text{csch}(a+bx)dx\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{4}\left(\frac{1}{2}i\int \text{csch}(a+bx)dx+\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{1}{4}\left(\frac{1}{2}i\int i\csc(ia+ibx)dx+\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{4}\left(\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}-\frac{1}{2}\int \csc(ia+ibx)dx\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{1}{4}\left(\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}-\frac{i\text{arctanh}(\cosh(a+bx))}{2b}\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

input
```
Int[Coth[a + b*x]^2*Csch[a + b*x]^3,x]
```

output
```
I*(((I/4)*Coth[a + b*x]*Csch[a + b*x]^3)/b + (((-1/2*I)*ArcTanh[Cosh[a + b
*x]])/b + ((I/2)*Coth[a + b*x]*Csch[a + b*x])/b)/4)
```

3.122.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.122.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{-\frac{\cosh(bx+a)}{3\sinh(bx+a)^4}-\frac{\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)}{3}+\frac{\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	58
default	$\frac{-\frac{\cosh(bx+a)}{3\sinh(bx+a)^4}-\frac{\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)}{3}+\frac{\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	58
risch	$-\frac{e^{bx+a}\left(e^{6bx+6a}+7e^{4bx+4a}+7e^{2bx+2a}+1\right)}{4b\left(e^{2bx+2a}-1\right)^4}+\frac{\ln\left(e^{bx+a}+1\right)}{8b}-\frac{\ln\left(e^{bx+a}-1\right)}{8b}$	87

input
```
int(coth(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sinh(b*x+a)^4*cosh(b*x+a)-1/3*(-1/4*csch(b*x+a)^3+3/8*csch(b*x+a
))*coth(b*x+a)+1/4*arctanh(exp(b*x+a)))
```

3.122.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1109 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 1109, normalized size of antiderivative = 20.16

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*cosh(b*x + a)^7 + 14*cosh(b*x + a)*sinh(b*x + a)^6 + 2*sinh(b*x +
a)^7 + 14*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^5 + 14*cosh(b*x + a)^5 + 7
0*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^4 + 14*(5*cosh(b*x + a)^
4 + 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 14*cosh(b*x + a)^3 + 14*(3*c
osh(b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - (
cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7
*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x
+ a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cos
h(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)
^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x
 + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*
cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)
^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) +
 1) + (cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8
 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*co
sh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 -
 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b
*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*c
osh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cos...
```

3.122.6 Sympy [F]

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx=\int \coth^2{(a+bx)}\operatorname{csch}^3{(a+bx)}\,dx$$

input
```
integrate(coth(b*x+a)**2*csch(b*x+a)**3,x)
```

output
```
Integral(coth(a + b*x)**2*csch(a + b*x)**3, x)
```

3.122.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(49) = 98$.

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.35

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{8\,b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{8\,b} + \frac{e^{(-bx-a)}+7\,e^{(-3\,bx-3\,a)}+7\,e^{(-5\,bx-5\,a)}+e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1)}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*log(e^(-b*x - a) + 1)/b - 1/8*log(e^(-b*x - a) - 1)/b + 1/4*(e^(-b*x -
 a) + 7*e^(-3*b*x - 3*a) + 7*e^(-5*b*x - 5*a) + e^(-7*b*x - 7*a))/(b*(4*e^
(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a
) - 1))
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(49) = 98$.

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.93

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = -\frac{\frac{4\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3+4\,e^{(bx+a)}+4\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2} - \log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + \log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{16\,b}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/16*(4*((e^(b*x + a) + e^(-b*x - a))^3 + 4*e^(b*x + a) + 4*e^(-b*x - a))
/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 - log(e^(b*x + a) + e^(-b*x - a) +
 2) + log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

3.122.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.98

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{4\sqrt{-b^2}} - \frac{\frac{e^{a+bx}}{b}+\frac{2\,e^{3a+3bx}}{b}+\frac{e^{5a+5bx}}{b}}{6\,e^{4a+4bx}-4\,e^{2a+2bx}-4\,e^{6a+6bx}+e^{8a+8bx}+1} - \frac{3\,e^{a+bx}}{2\,b\,\left(e^{4a+4bx}-2\,e^{2a+2bx}+1\right)} - \frac{2\,e^{a+bx}}{b\,\left(3\,e^{2a+2bx}-3\,e^{4a+4bx}+e^{6a+6bx}-1\right)} - \frac{e^{a+bx}}{4\,b\,\left(e^{2a+2bx}-1\right)}$$

input
```
int(coth(a + b*x)^2/sinh(a + b*x)^3,x)
```

output
```
atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(4*(-b^2)^(1/2)) - (exp(a + b*x)/b
+ (2*exp(3*a + 3*b*x))/b + exp(5*a + 5*b*x)/b)/(6*exp(4*a + 4*b*x) - 4*exp
(2*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1) - (3*exp(a + b*
x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (2*exp(a + b*x))/(
b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - exp(
a + b*x)/(4*b*(exp(2*a + 2*b*x) - 1))
```

3.123 $\int \coth^4(a+bx)\mathbf{csch}(a+bx)\,dx$

3.123.1 Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \coth^4(a+bx)\text{csch}(a+bx)\,dx=-\frac{3\text{arctanh}(\cosh(a+bx))}{8b}-\frac{3\coth(a+bx)\text{csch}(a+bx)}{8b}-\frac{\coth^3(a+bx)\text{csch}(a+bx)}{4b}$$

output
```
-3/8*arctanh(cosh(b*x+a))/b-3/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)^
3*csch(b*x+a)/b
```

3.123.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \coth^4(a+bx)\text{csch}(a+bx)\,dx=-\frac{5\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\text{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}-\frac{5\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\text{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}$$

input
```
Integrate[Coth[a + b*x]^4*Csch[a + b*x],x]
```

output
```
(-5*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) - (3*Log[Cosh
[(a + b*x)/2]])/(8*b) + (3*Log[Sinh[(a + b*x)/2]])/(8*b) - (5*Sech[(a + b*
x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/(64*b)
```

3.123.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {3042, 26, 3091, 26, 3042, 26, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan\left(ia+ibx-\frac{\pi}{2}\right)^4\sec\left(ia+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^4dx$$

$$\Big\downarrow \text{3091}$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\int i\coth^2(a+bx)\text{csch}(a+bx)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}i\int \coth^2(a+bx)\text{csch}(a+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}i\int -i\sec\left(ia+ibx-\frac{\pi}{2}\right)\tan\left(ia+ibx-\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2dx\right)$$

$$\Big\downarrow \text{3091}$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\left(-\frac{1}{2}\int -i\text{csch}(a+bx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\left(\frac{1}{2}i\int \text{csch}(a+bx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\left(\frac{1}{2}i\int i\csc(ia+ibx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\left(-\frac{1}{2}\int \csc(ia+ibx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\left(-\frac{i\text{arctanh}(\cosh(a+bx))}{2b}-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)\right)$$

input
```
Int[Coth[a + b*x]^4*Csch[a + b*x],x]
```

output
```
I*(((I/4)*Coth[a + b*x]^3*Csch[a + b*x])/b - (3*(((-1/2*I)*ArcTanh[Cosh[a
+ b*x]])/b - ((I/2)*Coth[a + b*x]*Csch[a + b*x])/b))/4)
```

3.123.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.123.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{-\frac{\cosh(bx+a)^3}{\sinh(bx+a)^4}+\frac{\cosh(bx+a)}{\sinh(bx+a)^4}+\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	74
default	$\frac{-\frac{\cosh(bx+a)^3}{\sinh(bx+a)^4}+\frac{\cosh(bx+a)}{\sinh(bx+a)^4}+\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	74
risch	$-\frac{e^{bx+a}\left(5\,e^{6bx+6a}+3\,e^{4bx+4a}+3\,e^{2bx+2a}+5\right)}{4b\left(e^{2bx+2a}-1\right)^4}+\frac{3\ln\left(e^{bx+a}-1\right)}{8b}-\frac{3\ln\left(e^{bx+a}+1\right)}{8b}$	89

input
```
int(coth(b*x+a)^4*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-cosh(b*x+a)^3/sinh(b*x+a)^4+1/sinh(b*x+a)^4*cosh(b*x+a)+(-1/4*csch(b
*x+a)^3+3/8*csch(b*x+a))*coth(b*x+a)-3/4*arctanh(exp(b*x+a)))
```

3.123.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 1114, normalized size of antiderivative = 20.25

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(10*cosh(b*x + a)^7 + 70*cosh(b*x + a)*sinh(b*x + a)^6 + 10*sinh(b*x
+ a)^7 + 6*(35*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^5 + 6*cosh(b*x + a)^5 +
10*(35*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^4 + 2*(175*cosh(b*
x + a)^4 + 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^3 + 6*cosh(b*x + a)^3 + 6
*(35*cosh(b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)
^2 + 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^
8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*c
osh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4
- 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(
b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*
cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a
)^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(
b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*
x + a) + 1) - 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(
b*x + a)^8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6
 + 8*(7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*
x + a)^4 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8
*(7*cosh(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^
3 + 4*(7*cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sin
h(b*x + a)^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)...
```

3.123.6 Sympy [F]

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = \int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx$$

input
```
integrate(coth(b*x+a)**4*csch(b*x+a),x)
```

output
```
Integral(coth(a + b*x)**4*csch(a + b*x), x)
```

3.123.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(49) = 98$.

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.42

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{3\log\left(e^{(-bx-a)}+1\right)}{8b} + \frac{3\log\left(e^{(-bx-a)}-1\right)}{8b} + \frac{5e^{(-bx-a)}+3e^{(-3bx-3a)}+3e^{(-5bx-5a)}+5e^{(-7bx-7a)}}{4b(4e^{(-2bx-2a)}-6e^{(-4bx-4a)}+4e^{(-6bx-6a)}-e^{(-8bx-8a)}-1)}$$

input
```
integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="maxima")
```

output
```
-3/8*log(e^(-b*x - a) + 1)/b + 3/8*log(e^(-b*x - a) - 1)/b + 1/4*(5*e^(-b*
x - a) + 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a) + 5*e^(-7*b*x - 7*a))/(b*
(4*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x
- 8*a) - 1))
```

3.123.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(49) = 98$.

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.00

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\frac{4\left(5\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-12e^{(bx+a)}-12e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2} + 3\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) - 3\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{16b}$$

input
```
integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="giac")
```

output
```
-1/16*(4*(5*(e^(b*x + a) + e^(-b*x - a))^3 - 12*e^(b*x + a) - 12*e^(-b*x -
 a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 + 3*log(e^(b*x + a) + e^(-b*x
- a) + 2) - 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

3.123.9 Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.45

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}} - \frac{9\,\mathrm{e}^{a+b\,x}}{2\,b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{6\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}-1\right)} - \frac{4\,\mathrm{e}^{a+b\,x}}{b\,\left(6\,\mathrm{e}^{4\,a+4\,b\,x}-4\,\mathrm{e}^{2\,a+2\,b\,x}-4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)} - \frac{5\,\mathrm{e}^{a+b\,x}}{4\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input

```
int(coth(a + b*x)^4/sinh(a + b*x),x)
```

output

```
- (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2)) - (9*exp(a +
 b*x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (6*exp(a + b*x)
)/(b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (
4*exp(a + b*x))/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a +
6*b*x) + exp(8*a + 8*b*x) + 1)) - (5*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x)
- 1))
```

3.124 $\int \coth^2(x)\mathbf{csch}^4(x)\,dx$

3.124.1 Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \coth^2(x)\text{csch}^4(x)\,dx = \frac{\coth^3(x)}{3} - \frac{\coth^5(x)}{5}$$

output
```
1/3*coth(x)^3-1/5*coth(x)^5
```

3.124.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \coth^2(x)\text{csch}^4(x)\,dx = \frac{2\coth(x)}{15} - \frac{1}{15}\coth(x)\text{csch}^2(x) - \frac{1}{5}\coth(x)\text{csch}^4(x)$$

input
```
Integrate[Coth[x]^2*Csch[x]^4,x]
```

output
```
(2*Coth[x])/15 - (Coth[x]*Csch[x]^2)/15 - (Coth[x]*Csch[x]^4)/5
```

3.124.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 25, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(x)\text{csch}^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(-\frac{\pi}{2}+ix\right)^2\left(-\sec\left(-\frac{\pi}{2}+ix\right)^4\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(ix-\frac{\pi}{2}\right)^4\tan\left(ix-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3087}$$

$$i\int -\coth^2(x)\left(1-\coth^2(x)\right)d(i\coth(x))$$

$$\Big\downarrow \text{244}$$

$$i\int \left(\coth^4(x)-\coth^2(x)\right)d(i\coth(x))$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{1}{5}i\coth^5(x)-\frac{1}{3}i\coth^3(x)\right)$$

input
```
Int[Coth[x]^2*Csch[x]^4,x]
```

output
```
I*((-1/3*I)*Coth[x]^3 + (I/5)*Coth[x]^5)
```

3.124.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.124.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\coth(x)^3}{3}-\frac{\coth(x)^5}{5}$	14
default	$\frac{\coth(x)^3}{3}-\frac{\coth(x)^5}{5}$	14
risch	$-\frac{4(15\,e^{6x}+5\,e^{4x}+5\,e^{2x}-1)}{15(e^{2x}-1)^5}$	31

input
```
int(coth(x)^2*csch(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*coth(x)^3-1/5*coth(x)^5
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 9.65

$$\int \coth^2(x)\text{csch}^4(x)\,dx =$$

$$-\frac{}{15\left(\cosh\left(x\right)^{7}+7\,\cosh\left(x\right)\sinh\left(x\right)^{6}+\sinh\left(x\right)^{7}+\left(21\,\cosh\left(x\right)^{2}-5\right)\sinh\left(x\right)^{5}-5\,\cosh\left(x\right)^{5}+5\left(7\,\cosh\right.\right.}$$

input
```
integrate(coth(x)^2*csch(x)^4,x, algorithm="fricas")
```

output
```
-8/15*(7*cosh(x)^3 + 24*cosh(x)^2*sinh(x) + 21*cosh(x)*sinh(x)^2 + 8*sinh(
x)^3 + 5*cosh(x))/(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(
x)^2 - 5)*sinh(x)^5 - 5*cosh(x)^5 + 5*(7*cosh(x)^3 - 5*cosh(x))*sinh(x)^4
+ (35*cosh(x)^4 - 50*cosh(x)^2 + 11)*sinh(x)^3 + 9*cosh(x)^3 + (21*cosh(x)
^5 - 50*cosh(x)^3 + 27*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 - 25*cosh(x)^4 +
33*cosh(x)^2 - 15)*sinh(x) - 5*cosh(x))
```

3.124.6 Sympy [F]

$$\int \coth^2(x)\text{csch}^4(x)\,dx = \int \coth^2\left(x\right)\text{csch}^4\left(x\right)dx$$

input
```
integrate(coth(x)**2*csch(x)**4,x)
```

output
```
Integral(coth(x)**2*csch(x)**4, x)
```

3.124.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(13) = 26.

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 8.76

$$\int \coth^2(x)\text{csch}^4(x)\,dx = \frac{4\,e^{(-2\,x)}}{3\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)} + \frac{4\,e^{(-4\,x)}}{3\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)} + \frac{4\,e^{(-6\,x)}}{5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1} - \frac{4}{15\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)}$$

input
```
integrate(coth(x)^2*csch(x)^4,x, algorithm="maxima")
```

output
```
4/3*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10
*x) - 1) + 4/3*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*
x) + e^(-10*x) - 1) + 4*e^(-6*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) -
 5*e^(-8*x) + e^(-10*x) - 1) - 4/15/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x
) - 5*e^(-8*x) + e^(-10*x) - 1)
```

3.124.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \coth^2(x)\text{csch}^4(x)\,dx = -\frac{4\,\left(15\,e^{(6\,x)} + 5\,e^{(4\,x)} + 5\,e^{(2\,x)} - 1\right)}{15\,\left(e^{(2\,x)} - 1\right)^5}$$

input
```
integrate(coth(x)^2*csch(x)^4,x, algorithm="giac")
```

output
```
-4/15*(15*e^(6*x) + 5*e^(4*x) + 5*e^(2*x) - 1)/(e^(2*x) - 1)^5
```

3.124.9 Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 8.47

$$\int \coth^2(x)\operatorname{csch}^4(x)\,dx = -\frac{\frac{8\,e^{2\,x}}{5}+\frac{16\,e^{4\,x}}{5}+\frac{8\,e^{6\,x}}{5}}{5\,e^{2\,x}-10\,e^{4\,x}+10\,e^{6\,x}-5\,e^{8\,x}+e^{10\,x}-1} - \frac{\frac{4\,e^{2\,x}}{5}+\frac{8}{15}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1} - \frac{2}{5\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)} - \frac{\frac{8\,e^{2\,x}}{5}+\frac{6\,e^{4\,x}}{5}+\frac{2}{5}}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1}$$

input
```
int(coth(x)^2/sinh(x)^4,x)
```

output
```
- ((8*exp(2*x))/5 + (16*exp(4*x))/5 + (8*exp(6*x))/5)/(5*exp(2*x) - 10*exp
(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1) - ((4*exp(2*x))/5 + 8/15
)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - 2/(5*(exp(4*x) - 2*exp(2*x) +
 1)) - ((8*exp(2*x))/5 + (6*exp(4*x))/5 + 2/5)/(6*exp(4*x) - 4*exp(2*x) -
4*exp(6*x) + exp(8*x) + 1)
```

3.125 $\int \coth^3(x)\operatorname{csch}^4(x)\,dx$

3.125.1 Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx = -\frac{1}{4}\operatorname{csch}^4(x) - \frac{\operatorname{csch}^6(x)}{6}$$

output
```
-1/4*csch(x)^4-1/6*csch(x)^6
```

3.125.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx = -\frac{1}{4}\operatorname{csch}^4(x) - \frac{\operatorname{csch}^6(x)}{6}$$

input
```
Integrate[Coth[x]^3*Csch[x]^4,x]
```

output
```
-1/4*Csch[x]^4 - Csch[x]^6/6
```

3.125.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 26, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(x)\text{csch}^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan\left(-\frac{\pi}{2}+ix\right)^3\sec\left(-\frac{\pi}{2}+ix\right)^4dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec\left(ix-\frac{\pi}{2}\right)^4\tan\left(ix-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\int -i\text{csch}^3(x)\left(\text{csch}^2(x)+1\right)d(-i\text{csch}(x))$$

$$\Big\downarrow \text{25}$$

$$-\int i\text{csch}^3(x)\left(\text{csch}^2(x)+1\right)d(-i\text{csch}(x))$$

$$\Big\downarrow \text{244}$$

$$-\int \left(i\text{csch}^5(x)+i\text{csch}^3(x)\right)d(-i\text{csch}(x))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}\text{csch}^6(x)-\frac{\text{csch}^4(x)}{4}$$

input
```
Int[Coth[x]^3*Csch[x]^4,x]
```

output
```
-1/4*Csch[x]^4 - Csch[x]^6/6
```

3.125.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.125.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

method	result	size
derivativedivides	$-\frac{\left(1+\operatorname{csch}(x)^{2}\right)^{3}}{6}+\frac{\left(1+\operatorname{csch}(x)^{2}\right)^{2}}{4}$	22
default	$-\frac{\left(1+\operatorname{csch}(x)^{2}\right)^{3}}{6}+\frac{\left(1+\operatorname{csch}(x)^{2}\right)^{2}}{4}$	22
risch	$-\frac{4\,{\mathrm e}^{4x}\left(3\,{\mathrm e}^{4x}+2\,{\mathrm e}^{2x}+3\right)}{3\left({\mathrm e}^{2x}-1\right)^{6}}$	29

input
```
int(coth(x)^3*csch(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(1+csch(x)^2)^3+1/4*(1+csch(x)^2)^2
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 222, normalized size of antiderivative = 13.06

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx =$$

$$-\frac{}{3\left(\cosh\left(x\right)^{8}+8\cosh\left(x\right)\sinh\left(x\right)^{7}+\sinh\left(x\right)^{8}+2\left(14\cosh\left(x\right)^{2}-3\right)\sinh\left(x\right)^{6}-6\cosh\left(x\right)^{6}+4\left(14\operatorname{co}\right.\right.}$$

input
```
integrate(coth(x)^3*csch(x)^4,x, algorithm="fricas")
```

output
```
-4/3*(3*cosh(x)^4 + 12*cosh(x)*sinh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 +
1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(3*cosh(x)^3 + cosh(x))*sinh(x) + 3)/(cosh(
x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(14*cosh(x)^2 - 3)*sinh(x)^6 -
6*cosh(x)^6 + 4*(14*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 4
5*cosh(x)^2 + 8)*sinh(x)^4 + 16*cosh(x)^4 + 8*(7*cosh(x)^5 - 15*cosh(x)^3
+ 7*cosh(x))*sinh(x)^3 + 2*(14*cosh(x)^6 - 45*cosh(x)^4 + 48*cosh(x)^2 - 1
3)*sinh(x)^2 - 26*cosh(x)^2 + 4*(2*cosh(x)^7 - 9*cosh(x)^5 + 14*cosh(x)^3
- 7*cosh(x))*sinh(x) + 15)
```

3.125.6 Sympy [F]

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx = \int \coth^3(x)\operatorname{csch}^4(x)\,dx$$

input
```
integrate(coth(x)**3*csch(x)**4,x)
```

output
```
Integral(coth(x)**3*csch(x)**4, x)
```

3.125.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(13) = 26.

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 8.18

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx$$
$$= \frac{4\,e^{(-4\,x)}}{6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1}$$
$$+ \frac{8\,e^{(-6\,x)}}{3\,\left(6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1\right)}$$
$$+ \frac{4\,e^{(-8\,x)}}{6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1}$$

input
```
integrate(coth(x)^3*csch(x)^4,x, algorithm="maxima")
```

output
```
4*e^(-4*x)/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-1
0*x) - e^(-12*x) - 1) + 8/3*e^(-6*x)/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*
x) - 15*e^(-8*x) + 6*e^(-10*x) - e^(-12*x) - 1) + 4*e^(-8*x)/(6*e^(-2*x) -
 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-10*x) - e^(-12*x) - 1)
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx = -\frac{4\,\left(3\,e^{(8\,x)} + 2\,e^{(6\,x)} + 3\,e^{(4\,x)}\right)}{3\,\left(e^{(2\,x)} - 1\right)^6}$$

input
```
integrate(coth(x)^3*csch(x)^4,x, algorithm="giac")
```

output
```
-4/3*(3*e^(8*x) + 2*e^(6*x) + 3*e^(4*x))/(e^(2*x) - 1)^6
```

3.125.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 210, normalized size of antiderivative = 12.35

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx = -\frac{\frac{8\,e^{2\,x}}{5}+\frac{12\,e^{4\,x}}{5}+\frac{16\,e^{6\,x}}{15}+\frac{4}{15}}{5\,e^{2\,x}-10\,e^{4\,x}+10\,e^{6\,x}-5\,e^{8\,x}+e^{10\,x}-1} - \frac{\frac{4\,e^{2\,x}}{3}+4\,e^{4\,x}+4\,e^{6\,x}+\frac{4\,e^{8\,x}}{3}}{15\,e^{4\,x}-6\,e^{2\,x}-20\,e^{6\,x}+15\,e^{8\,x}-6\,e^{10\,x}+e^{12\,x}+1} - \frac{\frac{8\,e^{2\,x}}{15}+\frac{2}{5}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1} - \frac{4}{15\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)} - \frac{\frac{6\,e^{2\,x}}{5}+\frac{4\,e^{4\,x}}{5}+\frac{2}{5}}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1}$$

input
```
int(coth(x)^3/sinh(x)^4,x)
```

output
```
- ((8*exp(2*x))/5 + (12*exp(4*x))/5 + (16*exp(6*x))/15 + 4/15)/(5*exp(2*x)
 - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1) - ((4*exp(2*x))
/3 + 4*exp(4*x) + 4*exp(6*x) + (4*exp(8*x))/3)/(15*exp(4*x) - 6*exp(2*x) -
 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) + exp(12*x) + 1) - ((8*exp(2*x))/
15 + 2/5)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - 4/(15*(exp(4*x) - 2*e
xp(2*x) + 1)) - ((6*exp(2*x))/5 + (4*exp(4*x))/5 + 2/5)/(6*exp(4*x) - 4*ex
p(2*x) - 4*exp(6*x) + exp(8*x) + 1)
```

3.126 $\int \coth^n(x)\operatorname{csch}^4(x)\,dx$

3.126.1 Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \coth^n(x)\operatorname{csch}^4(x)\,dx = \frac{\coth^{1+n}(x)}{1+n} - \frac{\coth^{3+n}(x)}{3+n}$$

output
```
coth(x)^(1+n)/(1+n)-coth(x)^(3+n)/(3+n)
```

3.126.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \coth^n(x)\operatorname{csch}^4(x)\,dx = \frac{(-2-n+\cosh(2x))\coth^{1+n}(x)\operatorname{csch}^2(x)}{(1+n)(3+n)}$$

input
```
Integrate[Coth[x]^n*Csch[x]^4,x]
```

output
```
((-2 - n + Cosh[2*x])*Coth[x]^(1 + n)*Csch[x]^2)/((1 + n)*(3 + n))
```

3.126.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(x)\coth^n(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec\left(-\frac{\pi}{2}+ix\right)^4\left(-i\tan\left(-\frac{\pi}{2}+ix\right)\right)^n dx$$

$$\Big\downarrow \text{3087}$$

$$-i\int \coth^n(x)\left(1-\coth^2(x)\right)d(i\coth(x))$$

$$\Big\downarrow \text{244}$$

$$-i\int \left(\coth^n(x)-\coth^{n+2}(x)\right)d(i\coth(x))$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{i\coth^{n+1}(x)}{n+1}-\frac{i\coth^{n+3}(x)}{n+3}\right)$$

input
```
Int[Coth[x]^n*Csch[x]^4,x]
```

output
```
(-I)*((I*Coth[x]^(1 + n))/(1 + n) - (I*Coth[x]^(3 + n))/(3 + n))
```

3.126.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.126.4 Maple [A] (verified)

Time = 34.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\coth(x)\mathrm{e}^{n\ln(\coth(x))}}{n+1}-\frac{\coth(x)^{3}\mathrm{e}^{n\ln(\coth(x))}}{3+n}$
default	$\frac{\coth(x)\mathrm{e}^{n\ln(\coth(x))}}{n+1}-\frac{\coth(x)^{3}\mathrm{e}^{n\ln(\coth(x))}}{3+n}$
risch	$-\frac{2\left(-\mathrm{e}^{6x}+2n\,\mathrm{e}^{4x}+3\,\mathrm{e}^{4x}+2n\,\mathrm{e}^{2x}+3\,\mathrm{e}^{2x}-1\right)\left(\mathrm{e}^{x}-1\right)^{-n}\left(\mathrm{e}^{x}+1\right)^{-n}\left(1+\mathrm{e}^{2x}\right)^{n}\mathrm{e}^{-\frac{i\pi n\left(-\operatorname{csgn}\left(i\left(1+\mathrm{e}^{2x}\right)\right)\operatorname{csgn}\left(\frac{i\left(1+\mathrm{e}^{2x}\right)}{\mathrm{e}^{x}+1}\right)^{2}+\cdots\right.}{}}}{}$

input
```
int(coth(x)^n*csch(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/(n+1)*coth(x)*exp(n*ln(coth(x)))-1/(3+n)*coth(x)^3*exp(n*ln(coth(x)))
```

3.126.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.38

$$\int \coth^n(x)\operatorname{csch}^4(x)\,dx$$
$$=\frac{2\left(\left(\cosh(x)^3+3\cosh(x)\sinh(x)^2-(2n+3)\cosh(x)\right)\cosh\left(n\log\left(\frac{\cosh(x)}{\sinh(x)}\right)\right)+\left(\cosh(x)^3+3\cosh(x)\cdots\right.}{(n^2+4n+3)\sinh(x)^3+3\left((n^2+4n+3)\cosh(x)^2-n^2-4n\cdots\right.}$$

input
```
integrate(coth(x)^n*csch(x)^4,x, algorithm="fricas")
```

output
```
2*((cosh(x)^3 + 3*cosh(x)*sinh(x)^2 - (2*n + 3)*cosh(x))*cosh(n*log(cosh(x
)/sinh(x))) + (cosh(x)^3 + 3*cosh(x)*sinh(x)^2 - (2*n + 3)*cosh(x))*sinh(n
*log(cosh(x)/sinh(x))))/((n^2 + 4*n + 3)*sinh(x)^3 + 3*((n^2 + 4*n + 3)*co
sh(x)^2 - n^2 - 4*n - 3)*sinh(x))
```

3.126.6 Sympy [F]

$$\int \coth^n(x)\text{csch}^4(x)\,dx = \int \coth^n(x)\,\text{csch}^4(x)\,dx$$

input
```
integrate(coth(x)**n*csch(x)**4,x)
```

output
```
Integral(coth(x)**n*csch(x)**4, x)
```

3.126.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(26) = 52.

Time = 0.32 (sec) , antiderivative size = 368, normalized size of antiderivative = 14.15

$$\begin{aligned}
&\int \coth^n(x)\text{csch}^4(x)\,dx \\
&= -\frac{2\,(2\,n+3)e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2\,x)}+1\right)-2\,x\right)}}{n^2-3\,(n^2+4\,n+3)e^{(-2\,x)}+3\,(n^2+4\,n+3)e^{(-4\,x)}-(n^2+4\,n+3)e^{(-6\,x)}+4\,n+3} \\
&\quad -\frac{2\,(2\,n+3)e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2\,x)}+1\right)-4\,x\right)}}{n^2-3\,(n^2+4\,n+3)e^{(-2\,x)}+3\,(n^2+4\,n+3)e^{(-4\,x)}-(n^2+4\,n+3)e^{(-6\,x)}+4\,n+3} \\
&\quad +\frac{2\,e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2\,x)}+1\right)-6\,x\right)}}{n^2-3\,(n^2+4\,n+3)e^{(-2\,x)}+3\,(n^2+4\,n+3)e^{(-4\,x)}-(n^2+4\,n+3)e^{(-6\,x)}+4\,n+3} \\
&\quad +\frac{2\,e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2\,x)}+1\right)\right)}}{n^2-3\,(n^2+4\,n+3)e^{(-2\,x)}+3\,(n^2+4\,n+3)e^{(-4\,x)}-(n^2+4\,n+3)e^{(-6\,x)}+4\,n+3}
\end{aligned}$$

input
```
integrate(coth(x)^n*csch(x)^4,x, algorithm="maxima")
```

output
```
-2*(2*n + 3)*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n*log(e^(-2*x) +
 1) - 2*x)/(n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x)
- (n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3) - 2*(2*n + 3)*e^(-n*log(e^(-x) + 1)
- n*log(-e^(-x) + 1) + n*log(e^(-2*x) + 1) - 4*x)/(n^2 - 3*(n^2 + 4*n + 3)
*e^(-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) - (n^2 + 4*n + 3)*e^(-6*x) + 4*n +
3) + 2*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n*log(e^(-2*x) + 1) -
6*x)/(n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) - (n^2
 + 4*n + 3)*e^(-6*x) + 4*n + 3) + 2*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x)
+ 1) + n*log(e^(-2*x) + 1))/(n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4
*n + 3)*e^(-4*x) - (n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3)
```

3.126.8 Giac [F]

$$\int \coth^n(x)\operatorname{csch}^4(x)\,dx = \int \coth(x)^n \operatorname{csch}(x)^4\,dx$$

input
```
integrate(coth(x)^n*csch(x)^4,x, algorithm="giac")
```

output
```
integrate(coth(x)^n*csch(x)^4, x)
```

3.126.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int \coth^n(x)\operatorname{csch}^4(x)\,dx = \frac{\left(\frac{4\cosh(3x)}{n^2+4n+3} - \frac{2\cosh(x)\,(4n+6)}{n^2+4n+3}\right)\left(\frac{e^{2x}+1}{e^{2x}-1}\right)^n}{2\sinh(3x) - \frac{2\sinh(x)\,(3n^2+12n+9)}{n^2+4n+3}}$$

input
```
int(coth(x)^n/sinh(x)^4,x)
```

output
```
(((4*cosh(3*x))/(4*n + n^2 + 3) - (2*cosh(x)*(4*n + 6))/(4*n + n^2 + 3))*(
(exp(2*x) + 1)/(exp(2*x) - 1))^n)/(2*sinh(3*x) - (2*sinh(x)*(12*n + 3*n^2
+ 9))/(4*n + n^2 + 3))
```

3.127 $\int \coth^4(x)\operatorname{csch}^3(x)\,dx$

3.127.1 Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \frac{1}{16}\operatorname{arctanh}(\cosh(x)) - \frac{1}{16}\coth(x)\operatorname{csch}(x) - \frac{1}{8}\coth(x)\operatorname{csch}^3(x) - \frac{1}{6}\coth^3(x)\operatorname{csch}^3(x)$$

output
```
1/16*arctanh(cosh(x))-1/16*coth(x)*csch(x)-1/8*coth(x)*csch(x)^3-1/6*coth(
x)^3*csch(x)^3
```

3.127.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(38) = 76.

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = -\frac{1}{64}\operatorname{csch}^2\left(\frac{x}{2}\right) - \frac{1}{64}\operatorname{csch}^4\left(\frac{x}{2}\right) - \frac{1}{384}\operatorname{csch}^6\left(\frac{x}{2}\right) + \frac{1}{16}\log\left(\cosh\left(\frac{x}{2}\right)\right) - \frac{1}{16}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{1}{64}\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{1}{64}\operatorname{sech}^4\left(\frac{x}{2}\right) - \frac{1}{384}\operatorname{sech}^6\left(\frac{x}{2}\right)$$

input
```
Integrate[Coth[x]^4*Csch[x]^3,x]
```

output
```
-1/64*Csch[x/2]^2 - Csch[x/2]^4/64 - Csch[x/2]^6/384 + Log[Cosh[x/2]]/16 -
 Log[Sinh[x/2]]/16 - Sech[x/2]^2/64 + Sech[x/2]^4/64 - Sech[x/2]^6/384
```

3.127.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.667, Rules used = {3042, 26, 3091, 26, 3042, 26, 3091, 26, 3042, 26, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(x)\text{csch}^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan\left(-\frac{\pi}{2}+ix\right)^4\sec\left(-\frac{\pi}{2}+ix\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec\left(ix-\frac{\pi}{2}\right)^3\tan\left(ix-\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3091}$$

$$-i\left(-\frac{1}{2}\int -i\coth^2(x)\text{csch}^3(x)dx-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}i\int \coth^2(x)\text{csch}^3(x)dx-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{2}i\int i\sec\left(ix-\frac{\pi}{2}\right)^3\tan\left(ix-\frac{\pi}{2}\right)^2dx-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{1}{2}\int \sec\left(ix-\frac{\pi}{2}\right)^3\tan\left(ix-\frac{\pi}{2}\right)^2dx-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{3091}$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\int i\text{csch}^3(x)dx-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}i\int \text{csch}^3(x)dx-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}i\int -i\csc(ix)^3dx-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\int \csc(ix)^3dx-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{4255}$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

input

```
Int[Coth[x]^4*Csch[x]^3,x]
```

output

```
(-I)*((-1/6*I)*Coth[x]^3*Csch[x]^3 + ((-1/4*I)*Coth[x]*Csch[x]^3 + ((I/2)*
ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csch[x])/4)/2)
```

3.127. $\int \coth^4(x)\text{csch}^3(x)\,dx$

3.127.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.127.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{\cosh(x)^3}{3\sinh(x)^6}+\frac{\cosh(x)}{5\sinh(x)^6}+\frac{\left(-\frac{\operatorname{csch}(x)^5}{6}+\frac{5\operatorname{csch}(x)^3}{24}-\frac{5\operatorname{csch}(x)}{16}\right)\coth(x)}{5}+\frac{\operatorname{arctanh}(e^x)}{8}$	46
risch	$-\frac{e^x(3e^{10x}+47e^{8x}+78e^{6x}+78e^{4x}+47e^{2x}+3)}{24(e^{2x}-1)^6}-\frac{\ln(e^x-1)}{16}+\frac{\ln(e^x+1)}{16}$	60

input
```
int(coth(x)^4*csch(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/sinh(x)^6*cosh(x)^3+1/5/sinh(x)^6*cosh(x)+1/5*(-1/6*csch(x)^5+5/24*cs
ch(x)^3-5/16*csch(x))*coth(x)+1/8*arctanh(exp(x))
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1260 vs. $2(30) = 60$.

Time = 0.25 (sec) , antiderivative size = 1260, normalized size of antiderivative = 33.16

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4*csch(x)^3,x, algorithm="fricas")
```

output
```
-1/48*(6*cosh(x)^11 + 66*cosh(x)*sinh(x)^10 + 6*sinh(x)^11 + 2*(165*cosh(x
)^2 + 47)*sinh(x)^9 + 94*cosh(x)^9 + 18*(55*cosh(x)^3 + 47*cosh(x))*sinh(x
)^8 + 12*(165*cosh(x)^4 + 282*cosh(x)^2 + 13)*sinh(x)^7 + 156*cosh(x)^7 +
84*(33*cosh(x)^5 + 94*cosh(x)^3 + 13*cosh(x))*sinh(x)^6 + 12*(231*cosh(x)^
6 + 987*cosh(x)^4 + 273*cosh(x)^2 + 13)*sinh(x)^5 + 156*cosh(x)^5 + 12*(16
5*cosh(x)^7 + 987*cosh(x)^5 + 455*cosh(x)^3 + 65*cosh(x))*sinh(x)^4 + 2*(4
95*cosh(x)^8 + 3948*cosh(x)^6 + 2730*cosh(x)^4 + 780*cosh(x)^2 + 47)*sinh(
x)^3 + 94*cosh(x)^3 + 6*(55*cosh(x)^9 + 564*cosh(x)^7 + 546*cosh(x)^5 + 26
0*cosh(x)^3 + 47*cosh(x))*sinh(x)^2 - 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^1
1 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*c
osh(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sin
h(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(
x)^7 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 2
0*cosh(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*
sinh(x)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2
+ 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(
x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)
^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)
^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x
)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 3*(cosh(x)^12 ...
```

3.127.6 Sympy [F]

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \int \coth^4(x)\operatorname{csch}^3(x)\,dx$$

input
```
integrate(coth(x)**4*csch(x)**3,x)
```

output
```
Integral(coth(x)**4*csch(x)**3, x)
```

3.127.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(30) = 60$.

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.58

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \frac{3\,e^{(-x)} + 47\,e^{(-3\,x)} + 78\,e^{(-5\,x)} + 78\,e^{(-7\,x)} + 47\,e^{(-9\,x)} + 3\,e^{(-11\,x)}}{24\,\left(6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1\right)} + \frac{1}{16}\,\log\left(e^{(-x)} + 1\right) - \frac{1}{16}\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(coth(x)^4*csch(x)^3,x, algorithm="maxima")
```

output
```
1/24*(3*e^(-x) + 47*e^(-3*x) + 78*e^(-5*x) + 78*e^(-7*x) + 47*e^(-9*x) + 3
*e^(-11*x))/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-
10*x) - e^(-12*x) - 1) + 1/16*log(e^(-x) + 1) - 1/16*log(e^(-x) - 1)
```

3.127.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = -\frac{3\,\left(e^{(-x)} + e^{x}\right)^5 + 32\,\left(e^{(-x)} + e^{x}\right)^3 - 48\,e^{(-x)} - 48\,e^{x}}{24\,\left(\left(e^{(-x)} + e^{x}\right)^2 - 4\right)^3} + \frac{1}{32}\,\log\left(e^{(-x)} + e^{x} + 2\right) - \frac{1}{32}\,\log\left(e^{(-x)} + e^{x} - 2\right)$$

input
```
integrate(coth(x)^4*csch(x)^3,x, algorithm="giac")
```

output
```
-1/24*(3*(e^(-x) + e^x)^5 + 32*(e^(-x) + e^x)^3 - 48*e^(-x) - 48*e^x)/((e^
(-x) + e^x)^2 - 4)^3 + 1/32*log(e^(-x) + e^x + 2) - 1/32*log(e^(-x) + e^x
- 2)
```

3.127.9 Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 5.63

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \frac{\ln\left(\frac{e^x}{8}+\frac{1}{8}\right)}{16} - \frac{\ln\left(\frac{e^x}{8}-\frac{1}{8}\right)}{16} - \frac{10\,e^x}{6\,e^{4x}-4\,e^{2x}-4\,e^{6x}+e^{8x}+1} - \frac{e^x}{8\,(e^{2x}-1)} - \frac{7\,e^x}{3\,e^{2x}-3\,e^{4x}+e^{6x}-1} - \frac{\frac{8\,e^{3x}}{3}+4\,e^{5x}+\frac{8\,e^{7x}}{3}+\frac{2\,e^{9x}}{3}+\frac{2\,e^x}{3}}{15\,e^{4x}-6\,e^{2x}-20\,e^{6x}+15\,e^{8x}-6\,e^{10x}+e^{12x}+1} - \frac{16\,e^x}{3\,(5\,e^{2x}-10\,e^{4x}+10\,e^{6x}-5\,e^{8x}+e^{10x}-1)} - \frac{23\,e^x}{12\,(e^{4x}-2\,e^{2x}+1)}$$

input
```
int(coth(x)^4/sinh(x)^3,x)
```

output
```
log(exp(x)/8 + 1/8)/16 - log(exp(x)/8 - 1/8)/16 - (10*exp(x))/(6*exp(4*x)
- 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - exp(x)/(8*(exp(2*x) - 1)) - (7
*exp(x))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - ((8*exp(3*x))/3 + 4*ex
p(5*x) + (8*exp(7*x))/3 + (2*exp(9*x))/3 + (2*exp(x))/3)/(15*exp(4*x) - 6*
exp(2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) + exp(12*x) + 1) - (16*
exp(x))/(3*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x
) - 1)) - (23*exp(x))/(12*(exp(4*x) - 2*exp(2*x) + 1))
```

3.128 $\int \coth^4(x)\mathrm{csch}^6(x)\,dx$

3.128.1 Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \coth^4(x)\mathrm{csch}^6(x)\,dx = -\frac{1}{5}\coth^5(x) + \frac{2\coth^7(x)}{7} - \frac{\coth^9(x)}{9}$$

output
```
-1/5*coth(x)^5+2/7*coth(x)^7-1/9*coth(x)^9
```

3.128.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \coth^4(x)\mathrm{csch}^6(x)\,dx = -\frac{8\coth(x)}{315} + \frac{4}{315}\coth(x)\mathrm{csch}^2(x) - \frac{1}{105}\coth(x)\mathrm{csch}^4(x) - \frac{10}{63}\coth(x)\mathrm{csch}^6(x) - \frac{1}{9}\coth(x)\mathrm{csch}^8(x)$$

input
```
Integrate[Coth[x]^4*Csch[x]^6,x]
```

output
```
(-8*Coth[x])/315 + (4*Coth[x]*Csch[x]^2)/315 - (Coth[x]*Csch[x]^4)/105 - (
10*Coth[x]*Csch[x]^6)/63 - (Coth[x]*Csch[x]^8)/9
```

3.128.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 25, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(x)\text{csch}^6(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan\left(-\frac{\pi}{2}+ix\right)^4\left(-\sec\left(-\frac{\pi}{2}+ix\right)^6\right)dx$$

$$\Big\downarrow \; 25$$

$$-\int \sec\left(ix-\frac{\pi}{2}\right)^6\tan\left(ix-\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \; 3087$$

$$i\int \coth^4(x)\left(1-\coth^2(x)\right)^2d(i\coth(x))$$

$$\Big\downarrow \; 244$$

$$i\int \left(\coth^8(x)-2\coth^6(x)+\coth^4(x)\right)d(i\coth(x))$$

$$\Big\downarrow \; 2009$$

$$i\left(\frac{1}{9}i\coth^9(x)-\frac{2}{7}i\coth^7(x)+\frac{1}{5}i\coth^5(x)\right)$$

input
```
Int[Coth[x]^4*Csch[x]^6,x]
```

output
```
I*((I/5)*Coth[x]^5 - ((2*I)/7)*Coth[x]^7 + (I/9)*Coth[x]^9)
```

3.128.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.128.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\coth\left(x\right)^{5}}{5}+\frac{2\coth\left(x\right)^{7}}{7}-\frac{\coth\left(x\right)^{9}}{9}$	20
default	$-\frac{\coth\left(x\right)^{5}}{5}+\frac{2\coth\left(x\right)^{7}}{7}-\frac{\coth\left(x\right)^{9}}{9}$	20
risch	$-\frac{16\left(210\,{\mathrm e}^{12x}+315\,{\mathrm e}^{10x}+441\,{\mathrm e}^{8x}+126\,{\mathrm e}^{6x}+36\,{\mathrm e}^{4x}-9\,{\mathrm e}^{2x}+1\right)}{315\left({\mathrm e}^{2x}-1\right)^{9}}$	49

input
```
int(coth(x)^4*csch(x)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*coth(x)^5+2/7*coth(x)^7-1/9*coth(x)^9
```

3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 430, normalized size of antiderivative = 17.20

$$\int \coth^4(x)\mathrm{csch}^6(x)\,dx =$$

$$-\frac{}{315\left(\cosh(x)^{12}+12\cosh(x)\sinh(x)^{11}+\sinh(x)^{12}+3\left(22\cosh(x)^{2}-3\right)\sinh(x)^{10}-9\cosh(x)^{10}+\right.}$$

input
```
integrate(coth(x)^4*csch(x)^6,x, algorithm="fricas")
```

output
```
-16/315*(211*cosh(x)^6 + 1254*cosh(x)*sinh(x)^5 + 211*sinh(x)^6 + 3*(1055*
cosh(x)^2 + 102)*sinh(x)^4 + 306*cosh(x)^4 + 4*(1045*cosh(x)^3 + 324*cosh(
x))*sinh(x)^3 + 3*(1055*cosh(x)^4 + 612*cosh(x)^2 + 159)*sinh(x)^2 + 477*c
osh(x)^2 + 6*(209*cosh(x)^5 + 216*cosh(x)^3 + 135*cosh(x))*sinh(x) + 126)/
(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(22*cosh(x)^2 - 3)*si
nh(x)^10 - 9*cosh(x)^10 + 10*(22*cosh(x)^3 - 9*cosh(x))*sinh(x)^9 + 9*(55*
cosh(x)^4 - 45*cosh(x)^2 + 4)*sinh(x)^8 + 36*cosh(x)^8 + 72*(11*cosh(x)^5
- 15*cosh(x)^3 + 4*cosh(x))*sinh(x)^7 + (924*cosh(x)^6 - 1890*cosh(x)^4 +
1008*cosh(x)^2 - 85)*sinh(x)^6 - 85*cosh(x)^6 + 6*(132*cosh(x)^7 - 378*cos
h(x)^5 + 336*cosh(x)^3 - 83*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 126*co
sh(x)^6 + 168*cosh(x)^4 - 85*cosh(x)^2 + 9)*sinh(x)^4 + 135*cosh(x)^4 + 4*
(55*cosh(x)^9 - 270*cosh(x)^7 + 504*cosh(x)^5 - 415*cosh(x)^3 + 117*cosh(x
))*sinh(x)^3 + 3*(22*cosh(x)^10 - 135*cosh(x)^8 + 336*cosh(x)^6 - 425*cosh
(x)^4 + 270*cosh(x)^2 - 54)*sinh(x)^2 - 162*cosh(x)^2 + 6*(2*cosh(x)^11 -
15*cosh(x)^9 + 48*cosh(x)^7 - 83*cosh(x)^5 + 78*cosh(x)^3 - 30*cosh(x))*si
nh(x) + 84)
```

3.128.6 Sympy [F]

$$\int \coth^4(x)\mathrm{csch}^6(x)\,dx = \int \coth^4(x)\,\mathrm{csch}^6(x)\,dx$$

input
```
integrate(coth(x)**4*csch(x)**6,x)
```

output
```
Integral(coth(x)**4*csch(x)**6, x)
```

3.128.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(19) = 38$.

Time = 0.20 (sec) , antiderivative size = 431, normalized size of antiderivative = 17.24

$$\int \coth^4(x)\text{csch}^6(x)\,dx =$$

$$-\frac{16\,e^{(-2\,x)}}{35\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(\cdots}}$$
$$+\frac{64\,e^{(-4\,x)}}{35\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(\cdots}}$$
$$+\frac{32\,e^{(-6\,x)}}{5\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1\cdots}}$$
$$+\frac{112\,e^{(-8\,x)}}{5\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1\cdots}}$$
$$+\frac{16\,e^{(-10\,x)}}{9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-18\,x\cdots}}$$
$$+\frac{32\,e^{(-12\,x)}}{3\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1\cdots}}$$
$$+\frac{16}{315\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{\cdots}}$$

input
```
integrate(coth(x)^4*csch(x)^6,x, algorithm="maxima")
```

output
```
-16/35*e^(-2*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 1
26*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
+ 64/35*e^(-4*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) +
126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
 + 32/5*e^(-6*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) +
126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
 + 112/5*e^(-8*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) +
 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1
) + 16*e^(-10*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) +
126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
 + 32/3*e^(-12*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) +
 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1
) + 16/315/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^
(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
```

3.128.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \coth^4(x)\mathrm{csch}^6(x)\,dx$$
$$= -\frac{16\left(210\,e^{(12\,x)} + 315\,e^{(10\,x)} + 441\,e^{(8\,x)} + 126\,e^{(6\,x)} + 36\,e^{(4\,x)} - 9\,e^{(2\,x)} + 1\right)}{315\,\left(e^{(2\,x)} - 1\right)^9}$$

input
```
integrate(coth(x)^4*csch(x)^6,x, algorithm="giac")
```

output
```
-16/315*(210*e^(12*x) + 315*e^(10*x) + 441*e^(8*x) + 126*e^(6*x) + 36*e^(4
*x) - 9*e^(2*x) + 1)/(e^(2*x) - 1)^9
```

3.128.9 Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 413, normalized size of antiderivative = 16.52

$$\int \coth^4(x)\mathrm{csch}^6(x)\,dx$$
$$= -\frac{\frac{8\,e^{2\,x}}{9} + \frac{16\,e^{4\,x}}{3} + \frac{32\,e^{6\,x}}{3} + \frac{80\,e^{8\,x}}{9} + \frac{8\,e^{10\,x}}{3}}{28\,e^{4\,x} - 8\,e^{2\,x} - 56\,e^{6\,x} + 70\,e^{8\,x} - 56\,e^{10\,x} + 28\,e^{12\,x} - 8\,e^{14\,x} + e^{16\,x} + 1}$$
$$- \frac{\frac{8\,e^{2\,x}}{21} + \frac{16}{63}}{6\,e^{4\,x} - 4\,e^{2\,x} - 4\,e^{6\,x} + e^{8\,x} + 1} - \frac{8}{63\,\left(3\,e^{2\,x} - 3\,e^{4\,x} + e^{6\,x} - 1\right)}$$
$$- \frac{\frac{64\,e^{2\,x}}{63} + \frac{16\,e^{4\,x}}{21} + \frac{32}{105}}{5\,e^{2\,x} - 10\,e^{4\,x} + 10\,e^{6\,x} - 5\,e^{8\,x} + e^{10\,x} - 1}$$
$$- \frac{\frac{32\,e^{2\,x}}{21} + \frac{160\,e^{4\,x}}{63} + \frac{80\,e^{6\,x}}{63} + \frac{16}{63}}{15\,e^{4\,x} - 6\,e^{2\,x} - 20\,e^{6\,x} + 15\,e^{8\,x} - 6\,e^{10\,x} + e^{12\,x} + 1}$$
$$- \frac{\frac{32\,e^{4\,x}}{9} + \frac{128\,e^{6\,x}}{9} + \frac{64\,e^{8\,x}}{3} + \frac{128\,e^{10\,x}}{9} + \frac{32\,e^{12\,x}}{9}}{9\,e^{2\,x} - 36\,e^{4\,x} + 84\,e^{6\,x} - 126\,e^{8\,x} + 126\,e^{10\,x} - 84\,e^{12\,x} + 36\,e^{14\,x} - 9\,e^{16\,x} + e^{18\,x} - 1}$$
$$- \frac{\frac{32\,e^{2\,x}}{21} + \frac{32\,e^{4\,x}}{7} + \frac{320\,e^{6\,x}}{63} + \frac{40\,e^{8\,x}}{21} + \frac{8}{63}}{7\,e^{2\,x} - 21\,e^{4\,x} + 35\,e^{6\,x} - 35\,e^{8\,x} + 21\,e^{10\,x} - 7\,e^{12\,x} + e^{14\,x} - 1}$$

input
```
int(coth(x)^4/sinh(x)^6,x)
```

output

```
- ((8*exp(2*x))/9 + (16*exp(4*x))/3 + (32*exp(6*x))/3 + (80*exp(8*x))/9 +
(8*exp(10*x))/3)/(28*exp(4*x) - 8*exp(2*x) - 56*exp(6*x) + 70*exp(8*x) - 5
6*exp(10*x) + 28*exp(12*x) - 8*exp(14*x) + exp(16*x) + 1) - ((8*exp(2*x))/
21 + 16/63)/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - 8/(63*
(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - ((64*exp(2*x))/63 + (16*exp(4*
x))/21 + 32/105)/(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + ex
p(10*x) - 1) - ((32*exp(2*x))/21 + (160*exp(4*x))/63 + (80*exp(6*x))/63 +
16/63)/(15*exp(4*x) - 6*exp(2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x)
 + exp(12*x) + 1) - ((32*exp(4*x))/9 + (128*exp(6*x))/9 + (64*exp(8*x))/3
+ (128*exp(10*x))/9 + (32*exp(12*x))/9)/(9*exp(2*x) - 36*exp(4*x) + 84*exp
(6*x) - 126*exp(8*x) + 126*exp(10*x) - 84*exp(12*x) + 36*exp(14*x) - 9*exp
(16*x) + exp(18*x) - 1) - ((32*exp(2*x))/21 + (32*exp(4*x))/7 + (320*exp(6
*x))/63 + (40*exp(8*x))/21 + 8/63)/(7*exp(2*x) - 21*exp(4*x) + 35*exp(6*x)
 - 35*exp(8*x) + 21*exp(10*x) - 7*exp(12*x) + exp(14*x) - 1)
```

3.128. $\int \coth^4(x)\text{csch}^6(x)\,dx$

3.129 $\int \coth^5(6x)\mathbf{csch}(6x)\,dx$

3.129.1 Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx = -\frac{1}{6}\operatorname{csch}(6x) - \frac{1}{9}\operatorname{csch}^3(6x) - \frac{1}{30}\operatorname{csch}^5(6x)$$

output
```
-1/6*csch(6*x)-1/9*csch(6*x)^3-1/30*csch(6*x)^5
```

3.129.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx = -\frac{1}{6}\operatorname{csch}(6x) - \frac{1}{9}\operatorname{csch}^3(6x) - \frac{1}{30}\operatorname{csch}^5(6x)$$

input
```
Integrate[Coth[6*x]^5*Csch[6*x],x]
```

output
```
-1/6*Csch[6*x] - Csch[6*x]^3/9 - Csch[6*x]^5/30
```

3.129.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3086, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(6x)\text{csch}(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(-\frac{\pi}{2}+6ix\right)^5 \sec\left(-\frac{\pi}{2}+6ix\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{1}{6}i\int \left(-\text{csch}^2(6x)-1\right)^2 d(-i\text{csch}(6x))$$

$$\Big\downarrow \text{210}$$

$$-\frac{1}{6}i\int \left(\text{csch}^4(6x)+2\text{csch}^2(6x)+1\right) d(-i\text{csch}(6x))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}i\left(-\frac{1}{5}i\text{csch}^5(6x)-\frac{2}{3}i\text{csch}^3(6x)-i\text{csch}(6x)\right)$$

input
```
Int[Coth[6*x]^5*Csch[6*x],x]
```

output
```
(-1/6*I)*((-I)*Csch[6*x] - ((2*I)/3)*Csch[6*x]^3 - (I/5)*Csch[6*x]^5)
```

3.129.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.129.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(6x)}{6}-\frac{\operatorname{csch}(6x)^{3}}{9}-\frac{\operatorname{csch}(6x)^{5}}{30}$	24
default	$-\frac{\operatorname{csch}(6x)}{6}-\frac{\operatorname{csch}(6x)^{3}}{9}-\frac{\operatorname{csch}(6x)^{5}}{30}$	24
risch	$-\frac{{\mathrm e}^{6x}\left(15\,{\mathrm e}^{48x}-20\,{\mathrm e}^{36x}+58\,{\mathrm e}^{24x}-20\,{\mathrm e}^{12x}+15\right)}{45\left({\mathrm e}^{12x}-1\right)^{5}}$	41

input
```
int(coth(6*x)^5*csch(6*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*csch(6*x)-1/9*csch(6*x)^3-1/30*csch(6*x)^5
```

3.129.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 250, normalized size of antiderivative = 8.62

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx =$$
$$-\frac{15\cosh\left(6\,x\right)^{5}+75\cosh\left(6\,x\right)\sinh\left(6\,x\right)^{4}+15\sinh\left(6\,x\right)^{5}+5\left(30\cosh\left(6\right.\right.}{45\left(\cosh\left(6\,x\right)^{6}+6\cosh\left(6\,x\right)\sinh\left(6\,x\right)^{5}+\sinh\left(6\,x\right)^{6}+3\left(5\cosh\left(6\,x\right)^{2}-2\right)\sinh\left(6\,x\right)^{4}-6\cosh\left(6\,x\right)^{4}\right.}$$

input
```
integrate(coth(6*x)^5*csch(6*x),x, algorithm="fricas")
```

output
```
-1/45*(15*cosh(6*x)^5 + 75*cosh(6*x)*sinh(6*x)^4 + 15*sinh(6*x)^5 + 5*(30*
cosh(6*x)^2 - 7)*sinh(6*x)^3 - 5*cosh(6*x)^3 + 15*(10*cosh(6*x)^3 - cosh(6
*x))*sinh(6*x)^2 + 3*(25*cosh(6*x)^4 - 35*cosh(6*x)^2 + 26)*sinh(6*x) + 38
*cosh(6*x))/(cosh(6*x)^6 + 6*cosh(6*x)*sinh(6*x)^5 + sinh(6*x)^6 + 3*(5*co
sh(6*x)^2 - 2)*sinh(6*x)^4 - 6*cosh(6*x)^4 + 4*(5*cosh(6*x)^3 - 4*cosh(6*x
))*sinh(6*x)^3 + 3*(5*cosh(6*x)^4 - 12*cosh(6*x)^2 + 5)*sinh(6*x)^2 + 15*c
osh(6*x)^2 + 2*(3*cosh(6*x)^5 - 8*cosh(6*x)^3 + 5*cosh(6*x))*sinh(6*x) - 1
0)
```

3.129.6 Sympy [F]

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx = \int \coth^5(6x)\operatorname{csch}(6x)\,dx$$

input
```
integrate(coth(6*x)**5*csch(6*x),x)
```

output
```
Integral(coth(6*x)**5*csch(6*x), x)
```

3.129.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(23) = 46$.

Time = 0.20 (sec) , antiderivative size = 191, normalized size of antiderivative = 6.59

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx = \frac{e^{(-6x)}}{3\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - 1\right)} - \frac{4e^{(-18x)}}{9\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - 1\right)} + \frac{58e^{(-30x)}}{45\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - 1\right)} - \frac{4e^{(-42x)}}{9\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - 1\right)} + \frac{e^{(-54x)}}{3\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - 1\right)}$$

input
```
integrate(coth(6*x)^5*csch(6*x),x, algorithm="maxima")
```

output
```
1/3*e^(-6*x)/(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^
(-60*x) - 1) - 4/9*e^(-18*x)/(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) -
5*e^(-48*x) + e^(-60*x) - 1) + 58/45*e^(-30*x)/(5*e^(-12*x) - 10*e^(-24*x)
 + 10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1) - 4/9*e^(-42*x)/(5*e^(-12*x
) - 10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1) + 1/3*e^(-5
4*x)/(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x)
- 1)
```

3.129.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx = -\frac{15\left(e^{(6x)} - e^{(-6x)}\right)^4 + 40\left(e^{(6x)} - e^{(-6x)}\right)^2 + 48}{45\left(e^{(6x)} - e^{(-6x)}\right)^5}$$

input
```
integrate(coth(6*x)^5*csch(6*x),x, algorithm="giac")
```

output
```
-1/45*(15*(e^(6*x) - e^(-6*x))^4 + 40*(e^(6*x) - e^(-6*x))^2 + 48)/(e^(6*x
) - e^(-6*x))^5
```

3.129.9 Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx = -\frac{e^{6x}\left(58\,e^{24x} - 20\,e^{12x} - 20\,e^{36x} + 15\,e^{48x} + 15\right)}{45\left(e^{12x} - 1\right)^5}$$

input
```
int(coth(6*x)^5/sinh(6*x),x)
```

output
```
-(exp(6*x)*(58*exp(24*x) - 20*exp(12*x) - 20*exp(36*x) + 15*exp(48*x) + 15
))/(45*(exp(12*x) - 1)^5)
```

3.130 $\int \coth^7(x)\mathrm{csch}^3(x)\,dx$

3.130.1 Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \coth^7(x)\mathrm{csch}^3(x)\,dx = -\frac{1}{3}\mathrm{csch}^3(x) - \frac{3\mathrm{csch}^5(x)}{5} - \frac{3\mathrm{csch}^7(x)}{7} - \frac{\mathrm{csch}^9(x)}{9}$$

output
```
-1/3*csch(x)^3-3/5*csch(x)^5-3/7*csch(x)^7-1/9*csch(x)^9
```

3.130.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \coth^7(x)\mathrm{csch}^3(x)\,dx = -\frac{1}{3}\mathrm{csch}^3(x) - \frac{3\mathrm{csch}^5(x)}{5} - \frac{3\mathrm{csch}^7(x)}{7} - \frac{\mathrm{csch}^9(x)}{9}$$

input
```
Integrate[Coth[x]^7*Csch[x]^3,x]
```

output
```
-1/3*Csch[x]^3 - (3*Csch[x]^5)/5 - (3*Csch[x]^7)/7 - Csch[x]^9/9
```

3.130.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^7(x)\text{csch}^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(-\frac{\pi}{2}+ix\right)^7\sec\left(-\frac{\pi}{2}+ix\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$-i\int \text{csch}^2(x)\left(\text{csch}^2(x)+1\right)^3d(-i\text{csch}(x))$$

$$\Big\downarrow \text{25}$$

$$i\int -\text{csch}^2(x)\left(\text{csch}^2(x)+1\right)^3d(-i\text{csch}(x))$$

$$\Big\downarrow \text{244}$$

$$i\int \left(-\text{csch}^8(x)-3\text{csch}^6(x)-3\text{csch}^4(x)-\text{csch}^2(x)\right)d(-i\text{csch}(x))$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{1}{9}i\text{csch}^9(x)-\frac{3}{7}i\text{csch}^7(x)-\frac{3}{5}i\text{csch}^5(x)-\frac{1}{3}i\text{csch}^3(x)\right)$$

input
```
Int[Coth[x]^7*Csch[x]^3,x]
```

output
```
(-I)*((-1/3*I)*Csch[x]^3 - ((3*I)/5)*Csch[x]^5 - ((3*I)/7)*Csch[x]^7 - (I/
9)*Csch[x]^9)
```

3.130.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.130.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(x)^{3}}{3}-\frac{3 \operatorname{csch}(x)^{5}}{5}-\frac{3 \operatorname{csch}(x)^{7}}{7}-\frac{\operatorname{csch}(x)^{9}}{9}$	26
default	$-\frac{\operatorname{csch}(x)^{3}}{3}-\frac{3 \operatorname{csch}(x)^{5}}{5}-\frac{3 \operatorname{csch}(x)^{7}}{7}-\frac{\operatorname{csch}(x)^{9}}{9}$	26
risch	$-\frac{8 \,{\mathrm e}^{3 x} \left(105 \,{\mathrm e}^{12 x}+126 \,{\mathrm e}^{10 x}+711 \,{\mathrm e}^{8 x}+356 \,{\mathrm e}^{6 x}+711 \,{\mathrm e}^{4 x}+126 \,{\mathrm e}^{2 x}+105\right)}{315 \left({\mathrm e}^{2 x}-1\right)^{9}}$	53

input
```
int(coth(x)^7*csch(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*csch(x)^3-3/5*csch(x)^5-3/7*csch(x)^7-1/9*csch(x)^9
```

3.130.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(25) = 50$.

Time = 0.24 (sec) , antiderivative size = 442, normalized size of antiderivative = 13.39

$$\int \coth^7(x)\text{csch}^3(x)\,dx =$$

$$-\frac{}{315\left(\cosh(x)^{11} + 11\cosh(x)\sinh(x)^{10} + \sinh(x)^{11} + \left(55\cosh(x)^2 - 9\right)\sinh(x)^9 - 9\cosh(x)^9 + 3\,(5\right.}$$

input
```
integrate(coth(x)^7*csch(x)^3,x, algorithm="fricas")
```

output
```
-8/315*(105*cosh(x)^8 + 840*cosh(x)*sinh(x)^7 + 105*sinh(x)^8 + 42*(70*cos
h(x)^2 + 3)*sinh(x)^6 + 126*cosh(x)^6 + 84*(70*cosh(x)^3 + 9*cosh(x))*sinh
(x)^5 + 6*(1225*cosh(x)^4 + 315*cosh(x)^2 + 136)*sinh(x)^4 + 816*cosh(x)^4
 + 24*(245*cosh(x)^5 + 105*cosh(x)^3 + 101*cosh(x))*sinh(x)^3 + 2*(1470*co
sh(x)^6 + 945*cosh(x)^4 + 2448*cosh(x)^2 + 241)*sinh(x)^2 + 482*cosh(x)^2
+ 4*(210*cosh(x)^7 + 189*cosh(x)^5 + 606*cosh(x)^3 + 115*cosh(x))*sinh(x)
+ 711)/(cosh(x)^11 + 11*cosh(x)*sinh(x)^10 + sinh(x)^11 + (55*cosh(x)^2 -
9)*sinh(x)^9 - 9*cosh(x)^9 + 3*(55*cosh(x)^3 - 27*cosh(x))*sinh(x)^8 + (33
0*cosh(x)^4 - 324*cosh(x)^2 + 37)*sinh(x)^7 + 35*cosh(x)^7 + 7*(66*cosh(x)
^5 - 108*cosh(x)^3 + 35*cosh(x))*sinh(x)^6 + 3*(154*cosh(x)^6 - 378*cosh(x
)^4 + 259*cosh(x)^2 - 31)*sinh(x)^5 - 75*cosh(x)^5 + (330*cosh(x)^7 - 1134
*cosh(x)^5 + 1225*cosh(x)^3 - 375*cosh(x))*sinh(x)^4 + (165*cosh(x)^8 - 75
6*cosh(x)^6 + 1295*cosh(x)^4 - 930*cosh(x)^2 + 162)*sinh(x)^3 + 90*cosh(x)
^3 + (55*cosh(x)^9 - 324*cosh(x)^7 + 735*cosh(x)^5 - 750*cosh(x)^3 + 270*c
osh(x))*sinh(x)^2 + (11*cosh(x)^10 - 81*cosh(x)^8 + 259*cosh(x)^6 - 465*co
sh(x)^4 + 486*cosh(x)^2 - 210)*sinh(x) - 42*cosh(x))
```

3.130.6 Sympy [F]

$$\int \coth^7(x)\text{csch}^3(x)\,dx = \int \coth^7(x)\,\text{csch}^3(x)\,dx$$

input
```
integrate(coth(x)**7*csch(x)**3,x)
```

output
```
Integral(coth(x)**7*csch(x)**3, x)
```

3.130.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 435, normalized size of antiderivative = 13.18

$$
\int \coth^7(x)\operatorname{csch}^3(x)\,dx
$$

$$
= \frac{8\,e^{(-3\,x)}}{3\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-18\,x}}
$$

$$
+ \frac{16\,e^{(-5\,x)}}{5\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1}}
$$

$$
+ \frac{632\,e^{(-7\,x)}}{35\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-}}
$$

$$
+ \frac{2848\,e^{(-9\,x)}}{315\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e}
$$

$$
+ \frac{632\,e^{(-11\,x)}}{35\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-}}
$$

$$
+ \frac{16\,e^{(-13\,x)}}{5\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1}}
$$

$$
+ \frac{8\,e^{(-15\,x)}}{3\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1}}
$$

input
```
integrate(coth(x)^7*csch(x)^3,x, algorithm="maxima")
```

output
```
8/3*e^(-3*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*
e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 1
6/5*e^(-5*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*
e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 6
32/35*e^(-7*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 12
6*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) +
 2848/315*e^(-9*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x)
+ 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) -
1) + 632/35*e^(-11*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*
x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x)
 - 1) + 16/5*e^(-13*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8
*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x
) - 1) + 8/3*e^(-15*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8
*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x
) - 1)
```

3.130.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

$$\int \coth^7(x)\mathrm{csch}^3(x)\,dx = \frac{8\left(105\left(e^{(-x)}-e^x\right)^6+756\left(e^{(-x)}-e^x\right)^4+2160\left(e^{(-x)}-e^x\right)^2+2240\right)}{315\left(e^{(-x)}-e^x\right)^9}$$

input
```
integrate(coth(x)^7*csch(x)^3,x, algorithm="giac")
```

output
```
8/315*(105*(e^(-x) - e^x)^6 + 756*(e^(-x) - e^x)^4 + 2160*(e^(-x) - e^x)^2
 + 2240)/(e^(-x) - e^x)^9
```

3.130.9 Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 372, normalized size of antiderivative = 11.27

$$\int \coth^7(x)\mathrm{csch}^3(x)\,dx = -\frac{5872\,\mathrm{e}^x}{105\,\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)} - \frac{\frac{28\,\mathrm{e}^{3\,x}}{9}+\frac{28\,\mathrm{e}^{5\,x}}{3}+\frac{140\,\mathrm{e}^{7\,x}}{9}+\frac{140\,\mathrm{e}^{9\,x}}{9}+\frac{28\,\mathrm{e}^{11\,x}}{3}+\frac{28\,\mathrm{e}^{13\,x}}{9}+\frac{4\,\mathrm{e}^{15\,x}}{9}+\frac{4\,\mathrm{e}^{x}}{9}}{9\,\mathrm{e}^{2\,x}-36\,\mathrm{e}^{4\,x}+84\,\mathrm{e}^{6\,x}-126\,\mathrm{e}^{8\,x}+126\,\mathrm{e}^{10\,x}-84\,\mathrm{e}^{12\,x}+36\,\mathrm{e}^{14\,x}-9\,\mathrm{e}^{16\,x}+\mathrm{e}^{18\,x}-1} - \frac{3008\,\mathrm{e}^x}{21\,\left(15\,\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}-20\,\mathrm{e}^{6\,x}+15\,\mathrm{e}^{8\,x}-6\,\mathrm{e}^{10\,x}+\mathrm{e}^{12\,x}+1\right)} - \frac{704\,\mathrm{e}^x}{45\,\left(3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1\right)} - \frac{256\,\mathrm{e}^x}{9\,\left(28\,\mathrm{e}^{4\,x}-8\,\mathrm{e}^{2\,x}-56\,\mathrm{e}^{6\,x}+70\,\mathrm{e}^{8\,x}-56\,\mathrm{e}^{10\,x}+28\,\mathrm{e}^{12\,x}-8\,\mathrm{e}^{14\,x}+\mathrm{e}^{16\,x}+1\right)} - \frac{36608\,\mathrm{e}^x}{315\,\left(5\,\mathrm{e}^{2\,x}-10\,\mathrm{e}^{4\,x}+10\,\mathrm{e}^{6\,x}-5\,\mathrm{e}^{8\,x}+\mathrm{e}^{10\,x}-1\right)} - \frac{20\,\mathrm{e}^x}{9\,\left(\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)} - \frac{2048\,\mathrm{e}^x}{21\,\left(7\,\mathrm{e}^{2\,x}-21\,\mathrm{e}^{4\,x}+35\,\mathrm{e}^{6\,x}-35\,\mathrm{e}^{8\,x}+21\,\mathrm{e}^{10\,x}-7\,\mathrm{e}^{12\,x}+\mathrm{e}^{14\,x}-1\right)}$$

input
```
int(coth(x)^7/sinh(x)^3,x)
```

output
```
- (5872*exp(x))/(105*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1)
) - ((28*exp(3*x))/9 + (28*exp(5*x))/3 + (140*exp(7*x))/9 + (140*exp(9*x))
/9 + (28*exp(11*x))/3 + (28*exp(13*x))/9 + (4*exp(15*x))/9 + (4*exp(x))/9)
/(9*exp(2*x) - 36*exp(4*x) + 84*exp(6*x) - 126*exp(8*x) + 126*exp(10*x) -
84*exp(12*x) + 36*exp(14*x) - 9*exp(16*x) + exp(18*x) - 1) - (3008*exp(x))
/(21*(15*exp(4*x) - 6*exp(2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) +
 exp(12*x) + 1)) - (704*exp(x))/(45*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) -
1)) - (256*exp(x))/(9*(28*exp(4*x) - 8*exp(2*x) - 56*exp(6*x) + 70*exp(8*x
) - 56*exp(10*x) + 28*exp(12*x) - 8*exp(14*x) + exp(16*x) + 1)) - (36608*e
xp(x))/(315*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*
x) - 1)) - (20*exp(x))/(9*(exp(4*x) - 2*exp(2*x) + 1)) - (2048*exp(x))/(21
*(7*exp(2*x) - 21*exp(4*x) + 35*exp(6*x) - 35*exp(8*x) + 21*exp(10*x) - 7*
exp(12*x) + exp(14*x) - 1))
```

3.130. $\int \coth^7(x)\text{csch}^3(x)\,dx$

3.131 $\int \sinh(a+bx)\sinh(c+bx)\,dx$

3.131.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \sinh(a+bx)\sinh(c+bx)\,dx = -\frac{1}{2}x\cosh(a-c)+\frac{\sinh(a+c+2bx)}{4b}$$

output
```
-1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

3.131.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sinh(a+bx)\sinh(c+bx)\,dx = \frac{-2bx\cosh(a-c)+\sinh(a+c+2bx)}{4b}$$

input
```
Integrate[Sinh[a + b*x]*Sinh[c + b*x],x]
```

output
```
(-2*b*x*Cosh[a - c] + Sinh[a + c + 2*b*x])/(4*b)
```

3.131.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\sinh(bx+c)\,dx$$

$$\Big\downarrow \text{6147}$$

$$\int \left(\frac{1}{2}\cosh(a+2bx+c)-\frac{1}{2}\cosh(a-c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(a+2bx+c)}{4b}-\frac{1}{2}x\cosh(a-c)$$

input
```
Int[Sinh[a + b*x]*Sinh[c + b*x],x]
```

output
```
-1/2*(x*Cosh[a - c]) + Sinh[a + c + 2*b*x]/(4*b)
```

3.131.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6147
```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.131.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$-\frac{x\cosh(a-c)}{2}+\frac{\sinh(2bx+a+c)}{4b}$
risch	$-\frac{x\,\mathrm{e}^{a-c}}{4}-\frac{x\,\mathrm{e}^{-a+c}}{4}+\frac{\mathrm{e}^{2bx+a+c}}{8b}-\frac{\mathrm{e}^{-2bx-a-c}}{8b}$
parallelrisch	$\frac{-bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\tanh\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}+2\left(2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)xb+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1\right)\tanh\left(\frac{bx}{2}+\frac{c}{2}\right)-bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}{2b\left(1+\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\tanh\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}$

input
```
int(sinh(b*x+a)*sinh(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(23) = 46.

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.22

$$\int \sinh(a+bx)\sinh(c+bx)\,dx =$$
$$-\frac{2\,bx\cosh\left(-a+c\right)-2\,\cosh\left(bx+c\right)\cosh\left(-a+c\right)\sinh\left(bx+c\right)+\cosh\left(bx+c\right)^{2}\sinh\left(-a+c\right)+\sinh\left(\right.}{4\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)}$$

input
```
integrate(sinh(b*x+a)*sinh(b*x+c),x, algorithm="fricas")
```

output
```
-1/4*(2*b*x*cosh(-a + c) - 2*cosh(b*x + c)*cosh(-a + c)*sinh(b*x + c) + co
sh(b*x + c)^2*sinh(-a + c) + sinh(b*x + c)^2*sinh(-a + c))/(b*cosh(-a + c)
^2 - b*sinh(-a + c)^2)
```

3.131.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \sinh(a+bx)\sinh(c+bx)\,dx = \begin{cases} \frac{x\sinh{(a+bx)}\sinh{(bx+c)}}{2} - \frac{x\cosh{(a+bx)}\cosh{(bx+c)}}{2} + \frac{\sinh{(a+bx)}\cosh{(bx+c)}}{2b} & \text{for } b \neq 0 \\ x\sinh{(a)}\sinh{(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(b*x+a)*sinh(b*x+c),x)
```

output
```
Piecewise((x*sinh(a + b*x)*sinh(b*x + c)/2 - x*cosh(a + b*x)*cosh(b*x + c)
/2 + sinh(a + b*x)*cosh(b*x + c)/(2*b), Ne(b, 0)), (x*sinh(a)*sinh(c), Tru
e))
```

3.131.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(23) = 46$.

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \sinh(a+bx)\sinh(c+bx)\,dx = -\frac{(bx+a)\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}}{4b} + \frac{e^{(2bx+a+c)}}{8b} - \frac{e^{(-2bx-a-c)}}{8b}$$

input
```
integrate(sinh(b*x+a)*sinh(b*x+c),x, algorithm="maxima")
```

output
```
-1/4*(b*x + a)*(e^(2*a) + e^(2*c))*e^(-a - c)/b + 1/8*e^(2*b*x + a + c)/b
- 1/8*e^(-2*b*x - a - c)/b
```

3.131.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(23) = 46.

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.63

$$\int \sinh(a+bx)\sinh(c+bx)\,dx$$
$$= -\frac{2\,bx\left(e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}-\left(e^{(2\,bx+2\,a)}+e^{(2\,bx+2\,c)}-1\right)e^{(-2\,bx-a-c)}-e^{(2\,bx+a+c)}}{8\,b}$$

input
```
integrate(sinh(b*x+a)*sinh(b*x+c),x, algorithm="giac")
```

output
```
-1/8*(2*b*x*(e^(2*a) + e^(2*c))*e^(-a - c) - (e^(2*b*x + 2*a) + e^(2*b*x +
 2*c) - 1)*e^(-2*b*x - a - c) - e^(2*b*x + a + c))/b
```

3.131.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sinh(a+bx)\sinh(c+bx)\,dx = \frac{\sinh(a+c+2\,b\,x)}{4\,b} - \frac{x\cosh(a-c)}{2}$$

input
```
int(sinh(a + b*x)*sinh(c + b*x),x)
```

output
```
sinh(a + c + 2*b*x)/(4*b) - (x*cosh(a - c))/2
```

3.132 $\int \sinh(c-bx)\sinh(a+bx)\,dx$

3.132.1 Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \sinh(c-bx)\sinh(a+bx)\,dx=\frac{1}{2}x\cosh(a+c)-\frac{\sinh(a-c+2bx)}{4b}$$

output
```
1/2*x*cosh(a+c)-1/4*sinh(2*b*x+a-c)/b
```

3.132.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sinh(c-bx)\sinh(a+bx)\,dx=\frac{1}{2}x\cosh(a+c)-\frac{\sinh(a-c+2bx)}{4b}$$

input
```
Integrate[Sinh[c - b*x]*Sinh[a + b*x],x]
```

output
```
(x*Cosh[a + c])/2 - Sinh[a - c + 2*b*x]/(4*b)
```

3.132.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\sinh(c-bx)\,dx$$

$$\Big\downarrow \text{6147}$$

$$\int \left(\frac{1}{2}\cosh(a+c)-\frac{1}{2}\cosh(a+2bx-c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x\cosh(a+c)-\frac{\sinh(a+2bx-c)}{4b}$$

input
```
Int[Sinh[c - b*x]*Sinh[a + b*x],x]
```

output
```
(x*Cosh[a + c])/2 - Sinh[a - c + 2*b*x]/(4*b)
```

3.132.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6147
```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.132.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cosh(a+c)}{2}-\frac{\sinh(2bx+a-c)}{4b}$
risch	$\frac{x\,\mathrm{e}^{-a-c}}{4}+\frac{x\,\mathrm{e}^{a+c}}{4}-\frac{\mathrm{e}^{2bx+a-c}}{8b}+\frac{\mathrm{e}^{-2bx-a+c}}{8b}$
parallelrisch	$-\frac{-bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\tanh\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}+2\left(2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)xb+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1\right)\tanh\left(\frac{bx}{2}-\frac{c}{2}\right)-bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right.}{2b\left(1+\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\tanh\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}$

input
```
int(-sinh(b*x-c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*cosh(a+c)-1/4*sinh(2*b*x+a-c)/b
```

3.132.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(23) = 46.

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.78

$$\int \sinh(c-bx)\sinh(a+bx)\,dx$$
$$=\frac{2\,bx\cosh\left(a+c\right)-2\,\cosh\left(bx+a\right)\cosh\left(a+c\right)\sinh\left(bx+a\right)+\cosh\left(bx+a\right)^{2}\sinh\left(a+c\right)+\sinh\left(bx+a\right)}{4\left(b\cosh\left(a+c\right)^{2}-b\sinh\left(a+c\right)^{2}\right)}$$

input
```
integrate(-sinh(b*x-c)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*b*x*cosh(a + c) - 2*cosh(b*x + a)*cosh(a + c)*sinh(b*x + a) + cosh(
b*x + a)^2*sinh(a + c) + sinh(b*x + a)^2*sinh(a + c))/(b*cosh(a + c)^2 - b
*sinh(a + c)^2)
```

3.132.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = -\begin{cases} \frac{x\sinh(a+bx)\sinh(bx-c)}{2} - \frac{x\cosh(a+bx)\cosh(bx-c)}{2} + \frac{\sinh(a+bx)\cosh(bx-c)}{2b} & \text{for } b\neq 0 \\ -x\sinh(a)\sinh(c) & \text{otherwise} \end{cases}$$

input
```
integrate(-sinh(b*x-c)*sinh(b*x+a),x)
```

output
```
-Piecewise((x*sinh(a + b*x)*sinh(b*x - c)/2 - x*cosh(a + b*x)*cosh(b*x - c
)/2 + sinh(a + b*x)*cosh(b*x - c)/(2*b), Ne(b, 0)), (-x*sinh(a)*sinh(c), T
rue))
```

3.132.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(23) = 46$.

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = \frac{(bx+a)\left(e^{(2a+2c)}+1\right)e^{(-a-c)}}{4b} - \frac{e^{(2bx+a-c)}}{8b} + \frac{e^{(-2bx-a+c)}}{8b}$$

input
```
integrate(-sinh(b*x-c)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(b*x + a)*(e^(2*a + 2*c) + 1)*e^(-a - c)/b - 1/8*e^(2*b*x + a - c)/b +
 1/8*e^(-2*b*x - a + c)/b
```

3.132.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(23) = 46$.

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.89

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = \frac{2\,bx\left(e^{(2a+2c)}+1\right)e^{(-a-c)} - \left(e^{(2bx)} + e^{(2bx+2a+2c)} - e^{(2c)}\right)e^{(-2bx-a-c)} - e^{(2bx+a-c)}}{8b}$$

input
```
integrate(-sinh(b*x-c)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/8*(2*b*x*(e^(2*a + 2*c) + 1)*e^(-a - c) - (e^(2*b*x) + e^(2*b*x + 2*a +
2*c) - e^(2*c))*e^(-2*b*x - a - c) - e^(2*b*x + a - c))/b
```

3.132.9 Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = \frac{x\cosh(a+c)}{2} - \frac{\sinh(a-c+2\,b\,x)}{4\,b}$$

input
```
int(sinh(a + b*x)*sinh(c - b*x),x)
```

output
```
(x*cosh(a + c))/2 - sinh(a - c + 2*b*x)/(4*b)
```

3.133 $\int \cosh(a+bx)\cosh(c+bx)\,dx$

3.133.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \frac{1}{2}x\cosh(a-c)+\frac{\sinh(a+c+2bx)}{4b}$$

output
```
1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

3.133.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \frac{2bx\cosh(a-c)+\sinh(a+c+2bx)}{4b}$$

input
```
Integrate[Cosh[a + b*x]*Cosh[c + b*x],x]
```

output
```
(2*b*x*Cosh[a - c] + Sinh[a + c + 2*b*x])/(4*b)
```

3.133.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6148, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\cosh(bx+c)\,dx$$

$$\Big\downarrow \; 6148$$

$$\int \left(\frac{1}{2}\cosh(a+2bx+c)+\frac{1}{2}\cosh(a-c)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sinh(a+2bx+c)}{4b}+\frac{1}{2}x\cosh(a-c)$$

input
```
Int[Cosh[a + b*x]*Cosh[c + b*x],x]
```

output
```
(x*Cosh[a - c])/2 + Sinh[a + c + 2*b*x]/(4*b)
```

3.133.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6148
```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.133.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cosh(a-c)}{2}+\frac{\sinh(2bx+a+c)}{4b}$
risch	$\frac{x\,\mathrm{e}^{a-c}}{4}+\frac{x\,\mathrm{e}^{-a+c}}{4}+\frac{\mathrm{e}^{2bx+a+c}}{8b}-\frac{\mathrm{e}^{-2bx-a-c}}{8b}$
parallelrisch	$\frac{bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\tanh\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}+2\left(-2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)xb+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1\right)\tanh\left(\frac{bx}{2}+\frac{c}{2}\right)+bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}{2b\left(1+\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\tanh\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}$

input
```
int(cosh(b*x+a)*cosh(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

3.133.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.30

$$\int \cosh(a+bx)\cosh(c+bx)\,dx$$
$$=\frac{2\,bx\cosh\left(-a+c\right)+2\,\cosh\left(bx+c\right)\cosh\left(-a+c\right)\sinh\left(bx+c\right)-\cosh\left(bx+c\right)^{2}\sinh\left(-a+c\right)-\sinh\left(bx\right.}{4\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)}$$

input
```
integrate(cosh(b*x+a)*cosh(b*x+c),x, algorithm="fricas")
```

output
```
1/4*(2*b*x*cosh(-a + c) + 2*cosh(b*x + c)*cosh(-a + c)*sinh(b*x + c) - cos
h(b*x + c)^2*sinh(-a + c) - sinh(b*x + c)^2*sinh(-a + c))/(b*cosh(-a + c)^
2 - b*sinh(-a + c)^2)
```

3.133.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \begin{cases} -\frac{x\sinh(a+bx)\sinh(bx+c)}{2} + \frac{x\cosh(a+bx)\cosh(bx+c)}{2} + \frac{\sinh(a+bx)\cosh(bx+c)}{2b} & \text{for } b \neq 0 \\ x\cosh(a)\cosh(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)*cosh(b*x+c),x)
```

output
```
Piecewise((-x*sinh(a + b*x)*sinh(b*x + c)/2 + x*cosh(a + b*x)*cosh(b*x + c
)/2 + sinh(a + b*x)*cosh(b*x + c)/(2*b), Ne(b, 0)), (x*cosh(a)*cosh(c), Tr
ue))
```

3.133.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(23) = 46$.

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \frac{(bx+a)\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}}{4b} + \frac{e^{(2bx+a+c)}}{8b} - \frac{e^{(-2bx-a-c)}}{8b}$$

input
```
integrate(cosh(b*x+a)*cosh(b*x+c),x, algorithm="maxima")
```

output
```
1/4*(b*x + a)*(e^(2*a) + e^(2*c))*e^(-a - c)/b + 1/8*e^(2*b*x + a + c)/b -
 1/8*e^(-2*b*x - a - c)/b
```

3.133.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(23) = 46$.

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.56

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \frac{2\,bx\left(e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}-\left(e^{(2\,bx+2\,a)}+e^{(2\,bx+2\,c)}+1\right)e^{(-2\,bx-a-c)}+e^{(2\,bx+a+c)}}{8\,b}$$

input
```
integrate(cosh(b*x+a)*cosh(b*x+c),x, algorithm="giac")
```

output
```
1/8*(2*b*x*(e^(2*a) + e^(2*c))*e^(-a - c) - (e^(2*b*x + 2*a) + e^(2*b*x +
2*c) + 1)*e^(-2*b*x - a - c) + e^(2*b*x + a + c))/b
```

3.133.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \frac{x\cosh(a-c)}{2}+\frac{\sinh(a+c+2\,b\,x)}{4\,b}$$

input
```
int(cosh(a + b*x)*cosh(c + b*x),x)
```

output
```
(x*cosh(a - c))/2 + sinh(a + c + 2*b*x)/(4*b)
```

3.134 $\int \cosh(c-bx)\cosh(a+bx)\,dx$

3.134.1 Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \cosh(c-bx)\cosh(a+bx)\,dx = \frac{1}{2}x\cosh(a+c)+\frac{\sinh(a-c+2bx)}{4b}$$

output
```
1/2*x*cosh(a+c)+1/4*sinh(2*b*x+a-c)/b
```

3.134.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \cosh(c-bx)\cosh(a+bx)\,dx = \frac{2bx\cosh(a+c)+\sinh(a-c+2bx)}{4b}$$

input
```
Integrate[Cosh[c - b*x]*Cosh[a + b*x],x]
```

output
```
(2*b*x*Cosh[a + c] + Sinh[a - c + 2*b*x])/(4*b)
```

3.134.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6148, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\cosh(c-bx)\,dx$$

$$\Big\downarrow \text{6148}$$

$$\int \left(\frac{1}{2}\cosh(a+2bx-c)+\frac{1}{2}\cosh(a+c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(a+2bx-c)}{4b}+\frac{1}{2}x\cosh(a+c)$$

input
```
Int[Cosh[c - b*x]*Cosh[a + b*x],x]
```

output
```
(x*Cosh[a + c])/2 + Sinh[a - c + 2*b*x]/(4*b)
```

3.134.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6148
```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.134.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cosh(a+c)}{2}+\frac{\sinh(2bx+a-c)}{4b}$
risch	$\frac{x\,\mathrm{e}^{a+c}}{4}+\frac{x\,\mathrm{e}^{-a-c}}{4}+\frac{\mathrm{e}^{2bx+a-c}}{8b}-\frac{\mathrm{e}^{-2bx-a+c}}{8b}$
parallelrisch	$\frac{bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\tanh\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}+2\left(-2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)xb+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1\right)\tanh\left(\frac{bx}{2}-\frac{c}{2}\right)+bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}{2b\left(1+\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\tanh\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}$

input
```
int(cosh(b*x-c)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*cosh(a+c)+1/4*sinh(2*b*x+a-c)/b
```

3.134.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.85

$$\int\cosh(c-bx)\cosh(a+bx)\,dx$$
$$=\frac{2\,bx\cosh\left(a+c\right)+2\,\cosh\left(bx+a\right)\cosh\left(a+c\right)\sinh\left(bx+a\right)-\cosh\left(bx+a\right)^{2}\sinh\left(a+c\right)-\sinh\left(bx+a\right)}{4\left(b\cosh\left(a+c\right)^{2}-b\sinh\left(a+c\right)^{2}\right)}$$

input
```
integrate(cosh(b*x-c)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*b*x*cosh(a + c) + 2*cosh(b*x + a)*cosh(a + c)*sinh(b*x + a) - cosh(
b*x + a)^2*sinh(a + c) - sinh(b*x + a)^2*sinh(a + c))/(b*cosh(a + c)^2 - b
*sinh(a + c)^2)
```

3.134.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cosh(c - bx)\cosh(a + bx)\,dx$$
$$= \begin{cases} -\frac{x\sinh(a+bx)\sinh(bx-c)}{2} + \frac{x\cosh(a+bx)\cosh(bx-c)}{2} + \frac{\sinh(a+bx)\cosh(bx-c)}{2b} & \text{for } b \neq 0 \\ x\cosh(a)\cosh(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x-c)*cosh(b*x+a),x)
```

output
```
Piecewise((-x*sinh(a + b*x)*sinh(b*x - c)/2 + x*cosh(a + b*x)*cosh(b*x - c
)/2 + sinh(a + b*x)*cosh(b*x - c)/(2*b), Ne(b, 0)), (x*cosh(a)*cosh(c), Tr
ue))
```

3.134.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(23) = 46$.

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \cosh(c - bx)\cosh(a + bx)\,dx = \frac{(bx + a)\left(e^{(2a+2c)} + 1\right)e^{(-a-c)}}{4b} + \frac{e^{(2bx+a-c)}}{8b} - \frac{e^{(-2bx-a+c)}}{8b}$$

input
```
integrate(cosh(b*x-c)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(b*x + a)*(e^(2*a + 2*c) + 1)*e^(-a - c)/b + 1/8*e^(2*b*x + a - c)/b -
 1/8*e^(-2*b*x - a + c)/b
```

3.134.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(23) = 46$.

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.74

$$\int \cosh(c - bx)\cosh(a + bx)\,dx = \frac{2\,bx\left(e^{(2\,a+2\,c)}+1\right)e^{(-a-c)} - \left(e^{(2\,bx)}+e^{(2\,bx+2\,a+2\,c)}+e^{(2\,c)}\right)e^{(-2\,bx-a-c)}+e^{(2\,bx+a-c)}}{8\,b}$$

input
```
integrate(cosh(b*x-c)*cosh(b*x+a),x, algorithm="giac")
```

output
```
1/8*(2*b*x*(e^(2*a + 2*c) + 1)*e^(-a - c) - (e^(2*b*x) + e^(2*b*x + 2*a +
2*c) + e^(2*c))*e^(-2*b*x - a - c) + e^(2*b*x + a - c))/b
```

3.134.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cosh(c - bx)\cosh(a + bx)\,dx = \frac{\sinh(a - c + 2\,b\,x)}{4\,b} + \frac{x\cosh(a + c)}{2}$$

input
```
int(cosh(a + b*x)*cosh(c - b*x),x)
```

output
```
sinh(a - c + 2*b*x)/(4*b) + (x*cosh(a + c))/2
```

3.135 $\int \tanh(a+bx)\tanh(c+bx)\,dx$

3.135.1 Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = x-\frac{\coth(a-c)\log(\cosh(a+bx))}{b}+\frac{\coth(a-c)\log(\cosh(c+bx))}{b}$$

output
```
x-coth(a-c)*ln(cosh(b*x+a))/b+coth(a-c)*ln(cosh(b*x+c))/b
```

3.135.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = x+\frac{\coth(a-c)(-\log(\cosh(a+bx))+\log(\cosh(c+bx)))}{b}$$

input
```
Integrate[Tanh[a + b*x]*Tanh[c + b*x],x]
```

output
```
x + (Coth[a - c]*(-Log[Cosh[a + b*x]] + Log[Cosh[c + b*x]]))/b
```

3.135.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6180, 6178, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(a+bx)\tanh(bx+c)\,dx$$

$$\Big\downarrow \text{6180}$$

$$x-\cosh(a-c)\int \text{sech}(a+bx)\text{sech}(c+bx)dx$$

$$\Big\downarrow \text{6178}$$

$$x-\cosh(a-c)\left(\text{csch}(a-c)\int \tanh(a+bx)dx-\text{csch}(a-c)\int \tanh(c+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$x-\cosh(a-c)\left(\text{csch}(a-c)\int -i\tan(ia+ibx)dx-\text{csch}(a-c)\int -i\tan(ic+ibx)dx\right)$$

$$\Big\downarrow \text{26}$$

$$x-\cosh(a-c)\left(i\text{csch}(a-c)\int \tan(ic+ibx)dx-i\text{csch}(a-c)\int \tan(ia+ibx)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$x-\cosh(a-c)\left(\frac{\text{csch}(a-c)\log(\cosh(a+bx))}{b}-\frac{\text{csch}(a-c)\log(\cosh(bx+c))}{b}\right)$$

input
```
Int[Tanh[a + b*x]*Tanh[c + b*x],x]
```

output
```
x - Cosh[a - c]*((Csch[a - c]*Log[Cosh[a + b*x]])/b - (Csch[a - c]*Log[Cos
h[c + b*x]])/b)
```

3.135.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 6178
```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Cs
ch[(b*c - a*d)/d]   Int[Tanh[a + b*x], x], x] + Simp[Csch[(b*c - a*d)/b]
Int[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] &&
 NeQ[b*c - a*d, 0]
```

rule 6180
```
Int[Tanh[(a_.) + (b_.)*(x_)]*Tanh[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[b*(
x/d), x] - Simp[(b/d)*Cosh[(b*c - a*d)/d]   Int[Sech[a + b*x]*Sech[c + d*x]
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0
]
```

3.135.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(37) = 74$.

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 4.08

method	result	size
risch	$x-\frac{\ln\left(1+{\mathrm e}^{2 b x+2 a}\right) {\mathrm e}^{2 a}}{b \left({\mathrm e}^{2 a}-{\mathrm e}^{2 c}\right)}-\frac{\ln\left(1+{\mathrm e}^{2 b x+2 a}\right) {\mathrm e}^{2 c}}{b \left({\mathrm e}^{2 a}-{\mathrm e}^{2 c}\right)}+\frac{\ln\left({\mathrm e}^{2 b x+2 a}+{\mathrm e}^{2 a-2 c}\right) {\mathrm e}^{2 a}}{b \left({\mathrm e}^{2 a}-{\mathrm e}^{2 c}\right)}+\frac{\ln\left({\mathrm e}^{2 b x+2 a}+{\mathrm e}^{2 a-2 c}\right) {\mathrm e}^{2 c}}{b \left({\mathrm e}^{2 a}-{\mathrm e}^{2 c}\right)}$	151

input
```
int(tanh(b*x+a)*tanh(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
x-1/b/(exp(2*a)-exp(2*c))*ln(1+exp(2*b*x+2*a))*exp(2*a)-1/b/(exp(2*a)-exp(
2*c))*ln(1+exp(2*b*x+2*a))*exp(2*c)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2
*a)+exp(2*a-2*c))*exp(2*a)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)+exp(2
*a-2*c))*exp(2*c)
```

3.135.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 259, normalized size of antiderivative = 7.00

$$\int \tanh(a+bx)\tanh(c+bx)\,dx$$
$$= \frac{bx\cosh\left(-a+c\right)^2 - 2\,bx\cosh\left(-a+c\right)\sinh\left(-a+c\right) + bx\sinh\left(-a+c\right)^2 - bx - \left(\cosh\left(-a+c\right)^2 - 2\cos\right.}{}$$

input
```
integrate(tanh(b*x+a)*tanh(b*x+c),x, algorithm="fricas")
```

output
```
(b*x*cosh(-a + c)^2 - 2*b*x*cosh(-a + c)*sinh(-a + c) + b*x*sinh(-a + c)^2
 - b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 +
1)*log(2*(cosh(b*x + c)*cosh(-a + c) - sinh(b*x + c)*sinh(-a + c))/(cosh(b
*x + c)*cosh(-a + c) - (cosh(-a + c) + sinh(-a + c))*sinh(b*x + c) + cosh(
b*x + c)*sinh(-a + c))) + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) +
sinh(-a + c)^2 + 1)*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/
(b*cosh(-a + c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

3.135.6 Sympy [F]

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = \int \tanh\left(a+bx\right)\tanh\left(bx+c\right)\,dx$$

input
```
integrate(tanh(b*x+a)*tanh(b*x+c),x)
```

output
```
Integral(tanh(a + b*x)*tanh(b*x + c), x)
```

3.135.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(37) = 74$.

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.24

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = x+\frac{a}{b}-\frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-2bx-2a)}+1\right)}{b\left(e^{(2a)}-e^{(2c)}\right)} + \frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-2bx)}+e^{(2c)}\right)}{b\left(e^{(2a)}-e^{(2c)}\right)}$$

input
```
integrate(tanh(b*x+a)*tanh(b*x+c),x, algorithm="maxima")
```

output
```
x + a/b - (e^(2*a) + e^(2*c))*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a) - e^(2
*c))) + (e^(2*a) + e^(2*c))*log(e^(-2*b*x) + e^(2*c))/(b*(e^(2*a) - e^(2*c
)))
```

3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(37) = 74$.

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.57

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = \frac{bx-\frac{\left(e^{(4a)}+e^{(2a+2c)}\right)\log\left(e^{(2bx+2a)}+1\right)}{e^{(4a)}-e^{(2a+2c)}}+\frac{\left(e^{(2a+2c)}+e^{(4c)}\right)\log\left(e^{(2bx+2c)}+1\right)}{e^{(2a+2c)}-e^{(4c)}}}{b}$$

input
```
integrate(tanh(b*x+a)*tanh(b*x+c),x, algorithm="giac")
```

output
```
(b*x - (e^(4*a) + e^(2*a + 2*c))*log(e^(2*b*x + 2*a) + 1)/(e^(4*a) - e^(2*
a + 2*c)) + (e^(2*a + 2*c) + e^(4*c))*log(e^(2*b*x + 2*c) + 1)/(e^(2*a + 2
*c) - e^(4*c)))/b
```

3.135.9 Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.11

$$\int \tanh(a+bx)\tanh(c+bx)\,dx$$
$$= x - \frac{\ln\left(4\,e^{4\,a}+4\,e^{6\,a}\,e^{2\,b\,x}+4\,e^{2\,a}\,e^{2\,c}+4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}\right)\,\coth(a-c)}{b}$$
$$+ \frac{\ln\left(4\,e^{4\,a}+4\,e^{2\,a}\,e^{2\,c}+4\,e^{2\,a}\,e^{4\,c}\,e^{2\,b\,x}+4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}\right)\,\coth(a-c)}{b}$$

input
```
int(tanh(a + b*x)*tanh(c + b*x),x)
```

output
```
x - (log(4*exp(4*a) + 4*exp(6*a)*exp(2*b*x) + 4*exp(2*a)*exp(2*c) + 4*exp(
4*a)*exp(2*c)*exp(2*b*x))*coth(a - c))/b + (log(4*exp(4*a) + 4*exp(2*a)*ex
p(2*c) + 4*exp(2*a)*exp(4*c)*exp(2*b*x) + 4*exp(4*a)*exp(2*c)*exp(2*b*x))*
coth(a - c))/b
```

3.136 $\int \tanh(c-bx)\tanh(a+bx)\,dx$

3.136.1 Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = -x - \frac{\coth(a+c)\log(\cosh(c-bx))}{b} + \frac{\coth(a+c)\log(\cosh(a+bx))}{b}$$

output
```
-x-coth(a+c)*ln(cosh(b*x-c))/b+coth(a+c)*ln(cosh(b*x+a))/b
```

3.136.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = -x + \frac{\coth(a+c)(-\log(\cosh(c-bx)) + \log(\cosh(a+bx)))}{b}$$

input
```
Integrate[Tanh[c - b*x]*Tanh[a + b*x],x]
```

output
```
-x + (Coth[a + c]*(-Log[Cosh[c - b*x]] + Log[Cosh[a + b*x]]))/b
```

3.136.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6180, 6178, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(a+bx)\tanh(c-bx)\,dx$$

$$\Big\downarrow \text{ 6180}$$

$$\cosh(a+c)\int \text{sech}(c-bx)\text{sech}(a+bx)dx-x$$

$$\Big\downarrow \text{ 6178}$$

$$\cosh(a+c)\left(\text{csch}(a+c)\int \tanh(c-bx)dx+\text{csch}(a+c)\int \tanh(a+bx)dx\right)-x$$

$$\Big\downarrow \text{ 3042}$$

$$-x+\cosh(a+c)\left(\text{csch}(a+c)\int -i\tan(ic-ibx)dx+\text{csch}(a+c)\int -i\tan(ia+ibx)dx\right)$$

$$\Big\downarrow \text{ 26}$$

$$-x+\cosh(a+c)\left(-i\text{csch}(a+c)\int \tan(ic-ibx)dx-i\text{csch}(a+c)\int \tan(ia+ibx)dx\right)$$

$$\Big\downarrow \text{ 3956}$$

$$\cosh(a+c)\left(\frac{\text{csch}(a+c)\log(\cosh(a+bx))}{b}-\frac{\text{csch}(a+c)\log(\cosh(c-bx))}{b}\right)-x$$

input
```
Int[Tanh[c - b*x]*Tanh[a + b*x],x]
```

output
```
-x + Cosh[a + c]*(-((Csch[a + c]*Log[Cosh[c - b*x]])/b) + (Csch[a + c]*Log
[Cosh[a + b*x]])/b)
```

3.136.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 6178
```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Cs
ch[(b*c - a*d)/d]   Int[Tanh[a + b*x], x], x] + Simp[Csch[(b*c - a*d)/b]
Int[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] &&
 NeQ[b*c - a*d, 0]
```

rule 6180
```
Int[Tanh[(a_.) + (b_.)*(x_)]*Tanh[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[b*(
x/d), x] - Simp[(b/d)*Cosh[(b*c - a*d)/d]   Int[Sech[a + b*x]*Sech[c + d*x]
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0
]
```

3.136.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(37) = 74$.

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 4.14

method	result	size
risch	$-x-\frac{\ln\left(\mathrm{e}^{2a+2c}+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{2a+2c}}{b\left(\mathrm{e}^{2a+2c}-1\right)}-\frac{\ln\left(\mathrm{e}^{2a+2c}+\mathrm{e}^{2bx+2a}\right)}{b\left(\mathrm{e}^{2a+2c}-1\right)}+\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{2a+2c}}{b\left(\mathrm{e}^{2a+2c}-1\right)}+\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b\left(\mathrm{e}^{2a+2c}-1\right)}$	149

input
```
int(-tanh(b*x-c)*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-x-1/b/(exp(2*a+2*c)-1)*ln(exp(2*a+2*c)+exp(2*b*x+2*a))*exp(2*a+2*c)-1/b/(
exp(2*a+2*c)-1)*ln(exp(2*a+2*c)+exp(2*b*x+2*a))+1/b/(exp(2*a+2*c)-1)*ln(1+
exp(2*b*x+2*a))*exp(2*a+2*c)+1/b/(exp(2*a+2*c)-1)*ln(1+exp(2*b*x+2*a))
```

3.136.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(37) = 74$.

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 6.00

$$\int \tanh(c-bx)\tanh(a+bx)\,dx =$$

$$-\frac{bx\cosh{(a+c)}^2 - 2\,bx\cosh{(a+c)}\sinh{(a+c)} + bx\sinh{(a+c)}^2 - bx - \left(\cosh{(a+c)}^2 - 2\,\cosh{(a+c}\right.}{}$$

input
```
integrate(-tanh(b*x-c)*tanh(b*x+a),x, algorithm="fricas")
```

output
```
-(b*x*cosh(a + c)^2 - 2*b*x*cosh(a + c)*sinh(a + c) + b*x*sinh(a + c)^2 -
b*x - (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*log(
2*(cosh(b*x + a)*cosh(a + c) - sinh(b*x + a)*sinh(a + c))/(cosh(b*x + a)*c
osh(a + c) - (cosh(a + c) + sinh(a + c))*sinh(b*x + a) + cosh(b*x + a)*sin
h(a + c))) + (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 +
1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2
- 2*b*cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

3.136.6 Sympy [F]

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = -\int \tanh{(a+bx)}\tanh{(bx-c)}\,dx$$

input
```
integrate(-tanh(b*x-c)*tanh(b*x+a),x)
```

output
```
-Integral(tanh(a + b*x)*tanh(b*x - c), x)
```

3.136.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(37) = 74$.

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.42

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = -x - \frac{a}{b} + \frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-2bx-2a)}+1\right)}{b(e^{(2a+2c)}-1)} - \frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-2bx+2c)}+1\right)}{b(e^{(2a+2c)}-1)}$$

input
```
integrate(-tanh(b*x-c)*tanh(b*x+a),x, algorithm="maxima")
```

output
```
-x - a/b + (e^(2*a + 2*c) + 1)*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a + 2*c)
 - 1)) - (e^(2*a + 2*c) + 1)*log(e^(-2*b*x + 2*c) + 1)/(b*(e^(2*a + 2*c) -
 1))
```

3.136.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.39

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = -\frac{bx + \frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(2bx)}+e^{(2c)}\right)}{e^{(2a+2c)}-1} + \frac{\left(e^{(2a)}+e^{(4a+2c)}\right)\log\left(e^{(2bx+2a)}+1\right)}{e^{(2a)}-e^{(4a+2c)}}}{b}$$

input
```
integrate(-tanh(b*x-c)*tanh(b*x+a),x, algorithm="giac")
```

output
```
-(b*x + (e^(2*a + 2*c) + 1)*log(e^(2*b*x) + e^(2*c))/(e^(2*a + 2*c) - 1) +
 (e^(2*a) + e^(4*a + 2*c))*log(e^(2*b*x + 2*a) + 1)/(e^(2*a) - e^(4*a + 2*
c)))/b
```

3.136.9 Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.36

$$\int \tanh(c-bx)\tanh(a+bx)\,dx$$
$$= \frac{\coth(a+c)\,\ln\left(4\,e^{2\,a}\,e^{2\,c}+4\,e^{4\,a}\,e^{4\,c}+4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}+4\,e^{6\,a}\,e^{4\,c}\,e^{2\,b\,x}\right)}{b} - \frac{\coth(a+c)\,\ln\left(4\,e^{2\,a}\,e^{2\,b\,x}+4\,e^{2\,a}\,e^{2\,c}+4\,e^{4\,a}\,e^{4\,c}+4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}\right)}{b} - x$$

input
```
int(tanh(a + b*x)*tanh(c - b*x),x)
```

output
```
(coth(a + c)*log(4*exp(2*a)*exp(2*c) + 4*exp(4*a)*exp(4*c) + 4*exp(4*a)*ex
p(2*c)*exp(2*b*x) + 4*exp(6*a)*exp(4*c)*exp(2*b*x)))/b - (coth(a + c)*log(
4*exp(2*a)*exp(2*b*x) + 4*exp(2*a)*exp(2*c) + 4*exp(4*a)*exp(4*c) + 4*exp(
4*a)*exp(2*c)*exp(2*b*x)))/b - x
```

3.137 $\int \coth(a+bx)\coth(c+bx)\,dx$

3.137.1 Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \coth(a+bx)\coth(c+bx)\,dx = x - \frac{\coth(a-c)\log(\sinh(a+bx))}{b} + \frac{\coth(a-c)\log(\sinh(c+bx))}{b}$$

output
```
x-coth(a-c)*ln(sinh(b*x+a))/b+coth(a-c)*ln(sinh(b*x+c))/b
```

3.137.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \coth(a+bx)\coth(c+bx)\,dx = x + \frac{\coth(a-c)(-\log(\sinh(a+bx))+\log(\sinh(c+bx)))}{b}$$

input
```
Integrate[Coth[a + b*x]*Coth[c + b*x],x]
```

output
```
x + (Coth[a - c]*(-Log[Sinh[a + b*x]] + Log[Sinh[c + b*x]]))/b
```

3.137.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6181, 6179, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(a+bx)\coth(bx+c)\,dx$$

$$\Big\downarrow \text{ 6181}$$

$$\cosh(a-c)\int \text{csch}(a+bx)\text{csch}(c+bx)dx+x$$

$$\Big\downarrow \text{ 6179}$$

$$\cosh(a-c)\left(\text{csch}(a-c)\int \coth(c+bx)dx-\text{csch}(a-c)\int \coth(a+bx)dx\right)+x$$

$$\Big\downarrow \text{ 3042}$$

$$x+\cosh(a-c)\left(\text{csch}(a-c)\int -i\tan\left(ic+ibx+\frac{\pi}{2}\right)dx-\text{csch}(a-c)\int -i\tan\left(ia+ibx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{ 26}$$

$$x+\cosh(a-c)\left(i\text{csch}(a-c)\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx-i\text{csch}(a-c)\int \tan\left(\frac{1}{2}(2ic+\pi)+ibx\right)dx\right)$$

$$\Big\downarrow \text{ 3956}$$

$$x+\cosh(a-c)\left(\frac{\text{csch}(a-c)\log(-i\sinh(bx+c))}{b}-\frac{\text{csch}(a-c)\log(-i\sinh(a+bx))}{b}\right)$$

input
```
Int[Coth[a + b*x]*Coth[c + b*x],x]
```

output
```
x + Cosh[a - c]*(-((Csch[a - c]*Log[(-I)*Sinh[a + b*x]])/b) + (Csch[a - c]
*Log[(-I)*Sinh[c + b*x]])/b)
```

3.137.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 6179
```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc
h[(b*c - a*d)/b]   Int[Coth[a + b*x], x], x] - Simp[Csch[(b*c - a*d)/d]   I
nt[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] &&
NeQ[b*c - a*d, 0]
```

rule 6181
```
Int[Coth[(a_.) + (b_.)*(x_)]*Coth[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[b*(
x/d), x] + Simp[Cosh[(b*c - a*d)/d]   Int[Csch[a + b*x]*Csch[c + d*x], x],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

3.137.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(37) = 74$.

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.19

method	result	size
risch	$x-\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)\mathrm{e}^{2a}}{b\left(\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)}-\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)\mathrm{e}^{2c}}{b\left(\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2c}\right)\mathrm{e}^{2a}}{b\left(\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2c}\right)\mathrm{e}^{2c}}{b\left(\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)}$	155

input
```
int(coth(b*x+a)*coth(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
x-1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)-1)*exp(2*a)-1/b/(exp(2*a)-exp(
2*c))*ln(exp(2*b*x+2*a)-1)*exp(2*c)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2
*a)-exp(2*a-2*c))*exp(2*a)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)-exp(2
*a-2*c))*exp(2*c)
```

3.137.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(37) = 74.

Time = 0.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 7.00

$$\int \coth(a+bx)\coth(c+bx)\,dx$$
$$= \frac{bx\cosh\left(-a+c\right)^2 - 2\,bx\cosh\left(-a+c\right)\sinh\left(-a+c\right) + bx\sinh\left(-a+c\right)^2 - bx - \left(\cosh\left(-a+c\right)^2 - 2\cosh\right.}{}$$

input
```
integrate(coth(b*x+a)*coth(b*x+c),x, algorithm="fricas")
```

output
```
(b*x*cosh(-a + c)^2 - 2*b*x*cosh(-a + c)*sinh(-a + c) + b*x*sinh(-a + c)^2
 - b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 +
1)*log(2*(cosh(-a + c)*sinh(b*x + c) - cosh(b*x + c)*sinh(-a + c))/(cosh(b
*x + c)*cosh(-a + c) - (cosh(-a + c) + sinh(-a + c))*sinh(b*x + c) + cosh(
b*x + c)*sinh(-a + c))) + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) +
sinh(-a + c)^2 + 1)*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/
(b*cosh(-a + c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

3.137.6 Sympy [F]

$$\int \coth(a+bx)\coth(c+bx)\,dx = \int \coth\left(a+bx\right)\coth\left(bx+c\right)\,dx$$

input
```
integrate(coth(b*x+a)*coth(b*x+c),x)
```

output
```
Integral(coth(a + b*x)*coth(b*x + c), x)
```

3.137.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(37) = 74$.

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 4.24

$$\int \coth(a+bx)\coth(c+bx)\,dx = x + \frac{a}{b} - \frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-bx-a)}+1\right)}{b\left(e^{(2a)}-e^{(2c)}\right)} - \frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-bx-a)}-1\right)}{b\left(e^{(2a)}-e^{(2c)}\right)} + \frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-bx)}+e^{c}\right)}{b\left(e^{(2a)}-e^{(2c)}\right)} + \frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-bx)}-e^{c}\right)}{b\left(e^{(2a)}-e^{(2c)}\right)}$$

input
```
integrate(coth(b*x+a)*coth(b*x+c),x, algorithm="maxima")
```

output
```
x + a/b - (e^(2*a) + e^(2*c))*log(e^(-b*x - a) + 1)/(b*(e^(2*a) - e^(2*c))
) - (e^(2*a) + e^(2*c))*log(e^(-b*x - a) - 1)/(b*(e^(2*a) - e^(2*c))) + (e
^(2*a) + e^(2*c))*log(e^(-b*x) + e^c)/(b*(e^(2*a) - e^(2*c))) + (e^(2*a) +
 e^(2*c))*log(e^(-b*x) - e^c)/(b*(e^(2*a) - e^(2*c)))
```

3.137.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.62

$$\int \coth(a+bx)\coth(c+bx)\,dx = \frac{bx - \frac{\left(e^{(4a)}+e^{(2a+2c)}\right)\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{e^{(4a)}-e^{(2a+2c)}} + \frac{\left(e^{(2a+2c)}+e^{(4c)}\right)\log\left(\left|e^{(2bx+2c)}-1\right|\right)}{e^{(2a+2c)}-e^{(4c)}}}{b}$$

input
```
integrate(coth(b*x+a)*coth(b*x+c),x, algorithm="giac")
```

output
```
(b*x - (e^(4*a) + e^(2*a + 2*c))*log(abs(e^(2*b*x + 2*a) - 1))/(e^(4*a) -
e^(2*a + 2*c)) + (e^(2*a + 2*c) + e^(4*c))*log(abs(e^(2*b*x + 2*c) - 1))/(
e^(2*a + 2*c) - e^(4*c)))/b
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.11

$$\int \coth(a+bx)\coth(c+bx)\,dx$$
$$= x - \frac{\ln\left(4\,\mathrm{e}^{4\,a} - 4\,\mathrm{e}^{6\,a}\,\mathrm{e}^{2\,b\,x} + 4\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c} - 4\,\mathrm{e}^{4\,a}\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,b\,x}\right)\,\coth(a-c)}{b}$$
$$+ \frac{\ln\left(4\,\mathrm{e}^{4\,a} + 4\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c} - 4\,\mathrm{e}^{2\,a}\,\mathrm{e}^{4\,c}\,\mathrm{e}^{2\,b\,x} - 4\,\mathrm{e}^{4\,a}\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,b\,x}\right)\,\coth(a-c)}{b}$$

input
```
int(coth(a + b*x)*coth(c + b*x),x)
```

output
```
x - (log(4*exp(4*a) - 4*exp(6*a)*exp(2*b*x) + 4*exp(2*a)*exp(2*c) - 4*exp(
4*a)*exp(2*c)*exp(2*b*x))*coth(a - c))/b + (log(4*exp(4*a) + 4*exp(2*a)*ex
p(2*c) - 4*exp(2*a)*exp(4*c)*exp(2*b*x) - 4*exp(4*a)*exp(2*c)*exp(2*b*x))*
coth(a - c))/b
```

3.138 $\int \coth(c-bx)\coth(a+bx)\,dx$

3.138.1 Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \coth(c-bx)\coth(a+bx)\,dx = -x - \frac{\coth(a+c)\log(\sinh(c-bx))}{b} + \frac{\coth(a+c)\log(\sinh(a+bx))}{b}$$

output
```
-x-coth(a+c)*ln(-sinh(b*x-c))/b+coth(a+c)*ln(sinh(b*x+a))/b
```

3.138.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \coth(c-bx)\coth(a+bx)\,dx = -x + \frac{\coth(a+c)(-\log(\sinh(c-bx)) + \log(-\sinh(a+bx)))}{b}$$

input
```
Integrate[Coth[c - b*x]*Coth[a + b*x],x]
```

output
```
-x + (Coth[a + c]*(-Log[Sinh[c - b*x]] + Log[-Sinh[a + b*x]]))/b
```

3.138.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6181, 6179, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(a+bx)\coth(c-bx)\,dx$$

$$\Big\downarrow \text{6181}$$

$$\cosh(a+c)\int \text{csch}(c-bx)\text{csch}(a+bx)dx-x$$

$$\Big\downarrow \text{6179}$$

$$\cosh(a+c)\left(\text{csch}(a+c)\int \coth(c-bx)dx+\text{csch}(a+c)\int \coth(a+bx)dx\right)-x$$

$$\Big\downarrow \text{3042}$$

$$-x+\cosh(a+c)\left(\text{csch}(a+c)\int -i\tan\left(ic-ibx+\frac{\pi}{2}\right)dx+\text{csch}(a+c)\int -i\tan\left(ia+ibx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{26}$$

$$-x+\cosh(a+c)\left(-i\text{csch}(a+c)\int \tan\left(\frac{1}{2}(2ic+\pi)-ibx\right)dx-i\text{csch}(a+c)\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$-x+\cosh(a+c)\left(\frac{\text{csch}(a+c)\log(-i\sinh(a+bx))}{b}-\frac{\text{csch}(a+c)\log(-i\sinh(c-bx))}{b}\right)$$

input
```
Int[Coth[c - b*x]*Coth[a + b*x],x]
```

output
```
-x + Cosh[a + c]*(-((Csch[a + c]*Log[(-I)*Sinh[c - b*x]])/b) + (Csch[a + c
]*Log[(-I)*Sinh[a + b*x]])/b)
```

3.138.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 6179
```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc
h[(b*c - a*d)/b]   Int[Coth[a + b*x], x], x] - Simp[Csch[(b*c - a*d)/d]   I
nt[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] &&
NeQ[b*c - a*d, 0]
```

rule 6181
```
Int[Coth[(a_.) + (b_.)*(x_)]*Coth[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[b*(
x/d), x] + Simp[Cosh[(b*c - a*d)/d]   Int[Csch[a + b*x]*Csch[c + d*x], x],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

3.138.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(39) = 78$.

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 4.25

method	result	size
risch	$-x-\frac{\ln\left(-{\rm e}^{2a+2c}+{\rm e}^{2bx+2a}\right){\rm e}^{2a+2c}}{b\left({\rm e}^{2a+2c}-1\right)}-\frac{\ln\left(-{\rm e}^{2a+2c}+{\rm e}^{2bx+2a}\right)}{b\left({\rm e}^{2a+2c}-1\right)}+\frac{\ln\left({\rm e}^{2bx+2a}-1\right){\rm e}^{2a+2c}}{b\left({\rm e}^{2a+2c}-1\right)}+\frac{\ln\left({\rm e}^{2bx+2a}-1\right)}{b\left({\rm e}^{2a+2c}-1\right)}$	153

input
```
int(-coth(b*x-c)*coth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-x-1/b/(exp(2*a+2*c)-1)*ln(-exp(2*a+2*c)+exp(2*b*x+2*a))*exp(2*a+2*c)-1/b/
(exp(2*a+2*c)-1)*ln(-exp(2*a+2*c)+exp(2*b*x+2*a))+1/b/(exp(2*a+2*c)-1)*ln(
exp(2*b*x+2*a)-1)*exp(2*a+2*c)+1/b/(exp(2*a+2*c)-1)*ln(exp(2*b*x+2*a)-1)
```

3.138.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(39) = 78.

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 6.00

$$\int \coth(c-bx)\coth(a+bx)\,dx =$$
$$-\frac{bx\cosh\left(a+c\right)^2-2\,bx\cosh\left(a+c\right)\sinh\left(a+c\right)+bx\sinh\left(a+c\right)^2-bx-\left(\cosh\left(a+c\right)^2-2\,\cosh\left(a+c\right.}{}$$

input
```
integrate(-coth(b*x-c)*coth(b*x+a),x, algorithm="fricas")
```

output
```
-(b*x*cosh(a + c)^2 - 2*b*x*cosh(a + c)*sinh(a + c) + b*x*sinh(a + c)^2 -
b*x - (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*log(
2*(cosh(a + c)*sinh(b*x + a) - cosh(b*x + a)*sinh(a + c))/(cosh(b*x + a)*c
osh(a + c) - (cosh(a + c) + sinh(a + c))*sinh(b*x + a) + cosh(b*x + a)*sin
h(a + c))) + (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 +
1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2
- 2*b*cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

3.138.6 Sympy [F]

$$\int \coth(c-bx)\coth(a+bx)\,dx = -\int \coth\left(a+bx\right)\coth\left(bx-c\right)dx$$

input
```
integrate(-coth(b*x-c)*coth(b*x+a),x)
```

output
```
-Integral(coth(a + b*x)*coth(b*x - c), x)
```

3.138.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(39) = 78.

Time = 0.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.44

$$\int \coth(c-bx)\coth(a+bx)\,dx = -x - \frac{a}{b} + \frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx-a)}+1\right)}{b(e^{(2a+2c)}-1)} + \frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx-a)}-1\right)}{b(e^{(2a+2c)}-1)} - \frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx+c)}+1\right)}{b(e^{(2a+2c)}-1)} - \frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx+c)}-1\right)}{b(e^{(2a+2c)}-1)}$$

input
```
integrate(-coth(b*x-c)*coth(b*x+a),x, algorithm="maxima")
```

output
```
-x - a/b + (e^(2*a + 2*c) + 1)*log(e^(-b*x - a) + 1)/(b*(e^(2*a + 2*c) - 1
)) + (e^(2*a + 2*c) + 1)*log(e^(-b*x - a) - 1)/(b*(e^(2*a + 2*c) - 1)) - (
e^(2*a + 2*c) + 1)*log(e^(-b*x + c) + 1)/(b*(e^(2*a + 2*c) - 1)) - (e^(2*a
 + 2*c) + 1)*log(e^(-b*x + c) - 1)/(b*(e^(2*a + 2*c) - 1))
```

3.138.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\int \coth(c-bx)\coth(a+bx)\,dx = -\frac{bx + \frac{\left(e^{(2a+2c)}+1\right)\log\left(\left|e^{(2bx)}-e^{(2c)}\right|\right)}{e^{(2a+2c)}-1} + \frac{\left(e^{(2a)}+e^{(4a+2c)}\right)\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{e^{(2a)}-e^{(4a+2c)}}}{b}$$

input
```
integrate(-coth(b*x-c)*coth(b*x+a),x, algorithm="giac")
```

output
```
-(b*x + (e^(2*a + 2*c) + 1)*log(abs(e^(2*b*x) - e^(2*c)))/(e^(2*a + 2*c) -
 1) + (e^(2*a) + e^(4*a + 2*c))*log(abs(e^(2*b*x + 2*a) - 1))/(e^(2*a) - e
^(4*a + 2*c)))/b
```

3.138.9 Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.36

$$\int \coth(c-bx)\coth(a+bx)\,dx$$
$$=\frac{\coth(a+c)\,\ln\left(4\,e^{2\,a}\,e^{2\,c}+4\,e^{4\,a}\,e^{4\,c}-4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}-4\,e^{6\,a}\,e^{4\,c}\,e^{2\,b\,x}\right)}{b}$$
$$-\frac{\coth(a+c)\,\ln\left(4\,e^{2\,a}\,e^{2\,b\,x}-4\,e^{2\,a}\,e^{2\,c}-4\,e^{4\,a}\,e^{4\,c}+4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}\right)}{b}-x$$

input
```
int(coth(a + b*x)*coth(c - b*x),x)
```

output
```
(coth(a + c)*log(4*exp(2*a)*exp(2*c) + 4*exp(4*a)*exp(4*c) - 4*exp(4*a)*ex
p(2*c)*exp(2*b*x) - 4*exp(6*a)*exp(4*c)*exp(2*b*x)))/b - (coth(a + c)*log(
4*exp(2*a)*exp(2*b*x) - 4*exp(2*a)*exp(2*c) - 4*exp(4*a)*exp(4*c) + 4*exp(
4*a)*exp(2*c)*exp(2*b*x)))/b - x
```

3.139 $\int \text{sech}(a+bx)\text{sech}(c+bx)\,dx$

3.139.1 Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \text{sech}(a+bx)\text{sech}(c+bx)\,dx = \frac{\text{csch}(a-c)\log(\cosh(a+bx))}{b} - \frac{\text{csch}(a-c)\log(\cosh(c+bx))}{b}$$

output
```
csch(a-c)*ln(cosh(b*x+a))/b-csch(a-c)*ln(cosh(b*x+c))/b
```

3.139.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \text{sech}(a+bx)\text{sech}(c+bx)\,dx = \frac{\text{csch}(a-c)(\log(\cosh(a+bx)) - \log(\cosh(c+bx)))}{b}$$

input
```
Integrate[Sech[a + b*x]*Sech[c + b*x],x]
```

output
```
(Csch[a - c]*(Log[Cosh[a + b*x]] - Log[Cosh[c + b*x]]))/b
```

3.139.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6178, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(a+bx)\operatorname{sech}(bx+c)\,dx$$

$$\Big\downarrow \ 6178$$

$$\operatorname{csch}(a-c)\int \tanh(a+bx)dx-\operatorname{csch}(a-c)\int \tanh(c+bx)dx$$

$$\Big\downarrow \ 3042$$

$$\operatorname{csch}(a-c)\int -i\tan(ia+ibx)dx-\operatorname{csch}(a-c)\int -i\tan(ic+ibx)dx$$

$$\Big\downarrow \ 26$$

$$i\operatorname{csch}(a-c)\int \tan(ic+ibx)dx-i\operatorname{csch}(a-c)\int \tan(ia+ibx)dx$$

$$\Big\downarrow \ 3956$$

$$\frac{\operatorname{csch}(a-c)\log(\cosh(a+bx))}{b}-\frac{\operatorname{csch}(a-c)\log(\cosh(bx+c))}{b}$$

input
```
Int[Sech[a + b*x]*Sech[c + b*x],x]
```

output
```
(Csch[a - c]*Log[Cosh[a + b*x]])/b - (Csch[a - c]*Log[Cosh[c + b*x]])/b
```

3.139.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 6178
```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Cs
ch[(b*c - a*d)/d]   Int[Tanh[a + b*x], x], x] + Simp[Csch[(b*c - a*d)/b]
Int[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] &&
 NeQ[b*c - a*d, 0]
```

3.139.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(36) = 72$.

Time = 0.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.14

method	result	size
risch	$-\frac{2\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2c}\right)\mathrm{e}^{a+c}}{\left(\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)b}+\frac{2\ln\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{a+c}}{\left(\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)b}$	77

input
```
int(sech(b*x+a)*sech(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/(exp(2*a)-exp(2*c))/b*exp(a+c)+2*ln(1+e
xp(2*b*x+2*a))/(exp(2*a)-exp(2*c))/b*exp(a+c)
```

3.139.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(36) = 72$.

Time = 0.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 5.11

$$\int \operatorname{sech}(a+bx)\operatorname{sech}(c+bx)\,dx$$
$$= \frac{2\left(\left(\cosh\left(-a+c\right)-\sinh\left(-a+c\right)\right)\log\left(\frac{2\left(\cosh(bx+c)\cosh(-a+c)-\sinh(bx+c)\sinh(-a+c)\right)}{\cosh(bx+c)\cosh(-a+c)-\left(\cosh(-a+c)+\sinh(-a+c)\right)\sinh(bx+c)+\cosh(bx+c)\sinh(-a+}\right.\right.}{b\cosh\left(-a+c\right)^2-2\,b\cosh\left(-a+c\right)\sinh\left(-a+c\right)+b\sin}$$

input
```
integrate(sech(b*x+a)*sech(b*x+c),x, algorithm="fricas")
```

output
```
2*((cosh(-a + c) - sinh(-a + c))*log(2*(cosh(b*x + c)*cosh(-a + c) - sinh(
b*x + c)*sinh(-a + c))/(cosh(b*x + c)*cosh(-a + c) - (cosh(-a + c) + sinh(
-a + c))*sinh(b*x + c) + cosh(b*x + c)*sinh(-a + c))) - (cosh(-a + c) - si
nh(-a + c))*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(
-a + c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

3.139.6 Sympy [F]

$$\int \mathrm{sech}(a+bx)\mathrm{sech}(c+bx)\,dx = \int \mathrm{sech}\,(a+bx)\,\mathrm{sech}\,(bx+c)\,dx$$

input
```
integrate(sech(b*x+a)*sech(b*x+c),x)
```

output
```
Integral(sech(a + b*x)*sech(b*x + c), x)
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.89

$$\int \mathrm{sech}(a+bx)\mathrm{sech}(c+bx)\,dx = \frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b(e^{(2\,a)}-e^{(2\,c)})} - \frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx)}+e^{(2\,c)}\right)}{b(e^{(2\,a)}-e^{(2\,c)})}$$

input
```
integrate(sech(b*x+a)*sech(b*x+c),x, algorithm="maxima")
```

output
```
2*e^(a + c)*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a) - e^(2*c))) - 2*e^(a + c
)*log(e^(-2*b*x) + e^(2*c))/(b*(e^(2*a) - e^(2*c)))
```

3.139.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(36) = 72.

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

$$\int \mathrm{sech}(a+bx)\mathrm{sech}(c+bx)\,dx = \frac{2\left(\frac{e^{(3\,a+c)}\log\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(4\,a)}-e^{(2\,a+2\,c)}} - \frac{e^{(a+3\,c)}\log\left(e^{(2\,bx+2\,c)}+1\right)}{e^{(2\,a+2\,c)}-e^{(4\,c)}}\right)}{b}$$

input
```
integrate(sech(b*x+a)*sech(b*x+c),x, algorithm="giac")
```

output
```
2*(e^(3*a + c)*log(e^(2*b*x + 2*a) + 1)/(e^(4*a) - e^(2*a + 2*c)) - e^(a +
 3*c)*log(e^(2*b*x + 2*c) + 1)/(e^(2*a + 2*c) - e^(4*c)))/b
```

3.139.9 Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 266, normalized size of antiderivative = 7.39

$$\int \mathrm{sech}(a+bx)\mathrm{sech}(c+bx)\,dx$$

$$= \frac{4\sqrt{e^{2a-2c}}\,\mathrm{atan}\left(\frac{b\left(e^{-a}\,e^{c}+e^{-3a}\,e^{3c}\right)\left(e^{2a}\,e^{-2c}\right)^{3/2}}{\sqrt{-b^2\left(e^{2a}\,e^{-2c}-1\right)^2}} + \frac{e^{2a}\,e^{2bx}\left(\frac{2\,e^{-c}\,e^{a}}{b\left(e^{2a}\,e^{-2c}\right)^{3/2}} + \frac{2\left(e^{-a}\,e^{c}+e^{-3a}\,e^{3c}\right)\left(b\sqrt{e^{2a}\,e^{-2c}}+b\left(e^{2a}\,e^{-2c}\right)^{3/2}\cdots\right.}{\sqrt{-b^2\left(e^{2a}\,e^{-2c}-1\right)^2}\,\sqrt{2\,b^2\,e^{2a}\,e^{-2c}-b^2-b^2\,e^{4a}\,e^{-4c}\cdots}}\cdots\right.}{4}\cdots\right.}{\sqrt{2\,b^2\,e^{2a-2c}-b^2\,e^{4a-4c}-b^2}}$$

input
```
int(1/(cosh(a + b*x)*cosh(c + b*x)),x)
```

output
```
(4*exp(2*a - 2*c)^(1/2)*atan((b*(exp(-a)*exp(c) + exp(-3*a)*exp(3*c))*(exp
(2*a)*exp(-2*c))^(3/2))/(-b^2*(exp(2*a)*exp(-2*c) - 1)^2)^(1/2) + (exp(2*a
)*exp(2*b*x)*((2*exp(-c)*exp(a))/(b*(exp(2*a)*exp(-2*c))^(3/2)) + (2*(exp(
-a)*exp(c) + exp(-3*a)*exp(3*c))*(b*(exp(2*a)*exp(-2*c))^(1/2) + b*(exp(2*
a)*exp(-2*c))^(3/2)))/((-b^2*(exp(2*a)*exp(-2*c) - 1)^2)^(1/2)*(2*b^2*exp(
2*a)*exp(-2*c) - b^2 - b^2*exp(4*a)*exp(-4*c))^(1/2)))*(2*b^2*exp(2*a)*exp
(-2*c) - b^2 - b^2*exp(4*a)*exp(-4*c))^(1/2))/4))/(2*b^2*exp(2*a - 2*c) -
b^2*exp(4*a - 4*c) - b^2)^(1/2)
```

3.140 $\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx$

3.140.1 Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx = -\frac{\operatorname{csch}(a+c)\log(\cosh(c-bx))}{b} + \frac{\operatorname{csch}(a+c)\log(\cosh(a+bx))}{b}$$

output
```
-csch(a+c)*ln(cosh(b*x-c))/b+csch(a+c)*ln(cosh(b*x+a))/b
```

3.140.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx = -\frac{\operatorname{csch}(a+c)(\log(\cosh(c-bx))-\log(\cosh(a+bx)))}{b}$$

input
```
Integrate[Sech[c - b*x]*Sech[a + b*x],x]
```

output
```
-((Csch[a + c]*(Log[Cosh[c - b*x]] - Log[Cosh[a + b*x]]))/b)
```

3.140.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6178, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}(a+bx)\text{sech}(c-bx)\,dx$$

$$\Big\downarrow \text{6178}$$

$$\text{csch}(a+c)\int \tanh(c-bx)dx+\text{csch}(a+c)\int \tanh(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\text{csch}(a+c)\int -i\tan(ic-ibx)dx+\text{csch}(a+c)\int -i\tan(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\text{csch}(a+c)\int \tan(ic-ibx)dx-i\text{csch}(a+c)\int \tan(ia+ibx)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\text{csch}(a+c)\log(\cosh(a+bx))}{b}-\frac{\text{csch}(a+c)\log(\cosh(c-bx))}{b}$$

input
```
Int[Sech[c - b*x]*Sech[a + b*x],x]
```

output
```
-((Csch[a + c]*Log[Cosh[c - b*x]])/b) + (Csch[a + c]*Log[Cosh[a + b*x]])/b
```

3.140.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 6178
```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Cs
ch[(b*c - a*d)/d]   Int[Tanh[a + b*x], x], x] + Simp[Csch[(b*c - a*d)/b]
Int[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] &&
 NeQ[b*c - a*d, 0]
```

3.140.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(34) = 68.

Time = 0.46 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.27

method	result	size
risch	$-\frac{2\ln\left(\mathrm{e}^{2a+2c}+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{a+c}}{\left(\mathrm{e}^{2a+2c}-1\right)b}+\frac{2\ln\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{a+c}}{b\left(\mathrm{e}^{2a+2c}-1\right)}$	75

input
```
int(sech(b*x-c)*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(exp(2*a+2*c)+exp(2*b*x+2*a))/(exp(2*a+2*c)-1)/b*exp(a+c)+2/b/(exp(2*
a+2*c)-1)*ln(1+exp(2*b*x+2*a))*exp(a+c)
```

3.140.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.73

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx$$
$$= \frac{2\left(\left(\cosh\left(a+c\right)-\sinh\left(a+c\right)\right)\log\left(\frac{2\left(\cosh(bx+a)\cosh(a+c)-\sinh(bx+a)\sinh(a+c)\right)}{\cosh(bx+a)\cosh(a+c)-\left(\cosh(a+c)+\sinh(a+c)\right)\sinh(bx+a)+\cosh(bx+a)\sinh(a+c)}\right)-(\mathrm{c}}{b\cosh\left(a+c\right)^{2}-2\,b\cosh\left(a+c\right)\sinh\left(a+c\right)+b\sinh\left(a+\mathrm{c}\right.}$$

input
```
integrate(sech(b*x-c)*sech(b*x+a),x, algorithm="fricas")
```

output
```
2*((cosh(a + c) - sinh(a + c))*log(2*(cosh(b*x + a)*cosh(a + c) - sinh(b*x
 + a)*sinh(a + c))/(cosh(b*x + a)*cosh(a + c) - (cosh(a + c) + sinh(a + c)
)*sinh(b*x + a) + cosh(b*x + a)*sinh(a + c))) - (cosh(a + c) - sinh(a + c)
)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 -
 2*b*cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

3.140.6 Sympy [F]

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx=\int \operatorname{sech}\left(a+bx\right)\operatorname{sech}\left(bx-c\right)dx$$

input
```
integrate(sech(b*x-c)*sech(b*x+a),x)
```

output
```
Integral(sech(a + b*x)*sech(b*x - c), x)
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx=\frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b(e^{(2\,a+2\,c)}-1)}-\frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx+2\,c)}+1\right)}{b(e^{(2\,a+2\,c)}-1)}$$

input
```
integrate(sech(b*x-c)*sech(b*x+a),x, algorithm="maxima")
```

output
```
2*e^(a + c)*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a + 2*c) - 1)) - 2*e^(a + c
)*log(e^(-2*b*x + 2*c) + 1)/(b*(e^(2*a + 2*c) - 1))
```

3.140.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx=-\frac{2\left(\frac{e^{(a+c)}\log\left(e^{(2\,bx)}+e^{(2\,c)}\right)}{e^{(2\,a+2\,c)}-1}+\frac{e^{(3\,a+c)}\log\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(2\,a)}-e^{(4\,a+2\,c)}}\right)}{b}$$

input
```
integrate(sech(b*x-c)*sech(b*x+a),x, algorithm="giac")
```

output
```
-2*(e^(a + c)*log(e^(2*b*x) + e^(2*c))/(e^(2*a + 2*c) - 1) + e^(3*a + c)*l
og(e^(2*b*x + 2*a) + 1)/(e^(2*a) - e^(4*a + 2*c)))/b
```

3.140.9 Mupad [B] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 268, normalized size of antiderivative = 8.12

$$\int \mathrm{sech}(c-bx)\mathrm{sech}(a+bx)\,dx = \frac{4\,\mathrm{atan}\left(\frac{e^{2a}\,e^{2bx}\left(\frac{2\,e^{a}\,e^{c}}{b\,\left(e^{2a}\,e^{2c}\right)^{3/2}}+\frac{2\,e^{-3a}\,e^{-3c}\,\left(e^{2a}\,e^{2c}+1\right)\left(b\,\sqrt{e^{2a}\,e^{2c}}+b\,\left(e^{2a}\,e^{2c}\right)^{3/2}\right)}{\sqrt{-b^2\,\left(e^{2a}\,e^{2c}-1\right)^2}\,\sqrt{2\,b^2\,e^{2a}\,e^{2c}-b^2-b^2\,e^{4a}\,e^{4c}}}\right)\sqrt{2\,b^2\,e^{2a}\,e^{2c}-b^2-b^2\,e^{4a}\,e^{4c}}}{4}+\frac{b\,e^{-3a}\,e^{-3c}\,(e\cdots}{\sqrt{-b\cdots}}\cdots\right)}{\sqrt{2\,b^2\,e^{2a+2c}-b^2\,e^{4a+4c}-b^2}}$$

input
```
int(1/(cosh(a + b*x)*cosh(c - b*x)),x)
```

output
```
(4*atan((exp(2*a)*exp(2*b*x)*((2*exp(a)*exp(c))/(b*(exp(2*a)*exp(2*c))^(3/
2)) + (2*exp(-3*a)*exp(-3*c)*(exp(2*a)*exp(2*c) + 1)*(b*(exp(2*a)*exp(2*c)
)^(1/2) + b*(exp(2*a)*exp(2*c))^(3/2)))/((-b^2*(exp(2*a)*exp(2*c) - 1)^2)^
(1/2)*(2*b^2*exp(2*a)*exp(2*c) - b^2 - b^2*exp(4*a)*exp(4*c))^(1/2)))*(2*b
^2*exp(2*a)*exp(2*c) - b^2 - b^2*exp(4*a)*exp(4*c))^(1/2))/4 + (b*exp(-3*a
)*exp(-3*c)*(exp(2*a)*exp(2*c) + 1)*(exp(2*a)*exp(2*c))^(3/2))/(-b^2*(exp(
2*a)*exp(2*c) - 1)^2)^(1/2))*exp(2*a + 2*c)^(1/2))/(2*b^2*exp(2*a + 2*c) -
 b^2*exp(4*a + 4*c) - b^2)^(1/2)
```

3.141 $\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx$

3.141.1 Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = -\frac{\operatorname{csch}(a-c)\log(\sinh(a+bx))}{b} + \frac{\operatorname{csch}(a-c)\log(\sinh(c+bx))}{b}$$

output
```
-csch(a-c)*ln(sinh(b*x+a))/b+csch(a-c)*ln(sinh(b*x+c))/b
```

3.141.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = -\frac{\operatorname{csch}(a-c)(\log(\sinh(a+bx))-\log(\sinh(c+bx)))}{b}$$

input
```
Integrate[Csch[a + b*x]*Csch[c + b*x],x]
```

output
```
-((Csch[a - c]*(Log[Sinh[a + b*x]] - Log[Sinh[c + b*x]]))/b)
```

3.141.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6179, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(a+bx)\text{csch}(bx+c)\,dx$$

$$\Big\downarrow \; 6179$$

$$\text{csch}(a-c)\int \coth(c+bx)dx-\text{csch}(a-c)\int \coth(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\text{csch}(a-c)\int -i\tan\left(ic+ibx+\frac{\pi}{2}\right)dx-\text{csch}(a-c)\int -i\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 26$$

$$i\text{csch}(a-c)\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx-i\text{csch}(a-c)\int \tan\left(\frac{1}{2}(2ic+\pi)+ibx\right)dx$$

$$\Big\downarrow \; 3956$$

$$\frac{\text{csch}(a-c)\log(-i\sinh(bx+c))}{b}-\frac{\text{csch}(a-c)\log(-i\sinh(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]*Csch[c + b*x],x]
```

output
```
-((Csch[a - c]*Log[(-I)*Sinh[a + b*x]])/b) + (Csch[a - c]*Log[(-I)*Sinh[c
+ b*x]])/b
```

3.141.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 6179
```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc
h[(b*c - a*d)/b]   Int[Coth[a + b*x], x], x] - Simp[Csch[(b*c - a*d)/d]   I
nt[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] &&
NeQ[b*c - a*d, 0]
```

3.141.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

method	result	size
risch	$-\frac{2\ln\left({\mathrm e}^{2bx+2a}-1\right){\mathrm e}^{a+c}}{\left({\mathrm e}^{2a}-{\mathrm e}^{2c}\right)b}+\frac{2\ln\left({\mathrm e}^{2bx+2a}-{\mathrm e}^{2a-2c}\right){\mathrm e}^{a+c}}{\left({\mathrm e}^{2a}-{\mathrm e}^{2c}\right)b}$	79

input
```
int(csch(b*x+a)*csch(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(exp(2*b*x+2*a)-1)/(exp(2*a)-exp(2*c))/b*exp(a+c)+2*ln(exp(2*b*x+2*a)
-exp(2*a-2*c))/(exp(2*a)-exp(2*c))/b*exp(a+c)
```

3.141.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 5.11

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = -\frac{2\left(\left(\cosh\left(-a+c\right)-\sinh\left(-a+c\right)\right)\log\left(\frac{2\left(\cosh(-a+c)\sinh(bx+c)-\cosh(bx+c)\sinh(-a+c)\right)}{\cosh(bx+c)\cosh(-a+c)-\left(\cosh(-a+c)+\sinh(-a+c)\right)\sinh(bx+c)+\cosh(bx+c)\sinh(-a+c)}\right)\right.}{b\cosh\left(-a+c\right)^2-2\,b\cosh\left(-a+c\right)\sinh\left(-a+c\right)+b}$$

input
```
integrate(csch(b*x+a)*csch(b*x+c),x, algorithm="fricas")
```

output
```
-2*((cosh(-a + c) - sinh(-a + c))*log(2*(cosh(-a + c)*sinh(b*x + c) - cosh
(b*x + c)*sinh(-a + c))/(cosh(b*x + c)*cosh(-a + c) - (cosh(-a + c) + sinh
(-a + c))*sinh(b*x + c) + cosh(b*x + c)*sinh(-a + c))) - (cosh(-a + c) - s
inh(-a + c))*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh
(-a + c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

3.141.6 Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = \int \operatorname{csch}\left(a+bx\right)\operatorname{csch}\left(bx+c\right)dx$$

input
```
integrate(csch(b*x+a)*csch(b*x+c),x)
```

output
```
Integral(csch(a + b*x)*csch(b*x + c), x)
```

3.141.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(36) = 72.

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.69

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = -\frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}+1\right)}{b(e^{(2\,a)}-e^{(2\,c)})} - \frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}-1\right)}{b(e^{(2\,a)}-e^{(2\,c)})} + \frac{2\,e^{(a+c)}\log\left(e^{(-bx)}+e^{c}\right)}{b(e^{(2\,a)}-e^{(2\,c)})} + \frac{2\,e^{(a+c)}\log\left(e^{(-bx)}-e^{c}\right)}{b(e^{(2\,a)}-e^{(2\,c)})}$$

input
```
integrate(csch(b*x+a)*csch(b*x+c),x, algorithm="maxima")
```

output
```
-2*e^(a + c)*log(e^(-b*x - a) + 1)/(b*(e^(2*a) - e^(2*c))) - 2*e^(a + c)*l
og(e^(-b*x - a) - 1)/(b*(e^(2*a) - e^(2*c))) + 2*e^(a + c)*log(e^(-b*x) +
e^c)/(b*(e^(2*a) - e^(2*c))) + 2*e^(a + c)*log(e^(-b*x) - e^c)/(b*(e^(2*a)
 - e^(2*c)))
```

3.141.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(36) = 72$.

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = -\frac{2\left(\frac{e^{(3a+c)}\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{e^{(4a)}-e^{(2a+2c)}} - \frac{e^{(a+3c)}\log\left(\left|e^{(2bx+2c)}-1\right|\right)}{e^{(2a+2c)}-e^{(4c)}}\right)}{b}$$

input
```
integrate(csch(b*x+a)*csch(b*x+c),x, algorithm="giac")
```

output
```
-2*(e^(3*a + c)*log(abs(e^(2*b*x + 2*a) - 1))/(e^(4*a) - e^(2*a + 2*c)) -
e^(a + 3*c)*log(abs(e^(2*b*x + 2*c) - 1))/(e^(2*a + 2*c) - e^(4*c)))/b
```

3.141.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 266, normalized size of antiderivative = 7.39

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx =$$

$$-\frac{4\sqrt{e^{2a-2c}}\operatorname{atan}\left(\frac{b\left(e^{-a}\,e^{c}+e^{-3a}\,e^{3c}\right)\left(e^{2a}\,e^{-2c}\right)^{3/2}}{\sqrt{-b^2\left(e^{2a}\,e^{-2c}-1\right)^2}} - \frac{e^{2a}\,e^{2bx}\left(\frac{2\,e^{-c}\,e^{a}}{b\left(e^{2a}\,e^{-2c}\right)^{3/2}}+\frac{2\left(e^{-a}\,e^{c}+e^{-3a}\,e^{3c}\right)\left(b\sqrt{e^{2a}\,e^{-2c}}+b\left(e^{2a}\,e^{-2c}\cdots\right.\right.}{\sqrt{-b^2\left(e^{2a}\,e^{-2c}-1\right)^2}\sqrt{2\,b^2\,e^{2a}\,e^{-2c}-b^2-b^2\,e^{4a}\cdots}}\cdots\right.}{4}\cdots\right.}{\sqrt{2\,b^2\,e^{2a-2c}-b^2\,e^{4a-4c}-b^2}\cdots}$$

input
```
int(1/(sinh(a + b*x)*sinh(c + b*x)),x)
```

output
```
-(4*exp(2*a - 2*c)^(1/2)*atan((b*(exp(-a)*exp(c) + exp(-3*a)*exp(3*c))*(ex
p(2*a)*exp(-2*c))^(3/2))/(-b^2*(exp(2*a)*exp(-2*c) - 1)^2)^(1/2) - (exp(2*
a)*exp(2*b*x)*((2*exp(-c)*exp(a))/(b*(exp(2*a)*exp(-2*c))^(3/2)) + (2*(exp
(-a)*exp(c) + exp(-3*a)*exp(3*c))*(b*(exp(2*a)*exp(-2*c))^(1/2) + b*(exp(2
*a)*exp(-2*c))^(3/2)))/((-b^2*(exp(2*a)*exp(-2*c) - 1)^2)^(1/2)*(2*b^2*exp
(2*a)*exp(-2*c) - b^2 - b^2*exp(4*a)*exp(-4*c))^(1/2)))*(2*b^2*exp(2*a)*ex
p(-2*c) - b^2 - b^2*exp(4*a)*exp(-4*c))^(1/2))/4))/(2*b^2*exp(2*a - 2*c) -
 b^2*exp(4*a - 4*c) - b^2)^(1/2)
```

3.141. $\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx$

3.142 $\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx$

3.142.1 Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx = -\frac{\operatorname{csch}(a+c)\log(\sinh(c-bx))}{b} + \frac{\operatorname{csch}(a+c)\log(\sinh(a+bx))}{b}$$

output
```
-csch(a+c)*ln(-sinh(b*x-c))/b+csch(a+c)*ln(sinh(b*x+a))/b
```

3.142.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx = -\frac{\operatorname{csch}(a+c)(\log(\sinh(c-bx)) - \log(-\sinh(a+bx)))}{b}$$

input
```
Integrate[Csch[c - b*x]*Csch[a + b*x],x]
```

output
```
-((Csch[a + c]*(Log[Sinh[c - b*x]] - Log[-Sinh[a + b*x]]))/b)
```

3.142.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6179, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c-bx)\,dx$$

$$\Big\downarrow \; 6179$$

$$\operatorname{csch}(a+c)\int \coth(c-bx)dx+\operatorname{csch}(a+c)\int \coth(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\operatorname{csch}(a+c)\int -i\tan\left(ic-ibx+\frac{\pi}{2}\right)dx+\operatorname{csch}(a+c)\int -i\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 26$$

$$-i\operatorname{csch}(a+c)\int \tan\left(\frac{1}{2}(2ic+\pi)-ibx\right)dx-i\operatorname{csch}(a+c)\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \; 3956$$

$$\frac{\operatorname{csch}(a+c)\log(-i\sinh(a+bx))}{b}-\frac{\operatorname{csch}(a+c)\log(-i\sinh(c-bx))}{b}$$

input
```
Int[Csch[c - b*x]*Csch[a + b*x],x]
```

output
```
-((Csch[a + c]*Log[(-I)*Sinh[c - b*x]])/b) + (Csch[a + c]*Log[(-I)*Sinh[a
+ b*x]])/b
```

3.142.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 6179
```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc
h[(b*c - a*d)/b]   Int[Coth[a + b*x], x], x] - Simp[Csch[(b*c - a*d)/d]   I
nt[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] &&
NeQ[b*c - a*d, 0]
```

3.142.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(36) = 72$.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

method	result	size
risch	$\frac{2\ln\left(\mathrm{e}^{2bx+2a}-1\right)\mathrm{e}^{a+c}}{b\left(\mathrm{e}^{2a+2c}-1\right)}-\frac{2\ln\left(-\mathrm{e}^{2a+2c}+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{a+c}}{b\left(\mathrm{e}^{2a+2c}-1\right)}$	77

input
```
int(-csch(b*x-c)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/b/(exp(2*a+2*c)-1)*ln(exp(2*b*x+2*a)-1)*exp(a+c)-2/b/(exp(2*a+2*c)-1)*ln
(-exp(2*a+2*c)+exp(2*b*x+2*a))*exp(a+c)
```

3.142.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.73

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx = \frac{2\left(\left(\cosh\left(a+c\right)-\sinh\left(a+c\right)\right)\log\left(\frac{2\left(\cosh(a+c)\sinh(bx+a)-\cosh(bx+a)\sinh(a+c)\right)}{\cosh(bx+a)\cosh(a+c)-(\cosh(a+c)+\sinh(a+c))\sinh(bx+a)+\cosh(bx+a)\sinh(a+c)}\right)-(c}{b\cosh\left(a+c\right)^{2}-2\,b\cosh\left(a+c\right)\sinh\left(a+c\right)+b\sinh\left(a+\right.}$$

input
```
integrate(-csch(b*x-c)*csch(b*x+a),x, algorithm="fricas")
```

output
```
2*((cosh(a + c) - sinh(a + c))*log(2*(cosh(a + c)*sinh(b*x + a) - cosh(b*x
 + a)*sinh(a + c))/(cosh(b*x + a)*cosh(a + c) - (cosh(a + c) + sinh(a + c)
)*sinh(b*x + a) + cosh(b*x + a)*sinh(a + c))) - (cosh(a + c) - sinh(a + c)
)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 -
 2*b*cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

3.142.6 Sympy [F]

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx = -\int \operatorname{csch}\left(a+bx\right)\operatorname{csch}\left(bx-c\right)\,dx$$

input
```
integrate(-csch(b*x-c)*csch(b*x+a),x)
```

output
```
-Integral(csch(a + b*x)*csch(b*x - c), x)
```

3.142.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(36) = 72.

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.91

$$\begin{aligned}\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx &= \frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}+1\right)}{b(e^{(2\,a+2\,c)}-1)}+\frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}-1\right)}{b(e^{(2\,a+2\,c)}-1)}\\ &\quad-\frac{2\,e^{(a+c)}\log\left(e^{(-bx+c)}+1\right)}{b(e^{(2\,a+2\,c)}-1)}-\frac{2\,e^{(a+c)}\log\left(e^{(-bx+c)}-1\right)}{b(e^{(2\,a+2\,c)}-1)}\end{aligned}$$

input
```
integrate(-csch(b*x-c)*csch(b*x+a),x, algorithm="maxima")
```

output
```
2*e^(a + c)*log(e^(-b*x - a) + 1)/(b*(e^(2*a + 2*c) - 1)) + 2*e^(a + c)*lo
g(e^(-b*x - a) - 1)/(b*(e^(2*a + 2*c) - 1)) - 2*e^(a + c)*log(e^(-b*x + c)
 + 1)/(b*(e^(2*a + 2*c) - 1)) - 2*e^(a + c)*log(e^(-b*x + c) - 1)/(b*(e^(2
*a + 2*c) - 1))
```

3.142.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\left(\frac{e^{(a+c)}\log\left(\left|e^{(2\,bx)}-e^{(2\,c)}\right|\right)}{e^{(2\,a+2\,c)}-1}+\frac{e^{(3\,a+c)}\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{e^{(2\,a)}-e^{(4\,a+2\,c)}}\right)}{b}$$

input
```
integrate(-csch(b*x-c)*csch(b*x+a),x, algorithm="giac")
```

output
```
-2*(e^(a + c)*log(abs(e^(2*b*x) - e^(2*c)))/(e^(2*a + 2*c) - 1) + e^(3*a +
 c)*log(abs(e^(2*b*x + 2*a) - 1))/(e^(2*a) - e^(4*a + 2*c)))/b
```

3.142.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 269, normalized size of antiderivative = 8.15

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx =$$

$$-\frac{4\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\left(\frac{2\,\mathrm{e}^{a}\,\mathrm{e}^{c}}{b\,{\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}\right)}^{3/2}}+\frac{2\,\mathrm{e}^{-3\,a}\,\mathrm{e}^{-3\,c}\,\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}+1\right)\,\left(b\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}}+b\,{\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}\right)}^{3/2}\right)}{\sqrt{-b^2\,{\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}-1\right)}^2}\,\sqrt{2\,b^2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}-b^2-b^2\,\mathrm{e}^{4\,a}\,\mathrm{e}^{4\,c}}}\right)\,\sqrt{2\,b^2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}-b^2-b^2\,\mathrm{e}^{4\,a}\,\mathrm{e}^{4\,c}}}{4}-\frac{b\,\mathrm{e}^{-3\,a}\,\mathrm{e}^{-3}}{\sqrt{}}\right.}{\sqrt{2\,b^2\,\mathrm{e}^{2\,a+2\,c}-b^2\,\mathrm{e}^{4\,a+4\,c}-b^2}}$$

input
```
int(1/(sinh(a + b*x)*sinh(c - b*x)),x)
```

output
```
-(4*atan((exp(2*a)*exp(2*b*x)*((2*exp(a)*exp(c))/(b*(exp(2*a)*exp(2*c))^(3
/2)) + (2*exp(-3*a)*exp(-3*c)*(exp(2*a)*exp(2*c) + 1)*(b*(exp(2*a)*exp(2*c
))^(1/2) + b*(exp(2*a)*exp(2*c))^(3/2)))/((-b^2*(exp(2*a)*exp(2*c) - 1)^2)
^(1/2)*(2*b^2*exp(2*a)*exp(2*c) - b^2 - b^2*exp(4*a)*exp(4*c))^(1/2)))*(2*
b^2*exp(2*a)*exp(2*c) - b^2 - b^2*exp(4*a)*exp(4*c))^(1/2))/4 - (b*exp(-3*
a)*exp(-3*c)*(exp(2*a)*exp(2*c) + 1)*(exp(2*a)*exp(2*c))^(3/2))/(-b^2*(exp
(2*a)*exp(2*c) - 1)^2)^(1/2))*exp(2*a + 2*c)^(1/2))/(2*b^2*exp(2*a + 2*c)
- b^2*exp(4*a + 4*c) - b^2)^(1/2)
```

3.142. $\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx$

3.143 $\int \sinh(a+bx)\tanh(c+bx)\,dx$

3.143.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \sinh(a+bx)\tanh(c+bx)\,dx=-\frac{\arctan(\sinh(c+bx))\cosh(a-c)}{b}+\frac{\sinh(a+bx)}{b}$$

output
```
-arctan(sinh(b*x+c))*cosh(a-c)/b+sinh(b*x+a)/b
```

3.143.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 86 vs. 2(29) = 58.

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.97

$$\int \sinh(a+bx)\tanh(c+bx)\,dx$$
$$=-\frac{2\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh\left(\frac{bx}{2}\right)\sinh(c)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\cosh(a-c)}{b}+\frac{\cosh(bx)\sinh(a)}{b}+\frac{\cosh(a)\sinh(bx)}{b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[c + b*x],x]
```

output
```
(-2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x
)/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])/b + (
Cosh[b*x]*Sinh[a])/b + (Cosh[a]*Sinh[b*x])/b
```

3.143.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6154, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh(bx+c)\,dx$$

$$\Big\downarrow \text{6154}$$

$$\int \cosh(a+bx)dx-\cosh(a-c)\int \operatorname{sech}(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sinh(a+bx)}{b}-\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sinh(a+bx)}{b}-\frac{\cosh(a-c)\arctan(\sinh(bx+c))}{b}$$

input
```
Int[Sinh[a + b*x]*Tanh[c + b*x],x]
```

output
```
-((ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b) + Sinh[a + b*x]/b
```

3.143.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6154
```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Simp[Cosh[v - w]   Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

3.143.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 5.76

method	result
risch	$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} + \frac{i\ln(e^{bx+a}-ie^{a-c})e^{-a-c}e^{2a}}{2b} + \frac{i\ln(e^{bx+a}-ie^{a-c})e^{-a-c}e^{2c}}{2b} - \frac{i\ln(e^{bx+a}+ie^{a-c})e^{-a-c}e^{2a}}{2b} - \frac{i\ln(e^{bx+a}}{}$

input
```
int(sinh(b*x+a)*tanh(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-
a-c)*exp(2*a)+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*I*l
n(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*I*ln(exp(b*x+a)+I*exp(a-
c))/b*exp(-a-c)*exp(2*c)
```

3.143.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(29) = 58.

Time = 0.25 (sec) , antiderivative size = 327, normalized size of antiderivative = 11.28

$$\int \sinh(a+bx)\tanh(c+bx)\,dx$$
$$= \frac{\cosh(bx+c)^2\cosh(-a+c)^2 - 2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c) + \cosh(bx+c)^2\sinh(-a+}{}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+c),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(
-a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c
)*sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + 2*(2*cosh(b*x + c)*cosh
(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1
)*cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a
+ c)^2 + 1)*sinh(b*x + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(cosh
(b*x + c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cos
h(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c) - 1)/(b*cosh(b*x + c)*cosh(-a + c
) - b*cosh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(
b*x + c))
```

3.143.6 Sympy [F]

$$\int \sinh(a+bx)\tanh(c+bx)\,dx = \int \sinh\left(a+bx\right)\tanh\left(bx+c\right)dx$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+c),x)
```

output
```
Integral(sinh(a + b*x)*tanh(b*x + c), x)
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

$$\int \sinh(a+bx)\tanh(c+bx)\,dx = \frac{\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} + \frac{e^{(bx+a)}}{2b} - \frac{e^{(-bx-a)}}{2b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+c),x, algorithm="maxima")
```

output
```
(e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(b*x + a)/b
- 1/2*e^(-b*x - a)/b
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \sinh(a+bx)\tanh(c+bx)\,dx = -\frac{2\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-e^{(bx+a)}+e^{(-bx-a)}}{2b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+c),x, algorithm="giac")
```

output
```
-1/2*(2*(e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - e^(b*x + a) +
 e^(-b*x - a))/b
```

3.143.9 Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.59

$$\int \sinh(a+bx)\tanh(c+bx)\,dx$$
$$= \frac{e^{a+b\,x}}{2\,b}-\frac{e^{-a-b\,x}}{2\,b}-\frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{b^2}+e^{2\,a}\,e^{-2\,c}\,\sqrt{b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(2\,e^{2\,a}\,e^{-2\,c}+e^{4\,a}\,e^{-4\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(2\,e^{2\,a-2\,c}+e^{4\,a-4\,c}+1\right)}}{\sqrt{b^2}}$$

input
```
int(sinh(a + b*x)*tanh(c + b*x),x)
```

output
```
exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (atan((exp(-a)*exp(2*c)*exp(b*
x)*((b^2)^(1/2) + exp(2*a)*exp(-2*c)*(b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*
(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(
2*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/(b^2)^(1/2)
```

3.144 $\int \sinh(a+bx)\tanh^2(c+bx)\,dx$

3.144.1 Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = \frac{\cosh(a+bx)}{b} + \frac{\cosh(a-c)\text{sech}(c+bx)}{b} - \frac{\arctan(\sinh(c+bx))\sinh(a-c)}{b}$$

output
```
cosh(b*x+a)/b+cosh(a-c)*sech(b*x+c)/b-arctan(sinh(b*x+c))*sinh(a-c)/b
```

3.144.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 102 vs. 2(45) = 90.

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.27

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = \frac{\cosh(a)\cosh(bx)}{b} + \frac{\cosh(a-c)\text{sech}(c+bx)}{b} - \frac{2\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh\left(\frac{bx}{2}\right)\sinh(c)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\sinh(a-c)}{b} + \frac{\sinh(a)\sinh(bx)}{b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[c + b*x]^2,x]
```

output
```
(Cosh[a]*Cosh[b*x])/b + (Cosh[a - c]*Sech[c + b*x])/b - (2*ArcTan[((Cosh[c
] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)/2]))/(Cosh[c]*Cos
h[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Sinh[a - c])/b + (Sinh[a]*Sinh[b*x])/
b
```

3.144.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6154, 3042, 26, 3086, 24, 6157, 3042, 26, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh^2(bx+c)\,dx$$

$$\Big\downarrow \text{6154}$$

$$\int \cosh(a+bx)\tanh(c+bx)dx-\cosh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cosh(a+bx)\tanh(c+bx)dx-\cosh(a-c)\int -i\sec(ic+ibx)\tan(ic+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$\int \cosh(a+bx)\tanh(c+bx)dx+i\cosh(a-c)\int \sec(ic+ibx)\tan(ic+ibx)dx$$

$$\Big\downarrow \text{3086}$$

$$\int \cosh(a+bx)\tanh(c+bx)dx+\frac{\cosh(a-c)\int 1d\operatorname{sech}(c+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$\int \cosh(a+bx)\tanh(c+bx)dx+\frac{\cosh(a-c)\operatorname{sech}(bx+c)}{b}$$

$$\Big\downarrow \text{6157}$$

$$-\sinh(a-c)\int \operatorname{sech}(c+bx)dx+\int \sinh(a+bx)dx+\frac{\cosh(a-c)\operatorname{sech}(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx+\int -i\sin(ia+ibx)dx+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx-i\int \sin(ia+ibx)dx+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}$$

$$\Big\downarrow \text{3118}$$

$$-\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sinh(a-c)\arctan(\sinh(bx+c))}{b}+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]*Tanh[c + b*x]^2,x]
```

output
```
Cosh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x])/b - (ArcTan[Sinh[c + b*x]]*S
inh[a - c])/b
```

3.144.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6154
```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Simp[Cosh[v - w]   Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

rule 6157
```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Simp[Sinh[v - w]   Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

3.144.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 205, normalized size of antiderivative = 4.56

method	result
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)}+\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)}{2b}$

input
```
int(sinh(b*x+a)*tanh(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(exp
(2*b*x+2*a+2*c)+exp(2*a))+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(
a)^2-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(c)^2-1/2*I*ln(exp(b*x
+a)+I*exp(a-c))/b*exp(-a-c)*exp(a)^2+1/2*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp
(-a-c)*exp(c)^2
```

3.144.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 902, normalized size of antiderivative = 20.04

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sin
h(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c
)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a +
c)^2)*sinh(b*x + c)^3 + 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + 3*(2*cosh
(b*x + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh
(-a + c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c
) + 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^4 + 3*cosh(b*x + c)^2)*sinh(-a + c
)^2 - 2*((cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-
a + c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x
+ c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a +
 c)^2 - 1)*cosh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 + cosh(b*x +
c))*sinh(-a + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (3*(cosh(-a + c)
^2 - 1)*cosh(b*x + c)^2 + (3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a
 + c)^2 - 2*(3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) -
 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-
a + c))*sinh(-a + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(2*cosh(b*
x + c)^3*cosh(-a + c)^2 + (2*cosh(b*x + c)^3 + 3*cosh(b*x + c))*sinh(-a +
c)^2 + 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c) - 2*(2*cosh(b*x + c)^3*cosh(-a
 + c) + 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cos
h(b*x + c)^4*cosh(-a + c) + 3*cosh(b*x + c)^2*cosh(-a + c))*sinh(-a + c...
```

3.144.6 Sympy [F]

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = \int \sinh(a+bx)\tanh^2(bx+c)\,dx$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+c)**2,x)
```

output
```
Integral(sinh(a + b*x)*tanh(b*x + c)**2, x)
```

3.144.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(45) = 90.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.33

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = \frac{\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} + \frac{e^{(-bx-a)}}{2b} + \frac{\left(3e^{(2a)}+2e^{(2c)}\right)e^{(-2bx-2a)}+e^{(2c)}}{2b\left(e^{(-bx-a+2c)}+e^{(-3bx-a)}\right)}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+c)^2,x, algorithm="maxima")
```

output
```
(e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(-b*x - a)/b
 + 1/2*((3*e^(2*a) + 2*e^(2*c))*e^(-2*b*x - 2*a) + e^(2*c))/(b*(e^(-b*x -
a + 2*c) + e^(-3*b*x - a)))
```

3.144.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(45) = 90.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.16

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = -\frac{2\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{2e^{(2bx+4a)}+3e^{(2bx+2a+2c)}+e^{(2a)}}{e^{(3bx+3a+2c)}+e^{(bx+3a)}} - e^{(bx+a)}}{2b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(2*(e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (2*e^(2*b*x +
 4*a) + 3*e^(2*b*x + 2*a + 2*c) + e^(2*a))/(e^(3*b*x + 3*a + 2*c) + e^(b*x
 + 3*a)) - e^(b*x + a))/b
```

3.144.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.84

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx$$
$$=\frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}}{2b}+\frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\,\left(\sqrt{b^2}-e^{2a}\,e^{-2c}\,\sqrt{b^2}\right)}{b\,\sqrt{e^{-2a}\,e^{2c}\,(e^{4a}\,e^{-4c}-2\,e^{2a}\,e^{-2c}+1)}}\right)\,\sqrt{e^{2c-2a}\,\left(e^{4a-4c}-2\,e^{2a-2c}+1\right)}}{\sqrt{b^2}}$$
$$+\frac{e^{a+bx}\,\left(e^{2a-2c}+1\right)}{b\,\left(e^{2a-2c}+e^{2a+2bx}\right)}$$

input
```
int(sinh(a + b*x)*tanh(c + b*x)^2,x)
```

output
```
exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) + (atan((exp(-a)*exp(2*c)*exp(b*
x)*((b^2)^(1/2) - exp(2*a)*exp(-2*c)*(b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*
(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(
exp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2))/(b^2)^(1/2) + (exp(a + b*x)
*(exp(2*a - 2*c) + 1))/(b*(exp(2*a - 2*c) + exp(2*a + 2*b*x)))
```

3.145 $\int \sinh(a+bx)\tanh^3(c+bx)\,dx$

3.145.1 Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx = -\frac{3\arctan(\sinh(c+bx))\cosh(a-c)}{2b} + \frac{\operatorname{sech}(c+bx)\sinh(a-c)}{b} + \frac{\sinh(a+bx)}{b} + \frac{\cosh(a-c)\operatorname{sech}(c+bx)\tanh(c+bx)}{2b}$$

output
```
-3/2*arctan(sinh(b*x+c))*cosh(a-c)/b+sech(b*x+c)*sinh(a-c)/b+sinh(b*x+a)/b
+1/2*cosh(a-c)*sech(b*x+c)*tanh(b*x+c)/b
```

3.145.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx = \frac{-12\arctan\left(\sinh(c)+\cosh(c)\tanh\left(\frac{bx}{2}\right)\right)\cosh(a-c)+\operatorname{sech}^2(c+bx)(2\sinh(a-2c-bx)+5\sinh(a+bx}{4b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[c + b*x]^3,x]
```

output
```
(-12*ArcTan[Sinh[c] + Cosh[c]*Tanh[(b*x)/2]]*Cosh[a - c] + Sech[c + b*x]^2
*(2*Sinh[a - 2*c - b*x] + 5*Sinh[a + b*x] + Sinh[a + 2*c + 3*b*x]))/(4*b)
```

3.145.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6154, 3042, 25, 3091, 3042, 4257, 6157, 3042, 26, 3086, 24, 6154, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh^3(bx+c)\,dx$$

$$\Big\downarrow \text{6154}$$

$$\int \cosh(a+bx)\tanh^2(c+bx)dx-\cosh(a-c)\int \text{sech}(c+bx)\tanh^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cosh(a+bx)\tanh^2(c+bx)dx-\cosh(a-c)\int -\sec(ic+ibx)\tan(ic+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$\int \cosh(a+bx)\tanh^2(c+bx)dx+\cosh(a-c)\int \sec(ic+ibx)\tan(ic+ibx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\int \cosh(a+bx)\tanh^2(c+bx)dx+\cosh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{1}{2}\int \text{sech}(c+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \cosh(a+bx)\tanh^2(c+bx)dx+\cosh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{1}{2}\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{4257}$$

$$\int \cosh(a+bx)\tanh^2(c+bx)dx+\cosh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{6157}$$

$$\int \sinh(a+bx)\tanh(c+bx)dx-\sinh(a-c)\int \text{sech}(c+bx)\tanh(c+bx)dx+\cosh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sinh(a+bx)\tanh(c+bx)dx - \sinh(a-c)\int -i\sec(ic+ibx)\tan(ic+ibx)dx + \cosh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b} - \frac{\arctan(\sinh(bx+c))}{2b}\right)$$

$$\Big\downarrow \; 26$$

$$\int \sinh(a+bx)\tanh(c+bx)dx + i\sinh(a-c)\int \sec(ic+ibx)\tan(ic+ibx)dx + \cosh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b} - \frac{\arctan(\sinh(bx+c))}{2b}\right)$$

$$\Big\downarrow \; 3086$$

$$\int \sinh(a+bx)\tanh(c+bx)dx + \frac{\sinh(a-c)\int 1d\text{sech}(c+bx)}{b} + \cosh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b} - \frac{\arctan(\sinh(bx+c))}{2b}\right)$$

$$\Big\downarrow \; 24$$

$$\int \sinh(a+bx)\tanh(c+bx)dx + \cosh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b} - \frac{\arctan(\sinh(bx+c))}{2b}\right) + \frac{\sinh(a-c)\text{sech}(bx+c)}{b}$$

$$\Big\downarrow \; 6154$$

$$-\cosh(a-c)\int \text{sech}(c+bx)dx + \int \cosh(a+bx)dx + \cosh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b} - \frac{\arctan(\sinh(bx+c))}{2b}\right) + \frac{\sinh(a-c)\text{sech}(bx+c)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx + \int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx + \cosh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b} - \frac{\arctan(\sinh(bx+c))}{2b}\right) + \frac{\sinh(a-c)\text{sech}(bx+c)}{b}$$

$$\Big\downarrow \; 3117$$

$$-\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx + \cosh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b} - \frac{\arctan(\sinh(bx+c))}{2b}\right) + \frac{\sinh(a-c)\text{sech}(bx+c)}{b} + \frac{\sinh(a+bx)}{b}$$

$$\Big\downarrow \; 4257$$

$$-\frac{\cosh(a-c)\arctan(\sinh(bx+c))}{b} + \cosh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b} - \frac{\arctan(\sinh(bx+c))}{2b}\right) + \frac{\sinh(a-c)\text{sech}(bx+c)}{b} + \frac{\sinh(a+bx)}{b}$$

3.145. $\int \sinh(a+bx)\tanh^3(c+bx)\,dx$

input
```
Int[Sinh[a + b*x]*Tanh[c + b*x]^3,x]
```

output
```
-((ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b) + (Sech[c + b*x]*Sinh[a - c])/b +
 Sinh[a + b*x]/b + Cosh[a - c]*(-1/2*ArcTan[Sinh[c + b*x]]/b + (Sech[c + b
*x]*Tanh[c + b*x])/(2*b))
```

3.145.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6154
```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Simp[Cosh[v - w]   Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

rule 6157
```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Simp[Sinh[v - w]   Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

3.145.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.33

method	result
risch	$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}\left(3\,e^{2bx+4a+2c} - e^{2bx+2a+4c} + e^{4a} - 3\,e^{2a+2c}\right)}{2b\left(e^{2bx+2a+2c} + e^{2a}\right)^2} + \frac{3i\ln\left(e^{bx+a} - ie^{a-c}\right)e^{-a-c}e^{2a}}{4b} + \frac{3i\ln\left(e^{bx+a} - ie^{a-c}\right)}{4b}$

input
```
int(sinh(b*x+a)*tanh(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(3*exp(2*b*x+4*a+2*c)-ex
p(2*b*x+2*a+4*c)+exp(4*a)-3*exp(2*a+2*c))/b/(exp(2*b*x+2*a+2*c)+exp(2*a))^
2+3/4*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*a)+3/4*I*ln(exp(b*x+a)
-I*exp(a-c))/b*exp(-a-c)*exp(2*c)-3/4*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a
-c)*exp(2*a)-3/4*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(2*c)
```

3.145.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1737 vs. $2(68) = 136$.

Time = 0.28 (sec) , antiderivative size = 1737, normalized size of antiderivative = 24.12

$$\int \sinh(a + bx)\tanh^3(c + bx)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + c)^6*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sin
h(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 6*(cosh(b*x + c)*cosh(-a + c
)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a +
c)^2)*sinh(b*x + c)^5 + (5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^4 + (15*cosh(
b*x + c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + 5*c
osh(-a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a
 + c) - 2)*sinh(b*x + c)^4 + 4*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(cosh
(b*x + c)^3 + cosh(b*x + c))*sinh(-a + c)^2 + (5*cosh(-a + c)^2 - 2)*cosh(
b*x + c) - 10*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*
sinh(-a + c))*sinh(b*x + c)^3 + (2*cosh(-a + c)^2 - 5)*cosh(b*x + c)^2 + (
15*cosh(b*x + c)^4*cosh(-a + c)^2 + 6*(5*cosh(-a + c)^2 - 2)*cosh(b*x + c)
^2 + (15*cosh(b*x + c)^4 + 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cosh
(-a + c)^2 - 2*(15*cosh(b*x + c)^4*cosh(-a + c) + 30*cosh(b*x + c)^2*cosh(
-a + c) + 2*cosh(-a + c))*sinh(-a + c) - 5)*sinh(b*x + c)^2 + (cosh(b*x +
c)^6 + 5*cosh(b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a
 + c)^2 + 1)*cosh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a +
c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)
*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b
*x + c))*sinh(b*x + c)^4 + 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + 2*(5*(
cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 + 1)*sinh(-a +...
```

3.145.6 Sympy [F]

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx = \int \sinh\left(a+bx\right)\tanh^3\left(bx+c\right)dx$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+c)**3,x)
```

output
```
Integral(sinh(a + b*x)*tanh(b*x + c)**3, x)
```

3.145.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(68) = 136$.

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.07

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx = \frac{3\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{2b} - \frac{e^{(-bx-a)}}{2b} + \frac{\left(5e^{(2a+2c)}-e^{(4c)}\right)e^{(-2bx-2a)} + \left(2e^{(4a)}-3e^{(2a+2c)}\right)e^{(-4bx-4a)} + e^{(4c)}}{2b\left(e^{(-bx-a+4c)}+2e^{(-3bx-a+2c)}+e^{(-5bx-a)}\right)}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+c)^3,x, algorithm="maxima")
```

output
```
3/2*(e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - 1/2*e^(-b*x -
a)/b + 1/2*((5*e^(2*a + 2*c) - e^(4*c))*e^(-2*b*x - 2*a) + (2*e^(4*a) - 3*
e^(2*a + 2*c))*e^(-4*b*x - 4*a) + e^(4*c))/(b*(e^(-b*x - a + 4*c) + 2*e^(-
3*b*x - a + 2*c) + e^(-5*b*x - a)))
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.67

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx = -\frac{3\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{3e^{(3bx+5a+2c)}-e^{(3bx+3a+4c)}+e^{(bx+5a)}-3e^{(bx+3a+2c)}}{\left(e^{(2bx+2a+2c)}+e^{(2a)}\right)^2} - e^{(bx+a)} + e^{(-bx-}}{2b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/2*(3*(e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (3*e^(3*b*x +
 5*a + 2*c) - e^(3*b*x + 3*a + 4*c) + e^(b*x + 5*a) - 3*e^(b*x + 3*a + 2*c
))/(e^(2*b*x + 2*a + 2*c) + e^(2*a))^2 - e^(b*x + a) + e^(-b*x - a))/b
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx = \int \sinh(a+b\,x)\,\tanh(c+b\,x)^3\,dx$$

input
```
int(sinh(a + b*x)*tanh(c + b*x)^3,x)
```

output
```
int(sinh(a + b*x)*tanh(c + b*x)^3, x)
```

3.146 $\int \coth(c+bx)\sinh(a+bx)\,dx$

3.146.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \coth(c+bx)\sinh(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(c+bx))\sinh(a-c)}{b}+\frac{\sinh(a+bx)}{b}$$

output
```
-arctanh(cosh(b*x+c))*sinh(a-c)/b+sinh(b*x+a)/b
```

3.146.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$\int \coth(c+bx)\sinh(a+bx)\,dx$$
$$=\frac{\cosh(bx)\sinh(a)}{b}-\frac{2i\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh(c)\cosh\left(\frac{bx}{2}\right)+\sinh(c)\sinh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\sinh(a-c)}{b}$$
$$+\frac{\cosh(a)\sinh(bx)}{b}$$

input
```
Integrate[Coth[c + b*x]*Sinh[a + b*x],x]
```

output
```
(Cosh[b*x]*Sinh[a])/b - ((2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(
b*x)/2] + Sinh[c]*Sinh[(b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/
2]*Sinh[c])]*Sinh[a - c])/b + (Cosh[a]*Sinh[b*x])/b
```

3.146.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6156, 3042, 26, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\coth(bx+c)\,dx$$

$$\Big\downarrow \text{6156}$$

$$\sinh(a-c)\int \operatorname{csch}(c+bx)dx+\int \cosh(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sinh(a-c)\int i\csc(ic+ibx)dx+\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$i\sinh(a-c)\int \csc(ic+ibx)dx+\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sinh(a+bx)}{b}+i\sinh(a-c)\int \csc(ic+ibx)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sinh(a+bx)}{b}-\frac{\sinh(a-c)\text{arctanh}(\cosh(bx+c))}{b}$$

input
```
Int[Coth[c + b*x]*Sinh[a + b*x],x]
```

output
```
-((ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b) + Sinh[a + b*x]/b
```

3.146.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6156
```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Simp[Sinh[v - w]   Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

3.146.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(29) = 58$.

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 5.34

method	result
risch	$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} + \frac{\ln(e^{bx+a}-e^{a-c})e^{-a-c}e^{2a}}{2b} - \frac{\ln(e^{bx+a}-e^{a-c})e^{-a-c}e^{2c}}{2b} - \frac{\ln(e^{bx+a}+e^{a-c})e^{-a-c}e^{2a}}{2b} + \frac{\ln(e^{bx+a}+e^{a-c}}{2b}$

input
```
int(coth(b*x+c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)
*exp(2*a)-1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*ln(exp(b*x+
a)+exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*
exp(2*c)
```

3.146.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(29) = 58.

Time = 0.25 (sec) , antiderivative size = 439, normalized size of antiderivative = 15.14

$$\int \coth(c+bx)\sinh(a+bx)\,dx$$
$$= \frac{\cosh(bx+c)^2\cosh(-a+c)^2 - 2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c) + \cosh(bx+c)^2\sinh(-a+}{}$$

input
```
integrate(coth(b*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(
-a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c
)*sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + (2*cosh(b*x + c)*cosh(-
a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*
cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a +
c)^2 - 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) - (2*cosh(
b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(
-a + c)^2 - 1)*cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a +
c) + sinh(-a + c)^2 - 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c)
- 1) + 2*(cosh(b*x + c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh
(-a + c) + cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c) - 1)/(b*cosh(b*x +
c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(
-a + c))*sinh(b*x + c))
```

3.146.6 Sympy [F]

$$\int \coth(c+bx)\sinh(a+bx)\,dx = \int \sinh(a+bx)\coth(bx+c)\,dx$$

input
```
integrate(coth(b*x+c)*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*coth(b*x + c), x)
```

3.146.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(29) = 58$.

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.24

$$\int \coth(c+bx)\sinh(a+bx)\,dx = -\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b} + \frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b} + \frac{e^{(bx+a)}}{2b} - \frac{e^{(-bx-a)}}{2b}$$

input
```
integrate(coth(b*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) -
 e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b
*x - a)/b
```

3.146.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$\int \coth(c+bx)\sinh(a+bx)\,dx = -\frac{\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+a+c)}+e^{a}\right)-\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+a+c)}-e^{a}\right|\right)-e^{(bx+a)}}{2b}$$

input
```
integrate(coth(b*x+c)*sinh(b*x+a),x, algorithm="giac")
```

output
```
-1/2*((e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + a + c) + e^a) - (e
^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + a + c) - e^a)) - e^(b*
x + a) + e^(-b*x - a))/b
```

3.146.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.79

$$\int \coth(c+bx)\sinh(a+bx)\,dx$$
$$=\frac{e^{a+b\,x}}{2\,b}-\frac{e^{-a-b\,x}}{2\,b}+\frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{-b^2}-e^{2\,a}\,e^{-2\,c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(e^{4\,a}\,e^{-4\,c}-2\,e^{2\,a}\,e^{-2\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(e^{4\,a-4\,c}-2\,e^{2\,a-2\,c}+1\right)}}{\sqrt{-b^2}}$$

input
```
int(coth(c + b*x)*sinh(a + b*x),x)
```

output
```
exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) + (atan((exp(-a)*exp(2*c)*exp(b*
x)*((-b^2)^(1/2) - exp(2*a)*exp(-2*c)*(-b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c
)*(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) + 1))^(1/2)))*(exp(2*c - 2*a)
*(exp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2))/(-b^2)^(1/2)
```

3.147 $\int \coth^2(c+bx)\sinh(a+bx)\,dx$

3.147.1 Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(c+bx))\cosh(a-c)}{b}+\frac{\cosh(a+bx)}{b}-\frac{\operatorname{csch}(c+bx)\sinh(a-c)}{b}$$

output
```
-arctanh(cosh(b*x+c))*cosh(a-c)/b+cosh(b*x+a)/b-csch(b*x+c)*sinh(a-c)/b
```

3.147.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.39

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = -\frac{2i\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh(c)\cosh\left(\frac{bx}{2}\right)+\sinh(c)\sinh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\cosh(a-c)}{b}+\frac{\cosh(a)\cosh(bx)}{b}-\frac{\operatorname{csch}(c+bx)\sinh(a-c)}{b}+\frac{\sinh(a)\sinh(bx)}{b}$$

input
```
Integrate[Coth[c + b*x]^2*Sinh[a + b*x],x]
```

output
```
((-2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[
(b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c
])/b + (Cosh[a]*Cosh[b*x])/b - (Csch[c + b*x]*Sinh[a - c])/b + (Sinh[a]*Si
nh[b*x])/b
```

3.147.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6156, 3042, 3086, 24, 6155, 3042, 26, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\coth^2(bx+c)\,dx$$

$$\Big\downarrow \text{6156}$$

$$\int \cosh(a+bx)\coth(c+bx)dx+\sinh(a-c)\int \coth(c+bx)\text{csch}(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cosh(a+bx)\coth(c+bx)dx+\sinh(a-c)\int \sec\left(ic+ibx-\frac{\pi}{2}\right)\tan\left(ic+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\int \cosh(a+bx)\coth(c+bx)dx-\frac{i\sinh(a-c)\int 1d(-i\text{csch}(c+bx))}{b}$$

$$\Big\downarrow \text{24}$$

$$\int \cosh(a+bx)\coth(c+bx)dx-\frac{\sinh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{6155}$$

$$\cosh(a-c)\int \text{csch}(c+bx)dx+\int \sinh(a+bx)dx-\frac{\sinh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\cosh(a-c)\int i\csc(ic+ibx)dx+\int -i\sin(ia+ibx)dx-\frac{\sinh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{26}$$

$$i\cosh(a-c)\int\csc(ic+ibx)dx-i\int\sin(ia+ibx)dx-\frac{\sinh(a-c)\operatorname{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{3118}$$

$$i\cosh(a-c)\int\csc(ic+ibx)dx-\frac{\sinh(a-c)\operatorname{csch}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\cosh(a-c)\operatorname{arctanh}(\cosh(bx+c))}{b}-\frac{\sinh(a-c)\operatorname{csch}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

input
```
Int[Coth[c + b*x]^2*Sinh[a + b*x],x]
```

output
```
-((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) + Cosh[a + b*x]/b - (Csch[c + b*
x]*Sinh[a - c])/b
```

3.147.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6155
```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Simp[Cosh[v - w]   Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

rule 6156
```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Simp[Sinh[v - w]   Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

3.147.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(46) = 92$.

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.28

method	result
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a}}{2b}$

input
```
int(coth(b*x+c)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(-ex
p(2*b*x+2*a+2*c)+exp(2*a))+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a
)+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*ln(exp(b*x+a)+exp(a
-c))/b*exp(-a-c)*exp(2*a)-1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)
```

3.147.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1237 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 1237, normalized size of antiderivative = 26.89

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(coth(b*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sin
h(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c
)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a +
c)^2)*sinh(b*x + c)^3 - 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + 3*(2*cosh
(b*x + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh
(-a + c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c
) + 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^4 - 3*cosh(b*x + c)^2)*sinh(-a + c
)^2 - ((cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a
+ c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x +
c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c
)^2 + 1)*cosh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 - cosh(b*x + c)
)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2
 + 1)*cosh(b*x + c)^2 + (3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a +
 c)^2 - 2*(3*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1
)*sinh(b*x + c) - 2*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a
+ c))*sinh(-a + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + ((cosh(-a + c
)^2 + 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) +
 sinh(-a + c)^2 + 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-a + c)*sin
h(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x +
 c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2...
```

3.147.6 Sympy [F]

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = \int \sinh(a+bx)\coth^2(bx+c)\,dx$$

input
```
integrate(coth(b*x+c)**2*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*coth(b*x + c)**2, x)
```

3.147.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(46) = 92$.

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.04

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = -\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b} + \frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b} + \frac{e^{(-bx-a)}}{2b} - \frac{\left(3e^{(2a)}-2e^{(2c)}\right)e^{(-2bx-2a)}-e^{(2c)}}{2b\left(e^{(-bx-a+2c)}-e^{(-3bx-a)}\right)}$$

input
```
integrate(coth(b*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
 e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(-b*x - a)/b - 1/2*((3*
e^(2*a) - 2*e^(2*c))*e^(-2*b*x - 2*a) - e^(2*c))/(b*(e^(-b*x - a + 2*c) -
e^(-3*b*x - a)))
```

3.147.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.96

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = -\frac{\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+a+c)}+e^{a}\right)-\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+a+c)}-e^{a}\right|\right)+\frac{2e^{(2bx+4}}{e^{(3b}}}{2b}$$

input
```
integrate(coth(b*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
-1/2*((e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + a + c) + e^a) - (e
^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + a + c) - e^a)) + (2*e^
(2*b*x + 4*a) - 3*e^(2*b*x + 2*a + 2*c) + e^(2*a))/(e^(3*b*x + 3*a + 2*c)
- e^(b*x + 3*a)) - e^(b*x + a))/b
```

3.147.9 Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.93

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx$$
$$=\frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}}{2b}-\frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\,\left(\sqrt{-b^2}+e^{2a}\,e^{-2c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2a}\,e^{2c}\,(2\,e^{2a}\,e^{-2c}+e^{4a}\,e^{-4c}+1)}}\right)\,\sqrt{e^{2c-2a}\,\left(2\,e^{2a-2c}+e^{4a-4c}+1\right)}}{\sqrt{-b^2}}$$
$$+\frac{e^{a+bx}\,\left(e^{2a-2c}-1\right)}{b\,\left(e^{2a-2c}-e^{2a+2bx}\right)}$$

input
```
int(coth(c + b*x)^2*sinh(a + b*x),x)
```

output
```
exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) - (atan((exp(-a)*exp(2*c)*exp(b*
x)*((-b^2)^(1/2) + exp(2*a)*exp(-2*c)*(-b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c
)*(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1))^(1/2)))*(exp(2*c - 2*a)
*(2*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/(-b^2)^(1/2) + (exp(a + b
*x)*(exp(2*a - 2*c) - 1))/(b*(exp(2*a - 2*c) - exp(2*a + 2*b*x)))
```

3.148 $\int \coth^3(c+bx)\sinh(a+bx)\,dx$

3.148.1 Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx = -\frac{\cosh(a-c)\operatorname{csch}(c+bx)}{b} - \frac{3\operatorname{arctanh}(\cosh(c+bx))\sinh(a-c)}{2b} - \frac{\coth(c+bx)\operatorname{csch}(c+bx)\sinh(a-c)}{2b} + \frac{\sinh(a+bx)}{b}$$

output
```
-cosh(a-c)*csch(b*x+c)/b-3/2*arctanh(cosh(b*x+c))*sinh(a-c)/b-1/2*coth(b*x
+c)*csch(b*x+c)*sinh(a-c)/b+sinh(b*x+a)/b
```

3.148.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx = \frac{-12\operatorname{arctanh}\left(\cosh(c)+\sinh(c)\tanh\left(\frac{bx}{2}\right)\right)\sinh(a-c)+\operatorname{csch}^2(c+bx)(2\sinh(a-2c-bx)-5\sinh(a+b}{4b}$$

input
```
Integrate[Coth[c + b*x]^3*Sinh[a + b*x],x]
```

output
```
(-12*ArcTanh[Cosh[c] + Sinh[c]*Tanh[(b*x)/2]]*Sinh[a - c] + Csch[c + b*x]^
2*(2*Sinh[a - 2*c - b*x] - 5*Sinh[a + b*x] + Sinh[a + 2*c + 3*b*x]))/(4*b)
```

3.148.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.133, Rules used = {6156, 3042, 26, 3091, 26, 3042, 26, 4257, 6155, 3042, 3086, 24, 6156, 3042, 26, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\coth^3(bx+c)\,dx$$

$$\Big\downarrow \text{6156}$$

$$\int \cosh(a+bx)\coth^2(c+bx)dx+\sinh(a-c)\int \coth^2(c+bx)\text{csch}(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cosh(a+bx)\coth^2(c+bx)dx+\sinh(a-c)\int -i\sec\left(ic+ibx-\frac{\pi}{2}\right)\tan\left(ic+ibx-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{26}$$

$$\int \cosh(a+bx)\coth^2(c+bx)dx-i\sinh(a-c)\int \sec\left(\frac{1}{2}(2ic-\pi)+ibx\right)\tan\left(\frac{1}{2}(2ic-\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\int \cosh(a+bx)\coth^2(c+bx)dx-i\sinh(a-c)\left(-\frac{1}{2}\int -i\text{csch}(c+bx)dx-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$\int \cosh(a+bx)\coth^2(c+bx)dx-i\sinh(a-c)\left(\frac{1}{2}i\int \text{csch}(c+bx)dx-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \cosh(a+bx)\coth^2(c+bx)dx-i\sinh(a-c)\left(\frac{1}{2}i\int i\csc(ic+ibx)dx-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$\int \cosh(a+bx)\coth^2(c+bx)dx-i\sinh(a-c)\left(-\frac{1}{2}\int \csc(ic+ibx)dx-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{4257}$$

$$\int \cosh(a+bx)\coth^2(c+bx)dx - i\sinh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b}-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{6155}$$

$$\int \coth(c+bx)\sinh(a+bx)dx+\cosh(a-c)\int \coth(c+bx)\text{csch}(c+bx)dx - i\sinh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b}-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \coth(c+bx)\sinh(a+bx)dx+\cosh(a-c)\int \sec\left(ic+ibx-\frac{\pi}{2}\right)\tan\left(ic+ibx-\frac{\pi}{2}\right)dx - i\sinh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b}-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3086}$$

$$-\frac{i\cosh(a-c)\int 1d(-i\text{csch}(c+bx))}{b}+\int \coth(c+bx)\sinh(a+bx)dx - i\sinh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b}-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \coth(c+bx)\sinh(a+bx)dx - i\sinh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b}-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{6156}$$

$$\sinh(a-c)\int \text{csch}(c+bx)dx+\int \cosh(a+bx)dx - i\sinh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b}-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\sinh(a-c)\int i\csc(ic+ibx)dx+\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx - i\sinh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b}-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{26}$$

$$i\sinh(a-c)\int \csc(ic+ibx)dx+\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx - i\sinh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b}-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}$$

3.148. $\int \coth^3(c+bx)\sinh(a+bx)\,dx$

$$\Big\downarrow \text{3117}$$

$$i\sinh(a-c)\int \csc(ic+ibx)dx - i\sinh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b} - \frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right) - \frac{\cosh(a-c)\text{csch}(bx+c)}{b} + \frac{\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sinh(a-c)\text{arctanh}(\cosh(bx+c))}{b} - i\sinh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b} - \frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right) - \frac{\cosh(a-c)\text{csch}(bx+c)}{b} + \frac{\sinh(a+bx)}{b}$$

input
```
Int[Coth[c + b*x]^3*Sinh[a + b*x],x]
```

output
```
-((Cosh[a - c]*Csch[c + b*x])/b) - (ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b
- I*(((-1/2*I)*ArcTanh[Cosh[c + b*x]])/b - ((I/2)*Coth[c + b*x]*Csch[c + b
*x])/b)*Sinh[a - c] + Sinh[a + b*x]/b
```

3.148.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6155
```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Simp[Cosh[v - w]   Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

rule 6156
```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Simp[Sinh[v - w]   Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

3.148.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(69) = 138$.

Time = 0.39 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.15

method	result
risch	$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(-3\,e^{2bx+4a+2c}-e^{2bx+2a+4c}+e^{4a}+3\,e^{2a+2c}\right)}{2b\left(-e^{2bx+2a+2c}+e^{2a}\right)^{2}}+\frac{3\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{4b}-\frac{3\ln\left(e^{bx+a}-e^{a-c}\right)e}{4b}$

input
```
int(coth(b*x+c)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(-3*exp(2*b*x+4*a+2*c)-e
xp(2*b*x+2*a+4*c)+exp(4*a)+3*exp(2*a+2*c))/b/(-exp(2*b*x+2*a+2*c)+exp(2*a)
)^2+3/4*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)-3/4*ln(exp(b*x+a)-exp
(a-c))/b*exp(-a-c)*exp(2*c)-3/4*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*
a)+3/4*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2372 vs. $2(69) = 138$.

Time = 0.27 (sec) , antiderivative size = 2372, normalized size of antiderivative = 32.49

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(coth(b*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*cosh(b*x + c)^6*cosh(-a + c)^2 + 2*(cosh(-a + c)^2 - 2*cosh(-a + c)
*sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 12*(cosh(b*x + c)*cosh(-
a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(
-a + c)^2)*sinh(b*x + c)^5 - 2*(5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^4 + 2*
(15*cosh(b*x + c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 - 1)*sinh(-a + c
)^2 - 5*cosh(-a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c)
)*sinh(-a + c) - 2)*sinh(b*x + c)^4 + 8*(5*cosh(b*x + c)^3*cosh(-a + c)^2
+ 5*(cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (5*cosh(-a + c)^2 +
 2)*cosh(b*x + c) - 10*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(
-a + c))*sinh(-a + c))*sinh(b*x + c)^3 + 2*(2*cosh(-a + c)^2 + 5)*cosh(b*x
 + c)^2 + 2*(15*cosh(b*x + c)^4*cosh(-a + c)^2 - 6*(5*cosh(-a + c)^2 + 2)*
cosh(b*x + c)^2 + (15*cosh(b*x + c)^4 - 30*cosh(b*x + c)^2 + 2)*sinh(-a +
c)^2 + 2*cosh(-a + c)^2 - 2*(15*cosh(b*x + c)^4*cosh(-a + c) - 30*cosh(b*x
 + c)^2*cosh(-a + c) + 2*cosh(-a + c))*sinh(-a + c) + 5)*sinh(b*x + c)^2 +
 2*(cosh(b*x + c)^6 - 5*cosh(b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^
2 - 3*((cosh(-a + c)^2 - 1)*cosh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a
+ c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x +
c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c
)^2 - 1)*cosh(b*x + c))*sinh(b*x + c)^4 - 2*(cosh(-a + c)^2 - 1)*cosh(b*x
+ c)^3 + 2*(5*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2...
```

3.148.6 Sympy [F]

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx = \int \sinh(a+bx)\coth^3(bx+c)\,dx$$

input
```
integrate(coth(b*x+c)**3*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*coth(b*x + c)**3, x)
```

3.148.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(69) = 138.

Time = 0.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.55

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx$$
$$= -\frac{3\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{4b} + \frac{3\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{4b} - \frac{e^{(-bx-a)}}{2b} - \frac{\left(5e^{(2a+2c)}+e^{(4c)}\right)e^{(-2bx-2a)}-\left(2e^{(4a)}+3e^{(2a+2c)}\right)e^{(-4bx-4a)}-e^{(4c)}}{2b\left(e^{(-bx-a+4c)}-2e^{(-3bx-a+2c)}+e^{(-5bx-a)}\right)}$$

input
```
integrate(coth(b*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-3/4*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 3/4*(e^(2*a) -
 e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b - 1/2*e^(-b*x - a)/b - 1/2*((5*
e^(2*a + 2*c) + e^(4*c))*e^(-2*b*x - 2*a) - (2*e^(4*a) + 3*e^(2*a + 2*c))*
e^(-4*b*x - 4*a) - e^(4*c))/(b*(e^(-b*x - a + 4*c) - 2*e^(-3*b*x - a + 2*c
) + e^(-5*b*x - a)))
```

3.148.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(69) = 138.

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.32

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx =$$

$$-\frac{3\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+a+c)}+e^{a}\right)-3\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+a+c)}-e^{a}\right|\right)+\frac{2\,(3\,e^{\cdots}}{\cdots}}{4\,b}$$

input
```
integrate(coth(b*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
-1/4*(3*(e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + a + c) + e^a) -
3*(e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + a + c) - e^a)) + 2
*(3*e^(3*b*x + 5*a + 2*c) + e^(3*b*x + 3*a + 4*c) - e^(b*x + 5*a) - 3*e^(b
*x + 3*a + 2*c))/(e^(2*b*x + 2*a + 2*c) - e^(2*a))^2 - 2*e^(b*x + a) + 2*e
^(-b*x - a))/b
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx = \int \coth(c+b\,x)^3\sinh(a+b\,x)\;dx$$

input
```
int(coth(c + b*x)^3*sinh(a + b*x),x)
```

output
```
int(coth(c + b*x)^3*sinh(a + b*x), x)
```

3.149 $\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx$

3.149.1 Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx=\frac{\cosh(a-c)\log(\cosh(c+bx))}{b}+x\sinh(a-c)$$

output
```
cosh(a-c)*ln(cosh(b*x+c))/b+x*sinh(a-c)
```

3.149.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx=\frac{\cosh(a-c)\log(\cosh(c+bx))}{b}+x\sinh(a-c)$$

input
```
Integrate[Sech[c + b*x]*Sinh[a + b*x],x]
```

output
```
(Cosh[a - c]*Log[Cosh[c + b*x]])/b + x*Sinh[a - c]
```

3.149.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6158, 24, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\text{sech}(bx+c)\,dx$$

$$\Big\downarrow \text{ 6158}$$

$$\cosh(a-c)\int \tanh(c+bx)dx+\sinh(a-c)\int 1dx$$

$$\Big\downarrow \text{ 24}$$

$$\cosh(a-c)\int \tanh(c+bx)dx+x\sinh(a-c)$$

$$\Big\downarrow \text{ 3042}$$

$$x\sinh(a-c)+\cosh(a-c)\int -i\tan(ic+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$x\sinh(a-c)-i\cosh(a-c)\int \tan(ic+ibx)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\cosh(a-c)\log(\cosh(bx+c))}{b}+x\sinh(a-c)$$

input
```
Int[Sech[c + b*x]*Sinh[a + b*x],x]
```

output
```
(Cosh[a - c]*Log[Cosh[c + b*x]])/b + x*Sinh[a - c]
```

3.149.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 6158
```
Int[Sech[w_]^(n_.)*Sinh[v_], x_Symbol] :> Simp[Cosh[v - w]   Int[Tanh[w]*Se
ch[w]^(n - 1), x], x] + Simp[Sinh[v - w]   Int[Sech[w]^(n - 1), x], x] /; G
tQ[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

3.149.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(26) = 52$.

Time = 0.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 5.69

method	result
risch	$x\,\mathrm{e}^{a-c}-\mathrm{e}^{-a-c}\mathrm{e}^{2a}x-\mathrm{e}^{-a-c}\mathrm{e}^{2c}x-\frac{\mathrm{e}^{-a-c}\mathrm{e}^{2a}a}{b}-\frac{\mathrm{e}^{-a-c}\mathrm{e}^{2c}a}{b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2c}\right)\mathrm{e}^{-a-c}\mathrm{e}^{2a}}{2b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2}\right.}{2b}$

input
```
int(sech(b*x+c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x*exp(a-c)-exp(-a-c)*exp(2*a)*x-exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a
)*a-1/b*exp(-a-c)*exp(2*c)*a+1/2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/b*exp(-a-
c)*exp(2*a)+1/2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/b*exp(-a-c)*exp(2*c)
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx = -\frac{2\,bx-\left(\cosh\left(-a+c\right)^{2}-2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right)+\sinh\left(-a+c\right)^{2}+1\right)\log\left(\frac{2\,\cosh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right)-b\sinh\left(-a+c\right)\right)}$$

input
```
integrate(sech(b*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)
^2 + 1)*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a +
 c) - b*sinh(-a + c))
```

3.149.6 Sympy [F]

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx = \int \sinh\left(a+bx\right)\operatorname{sech}\left(bx+c\right)dx$$

input
```
integrate(sech(b*x+c)*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*sech(b*x + c), x)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx = \frac{\left(e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-2\,bx)}+e^{(2\,c)}\right)}{2\,b}+\frac{(bx+a)e^{(a-c)}}{b}$$

input
```
integrate(sech(b*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-2*b*x) + e^(2*c))/b + (b*x + a)
*e^(a - c)/b
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx = -\frac{2\,bxe^{(-a+c)} - \left(e^{(2\,a+c)} + e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(2\,bx+2\,c)}+1\right)}{2\,b}$$

input
```
integrate(sech(b*x+c)*sinh(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(2*b*x*e^(-a + c) - (e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(2*b*x
 + 2*c) + 1))/b
```

3.149.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx = \frac{\mathrm{e}^{2\,c-2\,a}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,c}\right)\,\left(2\,b\,\mathrm{e}^{3\,a-3\,c}+2\,b\,\mathrm{e}^{a-c}\right)}{4\,b^2} - x\,\mathrm{e}^{c-a}$$

input
```
int(sinh(a + b*x)/cosh(c + b*x),x)
```

output
```
(exp(2*c - 2*a)*log(exp(2*a)*exp(2*b*x) + exp(2*a)*exp(-2*c))*(2*b*exp(3*a
 - 3*c) + 2*b*exp(a - c)))/(4*b^2) - x*exp(c - a)
```

3.150 $\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx$

3.150.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx = -\frac{\cosh(a-c)\operatorname{sech}(c+bx)}{b}+\frac{\arctan(\sinh(c+bx))\sinh(a-c)}{b}$$

output
```
-cosh(a-c)*sech(b*x+c)/b+arctan(sinh(b*x+c))*sinh(a-c)/b
```

3.150.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(35) = 70.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx = -\frac{\cosh(a-c)\operatorname{sech}(c+bx)}{b} + \frac{2\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh\left(\frac{bx}{2}\right)\sinh(c)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\sinh(a-c)}{b}$$

input
```
Integrate[Sech[c + b*x]^2*Sinh[a + b*x],x]
```

output
```
-((Cosh[a - c]*Sech[c + b*x])/b) + (2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b
*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*
x)/2]*Sinh[c])]*Sinh[a - c])/b
```

3.150.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6158, 3042, 26, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\text{sech}^2(bx+c)\,dx$$

$$\Big\downarrow \text{ 6158}$$

$$\sinh(a-c)\int \text{sech}(c+bx)dx+\cosh(a-c)\int \text{sech}(c+bx)\tanh(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx+\cosh(a-c)\int -i\sec(ic+ibx)\tan(ic+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx-i\cosh(a-c)\int \sec(ic+ibx)\tan(ic+ibx)dx$$

$$\Big\downarrow \text{ 3086}$$

$$-\frac{\cosh(a-c)\int 1d\text{sech}(c+bx)}{b}+\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\cosh(a-c)\text{sech}(bx+c)}{b}+\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sinh(a-c)\arctan(\sinh(bx+c))}{b}-\frac{\cosh(a-c)\text{sech}(bx+c)}{b}$$

input
```
Int[Sech[c + b*x]^2*Sinh[a + b*x],x]
```

output
```
-((Cosh[a - c]*Sech[c + b*x])/b) + (ArcTan[Sinh[c + b*x]]*Sinh[a - c])/b
```

3.150.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6158
```
Int[Sech[w_]^(n_.)*Sinh[v_], x_Symbol] :> Simp[Cosh[v - w]   Int[Tanh[w]*Se
ch[w]^(n - 1), x], x] + Simp[Sinh[v - w]   Int[Sech[w]^(n - 1), x], x] /; G
tQ[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

3.150.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 5.17

method	result
risch	$-\frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)}+\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{i\ln(e}{}$

input
```
int(sech(b*x+c)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(exp(2*b*x+2*a+2*c)+exp(2*a))+1/2*I*ln
(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(a)^2-1/2*I*ln(exp(b*x+a)+I*exp(a-c
))/b*exp(-a-c)*exp(c)^2-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(a)
^2+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(c)^2
```

3.150.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(35) = 70$.

Time = 0.26 (sec) , antiderivative size = 405, normalized size of antiderivative = 11.57

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx$$
$$= \frac{2\cosh(bx+c)\cosh(-a+c)\sinh(-a+c)-\cosh(bx+c)\sinh(-a+c)^2+\left(\left(\cosh(-a+c)^2-1\right)\cosh(\right.}{}$$

input
```
integrate(sech(b*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2
+ ((cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)
*sinh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 + 1
)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-
a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c)
)*sinh(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a
+ c) - 1)*arctan(cosh(b*x + c) + sinh(b*x + c)) - (cosh(-a + c)^2 + 1)*cos
h(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 + 1)*sinh(b*x + c))/(b*cosh(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) -
b*sinh(-a + c))*sinh(b*x + c)^2 + b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh
(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^
2 + b)*sinh(-a + c))
```

3.150.6 Sympy [F]

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx = \int \sinh(a+bx)\operatorname{sech}^2(bx+c)\,dx$$

input
```
integrate(sech(b*x+c)**2*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*sech(b*x + c)**2, x)
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx = -\frac{\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} - \frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}+e^{(2c)}\right)}$$

input
```
integrate(sech(b*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-(e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - (e^(2*a) + e^(2*c
))*e^(-b*x - a)/(b*(e^(-2*b*x) + e^(2*c)))
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx = \frac{\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{\left(e^{(bx+2a)}+e^{(bx+2c)}\right)e^{(-a)}}{e^{(2bx+2c)}+1}}{b}$$

input
```
integrate(sech(b*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
((e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (e^(b*x + 2*a) + e^(
b*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) + 1))/b
```

3.150.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.29

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx$$
$$= -\frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{b^2}-e^{2\,a}\,e^{-2\,c}\,\sqrt{b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(e^{4\,a}\,e^{-4\,c}-2\,e^{2\,a}\,e^{-2\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(e^{4\,a-4\,c}-2\,e^{2\,a-2\,c}+1\right)}}{\sqrt{b^2}}$$
$$-\frac{e^{a+b\,x}\,\left(e^{2\,a-2\,c}+1\right)}{b\,\left(e^{2\,a-2\,c}+e^{2\,a+2\,b\,x}\right)}$$

input
```
int(sinh(a + b*x)/cosh(c + b*x)^2,x)
```

output
```
- (atan((exp(-a)*exp(2*c)*exp(b*x)*((b^2)^(1/2) - exp(2*a)*exp(-2*c)*(b^2)
^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c)
 + 1))^(1/2)))*(exp(2*c - 2*a)*(exp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1
/2))/(b^2)^(1/2) - (exp(a + b*x)*(exp(2*a - 2*c) + 1))/(b*(exp(2*a - 2*c)
+ exp(2*a + 2*b*x)))
```

3.151 $\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx$

3.151.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx=-\frac{\cosh(a-c)\operatorname{sech}^2(c+bx)}{2b}+\frac{\sinh(a-c)\tanh(c+bx)}{b}$$

output
```
-1/2*cosh(a-c)*sech(b*x+c)^2/b+sinh(a-c)*tanh(b*x+c)/b
```

3.151.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx=-\frac{\operatorname{sech}(c)\operatorname{sech}^2(c+bx)(\cosh(a)-\sinh(a-c)\sinh(c+2bx))}{2b}$$

input
```
Integrate[Sech[c + b*x]^3*Sinh[a + b*x],x]
```

output
```
-1/2*(Sech[c]*Sech[c + b*x]^2*(Cosh[a] - Sinh[a - c]*Sinh[c + 2*b*x]))/b
```

3.151.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6158, 3042, 26, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\text{sech}^3(bx+c)\,dx$$

$$\Big\downarrow \text{6158}$$

$$\sinh(a-c)\int \text{sech}^2(c+bx)dx+\cosh(a-c)\int \text{sech}^2(c+bx)\tanh(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)^2dx+\cosh(a-c)\int -i\sec(ic+ibx)^2\tan(ic+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)^2dx-i\cosh(a-c)\int \sec(ic+ibx)^2\tan(ic+ibx)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cosh(a-c)\int \text{sech}(c+bx)d\text{sech}(c+bx)}{b}+\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{15}$$

$$-\frac{\cosh(a-c)\text{sech}^2(bx+c)}{2b}+\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\cosh(a-c)\text{sech}^2(bx+c)}{2b}+\frac{i\sinh(a-c)\int 1d(-i\tanh(c+bx))}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(a-c)\tanh(bx+c)}{b}-\frac{\cosh(a-c)\text{sech}^2(bx+c)}{2b}$$

input
```
Int[Sech[c + b*x]^3*Sinh[a + b*x],x]
```

output
```
-1/2*(Cosh[a - c]*Sech[c + b*x]^2)/b + (Sinh[a - c]*Tanh[c + b*x])/b
```

3.151.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 6158
```
Int[Sech[w_]^(n_.)*Sinh[v_], x_Symbol] :> Simp[Cosh[v - w]   Int[Tanh[w]*Se
ch[w]^(n - 1), x], x] + Simp[Sinh[v - w]   Int[Sech[w]^(n - 1), x], x] /; G
tQ[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

3.151.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

method	result	size
parallelrisch	$\frac{-1-\cosh(2bx+2c)+2\cosh(2bx+a+c)}{2b(1+\cosh(2bx+2c))}$	42
risch	$-\frac{\left(2\,\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)\mathrm{e}^{3a-c}}{\left(\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}\right)^{2}b}$	58

input
```
int(sech(b*x+c)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(-1-cosh(2*b*x+2*c)+2*cosh(2*b*x+a+c))/(1+cosh(2*b*x+2*c))
```

3.151.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 6.47

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx =$$

$$-\frac{}{b\cosh(bx+c)^3\cosh(-a+c)^2+3\,b\cosh(bx+c)\cosh(-a+c)^2+\left(b\cosh(-a+c)^2-b\sinh(-a+c)\right.}$$

input
```
integrate(sech(b*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + c)*cosh(-a + c) + cosh(-a + c)*sinh(b*x + c) - 2*cosh(b*x +
 c)*sinh(-a + c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 + 3*b*cosh(b*x + c)*co
sh(-a + c)^2 + (b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(
b*cosh(b*x + c)*cosh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x
+ c)^2 - (b*cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c)^2 + (3*b*cos
h(b*x + c)^2*cosh(-a + c)^2 + b*cosh(-a + c)^2 - (3*b*cosh(b*x + c)^2 + b)
*sinh(-a + c)^2)*sinh(b*x + c))
```

3.151.6 Sympy [F]

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx = \int \sinh(a+bx)\operatorname{sech}^3(bx+c)\,dx$$

input
```
integrate(sech(b*x+c)**3*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*sech(b*x + c)**3, x)
```

3.151.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(36) = 72.

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.16

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx = -\frac{2\,e^{(-2\,bx+3\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)})} + \frac{e^{(2\,a+3\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)})} - \frac{e^{(5\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)})}$$

input
```
integrate(sech(b*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4
*c))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a +
 4*c))) - e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*c
)))
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx = -\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)}+e^{(2\,a)}-e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)}+1\right)^2}$$

input
```
integrate(sech(b*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
-(2*e^(2*b*x + 2*a + 2*c) + e^(2*a) - e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2
*c) + 1)^2)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx=\int \frac{\sinh(a+b\,x)}{\cosh{(c+b\,x)}^3}\,dx$$

input
```
int(sinh(a + b*x)/cosh(c + b*x)^3,x)
```

output
```
int(sinh(a + b*x)/cosh(c + b*x)^3, x)
```

3.152 $\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx$

3.152.1 Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = x\cosh(a-c)+\frac{\log(\sinh(c+bx))\sinh(a-c)}{b}$$

output
```
x*cosh(a-c)+ln(sinh(b*x+c))*sinh(a-c)/b
```

3.152.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = x\cosh(a-c)+\frac{\log(\sinh(c+bx))\sinh(a-c)}{b}$$

input
```
Integrate[Csch[c + b*x]*Sinh[a + b*x],x]
```

output
```
x*Cosh[a - c] + (Log[Sinh[c + b*x]]*Sinh[a - c])/b
```

3.152.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6160, 24, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\text{csch}(bx+c)\,dx$$

$$\Big\downarrow \text{6160}$$

$$\sinh(a-c)\int \coth(c+bx)dx+\cosh(a-c)\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\sinh(a-c)\int \coth(c+bx)dx+x\cosh(a-c)$$

$$\Big\downarrow \text{3042}$$

$$x\cosh(a-c)+\sinh(a-c)\int -i\tan\left(ic+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$x\cosh(a-c)-i\sinh(a-c)\int \tan\left(\frac{1}{2}(2ic+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$x\cosh(a-c)+\frac{\sinh(a-c)\log(-i\sinh(bx+c))}{b}$$

input
```
Int[Csch[c + b*x]*Sinh[a + b*x],x]
```

output
```
x*Cosh[a - c] + (Log[(-I)*Sinh[c + b*x]]*Sinh[a - c])/b
```

3.152.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 6160
```
Int[Csch[w_]^(n_.)*Sinh[v_], x_Symbol] :> Simp[Sinh[v - w]   Int[Coth[w]*Cs
ch[w]^(n - 1), x], x] + Simp[Cosh[v - w]   Int[Csch[w]^(n - 1), x], x] /; G
tQ[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

3.152.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(26) = 52$.

Time = 0.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 5.77

method	result
risch	$x\,{\mathrm e}^{a-c}-{\mathrm e}^{-a-c}{\mathrm e}^{2a}x+{\mathrm e}^{-a-c}{\mathrm e}^{2c}x-\frac{{\mathrm e}^{-a-c}{\mathrm e}^{2a}a}{b}+\frac{{\mathrm e}^{-a-c}{\mathrm e}^{2c}a}{b}+\frac{\ln\left({\mathrm e}^{2bx+2a}-{\mathrm e}^{2a-2c}\right){\mathrm e}^{-a-c}{\mathrm e}^{2a}}{2b}-\frac{\ln\left({\mathrm e}^{2bx+2a}-{\mathrm e}^{2a-2}\right.}{2b}$

input
```
int(csch(b*x+c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x*exp(a-c)-exp(-a-c)*exp(2*a)*x+exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a
)*a+1/b*exp(-a-c)*exp(2*c)*a+1/2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/b*exp(-a-
c)*exp(2*a)-1/2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/b*exp(-a-c)*exp(2*c)
```

3.152.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.31

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = \frac{2\,bx + \left(\cosh\left(-a+c\right)^2 - 2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right) + \sinh\left(-a+c\right)^2 - 1\right)\log\left(\frac{2\,\sinh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\,\left(b\cosh\left(-a+c\right) - b\sinh\left(-a+c\right)\right)}$$

input
```
integrate(csch(b*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b*x + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 - 1)*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a +
c) - b*sinh(-a + c))
```

3.152.6 Sympy [F]

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = \int \sinh\left(a+bx\right)\operatorname{csch}\left(bx+c\right)\,dx$$

input
```
integrate(csch(b*x+c)*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*csch(b*x + c), x)
```

3.152.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(26) = 52.

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.23

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = \frac{\left(e^{(2\,a)} - e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} + e^{c}\right)}{2\,b} + \frac{\left(e^{(2\,a)} - e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} - e^{c}\right)}{2\,b} + \frac{(bx+a)e^{(a-c)}}{b}$$

input
```
integrate(csch(b*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) -
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (b*x + a)*e^(a - c)/b
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = \frac{2\,bxe^{(-a+c)} + \left(e^{(2\,a+c)} - e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(2\,bx+2\,c)} - 1\right|\right)}{2\,b}$$

input
```
integrate(csch(b*x+c)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(2*b*x*e^(-a + c) + (e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(2*
b*x + 2*c) - 1)))/b
```

3.152.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = x\,\mathrm{e}^{c-a} + \frac{\mathrm{e}^{2\,c-2\,a}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x} - \mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,c}\right)\,\left(2\,b\,\mathrm{e}^{3\,a-3\,c} - 2\,b\,\mathrm{e}^{a-c}\right)}{4\,b^2}$$

input
```
int(sinh(a + b*x)/sinh(c + b*x),x)
```

output
```
x*exp(c - a) + (exp(2*c - 2*a)*log(exp(2*a)*exp(2*b*x) - exp(2*a)*exp(-2*c
))*(2*b*exp(3*a - 3*c) - 2*b*exp(a - c)))/(4*b^2)
```

3.153 $\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx$

3.153.1 Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(c+bx))\cosh(a-c)}{b} - \frac{\operatorname{csch}(c+bx)\sinh(a-c)}{b}$$

output
```
-arctanh(cosh(b*x+c))*cosh(a-c)/b-csch(b*x+c)*sinh(a-c)/b
```

3.153.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx = -\frac{2i\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh(c)\cosh\left(\frac{bx}{2}\right)+\sinh(c)\sinh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\cosh(a-c)}{b} - \frac{\operatorname{csch}(c+bx)\sinh(a-c)}{b}$$

input
```
Integrate[Csch[c + b*x]^2*Sinh[a + b*x],x]
```

output
```
((-2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[
(b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c
])/b - (Csch[c + b*x]*Sinh[a - c])/b
```

3.153.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6160, 3042, 26, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\text{csch}^2(bx+c)\,dx$$

$$\Big\downarrow \text{6160}$$

$$\cosh(a-c)\int \text{csch}(c+bx)dx+\sinh(a-c)\int \coth(c+bx)\text{csch}(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cosh(a-c)\int i\csc(ic+ibx)dx+\sinh(a-c)\int \sec\left(ic+ibx-\frac{\pi}{2}\right)\tan\left(ic+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$i\cosh(a-c)\int \csc(ic+ibx)dx+\sinh(a-c)\int \sec\left(ic+ibx-\frac{\pi}{2}\right)\tan\left(ic+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3086}$$

$$i\cosh(a-c)\int \csc(ic+ibx)dx-\frac{i\sinh(a-c)\int 1d(-i\text{csch}(c+bx))}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\sinh(a-c)\text{csch}(bx+c)}{b}+i\cosh(a-c)\int \csc(ic+ibx)dx$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\cosh(a-c)\text{arctanh}(\cosh(bx+c))}{b}-\frac{\sinh(a-c)\text{csch}(bx+c)}{b}$$

input
```
Int[Csch[c + b*x]^2*Sinh[a + b*x],x]
```

output
```
-((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) - (Csch[c + b*x]*Sinh[a - c])/b
```

3.153.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6160
```
Int[Csch[w_]^(n_.)*Sinh[v_], x_Symbol] :> Simp[Sinh[v - w]   Int[Coth[w]*Cs
ch[w]^(n - 1), x], x] + Simp[Cosh[v - w]   Int[Csch[w]^(n - 1), x], x] /; G
tQ[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

3.153.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(36) = 72$.

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 4.78

method	result
risch	$\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2c}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}-e\right.}{}$

input
```
int(csch(b*x+c)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(-exp(2*b*x+2*a+2*c)+exp(2*a))-1/2*ln(e
xp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*ln(exp(b*x+a)+exp(a-c))/b*exp
(-a-c)*exp(2*c)+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*ln(ex
p(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)
```

3.153.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 617, normalized size of antiderivative = 17.14

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx$$
$$= \frac{4\cosh(bx+c)\cosh(-a+c)\sinh(-a+c) - 2\cosh(bx+c)\sinh(-a+c)^2 - 2\left(\cosh(-a+c)^2-1\right)\cos}{}$$

input
```
integrate(csch(b*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(4*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - 2*cosh(b*x + c)*sinh(-a +
 c)^2 - 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c) - ((cosh(-a + c)^2 + 1)*cosh(
b*x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 + 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a +
c)^2 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-
a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x
 + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)*log(cosh(b*x + c) +
 sinh(b*x + c) + 1) + ((cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (cosh(-a + c
)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^2 +
(cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(2*cosh(b*x + c)
*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^
2 + 1)*cosh(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) - co
sh(-a + c))*sinh(-a + c) - 1)*log(cosh(b*x + c) + sinh(b*x + c) - 1) - 2*(
cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*
x + c))/(b*cosh(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c)
)*sinh(b*x + c)^2 - b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) - b*c
osh(b*x + c)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 - b)*sinh(-a
 + c))
```

3.153.6 Sympy [F]

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx=\int \sinh(a+bx)\operatorname{csch}^2(bx+c)\,dx$$

input
```
integrate(csch(b*x+c)**2*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*csch(b*x + c)**2, x)
```

3.153.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(36) = 72$.

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.86

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx=-\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b}+\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b}+\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}-e^{(2c)}\right)}$$

input
```
integrate(csch(b*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
 e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (e^(2*a) - e^(2*c))*e^(-b*x -
 a)/(b*(e^(-2*b*x) - e^(2*c)))
```

3.153.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(36) = 72$.

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.89

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx=$$
$$-\frac{\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{2\left(e^{(bx+2a)}-e^{(bx+}\right.}{e^{(2bx+2c)}-}}{2b}$$

input
```
integrate(csch(b*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
-1/2*((e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a
+ c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + 2*(e^(b*x + 2*a)
- e^(b*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) - 1))/b
```

3.153.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.33

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx$$
$$= \frac{e^{a+bx}\left(e^{2a-2c}-1\right)}{b\left(e^{2a-2c}-e^{2a+2bx}\right)} - \frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{-b^2}+e^{2a}\,e^{-2c}\sqrt{-b^2}\right)}{b\sqrt{e^{-2a}\,e^{2c}\left(2\,e^{2a}\,e^{-2c}+e^{4a}\,e^{-4c}+1\right)}}\right)\sqrt{e^{2c-2a}\left(2\,e^{2a-2c}+e^{4a-4c}+1\right)}}{\sqrt{-b^2}}$$

input
```
int(sinh(a + b*x)/sinh(c + b*x)^2,x)
```

output
```
(exp(a + b*x)*(exp(2*a - 2*c) - 1))/(b*(exp(2*a - 2*c) - exp(2*a + 2*b*x))
) - (atan((exp(-a)*exp(2*c)*exp(b*x)*((-b^2)^(1/2) + exp(2*a)*exp(-2*c)*(-
b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-
4*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(2*exp(2*a - 2*c) + exp(4*a - 4*c) + 1)
)^(1/2))/(-b^2)^(1/2)
```

3.154 $\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx$

3.154.1 Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx=-\frac{\cosh(a-c)\coth(c+bx)}{b}-\frac{\operatorname{csch}^2(c+bx)\sinh(a-c)}{2b}$$

output
```
-cosh(a-c)*coth(b*x+c)/b-1/2*csch(b*x+c)^2*sinh(a-c)/b
```

3.154.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx=-\frac{(\cosh(a)-\cosh(a-c)\cosh(c+2bx))\operatorname{csch}(c)\operatorname{csch}^2(c+bx)}{2b}$$

input
```
Integrate[Csch[c + b*x]^3*Sinh[a + b*x],x]
```

output
```
-1/2*((Cosh[a] - Cosh[a - c]*Cosh[c + 2*b*x])*Csch[c]*Csch[c + b*x]^2)/b
```

3.154.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6160, 3042, 25, 26, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\text{csch}^3(bx+c)\,dx$$

$$\Big\downarrow \text{6160}$$

$$\cosh(a-c)\int \text{csch}^2(c+bx)dx+\sinh(a-c)\int \coth(c+bx)\text{csch}^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cosh(a-c)\int -\csc(ic+ibx)^2dx+\sinh(a-c)\int i\sec\left(ic+ibx-\frac{\pi}{2}\right)^2\tan\left(ic+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\sinh(a-c)\int i\sec\left(ic+ibx-\frac{\pi}{2}\right)^2\tan\left(ic+ibx-\frac{\pi}{2}\right)dx-\cosh(a-c)\int \csc(ic+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$i\sinh(a-c)\int \sec\left(\frac{1}{2}(2ic-\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ic-\pi)+ibx\right)dx-\cosh(a-c)\int \csc(ic+ibx)^2dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\sinh(a-c)\int -i\text{csch}(c+bx)d(-i\text{csch}(c+bx))}{b}-\cosh(a-c)\int \csc(ic+ibx)^2dx$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sinh(a-c)\text{csch}^2(bx+c)}{2b}-\cosh(a-c)\int \csc(ic+ibx)^2dx$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sinh(a-c)\text{csch}^2(bx+c)}{2b}-\frac{i\cosh(a-c)\int 1d(-i\coth(c+bx))}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\cosh(a-c)\coth(bx+c)}{b}-\frac{\sinh(a-c)\text{csch}^2(bx+c)}{2b}$$

input
```
Int[Csch[c + b*x]^3*Sinh[a + b*x],x]
```

output
```
-((Cosh[a - c]*Coth[c + b*x])/b) - (Csch[c + b*x]^2*Sinh[a - c])/(2*b)
```

3.154.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 6160
```
Int[Csch[w_]^(n_.)*Sinh[v_], x_Symbol] :> Simp[Sinh[v - w]   Int[Coth[w]*Cs
ch[w]^(n - 1), x], x] + Simp[Cosh[v - w]   Int[Csch[w]^(n - 1), x], x] /; G
tQ[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

3.154.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

method	result	size
risch	$\frac{\left(-2\,\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}+\mathrm{e}^{2c}\right)\mathrm{e}^{3a-c}}{\left(-\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}\right)^{2}b}$	57
parallelrisch	$-\frac{\left(\sinh(bx+a)\left(-\frac{\operatorname{sech}\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}}{2}+1\right)\operatorname{csch}\left(\frac{bx}{2}+\frac{c}{2}\right)+\operatorname{sech}\left(\frac{bx}{2}+\frac{c}{2}\right)\cosh(bx+a)\right)\operatorname{csch}\left(\frac{bx}{2}+\frac{c}{2}\right)}{4b}$	63

input
```
int(csch(b*x+c)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(-exp(2*b*x+2*a+2*c)+exp(2*a))^2/b*(-2*exp(2*b*x+2*a+2*c)+exp(2*a)+exp(2
*c))*exp(3*a-c)
```

3.154.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(37) = 74.

Time = 0.24 (sec) , antiderivative size = 246, normalized size of antiderivative = 6.31

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx =$$

$$-\frac{}{b\cosh\left(bx+c\right)^{3}\cosh\left(-a+c\right)^{2}-b\cosh\left(bx+c\right)\cosh\left(-a+c\right)^{2}+\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)}$$

input
```
integrate(csch(b*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-2*((2*cosh(-a + c) - sinh(-a + c))*sinh(b*x + c) - cosh(b*x + c)*sinh(-a
+ c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 - b*cosh(b*x + c)*cosh(-a + c)^2 +
 (b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*cosh(b*x + c
)*cosh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)^2 - (b*co
sh(b*x + c)^3 - b*cosh(b*x + c))*sinh(-a + c)^2 + 3*(b*cosh(b*x + c)^2*cos
h(-a + c)^2 - b*cosh(-a + c)^2 - (b*cosh(b*x + c)^2 - b)*sinh(-a + c)^2)*s
inh(b*x + c))
```

3.154.6 Sympy [F]

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx = \int \sinh(a+bx)\operatorname{csch}^3(bx+c)\,dx$$

input
```
integrate(csch(b*x+c)**3*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*csch(b*x + c)**3, x)
```

3.154.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(37) = 74.

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.36

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx = -\frac{2\,e^{(-2\,bx+3\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)})} + \frac{e^{(2\,a+3\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)})} + \frac{e^{(5\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)})}$$

input
```
integrate(csch(b*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4
*c))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a +
 4*c))) + e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*c
)))
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx = -\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)}-e^{(2\,a)}-e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)}-1\right)^2}$$

input
```
integrate(csch(b*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
-(2*e^(2*b*x + 2*a + 2*c) - e^(2*a) - e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2
*c) - 1)^2)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx = \int \frac{\sinh(a+b\,x)}{\sinh\left(c+b\,x\right)^3}\,dx$$

input
```
int(sinh(a + b*x)/sinh(c + b*x)^3,x)
```

output
```
int(sinh(a + b*x)/sinh(c + b*x)^3, x)
```

3.155 $\int \cosh(a+bx)\tanh(c+bx)\,dx$

3.155.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \cosh(a+bx)\tanh(c+bx)\,dx=\frac{\cosh(a+bx)}{b}-\frac{\arctan(\sinh(c+bx))\sinh(a-c)}{b}$$

output
```
cosh(b*x+a)/b-arctan(sinh(b*x+c))*sinh(a-c)/b
```

3.155.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 86 vs. 2(29) = 58.

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.97

$$\int \cosh(a+bx)\tanh(c+bx)\,dx =\frac{\cosh(a)\cosh(bx)}{b}-\frac{2\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh\left(\frac{bx}{2}\right)\sinh(c)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\sinh(a-c)}{b}+\frac{\sinh(a)\sinh(bx)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Tanh[c + b*x],x]
```

output
```
(Cosh[a]*Cosh[b*x])/b - (2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh
[c] + Cosh[c]*Sinh[(b*x)/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[
c])]*Sinh[a - c])/b + (Sinh[a]*Sinh[b*x])/b
```

3.155.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6157, 3042, 26, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\tanh(bx+c)\,dx$$

$$\Big\downarrow \text{ 6157}$$

$$\int \sinh(a+bx)dx-\sinh(a-c)\int \text{sech}(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sin(ia+ibx)dx-\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 26}$$

$$-\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx-i\int \sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{\cosh(a+bx)}{b}-\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\cosh(a+bx)}{b}-\frac{\sinh(a-c)\arctan(\sinh(bx+c))}{b}$$

input
```
Int[Cosh[a + b*x]*Tanh[c + b*x],x]
```

output
```
Cosh[a + b*x]/b - (ArcTan[Sinh[c + b*x]]*Sinh[a - c])/b
```

3.155.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6157
```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Simp[Sinh[v - w]   Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

3.155.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 5.76

method	result
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{i\ln\left(e^{bx+a}\right.}{}$

input
```
int(cosh(b*x+a)*tanh(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-
a-c)*exp(a)^2-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(c)^2-1/2*I*l
n(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(a)^2+1/2*I*ln(exp(b*x+a)+I*exp(a-
c))/b*exp(-a-c)*exp(c)^2
```

3.155.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 327, normalized size of antiderivative = 11.28

$$\int \cosh(a + bx)\tanh(c + bx)\,dx$$
$$= \frac{\cosh(bx+c)^2\cosh(-a+c)^2 - 2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c) + \cosh(bx+c)^2\sinh(-a+}{}$$

input
```
integrate(cosh(b*x+a)*tanh(b*x+c),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(
-a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c
)*sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + 2*(2*cosh(b*x + c)*cosh
(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1
)*cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a
+ c)^2 - 1)*sinh(b*x + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(cosh
(b*x + c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cos
h(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c) + 1)/(b*cosh(b*x + c)*cosh(-a + c
) - b*cosh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(
b*x + c))
```

3.155.6 Sympy [F]

$$\int \cosh(a + bx)\tanh(c + bx)\,dx = \int \cosh(a + bx)\tanh(bx + c)\,dx$$

input
```
integrate(cosh(b*x+a)*tanh(b*x+c),x)
```

output
```
Integral(cosh(a + b*x)*tanh(b*x + c), x)
```

3.155.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(29) = 58$.

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int \cosh(a+bx)\tanh(c+bx)\,dx = \frac{\left(e^{(2a)} - e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} + \frac{e^{(bx+a)}}{2b} + \frac{e^{(-bx-a)}}{2b}$$

input
```
integrate(cosh(b*x+a)*tanh(b*x+c),x, algorithm="maxima")
```

output
```
(e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(b*x + a)/b
+ 1/2*e^(-b*x - a)/b
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \cosh(a+bx)\tanh(c+bx)\,dx$$
$$= -\frac{2\left(e^{(2a)} - e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - e^{(bx+a)} - e^{(-bx-a)}}{2b}$$

input
```
integrate(cosh(b*x+a)*tanh(b*x+c),x, algorithm="giac")
```

output
```
-1/2*(2*(e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - e^(b*x + a) -
 e^(-b*x - a))/b
```

3.155.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.59

$$\int \cosh(a+bx)\tanh(c+bx)\,dx$$
$$= \frac{e^{a+bx}}{2b} + \frac{e^{-a-bx}}{2b} + \frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{b^2} - e^{2a}\,e^{-2c}\sqrt{b^2}\right)}{b\sqrt{e^{-2a}\,e^{2c}\left(e^{4a}\,e^{-4c} - 2\,e^{2a}\,e^{-2c} + 1\right)}}\right)\sqrt{e^{2c-2a}\left(e^{4a-4c} - 2\,e^{2a-2c} + 1\right)}}{\sqrt{b^2}}$$

input
```
int(cosh(a + b*x)*tanh(c + b*x),x)
```

output
```
exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) + (atan((exp(-a)*exp(2*c)*exp(b*
x)*((b^2)^(1/2) - exp(2*a)*exp(-2*c)*(b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*
(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(
exp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2))/(b^2)^(1/2)
```

3.155. $\int \cosh(a+bx)\tanh(c+bx)\,dx$

3.156 $\int \cosh(a+bx)\tanh^2(c+bx)\,dx$

3.156.1 Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \cosh(a+bx)\tanh^2(c+bx)\,dx=-\frac{\arctan(\sinh(c+bx))\cosh(a-c)}{b}+\frac{\operatorname{sech}(c+bx)\sinh(a-c)}{b}+\frac{\sinh(a+bx)}{b}$$

output
```
-arctan(sinh(b*x+c))*cosh(a-c)/b+sech(b*x+c)*sinh(a-c)/b+sinh(b*x+a)/b
```

3.156.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 102 vs. 2(45) = 90.

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.27

$$\int \cosh(a+bx)\tanh^2(c+bx)\,dx = -\frac{2\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh\left(\frac{bx}{2}\right)\sinh(c)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\cosh(a-c)}{b}+\frac{\cosh(bx)\sinh(a)}{b}+\frac{\operatorname{sech}(c+bx)\sinh(a-c)}{b}+\frac{\cosh(a)\sinh(bx)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Tanh[c + b*x]^2,x]
```

output
```
(-2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x
)/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])/b + (
Cosh[b*x]*Sinh[a])/b + (Sech[c + b*x]*Sinh[a - c])/b + (Cosh[a]*Sinh[b*x])
/b
```

3.156.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6157, 3042, 26, 3086, 24, 6154, 3042, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\tanh^2(bx+c)\,dx$$

$$\Big\downarrow \text{6157}$$

$$\int \sinh(a+bx)\tanh(c+bx)dx-\sinh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sinh(a+bx)\tanh(c+bx)dx-\sinh(a-c)\int -i\sec(ic+ibx)\tan(ic+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$\int \sinh(a+bx)\tanh(c+bx)dx+i\sinh(a-c)\int \sec(ic+ibx)\tan(ic+ibx)dx$$

$$\Big\downarrow \text{3086}$$

$$\int \sinh(a+bx)\tanh(c+bx)dx+\frac{\sinh(a-c)\int 1d\operatorname{sech}(c+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$\int \sinh(a+bx)\tanh(c+bx)dx+\frac{\sinh(a-c)\operatorname{sech}(bx+c)}{b}$$

$$\Big\downarrow \text{6154}$$

$$-\cosh(a-c)\int \operatorname{sech}(c+bx)dx+\int \cosh(a+bx)dx+\frac{\sinh(a-c)\operatorname{sech}(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx+\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{\sinh(a-c)\text{sech}(bx+c)}{b}$$

$$\Big\downarrow \text{3117}$$

$$-\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx+\frac{\sinh(a-c)\text{sech}(bx+c)}{b}+\frac{\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\cosh(a-c)\arctan(\sinh(bx+c))}{b}+\frac{\sinh(a-c)\text{sech}(bx+c)}{b}+\frac{\sinh(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x]*Tanh[c + b*x]^2,x]
```

output
```
-((ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b) + (Sech[c + b*x]*Sinh[a - c])/b +
 Sinh[a + b*x]/b
```

3.156.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6154
```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Simp[Cosh[v - w]   Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

rule 6157
```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Simp[Sinh[v - w]   Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

3.156.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 4.60

method	result
risch	$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)} + \frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b} + \frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2c}}{2b} - \frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)}{2b}$

input
```
int(cosh(b*x+a)*tanh(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(exp
(2*b*x+2*a+2*c)+exp(2*a))+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(
2*a)+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*I*ln(exp(b*x
+a)+I*exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp
(-a-c)*exp(2*c)
```

3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(45) = 90.

Time = 0.26 (sec) , antiderivative size = 902, normalized size of antiderivative = 20.04

$$\int \cosh(a+bx)\tanh^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)*tanh(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sin
h(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c
)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a +
c)^2)*sinh(b*x + c)^3 + 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + 3*(2*cosh
(b*x + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh
(-a + c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c
) - 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^4 + 3*cosh(b*x + c)^2)*sinh(-a + c
)^2 - 2*((cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-
a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x
+ c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a +
 c)^2 + 1)*cosh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 + cosh(b*x +
c))*sinh(-a + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (3*(cosh(-a + c)
^2 + 1)*cosh(b*x + c)^2 + (3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a
 + c)^2 - 2*(3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) +
 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-
a + c))*sinh(-a + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(2*cosh(b*
x + c)^3*cosh(-a + c)^2 + (2*cosh(b*x + c)^3 + 3*cosh(b*x + c))*sinh(-a +
c)^2 + 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c) - 2*(2*cosh(b*x + c)^3*cosh(-a
 + c) + 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cos
h(b*x + c)^4*cosh(-a + c) + 3*cosh(b*x + c)^2*cosh(-a + c))*sinh(-a + c...
```

3.156.6 Sympy [F]

$$\int \cosh(a+bx)\tanh^2(c+bx)\,dx = \int \cosh(a+bx)\tanh^2(bx+c)\,dx$$

input
```
integrate(cosh(b*x+a)*tanh(b*x+c)**2,x)
```

output
```
Integral(cosh(a + b*x)*tanh(b*x + c)**2, x)
```

3.156.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(45) = 90$.

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

$$\int \cosh(a+bx)\tanh^2(c+bx)\,dx = \frac{\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} - \frac{e^{(-bx-a)}}{2b} + \frac{\left(3e^{(2a)}-2e^{(2c)}\right)e^{(-2bx-2a)}+e^{(2c)}}{2b\left(e^{(-bx-a+2c)}+e^{(-3bx-a)}\right)}$$

input
```
integrate(cosh(b*x+a)*tanh(b*x+c)^2,x, algorithm="maxima")
```

output
```
(e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - 1/2*e^(-b*x - a)/b
 + 1/2*((3*e^(2*a) - 2*e^(2*c))*e^(-2*b*x - 2*a) + e^(2*c))/(b*(e^(-b*x -
a + 2*c) + e^(-3*b*x - a)))
```

3.156.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.16

$$\int \cosh(a+bx)\tanh^2(c+bx)\,dx = -\frac{2\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{2e^{(2bx+4a)}-3e^{(2bx+2a+2c)}-e^{(2a)}}{e^{(3bx+3a+2c)}+e^{(bx+3a)}} - e^{(bx+a)}}{2b}$$

input
```
integrate(cosh(b*x+a)*tanh(b*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(2*(e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (2*e^(2*b*x +
 4*a) - 3*e^(2*b*x + 2*a + 2*c) - e^(2*a))/(e^(3*b*x + 3*a + 2*c) + e^(b*x
 + 3*a)) - e^(b*x + a))/b
```

3.156.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.84

$$\int \cosh(a+bx)\tanh^2(c+bx)\,dx$$
$$= \frac{e^{a+bx}}{2b} - \frac{e^{-a-bx}}{2b} - \frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{b^2}+e^{2a}\,e^{-2c}\,\sqrt{b^2}\right)}{b\sqrt{e^{-2a}\,e^{2c}\,(2\,e^{2a}\,e^{-2c}+e^{4a}\,e^{-4c}+1)}}\right)\sqrt{e^{2c-2a}\,\left(2\,e^{2a-2c}+e^{4a-4c}+1\right)}}{\sqrt{b^2}} + \frac{e^{a+bx}\,\left(e^{2a-2c}-1\right)}{b\,\left(e^{2a-2c}+e^{2a+2bx}\right)}$$

input
```
int(cosh(a + b*x)*tanh(c + b*x)^2,x)
```

output
```
exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (atan((exp(-a)*exp(2*c)*exp(b*
x)*((b^2)^(1/2) + exp(2*a)*exp(-2*c)*(b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*
(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(
2*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/(b^2)^(1/2) + (exp(a + b*x)
*(exp(2*a - 2*c) - 1))/(b*(exp(2*a - 2*c) + exp(2*a + 2*b*x)))
```

3.157 $\int \cosh(a+bx)\tanh^3(c+bx)\,dx$

3.157.1 Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx = \frac{\cosh(a+bx)}{b}+\frac{\cosh(a-c)\text{sech}(c+bx)}{b} - \frac{3\arctan(\sinh(c+bx))\sinh(a-c)}{2b} + \frac{\text{sech}(c+bx)\sinh(a-c)\tanh(c+bx)}{2b}$$

output
```
cosh(b*x+a)/b+cosh(a-c)*sech(b*x+c)/b-3/2*arctan(sinh(b*x+c))*sinh(a-c)/b+
1/2*sech(b*x+c)*sinh(a-c)*tanh(b*x+c)/b
```

3.157.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.60

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx$$
$$= \frac{\cosh(a-2c)\text{sech}(c)\text{sech}(c+bx)-\cosh(a-c-bx)\text{sech}(c)\text{sech}^2(c+bx)+\cosh(a-c+bx)\text{sech}(c)\text{sech}^2(}{}$$

input
```
Integrate[Cosh[a + b*x]*Tanh[c + b*x]^3,x]
```

output
```
(Cosh[a - 2*c]*Sech[c]*Sech[c + b*x] - Cosh[a - c - b*x]*Sech[c]*Sech[c +
b*x]^2 + Cosh[a - c + b*x]*Sech[c]*Sech[c + b*x]^2 + Cosh[a]*(4*Cosh[b*x]
+ 3*Sech[c]*Sech[c + b*x]) - 12*ArcTan[Sinh[c] + Cosh[c]*Tanh[(b*x)/2]]*Si
nh[a - c] + 4*Sinh[a]*Sinh[b*x])/(4*b)
```

3.157.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.067, Rules used = {6157, 3042, 25, 3091, 3042, 4257, 6154, 3042, 26, 3086, 24, 6157, 3042, 26, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\tanh^3(bx+c)\,dx$$

$$\Big\downarrow \text{6157}$$

$$\int \sinh(a+bx)\tanh^2(c+bx)dx-\sinh(a-c)\int \text{sech}(c+bx)\tanh^2(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sinh(a+bx)\tanh^2(c+bx)dx-\sinh(a-c)\int -\sec(ic+ibx)\tan(ic+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$\int \sinh(a+bx)\tanh^2(c+bx)dx+\sinh(a-c)\int \sec(ic+ibx)\tan(ic+ibx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\int \sinh(a+bx)\tanh^2(c+bx)dx+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{1}{2}\int \text{sech}(c+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sinh(a+bx)\tanh^2(c+bx)dx+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{1}{2}\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{4257}$$

$$\int \sinh(a+bx)\tanh^2(c+bx)dx+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)$$

$$\Big\downarrow \text{6154}$$

$$\int \cosh(a+bx)\tanh(c+bx)dx-\cosh(a-c)\int \text{sech}(c+bx)\tanh(c+bx)dx+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)$$

$\Big\downarrow$ 3042

$$\int \cosh(a+bx)\tanh(c+bx)dx-\cosh(a-c)\int -i\sec(ic+ibx)\tan(ic+ibx)dx+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)$$

$\Big\downarrow$ 26

$$\int \cosh(a+bx)\tanh(c+bx)dx+i\cosh(a-c)\int \sec(ic+ibx)\tan(ic+ibx)dx+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)$$

$\Big\downarrow$ 3086

$$\int \cosh(a+bx)\tanh(c+bx)dx+\frac{\cosh(a-c)\int 1d\text{sech}(c+bx)}{b}+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)$$

$\Big\downarrow$ 24

$$\int \cosh(a+bx)\tanh(c+bx)dx+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}$$

$\Big\downarrow$ 6157

$$-\sinh(a-c)\int \text{sech}(c+bx)dx+\int \sinh(a+bx)dx+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}$$

$\Big\downarrow$ 3042

$$-\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx+\int -i\sin(ia+ibx)dx+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}$$

$\Big\downarrow$ 26

$$-\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx-i\int \sin(ia+ibx)dx+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}$$

3.157. $\int \cosh(a+bx)\tanh^3(c+bx)\,dx$

$$\Big\downarrow \text{3118}$$

$$-\sinh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sinh(a-c)\arctan(\sinh(bx+c))}{b}+\sinh(a-c)\left(\frac{\tanh(bx+c)\text{sech}(bx+c)}{2b}-\frac{\arctan(\sinh(bx+c))}{2b}\right)+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x]*Tanh[c + b*x]^3,x]
```

output
```
Cosh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x])/b - (ArcTan[Sinh[c + b*x]]*S
inh[a - c])/b + Sinh[a - c]*(-1/2*ArcTan[Sinh[c + b*x]]/b + (Sech[c + b*x]
*Tanh[c + b*x])/(2*b))
```

3.157.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6154
```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Simp[Cosh[v - w]   Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

rule 6157
```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Simp[Sinh[v - w]   Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

3.157.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.31

method	result
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(3\,e^{2bx+4a+2c}+e^{2bx+2a+4c}+e^{4a}+3\,e^{2a+2c}\right)}{2b\left(e^{2bx+2a+2c}+e^{2a}\right)^{2}}+\frac{3i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{4b}-\frac{3i\ln\left(e^{bx+a}-ie^{a-c}\right)}{4b}$

input
```
int(cosh(b*x+a)*tanh(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(3*exp(2*b*x+4*a+2*c)+ex
p(2*b*x+2*a+4*c)+exp(4*a)+3*exp(2*a+2*c))/b/(exp(2*b*x+2*a+2*c)+exp(2*a))^
2+3/4*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(a)^2-3/4*I*ln(exp(b*x+a)
-I*exp(a-c))/b*exp(-a-c)*exp(c)^2-3/4*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a
-c)*exp(a)^2+3/4*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(c)^2
```

3.157.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1737 vs. $2(68) = 136$.

Time = 0.27 (sec) , antiderivative size = 1737, normalized size of antiderivative = 24.12

$$\int \cosh(a + bx) \tanh^3(c + bx)\, dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)*tanh(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + c)^6*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sin
h(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 6*(cosh(b*x + c)*cosh(-a + c
)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a +
c)^2)*sinh(b*x + c)^5 + (5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^4 + (15*cosh(
b*x + c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + 5*c
osh(-a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a
 + c) + 2)*sinh(b*x + c)^4 + 4*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(cosh
(b*x + c)^3 + cosh(b*x + c))*sinh(-a + c)^2 + (5*cosh(-a + c)^2 + 2)*cosh(
b*x + c) - 10*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*
sinh(-a + c))*sinh(b*x + c)^3 + (2*cosh(-a + c)^2 + 5)*cosh(b*x + c)^2 + (
15*cosh(b*x + c)^4*cosh(-a + c)^2 + 6*(5*cosh(-a + c)^2 + 2)*cosh(b*x + c)
^2 + (15*cosh(b*x + c)^4 + 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cosh
(-a + c)^2 - 2*(15*cosh(b*x + c)^4*cosh(-a + c) + 30*cosh(b*x + c)^2*cosh(
-a + c) + 2*cosh(-a + c))*sinh(-a + c) + 5)*sinh(b*x + c)^2 + (cosh(b*x +
c)^6 + 5*cosh(b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a
 + c)^2 - 1)*cosh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a +
c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)
*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b
*x + c))*sinh(b*x + c)^4 + 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + 2*(5*(
cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 + 1)*sinh(-a +...
```

3.157.6 Sympy [F]

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx = \int \cosh\left(a+bx\right)\tanh^3\left(bx+c\right)dx$$

input
```
integrate(cosh(b*x+a)*tanh(b*x+c)**3,x)
```

output
```
Integral(cosh(a + b*x)*tanh(b*x + c)**3, x)
```

3.157.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(68) = 136$.

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.07

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx = \frac{3\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{2b} + \frac{e^{(-bx-a)}}{2b} + \frac{\left(5e^{(2a+2c)}+e^{(4c)}\right)e^{(-2bx-2a)} + \left(2e^{(4a)}+3e^{(2a+2c)}\right)e^{(-4bx-4a)} + e^{(4c)}}{2b\left(e^{(-bx-a+4c)}+2e^{(-3bx-a+2c)}+e^{(-5bx-a)}\right)}$$

input
```
integrate(cosh(b*x+a)*tanh(b*x+c)^3,x, algorithm="maxima")
```

output
```
3/2*(e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(-b*x -
a)/b + 1/2*((5*e^(2*a + 2*c) + e^(4*c))*e^(-2*b*x - 2*a) + (2*e^(4*a) + 3*
e^(2*a + 2*c))*e^(-4*b*x - 4*a) + e^(4*c))/(b*(e^(-b*x - a + 4*c) + 2*e^(-
3*b*x - a + 2*c) + e^(-5*b*x - a)))
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.69

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx = -\frac{3\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{3e^{(3bx+5a+2c)}+e^{(3bx+3a+4c)}+e^{(bx+5a)}+3e^{(bx+3a+2c)}}{\left(e^{(2bx+2a+2c)}+e^{(2a)}\right)^2} - e^{(bx+a)} - e^{(-bx-}}{2b}$$

input
```
integrate(cosh(b*x+a)*tanh(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/2*(3*(e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (3*e^(3*b*x +
 5*a + 2*c) + e^(3*b*x + 3*a + 4*c) + e^(b*x + 5*a) + 3*e^(b*x + 3*a + 2*c
))/(e^(2*b*x + 2*a + 2*c) + e^(2*a))^2 - e^(b*x + a) - e^(-b*x - a))/b
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx=\int \cosh(a+b\,x)\,\tanh(c+b\,x)^3\,dx$$

input
```
int(cosh(a + b*x)*tanh(c + b*x)^3,x)
```

output
```
int(cosh(a + b*x)*tanh(c + b*x)^3, x)
```

3.158 $\int \cosh(a+bx)\coth(c+bx)\,dx$

3.158.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \cosh(a+bx)\coth(c+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(c+bx))\cosh(a-c)}{b}+\frac{\cosh(a+bx)}{b}$$

output
```
-arctanh(cosh(b*x+c))*cosh(a-c)/b+cosh(b*x+a)/b
```

3.158.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$\begin{aligned}&\int \cosh(a+bx)\coth(c+bx)\,dx\\&=-\frac{2i\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh(c)\cosh\left(\frac{bx}{2}\right)+\sinh(c)\sinh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\cosh(a-c)}{b}\\&\quad+\frac{\cosh(a)\cosh(bx)}{b}+\frac{\sinh(a)\sinh(bx)}{b}\end{aligned}$$

input
```
Integrate[Cosh[a + b*x]*Coth[c + b*x],x]
```

output
```
((-2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[
(b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c
])/b + (Cosh[a]*Cosh[b*x])/b + (Sinh[a]*Sinh[b*x])/b
```

3.158.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6155, 3042, 26, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth(bx+c)\,dx$$

$$\Big\downarrow \text{ 6155}$$

$$\cosh(a-c)\int \operatorname{csch}(c+bx)dx+\int \sinh(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cosh(a-c)\int i\csc(ic+ibx)dx+\int -i\sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$i\cosh(a-c)\int \csc(ic+ibx)dx-i\int \sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{\cosh(a+bx)}{b}+i\cosh(a-c)\int \csc(ic+ibx)dx$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\cosh(a+bx)}{b}-\frac{\cosh(a-c)\text{arctanh}(\cosh(bx+c))}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[c + b*x],x]
```

output
```
-((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) + Cosh[a + b*x]/b
```

3.158.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6155
```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Simp[Cosh[v - w]   Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

3.158.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 5.34

method	result
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{\ln(e^{bx+a}-e^{a-c})e^{-a-c}e^{2a}}{2b}+\frac{\ln(e^{bx+a}-e^{a-c})e^{-a-c}e^{2c}}{2b}-\frac{\ln(e^{bx+a}+e^{a-c})e^{-a-c}e^{2a}}{2b}-\frac{\ln(e^{bx+a}+e^{a-c}}{2b}$

input
```
int(cosh(b*x+a)*coth(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)
*exp(2*a)+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*ln(exp(b*x+
a)+exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*
exp(2*c)
```

3.158.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(29) = 58.

Time = 0.26 (sec) , antiderivative size = 439, normalized size of antiderivative = 15.14

$$\int \cosh(a+bx)\coth(c+bx)\,dx$$
$$= \frac{\cosh(bx+c)^2\cosh(-a+c)^2 - 2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c) + \cosh(bx+c)^2\sinh(-a+}{}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+c),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(
-a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c
)*sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + (2*cosh(b*x + c)*cosh(-
a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*
cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a +
c)^2 + 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) - (2*cosh(
b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(
-a + c)^2 + 1)*cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a +
c) + sinh(-a + c)^2 + 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c)
- 1) + 2*(cosh(b*x + c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh
(-a + c) + cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c) + 1)/(b*cosh(b*x +
c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(
-a + c))*sinh(b*x + c))
```

3.158.6 Sympy [F]

$$\int \cosh(a+bx)\coth(c+bx)\,dx = \int \cosh(a+bx)\coth(bx+c)\,dx$$

input
```
integrate(cosh(b*x+a)*coth(b*x+c),x)
```

output
```
Integral(cosh(a + b*x)*coth(b*x + c), x)
```

3.158.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(29) = 58.

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.10

$$\int \cosh(a+bx)\coth(c+bx)\,dx = -\frac{\left(e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2\,b} + \frac{\left(e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2\,b} + \frac{e^{(bx+a)}}{2\,b} + \frac{e^{(-bx-a)}}{2\,b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+c),x, algorithm="maxima")
```

output
```
-1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
 e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(b*x + a)/b + 1/2*e^(-b
*x - a)/b
```

3.158.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.14

$$\int \cosh(a+bx)\coth(c+bx)\,dx = -\frac{\left(e^{(2\,a+c)}+e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(bx+a+c)}+e^{a}\right)-\left(e^{(2\,a+c)}+e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(bx+a+c)}-e^{a}\right|\right)-e^{(bx+a)}}{2\,b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+c),x, algorithm="giac")
```

output
```
-1/2*((e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + a + c) + e^a) - (e
^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + a + c) - e^a)) - e^(b*
x + a) - e^(-b*x - a))/b
```

3.158.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.79

$$\int \cosh(a+bx)\coth(c+bx)\,dx$$
$$=\frac{e^{a+b\,x}}{2\,b}+\frac{e^{-a-b\,x}}{2\,b}-\frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{-b^2}+e^{2\,a}\,e^{-2\,c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(2\,e^{2\,a}\,e^{-2\,c}+e^{4\,a}\,e^{-4\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(2\,e^{2\,a-2\,c}+e^{4\,a-4\,c}+1\right)}}{\sqrt{-b^2}}$$

input
```
int(cosh(a + b*x)*coth(c + b*x),x)
```

output
```
exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) - (atan((exp(-a)*exp(2*c)*exp(b*
x)*((-b^2)^(1/2) + exp(2*a)*exp(-2*c)*(-b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c
)*(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1))^(1/2)))*(exp(2*c - 2*a)
*(2*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/(-b^2)^(1/2)
```

3.159 $\int \cosh(a+bx)\coth^2(c+bx)\,dx$

3.159.1 Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = -\frac{\cosh(a-c)\text{csch}(c+bx)}{b} - \frac{\text{arctanh}(\cosh(c+bx))\sinh(a-c)}{b} + \frac{\sinh(a+bx)}{b}$$

output
```
-cosh(a-c)*csch(b*x+c)/b-arctanh(cosh(b*x+c))*sinh(a-c)/b+sinh(b*x+a)/b
```

3.159.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.39

$$\begin{aligned}\int \cosh(a+bx)\coth^2(c+bx)\,dx &= -\frac{\cosh(a-c)\text{csch}(c+bx)}{b} + \frac{\cosh(bx)\sinh(a)}{b} \\ &- \frac{2i\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh(c)\cosh\left(\frac{bx}{2}\right)+\sinh(c)\sinh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\sinh(a-c)}{b} + \frac{\cosh(a)\sinh(bx)}{b}\end{aligned}$$

input
```
Integrate[Cosh[a + b*x]*Coth[c + b*x]^2,x]
```

output
```
-((Cosh[a - c]*Csch[c + b*x])/b) + (Cosh[b*x]*Sinh[a])/b - ((2*I)*ArcTan[(
(Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(b*x)/2]))/(I*Co
sh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Sinh[a - c])/b + (Cosh[a]*
Sinh[b*x])/b
```

3.159.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6155, 3042, 3086, 24, 6156, 3042, 26, 3117, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth^2(bx+c)\,dx$$

$$\Big\downarrow \text{6155}$$

$$\int \coth(c+bx)\sinh(a+bx)dx+\cosh(a-c)\int \coth(c+bx)\text{csch}(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \coth(c+bx)\sinh(a+bx)dx+\cosh(a-c)\int \sec\left(ic+ibx-\frac{\pi}{2}\right)\tan\left(ic+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\int \coth(c+bx)\sinh(a+bx)dx-\frac{i\cosh(a-c)\int 1d(-i\text{csch}(c+bx))}{b}$$

$$\Big\downarrow \text{24}$$

$$\int \coth(c+bx)\sinh(a+bx)dx-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{6156}$$

$$\sinh(a-c)\int \text{csch}(c+bx)dx+\int \cosh(a+bx)dx-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\sinh(a-c)\int i\csc(ic+ibx)dx+\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{26}$$

$$i\sinh(a-c)\int\csc(ic+ibx)dx+\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{3117}$$

$$i\sinh(a-c)\int\csc(ic+ibx)dx-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}+\frac{\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sinh(a-c)\text{arctanh}(\cosh(bx+c))}{b}-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}+\frac{\sinh(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[c + b*x]^2,x]
```

output
```
-((Cosh[a - c]*Csch[c + b*x])/b) - (ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b
+ Sinh[a + b*x]/b
```

3.159.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6155
```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Simp[Cosh[v - w]   Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

rule 6156
```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Simp[Sinh[v - w]   Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

3.159.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(46) = 92$.

Time = 0.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 4.24

method	result
risch	$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)} + \frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b} - \frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2c}}{2b} - \frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a}}{2b}$

input
```
int(cosh(b*x+a)*coth(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(-ex
p(2*b*x+2*a+2*c)+exp(2*a))+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a
)-1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*ln(exp(b*x+a)+exp(a
-c))/b*exp(-a-c)*exp(2*a)+1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)
```

3.159.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1237 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 1237, normalized size of antiderivative = 26.89

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sin
h(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c
)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a +
c)^2)*sinh(b*x + c)^3 - 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + 3*(2*cosh
(b*x + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh
(-a + c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c
) - 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^4 - 3*cosh(b*x + c)^2)*sinh(-a + c
)^2 - ((cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a
+ c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x +
c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c
)^2 - 1)*cosh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 - cosh(b*x + c)
)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2
 - 1)*cosh(b*x + c)^2 + (3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a +
 c)^2 - 2*(3*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1
)*sinh(b*x + c) - 2*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a
+ c))*sinh(-a + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + ((cosh(-a + c
)^2 - 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) +
 sinh(-a + c)^2 - 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-a + c)*sin
h(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x +
 c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2...
```

3.159.6 Sympy [F]

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = \int \cosh\left(a+bx\right)\coth^2\left(bx+c\right)dx$$

input
```
integrate(cosh(b*x+a)*coth(b*x+c)**2,x)
```

output
```
Integral(cosh(a + b*x)*coth(b*x + c)**2, x)
```

3.159.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(46) = 92$.

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.13

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = -\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b} + \frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b} - \frac{e^{(-bx-a)}}{2b} - \frac{\left(3e^{(2a)}+2e^{(2c)}\right)e^{(-2bx-2a)}-e^{(2c)}}{2b\left(e^{(-bx-a+2c)}-e^{(-3bx-a)}\right)}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) -
 e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b - 1/2*e^(-b*x - a)/b - 1/2*((3*
e^(2*a) + 2*e^(2*c))*e^(-2*b*x - 2*a) - e^(2*c))/(b*(e^(-b*x - a + 2*c) -
e^(-3*b*x - a)))
```

3.159.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.09

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = -\frac{\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+a+c)}+e^{a}\right)-\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+a+c)}-e^{a}\right|\right)+\frac{2e^{(2bx+4}}{e^{(3b}}}{2b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*((e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + a + c) + e^a) - (e
^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + a + c) - e^a)) + (2*e^
(2*b*x + 4*a) + 3*e^(2*b*x + 2*a + 2*c) - e^(2*a))/(e^(3*b*x + 3*a + 2*c)
- e^(b*x + 3*a)) - e^(b*x + a))/b
```

3.159.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.98

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx$$
$$= \frac{e^{a+bx}}{2b} - \frac{e^{-a-bx}}{2b} - \frac{\operatorname{atan}\left(-\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{-b^2}-e^{2a}\,e^{-2c}\,\sqrt{-b^2}\right)}{b\sqrt{e^{-2a}\,e^{2c}\,(e^{4a}\,e^{-4c}-2\,e^{2a}\,e^{-2c}+1)}}\right)\sqrt{e^{2c-2a}\,\left(e^{4a-4c}-2\,e^{2a-2c}+1\right)}}{\sqrt{-b^2}} + \frac{e^{a+bx}\,\left(e^{2a-2c}+1\right)}{b\,\left(e^{2a-2c}-e^{2a+2bx}\right)}$$

input
```
int(cosh(a + b*x)*coth(c + b*x)^2,x)
```

output
```
exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (atan(-(exp(-a)*exp(2*c)*exp(b
*x)*((-b^2)^(1/2) - exp(2*a)*exp(-2*c)*(-b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*
c)*(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) + 1))^(1/2)))*(exp(2*c - 2*a
)*(exp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2))/(-b^2)^(1/2) + (exp(a +
b*x)*(exp(2*a - 2*c) + 1))/(b*(exp(2*a - 2*c) - exp(2*a + 2*b*x)))
```

3.160 $\int \cosh(a+bx)\coth^3(c+bx)\,dx$

3.160.1 Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = -\frac{3\text{arctanh}(\cosh(c+bx))\cosh(a-c)}{2b}+\frac{\cosh(a+bx)}{b}-\frac{\cosh(a-c)\coth(c+bx)\text{csch}(c+bx)}{2b}-\frac{\text{csch}(c+bx)\sinh(a-c)}{b}$$

output
```
-3/2*arctanh(cosh(b*x+c))*cosh(a-c)/b+cosh(b*x+a)/b-1/2*cosh(a-c)*coth(b*x
+c)*csch(b*x+c)/b-csch(b*x+c)*sinh(a-c)/b
```

3.160.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = \frac{-12\text{arctanh}\left(\cosh(c)+\sinh(c)\tanh\left(\frac{bx}{2}\right)\right)\cosh(a-c)+(2\cosh(a-2c-bx)-5\cosh(a+bx)+\cosh(a-}{4b}$$

input
```
Integrate[Cosh[a + b*x]*Coth[c + b*x]^3,x]
```

output
```
(-12*ArcTanh[Cosh[c] + Sinh[c]*Tanh[(b*x)/2]]*Cosh[a - c] + (2*Cosh[a - 2*
c - b*x] - 5*Cosh[a + b*x] + Cosh[a + 2*c + 3*b*x])*Csch[c + b*x]^2)/(4*b)
```

3.160.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.133$, Rules used = {6155, 3042, 26, 3091, 26, 3042, 26, 4257, 6156, 3042, 3086, 24, 6155, 3042, 26, 3118, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth^3(bx+c)\,dx$$

$$\Big\downarrow \text{6155}$$

$$\int \coth^2(c+bx)\sinh(a+bx)dx+\cosh(a-c)\int \coth^2(c+bx)\text{csch}(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \coth^2(c+bx)\sinh(a+bx)dx+\cosh(a-c)\int -i\sec\left(ic+ibx-\frac{\pi}{2}\right)\tan\left(ic+ibx-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{26}$$

$$\int \coth^2(c+bx)\sinh(a+bx)dx-i\cosh(a-c)\int \sec\left(\frac{1}{2}(2ic-\pi)+ibx\right)\tan\left(\frac{1}{2}(2ic-\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\int \coth^2(c+bx)\sinh(a+bx)dx-i\cosh(a-c)\left(-\frac{1}{2}\int -i\text{csch}(c+bx)dx-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$\int \coth^2(c+bx)\sinh(a+bx)dx-i\cosh(a-c)\left(\frac{1}{2}i\int \text{csch}(c+bx)dx-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \coth^2(c+bx)\sinh(a+bx)dx-i\cosh(a-c)\left(\frac{1}{2}i\int i\csc(ic+ibx)dx-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$\int \coth^2(c+bx)\sinh(a+bx)dx-i\cosh(a-c)\left(-\frac{1}{2}\int \csc(ic+ibx)dx-\frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{4257}$$

$$\int \coth^2(c+bx)\sinh(a+bx)dx - i\cosh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b} - \frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{6156}$$

$$\int \cosh(a+bx)\coth(c+bx)dx + \sinh(a-c)\int \coth(c+bx)\text{csch}(c+bx)dx - i\cosh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b} - \frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \cosh(a+bx)\coth(c+bx)dx + \sinh(a-c)\int \sec\left(ic+ibx-\frac{\pi}{2}\right)\tan\left(ic+ibx-\frac{\pi}{2}\right)dx - i\cosh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b} - \frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{3086}$$

$$\int \cosh(a+bx)\coth(c+bx)dx - \frac{i\sinh(a-c)\int 1d(-i\text{csch}(c+bx))}{b} - i\cosh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b} - \frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \cosh(a+bx)\coth(c+bx)dx - i\cosh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b} - \frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right) - \frac{\sinh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{6155}$$

$$\cosh(a-c)\int \text{csch}(c+bx)dx + \int \sinh(a+bx)dx - i\cosh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b} - \frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right) - \frac{\sinh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\cosh(a-c)\int i\csc(ic+ibx)dx + \int -i\sin(ia+ibx)dx - i\cosh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b} - \frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right) - \frac{\sinh(a-c)\text{csch}(bx+c)}{b}$$

$$\Big\downarrow \text{26}$$

$$i\cosh(a-c)\int \csc(ic+ibx)dx - i\int \sin(ia+ibx)dx - i\cosh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b} - \frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right) - \frac{\sinh(a-c)\text{csch}(bx+c)}{b}$$

3.160. $\int \cosh(a+bx)\coth^3(c+bx)\,dx$

$$\Big\downarrow \text{3118}$$

$$i\cosh(a-c)\int \csc(ic+ibx)dx - i\cosh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b} - \frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right) - \frac{\sinh(a-c)\text{csch}(bx+c)}{b} + \frac{\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\cosh(a-c)\text{arctanh}(\cosh(bx+c))}{b} - i\cosh(a-c)\left(-\frac{i\text{arctanh}(\cosh(bx+c))}{2b} - \frac{i\coth(bx+c)\text{csch}(bx+c)}{2b}\right) - \frac{\sinh(a-c)\text{csch}(bx+c)}{b} + \frac{\cosh(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[c + b*x]^3,x]
```

output
```
-((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) + Cosh[a + b*x]/b - I*Cosh[a - c
]*(((-1/2*I)*ArcTanh[Cosh[c + b*x]])/b - ((I/2)*Coth[c + b*x]*Csch[c + b*x
])/b) - (Csch[c + b*x]*Sinh[a - c])/b
```

3.160.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6155
```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Simp[Cosh[v - w]   Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

rule 6156
```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Simp[Sinh[v - w]   Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ
[w, v] && FreeQ[v - w, x]
```

3.160.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(69) = 138$.

Time = 0.43 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.12

method	result
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(-3\,e^{2bx+4a+2c}+e^{2bx+2a+4c}+e^{4a}-3\,e^{2a+2c}\right)}{2b\left(-e^{2bx+2a+2c}+e^{2a}\right)^{2}}+\frac{3\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{4b}+\frac{3\ln\left(e^{bx+a}-e^{a-c}\right)e^{-}}{4b}$

input
```
int(cosh(b*x+a)*coth(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(-3*exp(2*b*x+4*a+2*c)+e
xp(2*b*x+2*a+4*c)+exp(4*a)-3*exp(2*a+2*c))/b/(-exp(2*b*x+2*a+2*c)+exp(2*a)
)^2+3/4*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)+3/4*ln(exp(b*x+a)-exp
(a-c))/b*exp(-a-c)*exp(2*c)-3/4*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*
a)-3/4*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)
```

3.160.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2372 vs. $2(69) = 138$.

Time = 0.28 (sec) , antiderivative size = 2372, normalized size of antiderivative = 32.49

$$\int \cosh(a + bx) \coth^3(c + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(2*cosh(b*x + c)^6*cosh(-a + c)^2 + 2*(cosh(-a + c)^2 - 2*cosh(-a + c)
*sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 12*(cosh(b*x + c)*cosh(-
a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(
-a + c)^2)*sinh(b*x + c)^5 - 2*(5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^4 + 2*
(15*cosh(b*x + c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 - 1)*sinh(-a + c
)^2 - 5*cosh(-a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c)
)*sinh(-a + c) + 2)*sinh(b*x + c)^4 + 8*(5*cosh(b*x + c)^3*cosh(-a + c)^2
+ 5*(cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (5*cosh(-a + c)^2 -
 2)*cosh(b*x + c) - 10*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(
-a + c))*sinh(-a + c))*sinh(b*x + c)^3 + 2*(2*cosh(-a + c)^2 - 5)*cosh(b*x
 + c)^2 + 2*(15*cosh(b*x + c)^4*cosh(-a + c)^2 - 6*(5*cosh(-a + c)^2 - 2)*
cosh(b*x + c)^2 + (15*cosh(b*x + c)^4 - 30*cosh(b*x + c)^2 + 2)*sinh(-a +
c)^2 + 2*cosh(-a + c)^2 - 2*(15*cosh(b*x + c)^4*cosh(-a + c) - 30*cosh(b*x
 + c)^2*cosh(-a + c) + 2*cosh(-a + c))*sinh(-a + c) - 5)*sinh(b*x + c)^2 +
 2*(cosh(b*x + c)^6 - 5*cosh(b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^
2 - 3*((cosh(-a + c)^2 + 1)*cosh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a
+ c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x +
c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c
)^2 + 1)*cosh(b*x + c))*sinh(b*x + c)^4 - 2*(cosh(-a + c)^2 + 1)*cosh(b*x
+ c)^3 + 2*(5*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2...
```

3.160.6 Sympy [F]

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = \int \cosh\left(a+bx\right)\coth^3\left(bx+c\right)dx$$

input
```
integrate(cosh(b*x+a)*coth(b*x+c)**3,x)
```

output
```
Integral(cosh(a + b*x)*coth(b*x + c)**3, x)
```

3.160.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(69) = 138.

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.52

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = -\frac{3\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{4b} + \frac{3\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{4b} + \frac{e^{(-bx-a)}}{2b} - \frac{\left(5e^{(2a+2c)}-e^{(4c)}\right)e^{(-2bx-2a)}-\left(2e^{(4a)}-3e^{(2a+2c)}\right)e^{(-4bx-4a)}-e^{(4c)}}{2b\left(e^{(-bx-a+4c)}-2e^{(-3bx-a+2c)}+e^{(-5bx-a)}\right)}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+c)^3,x, algorithm="maxima")
```

output
```
-3/4*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 3/4*(e^(2*a) +
 e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(-b*x - a)/b - 1/2*((5*
e^(2*a + 2*c) - e^(4*c))*e^(-2*b*x - 2*a) - (2*e^(4*a) - 3*e^(2*a + 2*c))*
e^(-4*b*x - 4*a) - e^(4*c))/(b*(e^(-b*x - a + 4*c) - 2*e^(-3*b*x - a + 2*c
) + e^(-5*b*x - a)))
```

3.160.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(69) = 138$.

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.29

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = -\frac{3\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+a+c)}+e^{a}\right)-3\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+a+c)}-e^{a}\right|\right)+\frac{2(3e^{(}}{}}{4b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(3*(e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + a + c) + e^a) -
3*(e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + a + c) - e^a)) + 2
*(3*e^(3*b*x + 5*a + 2*c) - e^(3*b*x + 3*a + 4*c) - e^(b*x + 5*a) + 3*e^(b
*x + 3*a + 2*c))/(e^(2*b*x + 2*a + 2*c) - e^(2*a))^2 - 2*e^(b*x + a) - 2*e
^(-b*x - a))/b
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = \int \cosh(a+b\,x)\,\coth(c+b\,x)^3\,dx$$

input
```
int(cosh(a + b*x)*coth(c + b*x)^3,x)
```

output
```
int(cosh(a + b*x)*coth(c + b*x)^3, x)
```

3.161 $\int \cosh(a+bx)\mathbf{sech}(c+bx)\,dx$

3.161.1 Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \cosh(a+bx)\text{sech}(c+bx)\,dx = x\cosh(a-c)+\frac{\log(\cosh(c+bx))\sinh(a-c)}{b}$$

output
```
x*cosh(a-c)+ln(cosh(b*x+c))*sinh(a-c)/b
```

3.161.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\text{sech}(c+bx)\,dx = x\cosh(a-c)+\frac{\log(\cosh(c+bx))\sinh(a-c)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Sech[c + b*x],x]
```

output
```
x*Cosh[a - c] + (Log[Cosh[c + b*x]]*Sinh[a - c])/b
```

3.161.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6161, 24, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\text{sech}(bx+c)\,dx$$

$$\Big\downarrow \text{ 6161}$$

$$\sinh(a-c)\int \tanh(c+bx)dx+\cosh(a-c)\int 1dx$$

$$\Big\downarrow \text{ 24}$$

$$\sinh(a-c)\int \tanh(c+bx)dx+x\cosh(a-c)$$

$$\Big\downarrow \text{ 3042}$$

$$x\cosh(a-c)+\sinh(a-c)\int -i\tan(ic+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$x\cosh(a-c)-i\sinh(a-c)\int \tan(ic+ibx)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\sinh(a-c)\log(\cosh(bx+c))}{b}+x\cosh(a-c)$$

input
```
Int[Cosh[a + b*x]*Sech[c + b*x],x]
```

output
```
x*Cosh[a - c] + (Log[Cosh[c + b*x]]*Sinh[a - c])/b
```

3.161.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 6161
```
Int[Cosh[v_]*Sech[w_]^(n_.), x_Symbol] :> Simp[Sinh[v - w]   Int[Tanh[w]*Se
ch[w]^(n - 1), x], x] + Simp[Cosh[v - w]   Int[Sech[w]^(n - 1), x], x] /; G
tQ[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

3.161.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(26) = 52$.

Time = 0.43 (sec) , antiderivative size = 146, normalized size of antiderivative = 5.62

method	result
risch	$x\,\mathrm{e}^{a-c}-\mathrm{e}^{-a-c}\mathrm{e}^{2a}x+\mathrm{e}^{-a-c}\mathrm{e}^{2c}x-\frac{\mathrm{e}^{-a-c}\mathrm{e}^{2a}a}{b}+\frac{\mathrm{e}^{-a-c}\mathrm{e}^{2c}a}{b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2c}\right)\mathrm{e}^{-a-c}\mathrm{e}^{2a}}{2b}-\frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2}\right.}{2b}$

input
```
int(cosh(b*x+a)*sech(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
x*exp(a-c)-exp(-a-c)*exp(2*a)*x+exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a
)*a+1/b*exp(-a-c)*exp(2*c)*a+1/2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/b*exp(-a-
c)*exp(2*a)-1/2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/b*exp(-a-c)*exp(2*c)
```

3.161.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.31

$$\int \cosh(a+bx)\operatorname{sech}(c+bx)\,dx = \frac{2\,bx + \left(\cosh\left(-a+c\right)^2 - 2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right) + \sinh\left(-a+c\right)^2 - 1\right)\log\left(\frac{2\,\cosh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right) - b\sinh\left(-a+c\right)\right)}$$

input
```
integrate(cosh(b*x+a)*sech(b*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*b*x + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 - 1)*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a +
c) - b*sinh(-a + c))
```

3.161.6 Sympy [F]

$$\int \cosh(a+bx)\operatorname{sech}(c+bx)\,dx = \int \cosh\left(a+bx\right)\operatorname{sech}\left(bx+c\right)dx$$

input
```
integrate(cosh(b*x+a)*sech(b*x+c),x)
```

output
```
Integral(cosh(a + b*x)*sech(b*x + c), x)
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \cosh(a+bx)\operatorname{sech}(c+bx)\,dx = \frac{\left(e^{(2\,a)} - e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-2\,bx)} + e^{(2\,c)}\right)}{2\,b} + \frac{(bx+a)e^{(a-c)}}{b}$$

input
```
integrate(cosh(b*x+a)*sech(b*x+c),x, algorithm="maxima")
```

output
```
1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-2*b*x) + e^(2*c))/b + (b*x + a)
*e^(a - c)/b
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \cosh(a+bx)\text{sech}(c+bx)\,dx = \frac{2\,bxe^{(-a+c)} + \left(e^{(2\,a+c)} - e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(2\,bx+2\,c)}+1\right)}{2\,b}$$

input
```
integrate(cosh(b*x+a)*sech(b*x+c),x, algorithm="giac")
```

output
```
1/2*(2*b*x*e^(-a + c) + (e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(2*b*x
+ 2*c) + 1))/b
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \cosh(a+bx)\text{sech}(c+bx)\,dx = x\,\mathrm{e}^{c-a} + \frac{\mathrm{e}^{2\,c-2\,a}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,c}\right)\,\left(2\,b\,\mathrm{e}^{3\,a-3\,c}-2\,b\,\mathrm{e}^{a-c}\right)}{4\,b^2}$$

input
```
int(cosh(a + b*x)/cosh(c + b*x),x)
```

output
```
x*exp(c - a) + (exp(2*c - 2*a)*log(exp(2*a)*exp(2*b*x) + exp(2*a)*exp(-2*c
))*(2*b*exp(3*a - 3*c) - 2*b*exp(a - c)))/(4*b^2)
```

3.162 $\int \cosh(a+bx)\mathbf{sech}^2(c+bx)\,dx$

3.162.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \cosh(a+bx)\text{sech}^2(c+bx)\,dx=\frac{\arctan(\sinh(c+bx))\cosh(a-c)}{b}-\frac{\text{sech}(c+bx)\sinh(a-c)}{b}$$

output
```
arctan(sinh(b*x+c))*cosh(a-c)/b-sech(b*x+c)*sinh(a-c)/b
```

3.162.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(35) = 70.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\int \cosh(a+bx)\text{sech}^2(c+bx)\,dx = \frac{2\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh\left(\frac{bx}{2}\right)\sinh(c)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\cosh(a-c)}{b} - \frac{\text{sech}(c+bx)\sinh(a-c)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Sech[c + b*x]^2,x]
```

output
```
(2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)
/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])/b - (S
ech[c + b*x]*Sinh[a - c])/b
```

3.162.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6161, 3042, 26, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\text{sech}^2(bx+c)\,dx$$

$$\Big\downarrow \text{6161}$$

$$\cosh(a-c)\int \text{sech}(c+bx)dx+\sinh(a-c)\int \text{sech}(c+bx)\tanh(c+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx+\sinh(a-c)\int -i\sec(ic+ibx)\tan(ic+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx-i\sinh(a-c)\int \sec(ic+ibx)\tan(ic+ibx)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\sinh(a-c)\int 1d\text{sech}(c+bx)}{b}+\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{24}$$

$$-\frac{\sinh(a-c)\text{sech}(bx+c)}{b}+\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cosh(a-c)\arctan(\sinh(bx+c))}{b}-\frac{\sinh(a-c)\text{sech}(bx+c)}{b}$$

input
```
Int[Cosh[a + b*x]*Sech[c + b*x]^2,x]
```

output
```
(ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b - (Sech[c + b*x]*Sinh[a - c])/b
```

3.162.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6161
```
Int[Cosh[v_]*Sech[w_]^(n_.), x_Symbol] :> Simp[Sinh[v - w]   Int[Tanh[w]*Se
ch[w]^(n - 1), x], x] + Simp[Cosh[v - w]   Int[Sech[w]^(n - 1), x], x] /; G
tQ[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

3.162.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 183, normalized size of antiderivative = 5.23

method	result
risch	$-\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)}+\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{i\ln(e}{}$

input
```
int(cosh(b*x+a)*sech(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(exp(2*b*x+2*a+2*c)+exp(2*a))+1/2*I*ln
(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*I*ln(exp(b*x+a)+I*exp(a-c
))/b*exp(-a-c)*exp(2*c)-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*
a)-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*c)
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(35) = 70$.

Time = 0.25 (sec) , antiderivative size = 405, normalized size of antiderivative = 11.57

$$\int \cosh(a+bx)\operatorname{sech}^2(c+bx)\,dx$$
$$= \frac{2\cosh(bx+c)\cosh(-a+c)\sinh(-a+c) - \cosh(bx+c)\sinh(-a+c)^2 + \left(\left(\cosh(-a+c)^2+1\right)\cosh(\right.}{}$$

input
```
integrate(cosh(b*x+a)*sech(b*x+c)^2,x, algorithm="fricas")
```

output
```
(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2
+ ((cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)
*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 + 1
)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-
a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c)
)*sinh(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a
+ c) + 1)*arctan(cosh(b*x + c) + sinh(b*x + c)) - (cosh(-a + c)^2 - 1)*cos
h(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 - 1)*sinh(b*x + c))/(b*cosh(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) -
b*sinh(-a + c))*sinh(b*x + c)^2 + b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh
(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^
2 + b)*sinh(-a + c))
```

3.162.6 Sympy [F]

$$\int \cosh(a+bx)\text{sech}^2(c+bx)\,dx = \int \cosh(a+bx)\,\text{sech}^2(bx+c)\,dx$$

input
```
integrate(cosh(b*x+a)*sech(b*x+c)**2,x)
```

output
```
Integral(cosh(a + b*x)*sech(b*x + c)**2, x)
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.00

$$\int \cosh(a+bx)\text{sech}^2(c+bx)\,dx = -\frac{\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} - \frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}+e^{(2c)}\right)}$$

input
```
integrate(cosh(b*x+a)*sech(b*x+c)^2,x, algorithm="maxima")
```

output
```
-(e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - (e^(2*a) - e^(2*c
))*e^(-b*x - a)/(b*(e^(-2*b*x) + e^(2*c)))
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int \cosh(a+bx)\text{sech}^2(c+bx)\,dx = \frac{\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{\left(e^{(bx+2a)}-e^{(bx+2c)}\right)e^{(-a)}}{e^{(2bx+2c)}+1}}{b}$$

input
```
integrate(cosh(b*x+a)*sech(b*x+c)^2,x, algorithm="giac")
```

output
```
((e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (e^(b*x + 2*a) - e^(
b*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) + 1))/b
```

3.162.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.23

$$\int \cosh(a+bx)\mathrm{sech}^2(c+bx)\,dx$$
$$= \frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{b^2}+e^{2\,a}\,e^{-2\,c}\,\sqrt{b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,(2\,e^{2\,a}\,e^{-2\,c}+e^{4\,a}\,e^{-4\,c}+1)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(2\,e^{2\,a-2\,c}+e^{4\,a-4\,c}+1\right)}}{\sqrt{b^2}} - \frac{e^{a+b\,x}\,\left(e^{2\,a-2\,c}-1\right)}{b\,\left(e^{2\,a-2\,c}+e^{2\,a+2\,b\,x}\right)}$$

input
```
int(cosh(a + b*x)/cosh(c + b*x)^2,x)
```

output
```
(atan((exp(-a)*exp(2*c)*exp(b*x)*((b^2)^(1/2) + exp(2*a)*exp(-2*c)*(b^2)^(
1/2)))/(b*(exp(-2*a)*exp(2*c)*(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) +
 1))^(1/2)))*(exp(2*c - 2*a)*(2*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2
))/(b^2)^(1/2) - (exp(a + b*x)*(exp(2*a - 2*c) - 1))/(b*(exp(2*a - 2*c) +
exp(2*a + 2*b*x)))
```

3.163 $\int \cosh(a+bx)\mathbf{sech}^3(c+bx)\,dx$

3.163.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cosh(a+bx)\text{sech}^3(c+bx)\,dx = -\frac{\text{sech}^2(c+bx)\sinh(a-c)}{2b} + \frac{\cosh(a-c)\tanh(c+bx)}{b}$$

output
```
-1/2*sech(b*x+c)^2*sinh(a-c)/b+cosh(a-c)*tanh(b*x+c)/b
```

3.163.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cosh(a+bx)\text{sech}^3(c+bx)\,dx = -\frac{\text{sech}(c)\text{sech}^2(c+bx)(\sinh(a)-\cosh(a-c)\sinh(c+2bx))}{2b}$$

input
```
Integrate[Cosh[a + b*x]*Sech[c + b*x]^3,x]
```

output
```
-1/2*(Sech[c]*Sech[c + b*x]^2*(Sinh[a] - Cosh[a - c]*Sinh[c + 2*b*x]))/b
```

3.163.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6161, 3042, 26, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\text{sech}^3(bx+c)\,dx$$

$$\Big\downarrow \text{ 6161}$$

$$\cosh(a-c)\int \text{sech}^2(c+bx)dx+\sinh(a-c)\int \text{sech}^2(c+bx)\tanh(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)^2dx+\sinh(a-c)\int -i\sec(ic+ibx)^2\tan(ic+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)^2dx-i\sinh(a-c)\int \sec(ic+ibx)^2\tan(ic+ibx)dx$$

$$\Big\downarrow \text{ 3086}$$

$$-\frac{\sinh(a-c)\int \text{sech}(c+bx)d\text{sech}(c+bx)}{b}+\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\sinh(a-c)\text{sech}^2(bx+c)}{2b}+\cosh(a-c)\int \csc\left(ic+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{\sinh(a-c)\text{sech}^2(bx+c)}{2b}+\frac{i\cosh(a-c)\int 1d(-i\tanh(c+bx))}{b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\cosh(a-c)\tanh(bx+c)}{b}-\frac{\sinh(a-c)\text{sech}^2(bx+c)}{2b}$$

input
```
Int[Cosh[a + b*x]*Sech[c + b*x]^3,x]
```

output
```
-1/2*(Sech[c + b*x]^2*Sinh[a - c])/b + (Cosh[a - c]*Tanh[c + b*x])/b
```

3.163.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 6161
```
Int[Cosh[v_]*Sech[w_]^(n_.), x_Symbol] :> Simp[Sinh[v - w]   Int[Tanh[w]*Se
ch[w]^(n - 1), x], x] + Simp[Cosh[v - w]   Int[Sech[w]^(n - 1), x], x] /; G
tQ[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

3.163.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{\sinh(2bx+a+c)}{b(1+\cosh(2bx+2c))}$	26
risch	$-\frac{\left(2\,\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}+\mathrm{e}^{2c}\right)\mathrm{e}^{3a-c}}{\left(\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}\right)^{2}b}$	56

input
```
int(cosh(b*x+a)*sech(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*sinh(2*b*x+a+c)/(1+cosh(2*b*x+2*c))
```

3.163.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(36) = 72.

Time = 0.25 (sec) , antiderivative size = 248, normalized size of antiderivative = 6.53

$$\int \cosh(a+bx)\text{sech}^3(c+bx)\,dx =$$

$$-\frac{}{b\cosh\left(bx+c\right)^3\cosh\left(-a+c\right)^2+3\,b\cosh\left(bx+c\right)\cosh\left(-a+c\right)^2+\left(b\cosh\left(-a+c\right)^2-b\sinh\left(-a+c\right)\right.}$$

input
```
integrate(cosh(b*x+a)*sech(b*x+c)^3,x, algorithm="fricas")
```

output
```
-2*(2*cosh(b*x + c)*cosh(-a + c) - cosh(b*x + c)*sinh(-a + c) - sinh(b*x +
 c)*sinh(-a + c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 + 3*b*cosh(b*x + c)*co
sh(-a + c)^2 + (b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(
b*cosh(b*x + c)*cosh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x
+ c)^2 - (b*cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c)^2 + (3*b*cos
h(b*x + c)^2*cosh(-a + c)^2 + b*cosh(-a + c)^2 - (3*b*cosh(b*x + c)^2 + b)
*sinh(-a + c)^2)*sinh(b*x + c))
```

3.163.6 Sympy [F]

$$\int \cosh(a+bx)\text{sech}^3(c+bx)\,dx = \int \cosh\left(a+bx\right)\text{sech}^3\left(bx+c\right)dx$$

input
```
integrate(cosh(b*x+a)*sech(b*x+c)**3,x)
```

output
```
Integral(cosh(a + b*x)*sech(b*x + c)**3, x)
```

3.163.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(36) = 72.

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.13

$$\int \cosh(a+bx)\mathrm{sech}^3(c+bx)\,dx = \frac{2\,e^{(-2\,bx+3\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)})} + \frac{e^{(2\,a+3\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)})} + \frac{e^{(5\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)})}$$

input
```
integrate(cosh(b*x+a)*sech(b*x+c)^3,x, algorithm="maxima")
```

output
```
2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*
c))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a +
4*c))) + e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*c)
))
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \cosh(a+bx)\mathrm{sech}^3(c+bx)\,dx = -\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)}+e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}}{b(e^{(2\,bx+2\,c)}+1)^2}$$

input
```
integrate(cosh(b*x+a)*sech(b*x+c)^3,x, algorithm="giac")
```

output
```
-(2*e^(2*b*x + 2*a + 2*c) + e^(2*a) + e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2
*c) + 1)^2)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\text{sech}^3(c+bx)\,dx = \int \frac{\cosh(a+b\,x)}{\cosh{(c+b\,x)}^3}\,dx$$

input
```
int(cosh(a + b*x)/cosh(c + b*x)^3,x)
```

output
```
int(cosh(a + b*x)/cosh(c + b*x)^3, x)
```

3.164 $\int \cosh(a+bx)\operatorname{csch}(c+bx)\,dx$

3.164.1 Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \cosh(a+bx)\operatorname{csch}(c+bx)\,dx=\frac{\cosh(a-c)\log(\sinh(c+bx))}{b}+x\sinh(a-c)$$

output
```
cosh(a-c)*ln(sinh(b*x+c))/b+x*sinh(a-c)
```

3.164.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\operatorname{csch}(c+bx)\,dx=\frac{\cosh(a-c)\log(\sinh(c+bx))}{b}+x\sinh(a-c)$$

input
```
Integrate[Cosh[a + b*x]*Csch[c + b*x],x]
```

output
```
(Cosh[a - c]*Log[Sinh[c + b*x]])/b + x*Sinh[a - c]
```

3.164.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6159, 24, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\text{csch}(bx+c)\,dx$$

$$\Big\downarrow \text{6159}$$

$$\cosh(a-c)\int \coth(c+bx)dx+\sinh(a-c)\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\cosh(a-c)\int \coth(c+bx)dx+x\sinh(a-c)$$

$$\Big\downarrow \text{3042}$$

$$x\sinh(a-c)+\cosh(a-c)\int -i\tan\left(ic+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$x\sinh(a-c)-i\cosh(a-c)\int \tan\left(\frac{1}{2}(2ic+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$x\sinh(a-c)+\frac{\cosh(a-c)\log(-i\sinh(bx+c))}{b}$$

input
```
Int[Cosh[a + b*x]*Csch[c + b*x],x]
```

output
```
(Cosh[a - c]*Log[(-I)*Sinh[c + b*x]])/b + x*Sinh[a - c]
```

3.164.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 6159
```
Int[Cosh[v_]*Csch[w_]^(n_.), x_Symbol] :> Simp[Cosh[v - w]   Int[Coth[w]*Cs
ch[w]^(n - 1), x], x] + Simp[Sinh[v - w]   Int[Csch[w]^(n - 1), x], x] /; G
tQ[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

3.164.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 5.85

method	result
risch	$x\,\mathrm{e}^{a-c}-\mathrm{e}^{-a-c}\mathrm{e}^{2a}x-\mathrm{e}^{-a-c}\mathrm{e}^{2c}x-\frac{\mathrm{e}^{-a-c}\mathrm{e}^{2a}a}{b}-\frac{\mathrm{e}^{-a-c}\mathrm{e}^{2c}a}{b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2c}\right)\mathrm{e}^{-a-c}\mathrm{e}^{2a}}{2b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2}\right.}{2b}$

input
```
int(cosh(b*x+a)*csch(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
x*exp(a-c)-exp(-a-c)*exp(2*a)*x-exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a
)*a-1/b*exp(-a-c)*exp(2*c)*a+1/2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/b*exp(-a-
c)*exp(2*a)+1/2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/b*exp(-a-c)*exp(2*c)
```

3.164.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int \cosh(a+bx)\mathrm{csch}(c+bx)\,dx = -\frac{2\,bx - \left(\cosh\left(-a+c\right)^{2} - 2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right) + \sinh\left(-a+c\right)^{2} + 1\right)\log\left(\frac{2\,\sinh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right) - b\sinh\left(-a+c\right)\right)}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(2*b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)
^2 + 1)*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a +
 c) - b*sinh(-a + c))
```

3.164.6 Sympy [F]

$$\int \cosh(a+bx)\mathrm{csch}(c+bx)\,dx = \int \cosh\left(a+bx\right)\mathrm{csch}\left(bx+c\right)dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+c),x)
```

output
```
Integral(cosh(a + b*x)*csch(b*x + c), x)
```

3.164.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(26) = 52.

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.08

$$\int \cosh(a+bx)\mathrm{csch}(c+bx)\,dx = \frac{\left(e^{(2\,a)} + e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} + e^{c}\right)}{2\,b} + \frac{\left(e^{(2\,a)} + e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} - e^{c}\right)}{2\,b} + \frac{(bx+a)e^{(a-c)}}{b}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+c),x, algorithm="maxima")
```

output
```
1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (b*x + a)*e^(a - c)/b
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \cosh(a+bx)\text{csch}(c+bx)\,dx = -\frac{2\,bxe^{(-a+c)} - \left(e^{(2\,a+c)} + e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(2\,bx+2\,c)} - 1\right|\right)}{2\,b}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+c),x, algorithm="giac")
```

output
```
-1/2*(2*b*x*e^(-a + c) - (e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(2
*b*x + 2*c) - 1)))/b
```

3.164.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \cosh(a+bx)\text{csch}(c+bx)\,dx = \frac{e^{2\,c-2\,a}\,\ln\left(e^{2\,a}\,e^{2\,b\,x} - e^{2\,a}\,e^{-2\,c}\right)\,\left(2\,b\,e^{3\,a-3\,c} + 2\,b\,e^{a-c}\right)}{4\,b^2} - x\,e^{c-a}$$

input
```
int(cosh(a + b*x)/sinh(c + b*x),x)
```

output
```
(exp(2*c - 2*a)*log(exp(2*a)*exp(2*b*x) - exp(2*a)*exp(-2*c))*(2*b*exp(3*a
 - 3*c) + 2*b*exp(a - c)))/(4*b^2) - x*exp(c - a)
```

3.165 $\int \cosh(a+bx)\mathbf{csch}^2(c+bx)\,dx$

3.165.1 Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \cosh(a+bx)\operatorname{csch}^2(c+bx)\,dx = -\frac{\cosh(a-c)\operatorname{csch}(c+bx)}{b} - \frac{\operatorname{arctanh}(\cosh(c+bx))\sinh(a-c)}{b}$$

output
```
-cosh(a-c)*csch(b*x+c)/b-arctanh(cosh(b*x+c))*sinh(a-c)/b
```

3.165.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\int \cosh(a+bx)\operatorname{csch}^2(c+bx)\,dx = -\frac{\cosh(a-c)\operatorname{csch}(c+bx)}{b} - \frac{2i\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh(c)\cosh\left(\frac{bx}{2}\right)+\sinh(c)\sinh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\sinh(a-c)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Csch[c + b*x]^2,x]
```

output
```
-((Cosh[a - c]*Csch[c + b*x])/b) - ((2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cos
h[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*
Cosh[(b*x)/2]*Sinh[c])]*Sinh[a - c])/b
```

3.165.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6159, 3042, 26, 3086, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\text{csch}^2(bx+c)\,dx$$

$$\Big\downarrow \text{ 6159}$$

$$\sinh(a-c)\int \text{csch}(c+bx)dx+\cosh(a-c)\int \coth(c+bx)\text{csch}(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sinh(a-c)\int i\csc(ic+ibx)dx+\cosh(a-c)\int \sec\left(ic+ibx-\frac{\pi}{2}\right)\tan\left(ic+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 26}$$

$$i\sinh(a-c)\int \csc(ic+ibx)dx+\cosh(a-c)\int \sec\left(ic+ibx-\frac{\pi}{2}\right)\tan\left(ic+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 3086}$$

$$i\sinh(a-c)\int \csc(ic+ibx)dx-\frac{i\cosh(a-c)\int 1d(-i\text{csch}(c+bx))}{b}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}+i\sinh(a-c)\int \csc(ic+ibx)dx$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{\sinh(a-c)\text{arctanh}(\cosh(bx+c))}{b}-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}$$

input
```
Int[Cosh[a + b*x]*Csch[c + b*x]^2,x]
```

output
```
-((Cosh[a - c]*Csch[c + b*x])/b) - (ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b
```

3.165.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 6159
```
Int[Cosh[v_]*Csch[w_]^(n_.), x_Symbol] :> Simp[Cosh[v - w]   Int[Coth[w]*Cs
ch[w]^(n - 1), x], x] + Simp[Sinh[v - w]   Int[Csch[w]^(n - 1), x], x] /; G
tQ[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

3.165.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(36) = 72$.

Time = 0.44 (sec) , antiderivative size = 170, normalized size of antiderivative = 4.72

method	result
risch	$\frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}+e^{\cdots}\right.}{}$

input
```
int(cosh(b*x+a)*csch(b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(-exp(2*b*x+2*a+2*c)+exp(2*a))+1/2*ln(e
xp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*ln(exp(b*x+a)-exp(a-c))/b*exp
(-a-c)*exp(2*c)-1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*ln(ex
p(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)
```

3.165.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(36) = 72$.

Time = 0.25 (sec) , antiderivative size = 617, normalized size of antiderivative = 17.14

$$\int \cosh(a+bx)\operatorname{csch}^2(c+bx)\,dx$$
$$= \frac{4\cosh(bx+c)\cosh(-a+c)\sinh(-a+c) - 2\cosh(bx+c)\sinh(-a+c)^2 - 2\left(\cosh(-a+c)^2+1\right)\cos}{}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(4*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - 2*cosh(b*x + c)*sinh(-a +
 c)^2 - 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c) - ((cosh(-a + c)^2 - 1)*cosh(
b*x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 - 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a +
c)^2 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-
a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x
 + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)*log(cosh(b*x + c) +
 sinh(b*x + c) + 1) + ((cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (cosh(-a + c
)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^2 +
(cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(2*cosh(b*x + c)
*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^
2 - 1)*cosh(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) - co
sh(-a + c))*sinh(-a + c) + 1)*log(cosh(b*x + c) + sinh(b*x + c) - 1) - 2*(
cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*
x + c))/(b*cosh(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c)
)*sinh(b*x + c)^2 - b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) - b*c
osh(b*x + c)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 - b)*sinh(-a
 + c))
```

3.165.6 Sympy [F]

$$\int \cosh(a+bx)\text{csch}^2(c+bx)\,dx = \int \cosh\left(a+bx\right)\text{csch}^2\left(bx+c\right)dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+c)**2,x)
```

output
```
Integral(cosh(a + b*x)*csch(b*x + c)**2, x)
```

3.165.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(36) = 72$.

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.92

$$\begin{aligned}\int \cosh(a+bx)\text{csch}^2(c+bx)\,dx = &-\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b} \\ &+\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b} \\ &+\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}-e^{(2c)}\right)}\end{aligned}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) -
 e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (e^(2*a) + e^(2*c))*e^(-b*x -
 a)/(b*(e^(-2*b*x) - e^(2*c)))
```

3.165.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.94

$$\int \cosh(a+bx)\text{csch}^2(c+bx)\,dx =$$

$$-\frac{\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{2\left(e^{(bx+2a)}+e^{(bx+}\right.}{e^{(2bx+2c)}-}}{2b}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*((e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a
+ c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + 2*(e^(b*x + 2*a)
+ e^(b*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) - 1))/b
```

3.165.9 Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.33

$$\int \cosh(a+bx)\operatorname{csch}^2(c+bx)\,dx$$
$$=\frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{-b^2}-e^{2\,a}\,e^{-2\,c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(e^{4\,a}\,e^{-4\,c}-2\,e^{2\,a}\,e^{-2\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(e^{4\,a-4\,c}-2\,e^{2\,a-2\,c}+1\right)}}{\sqrt{-b^2}}$$
$$+\frac{e^{a+b\,x}\,\left(e^{2\,a-2\,c}+1\right)}{b\,\left(e^{2\,a-2\,c}-e^{2\,a+2\,b\,x}\right)}$$

input
```
int(cosh(a + b*x)/sinh(c + b*x)^2,x)
```

output
```
(atan((exp(-a)*exp(2*c)*exp(b*x)*((-b^2)^(1/2) - exp(2*a)*exp(-2*c)*(-b^2)
^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c)
 + 1))^(1/2)))*(exp(2*c - 2*a)*(exp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1
/2))/(-b^2)^(1/2) + (exp(a + b*x)*(exp(2*a - 2*c) + 1))/(b*(exp(2*a - 2*c)
 - exp(2*a + 2*b*x)))
```

3.166 $\int \cosh(a+bx)\operatorname{csch}^3(c+bx)\,dx$

3.166.1 Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \cosh(a+bx)\operatorname{csch}^3(c+bx)\,dx = -\frac{\cosh(a-c)\operatorname{csch}^2(c+bx)}{2b} - \frac{\coth(c+bx)\sinh(a-c)}{b}$$

output
```
-1/2*cosh(a-c)*csch(b*x+c)^2/b-coth(b*x+c)*sinh(a-c)/b
```

3.166.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \cosh(a+bx)\operatorname{csch}^3(c+bx)\,dx = -\frac{\operatorname{csch}(c)\operatorname{csch}^2(c+bx)(\sinh(a)-\cosh(c+2bx)\sinh(a-c))}{2b}$$

input
```
Integrate[Cosh[a + b*x]*Csch[c + b*x]^3,x]
```

output
```
-1/2*(Csch[c]*Csch[c + b*x]^2*(Sinh[a] - Cosh[c + 2*b*x]*Sinh[a - c]))/b
```

3.166.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6159, 3042, 25, 26, 3086, 15, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\text{csch}^3(bx+c)\,dx$$

$$\Big\downarrow \text{ 6159}$$

$$\sinh(a-c)\int \text{csch}^2(c+bx)dx+\cosh(a-c)\int \coth(c+bx)\text{csch}^2(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sinh(a-c)\int -\csc(ic+ibx)^2dx+\cosh(a-c)\int i\sec\left(ic+ibx-\frac{\pi}{2}\right)^2\tan\left(ic+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$\cosh(a-c)\int i\sec\left(ic+ibx-\frac{\pi}{2}\right)^2\tan\left(ic+ibx-\frac{\pi}{2}\right)dx-\sinh(a-c)\int \csc(ic+ibx)^2dx$$

$$\Big\downarrow \text{ 26}$$

$$i\cosh(a-c)\int \sec\left(\frac{1}{2}(2ic-\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ic-\pi)+ibx\right)dx-\sinh(a-c)\int \csc(ic+ibx)^2dx$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{\cosh(a-c)\int -i\text{csch}(c+bx)d(-i\text{csch}(c+bx))}{b}-\sinh(a-c)\int \csc(ic+ibx)^2dx$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\cosh(a-c)\text{csch}^2(bx+c)}{2b}-\sinh(a-c)\int \csc(ic+ibx)^2dx$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{\cosh(a-c)\text{csch}^2(bx+c)}{2b}-\frac{i\sinh(a-c)\int 1d(-i\coth(c+bx))}{b}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\cosh(a-c)\text{csch}^2(bx+c)}{2b}-\frac{\sinh(a-c)\coth(bx+c)}{b}$$

input
```
Int[Cosh[a + b*x]*Csch[c + b*x]^3,x]
```

output
```
-1/2*(Cosh[a - c]*Csch[c + b*x]^2)/b - (Coth[c + b*x]*Sinh[a - c])/b
```

3.166.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 6159
```
Int[Cosh[v_]*Csch[w_]^(n_.), x_Symbol] :> Simp[Cosh[v - w]   Int[Coth[w]*Cs
ch[w]^(n - 1), x], x] + Simp[Sinh[v - w]   Int[Csch[w]^(n - 1), x], x] /; G
tQ[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

3.166.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$-\frac{\operatorname{sech}\left(\frac{bx}{2}+\frac{c}{2}\right)^{2} \operatorname{csch}\left(\frac{bx}{2}+\frac{c}{2}\right)^{2} \cosh(2bx+a+c)}{8b}$	36
risch	$\frac{\left(-2\,\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)\mathrm{e}^{3a-c}}{\left(-\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}\right)^{2} b}$	59

input
```
int(cosh(b*x+a)*csch(b*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/b*sech(1/2*b*x+1/2*c)^2*csch(1/2*b*x+1/2*c)^2*cosh(2*b*x+a+c)
```

3.166.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(37) = 74.

Time = 0.24 (sec) , antiderivative size = 243, normalized size of antiderivative = 6.23

$$\int \cosh(a+bx)\operatorname{csch}^3(c+bx)\,dx =$$

$$-\frac{}{b\cosh\left(bx+c\right)^{3}\cosh\left(-a+c\right)^{2}-b\cosh\left(bx+c\right)\cosh\left(-a+c\right)^{2}+\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+c)^3,x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + c)*cosh(-a + c) + (cosh(-a + c) - 2*sinh(-a + c))*sinh(b*x
+ c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 - b*cosh(b*x + c)*cosh(-a + c)^2 +
 (b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*cosh(b*x + c
)*cosh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)^2 - (b*co
sh(b*x + c)^3 - b*cosh(b*x + c))*sinh(-a + c)^2 + 3*(b*cosh(b*x + c)^2*cos
h(-a + c)^2 - b*cosh(-a + c)^2 - (b*cosh(b*x + c)^2 - b)*sinh(-a + c)^2)*s
inh(b*x + c))
```

3.166.6 Sympy [F]

$$\int \cosh(a+bx)\mathrm{csch}^3(c+bx)\,dx = \int \cosh(a+bx)\,\mathrm{csch}^3(bx+c)\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+c)**3,x)
```

output
```
Integral(cosh(a + b*x)*csch(b*x + c)**3, x)
```

3.166.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(37) = 74.

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.38

$$\int \cosh(a+bx)\mathrm{csch}^3(c+bx)\,dx = \frac{2\,e^{(-2\,bx+3\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)} - e^{(-4\,bx+a)} - e^{(a+4\,c)})} + \frac{e^{(2\,a+3\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)} - e^{(-4\,bx+a)} - e^{(a+4\,c)})} - \frac{e^{(5\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)} - e^{(-4\,bx+a)} - e^{(a+4\,c)})}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+c)^3,x, algorithm="maxima")
```

output
```
2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*
c))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a +
4*c))) - e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*c)
))
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \cosh(a+bx)\mathrm{csch}^3(c+bx)\,dx = -\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)} - e^{(2\,a)} + e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)} - 1\right)^2}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+c)^3,x, algorithm="giac")
```

output
```
-(2*e^(2*b*x + 2*a + 2*c) - e^(2*a) + e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2
*c) - 1)^2)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\operatorname{csch}^3(c+bx)\,dx = \int \frac{\cosh(a+b\,x)}{\sinh{(c+b\,x)}^3}\,dx$$

input
```
int(cosh(a + b*x)/sinh(c + b*x)^3,x)
```

output
```
int(cosh(a + b*x)/sinh(c + b*x)^3, x)
```

3.167 $\int \sinh(a+bx)\sinh(c+dx)\,dx$

3.167.1 Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = -\frac{\sinh(a-c+(b-d)x)}{2(b-d)} + \frac{\sinh(a+c+(b+d)x)}{2(b+d)}$$

output
```
-1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

3.167.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = -\frac{\sinh(a-c+(b-d)x)}{2(b-d)} + \frac{\sinh(a+c+(b+d)x)}{2(b+d)}$$

input
```
Integrate[Sinh[a + b*x]*Sinh[c + d*x],x]
```

output
```
-1/2*Sinh[a - c + (b - d)*x]/(b - d) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

3.167.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\sinh(c+dx)\,dx$$

$$\Big\downarrow \text{6147}$$

$$\int \left(\frac{1}{2}\cosh(a+x(b+d)+c)-\frac{1}{2}\cosh(a+x(b-d)-c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(a+x(b+d)+c)}{2(b+d)}-\frac{\sinh(a+x(b-d)-c)}{2(b-d)}$$

input
```
Int[Sinh[a + b*x]*Sinh[c + d*x],x]
```

output
```
-1/2*Sinh[a - c + (b - d)*x]/(b - d) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

3.167.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6147
```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.167.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}$	40
parallelrisch	$\frac{(-b-d)\sinh(a-c+(b-d)x)+\sinh(a+c+(b+d)x)(b-d)}{2b^2-2d^2}$	52
risch	$\frac{\left(b\,\mathrm{e}^{2bx+2a}-\mathrm{e}^{2bx+2a}d+b+d\right)\mathrm{e}^{-bx+dx-a+c}}{4(b+d)(b-d)}-\frac{\left(b\,\mathrm{e}^{2bx+2a}+\mathrm{e}^{2bx+2a}d+b-d\right)\mathrm{e}^{-bx-dx-a-c}}{4(b+d)(b-d)}$	112

input
```
int(sinh(b*x+a)*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = -\frac{d\cosh(dx+c)\sinh(bx+a)-b\cosh(bx+a)\sinh(dx+c)}{(b^2-d^2)\cosh(bx+a)^2-(b^2-d^2)\sinh(bx+a)^2}$$

input
```
integrate(sinh(b*x+a)*sinh(d*x+c),x, algorithm="fricas")
```

output
```
-(d*cosh(d*x + c)*sinh(b*x + a) - b*cosh(b*x + a)*sinh(d*x + c))/((b^2 - d
^2)*cosh(b*x + a)^2 - (b^2 - d^2)*sinh(b*x + a)^2)
```

3.167.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(32) = 64$.

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.56

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = \begin{cases} x\sinh(a)\sinh(c) & \text{for } b=0 \wedge d=0 \\ \frac{x\sinh(a-dx)\sinh(c+dx)}{2}+\frac{x\cosh(a-dx)\cosh(c+dx)}{2}+\frac{\sinh(a-dx)\cosh(c+dx)}{2d} & \text{for } b=-d \\ \frac{x\sinh(a+dx)\sinh(c+dx)}{2}-\frac{x\cosh(a+dx)\cosh(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(a+dx)}{2d} & \text{for } b=d \\ \frac{b\sinh(c+dx)\cosh(a+bx)}{b^2-d^2}-\frac{d\sinh(a+bx)\cosh(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(b*x+a)*sinh(d*x+c),x)
```

output
```
Piecewise((x*sinh(a)*sinh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x)*sinh(
c + d*x)/2 + x*cosh(a - d*x)*cosh(c + d*x)/2 + sinh(a - d*x)*cosh(c + d*x)
/(2*d), Eq(b, -d)), (x*sinh(a + d*x)*sinh(c + d*x)/2 - x*cosh(a + d*x)*cos
h(c + d*x)/2 + sinh(c + d*x)*cosh(a + d*x)/(2*d), Eq(b, d)), (b*sinh(c + d
*x)*cosh(a + b*x)/(b**2 - d**2) - d*sinh(a + b*x)*cosh(c + d*x)/(b**2 - d*
*2), True))
```

3.167.7 Maxima [F(-2)]

Exception generated.

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(b*x+a)*sinh(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more
 details)I
```

3.167.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(39) = 78$.

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = \frac{e^{(bx+dx+a+c)}}{4\,(b+d)} - \frac{e^{(bx-dx+a-c)}}{4\,(b-d)} + \frac{e^{(-bx+dx-a+c)}}{4\,(b-d)} - \frac{e^{(-bx-dx-a-c)}}{4\,(b+d)}$$

input
```
integrate(sinh(b*x+a)*sinh(d*x+c),x, algorithm="giac")
```

output
```
1/4*e^(b*x + d*x + a + c)/(b + d) - 1/4*e^(b*x - d*x + a - c)/(b - d) + 1/
4*e^(-b*x + d*x - a + c)/(b - d) - 1/4*e^(-b*x - d*x - a - c)/(b + d)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = \frac{b\cosh(a+b\,x)\,\sinh(c+d\,x)-d\cosh(c+d\,x)\,\sinh(a+b\,x)}{b^2-d^2}$$

input
```
int(sinh(a + b*x)*sinh(c + d*x),x)
```

output
```
(b*cosh(a + b*x)*sinh(c + d*x) - d*cosh(c + d*x)*sinh(a + b*x))/(b^2 - d^2
)
```

3.168 $\int \sinh(a+bx)\sinh^2(c+dx)\,dx$

3.168.1 Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = -\frac{\cosh(a+bx)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cosh(a+2c+(b+2d)x)}{4(b+2d)}$$

output
```
-1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d
)*x)/(b+2*d)
```

3.168.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = \frac{1}{4}\left(-\frac{2\cosh(a)\cosh(bx)}{b}+\frac{\cosh(a-2c+bx-2dx)}{b-2d}+\frac{\cosh(a+2c+bx+2dx)}{b+2d}-\frac{2\sinh(a)\sinh(bx)}{b}\right)$$

input
```
Integrate[Sinh[a + b*x]*Sinh[c + d*x]^2,x]
```

output
```
((-2*Cosh[a]*Cosh[b*x])/b + Cosh[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cosh[a
 + 2*c + b*x + 2*d*x]/(b + 2*d) - (2*Sinh[a]*Sinh[b*x])/b)/4
```

3.168.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx$$

$$\Big\downarrow \text{6147}$$

$$\int \left(\frac{1}{4}\sinh(a+x(b-2d)-2c)+\frac{1}{4}\sinh(a+x(b+2d)+2c)-\frac{1}{2}\sinh(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cosh(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cosh(a+bx)}{2b}$$

input
```
Int[Sinh[a + b*x]*Sinh[c + d*x]^2,x]
```

output
```
-1/2*Cosh[a + b*x]/b + Cosh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cosh[a
+ 2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

3.168.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6147
```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.168.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4b-8d}+\frac{\cosh(a+2c+(b+2d)x)}{4b+8d}$
parallelrisch	$\frac{b(b+2d)\cosh(a-2c+(b-2d)x)+b(b-2d)\cosh(a+2c+(b+2d)x)+(-2b^2+8d^2)\cosh(bx+a)-8d^2}{4b^3-16b\,d^2}$
risch	$-\frac{\mathrm{e}^{bx+a}}{4b}-\frac{\mathrm{e}^{-bx-a}}{4b}+\frac{\left(b\,\mathrm{e}^{2bx+2a}-2\,\mathrm{e}^{2bx+2a}d+b+2d\right)\mathrm{e}^{-bx+2dx-a+2c}}{8(b+2d)(b-2d)}+\frac{\left(b\,\mathrm{e}^{2bx+2a}+2\,\mathrm{e}^{2bx+2a}d+b-2d\right)\mathrm{e}^{-bx-2dx-a-2c}}{8(b+2d)(b-2d)}$

input
```
int(sinh(b*x+a)*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d
)*x)/(b+2*d)
```

3.168.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(56) = 112.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.94

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = \frac{b^2\cosh(bx+a)\cosh(dx+c)^2-4\,bd\cosh(dx+c)\sinh(bx+a)\sinh(dx+c)+b^2\cosh(bx+a)\sinh(dx+c)^2-(b^2-4\,d^2)\cosh(bx+a)}{2\left((b^3-4\,bd^2)\cosh(bx+a)^2-(b^3-4\,bd^2)\sinh(bx+a)^2\right)}$$

input
```
integrate(sinh(b*x+a)*sinh(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 - 4*b*d*cosh(d*x + c)*sinh(b*x + a)
*sinh(d*x + c) + b^2*cosh(b*x + a)*sinh(d*x + c)^2 - (b^2 - 4*d^2)*cosh(b*
x + a))/((b^3 - 4*b*d^2)*cosh(b*x + a)^2 - (b^3 - 4*b*d^2)*sinh(b*x + a)^2
)
```

3.168.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(49) = 98$.

Time = 0.72 (sec) , antiderivative size = 405, normalized size of antiderivative = 6.53

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx$$
$$= \begin{cases} x\sinh(a)\sinh^2(c) \\ \left(\frac{x\sinh^2(c+dx)}{2} - \frac{x\cosh^2(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)\sinh(a) \\ \frac{x\sinh(a-2dx)\sinh^2(c+dx)}{4} + \frac{x\sinh(a-2dx)\cosh^2(c+dx)}{4} + \frac{x\sinh(c+dx)\cosh(a-2dx)\cosh(c+dx)}{2} - \frac{\sinh(a-2dx)\sinh(c+dx)\cosh(}{4d} \\ \frac{x\sinh(a+2dx)\sinh^2(c+dx)}{4} + \frac{x\sinh(a+2dx)\cosh^2(c+dx)}{4} - \frac{x\sinh(c+dx)\cosh(a+2dx)\cosh(c+dx)}{2} - \frac{\sinh(a+2dx)\sinh(c+dx)\cosh(}{4d} \\ \frac{b^2\sinh^2(c+dx)\cosh(a+bx)}{b^3-4bd^2} - \frac{2bd\sinh(a+bx)\sinh(c+dx)\cosh(c+dx)}{b^3-4bd^2} - \frac{2d^2\sinh^2(c+dx)\cosh(a+bx)}{b^3-4bd^2} + \frac{2d^2\cosh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

input
```
integrate(sinh(b*x+a)*sinh(d*x+c)**2,x)
```

output
```
Piecewise((x*sinh(a)*sinh(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sinh(c + d*x)**
2/2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a), E
q(b, 0)), (x*sinh(a - 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a - 2*d*x)*cosh(c
 + d*x)**2/4 + x*sinh(c + d*x)*cosh(a - 2*d*x)*cosh(c + d*x)/2 - sinh(a -
2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) - sinh(c + d*x)**2*cosh(a - 2*d*x
)/(2*d), Eq(b, -2*d)), (x*sinh(a + 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a +
2*d*x)*cosh(c + d*x)**2/4 - x*sinh(c + d*x)*cosh(a + 2*d*x)*cosh(c + d*x)/
2 - sinh(a + 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) + sinh(c + d*x)**2*c
osh(a + 2*d*x)/(2*d), Eq(b, 2*d)), (b**2*sinh(c + d*x)**2*cosh(a + b*x)/(b
**3 - 4*b*d**2) - 2*b*d*sinh(a + b*x)*sinh(c + d*x)*cosh(c + d*x)/(b**3 -
4*b*d**2) - 2*d**2*sinh(c + d*x)**2*cosh(a + b*x)/(b**3 - 4*b*d**2) + 2*d*
*2*cosh(a + b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2), True))
```

3.168.7 Maxima [F(-2)]

Exception generated.

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(b*x+a)*sinh(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for
more detai
```

3.168.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(56) = 112$.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.94

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = \frac{e^{(bx+2\,dx+a+2\,c)}}{8\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{8\,(b-2\,d)} - \frac{e^{(bx+a)}}{4\,b} + \frac{e^{(-bx+2\,dx-a+2\,c)}}{8\,(b-2\,d)} + \frac{e^{(-bx-2\,dx-a-2\,c)}}{8\,(b+2\,d)} - \frac{e^{(-bx-a)}}{4\,b}$$

input
```
integrate(sinh(b*x+a)*sinh(d*x+c)^2,x, algorithm="giac")
```

output
```
1/8*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/8*e^(b*x - 2*d*x + a - 2*c)/(b
 - 2*d) - 1/4*e^(b*x + a)/b + 1/8*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) + 1
/8*e^(-b*x - 2*d*x - a - 2*c)/(b + 2*d) - 1/4*e^(-b*x - a)/b
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx$$
$$= \frac{b^2\left(\cosh(a+b\,x) - \cosh(a+b\,x)\,\cosh(c+d\,x)^2\right) - 2\,d^2\cosh(a+b\,x) + 2\,b\,d\cosh(c+d\,x)\,\sinh(a+b\,x)\,\mathrm{s}}{4\,b\,d^2 - b^3}$$

input
```
int(sinh(a + b*x)*sinh(c + d*x)^2,x)
```

output
```
(b^2*(cosh(a + b*x) - cosh(a + b*x)*cosh(c + d*x)^2) - 2*d^2*cosh(a + b*x)
 + 2*b*d*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x))/(4*b*d^2 - b^3)
```

3.169 $\int \sinh(a+bx)\sinh^3(c+dx)\,dx$

3.169.1 Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = -\frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}-\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8(b+3d)}$$

output
```
-1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)-3/8*sinh(a+
c+(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.169.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = \frac{1}{8}\left(-\frac{\sinh(a-3c+bx-3dx)}{b-3d}+\frac{3\sinh(a-c+bx-dx)}{b-d}+\frac{\sinh(a+3c+bx+3dx)}{b+3d}-\frac{3\sinh(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Sinh[a + b*x]*Sinh[c + d*x]^3,x]
```

output
```
(-(Sinh[a - 3*c + b*x - 3*d*x]/(b - 3*d)) + (3*Sinh[a - c + b*x - d*x])/(b
 - d) + Sinh[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Sinh[a + c + (b + d)*x]
)/(b + d))/8
```

3.169.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx$$

$$\Big\downarrow \text{6147}$$

$$\int \left(-\frac{1}{8}\cosh(a+x(b-3d)-3c)+\frac{3}{8}\cosh(a+x(b-d)-c)-\frac{3}{8}\cosh(a+x(b+d)+c)+\frac{1}{8}\cosh(a+x(b+3d)-$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sinh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sinh(a+x(b-d)-c)}{8(b-d)}-\frac{3\sinh(a+x(b+d)+c)}{8(b+d)}+\frac{\sinh(a+x(b+3d)+3c)}{8(b+3d)}$$

input
```
Int[Sinh[a + b*x]*Sinh[c + d*x]^3,x]
```

output
```
-1/8*Sinh[a - 3*c + (b - 3*d)*x]/(b - 3*d) + (3*Sinh[a - c + (b - d)*x])/(
8*(b - d)) - (3*Sinh[a + c + (b + d)*x])/(8*(b + d)) + Sinh[a + 3*c + (b +
 3*d)*x]/(8*(b + 3*d))
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6147
```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.169.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}-\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}$
risch	$\frac{(b\,e^{2bx+2a}-3\,e^{2bx+2a}d+b+3d)e^{-bx+3dx-a+3c}}{16(b+3d)(b-3d)}-\frac{3(b\,e^{2bx+2a}-e^{2bx+2a}d+b+d)e^{-bx+dx-a+c}}{16(b+d)(b-d)}+\frac{3(b\,e^{2bx+2a}+e^{2bx+2a}d+b}{16(b+d)(b-}$
parallelrisch	$\frac{-12d^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+12d^2b\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+(-24b^2d+36d^3)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{(b-d)(b+3d)(b-3d)(b+d)\left(\tanh\right.}$

input
```
int(sinh(b*x+a)*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)-3/8*sinh(a+
c+(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.169.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(83) = 166.

Time = 0.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.40

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx =$$

$$-\frac{9\,(b^2d-d^3)\cosh(dx+c)\sinh(bx+a)\sinh(dx+c)^2-(b^3-bd^2)\cosh(bx+a)\sinh(dx+c)^3+3\,((b^2}{4\,((b^4-10\,b^2d}$$

input
```
integrate(sinh(b*x+a)*sinh(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(9*(b^2*d - d^3)*cosh(d*x + c)*sinh(b*x + a)*sinh(d*x + c)^2 - (b^3 -
 b*d^2)*cosh(b*x + a)*sinh(d*x + c)^3 + 3*((b^2*d - d^3)*cosh(d*x + c)^3 -
 (b^2*d - 9*d^3)*cosh(d*x + c))*sinh(b*x + a) - 3*((b^3 - b*d^2)*cosh(b*x
+ a)*cosh(d*x + c)^2 - (b^3 - 9*b*d^2)*cosh(b*x + a))*sinh(d*x + c))/((b^4
 - 10*b^2*d^2 + 9*d^4)*cosh(b*x + a)^2 - (b^4 - 10*b^2*d^2 + 9*d^4)*sinh(b
*x + a)^2)
```

3.169.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(76) = 152$.

Time = 1.91 (sec) , antiderivative size = 918, normalized size of antiderivative = 10.09

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)*sinh(d*x+c)**3,x)
```

output
```
Piecewise((x*sinh(a)*sinh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sinh(a - 3*d*x)*
sinh(c + d*x)**3/8 + 3*x*sinh(a - 3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8
+ 3*x*sinh(c + d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)/8 + x*cosh(a - 3*d*x)
*cosh(c + d*x)**3/8 - sinh(a - 3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d)
 + sinh(a - 3*d*x)*cosh(c + d*x)**3/(24*d) - 3*sinh(c + d*x)**3*cosh(a - 3
*d*x)/(8*d), Eq(b, -3*d)), (3*x*sinh(a - d*x)*sinh(c + d*x)**3/8 - 3*x*sin
h(a - d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + 3*x*sinh(c + d*x)**2*cosh(a
- d*x)*cosh(c + d*x)/8 - 3*x*cosh(a - d*x)*cosh(c + d*x)**3/8 + 3*sinh(a -
 d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) - 3*sinh(a - d*x)*cosh(c + d*x)
**3/(8*d) + sinh(c + d*x)**3*cosh(a - d*x)/(8*d), Eq(b, -d)), (3*x*sinh(a
+ d*x)*sinh(c + d*x)**3/8 - 3*x*sinh(a + d*x)*sinh(c + d*x)*cosh(c + d*x)*
*2/8 - 3*x*sinh(c + d*x)**2*cosh(a + d*x)*cosh(c + d*x)/8 + 3*x*cosh(a + d
*x)*cosh(c + d*x)**3/8 + 3*sinh(a + d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4
*d) - 3*sinh(a + d*x)*cosh(c + d*x)**3/(8*d) - sinh(c + d*x)**3*cosh(a + d
*x)/(8*d), Eq(b, d)), (x*sinh(a + 3*d*x)*sinh(c + d*x)**3/8 + 3*x*sinh(a +
 3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(a + 3
*d*x)*cosh(c + d*x)/8 - x*cosh(a + 3*d*x)*cosh(c + d*x)**3/8 - sinh(a + 3*
d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + sinh(a + 3*d*x)*cosh(c + d*x)*
*3/(24*d) + 3*sinh(c + d*x)**3*cosh(a + 3*d*x)/(8*d), Eq(b, 3*d)), (b**3*s
inh(c + d*x)**3*cosh(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d...
```

3.169.7 Maxima [F(-2)]

Exception generated.

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(b*x+a)*sinh(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for
more detai
```

3.169.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(83) = 166$.

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.97

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = \frac{e^{(bx+3\,dx+a+3\,c)}}{16\,(b+3\,d)} - \frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)} + \frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)} - \frac{e^{(bx-3\,dx+a-3\,c)}}{16\,(b-3\,d)} + \frac{e^{(-bx+3\,dx-a+3\,c)}}{16\,(b-3\,d)} - \frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)} + \frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)} - \frac{e^{(-bx-3\,dx-a-3\,c)}}{16\,(b+3\,d)}$$

input
```
integrate(sinh(b*x+a)*sinh(d*x+c)^3,x, algorithm="giac")
```

output
```
1/16*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/16*e^(b*x + d*x + a + c)/(b +
 d) + 3/16*e^(b*x - d*x + a - c)/(b - d) - 1/16*e^(b*x - 3*d*x + a - 3*c)/
(b - 3*d) + 1/16*e^(-b*x + 3*d*x - a + 3*c)/(b - 3*d) - 3/16*e^(-b*x + d*x
 - a + c)/(b - d) + 3/16*e^(-b*x - d*x - a - c)/(b + d) - 1/16*e^(-b*x - 3
*d*x - a - 3*c)/(b + 3*d)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.00

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = \frac{6\,b\,d^2\cosh(a+b\,x)\,\cosh(c+d\,x)^2\sinh(c+d\,x)}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{6\,d^3\cosh(c+d\,x)^3\sinh(a+b\,x)}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{3\,d\cosh(c+d\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)^2\,(b^2-3\,d^2)}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{\cosh(a+b\,x)\,\sinh(c+d\,x)^3\,(7\,b\,d^2-b^3)}{b^4-10\,b^2\,d^2+9\,d^4}$$

input
```
int(sinh(a + b*x)*sinh(c + d*x)^3,x)
```

output
```
(6*b*d^2*cosh(a + b*x)*cosh(c + d*x)^2*sinh(c + d*x))/(b^4 + 9*d^4 - 10*b^
2*d^2) - (6*d^3*cosh(c + d*x)^3*sinh(a + b*x))/(b^4 + 9*d^4 - 10*b^2*d^2)
- (3*d*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x)^2*(b^2 - 3*d^2))/(b^4 + 9
*d^4 - 10*b^2*d^2) - (cosh(a + b*x)*sinh(c + d*x)^3*(7*b*d^2 - b^3))/(b^4
+ 9*d^4 - 10*b^2*d^2)
```

3.169. $\int \sinh(a+bx)\sinh^3(c+dx)\,dx$

3.170 $\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx$

3.170.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx = \frac{x}{4} - \frac{\sinh(2a+2bx)}{8b} + \frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)} - \frac{\sinh(2c+2dx)}{8d} + \frac{\sinh(2(a+c)+2(b+d)x)}{16(b+d)}$$

output
```
1/4*x-1/8*sinh(2*b*x+2*a)/b+1/16*sinh(2*a-2*c+2*(b-d)*x)/(b-d)-1/8*sinh(2*
d*x+2*c)/d+1/16*sinh(2*a+2*c+2*(b+d)*x)/(b+d)
```

3.170.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx$$
$$= \frac{(-2b^2d+2d^3)\sinh(2(a+bx)) + bd(b+d)\sinh(2(a-c+(b-d)x)) + b(b-d)(-2(b+d)\sinh(2(c+dx)}{16b(b-d)d(b+d)}$$

input
```
Integrate[Sinh[a + b*x]^2*Sinh[c + d*x]^2,x]
```

output
```
((-2*b^2*d + 2*d^3)*Sinh[2*(a + b*x)] + b*d*(b + d)*Sinh[2*(a - c + (b - d
)*x)] + b*(b - d)*(-2*(b + d)*Sinh[2*(c + d*x)] + d*(4*(b + d)*x + Sinh[2*
(a + c + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

3.170.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx$$

$$\Big\downarrow \ 6147$$

$$\int \left(\frac{1}{8}\cosh(2(a-c)+2x(b-d))+\frac{1}{8}\cosh(2(a+c)+2x(b+d))-\frac{1}{4}\cosh(2a+2bx)-\frac{1}{4}\cosh(2c+2dx)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sinh(2a+2bx)}{8b}-\frac{\sinh(2c+2dx)}{8d}+\frac{x}{4}$$

input
```
Int[Sinh[a + b*x]^2*Sinh[c + d*x]^2,x]
```

output
```
x/4 - Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
 - Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

3.170.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6147
```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.170.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$\frac{x}{4}-\frac{\sinh(2bx+2a)}{8b}-\frac{\sinh(2dx+2c)}{8d}+\frac{\sinh((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sinh((2b+2d)x+2a+2c)}{16b+16d}$
parallelrisch	$\frac{bd(b+d)\sinh((2b-2d)x+2a-2c)+4\left(\frac{bd\sinh((2b+2d)x+2a+2c)}{4}+\left(-\frac{d\sinh(2bx+2a)}{2}+b\left(dx-\frac{\sinh(2dx+2c)}{2}\right)\right)(b+d)\right)(b-d)}{16b^3d-16d^3b}$
risch	$\frac{x}{4}-\frac{e^{2bx+2a}}{16b}+\frac{e^{-2bx-2a}}{16b}-\frac{(-d\,e^{4bx+4a}b+d^2e^{4bx+4a}+2b^2e^{2bx+2a}-2\,e^{2bx+2a}d^2+bd+d^2)e^{-2bx+2dx-2a+2c}}{32(b+d)(b-d)d}+\frac{(d\,e^{4bx+}$

input
```
int(sinh(b*x+a)^2*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x-1/8*sinh(2*b*x+2*a)/b-1/8*sinh(2*d*x+2*c)/d+1/8/(2*b-2*d)*sinh((2*b-
2*d)*x+2*a-2*c)+1/8/(2*b+2*d)*sinh((2*b+2*d)*x+2*a+2*c)
```

3.170.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(78) = 156.

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.18

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx = \frac{b^2d\cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^2+(b^3d-bd^3)x+\left(b^2d\cosh(bx+a)\cosh(dx+c)^2-(b^2d-\right.}{4\left((b^3d-bd^3)\cos\right.}$$

input
```
integrate(sinh(b*x+a)^2*sinh(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(b^2*d*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^2 + (b^3*d - b*d^3)*x
 + (b^2*d*cosh(b*x + a)*cosh(d*x + c)^2 - (b^2*d - d^3)*cosh(b*x + a))*sin
h(b*x + a) - (b*d^2*cosh(d*x + c)*sinh(b*x + a)^2 + (b*d^2*cosh(b*x + a)^2
 + b^3 - b*d^2)*cosh(d*x + c))*sinh(d*x + c))/((b^3*d - b*d^3)*cosh(b*x +
a)^2 - (b^3*d - b*d^3)*sinh(b*x + a)^2)
```

3.170.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(76) = 152$.

Time = 1.56 (sec) , antiderivative size = 1027, normalized size of antiderivative = 11.67

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)**2*sinh(d*x+c)**2,x)
```

output
```
Piecewise((x*sinh(a)**2*sinh(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sinh(c + d*x
)**2/2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a)
**2, Eq(b, 0)), (3*x*sinh(a - d*x)**2*sinh(c + d*x)**2/8 - x*sinh(a - d*x)
**2*cosh(c + d*x)**2/8 + x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)*cosh(
c + d*x)/2 - x*sinh(c + d*x)**2*cosh(a - d*x)**2/8 + 3*x*cosh(a - d*x)**2*
cosh(c + d*x)**2/8 + sinh(a - d*x)**2*sinh(c + d*x)*cosh(c + d*x)/(8*d) -
sinh(a - d*x)*sinh(c + d*x)**2*cosh(a - d*x)/(2*d) - 3*sinh(c + d*x)*cosh(
a - d*x)**2*cosh(c + d*x)/(8*d), Eq(b, -d)), (3*x*sinh(a + d*x)**2*sinh(c
+ d*x)**2/8 - x*sinh(a + d*x)**2*cosh(c + d*x)**2/8 - x*sinh(a + d*x)*sinh
(c + d*x)*cosh(a + d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)**2*cosh(a + d*x)
**2/8 + 3*x*cosh(a + d*x)**2*cosh(c + d*x)**2/8 + 5*sinh(a + d*x)*sinh(c +
 d*x)**2*cosh(a + d*x)/(8*d) + sinh(a + d*x)*cosh(a + d*x)*cosh(c + d*x)**
2/(8*d) - sinh(c + d*x)*cosh(a + d*x)**2*cosh(c + d*x)/(2*d), Eq(b, d)), (
(x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)
/(2*b))*sinh(c)**2, Eq(d, 0)), (b**3*d*x*sinh(a + b*x)**2*sinh(c + d*x)**2
/(4*b**3*d - 4*b*d**3) - b**3*d*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4*b**
3*d - 4*b*d**3) - b**3*d*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d - 4
*b*d**3) + b**3*d*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3
) + b**3*sinh(a + b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*d**3
) - b**3*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)/(4*b**3*d - 4*b*d...
```

3.170.7 Maxima [F(-2)]

Exception generated.

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(b*x+a)^2*sinh(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-(2*d)/b>0)', see `assume?` for
 more deta
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx = \frac{1}{4}\,x + \frac{e^{(2\,bx+2\,dx+2\,a+2\,c)}}{32\,(b+d)} + \frac{e^{(2\,bx-2\,dx+2\,a-2\,c)}}{32\,(b-d)} - \frac{e^{(2\,bx+2\,a)}}{16\,b} - \frac{e^{(-2\,bx+2\,dx-2\,a+2\,c)}}{32\,(b-d)} - \frac{e^{(-2\,bx-2\,dx-2\,a-2\,c)}}{32\,(b+d)} + \frac{e^{(-2\,bx-2\,a)}}{16\,b} - \frac{e^{(2\,dx+2\,c)}}{16\,d} + \frac{e^{(-2\,dx-2\,c)}}{16\,d}$$

input
```
integrate(sinh(b*x+a)^2*sinh(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*x + 1/32*e^(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/32*e^(2*b*x - 2*d*x
 + 2*a - 2*c)/(b - d) - 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x + 2*d*x -
2*a + 2*c)/(b - d) - 1/32*e^(-2*b*x - 2*d*x - 2*a - 2*c)/(b + d) + 1/16*e^
(-2*b*x - 2*a)/b - 1/16*e^(2*d*x + 2*c)/d + 1/16*e^(-2*d*x - 2*c)/d
```

3.170.9 Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.73

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx = -\frac{d^3\cosh(a+b\,x)\,\sinh(a+b\,x) - b^3\cosh(c+d\,x)\,\sinh(c+d\,x) - b\,d^3\,x + b^3\,d\,x - 2\,b^2\,d\cosh(a+b\,x)\,\mathrm{si}}{}$$

input
```
int(sinh(a + b*x)^2*sinh(c + d*x)^2,x)
```

output
```
-(d^3*cosh(a + b*x)*sinh(a + b*x) - b^3*cosh(c + d*x)*sinh(c + d*x) - b*d^
3*x + b^3*d*x - 2*b^2*d*cosh(a + b*x)*sinh(a + b*x) + 2*b*d^2*cosh(c + d*x
)*sinh(c + d*x) + 2*b^2*d*cosh(a + b*x)*cosh(c + d*x)^2*sinh(a + b*x) - 2*
b*d^2*cosh(a + b*x)^2*cosh(c + d*x)*sinh(c + d*x))/(4*b*d^3 - 4*b^3*d)
```

3.171 $\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx$

3.171.1 Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = -\frac{\cosh(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\cosh(2a-c+(2b-d)x)}{16(2b-d)} + \frac{3\cosh(c+dx)}{8d} - \frac{\cosh(3c+3dx)}{24d} - \frac{3\cosh(2a+c+(2b+d)x)}{16(2b+d)} + \frac{\cosh(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

output
```
-1/16*cosh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*cosh(2*a-c+(2*b-d)*x)/(2*b-
d)+3/8*cosh(d*x+c)/d-1/24*cosh(3*d*x+3*c)/d-3/16*cosh(2*a+c+(2*b+d)*x)/(2*
b+d)+1/16*cosh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

3.171.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = \frac{1}{48}\left(\frac{18\cosh(c)\cosh(dx)}{d} - \frac{2\cosh(3c)\cosh(3dx)}{d} - \frac{3\cosh(2a-3c+2bx-3dx)}{2b-3d} + \frac{9\cosh(2a-c+2bx-dx)}{2b-d} - \frac{9\cosh(2a+c+2bx+dx)}{2b+d} + \frac{3\cosh(2a+3c+2bx+3dx)}{2b+3d} + \frac{18\sinh(c)\sinh(dx)}{d} - \frac{2\sinh(3c)\sinh(3dx)}{d}\right)$$

input
```
Integrate[Sinh[a + b*x]^2*Sinh[c + d*x]^3,x]
```

output
```
((18*Cosh[c]*Cosh[d*x])/d - (2*Cosh[3*c]*Cosh[3*d*x])/d - (3*Cosh[2*a - 3*
c + 2*b*x - 3*d*x])/(2*b - 3*d) + (9*Cosh[2*a - c + 2*b*x - d*x])/(2*b - d
) - (9*Cosh[2*a + c + 2*b*x + d*x])/(2*b + d) + (3*Cosh[2*a + 3*c + 2*b*x
+ 3*d*x])/(2*b + 3*d) + (18*Sinh[c]*Sinh[d*x])/d - (2*Sinh[3*c]*Sinh[3*d*x
])/d)/48
```

3.171.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx$$

$$\Big\downarrow \; 6147$$

$$\int \left(-\frac{1}{16}\sinh(2a+x(2b-3d)-3c) + \frac{3}{16}\sinh(2a+x(2b-d)-c) - \frac{3}{16}\sinh(2a+x(2b+d)+c) + \frac{1}{16}\sinh(2a+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cosh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\cosh(2a+x(2b-d)-c)}{16(2b-d)}-\frac{3\cosh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\cosh(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\cosh(c+dx)}{8d}-\frac{\cosh(3c+3dx)}{24d}$$

input
```
Int[Sinh[a + b*x]^2*Sinh[c + d*x]^3,x]
```

output
```
-1/16*Cosh[2*a - 3*c + (2*b - 3*d)*x]/(2*b - 3*d) + (3*Cosh[2*a - c + (2*b
 - d)*x])/(16*(2*b - d)) + (3*Cosh[c + d*x])/(8*d) - Cosh[3*c + 3*d*x]/(24
*d) - (3*Cosh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Cosh[2*a + 3*c + (2
*b + 3*d)*x]/(16*(2*b + 3*d))
```

3.171.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6147
```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.171.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\cosh(2a-3c+(2b-3d)x)}{16(2b-3d)}+\frac{3\cosh(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\cosh(dx+c)}{8d}-\frac{\cosh(3dx+3c)}{24d}-\frac{3\cosh(2a+c+(2b+d)x)}{16(2b+d)}+$ c
parallelrisch	$\left(-24b^3d-36d^2b^2+6d^3b+9d^4\right)\cosh(2a-3c+(2b-3d)x)+\left(72b^3d+36d^2b^2-162d^3b-81d^4\right)\cosh(2a-c+(2b-d)x)+\left(24b^3d-36d^2\right.$
risch	$-\frac{\left(-6d\,e^{4bx+4a}b+9d^2e^{4bx+4a}+8b^2e^{2bx+2a}-18\,e^{2bx+2a}d^2+6bd+9d^2\right)e^{-2bx+3dx-2a+3c}}{96(2b+3d)(2b-3d)d}+\frac{3\left(-2d\,e^{4bx+4a}b+d^2e^{4bx+4a}+8b^2\right.}{3}$

input
```
int(sinh(b*x+a)^2*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*cosh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*cosh(2*a-c+(2*b-d)*x)/(2*b-
d)+3/8*cosh(d*x+c)/d-1/24*cosh(3*d*x+3*c)/d-3/16*cosh(2*a+c+(2*b+d)*x)/(2*
b+d)+1/16*cosh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

3.171.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(132) = 264$.

Time = 0.25 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.88

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx$$
$$= \frac{12\left(4\,b^3d-bd^3\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^3-\left(16\,b^4-40\,b^2d^2+9\,d^4+9\left(4\,b^2d^2-d^4\right)\cosh\left(\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)^2*sinh(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/24*(12*(4*b^3*d - b*d^3)*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^3 - (
16*b^4 - 40*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cosh(b*x + a)^2)*cosh(d*
x + c)^3 - 9*((4*b^2*d^2 - d^4)*cosh(d*x + c)^3 - (4*b^2*d^2 - 9*d^4)*cosh
(d*x + c))*sinh(b*x + a)^2 + 36*((4*b^3*d - b*d^3)*cosh(b*x + a)*cosh(d*x
+ c)^2 - (4*b^3*d - 9*b*d^3)*cosh(b*x + a))*sinh(b*x + a)*sinh(d*x + c) -
3*(9*(4*b^2*d^2 - d^4)*cosh(d*x + c)*sinh(b*x + a)^2 + (16*b^4 - 40*b^2*d^
2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cosh(b*x + a)^2)*cosh(d*x + c))*sinh(d*x +
 c)^2 + 9*(16*b^4 - 40*b^2*d^2 + 9*d^4 + (4*b^2*d^2 - 9*d^4)*cosh(b*x + a)
^2)*cosh(d*x + c))/((16*b^4*d - 40*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 - (16*
b^4*d - 40*b^2*d^3 + 9*d^5)*sinh(b*x + a)^2)
```

3.171.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. $2(116) = 232$.

Time = 5.48 (sec) , antiderivative size = 2001, normalized size of antiderivative = 13.90

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)**2*sinh(d*x+c)**3,x)
```

output
```
Piecewise((x*sinh(a)**2*sinh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sinh(a - 3*d*
x/2)**2*sinh(c + d*x)**3/16 + 3*x*sinh(a - 3*d*x/2)**2*sinh(c + d*x)*cosh(
c + d*x)**2/16 + 3*x*sinh(a - 3*d*x/2)*sinh(c + d*x)**2*cosh(a - 3*d*x/2)*
cosh(c + d*x)/8 + x*sinh(a - 3*d*x/2)*cosh(a - 3*d*x/2)*cosh(c + d*x)**3/8
 + x*sinh(c + d*x)**3*cosh(a - 3*d*x/2)**2/16 + 3*x*sinh(c + d*x)*cosh(a -
 3*d*x/2)**2*cosh(c + d*x)**2/16 - 7*sinh(a - 3*d*x/2)**2*cosh(c + d*x)**3
/(16*d) - 5*sinh(a - 3*d*x/2)*sinh(c + d*x)**3*cosh(a - 3*d*x/2)/(8*d) - 3
*sinh(a - 3*d*x/2)*sinh(c + d*x)*cosh(a - 3*d*x/2)*cosh(c + d*x)**2/(4*d)
- sinh(c + d*x)**2*cosh(a - 3*d*x/2)**2*cosh(c + d*x)/d + 11*cosh(a - 3*d*
x/2)**2*cosh(c + d*x)**3/(48*d), Eq(b, -3*d/2)), (3*x*sinh(a - d*x/2)**2*s
inh(c + d*x)**3/16 - 3*x*sinh(a - d*x/2)**2*sinh(c + d*x)*cosh(c + d*x)**2
/16 + 3*x*sinh(a - d*x/2)*sinh(c + d*x)**2*cosh(a - d*x/2)*cosh(c + d*x)/8
 - 3*x*sinh(a - d*x/2)*cosh(a - d*x/2)*cosh(c + d*x)**3/8 + 3*x*sinh(c + d
*x)**3*cosh(a - d*x/2)**2/16 - 3*x*sinh(c + d*x)*cosh(a - d*x/2)**2*cosh(c
 + d*x)**2/16 + sinh(a - d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 31*s
inh(a - d*x/2)**2*cosh(c + d*x)**3/(48*d) + 3*sinh(a - d*x/2)*sinh(c + d*x
)**3*cosh(a - d*x/2)/(8*d) - sinh(a - d*x/2)*sinh(c + d*x)*cosh(a - d*x/2)
*cosh(c + d*x)**2/(4*d) + cosh(a - d*x/2)**2*cosh(c + d*x)**3/(48*d), Eq(b
, -d/2)), (3*x*sinh(a + d*x/2)**2*sinh(c + d*x)**3/16 - 3*x*sinh(a + d*x/2
)**2*sinh(c + d*x)*cosh(c + d*x)**2/16 - 3*x*sinh(a + d*x/2)*sinh(c + d...
```

3.171.7 Maxima [F(-2)]

Exception generated.

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(b*x+a)^2*sinh(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-(3*d)/b>0)', see `assume?` for
 more deta
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.81

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = \frac{e^{(2\,bx+3\,dx+2\,a+3\,c)}}{32\,(2\,b+3\,d)} - \frac{3\,e^{(2\,bx+dx+2\,a+c)}}{32\,(2\,b+d)} + \frac{3\,e^{(2\,bx-dx+2\,a-c)}}{32\,(2\,b-d)} - \frac{e^{(2\,bx-3\,dx+2\,a-3\,c)}}{32\,(2\,b-3\,d)} - \frac{e^{(-2\,bx+3\,dx-2\,a+3\,c)}}{32\,(2\,b-3\,d)} + \frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{32\,(2\,b-d)} - \frac{3\,e^{(-2\,bx-dx-2\,a-c)}}{32\,(2\,b+d)} + \frac{e^{(-2\,bx-3\,dx-2\,a-3\,c)}}{32\,(2\,b+3\,d)} - \frac{e^{(3\,dx+3\,c)}}{48\,d} + \frac{3\,e^{(dx+c)}}{16\,d} + \frac{3\,e^{(-dx-c)}}{16\,d} - \frac{e^{(-3\,dx-3\,c)}}{48\,d}$$

input
```
integrate(sinh(b*x+a)^2*sinh(d*x+c)^3,x, algorithm="giac")
```

output
```
1/32*e^(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) - 3/32*e^(2*b*x + d*x + 2*a
 + c)/(2*b + d) + 3/32*e^(2*b*x - d*x + 2*a - c)/(2*b - d) - 1/32*e^(2*b*x
 - 3*d*x + 2*a - 3*c)/(2*b - 3*d) - 1/32*e^(-2*b*x + 3*d*x - 2*a + 3*c)/(2
*b - 3*d) + 3/32*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 3/32*e^(-2*b*x - d
*x - 2*a - c)/(2*b + d) + 1/32*e^(-2*b*x - 3*d*x - 2*a - 3*c)/(2*b + 3*d)
- 1/48*e^(3*d*x + 3*c)/d + 3/16*e^(d*x + c)/d + 3/16*e^(-d*x - c)/d - 1/48
*e^(-3*d*x - 3*c)/d
```

3.171.9 Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.34

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx$$
$$= \frac{\cosh(c+d\,x)\,\sinh(a+b\,x)^2\sinh(c+d\,x)^2\,(8\,b^4-26\,b^2\,d^2+9\,d^4)}{d\,(16\,b^4-40\,b^2\,d^2+9\,d^4)}$$
$$-\cosh(c+d\,x)^3\sinh(a+b\,x)^2\left(\frac{3\,d^3}{16\,b^4-40\,b^2\,d^2+9\,d^4}+\frac{1}{3\,d}\right)$$
$$-\frac{2\cosh(a+b\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)^3\,(7\,b\,d^2-4\,b^3)}{16\,b^4-40\,b^2\,d^2+9\,d^4}$$
$$-\frac{\cosh(a+b\,x)^2\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(8\,b^4-14\,b^2\,d^2)}{d\,(16\,b^4-40\,b^2\,d^2+9\,d^4)}$$
$$-\cosh(a+b\,x)^2\cosh(c+d\,x)^3\left(\frac{3\,d^3}{16\,b^4-40\,b^2\,d^2+9\,d^4}-\frac{1}{3\,d}\right)$$
$$+\frac{12\,b\,d^2\cosh(a+b\,x)\,\cosh(c+d\,x)^2\sinh(a+b\,x)\,\sinh(c+d\,x)}{16\,b^4-40\,b^2\,d^2+9\,d^4}$$

input
```
int(sinh(a + b*x)^2*sinh(c + d*x)^3,x)
```

output
```
(cosh(c + d*x)*sinh(a + b*x)^2*sinh(c + d*x)^2*(8*b^4 + 9*d^4 - 26*b^2*d^2
))/(d*(16*b^4 + 9*d^4 - 40*b^2*d^2)) - cosh(c + d*x)^3*sinh(a + b*x)^2*((3
*d^3)/(16*b^4 + 9*d^4 - 40*b^2*d^2) + 1/(3*d)) - (2*cosh(a + b*x)*sinh(a +
 b*x)*sinh(c + d*x)^3*(7*b*d^2 - 4*b^3))/(16*b^4 + 9*d^4 - 40*b^2*d^2) - (
cosh(a + b*x)^2*cosh(c + d*x)*sinh(c + d*x)^2*(8*b^4 - 14*b^2*d^2))/(d*(16
*b^4 + 9*d^4 - 40*b^2*d^2)) - cosh(a + b*x)^2*cosh(c + d*x)^3*((3*d^3)/(16
*b^4 + 9*d^4 - 40*b^2*d^2) - 1/(3*d)) + (12*b*d^2*cosh(a + b*x)*cosh(c + d
*x)^2*sinh(a + b*x)*sinh(c + d*x))/(16*b^4 + 9*d^4 - 40*b^2*d^2)
```

3.172 $\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx$

3.172.1 Optimal result

Integrand size = 17, antiderivative size = 195

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)} - \frac{9\sinh(a-c+(b-d)x)}{32(b-d)} - \frac{\sinh(3(a-c)+3(b-d)x)}{96(b-d)} + \frac{3\sinh(3a-c+(3b-d)x)}{32(3b-d)} + \frac{9\sinh(a+c+(b+d)x)}{32(b+d)} + \frac{\sinh(3(a+c)+3(b+d)x)}{96(b+d)} - \frac{3\sinh(3a+c+(3b+d)x)}{32(3b+d)} - \frac{3\sinh(a+3c+(b+3d)x)}{32(b+3d)}$$

output
```
3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*sinh(a-c+(b-d)*x)/(b-d)-1/96*sinh(
3*a-3*c+3*(b-d)*x)/(b-d)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)+9/32*sinh(a+c+
(b+d)*x)/(b+d)+1/96*sinh(3*a+3*c+3*(b+d)*x)/(b+d)-3/32*sinh(3*a+c+(3*b+d)*
x)/(3*b+d)-3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.172.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \frac{1}{96}\left(\frac{9\sinh(a-3c+bx-3dx)}{b-3d} - \frac{27\sinh(a-c+bx-dx)}{b-d} - \frac{\sinh(3(a-c+bx-dx))}{b-d} + \frac{9\sinh(3a-c+3bx-dx)}{3b-d} - \frac{9\sinh(3a+c+3bx+dx)}{3b+d} - \frac{9\sinh(a+3c+bx+3dx)}{b+3d} + \frac{27\sinh(a+c+(b+d)x)}{b+d} + \frac{\sinh(3(a+c+(b+d)x))}{b+d}\right)$$

input
```
Integrate[Sinh[a + b*x]^3*Sinh[c + d*x]^3,x]
```

output
```
((9*Sinh[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Sinh[a - c + b*x - d*x])/
(b - d) - Sinh[3*(a - c + b*x - d*x)]/(b - d) + (9*Sinh[3*a - c + 3*b*x -
d*x])/(3*b - d) - (9*Sinh[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Sinh[a +
3*c + b*x + 3*d*x])/(b + 3*d) + (27*Sinh[a + c + (b + d)*x])/(b + d) + Sin
h[3*(a + c + (b + d)*x)]/(b + d))/96
```

3.172.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx$$

$$\Big\downarrow \text{6147}$$

$$\int \left(\frac{3}{32}\cosh(a+x(b-3d)-3c) - \frac{9}{32}\cosh(a+x(b-d)-c) - \frac{1}{32}\cosh(3(a-c)+3x(b-d)) + \frac{3}{32}\cosh(3a+x(\right.$$

$$\downarrow 2009$$

$$\frac{3\sinh(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\sinh(a+x(b-d)-c)}{32(b-d)}-\frac{\sinh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sinh(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sinh(a+x(b+d)+c)}{32(b+d)}+\frac{\sinh(3(a+c)+3x(b+d))}{96(b+d)}-\frac{3\sinh(3a+x(3b+d)+c)}{32(3b+d)}-\frac{3\sinh(a+x(b+3d)+3c)}{32(b+3d)}$$

input
```
Int[Sinh[a + b*x]^3*Sinh[c + d*x]^3,x]
```

output
```
(3*Sinh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Sinh[a - c + (b - d)*x
])/(32*(b - d)) - Sinh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sinh[3*a
 - c + (3*b - d)*x])/(32*(3*b - d)) + (9*Sinh[a + c + (b + d)*x])/(32*(b +
 d)) + Sinh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) - (3*Sinh[3*a + c + (3*b
 + d)*x])/(32*(3*b + d)) - (3*Sinh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

3.172.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6147
```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.172.4 Maple [A] (verified)

Time = 10.56 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

method	result
default	$\frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\sinh(a-c+(b-d)x)}{32(b-d)}+\frac{9\sinh(a+c+(b+d)x)}{32(b+d)}-\frac{3\sinh(a+3c+(b+3d)x)}{32(b+3d)}-\frac{\sinh((3b-3d)x+3a}{32(3b-3d)}$
parallelrisch	$\frac{\frac{9(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b-d)(b+d)\sinh(3a-c+(3b-d)x)}{32}-\frac{9\left(\frac{(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b+d)\sinh((3b-3d)x+3a-3c)}{3}-\frac{(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b-d}{3}\right.}{}}{}$
risch	$\frac{(b^3 e^{6bx+6a}-b^2 d\,e^{6bx+6a}-9b\,d^2 e^{6bx+6a}+9d^3 e^{6bx+6a}-9b^3 e^{4bx+4a}+27b^2 d\,e^{4bx+4a}+9b\,d^2 e^{4bx+4a}-27d^3 e^{4bx+4a}-9b^3 e^{2bx+2a}-}{192(b+d)(b+3d)(b-d)(b-3d)}$

input
```
int(sinh(b*x+a)^3*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*sinh(a-c+(b-d)*x)/(b-d)+9/32*sinh(
a+c+(b+d)*x)/(b+d)-3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)-1/32/(3*b-3*d)*sinh(
(3*b-3*d)*x+3*a-3*c)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)-3/32*sinh(3*a+c+(3
*b+d)*x)/(3*b+d)+1/32/(3*b+3*d)*sinh((3*b+3*d)*x+3*a+3*c)
```

3.172.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(179) = 358$.

Time = 0.27 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.75

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = -\frac{\left((9\,b^4d-82\,b^2d^3+9\,d^5)\cosh{(dx+c)}^3-9\,(b^4d-10\,b^2d^3+9\,d^5)\cosh{(dx+c)}\right)\sinh{(bx+a)}^3-\left((9\,b^5\right.}{}$$

input
```
integrate(sinh(b*x+a)^3*sinh(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/48*(((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(d*x + c)^3 - 9*(b^4*d - 10*b^2
*d^3 + 9*d^5)*cosh(d*x + c))*sinh(b*x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*
d^4)*cosh(b*x + a)^3 + 3*(9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)*sinh
(b*x + a)^2 - 9*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a))*sinh(d*x + c)^
3 + 3*((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(d*x + c)*sinh(b*x + a)^3 - 3*(8
1*b^4*d - 90*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a
)^2)*cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 - 3*((81*b^4*d - 90*b^2*
d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2)*cosh(d*x + c
)^3 - 9*(9*b^4*d - 82*b^2*d^3 + 9*d^5 - (b^4*d - 10*b^2*d^3 + 9*d^5)*cosh(
b*x + a)^2)*cosh(d*x + c))*sinh(b*x + a) + 3*(9*(b^5 - 10*b^3*d^2 + 9*b*d^
4)*cosh(b*x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 - 9*(
9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c)^2 - 3*((9*b^5 - 8
2*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)*cosh(d*x + c)^2 - 9*(b^5 - 10*b^3*d^2 +
 9*b*d^4)*cosh(b*x + a))*sinh(b*x + a)^2 - 9*(9*b^5 - 82*b^3*d^2 + 9*b*d^4
)*cosh(b*x + a))*sinh(d*x + c))/((9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)
*cosh(b*x + a)^4 - 2*(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x +
a)^2*sinh(b*x + a)^2 + (9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*sinh(b*x
+ a)^4)
```

3.172.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. $2(172) = 344$.

Time = 16.69 (sec) , antiderivative size = 3580, normalized size of antiderivative = 18.36

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)**3*sinh(d*x+c)**3,x)
```

output
```
Piecewise((x*sinh(a)**3*sinh(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - 3*
d*x)**3*sinh(c + d*x)**3/32 + 9*x*sinh(a - 3*d*x)**3*sinh(c + d*x)*cosh(c
+ d*x)**2/32 + 9*x*sinh(a - 3*d*x)**2*sinh(c + d*x)**2*cosh(a - 3*d*x)*cos
h(c + d*x)/32 + 3*x*sinh(a - 3*d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)**3/32
 - 3*x*sinh(a - 3*d*x)*sinh(c + d*x)**3*cosh(a - 3*d*x)**2/32 - 9*x*sinh(a
 - 3*d*x)*sinh(c + d*x)*cosh(a - 3*d*x)**2*cosh(c + d*x)**2/32 - 9*x*sinh(
c + d*x)**2*cosh(a - 3*d*x)**3*cosh(c + d*x)/32 - 3*x*cosh(a - 3*d*x)**3*c
osh(c + d*x)**3/32 - 13*sinh(a - 3*d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/
(320*d) + sinh(a - 3*d*x)**3*cosh(c + d*x)**3/(12*d) - 101*sinh(a - 3*d*x)
**2*sinh(c + d*x)**3*cosh(a - 3*d*x)/(320*d) + 3*sinh(a - 3*d*x)**2*sinh(c
 + d*x)*cosh(a - 3*d*x)*cosh(c + d*x)**2/(20*d) - 27*sinh(a - 3*d*x)*cosh(
a - 3*d*x)**2*cosh(c + d*x)**3/(320*d) + sinh(c + d*x)**3*cosh(a - 3*d*x)*
*3/(5*d) - 51*sinh(c + d*x)*cosh(a - 3*d*x)**3*cosh(c + d*x)**2/(320*d), E
q(b, -3*d)), (5*x*sinh(a - d*x)**3*sinh(c + d*x)**3/16 - 3*x*sinh(a - d*x)
**3*sinh(c + d*x)*cosh(c + d*x)**2/16 + 9*x*sinh(a - d*x)**2*sinh(c + d*x)
**2*cosh(a - d*x)*cosh(c + d*x)/16 - 3*x*sinh(a - d*x)**2*cosh(a - d*x)*co
sh(c + d*x)**3/16 - 3*x*sinh(a - d*x)*sinh(c + d*x)**3*cosh(a - d*x)**2/16
 + 9*x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)**2*cosh(c + d*x)**2/16 -
3*x*sinh(c + d*x)**2*cosh(a - d*x)**3*cosh(c + d*x)/16 + 5*x*cosh(a - d*x)
**3*cosh(c + d*x)**3/16 + sinh(a - d*x)**3*sinh(c + d*x)**2*cosh(c + d*...
```

3.172.7 Maxima [F(-2)]

Exception generated.

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(b*x+a)^3*sinh(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for
more detai
```

3.172.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(179) = 358$.

Time = 0.28 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.91

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \frac{e^{(3\,bx+3\,dx+3\,a+3\,c)}}{192\,(b+d)} - \frac{3\,e^{(3\,bx+dx+3\,a+c)}}{64\,(3\,b+d)} + \frac{3\,e^{(3\,bx-dx+3\,a-c)}}{64\,(3\,b-d)} - \frac{e^{(3\,bx-3\,dx+3\,a-3\,c)}}{192\,(b-d)} - \frac{3\,e^{(bx+3\,dx+a+3\,c)}}{64\,(b+3\,d)} + \frac{9\,e^{(bx+dx+a+c)}}{64\,(b+d)} - \frac{9\,e^{(bx-dx+a-c)}}{64\,(b-d)} + \frac{3\,e^{(bx-3\,dx+a-3\,c)}}{64\,(b-3\,d)} - \frac{3\,e^{(-bx+3\,dx-a+3\,c)}}{64\,(b-3\,d)} + \frac{9\,e^{(-bx+dx-a+c)}}{64\,(b-d)} - \frac{9\,e^{(-bx-dx-a-c)}}{64\,(b+d)} + \frac{3\,e^{(-bx-3\,dx-a-3\,c)}}{64\,(b+3\,d)} + \frac{e^{(-3\,bx+3\,dx-3\,a+3\,c)}}{192\,(b-d)} - \frac{3\,e^{(-3\,bx+dx-3\,a+c)}}{64\,(3\,b-d)} + \frac{3\,e^{(-3\,bx-dx-3\,a-c)}}{64\,(3\,b+d)} - \frac{e^{(-3\,bx-3\,dx-3\,a-3\,c)}}{192\,(b+d)}$$

input
```
integrate(sinh(b*x+a)^3*sinh(d*x+c)^3,x, algorithm="giac")
```

output
```
1/192*e^(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) - 3/64*e^(3*b*x + d*x + 3*a +
c)/(3*b + d) + 3/64*e^(3*b*x - d*x + 3*a - c)/(3*b - d) - 1/192*e^(3*b*x -
 3*d*x + 3*a - 3*c)/(b - d) - 3/64*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 9
/64*e^(b*x + d*x + a + c)/(b + d) - 9/64*e^(b*x - d*x + a - c)/(b - d) + 3
/64*e^(b*x - 3*d*x + a - 3*c)/(b - 3*d) - 3/64*e^(-b*x + 3*d*x - a + 3*c)/
(b - 3*d) + 9/64*e^(-b*x + d*x - a + c)/(b - d) - 9/64*e^(-b*x - d*x - a -
 c)/(b + d) + 3/64*e^(-b*x - 3*d*x - a - 3*c)/(b + 3*d) + 1/192*e^(-3*b*x
+ 3*d*x - 3*a + 3*c)/(b - d) - 3/64*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) +
 3/64*e^(-3*b*x - d*x - 3*a - c)/(3*b + d) - 1/192*e^(-3*b*x - 3*d*x - 3*a
 - 3*c)/(b + d)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 906, normalized size of antiderivative = 4.65

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = e^{3a+c+3bx+dx}\left(\frac{-9b^3+3b^2d+9bd^2-3d^3}{576b^4-640b^2d^2+64d^4} + \frac{e^{-6a-6bx}\left(-9b^3-3b^2d+9bd^2+3d^3\right)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-2a-2bx}\left(-81b^3+81b^2d+9bd^2-9d^3\right)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-4a-4bx}\left(-81b^3-81b^2d+9bd^2+9d^3\right)}{576b^4-640b^2d^2+64d^4}\right) - e^{3a-c+3bx-dx}\left(\frac{-9b^3-3b^2d+9bd^2+3d^3}{576b^4-640b^2d^2+64d^4} + \frac{e^{-6a-6bx}\left(-9b^3+3b^2d+9bd^2-3d^3\right)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-2a-2bx}\left(-81b^3-81b^2d+9bd^2+9d^3\right)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-4a-4bx}\left(-81b^3+81b^2d+9bd^2-9d^3\right)}{576b^4-640b^2d^2+64d^4}\right) + e^{3a-3c+3bx-3dx}\left(\frac{-b^3-b^2d+9bd^2+9d^3}{192b^4-1920b^2d^2+1728d^4} + \frac{e^{-6a-6bx}\left(-b^3+b^2d+9bd^2-9d^3\right)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-2a-2bx}\left(-9b^3-27b^2d+9bd^2+27d^3\right)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-4a-4bx}\left(-9b^3+27b^2d+9bd^2-27d^3\right)}{192b^4-1920b^2d^2+1728d^4}\right) - e^{3a+3c+3bx+3dx}\left(\frac{-b^3+b^2d+9bd^2-9d^3}{192b^4-1920b^2d^2+1728d^4} + \frac{e^{-6a-6bx}\left(-b^3-b^2d+9bd^2+9d^3\right)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-2a-2bx}\left(-9b^3+27b^2d+9bd^2-27d^3\right)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-4a-4bx}\left(-9b^3-27b^2d+9bd^2+27d^3\right)}{192b^4-1920b^2d^2+1728d^4}\right)$$

input
```
int(sinh(a + b*x)^3*sinh(c + d*x)^3,x)
```

output
```
exp(3*a + c + 3*b*x + d*x)*((9*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(576*b^4 +
 64*d^4 - 640*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 - 3*b^2*d - 9*b^3 +
3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 2*a - 2*b*x)*(9*b*d^2 +
81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4*a
- 4*b*x)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^
2*d^2)) - exp(3*a - c + 3*b*x - d*x)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/
(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 + 3*b^2*d
- 9*b^3 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 2*a - 2*b*x)*(
9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (
exp(- 4*a - 4*b*x)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^
4 - 640*b^2*d^2)) + exp(3*a - 3*c + 3*b*x - 3*d*x)*((9*b*d^2 - b^2*d - b^3
 + 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d
^2 + b^2*d - b^3 - 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 2*
a - 2*b*x)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(192*b^4 + 1728*d^4 - 19
20*b^2*d^2) - (exp(- 4*a - 4*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(
192*b^4 + 1728*d^4 - 1920*b^2*d^2)) - exp(3*a + 3*c + 3*b*x + 3*d*x)*((9*b
*d^2 + b^2*d - b^3 - 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 6
*a - 6*b*x)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^
2*d^2) - (exp(- 2*a - 2*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b
^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 4*a - 4*b*x)*(9*b*d^2 - 27*b^2*d...
```

3.172. $\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx$

3.173 $\int \cosh(a+bx)\cosh(c+dx)\,dx$

3.173.1 Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \cosh(a+bx)\cosh(c+dx)\,dx=\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2(b+d)}$$

output
```
1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

3.173.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\cosh(c+dx)\,dx=\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2(b+d)}$$

input
```
Integrate[Cosh[a + b*x]*Cosh[c + d*x],x]
```

output
```
Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

3.173.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6148, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\cosh(c+dx)\,dx$$

$$\Big\downarrow \text{6148}$$

$$\int \left(\frac{1}{2}\cosh(a+x(b-d)-c)+\frac{1}{2}\cosh(a+x(b+d)+c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(a+x(b-d)-c)}{2(b-d)}+\frac{\sinh(a+x(b+d)+c)}{2(b+d)}$$

input
```
Int[Cosh[a + b*x]*Cosh[c + d*x],x]
```

output
```
Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

3.173.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6148
```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.173.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\sinh(a-c+(b-d)x)}{2b-2d}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}$	40
parallelrisch	$\frac{(b+d)\sinh(a-c+(b-d)x)+\sinh(a+c+(b+d)x)(b-d)}{2b^2-2d^2}$	48
risch	$\frac{\left(b\,\mathrm{e}^{2bx+2a}-\mathrm{e}^{2bx+2a}d-b-d\right)\mathrm{e}^{-bx+dx-a+c}}{4(b+d)(b-d)}+\frac{\left(b\,\mathrm{e}^{2bx+2a}+\mathrm{e}^{2bx+2a}d-b+d\right)\mathrm{e}^{-bx-dx-a-c}}{4(b+d)(b-d)}$	116

input
```
int(cosh(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \cosh(a+bx)\cosh(c+dx)\,dx = \frac{b\cosh{(dx+c)}\sinh{(bx+a)}-d\cosh{(bx+a)}\sinh{(dx+c)}}{(b^2-d^2)\cosh{(bx+a)}^2-(b^2-d^2)\sinh{(bx+a)}^2}$$

input
```
integrate(cosh(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
(b*cosh(d*x + c)*sinh(b*x + a) - d*cosh(b*x + a)*sinh(d*x + c))/((b^2 - d^
2)*cosh(b*x + a)^2 - (b^2 - d^2)*sinh(b*x + a)^2)
```

3.173.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(32) = 64$.

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.56

$$\int \cosh(a+bx)\cosh(c+dx)\,dx$$
$$= \begin{cases} x\cosh{(a)}\cosh{(c)} & \text{for}\ b=0 \wedge d=0 \\ \frac{x\sinh{(a-dx)}\sinh{(c+dx)}}{2}+\frac{x\cosh{(a-dx)}\cosh{(c+dx)}}{2}-\frac{\sinh{(a-dx)}\cosh{(c+dx)}}{2d} & \text{for}\ b=-d \\ -\frac{x\sinh{(a+dx)}\sinh{(c+dx)}}{2}+\frac{x\cosh{(a+dx)}\cosh{(c+dx)}}{2}+\frac{\sinh{(c+dx)}\cosh{(a+dx)}}{2d} & \text{for}\ b=d \\ \frac{b\sinh{(a+bx)}\cosh{(c+dx)}}{b^2-d^2}-\frac{d\sinh{(c+dx)}\cosh{(a+bx)}}{b^2-d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)*cosh(d*x+c),x)
```

output
```
Piecewise((x*cosh(a)*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x)*sinh(
c + d*x)/2 + x*cosh(a - d*x)*cosh(c + d*x)/2 - sinh(a - d*x)*cosh(c + d*x)
/(2*d), Eq(b, -d)), (-x*sinh(a + d*x)*sinh(c + d*x)/2 + x*cosh(a + d*x)*co
sh(c + d*x)/2 + sinh(c + d*x)*cosh(a + d*x)/(2*d), Eq(b, d)), (b*sinh(a +
b*x)*cosh(c + d*x)/(b**2 - d**2) - d*sinh(c + d*x)*cosh(a + b*x)/(b**2 - d
**2), True))
```

3.173.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh(a+bx)\cosh(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more
 details)I
```

3.173.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \cosh(a+bx)\cosh(c+dx)\,dx = \frac{e^{(bx+dx+a+c)}}{4\,(b+d)} + \frac{e^{(bx-dx+a-c)}}{4\,(b-d)} - \frac{e^{(-bx+dx-a+c)}}{4\,(b-d)} - \frac{e^{(-bx-dx-a-c)}}{4\,(b+d)}$$

input
```
integrate(cosh(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/4*e^(b*x + d*x + a + c)/(b + d) + 1/4*e^(b*x - d*x + a - c)/(b - d) - 1/
4*e^(-b*x + d*x - a + c)/(b - d) - 1/4*e^(-b*x - d*x - a - c)/(b + d)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \cosh(a+bx)\cosh(c+dx)\,dx = \frac{b\cosh(c+d\,x)\,\sinh(a+b\,x)-d\cosh(a+b\,x)\,\sinh(c+d\,x)}{b^2-d^2}$$

input
```
int(cosh(a + b*x)*cosh(c + d*x),x)
```

output
```
(b*cosh(c + d*x)*sinh(a + b*x) - d*cosh(a + b*x)*sinh(c + d*x))/(b^2 - d^2
)
```

3.174 $\int \cosh(a+bx)\cosh^2(c+dx)\,dx$

3.174.1 Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx=\frac{\sinh(a+bx)}{2b}+\frac{\sinh(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\sinh(a+2c+(b+2d)x)}{4(b+2d)}$$

output
```
1/2*sinh(b*x+a)/b+1/4*sinh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sinh(a+2*c+(b+2*d)
*x)/(b+2*d)
```

3.174.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx=\frac{1}{4}\left(\frac{2\cosh(bx)\sinh(a)}{b}+\frac{2\cosh(a)\sinh(bx)}{b}+\frac{\sinh(a-2c+bx-2dx)}{b-2d}+\frac{\sinh(a+2c+bx+2dx)}{b+2d}\right)$$

input
```
Integrate[Cosh[a + b*x]*Cosh[c + d*x]^2,x]
```

output
```
((2*Cosh[b*x]*Sinh[a])/b + (2*Cosh[a]*Sinh[b*x])/b + Sinh[a - 2*c + b*x -
2*d*x]/(b - 2*d) + Sinh[a + 2*c + b*x + 2*d*x]/(b + 2*d))/4
```

3.174.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6148, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx$$

$$\Big\downarrow \text{6148}$$

$$\int \left(\frac{1}{4}\cosh(a+x(b-2d)-2c)+\frac{1}{4}\cosh(a+x(b+2d)+2c)+\frac{1}{2}\cosh(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\sinh(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\sinh(a+bx)}{2b}$$

input
```
Int[Cosh[a + b*x]*Cosh[c + d*x]^2,x]
```

output
```
Sinh[a + b*x]/(2*b) + Sinh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Sinh[a +
 2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

3.174.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6148
```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.174.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sinh(bx+a)}{2b}+\frac{\sinh(a-2c+(b-2d)x)}{4b-8d}+\frac{\sinh(a+2c+(b+2d)x)}{4b+8d}$
parallelrisch	$\frac{b(b+2d)\sinh(a-2c+(b-2d)x)+2(b-2d)\left(\frac{b\sinh(a+2c+(b+2d)x)}{2}+\sinh(bx+a)(b+2d)\right)}{4b^{3}-16b\,d^{2}}$
risch	$\frac{e^{bx+a}}{4b}-\frac{e^{-bx-a}}{4b}+\frac{\left(b\,e^{2bx+2a}-2\,e^{2bx+2a}d-b-2d\right)e^{-bx+2dx-a+2c}}{8(b+2d)(b-2d)}+\frac{\left(b\,e^{2bx+2a}+2\,e^{2bx+2a}d-b+2d\right)e^{-bx-2dx-a-2c}}{8(b+2d)(b-2d)}$

input
```
int(cosh(b*x+a)*cosh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*sinh(b*x+a)/b+1/4*sinh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sinh(a+2*c+(b+2*d)
*x)/(b+2*d)
```

3.174.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(56) = 112.

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.85

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = -\frac{4\,bd\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)-b^2\sinh\left(bx+a\right)\sinh\left(dx+c\right)^2-\left(b^2\cosh\left(dx+c\right)^2+b^2\right.}{2\left(\left(b^3-4\,bd^2\right)\cosh\left(bx+a\right)^2-\left(b^3-4\,bd^2\right)\sinh\left(bx+a\right)^2\right)}$$

input
```
integrate(cosh(b*x+a)*cosh(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(4*b*d*cosh(b*x + a)*cosh(d*x + c)*sinh(d*x + c) - b^2*sinh(b*x + a)*
sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^2 + b^2 - 4*d^2)*sinh(b*x + a))/((b^3
 - 4*b*d^2)*cosh(b*x + a)^2 - (b^3 - 4*b*d^2)*sinh(b*x + a)^2)
```

3.174.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(49) = 98$.

Time = 0.70 (sec) , antiderivative size = 408, normalized size of antiderivative = 6.58

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx$$
$$= \begin{cases} x\cosh(a)\cosh^2(c) \\ \left(-\frac{x\sinh^2(c+dx)}{2}+\frac{x\cosh^2(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)\cosh(a) \\ \frac{x\sinh(a-2dx)\sinh(c+dx)\cosh(c+dx)}{2}+\frac{x\sinh^2(c+dx)\cosh(a-2dx)}{4}+\frac{x\cosh(a-2dx)\cosh^2(c+dx)}{4}+\frac{\sinh(a-2dx)\sinh^2(c+dx)}{2d}+ \\ -\frac{x\sinh(a+2dx)\sinh(c+dx)\cosh(c+dx)}{2}+\frac{x\sinh^2(c+dx)\cosh(a+2dx)}{4}+\frac{x\cosh(a+2dx)\cosh^2(c+dx)}{4}-\frac{\sinh(a+2dx)\sinh^2(c+dx)}{2d} \\ \frac{b^2\sinh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2}-\frac{2bd\sinh(c+dx)\cosh(a+bx)\cosh(c+dx)}{b^3-4bd^2}+\frac{2d^2\sinh(a+bx)\sinh^2(c+dx)}{b^3-4bd^2}-\frac{2d^2\sinh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

input
```
integrate(cosh(b*x+a)*cosh(d*x+c)**2,x)
```

output
```
Piecewise((x*cosh(a)*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x)*
*2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*cosh(a),
Eq(b, 0)), (x*sinh(a - 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*sinh(c + d
*x)**2*cosh(a - 2*d*x)/4 + x*cosh(a - 2*d*x)*cosh(c + d*x)**2/4 + sinh(a -
 2*d*x)*sinh(c + d*x)**2/(2*d) + 3*sinh(c + d*x)*cosh(a - 2*d*x)*cosh(c +
d*x)/(4*d), Eq(b, -2*d)), (-x*sinh(a + 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/
2 + x*sinh(c + d*x)**2*cosh(a + 2*d*x)/4 + x*cosh(a + 2*d*x)*cosh(c + d*x)
**2/4 - sinh(a + 2*d*x)*sinh(c + d*x)**2/(2*d) + 3*sinh(c + d*x)*cosh(a +
2*d*x)*cosh(c + d*x)/(4*d), Eq(b, 2*d)), (b**2*sinh(a + b*x)*cosh(c + d*x)
**2/(b**3 - 4*b*d**2) - 2*b*d*sinh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)/(b
**3 - 4*b*d**2) + 2*d**2*sinh(a + b*x)*sinh(c + d*x)**2/(b**3 - 4*b*d**2)
- 2*d**2*sinh(a + b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2), True))
```

3.174.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(b*x+a)*cosh(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for
more detai
```

3.174.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(56) = 112$.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.94

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = \frac{e^{(bx+2\,dx+a+2\,c)}}{8\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{8\,(b-2\,d)} + \frac{e^{(bx+a)}}{4\,b} - \frac{e^{(-bx+2\,dx-a+2\,c)}}{8\,(b-2\,d)} - \frac{e^{(-bx-2\,dx-a-2\,c)}}{8\,(b+2\,d)} - \frac{e^{(-bx-a)}}{4\,b}$$

input
```
integrate(cosh(b*x+a)*cosh(d*x+c)^2,x, algorithm="giac")
```

output
```
1/8*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/8*e^(b*x - 2*d*x + a - 2*c)/(b
 - 2*d) + 1/4*e^(b*x + a)/b - 1/8*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) - 1
/8*e^(-b*x - 2*d*x - a - 2*c)/(b + 2*d) - 1/4*e^(-b*x - a)/b
```

3.174.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = \frac{2\,d^2\sinh(a+b\,x) - b^2\cosh(c+d\,x)^2\sinh(a+b\,x) + 2\,b\,d\cosh(a+b\,x)\,\cosh(c+d\,x)\,\sinh(c+d\,x)}{4\,b\,d^2 - b^3}$$

input
```
int(cosh(a + b*x)*cosh(c + d*x)^2,x)
```

output
```
(2*d^2*sinh(a + b*x) - b^2*cosh(c + d*x)^2*sinh(a + b*x) + 2*b*d*cosh(a +
b*x)*cosh(c + d*x)*sinh(c + d*x))/(4*b*d^2 - b^3)
```

3.175 $\int \cosh(a+bx)\cosh^3(c+dx)\,dx$

3.175.1 Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}+\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8(b+3d)}$$

output
```
1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)+3/8*sinh(a+c
+(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.175.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \frac{1}{8}\left(\frac{\sinh(a-3c+bx-3dx)}{b-3d}+\frac{3\sinh(a-c+bx-dx)}{b-d}+\frac{\sinh(a+3c+bx+3dx)}{b+3d}+\frac{3\sinh(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Cosh[a + b*x]*Cosh[c + d*x]^3,x]
```

output
```
(Sinh[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Sinh[a - c + b*x - d*x])/(b -
d) + Sinh[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Sinh[a + c + (b + d)*x])/(
b + d))/8
```

3.175.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6148, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx$$

$$\Big\downarrow \text{6148}$$

$$\int \left(\frac{1}{8}\cosh(a+x(b-3d)-3c)+\frac{3}{8}\cosh(a+x(b-d)-c)+\frac{3}{8}\cosh(a+x(b+d)+c)+\frac{1}{8}\cosh(a+x(b+3d)+$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sinh(a+x(b-d)-c)}{8(b-d)}+\frac{3\sinh(a+x(b+d)+c)}{8(b+d)}+\frac{\sinh(a+x(b+3d)+3c)}{8(b+3d)}$$

input
```
Int[Cosh[a + b*x]*Cosh[c + d*x]^3,x]
```

output
```
Sinh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sinh[a - c + (b - d)*x])/(8
*(b - d)) + (3*Sinh[a + c + (b + d)*x])/(8*(b + d)) + Sinh[a + 3*c + (b +
3*d)*x]/(8*(b + 3*d))
```

3.175.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6148
```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.175.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sinh(a-3c+(b-3d)x)}{8b-24d}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}+\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}$
risch	$\frac{(b\,e^{2bx+2a}-3\,e^{2bx+2a}d-b-3d)\,e^{-bx+3dx-a+3c}}{16(b+3d)(b-3d)}+\frac{3(b\,e^{2bx+2a}-e^{2bx+2a}d-b-d)\,e^{-bx+dx-a+c}}{16(b+d)(b-d)}+\frac{3(b\,e^{2bx+2a}+e^{2bx+2a}d-b\cdots}{16(b+d)(b-\cdots}$
parallelrisch	$\frac{2b\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)(b^2-7d^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6-6d\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)(b^2-3d^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+6b\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)(b^2+d^2)\tanh\left(\cdots\right.}{(b-d)(b+3d)(b-3d)(b+d)\cdots}$

input
```
int(cosh(b*x+a)*cosh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)+3/8*sinh(a+c
+(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.175.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(83) = 166.

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.38

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx$$
$$=\frac{3\left(b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^2-3\left(b^2d-d^3\right)\cosh\left(bx+a\right)\sinh\left(dx+c\right)^3+\left(\left(b^3-\cdots\right.\right.}{4\left(\left(b^4-10\,b^2d^2\cdots\right.\right.}$$

input
```
integrate(cosh(b*x+a)*cosh(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(3*(b^3 - b*d^2)*cosh(d*x + c)*sinh(b*x + a)*sinh(d*x + c)^2 - 3*(b^2*
d - d^3)*cosh(b*x + a)*sinh(d*x + c)^3 + ((b^3 - b*d^2)*cosh(d*x + c)^3 +
3*(b^3 - 9*b*d^2)*cosh(d*x + c))*sinh(b*x + a) - 3*(3*(b^2*d - d^3)*cosh(b
*x + a)*cosh(d*x + c)^2 + (b^2*d - 9*d^3)*cosh(b*x + a))*sinh(d*x + c))/((
b^4 - 10*b^2*d^2 + 9*d^4)*cosh(b*x + a)^2 - (b^4 - 10*b^2*d^2 + 9*d^4)*sin
h(b*x + a)^2)
```

3.175.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(76) = 152$.

Time = 1.88 (sec) , antiderivative size = 921, normalized size of antiderivative = 10.12

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)*cosh(d*x+c)**3,x)
```

output
```
Piecewise((x*cosh(a)*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sinh(a - 3*d*x)*
sinh(c + d*x)**3/8 + 3*x*sinh(a - 3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8
+ 3*x*sinh(c + d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)/8 + x*cosh(a - 3*d*x)
*cosh(c + d*x)**3/8 + sinh(a - 3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d)
 - 7*sinh(a - 3*d*x)*cosh(c + d*x)**3/(24*d) + sinh(c + d*x)**3*cosh(a - 3
*d*x)/(8*d), Eq(b, -3*d)), (-3*x*sinh(a - d*x)*sinh(c + d*x)**3/8 + 3*x*si
nh(a - d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(a
 - d*x)*cosh(c + d*x)/8 + 3*x*cosh(a - d*x)*cosh(c + d*x)**3/8 + 3*sinh(a
- d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) - 5*sinh(a - d*x)*cosh(c + d*x
)**3/(8*d) + 3*sinh(c + d*x)**3*cosh(a - d*x)/(8*d), Eq(b, -d)), (3*x*sinh
(a + d*x)*sinh(c + d*x)**3/8 - 3*x*sinh(a + d*x)*sinh(c + d*x)*cosh(c + d*
x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(a + d*x)*cosh(c + d*x)/8 + 3*x*cosh(a
+ d*x)*cosh(c + d*x)**3/8 - 3*sinh(a + d*x)*sinh(c + d*x)**2*cosh(c + d*x)
/(4*d) + 5*sinh(a + d*x)*cosh(c + d*x)**3/(8*d) + 3*sinh(c + d*x)**3*cosh(
a + d*x)/(8*d), Eq(b, d)), (-x*sinh(a + 3*d*x)*sinh(c + d*x)**3/8 - 3*x*si
nh(a + 3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + 3*x*sinh(c + d*x)**2*cosh
(a + 3*d*x)*cosh(c + d*x)/8 + x*cosh(a + 3*d*x)*cosh(c + d*x)**3/8 - sinh(
a + 3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + 7*sinh(a + 3*d*x)*cosh(c
 + d*x)**3/(24*d) + sinh(c + d*x)**3*cosh(a + 3*d*x)/(8*d), Eq(b, 3*d)), (
b**3*sinh(a + b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*...
```

3.175.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(b*x+a)*cosh(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for
more detai
```

3.175.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(83) = 166$.

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.97

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \frac{e^{(bx+3\,dx+a+3\,c)}}{16\,(b+3\,d)} + \frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)} + \frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)} + \frac{e^{(bx-3\,dx+a-3\,c)}}{16\,(b-3\,d)} - \frac{e^{(-bx+3\,dx-a+3\,c)}}{16\,(b-3\,d)} - \frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)} - \frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)} - \frac{e^{(-bx-3\,dx-a-3\,c)}}{16\,(b+3\,d)}$$

input
```
integrate(cosh(b*x+a)*cosh(d*x+c)^3,x, algorithm="giac")
```

output
```
1/16*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/16*e^(b*x + d*x + a + c)/(b +
 d) + 3/16*e^(b*x - d*x + a - c)/(b - d) + 1/16*e^(b*x - 3*d*x + a - 3*c)/
(b - 3*d) - 1/16*e^(-b*x + 3*d*x - a + 3*c)/(b - 3*d) - 3/16*e^(-b*x + d*x
 - a + c)/(b - d) - 3/16*e^(-b*x - d*x - a - c)/(b + d) - 1/16*e^(-b*x - 3
*d*x - a - 3*c)/(b + 3*d)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.98

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \frac{b\cosh(c+d\,x)^3\sinh(a+b\,x)\,\left(b^2-7\,d^2\right)}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{3\cosh(a+b\,x)\,\cosh(c+d\,x)^2\sinh(c+d\,x)\,\left(b^2\,d-3\,d^3\right)}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{6\,d^3\cosh(a+b\,x)\,\sinh(c+d\,x)^3}{b^4-10\,b^2\,d^2+9\,d^4} + \frac{6\,b\,d^2\cosh(c+d\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)^2}{b^4-10\,b^2\,d^2+9\,d^4}$$

input
```
int(cosh(a + b*x)*cosh(c + d*x)^3,x)
```

output
```
(b*cosh(c + d*x)^3*sinh(a + b*x)*(b^2 - 7*d^2))/(b^4 + 9*d^4 - 10*b^2*d^2)
 - (3*cosh(a + b*x)*cosh(c + d*x)^2*sinh(c + d*x)*(b^2*d - 3*d^3))/(b^4 +
9*d^4 - 10*b^2*d^2) - (6*d^3*cosh(a + b*x)*sinh(c + d*x)^3)/(b^4 + 9*d^4 -
 10*b^2*d^2) + (6*b*d^2*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x)^2)/(b^4
+ 9*d^4 - 10*b^2*d^2)
```

3.175. $\int \cosh(a+bx)\cosh^3(c+dx)\,dx$

3.176 $\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx$

3.176.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \frac{x}{4}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)} + \frac{\sinh(2c+2dx)}{8d}+\frac{\sinh(2(a+c)+2(b+d)x)}{16(b+d)}$$

output
```
1/4*x+1/8*sinh(2*b*x+2*a)/b+1/16*sinh(2*a-2*c+2*(b-d)*x)/(b-d)+1/8*sinh(2*
d*x+2*c)/d+1/16*sinh(2*a+2*c+2*(b+d)*x)/(b+d)
```

3.176.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \frac{2d(b^2-d^2)\sinh(2(a+bx))+bd(b+d)\sinh(2(a-c+(b-d)x))+b(b-d)(2(b+d)\sinh(2(c+dx))+}{16b(b-d)d(b+d)}$$

input
```
Integrate[Cosh[a + b*x]^2*Cosh[c + d*x]^2,x]
```

output
```
(2*d*(b^2 - d^2)*Sinh[2*(a + b*x)] + b*d*(b + d)*Sinh[2*(a - c + (b - d)*x
)] + b*(b - d)*(2*(b + d)*Sinh[2*(c + d*x)] + d*(4*(b + d)*x + Sinh[2*(a +
 c + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

3.176.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6148, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx$$

$$\Big\downarrow \text{6148}$$

$$\int \left(\frac{1}{8}\cosh(2(a-c)+2x(b-d))+\frac{1}{8}\cosh(2(a+c)+2x(b+d))+\frac{1}{4}\cosh(2a+2bx)+\frac{1}{4}\cosh(2c+2dx)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2c+2dx)}{8d}+\frac{x}{4}$$

input
```
Int[Cosh[a + b*x]^2*Cosh[c + d*x]^2,x]
```

output
```
x/4 + Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
 + Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

3.176.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6148
```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.176.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$\frac{x}{4}+\frac{\sinh(2bx+2a)}{8b}+\frac{\sinh(2dx+2c)}{8d}+\frac{\sinh((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sinh((2b+2d)x+2a+2c)}{16b+16d}$
parallelrisch	$\frac{bd(b+d)\sinh((2b-2d)x+2a-2c)+4\left(\frac{bd\sinh((2b+2d)x+2a+2c)}{4}+\left(\frac{d\sinh(2bx+2a)}{2}+b\left(dx+\frac{\sinh(2dx+2c)}{2}\right)\right)(b+d)\right)(b-d)}{16b^3d-16d^3b}$
risch	$\frac{x}{4}+\frac{e^{2bx+2a}}{16b}-\frac{e^{-2bx-2a}}{16b}+\frac{\left(d\,e^{4bx+4a}b-d^2e^{4bx+4a}+2b^2e^{2bx+2a}-2\,e^{2bx+2a}d^2-bd-d^2\right)e^{-2bx+2dx-2a+2c}}{32(b+d)(b-d)d}-\frac{(-d\,e^{4bx+}}{}$

input
```
int(cosh(b*x+a)^2*cosh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/8*sinh(2*b*x+2*a)/b+1/8*sinh(2*d*x+2*c)/d+1/8/(2*b-2*d)*sinh((2*b-
2*d)*x+2*a-2*c)+1/8/(2*b+2*d)*sinh((2*b+2*d)*x+2*a+2*c)
```

3.176.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(78) = 156.

Time = 0.24 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.18

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx$$
$$=\frac{b^2d\cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^2+(b^3d-bd^3)x+\left(b^2d\cosh(bx+a)\cosh(dx+c)^2+(b^2d-\right.}{4\left((b^3d-bd^3)\cos\right.}$$

input
```
integrate(cosh(b*x+a)^2*cosh(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(b^2*d*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^2 + (b^3*d - b*d^3)*x
 + (b^2*d*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - d^3)*cosh(b*x + a))*sin
h(b*x + a) - (b*d^2*cosh(d*x + c)*sinh(b*x + a)^2 + (b*d^2*cosh(b*x + a)^2
 - b^3 + b*d^2)*cosh(d*x + c))*sinh(d*x + c))/((b^3*d - b*d^3)*cosh(b*x +
a)^2 - (b^3*d - b*d^3)*sinh(b*x + a)^2)
```

3.176.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(76) = 152$.

Time = 1.58 (sec) , antiderivative size = 1027, normalized size of antiderivative = 11.67

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)**2*cosh(d*x+c)**2,x)
```

output
```
Piecewise((x*cosh(a)**2*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*
x)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*cosh(a
)**2, Eq(b, 0)), (3*x*sinh(a - d*x)**2*sinh(c + d*x)**2/8 - x*sinh(a - d*x
)**2*cosh(c + d*x)**2/8 + x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)*cosh
(c + d*x)/2 - x*sinh(c + d*x)**2*cosh(a - d*x)**2/8 + 3*x*cosh(a - d*x)**2
*cosh(c + d*x)**2/8 + sinh(a - d*x)**2*sinh(c + d*x)*cosh(c + d*x)/(8*d) +
 sinh(a - d*x)*sinh(c + d*x)**2*cosh(a - d*x)/(2*d) + 5*sinh(c + d*x)*cosh
(a - d*x)**2*cosh(c + d*x)/(8*d), Eq(b, -d)), (3*x*sinh(a + d*x)**2*sinh(c
 + d*x)**2/8 - x*sinh(a + d*x)**2*cosh(c + d*x)**2/8 - x*sinh(a + d*x)*sin
h(c + d*x)*cosh(a + d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)**2*cosh(a + d*x
)**2/8 + 3*x*cosh(a + d*x)**2*cosh(c + d*x)**2/8 + sinh(a + d*x)**2*sinh(c
 + d*x)*cosh(c + d*x)/(8*d) - sinh(a + d*x)*sinh(c + d*x)**2*cosh(a + d*x)
/(2*d) + 5*sinh(c + d*x)*cosh(a + d*x)**2*cosh(c + d*x)/(8*d), Eq(b, d)),
((-x*sinh(a + b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*
x)/(2*b))*cosh(c)**2, Eq(d, 0)), (b**3*d*x*sinh(a + b*x)**2*sinh(c + d*x)*
*2/(4*b**3*d - 4*b*d**3) - b**3*d*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4*b
**3*d - 4*b*d**3) - b**3*d*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d -
 4*b*d**3) + b**3*d*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d*
*3) - b**3*sinh(a + b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*d*
*3) + b**3*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)/(4*b**3*d - 4*b...
```

3.176.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(b*x+a)^2*cosh(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-(2*d)/b>0)', see `assume?` for
 more deta
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \frac{1}{4}\,x + \frac{e^{(2\,bx+2\,dx+2\,a+2\,c)}}{32\,(b+d)} + \frac{e^{(2\,bx-2\,dx+2\,a-2\,c)}}{32\,(b-d)} + \frac{e^{(2\,bx+2\,a)}}{16\,b} - \frac{e^{(-2\,bx+2\,dx-2\,a+2\,c)}}{32\,(b-d)} - \frac{e^{(-2\,bx-2\,dx-2\,a-2\,c)}}{32\,(b+d)} - \frac{e^{(-2\,bx-2\,a)}}{16\,b} + \frac{e^{(2\,dx+2\,c)}}{16\,d} - \frac{e^{(-2\,dx-2\,c)}}{16\,d}$$

input
```
integrate(cosh(b*x+a)^2*cosh(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*x + 1/32*e^(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/32*e^(2*b*x - 2*d*x
 + 2*a - 2*c)/(b - d) + 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x + 2*d*x -
2*a + 2*c)/(b - d) - 1/32*e^(-2*b*x - 2*d*x - 2*a - 2*c)/(b + d) - 1/16*e^
(-2*b*x - 2*a)/b + 1/16*e^(2*d*x + 2*c)/d - 1/16*e^(-2*d*x - 2*c)/d
```

3.176.9 Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \frac{d^3\cosh(a+b\,x)\,\sinh(a+b\,x) - b^3\cosh(c+d\,x)\,\sinh(c+d\,x) + b\,d^3\,x - b^3\,d\,x - 2\,b^2\,d\cosh(a+b\,x)\,\cosh}{4\,b\,d^3 - 4\,b^3\,d}$$

input
```
int(cosh(a + b*x)^2*cosh(c + d*x)^2,x)
```

output
```
(d^3*cosh(a + b*x)*sinh(a + b*x) - b^3*cosh(c + d*x)*sinh(c + d*x) + b*d^3
*x - b^3*d*x - 2*b^2*d*cosh(a + b*x)*cosh(c + d*x)^2*sinh(a + b*x) + 2*b*d
^2*cosh(a + b*x)^2*cosh(c + d*x)*sinh(c + d*x))/(4*b*d^3 - 4*b^3*d)
```

3.177 $\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx$

3.177.1 Optimal result

Integrand size = 17, antiderivative size = 144

$$\begin{aligned}\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx =& \frac{\sinh(2a-3c+(2b-3d)x)}{16(2b-3d)}\\ &+\frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sinh(c+dx)}{8d}\\ &+\frac{\sinh(3c+3dx)}{24d}+\frac{3\sinh(2a+c+(2b+d)x)}{16(2b+d)}\\ &+\frac{\sinh(2a+3c+(2b+3d)x)}{16(2b+3d)}\end{aligned}$$

output
```
1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d
)+3/8*sinh(d*x+c)/d+1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b
+d)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

3.177.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \frac{1}{48}\left(\frac{18\cosh(dx)\sinh(c)}{d}+\frac{2\cosh(3dx)\sinh(3c)}{d}+\frac{18\cosh(c)\sinh(dx)}{d}+\frac{2\cosh(3c)\sinh(3dx)}{d}+\frac{3\sinh(2a-3c+2bx-3dx)}{2b-3d}+\frac{9\sinh(2a-c+2bx-dx)}{2b-d}+\frac{9\sinh(2a+c+2bx+dx)}{2b+d}+\frac{3\sinh(2a+3c+2bx+3dx)}{2b+3d}\right)$$

input
```
Integrate[Cosh[a + b*x]^2*Cosh[c + d*x]^3,x]
```

output
```
((18*Cosh[d*x]*Sinh[c])/d + (2*Cosh[3*d*x]*Sinh[3*c])/d + (18*Cosh[c]*Sinh
[d*x])/d + (2*Cosh[3*c]*Sinh[3*d*x])/d + (3*Sinh[2*a - 3*c + 2*b*x - 3*d*x
])/(2*b - 3*d) + (9*Sinh[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sinh[2*a +
 c + 2*b*x + d*x])/(2*b + d) + (3*Sinh[2*a + 3*c + 2*b*x + 3*d*x])/(2*b +
3*d))/48
```

3.177.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6148, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx$$

$$\Big\downarrow \text{6148}$$

$$\int \left(\frac{1}{16}\cosh(2a+x(2b-3d)-3c)+\frac{3}{16}\cosh(2a+x(2b-d)-c)+\frac{3}{16}\cosh(2a+x(2b+d)+c)+\frac{1}{16}\cosh(2a+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sinh(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sinh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sinh(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\sinh(c+dx)}{8d}+\frac{\sinh(3c+3dx)}{24d}$$

input
```
Int[Cosh[a + b*x]^2*Cosh[c + d*x]^3,x]
```

output
```
Sinh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sinh[2*a - c + (2*b
- d)*x])/(16*(2*b - d)) + (3*Sinh[c + d*x])/(8*d) + Sinh[3*c + 3*d*x]/(24*
d) + (3*Sinh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sinh[2*a + 3*c + (2*
b + 3*d)*x]/(16*(2*b + 3*d))
```

3.177.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6148
```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.177.4 Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sinh(2a-3c+(2b-3d)x)}{32b-48d}+\frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sinh(dx+c)}{8d}+\frac{\sinh(3dx+3c)}{24d}+\frac{3\sinh(2a+c+(2b+d)x)}{16(2b+d)}+\sinh($
parallelrisch	$\frac{(24b^3d+36d^2b^2-6d^3b-9d^4)\sinh(2a-3c+(2b-3d)x)+72\left(d\left(b+\frac{3d}{2}\right)\left(b+\frac{d}{2}\right)\sinh(2a-c+(2b-d)x)+\left(\frac{d\left(b+\frac{d}{2}\right)\sinh(2a+3c+(2b+}{3}\right.\right.}{768d\,b^4-1920b^2d^3+432d^5}$
risch	$\frac{\left(6d\,e^{4bx+4a}b-9d^2e^{4bx+4a}+8b^2e^{2bx+2a}-18\,e^{2bx+2a}d^2-6bd-9d^2\right)e^{-2bx+3dx-2a+3c}}{96(2b+3d)(2b-3d)d}+\frac{3\left(2d\,e^{4bx+4a}b-d^2e^{4bx+4a}+8b^2e^{2bx+}\right.}{32(2b-}$

input
```
int(cosh(b*x+a)^2*cosh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d
)+3/8*sinh(d*x+c)/d+1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b
+d)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

3.177.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(132) = 264$.

Time = 0.25 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.76

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx$$
$$= \frac{36\,(4\,b^3d - bd^3)\cosh(bx+a)\cosh(dx+c)\sinh(bx+a)\sinh(dx+c)^2 + (16\,b^4 - 40\,b^2d^2 + 9\,d^4 - 9\,(4\,b^2}{}$$

input
```
integrate(cosh(b*x+a)^2*cosh(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/24*(36*(4*b^3*d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)*sinh(
d*x + c)^2 + (16*b^4 - 40*b^2*d^2 + 9*d^4 - 9*(4*b^2*d^2 - d^4)*cosh(b*x +
 a)^2 - 9*(4*b^2*d^2 - d^4)*sinh(b*x + a)^2)*sinh(d*x + c)^3 + 12*((4*b^3*
d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)^3 + 3*(4*b^3*d - 9*b*d^3)*cosh(b*x
+ a)*cosh(d*x + c))*sinh(b*x + a) + 3*(48*b^4 - 120*b^2*d^2 + 27*d^4 - 3*(
4*b^2*d^2 - 9*d^4)*cosh(b*x + a)^2 + (16*b^4 - 40*b^2*d^2 + 9*d^4 - 9*(4*b
^2*d^2 - d^4)*cosh(b*x + a)^2)*cosh(d*x + c)^2 - 3*(4*b^2*d^2 - 9*d^4 + 3*
(4*b^2*d^2 - d^4)*cosh(d*x + c)^2)*sinh(b*x + a)^2)*sinh(d*x + c))/((16*b^
4*d - 40*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 - (16*b^4*d - 40*b^2*d^3 + 9*d^5
)*sinh(b*x + a)^2)
```

3.177.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2008 vs. $2(116) = 232$.

Time = 5.29 (sec) , antiderivative size = 2008, normalized size of antiderivative = 13.94

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)**2*cosh(d*x+c)**3,x)
```

output
```
Piecewise((x*cosh(a)**2*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - 3*
d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/16 + x*sinh(a - 3*d*x/2)**2*cosh(
c + d*x)**3/16 + x*sinh(a - 3*d*x/2)*sinh(c + d*x)**3*cosh(a - 3*d*x/2)/8
+ 3*x*sinh(a - 3*d*x/2)*sinh(c + d*x)*cosh(a - 3*d*x/2)*cosh(c + d*x)**2/8
 + 3*x*sinh(c + d*x)**2*cosh(a - 3*d*x/2)**2*cosh(c + d*x)/16 + x*cosh(a -
 3*d*x/2)**2*cosh(c + d*x)**3/16 + 9*sinh(a - 3*d*x/2)**2*sinh(c + d*x)**3
/(16*d) + 5*sinh(a - 3*d*x/2)*sinh(c + d*x)**2*cosh(a - 3*d*x/2)*cosh(c +
d*x)/(4*d) + sinh(a - 3*d*x/2)*cosh(a - 3*d*x/2)*cosh(c + d*x)**3/(24*d) -
 5*sinh(c + d*x)**3*cosh(a - 3*d*x/2)**2/(48*d) + sinh(c + d*x)*cosh(a - 3
*d*x/2)**2*cosh(c + d*x)**2/d, Eq(b, -3*d/2)), (-3*x*sinh(a - d*x/2)**2*si
nh(c + d*x)**2*cosh(c + d*x)/16 + 3*x*sinh(a - d*x/2)**2*cosh(c + d*x)**3/
16 - 3*x*sinh(a - d*x/2)*sinh(c + d*x)**3*cosh(a - d*x/2)/8 + 3*x*sinh(a -
 d*x/2)*sinh(c + d*x)*cosh(a - d*x/2)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*
x)**2*cosh(a - d*x/2)**2*cosh(c + d*x)/16 + 3*x*cosh(a - d*x/2)**2*cosh(c
+ d*x)**3/16 + 49*sinh(a - d*x/2)**2*sinh(c + d*x)**3/(48*d) - sinh(a - d*
x/2)**2*sinh(c + d*x)*cosh(c + d*x)**2/d + 7*sinh(a - d*x/2)*sinh(c + d*x)
**2*cosh(a - d*x/2)*cosh(c + d*x)/(4*d) - 13*sinh(a - d*x/2)*cosh(a - d*x/
2)*cosh(c + d*x)**3/(8*d) + 17*sinh(c + d*x)**3*cosh(a - d*x/2)**2/(48*d),
 Eq(b, -d/2)), (-3*x*sinh(a + d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/16
+ 3*x*sinh(a + d*x/2)**2*cosh(c + d*x)**3/16 + 3*x*sinh(a + d*x/2)*sinh...
```

3.177.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(b*x+a)^2*cosh(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-(3*d)/b>0)', see `assume?` for
 more deta
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.81

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \frac{e^{(2\,bx+3\,dx+2\,a+3\,c)}}{32\,(2\,b+3\,d)} + \frac{3\,e^{(2\,bx+dx+2\,a+c)}}{32\,(2\,b+d)} + \frac{3\,e^{(2\,bx-dx+2\,a-c)}}{32\,(2\,b-d)} + \frac{e^{(2\,bx-3\,dx+2\,a-3\,c)}}{32\,(2\,b-3\,d)} - \frac{e^{(-2\,bx+3\,dx-2\,a+3\,c)}}{32\,(2\,b-3\,d)} - \frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{32\,(2\,b-d)} - \frac{3\,e^{(-2\,bx-dx-2\,a-c)}}{32\,(2\,b+d)} - \frac{e^{(-2\,bx-3\,dx-2\,a-3\,c)}}{32\,(2\,b+3\,d)} + \frac{e^{(3\,dx+3\,c)}}{48\,d} + \frac{3\,e^{(dx+c)}}{16\,d} - \frac{3\,e^{(-dx-c)}}{16\,d} - \frac{e^{(-3\,dx-3\,c)}}{48\,d}$$

input
```
integrate(cosh(b*x+a)^2*cosh(d*x+c)^3,x, algorithm="giac")
```

output
```
1/32*e^(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/32*e^(2*b*x + d*x + 2*a
 + c)/(2*b + d) + 3/32*e^(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/32*e^(2*b*x
 - 3*d*x + 2*a - 3*c)/(2*b - 3*d) - 1/32*e^(-2*b*x + 3*d*x - 2*a + 3*c)/(2
*b - 3*d) - 3/32*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 3/32*e^(-2*b*x - d
*x - 2*a - c)/(2*b + d) - 1/32*e^(-2*b*x - 3*d*x - 2*a - 3*c)/(2*b + 3*d)
+ 1/48*e^(3*d*x + 3*c)/d + 3/16*e^(d*x + c)/d - 3/16*e^(-d*x - c)/d - 1/48
*e^(-3*d*x - 3*c)/d
```

3.177.9 Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.34

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx$$
$$= \frac{\cosh(a+b\,x)^2\cosh(c+d\,x)^2\sinh(c+d\,x)\,\left(8\,b^4-26\,b^2\,d^2+9\,d^4\right)}{d\,\left(16\,b^4-40\,b^2\,d^2+9\,d^4\right)}$$
$$-\sinh(a+b\,x)^2\sinh(c+d\,x)^3\left(\frac{3\,d^3}{16\,b^4-40\,b^2\,d^2+9\,d^4}-\frac{1}{3\,d}\right)$$
$$-\frac{2\cosh(a+b\,x)\,\cosh(c+d\,x)^3\sinh(a+b\,x)\,\left(7\,b\,d^2-4\,b^3\right)}{16\,b^4-40\,b^2\,d^2+9\,d^4}$$
$$-\cosh(a+b\,x)^2\sinh(c+d\,x)^3\left(\frac{3\,d^3}{16\,b^4-40\,b^2\,d^2+9\,d^4}+\frac{1}{3\,d}\right)$$
$$+\frac{12\,b\,d^2\cosh(a+b\,x)\,\cosh(c+d\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)^2}{16\,b^4-40\,b^2\,d^2+9\,d^4}$$
$$-\frac{2\,b^2\cosh(c+d\,x)^2\sinh(a+b\,x)^2\sinh(c+d\,x)\,\left(4\,b^2-7\,d^2\right)}{d\,\left(16\,b^4-40\,b^2\,d^2+9\,d^4\right)}$$

input
```
int(cosh(a + b*x)^2*cosh(c + d*x)^3,x)
```

output
```
(cosh(a + b*x)^2*cosh(c + d*x)^2*sinh(c + d*x)*(8*b^4 + 9*d^4 - 26*b^2*d^2
))/(d*(16*b^4 + 9*d^4 - 40*b^2*d^2)) - sinh(a + b*x)^2*sinh(c + d*x)^3*((3
*d^3)/(16*b^4 + 9*d^4 - 40*b^2*d^2) - 1/(3*d)) - (2*cosh(a + b*x)*cosh(c +
 d*x)^3*sinh(a + b*x)*(7*b*d^2 - 4*b^3))/(16*b^4 + 9*d^4 - 40*b^2*d^2) - c
osh(a + b*x)^2*sinh(c + d*x)^3*((3*d^3)/(16*b^4 + 9*d^4 - 40*b^2*d^2) + 1/
(3*d)) + (12*b*d^2*cosh(a + b*x)*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x)
^2)/(16*b^4 + 9*d^4 - 40*b^2*d^2) - (2*b^2*cosh(c + d*x)^2*sinh(a + b*x)^2
*sinh(c + d*x)*(4*b^2 - 7*d^2))/(d*(16*b^4 + 9*d^4 - 40*b^2*d^2))
```

3.178 $\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx$

3.178.1 Optimal result

Integrand size = 17, antiderivative size = 195

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = \frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sinh(a-c+(b-d)x)}{32(b-d)}+\frac{\sinh(3(a-c)+3(b-d)x)}{96(b-d)}+\frac{3\sinh(3a-c+(3b-d)x)}{32(3b-d)}+\frac{9\sinh(a+c+(b+d)x)}{32(b+d)}+\frac{\sinh(3(a+c)+3(b+d)x)}{96(b+d)}+\frac{3\sinh(3a+c+(3b+d)x)}{32(3b+d)}+\frac{3\sinh(a+3c+(b+3d)x)}{32(b+3d)}$$

output
```
3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sinh(a-c+(b-d)*x)/(b-d)+1/96*sinh(
3*a-3*c+3*(b-d)*x)/(b-d)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)+9/32*sinh(a+c+
(b+d)*x)/(b+d)+1/96*sinh(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*sinh(3*a+c+(3*b+d)*
x)/(3*b+d)+3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.178.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = \frac{1}{96}\left(\frac{9\sinh(a-3c+bx-3dx)}{b-3d} + \frac{27\sinh(a-c+bx-dx)}{b-d} + \frac{\sinh(3(a-c+bx-dx))}{b-d} + \frac{9\sinh(3a-c+3bx-dx)}{3b-d} + \frac{9\sinh(3a+c+3bx+dx)}{3b+d} + \frac{9\sinh(a+3c+bx+3dx)}{b+3d} + \frac{27\sinh(a+c+(b+d)x)}{b+d} + \frac{\sinh(3(a+c+(b+d)x))}{b+d}\right)$$

input
```
Integrate[Cosh[a + b*x]^3*Cosh[c + d*x]^3,x]
```

output
```
((9*Sinh[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sinh[a - c + b*x - d*x])/
(b - d) + Sinh[3*(a - c + b*x - d*x)]/(b - d) + (9*Sinh[3*a - c + 3*b*x -
d*x])/(3*b - d) + (9*Sinh[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sinh[a +
3*c + b*x + 3*d*x])/(b + 3*d) + (27*Sinh[a + c + (b + d)*x])/(b + d) + Sin
h[3*(a + c + (b + d)*x)]/(b + d))/96
```

3.178.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6148, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx$$

$$\Big\downarrow \; 6148$$

$$\int \left(\frac{3}{32}\cosh(a+x(b-3d)-3c)+\frac{9}{32}\cosh(a+x(b-d)-c)+\frac{1}{32}\cosh(3(a-c)+3x(b-d))+\frac{3}{32}\cosh(3a+x(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sinh(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sinh(a+x(b-d)-c)}{32(b-d)}+\frac{\sinh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sinh(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sinh(a+x(b+d)+c)}{32(b+d)}+\frac{\sinh(3(a+c)+3x(b+d))}{96(b+d)}+\frac{3\sinh(3a+x(3b+d)+c)}{32(3b+d)}+\frac{3\sinh(a+x(b+3d)+3c)}{32(b+3d)}$$

input
```
Int[Cosh[a + b*x]^3*Cosh[c + d*x]^3,x]
```

output
```
(3*Sinh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sinh[a - c + (b - d)*x
])/(32*(b - d)) + Sinh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sinh[3*a
 - c + (3*b - d)*x])/(32*(3*b - d)) + (9*Sinh[a + c + (b + d)*x])/(32*(b +
 d)) + Sinh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sinh[3*a + c + (3*b
 + d)*x])/(32*(3*b + d)) + (3*Sinh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

3.178.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6148
```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.178.4 Maple [A] (verified)

Time = 11.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

method	result
default	$\frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sinh(a-c+(b-d)x)}{32(b-d)}+\frac{9\sinh(a+c+(b+d)x)}{32(b+d)}+\frac{3\sinh(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\sinh((3b-3d)x+3a}{96b-96d}$
parallelrisch	$\frac{\frac{9(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b-d)(b+d)\sinh(3a-c+(3b-d)x)}{32}+\frac{9\left(\frac{(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b+d)\sinh((3b-3d)x+3a-3c)}{3}+\frac{(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b-d}{3}\right.}{}}{}$
risch	$\frac{(b^3e^{6bx+6a}-b^2d\,e^{6bx+6a}-9b\,d^2e^{6bx+6a}+9d^3e^{6bx+6a}+9b^3e^{4bx+4a}-27b^2d\,e^{4bx+4a}-9b\,d^2e^{4bx+4a}+27d^3e^{4bx+4a}-9b^3e^{2bx+2a}}{192(b+d)(b+3d)(b-d)(b-3d)}$

input
```
int(cosh(b*x+a)^3*cosh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sinh(a-c+(b-d)*x)/(b-d)+9/32*sinh(
a+c+(b+d)*x)/(b+d)+3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)+1/32/(3*b-3*d)*sinh(
(3*b-3*d)*x+3*a-3*c)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*sinh(3*a+c+(3
*b+d)*x)/(3*b+d)+1/32/(3*b+3*d)*sinh((3*b+3*d)*x+3*a+3*c)
```

3.178.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(179) = 358.

Time = 0.27 (sec) , antiderivative size = 726, normalized size of antiderivative = 3.72

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx$$
$$=\frac{\left((9\,b^5-82\,b^3d^2+9\,bd^4)\cosh\left(dx+c\right)^3+27\left(b^5-10\,b^3d^2+9\,bd^4\right)\cosh\left(dx+c\right)\right)\sinh\left(bx+a\right)^3-\left((9\,b^4d\right.}{}$$

input
```
integrate(cosh(b*x+a)^3*cosh(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/48*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(d*x + c)^3 + 27*(b^5 - 10*b^3*d
^2 + 9*b*d^4)*cosh(d*x + c))*sinh(b*x + a)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*
d^5)*cosh(b*x + a)^3 + 3*(9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)*sinh
(b*x + a)^2 + 27*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a))*sinh(d*x + c)
^3 + 3*((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(d*x + c)*sinh(b*x + a)^3 + 3*(
27*b^5 - 30*b^3*d^2 + 3*b*d^4 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x +
a)^2)*cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 + 3*((27*b^5 - 30*b^3*d
^2 + 3*b*d^4 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^2)*cosh(d*x +
c)^3 + 9*(9*b^5 - 82*b^3*d^2 + 9*b*d^4 + 3*(b^5 - 10*b^3*d^2 + 9*b*d^4)*co
sh(b*x + a)^2)*cosh(d*x + c))*sinh(b*x + a) - 3*(3*(b^4*d - 10*b^2*d^3 + 9
*d^5)*cosh(b*x + a)^3 + ((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^3 +
27*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a))*cosh(d*x + c)^2 + 3*((9*b^4
*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)*cosh(d*x + c)^2 + 3*(b^4*d - 10*b^2
*d^3 + 9*d^5)*cosh(b*x + a))*sinh(b*x + a)^2 + 9*(9*b^4*d - 82*b^2*d^3 + 9
*d^5)*cosh(b*x + a))*sinh(d*x + c))/((9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*
d^6)*cosh(b*x + a)^4 - 2*(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*
x + a)^2*sinh(b*x + a)^2 + (9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*sinh(
b*x + a)^4)
```

3.178.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3582 vs. $2(172) = 344$.

Time = 16.83 (sec) , antiderivative size = 3582, normalized size of antiderivative = 18.37

$$\int \cosh^3(a + bx) \cosh^3(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)**3*cosh(d*x+c)**3,x)
```

output
```
Piecewise((x*cosh(a)**3*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*sinh(a - 3
*d*x)**3*sinh(c + d*x)**3/32 - 9*x*sinh(a - 3*d*x)**3*sinh(c + d*x)*cosh(c
 + d*x)**2/32 - 9*x*sinh(a - 3*d*x)**2*sinh(c + d*x)**2*cosh(a - 3*d*x)*co
sh(c + d*x)/32 - 3*x*sinh(a - 3*d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)**3/3
2 + 3*x*sinh(a - 3*d*x)*sinh(c + d*x)**3*cosh(a - 3*d*x)**2/32 + 9*x*sinh(
a - 3*d*x)*sinh(c + d*x)*cosh(a - 3*d*x)**2*cosh(c + d*x)**2/32 + 9*x*sinh
(c + d*x)**2*cosh(a - 3*d*x)**3*cosh(c + d*x)/32 + 3*x*cosh(a - 3*d*x)**3*
cosh(c + d*x)**3/32 - 3*sinh(a - 3*d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/
(320*d) + sinh(a - 3*d*x)**3*cosh(c + d*x)**3/(4*d) - 11*sinh(a - 3*d*x)**
2*sinh(c + d*x)**3*cosh(a - 3*d*x)/(320*d) + 3*sinh(a - 3*d*x)**2*sinh(c +
 d*x)*cosh(a - 3*d*x)*cosh(c + d*x)**2/(20*d) - 117*sinh(a - 3*d*x)*cosh(a
 - 3*d*x)**2*cosh(c + d*x)**3/(320*d) + sinh(c + d*x)**3*cosh(a - 3*d*x)**
3/(30*d) - 61*sinh(c + d*x)*cosh(a - 3*d*x)**3*cosh(c + d*x)**2/(320*d), E
q(b, -3*d)), (5*x*sinh(a - d*x)**3*sinh(c + d*x)**3/16 - 3*x*sinh(a - d*x)
**3*sinh(c + d*x)*cosh(c + d*x)**2/16 + 9*x*sinh(a - d*x)**2*sinh(c + d*x)
**2*cosh(a - d*x)*cosh(c + d*x)/16 - 3*x*sinh(a - d*x)**2*cosh(a - d*x)*co
sh(c + d*x)**3/16 - 3*x*sinh(a - d*x)*sinh(c + d*x)**3*cosh(a - d*x)**2/16
 + 9*x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)**2*cosh(c + d*x)**2/16 -
3*x*sinh(c + d*x)**2*cosh(a - d*x)**3*cosh(c + d*x)/16 + 5*x*cosh(a - d*x)
**3*cosh(c + d*x)**3/16 - sinh(a - d*x)**3*sinh(c + d*x)**2*cosh(c + d*...
```

3.178.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(b*x+a)^3*cosh(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for
more detai
```

3.178.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(179) = 358$.

Time = 0.28 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.91

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = \frac{e^{(3\,bx+3\,dx+3\,a+3\,c)}}{192\,(b+d)} + \frac{3\,e^{(3\,bx+dx+3\,a+c)}}{64\,(3\,b+d)} + \frac{3\,e^{(3\,bx-dx+3\,a-c)}}{64\,(3\,b-d)} + \frac{e^{(3\,bx-3\,dx+3\,a-3\,c)}}{192\,(b-d)} + \frac{3\,e^{(bx+3\,dx+a+3\,c)}}{64\,(b+3\,d)} + \frac{9\,e^{(bx+dx+a+c)}}{64\,(b+d)} + \frac{9\,e^{(bx-dx+a-c)}}{64\,(b-d)} + \frac{3\,e^{(bx-3\,dx+a-3\,c)}}{64\,(b-3\,d)} - \frac{3\,e^{(-bx+3\,dx-a+3\,c)}}{64\,(b-3\,d)} - \frac{9\,e^{(-bx+dx-a+c)}}{64\,(b-d)} - \frac{9\,e^{(-bx-dx-a-c)}}{64\,(b+d)} - \frac{3\,e^{(-bx-3\,dx-a-3\,c)}}{64\,(b+3\,d)} - \frac{e^{(-3\,bx+3\,dx-3\,a+3\,c)}}{192\,(b-d)} - \frac{3\,e^{(-3\,bx+dx-3\,a+c)}}{64\,(3\,b-d)} - \frac{3\,e^{(-3\,bx-dx-3\,a-c)}}{64\,(3\,b+d)} - \frac{e^{(-3\,bx-3\,dx-3\,a-3\,c)}}{192\,(b+d)}$$

input
```
integrate(cosh(b*x+a)^3*cosh(d*x+c)^3,x, algorithm="giac")
```

output
```
1/192*e^(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/64*e^(3*b*x + d*x + 3*a +
c)/(3*b + d) + 3/64*e^(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/192*e^(3*b*x -
 3*d*x + 3*a - 3*c)/(b - d) + 3/64*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 9
/64*e^(b*x + d*x + a + c)/(b + d) + 9/64*e^(b*x - d*x + a - c)/(b - d) + 3
/64*e^(b*x - 3*d*x + a - 3*c)/(b - 3*d) - 3/64*e^(-b*x + 3*d*x - a + 3*c)/
(b - 3*d) - 9/64*e^(-b*x + d*x - a + c)/(b - d) - 9/64*e^(-b*x - d*x - a -
 c)/(b + d) - 3/64*e^(-b*x - 3*d*x - a - 3*c)/(b + 3*d) - 1/192*e^(-3*b*x
+ 3*d*x - 3*a + 3*c)/(b - d) - 3/64*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) -
 3/64*e^(-3*b*x - d*x - 3*a - c)/(3*b + d) - 1/192*e^(-3*b*x - 3*d*x - 3*a
 - 3*c)/(b + d)
```

3.178.9 Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 908, normalized size of antiderivative = 4.66

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = -e^{3a+c+3bx+dx}\left(\frac{-9b^3+3b^2d+9bd^2-3d^3}{576b^4-640b^2d^2+64d^4} - \frac{e^{-6a-6bx}\left(-9b^3-3b^2d+9bd^2+3d^3\right)}{576b^4-640b^2d^2+64d^4} + \frac{e^{-2a-2bx}\left(-81b^3+81b^2d+9bd^2-9d^3\right)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-4a-4bx}\left(-81b^3-81b^2d+9bd^2+9d^3\right)}{576b^4-640b^2d^2+64d^4}\right) - e^{3a-c+3bx-dx}\left(\frac{-9b^3-3b^2d+9bd^2+3d^3}{576b^4-640b^2d^2+64d^4} - \frac{e^{-6a-6bx}\left(-9b^3+3b^2d+9bd^2-3d^3\right)}{576b^4-640b^2d^2+64d^4} + \frac{e^{-2a-2bx}\left(-81b^3-81b^2d+9bd^2+9d^3\right)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-4a-4bx}\left(-81b^3+81b^2d+9bd^2-9d^3\right)}{576b^4-640b^2d^2+64d^4}\right) - e^{3a-3c+3bx-3dx}\left(\frac{-b^3-b^2d+9bd^2+9d^3}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-6a-6bx}\left(-b^3+b^2d+9bd^2-9d^3\right)}{192b^4-1920b^2d^2+1728d^4} + \frac{e^{-2a-2bx}\left(-9b^3-27b^2d+9bd^2+27d^3\right)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-4a-4bx}\left(-9b^3+27b^2d+9bd^2-27d^3\right)}{192b^4-1920b^2d^2+1728d^4}\right) - e^{3a+3c+3bx+3dx}\left(\frac{-b^3+b^2d+9bd^2-9d^3}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-6a-6bx}\left(-b^3-b^2d+9bd^2+9d^3\right)}{192b^4-1920b^2d^2+1728d^4} + \frac{e^{-2a-2bx}\left(-9b^3+27b^2d+9bd^2-27d^3\right)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-4a-4bx}\left(-9b^3-27b^2d+9bd^2+27d^3\right)}{192b^4-1920b^2d^2+1728d^4}\right)$$

input
```
int(cosh(a + b*x)^3*cosh(c + d*x)^3,x)
```

output
```
- exp(3*a + c + 3*b*x + d*x)*((9*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(576*b^4
 + 64*d^4 - 640*b^2*d^2) - (exp(- 6*a - 6*b*x)*(9*b*d^2 - 3*b^2*d - 9*b^3
+ 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- 2*a - 2*b*x)*(9*b*d^2
+ 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4*
a - 4*b*x)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*
b^2*d^2)) - exp(3*a - c + 3*b*x - d*x)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3
)/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 6*a - 6*b*x)*(9*b*d^2 + 3*b^2*
d - 9*b^3 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- 2*a - 2*b*x)
*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) -
 (exp(- 4*a - 4*b*x)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*
d^4 - 640*b^2*d^2)) - exp(3*a - 3*c + 3*b*x - 3*d*x)*((9*b*d^2 - b^2*d - b
^3 + 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 6*a - 6*b*x)*(9*b
*d^2 + b^2*d - b^3 - 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(-
2*a - 2*b*x)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(192*b^4 + 1728*d^4 -
1920*b^2*d^2) - (exp(- 4*a - 4*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))
/(192*b^4 + 1728*d^4 - 1920*b^2*d^2)) - exp(3*a + 3*c + 3*b*x + 3*d*x)*((9
*b*d^2 + b^2*d - b^3 - 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(-
 6*a - 6*b*x)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(192*b^4 + 1728*d^4 - 1920*
b^2*d^2) + (exp(- 2*a - 2*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192
*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 4*a - 4*b*x)*(9*b*d^2 - 27*b^2...
```

3.178. $\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx$

3.179 $\int \cosh(c+dx)\sinh(a+bx)\,dx$

3.179.1 Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \cosh(c+dx)\sinh(a+bx)\,dx=\frac{\cosh(a-c+(b-d)x)}{2(b-d)}+\frac{\cosh(a+c+(b+d)x)}{2(b+d)}$$

output
```
1/2*cosh(a-c+(b-d)*x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

3.179.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \cosh(c+dx)\sinh(a+bx)\,dx=\frac{\cosh(a-c+(b-d)x)}{2(b-d)}+\frac{\cosh(a+c+(b+d)x)}{2(b+d)}$$

input
```
Integrate[Cosh[c + d*x]*Sinh[a + b*x],x]
```

output
```
Cosh[a - c + (b - d)*x]/(2*(b - d)) + Cosh[a + c + (b + d)*x]/(2*(b + d))
```

3.179.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6152, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\cosh(c+dx)\,dx$$

$$\Big\downarrow \; 6152$$

$$\int \left(\frac{1}{2}\sinh(a+x(b-d)-c)+\frac{1}{2}\sinh(a+x(b+d)+c)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\cosh(a+x(b-d)-c)}{2(b-d)}+\frac{\cosh(a+x(b+d)+c)}{2(b+d)}$$

input
```
Int[Cosh[c + d*x]*Sinh[a + b*x],x]
```

output
```
Cosh[a - c + (b - d)*x]/(2*(b - d)) + Cosh[a + c + (b + d)*x]/(2*(b + d))
```

3.179.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6152
```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.179.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\cosh(a-c+(b-d)x)}{2b-2d}+\frac{\cosh(a+c+(b+d)x)}{2b+2d}$	40
risch	$\frac{\left(b\,{\rm e}^{2bx+2a}-{\rm e}^{2bx+2a}d+b+d\right){\rm e}^{-bx+dx-a+c}}{4(b+d)(b-d)}+\frac{\left(b\,{\rm e}^{2bx+2a}+{\rm e}^{2bx+2a}d+b-d\right){\rm e}^{-bx-dx-a-c}}{4(b+d)(b-d)}$	112
parallelrisch	$\frac{2b-4d\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b}{(b^{2}-d^{2})\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)}$	116

input
```
int(cosh(d*x+c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*cosh(a-c+(b-d)*x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int\cosh(c+dx)\sinh(a+bx)\,dx=\frac{b\cosh(bx+a)\cosh(dx+c)-d\sinh(bx+a)\sinh(dx+c)}{(b^2-d^2)\cosh(bx+a)^2-(b^2-d^2)\sinh(bx+a)^2}$$

input
```
integrate(cosh(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
(b*cosh(b*x + a)*cosh(d*x + c) - d*sinh(b*x + a)*sinh(d*x + c))/((b^2 - d^
2)*cosh(b*x + a)^2 - (b^2 - d^2)*sinh(b*x + a)^2)
```

3.179.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(32)=64$.

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.56

$$\int\cosh(c+dx)\sinh(a+bx)\,dx$$
$$=\begin{cases}x\sinh(a)\cosh(c) & \text{for } b=0\wedge d=0\\ \frac{x\sinh(a-dx)\cosh(c+dx)}{2}+\frac{x\sinh(c+dx)\cosh(a-dx)}{2}+\frac{\sinh(a-dx)\sinh(c+dx)}{2d} & \text{for } b=-d\\ \frac{x\sinh(a+dx)\cosh(c+dx)}{2}-\frac{x\sinh(c+dx)\cosh(a+dx)}{2}+\frac{\cosh(a+dx)\cosh(c+dx)}{2d} & \text{for } b=d\\ \frac{b\cosh(a+bx)\cosh(c+dx)}{b^2-d^2}-\frac{d\sinh(a+bx)\sinh(c+dx)}{b^2-d^2} & \text{otherwise}\end{cases}$$

input
```
integrate(cosh(d*x+c)*sinh(b*x+a),x)
```

output
```
Piecewise((x*sinh(a)*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x)*cosh(
c + d*x)/2 + x*sinh(c + d*x)*cosh(a - d*x)/2 + sinh(a - d*x)*sinh(c + d*x)
/(2*d), Eq(b, -d)), (x*sinh(a + d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)*cos
h(a + d*x)/2 + cosh(a + d*x)*cosh(c + d*x)/(2*d), Eq(b, d)), (b*cosh(a + b
*x)*cosh(c + d*x)/(b**2 - d**2) - d*sinh(a + b*x)*sinh(c + d*x)/(b**2 - d*
*2), True))
```

3.179.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh(c+dx)\sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more
 details)I
```

3.179.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \cosh(c+dx)\sinh(a+bx)\,dx = \frac{e^{(bx+dx+a+c)}}{4\,(b+d)} + \frac{e^{(bx-dx+a-c)}}{4\,(b-d)} + \frac{e^{(-bx+dx-a+c)}}{4\,(b-d)} + \frac{e^{(-bx-dx-a-c)}}{4\,(b+d)}$$

input
```
integrate(cosh(d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/4*e^(b*x + d*x + a + c)/(b + d) + 1/4*e^(b*x - d*x + a - c)/(b - d) + 1/
4*e^(-b*x + d*x - a + c)/(b - d) + 1/4*e^(-b*x - d*x - a - c)/(b + d)
```

3.179.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \cosh(c+dx)\sinh(a+bx)\,dx = \frac{b\cosh(a+b\,x)\,\cosh(c+d\,x)-d\sinh(a+b\,x)\,\sinh(c+d\,x)}{b^2-d^2}$$

input
```
int(cosh(c + d*x)*sinh(a + b*x),x)
```

output
```
(b*cosh(a + b*x)*cosh(c + d*x) - d*sinh(a + b*x)*sinh(c + d*x))/(b^2 - d^2
)
```

3.180 $\int \cosh^2(c+dx)\sinh(a+bx)\,dx$

3.180.1 Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx = \frac{\cosh(a+bx)}{2b} + \frac{\cosh(a-2c+(b-2d)x)}{4(b-2d)} + \frac{\cosh(a+2c+(b+2d)x)}{4(b+2d)}$$

output
```
1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d)
*x)/(b+2*d)
```

3.180.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx = \frac{1}{4}\left(\frac{2\cosh(a)\cosh(bx)}{b} + \frac{\cosh(a-2c+bx-2dx)}{b-2d} + \frac{\cosh(a+2c+bx+2dx)}{b+2d} + \frac{2\sinh(a)\sinh(bx)}{b}\right)$$

input
```
Integrate[Cosh[c + d*x]^2*Sinh[a + b*x],x]
```

output
```
((2*Cosh[a]*Cosh[b*x])/b + Cosh[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cosh[a
+ 2*c + b*x + 2*d*x]/(b + 2*d) + (2*Sinh[a]*Sinh[b*x])/b)/4
```

3.180.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6152, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\cosh^2(c+dx)\,dx$$

$$\Big\downarrow \text{6152}$$

$$\int \left(\frac{1}{4}\sinh(a+x(b-2d)-2c)+\frac{1}{4}\sinh(a+x(b+2d)+2c)+\frac{1}{2}\sinh(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cosh(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\cosh(a+bx)}{2b}$$

input
```
Int[Cosh[c + d*x]^2*Sinh[a + b*x],x]
```

output
```
Cosh[a + b*x]/(2*b) + Cosh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cosh[a +
 2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

3.180.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6152
```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.180.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result
default	$\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4b-8d}+\frac{\cosh(a+2c+(b+2d)x)}{4b+8d}$
risch	$\frac{e^{bx+a}}{4b}+\frac{e^{-bx-a}}{4b}+\frac{(b\,e^{2bx+2a}-2\,e^{2bx+2a}d+b+2d)\,e^{-bx+2dx-a+2c}}{8(b+2d)(b-2d)}+\frac{(b\,e^{2bx+2a}+2\,e^{2bx+2a}d+b-2d)\,e^{-bx-2dx-a-2c}}{8(b+2d)(b-2d)}$
parallelrisch	$\frac{(-2b^2+4d^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+8d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)b+\left(-4\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2b^2-8d^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+8d\tanh\left(\frac{bx}{2}+\right.}{b(b-2d)(b+2d)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$

input
```
int(cosh(d*x+c)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d)
*x)/(b+2*d)
```

3.180.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(56) = 112.

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.92

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx$$
$$=\frac{b^2\cosh(bx+a)\cosh(dx+c)^2-4\,bd\cosh(dx+c)\sinh(bx+a)\sinh(dx+c)+b^2\cosh(bx+a)\sinh(dx+}{2\left((b^3-4\,bd^2)\cosh(bx+a)^2-(b^3-4\,bd^2)\sinh(bx+a)^2\right)}$$

input
```
integrate(cosh(d*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 - 4*b*d*cosh(d*x + c)*sinh(b*x + a)
*sinh(d*x + c) + b^2*cosh(b*x + a)*sinh(d*x + c)^2 + (b^2 - 4*d^2)*cosh(b*
x + a))/((b^3 - 4*b*d^2)*cosh(b*x + a)^2 - (b^3 - 4*b*d^2)*sinh(b*x + a)^2
)
```

3.180.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(49) = 98.

Time = 0.71 (sec) , antiderivative size = 408, normalized size of antiderivative = 6.58

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx$$
$$= \begin{cases} x\sinh(a)\cosh^2(c) \\ \left(-\frac{x\sinh^2(c+dx)}{2} + \frac{x\cosh^2(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)\sinh(a) \\ \frac{x\sinh(a-2dx)\sinh^2(c+dx)}{4} + \frac{x\sinh(a-2dx)\cosh^2(c+dx)}{4} + \frac{x\sinh(c+dx)\cosh(a-2dx)\cosh(c+dx)}{2} + \frac{3\sinh(a-2dx)\sinh(c+dx)\cosh}{4d} \\ \frac{x\sinh(a+2dx)\sinh^2(c+dx)}{4} + \frac{x\sinh(a+2dx)\cosh^2(c+dx)}{4} - \frac{x\sinh(c+dx)\cosh(a+2dx)\cosh(c+dx)}{2} + \frac{3\sinh(a+2dx)\sinh(c+dx)\cosh}{4d} \\ \frac{b^2\cosh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} - \frac{2bd\sinh(a+bx)\sinh(c+dx)\cosh(c+dx)}{b^3-4bd^2} + \frac{2d^2\sinh^2(c+dx)\cosh(a+bx)}{b^3-4bd^2} - \frac{2d^2\cosh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

input
```
integrate(cosh(d*x+c)**2*sinh(b*x+a),x)
```

output
```
Piecewise((x*sinh(a)*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x)*
*2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a),
Eq(b, 0)), (x*sinh(a - 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a - 2*d*x)*cosh(
c + d*x)**2/4 + x*sinh(c + d*x)*cosh(a - 2*d*x)*cosh(c + d*x)/2 + 3*sinh(a
 - 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) + sinh(c + d*x)**2*cosh(a - 2*
d*x)/(2*d), Eq(b, -2*d)), (x*sinh(a + 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a
 + 2*d*x)*cosh(c + d*x)**2/4 - x*sinh(c + d*x)*cosh(a + 2*d*x)*cosh(c + d*
x)/2 + 3*sinh(a + 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) - sinh(c + d*x)
**2*cosh(a + 2*d*x)/(2*d), Eq(b, 2*d)), (b**2*cosh(a + b*x)*cosh(c + d*x)*
*2/(b**3 - 4*b*d**2) - 2*b*d*sinh(a + b*x)*sinh(c + d*x)*cosh(c + d*x)/(b*
*3 - 4*b*d**2) + 2*d**2*sinh(c + d*x)**2*cosh(a + b*x)/(b**3 - 4*b*d**2) -
 2*d**2*cosh(a + b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2), True))
```

3.180.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for
more detai
```

3.180.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(56) = 112.

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.94

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx = \frac{e^{(bx+2\,dx+a+2\,c)}}{8\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{8\,(b-2\,d)} + \frac{e^{(bx+a)}}{4\,b} + \frac{e^{(-bx+2\,dx-a+2\,c)}}{8\,(b-2\,d)} + \frac{e^{(-bx-2\,dx-a-2\,c)}}{8\,(b+2\,d)} + \frac{e^{(-bx-a)}}{4\,b}$$

input
```
integrate(cosh(d*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/8*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/8*e^(b*x - 2*d*x + a - 2*c)/(b
 - 2*d) + 1/4*e^(b*x + a)/b + 1/8*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) + 1
/8*e^(-b*x - 2*d*x - a - 2*c)/(b + 2*d) + 1/4*e^(-b*x - a)/b
```

3.180.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx = \frac{2\,d^2\cosh(a+b\,x)-b^2\cosh(a+b\,x)\,\cosh(c+d\,x)^2+2\,b\,d\cosh(c+d\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)}{4\,b\,d^2-b^3}$$

input
```
int(cosh(c + d*x)^2*sinh(a + b*x),x)
```

output
```
(2*d^2*cosh(a + b*x) - b^2*cosh(a + b*x)*cosh(c + d*x)^2 + 2*b*d*cosh(c +
d*x)*sinh(a + b*x)*sinh(c + d*x))/(4*b*d^2 - b^3)
```

3.181 $\int \cosh^3(c+dx)\sinh(a+bx)\,dx$

3.181.1 Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx=\frac{\cosh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}+\frac{3\cosh(a+c+(b+d)x)}{8(b+d)}+\frac{\cosh(a+3c+(b+3d)x)}{8(b+3d)}$$

output
```
1/8*cosh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*cosh(a-c+(b-d)*x)/(b-d)+3/8*cosh(a+c
+(b+d)*x)/(b+d)+1/8*cosh(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.181.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx=\frac{1}{8}\left(\frac{\cosh(a-3c+bx-3dx)}{b-3d}+\frac{3\cosh(a-c+bx-dx)}{b-d}+\frac{\cosh(a+3c+bx+3dx)}{b+3d}+\frac{3\cosh(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Cosh[c + d*x]^3*Sinh[a + b*x],x]
```

output
```
(Cosh[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Cosh[a - c + b*x - d*x])/(b -
d) + Cosh[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Cosh[a + c + (b + d)*x])/(
b + d))/8
```

3.181.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6152, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\cosh^3(c+dx)\,dx$$

$$\Big\downarrow \text{6152}$$

$$\int \left(\frac{1}{8}\sinh(a+x(b-3d)-3c)+\frac{3}{8}\sinh(a+x(b-d)-c)+\frac{3}{8}\sinh(a+x(b+d)+c)+\frac{1}{8}\sinh(a+x(b+3d)+3$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\cosh(a+x(b-d)-c)}{8(b-d)}+\frac{3\cosh(a+x(b+d)+c)}{8(b+d)}+\frac{\cosh(a+x(b+3d)+3c)}{8(b+3d)}$$

input
```
Int[Cosh[c + d*x]^3*Sinh[a + b*x],x]
```

output
```
Cosh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Cosh[a - c + (b - d)*x])/(8
*(b - d)) + (3*Cosh[a + c + (b + d)*x])/(8*(b + d)) + Cosh[a + 3*c + (b +
3*d)*x]/(8*(b + 3*d))
```

3.181.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6152
```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.181.4 Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result
default	$\frac{\cosh(a-3c+(b-3d)x)}{8b-24d}+\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}+\frac{3\cosh(a+c+(b+d)x)}{8(b+d)}+\frac{\cosh(a+3c+(b+3d)x)}{8b+24d}$
risch	$\frac{\left(b\,\mathrm{e}^{2bx+2a}-3\,\mathrm{e}^{2bx+2a}d+b+3d\right)\mathrm{e}^{-bx+3dx-a+3c}}{16(b+3d)(b-3d)}+\frac{3\left(b\,\mathrm{e}^{2bx+2a}-\mathrm{e}^{2bx+2a}d+b+d\right)\mathrm{e}^{-bx+dx-a+c}}{16(b+d)(b-d)}+\frac{3\left(b\,\mathrm{e}^{2bx+2a}+\mathrm{e}^{2bx+2a}d+b\right.}{16(b+d)(b-}$
parallelrisch	$\frac{2b\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}(b^{2}-7d^{2})\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12d\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)(b^{2}-3d^{2})\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+6b\left(4\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d^{2}+b^{2}-3d^{2}\right)\mathrm{ta}}{(b-d)(b+3d)(b-3d)(b+d)\left(\tanh\right.}$

input
```
int(cosh(d*x+c)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*cosh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*cosh(a-c+(b-d)*x)/(b-d)+3/8*cosh(a+c
+(b+d)*x)/(b+d)+1/8*cosh(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.181.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(83) = 166$.

Time = 0.26 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.34

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx$$
$$=\frac{(b^3-bd^2)\cosh(bx+a)\cosh(dx+c)^3+3(b^3-bd^2)\cosh(bx+a)\cosh(dx+c)\sinh(dx+c)^2-3(b^2d-}{4\left((b^4-10\,b^2d^2+\right.}$$

input
```
integrate(cosh(d*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)^3 + 3*(b^3 - b*d^2)*cosh(b*
x + a)*cosh(d*x + c)*sinh(d*x + c)^2 - 3*(b^2*d - d^3)*sinh(b*x + a)*sinh(
d*x + c)^3 + 3*(b^3 - 9*b*d^2)*cosh(b*x + a)*cosh(d*x + c) - 3*(b^2*d - 9*
d^3 + 3*(b^2*d - d^3)*cosh(d*x + c)^2)*sinh(b*x + a)*sinh(d*x + c))/((b^4
- 10*b^2*d^2 + 9*d^4)*cosh(b*x + a)^2 - (b^4 - 10*b^2*d^2 + 9*d^4)*sinh(b*
x + a)^2)
```

3.181.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(76) = 152$.

Time = 1.96 (sec) , antiderivative size = 921, normalized size of antiderivative = 10.12

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)**3*sinh(b*x+a),x)
```

output
```
Piecewise((x*sinh(a)*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - 3*d*x
)*sinh(c + d*x)**2*cosh(c + d*x)/8 + x*sinh(a - 3*d*x)*cosh(c + d*x)**3/8
+ x*sinh(c + d*x)**3*cosh(a - 3*d*x)/8 + 3*x*sinh(c + d*x)*cosh(a - 3*d*x)
*cosh(c + d*x)**2/8 + sinh(a - 3*d*x)*sinh(c + d*x)**3/(8*d) + sinh(c + d*
x)**2*cosh(a - 3*d*x)*cosh(c + d*x)/(4*d) - 7*cosh(a - 3*d*x)*cosh(c + d*x
)**3/(24*d), Eq(b, -3*d)), (-3*x*sinh(a - d*x)*sinh(c + d*x)**2*cosh(c + d
*x)/8 + 3*x*sinh(a - d*x)*cosh(c + d*x)**3/8 - 3*x*sinh(c + d*x)**3*cosh(a
 - d*x)/8 + 3*x*sinh(c + d*x)*cosh(a - d*x)*cosh(c + d*x)**2/8 + 3*sinh(a
- d*x)*sinh(c + d*x)**3/(8*d) + 3*sinh(c + d*x)**2*cosh(a - d*x)*cosh(c +
d*x)/(4*d) - 5*cosh(a - d*x)*cosh(c + d*x)**3/(8*d), Eq(b, -d)), (-3*x*sin
h(a + d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 + 3*x*sinh(a + d*x)*cosh(c + d
*x)**3/8 + 3*x*sinh(c + d*x)**3*cosh(a + d*x)/8 - 3*x*sinh(c + d*x)*cosh(a
 + d*x)*cosh(c + d*x)**2/8 + 3*sinh(a + d*x)*sinh(c + d*x)**3/(8*d) - 3*si
nh(c + d*x)**2*cosh(a + d*x)*cosh(c + d*x)/(4*d) + 5*cosh(a + d*x)*cosh(c
+ d*x)**3/(8*d), Eq(b, d)), (3*x*sinh(a + 3*d*x)*sinh(c + d*x)**2*cosh(c +
 d*x)/8 + x*sinh(a + 3*d*x)*cosh(c + d*x)**3/8 - x*sinh(c + d*x)**3*cosh(a
 + 3*d*x)/8 - 3*x*sinh(c + d*x)*cosh(a + 3*d*x)*cosh(c + d*x)**2/8 + sinh(
a + 3*d*x)*sinh(c + d*x)**3/(8*d) - sinh(c + d*x)**2*cosh(a + 3*d*x)*cosh(
c + d*x)/(4*d) + 7*cosh(a + 3*d*x)*cosh(c + d*x)**3/(24*d), Eq(b, 3*d)), (
b**3*cosh(a + b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*...
```

3.181.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for
more detai
```

3.181.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(83) = 166$.

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.97

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx = \frac{e^{(bx+3\,dx+a+3\,c)}}{16\,(b+3\,d)} + \frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)} + \frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)} + \frac{e^{(bx-3\,dx+a-3\,c)}}{16\,(b-3\,d)} + \frac{e^{(-bx+3\,dx-a+3\,c)}}{16\,(b-3\,d)} + \frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)} + \frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)} + \frac{e^{(-bx-3\,dx-a-3\,c)}}{16\,(b+3\,d)}$$

input
```
integrate(cosh(d*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/16*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/16*e^(b*x + d*x + a + c)/(b +
 d) + 3/16*e^(b*x - d*x + a - c)/(b - d) + 1/16*e^(b*x - 3*d*x + a - 3*c)/
(b - 3*d) + 1/16*e^(-b*x + 3*d*x - a + 3*c)/(b - 3*d) + 3/16*e^(-b*x + d*x
 - a + c)/(b - d) + 3/16*e^(-b*x - d*x - a - c)/(b + d) + 1/16*e^(-b*x - 3
*d*x - a - 3*c)/(b + 3*d)
```

3.181.9 Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.00

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx = \frac{6\,b\,d^2\cosh(a+b\,x)\,\cosh(c+d\,x)\,\sinh(c+d\,x)^2}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{6\,d^3\sinh(a+b\,x)\,\sinh(c+d\,x)^3}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{3\,d\cosh(c+d\,x)^2\sinh(a+b\,x)\,\sinh(c+d\,x)\,\left(b^2-3\,d^2\right)}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{\cosh(a+b\,x)\,\cosh(c+d\,x)^3\,\left(7\,b\,d^2-b^3\right)}{b^4-10\,b^2\,d^2+9\,d^4}$$

input
```
int(cosh(c + d*x)^3*sinh(a + b*x),x)
```

output
```
(6*b*d^2*cosh(a + b*x)*cosh(c + d*x)*sinh(c + d*x)^2)/(b^4 + 9*d^4 - 10*b^
2*d^2) - (6*d^3*sinh(a + b*x)*sinh(c + d*x)^3)/(b^4 + 9*d^4 - 10*b^2*d^2)
- (3*d*cosh(c + d*x)^2*sinh(a + b*x)*sinh(c + d*x)*(b^2 - 3*d^2))/(b^4 + 9
*d^4 - 10*b^2*d^2) - (cosh(a + b*x)*cosh(c + d*x)^3*(7*b*d^2 - b^3))/(b^4
+ 9*d^4 - 10*b^2*d^2)
```

3.181. $\int \cosh^3(c+dx)\sinh(a+bx)\,dx$

3.182 $\int \cosh(c+dx)\sinh^2(a+bx)\,dx$

3.182.1 Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx = \frac{\sinh(2a-c+(2b-d)x)}{4(2b-d)} - \frac{\sinh(c+dx)}{2d} + \frac{\sinh(2a+c+(2b+d)x)}{4(2b+d)}$$

output
```
1/4*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*sinh(d*x+c)/d+1/4*sinh(2*a+c+(2*b+d)
*x)/(2*b+d)
```

3.182.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx = \frac{1}{4}\left(-\frac{2\cosh(dx)\sinh(c)}{d} - \frac{2\cosh(c)\sinh(dx)}{d} + \frac{\sinh(2a-c+2bx-dx)}{2b-d} + \frac{\sinh(2a+c+2bx+dx)}{2b+d}\right)$$

input
```
Integrate[Cosh[c + d*x]*Sinh[a + b*x]^2,x]
```

output
```
((-2*Cosh[d*x]*Sinh[c])/d - (2*Cosh[c]*Sinh[d*x])/d + Sinh[2*a - c + 2*b*x
 - d*x]/(2*b - d) + Sinh[2*a + c + 2*b*x + d*x]/(2*b + d))/4
```

3.182.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6152, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\cosh(c+dx)\,dx$$

$$\Big\downarrow \text{6152}$$

$$\int \left(\frac{1}{4}\cosh(2a+x(2b-d)-c)+\frac{1}{4}\cosh(2a+x(2b+d)+c)-\frac{1}{2}\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\sinh(2a+x(2b+d)+c)}{4(2b+d)}-\frac{\sinh(c+dx)}{2d}$$

input
```
Int[Cosh[c + d*x]*Sinh[a + b*x]^2,x]
```

output
```
Sinh[2*a - c + (2*b - d)*x]/(4*(2*b - d)) - Sinh[c + d*x]/(2*d) + Sinh[2*a
 + c + (2*b + d)*x]/(4*(2*b + d))
```

3.182.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6152
```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.182.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

method	result
default	$\frac{\sinh(2a-c+(2b-d)x)}{8b-4d}-\frac{\sinh(dx+c)}{2d}+\frac{\sinh(2a+c+(2b+d)x)}{8b+4d}$
parallelrisch	$\frac{(2bd+d^2)\sinh(2a-c+(2b-d)x)+(2bd-d^2)\sinh(2a+c+(2b+d)x)+(-8b^2+2d^2)\sinh(dx+c)}{16b^2d-4d^3}$
risch	$-\frac{(-2d\,e^{4bx+4a}b+d^2e^{4bx+4a}+8b^2e^{2bx+2a}-2\,e^{2bx+2a}d^2+2bd+d^2)e^{-2bx+dx-2a+c}}{8(2b+d)(2b-d)d}+\frac{(2d\,e^{4bx+4a}b+d^2e^{4bx+4a}+8b^2e^{2bx+2a}}{8(2b+d}$

input
```
int(cosh(d*x+c)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*sinh(d*x+c)/d+1/4*sinh(2*a+c+(2*b+d)
*x)/(2*b+d)
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.68

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx$$
$$=\frac{4\,bd\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)-\left(d^2\cosh\left(bx+a\right)^2+d^2\sinh\left(bx+a\right)^2+4\,b^2-d^2\right)\sinh\left(dx\right.}{2\left(\left(4\,b^2d-d^3\right)\cosh\left(bx+a\right)^2-\left(4\,b^2d-d^3\right)\sinh\left(bx+a\right)^2\right)}$$

input
```
integrate(cosh(d*x+c)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(4*b*d*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a) - (d^2*cosh(b*x + a)^
2 + d^2*sinh(b*x + a)^2 + 4*b^2 - d^2)*sinh(d*x + c))/((4*b^2*d - d^3)*cos
h(b*x + a)^2 - (4*b^2*d - d^3)*sinh(b*x + a)^2)
```

3.182.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(49) = 98$.

Time = 0.74 (sec) , antiderivative size = 408, normalized size of antiderivative = 6.00

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx$$

$$= \begin{cases} x\sinh^2(a)\cosh(c) \\ \frac{x\sinh^2\left(a-\frac{dx}{2}\right)\cosh(c+dx)}{4} + \frac{x\sinh\left(a-\frac{dx}{2}\right)\sinh(c+dx)\cosh\left(a-\frac{dx}{2}\right)}{2} + \frac{x\cosh^2\left(a-\frac{dx}{2}\right)\cosh(c+dx)}{4} + \frac{\sinh^2\left(a-\frac{dx}{2}\right)\sinh(c+dx)}{d} \\ \frac{x\sinh^2\left(a+\frac{dx}{2}\right)\cosh(c+dx)}{4} - \frac{x\sinh\left(a+\frac{dx}{2}\right)\sinh(c+dx)\cosh\left(a+\frac{dx}{2}\right)}{2} + \frac{x\cosh^2\left(a+\frac{dx}{2}\right)\cosh(c+dx)}{4} + \frac{\sinh^2\left(a+\frac{dx}{2}\right)\sinh(c+dx)}{d} \\ \left(\frac{x\sinh^2(a+bx)}{2} - \frac{x\cosh^2(a+bx)}{2} + \frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)\cosh(c) \\ \frac{2b^2\sinh^2(a+bx)\sinh(c+dx)}{4b^2d-d^3} - \frac{2b^2\sinh(c+dx)\cosh^2(a+bx)}{4b^2d-d^3} + \frac{2bd\sinh(a+bx)\cosh(a+bx)\cosh(c+dx)}{4b^2d-d^3} - \frac{d^2\sinh^2(a+bx)\sinh(c+dx)}{4b^2d-d^3} \end{cases}$$

input
```
integrate(cosh(d*x+c)*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x*sinh(a)**2*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x/2)*
*2*cosh(c + d*x)/4 + x*sinh(a - d*x/2)*sinh(c + d*x)*cosh(a - d*x/2)/2 + x
*cosh(a - d*x/2)**2*cosh(c + d*x)/4 + sinh(a - d*x/2)**2*sinh(c + d*x)/d +
 sinh(a - d*x/2)*cosh(a - d*x/2)*cosh(c + d*x)/(2*d), Eq(b, -d/2)), (x*sin
h(a + d*x/2)**2*cosh(c + d*x)/4 - x*sinh(a + d*x/2)*sinh(c + d*x)*cosh(a +
 d*x/2)/2 + x*cosh(a + d*x/2)**2*cosh(c + d*x)/4 + sinh(a + d*x/2)**2*sinh
(c + d*x)/d - sinh(a + d*x/2)*cosh(a + d*x/2)*cosh(c + d*x)/(2*d), Eq(b, d
/2)), ((x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a
 + b*x)/(2*b))*cosh(c), Eq(d, 0)), (2*b**2*sinh(a + b*x)**2*sinh(c + d*x)/
(4*b**2*d - d**3) - 2*b**2*sinh(c + d*x)*cosh(a + b*x)**2/(4*b**2*d - d**3
) + 2*b*d*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d*x)/(4*b**2*d - d**3) - d*
*2*sinh(a + b*x)**2*sinh(c + d*x)/(4*b**2*d - d**3), True))
```

3.182.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for mor
e details)
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.82

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx = \frac{e^{(2\,bx+dx+2\,a+c)}}{8\,(2\,b+d)} + \frac{e^{(2\,bx-dx+2\,a-c)}}{8\,(2\,b-d)} - \frac{e^{(-2\,bx+dx-2\,a+c)}}{8\,(2\,b-d)} - \frac{e^{(-2\,bx-dx-2\,a-c)}}{8\,(2\,b+d)} - \frac{e^{(dx+c)}}{4\,d} + \frac{e^{(-dx-c)}}{4\,d}$$

input
```
integrate(cosh(d*x+c)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/8*e^(2*b*x + d*x + 2*a + c)/(2*b + d) + 1/8*e^(2*b*x - d*x + 2*a - c)/(2
*b - d) - 1/8*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 1/8*e^(-2*b*x - d*x -
 2*a - c)/(2*b + d) - 1/4*e^(d*x + c)/d + 1/4*e^(-d*x - c)/d
```

3.182.9 Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx$$
$$= \frac{d^2\left(\sinh(c+d\,x)-\cosh(a+b\,x)^2\sinh(c+d\,x)\right)-2\,b^2\sinh(c+d\,x)+2\,b\,d\cosh(a+b\,x)\,\cosh(c+d\,x)\,\mathrm{si}}{4\,b^2\,d-d^3}$$

input
```
int(cosh(c + d*x)*sinh(a + b*x)^2,x)
```

output
```
(d^2*(sinh(c + d*x) - cosh(a + b*x)^2*sinh(c + d*x)) - 2*b^2*sinh(c + d*x)
 + 2*b*d*cosh(a + b*x)*cosh(c + d*x)*sinh(a + b*x))/(4*b^2*d - d^3)
```

3.183 $\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx$

3.183.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = -\frac{x}{4}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)} -\frac{\sinh(2c+2dx)}{8d}+\frac{\sinh(2(a+c)+2(b+d)x)}{16(b+d)}$$

output
```
-1/4*x+1/8*sinh(2*b*x+2*a)/b+1/16*sinh(2*a-2*c+2*(b-d)*x)/(b-d)-1/8*sinh(2
*d*x+2*c)/d+1/16*sinh(2*a+2*c+2*(b+d)*x)/(b+d)
```

3.183.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = \frac{2d(b^2-d^2)\sinh(2(a+bx))+bd(b+d)\sinh(2(a-c+(b-d)x))-b(b-d)(4d(b+d)x+2(b+d)\sinh(2}{16b(b-d)d(b+d)}$$

input
```
Integrate[Cosh[c + d*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(2*d*(b^2 - d^2)*Sinh[2*(a + b*x)] + b*d*(b + d)*Sinh[2*(a - c + (b - d)*x
)] - b*(b - d)*(4*d*(b + d)*x + 2*(b + d)*Sinh[2*(c + d*x)] - d*Sinh[2*(a
+ c + (b + d)*x)]))/(16*b*(b - d)*d*(b + d))
```

3.183.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6152, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\cosh^2(c+dx)\,dx$$

$$\Big\downarrow \text{6152}$$

$$\int \left(\frac{1}{8}\cosh(2(a-c)+2x(b-d))+\frac{1}{8}\cosh(2(a+c)+2x(b+d))+\frac{1}{4}\cosh(2a+2bx)-\frac{1}{4}\cosh(2c+2dx)-\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sinh(2a+2bx)}{8b}-\frac{\sinh(2c+2dx)}{8d}-\frac{x}{4}$$

input
```
Int[Cosh[c + d*x]^2*Sinh[a + b*x]^2,x]
```

output
```
-1/4*x + Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b -
d)) - Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

3.183.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6152
```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.183.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$-\frac{x}{4}+\frac{\sinh(2bx+2a)}{8b}-\frac{\sinh(2dx+2c)}{8d}+\frac{\sinh((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sinh((2b+2d)x+2a+2c)}{16b+16d}$
parallelrisch	$\frac{bd(b+d)\sinh((2b-2d)x+2a-2c)-4\left(-\frac{bd\sinh((2b+2d)x+2a+2c)}{4}+\left(-\frac{d\sinh(2bx+2a)}{2}+b\left(dx+\frac{\sinh(2dx+2c)}{2}\right)\right)(b+d)\right)(b-d)}{16b^3d-16d^3b}$
risch	$-\frac{x}{4}+\frac{e^{2bx+2a}}{16b}-\frac{e^{-2bx-2a}}{16b}-\frac{(-d\,e^{4bx+4a}b+d^2e^{4bx+4a}+2b^2e^{2bx+2a}-2\,e^{2bx+2a}d^2+bd+d^2)e^{-2bx+2dx-2a+2c}}{32(b+d)(b-d)d}+\frac{(d\,e^{4b}}{}$

input
```
int(cosh(d*x+c)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*x+1/8*sinh(2*b*x+2*a)/b-1/8*sinh(2*d*x+2*c)/d+1/8/(2*b-2*d)*sinh((2*b
-2*d)*x+2*a-2*c)+1/8/(2*b+2*d)*sinh((2*b+2*d)*x+2*a+2*c)
```

3.183.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(78) = 156.

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.18

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx$$
$$=\frac{b^2d\cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^2-(b^3d-bd^3)x+\left(b^2d\cosh(bx+a)\cosh(dx+c)^2+(b^2d-\right.}{4\left((b^3d-bd^3)\cos\right.}$$

input
```
integrate(cosh(d*x+c)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(b^2*d*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^2 - (b^3*d - b*d^3)*x
 + (b^2*d*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - d^3)*cosh(b*x + a))*sin
h(b*x + a) - (b*d^2*cosh(d*x + c)*sinh(b*x + a)^2 + (b*d^2*cosh(b*x + a)^2
 + b^3 - b*d^2)*cosh(d*x + c))*sinh(d*x + c))/((b^3*d - b*d^3)*cosh(b*x +
a)^2 - (b^3*d - b*d^3)*sinh(b*x + a)^2)
```

3.183.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(76) = 152$.

Time = 1.56 (sec) , antiderivative size = 1027, normalized size of antiderivative = 11.67

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)**2*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x*sinh(a)**2*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*
x)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a
)**2, Eq(b, 0)), (-x*sinh(a - d*x)**2*sinh(c + d*x)**2/8 + 3*x*sinh(a - d*
x)**2*cosh(c + d*x)**2/8 + x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)*cos
h(c + d*x)/2 + 3*x*sinh(c + d*x)**2*cosh(a - d*x)**2/8 - x*cosh(a - d*x)**
2*cosh(c + d*x)**2/8 + 5*sinh(a - d*x)**2*sinh(c + d*x)*cosh(c + d*x)/(8*d
) + sinh(a - d*x)*sinh(c + d*x)**2*cosh(a - d*x)/(2*d) + sinh(c + d*x)*cos
h(a - d*x)**2*cosh(c + d*x)/(8*d), Eq(b, -d)), (-x*sinh(a + d*x)**2*sinh(c
 + d*x)**2/8 + 3*x*sinh(a + d*x)**2*cosh(c + d*x)**2/8 - x*sinh(a + d*x)*s
inh(c + d*x)*cosh(a + d*x)*cosh(c + d*x)/2 + 3*x*sinh(c + d*x)**2*cosh(a +
 d*x)**2/8 - x*cosh(a + d*x)**2*cosh(c + d*x)**2/8 + sinh(a + d*x)*sinh(c
+ d*x)**2*cosh(a + d*x)/(8*d) + 5*sinh(a + d*x)*cosh(a + d*x)*cosh(c + d*x
)**2/(8*d) - sinh(c + d*x)*cosh(a + d*x)**2*cosh(c + d*x)/(2*d), Eq(b, d))
, ((x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b
*x)/(2*b))*cosh(c)**2, Eq(d, 0)), (-b**3*d*x*sinh(a + b*x)**2*sinh(c + d*x
)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4
*b**3*d - 4*b*d**3) + b**3*d*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d
 - 4*b*d**3) - b**3*d*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*
d**3) + b**3*sinh(a + b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*
d**3) - b**3*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)/(4*b**3*d - 4...
```

3.183.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-(2*d)/b>0)', see `assume?` for
 more deta
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = -\frac{1}{4}x + \frac{e^{(2\,bx+2\,dx+2\,a+2\,c)}}{32\,(b+d)} + \frac{e^{(2\,bx-2\,dx+2\,a-2\,c)}}{32\,(b-d)} + \frac{e^{(2\,bx+2\,a)}}{16\,b} - \frac{e^{(-2\,bx+2\,dx-2\,a+2\,c)}}{32\,(b-d)} - \frac{e^{(-2\,bx-2\,dx-2\,a-2\,c)}}{32\,(b+d)} - \frac{e^{(-2\,bx-2\,a)}}{16\,b} - \frac{e^{(2\,dx+2\,c)}}{16\,d} + \frac{e^{(-2\,dx-2\,c)}}{16\,d}$$

input
```
integrate(cosh(d*x+c)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*x + 1/32*e^(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/32*e^(2*b*x - 2*d*
x + 2*a - 2*c)/(b - d) + 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x + 2*d*x -
 2*a + 2*c)/(b - d) - 1/32*e^(-2*b*x - 2*d*x - 2*a - 2*c)/(b + d) - 1/16*e
^(-2*b*x - 2*a)/b - 1/16*e^(2*d*x + 2*c)/d + 1/16*e^(-2*d*x - 2*c)/d
```

3.183.9 Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.53

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx =$$

$$-\frac{d^3\cosh(a+b\,x)\,\sinh(a+b\,x)+b^3\cosh(c+d\,x)\,\sinh(c+d\,x)-b\,d^3\,x+b^3\,d\,x-2\,b\,d^2\cosh(c+d\,x)\,\mathrm{si}}{4\,b}$$

input
```
int(cosh(c + d*x)^2*sinh(a + b*x)^2,x)
```

output
```
-(d^3*cosh(a + b*x)*sinh(a + b*x) + b^3*cosh(c + d*x)*sinh(c + d*x) - b*d^
3*x + b^3*d*x - 2*b*d^2*cosh(c + d*x)*sinh(c + d*x) - 2*b^2*d*cosh(a + b*x
)*cosh(c + d*x)^2*sinh(a + b*x) + 2*b*d^2*cosh(a + b*x)^2*cosh(c + d*x)*si
nh(c + d*x))/(4*b*d*(b^2 - d^2))
```

3.184 $\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx$

3.184.1 Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \frac{\sinh(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)} - \frac{3\sinh(c+dx)}{8d} - \frac{\sinh(3c+3dx)}{24d} + \frac{3\sinh(2a+c+(2b+d)x)}{16(2b+d)} + \frac{\sinh(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

output
```
1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d
)-3/8*sinh(d*x+c)/d-1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b
+d)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

3.184.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \frac{1}{48}\left(-\frac{18\cosh(dx)\sinh(c)}{d}-\frac{2\cosh(3dx)\sinh(3c)}{d}-\frac{18\cosh(c)\sinh(dx)}{d}-\frac{2\cosh(3c)\sinh(3dx)}{d}+\frac{3\sinh(2a-3c+2bx-3dx)}{2b-3d}+\frac{9\sinh(2a-c+2bx-dx)}{2b-d}+\frac{9\sinh(2a+c+2bx+dx)}{2b+d}+\frac{3\sinh(2a+3c+2bx+3dx)}{2b+3d}\right)$$

input
```
Integrate[Cosh[c + d*x]^3*Sinh[a + b*x]^2,x]
```

output
```
((-18*Cosh[d*x]*Sinh[c])/d - (2*Cosh[3*d*x]*Sinh[3*c])/d - (18*Cosh[c]*Sin
h[d*x])/d - (2*Cosh[3*c]*Sinh[3*d*x])/d + (3*Sinh[2*a - 3*c + 2*b*x - 3*d*
x])/(2*b - 3*d) + (9*Sinh[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sinh[2*a
+ c + 2*b*x + d*x])/(2*b + d) + (3*Sinh[2*a + 3*c + 2*b*x + 3*d*x])/(2*b +
 3*d))/48
```

3.184.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6152, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\cosh^3(c+dx)\,dx$$

$$\Big\downarrow \text{6152}$$

$$\int \left(\frac{1}{16}\cosh(2a+x(2b-3d)-3c)+\frac{3}{16}\cosh(2a+x(2b-d)-c)+\frac{3}{16}\cosh(2a+x(2b+d)+c)+\frac{1}{16}\cosh(2a+\right.$$

$$\downarrow \text{2009}$$

$$\frac{\sinh(2a + x(2b - 3d) - 3c)}{16(2b - 3d)} + \frac{3\sinh(2a + x(2b - d) - c)}{16(2b - d)} + \frac{3\sinh(2a + x(2b + d) + c)}{16(2b + d)} + \frac{\sinh(2a + x(2b + 3d) + 3c)}{16(2b + 3d)} - \frac{3\sinh(c + dx)}{8d} - \frac{\sinh(3c + 3dx)}{24d}$$

input
```
Int[Cosh[c + d*x]^3*Sinh[a + b*x]^2,x]
```

output
```
Sinh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sinh[2*a - c + (2*b
- d)*x])/(16*(2*b - d)) - (3*Sinh[c + d*x])/(8*d) - Sinh[3*c + 3*d*x]/(24*
d) + (3*Sinh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sinh[2*a + 3*c + (2*
b + 3*d)*x]/(16*(2*b + 3*d))
```

3.184.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6152
```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.184.4 Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sinh(2a-3c+(2b-3d)x)}{32b-48d}+\frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)}-\frac{3\sinh(dx+c)}{8d}-\frac{\sinh(3dx+3c)}{24d}+\frac{3\sinh(2a+c+(2b+d)x)}{16(2b+d)}+\sinh($
parallelrisch	$\frac{(24b^3d+36d^2b^2-6d^3b-9d^4)\sinh(2a-3c+(2b-3d)x)+72\left(d\left(b+\frac{3d}{2}\right)\left(b+\frac{d}{2}\right)\sinh(2a-c+(2b-d)x)+\left(\frac{d\left(b+\frac{d}{2}\right)\sinh(2a+3c+(2b+}{3}\right.\right.}{768d\,b^4-1920b^2d^3+432d^5}$
risch	$-\frac{(-6d\,e^{4bx+4a}b+9d^2e^{4bx+4a}+8b^2e^{2bx+2a}-18\,e^{2bx+2a}d^2+6bd+9d^2)e^{-2bx+3dx-2a+3c}}{96(2b+3d)(2b-3d)d}-\frac{3(-2d\,e^{4bx+4a}b+d^2e^{4bx+4a}+8b^2}{3}$

input
```
int(cosh(d*x+c)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d
)-3/8*sinh(d*x+c)/d-1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b
+d)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

3.184.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(132) = 264$.

Time = 0.26 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.76

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx$$
$$= \frac{36\left(4\,b^3d-bd^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^2-\left(16\,b^4-40\,b^2d^2+9\,d^4+9\left(4\,b^2\right.\right.}{}$$

input
```
integrate(cosh(d*x+c)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/24*(36*(4*b^3*d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)*sinh(
d*x + c)^2 - (16*b^4 - 40*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cosh(b*x +
 a)^2 + 9*(4*b^2*d^2 - d^4)*sinh(b*x + a)^2)*sinh(d*x + c)^3 + 12*((4*b^3*
d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)^3 + 3*(4*b^3*d - 9*b*d^3)*cosh(b*x
+ a)*cosh(d*x + c))*sinh(b*x + a) - 3*(48*b^4 - 120*b^2*d^2 + 27*d^4 + 3*(
4*b^2*d^2 - 9*d^4)*cosh(b*x + a)^2 + (16*b^4 - 40*b^2*d^2 + 9*d^4 + 9*(4*b
^2*d^2 - d^4)*cosh(b*x + a)^2)*cosh(d*x + c)^2 + 3*(4*b^2*d^2 - 9*d^4 + 3*
(4*b^2*d^2 - d^4)*cosh(d*x + c)^2)*sinh(b*x + a)^2)*sinh(d*x + c))/((16*b^
4*d - 40*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 - (16*b^4*d - 40*b^2*d^3 + 9*d^5
)*sinh(b*x + a)^2)
```

3.184.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. $2(116) = 232$.

Time = 5.55 (sec) , antiderivative size = 2001, normalized size of antiderivative = 13.90

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)**3*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x*sinh(a)**2*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - 3*
d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/16 + x*sinh(a - 3*d*x/2)**2*cosh(
c + d*x)**3/16 + x*sinh(a - 3*d*x/2)*sinh(c + d*x)**3*cosh(a - 3*d*x/2)/8
+ 3*x*sinh(a - 3*d*x/2)*sinh(c + d*x)*cosh(a - 3*d*x/2)*cosh(c + d*x)**2/8
 + 3*x*sinh(c + d*x)**2*cosh(a - 3*d*x/2)**2*cosh(c + d*x)/16 + x*cosh(a -
 3*d*x/2)**2*cosh(c + d*x)**3/16 - 7*sinh(a - 3*d*x/2)**2*sinh(c + d*x)**3
/(16*d) - 3*sinh(a - 3*d*x/2)*sinh(c + d*x)**2*cosh(a - 3*d*x/2)*cosh(c +
d*x)/(4*d) - 5*sinh(a - 3*d*x/2)*cosh(a - 3*d*x/2)*cosh(c + d*x)**3/(8*d)
+ 11*sinh(c + d*x)**3*cosh(a - 3*d*x/2)**2/(48*d) - sinh(c + d*x)*cosh(a -
 3*d*x/2)**2*cosh(c + d*x)**2/d, Eq(b, -3*d/2)), (-3*x*sinh(a - d*x/2)**2*
sinh(c + d*x)**2*cosh(c + d*x)/16 + 3*x*sinh(a - d*x/2)**2*cosh(c + d*x)**
3/16 - 3*x*sinh(a - d*x/2)*sinh(c + d*x)**3*cosh(a - d*x/2)/8 + 3*x*sinh(a
 - d*x/2)*sinh(c + d*x)*cosh(a - d*x/2)*cosh(c + d*x)**2/8 - 3*x*sinh(c +
d*x)**2*cosh(a - d*x/2)**2*cosh(c + d*x)/16 + 3*x*cosh(a - d*x/2)**2*cosh(
c + d*x)**3/16 - 31*sinh(a - d*x/2)**2*sinh(c + d*x)**3/(48*d) + sinh(a -
d*x/2)**2*sinh(c + d*x)*cosh(c + d*x)**2/d - sinh(a - d*x/2)*sinh(c + d*x)
**2*cosh(a - d*x/2)*cosh(c + d*x)/(4*d) + 3*sinh(a - d*x/2)*cosh(a - d*x/2
)*cosh(c + d*x)**3/(8*d) + sinh(c + d*x)**3*cosh(a - d*x/2)**2/(48*d), Eq(
b, -d/2)), (-3*x*sinh(a + d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/16 + 3*
x*sinh(a + d*x/2)**2*cosh(c + d*x)**3/16 + 3*x*sinh(a + d*x/2)*sinh(c +...
```

3.184.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-(3*d)/b>0)', see `assume?` for
 more deta
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.81

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \frac{e^{(2bx+3dx+2a+3c)}}{32(2b+3d)} + \frac{3e^{(2bx+dx+2a+c)}}{32(2b+d)} + \frac{3e^{(2bx-dx+2a-c)}}{32(2b-d)} + \frac{e^{(2bx-3dx+2a-3c)}}{32(2b-3d)} - \frac{e^{(-2bx+3dx-2a+3c)}}{32(2b-3d)} - \frac{3e^{(-2bx+dx-2a+c)}}{32(2b-d)} - \frac{3e^{(-2bx-dx-2a-c)}}{32(2b+d)} - \frac{e^{(-2bx-3dx-2a-3c)}}{32(2b+3d)} - \frac{e^{(3dx+3c)}}{48d} - \frac{3e^{(dx+c)}}{16d} + \frac{3e^{(-dx-c)}}{16d} + \frac{e^{(-3dx-3c)}}{48d}$$

input
```
integrate(cosh(d*x+c)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/32*e^(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/32*e^(2*b*x + d*x + 2*a
 + c)/(2*b + d) + 3/32*e^(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/32*e^(2*b*x
 - 3*d*x + 2*a - 3*c)/(2*b - 3*d) - 1/32*e^(-2*b*x + 3*d*x - 2*a + 3*c)/(2
*b - 3*d) - 3/32*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 3/32*e^(-2*b*x - d
*x - 2*a - c)/(2*b + d) - 1/32*e^(-2*b*x - 3*d*x - 2*a - 3*c)/(2*b + 3*d)
- 1/48*e^(3*d*x + 3*c)/d - 3/16*e^(d*x + c)/d + 3/16*e^(-d*x - c)/d + 1/48
*e^(-3*d*x - 3*c)/d
```

3.184.9 Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.34

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx$$
$$= \frac{\cosh(c+d\,x)^2\sinh(a+b\,x)^2\sinh(c+d\,x)\,\left(8\,b^4-26\,b^2\,d^2+9\,d^4\right)}{d\,\left(16\,b^4-40\,b^2\,d^2+9\,d^4\right)}$$
$$-\sinh(a+b\,x)^2\sinh(c+d\,x)^3\left(\frac{3\,d^3}{16\,b^4-40\,b^2\,d^2+9\,d^4}+\frac{1}{3\,d}\right)$$
$$-\frac{2\cosh(a+b\,x)\,\cosh(c+d\,x)^3\sinh(a+b\,x)\,\left(7\,b\,d^2-4\,b^3\right)}{16\,b^4-40\,b^2\,d^2+9\,d^4}$$
$$-\frac{2\cosh(a+b\,x)^2\cosh(c+d\,x)^2\sinh(c+d\,x)\,\left(4\,b^4-7\,b^2\,d^2\right)}{d\,\left(16\,b^4-40\,b^2\,d^2+9\,d^4\right)}$$
$$-\cosh(a+b\,x)^2\sinh(c+d\,x)^3\left(\frac{3\,d^3}{16\,b^4-40\,b^2\,d^2+9\,d^4}-\frac{1}{3\,d}\right)$$
$$+\frac{12\,b\,d^2\cosh(a+b\,x)\,\cosh(c+d\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)^2}{16\,b^4-40\,b^2\,d^2+9\,d^4}$$

input
```
int(cosh(c + d*x)^3*sinh(a + b*x)^2,x)
```

output
```
(cosh(c + d*x)^2*sinh(a + b*x)^2*sinh(c + d*x)*(8*b^4 + 9*d^4 - 26*b^2*d^2
))/(d*(16*b^4 + 9*d^4 - 40*b^2*d^2)) - sinh(a + b*x)^2*sinh(c + d*x)^3*((3
*d^3)/(16*b^4 + 9*d^4 - 40*b^2*d^2) + 1/(3*d)) - (2*cosh(a + b*x)*cosh(c +
 d*x)^3*sinh(a + b*x)*(7*b*d^2 - 4*b^3))/(16*b^4 + 9*d^4 - 40*b^2*d^2) - (
2*cosh(a + b*x)^2*cosh(c + d*x)^2*sinh(c + d*x)*(4*b^4 - 7*b^2*d^2))/(d*(1
6*b^4 + 9*d^4 - 40*b^2*d^2)) - cosh(a + b*x)^2*sinh(c + d*x)^3*((3*d^3)/(1
6*b^4 + 9*d^4 - 40*b^2*d^2) - 1/(3*d)) + (12*b*d^2*cosh(a + b*x)*cosh(c +
d*x)*sinh(a + b*x)*sinh(c + d*x)^2)/(16*b^4 + 9*d^4 - 40*b^2*d^2)
```

3.185 $\int \cosh(c+dx)\sinh^3(a+bx)\,dx$

3.185.1 Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx = -\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}+\frac{\cosh(3a-c+(3b-d)x)}{8(3b-d)} -\frac{3\cosh(a+c+(b+d)x)}{8(b+d)}+\frac{\cosh(3a+c+(3b+d)x)}{8(3b+d)}$$

output
```
-3/8*cosh(a-c+(b-d)*x)/(b-d)+1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cosh(a+
c+(b+d)*x)/(b+d)+1/8*cosh(3*a+c+(3*b+d)*x)/(3*b+d)
```

3.185.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx = \frac{1}{8}\left(-\frac{3\cosh(a-c+bx-dx)}{b-d}+\frac{\cosh(3a-c+3bx-dx)}{3b-d} +\frac{\cosh(3a+c+3bx+dx)}{3b+d}-\frac{3\cosh(a+c+(b+d)x)}{b+d}\right)$$

input
```
Integrate[Cosh[c + d*x]*Sinh[a + b*x]^3,x]
```

output
```
((-3*Cosh[a - c + b*x - d*x])/(b - d) + Cosh[3*a - c + 3*b*x - d*x]/(3*b -
 d) + Cosh[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Cosh[a + c + (b + d)*x])/
(b + d))/8
```

3.185.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6152, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\cosh(c+dx)\,dx$$

$$\Big\downarrow \text{6152}$$

$$\int \left(-\frac{3}{8}\sinh(a+x(b-d)-c)+\frac{1}{8}\sinh(3a+x(3b-d)-c)-\frac{3}{8}\sinh(a+x(b+d)+c)+\frac{1}{8}\sinh(3a+x(3b+d)+$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\cosh(a+x(b-d)-c)}{8(b-d)}+\frac{\cosh(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cosh(a+x(b+d)+c)}{8(b+d)}+\frac{\cosh(3a+x(3b+d)+c)}{8(3b+d)}$$

input
```
Int[Cosh[c + d*x]*Sinh[a + b*x]^3,x]
```

output
```
(-3*Cosh[a - c + (b - d)*x])/(8*(b - d)) + Cosh[3*a - c + (3*b - d)*x]/(8*
(3*b - d)) - (3*Cosh[a + c + (b + d)*x])/(8*(b + d)) + Cosh[3*a + c + (3*b
 + d)*x]/(8*(3*b + d))
```

3.185.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6152
```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.185.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
default	$-\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}+\frac{\cosh(3a-c+(3b-d)x)}{24b-8d}-\frac{3\cosh(a+c+(b+d)x)}{8(b+d)}+\frac{\cosh(3a+c+(3b+d)x)}{24b+8d}$
parallelrisch	$\frac{-12\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^6 b^3+24d b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^5+\left(-12\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b d^2+36b^3-12b d^2\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^4+(-64b}{9\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(b-\frac{d}{3}\right)\left(1+\tanh\right(}$
risch	$\frac{(3b^3 e^{6bx+6a}-b^2 d\, e^{6bx+6a}-3b\, d^2 e^{6bx+6a}+d^3 e^{6bx+6a}-27b^3 e^{4bx+4a}+27b^2 d\, e^{4bx+4a}+3b\, d^2 e^{4bx+4a}-3d^3 e^{4bx+4a}-27b^3 e^{2bx+2a}}{16(3b+d)(b+d)(3b-d)(b-d)}$

input
```
int(cosh(d*x+c)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cosh(a-c+(b-d)*x)/(b-d)+1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cosh(a+
c+(b+d)*x)/(b+d)+1/8*cosh(3*a+c+(3*b+d)*x)/(3*b+d)
```

3.185.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(89) = 178.

Time = 0.25 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.51

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx$$
$$=\frac{9\left(b^3-bd^2\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2+3\left(\left(b^3-bd^2\right)\cosh\left(bx+a\right)^3-\left(9\,b^3-bd^2\right)\cosh\left(b\right.\right.}{4\left(\left(9\,b^4-10\,b^2d^2+d^4\right)\cosh\left(bx+a\right)^4-2\left(9\,b^4-10\,b^2d^2\right.\right.}$$

input
```
integrate(cosh(d*x+c)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(9*(b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 + 3*((b^3
 - b*d^2)*cosh(b*x + a)^3 - (9*b^3 - b*d^2)*cosh(b*x + a))*cosh(d*x + c) -
 ((b^2*d - d^3)*sinh(b*x + a)^3 - 3*(9*b^2*d - d^3 - (b^2*d - d^3)*cosh(b*
x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^2*d^2 + d^4)*cosh(b
*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 +
 (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

3.185.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(76) = 152$.

Time = 1.94 (sec) , antiderivative size = 935, normalized size of antiderivative = 9.64

$$\int \cosh(c + dx) \sinh^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)*sinh(b*x+a)**3,x)
```

output
```
Piecewise((x*sinh(a)**3*cosh(c), Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - d*x)*
*3*cosh(c + d*x)/8 + 3*x*sinh(a - d*x)**2*sinh(c + d*x)*cosh(a - d*x)/8 -
3*x*sinh(a - d*x)*cosh(a - d*x)**2*cosh(c + d*x)/8 - 3*x*sinh(c + d*x)*cos
h(a - d*x)**3/8 - sinh(a - d*x)**3*sinh(c + d*x)/(8*d) - 3*sinh(a - d*x)**
2*cosh(a - d*x)*cosh(c + d*x)/(4*d) + 3*cosh(a - d*x)**3*cosh(c + d*x)/(8*
d), Eq(b, -d)), (x*sinh(a - d*x/3)**3*cosh(c + d*x)/8 + 3*x*sinh(a - d*x/3
)**2*sinh(c + d*x)*cosh(a - d*x/3)/8 + 3*x*sinh(a - d*x/3)*cosh(a - d*x/3)
**2*cosh(c + d*x)/8 + x*sinh(c + d*x)*cosh(a - d*x/3)**3/8 + 9*sinh(a - d*
x/3)**3*sinh(c + d*x)/(8*d) + 3*sinh(a - d*x/3)**2*cosh(a - d*x/3)*cosh(c
+ d*x)/(4*d) - cosh(a - d*x/3)**3*cosh(c + d*x)/(8*d), Eq(b, -d/3)), (x*si
nh(a + d*x/3)**3*cosh(c + d*x)/8 - 3*x*sinh(a + d*x/3)**2*sinh(c + d*x)*co
sh(a + d*x/3)/8 + 3*x*sinh(a + d*x/3)*cosh(a + d*x/3)**2*cosh(c + d*x)/8 -
 x*sinh(c + d*x)*cosh(a + d*x/3)**3/8 + 7*sinh(a + d*x/3)**3*sinh(c + d*x)
/(8*d) - 3*sinh(a + d*x/3)*sinh(c + d*x)*cosh(a + d*x/3)**2/(4*d) + 3*cosh
(a + d*x/3)**3*cosh(c + d*x)/(8*d), Eq(b, d/3)), (3*x*sinh(a + d*x)**3*cos
h(c + d*x)/8 - 3*x*sinh(a + d*x)**2*sinh(c + d*x)*cosh(a + d*x)/8 - 3*x*si
nh(a + d*x)*cosh(a + d*x)**2*cosh(c + d*x)/8 + 3*x*sinh(c + d*x)*cosh(a +
d*x)**3/8 + 5*sinh(a + d*x)**3*sinh(c + d*x)/(8*d) - 3*sinh(a + d*x)*sinh(
c + d*x)*cosh(a + d*x)**2/(4*d) + 3*cosh(a + d*x)**3*cosh(c + d*x)/(8*d),
Eq(b, d)), (9*b**3*sinh(a + b*x)**2*cosh(a + b*x)*cosh(c + d*x)/(9*b**4...
```

3.185.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh(c + dx) \sinh^3(a + bx) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more
 details)I
```

3.185.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(89) = 178$.

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.89

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx = \frac{e^{(3\,bx+dx+3\,a+c)}}{16\,(3\,b+d)} + \frac{e^{(3\,bx-dx+3\,a-c)}}{16\,(3\,b-d)} - \frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)} - \frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)} - \frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)} - \frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)} + \frac{e^{(-3\,bx+dx-3\,a+c)}}{16\,(3\,b-d)} + \frac{e^{(-3\,bx-dx-3\,a-c)}}{16\,(3\,b+d)}$$

input
```
integrate(cosh(d*x+c)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/16*e^(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/16*e^(3*b*x - d*x + 3*a - c)/
(3*b - d) - 3/16*e^(b*x + d*x + a + c)/(b + d) - 3/16*e^(b*x - d*x + a - c
)/(b - d) - 3/16*e^(-b*x + d*x - a + c)/(b - d) - 3/16*e^(-b*x - d*x - a -
 c)/(b + d) + 1/16*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 1/16*e^(-3*b*x -
 d*x - 3*a - c)/(3*b + d)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.89

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx = \frac{6\,b^2\,d\cosh(a+b\,x)^2\sinh(a+b\,x)\,\sinh(c+d\,x)}{9\,b^4-10\,b^2\,d^2+d^4} - \frac{d\sinh(a+b\,x)^3\sinh(c+d\,x)\,\left(7\,b^2-d^2\right)}{9\,b^4-10\,b^2\,d^2+d^4} - \frac{3\cosh(a+b\,x)\,\cosh(c+d\,x)\,\sinh(a+b\,x)^2\,\left(b\,d^2-3\,b^3\right)}{9\,b^4-10\,b^2\,d^2+d^4} - \frac{6\,b^3\cosh(a+b\,x)^3\cosh(c+d\,x)}{9\,b^4-10\,b^2\,d^2+d^4}$$

input
```
int(cosh(c + d*x)*sinh(a + b*x)^3,x)
```

output
```
(6*b^2*d*cosh(a + b*x)^2*sinh(a + b*x)*sinh(c + d*x))/(9*b^4 + d^4 - 10*b^
2*d^2) - (d*sinh(a + b*x)^3*sinh(c + d*x)*(7*b^2 - d^2))/(9*b^4 + d^4 - 10
*b^2*d^2) - (3*cosh(a + b*x)*cosh(c + d*x)*sinh(a + b*x)^2*(b*d^2 - 3*b^3)
)/(9*b^4 + d^4 - 10*b^2*d^2) - (6*b^3*cosh(a + b*x)^3*cosh(c + d*x))/(9*b^
4 + d^4 - 10*b^2*d^2)
```

3.185. $\int \cosh(c+dx)\sinh^3(a+bx)\,dx$

3.186 $\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx$

3.186.1 Optimal result

Integrand size = 17, antiderivative size = 138

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = -\frac{3\cosh(a+bx)}{8b}+\frac{\cosh(3a+3bx)}{24b} - \frac{3\cosh(a-2c+(b-2d)x)}{16(b-2d)} + \frac{\cosh(3a-2c+(3b-2d)x)}{16(3b-2d)} - \frac{3\cosh(a+2c+(b+2d)x)}{16(b+2d)} + \frac{\cosh(3a+2c+(3b+2d)x)}{16(3b+2d)}$$

output
```
-3/8*cosh(b*x+a)/b+1/24*cosh(3*b*x+3*a)/b-3/16*cosh(a-2*c+(b-2*d)*x)/(b-2*
d)+1/16*cosh(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cosh(a+2*c+(b+2*d)*x)/(b+
2*d)+1/16*cosh(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

3.186.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = \frac{1}{48}\left(-\frac{18\cosh(a)\cosh(bx)}{b}+\frac{2\cosh(3a)\cosh(3bx)}{b}-\frac{9\cosh(a-2c+bx-2dx)}{b-2d}+\frac{3\cosh(3a-2c+3bx-2dx)}{3b-2d}-\frac{9\cosh(a+2c+bx+2dx)}{b+2d}+\frac{3\cosh(3a+2c+3bx+2dx)}{3b+2d}-\frac{18\sinh(a)\sinh(bx)}{b}+\frac{2\sinh(3a)\sinh(3bx)}{b}\right)$$

input
```
Integrate[Cosh[c + d*x]^2*Sinh[a + b*x]^3,x]
```

output
```
((-18*Cosh[a]*Cosh[b*x])/b + (2*Cosh[3*a]*Cosh[3*b*x])/b - (9*Cosh[a - 2*c
 + b*x - 2*d*x])/(b - 2*d) + (3*Cosh[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*
d) - (9*Cosh[a + 2*c + b*x + 2*d*x])/(b + 2*d) + (3*Cosh[3*a + 2*c + 3*b*x
 + 2*d*x])/(3*b + 2*d) - (18*Sinh[a]*Sinh[b*x])/b + (2*Sinh[3*a]*Sinh[3*b*
x])/b)/48
```

3.186.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6152, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\cosh^2(c+dx)\,dx$$

$$\Big\downarrow \text{6152}$$

$$\int \left(-\frac{3}{16}\sinh(a+x(b-2d)-2c)+\frac{1}{16}\sinh(3a+x(3b-2d)-2c)-\frac{3}{16}\sinh(a+x(b+2d)+2c)+\frac{1}{16}\sinh(3a+\right.$$

$$\downarrow 2009$$

$$-\frac{3\cosh(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\cosh(3a+x(3b-2d)-2c)}{16(3b-2d)}-\frac{3\cosh(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\cosh(3a+x(3b+2d)+2c)}{16(3b+2d)}-\frac{3\cosh(a+bx)}{8b}+\frac{\cosh(3a+3bx)}{24b}$$

input
```
Int[Cosh[c + d*x]^2*Sinh[a + b*x]^3,x]
```

output
```
(-3*Cosh[a + b*x])/(8*b) + Cosh[3*a + 3*b*x]/(24*b) - (3*Cosh[a - 2*c + (b
 - 2*d)*x])/(16*(b - 2*d)) + Cosh[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*
d)) - (3*Cosh[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) + Cosh[3*a + 2*c + (3
*b + 2*d)*x]/(16*(3*b + 2*d))
```

3.186.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6152
```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.186.4 Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

method	result
default	$-\frac{3\cosh(bx+a)}{8b}+\frac{\cosh(3bx+3a)}{24b}-\frac{3\cosh(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\cosh(3a-2c+(3b-2d)x)}{48b-32d}-\frac{3\cosh(a+2c+(b+2d)x)}{16(b+2d)}+$ c
parallelrisch	$\frac{9(b+2d)\left(b+\frac{2d}{3}\right)(b-2d)b\cosh(3a-2c+(3b-2d)x)+9(b+2d)\left(b-\frac{2d}{3}\right)(b-2d)b\cosh(3a+2c+(3b+2d)x)-81(b+2d)\left(b+\frac{2d}{3}\right)\left(b-\frac{2}{3}\right.}{}$
risch	$\frac{e^{3bx+3a}}{48b}-\frac{3\,e^{bx+a}}{16b}-\frac{3\,e^{-bx-a}}{16b}+\frac{e^{-3bx-3a}}{48b}+\frac{(3b^3e^{6bx+6a}-2b^2d\,e^{6bx+6a}-12b\,d^2e^{6bx+6a}+8d^3e^{6bx+6a}-27b^3e^{4bx+4a}+}{}$

input
```
int(cosh(d*x+c)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cosh(b*x+a)/b+1/24*cosh(3*b*x+3*a)/b-3/16*cosh(a-2*c+(b-2*d)*x)/(b-2*
d)+1/16*cosh(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cosh(a+2*c+(b+2*d)*x)/(b+
2*d)+1/16*cosh(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

3.186.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(126) = 252$.

Time = 0.26 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.21

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx$$
$$= \frac{(9\,b^4-40\,b^2d^2+16\,d^4)\cosh{(bx+a)}^3+9\left((b^4-4\,b^2d^2)\cosh{(bx+a)}^3-(9\,b^4-4\,b^2d^2)\cosh{(bx+a)}\right)\cos}{}$$

input
```
integrate(cosh(d*x+c)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/24*((9*b^4 - 40*b^2*d^2 + 16*d^4)*cosh(b*x + a)^3 + 9*((b^4 - 4*b^2*d^2)
*cosh(b*x + a)^3 - (9*b^4 - 4*b^2*d^2)*cosh(b*x + a))*cosh(d*x + c)^2 + 3*
(9*(b^4 - 4*b^2*d^2)*cosh(b*x + a)*cosh(d*x + c)^2 + (9*b^4 - 40*b^2*d^2 +
 16*d^4)*cosh(b*x + a))*sinh(b*x + a)^2 + 9*((b^4 - 4*b^2*d^2)*cosh(b*x +
a)^3 + 3*(b^4 - 4*b^2*d^2)*cosh(b*x + a)*sinh(b*x + a)^2 - (9*b^4 - 4*b^2*
d^2)*cosh(b*x + a))*sinh(d*x + c)^2 - 9*(9*b^4 - 40*b^2*d^2 + 16*d^4)*cosh
(b*x + a) - 12*((b^3*d - 4*b*d^3)*cosh(d*x + c)*sinh(b*x + a)^3 - 3*(9*b^3
*d - 4*b*d^3 - (b^3*d - 4*b*d^3)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x +
 a))*sinh(d*x + c))/((9*b^5 - 40*b^3*d^2 + 16*b*d^4)*cosh(b*x + a)^4 - 2*(
9*b^5 - 40*b^3*d^2 + 16*b*d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (9*b^5 -
40*b^3*d^2 + 16*b*d^4)*sinh(b*x + a)^4)
```

3.186.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. $2(116) = 232$.

Time = 5.34 (sec) , antiderivative size = 2030, normalized size of antiderivative = 14.71

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)**2*sinh(b*x+a)**3,x)
```

output
```
Piecewise((x*sinh(a)**3*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*
x)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a
)**3, Eq(b, 0)), (3*x*sinh(a - 2*d*x)**3*sinh(c + d*x)**2/16 + 3*x*sinh(a
- 2*d*x)**3*cosh(c + d*x)**2/16 + 3*x*sinh(a - 2*d*x)**2*sinh(c + d*x)*cos
h(a - 2*d*x)*cosh(c + d*x)/8 - 3*x*sinh(a - 2*d*x)*sinh(c + d*x)**2*cosh(a
 - 2*d*x)**2/16 - 3*x*sinh(a - 2*d*x)*cosh(a - 2*d*x)**2*cosh(c + d*x)**2/
16 - 3*x*sinh(c + d*x)*cosh(a - 2*d*x)**3*cosh(c + d*x)/8 + 13*sinh(a - 2*
d*x)**3*sinh(c + d*x)*cosh(c + d*x)/(16*d) + sinh(a - 2*d*x)**2*sinh(c + d
*x)**2*cosh(a - 2*d*x)/(2*d) - 7*sinh(a - 2*d*x)*sinh(c + d*x)*cosh(a - 2*
d*x)**2*cosh(c + d*x)/(8*d) - 49*sinh(c + d*x)**2*cosh(a - 2*d*x)**3/(96*d
) - 17*cosh(a - 2*d*x)**3*cosh(c + d*x)**2/(96*d), Eq(b, -2*d)), (x*sinh(a
 - 2*d*x/3)**3*sinh(c + d*x)**2/16 + x*sinh(a - 2*d*x/3)**3*cosh(c + d*x)*
*2/16 + 3*x*sinh(a - 2*d*x/3)**2*sinh(c + d*x)*cosh(a - 2*d*x/3)*cosh(c +
d*x)/8 + 3*x*sinh(a - 2*d*x/3)*sinh(c + d*x)**2*cosh(a - 2*d*x/3)**2/16 +
3*x*sinh(a - 2*d*x/3)*cosh(a - 2*d*x/3)**2*cosh(c + d*x)**2/16 + x*sinh(c
+ d*x)*cosh(a - 2*d*x/3)**3*cosh(c + d*x)/8 + 15*sinh(a - 2*d*x/3)**3*sinh
(c + d*x)*cosh(c + d*x)/(16*d) + 3*sinh(a - 2*d*x/3)**2*sinh(c + d*x)**2*c
osh(a - 2*d*x/3)/(2*d) + 9*sinh(a - 2*d*x/3)*sinh(c + d*x)*cosh(a - 2*d*x/
3)**2*cosh(c + d*x)/(8*d) - 11*sinh(c + d*x)**2*cosh(a - 2*d*x/3)**3/(32*d
) + 21*cosh(a - 2*d*x/3)**3*cosh(c + d*x)**2/(32*d), Eq(b, -2*d/3)), (x...
```

3.186.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for
more detai
```

3.186.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(126) = 252$.

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.86

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = \frac{e^{(3\,bx+2\,dx+3\,a+2\,c)}}{32\,(3\,b+2\,d)} + \frac{e^{(3\,bx-2\,dx+3\,a-2\,c)}}{32\,(3\,b-2\,d)} + \frac{e^{(3\,bx+3\,a)}}{48\,b} - \frac{3\,e^{(bx+2\,dx+a+2\,c)}}{32\,(b+2\,d)} - \frac{3\,e^{(bx-2\,dx+a-2\,c)}}{32\,(b-2\,d)} - \frac{3\,e^{(bx+a)}}{16\,b} - \frac{3\,e^{(-bx+2\,dx-a+2\,c)}}{32\,(b-2\,d)} - \frac{3\,e^{(-bx-2\,dx-a-2\,c)}}{32\,(b+2\,d)} - \frac{3\,e^{(-bx-a)}}{16\,b} + \frac{e^{(-3\,bx+2\,dx-3\,a+2\,c)}}{32\,(3\,b-2\,d)} + \frac{e^{(-3\,bx-2\,dx-3\,a-2\,c)}}{32\,(3\,b+2\,d)} + \frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

input
```
integrate(cosh(d*x+c)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/32*e^(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) + 1/32*e^(3*b*x - 2*d*x + 3
*a - 2*c)/(3*b - 2*d) + 1/48*e^(3*b*x + 3*a)/b - 3/32*e^(b*x + 2*d*x + a +
 2*c)/(b + 2*d) - 3/32*e^(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/16*e^(b*x +
 a)/b - 3/32*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) - 3/32*e^(-b*x - 2*d*x -
 a - 2*c)/(b + 2*d) - 3/16*e^(-b*x - a)/b + 1/32*e^(-3*b*x + 2*d*x - 3*a +
 2*c)/(3*b - 2*d) + 1/32*e^(-3*b*x - 2*d*x - 3*a - 2*c)/(3*b + 2*d) + 1/48
*e^(-3*b*x - 3*a)/b
```

3.186.9 Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.44

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = \frac{\cosh(a+b\,x)\,\cosh(c+d\,x)^2\sinh(a+b\,x)^2\,(9\,b^4-26\,b^2\,d^2+8\,d^4)}{b\,(9\,b^4-40\,b^2\,d^2+16\,d^4)} - \cosh(a+b\,x)^3\sinh(c+d\,x)^2\left(\frac{3\,b^3}{9\,b^4-40\,b^2\,d^2+16\,d^4}-\frac{1}{3\,b}\right) - \cosh(a+b\,x)^3\cosh(c+d\,x)^2\left(\frac{3\,b^3}{9\,b^4-40\,b^2\,d^2+16\,d^4}+\frac{1}{3\,b}\right) - \frac{2\,d\cosh(c+d\,x)\,\sinh(a+b\,x)^3\sinh(c+d\,x)\,(7\,b^2-4\,d^2)}{9\,b^4-40\,b^2\,d^2+16\,d^4} + \frac{12\,b^2\,d\cosh(a+b\,x)^2\cosh(c+d\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)}{9\,b^4-40\,b^2\,d^2+16\,d^4} + \frac{2\,d^2\cosh(a+b\,x)\,\sinh(a+b\,x)^2\sinh(c+d\,x)^2\,(7\,b^2-4\,d^2)}{b\,(9\,b^4-40\,b^2\,d^2+16\,d^4)}$$

input
```
int(cosh(c + d*x)^2*sinh(a + b*x)^3,x)
```

output
```
(cosh(a + b*x)*cosh(c + d*x)^2*sinh(a + b*x)^2*(9*b^4 + 8*d^4 - 26*b^2*d^2
))/(b*(9*b^4 + 16*d^4 - 40*b^2*d^2)) - cosh(a + b*x)^3*sinh(c + d*x)^2*((3
*b^3)/(9*b^4 + 16*d^4 - 40*b^2*d^2) - 1/(3*b)) - cosh(a + b*x)^3*cosh(c +
d*x)^2*((3*b^3)/(9*b^4 + 16*d^4 - 40*b^2*d^2) + 1/(3*b)) - (2*d*cosh(c + d
*x)*sinh(a + b*x)^3*sinh(c + d*x)*(7*b^2 - 4*d^2))/(9*b^4 + 16*d^4 - 40*b^
2*d^2) + (12*b^2*d*cosh(a + b*x)^2*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*
x))/(9*b^4 + 16*d^4 - 40*b^2*d^2) + (2*d^2*cosh(a + b*x)*sinh(a + b*x)^2*s
inh(c + d*x)^2*(7*b^2 - 4*d^2))/(b*(9*b^4 + 16*d^4 - 40*b^2*d^2))
```

3.187 $\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx$

3.187.1 Optimal result

Integrand size = 17, antiderivative size = 195

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = -\frac{3\cosh(a-3c+(b-3d)x)}{32(b-3d)} - \frac{9\cosh(a-c+(b-d)x)}{32(b-d)} + \frac{\cosh(3(a-c)+3(b-d)x)}{96(b-d)} + \frac{3\cosh(3a-c+(3b-d)x)}{32(3b-d)} - \frac{9\cosh(a+c+(b+d)x)}{32(b+d)} + \frac{\cosh(3(a+c)+3(b+d)x)}{96(b+d)} + \frac{3\cosh(3a+c+(3b+d)x)}{32(3b+d)} - \frac{3\cosh(a+3c+(b+3d)x)}{32(b+3d)}$$

output
```
-3/32*cosh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cosh(a-c+(b-d)*x)/(b-d)+1/96*cosh
(3*a-3*c+3*(b-d)*x)/(b-d)+3/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-9/32*cosh(a+c
+(b+d)*x)/(b+d)+1/96*cosh(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*cosh(3*a+c+(3*b+d)
*x)/(3*b+d)-3/32*cosh(a+3*c+(b+3*d)*x)/(b+3*d)
```

3.187.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = \frac{1}{96}\left(-\frac{9\cosh(a-3c+bx-3dx)}{b-3d} - \frac{27\cosh(a-c+bx-dx)}{b-d} + \frac{\cosh(3(a-c+bx-dx))}{b-d} + \frac{9\cosh(3a-c+3bx-dx)}{3b-d} + \frac{9\cosh(3a+c+3bx+dx)}{3b+d} - \frac{9\cosh(a+3c+bx+3dx)}{b+3d} - \frac{27\cosh(a+c+(b+d)x)}{b+d} + \frac{\cosh(3(a+c+(b+d)x))}{b+d}\right)$$

input
```
Integrate[Cosh[c + d*x]^3*Sinh[a + b*x]^3,x]
```

output
```
((-9*Cosh[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Cosh[a - c + b*x - d*x])
/(b - d) + Cosh[3*(a - c + b*x - d*x)]/(b - d) + (9*Cosh[3*a - c + 3*b*x -
 d*x])/(3*b - d) + (9*Cosh[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Cosh[a +
 3*c + b*x + 3*d*x])/(b + 3*d) - (27*Cosh[a + c + (b + d)*x])/(b + d) + Co
sh[3*(a + c + (b + d)*x)]/(b + d))/96
```

3.187.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6152, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\cosh^3(c+dx)\,dx$$

$$\Big\downarrow \text{ 6152}$$

$$\int \left(-\frac{3}{32}\sinh(a+x(b-3d)-3c)-\frac{9}{32}\sinh(a+x(b-d)-c)+\frac{1}{32}\sinh(3(a-c)+3x(b-d))+\frac{3}{32}\sinh(3a+x\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\cosh(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cosh(a+x(b-d)-c)}{32(b-d)}+\frac{\cosh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cosh(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\cosh(a+x(b+d)+c)}{32(b+d)}+\frac{\cosh(3(a+c)+3x(b+d))}{96(b+d)}+\frac{3\cosh(3a+x(3b+d)+c)}{32(3b+d)}-\frac{3\cosh(a+x(b+3d)+3c)}{32(b+3d)}$$

input
```
Int[Cosh[c + d*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(-3*Cosh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Cosh[a - c + (b - d)*
x])/(32*(b - d)) + Cosh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Cosh[3*
a - c + (3*b - d)*x])/(32*(3*b - d)) - (9*Cosh[a + c + (b + d)*x])/(32*(b
+ d)) + Cosh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Cosh[3*a + c + (3*
b + d)*x])/(32*(3*b + d)) - (3*Cosh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

3.187.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6152
```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.187.4 Maple [A] (verified)

Time = 11.36 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

method	result
default	$-\frac{3\cosh(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\cosh(a-c+(b-d)x)}{32(b-d)}-\frac{9\cosh(a+c+(b+d)x)}{32(b+d)}-\frac{3\cosh(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\cosh((3b-3d)x}{96b-96}$
parallelrisch	$\frac{9(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b+d)\left(b-\frac{d}{3}\right)\cosh((3b-3d)x+3a-3c)+27(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b-d)(b+d)\cosh(3a-c+(3b-d)x)+9(b}{}$
risch	$\frac{(b^3e^{6bx+6a}-b^2d\,e^{6bx+6a}-9b\,d^2e^{6bx+6a}+9d^3e^{6bx+6a}-9b^3e^{4bx+4a}+27b^2d\,e^{4bx+4a}+9b\,d^2e^{4bx+4a}-27d^3e^{4bx+4a}-9b^3e^{2bx+2a}-}{192(b+d)(b+3d)(b-d)(b-3d)}$

input
```
int(cosh(d*x+c)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/32*cosh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cosh(a-c+(b-d)*x)/(b-d)-9/32*cosh
(a+c+(b+d)*x)/(b+d)-3/32*cosh(a+3*c+(b+3*d)*x)/(b+3*d)+1/32/(3*b-3*d)*cosh
((3*b-3*d)*x+3*a-3*c)+3/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*cosh(3*a+c+(
3*b+d)*x)/(3*b+d)+1/32/(3*b+3*d)*cosh((3*b+3*d)*x+3*a+3*c)
```

3.187.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. 2(179) = 358.

Time = 0.27 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.74

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx$$
$$=\frac{\left((9\,b^5-82\,b^3d^2+9\,bd^4)\cosh{(bx+a)}^3-9\,(9\,b^5-10\,b^3d^2+bd^4)\cosh{(bx+a)}\right)\cosh{(dx+c)}^3-\left((9\,b^4d-\right.}{}$$

input
```
integrate(cosh(d*x+c)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/48*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 - 9*(9*b^5 - 10*b^3*
d^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*d
^5)*sinh(b*x + a)^3 - 3*(81*b^4*d - 90*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2
*d^3 + 9*d^5)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c)^3 + 3*((9*b^5
- 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)*cosh(d*x + c)^3 + 27*(b^5 - 10*b^3*d
^2 + 9*b*d^4)*cosh(b*x + a)*cosh(d*x + c))*sinh(b*x + a)^2 + 3*(3*(9*b^5 -
 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 + ((9*b
^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 - 9*(9*b^5 - 10*b^3*d^2 + b*d^4
)*cosh(b*x + a))*cosh(d*x + c))*sinh(d*x + c)^2 + 27*((b^5 - 10*b^3*d^2 +
9*b*d^4)*cosh(b*x + a)^3 - (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a))*c
osh(d*x + c) - 3*((3*b^4*d - 30*b^2*d^3 + 27*d^5 + (9*b^4*d - 82*b^2*d^3 +
 9*d^5)*cosh(d*x + c)^2)*sinh(b*x + a)^3 - 3*(27*b^4*d - 246*b^2*d^3 + 27*
d^5 - 3*(b^4*d - 10*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 + (81*b^4*d - 90*b^2*
d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2)*cosh(d*x + c
)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d
^6)*cosh(b*x + a)^4 - 2*(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x
 + a)^2*sinh(b*x + a)^2 + (9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*sinh(b
*x + a)^4)
```

3.187.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. $2(172) = 344$.

Time = 16.87 (sec) , antiderivative size = 3580, normalized size of antiderivative = 18.36

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)**3*sinh(b*x+a)**3,x)
```

output
```
Piecewise((x*sinh(a)**3*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (9*x*sinh(a - 3*
d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/32 + 3*x*sinh(a - 3*d*x)**3*cosh(c
+ d*x)**3/32 + 3*x*sinh(a - 3*d*x)**2*sinh(c + d*x)**3*cosh(a - 3*d*x)/32
+ 9*x*sinh(a - 3*d*x)**2*sinh(c + d*x)*cosh(a - 3*d*x)*cosh(c + d*x)**2/32
 - 9*x*sinh(a - 3*d*x)*sinh(c + d*x)**2*cosh(a - 3*d*x)**2*cosh(c + d*x)/3
2 - 3*x*sinh(a - 3*d*x)*cosh(a - 3*d*x)**2*cosh(c + d*x)**3/32 - 3*x*sinh(
c + d*x)**3*cosh(a - 3*d*x)**3/32 - 9*x*sinh(c + d*x)*cosh(a - 3*d*x)**3*c
osh(c + d*x)**2/32 + sinh(a - 3*d*x)**3*sinh(c + d*x)**3/(12*d) - 13*sinh(
a - 3*d*x)**3*sinh(c + d*x)*cosh(c + d*x)**2/(320*d) + 3*sinh(a - 3*d*x)**
2*sinh(c + d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)/(20*d) - 101*sinh(a - 3*d
*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)**3/(320*d) - 27*sinh(a - 3*d*x)*sinh(
c + d*x)**3*cosh(a - 3*d*x)**2/(320*d) - 51*sinh(c + d*x)**2*cosh(a - 3*d*
x)**3*cosh(c + d*x)/(320*d) + cosh(a - 3*d*x)**3*cosh(c + d*x)**3/(5*d), E
q(b, -3*d)), (-3*x*sinh(a - d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/16 + 5*
x*sinh(a - d*x)**3*cosh(c + d*x)**3/16 - 3*x*sinh(a - d*x)**2*sinh(c + d*x
)**3*cosh(a - d*x)/16 + 9*x*sinh(a - d*x)**2*sinh(c + d*x)*cosh(a - d*x)*c
osh(c + d*x)**2/16 + 9*x*sinh(a - d*x)*sinh(c + d*x)**2*cosh(a - d*x)**2*c
osh(c + d*x)/16 - 3*x*sinh(a - d*x)*cosh(a - d*x)**2*cosh(c + d*x)**3/16 +
 5*x*sinh(c + d*x)**3*cosh(a - d*x)**3/16 - 3*x*sinh(c + d*x)*cosh(a - d*x
)**3*cosh(c + d*x)**2/16 - sinh(a - d*x)**3*sinh(c + d*x)**3/(48*d) + s...
```

3.187.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for
more detai
```

3.187.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(179) = 358$.

Time = 0.30 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.91

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = \frac{e^{(3\,bx+3\,dx+3\,a+3\,c)}}{192\,(b+d)} + \frac{3\,e^{(3\,bx+dx+3\,a+c)}}{64\,(3\,b+d)} + \frac{3\,e^{(3\,bx-dx+3\,a-c)}}{64\,(3\,b-d)} + \frac{e^{(3\,bx-3\,dx+3\,a-3\,c)}}{192\,(b-d)} - \frac{3\,e^{(bx+3\,dx+a+3\,c)}}{64\,(b+3\,d)} - \frac{9\,e^{(bx+dx+a+c)}}{64\,(b+d)} - \frac{9\,e^{(bx-dx+a-c)}}{64\,(b-d)} - \frac{3\,e^{(bx-3\,dx+a-3\,c)}}{64\,(b-3\,d)} - \frac{3\,e^{(-bx+3\,dx-a+3\,c)}}{64\,(b-3\,d)} - \frac{9\,e^{(-bx+dx-a+c)}}{64\,(b-d)} - \frac{9\,e^{(-bx-dx-a-c)}}{64\,(b+d)} - \frac{3\,e^{(-bx-3\,dx-a-3\,c)}}{64\,(b+3\,d)} + \frac{e^{(-3\,bx+3\,dx-3\,a+3\,c)}}{192\,(b-d)} + \frac{3\,e^{(-3\,bx+dx-3\,a+c)}}{64\,(3\,b-d)} + \frac{3\,e^{(-3\,bx-dx-3\,a-c)}}{64\,(3\,b+d)} + \frac{e^{(-3\,bx-3\,dx-3\,a-3\,c)}}{192\,(b+d)}$$

input
```
integrate(cosh(d*x+c)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/192*e^(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/64*e^(3*b*x + d*x + 3*a +
c)/(3*b + d) + 3/64*e^(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/192*e^(3*b*x -
 3*d*x + 3*a - 3*c)/(b - d) - 3/64*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 9
/64*e^(b*x + d*x + a + c)/(b + d) - 9/64*e^(b*x - d*x + a - c)/(b - d) - 3
/64*e^(b*x - 3*d*x + a - 3*c)/(b - 3*d) - 3/64*e^(-b*x + 3*d*x - a + 3*c)/
(b - 3*d) - 9/64*e^(-b*x + d*x - a + c)/(b - d) - 9/64*e^(-b*x - d*x - a -
 c)/(b + d) - 3/64*e^(-b*x - 3*d*x - a - 3*c)/(b + 3*d) + 1/192*e^(-3*b*x
+ 3*d*x - 3*a + 3*c)/(b - d) + 3/64*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) +
 3/64*e^(-3*b*x - d*x - 3*a - c)/(3*b + d) + 1/192*e^(-3*b*x - 3*d*x - 3*a
 - 3*c)/(b + d)
```

3.187.9 Mupad [B] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 908, normalized size of antiderivative = 4.66

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = -e^{3a+c+3bx+dx}\left(\frac{-9b^3+3b^2d+9bd^2-3d^3}{576b^4-640b^2d^2+64d^4} + \frac{e^{-6a-6bx}(-9b^3-3b^2d+9bd^2+3d^3)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-2a-2bx}(-81b^3+81b^2d+9bd^2-9d^3)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-4a-4bx}(-81b^3-81b^2d+9bd^2+9d^3)}{576b^4-640b^2d^2+64d^4}\right) - e^{3a-c+3bx-dx}\left(\frac{-9b^3-3b^2d+9bd^2+3d^3}{576b^4-640b^2d^2+64d^4} + \frac{e^{-6a-6bx}(-9b^3+3b^2d+9bd^2-3d^3)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-2a-2bx}(-81b^3-81b^2d+9bd^2+9d^3)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-4a-4bx}(-81b^3+81b^2d+9bd^2-9d^3)}{576b^4-640b^2d^2+64d^4}\right) - e^{3a-3c+3bx-3dx}\left(\frac{-b^3-b^2d+9bd^2+9d^3}{192b^4-1920b^2d^2+1728d^4} + \frac{e^{-6a-6bx}(-b^3+b^2d+9bd^2-9d^3)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-2a-2bx}(-9b^3-27b^2d+9bd^2+27d^3)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-4a-4bx}(-9b^3+27b^2d+9bd^2-27d^3)}{192b^4-1920b^2d^2+1728d^4}\right) - e^{3a+3c+3bx+3dx}\left(\frac{-b^3+b^2d+9bd^2-9d^3}{192b^4-1920b^2d^2+1728d^4} + \frac{e^{-6a-6bx}(-b^3-b^2d+9bd^2+9d^3)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-2a-2bx}(-9b^3+27b^2d+9bd^2-27d^3)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-4a-4bx}(-9b^3-27b^2d+9bd^2+27d^3)}{192b^4-1920b^2d^2+1728d^4}\right)$$

input
```
int(cosh(c + d*x)^3*sinh(a + b*x)^3,x)
```

output

```
- exp(3*a + c + 3*b*x + d*x)*((9*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(576*b^4
 + 64*d^4 - 640*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 - 3*b^2*d - 9*b^3
+ 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 2*a - 2*b*x)*(9*b*d^2
+ 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4*
a - 4*b*x)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*
b^2*d^2)) - exp(3*a - c + 3*b*x - d*x)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3
)/(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 + 3*b^2*
d - 9*b^3 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 2*a - 2*b*x)
*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) -
 (exp(- 4*a - 4*b*x)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*
d^4 - 640*b^2*d^2)) - exp(3*a - 3*c + 3*b*x - 3*d*x)*((9*b*d^2 - b^2*d - b
^3 + 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b
*d^2 + b^2*d - b^3 - 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(-
2*a - 2*b*x)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(192*b^4 + 1728*d^4 -
1920*b^2*d^2) - (exp(- 4*a - 4*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))
/(192*b^4 + 1728*d^4 - 1920*b^2*d^2)) - exp(3*a + 3*c + 3*b*x + 3*d*x)*((9
*b*d^2 + b^2*d - b^3 - 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(-
 6*a - 6*b*x)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(192*b^4 + 1728*d^4 - 1920*
b^2*d^2) - (exp(- 2*a - 2*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192
*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 4*a - 4*b*x)*(9*b*d^2 - 27*b^2...
```

3.187. $\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx$

3.188 $\int \sinh(a+bx)\tanh(c+dx)\,dx$

3.188.1 Optimal result

Integrand size = 13, antiderivative size = 121

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = \frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b} - \frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},-e^{2(c+dx)}\right)}{b}$$

output
```
1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)/b-exp(-b*x-a)*hypergeom([1, -1/2*b/d],[1-
1/2*b/d],-exp(2*d*x+2*c))/b-exp(b*x+a)*hypergeom([1, 1/2*b/d],[1+1/2*b/d],
-exp(2*d*x+2*c))/b
```

3.188.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = \frac{e^{-a-bx}\left(1+e^{2(a+bx)}-2\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)-2e^{2(a+bx)}\,\text{Hypergeometric2F1}\left(1\right.\right.}{2b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[c + d*x],x]
```

output
```
(E^(-a - b*x)*(1 + E^(2*(a + b*x)) - 2*Hypergeometric2F1[1, -1/2*b/d, 1 -
b/(2*d), -E^(2*(c + d*x))] - 2*E^(2*(a + b*x))*Hypergeometric2F1[1, b/(2*d
), 1 + b/(2*d), -E^(2*(c + d*x))]))/(2*b)
```

3.188.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6135, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh(c+dx)\,dx$$

$$\Big\downarrow \text{6135}$$

$$\int \left(\frac{e^{-a-bx}}{e^{2(c+dx)}+1}-\frac{e^{a+bx}}{e^{2(c+dx)}+1}-\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,-e^{2(c+dx)}\right)}{b}+\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

input
```
Int[Sinh[a + b*x]*Tanh[c + d*x],x]
```

output
```
E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) - (E^(-a - b*x)*Hypergeometric2F1[1
, -1/2*b/d, 1 - b/(2*d), -E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometri
c2F1[1, b/(2*d), 1 + b/(2*d), -E^(2*(c + d*x))])/b
```

3.188.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6135
```
Int[Sinh[(a_.) + (b_.)*(x_)]*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[-E^
(-(a + b*x))/2 + E^(a + b*x)/2 + 1/(E^(a + b*x)*(1 + E^(2*(c + d*x)))) - E^
(a + b*x)/(1 + E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

3.188.4 Maple [F]

$$\int \sinh (bx + a) \tanh (dx + c) \, dx$$

input
```
int(sinh(b*x+a)*tanh(d*x+c),x)
```

output
```
int(sinh(b*x+a)*tanh(d*x+c),x)
```

3.188.5 Fricas [F]

$$\int \sinh(a + bx) \tanh(c + dx) \, dx = \int \sinh (bx + a) \tanh (dx + c) \, dx$$

input
```
integrate(sinh(b*x+a)*tanh(d*x+c),x, algorithm="fricas")
```

output
```
integral(sinh(b*x + a)*tanh(d*x + c), x)
```

3.188.6 Sympy [F]

$$\int \sinh(a + bx) \tanh(c + dx) \, dx = \int \sinh (a + bx) \tanh (c + dx) \, dx$$

input
```
integrate(sinh(b*x+a)*tanh(d*x+c),x)
```

output
```
Integral(sinh(a + b*x)*tanh(c + d*x), x)
```

3.188.7 Maxima [F]

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = \int \sinh\left(bx+a\right)\tanh\left(dx+c\right)\,dx$$

input
```
integrate(sinh(b*x+a)*tanh(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)/b - 1/2*integrate(2*(e^(2*b*x + 2*a
) - 1)/(e^(b*x + 2*d*x + a + 2*c) + e^(b*x + a)), x)
```

3.188.8 Giac [F]

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = \int \sinh\left(bx+a\right)\tanh\left(dx+c\right)\,dx$$

input
```
integrate(sinh(b*x+a)*tanh(d*x+c),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)*tanh(d*x + c), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = \int \sinh(a+b\,x)\,\tanh(c+d\,x)\,dx$$

input
```
int(sinh(a + b*x)*tanh(c + d*x),x)
```

output
```
int(sinh(a + b*x)*tanh(c + d*x), x)
```

3.189 $\int \coth(c+dx)\sinh(a+bx)\,dx$

3.189.1 Optimal result

Integrand size = 13, antiderivative size = 117

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \frac{e^{-a-bx}}{2b} + \frac{e^{a+bx}}{2b} - \frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},e^{2(c+dx)}\right)}{b}$$

output
```
1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)/b-exp(-b*x-a)*hypergeom([1, -1/2*b/d],[1-
1/2*b/d],exp(2*d*x+2*c))/b-exp(b*x+a)*hypergeom([1, 1/2*b/d],[1+1/2*b/d],e
xp(2*d*x+2*c))/b
```

3.189.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(117) = 234.

Time = 2.15 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.05

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \frac{\cosh(a)\cosh(bx)\coth(c)}{b} + \frac{e^{-a+2c-bx}\left(be^{2dx}\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{2d},2-\frac{b}{2d},e^{2(c+dx)}\right)-(b-2d)\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d}\right.\right.}{b(b-2d)\left(-1+e^{2c}\right)} - \frac{e^{a+2c}\left(-\frac{e^{(b+2d)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{b}{2d},2+\frac{b}{2d},e^{2(c+dx)}\right)}{b+2d}+\frac{e^{bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},e^{2(c+dx)}\right)}{b}\right)}{-1+e^{2c}} + \frac{\coth(c)\sinh(a)\sinh(bx)}{b}$$

input
```
Integrate[Coth[c + d*x]*Sinh[a + b*x],x]
```

output
```
(Cosh[a]*Cosh[b*x]*Coth[c])/b + (E^(-a + 2*c - b*x)*(b*E^(2*d*x)*Hypergeom
etric2F1[1, 1 - b/(2*d), 2 - b/(2*d), E^(2*(c + d*x))] - (b - 2*d)*Hyperge
ometric2F1[1, -1/2*b/d, 1 - b/(2*d), E^(2*(c + d*x))]))/(b*(b - 2*d)*(-1 +
 E^(2*c))) - (E^(a + 2*c)*(-((E^((b + 2*d)*x)*Hypergeometric2F1[1, 1 + b/(
2*d), 2 + b/(2*d), E^(2*(c + d*x))])/(b + 2*d)) + (E^(b*x)*Hypergeometric2
F1[1, b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b))/(-1 + E^(2*c)) + (Coth[c
]*Sinh[a]*Sinh[b*x])/b
```

3.189.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6137, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\coth(c+dx)\,dx$$

$$\downarrow \text{ 6137}$$

$$\int \left(\frac{e^{-a-bx}}{1-e^{2(c+dx)}}-\frac{e^{a+bx}}{1-e^{2(c+dx)}}-\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{-a-bx}\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,e^{2(c+dx)}\right)}{b}+\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

input
```
Int[Coth[c + d*x]*Sinh[a + b*x],x]
```

output
```
E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) - (E^(-a - b*x)*Hypergeometric2F1[1
, -1/2*b/d, 1 - b/(2*d), E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric
2F1[1, b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b
```

3.189.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6137
```
Int[Coth[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :> Int[-E^
(-(a + b*x))/2 + E^(a + b*x)/2 + 1/(E^(a + b*x)*(1 - E^(2*(c + d*x)))) - E^
(a + b*x)/(1 - E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

3.189.4 Maple [F]

$$\int \coth(dx+c)\sinh(bx+a)\,dx$$

input
```
int(coth(d*x+c)*sinh(b*x+a),x)
```

output
```
int(coth(d*x+c)*sinh(b*x+a),x)
```

3.189.5 Fricas [F]

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \int \coth\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(coth(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(coth(d*x + c)*sinh(b*x + a), x)
```

3.189.6 Sympy [F]

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \int \sinh\left(a+bx\right)\coth\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*coth(c + d*x), x)
```

3.189.7 Maxima [F]

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \int \coth\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(coth(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)/b - 1/2*integrate((e^(2*b*x + 2*a)
- 1)/(e^(b*x + d*x + a + c) + e^(b*x + a)), x) + 1/2*integrate((e^(2*b*x +
 2*a) - 1)/(e^(b*x + d*x + a + c) - e^(b*x + a)), x)
```

3.189.8 Giac [F]

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \int \coth\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(coth(d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(coth(d*x + c)*sinh(b*x + a), x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \int \coth(c+d\,x)\,\sinh(a+b\,x)\,dx$$

input
```
int(coth(c + d*x)*sinh(a + b*x),x)
```

output
```
int(coth(c + d*x)*sinh(a + b*x), x)
```

3.190 $\int \cosh(a+bx)\coth(c+dx)\,dx$

3.190.1 Optimal result

Integrand size = 13, antiderivative size = 116

$$\int \cosh(a+bx)\coth(c+dx)\,dx = -\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b} + \frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},e^{2(c+dx)}\right)}{b}$$

output
```
-1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)/b+exp(-b*x-a)*hypergeom([1, -1/2*b/d],[1
-1/2*b/d],exp(2*d*x+2*c))/b-exp(b*x+a)*hypergeom([1, 1/2*b/d],[1+1/2*b/d],
exp(2*d*x+2*c))/b
```

3.190.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \cosh(a+bx)\coth(c+dx)\,dx = \frac{e^{-a-bx}\left(-1+e^{2(a+bx)}+2\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2(c+dx)}\right)-2e^{2(a+bx)}\,\text{Hypergeometric2F1}\left(1\right.\right.}{2b}$$

input
```
Integrate[Cosh[a + b*x]*Coth[c + d*x],x]
```

output
```
(E^(-a - b*x)*(-1 + E^(2*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 -
 b/(2*d), E^(2*(c + d*x))] - 2*E^(2*(a + b*x))*Hypergeometric2F1[1, b/(2*d
), 1 + b/(2*d), E^(2*(c + d*x))]))/(2*b)
```

3.190.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6136, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth(c+dx)\,dx$$

$$\Big\downarrow \text{6136}$$

$$\int \left(-\frac{e^{-a-bx}}{1-e^{2(c+dx)}}-\frac{e^{a+bx}}{1-e^{2(c+dx)}}+\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,e^{2(c+dx)}\right)}{b}-\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

input
```
Int[Cosh[a + b*x]*Coth[c + d*x],x]
```

output
```
-1/2*E^(-a - b*x)/b + E^(a + b*x)/(2*b) + (E^(-a - b*x)*Hypergeometric2F1[
1, -1/2*b/d, 1 - b/(2*d), E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometri
c2F1[1, b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b
```

3.190.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6136
```
Int[Cosh[(a_.) + (b_.)*(x_)]*Coth[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(
E^(a + b*x)*2) + E^(a + b*x)/2 - 1/(E^(a + b*x)*(1 - E^(2*(c + d*x)))) - E^
(a + b*x)/(1 - E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

3.190.4 Maple [F]

$$\int \cosh(bx+a)\coth(dx+c)\,dx$$

input
```
int(cosh(b*x+a)*coth(d*x+c),x)
```

output
```
int(cosh(b*x+a)*coth(d*x+c),x)
```

3.190.5 Fricas [F]

$$\int \cosh(a+bx)\coth(c+dx)\,dx = \int \cosh(bx+a)\coth(dx+c)\,dx$$

input
```
integrate(cosh(b*x+a)*coth(d*x+c),x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*coth(d*x + c), x)
```

3.190.6 Sympy [F]

$$\int \cosh(a+bx)\coth(c+dx)\,dx = \int \cosh(a+bx)\coth(c+dx)\,dx$$

input
```
integrate(cosh(b*x+a)*coth(d*x+c),x)
```

output
```
Integral(cosh(a + b*x)*coth(c + d*x), x)
```

3.190.7 Maxima [F]

$$\int \cosh(a+bx)\coth(c+dx)\,dx = \int \cosh{(bx+a)}\coth{(dx+c)}\,dx$$

input
```
integrate(cosh(b*x+a)*coth(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)/b - 1/2*integrate((e^(2*b*x + 2*a)
+ 1)/(e^(b*x + d*x + a + c) + e^(b*x + a)), x) + 1/2*integrate((e^(2*b*x +
 2*a) + 1)/(e^(b*x + d*x + a + c) - e^(b*x + a)), x)
```

3.190.8 Giac [F]

$$\int \cosh(a+bx)\coth(c+dx)\,dx = \int \cosh{(bx+a)}\coth{(dx+c)}\,dx$$

input
```
integrate(cosh(b*x+a)*coth(d*x+c),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)*coth(d*x + c), x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\coth(c+dx)\,dx = \int \cosh(a+b\,x)\,\coth(c+d\,x)\,dx$$

input
```
int(cosh(a + b*x)*coth(c + d*x),x)
```

output
```
int(cosh(a + b*x)*coth(c + d*x), x)
```

3.191 $\int \cosh(a+bx)\tanh(c+dx)\,dx$

3.191.1 Optimal result

Integrand size = 13, antiderivative size = 120

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = -\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b} + \frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},-e^{2(c+dx)}\right)}{b}$$

output
```
-1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)/b+exp(-b*x-a)*hypergeom([1, -1/2*b/d],[1
-1/2*b/d],-exp(2*d*x+2*c))/b-exp(b*x+a)*hypergeom([1, 1/2*b/d],[1+1/2*b/d]
,-exp(2*d*x+2*c))/b
```

3.191.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = \frac{e^{-a-bx}\left(-1+e^{2(a+bx)}+2\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)-2e^{2(a+bx)}\,\text{Hypergeometric2F1}\left(\right.\right.}{2b}$$

input
```
Integrate[Cosh[a + b*x]*Tanh[c + d*x],x]
```

output
```
(E^(-a - b*x)*(-1 + E^(2*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 -
 b/(2*d), -E^(2*(c + d*x))] - 2*E^(2*(a + b*x))*Hypergeometric2F1[1, b/(2*
d), 1 + b/(2*d), -E^(2*(c + d*x))]))/(2*b)
```

3.191.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6138, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\tanh(c+dx)\,dx$$

$$\Big\downarrow \text{6138}$$

$$\int \left(-\frac{e^{-a-bx}}{e^{2(c+dx)}+1}-\frac{e^{a+bx}}{e^{2(c+dx)}+1}+\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,-e^{2(c+dx)}\right)}{b}-\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

input
```
Int[Cosh[a + b*x]*Tanh[c + d*x],x]
```

output
```
-1/2*E^(-a - b*x)/b + E^(a + b*x)/(2*b) + (E^(-a - b*x)*Hypergeometric2F1[
1, -1/2*b/d, 1 - b/(2*d), -E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometr
ic2F1[1, b/(2*d), 1 + b/(2*d), -E^(2*(c + d*x))])/b
```

3.191.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6138
```
Int[Cosh[(a_.) + (b_.)*(x_)]*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(
E^(a + b*x)*2) + E^(a + b*x)/2 - 1/(E^(a + b*x)*(1 + E^(2*(c + d*x)))) - E^
(a + b*x)/(1 + E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

3.191.4 Maple [F]

$$\int \cosh(bx+a)\tanh(dx+c)\,dx$$

input
```
int(cosh(b*x+a)*tanh(d*x+c),x)
```

output
```
int(cosh(b*x+a)*tanh(d*x+c),x)
```

3.191.5 Fricas [F]

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = \int \cosh(bx+a)\tanh(dx+c)\,dx$$

input
```
integrate(cosh(b*x+a)*tanh(d*x+c),x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*tanh(d*x + c), x)
```

3.191.6 Sympy [F]

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = \int \cosh(a+bx)\tanh(c+dx)\,dx$$

input
```
integrate(cosh(b*x+a)*tanh(d*x+c),x)
```

output
```
Integral(cosh(a + b*x)*tanh(c + d*x), x)
```

3.191.7 Maxima [F]

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = \int \cosh\left(bx+a\right)\tanh\left(dx+c\right)\,dx$$

input
```
integrate(cosh(b*x+a)*tanh(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)/b - 1/2*integrate(2*(e^(2*b*x + 2*a
) + 1)/(e^(b*x + 2*d*x + a + 2*c) + e^(b*x + a)), x)
```

3.191.8 Giac [F]

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = \int \cosh\left(bx+a\right)\tanh\left(dx+c\right)\,dx$$

input
```
integrate(cosh(b*x+a)*tanh(d*x+c),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)*tanh(d*x + c), x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = \int \cosh(a+b\,x)\,\tanh(c+d\,x)\,dx$$

input
```
int(cosh(a + b*x)*tanh(c + d*x),x)
```

output
```
int(cosh(a + b*x)*tanh(c + d*x), x)
```

3.192 $\int \sinh(x)\sinh(2x)\,dx$

3.192.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sinh(x)\sinh(2x)\,dx = \frac{2\sinh^3(x)}{3}$$

output
```
2/3*sinh(x)^3
```

3.192.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \sinh(x)\sinh(2x)\,dx = -\frac{\sinh(x)}{2} + \frac{1}{6}\sinh(3x)$$

input
```
Integrate[Sinh[x]*Sinh[2*x],x]
```

output
```
-1/2*Sinh[x] + Sinh[3*x]/6
```

3.192.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 25, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\sinh(2x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\sin(ix)\sin(2ix)dx$$

$$\Big\downarrow \; 25$$

$$-\int \sin(ix)\sin(2ix)dx$$

$$\Big\downarrow \; 4770$$

$$\frac{1}{6}\sinh(3x)-\frac{\sinh(x)}{2}$$

input
```
Int[Sinh[x]*Sinh[2*x],x]
```

output
```
-1/2*Sinh[x] + Sinh[3*x]/6
```

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.192.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

method	result	size
default	$-\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}$	12
parallelrisch	$-\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}$	12
risch	$\frac{e^{3x}}{12}-\frac{e^{x}}{4}+\frac{e^{-x}}{4}-\frac{e^{-3x}}{12}$	24

input
```
int(sinh(x)*sinh(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*sinh(x)+1/6*sinh(3*x)
```

3.192.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \sinh(x)\sinh(2x)\,dx = \frac{1}{6}\sinh{(x)}^3 + \frac{1}{2}\left(\cosh{(x)}^2 - 1\right)\sinh{(x)}$$

input
```
integrate(sinh(x)*sinh(2*x),x, algorithm="fricas")
```

output
```
1/6*sinh(x)^3 + 1/2*(cosh(x)^2 - 1)*sinh(x)
```

3.192.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(7) = 14.

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \sinh(x)\sinh(2x)\,dx = \frac{2\sinh{(x)}\cosh{(2x)}}{3} - \frac{\sinh{(2x)}\cosh{(x)}}{3}$$

input
```
integrate(sinh(x)*sinh(2*x),x)
```

output
```
2*sinh(x)*cosh(2*x)/3 - sinh(2*x)*cosh(x)/3
```

3.192.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \sinh(x)\sinh(2x)\,dx = -\frac{1}{12}\left(3\,e^{(-2\,x)} - 1\right)e^{(3\,x)} + \frac{1}{4}\,e^{(-x)} - \frac{1}{12}\,e^{(-3\,x)}$$

input
```
integrate(sinh(x)*sinh(2*x),x, algorithm="maxima")
```

output
```
-1/12*(3*e^(-2*x) - 1)*e^(3*x) + 1/4*e^(-x) - 1/12*e^(-3*x)
```

3.192.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 3.12

$$\int \sinh(x)\sinh(2x)\,dx = \frac{1}{12}\left(3\,e^{(2\,x)} - 1\right)e^{(-3\,x)} + \frac{1}{12}\,e^{(3\,x)} - \frac{1}{4}\,e^{x}$$

input
```
integrate(sinh(x)*sinh(2*x),x, algorithm="giac")
```

output
```
1/12*(3*e^(2*x) - 1)*e^(-3*x) + 1/12*e^(3*x) - 1/4*e^x
```

3.192.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sinh(x)\sinh(2x)\,dx = \frac{2\sinh(x)^3}{3}$$

input
```
int(sinh(2*x)*sinh(x),x)
```

output
```
(2*sinh(x)^3)/3
```

3.193 $\int \sinh(x)\sinh(3x)\,dx$

3.193.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \sinh(x)\sinh(3x)\,dx = -\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

output
```
-1/4*sinh(2*x)+1/8*sinh(4*x)
```

3.193.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sinh(x)\sinh(3x)\,dx = -\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

input
```
Integrate[Sinh[x]*Sinh[3*x],x]
```

output
```
-1/4*Sinh[2*x] + Sinh[4*x]/8
```

3.193.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 25, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\sinh(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ix)\sin(3ix)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ix)\sin(3ix)dx$$

$$\Big\downarrow \text{4770}$$

$$\frac{1}{8}\sinh(4x)-\frac{1}{4}\sinh(2x)$$

input
```
Int[Sinh[x]*Sinh[3*x],x]
```

output
```
-1/4*Sinh[2*x] + Sinh[4*x]/8
```

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.193.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}$	14
risch	$\frac{e^{4x}}{16}-\frac{e^{2x}}{8}+\frac{e^{-2x}}{8}-\frac{e^{-4x}}{16}$	26
parallelrisch	$\frac{\sinh(5x)-2\sinh(x)-\sinh(3x)+4\sinh(2x)-2\sinh(4x)}{16\cosh(x)-16}$	37

input
```
int(sinh(x)*sinh(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*sinh(2*x)+1/8*sinh(4*x)
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \sinh(x)\sinh(3x)\,dx = \frac{1}{2}\cosh(x)\sinh(x)^3 + \frac{1}{2}\left(\cosh(x)^3 - \cosh(x)\right)\sinh(x)$$

input
```
integrate(sinh(x)*sinh(3*x),x, algorithm="fricas")
```

output
```
1/2*cosh(x)*sinh(x)^3 + 1/2*(cosh(x)^3 - cosh(x))*sinh(x)
```

3.193.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sinh(x)\sinh(3x)\,dx = \frac{3\sinh(x)\cosh(3x)}{8} - \frac{\sinh(3x)\cosh(x)}{8}$$

input
```
integrate(sinh(x)*sinh(3*x),x)
```

output
```
3*sinh(x)*cosh(3*x)/8 - sinh(3*x)*cosh(x)/8
```

3.193.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sinh(x)\sinh(3x)\,dx = -\frac{1}{16}\left(2\,e^{(-2\,x)} - 1\right)e^{(4\,x)} + \frac{1}{8}\,e^{(-2\,x)} - \frac{1}{16}\,e^{(-4\,x)}$$

input
```
integrate(sinh(x)*sinh(3*x),x, algorithm="maxima")
```

output
```
-1/16*(2*e^(-2*x) - 1)*e^(4*x) + 1/8*e^(-2*x) - 1/16*e^(-4*x)
```

3.193.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sinh(x)\sinh(3x)\,dx = \frac{1}{16}\left(2\,e^{(2\,x)} - 1\right)e^{(-4\,x)} + \frac{1}{16}\,e^{(4\,x)} - \frac{1}{8}\,e^{(2\,x)}$$

input
```
integrate(sinh(x)*sinh(3*x),x, algorithm="giac")
```

output
```
1/16*(2*e^(2*x) - 1)*e^(-4*x) + 1/16*e^(4*x) - 1/8*e^(2*x)
```

3.193.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sinh(x)\sinh(3x)\,dx = \frac{\sinh(4\,x)}{8} - \frac{\sinh(2\,x)}{4}$$

input
```
int(sinh(3*x)*sinh(x),x)
```

output
```
sinh(4*x)/8 - sinh(2*x)/4
```

3.194 $\int \sinh(x)\sinh(4x)\,dx$

3.194.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \sinh(x)\sinh(4x)\,dx = -\frac{1}{6}\sinh(3x)+\frac{1}{10}\sinh(5x)$$

output
```
-1/6*sinh(3*x)+1/10*sinh(5*x)
```

3.194.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sinh(x)\sinh(4x)\,dx = -\frac{1}{6}\sinh(3x)+\frac{1}{10}\sinh(5x)$$

input
```
Integrate[Sinh[x]*Sinh[4*x],x]
```

output
```
-1/6*Sinh[3*x] + Sinh[5*x]/10
```

3.194.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 25, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\sinh(4x)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\sin(ix)\sin(4ix)dx$$
$$\Big\downarrow \text{ 25}$$
$$-\int \sin(ix)\sin(4ix)dx$$
$$\Big\downarrow \text{ 4770}$$
$$\frac{1}{10}\sinh(5x)-\frac{1}{6}\sinh(3x)$$

input
```
Int[Sinh[x]*Sinh[4*x],x]
```

output
```
-1/6*Sinh[3*x] + Sinh[5*x]/10
```

3.194.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.194.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}$	14
parallelrisch	$-\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}$	14
risch	$\frac{e^{5x}}{20}-\frac{e^{3x}}{12}+\frac{e^{-3x}}{12}-\frac{e^{-5x}}{20}$	26

input
```
int(sinh(x)*sinh(4*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*sinh(3*x)+1/10*sinh(5*x)
```

3.194.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(13) = 26.

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \sinh(x)\sinh(4x)\,dx = \frac{1}{10}\sinh(x)^5+\frac{1}{6}\left(6\cosh(x)^2-1\right)\sinh(x)^3 + \frac{1}{2}\left(\cosh(x)^4-\cosh(x)^2\right)\sinh(x)$$

input
```
integrate(sinh(x)*sinh(4*x),x, algorithm="fricas")
```

output
```
1/10*sinh(x)^5 + 1/6*(6*cosh(x)^2 - 1)*sinh(x)^3 + 1/2*(cosh(x)^4 - cosh(x
)^2)*sinh(x)
```

3.194.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sinh(x)\sinh(4x)\,dx = \frac{4\sinh(x)\cosh(4x)}{15}-\frac{\sinh(4x)\cosh(x)}{15}$$

input
```
integrate(sinh(x)*sinh(4*x),x)
```

output
```
4*sinh(x)*cosh(4*x)/15 - sinh(4*x)*cosh(x)/15
```

3.194.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sinh(x)\sinh(4x)\,dx = -\frac{1}{60}\left(5\,e^{(-2\,x)} - 3\right)e^{(5\,x)} + \frac{1}{12}\,e^{(-3\,x)} - \frac{1}{20}\,e^{(-5\,x)}$$

input
```
integrate(sinh(x)*sinh(4*x),x, algorithm="maxima")
```

output
```
-1/60*(5*e^(-2*x) - 3)*e^(5*x) + 1/12*e^(-3*x) - 1/20*e^(-5*x)
```

3.194.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sinh(x)\sinh(4x)\,dx = \frac{1}{60}\left(5\,e^{(2\,x)} - 3\right)e^{(-5\,x)} + \frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(3\,x)}$$

input
```
integrate(sinh(x)*sinh(4*x),x, algorithm="giac")
```

output
```
1/60*(5*e^(2*x) - 3)*e^(-5*x) + 1/20*e^(5*x) - 1/12*e^(3*x)
```

3.194.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sinh(x)\sinh(4x)\,dx = \frac{4\sinh(x)^3\left(6\sinh(x)^2 + 5\right)}{15}$$

input
```
int(sinh(4*x)*sinh(x),x)
```

output
```
(4*sinh(x)^3*(6*sinh(x)^2 + 5))/15
```

3.195 $\int \sinh(x)\sinh(mx)\,dx$

3.195.1 Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \sinh(x)\sinh(mx)\,dx = -\frac{\sinh((1-m)x)}{2(1-m)} + \frac{\sinh((1+m)x)}{2(1+m)}$$

output
```
-1/2*sinh((1-m)*x)/(1-m)+1/2*sinh((1+m)*x)/(1+m)
```

3.195.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \sinh(x)\sinh(mx)\,dx = \frac{m\cosh(mx)\sinh(x) - \cosh(x)\sinh(mx)}{-1+m^2}$$

input
```
Integrate[Sinh[x]*Sinh[m*x],x]
```

output
```
(m*Cosh[m*x]*Sinh[x] - Cosh[x]*Sinh[m*x])/(-1 + m^2)
```

3.195.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\sinh(mx)\,dx$$

$$\Big\downarrow \text{ 6147}$$

$$\int \left(\frac{1}{2}\cosh((m+1)x)-\frac{1}{2}\cosh((1-m)x)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sinh((m+1)x)}{2(m+1)}-\frac{\sinh((1-m)x)}{2(1-m)}$$

input
```
Int[Sinh[x]*Sinh[m*x],x]
```

output
```
-1/2*Sinh[(1 - m)*x]/(1 - m) + Sinh[(1 + m)*x]/(2*(1 + m))
```

3.195.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6147
```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.195.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\sinh(x(-1+m))}{2(-1+m)}+\frac{\sinh((1+m)x)}{2+2m}$	28
parallelrisch	$\frac{(-1-m)\sinh(x(-1+m))+\sinh((1+m)x)(-1+m)}{2m^2-2}$	34
risch	$\frac{(m\,\mathrm{e}^{2x}-\mathrm{e}^{2x}-m-1)\mathrm{e}^{x(-1+m)}}{4(1+m)(-1+m)}+\frac{(m\,\mathrm{e}^{2x}+\mathrm{e}^{2x}-m+1)\mathrm{e}^{-(1+m)x}}{4(1+m)(-1+m)}$	71

input
```
int(sinh(x)*sinh(m*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-1+m)*sinh(x*(-1+m))+1/2*sinh((1+m)*x)/(1+m)
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \sinh(x)\sinh(mx)\,dx = \frac{m\cosh(mx)\sinh(x)-\cosh(x)\sinh(mx)}{(m^2-1)\cosh(x)^2-(m^2-1)\sinh(x)^2}$$

input
```
integrate(sinh(x)*sinh(m*x),x, algorithm="fricas")
```

output
```
(m*cosh(m*x)*sinh(x) - cosh(x)*sinh(m*x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)
*sinh(x)^2)
```

3.195.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(22) = 44$.

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.23

$$\int \sinh(x)\sinh(mx)\,dx = \begin{cases} -\frac{x\sinh^2(x)}{2}+\frac{x\cosh^2(x)}{2}-\frac{\sinh(x)\cosh(x)}{2} & \text{for } m=-1 \\ \frac{x\sinh^2(x)}{2}-\frac{x\cosh^2(x)}{2}+\frac{\sinh(x)\cosh(x)}{2} & \text{for } m=1 \\ \frac{m\sinh(x)\cosh(mx)}{m^2-1}-\frac{\sinh(mx)\cosh(x)}{m^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(x)*sinh(m*x),x)
```

output
```
Piecewise((-x*sinh(x)**2/2 + x*cosh(x)**2/2 - sinh(x)*cosh(x)/2, Eq(m, -1)
), (x*sinh(x)**2/2 - x*cosh(x)**2/2 + sinh(x)*cosh(x)/2, Eq(m, 1)), (m*sin
h(x)*cosh(m*x)/(m**2 - 1) - sinh(m*x)*cosh(x)/(m**2 - 1), True))
```

3.195.7 Maxima [F(-2)]

Exception generated.

$$\int \sinh(x)\sinh(mx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)*sinh(m*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-2>0)', see `assume?` for more
details)Is
```

3.195.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \sinh(x)\sinh(mx)\,dx = \frac{e^{(mx+x)}}{4\,(m+1)} - \frac{e^{(mx-x)}}{4\,(m-1)} + \frac{e^{(-mx+x)}}{4\,(m-1)} - \frac{e^{(-mx-x)}}{4\,(m+1)}$$

input
```
integrate(sinh(x)*sinh(m*x),x, algorithm="giac")
```

output
```
1/4*e^(m*x + x)/(m + 1) - 1/4*e^(m*x - x)/(m - 1) + 1/4*e^(-m*x + x)/(m -
1) - 1/4*e^(-m*x - x)/(m + 1)
```

3.195.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \sinh(x)\sinh(mx)\,dx = -\frac{\sinh(m\,x)\,\cosh(x) - m\cosh(m\,x)\,\sinh(x)}{m^2-1}$$

input
```
int(sinh(m*x)*sinh(x),x)
```

output
```
-(sinh(m*x)*cosh(x) - m*cosh(m*x)*sinh(x))/(m^2 - 1)
```

3.196 $\int \cosh(2x)\sinh(x)\,dx$

3.196.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cosh(2x)\sinh(x)\,dx = -\frac{\cosh(x)}{2} + \frac{1}{6}\cosh(3x)$$

output
```
-1/2*cosh(x)+1/6*cosh(3*x)
```

3.196.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh(2x)\sinh(x)\,dx = -\frac{\cosh(x)}{2} + \frac{1}{6}\cosh(3x)$$

input
```
Integrate[Cosh[2*x]*Sinh[x],x]
```

output
```
-1/2*Cosh[x] + Cosh[3*x]/6
```

3.196.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 26, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\cosh(2x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -i\sin(ix)\cos(2ix)dx$$

$$\Big\downarrow \; 26$$

$$-i\int \cos(2ix)\sin(ix)dx$$

$$\Big\downarrow \; 4772$$

$$-i\left(\frac{1}{6}i\cosh(3x)-\frac{1}{2}i\cosh(x)\right)$$

input
```
Int[Cosh[2*x]*Sinh[x],x]
```

output
```
(-I)*((-1/2*I)*Cosh[x] + (I/6)*Cosh[3*x])
```

3.196.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.196.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$	12
parallelrisch	$\frac{1}{3}-\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$	13
risch	$\frac{e^{3x}}{12}-\frac{e^{x}}{4}-\frac{e^{-x}}{4}+\frac{e^{-3x}}{12}$	24

input
```
int(sinh(x)*cosh(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cosh(x)+1/6*cosh(3*x)
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cosh(2x)\sinh(x)\,dx = \frac{1}{6}\cosh{(x)}^{3}+\frac{1}{2}\cosh{(x)}\sinh{(x)}^{2}-\frac{1}{2}\cosh{(x)}$$

input
```
integrate(cosh(2*x)*sinh(x),x, algorithm="fricas")
```

output
```
1/6*cosh(x)^3 + 1/2*cosh(x)*sinh(x)^2 - 1/2*cosh(x)
```

3.196.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cosh(2x)\sinh(x)\,dx = \frac{2\sinh(x)\sinh(2x)}{3} - \frac{\cosh(x)\cosh(2x)}{3}$$

input
```
integrate(cosh(2*x)*sinh(x),x)
```

output
```
2*sinh(x)*sinh(2*x)/3 - cosh(x)*cosh(2*x)/3
```

3.196.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \cosh(2x)\sinh(x)\,dx = -\frac{1}{12}\left(3e^{(-2x)} - 1\right)e^{(3x)} - \frac{1}{4}e^{(-x)} + \frac{1}{12}e^{(-3x)}$$

input
```
integrate(cosh(2*x)*sinh(x),x, algorithm="maxima")
```

output
```
-1/12*(3*e^(-2*x) - 1)*e^(3*x) - 1/4*e^(-x) + 1/12*e^(-3*x)
```

3.196.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \cosh(2x)\sinh(x)\,dx = -\frac{1}{12}\left(3e^{(2x)} - 1\right)e^{(-3x)} + \frac{1}{12}e^{(3x)} - \frac{1}{4}e^{x}$$

input
```
integrate(cosh(2*x)*sinh(x),x, algorithm="giac")
```

output
```
-1/12*(3*e^(2*x) - 1)*e^(-3*x) + 1/12*e^(3*x) - 1/4*e^x
```

3.196.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cosh(2x)\sinh(x)\,dx = \frac{2\cosh(x)^3}{3} - \cosh(x)$$

input
```
int(cosh(2*x)*sinh(x),x)
```

output
```
(2*cosh(x)^3)/3 - cosh(x)
```

3.197 $\int \cosh(3x)\sinh(x)\,dx$

3.197.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cosh(3x)\sinh(x)\,dx = -\frac{1}{4}\cosh(2x) + \frac{1}{8}\cosh(4x)$$

output
```
-1/4*cosh(2*x)+1/8*cosh(4*x)
```

3.197.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cosh(3x)\sinh(x)\,dx = -\frac{1}{2}\cosh^2(x) + \frac{1}{8}\cosh(4x)$$

input
```
Integrate[Cosh[3*x]*Sinh[x],x]
```

output
```
-1/2*Cosh[x]^2 + Cosh[4*x]/8
```

3.197.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 26, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\cosh(3x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -i\sin(ix)\cos(3ix)dx$$

$$\Big\downarrow \; 26$$

$$-i\int \cos(3ix)\sin(ix)dx$$

$$\Big\downarrow \; 4772$$

$$-i\left(\frac{1}{8}i\cosh(4x)-\frac{1}{4}i\cosh(2x)\right)$$

input
```
Int[Cosh[3*x]*Sinh[x],x]
```

output
```
(-I)*((-1/4*I)*Cosh[2*x] + (I/8)*Cosh[4*x])
```

3.197.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.197.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\cosh(2x)}{4}+\frac{\cosh(4x)}{8}$	14
risch	$\frac{e^{4x}}{16}-\frac{e^{2x}}{8}-\frac{e^{-2x}}{8}+\frac{e^{-4x}}{16}$	26
parallelrisch	$\frac{\cosh(5x)-\cosh(3x)+4\cosh(2x)-2-2\cosh(4x)}{16\cosh(x)-16}$	34

input
```
int(sinh(x)*cosh(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cosh(2*x)+1/8*cosh(4*x)
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \cosh(3x)\sinh(x)\,dx$$
$$=\frac{1}{8}\cosh(x)^4+\frac{1}{8}\sinh(x)^4+\frac{1}{4}\left(3\cosh(x)^2-1\right)\sinh(x)^2-\frac{1}{4}\cosh(x)^2$$

input
```
integrate(cosh(3*x)*sinh(x),x, algorithm="fricas")
```

output
```
1/8*cosh(x)^4 + 1/8*sinh(x)^4 + 1/4*(3*cosh(x)^2 - 1)*sinh(x)^2 - 1/4*cosh
(x)^2
```

3.197.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(3x)\sinh(x)\,dx = \frac{3\sinh{(x)}\sinh{(3x)}}{8} - \frac{\cosh{(x)}\cosh{(3x)}}{8}$$

input
```
integrate(cosh(3*x)*sinh(x),x)
```

output
```
3*sinh(x)*sinh(3*x)/8 - cosh(x)*cosh(3*x)/8
```

3.197.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(3x)\sinh(x)\,dx = -\frac{1}{16}\left(2\,e^{(-2\,x)} - 1\right)e^{(4\,x)} - \frac{1}{8}\,e^{(-2\,x)} + \frac{1}{16}\,e^{(-4\,x)}$$

input
```
integrate(cosh(3*x)*sinh(x),x, algorithm="maxima")
```

output
```
-1/16*(2*e^(-2*x) - 1)*e^(4*x) - 1/8*e^(-2*x) + 1/16*e^(-4*x)
```

3.197.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \cosh(3x)\sinh(x)\,dx = \frac{1}{16}\left(e^{(2\,x)} + e^{(-2\,x)}\right)^2 - \frac{1}{8}\,e^{(2\,x)} - \frac{1}{8}\,e^{(-2\,x)}$$

input
```
integrate(cosh(3*x)*sinh(x),x, algorithm="giac")
```

output
```
1/16*(e^(2*x) + e^(-2*x))^2 - 1/8*e^(2*x) - 1/8*e^(-2*x)
```

3.197.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \cosh(3x)\sinh(x)\,dx = \sinh(x)^4 + \frac{\sinh(x)^2}{2}$$

input
```
int(cosh(3*x)*sinh(x),x)
```

output
```
sinh(x)^2/2 + sinh(x)^4
```

3.198 $\int \cosh(4x)\sinh(x)\,dx$

3.198.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cosh(4x)\sinh(x)\,dx = -\frac{1}{6}\cosh(3x) + \frac{1}{10}\cosh(5x)$$

output
```
-1/6*cosh(3*x)+1/10*cosh(5*x)
```

3.198.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cosh(4x)\sinh(x)\,dx = -\frac{1}{6}\cosh(3x) + \frac{1}{10}\cosh(5x)$$

input
```
Integrate[Cosh[4*x]*Sinh[x],x]
```

output
```
-1/6*Cosh[3*x] + Cosh[5*x]/10
```

3.198.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 26, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\cosh(4x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sin(ix)\cos(4ix)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \cos(4ix)\sin(ix)dx$$

$$\Big\downarrow \text{ 4772}$$

$$-i\left(\frac{1}{10}i\cosh(5x)-\frac{1}{6}i\cosh(3x)\right)$$

input
```
Int[Cosh[4*x]*Sinh[x],x]
```

output
```
(-I)*((-1/6*I)*Cosh[3*x] + (I/10)*Cosh[5*x])
```

3.198.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.198.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$	14
parallelrisch	$\frac{1}{15}-\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$	15
risch	$\frac{e^{5x}}{20}-\frac{e^{3x}}{12}-\frac{e^{-3x}}{12}+\frac{e^{-5x}}{20}$	26

input
```
int(cosh(4*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*cosh(3*x)+1/10*cosh(5*x)
```

3.198.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

$$\int \cosh(4x)\sinh(x)\,dx = \frac{1}{10}\cosh(x)^5+\frac{1}{2}\cosh(x)\sinh(x)^4-\frac{1}{6}\cosh(x)^3 + \frac{1}{2}\left(2\cosh(x)^3-\cosh(x)\right)\sinh(x)^2$$

input
```
integrate(cosh(4*x)*sinh(x),x, algorithm="fricas")
```

output
```
1/10*cosh(x)^5 + 1/2*cosh(x)*sinh(x)^4 - 1/6*cosh(x)^3 + 1/2*(2*cosh(x)^3
- cosh(x))*sinh(x)^2
```

3.198.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(4x)\sinh(x)\,dx = \frac{4\sinh(x)\sinh(4x)}{15} - \frac{\cosh(x)\cosh(4x)}{15}$$

input
```
integrate(cosh(4*x)*sinh(x),x)
```

output
```
4*sinh(x)*sinh(4*x)/15 - cosh(x)*cosh(4*x)/15
```

3.198.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(4x)\sinh(x)\,dx = -\frac{1}{60}\left(5\,e^{(-2\,x)} - 3\right)e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{20}\,e^{(-5\,x)}$$

input
```
integrate(cosh(4*x)*sinh(x),x, algorithm="maxima")
```

output
```
-1/60*(5*e^(-2*x) - 3)*e^(5*x) - 1/12*e^(-3*x) + 1/20*e^(-5*x)
```

3.198.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(4x)\sinh(x)\,dx = -\frac{1}{60}\left(5\,e^{(2\,x)} - 3\right)e^{(-5\,x)} + \frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(3\,x)}$$

input
```
integrate(cosh(4*x)*sinh(x),x, algorithm="giac")
```

output
```
-1/60*(5*e^(2*x) - 3)*e^(-5*x) + 1/20*e^(5*x) - 1/12*e^(3*x)
```

3.198.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cosh(4x)\sinh(x)\,dx = \frac{8\cosh(x)^5}{5} - \frac{8\cosh(x)^3}{3} + \cosh(x)$$

input
```
int(cosh(4*x)*sinh(x),x)
```

output
```
cosh(x) - (8*cosh(x)^3)/3 + (8*cosh(x)^5)/5
```

3.199 $\int \cosh(mx)\sinh(x)\,dx$

3.199.1 Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cosh(mx)\sinh(x)\,dx = \frac{\cosh((1-m)x)}{2(1-m)} + \frac{\cosh((1+m)x)}{2(1+m)}$$

output
```
1/2*cosh((1-m)*x)/(1-m)+1/2*cosh((1+m)*x)/(1+m)
```

3.199.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cosh(mx)\sinh(x)\,dx = \frac{-\cosh(x)\cosh(mx) + m\sinh(x)\sinh(mx)}{-1+m^2}$$

input
```
Integrate[Cosh[m*x]*Sinh[x],x]
```

output
```
(-(Cosh[x]*Cosh[m*x]) + m*Sinh[x]*Sinh[m*x])/(-1 + m^2)
```

3.199.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6152, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\cosh(mx)\,dx$$

$$\Big\downarrow \text{6152}$$

$$\int \left(\frac{1}{2}\sinh((1-m)x)+\frac{1}{2}\sinh((m+1)x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh((1-m)x)}{2(1-m)}+\frac{\cosh((m+1)x)}{2(m+1)}$$

input
```
Int[Cosh[m*x]*Sinh[x],x]
```

output
```
Cosh[(1 - m)*x]/(2*(1 - m)) + Cosh[(1 + m)*x]/(2*(1 + m))
```

3.199.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6152
```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.199.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\cosh(x(-1+m))}{2(-1+m)}+\frac{\cosh((1+m)x)}{2+2m}$	28
parallelrisch	$\frac{(-1-m)\cosh(x(-1+m))+2+\cosh((1+m)x)(-1+m)}{2m^2-2}$	35
risch	$\frac{(m\,e^{2x}-e^{2x}-m-1)e^{x(-1+m)}}{4(1+m)(-1+m)}-\frac{(m\,e^{2x}+e^{2x}-m+1)e^{-(1+m)x}}{4(1+m)(-1+m)}$	71

input
```
int(cosh(m*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-1+m)*cosh(x*(-1+m))+1/2*cosh((1+m)*x)/(1+m)
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \cosh(mx)\sinh(x)\,dx = \frac{m\sinh(mx)\sinh(x)-\cosh(mx)\cosh(x)}{(m^2-1)\cosh(x)^2-(m^2-1)\sinh(x)^2}$$

input
```
integrate(cosh(m*x)*sinh(x),x, algorithm="fricas")
```

output
```
(m*sinh(m*x)*sinh(x) - cosh(m*x)*cosh(x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)
*sinh(x)^2)
```

3.199.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \cosh(mx)\sinh(x)\,dx = \begin{cases} \frac{\cosh^2(x)}{2} & \text{for}\ m=-1\vee m=1 \\ \frac{m\sinh(x)\sinh(mx)}{m^2-1}-\frac{\cosh(x)\cosh(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(m*x)*sinh(x),x)
```

output
```
Piecewise((cosh(x)**2/2, Eq(m, -1) | Eq(m, 1)), (m*sinh(x)*sinh(m*x)/(m**2
 - 1) - cosh(x)*cosh(m*x)/(m**2 - 1), True))
```

3.199.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh(mx)\sinh(x)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(m*x)*sinh(x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-2>0)', see `assume?` for more
details)Is
```

3.199.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \cosh(mx)\sinh(x)\,dx = \frac{e^{(mx+x)}}{4\,(m+1)} - \frac{e^{(mx-x)}}{4\,(m-1)} - \frac{e^{(-mx+x)}}{4\,(m-1)} + \frac{e^{(-mx-x)}}{4\,(m+1)}$$

input
```
integrate(cosh(m*x)*sinh(x),x, algorithm="giac")
```

output
```
1/4*e^(m*x + x)/(m + 1) - 1/4*e^(m*x - x)/(m - 1) - 1/4*e^(-m*x + x)/(m -
1) + 1/4*e^(-m*x - x)/(m + 1)
```

3.199.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \cosh(mx)\sinh(x)\,dx = -\frac{\cosh(m\,x)\,\cosh(x) - m\sinh(m\,x)\,\sinh(x)}{m^2-1}$$

input
```
int(cosh(m*x)*sinh(x),x)
```

output
```
-(cosh(m*x)*cosh(x) - m*sinh(m*x)*sinh(x))/(m^2 - 1)
```

3.200 $\int \sinh(x)\tanh(2x)\,dx$

3.200.1 Optimal result

Integrand size = 7, antiderivative size = 19

$$\int \sinh(x)\tanh(2x)\,dx = -\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}} + \sinh(x)$$

output
```
sinh(x)-1/2*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sinh(x)\tanh(2x)\,dx = -\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}} + \sinh(x)$$

input
```
Integrate[Sinh[x]*Tanh[2*x],x]
```

output
```
-(ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]) + Sinh[x]
```

3.200.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 25, 4878, 27, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\tanh(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ix)\tan(2ix)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ix)\tan(2ix)dx$$

$$\Big\downarrow \text{4878}$$

$$-\int -\frac{2\sinh^2(x)}{2\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\sinh^2(x)}{2\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{262}$$

$$2\left(\frac{\sinh(x)}{2}-\frac{1}{2}\int \frac{1}{2\sinh^2(x)+1}d\sinh(x)\right)$$

$$\Big\downarrow \text{216}$$

$$2\left(\frac{\sinh(x)}{2}-\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}\right)$$

input
```
Int[Sinh[x]*Tanh[2*x],x]
```

output
```
2*(-1/2*ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2] + Sinh[x]/2)
```

3.200.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.200.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\sinh(x)-\frac{\arctan\left(\sinh(x)\sqrt{2}\right)\sqrt{2}}{2}$	16
default	$\sinh(x)-\frac{\arctan\left(\sinh(x)\sqrt{2}\right)\sqrt{2}}{2}$	16
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i\sqrt{2}\,\ln\left(e^{2x}-i\sqrt{2}\,e^{x}-1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(e^{2x}+i\sqrt{2}\,e^{x}-1\right)}{4}$	54

input
```
int(sinh(x)*tanh(2*x),x,method=_RETURNVERBOSE)
```

output
```
sinh(x)-1/2*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(15) = 30.

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 6.05

$$\int \sinh(x)\tanh(2x)\,dx =$$
$$-\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\arctan\left(\frac{1}{2}\sqrt{2}\cosh\left(x\right)+\frac{1}{2}\sqrt{2}\sinh\left(x\right)\right)-\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\text{arct}}{2\left(\cosh\left(x\right)+\text{s}\right.}$$

input
```
integrate(sinh(x)*tanh(2*x),x, algorithm="fricas")
```

output
```
-1/2*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(1/2*sqrt(2)*cosh(x) + 1/2
*sqrt(2)*sinh(x)) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(-1/2*(sqrt(
2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))/(c
osh(x) - sinh(x))) - cosh(x)^2 - 2*cosh(x)*sinh(x) - sinh(x)^2 + 1)/(cosh(
x) + sinh(x))
```

3.200.6 Sympy [F]

$$\int \sinh(x)\tanh(2x)\,dx = \int \sinh\left(x\right)\tanh\left(2x\right)\,dx$$

input
```
integrate(sinh(x)*tanh(2*x),x)
```

output
```
Integral(sinh(x)*tanh(2*x), x)
```

3.200.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int \sinh(x)\tanh(2x)\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(-x)}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(-x)}\right)\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

input
```
integrate(sinh(x)*tanh(2*x),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x))) - 1/2*e^(-x) + 1/2*e^x
```

3.200.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \sinh(x)\tanh(2x)\,dx = -\frac{1}{4}\sqrt{2}\left(\pi + 2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

input
```
integrate(sinh(x)*tanh(2*x),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1/2*e^(-x
) + 1/2*e^x
```

3.200.9 Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47

$$\int \sinh(x)\tanh(2x)\,dx = \frac{e^{x}}{2} - \frac{e^{-x}}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{x}}{2} + \frac{\sqrt{2}\,e^{3x}}{2}\right)}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{x}}{2}\right)}{2}$$

input
```
int(tanh(2*x)*sinh(x),x)
```

output
```
exp(x)/2 - exp(-x)/2 - (2^(1/2)*atan((2^(1/2)*exp(x))/2 + (2^(1/2)*exp(3*x
))/2))/2 - (2^(1/2)*atan((2^(1/2)*exp(x))/2))/2
```

3.200. $\int \sinh(x)\tanh(2x)\,dx$

3.201 $\int \sinh(x)\tanh(3x)\,dx$

3.201.1 Optimal result

Integrand size = 7, antiderivative size = 19

$$\int \sinh(x)\tanh(3x)\,dx = -\frac{1}{3}\arctan(\sinh(x)) - \frac{1}{3}\arctan(2\sinh(x)) + \sinh(x)$$

output
```
-1/3*arctan(sinh(x))-1/3*arctan(2*sinh(x))+sinh(x)
```

3.201.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sinh(x)\tanh(3x)\,dx = -\frac{1}{3}\arctan(\sinh(x)) - \frac{1}{3}\arctan(2\sinh(x)) + \sinh(x)$$

input
```
Integrate[Sinh[x]*Tanh[3*x],x]
```

output
```
-1/3*ArcTan[Sinh[x]] - ArcTan[2*Sinh[x]]/3 + Sinh[x]
```

3.201.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3042, 25, 4878, 25, 1602, 27, 1477, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\tanh(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ix)\tan(3ix)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ix)\tan(3ix)dx$$

$$\Big\downarrow \text{4878}$$

$$-\int -\frac{\sinh^2(x)\left(4\sinh^2(x)+3\right)}{4\sinh^4(x)+5\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh^2(x)\left(4\sinh^2(x)+3\right)}{4\sinh^4(x)+5\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{1602}$$

$$\sinh(x)-\frac{1}{4}\int \frac{4\left(2\sinh^2(x)+1\right)}{4\sinh^4(x)+5\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{27}$$

$$\sinh(x)-\int \frac{2\sinh^2(x)+1}{4\sinh^4(x)+5\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{1477}$$

$$-\frac{2}{3}\int \frac{1}{4\sinh^2(x)+1}d\sinh(x)-\frac{4}{3}\int \frac{1}{4\sinh^2(x)+4}d\sinh(x)+\sinh(x)$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{3}\arctan(\sinh(x))-\frac{1}{3}\arctan(2\sinh(x))+\sinh(x)$$

input
```
Int[Sinh[x]*Tanh[3*x],x]
```

output
```
-1/3*ArcTan[Sinh[x]] - ArcTan[2*Sinh[x]]/3 + Sinh[x]
```

3.201.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1477
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

rule 1602
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.201.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.16

method	result	size
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i \ln \left(e^{x}-i\right)}{3}-\frac{i \ln \left(e^{x}+i\right)}{3}+\frac{i \ln \left(e^{2 x}-i e^{x}-1\right)}{6}-\frac{i \ln \left(e^{2 x}+i e^{x}-1\right)}{6}$	60

input
```
int(sinh(x)*tanh(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)-1/2*exp(-x)+1/3*I*ln(exp(x)-I)-1/3*I*ln(exp(x)+I)+1/6*I*ln(exp(
2*x)-I*exp(x)-1)-1/6*I*ln(exp(2*x)+I*exp(x)-1)
```

3.201.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(15) = 30.

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.00

$$\int \sinh(x)\tanh(3x)\,dx$$
$$= \frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(-\frac{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2}{\cosh(x)-\sinh(x)}\right)-6\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(\cosh\left(x\right)+\dots\right.}{6\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

input
```
integrate(sinh(x)*tanh(3*x),x, algorithm="fricas")
```

output
```
1/6*(2*(cosh(x) + sinh(x))*arctan(-(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x
)^2)/(cosh(x) - sinh(x))) - 6*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)
) + 3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 - 3)/(cosh(x) + sinh(x))
```

3.201.6 Sympy [F]

$$\int \sinh(x)\tanh(3x)\,dx = \int \sinh(x)\tanh(3x)\,dx$$

input
```
integrate(sinh(x)*tanh(3*x),x)
```

output
```
Integral(sinh(x)*tanh(3*x), x)
```

3.201.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \sinh(x)\tanh(3x)\,dx = \frac{1}{3}\arctan\left(\sqrt{3}+2e^{(-x)}\right)+\frac{1}{3}\arctan\left(-\sqrt{3}+2e^{(-x)}\right) + \frac{2}{3}\arctan\left(e^{(-x)}\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

input
```
integrate(sinh(x)*tanh(3*x),x, algorithm="maxima")
```

output
```
1/3*arctan(sqrt(3) + 2*e^(-x)) + 1/3*arctan(-sqrt(3) + 2*e^(-x)) + 2/3*arc
tan(e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

3.201.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \sinh(x)\tanh(3x)\,dx = -\frac{1}{3}\pi-\frac{1}{3}\arctan\left(\left(e^{(2x)}-1\right)e^{(-x)}\right) - \frac{1}{3}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

input
```
integrate(sinh(x)*tanh(3*x),x, algorithm="giac")
```

output
```
-1/3*pi - 1/3*arctan((e^(2*x) - 1)*e^(-x)) - 1/3*arctan(1/2*(e^(2*x) - 1)*
e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

3.201.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \sinh(x)\tanh(3x)\,dx = \frac{e^x}{2} - \operatorname{atan}(e^x) - \frac{\operatorname{atan}(e^{3\,x})}{3} - \frac{e^{-x}}{2}$$

input
```
int(tanh(3*x)*sinh(x),x)
```

output
```
exp(x)/2 - atan(exp(x)) - atan(exp(3*x))/3 - exp(-x)/2
```

3.202 $\int \sinh(x)\tanh(4x)\,dx$

3.202.1 Optimal result

Integrand size = 7, antiderivative size = 69

$$\int \sinh(x)\tanh(4x)\,dx = -\frac{1}{4}\sqrt{2-\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right) + \sinh(x)$$

output
```
sinh(x)-1/4*arctan(2*sinh(x)/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/4*arct
an(2*sinh(x)/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \sinh(x)\tanh(4x)\,dx = -\frac{1}{4}\sqrt{2-\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right) + \sinh(x)$$

input
```
Integrate[Sinh[x]*Tanh[4*x],x]
```

output
```
-1/4*(Sqrt[2 - Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]]) - (Sqrt[2 +
 Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[2]]])/4 + Sinh[x]
```

3.202.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3042, 25, 4878, 27, 1602, 27, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\tanh(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ix)\tan(4ix)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ix)\tan(4ix)dx$$

$$\Big\downarrow \text{4878}$$

$$-\int -\frac{4\sinh^2(x)\left(2\sinh^2(x)+1\right)}{8\sinh^4(x)+8\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{\sinh^2(x)\left(2\sinh^2(x)+1\right)}{8\sinh^4(x)+8\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{1602}$$

$$4\left(\frac{\sinh(x)}{4}-\frac{1}{8}\int \frac{2\left(4\sinh^2(x)+1\right)}{8\sinh^4(x)+8\sinh^2(x)+1}d\sinh(x)\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{\sinh(x)}{4}-\frac{1}{4}\int \frac{4\sinh^2(x)+1}{8\sinh^4(x)+8\sinh^2(x)+1}d\sinh(x)\right)$$

$$\Big\downarrow \text{1480}$$

$$4\left(\frac{1}{4}\left(-\left(\left(2-\sqrt{2}\right)\int \frac{1}{8\sinh^2(x)+2\left(2-\sqrt{2}\right)}d\sinh(x)\right)-\left(2+\sqrt{2}\right)\int \frac{1}{8\sinh^2(x)+2\left(2+\sqrt{2}\right)}d\sinh(x)\right)+\ldots\right.$$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{1}{4}\left(-\frac{1}{4}\sqrt{2-\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)\right)+\frac{\sinh(x)}{4}\right)$$

input
```
Int[Sinh[x]*Tanh[4*x],x]
```

output
```
4*((-1/4*(Sqrt[2 - Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]]) - (Sqrt
[2 + Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[2]]])/4)/4 + Sinh[x]/4)
```

3.202.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.202.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\left(\sum_{\_R=\mathrm{RootOf}\left(2048\_Z^{4}+128\_Z^{2}+1\right)}\_R\ln\left(-8\_R\,e^{x}+e^{2x}-1\right)\right)$	42

input
```
int(sinh(x)*tanh(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)-1/2*exp(-x)+sum(_R*ln(-8*_R*exp(x)+exp(2*x)-1),_R=RootOf(2048*_
Z^4+128*_Z^2+1))
```

3.202.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(49) = 98.

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.09

$$\int \sinh(x)\tanh(4x)\,dx =$$

$$-\frac{\sqrt{\sqrt{2}-2}(\cosh(x)+\sinh(x))\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+\sqrt{\sqrt{2}-2}(\cosh(x)+\mathrm{s}\right.}{}$$

input
```
integrate(sinh(x)*tanh(4*x),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(sqrt(2) - 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + sqrt(sqrt(2) - 2)*(cosh(x) + sinh(x)) - 1) - sqrt(sqrt(2
) - 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 -
 sqrt(sqrt(2) - 2)*(cosh(x) + sinh(x)) - 1) + sqrt(-sqrt(2) - 2)*(cosh(x)
+ sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(-sqrt(2) -
 2)*(cosh(x) + sinh(x)) - 1) - sqrt(-sqrt(2) - 2)*(cosh(x) + sinh(x))*log(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sqrt(-sqrt(2) - 2)*(cosh(x) +
sinh(x)) - 1) - 4*cosh(x)^2 - 8*cosh(x)*sinh(x) - 4*sinh(x)^2 + 4)/(cosh(x
) + sinh(x))
```

3.202.6 Sympy [F]

$$\int \sinh(x)\tanh(4x)\,dx = \int \sinh(x)\tanh(4x)\,dx$$

input
```
integrate(sinh(x)*tanh(4*x),x)
```

output
```
Integral(sinh(x)*tanh(4*x), x)
```

3.202.7 Maxima [F]

$$\int \sinh(x)\tanh(4x)\,dx = \int \sinh(x)\tanh(4x)\,dx$$

input
```
integrate(sinh(x)*tanh(4*x),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) - 1)*e^(-x) - 1/2*integrate(2*(e^(7*x) + e^x)/(e^(8*x) + 1),
x)
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \sinh(x)\tanh(4x)\,dx = -\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{\sqrt{2}+2}}\right) - \frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

input
```
integrate(sinh(x)*tanh(4*x),x, algorithm="giac")
```

output
```
-1/4*sqrt(sqrt(2) + 2)*arctan(-(e^(-x) - e^x)/sqrt(sqrt(2) + 2)) - 1/4*sqr
t(-sqrt(2) + 2)*arctan(-(e^(-x) - e^x)/sqrt(-sqrt(2) + 2)) - 1/2*e^(-x) +
1/2*e^x
```

3.202.9 Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \sinh(x)\tanh(4x)\,dx = \frac{e^{x}}{2} - \frac{e^{-x}}{2} - \frac{\operatorname{atan}\left(\frac{e^{-x}\,(e^{2\,x}-1)}{\sqrt{\sqrt{2}+2}}\right)\,\sqrt{\sqrt{2}+2}}{4} - \frac{\operatorname{atan}\left(\frac{e^{-x}\,(e^{2\,x}-1)}{\sqrt{2-\sqrt{2}}}\right)\,\sqrt{2-\sqrt{2}}}{4}$$

input
```
int(tanh(4*x)*sinh(x),x)
```

output
```
exp(x)/2 - exp(-x)/2 - (atan((exp(-x)*(exp(2*x) - 1))/(2^(1/2) + 2)^(1/2))
*(2^(1/2) + 2)^(1/2))/4 - (atan((exp(-x)*(exp(2*x) - 1))/(2 - 2^(1/2))^(1/
2))*(2 - 2^(1/2))^(1/2))/4
```

3.203 $\int \sinh(x)\tanh(5x)\,dx$

3.203.1 Optimal result

Integrand size = 7, antiderivative size = 87

$$\int \sinh(x)\tanh(5x)\,dx = -\frac{1}{5}\arctan(\sinh(x)) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\arctan\left(\sqrt{2\left(3+\sqrt{5}\right)}\sinh(x)\right) + \sinh(x)$$

output
```
-1/5*arctan(sinh(x))+sinh(x)-1/5*arctan(sinh(x)*(5^(1/2)+1))*(1/2*5^(1/2)-
1/2)-1/5*arctan(2*sinh(x)*2^(1/2)/(3+5^(1/2))^(1/2))*(1/2+1/2*5^(1/2))
```

3.203.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \sinh(x)\tanh(5x)\,dx = \frac{1}{10}\left(-2\arctan(\sinh(x)) - \sqrt{2\left(3+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right) - \sqrt{6-2\sqrt{5}}\arctan\left(\sqrt{2\left(3+\sqrt{5}\right)}\sinh(x)\right) + 10\sinh(x)\right)$$

input
```
Integrate[Sinh[x]*Tanh[5*x],x]
```

output
```
(-2*ArcTan[Sinh[x]] - Sqrt[2*(3 + Sqrt[5])]*ArcTan[2*Sqrt[2/(3 + Sqrt[5])]
*Sinh[x]] - Sqrt[6 - 2*Sqrt[5]]*ArcTan[Sqrt[2*(3 + Sqrt[5])]*Sinh[x]] + 10
*Sinh[x])/10
```

3.203.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 25, 4878, 25, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\tanh(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ix)\tan(5ix)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ix)\tan(5ix)dx$$

$$\Big\downarrow \text{4878}$$

$$-\int -\frac{\sinh^2(x)\left(16\sinh^4(x)+20\sinh^2(x)+5\right)}{16\sinh^6(x)+28\sinh^4(x)+13\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh^2(x)\left(16\sinh^4(x)+20\sinh^2(x)+5\right)}{16\sinh^6(x)+28\sinh^4(x)+13\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{2460}$$

$$\int \left(-\frac{1}{5\left(\sinh^2(x)+1\right)}-\frac{4\left(6\sinh^2(x)+1\right)}{5\left(16\sinh^4(x)+12\sinh^2(x)+1\right)}+1\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\arctan(\sinh(x))-\frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\arctan\left(\sqrt{2\left(3+\sqrt{5}\right)}\sinh(x)\right)+\sinh(x)$$

input
```
Int[Sinh[x]*Tanh[5*x],x]
```

output
```
-1/5*ArcTan[Sinh[x]] - (Sqrt[(3 + Sqrt[5])/2]*ArcTan[2*Sqrt[2/(3 + Sqrt[5]
)]*Sinh[x]])/5 - (Sqrt[(3 - Sqrt[5])/2]*ArcTan[Sqrt[2*(3 + Sqrt[5])]*Sinh[
x]])/5 + Sinh[x]
```

3.203.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.203.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

method	result	s
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i\ln(e^{x}-i)}{5}-\frac{i\ln(e^{x}+i)}{5}+\left(\sum_{\_R=\operatorname{RootOf}(10000\_Z^{4}+300\_Z^{2}+1)}\_R\ln\left(-10\_R\,e^{x}+e^{2x}-1\right)\right)$	6

input
```
int(sinh(x)*tanh(5*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)-1/2*exp(-x)+1/5*I*ln(exp(x)-I)-1/5*I*ln(exp(x)+I)+sum(_R*ln(-10
*_R*exp(x)+exp(2*x)-1),_R=RootOf(10000*_Z^4+300*_Z^2+1))
```

3.203.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(46) = 92.

Time = 0.27 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.52

$$\int\sinh(x)\tanh(5x)\,dx =$$

$$-\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\sqrt{5}-3}\log\left(2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}+\left(\sqrt{2}\cosh\left(\right.\right.\right.}{}$$

input
```
integrate(sinh(x)*tanh(5*x),x, algorithm="fricas")
```

output
```
-1/20*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) - 3)*log(2*cosh(x)
^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))
*sqrt(sqrt(5) - 3) - 2) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5)
 - 3)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - (sqrt(2)*cosh(x)
 + sqrt(2)*sinh(x))*sqrt(sqrt(5) - 3) - 2) + (sqrt(2)*cosh(x) + sqrt(2)*si
nh(x))*sqrt(-sqrt(5) - 3)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^
2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) - 3) - 2) - (sqrt(2)
*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) - 3)*log(2*cosh(x)^2 + 4*cosh(x)
*sinh(x) + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5)
 - 3) - 2) + 8*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - 10*cosh(x)^
2 - 20*cosh(x)*sinh(x) - 10*sinh(x)^2 + 10)/(cosh(x) + sinh(x))
```

3.203.6 Sympy [F]

$$\int \sinh(x)\tanh(5x)\,dx = \int \sinh{(x)}\tanh{(5x)}\,dx$$

input
```
integrate(sinh(x)*tanh(5*x),x)
```

output
```
Integral(sinh(x)*tanh(5*x), x)
```

3.203.7 Maxima [F]

$$\int \sinh(x)\tanh(5x)\,dx = \int \sinh\left(x\right)\tanh\left(5\,x\right)\,dx$$

input
```
integrate(sinh(x)*tanh(5*x),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) - 1)*e^(-x) - 2/5*arctan(e^x) - 1/2*integrate(2/5*(3*e^(7*x)
- e^(5*x) - e^(3*x) + 3*e^x)/(e^(8*x) - e^(6*x) + e^(4*x) - e^(2*x) + 1),
x)
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \sinh(x)\tanh(5x)\,dx = & -\frac{1}{10}\,\pi-\frac{1}{10}\left(\sqrt{5}+1\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{5}+1}\right)\\ & -\frac{1}{10}\left(\sqrt{5}-1\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{5}-1}\right)\\ & -\frac{1}{5}\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)-\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}\end{aligned}$$

input
```
integrate(sinh(x)*tanh(5*x),x, algorithm="giac")
```

output
```
-1/10*pi - 1/10*(sqrt(5) + 1)*arctan(-2*(e^(-x) - e^x)/(sqrt(5) + 1)) - 1/
10*(sqrt(5) - 1)*arctan(-2*(e^(-x) - e^x)/(sqrt(5) - 1)) - 1/5*arctan(1/2*
(e^(2*x) - 1)*e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

3.203.9 Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \sinh(x)\tanh(5x)\,dx = \frac{e^x}{2} - \frac{2\,\mathrm{atan}(e^x)}{5} - \frac{e^{-x}}{2} - 2\,\mathrm{atan}\left(\frac{e^{-x}\,(e^{2x}-1)}{10\sqrt{\frac{3}{200}-\frac{\sqrt{5}}{200}}}\right)\sqrt{\frac{3}{200}-\frac{\sqrt{5}}{200}} - 2\,\mathrm{atan}\left(\frac{e^{-x}\,(e^{2x}-1)}{10\sqrt{\frac{\sqrt{5}}{200}+\frac{3}{200}}}\right)\sqrt{\frac{\sqrt{5}}{200}+\frac{3}{200}}$$

input
```
int(tanh(5*x)*sinh(x),x)
```

output
```
exp(x)/2 - (2*atan(exp(x)))/5 - exp(-x)/2 - 2*atan((exp(-x)*(exp(2*x) - 1)
)/(10*(3/200 - 5^(1/2)/200)^(1/2)))*(3/200 - 5^(1/2)/200)^(1/2) - 2*atan((
exp(-x)*(exp(2*x) - 1))/(10*(5^(1/2)/200 + 3/200)^(1/2)))*(5^(1/2)/200 + 3
/200)^(1/2)
```

3.204 $\int \sinh(x)\tanh(6x)\,dx$

3.204.1 Optimal result

Integrand size = 7, antiderivative size = 87

$$\int \sinh(x)\tanh(6x)\,dx = -\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}} - \frac{1}{6}\sqrt{2-\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{6}\sqrt{2+\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right) + \sinh(x)$$

output
```
sinh(x)-1/6*arctan(sinh(x)*2^(1/2))*2^(1/2)-1/6*arctan(2*sinh(x)/(1/2*6^(1
/2)-1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))-1/6*arctan(2*sinh(x)/(1/2*6^(1
/2)+1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

3.204.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \sinh(x)\tanh(6x)\,dx = -\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}} - \frac{1}{6}\sqrt{2-\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{6}\sqrt{2+\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right) + \sinh(x)$$

input
```
Integrate[Sinh[x]*Tanh[6*x],x]
```

output
```
-1/3*ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2] - (Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sinh[x
])/Sqrt[2 - Sqrt[3]]])/6 - (Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[2 +
Sqrt[3]]])/6 + Sinh[x]
```

3.204.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 25, 4878, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\tanh(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ix)\tan(6ix)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ix)\tan(6ix)dx$$

$$\Big\downarrow \text{4878}$$

$$-\int -\frac{2\sinh^2(x)\left(16\sinh^4(x)+16\sinh^2(x)+3\right)}{32\sinh^6(x)+48\sinh^4(x)+18\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\sinh^2(x)\left(16\sinh^4(x)+16\sinh^2(x)+3\right)}{32\sinh^6(x)+48\sinh^4(x)+18\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{2460}$$

$$2\int \left(\frac{-8\sinh^2(x)-1}{3\left(16\sinh^4(x)+16\sinh^2(x)+1\right)}-\frac{1}{6\left(2\sinh^2(x)+1\right)}+\frac{1}{2}\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{6\sqrt{2}}-\frac{1}{12}\sqrt{2-\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{12}\sqrt{2+\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)+\frac{\sinh(x)}{2}\right)$$

input
```
Int[Sinh[x]*Tanh[6*x],x]
```

output
```
2*(-1/6*ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2] - (Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sin
h[x])/Sqrt[2 - Sqrt[3]]])/12 - (Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[
2 + Sqrt[3]]])/12 + Sinh[x]/2)
```

3.204.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.204.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

method	result
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i\sqrt{2}\,\ln\left(e^{2x}-i\sqrt{2}\,e^{x}-1\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2x}+i\sqrt{2}\,e^{x}-1\right)}{12}+\left(\sum_{\_R=\mathrm{RootOf}(20736\_Z^4+576\_Z^2+1)}\_R\ln\left(-12\_\right.\right.$

input
```
int(sinh(x)*tanh(6*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)-1/2*exp(-x)+1/12*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)-1/12
*I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*exp(x)-1)+sum(_R*ln(-12*_R*exp(x)+exp(2*x
)-1),_R=RootOf(20736*_Z^4+576*_Z^2+1))
```

3.204.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(65) = 130$.

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.45

$$\int \sinh(x)\tanh(6x)\,dx =$$
$$-\frac{\sqrt{\sqrt{3}-2}(\cosh(x)+\sinh(x))\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+\sqrt{\sqrt{3}-2}(\cosh(x)+\mathrm{s}\right.}{}$$

input
```
integrate(sinh(x)*tanh(6*x),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(sqrt(3) - 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 + sqrt(sqrt(3) - 2)*(cosh(x) + sinh(x)) - 1) - sqrt(sqrt(
3) - 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
- sqrt(sqrt(3) - 2)*(cosh(x) + sinh(x)) - 1) + sqrt(-sqrt(3) - 2)*(cosh(x)
 + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(-sqrt(3)
- 2)*(cosh(x) + sinh(x)) - 1) - sqrt(-sqrt(3) - 2)*(cosh(x) + sinh(x))*log
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sqrt(-sqrt(3) - 2)*(cosh(x) +
 sinh(x)) - 1) + 2*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(1/2*sqrt(2)*
cosh(x) + 1/2*sqrt(2)*sinh(x)) - 2*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arc
tan(-1/2*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^
2 + sqrt(2))/(cosh(x) - sinh(x))) - 6*cosh(x)^2 - 12*cosh(x)*sinh(x) - 6*s
inh(x)^2 + 6)/(cosh(x) + sinh(x))
```

3.204.6 Sympy [F]

$$\int \sinh(x)\tanh(6x)\,dx = \int \sinh\left(x\right)\tanh\left(6x\right)\,dx$$

input
```
integrate(sinh(x)*tanh(6*x),x)
```

output
```
Integral(sinh(x)*tanh(6*x), x)
```

3.204.7 Maxima [F]

$$\int \sinh(x)\tanh(6x)\,dx = \int \sinh\left(x\right)\tanh\left(6\,x\right)\,dx$$

input
```
integrate(sinh(x)*tanh(6*x),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) - 1)*e^(-x) - 1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x
)) - 1/6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/2*integrate(2/
3*(2*e^(7*x) - e^(5*x) - e^(3*x) + 2*e^x)/(e^(8*x) - e^(4*x) + 1), x)
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int \sinh(x)\tanh(6x)\,dx = -\frac{1}{12}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{12}\sqrt{2}\left(\pi+2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

input
```
integrate(sinh(x)*tanh(6*x),x, algorithm="giac")
```

output
```
-1/12*(sqrt(6) + sqrt(2))*arctan(-2*(e^(-x) - e^x)/(sqrt(6) + sqrt(2))) -
1/12*(sqrt(6) - sqrt(2))*arctan(-2*(e^(-x) - e^x)/(sqrt(6) - sqrt(2))) - 1
/12*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1/2*e^(-x)
 + 1/2*e^x
```

3.204.9 Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\int \sinh(x)\tanh(6x)\,dx = \frac{e^{x}}{2} - \frac{e^{-x}}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{-x}\left(e^{2x}-1\right)}{2}\right)}{6} - 2\,\mathrm{atan}\left(\frac{e^{-x}\left(e^{2x}-1\right)}{12\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}}\right)\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}} - 2\,\mathrm{atan}\left(\frac{e^{-x}\left(e^{2x}-1\right)}{12\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}}\right)\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}$$

input
```
int(tanh(6*x)*sinh(x),x)
```

output
```
exp(x)/2 - exp(-x)/2 - (2^(1/2)*atan((2^(1/2)*exp(-x)*(exp(2*x) - 1))/2))/
6 - 2*atan((exp(-x)*(exp(2*x) - 1))/(12*(1/72 - 3^(1/2)/144)^(1/2)))*(1/72
 - 3^(1/2)/144)^(1/2) - 2*atan((exp(-x)*(exp(2*x) - 1))/(12*(3^(1/2)/144 +
 1/72)^(1/2)))*(3^(1/2)/144 + 1/72)^(1/2)
```

3.205 $\int \sinh(x)\tanh(nx)\,dx$

3.205.1 Optimal result

Integrand size = 7, antiderivative size = 81

$$\int \sinh(x)\tanh(nx)\,dx = \frac{e^{-x}}{2}+\frac{e^{x}}{2}-e^{-x}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-e^{2nx}\right) - e^{x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-e^{2nx}\right)$$

output
```
1/2/exp(x)+1/2*exp(x)-hypergeom([1, -1/2/n],[1-1/2/n],-exp(2*n*x))/exp(x)-
exp(x)*hypergeom([1, 1/2/n],[1+1/2/n],-exp(2*n*x))
```

3.205.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \sinh(x)\tanh(nx)\,dx = \frac{1}{2}\left(e^{-x}+e^{x}-2e^{-x}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-e^{2nx}\right) - 2e^{x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},1+\frac{1}{2n},-e^{2nx}\right)\right)$$

input
```
Integrate[Sinh[x]*Tanh[n*x],x]
```

output
```
(E^(-x) + E^x - (2*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), -E^(2*n*x)]
)/E^x - 2*E^x*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -E^(2*n*x)])/2
```

3.205.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6135, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\tanh(nx)\,dx$$

$$\Big\downarrow \text{6135}$$

$$\int \left(\frac{e^{-x}}{e^{2nx}+1}-\frac{e^{x}}{e^{2nx}+1}-\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-e^{-x}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-e^{2nx}\right)-e^{x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-e^{2nx}\right)+\frac{e^{-x}}{2}+\frac{e^{x}}{2}$$

input
```
Int[Sinh[x]*Tanh[n*x],x]
```

output
```
1/(2*E^x) + E^x/2 - Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), -E^(2*n*x)
]/E^x - E^x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -E^(2*n*x)]
```

3.205.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6135
```
Int[Sinh[(a_.) + (b_.)*(x_)]*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[-E^
(-(a + b*x))/2 + E^(a + b*x)/2 + 1/(E^(a + b*x)*(1 + E^(2*(c + d*x)))) - E^
(a + b*x)/(1 + E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

3.205.4 Maple [F]

$$\int \sinh(x)\tanh(nx)\,dx$$

input
```
int(sinh(x)*tanh(n*x),x)
```

output
```
int(sinh(x)*tanh(n*x),x)
```

3.205.5 Fricas [F]

$$\int \sinh(x)\tanh(nx)\,dx = \int \sinh(x)\tanh(nx)\,dx$$

input
```
integrate(sinh(x)*tanh(n*x),x, algorithm="fricas")
```

output
```
integral(sinh(x)*tanh(n*x), x)
```

3.205.6 Sympy [F]

$$\int \sinh(x)\tanh(nx)\,dx = \int \sinh(x)\tanh(nx)\,dx$$

input
```
integrate(sinh(x)*tanh(n*x),x)
```

output
```
Integral(sinh(x)*tanh(n*x), x)
```

3.205.7 Maxima [F]

$$\int \sinh(x)\tanh(nx)\,dx = \int \sinh(x)\tanh(nx)\ dx$$

input
```
integrate(sinh(x)*tanh(n*x),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) + 1)*e^(-x) - 1/2*integrate(2*(e^(2*x) - 1)/(e^(2*n*x + x) +
e^x), x)
```

3.205.8 Giac [F]

$$\int \sinh(x)\tanh(nx)\,dx = \int \sinh(x)\tanh(nx)\ dx$$

input
```
integrate(sinh(x)*tanh(n*x),x, algorithm="giac")
```

output
```
integrate(sinh(x)*tanh(n*x), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \sinh(x)\tanh(nx)\,dx = \int \tanh(n\,x)\ \sinh(x)\ dx$$

input
```
int(tanh(n*x)*sinh(x),x)
```

output
```
int(tanh(n*x)*sinh(x), x)
```

3.206 $\int \coth(2x)\sinh(x)\,dx$

3.206.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \coth(2x)\sinh(x)\,dx = -\frac{1}{2}\arctan(\sinh(x)) + \sinh(x)$$

output
```
-1/2*arctan(sinh(x))+sinh(x)
```

3.206.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \coth(2x)\sinh(x)\,dx = -\frac{1}{2}\arctan(\sinh(x)) + \sinh(x)$$

input
```
Integrate[Coth[2*x]*Sinh[x],x]
```

output
```
-1/2*ArcTan[Sinh[x]] + Sinh[x]
```

3.206.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4878, 27, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\coth(2x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cot(2ix)}{\csc(ix)}dx$$

$$\Big\downarrow \; 4878$$

$$\int \frac{2\sinh^2(x)+1}{2\left(\sinh^2(x)+1\right)}d\sinh(x)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\int \frac{2\sinh^2(x)+1}{\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \; 299$$

$$\frac{1}{2}\left(2\sinh(x)-\int \frac{1}{\sinh^2(x)+1}d\sinh(x)\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}(2\sinh(x)-\arctan(\sinh(x)))$$

input
```
Int[Coth[2*x]*Sinh[x],x]
```

output
```
(-ArcTan[Sinh[x]] + 2*Sinh[x])/2
```

3.206.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.206.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{\arctan(\sinh(x))}{2}+\sinh\left(x\right)$	9
risch	$\frac{{\mathrm e}^{x}}{2}-\frac{{\mathrm e}^{-x}}{2}+\frac{i\ln({\mathrm e}^{x}-i)}{2}-\frac{i\ln({\mathrm e}^{x}+i)}{2}$	30

input
```
int(coth(2*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(sinh(x))+sinh(x)
```

3.206.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 4.20

$$\int \coth(2x)\sinh(x)\,dx = -\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-\cosh\left(x\right)^{2}-2\cosh\left(x\right)\sinh\left(x\right)-\sinh\left(x\right)^{2}+1}{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

input
```
integrate(coth(2*x)*sinh(x),x, algorithm="fricas")
```

output
```
-1/2*(2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - cosh(x)^2 - 2*cosh
(x)*sinh(x) - sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

3.206.6 Sympy [F]

$$\int \coth(2x)\sinh(x)\,dx = \int \sinh\left(x\right)\coth\left(2x\right)\,dx$$

input
```
integrate(coth(2*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*coth(2*x), x)
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \coth(2x)\sinh(x)\,dx = \arctan\left(e^{(-x)}\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

input
```
integrate(coth(2*x)*sinh(x),x, algorithm="maxima")
```

output
```
arctan(e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \coth(2x)\sinh(x)\,dx = -\arctan\left(e^{x}\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

input
```
integrate(coth(2*x)*sinh(x),x, algorithm="giac")
```

output
```
-arctan(e^x) - 1/2*e^(-x) + 1/2*e^x
```

3.206.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \coth(2x)\sinh(x)\,dx = \frac{e^{x}}{2} - \operatorname{atan}(e^{x}) - \frac{e^{-x}}{2}$$

input
```
int(coth(2*x)*sinh(x),x)
```

output
```
exp(x)/2 - atan(exp(x)) - exp(-x)/2
```

3.207 $\int \coth(3x)\sinh(x)\,dx$

3.207.1 Optimal result

Integrand size = 7, antiderivative size = 20

$$\int \coth(3x)\sinh(x)\,dx = -\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sinh(x)$$

output
```
sinh(x)-1/3*arctan(2/3*sinh(x)*3^(1/2))*3^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \coth(3x)\sinh(x)\,dx = -\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sinh(x)$$

input
```
Integrate[Coth[3*x]*Sinh[x],x]
```

output
```
-(ArcTan[(2*Sinh[x])/Sqrt[3]]/Sqrt[3]) + Sinh[x]
```

3.207.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4878, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\coth(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cot(3ix)}{\csc(ix)}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{4\sinh^2(x)+1}{4\sinh^2(x)+3}d\sinh(x)$$

$$\Big\downarrow \text{ 299}$$

$$\sinh(x)-2\int \frac{1}{4\sinh^2(x)+3}d\sinh(x)$$

$$\Big\downarrow \text{ 216}$$

$$\sinh(x)-\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[Coth[3*x]*Sinh[x],x]
```

output
```
-(ArcTan[(2*Sinh[x])/Sqrt[3]]/Sqrt[3]) + Sinh[x]
```

3.207.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.207.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\sinh(x) - \frac{\arctan\left(\frac{2\sinh(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17
default	$\sinh(x) - \frac{\arctan\left(\frac{2\sinh(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17
risch	$\frac{e^{x}}{2} - \frac{e^{-x}}{2} + \frac{i\sqrt{3}\,\ln\left(e^{2x} - i\sqrt{3}\,e^{x} - 1\right)}{6} - \frac{i\sqrt{3}\,\ln\left(e^{2x} + i\sqrt{3}\,e^{x} - 1\right)}{6}$	54

input
```
int(coth(3*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
sinh(x)-1/3*arctan(2/3*sinh(x)*3^(1/2))*3^(1/2)
```

3.207.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 5.90

$$\int \coth(3x)\sinh(x)\,dx =$$

$$-\frac{2\left(\sqrt{3}\cosh\left(x\right)+\sqrt{3}\sinh\left(x\right)\right)\arctan\left(\frac{1}{3}\sqrt{3}\cosh\left(x\right)+\frac{1}{3}\sqrt{3}\sinh\left(x\right)\right)-2\left(\sqrt{3}\cosh\left(x\right)+\sqrt{3}\sinh\left(x\right)\right)\text{a}}{6\left(\cosh\left(x\right)\right.}$$

input
```
integrate(coth(3*x)*sinh(x),x, algorithm="fricas")
```

output
```
-1/6*(2*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(1/3*sqrt(3)*cosh(x) + 1
/3*sqrt(3)*sinh(x)) - 2*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(-1/3*(s
qrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2 + 2*sqrt(
3))/(cosh(x) - sinh(x))) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 +
 3)/(cosh(x) + sinh(x))
```

3.207.6 Sympy [F]

$$\int \coth(3x)\sinh(x)\,dx = \int \sinh\left(x\right)\coth\left(3x\right)\,dx$$

input
```
integrate(coth(3*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*coth(3*x), x)
```

3.207.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.45

$$\int \coth(3x)\sinh(x)\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-x)}+1\right)\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-x)}-1\right)\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

input
```
integrate(coth(3*x)*sinh(x),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-x) + 1)) + 1/3*sqrt(3)*arctan(1/3*sq
rt(3)*(2*e^(-x) - 1)) - 1/2*e^(-x) + 1/2*e^x
```

3.207.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \coth(3x)\sinh(x)\,dx = -\frac{1}{6}\sqrt{3}\left(\pi + 2\arctan\left(\frac{1}{3}\sqrt{3}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

input
```
integrate(coth(3*x)*sinh(x),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*(pi + 2*arctan(1/3*sqrt(3)*(e^(2*x) - 1)*e^(-x))) - 1/2*e^(-x
) + 1/2*e^x
```

3.207.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \coth(3x)\sinh(x)\,dx = \frac{e^{x}}{2} - \frac{e^{-x}}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,e^{x}}{3} + \frac{\sqrt{3}\,e^{3x}}{3}\right)}{3} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,e^{x}}{3}\right)}{3}$$

input
```
int(coth(3*x)*sinh(x),x)
```

output
```
exp(x)/2 - exp(-x)/2 - (3^(1/2)*atan((2*3^(1/2)*exp(x))/3 + (3^(1/2)*exp(3
*x))/3))/3 - (3^(1/2)*atan((3^(1/2)*exp(x))/3))/3
```

3.208 $\int \coth(4x)\sinh(x)\,dx$

3.208.1 Optimal result

Integrand size = 7, antiderivative size = 28

$$\int \coth(4x)\sinh(x)\,dx = -\frac{1}{4}\arctan(\sinh(x)) - \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}} + \sinh(x)$$

output
```
-1/4*arctan(sinh(x))+sinh(x)-1/4*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \coth(4x)\sinh(x)\,dx = -\frac{1}{4}\arctan(\sinh(x)) - \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}} + \sinh(x)$$

input
```
Integrate[Coth[4*x]*Sinh[x],x]
```

output
```
-1/4*ArcTan[Sinh[x]] - ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2]) + Sinh[x]
```

3.208.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4878, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\coth(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(4ix)}{\csc(ix)}dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{8\sinh^4(x)+8\sinh^2(x)+1}{4\left(2\sinh^4(x)+3\sinh^2(x)+1\right)}d\sinh(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{8\sinh^4(x)+8\sinh^2(x)+1}{2\sinh^4(x)+3\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{4}\int \left(4-\frac{4\sinh^2(x)+3}{2\sinh^4(x)+3\sinh^2(x)+1}\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\arctan(\sinh(x))-\sqrt{2}\arctan\left(\sqrt{2}\sinh(x)\right)+4\sinh(x)\right)$$

input
```
Int[Coth[4*x]*Sinh[x],x]
```

output
```
(-ArcTan[Sinh[x]] - Sqrt[2]*ArcTan[Sqrt[2]*Sinh[x]] + 4*Sinh[x])/4
```

3.208.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.208.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.57

method	result	size
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i \ln\left(e^{x}-i\right)}{4}-\frac{i \ln\left(e^{x}+i\right)}{4}+\frac{i \sqrt{2}\, \ln\left(e^{2 x}-i \sqrt{2}\, e^{x}-1\right)}{8}-\frac{i \sqrt{2}\, \ln\left(e^{2 x}+i \sqrt{2}\, e^{x}-1\right)}{8}$	72

input
```
int(coth(4*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)-1/2*exp(-x)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)+1/8*I*2^(1/2)
*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)-1/8*I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*exp(x
)-1)
```

3.208.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.57

$$\int \coth(4x)\sinh(x)\,dx =$$
$$-\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\arctan\left(\frac{1}{2}\sqrt{2}\cosh\left(x\right)+\frac{1}{2}\sqrt{2}\sinh\left(x\right)\right)-\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\mathrm{arct}}{}$$

input
```
integrate(coth(4*x)*sinh(x),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(1/2*sqrt(2)*cosh(x) + 1/2
*sqrt(2)*sinh(x)) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(-1/2*(sqrt(
2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))/(c
osh(x) - sinh(x))) + 2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - 2*c
osh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 + 2)/(cosh(x) + sinh(x))
```

3.208.6 Sympy [F]

$$\int \coth(4x)\sinh(x)\,dx = \int \sinh\left(x\right)\coth\left(4x\right)\,dx$$

input
```
integrate(coth(4*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*coth(4*x), x)
```

3.208.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\begin{aligned}\int \coth(4x)\sinh(x)\,dx &= \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right)\\&\quad+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)\\&\quad+\frac{1}{2}\arctan\left(e^{(-x)}\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}\end{aligned}$$

input
```
integrate(coth(4*x)*sinh(x),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) + 1/4*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x))) + 1/2*arctan(e^(-x)) - 1/2*e^(-x) + 1/2
*e^x
```

3.208.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \coth(4x)\sinh(x)\,dx = -\frac{1}{8}\pi - \frac{1}{8}\sqrt{2}\left(\pi + 2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right) - \frac{1}{4}\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

input
```
integrate(coth(4*x)*sinh(x),x, algorithm="giac")
```

output
```
-1/8*pi - 1/8*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) -
1/4*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

3.208.9 Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \coth(4x)\sinh(x)\,dx = \frac{e^x}{2} - \frac{\operatorname{atan}(e^x)}{2} - \frac{e^{-x}}{2} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,e^x}{2} + \frac{\sqrt{2}\,e^{3x}}{2}\right)}{4} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,e^x}{2}\right)}{4}$$

input
```
int(coth(4*x)*sinh(x),x)
```

output
```
exp(x)/2 - atan(exp(x))/2 - exp(-x)/2 - (2^(1/2)*atan((2^(1/2)*exp(x))/2 +
 (2^(1/2)*exp(3*x))/2))/4 - (2^(1/2)*atan((2^(1/2)*exp(x))/2))/4
```

3.209 $\int \coth(5x)\sinh(x)\,dx$

3.209.1 Optimal result

Integrand size = 7, antiderivative size = 82

$$\int \coth(5x)\sinh(x)\,dx = -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sinh(x)\right) + \sinh(x)$$

output
```
sinh(x)-1/10*arctan(1/5*sinh(x)*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2
)-1/10*arctan(2*sinh(x)*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \coth(5x)\sinh(x)\,dx = \frac{1}{10}\left(-\sqrt{10-2\sqrt{5}}\arctan\left(\sqrt{2+\frac{2}{\sqrt{5}}}\sinh(x)\right) - \sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right) + 10\sinh(x)\right)$$

input
```
Integrate[Coth[5*x]*Sinh[x],x]
```

output
```
(-(Sqrt[10 - 2*Sqrt[5]]*ArcTan[Sqrt[2 + 2/Sqrt[5]]*Sinh[x]]) - Sqrt[2*(5 +
 Sqrt[5])]*ArcTan[2*Sqrt[2/(5 + Sqrt[5])]*Sinh[x]] + 10*Sinh[x])/10
```

3.209.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4878, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\coth(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(5ix)}{\csc(ix)}dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{16\sinh^4(x)+12\sinh^2(x)+1}{16\sinh^4(x)+20\sinh^2(x)+5}d\sinh(x)$$

$$\Big\downarrow \text{2205}$$

$$\int \left(1-\frac{4\left(2\sinh^2(x)+1\right)}{16\sinh^4(x)+20\sinh^2(x)+5}\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sinh(x)\right)+\sinh(x)$$

input
```
Int[Coth[5*x]*Sinh[x],x]
```

output
```
-1/5*(Sqrt[(5 + Sqrt[5])/2]*ArcTan[2*Sqrt[2/(5 + Sqrt[5])]*Sinh[x]]) - (Sq
rt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(2*(5 + Sqrt[5]))/5]*Sinh[x]])/5 + Sinh[x]
```

3.209.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.209.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\left(\sum_{\_R=\operatorname{RootOf}\left(2000\_Z^{4}+100\_Z^{2}+1\right)}\_R\ln\left(-10\_R\,e^{x}+e^{2x}-1\right)\right)$	42

input
```
int(coth(5*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)-1/2*exp(-x)+sum(_R*ln(-10*_R*exp(x)+exp(2*x)-1),_R=RootOf(2000*
_Z^4+100*_Z^2+1))
```

3.209.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(54) = 108$.

Time = 0.25 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.57

$$\int \coth(5x)\sinh(x)\,dx =$$

$$-\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\sqrt{5}-5}\log\left(2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}+\left(\sqrt{2}\cosh\left(\right.\right.\right.}{}$$

input
```
integrate(coth(5*x)*sinh(x),x, algorithm="fricas")
```

output
```
-1/20*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) - 5)*log(2*cosh(x)
^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))
*sqrt(sqrt(5) - 5) - 2) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5)
 - 5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - (sqrt(2)*cosh(x)
 + sqrt(2)*sinh(x))*sqrt(sqrt(5) - 5) - 2) + (sqrt(2)*cosh(x) + sqrt(2)*si
nh(x))*sqrt(-sqrt(5) - 5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^
2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) - 5) - 2) - (sqrt(2)
*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) - 5)*log(2*cosh(x)^2 + 4*cosh(x)
*sinh(x) + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5)
 - 5) - 2) - 10*cosh(x)^2 - 20*cosh(x)*sinh(x) - 10*sinh(x)^2 + 10)/(cosh(
x) + sinh(x))
```

3.209.6 Sympy [F]

$$\int \coth(5x)\sinh(x)\,dx = \int \sinh\left(x\right)\coth\left(5x\right)\,dx$$

input
```
integrate(coth(5*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*coth(5*x), x)
```

3.209.7 Maxima [F]

$$\int \coth(5x)\sinh(x)\,dx = \int \coth(5\,x)\sinh(x)\ dx$$

input
```
integrate(coth(5*x)*sinh(x),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) - 1)*e^(-x) - 1/2*integrate((e^(3*x) + e^(2*x) + e^x)/(e^(4*x
) + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/2*integrate((e^(3*x) - e^(2*x) +
e^x)/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x)
```

3.209.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \coth(5x)\sinh(x)\,dx = -\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}}\right) - \frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

input
```
integrate(coth(5*x)*sinh(x),x, algorithm="giac")
```

output
```
-1/10*sqrt(2*sqrt(5) + 10)*arctan(-(e^(-x) - e^x)/sqrt(1/2*sqrt(5) + 5/2))
 - 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(e^(-x) - e^x)/sqrt(-1/2*sqrt(5) + 5
/2)) - 1/2*e^(-x) + 1/2*e^x
```

3.209.9 Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.72

$$\int \coth(5x)\sinh(x)\,dx = \frac{e^x}{2} - \frac{e^{-x}}{2} + \ln\left(40\,e^x\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}} - 4\,e^{2\,x} + 4\right)\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}} + \ln\left(40\,e^x\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}} - 4\,e^{2\,x} + 4\right)\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}} - \ln\left(4\,e^{2\,x} + 40\,e^x\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}} - 4\right)\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}} - \ln\left(4\,e^{2\,x} + 40\,e^x\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}} - 4\right)\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}$$

input
```
int(coth(5*x)*sinh(x),x)
```

output
```
exp(x)/2 - exp(-x)/2 + log(40*exp(x)*(- 5^(1/2)/200 - 1/40)^(1/2) - 4*exp(
2*x) + 4)*(- 5^(1/2)/200 - 1/40)^(1/2) + log(40*exp(x)*(5^(1/2)/200 - 1/40
)^(1/2) - 4*exp(2*x) + 4)*(5^(1/2)/200 - 1/40)^(1/2) - log(4*exp(2*x) + 40
*exp(x)*(- 5^(1/2)/200 - 1/40)^(1/2) - 4)*(- 5^(1/2)/200 - 1/40)^(1/2) - l
og(4*exp(2*x) + 40*exp(x)*(5^(1/2)/200 - 1/40)^(1/2) - 4)*(5^(1/2)/200 - 1
/40)^(1/2)
```

3.210 $\int \coth(6x)\sinh(x)\,dx$

3.210.1 Optimal result

Integrand size = 7, antiderivative size = 38

$$\int \coth(6x)\sinh(x)\,dx = -\frac{1}{6}\arctan(\sinh(x)) - \frac{1}{6}\arctan(2\sinh(x)) - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \sinh(x)$$

output
```
-1/6*arctan(sinh(x))-1/6*arctan(2*sinh(x))+sinh(x)-1/6*arctan(2/3*sinh(x)*
3^(1/2))*3^(1/2)
```

3.210.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \coth(6x)\sinh(x)\,dx = -\frac{1}{6}\arctan(\sinh(x)) - \frac{1}{6}\arctan(2\sinh(x)) - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \sinh(x)$$

input
```
Integrate[Coth[6*x]*Sinh[x],x]
```

output
```
-1/6*ArcTan[Sinh[x]] - ArcTan[2*Sinh[x]]/6 - ArcTan[(2*Sinh[x])/Sqrt[3]]/(
2*Sqrt[3]) + Sinh[x]
```

3.210.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4878, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\coth(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(6ix)}{\csc(ix)}dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{32\sinh^6(x)+48\sinh^4(x)+18\sinh^2(x)+1}{2\left(16\sinh^6(x)+32\sinh^4(x)+19\sinh^2(x)+3\right)}d\sinh(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{32\sinh^6(x)+48\sinh^4(x)+18\sinh^2(x)+1}{16\sinh^6(x)+32\sinh^4(x)+19\sinh^2(x)+3}d\sinh(x)$$

$$\Big\downarrow \text{2460}$$

$$\frac{1}{2}\int \left(-\frac{2}{3\left(4\sinh^2(x)+1\right)}-\frac{2}{4\sinh^2(x)+3}+2-\frac{1}{3\left(\sinh^2(x)+1\right)}\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{3}\arctan(\sinh(x))-\frac{1}{3}\arctan(2\sinh(x))-\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}}+2\sinh(x)\right)$$

input
```
Int[Coth[6*x]*Sinh[x],x]
```

output
```
(-1/3*ArcTan[Sinh[x]] - ArcTan[2*Sinh[x]]/3 - ArcTan[(2*Sinh[x])/Sqrt[3]]/
Sqrt[3] + 2*Sinh[x])/2
```

3.210.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.210.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.68

method	result
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i \ln(e^{x}-i)}{6}-\frac{i \ln(e^{x}+i)}{6}+\frac{i \sqrt{3}\, \ln\left(e^{2x}-i \sqrt{3}\, e^{x}-1\right)}{12}-\frac{i \sqrt{3}\, \ln\left(e^{2x}+i \sqrt{3}\, e^{x}-1\right)}{12}+\frac{i \ln(e^{2x}-i e^{x}-1)}{12}-\frac{i \ln(e^{2x}$

input
```
int(coth(6*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)-1/2*exp(-x)+1/6*I*ln(exp(x)-I)-1/6*I*ln(exp(x)+I)+1/12*I*3^(1/2
)*ln(exp(2*x)-I*3^(1/2)*exp(x)-1)-1/12*I*3^(1/2)*ln(exp(2*x)+I*3^(1/2)*exp
(x)-1)+1/12*I*ln(exp(2*x)-I*exp(x)-1)-1/12*I*ln(exp(2*x)+I*exp(x)-1)
```

3.210.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(28) = 56$.

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 4.32

$$\int \coth(6x)\sinh(x)\,dx =$$
$$-\frac{\left(\sqrt{3}\cosh\left(x\right)+\sqrt{3}\sinh\left(x\right)\right)\arctan\left(\frac{1}{3}\sqrt{3}\cosh\left(x\right)+\frac{1}{3}\sqrt{3}\sinh\left(x\right)\right)-\left(\sqrt{3}\cosh\left(x\right)+\sqrt{3}\sinh\left(x\right)\right)\mathrm{arct}}{}$$

input
```
integrate(coth(6*x)*sinh(x),x, algorithm="fricas")
```

output
```
-1/6*((sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(1/3*sqrt(3)*cosh(x) + 1/3
*sqrt(3)*sinh(x)) - (sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(-1/3*(sqrt(
3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2 + 2*sqrt(3))/
(cosh(x) - sinh(x))) - (cosh(x) + sinh(x))*arctan(-(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))) + 3*(cosh(x) + sinh(x))*arctan(c
osh(x) + sinh(x)) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 + 3)/(co
sh(x) + sinh(x))
```

3.210.6 Sympy [F]

$$\int \coth(6x)\sinh(x)\,dx = \int \sinh\left(x\right)\coth\left(6x\right)\,dx$$

input
```
integrate(coth(6*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*coth(6*x), x)
```

3.210.7 Maxima [F]

$$\int \coth(6x)\sinh(x)\,dx = \int \coth(6x)\sinh(x)\,dx$$

input
```
integrate(coth(6*x)*sinh(x),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) - 1)*e^(-x) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1
/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x - 1)) - 1/3*arctan(e^x) - 1/2*integra
te(1/3*(e^(3*x) + e^x)/(e^(4*x) - e^(2*x) + 1), x)
```

3.210.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.79

$$\int \coth(6x)\sinh(x)\,dx = -\frac{1}{6}\pi - \frac{1}{12}\sqrt{3}\left(\pi + 2\arctan\left(\frac{1}{3}\sqrt{3}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right) - \frac{1}{6}\arctan\left(\left(e^{(2x)} - 1\right)e^{(-x)}\right) - \frac{1}{6}\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

input
```
integrate(coth(6*x)*sinh(x),x, algorithm="giac")
```

output
```
-1/6*pi - 1/12*sqrt(3)*(pi + 2*arctan(1/3*sqrt(3)*(e^(2*x) - 1)*e^(-x))) -
 1/6*arctan((e^(2*x) - 1)*e^(-x)) - 1/6*arctan(1/2*(e^(2*x) - 1)*e^(-x)) -
 1/2*e^(-x) + 1/2*e^x
```

3.210.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \coth(6x)\sinh(x)\,dx = \frac{e^x}{2} - \frac{\mathrm{atan}(e^x)}{3} - \frac{e^{-x}}{2} - \frac{\mathrm{atan}\left(36\,e^{-x}\left(\frac{e^{2x}}{36} - \frac{1}{36}\right)\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(4\sqrt{3}\,e^{-x}\left(\frac{e^{2x}}{12} - \frac{1}{12}\right)\right)}{6}$$

input
```
int(coth(6*x)*sinh(x),x)
```

output
```
exp(x)/2 - atan(exp(x))/3 - exp(-x)/2 - atan(36*exp(-x)*(exp(2*x)/36 - 1/3
6))/6 - (3^(1/2)*atan(4*3^(1/2)*exp(-x)*(exp(2*x)/12 - 1/12)))/6
```

3.210. $\int \coth(6x)\sinh(x)\,dx$

3.211 $\int \mathrm{sech}(2x)\sinh(x)\,dx$

3.211.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \mathrm{sech}(2x)\sinh(x)\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

3.211.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \mathrm{sech}(2x)\sinh(x)\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)+\operatorname{arctanh}\left(\sqrt{2}+i\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{2}}$$

input
```
Integrate[Sech[2*x]*Sinh[x],x]
```

output
```
-((ArcTanh[Sqrt[2] - I*Tanh[x/2]] + ArcTanh[Sqrt[2] + I*Tanh[x/2]])/Sqrt[2
])
```

3.211.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 26, 4857, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\text{sech}(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sin(ix)}{\cos(2ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ix)}{\cos(2ix)}dx$$

$$\Big\downarrow \text{ 4857}$$

$$\int \frac{1}{2\cosh^2(x)-1}d\cosh(x)$$

$$\Big\downarrow \text{ 220}$$

$$-\frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

input
```
Int[Sech[2*x]*Sinh[x],x]
```

output
```
-(ArcTanh[Sqrt[2]*Cosh[x]]/Sqrt[2])
```

3.211.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.211.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 1.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

method	result	size
risch	$\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{2}\right)\sqrt{2}}{4}-\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{2}\right)\sqrt{2}}{4}$	39

input
```
int(sech(2*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/4*ln(1+exp(2*x)+exp(x)*2^(1/2)
)*2^(1/2)
```

3.211.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \operatorname{sech}(2x)\sinh(x)\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\cosh(x)^2+\sinh(x)^2-2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right)$$

input
```
integrate(sech(2*x)*sinh(x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((cosh(x)^2 + sinh(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2
 + sinh(x)^2))
```

3.211.6 Sympy [F]

$$\int \operatorname{sech}(2x)\sinh(x)\,dx = \int \sinh(x)\operatorname{sech}(2x)\,dx$$

input
```
integrate(sech(2*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*sech(2*x), x)
```

3.211.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \operatorname{sech}(2x)\sinh(x)\,dx = -\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)$$

input
```
integrate(sech(2*x)*sinh(x),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)
*e^(-x) + e^(-2*x) + 1)
```

3.211.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \operatorname{sech}(2x)\sinh(x)\,dx = -\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)$$

input
```
integrate(sech(2*x)*sinh(x),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*e^x
 + e^(2*x) + 1)
```

3.211.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \operatorname{sech}(2x)\sinh(x)\,dx = -\frac{\sqrt{2}\left(\ln\left(e^{2x}+\sqrt{2}\,e^{x}+1\right)-\ln\left(e^{2x}-\sqrt{2}\,e^{x}+1\right)\right)}{4}$$

input
```
int(sinh(x)/cosh(2*x),x)
```

output
```
-(2^(1/2)*(log(exp(2*x) + 2^(1/2)*exp(x) + 1) - log(exp(2*x) - 2^(1/2)*exp
(x) + 1)))/4
```

3.212 $\int \mathbf{sech}(3x)\sinh(x)\,dx$

3.212.1 Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = -\frac{1}{3}\log(\cosh(x)) + \frac{1}{6}\log\left(3-4\cosh^2(x)\right)$$

output
```
-1/3*ln(cosh(x))+1/6*ln(3-4*cosh(x)^2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = -\frac{1}{3}\operatorname{arctanh}\left(\frac{1}{3}\left(5+8\sinh^2(x)\right)\right)$$

input
```
Integrate[Sech[3*x]*Sinh[x],x]
```

output
```
-1/3*ArcTanh[(5 + 8*Sinh[x]^2)/3]
```

3.212.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3042, 26, 4857, 25, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\text{sech}(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\cos(3ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\cos(3ix)}dx$$

$$\Big\downarrow \text{4857}$$

$$\int -\frac{\text{sech}(x)}{3-4\cosh^2(x)}d\cosh(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\text{sech}(x)}{3-4\cosh^2(x)}d\cosh(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\int \frac{\text{sech}(x)}{3-4\cosh^2(x)}d\cosh^2(x)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(-\frac{4}{3}\int \frac{1}{3-4\cosh^2(x)}d\cosh^2(x)-\frac{1}{3}\int \text{sech}(x)d\cosh^2(x)\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(-\frac{4}{3}\int \frac{1}{3-4\cosh^2(x)}d\cosh^2(x)-\frac{1}{3}\log\left(\cosh^2(x)\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\log\left(3-4\cosh^2(x)\right)-\frac{1}{3}\log\left(\cosh^2(x)\right)\right)$$

input
```
Int[Sech[3*x]*Sinh[x],x]
```

output
```
(-1/3*Log[Cosh[x]^2] + Log[3 - 4*Cosh[x]^2]/3)/2
```

3.212.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.212.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
risch	$-\frac{\ln(1+e^{2x})}{3}+\frac{\ln(1-e^{2x}+e^{4x})}{6}$	26

input
```
int(sech(3*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(1+exp(2*x))+1/6*ln(1-exp(2*x)+exp(4*x))
```

3.212.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = \frac{1}{6}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2-1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right) - \frac{1}{3}\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(sech(3*x)*sinh(x),x, algorithm="fricas")
```

output
```
1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 - 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) - 1/3*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

3.212.6 Sympy [F]

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = \int \sinh(x)\operatorname{sech}(3x)\,dx$$

input
```
integrate(sech(3*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*sech(3*x), x)
```

3.212.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.14

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = \frac{1}{6}\log\left(\sqrt{3}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{6}\log\left(-\sqrt{3}e^{(-x)} + e^{(-2x)} + 1\right) - \frac{1}{3}\log\left(e^{(-2x)} + 1\right)$$

input
```
integrate(sech(3*x)*sinh(x),x, algorithm="maxima")
```

output
```
1/6*log(sqrt(3)*e^(-x) + e^(-2*x) + 1) + 1/6*log(-sqrt(3)*e^(-x) + e^(-2*x
) + 1) - 1/3*log(e^(-2*x) + 1)
```

3.212.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = \frac{1}{6}\log\left(\sqrt{3}e^{x} + e^{(2x)} + 1\right) + \frac{1}{6}\log\left(-\sqrt{3}e^{x} + e^{(2x)} + 1\right) - \frac{1}{3}\log\left(e^{(2x)} + 1\right)$$

input
```
integrate(sech(3*x)*sinh(x),x, algorithm="giac")
```

output
```
1/6*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/6*log(-sqrt(3)*e^x + e^(2*x) + 1) -
 1/3*log(e^(2*x) + 1)
```

3.212.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = \frac{\ln\left(e^{2x} - e^{4x} - 1\right)}{6} - \frac{\ln\left(3\,e^{2x} + 3\right)}{3}$$

input
```
int(sinh(x)/cosh(3*x),x)
```

output
```
log(exp(2*x) - exp(4*x) - 1)/6 - log(3*exp(2*x) + 3)/3
```

3.213 $\int \operatorname{sech}(4x)\sinh(x)\,dx$

3.213.1 Optimal result

Integrand size = 7, antiderivative size = 71

$$\int \operatorname{sech}(4x)\sinh(x)\,dx=\frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

output
```
1/2*arctanh(2*cosh(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/2*arctanh(2
*cosh(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

3.213.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.55

$$\int \operatorname{sech}(4x)\sinh(x)\,dx=\frac{1}{16}\text{RootSum}\Bigg[1+\#1^8\&,\frac{-x-2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)+x\#1^2+2\log\left(-\cosh\left(\frac{x}{2}\right)-\right.}{\#1^5}$$

input
```
Integrate[Sech[4*x]*Sinh[x],x]
```

output
```
RootSum[1 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - S
inh[x/2]*#1] + x*#1^2 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh
[x/2]*#1]*#1^2)/#1^5 & ]/16
```

3.213.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 26, 4857, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\text{sech}(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\cos(4ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\cos(4ix)}dx$$

$$\Big\downarrow \text{4857}$$

$$\int \frac{1}{8\cosh^4(x)-8\cosh^2(x)+1}d\cosh(x)$$

$$\Big\downarrow \text{1406}$$

$$\sqrt{2}\int \frac{1}{8\cosh^2(x)-2\left(2+\sqrt{2}\right)}d\cosh(x)-\sqrt{2}\int \frac{1}{8\cosh^2(x)-2\left(2-\sqrt{2}\right)}d\cosh(x)$$

$$\Big\downarrow \text{220}$$

$$\frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

input
```
Int[Sech[4*x]*Sinh[x],x]
```

output
```
ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTanh
[(2*Cosh[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

3.213.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.213.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result	size
risch	$2\left(\sum_{\_R=\mathrm{RootOf}(32768\_Z^4-512\_Z^2+1)} \_R\ln\left(\mathrm{e}^{2x}+\left(4096\_R^3-48\_R\right)\mathrm{e}^{x}+1\right)\right)$	40

input
```
int(sech(4*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(exp(2*x)+(4096*_R^3-48*_R)*exp(x)+1),_R=RootOf(32768*_Z^4-512*
_Z^2+1))
```

3.213.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.03

$$\int \operatorname{sech}(4x)\sinh(x)\,dx = \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + \left(\left(\sqrt{2}-1\right)\cosh(x) + \left(\sqrt{2}-1\right)\sinh(x)\right)\sqrt{\sqrt{2}+2} + 1\right) - \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - \left(\left(\sqrt{2}-1\right)\cosh(x) + \left(\sqrt{2}-1\right)\sinh(x)\right)\sqrt{\sqrt{2}+2} + 1\right) - \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + \left(\left(\sqrt{2}+1\right)\cosh(x) + \left(\sqrt{2}+1\right)\sinh(x)\right)\sqrt{-\sqrt{2}+2} + 1\right) + \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - \left(\left(\sqrt{2}+1\right)\cosh(x) + \left(\sqrt{2}+1\right)\sinh(x)\right)\sqrt{-\sqrt{2}+2} + 1\right)$$

input
```
integrate(sech(4*x)*sinh(x),x, algorithm="fricas")
```

output
```
1/8*sqrt(sqrt(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sq
rt(2) - 1)*cosh(x) + (sqrt(2) - 1)*sinh(x))*sqrt(sqrt(2) + 2) + 1) - 1/8*s
qrt(sqrt(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(2)
 - 1)*cosh(x) + (sqrt(2) - 1)*sinh(x))*sqrt(sqrt(2) + 2) + 1) - 1/8*sqrt(-
sqrt(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(2) + 1
)*cosh(x) + (sqrt(2) + 1)*sinh(x))*sqrt(-sqrt(2) + 2) + 1) + 1/8*sqrt(-sqr
t(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(2) + 1)*c
osh(x) + (sqrt(2) + 1)*sinh(x))*sqrt(-sqrt(2) + 2) + 1)
```

3.213.6 Sympy [F]

$$\int \operatorname{sech}(4x)\sinh(x)\,dx = \int \sinh\left(x\right)\operatorname{sech}\left(4x\right)\,dx$$

input
```
integrate(sech(4*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*sech(4*x), x)
```

3.213.7 Maxima [F]

$$\int \operatorname{sech}(4x)\sinh(x)\,dx = \int \operatorname{sech}\left(4\,x\right)\sinh\left(x\right)\,dx$$

input
```
integrate(sech(4*x)*sinh(x),x, algorithm="maxima")
```

output
```
integrate(sech(4*x)*sinh(x), x)
```

3.213.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(49) = 98.

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.62

$$\begin{aligned}\int \operatorname{sech}(4x)\sinh(x)\,dx = &-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}e^{x}+e^{(2\,x)}+1\right)\\&+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}e^{x}+e^{(2\,x)}+1\right)\\&+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}e^{x}+e^{(2\,x)}+1\right)\\&-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}e^{x}+e^{(2\,x)}+1\right)\end{aligned}$$

input
```
integrate(sech(4*x)*sinh(x),x, algorithm="giac")
```

output
```
-1/8*sqrt(-sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/8*sqr
t(-sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/8*sqrt(sqrt(
2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/8*sqrt(sqrt(2) + 2)*
log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
```

3.213.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 251, normalized size of antiderivative = 3.54

$$\int \operatorname{sech}(4x)\sinh(x)\,dx = \ln\left(3\,\mathrm{e}^{2\,x}-2\,\sqrt{2}+8\,\mathrm{e}^{x}\,\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}-2\,\sqrt{2}\,\mathrm{e}^{2\,x}-8\,\sqrt{2}\,\mathrm{e}^{x}\,\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}+3\right)\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}-\ln\left(3\,\mathrm{e}^{2\,x}-2\,\sqrt{2}-8\,\mathrm{e}^{x}\,\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}-2\,\sqrt{2}\,\mathrm{e}^{2\,x}+8\,\sqrt{2}\,\mathrm{e}^{x}\,\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}+3\right)\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}-\ln\left(3\,\mathrm{e}^{2\,x}+2\,\sqrt{2}-8\,\mathrm{e}^{x}\,\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{32}}+2\,\sqrt{2}\,\mathrm{e}^{2\,x}-8\,\sqrt{2}\,\mathrm{e}^{x}\,\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{32}}+3\right)\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{32}}+\ln\left(3\,\mathrm{e}^{2\,x}+2\,\sqrt{2}+8\,\mathrm{e}^{x}\,\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{32}}+2\,\sqrt{2}\,\mathrm{e}^{2\,x}+8\,\sqrt{2}\,\mathrm{e}^{x}\,\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{32}}+3\right)\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{32}}$$

input
```
int(sinh(x)/cosh(4*x),x)
```

output
```
log(3*exp(2*x) - 2*2^(1/2) + 8*exp(x)*(1/32 - 2^(1/2)/64)^(1/2) - 2*2^(1/2
)*exp(2*x) - 8*2^(1/2)*exp(x)*(1/32 - 2^(1/2)/64)^(1/2) + 3)*(1/32 - 2^(1/
2)/64)^(1/2) - log(3*exp(2*x) - 2*2^(1/2) - 8*exp(x)*(1/32 - 2^(1/2)/64)^(
1/2) - 2*2^(1/2)*exp(2*x) + 8*2^(1/2)*exp(x)*(1/32 - 2^(1/2)/64)^(1/2) + 3
)*(1/32 - 2^(1/2)/64)^(1/2) - log(3*exp(2*x) + 2*2^(1/2) - 8*exp(x)*(2^(1/
2)/64 + 1/32)^(1/2) + 2*2^(1/2)*exp(2*x) - 8*2^(1/2)*exp(x)*(2^(1/2)/64 +
1/32)^(1/2) + 3)*(2^(1/2)/64 + 1/32)^(1/2) + log(3*exp(2*x) + 2*2^(1/2) +
8*exp(x)*(2^(1/2)/64 + 1/32)^(1/2) + 2*2^(1/2)*exp(2*x) + 8*2^(1/2)*exp(x)
*(2^(1/2)/64 + 1/32)^(1/2) + 3)*(2^(1/2)/64 + 1/32)^(1/2)
```

3.213. $\int \operatorname{sech}(4x)\sinh(x)\,dx$

3.214 $\int \text{sech}(5x)\sinh(x)\,dx$

3.214.1 Optimal result

Integrand size = 7, antiderivative size = 62

$$\int \text{sech}(5x)\sinh(x)\,dx = \frac{1}{5}\log(\cosh(x)) - \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\cosh^2(x)\right) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}-8\cosh^2(x)\right)$$

output
```
1/5*ln(cosh(x))-1/20*ln(5-8*cosh(x)^2+5^(1/2))*(-5^(1/2)+1)-1/20*ln(5-8*co
sh(x)^2-5^(1/2))*(5^(1/2)+1)
```

3.214.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \text{sech}(5x)\sinh(x)\,dx = \frac{1}{20}\left(4\log(\cosh(x)) + \left(-1+\sqrt{5}\right)\log\left(3-\sqrt{5}+8\sinh^2(x)\right) - \left(1+\sqrt{5}\right)\log\left(3+\sqrt{5}+8\sinh^2(x)\right)\right)$$

input
```
Integrate[Sech[5*x]*Sinh[x],x]
```

output
```
(4*Log[Cosh[x]] + (-1 + Sqrt[5])*Log[3 - Sqrt[5] + 8*Sinh[x]^2] - (1 + Sqr
t[5])*Log[3 + Sqrt[5] + 8*Sinh[x]^2])/20
```

3.214.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 26, 4857, 1434, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\text{sech}(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\cos(5ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\cos(5ix)}dx$$

$$\Big\downarrow \text{4857}$$

$$\int \frac{\text{sech}(x)}{16\cosh^4(x)-20\cosh^2(x)+5}d\cosh(x)$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{\text{sech}(x)}{16\cosh^4(x)-20\cosh^2(x)+5}d\cosh^2(x)$$

$$\Big\downarrow \text{1141}$$

$$8\int \left(\frac{\text{sech}(x)}{80}+\frac{1}{\sqrt{5}\left(5-\sqrt{5}\right)\left(-8\cosh^2(x)-\sqrt{5}+5\right)}-\frac{1}{\sqrt{5}\left(5+\sqrt{5}\right)\left(-8\cosh^2(x)+\sqrt{5}+5\right)}\right)d\cosh^2(x)$$

$$\Big\downarrow \text{2009}$$

$$8\left(\frac{1}{80}\log\left(\cosh^2(x)\right)-\frac{\log\left(-8\cosh^2(x)-\sqrt{5}+5\right)}{8\sqrt{5}\left(5-\sqrt{5}\right)}+\frac{\log\left(-8\cosh^2(x)+\sqrt{5}+5\right)}{8\sqrt{5}\left(5+\sqrt{5}\right)}\right)$$

input
```
Int[Sech[5*x]*Sinh[x],x]
```

output
```
8*(Log[Cosh[x]^2]/80 - Log[5 - Sqrt[5] - 8*Cosh[x]^2]/(8*Sqrt[5]*(5 - Sqrt
[5])) + Log[5 + Sqrt[5] - 8*Cosh[x]^2]/(8*Sqrt[5]*(5 + Sqrt[5])))
```

3.214.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.214.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(48) = 96$.

Time = 1.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

method	result
risch	$\frac{\ln(1+e^{2x})}{5}-\frac{\ln\left(e^{4x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)}{20}+\frac{\ln\left(e^{4x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)\sqrt{5}}{20}-\frac{\ln\left(e^{4x}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)e^{2x}+1\right)}{20}-\frac{\ln\left(e^{4x}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\right.}{20}$

input
```
int(sech(5*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
1/5*ln(1+exp(2*x))-1/20*ln(exp(4*x)+(-1/2-1/2*5^(1/2))*exp(2*x)+1)+1/20*ln
(exp(4*x)+(-1/2-1/2*5^(1/2))*exp(2*x)+1)*5^(1/2)-1/20*ln(exp(4*x)+(1/2*5^(
1/2)-1/2)*exp(2*x)+1)-1/20*ln(exp(4*x)+(1/2*5^(1/2)-1/2)*exp(2*x)+1)*5^(1/
2)
```

3.214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(46) = 92$.

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.94

$$\int \operatorname{sech}(5x)\sinh(x)\,dx$$

$$= \frac{1}{20}\sqrt{5}\log\left(\frac{4\cosh(x)^4 + 4\sinh(x)^4 - 4\left(\sqrt{5}+1\right)\cosh(x)^2 + 4\left(6\cosh(x)^2 - \sqrt{5} - 1\right)\sinh(x)^2 + \sqrt{5} +}{2\cosh(x)^4 + 2\sinh(x)^4 + 2\left(6\cosh(x)^2 - 1\right)\sinh(x)^2 - 2\cosh(x)^2 + 1}\right.$$
$$- \frac{1}{20}\log\left(\frac{2\cosh(x)^4 + 2\sinh(x)^4 + 2\left(6\cosh(x)^2 - 1\right)\sinh(x)^2 - 2\cosh(x)^2 + 1}{\cosh(x)^4 - 4\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 4\cosh(x)\sinh(x)^3 + \sinh(x)^4}\right)$$
$$+ \frac{1}{5}\log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

input
```
integrate(sech(5*x)*sinh(x),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((4*cosh(x)^4 + 4*sinh(x)^4 - 4*(sqrt(5) + 1)*cosh(x)^2 +
4*(6*cosh(x)^2 - sqrt(5) - 1)*sinh(x)^2 + sqrt(5) + 7)/(2*cosh(x)^4 + 2*si
nh(x)^4 + 2*(6*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 1)) - 1/20*log((2*
cosh(x)^4 + 2*sinh(x)^4 + 2*(6*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 1)
/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh
(x)^3 + sinh(x)^4)) + 1/5*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

3.214.6 Sympy [F]

$$\int \operatorname{sech}(5x)\sinh(x)\,dx = \int \sinh(x)\operatorname{sech}(5x)\,dx$$

input
```
integrate(sech(5*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*sech(5*x), x)
```

3.214.7 Maxima [F]

$$\int \operatorname{sech}(5x)\sinh(x)\,dx = \int \operatorname{sech}(5\,x)\sinh(x)\,\,dx$$

input
```
integrate(sech(5*x)*sinh(x),x, algorithm="maxima")
```

output
```
-2/5*integrate((e^(6*x) - e^(4*x) + e^(2*x) - 1)*e^(2*x)/(e^(8*x) - e^(6*x
) + e^(4*x) - e^(2*x) + 1), x) + 2/5*integrate(e^(6*x)/(e^(8*x) - e^(6*x)
+ e^(4*x) - e^(2*x) + 1), x) + 1/5*integrate(e^(4*x)/(e^(8*x) - e^(6*x) +
e^(4*x) - e^(2*x) + 1), x) - 4/5*integrate(e^(2*x)/(e^(8*x) - e^(6*x) + e^
(4*x) - e^(2*x) + 1), x) + 1/5*log(e^(2*x) + 1)
```

3.214.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(46) = 92$.

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.90

$$\begin{aligned}\int \operatorname{sech}(5x)\sinh(x)\,dx &= \frac{1}{20}\left(\sqrt{5}-1\right)\log\left(\frac{1}{2}\sqrt{2\sqrt{5}+10}e^{x}+e^{(2\,x)}+1\right) \\ &\quad+\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(-\frac{1}{2}\sqrt{2\sqrt{5}+10}e^{x}+e^{(2\,x)}+1\right) \\ &\quad-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(\frac{1}{2}\sqrt{-2\sqrt{5}+10}e^{x}+e^{(2\,x)}+1\right) \\ &\quad-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(-\frac{1}{2}\sqrt{-2\sqrt{5}+10}e^{x}+e^{(2\,x)}+1\right) \\ &\quad+\frac{1}{5}\log\left(e^{(2\,x)}+1\right)\end{aligned}$$

input
```
integrate(sech(5*x)*sinh(x),x, algorithm="giac")
```

output
```
1/20*(sqrt(5) - 1)*log(1/2*sqrt(2*sqrt(5) + 10)*e^x + e^(2*x) + 1) + 1/20*
(sqrt(5) - 1)*log(-1/2*sqrt(2*sqrt(5) + 10)*e^x + e^(2*x) + 1) - 1/20*(sqr
t(5) + 1)*log(1/2*sqrt(-2*sqrt(5) + 10)*e^x + e^(2*x) + 1) - 1/20*(sqrt(5)
 + 1)*log(-1/2*sqrt(-2*sqrt(5) + 10)*e^x + e^(2*x) + 1) + 1/5*log(e^(2*x)
+ 1)
```

3.214.9 Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \operatorname{sech}(5x)\sinh(x)\,dx = \frac{\ln\left(5\,e^{2\,x}+5\right)}{5} - \ln\left(e^{2\,x}+2\,e^{4\,x}+\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)\,\left(20\,e^{2\,x}+30\,e^{4\,x}+30\right)+2\right)\,\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right) + \ln\left(e^{2\,x}+2\,e^{4\,x}-\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)\,\left(20\,e^{2\,x}+30\,e^{4\,x}+30\right)+2\right)\,\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)$$

input
```
int(sinh(x)/cosh(5*x),x)
```

output
```
log(5*exp(2*x) + 5)/5 - log(exp(2*x) + 2*exp(4*x) + (5^(1/2)/20 + 1/20)*(2
0*exp(2*x) + 30*exp(4*x) + 30) + 2)*(5^(1/2)/20 + 1/20) + log(exp(2*x) + 2
*exp(4*x) - (5^(1/2)/20 - 1/20)*(20*exp(2*x) + 30*exp(4*x) + 30) + 2)*(5^(
1/2)/20 - 1/20)
```

3.215 $\int \text{sech}(6x)\sinh(x)\,dx$

3.215.1 Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \text{sech}(6x)\sinh(x)\,dx=\frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}-\frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

output
```
1/6*arctanh(cosh(x)*2^(1/2))*2^(1/2)-1/6*arctanh(2*cosh(x)/(1/2*6^(1/2)-1/
2*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))-1/6*arctanh(2*cosh(x)/(1/2*6^(1/2)+1
/2*2^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

3.215.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.16

$$\int \text{sech}(6x)\sinh(x)\,dx$$
$$=\frac{1}{24}\left(4\sqrt{2}\left(\text{arctanh}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)+\text{arctanh}\left(\sqrt{2}+i\tanh\left(\frac{x}{2}\right)\right)\right)+\text{RootSum}\left[1-\#1^4\right.\right.$$
$$+\#1^8\&,\frac{-x-2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)+x\#1^2+2\log\left(-\cosh\left(\frac{x}{2}\right)-\right.}{}$$

input
```
Integrate[Sech[6*x]*Sinh[x],x]
```

output
```
(4*Sqrt[2]*(ArcTanh[Sqrt[2] - I*Tanh[x/2]] + ArcTanh[Sqrt[2] + I*Tanh[x/2]
]) + RootSum[1 - #1^4 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh
[x/2]*#1 - Sinh[x/2]*#1] + x*#1^2 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/
2]*#1 - Sinh[x/2]*#1]*#1^2 - x*#1^4 - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[
x/2]*#1 - Sinh[x/2]*#1]*#1^4 + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cos
h[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-#1^3 + 2*#1^7) & ])/24
```

3.215.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 26, 4857, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\text{sech}(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\cos(6ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\cos(6ix)}dx$$

$$\Big\downarrow \text{4857}$$

$$\int \frac{1}{32\cosh^6(x)-48\cosh^4(x)+18\cosh^2(x)-1}d\cosh(x)$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{4\left(2\cosh^2(x)-1\right)}{3\left(16\cosh^4(x)-16\cosh^2(x)+1\right)}-\frac{1}{3\left(2\cosh^2(x)-1\right)}\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}-\frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

input
```
Int[Sech[6*x]*Sinh[x],x]
```

output
```
ArcTanh[Sqrt[2]*Cosh[x]]/(3*Sqrt[2]) - ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[3
]]]/(6*Sqrt[2 - Sqrt[3]]) - ArcTanh[(2*Cosh[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt
[2 + Sqrt[3]])
```

3.215.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.215.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{2}\right)\sqrt{2}}{12}-\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{2}\right)\sqrt{2}}{12}+2\left(\sum_{\_R=\mathrm{RootOf}\left(331776\_Z^{4}-2304\_Z^{2}+1\right)}\_R\ln\left(e^{2x}+\left(13824\_R^{3}\right.\right.\right.$

input
```
int(sech(6*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)-1/12*ln(1+exp(2*x)-exp(x)*2^(1/
2))*2^(1/2)+2*sum(_R*ln(exp(2*x)+(13824*_R^3-96*_R)*exp(x)+1),_R=RootOf(33
1776*_Z^4-2304*_Z^2+1))
```

3.215.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(67) = 134$.

Time = 0.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.94

$$\begin{aligned}\int \operatorname{sech}(6x)\sinh(x)\,dx =& \frac{1}{12}\sqrt{\sqrt{3}+2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\\&+\left(\left(\sqrt{3}-2\right)\cosh(x)+\left(\sqrt{3}-2\right)\sinh(x)\right)\sqrt{\sqrt{3}+2}+1\Big)\\&-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\\&-\left(\left(\sqrt{3}-2\right)\cosh(x)+\left(\sqrt{3}-2\right)\sinh(x)\right)\sqrt{\sqrt{3}+2}+1\Big)\\&-\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\\&+\left(\left(\sqrt{3}+2\right)\cosh(x)+\left(\sqrt{3}+2\right)\sinh(x)\right)\sqrt{-\sqrt{3}+2}+1\Big)\\&+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\\&-\left(\left(\sqrt{3}+2\right)\cosh(x)+\left(\sqrt{3}+2\right)\sinh(x)\right)\sqrt{-\sqrt{3}+2}+1\Big)\\&+\frac{1}{12}\sqrt{2}\log\left(\frac{\cosh(x)^2+\sinh(x)^2+2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right)\end{aligned}$$

input
```
integrate(sech(6*x)*sinh(x),x, algorithm="fricas")
```

output
```
1/12*sqrt(sqrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((s
qrt(3) - 2)*cosh(x) + (sqrt(3) - 2)*sinh(x))*sqrt(sqrt(3) + 2) + 1) - 1/12
*sqrt(sqrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(
3) - 2)*cosh(x) + (sqrt(3) - 2)*sinh(x))*sqrt(sqrt(3) + 2) + 1) - 1/12*sqr
t(-sqrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(3)
+ 2)*cosh(x) + (sqrt(3) + 2)*sinh(x))*sqrt(-sqrt(3) + 2) + 1) + 1/12*sqrt(
-sqrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(3) +
2)*cosh(x) + (sqrt(3) + 2)*sinh(x))*sqrt(-sqrt(3) + 2) + 1) + 1/12*sqrt(2)
*log((cosh(x)^2 + sinh(x)^2 + 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^
2))
```

3.215.6 Sympy [F]

$$\int \operatorname{sech}(6x)\sinh(x)\,dx = \int \sinh(x)\operatorname{sech}(6x)\,dx$$

input
```
integrate(sech(6*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*sech(6*x), x)
```

3.215.7 Maxima [F]

$$\int \operatorname{sech}(6x)\sinh(x)\,dx = \int \operatorname{sech}(6\,x)\sinh(x)\,\,dx$$

input
```
integrate(sech(6*x)*sinh(x),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) - 1/12*sqrt(2)*log(-sqrt(2)*e^
x + e^(2*x) + 1) + integrate(1/3*(e^(7*x) - e^(5*x) + e^(3*x) - e^x)/(e^(8
*x) - e^(4*x) + 1), x)
```

3.215.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(67) = 134.

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.81

$$\int \operatorname{sech}(6x)\sinh(x)\,dx = -\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\frac{1}{2}\left(\sqrt{6}+\sqrt{2}\right)e^{x}+e^{(2x)}+1\right) + \frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(-\frac{1}{2}\left(\sqrt{6}+\sqrt{2}\right)e^{x}+e^{(2x)}+1\right) - \frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\frac{1}{2}\left(\sqrt{6}-\sqrt{2}\right)e^{x}+e^{(2x)}+1\right) + \frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(-\frac{1}{2}\left(\sqrt{6}-\sqrt{2}\right)e^{x}+e^{(2x)}+1\right) + \frac{1}{12}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right) - \frac{1}{12}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)$$

input
```
integrate(sech(6*x)*sinh(x),x, algorithm="giac")
```

output
```
-1/24*(sqrt(6) - sqrt(2))*log(1/2*(sqrt(6) + sqrt(2))*e^x + e^(2*x) + 1) +
 1/24*(sqrt(6) - sqrt(2))*log(-1/2*(sqrt(6) + sqrt(2))*e^x + e^(2*x) + 1)
- 1/24*(sqrt(6) + sqrt(2))*log(1/2*(sqrt(6) - sqrt(2))*e^x + e^(2*x) + 1)
+ 1/24*(sqrt(6) + sqrt(2))*log(-1/2*(sqrt(6) - sqrt(2))*e^x + e^(2*x) + 1)
 + 1/12*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) - 1/12*sqrt(2)*log(-sqrt(2)
*e^x + e^(2*x) + 1)
```

3.215.9 Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.39

$$\int \operatorname{sech}(6x)\sinh(x)\,dx = \frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+\sqrt{2}\,\mathrm{e}^{x}+1\right)}{12} - \frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-\sqrt{2}\,\mathrm{e}^{x}+1\right)}{12}$$
$$+\ln\left(7\,\mathrm{e}^{2x}-4\sqrt{3}-24\,\mathrm{e}^{x}\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}-4\sqrt{3}\,\mathrm{e}^{2x} + 12\sqrt{3}\,\mathrm{e}^{x}\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}+7\right)\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}$$
$$-\ln\left(7\,\mathrm{e}^{2x}-4\sqrt{3}+24\,\mathrm{e}^{x}\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}-4\sqrt{3}\,\mathrm{e}^{2x} - 12\sqrt{3}\,\mathrm{e}^{x}\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}+7\right)\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}$$
$$+\ln\left(7\,\mathrm{e}^{2x}+4\sqrt{3}-24\,\mathrm{e}^{x}\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}+4\sqrt{3}\,\mathrm{e}^{2x} - 12\sqrt{3}\,\mathrm{e}^{x}\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}+7\right)\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}$$
$$-\ln\left(7\,\mathrm{e}^{2x}+4\sqrt{3}+24\,\mathrm{e}^{x}\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}+4\sqrt{3}\,\mathrm{e}^{2x} + 12\sqrt{3}\,\mathrm{e}^{x}\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}+7\right)\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}$$

input
```
int(sinh(x)/cosh(6*x),x)
```

output
```
(2^(1/2)*log(exp(2*x) + 2^(1/2)*exp(x) + 1))/12 - (2^(1/2)*log(exp(2*x) -
2^(1/2)*exp(x) + 1))/12 + log(7*exp(2*x) - 4*3^(1/2) - 24*exp(x)*(1/72 - 3
^(1/2)/144)^(1/2) - 4*3^(1/2)*exp(2*x) + 12*3^(1/2)*exp(x)*(1/72 - 3^(1/2)
/144)^(1/2) + 7)*(1/72 - 3^(1/2)/144)^(1/2) - log(7*exp(2*x) - 4*3^(1/2) +
 24*exp(x)*(1/72 - 3^(1/2)/144)^(1/2) - 4*3^(1/2)*exp(2*x) - 12*3^(1/2)*ex
p(x)*(1/72 - 3^(1/2)/144)^(1/2) + 7)*(1/72 - 3^(1/2)/144)^(1/2) + log(7*ex
p(2*x) + 4*3^(1/2) - 24*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 4*3^(1/2)*exp(
2*x) - 12*3^(1/2)*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 7)*(3^(1/2)/144 + 1/
72)^(1/2) - log(7*exp(2*x) + 4*3^(1/2) + 24*exp(x)*(3^(1/2)/144 + 1/72)^(1
/2) + 4*3^(1/2)*exp(2*x) + 12*3^(1/2)*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) +
7)*(3^(1/2)/144 + 1/72)^(1/2)
```

3.215. $\int \operatorname{sech}(6x)\sinh(x)\,dx$

3.216 $\int \operatorname{csch}(2x)\sinh(x)\,dx$

3.216.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = \frac{1}{2}\arctan(\sinh(x))$$

output
```
1/2*arctan(sinh(x))
```

3.216.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = \frac{1}{2}\arctan(\sinh(x))$$

input
```
Integrate[Csch[2*x]*Sinh[x],x]
```

output
```
ArcTan[Sinh[x]]/2
```

3.216.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4776, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\text{csch}(2x)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(ix)}{\sin(2ix)}dx$$
$$\Big\downarrow \text{ 4776}$$
$$\frac{\int \text{sech}(x)dx}{2}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{ 4257}$$
$$\frac{1}{2}\arctan(\sinh(x))$$

input
```
Int[Csch[2*x]*Sinh[x],x]
```

output
```
ArcTan[Sinh[x]]/2
```

3.216.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.216.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\arctan(\sinh(x))}{2}$	6
risch	$\frac{i\ln(\mathrm{e}^{x}+i)}{2}-\frac{i\ln(\mathrm{e}^{x}-i)}{2}$	20

input
```
int(csch(2*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(sinh(x))
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \operatorname{csch}(2x)\sinh(x)\,dx=\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)$$

input
```
integrate(csch(2*x)*sinh(x),x, algorithm="fricas")
```

output
```
arctan(cosh(x) + sinh(x))
```

3.216.6 Sympy [F]

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = \int \sinh(x)\operatorname{csch}(2x)\,dx$$

input
```
integrate(csch(2*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*csch(2*x), x)
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = -\arctan\left(e^{(-x)}\right)$$

input
```
integrate(csch(2*x)*sinh(x),x, algorithm="maxima")
```

output
```
-arctan(e^(-x))
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.43

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = \arctan\left(e^{x}\right)$$

input
```
integrate(csch(2*x)*sinh(x),x, algorithm="giac")
```

output
```
arctan(e^x)
```

3.216.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.43

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = \operatorname{atan}(e^x)$$

input
```
int(sinh(x)/sinh(2*x),x)
```

output
```
atan(exp(x))
```

3.217 $\int \operatorname{csch}(3x)\sinh(x)\,dx$

3.217.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \operatorname{csch}(3x)\sinh(x)\,dx=\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
1/3*arctan(1/3*tanh(x)*3^(1/2))*3^(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(3x)\sinh(x)\,dx=\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[Csch[3*x]*Sinh[x],x]
```

output
```
ArcTan[Tanh[x]/Sqrt[3]]/Sqrt[3]
```

3.217.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\text{csch}(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(ix)}{\sin(3ix)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{\tanh^2(x)+3}d\tanh(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[Csch[3*x]*Sinh[x],x]
```

output
```
ArcTan[Tanh[x]/Sqrt[3]]/Sqrt[3]
```

3.217.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.217.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.67

method	result	size
risch	$\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}$	40

input
```
int(csch(3*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*3^(1/2)*ln(exp(2*x)+1/2+1/2*I*3^(1/2))-1/6*I*3^(1/2)*ln(exp(2*x)+1/2
-1/2*I*3^(1/2))
```

3.217.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \operatorname{csch}(3x)\sinh(x)\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(-\frac{3\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3\left(\cosh(x)-\sinh(x)\right)}\right)$$

input
```
integrate(csch(3*x)*sinh(x),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*(3*sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) -
sinh(x)))
```

3.217.6 Sympy [F]

$$\int \operatorname{csch}(3x)\sinh(x)\,dx = \int \sinh(x)\operatorname{csch}(3x)\,dx$$

input
```
integrate(csch(3*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*csch(3*x), x)
```

3.217.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \operatorname{csch}(3x)\sinh(x)\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-x)}+1\right)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-x)}-1\right)\right)$$

input
```
integrate(csch(3*x)*sinh(x),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-x) + 1)) - 1/3*sqrt(3)*arctan(1/3*sq
rt(3)*(2*e^(-x) - 1))
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \operatorname{csch}(3x)\sinh(x)\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}+1\right)\right)$$

input
```
integrate(csch(3*x)*sinh(x),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) + 1))
```

3.217.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \operatorname{csch}(3x)\sinh(x)\,dx = \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,(2\,e^{2\,x}+1)}{3}\right)}{3}$$

input
```
int(sinh(x)/sinh(3*x),x)
```

output
```
(3^(1/2)*atan((3^(1/2)*(2*exp(2*x) + 1))/3))/3
```

3.218 $\int \text{csch}(4x)\sinh(x)\,dx$

3.218.1 Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \text{csch}(4x)\sinh(x)\,dx = -\frac{1}{4}\arctan(\sinh(x)) + \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}$$

output
```
-1/4*arctan(sinh(x))+1/4*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \text{csch}(4x)\sinh(x)\,dx = -\frac{1}{4}\arctan(\sinh(x)) + \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}$$

input
```
Integrate[Csch[4*x]*Sinh[x],x]
```

output
```
-1/4*ArcTan[Sinh[x]] + ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2])
```

3.218.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4878, 1406, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\text{csch}(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(ix)}{\sin(4ix)}dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{1}{8\sinh^4(x)+12\sinh^2(x)+4}d\sinh(x)$$

$$\Big\downarrow \text{1406}$$

$$2\int \frac{1}{8\sinh^2(x)+4}d\sinh(x)-2\int \frac{1}{8\sinh^2(x)+8}d\sinh(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}-\frac{1}{4}\arctan(\sinh(x))$$

input
```
Int[Csch[4*x]*Sinh[x],x]
```

output
```
-1/4*ArcTan[Sinh[x]] + ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2])
```

3.218.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.218.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

method	result	size
risch	$\frac{i\ln(\mathrm{e}^{x}-i)}{4}-\frac{i\ln(\mathrm{e}^{x}+i)}{4}+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+i\sqrt{2}\,\mathrm{e}^{x}-1\right)}{8}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-i\sqrt{2}\,\mathrm{e}^{x}-1\right)}{8}$	62

input
```
int(csch(4*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)+1/8*I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*
exp(x)-1)-1/8*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)
```

3.218.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.92

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\cosh(x) + \frac{1}{2}\sqrt{2}\sinh(x)\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 + \sqrt{2}}{2(\cosh(x) - \sinh(x))}\right) - \frac{1}{2}\arctan(\cosh(x) + \sinh(x))$$

input
```
integrate(csch(4*x)*sinh(x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*cosh(x) + 1/2*sqrt(2)*sinh(x)) - 1/4*sqrt(2
)*arctan(-1/2*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sin
h(x)^2 + sqrt(2))/(cosh(x) - sinh(x))) - 1/2*arctan(cosh(x) + sinh(x))
```

3.218.6 Sympy [F]

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = \int \sinh(x)\operatorname{csch}(4x)\,dx$$

input
```
integrate(csch(4*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*csch(4*x), x)
```

3.218.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right) + \frac{1}{2}\arctan\left(e^{(-x)}\right)$$

input
```
integrate(csch(4*x)*sinh(x),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - 1/4*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x))) + 1/2*arctan(e^(-x))
```

3.218.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = -\frac{1}{8}\pi + \frac{1}{8}\sqrt{2}\left(\pi + 2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right) - \frac{1}{4}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)$$

input
```
integrate(csch(4*x)*sinh(x),x, algorithm="giac")
```

output
```
-1/8*pi + 1/8*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) -
1/4*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

3.218.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = \frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\mathrm{e}^{x}}{2}+\frac{\sqrt{2}\,\mathrm{e}^{3\,x}}{2}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\mathrm{e}^{x}}{2}\right)\right)}{8}-\frac{\mathrm{atan}(\mathrm{e}^{x})}{2}$$

input
```
int(sinh(x)/sinh(4*x),x)
```

output
```
(2^(1/2)*(2*atan((2^(1/2)*exp(x))/2 + (2^(1/2)*exp(3*x))/2) + 2*atan((2^(1
/2)*exp(x))/2)))/8 - atan(exp(x))/2
```

3.219 $\int \operatorname{csch}(5x)\sinh(x)\,dx$

3.219.1 Optimal result

Integrand size = 7, antiderivative size = 75

$$\int \operatorname{csch}(5x)\sinh(x)\,dx=\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\tanh(x)}{\sqrt{5-2\sqrt{5}}}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\tanh(x)}{\sqrt{5+2\sqrt{5}}}\right)$$

output
```
1/10*arctan(tanh(x)/(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/10*arctan(
tanh(x)/(5+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int \operatorname{csch}(5x)\sinh(x)\,dx=\frac{\sqrt{5+\sqrt{5}}\arctan\left(\frac{\left(-3+\sqrt{5}\right)\tanh(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\arctan\left(\frac{\left(3+\sqrt{5}\right)\tanh(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

input
```
Integrate[Csch[5*x]*Sinh[x],x]
```

output
```
(Sqrt[5 + Sqrt[5]]*ArcTan[((-3 + Sqrt[5])*Tanh[x])/Sqrt[10 - 2*Sqrt[5]]] +
 Sqrt[5 - Sqrt[5]]*ArcTan[((3 + Sqrt[5])*Tanh[x])/Sqrt[2*(5 + Sqrt[5])]])/
(5*Sqrt[2])
```

3.219.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4889, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\text{csch}(5x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(ix)}{\sin(5ix)}dx$$

$$\Big\downarrow \; 4889$$

$$\int \frac{1-\tanh^2(x)}{\tanh^4(x)+10\tanh^2(x)+5}d\tanh(x)$$

$$\Big\downarrow \; 1480$$

$$-\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{\tanh^2(x)-2\sqrt{5}+5}d\tanh(x)-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\tanh^2(x)+2\sqrt{5}+5}d\tanh(x)$$

$$\Big\downarrow \; 216$$

$$-\frac{\left(5-3\sqrt{5}\right)\arctan\left(\frac{\tanh(x)}{\sqrt{5-2\sqrt{5}}}\right)}{10\sqrt{5-2\sqrt{5}}}-\frac{\left(5+3\sqrt{5}\right)\arctan\left(\frac{\tanh(x)}{\sqrt{5+2\sqrt{5}}}\right)}{10\sqrt{5+2\sqrt{5}}}$$

input
```
Int[Csch[5*x]*Sinh[x],x]
```

output
```
-1/10*((5 - 3*Sqrt[5])*ArcTan[Tanh[x]/Sqrt[5 - 2*Sqrt[5]]])/Sqrt[5 - 2*Sqr
t[5]] - ((5 + 3*Sqrt[5])*ArcTan[Tanh[x]/Sqrt[5 + 2*Sqrt[5]]])/(10*Sqrt[5 +
 2*Sqrt[5]])
```

3.219.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.219.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
risch	$2\left(\sum_{\_R=\operatorname{RootOf}\left(32000\_Z^4+400\_Z^2+1\right)}\_R\ln\left(4000\_R^3-200\_R^2+\mathrm{e}^{2x}+30\_R-1\right)\right)$	41

input
```
int(csch(5*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(4000*_R^3-200*_R^2+exp(2*x)+30*_R-1),_R=RootOf(32000*_Z^4+400*
_Z^2+1))
```

3.219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(49) = 98$.

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.24

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = \frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}-5}\log\Big(8\cosh(x)^2+16\cosh(x)\sinh(x)+8\sinh(x)^2+\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-5}-2\sqrt{5}+2\Big) - \frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}-5}\log\Big(8\cosh(x)^2+16\cosh(x)\sinh(x)+8\sinh(x)^2-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-5}-2\sqrt{5}+2\Big) - \frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}-5}\log\Big(8\cosh(x)^2+16\cosh(x)\sinh(x)+8\sinh(x)^2+\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-5}+2\sqrt{5}+2\Big) + \frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}-5}\log\Big(8\cosh(x)^2+16\cosh(x)\sinh(x)+8\sinh(x)^2-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-5}+2\sqrt{5}+2\Big)$$

input
```
integrate(csch(5*x)*sinh(x),x, algorithm="fricas")
```

output
```
1/20*sqrt(2)*sqrt(sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8*si
nh(x)^2 + (sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 5) - 2*sqrt(5) + 2) -
 1/20*sqrt(2)*sqrt(sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8*s
inh(x)^2 - (sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 5) - 2*sqrt(5) + 2)
- 1/20*sqrt(2)*sqrt(-sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8
*sinh(x)^2 + (sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(5) - 5) + 2*sqrt(5) +
2) + 1/20*sqrt(2)*sqrt(-sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x)
+ 8*sinh(x)^2 - (sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(5) - 5) + 2*sqrt(5)
 + 2)
```

3.219.6 Sympy [F]

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = \int \sinh(x)\operatorname{csch}(5x)\,dx$$

input
```
integrate(csch(5*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*csch(5*x), x)
```

3.219.7 Maxima [F]

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = \int \operatorname{csch}(5\,x)\sinh(x)\,dx$$

input
```
integrate(csch(5*x)*sinh(x),x, algorithm="maxima")
```

output
```
1/10*(-1)^(3/5)*log((-1)^(1/5) + e^(-2*x)) + 1/10*sqrt(5)*(-1)^(3/5)*log((
sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*e^(-
2*x))/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) -
 4*e^(-2*x)))/sqrt(2*sqrt(5) - 10) - 1/10*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*
(-1)^(1/5) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*e^(-2*x))/(
sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*e^(
-2*x)))/sqrt(-2*sqrt(5) - 10) - 1/10*log(-(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)
)*e^(-2*x) + 2*(-1)^(2/5) + 2*e^(-4*x))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5))
+ 1/10*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5))*e^(-2*x) + 2*(-1)^(2/5) + 2*e
^(-4*x))/(sqrt(5)*(-1)^(2/5) - (-1)^(2/5)) - 1/10*integrate((e^(3*x) + 2*e
^(2*x) + 3*e^x + 4)*e^x/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/10
*integrate((e^(3*x) - 2*e^(2*x) + 3*e^x - 4)*e^x/(e^(4*x) - e^(3*x) + e^(2
*x) - e^x + 1), x) + 1/10*log(e^x + 1) + 1/10*log(e^x - 1)
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = \frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4\,e^{(2\,x)}-1}{\sqrt{2\sqrt{5}+10}}\right) - \frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4\,e^{(2\,x)}+1}{\sqrt{-2\sqrt{5}+10}}\right)$$

input
```
integrate(csch(5*x)*sinh(x),x, algorithm="giac")
```

output
```
1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*e^(2*x) - 1)/sqrt(2*sqrt(5
) + 10)) - 1/10*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*e^(2*x) + 1)/sqrt
(-2*sqrt(5) + 10))
```

3.219.9 Mupad [B] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.76

$$\int \operatorname{csch}(5x)\sinh(x)\,dx$$

$$= 2\,\mathrm{atan}\left(\frac{\frac{e^{2x}}{5}+\frac{9\sqrt{5}}{25}+\frac{6\sqrt{5}\,e^{2x}}{25}+\frac{4}{5}}{5\,e^{2x}\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}+\frac{9\sqrt{5}\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}}{5}+\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}+\frac{9\sqrt{5}\,e^{2x}\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}}{5}}\right)\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}} + \sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\left(\ln\left(\frac{9\sqrt{5}}{25}-\frac{e^{2x}}{5}+\frac{6\sqrt{5}\,e^{2x}}{25}-\frac{4}{5}-e^{2x}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,5\mathrm{i}+\frac{\sqrt{5}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,9\mathrm{i}}{5} - \sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,1\mathrm{i}+\frac{\sqrt{5}\,e^{2x}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,9\mathrm{i}}{5}\right)1\mathrm{i}-\ln\left(\frac{9\sqrt{5}}{25}-\frac{e^{2x}}{5}+\frac{6\sqrt{5}\,e^{2x}}{25}-\frac{4}{5} + e^{2x}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,5\mathrm{i}-\frac{\sqrt{5}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,9\mathrm{i}}{5}+\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,1\mathrm{i}-\frac{\sqrt{5}\,e^{2x}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,9\mathrm{i}}{5}\right)1\mathrm{i}\right)$$

input
```
int(sinh(x)/sinh(5*x),x)
```

output
```
2*atan((exp(2*x)/5 + (9*5^(1/2))/25 + (6*5^(1/2)*exp(2*x))/25 + 4/5)/(5*ex
p(2*x)*(5^(1/2)/200 + 1/40)^(1/2) + (9*5^(1/2)*(5^(1/2)/200 + 1/40)^(1/2))
/5 + (5^(1/2)/200 + 1/40)^(1/2) + (9*5^(1/2)*exp(2*x)*(5^(1/2)/200 + 1/40)
^(1/2))/5))*(5^(1/2)/200 + 1/40)^(1/2) + (1/40 - 5^(1/2)/200)^(1/2)*(log((
5^(1/2)*(1/40 - 5^(1/2)/200)^(1/2)*9i)/5 - exp(2*x)*(1/40 - 5^(1/2)/200)^(
1/2)*5i - exp(2*x)/5 + (9*5^(1/2))/25 - (1/40 - 5^(1/2)/200)^(1/2)*1i + (6
*5^(1/2)*exp(2*x))/25 + (5^(1/2)*exp(2*x)*(1/40 - 5^(1/2)/200)^(1/2)*9i)/5
 - 4/5)*1i - log(exp(2*x)*(1/40 - 5^(1/2)/200)^(1/2)*5i - exp(2*x)/5 - (5^
(1/2)*(1/40 - 5^(1/2)/200)^(1/2)*9i)/5 + (9*5^(1/2))/25 + (1/40 - 5^(1/2)/
200)^(1/2)*1i + (6*5^(1/2)*exp(2*x))/25 - (5^(1/2)*exp(2*x)*(1/40 - 5^(1/2
)/200)^(1/2)*9i)/5 - 4/5)*1i)
```

3.219. $\int \operatorname{csch}(5x)\sinh(x)\,dx$

3.220 $\int \operatorname{csch}(6x)\sinh(x)\,dx$

3.220.1 Optimal result

Integrand size = 7, antiderivative size = 36

$$\int \operatorname{csch}(6x)\sinh(x)\,dx = \frac{1}{6}\arctan(\sinh(x)) + \frac{1}{6}\arctan(2\sinh(x)) - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
1/6*arctan(sinh(x))+1/6*arctan(2*sinh(x))-1/6*arctan(2/3*sinh(x)*3^(1/2))*
3^(1/2)
```

3.220.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \operatorname{csch}(6x)\sinh(x)\,dx = \frac{1}{6}\left(\arctan(\sinh(x)) + \arctan(2\sinh(x)) - \sqrt{3}\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)\right)$$

input
```
Integrate[Csch[6*x]*Sinh[x],x]
```

output
```
(ArcTan[Sinh[x]] + ArcTan[2*Sinh[x]] - Sqrt[3]*ArcTan[(2*Sinh[x])/Sqrt[3]]
)/6
```

3.220.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4878, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\text{csch}(6x)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(ix)}{\sin(6ix)}dx$$
$$\Big\downarrow \; 4878$$
$$\int \frac{1}{2\left(16\sinh^6(x)+32\sinh^4(x)+19\sinh^2(x)+3\right)}d\sinh(x)$$
$$\Big\downarrow \; 27$$
$$\frac{1}{2}\int \frac{1}{16\sinh^6(x)+32\sinh^4(x)+19\sinh^2(x)+3}d\sinh(x)$$
$$\Big\downarrow \; 2460$$
$$\frac{1}{2}\int \left(\frac{2}{3\left(4\sinh^2(x)+1\right)}-\frac{2}{4\sinh^2(x)+3}+\frac{1}{3\left(\sinh^2(x)+1\right)}\right)d\sinh(x)$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}\left(\frac{1}{3}\arctan(\sinh(x))+\frac{1}{3}\arctan(2\sinh(x))-\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

input
```
Int[Csch[6*x]*Sinh[x],x]
```

output
```
(ArcTan[Sinh[x]]/3 + ArcTan[2*Sinh[x]]/3 - ArcTan[(2*Sinh[x])/Sqrt[3]]/Sqr
t[3])/2
```

3.220.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.220.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.56

method	result	si
risch	$\frac{i\ln(\mathrm{e}^{x}+i)}{6}-\frac{i\ln(\mathrm{e}^{x}-i)}{6}+\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}-i\sqrt{3}\,\mathrm{e}^{x}-1\right)}{12}-\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}+i\sqrt{3}\,\mathrm{e}^{x}-1\right)}{12}+\frac{i\ln(\mathrm{e}^{2x}+i\mathrm{e}^{x}-1)}{12}-\frac{i\ln(\mathrm{e}^{2x}-i\mathrm{e}^{x}-1)}{12}$	92

input
```
int(csch(6*x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*ln(exp(x)+I)-1/6*I*ln(exp(x)-I)+1/12*I*3^(1/2)*ln(exp(2*x)-I*3^(1/2)
*exp(x)-1)-1/12*I*3^(1/2)*ln(exp(2*x)+I*3^(1/2)*exp(x)-1)+1/12*I*ln(exp(2*
x)+I*exp(x)-1)-1/12*I*ln(exp(2*x)-I*exp(x)-1)
```

3.220.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.97

$$\begin{aligned}
&\int \operatorname{csch}(6x)\sinh(x)\,dx \\
&= -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\cosh(x)+\frac{1}{3}\sqrt{3}\sinh(x)\right) \\
&\quad +\frac{1}{6}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh(x)^2+2\sqrt{3}\cosh(x)\sinh(x)+\sqrt{3}\sinh(x)^2+2\sqrt{3}}{3(\cosh(x)-\sinh(x))}\right) \\
&\quad -\frac{1}{6}\arctan\left(-\frac{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2}{\cosh(x)-\sinh(x)}\right) \\
&\quad +\frac{1}{2}\arctan(\cosh(x)+\sinh(x))
\end{aligned}$$

input
```
integrate(csch(6*x)*sinh(x),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*cosh(x) + 1/3*sqrt(3)*sinh(x)) + 1/6*sqrt(
3)*arctan(-1/3*(sqrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*si
nh(x)^2 + 2*sqrt(3))/(cosh(x) - sinh(x))) - 1/6*arctan(-(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))) + 1/2*arctan(cosh(x) + sinh
(x))
```

3.220.6 Sympy [F]

$$\int \operatorname{csch}(6x)\sinh(x)\,dx = \int \sinh(x)\operatorname{csch}(6x)\,dx$$

input
```
integrate(csch(6*x)*sinh(x),x)
```

output
```
Integral(sinh(x)*csch(6*x), x)
```

3.220.7 Maxima [F]

$$\int \operatorname{csch}(6x)\sinh(x)\,dx=\int \operatorname{csch}(6\,x)\sinh(x)\,dx$$

input
```
integrate(csch(6*x)*sinh(x),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt
(3)*(2*e^x - 1)) + 1/3*arctan(e^x) + integrate(1/6*(e^(3*x) + e^x)/(e^(4*x
) - e^(2*x) + 1), x)
```

3.220.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \operatorname{csch}(6x)\sinh(x)\,dx=\frac{1}{6}\pi-\frac{1}{12}\sqrt{3}\left(\pi+2\arctan\left(\frac{1}{3}\sqrt{3}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)+\frac{1}{6}\arctan\left(\left(e^{(2x)}-1\right)e^{(-x)}\right)+\frac{1}{6}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)$$

input
```
integrate(csch(6*x)*sinh(x),x, algorithm="giac")
```

output
```
1/6*pi - 1/12*sqrt(3)*(pi + 2*arctan(1/3*sqrt(3)*(e^(2*x) - 1)*e^(-x))) +
1/6*arctan((e^(2*x) - 1)*e^(-x)) + 1/6*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

3.220.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \operatorname{csch}(6x)\sinh(x)\,dx=\frac{\operatorname{atan}(e^x)}{3}-\frac{\operatorname{atan}(e^{-x}-e^x)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\,e^x}{3}-\frac{\sqrt{3}\,e^{-x}}{3}\right)}{6}$$

input
```
int(sinh(x)/sinh(6*x),x)
```

output
```
atan(exp(x))/3 - atan(exp(-x) - exp(x))/6 - (3^(1/2)*atan((3^(1/2)*exp(x))
/3 - (3^(1/2)*exp(-x))/3))/6
```

3.221 $\int \cosh(x)\sinh(2x)\,dx$

3.221.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \cosh(x)\sinh(2x)\,dx = \frac{2\cosh^3(x)}{3}$$

output
```
2/3*cosh(x)^3
```

3.221.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \cosh(x)\sinh(2x)\,dx = \frac{\cosh(x)}{2} + \frac{1}{6}\cosh(3x)$$

input
```
Integrate[Cosh[x]*Sinh[2*x],x]
```

output
```
Cosh[x]/2 + Cosh[3*x]/6
```

3.221.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 26, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(2x)\cosh(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -i\sin(2ix)\cos(ix)dx$$

$$\Big\downarrow \; 26$$

$$-i\int \cos(ix)\sin(2ix)dx$$

$$\Big\downarrow \; 4772$$

$$-i\left(\frac{1}{2}i\cosh(x)+\frac{1}{6}i\cosh(3x)\right)$$

input
```
Int[Cosh[x]*Sinh[2*x],x]
```

output
```
(-I)*((I/2)*Cosh[x] + (I/6)*Cosh[3*x])
```

3.221.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.221.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$	12
parallelrisch	$-\frac{2}{3}+\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$	13
risch	$\frac{{\rm e}^{3x}}{12}+\frac{{\rm e}^{x}}{4}+\frac{{\rm e}^{-x}}{4}+\frac{{\rm e}^{-3x}}{12}$	24

input
```
int(cosh(x)*sinh(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*cosh(x)+1/6*cosh(3*x)
```

3.221.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \cosh(x)\sinh(2x)\,dx=\frac{1}{6}\cosh{(x)}^3+\frac{1}{2}\cosh{(x)}\sinh{(x)}^2+\frac{1}{2}\cosh{(x)}$$

input
```
integrate(cosh(x)*sinh(2*x),x, algorithm="fricas")
```

output
```
1/6*cosh(x)^3 + 1/2*cosh(x)*sinh(x)^2 + 1/2*cosh(x)
```

3.221.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(7) = 14$.

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \cosh(x)\sinh(2x)\,dx = -\frac{\sinh(x)\sinh(2x)}{3} + \frac{2\cosh(x)\cosh(2x)}{3}$$

input
```
integrate(cosh(x)*sinh(2*x),x)
```

output
```
-sinh(x)*sinh(2*x)/3 + 2*cosh(x)*cosh(2*x)/3
```

3.221.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \cosh(x)\sinh(2x)\,dx = \frac{1}{12}\left(3\,e^{(-2\,x)} + 1\right)e^{(3\,x)} + \frac{1}{4}\,e^{(-x)} + \frac{1}{12}\,e^{(-3\,x)}$$

input
```
integrate(cosh(x)*sinh(2*x),x, algorithm="maxima")
```

output
```
1/12*(3*e^(-2*x) + 1)*e^(3*x) + 1/4*e^(-x) + 1/12*e^(-3*x)
```

3.221.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(6) = 12$.

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 3.12

$$\int \cosh(x)\sinh(2x)\,dx = \frac{1}{12}\left(3\,e^{(2\,x)} + 1\right)e^{(-3\,x)} + \frac{1}{12}\,e^{(3\,x)} + \frac{1}{4}\,e^{x}$$

input
```
integrate(cosh(x)*sinh(2*x),x, algorithm="giac")
```

output
```
1/12*(3*e^(2*x) + 1)*e^(-3*x) + 1/12*e^(3*x) + 1/4*e^x
```

3.221.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cosh(x)\sinh(2x)\,dx = \frac{2\cosh(x)^3}{3}$$

input
```
int(sinh(2*x)*cosh(x),x)
```

output
```
(2*cosh(x)^3)/3
```

3.222 $\int \cosh(x)\sinh(3x)\,dx$

3.222.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cosh(x)\sinh(3x)\,dx = \frac{1}{4}\cosh(2x) + \frac{1}{8}\cosh(4x)$$

output
```
1/4*cosh(2*x)+1/8*cosh(4*x)
```

3.222.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cosh(x)\sinh(3x)\,dx = \frac{\cosh^2(x)}{2} + \frac{1}{8}\cosh(4x)$$

input
```
Integrate[Cosh[x]*Sinh[3*x],x]
```

output
```
Cosh[x]^2/2 + Cosh[4*x]/8
```

3.222.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 26, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(3x)\cosh(x)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -i\sin(3ix)\cos(ix)dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \cos(ix)\sin(3ix)dx$$
$$\Big\downarrow \text{ 4772}$$
$$-i\left(\frac{1}{4}i\cosh(2x)+\frac{1}{8}i\cosh(4x)\right)$$

input
```
Int[Cosh[x]*Sinh[3*x],x]
```

output
```
(-I)*((I/4)*Cosh[2*x] + (I/8)*Cosh[4*x])
```

3.222.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.222.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\cosh(2x)}{4}+\frac{\cosh(4x)}{8}$	14
risch	$\frac{e^{4x}}{16}+\frac{e^{2x}}{8}+\frac{e^{-2x}}{8}+\frac{e^{-4x}}{16}$	26
parallelrisch	$\frac{\cosh(5x)-4\cosh(x)+3\cosh(3x)-4\cosh(2x)+6-2\cosh(4x)}{16\cosh(x)-16}$	38

input
```
int(cosh(x)*sinh(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*cosh(2*x)+1/8*cosh(4*x)
```

3.222.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \cosh(x)\sinh(3x)\,dx$$
$$=\frac{1}{8}\cosh(x)^4+\frac{1}{8}\sinh(x)^4+\frac{1}{4}\left(3\cosh(x)^2+1\right)\sinh(x)^2+\frac{1}{4}\cosh(x)^2$$

input
```
integrate(cosh(x)*sinh(3*x),x, algorithm="fricas")
```

output
```
1/8*cosh(x)^4 + 1/8*sinh(x)^4 + 1/4*(3*cosh(x)^2 + 1)*sinh(x)^2 + 1/4*cosh
(x)^2
```

3.222.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(x)\sinh(3x)\,dx = -\frac{\sinh{(x)}\sinh{(3x)}}{8} + \frac{3\cosh{(x)}\cosh{(3x)}}{8}$$

input
```
integrate(cosh(x)*sinh(3*x),x)
```

output
```
-sinh(x)*sinh(3*x)/8 + 3*cosh(x)*cosh(3*x)/8
```

3.222.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\sinh(3x)\,dx = \frac{1}{16}\left(2\,e^{(-2\,x)}+1\right)e^{(4\,x)} + \frac{1}{8}\,e^{(-2\,x)} + \frac{1}{16}\,e^{(-4\,x)}$$

input
```
integrate(cosh(x)*sinh(3*x),x, algorithm="maxima")
```

output
```
1/16*(2*e^(-2*x) + 1)*e^(4*x) + 1/8*e^(-2*x) + 1/16*e^(-4*x)
```

3.222.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \cosh(x)\sinh(3x)\,dx = \frac{1}{16}\left(e^{(2\,x)}+e^{(-2\,x)}\right)^2 + \frac{1}{8}\,e^{(2\,x)} + \frac{1}{8}\,e^{(-2\,x)}$$

input
```
integrate(cosh(x)*sinh(3*x),x, algorithm="giac")
```

output
```
1/16*(e^(2*x) + e^(-2*x))^2 + 1/8*e^(2*x) + 1/8*e^(-2*x)
```

3.222.9 Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \cosh(x)\sinh(3x)\,dx = \cosh(x)^4 - \frac{\cosh(x)^2}{2}$$

input
```
int(sinh(3*x)*cosh(x),x)
```

output
```
cosh(x)^4 - cosh(x)^2/2
```

3.223 $\int \cosh(x)\sinh(4x)\,dx$

3.223.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cosh(x)\sinh(4x)\,dx = \frac{1}{6}\cosh(3x) + \frac{1}{10}\cosh(5x)$$

output
```
1/6*cosh(3*x)+1/10*cosh(5*x)
```

3.223.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cosh(x)\sinh(4x)\,dx = \frac{1}{6}\cosh(3x) + \frac{1}{10}\cosh(5x)$$

input
```
Integrate[Cosh[x]*Sinh[4*x],x]
```

output
```
Cosh[3*x]/6 + Cosh[5*x]/10
```

3.223.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 26, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(4x)\cosh(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sin(4ix)\cos(ix)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \cos(ix)\sin(4ix)dx$$

$$\Big\downarrow \text{ 4772}$$

$$-i\left(\frac{1}{6}i\cosh(3x)+\frac{1}{10}i\cosh(5x)\right)$$

input
```
Int[Cosh[x]*Sinh[4*x],x]
```

output
```
(-I)*((I/6)*Cosh[3*x] + (I/10)*Cosh[5*x])
```

3.223.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.223.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$	14
parallelrisch	$-\frac{4}{15}+\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$	15
risch	$\frac{e^{5x}}{20}+\frac{e^{3x}}{12}+\frac{e^{-3x}}{12}+\frac{e^{-5x}}{20}$	26

input
```
int(cosh(x)*sinh(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*cosh(3*x)+1/10*cosh(5*x)
```

3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(13) = 26$.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \cosh(x)\sinh(4x)\,dx = \frac{1}{10}\cosh(x)^5 + \frac{1}{2}\cosh(x)\sinh(x)^4 + \frac{1}{6}\cosh(x)^3 + \frac{1}{2}\left(2\cosh(x)^3 + \cosh(x)\right)\sinh(x)^2$$

input
```
integrate(cosh(x)*sinh(4*x),x, algorithm="fricas")
```

output
```
1/10*cosh(x)^5 + 1/2*cosh(x)*sinh(x)^4 + 1/6*cosh(x)^3 + 1/2*(2*cosh(x)^3
+ cosh(x))*sinh(x)^2
```

3.223.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(x)\sinh(4x)\,dx = -\frac{\sinh(x)\sinh(4x)}{15} + \frac{4\cosh(x)\cosh(4x)}{15}$$

input
```
integrate(cosh(x)*sinh(4*x),x)
```

output
```
-sinh(x)*sinh(4*x)/15 + 4*cosh(x)*cosh(4*x)/15
```

3.223.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\sinh(4x)\,dx = \frac{1}{60}\left(5e^{(-2x)} + 3\right)e^{(5x)} + \frac{1}{12}e^{(-3x)} + \frac{1}{20}e^{(-5x)}$$

input
```
integrate(cosh(x)*sinh(4*x),x, algorithm="maxima")
```

output
```
1/60*(5*e^(-2*x) + 3)*e^(5*x) + 1/12*e^(-3*x) + 1/20*e^(-5*x)
```

3.223.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\sinh(4x)\,dx = \frac{1}{60}\left(5e^{(2x)} + 3\right)e^{(-5x)} + \frac{1}{20}e^{(5x)} + \frac{1}{12}e^{(3x)}$$

input
```
integrate(cosh(x)*sinh(4*x),x, algorithm="giac")
```

output
```
1/60*(5*e^(2*x) + 3)*e^(-5*x) + 1/20*e^(5*x) + 1/12*e^(3*x)
```

3.223.9 Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cosh(x)\sinh(4x)\,dx = \frac{4\cosh(x)^3\,\left(6\cosh(x)^2-5\right)}{15}$$

input
```
int(sinh(4*x)*cosh(x),x)
```

output
```
(4*cosh(x)^3*(6*cosh(x)^2 - 5))/15
```

3.224 $\int \cosh(x)\sinh(mx)\,dx$

3.224.1 Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cosh(x)\sinh(mx)\,dx = -\frac{\cosh((1-m)x)}{2(1-m)} + \frac{\cosh((1+m)x)}{2(1+m)}$$

output
```
-1/2*cosh((1-m)*x)/(1-m)+1/2*cosh((1+m)*x)/(1+m)
```

3.224.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cosh(x)\sinh(mx)\,dx = \frac{m\cosh(x)\cosh(mx) - \sinh(x)\sinh(mx)}{-1+m^2}$$

input
```
Integrate[Cosh[x]*Sinh[m*x],x]
```

output
```
(m*Cosh[x]*Cosh[m*x] - Sinh[x]*Sinh[m*x])/(-1 + m^2)
```

3.224.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6152, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\sinh(mx)\,dx$$

$$\Big\downarrow \text{6152}$$

$$\int \left(\frac{1}{2}\sinh((m+1)x)-\frac{1}{2}\sinh((1-m)x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh((m+1)x)}{2(m+1)}-\frac{\cosh((1-m)x)}{2(1-m)}$$

input
```
Int[Cosh[x]*Sinh[m*x],x]
```

output
```
-1/2*Cosh[(1 - m)*x]/(1 - m) + Cosh[(1 + m)*x]/(2*(1 + m))
```

3.224.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6152
```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.224.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\cosh(x(-1+m))}{-2+2m}+\frac{\cosh((1+m)x)}{2+2m}$	28
parallelrisch	$\frac{(1+m)\cosh(x(-1+m))+\cosh((1+m)x)(-1+m)-2m}{2m^2-2}$	35
risch	$\frac{(m\,\mathrm{e}^{2x}-\mathrm{e}^{2x}+m+1)\mathrm{e}^{x(-1+m)}}{4(1+m)(-1+m)}+\frac{(m\,\mathrm{e}^{2x}+\mathrm{e}^{2x}+m-1)\mathrm{e}^{-(1+m)x}}{4(1+m)(-1+m)}$	67

input
```
int(cosh(x)*sinh(m*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-1+m)*cosh(x*(-1+m))+1/2*cosh((1+m)*x)/(1+m)
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \cosh(x)\sinh(mx)\,dx = \frac{m\cosh(mx)\cosh(x)-\sinh(mx)\sinh(x)}{(m^2-1)\cosh(x)^2-(m^2-1)\sinh(x)^2}$$

input
```
integrate(cosh(x)*sinh(m*x),x, algorithm="fricas")
```

output
```
(m*cosh(m*x)*cosh(x) - sinh(m*x)*sinh(x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)
*sinh(x)^2)
```

3.224.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \cosh(x)\sinh(mx)\,dx = \begin{cases} -\frac{\cosh^2(x)}{2} & \text{for } m=-1 \\ \frac{\cosh^2(x)}{2} & \text{for } m=1 \\ \frac{m\cosh(x)\cosh(mx)}{m^2-1}-\frac{\sinh(x)\sinh(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(x)*sinh(m*x),x)
```

output
```
Piecewise((-cosh(x)**2/2, Eq(m, -1)), (cosh(x)**2/2, Eq(m, 1)), (m*cosh(x)
*cosh(m*x)/(m**2 - 1) - sinh(x)*sinh(m*x)/(m**2 - 1), True))
```

3.224.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh(x)\sinh(mx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)*sinh(m*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-2>0)', see `assume?` for more
details)Is
```

3.224.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \cosh(x)\sinh(mx)\,dx = \frac{e^{(mx+x)}}{4(m+1)} + \frac{e^{(mx-x)}}{4(m-1)} + \frac{e^{(-mx+x)}}{4(m-1)} + \frac{e^{(-mx-x)}}{4(m+1)}$$

input
```
integrate(cosh(x)*sinh(m*x),x, algorithm="giac")
```

output
```
1/4*e^(m*x + x)/(m + 1) + 1/4*e^(m*x - x)/(m - 1) + 1/4*e^(-m*x + x)/(m -
1) + 1/4*e^(-m*x - x)/(m + 1)
```

3.224.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \cosh(x)\sinh(mx)\,dx = -\frac{\sinh(m\,x)\,\sinh(x) - m\cosh(m\,x)\,\cosh(x)}{m^2-1}$$

input
```
int(sinh(m*x)*cosh(x),x)
```

output
```
-(sinh(m*x)*sinh(x) - m*cosh(m*x)*cosh(x))/(m^2 - 1)
```

3.225 $\int \cosh(x)\cosh(2x)\,dx$

3.225.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cosh(x)\cosh(2x)\,dx = \frac{\sinh(x)}{2} + \frac{1}{6}\sinh(3x)$$

output
```
1/2*sinh(x)+1/6*sinh(3*x)
```

3.225.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh(x)\cosh(2x)\,dx = \frac{\sinh(x)}{2} + \frac{1}{6}\sinh(3x)$$

input
```
Integrate[Cosh[x]*Cosh[2*x],x]
```

output
```
Sinh[x]/2 + Sinh[3*x]/6
```

3.225.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\cosh(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ix)\cos(2ix)dx$$

$$\Big\downarrow \text{4771}$$

$$\frac{\sinh(x)}{2}+\frac{1}{6}\sinh(3x)$$

input
```
Int[Cosh[x]*Cosh[2*x],x]
```

output
```
Sinh[x]/2 + Sinh[3*x]/6
```

3.225.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.225.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}$	12
parallelrisch	$\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}$	12
risch	$\frac{e^{3x}}{12}+\frac{e^{x}}{4}-\frac{e^{-x}}{4}-\frac{e^{-3x}}{12}$	24

input
```
int(cosh(x)*cosh(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sinh(x)+1/6*sinh(3*x)
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cosh(x)\cosh(2x)\,dx = \frac{1}{6}\sinh{(x)}^3 + \frac{1}{2}\left(\cosh{(x)}^2+1\right)\sinh{(x)}$$

input
```
integrate(cosh(x)*cosh(2*x),x, algorithm="fricas")
```

output
```
1/6*sinh(x)^3 + 1/2*(cosh(x)^2 + 1)*sinh(x)
```

3.225.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cosh(x)\cosh(2x)\,dx = -\frac{\sinh{(x)}\cosh{(2x)}}{3} + \frac{2\sinh{(2x)}\cosh{(x)}}{3}$$

input
```
integrate(cosh(x)*cosh(2*x),x)
```

output
```
-sinh(x)*cosh(2*x)/3 + 2*sinh(2*x)*cosh(x)/3
```

3.225.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \cosh(x)\cosh(2x)\,dx = \frac{1}{12}\left(3e^{(-2x)}+1\right)e^{(3x)} - \frac{1}{4}e^{(-x)} - \frac{1}{12}e^{(-3x)}$$

input
```
integrate(cosh(x)*cosh(2*x),x, algorithm="maxima")
```

output
```
1/12*(3*e^(-2*x) + 1)*e^(3*x) - 1/4*e^(-x) - 1/12*e^(-3*x)
```

3.225.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \cosh(x)\cosh(2x)\,dx = -\frac{1}{12}\left(3e^{(2x)}+1\right)e^{(-3x)} + \frac{1}{12}e^{(3x)} + \frac{1}{4}e^{x}$$

input
```
integrate(cosh(x)*cosh(2*x),x, algorithm="giac")
```

output
```
-1/12*(3*e^(2*x) + 1)*e^(-3*x) + 1/12*e^(3*x) + 1/4*e^x
```

3.225.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \cosh(x)\cosh(2x)\,dx = \frac{2\sinh(x)^3}{3} + \sinh(x)$$

input
```
int(cosh(2*x)*cosh(x),x)
```

output
```
sinh(x) + (2*sinh(x)^3)/3
```

3.226 $\int \cosh(x)\cosh(3x)\,dx$

3.226.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cosh(x)\cosh(3x)\,dx = \frac{1}{4}\sinh(2x) + \frac{1}{8}\sinh(4x)$$

output
```
1/4*sinh(2*x)+1/8*sinh(4*x)
```

3.226.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cosh(x)\cosh(3x)\,dx = \frac{1}{4}\sinh(2x) + \frac{1}{8}\sinh(4x)$$

input
```
Integrate[Cosh[x]*Cosh[3*x],x]
```

output
```
Sinh[2*x]/4 + Sinh[4*x]/8
```

3.226.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\cosh(3x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(ix)\cos(3ix)dx$$

$$\Big\downarrow \; 4771$$

$$\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

input
```
Int[Cosh[x]*Cosh[3*x],x]
```

output
```
Sinh[2*x]/4 + Sinh[4*x]/8
```

3.226.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.226.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}$	14
parallelrisch	$\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}$	14
risch	$\frac{e^{4x}}{16}+\frac{e^{2x}}{8}-\frac{e^{-2x}}{8}-\frac{e^{-4x}}{16}$	26

input
```
int(cosh(x)*cosh(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*sinh(2*x)+1/8*sinh(4*x)
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(x)\cosh(3x)\,dx = \frac{1}{2}\cosh(x)\sinh(x)^3 + \frac{1}{2}\left(\cosh(x)^3 + \cosh(x)\right)\sinh(x)$$

input
```
integrate(cosh(x)*cosh(3*x),x, algorithm="fricas")
```

output
```
1/2*cosh(x)*sinh(x)^3 + 1/2*(cosh(x)^3 + cosh(x))*sinh(x)
```

3.226.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(x)\cosh(3x)\,dx = -\frac{\sinh(x)\cosh(3x)}{8} + \frac{3\sinh(3x)\cosh(x)}{8}$$

input
```
integrate(cosh(x)*cosh(3*x),x)
```

output
```
-sinh(x)*cosh(3*x)/8 + 3*sinh(3*x)*cosh(x)/8
```

3.226.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\cosh(3x)\,dx = \frac{1}{16}\left(2\,e^{(-2\,x)} + 1\right)e^{(4\,x)} - \frac{1}{8}\,e^{(-2\,x)} - \frac{1}{16}\,e^{(-4\,x)}$$

input
```
integrate(cosh(x)*cosh(3*x),x, algorithm="maxima")
```

output
```
1/16*(2*e^(-2*x) + 1)*e^(4*x) - 1/8*e^(-2*x) - 1/16*e^(-4*x)
```

3.226.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\cosh(3x)\,dx = -\frac{1}{16}\left(2\,e^{(2\,x)} + 1\right)e^{(-4\,x)} + \frac{1}{16}\,e^{(4\,x)} + \frac{1}{8}\,e^{(2\,x)}$$

input
```
integrate(cosh(x)*cosh(3*x),x, algorithm="giac")
```

output
```
-1/16*(2*e^(2*x) + 1)*e^(-4*x) + 1/16*e^(4*x) + 1/8*e^(2*x)
```

3.226.9 Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(x)\cosh(3x)\,dx = \frac{\mathrm{e}^{-4\,x}\,\left(\mathrm{e}^{2\,x} - 1\right)\,\left(\mathrm{e}^{2\,x} + 1\right)^3}{16}$$

input
```
int(cosh(3*x)*cosh(x),x)
```

output
```
(exp(-4*x)*(exp(2*x) - 1)*(exp(2*x) + 1)^3)/16
```

3.227 $\int \cosh(x)\cosh(4x)\,dx$

3.227.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cosh(x)\cosh(4x)\,dx = \frac{1}{6}\sinh(3x) + \frac{1}{10}\sinh(5x)$$

output
```
1/6*sinh(3*x)+1/10*sinh(5*x)
```

3.227.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cosh(x)\cosh(4x)\,dx = \frac{1}{6}\sinh(3x) + \frac{1}{10}\sinh(5x)$$

input
```
Integrate[Cosh[x]*Cosh[4*x],x]
```

output
```
Sinh[3*x]/6 + Sinh[5*x]/10
```

3.227.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\cosh(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ix)\cos(4ix)dx$$

$$\Big\downarrow \text{4771}$$

$$\frac{1}{6}\sinh(3x)+\frac{1}{10}\sinh(5x)$$

input
```
Int[Cosh[x]*Cosh[4*x],x]
```

output
```
Sinh[3*x]/6 + Sinh[5*x]/10
```

3.227.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.227.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}$	14
parallelrisch	$\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}$	14
risch	$\frac{e^{5x}}{20}+\frac{e^{3x}}{12}-\frac{e^{-3x}}{12}-\frac{e^{-5x}}{20}$	26

input
```
int(cosh(x)*cosh(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*sinh(3*x)+1/10*sinh(5*x)
```

3.227.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int \cosh(x)\cosh(4x)\,dx = \frac{1}{10}\sinh(x)^5 + \frac{1}{6}\left(6\cosh(x)^2+1\right)\sinh(x)^3 + \frac{1}{2}\left(\cosh(x)^4+\cosh(x)^2\right)\sinh(x)$$

input
```
integrate(cosh(x)*cosh(4*x),x, algorithm="fricas")
```

output
```
1/10*sinh(x)^5 + 1/6*(6*cosh(x)^2 + 1)*sinh(x)^3 + 1/2*(cosh(x)^4 + cosh(x
)^2)*sinh(x)
```

3.227.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(x)\cosh(4x)\,dx = -\frac{\sinh(x)\cosh(4x)}{15} + \frac{4\sinh(4x)\cosh(x)}{15}$$

input
```
integrate(cosh(x)*cosh(4*x),x)
```

output
```
-sinh(x)*cosh(4*x)/15 + 4*sinh(4*x)*cosh(x)/15
```

3.227.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\cosh(4x)\,dx = \frac{1}{60}\left(5\,e^{(-2\,x)} + 3\right)e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} - \frac{1}{20}\,e^{(-5\,x)}$$

input
```
integrate(cosh(x)*cosh(4*x),x, algorithm="maxima")
```

output
```
1/60*(5*e^(-2*x) + 3)*e^(5*x) - 1/12*e^(-3*x) - 1/20*e^(-5*x)
```

3.227.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\cosh(4x)\,dx = -\frac{1}{60}\left(5\,e^{(2\,x)} + 3\right)e^{(-5\,x)} + \frac{1}{20}\,e^{(5\,x)} + \frac{1}{12}\,e^{(3\,x)}$$

input
```
integrate(cosh(x)*cosh(4*x),x, algorithm="giac")
```

output
```
-1/60*(5*e^(2*x) + 3)*e^(-5*x) + 1/20*e^(5*x) + 1/12*e^(3*x)
```

3.227.9 Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cosh(x)\cosh(4x)\,dx = \frac{8\sinh(x)^5}{5} + \frac{8\sinh(x)^3}{3} + \sinh(x)$$

input
```
int(cosh(4*x)*cosh(x),x)
```

output
```
sinh(x) + (8*sinh(x)^3)/3 + (8*sinh(x)^5)/5
```

3.228 $\int \cosh(x)\cosh(mx)\,dx$

3.228.1 Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cosh(x)\cosh(mx)\,dx = \frac{\sinh((1-m)x)}{2(1-m)} + \frac{\sinh((1+m)x)}{2(1+m)}$$

output
```
1/2*sinh((1-m)*x)/(1-m)+1/2*sinh((1+m)*x)/(1+m)
```

3.228.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cosh(x)\cosh(mx)\,dx = \frac{-\cosh(mx)\sinh(x) + m\cosh(x)\sinh(mx)}{-1+m^2}$$

input
```
Integrate[Cosh[x]*Cosh[m*x],x]
```

output
```
(-(Cosh[m*x]*Sinh[x]) + m*Cosh[x]*Sinh[m*x])/(-1 + m^2)
```

3.228.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6148, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\cosh(mx)\,dx$$

$$\Big\downarrow \; 6148$$

$$\int \left(\frac{1}{2}\cosh((1-m)x)+\frac{1}{2}\cosh((m+1)x)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sinh((1-m)x)}{2(1-m)}+\frac{\sinh((m+1)x)}{2(m+1)}$$

input
```
Int[Cosh[x]*Cosh[m*x],x]
```

output
```
Sinh[(1 - m)*x]/(2*(1 - m)) + Sinh[(1 + m)*x]/(2*(1 + m))
```

3.228.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6148
```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.228.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sinh(x(-1+m))}{-2+2m}+\frac{\sinh((1+m)x)}{2+2m}$	28
parallelrisch	$\frac{(1+m)\sinh(x(-1+m))+\sinh((1+m)x)(-1+m)}{2m^2-2}$	32
risch	$\frac{(m\,\mathrm{e}^{2x}-\mathrm{e}^{2x}+m+1)\,\mathrm{e}^{x(-1+m)}}{4(1+m)(-1+m)}-\frac{(m\,\mathrm{e}^{2x}+\mathrm{e}^{2x}+m-1)\,\mathrm{e}^{-(1+m)x}}{4(1+m)(-1+m)}$	67

input
```
int(cosh(x)*cosh(m*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-1+m)*sinh(x*(-1+m))+1/2*sinh((1+m)*x)/(1+m)
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \cosh(x)\cosh(mx)\,dx = \frac{m\cosh(x)\sinh(mx)-\cosh(mx)\sinh(x)}{(m^2-1)\cosh(x)^2-(m^2-1)\sinh(x)^2}$$

input
```
integrate(cosh(x)*cosh(m*x),x, algorithm="fricas")
```

output
```
(m*cosh(x)*sinh(m*x) - cosh(m*x)*sinh(x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)
*sinh(x)^2)
```

3.228.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(22) = 44.

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \cosh(x)\cosh(mx)\,dx = \begin{cases} -\frac{x\sinh^2(x)}{2}+\frac{x\cosh^2(x)}{2}+\frac{\sinh(x)\cosh(x)}{2} & \text{for } m=-1\vee m=1 \\ \frac{m\sinh(mx)\cosh(x)}{m^2-1}-\frac{\sinh(x)\cosh(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(x)*cosh(m*x),x)
```

output
```
Piecewise((-x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2, Eq(m, -1)
 | Eq(m, 1)), (m*sinh(m*x)*cosh(x)/(m**2 - 1) - sinh(x)*cosh(m*x)/(m**2 -
1), True))
```

3.228.7 Maxima [F(-2)]

Exception generated.

$$\int \cosh(x)\cosh(mx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)*cosh(m*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-2>0)', see `assume?` for more
details)Is
```

3.228.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \cosh(x)\cosh(mx)\,dx = \frac{e^{(mx+x)}}{4(m+1)} + \frac{e^{(mx-x)}}{4(m-1)} - \frac{e^{(-mx+x)}}{4(m-1)} - \frac{e^{(-mx-x)}}{4(m+1)}$$

input
```
integrate(cosh(x)*cosh(m*x),x, algorithm="giac")
```

output
```
1/4*e^(m*x + x)/(m + 1) + 1/4*e^(m*x - x)/(m - 1) - 1/4*e^(-m*x + x)/(m -
1) - 1/4*e^(-m*x - x)/(m + 1)
```

3.228.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \cosh(x)\cosh(mx)\,dx = -\frac{\cosh(m\,x)\,\sinh(x) - m\sinh(m\,x)\,\cosh(x)}{m^2 - 1}$$

input
```
int(cosh(m*x)*cosh(x),x)
```

output
```
-(cosh(m*x)*sinh(x) - m*sinh(m*x)*cosh(x))/(m^2 - 1)
```

3.229 $\int \cosh(x)\tanh(2x)\,dx$

3.229.1 Optimal result

Integrand size = 7, antiderivative size = 19

$$\int \cosh(x)\tanh(2x)\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}} + \cosh(x)$$

output
```
cosh(x)-1/2*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

3.229.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.68

$$\int \cosh(x)\tanh(2x)\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\sqrt{2}+i\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{2}} + \cosh(x)$$

input
```
Integrate[Cosh[x]*Tanh[2*x],x]
```

output
```
-(ArcTanh[Sqrt[2] - I*Tanh[x/2]]/Sqrt[2]) - ArcTanh[Sqrt[2] + I*Tanh[x/2]]
/Sqrt[2] + Cosh[x]
```

3.229.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 26, 4879, 27, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\tanh(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(2ix)}{\sec(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(2ix)}{\sec(ix)}dx$$

$$\Big\downarrow \text{4879}$$

$$\int -\frac{2\cosh^2(x)}{1-2\cosh^2(x)}d\cosh(x)$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{\cosh^2(x)}{1-2\cosh^2(x)}d\cosh(x)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{1}{2}\int \frac{1}{1-2\cosh^2(x)}d\cosh(x)-\frac{\cosh(x)}{2}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}-\frac{\cosh(x)}{2}\right)$$

input
```
Int[Cosh[x]*Tanh[2*x],x]
```

output
```
-2*(ArcTanh[Sqrt[2]*Cosh[x]]/(2*Sqrt[2]) - Cosh[x]/2)
```

3.229.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.229.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\cosh\left(x\right)-\frac{\operatorname{arctanh}\left(\cosh\left(x\right)\sqrt{2}\right)\sqrt{2}}{2}$	16
default	$\cosh\left(x\right)-\frac{\operatorname{arctanh}\left(\cosh\left(x\right)\sqrt{2}\right)\sqrt{2}}{2}$	16
risch	$\frac{{\mathrm e}^{x}}{2}+\frac{{\mathrm e}^{-x}}{2}+\frac{\ln\left(1+{\mathrm e}^{2 x}-{\mathrm e}^{x}\sqrt{2}\right)\sqrt{2}}{4}-\frac{\ln\left(1+{\mathrm e}^{2 x}+{\mathrm e}^{x}\sqrt{2}\right)\sqrt{2}}{4}$	49

input
```
int(cosh(x)*tanh(2*x),x,method=_RETURNVERBOSE)
```

output
```
cosh(x)-1/2*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

3.229.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.84

$$\int \cosh(x)\tanh(2x)\,dx$$
$$= \frac{2\cosh(x)^2 + \left(\sqrt{2}\cosh(x) + \sqrt{2}\sinh(x)\right)\log\left(\frac{\cosh(x)^2+\sinh(x)^2-2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right) + 4\cosh(x)\sinh(x) + 2\sin}{4(\cosh(x)+\sinh(x))}$$

input
```
integrate(cosh(x)*tanh(2*x),x, algorithm="fricas")
```

output
```
1/4*(2*cosh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log((cosh(x)^2 + si
nh(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2)) + 4*cosh(x)*sinh
(x) + 2*sinh(x)^2 + 2)/(cosh(x) + sinh(x))
```

3.229.6 Sympy [F]

$$\int \cosh(x)\tanh(2x)\,dx = \int \cosh(x)\tanh(2x)\,dx$$

input
```
integrate(cosh(x)*tanh(2*x),x)
```

output
```
Integral(cosh(x)*tanh(2*x), x)
```

3.229.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.74

$$\int \cosh(x)\tanh(2x)\,dx = -\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

input
```
integrate(cosh(x)*tanh(2*x),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)
*e^(-x) + e^(-2*x) + 1) + 1/2*e^(-x) + 1/2*e^x
```

3.229.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \cosh(x)\tanh(2x)\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2} - e^{(-x)} - e^{x}}{\sqrt{2} + e^{(-x)} + e^{x}}\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

input
```
integrate(cosh(x)*tanh(2*x),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) + 1/2*
e^(-x) + 1/2*e^x
```

3.229.9 Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \cosh(x)\tanh(2x)\,dx = \frac{e^{-x}}{2} + \frac{e^{x}}{2} - \frac{\sqrt{2}\ln\left(e^{2x} + \sqrt{2}\,e^{x} + 1\right)}{4} + \frac{\sqrt{2}\ln\left(e^{2x} - \sqrt{2}\,e^{x} + 1\right)}{4}$$

input
```
int(tanh(2*x)*cosh(x),x)
```

output
```
exp(-x)/2 + exp(x)/2 - (2^(1/2)*log(exp(2*x) + 2^(1/2)*exp(x) + 1))/4 + (2
^(1/2)*log(exp(2*x) - 2^(1/2)*exp(x) + 1))/4
```

3.230 $\int \cosh(x)\tanh(3x)\,dx$

3.230.1 Optimal result

Integrand size = 7, antiderivative size = 20

$$\int \cosh(x)\tanh(3x)\,dx = -\frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \cosh(x)$$

output
```
cosh(x)-1/3*arctanh(2/3*cosh(x)*3^(1/2))*3^(1/2)
```

3.230.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \cosh(x)\tanh(3x)\,dx = -\frac{\operatorname{arctanh}\left(\frac{2-i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{2+i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}} + \cosh(x)$$

input
```
Integrate[Cosh[x]*Tanh[3*x],x]
```

output
```
-(ArcTanh[(2 - I*Tanh[x/2])/Sqrt[3]]/Sqrt[3]) - ArcTanh[(2 + I*Tanh[x/2])/
Sqrt[3]]/Sqrt[3] + Cosh[x]
```

3.230.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 26, 4879, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\tanh(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(3ix)}{\sec(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(3ix)}{\sec(ix)}dx$$

$$\Big\downarrow \text{4879}$$

$$\int \frac{1-4\cosh^2(x)}{3-4\cosh^2(x)}d\cosh(x)$$

$$\Big\downarrow \text{299}$$

$$\cosh(x)-2\int \frac{1}{3-4\cosh^2(x)}d\cosh(x)$$

$$\Big\downarrow \text{219}$$

$$\cosh(x)-\frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[Cosh[x]*Tanh[3*x],x]
```

output
```
-(ArcTanh[(2*Cosh[x])/Sqrt[3]]/Sqrt[3]) + Cosh[x]
```

3.230.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.230.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\cosh\left(x\right)-\frac{\operatorname{arctanh}\left(\frac{2\cosh\left(x\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17
default	$\cosh\left(x\right)-\frac{\operatorname{arctanh}\left(\frac{2\cosh\left(x\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17
risch	$\frac{{\mathrm e}^{x}}{2}+\frac{{\mathrm e}^{-x}}{2}+\frac{\ln\left(1+{\mathrm e}^{2x}-{\mathrm e}^{x}\sqrt{3}\right)\sqrt{3}}{6}-\frac{\ln\left(1+{\mathrm e}^{2x}+{\mathrm e}^{x}\sqrt{3}\right)\sqrt{3}}{6}$	49

input
```
int(cosh(x)*tanh(3*x),x,method=_RETURNVERBOSE)
```

output
```
cosh(x)-1/3*arctanh(2/3*cosh(x)*3^(1/2))*3^(1/2)
```

3.230.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.10

$$\int \cosh(x)\tanh(3x)\,dx$$
$$= \frac{3\,\cosh(x)^2 + \left(\sqrt{3}\cosh(x) + \sqrt{3}\sinh(x)\right)\log\left(\frac{2\,\cosh(x)^2+2\,\sinh(x)^2-4\,\sqrt{3}\cosh(x)+5}{2\,\cosh(x)^2+2\,\sinh(x)^2-1}\right) + 6\,\cosh(x)\sinh(x) + 3\,\mathrm{s}}{6\left(\cosh(x) + \sinh(x)\right)}$$

input
```
integrate(cosh(x)*tanh(3*x),x, algorithm="fricas")
```

output
```
1/6*(3*cosh(x)^2 + (sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*log((2*cosh(x)^2 +
2*sinh(x)^2 - 4*sqrt(3)*cosh(x) + 5)/(2*cosh(x)^2 + 2*sinh(x)^2 - 1)) + 6*
cosh(x)*sinh(x) + 3*sinh(x)^2 + 3)/(cosh(x) + sinh(x))
```

3.230.6 Sympy [F]

$$\int \cosh(x)\tanh(3x)\,dx = \int \cosh(x)\tanh(3x)\,dx$$

input
```
integrate(cosh(x)*tanh(3*x),x)
```

output
```
Integral(cosh(x)*tanh(3*x), x)
```

3.230.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 7.65

$$\int \cosh(x)\tanh(3x)\,dx = -\frac{1}{12}\sqrt{3}\log\left(\sqrt{3}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{12}\sqrt{3}\log\left(-\sqrt{3}e^{(-x)} + e^{(-2x)} + 1\right) - \frac{1}{12}\sqrt{3}\log\left(\sqrt{3}e^{x} + e^{(2x)} + 1\right) + \frac{1}{12}\sqrt{3}\log\left(-\sqrt{3}e^{x} + e^{(2x)} + 1\right) + \frac{1}{6}\arctan\left(\sqrt{3} + 2e^{(-x)}\right) + \frac{1}{6}\arctan\left(\sqrt{3} + 2e^{x}\right) + \frac{1}{6}\arctan\left(-\sqrt{3} + 2e^{(-x)}\right) + \frac{1}{6}\arctan\left(-\sqrt{3} + 2e^{x}\right) + \frac{1}{3}\arctan\left(e^{(-x)}\right) + \frac{1}{3}\arctan\left(e^{x}\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

input
```
integrate(cosh(x)*tanh(3*x),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*log(sqrt(3)*e^(-x) + e^(-2*x) + 1) + 1/12*sqrt(3)*log(-sqrt(
3)*e^(-x) + e^(-2*x) + 1) - 1/12*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) +
1/12*sqrt(3)*log(-sqrt(3)*e^x + e^(2*x) + 1) + 1/6*arctan(sqrt(3) + 2*e^(-
x)) + 1/6*arctan(sqrt(3) + 2*e^x) + 1/6*arctan(-sqrt(3) + 2*e^(-x)) + 1/6*
arctan(-sqrt(3) + 2*e^x) + 1/3*arctan(e^(-x)) + 1/3*arctan(e^x) + 1/2*e^(-
x) + 1/2*e^x
```

3.230.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.25

$$\int \cosh(x)\tanh(3x)\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3} - e^{(-x)} - e^{x}}{\sqrt{3} + e^{(-x)} + e^{x}}\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

input
```
integrate(cosh(x)*tanh(3*x),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(-(sqrt(3) - e^(-x) - e^x)/(sqrt(3) + e^(-x) + e^x)) + 1/2*
e^(-x) + 1/2*e^x
```

3.230.9 Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \cosh(x)\tanh(3x)\,dx = \frac{e^{-x}}{2} + \frac{e^{x}}{2} + \frac{\sqrt{3}\,\ln\left(\frac{e^{2x}}{3} - \frac{\sqrt{3}\,e^{x}}{3} + \frac{1}{3}\right)}{6} - \frac{\sqrt{3}\,\ln\left(\frac{e^{2x}}{3} + \frac{\sqrt{3}\,e^{x}}{3} + \frac{1}{3}\right)}{6}$$

input
```
int(tanh(3*x)*cosh(x),x)
```

output
```
exp(-x)/2 + exp(x)/2 + (3^(1/2)*log(exp(2*x)/3 - (3^(1/2)*exp(x))/3 + 1/3)
)/6 - (3^(1/2)*log(exp(2*x)/3 + (3^(1/2)*exp(x))/3 + 1/3))/6
```

3.231 $\int \cosh(x) \tanh(4x) \, dx$

3.231.1 Optimal result

Integrand size = 7, antiderivative size = 69

$$\int \cosh(x) \tanh(4x) \, dx = -\frac{1}{4}\sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right) + \cosh(x)$$

output
```
cosh(x)-1/4*arctanh(2*cosh(x)/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/4*arc
tanh(2*cosh(x)/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

3.231.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int \cosh(x) \tanh(4x) \, dx = \cosh(x) + \frac{1}{16}\text{RootSum}\Bigg[1 + \#1^8\&, \frac{-x - 2\log\left(-\cosh\left(\frac{x}{2}\right) - \sinh\left(\frac{x}{2}\right) + \cosh\left(\frac{x}{2}\right)\#1 - \sinh\left(\frac{x}{2}\right)\#1\right) + x\#1^6 + 2\log\left(-\cosh\left(\frac{x}{2}\right) - \right.}{\#1^7}$$

input
```
Integrate[Cosh[x]*Tanh[4*x],x]
```

output
```
Cosh[x] + RootSum[1 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x
/2]*#1 - Sinh[x/2]*#1] + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]
*#1 - Sinh[x/2]*#1]*#1^6)/#1^7 & ]/16
```

3.231.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3042, 26, 4879, 27, 1602, 27, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\tanh(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(4ix)}{\sec(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(4ix)}{\sec(ix)}dx$$

$$\Big\downarrow \text{4879}$$

$$\int -\frac{4\cosh^2(x)\left(1-2\cosh^2(x)\right)}{8\cosh^4(x)-8\cosh^2(x)+1}d\cosh(x)$$

$$\Big\downarrow \text{27}$$

$$-4\int \frac{\cosh^2(x)\left(1-2\cosh^2(x)\right)}{8\cosh^4(x)-8\cosh^2(x)+1}d\cosh(x)$$

$$\Big\downarrow \text{1602}$$

$$-4\left(-\frac{1}{8}\int -\frac{2\left(1-4\cosh^2(x)\right)}{8\cosh^4(x)-8\cosh^2(x)+1}d\cosh(x)-\frac{\cosh(x)}{4}\right)$$

$$\Big\downarrow \text{27}$$

$$-4\left(\frac{1}{4}\int \frac{1-4\cosh^2(x)}{8\cosh^4(x)-8\cosh^2(x)+1}d\cosh(x)-\frac{\cosh(x)}{4}\right)$$

$$\Big\downarrow \text{1480}$$

$$-4\left(\frac{1}{4}\left(-\left(\left(2-\sqrt{2}\right)\int \frac{1}{8\cosh^2(x)-2\left(2-\sqrt{2}\right)}d\cosh(x)\right)-\left(2+\sqrt{2}\right)\int \frac{1}{8\cosh^2(x)-2\left(2+\sqrt{2}\right)}d\cosh(x)\right)\right.$$

$$\Big\downarrow \text{220}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)\right)-\frac{\cosh(x)}{4}\right)$$

input
```
Int[Cosh[x]*Tanh[4*x],x]
```

output
```
-4*(((Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[
2 + Sqrt[2]]*ArcTanh[(2*Cosh[x])/Sqrt[2 + Sqrt[2]]])/4)/4 - Cosh[x]/4)
```

3.231.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.231.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\left(\sum_{\_R=\mathrm{RootOf}\left(2048\_Z^{4}-128\_Z^{2}+1\right)}\_R\ln\left(-8\_R\,e^{x}+e^{2x}+1\right)\right)$	42

input
```
int(cosh(x)*tanh(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)+1/2*exp(-x)+sum(_R*ln(-8*_R*exp(x)+exp(2*x)+1),_R=RootOf(2048*_
Z^4-128*_Z^2+1))
```

3.231.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.09

$$\int \cosh(x)\tanh(4x)\,dx = -\frac{\sqrt{\sqrt{2}+2}(\cosh(x)+\sinh(x))\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+\sqrt{\sqrt{2}+2}(\cosh(x)+s\right.}{}$$

input
```
integrate(cosh(x)*tanh(4*x),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(sqrt(2) + 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + sqrt(sqrt(2) + 2)*(cosh(x) + sinh(x)) + 1) - sqrt(sqrt(2
) + 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 -
 sqrt(sqrt(2) + 2)*(cosh(x) + sinh(x)) + 1) + sqrt(-sqrt(2) + 2)*(cosh(x)
+ sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(-sqrt(2) +
 2)*(cosh(x) + sinh(x)) + 1) - sqrt(-sqrt(2) + 2)*(cosh(x) + sinh(x))*log(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sqrt(-sqrt(2) + 2)*(cosh(x) +
sinh(x)) + 1) - 4*cosh(x)^2 - 8*cosh(x)*sinh(x) - 4*sinh(x)^2 - 4)/(cosh(x
) + sinh(x))
```

3.231.6 Sympy [F]

$$\int \cosh(x)\tanh(4x)\,dx = \int \cosh(x)\tanh(4x)\,dx$$

input
```
integrate(cosh(x)*tanh(4*x),x)
```

output
```
Integral(cosh(x)*tanh(4*x), x)
```

3.231.7 Maxima [F]

$$\int \cosh(x)\tanh(4x)\,dx = \int \cosh\left(x\right)\tanh\left(4\,x\right)\,dx$$

input
```
integrate(cosh(x)*tanh(4*x),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) + 1)*e^(-x) + 1/2*integrate(2*(e^(7*x) - e^x)/(e^(8*x) + 1),
x)
```

3.231.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(49) = 98$.

Time = 0.35 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.72

$$\begin{aligned}\int \cosh(x)\tanh(4x)\,dx = &-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}+e^{(-x)}+e^{x}\right)\\&+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}+e^{(-x)}+e^{x}\right)\\&-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}+e^{(-x)}+e^{x}\right)\\&+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}+e^{(-x)}+e^{x}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}\end{aligned}$$

input
```
integrate(cosh(x)*tanh(4*x),x, algorithm="giac")
```

output
```
-1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + e^(-x) + e^x) + 1/8*sqrt(sq
rt(2) + 2)*log(-sqrt(sqrt(2) + 2) + e^(-x) + e^x) - 1/8*sqrt(-sqrt(2) + 2)
*log(sqrt(-sqrt(2) + 2) + e^(-x) + e^x) + 1/8*sqrt(-sqrt(2) + 2)*log(-sqrt
(-sqrt(2) + 2) + e^(-x) + e^x) + 1/2*e^(-x) + 1/2*e^x
```

3.231.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.93

$$\int \cosh(x)\tanh(4x)\,dx = \frac{e^{-x}}{2} + \frac{e^x}{2} + \ln\left(e^{2x} - 8\,e^x\sqrt{\frac{1}{32} - \frac{\sqrt{2}}{64}} + 1\right)\sqrt{\frac{1}{32} - \frac{\sqrt{2}}{64}} - \ln\left(e^{2x} + 8\,e^x\sqrt{\frac{1}{32} - \frac{\sqrt{2}}{64}} + 1\right)\sqrt{\frac{1}{32} - \frac{\sqrt{2}}{64}} + \ln\left(e^{2x} - 8\,e^x\sqrt{\frac{\sqrt{2}}{64} + \frac{1}{32}} + 1\right)\sqrt{\frac{\sqrt{2}}{64} + \frac{1}{32}} - \ln\left(e^{2x} + 8\,e^x\sqrt{\frac{\sqrt{2}}{64} + \frac{1}{32}} + 1\right)\sqrt{\frac{\sqrt{2}}{64} + \frac{1}{32}}$$

input
```
int(tanh(4*x)*cosh(x),x)
```

output
```
exp(-x)/2 + exp(x)/2 + log(exp(2*x) - 8*exp(x)*(1/32 - 2^(1/2)/64)^(1/2) +
 1)*(1/32 - 2^(1/2)/64)^(1/2) - log(exp(2*x) + 8*exp(x)*(1/32 - 2^(1/2)/64
)^(1/2) + 1)*(1/32 - 2^(1/2)/64)^(1/2) + log(exp(2*x) - 8*exp(x)*(2^(1/2)/
64 + 1/32)^(1/2) + 1)*(2^(1/2)/64 + 1/32)^(1/2) - log(exp(2*x) + 8*exp(x)*
(2^(1/2)/64 + 1/32)^(1/2) + 1)*(2^(1/2)/64 + 1/32)^(1/2)
```

3.232 $\int \cosh(x)\tanh(5x)\,dx$

3.232.1 Optimal result

Integrand size = 7, antiderivative size = 82

$$\int \cosh(x)\tanh(5x)\,dx = -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\text{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cosh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cosh(x)\right) + \cosh(x)$$

output
```
cosh(x)-1/10*arctanh(1/5*cosh(x)*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/
2)-1/10*arctanh(2*cosh(x)*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

3.232.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.04

$$\int \cosh(x)\tanh(5x)\,dx = \cosh(x) + \frac{1}{4}\text{RootSum}\Big[1-\#1^2+\#1^4-\#1^6 +\#1^8\&, \frac{-x-2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)+x\#1^2+2\log\left(-\cosh\left(\frac{x}{2}\right)-\right.}{}$$

input
```
Integrate[Cosh[x]*Tanh[5*x],x]
```

output
```
Cosh[x] + RootSum[1 - #1^2 + #1^4 - #1^6 + #1^8 & , (-x - 2*Log[-Cosh[x/2]
 - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1] + x*#1^2 + 2*Log[-Cosh[x/2] -
Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^2 - x*#1^4 - 2*Log[-Cosh[x/2]
- Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 + x*#1^6 + 2*Log[-Cosh[x/2
] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-#1 + 2*#1^3 - 3*#1^5
+ 4*#1^7) & ]/4
```

3.232.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 26, 4879, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\tanh(5x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\tan(5ix)}{\sec(ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\tan(5ix)}{\sec(ix)}dx$$

$$\Big\downarrow \text{ 4879}$$

$$\int \frac{16\cosh^4(x)-12\cosh^2(x)+1}{16\cosh^4(x)-20\cosh^2(x)+5}d\cosh(x)$$

$$\Big\downarrow \text{ 2205}$$

$$\int \left(1-\frac{4(1-2\cosh^2(x))}{16\cosh^4(x)-20\cosh^2(x)+5}\right)d\cosh(x)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\text{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cosh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cosh(x)\right)+\cosh(x)$$

input
```
Int[Cosh[x]*Tanh[5*x],x]
```

output
```
-1/5*(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Cosh[x]]) - (S
qrt[(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Cosh[x]])/5 + Cosh[
x]
```

3.232.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.232.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\left(\sum_{\_R =\mathrm{RootOf}\left(2000\_Z^{4}-100\_Z^{2}+1\right)}\_R \ln\left(-10\_R \,e^{x}+e^{2 x}+1\right)\right)$	42

input
```
int(cosh(x)*tanh(5*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)+1/2*exp(-x)+sum(_R*ln(-10*_R*exp(x)+exp(2*x)+1),_R=RootOf(2000*
_Z^4-100*_Z^2+1))
```

3.232.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(54) = 108$.

Time = 0.26 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.57

$$\int \cosh(x)\tanh(5x)\,dx =$$

$$-\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\sqrt{5}+5}\log\left(2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}+\left(\sqrt{2}\cosh\left(\right.\right.\right.}{}$$

input
```
integrate(cosh(x)*tanh(5*x),x, algorithm="fricas")
```

output
```
-1/20*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) + 5)*log(2*cosh(x)
^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))
*sqrt(sqrt(5) + 5) + 2) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5)
 + 5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - (sqrt(2)*cosh(x)
 + sqrt(2)*sinh(x))*sqrt(sqrt(5) + 5) + 2) + (sqrt(2)*cosh(x) + sqrt(2)*si
nh(x))*sqrt(-sqrt(5) + 5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^
2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) + 5) + 2) - (sqrt(2)
*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) + 5)*log(2*cosh(x)^2 + 4*cosh(x)
*sinh(x) + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5)
 + 5) + 2) - 10*cosh(x)^2 - 20*cosh(x)*sinh(x) - 10*sinh(x)^2 - 10)/(cosh(
x) + sinh(x))
```

3.232.6 Sympy [F]

$$\int \cosh(x)\tanh(5x)\,dx = \int \cosh\left(x\right)\tanh\left(5x\right)\,dx$$

input
```
integrate(cosh(x)*tanh(5*x),x)
```

output
```
Integral(cosh(x)*tanh(5*x), x)
```

3.232.7 Maxima [F]

$$\int \cosh(x)\tanh(5x)\,dx = \int \cosh\left(x\right)\tanh\left(5\,x\right)\,dx$$

input
```
integrate(cosh(x)*tanh(5*x),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) + 1)*e^(-x) + 1/2*integrate(2*(e^(7*x) - e^(5*x) + e^(3*x) -
e^x)/(e^(8*x) - e^(6*x) + e^(4*x) - e^(2*x) + 1), x)
```

3.232.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(54) = 108$.

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.55

$$\begin{aligned}\int \cosh(x)\tanh(5x)\,dx = &-\frac{1}{20}\,\sqrt{2\,\sqrt{5}+10}\log\left(\sqrt{\frac{1}{2}\,\sqrt{5}+\frac{5}{2}}+e^{(-x)}+e^{x}\right)\\&+\frac{1}{20}\,\sqrt{2\,\sqrt{5}+10}\log\left(-\sqrt{\frac{1}{2}\,\sqrt{5}+\frac{5}{2}}+e^{(-x)}+e^{x}\right)\\&-\frac{1}{20}\,\sqrt{-2\,\sqrt{5}+10}\log\left(\sqrt{-\frac{1}{2}\,\sqrt{5}+\frac{5}{2}}+e^{(-x)}+e^{x}\right)\\&+\frac{1}{20}\,\sqrt{-2\,\sqrt{5}+10}\log\left(-\sqrt{-\frac{1}{2}\,\sqrt{5}+\frac{5}{2}}+e^{(-x)}+e^{x}\right)\\&+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}\end{aligned}$$

input
```
integrate(cosh(x)*tanh(5*x),x, algorithm="giac")
```

output
```
-1/20*sqrt(2*sqrt(5) + 10)*log(sqrt(1/2*sqrt(5) + 5/2) + e^(-x) + e^x) + 1
/20*sqrt(2*sqrt(5) + 10)*log(-sqrt(1/2*sqrt(5) + 5/2) + e^(-x) + e^x) - 1/
20*sqrt(-2*sqrt(5) + 10)*log(sqrt(-1/2*sqrt(5) + 5/2) + e^(-x) + e^x) + 1/
20*sqrt(-2*sqrt(5) + 10)*log(-sqrt(-1/2*sqrt(5) + 5/2) + e^(-x) + e^x) + 1
/2*e^(-x) + 1/2*e^x
```

3.232.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.72

$$\int \cosh(x)\tanh(5x)\,dx = \frac{e^{-x}}{2} + \frac{e^{x}}{2} + \ln\left(4\,e^{2\,x} - 40\,e^{x}\sqrt{\frac{1}{40} - \frac{\sqrt{5}}{200}} + 4\right)\sqrt{\frac{1}{40} - \frac{\sqrt{5}}{200}} - \ln\left(4\,e^{2\,x} + 40\,e^{x}\sqrt{\frac{1}{40} - \frac{\sqrt{5}}{200}} + 4\right)\sqrt{\frac{1}{40} - \frac{\sqrt{5}}{200}} + \ln\left(4\,e^{2\,x} - 40\,e^{x}\sqrt{\frac{\sqrt{5}}{200} + \frac{1}{40}} + 4\right)\sqrt{\frac{\sqrt{5}}{200} + \frac{1}{40}} - \ln\left(4\,e^{2\,x} + 40\,e^{x}\sqrt{\frac{\sqrt{5}}{200} + \frac{1}{40}} + 4\right)\sqrt{\frac{\sqrt{5}}{200} + \frac{1}{40}}$$

input

```
int(tanh(5*x)*cosh(x),x)
```

output

```
exp(-x)/2 + exp(x)/2 + log(4*exp(2*x) - 40*exp(x)*(1/40 - 5^(1/2)/200)^(1/
2) + 4)*(1/40 - 5^(1/2)/200)^(1/2) - log(4*exp(2*x) + 40*exp(x)*(1/40 - 5^
(1/2)/200)^(1/2) + 4)*(1/40 - 5^(1/2)/200)^(1/2) + log(4*exp(2*x) - 40*exp
(x)*(5^(1/2)/200 + 1/40)^(1/2) + 4)*(5^(1/2)/200 + 1/40)^(1/2) - log(4*exp
(2*x) + 40*exp(x)*(5^(1/2)/200 + 1/40)^(1/2) + 4)*(5^(1/2)/200 + 1/40)^(1/
2)
```

3.233 $\int \cosh(x)\tanh(6x)\,dx$

3.233.1 Optimal result

Integrand size = 7, antiderivative size = 87

$$\int \cosh(x)\tanh(6x)\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}} - \frac{1}{6}\sqrt{2-\sqrt{3}}\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{6}\sqrt{2+\sqrt{3}}\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right) + \cosh(x)$$

output
```
cosh(x)-1/6*arctanh(cosh(x)*2^(1/2))*2^(1/2)-1/6*arctanh(2*cosh(x)/(1/2*6^
(1/2)-1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))-1/6*arctanh(2*cosh(x)/(1/2*6
^(1/2)+1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

3.233.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.23

$$\int \cosh(x)\tanh(6x)\,dx$$
$$= \frac{1}{24}\left(-4\left(\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right) + \sqrt{2}\operatorname{arctanh}\left(\sqrt{2}+i\tanh\left(\frac{x}{2}\right)\right) - 6\cosh(x)\right) + \text{RootSum}\left[1-\#1^4 + \#1^8\&, \frac{-2x-4\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)-x\#1^2-2\log\left(-\cosh\left(\frac{x}{2}\right)\right.}{}\right.\right.$$

input
```
Integrate[Cosh[x]*Tanh[6*x],x]
```

output
```
(-4*(Sqrt[2]*ArcTanh[Sqrt[2] - I*Tanh[x/2]] + Sqrt[2]*ArcTanh[Sqrt[2] + I*
Tanh[x/2]] - 6*Cosh[x]) + RootSum[1 - #1^4 + #1^8 & , (-2*x - 4*Log[-Cosh[
x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1] - x*#1^2 - 2*Log[-Cosh[x/2
] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^2 + x*#1^4 + 2*Log[-Cosh[x
/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 + 2*x*#1^6 + 4*Log[-Co
sh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-#1^3 + 2*#1^7)
& ])/24
```

3.233.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 26, 4879, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\tanh(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(6ix)}{\sec(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(6ix)}{\sec(ix)}dx$$

$$\Big\downarrow \text{4879}$$

$$\int -\frac{2\cosh^2(x)\left(16\cosh^4(x)-16\cosh^2(x)+3\right)}{-32\cosh^6(x)+48\cosh^4(x)-18\cosh^2(x)+1}d\cosh(x)$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{\cosh^2(x)\left(16\cosh^4(x)-16\cosh^2(x)+3\right)}{-32\cosh^6(x)+48\cosh^4(x)-18\cosh^2(x)+1}d\cosh(x)$$

$$\Big\downarrow \text{2460}$$

$$-2\int \left(\frac{1-8\cosh^2(x)}{3\left(16\cosh^4(x)-16\cosh^2(x)+1\right)}-\frac{1}{6\left(2\cosh^2(x)-1\right)}-\frac{1}{2}\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{6\sqrt{2}}+\frac{1}{12}\sqrt{2-\sqrt{3}}\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{12}\sqrt{2+\sqrt{3}}\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)-\frac{\cosh(x)}{2}\right)$$

input
```
Int[Cosh[x]*Tanh[6*x],x]
```

output
```
-2*(ArcTanh[Sqrt[2]*Cosh[x]]/(6*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*C
osh[x])/Sqrt[2 - Sqrt[3]]])/12 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Cosh[x])/Sq
rt[2 + Sqrt[3]]])/12 - Cosh[x]/2)
```

3.233.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.233.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

method	result
risch	$\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\left(\sum_{\_R=\operatorname{RootOf}\left(20736\_Z^{4}-576\_Z^{2}+1\right)}\_R\ln\left(-12\_R\,e^{x}+e^{2x}+1\right)\right)+\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{2}\right)\sqrt{2}}{12}-\frac{\ln(}{}$

input
```
int(cosh(x)*tanh(6*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)+1/2*exp(-x)+sum(_R*ln(-12*_R*exp(x)+exp(2*x)+1),_R=RootOf(20736
*_Z^4-576*_Z^2+1))+1/12*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/12*ln(1+ex
p(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

3.233.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(65) = 130.

Time = 0.26 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.97

$$\int \cosh(x)\tanh(6x)\,dx = -\frac{\sqrt{\sqrt{3}+2}(\cosh(x)+\sinh(x))\log\left(\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}+\sqrt{\sqrt{3}+2}(\cosh(x)+\mathrm{s}\right.}{}$$

input
```
integrate(cosh(x)*tanh(6*x),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(sqrt(3) + 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 + sqrt(sqrt(3) + 2)*(cosh(x) + sinh(x)) + 1) - sqrt(sqrt(
3) + 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
- sqrt(sqrt(3) + 2)*(cosh(x) + sinh(x)) + 1) + sqrt(-sqrt(3) + 2)*(cosh(x)
 + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(-sqrt(3)
+ 2)*(cosh(x) + sinh(x)) + 1) - sqrt(-sqrt(3) + 2)*(cosh(x) + sinh(x))*log
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sqrt(-sqrt(3) + 2)*(cosh(x) +
 sinh(x)) + 1) - 6*cosh(x)^2 - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log((co
sh(x)^2 + sinh(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2)) - 12
*cosh(x)*sinh(x) - 6*sinh(x)^2 - 6)/(cosh(x) + sinh(x))
```

3.233.6 Sympy [F]

$$\int \cosh(x)\tanh(6x)\,dx = \int \cosh{(x)}\tanh{(6x)}\,dx$$

input
```
integrate(cosh(x)*tanh(6*x),x)
```

output
```
Integral(cosh(x)*tanh(6*x), x)
```

3.233.7 Maxima [F]

$$\int \cosh(x)\tanh(6x)\,dx = \int \cosh{(x)}\tanh{(6\,x)}\,dx$$

input
```
integrate(cosh(x)*tanh(6*x),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) + 1)*e^(-x) - 1/12*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) + 1
/12*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) + 1/2*integrate(2/3*(2*e^(7*x)
 + e^(5*x) - e^(3*x) - 2*e^x)/(e^(8*x) - e^(4*x) + 1), x)
```

3.233.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(65) = 130.

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.80

$$\begin{aligned}\int \cosh(x)\tanh(6x)\,dx = &-\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}+e^{(-x)}+e^{x}\right)\\ &-\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}+e^{(-x)}+e^{x}\right)\\ &+\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}+e^{(-x)}+e^{x}\right)\\ &+\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(-\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}+e^{(-x)}+e^{x}\right)\\ &+\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-e^{x}}{\sqrt{2}+e^{(-x)}+e^{x}}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}\end{aligned}$$

input
```
integrate(cosh(x)*tanh(6*x),x, algorithm="giac")
```

output
```
-1/24*(sqrt(6) + sqrt(2))*log(1/2*sqrt(6) + 1/2*sqrt(2) + e^(-x) + e^x) -
1/24*(sqrt(6) - sqrt(2))*log(1/2*sqrt(6) - 1/2*sqrt(2) + e^(-x) + e^x) + 1
/24*(sqrt(6) - sqrt(2))*log(-1/2*sqrt(6) + 1/2*sqrt(2) + e^(-x) + e^x) + 1
/24*(sqrt(6) + sqrt(2))*log(-1/2*sqrt(6) - 1/2*sqrt(2) + e^(-x) + e^x) + 1
/12*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) + 1/2*
e^(-x) + 1/2*e^x
```

3.233.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.95

$$\int \cosh(x)\tanh(6x)\,dx = \frac{e^{-x}}{2} + \frac{e^{x}}{2} - \frac{\sqrt{2}\ln\left(e^{2x} + \sqrt{2}\,e^{x} + 1\right)}{12} + \frac{\sqrt{2}\ln\left(e^{2x} - \sqrt{2}\,e^{x} + 1\right)}{12} + \ln\left(e^{2x} - 12\,e^{x}\sqrt{\frac{1}{72} - \frac{\sqrt{3}}{144}} + 1\right)\sqrt{\frac{1}{72} - \frac{\sqrt{3}}{144}} - \ln\left(e^{2x} + 12\,e^{x}\sqrt{\frac{1}{72} - \frac{\sqrt{3}}{144}} + 1\right)\sqrt{\frac{1}{72} - \frac{\sqrt{3}}{144}} + \ln\left(e^{2x} - 12\,e^{x}\sqrt{\frac{\sqrt{3}}{144} + \frac{1}{72}} + 1\right)\sqrt{\frac{\sqrt{3}}{144} + \frac{1}{72}} - \ln\left(e^{2x} + 12\,e^{x}\sqrt{\frac{\sqrt{3}}{144} + \frac{1}{72}} + 1\right)\sqrt{\frac{\sqrt{3}}{144} + \frac{1}{72}}$$

input
```
int(tanh(6*x)*cosh(x),x)
```

output
```
exp(-x)/2 + exp(x)/2 - (2^(1/2)*log(exp(2*x) + 2^(1/2)*exp(x) + 1))/12 + (
2^(1/2)*log(exp(2*x) - 2^(1/2)*exp(x) + 1))/12 + log(exp(2*x) - 12*exp(x)*
(1/72 - 3^(1/2)/144)^(1/2) + 1)*(1/72 - 3^(1/2)/144)^(1/2) - log(exp(2*x)
+ 12*exp(x)*(1/72 - 3^(1/2)/144)^(1/2) + 1)*(1/72 - 3^(1/2)/144)^(1/2) + l
og(exp(2*x) - 12*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 1)*(3^(1/2)/144 + 1/7
2)^(1/2) - log(exp(2*x) + 12*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 1)*(3^(1/
2)/144 + 1/72)^(1/2)
```

3.234 $\int \cosh(x)\coth(2x)\,dx$

3.234.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \cosh(x)\coth(2x)\,dx=-\frac{1}{2}\text{arctanh}(\cosh(x))+\cosh(x)$$

output
```
-1/2*arctanh(cosh(x))+cosh(x)
```

3.234.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50

$$\int \cosh(x)\coth(2x)\,dx=\cosh(x)-\frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Cosh[x]*Coth[2*x],x]
```

output
```
Cosh[x] - Log[Cosh[x/2]]/2 + Log[Sinh[x/2]]/2
```

3.234.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 26, 4879, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\coth(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\cos(ix)\cot(2ix)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \cos(ix)\cot(2ix)dx$$

$$\Big\downarrow \text{4879}$$

$$-\int -\frac{1-2\cosh^2(x)}{2\left(1-\cosh^2(x)\right)}d\cosh(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1-2\cosh^2(x)}{1-\cosh^2(x)}d\cosh(x)$$

$$\Big\downarrow \text{299}$$

$$\frac{1}{2}\left(2\cosh(x)-\int \frac{1}{1-\cosh^2(x)}d\cosh(x)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}(2\cosh(x)-\text{arctanh}(\cosh(x)))$$

input
```
Int[Cosh[x]*Coth[2*x],x]
```

output
```
(-ArcTanh[Cosh[x]] + 2*Cosh[x])/2
```

3.234.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.234.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

method	result	size
default	$\cosh(x)-2\,\operatorname{arctanh}(\mathrm{e}^{x})-\frac{\ln(\tanh(\frac{x}{2}))}{2}$	16
risch	$\frac{\mathrm{e}^{x}}{2}+\frac{\mathrm{e}^{-x}}{2}+\frac{\ln(\mathrm{e}^{x}-1)}{2}-\frac{\ln(\mathrm{e}^{x}+1)}{2}$	26

input
```
int(cosh(x)*coth(2*x),x,method=_RETURNVERBOSE)
```

output
```
cosh(x)-2*arctanh(exp(x))-1/2*ln(tanh(1/2*x))
```

3.234.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 5.20

$$\int \cosh(x)\coth(2x)\,dx$$
$$= \frac{\cosh\left(x\right)^{2} - \left(\cosh\left(x\right) + \sinh\left(x\right)\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) + \left(\cosh\left(x\right) + \sinh\left(x\right)\right)\log\left(\cosh\left(x\right) + \sinh}{2\left(\cosh\left(x\right) + \sinh\left(x\right)\right)}$$

input
```
integrate(cosh(x)*coth(2*x),x, algorithm="fricas")
```

output
```
1/2*(cosh(x)^2 - (cosh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) + (cosh(x)
 + sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1
)/(cosh(x) + sinh(x))
```

3.234.6 Sympy [F]

$$\int \cosh(x)\coth(2x)\,dx = \int \cosh\left(x\right)\coth\left(2x\right)\,dx$$

input
```
integrate(cosh(x)*coth(2*x),x)
```

output
```
Integral(cosh(x)*coth(2*x), x)
```

3.234.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(8) = 16.

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \cosh(x)\coth(2x)\,dx = \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - \frac{1}{2}\,\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(cosh(x)*coth(2*x),x, algorithm="maxima")
```

output
```
1/2*e^(-x) + 1/2*e^x - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

3.234.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \cosh(x)\coth(2x)\,dx = \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - \frac{1}{2}\,\log\left(e^{x} + 1\right) + \frac{1}{2}\,\log\left(|e^{x} - 1|\right)$$

input
```
integrate(cosh(x)*coth(2*x),x, algorithm="giac")
```

output
```
1/2*e^(-x) + 1/2*e^x - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

3.234.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \cosh(x)\coth(2x)\,dx = \frac{\ln\left(1 - e^{x}\right)}{2} - \frac{\ln\left(-e^{x} - 1\right)}{2} + \frac{e^{-x}}{2} + \frac{e^{x}}{2}$$

input
```
int(coth(2*x)*cosh(x),x)
```

output
```
log(1 - exp(x))/2 - log(- exp(x) - 1)/2 + exp(-x)/2 + exp(x)/2
```

3.235 $\int \cosh(x)\coth(3x)\,dx$

3.235.1 Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \cosh(x)\coth(3x)\,dx=\cosh(x)+\frac{1}{6}\log(1-2\cosh(x))+\frac{1}{6}\log(1-\cosh(x))-\frac{1}{6}\log(1+\cosh(x))-\frac{1}{6}\log(1+2\cosh(x))$$

output
```
cosh(x)+1/6*ln(1-2*cosh(x))+1/6*ln(1-cosh(x))-1/6*ln(1+cosh(x))-1/6*ln(1+2
*cosh(x))
```

3.235.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \cosh(x)\coth(3x)\,dx=\cosh(x)-\frac{1}{3}\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{6}\log(1-2\cosh(x))-\frac{1}{6}\log(1+2\cosh(x))+\frac{1}{3}\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Cosh[x]*Coth[3*x],x]
```

output
```
Cosh[x] - Log[Cosh[x/2]]/3 + Log[1 - 2*Cosh[x]]/6 - Log[1 + 2*Cosh[x]]/6 +
 Log[Sinh[x/2]]/3
```

3.235.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3042, 26, 4879, 1602, 27, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\coth(3x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int i\cos(ix)\cot(3ix)dx$$

$$\Big\downarrow \; 26$$

$$i\int \cos(ix)\cot(3ix)dx$$

$$\Big\downarrow \; 4879$$

$$-\int \frac{\cosh^2(x)\left(3-4\cosh^2(x)\right)}{4\cosh^4(x)-5\cosh^2(x)+1}d\cosh(x)$$

$$\Big\downarrow \; 1602$$

$$\frac{1}{4}\int -\frac{4(1-2\cosh^2(x))}{4\cosh^4(x)-5\cosh^2(x)+1}d\cosh(x)+\cosh(x)$$

$$\Big\downarrow \; 27$$

$$\cosh(x)-\int \frac{1-2\cosh^2(x)}{4\cosh^4(x)-5\cosh^2(x)+1}d\cosh(x)$$

$$\Big\downarrow \; 1475$$

$$\frac{1}{4}\int \frac{1}{\cosh^2(x)-\frac{\cosh(x)}{2}-\frac{1}{2}}d\cosh(x)+\frac{1}{4}\int \frac{1}{\cosh^2(x)+\frac{\cosh(x)}{2}-\frac{1}{2}}d\cosh(x)+\cosh(x)$$

$$\Big\downarrow \; 1081$$

$$\frac{1}{4}\int \left(-\frac{2}{3(\cosh(x)+1)}-\frac{4}{3(1-2\cosh(x))}\right)d\cosh(x)+\frac{1}{4}\int \left(-\frac{4}{3(2\cosh(x)+1)}-\frac{2}{3(1-\cosh(x))}\right)d\cosh(x)+\cosh(x)$$

$$\Big\downarrow \; 2009$$

$$\cosh(x) + \frac{1}{4}\left(\frac{2}{3}\log(1-2\cosh(x)) - \frac{2}{3}\log(\cosh(x)+1)\right) + \frac{1}{4}\left(\frac{2}{3}\log(1-\cosh(x)) - \frac{2}{3}\log(2\cosh(x)+1)\right)$$

input
```
Int[Cosh[x]*Coth[3*x],x]
```

output
```
Cosh[x] + ((2*Log[1 - 2*Cosh[x]])/3 - (2*Log[1 + Cosh[x]])/3)/4 + ((2*Log[
1 - Cosh[x]])/3 - (2*Log[1 + 2*Cosh[x]])/3)/4
```

3.235.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.235.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{e^x}{2}+\frac{e^{-x}}{2}-\frac{\ln(e^x+1)}{3}+\frac{\ln(e^x-1)}{3}+\frac{\ln(e^{2x}-e^x+1)}{6}-\frac{\ln(1+e^x+e^{2x})}{6}$	50

input
```
int(cosh(x)*coth(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)+1/2*exp(-x)-1/3*ln(exp(x)+1)+1/3*ln(exp(x)-1)+1/6*ln(exp(2*x)-e
xp(x)+1)-1/6*ln(1+exp(x)+exp(2*x))
```

3.235.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(37) = 74$.

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.31

$$\int \cosh(x)\coth(3x)\,dx$$
$$= \frac{3\cosh(x)^2 - (\cosh(x)+\sinh(x))\log\left(\frac{2\cosh(x)+1}{\cosh(x)-\sinh(x)}\right) + (\cosh(x)+\sinh(x))\log\left(\frac{2\cosh(x)-1}{\cosh(x)-\sinh(x)}\right) - 2(\cos}{}$$

input
```
integrate(cosh(x)*coth(3*x),x, algorithm="fricas")
```

output
```
1/6*(3*cosh(x)^2 - (cosh(x) + sinh(x))*log((2*cosh(x) + 1)/(cosh(x) - sinh
(x))) + (cosh(x) + sinh(x))*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 2*(
cosh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) + 2*(cosh(x) + sinh(x))*log(
cosh(x) + sinh(x) - 1) + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 + 3)/(cosh(x) + s
inh(x))
```

3.235.6 Sympy [F]

$$\int \cosh(x)\coth(3x)\,dx = \int \cosh\left(x\right)\coth\left(3x\right)\,dx$$

input
```
integrate(cosh(x)*coth(3*x),x)
```

output
```
Integral(cosh(x)*coth(3*x), x)
```

3.235.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \cosh(x)\coth(3x)\,dx = \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - \frac{1}{6}\,\log\left(e^{(-x)} + e^{(-2\,x)} + 1\right) - \frac{1}{3}\,\log\left(e^{(-x)} + 1\right) + \frac{1}{3}\,\log\left(e^{(-x)} - 1\right) + \frac{1}{6}\,\log\left(-e^{(-x)} + e^{(-2\,x)} + 1\right)$$

input
```
integrate(cosh(x)*coth(3*x),x, algorithm="maxima")
```

output
```
1/2*e^(-x) + 1/2*e^x - 1/6*log(e^(-x) + e^(-2*x) + 1) - 1/3*log(e^(-x) + 1
) + 1/3*log(e^(-x) - 1) + 1/6*log(-e^(-x) + e^(-2*x) + 1)
```

3.235.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \cosh(x)\coth(3x)\,dx = \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - \frac{1}{6}\,\log\left(e^{(-x)} + e^{x} + 2\right) - \frac{1}{6}\,\log\left(e^{(-x)} + e^{x} + 1\right) + \frac{1}{6}\,\log\left(e^{(-x)} + e^{x} - 1\right) + \frac{1}{6}\,\log\left(e^{(-x)} + e^{x} - 2\right)$$

input
```
integrate(cosh(x)*coth(3*x),x, algorithm="giac")
```

output
```
1/2*e^(-x) + 1/2*e^x - 1/6*log(e^(-x) + e^x + 2) - 1/6*log(e^(-x) + e^x +
1) + 1/6*log(e^(-x) + e^x - 1) + 1/6*log(e^(-x) + e^x - 2)
```

3.235.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \cosh(x)\coth(3x)\,dx = \frac{\ln\left(6-6\,e^{x}\right)}{3} - \frac{\ln\left(-6\,e^{x}-6\right)}{3} + \frac{e^{-x}}{2} + \frac{\ln\left(e^{x}-e^{2\,x}-1\right)}{6} - \frac{\ln\left(-e^{2\,x}-e^{x}-1\right)}{6} + \frac{e^{x}}{2}$$

input
```
int(coth(3*x)*cosh(x),x)
```

output
```
log(6 - 6*exp(x))/3 - log(- 6*exp(x) - 6)/3 + exp(-x)/2 + log(exp(x) - exp
(2*x) - 1)/6 - log(- exp(2*x) - exp(x) - 1)/6 + exp(x)/2
```

3.236 $\int \cosh(x)\coth(4x)\,dx$

3.236.1 Optimal result

Integrand size = 7, antiderivative size = 28

$$\int \cosh(x)\coth(4x)\,dx = -\frac{1}{4}\text{arctanh}(\cosh(x)) - \frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}} + \cosh(x)$$

output
```
-1/4*arctanh(cosh(x))+cosh(x)-1/4*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

3.236.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int \cosh(x)\coth(4x)\,dx = \frac{1}{4}\left(-\sqrt{2}\text{arctanh}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right) - \sqrt{2}\text{arctanh}\left(\sqrt{2}+i\tanh\left(\frac{x}{2}\right)\right) + 4\cosh(x) - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cosh[x]*Coth[4*x],x]
```

output
```
(-(Sqrt[2]*ArcTanh[Sqrt[2] - I*Tanh[x/2]]) - Sqrt[2]*ArcTanh[Sqrt[2] + I*T
anh[x/2]] + 4*Cosh[x] - Log[Cosh[x/2]] + Log[Sinh[x/2]])/4
```

3.236.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 26, 4879, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\coth(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\cos(ix)\cot(4ix)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \cos(ix)\cot(4ix)dx$$

$$\Big\downarrow \text{4879}$$

$$-\int -\frac{8\cosh^4(x)-8\cosh^2(x)+1}{4\left(2\cosh^4(x)-3\cosh^2(x)+1\right)}d\cosh(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{8\cosh^4(x)-8\cosh^2(x)+1}{2\cosh^4(x)-3\cosh^2(x)+1}d\cosh(x)$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{4}\int \left(4-\frac{3-4\cosh^2(x)}{2\cosh^4(x)-3\cosh^2(x)+1}\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\text{arctanh}(\cosh(x))-\sqrt{2}\text{arctanh}\left(\sqrt{2}\cosh(x)\right)+4\cosh(x)\right)$$

input
```
Int[Cosh[x]*Coth[4*x],x]
```

output
```
(-ArcTanh[Cosh[x]] - Sqrt[2]*ArcTanh[Sqrt[2]*Cosh[x]] + 4*Cosh[x])/4
```

3.236.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.236.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(20) = 40$.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

method	result	size
risch	$\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{\ln(e^{x}-1)}{4}-\frac{\ln(e^{x}+1)}{4}+\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{2}\right)\sqrt{2}}{8}-\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{2}\right)\sqrt{2}}{8}$	63

input
```
int(cosh(x)*coth(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)+1/2*exp(-x)+1/4*ln(exp(x)-1)-1/4*ln(exp(x)+1)+1/8*ln(1+exp(2*x)
-exp(x)*2^(1/2))*2^(1/2)-1/8*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

3.236.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(20) = 40.

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.61

$$\int \cosh(x)\coth(4x)\,dx = \frac{4\cosh(x)^2 + \left(\sqrt{2}\cosh(x) + \sqrt{2}\sinh(x)\right)\log\left(\frac{\cosh(x)^2+\sinh(x)^2-2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right) - 2\left(\cosh(x)+\sinh(x)\right)\log}{8\,(\cos}$$

input
```
integrate(cosh(x)*coth(4*x),x, algorithm="fricas")
```

output
```
1/8*(4*cosh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log((cosh(x)^2 + si
nh(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2)) - 2*(cosh(x) + s
inh(x))*log(cosh(x) + sinh(x) + 1) + 2*(cosh(x) + sinh(x))*log(cosh(x) + s
inh(x) - 1) + 8*cosh(x)*sinh(x) + 4*sinh(x)^2 + 4)/(cosh(x) + sinh(x))
```

3.236.6 Sympy [F]

$$\int \cosh(x)\coth(4x)\,dx = \int \cosh(x)\coth(4x)\,dx$$

input
```
integrate(cosh(x)*coth(4*x),x)
```

output
```
Integral(cosh(x)*coth(4*x), x)
```

3.236.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(20) = 40.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.50

$$\begin{aligned}\int \cosh(x)\coth(4x)\,dx = &-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right) \\ &+ \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{2}e^{(-x)} \\ &+ \frac{1}{2}e^{x} - \frac{1}{4}\log\left(e^{(-x)} + 1\right) + \frac{1}{4}\log\left(e^{(-x)} - 1\right)\end{aligned}$$

input
```
integrate(cosh(x)*coth(4*x),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) + 1/8*sqrt(2)*log(-sqrt(2)
*e^(-x) + e^(-2*x) + 1) + 1/2*e^(-x) + 1/2*e^x - 1/4*log(e^(-x) + 1) + 1/4
*log(e^(-x) - 1)
```

3.236.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \cosh(x)\coth(4x)\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-e^{x}}{\sqrt{2}+e^{(-x)}+e^{x}}\right)+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x} - \frac{1}{8}\,\log\left(e^{(-x)}+e^{x}+2\right)+\frac{1}{8}\,\log\left(e^{(-x)}+e^{x}-2\right)$$

input
```
integrate(cosh(x)*coth(4*x),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) + 1/2*
e^(-x) + 1/2*e^x - 1/8*log(e^(-x) + e^x + 2) + 1/8*log(e^(-x) + e^x - 2)
```

3.236.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \cosh(x)\coth(4x)\,dx = \frac{\ln\left(\frac{1}{2}-\frac{e^{x}}{2}\right)}{4}-\frac{\ln\left(-\frac{e^{x}}{2}-\frac{1}{2}\right)}{4}+\frac{e^{-x}}{2}+\frac{e^{x}}{2} - \frac{\sqrt{2}\,\ln\left(-\frac{e^{2\,x}}{8}-\frac{\sqrt{2}\,e^{x}}{8}-\frac{1}{8}\right)}{8}+\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,e^{x}}{8}-\frac{e^{2\,x}}{8}-\frac{1}{8}\right)}{8}$$

input
```
int(coth(4*x)*cosh(x),x)
```

output
```
log(1/2 - exp(x)/2)/4 - log(- exp(x)/2 - 1/2)/4 + exp(-x)/2 + exp(x)/2 - (
2^(1/2)*log(- exp(2*x)/8 - (2^(1/2)*exp(x))/8 - 1/8))/8 + (2^(1/2)*log((2^
(1/2)*exp(x))/8 - exp(2*x)/8 - 1/8))/8
```

3.237 $\int \cosh(x)\coth(5x)\,dx$

3.237.1 Optimal result

Integrand size = 7, antiderivative size = 110

$$\begin{aligned}\int \cosh(x)\coth(5x)\,dx = &-\frac{1}{5}\operatorname{arctanh}(\cosh(x))+\cosh(x)\\ &+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cosh(x)\right)\\ &+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\cosh(x)\right)\\ &-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cosh(x)\right)\\ &-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cosh(x)\right)\end{aligned}$$

output
```
-1/5*arctanh(cosh(x))+cosh(x)+1/20*ln(1-4*cosh(x)-5^(1/2))*(-5^(1/2)+1)-1/
20*ln(1+4*cosh(x)-5^(1/2))*(-5^(1/2)+1)+1/20*ln(1-4*cosh(x)+5^(1/2))*(5^(1
/2)+1)-1/20*ln(1+4*cosh(x)+5^(1/2))*(5^(1/2)+1)
```

3.237.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\int \cosh(x)\coth(5x)\,dx = \frac{1}{100}\Big(100\cosh(x) - 20\log\left(\cosh\left(\frac{x}{2}\right)\right) + \sqrt{5}\left(-5+\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cosh(x)\right) + \sqrt{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\cosh(x)\right) - \sqrt{5}\left(-5+\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cosh(x)\right) - \sqrt{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cosh(x)\right) + 20\log\left(\sinh\left(\frac{x}{2}\right)\right)\Big)$$

input
```
Integrate[Cosh[x]*Coth[5*x],x]
```

output
```
(100*Cosh[x] - 20*Log[Cosh[x/2]] + Sqrt[5]*(-5 + Sqrt[5])*Log[1 - Sqrt[5]
- 4*Cosh[x]] + Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cosh[x]] - Sqrt[5
]*(-5 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cosh[x]] - Sqrt[5]*(5 + Sqrt[5])*Log[
1 + Sqrt[5] + 4*Cosh[x]] + 20*Log[Sinh[x/2]])/100
```

3.237.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 26, 4879, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\coth(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\cos(ix)\cot(5ix)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \cos(ix)\cot(5ix)dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \frac{\cosh^2(x) \left(16 \cosh^4(x)-20 \cosh^2(x)+5\right)}{-16 \cosh^6(x)+28 \cosh^4(x)-13 \cosh^2(x)+1}d\cosh(x)$$

$$\Big\downarrow \text{2460}$$

$$-\int \left(\frac{2(\cosh(x)-1)}{5 \left(4 \cosh^2(x)+2 \cosh(x)-1\right)}-\frac{1}{5 \left(\cosh^2(x)-1\right)}-\frac{2(\cosh(x)+1)}{5 \left(4 \cosh^2(x)-2 \cosh(x)-1\right)}-1\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\text{arctanh}(\cosh(x))+\cosh(x)+\frac{1}{20}\left(1-\sqrt{5}\right) \log \left(-4 \cosh(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right) \log \left(-4 \cosh(x)+\sqrt{5}+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right) \log \left(4 \cosh(x)-\sqrt{5}+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right) \log \left(4 \cosh(x)+\sqrt{5}+1\right)$$

input
```
Int[Cosh[x]*Coth[5*x],x]
```

output
```
-1/5*ArcTanh[Cosh[x]] + Cosh[x] + ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Cosh[
x]])/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cosh[x]])/20 - ((1 - Sqrt[5])
*Log[1 - Sqrt[5] + 4*Cosh[x]])/20 - ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cos
h[x]])/20
```

3.237.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.237.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(84) = 168$.

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.73

method	result
risch	$\frac{e^{x}}{2}+\frac{e^{-x}}{2}-\frac{\ln(e^{x}+1)}{5}+\frac{\ln(e^{x}-1)}{5}+\frac{\ln\left(e^{2x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{x}+1\right)}{20}+\frac{\ln\left(e^{2x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{x}+1\right)\sqrt{5}}{20}+\frac{\ln\left(e^{2x}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)e^{x}+\right.}{20}$

input
```
int(cosh(x)*coth(5*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)+1/2*exp(-x)-1/5*ln(exp(x)+1)+1/5*ln(exp(x)-1)+1/20*ln(exp(2*x)+
(-1/2-1/2*5^(1/2))*exp(x)+1)+1/20*ln(exp(2*x)+(-1/2-1/2*5^(1/2))*exp(x)+1)
*5^(1/2)+1/20*ln(exp(2*x)+(1/2*5^(1/2)-1/2)*exp(x)+1)-1/20*ln(exp(2*x)+(1/
2*5^(1/2)-1/2)*exp(x)+1)*5^(1/2)-1/20*ln(exp(2*x)+(1/2-1/2*5^(1/2))*exp(x)
+1)+1/20*ln(exp(2*x)+(1/2-1/2*5^(1/2))*exp(x)+1)*5^(1/2)-1/20*ln(exp(2*x)+
(1/2+1/2*5^(1/2))*exp(x)+1)-1/20*ln(exp(2*x)+(1/2+1/2*5^(1/2))*exp(x)+1)*5
^(1/2)
```

3.237.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(80) = 160$.

Time = 0.26 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.47

$$\int \cosh(x)\coth(5x)\,dx$$
$$= \frac{10\cosh(x)^2+\left(\sqrt{5}\cosh(x)+\sqrt{5}\sinh(x)\right)\log\left(-\frac{4\left(\sqrt{5}-1\right)\cosh(x)-4\cosh(x)^2-4\sinh(x)^2+\sqrt{5}-7}{2\cosh(x)^2+2\sinh(x)^2+2\cosh(x)+1}\right)+\left(\sqrt{5}\cosh(x\right.}{}$$

input
```
integrate(cosh(x)*coth(5*x),x, algorithm="fricas")
```

output
```
1/20*(10*cosh(x)^2 + (sqrt(5)*cosh(x) + sqrt(5)*sinh(x))*log(-(4*(sqrt(5)
- 1)*cosh(x) - 4*cosh(x)^2 - 4*sinh(x)^2 + sqrt(5) - 7)/(2*cosh(x)^2 + 2*s
inh(x)^2 + 2*cosh(x) + 1)) + (sqrt(5)*cosh(x) + sqrt(5)*sinh(x))*log(-(4*(
sqrt(5) + 1)*cosh(x) - 4*cosh(x)^2 - 4*sinh(x)^2 - sqrt(5) - 7)/(2*cosh(x)
^2 + 2*sinh(x)^2 - 2*cosh(x) + 1)) - (cosh(x) + sinh(x))*log((2*cosh(x)^2
+ 2*sinh(x)^2 + 2*cosh(x) + 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
) + (cosh(x) + sinh(x))*log((2*cosh(x)^2 + 2*sinh(x)^2 - 2*cosh(x) + 1)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*(cosh(x) + sinh(x))*log(cos
h(x) + sinh(x) + 1) + 4*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x) - 1) + 2
0*cosh(x)*sinh(x) + 10*sinh(x)^2 + 10)/(cosh(x) + sinh(x))
```

3.237.6 Sympy [F]

$$\int \cosh(x)\coth(5x)\,dx = \int \cosh{(x)}\coth{(5x)}\,dx$$

input
```
integrate(cosh(x)*coth(5*x),x)
```

output
```
Integral(cosh(x)*coth(5*x), x)
```

3.237.7 Maxima [F]

$$\int \cosh(x)\coth(5x)\,dx = \int \cosh{(x)}\coth{(5\,x)}\,\,dx$$

input
```
integrate(cosh(x)*coth(5*x),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) + 1)*e^(-x) - 1/5*integrate((e^(3*x) + e^(2*x) + e^x + 1)*e^x
/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) + 1/5*integrate((e^(3*x) - e^
(2*x) + e^x - 1)*e^x/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x) + 3/10*in
tegrate(e^(3*x)/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) + 3/10*integra
te(e^(3*x)/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x) + 1/10*integrate(e^
(2*x)/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/10*integrate(e^(2*x)
/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x) - 1/10*integrate(e^x/(e^(4*x)
 + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/10*integrate(e^x/(e^(4*x) - e^(3*x
) + e^(2*x) - e^x + 1), x) - 1/5*log(e^x + 1) + 1/5*log(e^x - 1)
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.43

$$\int \cosh(x)\coth(5x)\,dx = \frac{1}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\,e^{(-x)}-2\,e^{x}+1}{\sqrt{5}+2\,e^{(-x)}+2\,e^{x}-1}\right) + \frac{1}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\,e^{(-x)}-2\,e^{x}-1}{\sqrt{5}+2\,e^{(-x)}+2\,e^{x}+1}\right) + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - \frac{1}{20}\,\log\left(\left(e^{(-x)}+e^{x}\right)^{2}+e^{(-x)}+e^{x}-1\right) + \frac{1}{20}\,\log\left(\left(e^{(-x)}+e^{x}\right)^{2}-e^{(-x)}-e^{x}-1\right) - \frac{1}{10}\,\log\left(e^{(-x)}+e^{x}+2\right) + \frac{1}{10}\,\log\left(e^{(-x)}+e^{x}-2\right)$$

input
```
integrate(cosh(x)*coth(5*x),x, algorithm="giac")
```

output
```
1/20*sqrt(5)*log(-(sqrt(5) - 2*e^(-x) - 2*e^x + 1)/(sqrt(5) + 2*e^(-x) + 2
*e^x - 1)) + 1/20*sqrt(5)*log(-(sqrt(5) - 2*e^(-x) - 2*e^x - 1)/(sqrt(5) +
 2*e^(-x) + 2*e^x + 1)) + 1/2*e^(-x) + 1/2*e^x - 1/20*log((e^(-x) + e^x)^2
 + e^(-x) + e^x - 1) + 1/20*log((e^(-x) + e^x)^2 - e^(-x) - e^x - 1) - 1/1
0*log(e^(-x) + e^x + 2) + 1/10*log(e^(-x) + e^x - 2)
```

3.237.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.30

$$\int \cosh(x)\coth(5x)\,dx = \frac{\ln\left(10-10\,\mathrm{e}^{x}\right)}{5} - \frac{\ln\left(-10\,\mathrm{e}^{x}-10\right)}{5} + \frac{\mathrm{e}^{-x}}{2} + \frac{\mathrm{e}^{x}}{2} - \ln\left(-\mathrm{e}^{2\,x}-10\,\mathrm{e}^{x}\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)-1\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right) + \ln\left(10\,\mathrm{e}^{x}\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)-\mathrm{e}^{2\,x}-1\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right) - \ln\left(-\mathrm{e}^{2\,x}-10\,\mathrm{e}^{x}\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-1\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right) + \ln\left(10\,\mathrm{e}^{x}\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-\mathrm{e}^{2\,x}-1\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)$$

input
```
int(coth(5*x)*cosh(x),x)
```

output
```
log(10 - 10*exp(x))/5 - log(- 10*exp(x) - 10)/5 + exp(-x)/2 + exp(x)/2 - l
og(- exp(2*x) - 10*exp(x)*(5^(1/2)/20 - 1/20) - 1)*(5^(1/2)/20 - 1/20) + l
og(10*exp(x)*(5^(1/2)/20 - 1/20) - exp(2*x) - 1)*(5^(1/2)/20 - 1/20) - log
(- exp(2*x) - 10*exp(x)*(5^(1/2)/20 + 1/20) - 1)*(5^(1/2)/20 + 1/20) + log
(10*exp(x)*(5^(1/2)/20 + 1/20) - exp(2*x) - 1)*(5^(1/2)/20 + 1/20)
```

3.237. $\int \cosh(x) \coth(5x) \, dx$

3.238 $\int \cosh(x)\coth(6x)\,dx$

3.238.1 Optimal result

Integrand size = 7, antiderivative size = 38

$$\int \cosh(x)\coth(6x)\,dx = -\frac{1}{6}\text{arctanh}(\cosh(x)) - \frac{1}{6}\text{arctanh}(2\cosh(x)) - \frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \cosh(x)$$

output
```
-1/6*arctanh(cosh(x))-1/6*arctanh(2*cosh(x))+cosh(x)-1/6*arctanh(2/3*cosh(
x)*3^(1/2))*3^(1/2)
```

3.238.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int \cosh(x)\coth(6x)\,dx = \frac{1}{12}\left(-2\sqrt{3}\text{arctanh}\left(\frac{2-i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right) - 2\sqrt{3}\text{arctanh}\left(\frac{2+i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right) + 12\cosh(x) - 2\log\left(\cosh\left(\frac{x}{2}\right)\right) + \log(1-2\cosh(x)) - \log(1+2\cosh(x)) + 2\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cosh[x]*Coth[6*x],x]
```

output
```
(-2*Sqrt[3]*ArcTanh[(2 - I*Tanh[x/2])/Sqrt[3]] - 2*Sqrt[3]*ArcTanh[(2 + I*
Tanh[x/2])/Sqrt[3]] + 12*Cosh[x] - 2*Log[Cosh[x/2]] + Log[1 - 2*Cosh[x]] -
 Log[1 + 2*Cosh[x]] + 2*Log[Sinh[x/2]])/12
```

3.238.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 26, 4879, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\coth(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\cos(ix)\cot(6ix)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \cos(ix)\cot(6ix)dx$$

$$\Big\downarrow \text{4879}$$

$$-\int -\frac{-32\cosh^6(x)+48\cosh^4(x)-18\cosh^2(x)+1}{2\left(-16\cosh^6(x)+32\cosh^4(x)-19\cosh^2(x)+3\right)}d\cosh(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{-32\cosh^6(x)+48\cosh^4(x)-18\cosh^2(x)+1}{-16\cosh^6(x)+32\cosh^4(x)-19\cosh^2(x)+3}d\cosh(x)$$

$$\Big\downarrow \text{2460}$$

$$\frac{1}{2}\int \left(\frac{2}{4\cosh^2(x)-3}+\frac{2}{3\left(4\cosh^2(x)-1\right)}+2+\frac{1}{3\left(\cosh^2(x)-1\right)}\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{3}\text{arctanh}(\cosh(x))-\frac{1}{3}\text{arctanh}(2\cosh(x))-\frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}}+2\cosh(x)\right)$$

input
```
Int[Cosh[x]*Coth[6*x],x]
```

output
```
(-1/3*ArcTanh[Cosh[x]] - ArcTanh[2*Cosh[x]]/3 - ArcTanh[(2*Cosh[x])/Sqrt[3
]]/Sqrt[3] + 2*Cosh[x])/2
```

3.238.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.238.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(28) = 56.

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

method	result
risch	$\frac{e^{x}}{2}+\frac{e^{-x}}{2}-\frac{\ln(e^{x}+1)}{6}+\frac{\ln(e^{x}-1)}{6}+\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln(e^{2x}-e^{x}+1)}{12}-\frac{\ln(1+e^{x}+e^{2x})}{12}$

input
```
int(cosh(x)*coth(6*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)+1/2*exp(-x)-1/6*ln(exp(x)+1)+1/6*ln(exp(x)-1)+1/12*ln(1+exp(2*x
)-exp(x)*3^(1/2))*3^(1/2)-1/12*ln(1+exp(2*x)+exp(x)*3^(1/2))*3^(1/2)+1/12*
ln(exp(2*x)-exp(x)+1)-1/12*ln(1+exp(x)+exp(2*x))
```

3.238.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 4.13

$$\int \cosh(x)\coth(6x)\,dx$$

$$= \frac{6\cosh(x)^{2}+\left(\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)\right)\log\left(\frac{2\cosh(x)^{2}+2\sinh(x)^{2}-4\sqrt{3}\cosh(x)+5}{2\cosh(x)^{2}+2\sinh(x)^{2}-1}\right)-(\cosh(x)+\sinh(x))\log}{}$$

input
```
integrate(cosh(x)*coth(6*x),x, algorithm="fricas")
```

output
```
1/12*(6*cosh(x)^2 + (sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*log((2*cosh(x)^2 +
 2*sinh(x)^2 - 4*sqrt(3)*cosh(x) + 5)/(2*cosh(x)^2 + 2*sinh(x)^2 - 1)) - (
cosh(x) + sinh(x))*log((2*cosh(x) + 1)/(cosh(x) - sinh(x))) + (cosh(x) + s
inh(x))*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 2*(cosh(x) + sinh(x))*l
og(cosh(x) + sinh(x) + 1) + 2*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x) -
1) + 12*cosh(x)*sinh(x) + 6*sinh(x)^2 + 6)/(cosh(x) + sinh(x))
```

3.238.6 Sympy [F]

$$\int \cosh(x)\coth(6x)\,dx = \int \cosh{(x)}\coth{(6x)}\,dx$$

input
```
integrate(cosh(x)*coth(6*x),x)
```

output
```
Integral(cosh(x)*coth(6*x), x)
```

3.238.7 Maxima [F]

$$\int \cosh(x)\coth(6x)\,dx = \int \cosh{(x)}\coth{(6\,x)}\,dx$$

input
```
integrate(cosh(x)*coth(6*x),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) + 1)*e^(-x) + 1/2*integrate((e^(3*x) - e^x)/(e^(4*x) - e^(2*x
) + 1), x) - 1/12*log(e^(2*x) + e^x + 1) + 1/12*log(e^(2*x) - e^x + 1) - 1
/6*log(e^x + 1) + 1/6*log(e^x - 1)
```

3.238.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.34

$$\begin{aligned}\int \cosh(x)\coth(6x)\,dx &= \frac{1}{12}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}-e^{x}}{\sqrt{3}+e^{(-x)}+e^{x}}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}\\ &\quad-\frac{1}{12}\log\left(e^{(-x)}+e^{x}+2\right)-\frac{1}{12}\log\left(e^{(-x)}+e^{x}+1\right)\\ &\quad+\frac{1}{12}\log\left(e^{(-x)}+e^{x}-1\right)+\frac{1}{12}\log\left(e^{(-x)}+e^{x}-2\right)\end{aligned}$$

input
```
integrate(cosh(x)*coth(6*x),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(-(sqrt(3) - e^(-x) - e^x)/(sqrt(3) + e^(-x) + e^x)) + 1/2
*e^(-x) + 1/2*e^x - 1/12*log(e^(-x) + e^x + 2) - 1/12*log(e^(-x) + e^x + 1
) + 1/12*log(e^(-x) + e^x - 1) + 1/12*log(e^(-x) + e^x - 2)
```

3.238.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.66

$$\int \cosh(x)\coth(6x)\,dx = \frac{\ln\left(\frac{1}{3}-\frac{e^x}{3}\right)}{6} - \frac{\ln\left(-\frac{e^x}{3}-\frac{1}{3}\right)}{6} + \frac{e^{-x}}{2} - \frac{\ln\left(-\frac{e^{2x}}{36}-\frac{e^x}{36}-\frac{1}{36}\right)}{12} + \frac{\ln\left(\frac{e^x}{36}-\frac{e^{2x}}{36}-\frac{1}{36}\right)}{12} + \frac{e^x}{2} - \frac{\sqrt{3}\ln\left(-\frac{e^{2x}}{12}-\frac{\sqrt{3}\,e^x}{12}-\frac{1}{12}\right)}{12} + \frac{\sqrt{3}\ln\left(\frac{\sqrt{3}\,e^x}{12}-\frac{e^{2x}}{12}-\frac{1}{12}\right)}{12}$$

input
```
int(coth(6*x)*cosh(x),x)
```

output
```
log(1/3 - exp(x)/3)/6 - log(- exp(x)/3 - 1/3)/6 + exp(-x)/2 - log(- exp(2*
x)/36 - exp(x)/36 - 1/36)/12 + log(exp(x)/36 - exp(2*x)/36 - 1/36)/12 + ex
p(x)/2 - (3^(1/2)*log(- exp(2*x)/12 - (3^(1/2)*exp(x))/12 - 1/12))/12 + (3
^(1/2)*log((3^(1/2)*exp(x))/12 - exp(2*x)/12 - 1/12))/12
```

3.239 $\int \cosh(x)\coth(nx)\,dx$

3.239.1 Optimal result

Integrand size = 7, antiderivative size = 76

$$\int \cosh(x)\coth(nx)\,dx = -\frac{e^{-x}}{2}+\frac{e^{x}}{2}+e^{-x}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2nx}\right) - e^{x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),e^{2nx}\right)$$

output
```
-1/2/exp(x)+1/2*exp(x)+hypergeom([1, -1/2/n],[1-1/2/n],exp(2*n*x))/exp(x)-
exp(x)*hypergeom([1, 1/2/n],[1+1/2/n],exp(2*n*x))
```

3.239.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \cosh(x)\coth(nx)\,dx = \frac{1}{2}\left(-e^{-x}+e^{x}+2e^{-x}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2nx}\right) - 2e^{x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},1+\frac{1}{2n},e^{2nx}\right)\right)$$

input
```
Integrate[Cosh[x]*Coth[n*x],x]
```

output
```
(-E^(-x) + E^x + (2*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), E^(2*n*x)]
)/E^x - 2*E^x*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), E^(2*n*x)])/2
```

3.239.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6136, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\coth(nx)\,dx$$

$$\Big\downarrow \text{6136}$$

$$\int \left(-\frac{e^{-x}}{1-e^{2nx}}-\frac{e^{x}}{1-e^{2nx}}+\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^{-x}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2nx}\right)-e^{x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),e^{2nx}\right)-\frac{e^{-x}}{2}+\frac{e^{x}}{2}$$

input
```
Int[Cosh[x]*Coth[n*x],x]
```

output
```
-1/2*1/E^x + E^x/2 + Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), E^(2*n*x)
]/E^x - E^x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, E^(2*n*x)]
```

3.239.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6136
```
Int[Cosh[(a_.) + (b_.)*(x_)]*Coth[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(
E^(a + b*x)*2) + E^(a + b*x)/2 - 1/(E^(a + b*x)*(1 - E^(2*(c + d*x)))) - E^
(a + b*x)/(1 - E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

3.239.4 Maple [F]

$$\int \cosh(x)\coth(nx)\,dx$$

input
```
int(cosh(x)*coth(n*x),x)
```

output
```
int(cosh(x)*coth(n*x),x)
```

3.239.5 Fricas [F]

$$\int \cosh(x)\coth(nx)\,dx = \int \cosh(x)\coth(nx)\,dx$$

input
```
integrate(cosh(x)*coth(n*x),x, algorithm="fricas")
```

output
```
integral(cosh(x)*coth(n*x), x)
```

3.239.6 Sympy [F]

$$\int \cosh(x)\coth(nx)\,dx = \int \cosh(x)\coth(nx)\,dx$$

input
```
integrate(cosh(x)*coth(n*x),x)
```

output
```
Integral(cosh(x)*coth(n*x), x)
```

3.239.7 Maxima [F]

$$\int \cosh(x)\coth(nx)\,dx = \int \cosh\left(x\right)\coth\left(nx\right)\,dx$$

input
```
integrate(cosh(x)*coth(n*x),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) - 1)*e^(-x) - 1/2*integrate((e^(2*x) + 1)/(e^(n*x + x) + e^x)
, x) + 1/2*integrate((e^(2*x) + 1)/(e^(n*x + x) - e^x), x)
```

3.239.8 Giac [F]

$$\int \cosh(x)\coth(nx)\,dx = \int \cosh\left(x\right)\coth\left(nx\right)\,dx$$

input
```
integrate(cosh(x)*coth(n*x),x, algorithm="giac")
```

output
```
integrate(cosh(x)*coth(n*x), x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \cosh(x)\coth(nx)\,dx = \int \coth(n\,x)\,\cosh(x)\,dx$$

input
```
int(coth(n*x)*cosh(x),x)
```

output
```
int(coth(n*x)*cosh(x), x)
```

3.240 $\int \cosh(x)\mathbf{sech}(2x)\,dx$

3.240.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cosh(x)\text{sech}(2x)\,dx = \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

output
```
1/2*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh(x)\text{sech}(2x)\,dx = \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

input
```
Integrate[Cosh[x]*Sech[2*x],x]
```

output
```
ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

3.240.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4856, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\text{sech}(2x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ix)}{\cos(2ix)}dx$$

$$\Big\downarrow \; 4856$$

$$\int \frac{1}{2\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

input
```
Int[Cosh[x]*Sech[2*x],x]
```

output
```
ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

3.240.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.240.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

method	result	size
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+i\sqrt{2}\,\mathrm{e}^{x}-1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-i\sqrt{2}\,\mathrm{e}^{x}-1\right)}{4}$	44

input
```
int(cosh(x)*sech(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*exp(x)-1)-1/4*I*2^(1/2)*ln(exp(2*x)-I*
2^(1/2)*exp(x)-1)
```

3.240.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.53

$$\int \cosh(x)\mathrm{sech}(2x)\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\cosh(x)+\frac{1}{2}\sqrt{2}\sinh(x)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2+\sqrt{2}}{2(\cosh(x)-\sinh(x))}\right)$$

input
```
integrate(cosh(x)*sech(2*x),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*cosh(x) + 1/2*sqrt(2)*sinh(x)) - 1/2*sqrt(2
)*arctan(-1/2*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sin
h(x)^2 + sqrt(2))/(cosh(x) - sinh(x)))
```

3.240.6 Sympy [F]

$$\int \cosh(x)\text{sech}(2x)\,dx = \int \cosh(x)\,\text{sech}\,(2x)\,dx$$

input
```
integrate(cosh(x)*sech(2*x),x)
```

output
```
Integral(cosh(x)*sech(2*x), x)
```

3.240.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int \cosh(x)\text{sech}(2x)\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)$$

input
```
integrate(cosh(x)*sech(2*x),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x)))
```

3.240.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \cosh(x)\text{sech}(2x)\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)$$

input
```
integrate(cosh(x)*sech(2*x),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x))
```

3.240.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \cosh(x)\mathrm{sech}(2x)\,dx = \frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{\sqrt{2}\,\mathrm{e}^{x}}{2}+\frac{\sqrt{2}\,\mathrm{e}^{3\,x}}{2}\right)+\mathrm{atan}\left(\frac{\sqrt{2}\,\mathrm{e}^{x}}{2}\right)\right)}{2}$$

input
```
int(cosh(x)/cosh(2*x),x)
```

output
```
(2^(1/2)*(atan((2^(1/2)*exp(x))/2 + (2^(1/2)*exp(3*x))/2) + atan((2^(1/2)*
exp(x))/2)))/2
```

3.241 $\int \cosh(x)\textbf{sech}(3x)\,dx$

3.241.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cosh(x)\text{sech}(3x)\,dx = \frac{\arctan\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}}$$

output
```
1/3*arctan(tanh(x)*3^(1/2))*3^(1/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh(x)\text{sech}(3x)\,dx = \frac{\arctan\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}}$$

input
```
Integrate[Cosh[x]*Sech[3*x],x]
```

output
```
ArcTan[Sqrt[3]*Tanh[x]]/Sqrt[3]
```

3.241.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\text{sech}(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ix)}{\cos(3ix)}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{1}{3\tanh^2(x)+1}d\tanh(x)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}}$$

input
```
Int[Cosh[x]*Sech[3*x],x]
```

output
```
ArcTan[Sqrt[3]*Tanh[x]]/Sqrt[3]
```

3.241.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.241.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.67

method	result	size
risch	$\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}$	40

input
```
int(cosh(x)*sech(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*3^(1/2)*ln(exp(2*x)-1/2+1/2*I*3^(1/2))-1/6*I*3^(1/2)*ln(exp(2*x)-1/2
-1/2*I*3^(1/2))
```

3.241.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \cosh(x)\mathrm{sech}(3x)\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh(x)+3\sqrt{3}\sinh(x)}{3\left(\cosh(x)-\sinh(x)\right)}\right)$$

input
```
integrate(cosh(x)*sech(3*x),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + 3*sqrt(3)*sinh(x))/(cosh(x) -
sinh(x)))
```

3.241.6 Sympy [F]

$$\int \cosh(x)\mathrm{sech}(3x)\,dx = \int \cosh(x)\,\mathrm{sech}(3x)\,dx$$

input
```
integrate(cosh(x)*sech(3*x),x)
```

output
```
Integral(cosh(x)*sech(3*x), x)
```

3.241.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 7.60

$$\begin{aligned}\int \cosh(x)\mathrm{sech}(3x)\,dx =& -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{(-2x)}-1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\sqrt{3}+2e^{x}\right)\\ &+\frac{1}{6}\sqrt{3}\arctan\left(-\sqrt{3}+2e^{x}\right)+\frac{1}{12}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)\\ &+\frac{1}{12}\log\left(-\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{1}{6}\log\left(e^{(2x)}+1\right)\\ &+\frac{1}{6}\log\left(e^{(-2x)}+1\right)-\frac{1}{12}\log\left(-e^{(-2x)}+e^{(-4x)}+1\right)\end{aligned}$$

input
```
integrate(cosh(x)*sech(3*x),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-2*x) - 1)) - 1/6*sqrt(3)*arctan(sqr
t(3) + 2*e^x) + 1/6*sqrt(3)*arctan(-sqrt(3) + 2*e^x) + 1/12*log(sqrt(3)*e^
x + e^(2*x) + 1) + 1/12*log(-sqrt(3)*e^x + e^(2*x) + 1) - 1/6*log(e^(2*x)
+ 1) + 1/6*log(e^(-2*x) + 1) - 1/12*log(-e^(-2*x) + e^(-4*x) + 1)
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cosh(x)\text{sech}(3x)\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}-1\right)\right)$$

input
```
integrate(cosh(x)*sech(3*x),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) - 1))
```

3.241.9 Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cosh(x)\text{sech}(3x)\,dx = \frac{\sqrt{3}\,\text{atan}\left(\frac{\sqrt{3}\,(2\,e^{2\,x}-1)}{3}\right)}{3}$$

input
```
int(cosh(x)/cosh(3*x),x)
```

output
```
(3^(1/2)*atan((3^(1/2)*(2*exp(2*x) - 1))/3))/3
```

3.242 $\int \cosh(x)\mathbf{sech}(4x)\,dx$

3.242.1 Optimal result

Integrand size = 7, antiderivative size = 71

$$\int \cosh(x)\text{sech}(4x)\,dx = \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

output
```
1/2*arctan(2*sinh(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/2*arctan(2*s
inh(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \cosh(x)\text{sech}(4x)\,dx = \frac{1}{4}\sqrt{2+\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

input
```
Integrate[Cosh[x]*Sech[4*x],x]
```

output
```
(Sqrt[2 + Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]])/4 - ArcTan[(2*Si
nh[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

3.242.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4856, 1406, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\text{sech}(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)}{\cos(4ix)}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{1}{8\sinh^4(x)+8\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{1406}$$

$$\sqrt{2}\int \frac{1}{8\sinh^2(x)+2\left(2-\sqrt{2}\right)}d\sinh(x)-\sqrt{2}\int \frac{1}{8\sinh^2(x)+2\left(2+\sqrt{2}\right)}d\sinh(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

input
```
Int[Cosh[x]*Sech[4*x],x]
```

output
```
ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(
2*Sinh[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

3.242.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.242.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result	size
risch	$2\left(\sum_{\_R=\mathrm{RootOf}\left(32768\_Z^4+512\_Z^2+1\right)}\_R\ln\left(\mathrm{e}^{2x}+\left(-4096\_R^3-48\_R\right)\mathrm{e}^{x}-1\right)\right)$	40

input
```
int(cosh(x)*sech(4*x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(exp(2*x)+(-4096*_R^3-48*_R)*exp(x)-1),_R=RootOf(32768*_Z^4+512
*_Z^2+1))
```

3.242.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.03

$$\int \cosh(x)\mathrm{sech}(4x)\,dx = -\frac{1}{8}\sqrt{\sqrt{2}-2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + \left(\left(\sqrt{2}+1\right)\cosh(x) + \left(\sqrt{2}+1\right)\sinh(x)\right)\sqrt{\sqrt{2}-2} - 1\right) + \frac{1}{8}\sqrt{\sqrt{2}-2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - \left(\left(\sqrt{2}+1\right)\cosh(x) + \left(\sqrt{2}+1\right)\sinh(x)\right)\sqrt{\sqrt{2}-2} - 1\right) + \frac{1}{8}\sqrt{-\sqrt{2}-2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + \left(\left(\sqrt{2}-1\right)\cosh(x) + \left(\sqrt{2}-1\right)\sinh(x)\right)\sqrt{-\sqrt{2}-2} - 1\right) - \frac{1}{8}\sqrt{-\sqrt{2}-2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - \left(\left(\sqrt{2}-1\right)\cosh(x) + \left(\sqrt{2}-1\right)\sinh(x)\right)\sqrt{-\sqrt{2}-2} - 1\right)$$

input
```
integrate(cosh(x)*sech(4*x),x, algorithm="fricas")
```

output
```
-1/8*sqrt(sqrt(2) - 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((s
qrt(2) + 1)*cosh(x) + (sqrt(2) + 1)*sinh(x))*sqrt(sqrt(2) - 2) - 1) + 1/8*
sqrt(sqrt(2) - 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(2
) + 1)*cosh(x) + (sqrt(2) + 1)*sinh(x))*sqrt(sqrt(2) - 2) - 1) + 1/8*sqrt(
-sqrt(2) - 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(2) -
1)*cosh(x) + (sqrt(2) - 1)*sinh(x))*sqrt(-sqrt(2) - 2) - 1) - 1/8*sqrt(-sq
rt(2) - 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(2) - 1)*
cosh(x) + (sqrt(2) - 1)*sinh(x))*sqrt(-sqrt(2) - 2) - 1)
```

3.242.6 Sympy [F]

$$\int \cosh(x)\text{sech}(4x)\,dx = \int \cosh(x)\,\text{sech}(4x)\,dx$$

input
```
integrate(cosh(x)*sech(4*x),x)
```

output
```
Integral(cosh(x)*sech(4*x), x)
```

3.242.7 Maxima [F]

$$\int \cosh(x)\text{sech}(4x)\,dx = \int \cosh(x)\,\text{sech}(4\,x)\,\,dx$$

input
```
integrate(cosh(x)*sech(4*x),x, algorithm="maxima")
```

output
```
integrate(cosh(x)*sech(4*x), x)
```

3.242.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(49) = 98.

Time = 0.36 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.90

$$\begin{aligned}\int \cosh(x)\text{sech}(4x)\,dx &= \frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\,e^x}{\sqrt{-\sqrt{2}+2}}\right) \\ &\quad+\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\,e^x}{\sqrt{-\sqrt{2}+2}}\right) \\ &\quad-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2\,e^x}{\sqrt{\sqrt{2}+2}}\right) \\ &\quad-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2\,e^x}{\sqrt{\sqrt{2}+2}}\right)\end{aligned}$$

input
```
integrate(cosh(x)*sech(4*x),x, algorithm="giac")
```

output
```
1/4*sqrt(sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) + 2
)) + 1/4*sqrt(sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(
2) + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqrt
(sqrt(2) + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^
x)/sqrt(sqrt(2) + 2))
```

3.242.9 Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.77

$$\int \cosh(x)\text{sech}(4x)\,dx = \frac{\text{atan}\left(\frac{3\,e^{2\,x}-2\,\sqrt{2}+2\,\sqrt{2}\,e^{2\,x}-3}{e^x\,\sqrt{\sqrt{2}+2}+\sqrt{2}\,e^x\,\sqrt{\sqrt{2}+2}}\right)\,\sqrt{\sqrt{2}+2}}{4} + \frac{\text{atan}\left(\frac{3\,e^{2\,x}+2\,\sqrt{2}-2\,\sqrt{2}\,e^{2\,x}-3}{e^x\,\sqrt{2-\sqrt{2}}-\sqrt{2}\,e^x\,\sqrt{2-\sqrt{2}}}\right)\,\sqrt{2-\sqrt{2}}}{4}$$

input
```
int(cosh(x)/cosh(4*x),x)
```

output
```
(atan((3*exp(2*x) - 2*2^(1/2) + 2*2^(1/2)*exp(2*x) - 3)/(exp(x)*(2^(1/2) +
 2)^(1/2) + 2^(1/2)*exp(x)*(2^(1/2) + 2)^(1/2)))*(2^(1/2) + 2)^(1/2))/4 +
(atan((3*exp(2*x) + 2*2^(1/2) - 2*2^(1/2)*exp(2*x) - 3)/(exp(x)*(2 - 2^(1/
2))^(1/2) - 2^(1/2)*exp(x)*(2 - 2^(1/2))^(1/2)))*(2 - 2^(1/2))^(1/2))/4
```

3.243 $\int \cosh(x)\mathbf{sech}(5x)\,dx$

3.243.1 Optimal result

Integrand size = 7, antiderivative size = 75

$$\int \cosh(x)\text{sech}(5x)\,dx = -\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{5-2\sqrt{5}}\tanh(x)\right)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\sqrt{5+2\sqrt{5}}\tanh(x)\right)$$

output
```
-1/10*arctan((5-2*5^(1/2))^(1/2)*tanh(x))*(10-2*5^(1/2))^(1/2)+1/10*arctan
((5+2*5^(1/2))^(1/2)*tanh(x))*(10+2*5^(1/2))^(1/2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int \cosh(x)\text{sech}(5x)\,dx = \frac{\sqrt{5+\sqrt{5}}\arctan\left(\frac{\left(5+\sqrt{5}\right)\tanh(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\arctan\left(\frac{\left(-5+\sqrt{5}\right)\tanh(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

input
```
Integrate[Cosh[x]*Sech[5*x],x]
```

output
```
(Sqrt[5 + Sqrt[5]]*ArcTan[((5 + Sqrt[5])*Tanh[x])/Sqrt[10 - 2*Sqrt[5]]] +
Sqrt[5 - Sqrt[5]]*ArcTan[((-5 + Sqrt[5])*Tanh[x])/Sqrt[2*(5 + Sqrt[5])]])/
(5*Sqrt[2])
```

3.243.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4889, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\text{sech}(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)}{\cos(5ix)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1-\tanh^2(x)}{5\tanh^4(x)+10\tanh^2(x)+1}d\tanh(x)$$

$$\Big\downarrow \text{1480}$$

$$-\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{1}{5\tanh^2(x)-2\sqrt{5}+5}d\tanh(x)-\frac{1}{2}\left(1+\sqrt{5}\right)\int \frac{1}{5\tanh^2(x)+2\sqrt{5}+5}d\tanh(x)$$

$$\Big\downarrow \text{216}$$

$$-\frac{(1+\sqrt{5})\arctan\left(\sqrt{5-2\sqrt{5}}\tanh(x)\right)}{2\sqrt{5\left(5+2\sqrt{5}\right)}}-\frac{(1-\sqrt{5})\arctan\left(\sqrt{5+2\sqrt{5}}\tanh(x)\right)}{2\sqrt{5\left(5-2\sqrt{5}\right)}}$$

input
```
Int[Cosh[x]*Sech[5*x],x]
```

output
```
-1/2*((1 + Sqrt[5])*ArcTan[Sqrt[5 - 2*Sqrt[5]]*Tanh[x]])/Sqrt[5*(5 + 2*Sqr
t[5])] - ((1 - Sqrt[5])*ArcTan[Sqrt[5 + 2*Sqrt[5]]*Tanh[x]])/(2*Sqrt[5*(5
- 2*Sqrt[5])])
```

3.243.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.243.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
risch	$2\left(\sum_{\_R=\operatorname{RootOf}\left(32000\_Z^4+400\_Z^2+1\right)} \_R\ln\left(-4000\_R^3+200\_R^2+e^{2x}-30\_R+1\right)\right)$	41

input
```
int(cosh(x)*sech(5*x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(-4000*_R^3+200*_R^2+exp(2*x)-30*_R+1),_R=RootOf(32000*_Z^4+400
*_Z^2+1))
```

3.243.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(49) = 98.

Time = 0.25 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.24

$$\int \cosh(x)\mathrm{sech}(5x)\,dx = -\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}-5}\log\left(8\cosh(x)^2+16\cosh(x)\sinh(x)+8\sinh(x)^2+\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-5}+2\sqrt{5}-2\right)+\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}-5}\log\left(8\cosh(x)^2+16\cosh(x)\sinh(x)+8\sinh(x)^2-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-5}+2\sqrt{5}-2\right)+\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}-5}\log\left(8\cosh(x)^2+16\cosh(x)\sinh(x)+8\sinh(x)^2+\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-5}-2\sqrt{5}-2\right)-\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}-5}\log\left(8\cosh(x)^2+16\cosh(x)\sinh(x)+8\sinh(x)^2-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-5}-2\sqrt{5}-2\right)$$

input
```
integrate(cosh(x)*sech(5*x),x, algorithm="fricas")
```

output
```
-1/20*sqrt(2)*sqrt(sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8*s
inh(x)^2 + (sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 5) + 2*sqrt(5) - 2)
+ 1/20*sqrt(2)*sqrt(sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8*
sinh(x)^2 - (sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 5) + 2*sqrt(5) - 2)
 + 1/20*sqrt(2)*sqrt(-sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) +
8*sinh(x)^2 + (sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(5) - 5) - 2*sqrt(5) -
 2) - 1/20*sqrt(2)*sqrt(-sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x)
 + 8*sinh(x)^2 - (sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(5) - 5) - 2*sqrt(5
) - 2)
```

3.243.6 Sympy [F]

$$\int \cosh(x)\text{sech}(5x)\,dx = \int \cosh(x)\,\text{sech}\,(5x)\,dx$$

input
```
integrate(cosh(x)*sech(5*x),x)
```

output
```
Integral(cosh(x)*sech(5*x), x)
```

3.243.7 Maxima [F]

$$\int \cosh(x)\text{sech}(5x)\,dx = \int \cosh(x)\,\text{sech}\,(5\,x)\,\,dx$$

input
```
integrate(cosh(x)*sech(5*x),x, algorithm="maxima")
```

output
```
1/5*sqrt(5)*arctan((sqrt(5) + 4*e^(-2*x) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2
*sqrt(5) + 10) - 1/5*sqrt(5)*arctan(-(sqrt(5) - 4*e^(-2*x) + 1)/sqrt(-2*sq
rt(5) + 10))/sqrt(-2*sqrt(5) + 10) - 1/10*log(-(sqrt(5) + 1)*e^(-2*x) + 2*
e^(-4*x) + 2)/(sqrt(5) + 1) + 1/10*log((sqrt(5) - 1)*e^(-2*x) + 2*e^(-4*x)
 + 2)/(sqrt(5) - 1) - 1/5*integrate((e^(7*x) - 2*e^(5*x) - 2*e^(3*x) + e^x
)*e^x/(e^(8*x) - e^(6*x) + e^(4*x) - e^(2*x) + 1), x) + 1/10*log(e^(2*x) +
 1) - 1/10*log(e^(-2*x) + 1)
```

3.243.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \cosh(x)\text{sech}(5x)\,dx = &-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4\,e^{(2\,x)}-1}{\sqrt{2\sqrt{5}+10}}\right)\\&+\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4\,e^{(2\,x)}+1}{\sqrt{-2\sqrt{5}+10}}\right)\end{aligned}$$

input
```
integrate(cosh(x)*sech(5*x),x, algorithm="giac")
```

output
```
-1/10*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*e^(2*x) - 1)/sqrt(2*sqrt(5
) + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*e^(2*x) + 1)/sqr
t(-2*sqrt(5) + 10))
```

3.243.9 Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.96

$$\int \cosh(x)\mathrm{sech}(5x)\,dx = \ln\left(1-\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(4\,\mathrm{e}^{2\,x}+\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(48\,\mathrm{e}^{2\,x}+\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(360\,\mathrm{e}^{2\,x}-360\right)-72\right)-8\right)-\mathrm{e}^{2\,x}\right)\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}-\ln\left(\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(4\,\mathrm{e}^{2\,x}+\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(360\,\mathrm{e}^{2\,x}-360\right)-48\,\mathrm{e}^{2\,x}+72\right)-8\right)-\mathrm{e}^{2\,x}+1\right)\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}-\ln\left(\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(4\,\mathrm{e}^{2\,x}+\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(360\,\mathrm{e}^{2\,x}-360\right)-48\,\mathrm{e}^{2\,x}+72\right)-8\right)-\mathrm{e}^{2\,x}+1\right)\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}+\ln\left(1-\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(4\,\mathrm{e}^{2\,x}+\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(48\,\mathrm{e}^{2\,x}+\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(360\,\mathrm{e}^{2\,x}-360\right)-72\right)-8\right)-\mathrm{e}^{2\,x}\right)\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}$$

input
```
int(cosh(x)/cosh(5*x),x)
```

output
```
log(1 - (- 5^(1/2)/200 - 1/40)^(1/2)*(4*exp(2*x) + (- 5^(1/2)/200 - 1/40)^
(1/2)*(48*exp(2*x) + (- 5^(1/2)/200 - 1/40)^(1/2)*(360*exp(2*x) - 360) - 7
2) - 8) - exp(2*x))*(- 5^(1/2)/200 - 1/40)^(1/2) - log((5^(1/2)/200 - 1/40
)^(1/2)*(4*exp(2*x) + (5^(1/2)/200 - 1/40)^(1/2)*((5^(1/2)/200 - 1/40)^(1/
2)*(360*exp(2*x) - 360) - 48*exp(2*x) + 72) - 8) - exp(2*x) + 1)*(5^(1/2)/
200 - 1/40)^(1/2) - log((- 5^(1/2)/200 - 1/40)^(1/2)*(4*exp(2*x) + (- 5^(1
/2)/200 - 1/40)^(1/2)*((- 5^(1/2)/200 - 1/40)^(1/2)*(360*exp(2*x) - 360) -
 48*exp(2*x) + 72) - 8) - exp(2*x) + 1)*(- 5^(1/2)/200 - 1/40)^(1/2) + log
(1 - (5^(1/2)/200 - 1/40)^(1/2)*(4*exp(2*x) + (5^(1/2)/200 - 1/40)^(1/2)*(
48*exp(2*x) + (5^(1/2)/200 - 1/40)^(1/2)*(360*exp(2*x) - 360) - 72) - 8) -
 exp(2*x))*(5^(1/2)/200 - 1/40)^(1/2)
```

3.243. $\int \cosh(x)\text{sech}(5x)\,dx$

3.244 $\int \cosh(x)\text{sech}(6x)\,dx$

3.244.1 Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \cosh(x)\text{sech}(6x)\,dx=-\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

output
```
-1/6*arctan(sinh(x)*2^(1/2))*2^(1/2)+1/6*arctan(2*sinh(x)/(1/2*6^(1/2)-1/2
*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctan(2*sinh(x)/(1/2*6^(1/2)+1/2
*2^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

3.244.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \cosh(x)\text{sech}(6x)\,dx=\frac{1}{6}\left(-\sqrt{2}\arctan\left(\sqrt{2}\sinh(x)\right)+\sqrt{2+\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2-\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)\right)$$

input
```
Integrate[Cosh[x]*Sech[6*x],x]
```

output
```
(-(Sqrt[2]*ArcTan[Sqrt[2]*Sinh[x]]) + Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])
/Sqrt[2 - Sqrt[3]]] + Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[3
]]])/6
```

3.244.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4856, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\text{sech}(6x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)}{\cos(6ix)}dx$$
$$\Big\downarrow \text{4856}$$
$$\int \frac{1}{32\sinh^6(x)+48\sinh^4(x)+18\sinh^2(x)+1}d\sinh(x)$$
$$\Big\downarrow \text{2460}$$
$$\int \left(\frac{4(2\sinh^2(x)+1)}{3\left(16\sinh^4(x)+16\sinh^2(x)+1\right)}-\frac{1}{3\left(2\sinh^2(x)+1\right)}\right)d\sinh(x)$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

input
```
Int[Cosh[x]*Sech[6*x],x]
```

output
```
-1/3*ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2] + ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[3]
]]/(6*Sqrt[2 - Sqrt[3]]) + ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2
 + Sqrt[3]])
```

3.244.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.244.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

method	result
risch	$2\left(\sum_{\_R=\mathrm{RootOf}\left(331776\_Z^{4}+2304\_Z^{2}+1\right)}\_R\ln\left(\mathrm{e}^{2x}+\left(13824\_R^{3}+96\_R\right)\mathrm{e}^{x}-1\right)\right)+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-i\sqrt{2}\,\mathrm{e}^{x}-\right.}{12}$

input
```
int(cosh(x)*sech(6*x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(exp(2*x)+(13824*_R^3+96*_R)*exp(x)-1),_R=RootOf(331776*_Z^4+23
04*_Z^2+1))+1/12*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)-1/12*I*2^(1/2)*
ln(exp(2*x)+I*2^(1/2)*exp(x)-1)
```

3.244.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(67) = 134.

Time = 0.27 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.32

$$\int \cosh(x)\mathrm{sech}(6x)\,dx$$
$$= -\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\cosh(x) + \frac{1}{2}\sqrt{2}\sinh(x)\right) + \frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 + \sqrt{2}}{2(\cosh(x) - \sinh(x))}\right) + \frac{1}{12}\sqrt{\sqrt{3}-2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + \left(\left(\sqrt{3}+2\right)\cosh(x) + \left(\sqrt{3}+2\right)\sinh(x)\right)\sqrt{\sqrt{3}-2} - 1\right) - \frac{1}{12}\sqrt{\sqrt{3}-2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - \left(\left(\sqrt{3}+2\right)\cosh(x) + \left(\sqrt{3}+2\right)\sinh(x)\right)\sqrt{\sqrt{3}-2} - 1\right) - \frac{1}{12}\sqrt{-\sqrt{3}-2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + \left(\left(\sqrt{3}-2\right)\cosh(x) + \left(\sqrt{3}-2\right)\sinh(x)\right)\sqrt{-\sqrt{3}-2} - 1\right) + \frac{1}{12}\sqrt{-\sqrt{3}-2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - \left(\left(\sqrt{3}-2\right)\cosh(x) + \left(\sqrt{3}-2\right)\sinh(x)\right)\sqrt{-\sqrt{3}-2} - 1\right)$$

input
```
integrate(cosh(x)*sech(6*x),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*arctan(1/2*sqrt(2)*cosh(x) + 1/2*sqrt(2)*sinh(x)) + 1/6*sqrt(
2)*arctan(-1/2*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*si
nh(x)^2 + sqrt(2))/(cosh(x) - sinh(x))) + 1/12*sqrt(sqrt(3) - 2)*log(cosh(
x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(3) + 2)*cosh(x) + (sqrt(3) +
 2)*sinh(x))*sqrt(sqrt(3) - 2) - 1) - 1/12*sqrt(sqrt(3) - 2)*log(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(3) + 2)*cosh(x) + (sqrt(3) + 2)*
sinh(x))*sqrt(sqrt(3) - 2) - 1) - 1/12*sqrt(-sqrt(3) - 2)*log(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(3) - 2)*cosh(x) + (sqrt(3) - 2)*sin
h(x))*sqrt(-sqrt(3) - 2) - 1) + 1/12*sqrt(-sqrt(3) - 2)*log(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(3) - 2)*cosh(x) + (sqrt(3) - 2)*sinh(
x))*sqrt(-sqrt(3) - 2) - 1)
```

3.244.6 Sympy [F]

$$\int \cosh(x)\text{sech}(6x)\,dx = \int \cosh{(x)}\,\text{sech}\,(6x)\,dx$$

input
```
integrate(cosh(x)*sech(6*x),x)
```

output
```
Integral(cosh(x)*sech(6*x), x)
```

3.244.7 Maxima [F]

$$\int \cosh(x)\text{sech}(6x)\,dx = \int \cosh{(x)}\,\text{sech}\,(6\,x)\,\,dx$$

input
```
integrate(cosh(x)*sech(6*x),x, algorithm="maxima")
```

output
```
-1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/6*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + integrate(1/3*(e^(7*x) + e^(5*x) + e^(3*x)
 + e^x)/(e^(8*x) - e^(4*x) + 1), x)
```

3.244.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(67) = 134.

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.08

$$\int \cosh(x)\operatorname{sech}(6x)\,dx = \frac{1}{12}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{\sqrt{6}-\sqrt{2}+4\,e^{x}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{12}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(-\frac{\sqrt{6}-\sqrt{2}-4\,e^{x}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{12}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{\sqrt{6}+\sqrt{2}+4\,e^{x}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{12}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(-\frac{\sqrt{6}+\sqrt{2}-4\,e^{x}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{x}\right)\right) - \frac{1}{6}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{x}\right)\right)$$

input
```
integrate(cosh(x)*sech(6*x),x, algorithm="giac")
```

output
```
1/12*(sqrt(6) - sqrt(2))*arctan((sqrt(6) - sqrt(2) + 4*e^x)/(sqrt(6) + sqr
t(2))) + 1/12*(sqrt(6) - sqrt(2))*arctan(-(sqrt(6) - sqrt(2) - 4*e^x)/(sqr
t(6) + sqrt(2))) + 1/12*(sqrt(6) + sqrt(2))*arctan((sqrt(6) + sqrt(2) + 4*
e^x)/(sqrt(6) - sqrt(2))) + 1/12*(sqrt(6) + sqrt(2))*arctan(-(sqrt(6) + sq
rt(2) - 4*e^x)/(sqrt(6) - sqrt(2))) - 1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt
(2) + 2*e^x)) - 1/6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^x))
```

3.244.9 Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.42

$$\int \cosh(x)\operatorname{sech}(6x)\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{7\,\mathrm{e}^{2\,x}+4\,\sqrt{3}-4\,\sqrt{3}\,\mathrm{e}^{2\,x}-7}{\frac{5\,\sqrt{2}\,\mathrm{e}^{x}}{2}-\frac{3\,\sqrt{6}\,\mathrm{e}^{x}}{2}}\right)}{12} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{7\,\mathrm{e}^{2\,x}-4\,\sqrt{3}+4\,\sqrt{3}\,\mathrm{e}^{2\,x}-7}{\frac{5\,\sqrt{2}\,\mathrm{e}^{x}}{2}+\frac{3\,\sqrt{6}\,\mathrm{e}^{x}}{2}}\right)}{12} - \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{7\,\mathrm{e}^{2\,x}+4\,\sqrt{3}-4\,\sqrt{3}\,\mathrm{e}^{2\,x}-7}{\frac{5\,\sqrt{2}\,\mathrm{e}^{x}}{2}-\frac{3\,\sqrt{6}\,\mathrm{e}^{x}}{2}}\right)}{12} + \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{7\,\mathrm{e}^{2\,x}-4\,\sqrt{3}+4\,\sqrt{3}\,\mathrm{e}^{2\,x}-7}{\frac{5\,\sqrt{2}\,\mathrm{e}^{x}}{2}+\frac{3\,\sqrt{6}\,\mathrm{e}^{x}}{2}}\right)}{12} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\mathrm{e}^{-x}\,\left(\mathrm{e}^{2\,x}-1\right)}{2}\right)}{6}$$

input
```
int(cosh(x)/cosh(6*x),x)
```

output
```
(2^(1/2)*atan((7*exp(2*x) + 4*3^(1/2) - 4*3^(1/2)*exp(2*x) - 7)/((5*2^(1/2
)*exp(x))/2 - (3*6^(1/2)*exp(x))/2)))/12 + (2^(1/2)*atan((7*exp(2*x) - 4*3
^(1/2) + 4*3^(1/2)*exp(2*x) - 7)/((5*2^(1/2)*exp(x))/2 + (3*6^(1/2)*exp(x)
)/2)))/12 - (6^(1/2)*atan((7*exp(2*x) + 4*3^(1/2) - 4*3^(1/2)*exp(2*x) - 7
)/((5*2^(1/2)*exp(x))/2 - (3*6^(1/2)*exp(x))/2)))/12 + (6^(1/2)*atan((7*ex
p(2*x) - 4*3^(1/2) + 4*3^(1/2)*exp(2*x) - 7)/((5*2^(1/2)*exp(x))/2 + (3*6^
(1/2)*exp(x))/2)))/12 - (2^(1/2)*atan((2^(1/2)*exp(-x)*(exp(2*x) - 1))/2))
/6
```

3.244. $\int \cosh(x)\text{sech}(6x)\,dx$

3.245 $\int \cosh(x)\textbf{csch}(2x)\,dx$

3.245.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \cosh(x)\text{csch}(2x)\,dx = -\frac{1}{2}\text{arctanh}(\cosh(x))$$

output
```
-1/2*arctanh(cosh(x))
```

3.245.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 3.00

$$\int \cosh(x)\text{csch}(2x)\,dx = \frac{1}{2}\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cosh[x]*Csch[2*x],x]
```

output
```
(-Log[Cosh[x/2]] + Log[Sinh[x/2]])/2
```

3.245.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 26, 4775, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\text{csch}(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos(ix)}{\sin(2ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos(ix)}{\sin(2ix)}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{1}{2}i\int -i\text{csch}(x)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \text{csch}(x)dx}{2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int i\csc(ix)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}i\int \csc(ix)dx$$

$$\Big\downarrow \text{4257}$$

$$-\frac{1}{2}\text{arctanh}(\cosh(x))$$

input
```
Int[Cosh[x]*Csch[2*x],x]
```

output
```
-1/2*ArcTanh[Cosh[x]]
```

3.245.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.245.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$	8
risch	$\frac{\ln(e^{x}-1)}{2}-\frac{\ln(e^{x}+1)}{2}$	16

input
```
int(cosh(x)*csch(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(tanh(1/2*x))
```

3.245.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(5) = 10.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.71

$$\int \cosh(x)\operatorname{csch}(2x)\,dx = -\frac{1}{2}\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\frac{1}{2}\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)$$

input
```
integrate(cosh(x)*csch(2*x),x, algorithm="fricas")
```

output
```
-1/2*log(cosh(x) + sinh(x) + 1) + 1/2*log(cosh(x) + sinh(x) - 1)
```

3.245.6 Sympy [F]

$$\int \cosh(x)\operatorname{csch}(2x)\,dx = \int \cosh\left(x\right)\operatorname{csch}\left(2x\right)\,dx$$

input
```
integrate(cosh(x)*csch(2*x),x)
```

output
```
Integral(cosh(x)*csch(2*x), x)
```

3.245.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(5) = 10.

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.71

$$\int \cosh(x)\operatorname{csch}(2x)\,dx = -\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

input
```
integrate(cosh(x)*csch(2*x),x, algorithm="maxima")
```

output
```
-1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

3.245.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(5) = 10.

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.29

$$\int \cosh(x)\operatorname{csch}(2x)\,dx = -\frac{1}{2}\log\left(e^x+1\right)+\frac{1}{2}\log\left(|e^x-1|\right)$$

input
```
integrate(cosh(x)*csch(2*x),x, algorithm="giac")
```

output
```
-1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

3.245.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.71

$$\int \cosh(x)\operatorname{csch}(2x)\,dx = \frac{\ln\left(1-\mathrm{e}^x\right)}{2}-\frac{\ln\left(-\mathrm{e}^x-1\right)}{2}$$

input
```
int(cosh(x)/sinh(2*x),x)
```

output
```
log(1 - exp(x))/2 - log(- exp(x) - 1)/2
```

3.246 $\int \cosh(x)\mathbf{csch}(3x)\,dx$

3.246.1 Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \cosh(x)\text{csch}(3x)\,dx = \frac{1}{3}\log(\sinh(x)) - \frac{1}{6}\log\left(3+4\sinh^2(x)\right)$$

output
```
1/3*ln(sinh(x))-1/6*ln(3+4*sinh(x)^2)
```

3.246.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cosh(x)\text{csch}(3x)\,dx = \frac{1}{3}\log(\sinh(x)) - \frac{1}{6}\log\left(3+4\sinh^2(x)\right)$$

input
```
Integrate[Cosh[x]*Csch[3*x],x]
```

output
```
Log[Sinh[x]]/3 - Log[3 + 4*Sinh[x]^2]/6
```

3.246.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3042, 26, 4856, 26, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\text{csch}(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\cos(ix)}{\sin(3ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\cos(ix)}{\sin(3ix)}dx$$

$$\Big\downarrow \text{ 4856}$$

$$i\int -\frac{i\text{csch}(x)}{4\sinh^2(x)+3}d\sinh(x)$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\text{csch}(x)}{4\sinh^2(x)+3}d\sinh(x)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{\text{csch}(x)}{4\sinh^2(x)+3}d\sinh^2(x)$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \text{csch}(x)d\sinh^2(x)-\frac{4}{3}\int \frac{1}{4\sinh^2(x)+3}d\sinh^2(x)\right)$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2}\left(\frac{1}{3}\log\left(\sinh^2(x)\right)-\frac{4}{3}\int \frac{1}{4\sinh^2(x)+3}d\sinh^2(x)\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}\left(\frac{1}{3}\log\left(\sinh^2(x)\right)-\frac{1}{3}\log\left(4\sinh^2(x)+3\right)\right)$$

input
```
Int[Cosh[x]*Csch[3*x],x]
```

output
```
(Log[Sinh[x]^2]/3 - Log[3 + 4*Sinh[x]^2]/3)/2
```

3.246.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.246. $\int \cosh(x)\text{csch}(3x)\,dx$

3.246.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

method	result	size
risch	$\frac{\ln(e^{2x}-1)}{3}-\frac{\ln(e^{4x}+e^{2x}+1)}{6}$	24

input
```
int(cosh(x)*csch(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(exp(2*x)-1)-1/6*ln(exp(4*x)+exp(2*x)+1)
```

3.246.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \cosh(x)\mathrm{csch}(3x)\,dx = -\frac{1}{6}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2+1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right) + \frac{1}{3}\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(cosh(x)*csch(3*x),x, algorithm="fricas")
```

output
```
-1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2)) + 1/3*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

3.246.6 Sympy [F]

$$\int \cosh(x)\mathrm{csch}(3x)\,dx = \int \cosh(x)\,\mathrm{csch}(3x)\,dx$$

input
```
integrate(cosh(x)*csch(3*x),x)
```

output
```
Integral(cosh(x)*csch(3*x), x)
```

3.246.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.24

$$\int \cosh(x)\text{csch}(3x)\,dx = -\frac{1}{6}\log\left(e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{3}\log\left(e^{(-x)} + 1\right) + \frac{1}{3}\log\left(e^{(-x)} - 1\right) - \frac{1}{6}\log\left(-e^{(-x)} + e^{(-2x)} + 1\right)$$

input
```
integrate(cosh(x)*csch(3*x),x, algorithm="maxima")
```

output
```
-1/6*log(e^(-x) + e^(-2*x) + 1) + 1/3*log(e^(-x) + 1) + 1/3*log(e^(-x) - 1
) - 1/6*log(-e^(-x) + e^(-2*x) + 1)
```

3.246.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \cosh(x)\text{csch}(3x)\,dx = -\frac{1}{6}\log\left(e^{(2x)} + e^{x} + 1\right) - \frac{1}{6}\log\left(e^{(2x)} - e^{x} + 1\right) + \frac{1}{3}\log\left(e^{x} + 1\right) + \frac{1}{3}\log\left(|e^{x} - 1|\right)$$

input
```
integrate(cosh(x)*csch(3*x),x, algorithm="giac")
```

output
```
-1/6*log(e^(2*x) + e^x + 1) - 1/6*log(e^(2*x) - e^x + 1) + 1/3*log(e^x + 1
) + 1/3*log(abs(e^x - 1))
```

3.246.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \cosh(x)\text{csch}(3x)\,dx = \frac{\ln\left(3\,e^{2x} - 3\right)}{3} - \frac{\ln\left(-e^{2x} - e^{4x} - 1\right)}{6}$$

input
```
int(cosh(x)/sinh(3*x),x)
```

output
```
log(3*exp(2*x) - 3)/3 - log(- exp(2*x) - exp(4*x) - 1)/6
```

3.247 $\int \cosh(x)\mathbf{csch}(4x)\,dx$

3.247.1 Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \cosh(x)\text{csch}(4x)\,dx = -\frac{1}{4}\text{arctanh}(\cosh(x)) + \frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}$$

output
```
-1/4*arctanh(cosh(x))+1/4*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

3.247.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.58

$$\int \cosh(x)\text{csch}(4x)\,dx = \frac{1}{4}\left(\sqrt{2}\text{arctanh}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)+\sqrt{2}\text{arctanh}\left(\sqrt{2}+i\tanh\left(\frac{x}{2}\right)\right) - \log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cosh[x]*Csch[4*x],x]
```

output
```
(Sqrt[2]*ArcTanh[Sqrt[2] - I*Tanh[x/2]] + Sqrt[2]*ArcTanh[Sqrt[2] + I*Tanh
[x/2]] - Log[Cosh[x/2]] + Log[Sinh[x/2]])/4
```

3.247.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 26, 4879, 1406, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\text{csch}(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos(ix)}{\sin(4ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos(ix)}{\sin(4ix)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \frac{1}{-8\cosh^4(x)+12\cosh^2(x)-4}d\cosh(x)$$

$$\Big\downarrow \text{1406}$$

$$2\int \frac{1}{4-8\cosh^2(x)}d\cosh(x)-2\int \frac{1}{8-8\cosh^2(x)}d\cosh(x)$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}-\frac{1}{4}\text{arctanh}(\cosh(x))$$

input
```
Int[Cosh[x]*Csch[4*x],x]
```

output
```
-1/4*ArcTanh[Cosh[x]] + ArcTanh[Sqrt[2]*Cosh[x]]/(2*Sqrt[2])
```

3.247.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.247.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.04

method	result	size
risch	$\frac{\ln(e^x-1)}{4} - \frac{\ln(e^x+1)}{4} + \frac{\ln\left(1+e^{2x}+e^x\sqrt{2}\right)\sqrt{2}}{8} - \frac{\ln\left(1+e^{2x}-e^x\sqrt{2}\right)\sqrt{2}}{8}$	53

input
```
int(cosh(x)*csch(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(exp(x)-1)-1/4*ln(exp(x)+1)+1/8*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2
)-1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)
```

3.247.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(18) = 36$.

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \cosh(x)\operatorname{csch}(4x)\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{\cosh(x)^2+\sinh(x)^2+2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right) - \frac{1}{4}\log(\cosh(x)+\sinh(x)+1) + \frac{1}{4}\log(\cosh(x)+\sinh(x)-1)$$

input
```
integrate(cosh(x)*csch(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log((cosh(x)^2 + sinh(x)^2 + 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2
 + sinh(x)^2)) - 1/4*log(cosh(x) + sinh(x) + 1) + 1/4*log(cosh(x) + sinh(x
) - 1)
```

3.247.6 Sympy [F]

$$\int \cosh(x)\operatorname{csch}(4x)\,dx = \int \cosh(x)\operatorname{csch}(4x)\,dx$$

input
```
integrate(cosh(x)*csch(4*x),x)
```

output
```
Integral(cosh(x)*csch(4*x), x)
```

3.247.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \cosh(x)\operatorname{csch}(4x)\,dx = \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right) - \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right) - \frac{1}{4}\log\left(e^{(-x)}+1\right) + \frac{1}{4}\log\left(e^{(-x)}-1\right)$$

input
```
integrate(cosh(x)*csch(4*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) - 1/8*sqrt(2)*log(-sqrt(2)*
e^(-x) + e^(-2*x) + 1) - 1/4*log(e^(-x) + 1) + 1/4*log(e^(-x) - 1)
```

3.247.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int \cosh(x)\operatorname{csch}(4x)\,dx = -\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-e^{x}}{\sqrt{2}+e^{(-x)}+e^{x}}\right) - \frac{1}{8}\log\left(e^{(-x)}+e^{x}+2\right) + \frac{1}{8}\log\left(e^{(-x)}+e^{x}-2\right)$$

input
```
integrate(cosh(x)*csch(4*x),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) - 1/8
*log(e^(-x) + e^x + 2) + 1/8*log(e^(-x) + e^x - 2)
```

3.247.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \cosh(x)\operatorname{csch}(4x)\,dx = \frac{\ln\left(\frac{1}{2}-\frac{e^{x}}{2}\right)}{4} - \frac{\ln\left(-\frac{e^{x}}{2}-\frac{1}{2}\right)}{4} + \frac{\sqrt{2}\,\ln\left(-\frac{e^{2\,x}}{8}-\frac{\sqrt{2}\,e^{x}}{8}-\frac{1}{8}\right)}{8} - \frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,e^{x}}{8}-\frac{e^{2\,x}}{8}-\frac{1}{8}\right)}{8}$$

input
```
int(cosh(x)/sinh(4*x),x)
```

output
```
log(1/2 - exp(x)/2)/4 - log(- exp(x)/2 - 1/2)/4 + (2^(1/2)*log(- exp(2*x)/
8 - (2^(1/2)*exp(x))/8 - 1/8))/8 - (2^(1/2)*log((2^(1/2)*exp(x))/8 - exp(2
*x)/8 - 1/8))/8
```

3.248 $\int \cosh(x)\mathbf{csch}(5x)\,dx$

3.248.1 Optimal result

Integrand size = 7, antiderivative size = 62

$$\int \cosh(x)\operatorname{csch}(5x)\,dx = \frac{1}{5}\log(\sinh(x)) - \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}+8\sinh^2(x)\right) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}+8\sinh^2(x)\right)$$

output
```
1/5*ln(sinh(x))-1/20*ln(5+8*sinh(x)^2+5^(1/2))*(-5^(1/2)+1)-1/20*ln(5+8*si
nh(x)^2-5^(1/2))*(5^(1/2)+1)
```

3.248.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \cosh(x)\operatorname{csch}(5x)\,dx = \frac{1}{20}\left(-\left(\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cosh(2x)\right)\right) + \left(-1+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cosh(2x)\right) + 4\log(\sinh(x))\right)$$

input
```
Integrate[Cosh[x]*Csch[5*x],x]
```

output
```
(-((1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cosh[2*x]]) + (-1 + Sqrt[5])*Log[1 +
Sqrt[5] + 4*Cosh[2*x]] + 4*Log[Sinh[x]])/20
```

3.248.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 26, 4856, 26, 1434, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\text{csch}(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos(ix)}{\sin(5ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos(ix)}{\sin(5ix)}dx$$

$$\Big\downarrow \text{4856}$$

$$i\int -\frac{i\text{csch}(x)}{16\sinh^4(x)+20\sinh^2(x)+5}d\sinh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\text{csch}(x)}{16\sinh^4(x)+20\sinh^2(x)+5}d\sinh(x)$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{\text{csch}(x)}{16\sinh^4(x)+20\sinh^2(x)+5}d\sinh^2(x)$$

$$\Big\downarrow \text{1141}$$

$$8\int \left(\frac{\text{csch}(x)}{80}-\frac{1}{\sqrt{5}\left(5-\sqrt{5}\right)\left(8\sinh^2(x)-\sqrt{5}+5\right)}+\frac{1}{\sqrt{5}\left(5+\sqrt{5}\right)\left(8\sinh^2(x)+\sqrt{5}+5\right)}\right)d\sinh^2(x)$$

$$\Big\downarrow \text{2009}$$

$$8\left(\frac{1}{80}\log\left(\sinh^2(x)\right)-\frac{\log\left(8\sinh^2(x)-\sqrt{5}+5\right)}{8\sqrt{5}\left(5-\sqrt{5}\right)}+\frac{\log\left(8\sinh^2(x)+\sqrt{5}+5\right)}{8\sqrt{5}\left(5+\sqrt{5}\right)}\right)$$

input
```
Int[Cosh[x]*Csch[5*x],x]
```

output
```
8*(Log[Sinh[x]^2]/80 - Log[5 - Sqrt[5] + 8*Sinh[x]^2]/(8*Sqrt[5]*(5 - Sqrt
[5])) + Log[5 + Sqrt[5] + 8*Sinh[x]^2]/(8*Sqrt[5]*(5 + Sqrt[5])))
```

3.248.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.248.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(48) = 96.

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

method	result
risch	$\frac{\ln(e^{2x}-1)}{5}-\frac{\ln\left(e^{4x}+\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)}{20}+\frac{\ln\left(e^{4x}+\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)\sqrt{5}}{20}-\frac{\ln\left(e^{4x}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)}{20}-\frac{\ln\left(e^{4x}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2x}\right.}{20}$

input
```
int(cosh(x)*csch(5*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*ln(exp(2*x)-1)-1/20*ln(exp(4*x)+(1/2+1/2*5^(1/2))*exp(2*x)+1)+1/20*ln(
exp(4*x)+(1/2+1/2*5^(1/2))*exp(2*x)+1)*5^(1/2)-1/20*ln(exp(4*x)+(1/2-1/2*5
^(1/2))*exp(2*x)+1)-1/20*ln(exp(4*x)+(1/2-1/2*5^(1/2))*exp(2*x)+1)*5^(1/2)
```

3.248.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.90

$$\int \cosh(x)\operatorname{csch}(5x)\,dx$$
$$=\frac{1}{20}\sqrt{5}\log\left(\frac{4\cosh(x)^4+4\sinh(x)^4+4\left(\sqrt{5}+1\right)\cosh(x)^2+4\left(6\cosh(x)^2+\sqrt{5}+1\right)\sinh(x)^2+\sqrt{5}+}{2\cosh(x)^4+2\sinh(x)^4+2\left(6\cosh(x)^2+1\right)\sinh(x)^2+2\cosh(x)^2+1}\right.$$
$$-\frac{1}{20}\log\left(\frac{2\cosh(x)^4+2\sinh(x)^4+2\left(6\cosh(x)^2+1\right)\sinh(x)^2+2\cosh(x)^2+1}{\cosh(x)^4-4\cosh(x)^3\sinh(x)+6\cosh(x)^2\sinh(x)^2-4\cosh(x)\sinh(x)^3+\sinh(x)^4}\right)$$
$$+\frac{1}{5}\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(cosh(x)*csch(5*x),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((4*cosh(x)^4 + 4*sinh(x)^4 + 4*(sqrt(5) + 1)*cosh(x)^2 +
4*(6*cosh(x)^2 + sqrt(5) + 1)*sinh(x)^2 + sqrt(5) + 7)/(2*cosh(x)^4 + 2*si
nh(x)^4 + 2*(6*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 1)) - 1/20*log((2*
cosh(x)^4 + 2*sinh(x)^4 + 2*(6*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 1)
/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh
(x)^3 + sinh(x)^4)) + 1/5*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

3.248.6 Sympy [F]

$$\int \cosh(x)\mathrm{csch}(5x)\,dx = \int \cosh(x)\,\mathrm{csch}(5x)\,dx$$

input
```
integrate(cosh(x)*csch(5*x),x)
```

output
```
Integral(cosh(x)*csch(5*x), x)
```

3.248.7 Maxima [F]

$$\int \cosh(x)\mathrm{csch}(5x)\,dx = \int \cosh(x)\,\mathrm{csch}(5\,x)\;dx$$

input
```
integrate(cosh(x)*csch(5*x),x, algorithm="maxima")
```

output
```
-1/5*integrate((e^(3*x) + e^(2*x) + e^x + 1)*e^x/(e^(4*x) + e^(3*x) + e^(2
*x) + e^x + 1), x) - 1/5*integrate((e^(3*x) - e^(2*x) + e^x - 1)*e^x/(e^(4
*x) - e^(3*x) + e^(2*x) - e^x + 1), x) + 3/10*integrate(e^(3*x)/(e^(4*x) +
 e^(3*x) + e^(2*x) + e^x + 1), x) - 3/10*integrate(e^(3*x)/(e^(4*x) - e^(3
*x) + e^(2*x) - e^x + 1), x) + 1/10*integrate(e^(2*x)/(e^(4*x) + e^(3*x) +
 e^(2*x) + e^x + 1), x) + 1/10*integrate(e^(2*x)/(e^(4*x) - e^(3*x) + e^(2
*x) - e^x + 1), x) - 1/10*integrate(e^x/(e^(4*x) + e^(3*x) + e^(2*x) + e^x
 + 1), x) + 1/10*integrate(e^x/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x)
 + 1/5*log(e^x + 1) + 1/5*log(e^x - 1)
```

3.248.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(46) = 92$.

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.74

$$\int \cosh(x)\text{csch}(5x)\,dx = -\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(\frac{1}{2}\left(\sqrt{5}+1\right)e^{x}+e^{(2x)}+1\right) - \frac{1}{20}\left(\sqrt{5}+1\right)\log\left(-\frac{1}{2}\left(\sqrt{5}+1\right)e^{x}+e^{(2x)}+1\right) + \frac{1}{20}\left(\sqrt{5}-1\right)\log\left(\frac{1}{2}\left(\sqrt{5}-1\right)e^{x}+e^{(2x)}+1\right) + \frac{1}{20}\left(\sqrt{5}-1\right)\log\left(-\frac{1}{2}\left(\sqrt{5}-1\right)e^{x}+e^{(2x)}+1\right) + \frac{1}{5}\log\left(e^{x}+1\right) + \frac{1}{5}\log\left(|e^{x}-1|\right)$$

input
```
integrate(cosh(x)*csch(5*x),x, algorithm="giac")
```

output
```
-1/20*(sqrt(5) + 1)*log(1/2*(sqrt(5) + 1)*e^x + e^(2*x) + 1) - 1/20*(sqrt(
5) + 1)*log(-1/2*(sqrt(5) + 1)*e^x + e^(2*x) + 1) + 1/20*(sqrt(5) - 1)*log
(1/2*(sqrt(5) - 1)*e^x + e^(2*x) + 1) + 1/20*(sqrt(5) - 1)*log(-1/2*(sqrt(
5) - 1)*e^x + e^(2*x) + 1) + 1/5*log(e^x + 1) + 1/5*log(abs(e^x - 1))
```

3.248.9 Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.68

$$\int \cosh(x)\text{csch}(5x)\,dx = \frac{\ln\left(5-5\,\mathrm{e}^{2x}\right)}{5} - \ln\left(2\,\mathrm{e}^{4x}-\mathrm{e}^{2x}+\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)\left(30\,\mathrm{e}^{4x}-20\,\mathrm{e}^{2x}+30\right)+2\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right) + \ln\left(2\,\mathrm{e}^{4x}-\mathrm{e}^{2x}-\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)\left(30\,\mathrm{e}^{4x}-20\,\mathrm{e}^{2x}+30\right)+2\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)$$

input
```
int(cosh(x)/sinh(5*x),x)
```

output
```
log(5 - 5*exp(2*x))/5 - log(2*exp(4*x) - exp(2*x) + (5^(1/2)/20 + 1/20)*(3
0*exp(4*x) - 20*exp(2*x) + 30) + 2)*(5^(1/2)/20 + 1/20) + log(2*exp(4*x) -
 exp(2*x) - (5^(1/2)/20 - 1/20)*(30*exp(4*x) - 20*exp(2*x) + 30) + 2)*(5^(
1/2)/20 - 1/20)
```

3.249 $\int \cosh(x)\mathbf{csch}(6x)\,dx$

3.249.1 Optimal result

Integrand size = 7, antiderivative size = 36

$$\int \cosh(x)\operatorname{csch}(6x)\,dx = -\frac{1}{6}\operatorname{arctanh}(\cosh(x)) - \frac{1}{6}\operatorname{arctanh}(2\cosh(x)) + \frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctanh(cosh(x))-1/6*arctanh(2*cosh(x))+1/6*arctanh(2/3*cosh(x)*3^(1/
2))*3^(1/2)
```

3.249.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.53

$$\int \cosh(x)\operatorname{csch}(6x)\,dx = \frac{1}{12}\left(2\sqrt{3}\operatorname{arctanh}\left(\frac{2-i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right) + 2\sqrt{3}\operatorname{arctanh}\left(\frac{2+i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right) - 2\log\left(\cosh\left(\frac{x}{2}\right)\right) + \log(1-2\cosh(x)) - \log(1+2\cosh(x)) + 2\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cosh[x]*Csch[6*x],x]
```

output
```
(2*Sqrt[3]*ArcTanh[(2 - I*Tanh[x/2])/Sqrt[3]] + 2*Sqrt[3]*ArcTanh[(2 + I*T
anh[x/2])/Sqrt[3]] - 2*Log[Cosh[x/2]] + Log[1 - 2*Cosh[x]] - Log[1 + 2*Cos
h[x]] + 2*Log[Sinh[x/2]])/12
```

3.249.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 26, 4879, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\text{csch}(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos(ix)}{\sin(6ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos(ix)}{\sin(6ix)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \frac{1}{2\left(-16\cosh^6(x)+32\cosh^4(x)-19\cosh^2(x)+3\right)}d\cosh(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \frac{1}{-16\cosh^6(x)+32\cosh^4(x)-19\cosh^2(x)+3}d\cosh(x)$$

$$\Big\downarrow \text{2460}$$

$$-\frac{1}{2}\int \left(\frac{2}{4\cosh^2(x)-3}-\frac{2}{3\left(4\cosh^2(x)-1\right)}-\frac{1}{3\left(\cosh^2(x)-1\right)}\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{3}\text{arctanh}(\cosh(x))-\frac{1}{3}\text{arctanh}(2\cosh(x))+\frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

input
```
Int[Cosh[x]*Csch[6*x],x]
```

output
```
(-1/3*ArcTanh[Cosh[x]] - ArcTanh[2*Cosh[x]]/3 + ArcTanh[(2*Cosh[x])/Sqrt[3
]]/Sqrt[3])/2
```

3.249.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.249.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(26) = 52.

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.14

method	result	size
risch	$-\frac{\ln(e^x+1)}{6}+\frac{\ln(e^x-1)}{6}-\frac{\ln(1+e^x+e^{2x})}{12}+\frac{\ln\left(1+e^{2x}+e^x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+e^{2x}-e^x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln(e^{2x}-e^x+1)}{12}$	77

input
```
int(cosh(x)*csch(6*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(exp(x)+1)+1/6*ln(exp(x)-1)-1/12*ln(1+exp(x)+exp(2*x))+1/12*ln(1+ex
p(2*x)+exp(x)*3^(1/2))*3^(1/2)-1/12*ln(1+exp(2*x)-exp(x)*3^(1/2))*3^(1/2)+
1/12*ln(exp(2*x)-exp(x)+1)
```

3.249.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.81

$$\int \cosh(x)\operatorname{csch}(6x)\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2+4\sqrt{3}\cosh(x)+5}{2\cosh(x)^2+2\sinh(x)^2-1}\right) - \frac{1}{12}\log\left(\frac{2\cosh(x)+1}{\cosh(x)-\sinh(x)}\right)+\frac{1}{12}\log\left(\frac{2\cosh(x)-1}{\cosh(x)-\sinh(x)}\right) - \frac{1}{6}\log(\cosh(x)+\sinh(x)+1)+\frac{1}{6}\log(\cosh(x)+\sinh(x)-1)$$

input
```
integrate(cosh(x)*csch(6*x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log((2*cosh(x)^2 + 2*sinh(x)^2 + 4*sqrt(3)*cosh(x) + 5)/(2*co
sh(x)^2 + 2*sinh(x)^2 - 1)) - 1/12*log((2*cosh(x) + 1)/(cosh(x) - sinh(x))
) + 1/12*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 1/6*log(cosh(x) + sinh
(x) + 1) + 1/6*log(cosh(x) + sinh(x) - 1)
```

3.249.6 Sympy [F]

$$\int \cosh(x)\text{csch}(6x)\,dx = \int \cosh(x)\,\text{csch}(6x)\,dx$$

input
```
integrate(cosh(x)*csch(6*x),x)
```

output
```
Integral(cosh(x)*csch(6*x), x)
```

3.249.7 Maxima [F]

$$\int \cosh(x)\text{csch}(6x)\,dx = \int \cosh(x)\,\text{csch}(6\,x)\;dx$$

input
```
integrate(cosh(x)*csch(6*x),x, algorithm="maxima")
```

output
```
-integrate(1/2*(e^(3*x) - e^x)/(e^(4*x) - e^(2*x) + 1), x) - 1/12*log(e^(2
*x) + e^x + 1) + 1/12*log(e^(2*x) - e^x + 1) - 1/6*log(e^x + 1) + 1/6*log(
e^x - 1)
```

3.249.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

$$\begin{aligned}\int \cosh(x)\text{csch}(6x)\,dx = &-\frac{1}{12}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}-e^{x}}{\sqrt{3}+e^{(-x)}+e^{x}}\right)\\ &-\frac{1}{12}\log\left(e^{(-x)}+e^{x}+2\right)-\frac{1}{12}\log\left(e^{(-x)}+e^{x}+1\right)\\ &+\frac{1}{12}\log\left(e^{(-x)}+e^{x}-1\right)+\frac{1}{12}\log\left(e^{(-x)}+e^{x}-2\right)\end{aligned}$$

input
```
integrate(cosh(x)*csch(6*x),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*log(-(sqrt(3) - e^(-x) - e^x)/(sqrt(3) + e^(-x) + e^x)) - 1/
12*log(e^(-x) + e^x + 2) - 1/12*log(e^(-x) + e^x + 1) + 1/12*log(e^(-x) +
e^x - 1) + 1/12*log(e^(-x) + e^x - 2)
```

3.249.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.53

$$\int \cosh(x)\mathrm{csch}(6x)\,dx = \frac{\ln\left(\frac{1}{3}-\frac{e^x}{3}\right)}{6}-\frac{\ln\left(-\frac{e^x}{3}-\frac{1}{3}\right)}{6}-\frac{\ln\left(-\frac{e^{2x}}{36}-\frac{e^x}{36}-\frac{1}{36}\right)}{12}+\frac{\ln\left(\frac{e^x}{36}-\frac{e^{2x}}{36}-\frac{1}{36}\right)}{12}+\frac{\sqrt{3}\,\ln\left(-\frac{e^{2x}}{12}-\frac{\sqrt{3}\,e^x}{12}-\frac{1}{12}\right)}{12}-\frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,e^x}{12}-\frac{e^{2x}}{12}-\frac{1}{12}\right)}{12}$$

input
```
int(cosh(x)/sinh(6*x),x)
```

output
```
log(1/3 - exp(x)/3)/6 - log(- exp(x)/3 - 1/3)/6 - log(- exp(2*x)/36 - exp(
x)/36 - 1/36)/12 + log(exp(x)/36 - exp(2*x)/36 - 1/36)/12 + (3^(1/2)*log(-
 exp(2*x)/12 - (3^(1/2)*exp(x))/12 - 1/12))/12 - (3^(1/2)*log((3^(1/2)*exp
(x))/12 - exp(2*x)/12 - 1/12))/12
```

3.250 $\int x^m \cosh(a+bx)\sinh(a+bx)\,dx$

3.250.1 Optimal result

Integrand size = 16, antiderivative size = 70

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} + \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}$$

output
```
2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)+2^(-3-m)*x^m*GAMMA(1+
m,2*b*x)/b/exp(2*a)/((b*x)^m)
```

3.250.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \frac{2^{-3-m}e^{-2a}x^m(-b^2x^2)^{-m}\left(e^{4a}(bx)^m\Gamma(1+m,-2bx)+(-bx)^m\Gamma(1+m,2bx)\right)}{b}$$

input
```
Integrate[x^m*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
(2^(-3 - m)*x^m*(E^(4*a)*(b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*Gamma[1
 + m, 2*b*x]))/(b*E^(2*a)*(-(b^2*x^2))^m)
```

3.250.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5971, 27, 3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \frac{1}{2}x^m \sinh(2a+2bx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int x^m \sinh(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int -ix^m \sin(2ia+2ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\int x^m \sin(2ia+2ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-\frac{1}{2}i\left(\frac{1}{2}i\int e^{2(a+bx)}x^m dx-\frac{1}{2}i\int e^{-2(a+bx)}x^m dx\right)$$

$$\Big\downarrow \text{2612}$$

$$-\frac{1}{2}i\left(\frac{ie^{2a}2^{-m-2}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}+\frac{ie^{-2a}2^{-m-2}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}\right)$$

input
```
Int[x^m*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
(-1/2*I)*((I*2^(-2 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) +
 (I*2^(-2 - m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m))
```

3.250.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.250.4 Maple [F]

$$\int x^m \cosh(bx+a)\sinh(bx+a)\,dx$$

input
```
int(x^m*cosh(b*x+a)*sinh(b*x+a),x)
```

output
```
int(x^m*cosh(b*x+a)*sinh(b*x+a),x)
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.26

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx$$
$$= \frac{\cosh\left(m\log\left(2\,b\right)+2\,a\right)\Gamma(m+1,2\,bx)+\cosh\left(m\log\left(-2\,b\right)-2\,a\right)\Gamma(m+1,-2\,bx)-\Gamma(m+1,2\,bx)\sinh(}{8\,b}$$

input
```
integrate(x^m*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2*b*x) + cosh(m*log(-2*b) - 2*a)*
gamma(m + 1, -2*b*x) - gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) - gamma(
m + 1, -2*b*x)*sinh(m*log(-2*b) - 2*a))/b
```

3.250.6 Sympy [F(-2)]

Exception generated.

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**m*cosh(b*x+a)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.250.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \frac{1}{4}\left(2\,bx\right)^{-m-1}x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx) - \frac{1}{4}\left(-2\,bx\right)^{-m-1}x^{m+1}e^{(2\,a)}\Gamma(m+1,-2\,bx)$$

input
```
integrate(x^m*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(2*b*x)^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/4*(-2*b*x)
^(-m - 1)*x^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x)
```

3.250.8 Giac [F]

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^m*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)*sinh(b*x + a), x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \int x^m \cosh(a+b\,x)\,\sinh(a+b\,x)\,dx$$

input
```
int(x^m*cosh(a + b*x)*sinh(a + b*x),x)
```

output
```
int(x^m*cosh(a + b*x)*sinh(a + b*x), x)
```

3.251 $\int x^3 \cosh(a+bx)\sinh(a+bx)\,dx$

3.251.1 Optimal result

Integrand size = 16, antiderivative size = 94

$$\int x^3 \cosh(a+bx)\sinh(a+bx)\,dx = \frac{3x}{8b^3}+\frac{x^3}{4b}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4} - \frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{3x\sinh^2(a+bx)}{4b^3}+\frac{x^3\sinh^2(a+bx)}{2b}$$

output
```
3/8*x/b^3+1/4*x^3/b-3/8*cosh(b*x+a)*sinh(b*x+a)/b^4-3/4*x^2*cosh(b*x+a)*si
nh(b*x+a)/b^2+3/4*x*sinh(b*x+a)^2/b^3+1/2*x^3*sinh(b*x+a)^2/b
```

3.251.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.53

$$\int x^3 \cosh(a+bx)\sinh(a+bx)\,dx = \frac{(6bx+4b^3x^3)\cosh(2(a+bx))-3(1+2b^2x^2)\sinh(2(a+bx))}{16b^4}$$

input
```
Integrate[x^3*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
((6*b*x + 4*b^3*x^3)*Cosh[2*(a + b*x)] - 3*(1 + 2*b^2*x^2)*Sinh[2*(a + b*x
)])/(16*b^4)
```

3.251.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5895, 3042, 25, 3792, 15, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sinh(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{5895}$$

$$\frac{x^3\sinh^2(a+bx)}{2b}-\frac{3\int x^2\sinh^2(a+bx)dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^3\sinh^2(a+bx)}{2b}-\frac{3\int -x^2\sin(ia+ibx)^2dx}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^3\sinh^2(a+bx)}{2b}+\frac{3\int x^2\sin(ia+ibx)^2dx}{2b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{3\left(\frac{\int -\sinh^2(a+bx)dx}{2b^2}+\frac{\int x^2dx}{2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\left(\frac{\int -\sinh^2(a+bx)dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(-\frac{\int \sinh^2(a+bx)dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^3\sinh^2(a+bx)}{2b}+\frac{3\left(-\frac{\int -\sin(ia+ibx)^2dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^3\sinh^2(a+bx)}{2b}+\frac{3\left(\frac{\int \sin(ia+ibx)^2dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}$$

$$\Big\downarrow \; 3115$$

$$\frac{3\left(\frac{\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \; 24$$

$$\frac{3\left(\frac{x\sinh^2(a+bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

input
```
Int[x^3*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
(x^3*Sinh[a + b*x]^2)/(2*b) + (3*(x^3/6 - (x^2*Cosh[a + b*x]*Sinh[a + b*x]
)/(2*b) + (x*Sinh[a + b*x]^2)/(2*b^2) + (x/2 - (Cosh[a + b*x]*Sinh[a + b*x
])/(2*b))/(2*b^2)))/(2*b)
```

3.251.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.251.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

method	result
risch	$\frac{(4x^3b^3-6x^2b^2+6bx-3)\,\mathrm{e}^{2bx+2a}}{32b^4}+\frac{(4x^3b^3+6x^2b^2+6bx+3)\,\mathrm{e}^{-2bx-2a}}{32b^4}$
derivativedivides	$\frac{-\frac{a^3\cosh(bx+a)^2}{2}+3a^2\left(\frac{(bx+a)\cosh(bx+a)^2}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}\right)-3a\left(\frac{(bx+a)^2\cosh(bx+a)^2}{2}-\frac{(bx+a)\cosh(bx+a}{2}\right.}{}$
default	$\frac{-\frac{a^3\cosh(bx+a)^2}{2}+3a^2\left(\frac{(bx+a)\cosh(bx+a)^2}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}\right)-3a\left(\frac{(bx+a)^2\cosh(bx+a)^2}{2}-\frac{(bx+a)\cosh(bx+a}{2}\right.}{}$

input
```
int(x^3*cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/32*(4*b^3*x^3-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)+1/32*(4*b^3*x^3+6*b^
2*x^2+6*b*x+3)/b^4*exp(-2*b*x-2*a)
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

$$\int x^3\cosh(a+bx)\sinh(a+bx)\,dx$$
$$=\frac{(2\,b^3x^3+3\,bx)\cosh\left(bx+a\right)^2-3\left(2\,b^2x^2+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+(2\,b^3x^3+3\,bx)\sinh\left(bx+a\right)^2}{8\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/8*((2*b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 - 3*(2*b^2*x^2 + 1)*cosh(b*x + a)
*sinh(b*x + a) + (2*b^3*x^3 + 3*b*x)*sinh(b*x + a)^2)/b^4
```

3.251.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.27

$$\int x^3\cosh(a+bx)\sinh(a+bx)\,dx$$
$$= \begin{cases} \frac{x^3\sinh^2(a+bx)}{4b} + \frac{x^3\cosh^2(a+bx)}{4b} - \frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2} + \frac{3x\sinh^2(a+bx)}{8b^3} + \frac{3x\cosh^2(a+bx)}{8b^3} - \frac{3\sinh(a+bx)\cosh(a+bx)}{8b^4} \\ \frac{x^4\sinh(a)\cosh(a)}{4} \end{cases}$$

input
```
integrate(x**3*cosh(b*x+a)*sinh(b*x+a),x)
```

output
```
Piecewise((x**3*sinh(a + b*x)**2/(4*b) + x**3*cosh(a + b*x)**2/(4*b) - 3*x
**2*sinh(a + b*x)*cosh(a + b*x)/(4*b**2) + 3*x*sinh(a + b*x)**2/(8*b**3) +
 3*x*cosh(a + b*x)**2/(8*b**3) - 3*sinh(a + b*x)*cosh(a + b*x)/(8*b**4), N
e(b, 0)), (x**4*sinh(a)*cosh(a)/4, True))
```

3.251.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int x^3\cosh(a+bx)\sinh(a+bx)\,dx = \frac{(4\,b^3x^3e^{(2\,a)} - 6\,b^2x^2e^{(2\,a)} + 6\,bxe^{(2\,a)} - 3\,e^{(2\,a)})e^{(2\,bx)}}{32\,b^4} + \frac{(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3)e^{(-2\,bx-2\,a)}}{32\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/32*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a) - 3*e^(2*a))*e
^(2*b*x)/b^4 + 1/32*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b
^4
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int x^3\cosh(a+bx)\sinh(a+bx)\,dx = \frac{(4\,b^3x^3-6\,b^2x^2+6\,bx-3)e^{(2\,bx+2\,a)}}{32\,b^4} + \frac{(4\,b^3x^3+6\,b^2x^2+6\,bx+3)e^{(-2\,bx-2\,a)}}{32\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/32*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 + 1/32*(4*b^3
*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4
```

3.251.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.68

$$\int x^3\cosh(a+bx)\sinh(a+bx)\,dx = -\frac{\frac{3\sinh(2\,a+2\,b\,x)}{2}-2\,b^3\,x^3\cosh(2\,a+2\,b\,x)+3\,b^2\,x^2\sinh(2\,a+2\,b\,x)-3\,b\,x\cosh(2\,a+2\,b\,x)}{8\,b^4}$$

input
```
int(x^3*cosh(a + b*x)*sinh(a + b*x),x)
```

output
```
-((3*sinh(2*a + 2*b*x))/2 - 2*b^3*x^3*cosh(2*a + 2*b*x) + 3*b^2*x^2*sinh(2
*a + 2*b*x) - 3*b*x*cosh(2*a + 2*b*x))/(8*b^4)
```

3.252 $\int x^2 \cosh(a+bx)\sinh(a+bx)\,dx$

3.252.1 Optimal result

Integrand size = 16, antiderivative size = 64

$$\int x^2 \cosh(a+bx)\sinh(a+bx)\,dx = \frac{x^2}{4b} - \frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x^2\sinh^2(a+bx)}{2b}$$

output
```
1/4*x^2/b-1/2*x*cosh(b*x+a)*sinh(b*x+a)/b^2+1/4*sinh(b*x+a)^2/b^3+1/2*x^2*
sinh(b*x+a)^2/b
```

3.252.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int x^2 \cosh(a+bx)\sinh(a+bx)\,dx = \frac{(1+2b^2x^2)\cosh(2(a+bx)) - 2bx\sinh(2(a+bx))}{8b^3}$$

input
```
Integrate[x^2*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
((1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] - 2*b*x*Sinh[2*(a + b*x)])/(8*b^3)
```

3.252.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5895, 3042, 25, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sinh(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \; 5895$$

$$\frac{x^2\sinh^2(a+bx)}{2b}-\frac{\int x\sinh^2(a+bx)dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{x^2\sinh^2(a+bx)}{2b}-\frac{\int -x\sin(ia+ibx)^2dx}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{x^2\sinh^2(a+bx)}{2b}+\frac{\int x\sin(ia+ibx)^2dx}{b}$$

$$\Big\downarrow \; 3791$$

$$\frac{\frac{\int xdx}{2}+\frac{\sinh^2(a+bx)}{4b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \; 15$$

$$\frac{\frac{\sinh^2(a+bx)}{4b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

input
```
Int[x^2*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
(x^2*Sinh[a + b*x]^2)/(2*b) + (x^2/4 - (x*Cosh[a + b*x]*Sinh[a + b*x])/(2*
b) + Sinh[a + b*x]^2/(4*b^2))/b
```

3.252.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.252.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

method	result
risch	$\frac{(2x^2b^2-2bx+1)\mathrm{e}^{2bx+2a}}{16b^3}+\frac{(2x^2b^2+2bx+1)\mathrm{e}^{-2bx-2a}}{16b^3}$
derivativedivides	$\frac{\frac{a^2\cosh(bx+a)^2}{2}-2a\left(\frac{(bx+a)\cosh(bx+a)^2}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}\right)+\frac{(bx+a)^2\cosh(bx+a)^2}{2}-\frac{(bx+a)\cosh(bx+a)\sinh(b}{2}}{b^3}$
default	$\frac{\frac{a^2\cosh(bx+a)^2}{2}-2a\left(\frac{(bx+a)\cosh(bx+a)^2}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}\right)+\frac{(bx+a)^2\cosh(bx+a)^2}{2}-\frac{(bx+a)\cosh(bx+a)\sinh(b}{2}}{b^3}$

input
```
int(x^2*cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/16*(2*b^2*x^2-2*b*x+1)/b^3*exp(2*b*x+2*a)+1/16*(2*b^2*x^2+2*b*x+1)/b^3*e
xp(-2*b*x-2*a)
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int x^2 \cosh(a+bx)\sinh(a+bx)\,dx = -\frac{4\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)-\left(2\,b^2x^2+1\right)\cosh\left(bx+a\right)^2-\left(2\,b^2x^2+1\right)\sinh\left(bx+a\right)^2}{8\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh(b*x + a)*sinh(b*x + a) - (2*b^2*x^2 + 1)*cosh(b*x + a)^2
- (2*b^2*x^2 + 1)*sinh(b*x + a)^2)/b^3
```

3.252.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int x^2 \cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{x^2\sinh^2\left(a+bx\right)}{4b}+\frac{x^2\cosh^2\left(a+bx\right)}{4b}-\frac{x\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b^2}+\frac{\sinh^2\left(a+bx\right)}{4b^3} & \text{for } b\neq 0 \\ \frac{x^3\sinh\left(a\right)\cosh\left(a\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*cosh(b*x+a)*sinh(b*x+a),x)
```

output
```
Piecewise((x**2*sinh(a + b*x)**2/(4*b) + x**2*cosh(a + b*x)**2/(4*b) - x*s
inh(a + b*x)*cosh(a + b*x)/(2*b**2) + sinh(a + b*x)**2/(4*b**3), Ne(b, 0))
, (x**3*sinh(a)*cosh(a)/3, True))
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int x^2\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\left(2\,b^2x^2e^{(2\,a)}-2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^3} + \frac{(2\,b^2x^2+2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/16*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 + 1/16*(2
*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int x^2\cosh(a+bx)\sinh(a+bx)\,dx = \frac{(2\,b^2x^2-2\,bx+1)e^{(2\,bx+2\,a)}}{16\,b^3} + \frac{(2\,b^2x^2+2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/16*(2*b^2*x^2 - 2*b*x + 1)*e^(2*b*x + 2*a)/b^3 + 1/16*(2*b^2*x^2 + 2*b*x
 + 1)*e^(-2*b*x - 2*a)/b^3
```

3.252.9 Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int x^2\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\frac{\cosh(2\,a+2\,b\,x)}{2}-b\,x\sinh(2\,a+2\,b\,x)+b^2\,x^2\cosh(2\,a+2\,b\,x)}{4\,b^3}$$

input
```
int(x^2*cosh(a + b*x)*sinh(a + b*x),x)
```

output
```
(cosh(2*a + 2*b*x)/2 - b*x*sinh(2*a + 2*b*x) + b^2*x^2*cosh(2*a + 2*b*x))/
(4*b^3)
```

3.253 $\int x \cosh(a+bx) \sinh(a+bx) \, dx$

3.253.1 Optimal result

Integrand size = 14, antiderivative size = 44

$$\int x \cosh(a+bx) \sinh(a+bx) \, dx = \frac{x}{4b} - \frac{\cosh(a+bx) \sinh(a+bx)}{4b^2} + \frac{x \sinh^2(a+bx)}{2b}$$

output
```
1/4*x/b-1/4*cosh(b*x+a)*sinh(b*x+a)/b^2+1/2*x*sinh(b*x+a)^2/b
```

3.253.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int x \cosh(a+bx) \sinh(a+bx) \, dx = -\frac{-2bx \cosh(2(a+bx)) + \sinh(2(a+bx))}{8b^2}$$

input
```
Integrate[x*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
-1/8*(-2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])/b^2
```

3.253.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5895, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sinh(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{5895}$$

$$\frac{x\sinh^2(a+bx)}{2b}-\frac{\int \sinh^2(a+bx)dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x\sinh^2(a+bx)}{2b}-\frac{\int -\sin(ia+ibx)^2dx}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\sinh^2(a+bx)}{2b}+\frac{\int \sin(ia+ibx)^2dx}{2b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b}+\frac{x\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\sinh^2(a+bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b}$$

input
```
Int[x*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
(x*Sinh[a + b*x]^2)/(2*b) + (x/2 - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b))/(2
*b)
```

3.253.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.253.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{(2bx-1)\mathrm{e}^{2bx+2a}}{16b^2}+\frac{(2bx+1)\mathrm{e}^{-2bx-2a}}{16b^2}$	42
derivativedivides	$\frac{\frac{(bx+a)\cosh(bx+a)^2}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}-\frac{a\cosh(bx+a)^2}{2}}{b^2}$	53
default	$\frac{\frac{(bx+a)\cosh(bx+a)^2}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}-\frac{a\cosh(bx+a)^2}{2}}{b^2}$	53

input
```
int(x*cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/16*(2*b*x-1)/b^2*exp(2*b*x+2*a)+1/16*(2*b*x+1)/b^2*exp(-2*b*x-2*a)
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = \frac{bx\cosh(bx+a)^2+bx\sinh(bx+a)^2-\cosh(bx+a)\sinh(bx+a)}{4\,b^2}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(b*x*cosh(b*x + a)^2 + b*x*sinh(b*x + a)^2 - cosh(b*x + a)*sinh(b*x +
a))/b^2
```

3.253.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{x\sinh^2(a+bx)}{4b}+\frac{x\cosh^2(a+bx)}{4b}-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2} & \text{for } b\neq 0 \\ \frac{x^2\sinh(a)\cosh(a)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a),x)
```

output
```
Piecewise((x*sinh(a + b*x)**2/(4*b) + x*cosh(a + b*x)**2/(4*b) - sinh(a +
b*x)*cosh(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*sinh(a)*cosh(a)/2, True))
```

3.253.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^2}+\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/16*(2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + 1/16*(2*b*x + 1)*e^(-2*b*x
- 2*a)/b^2
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = \frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{16\,b^2} + \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/16*(2*b*x - 1)*e^(2*b*x + 2*a)/b^2 + 1/16*(2*b*x + 1)*e^(-2*b*x - 2*a)/b
^2
```

3.253.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = -\frac{\sinh(2\,a+2\,b\,x)-2\,b\,x\cosh(2\,a+2\,b\,x)}{8\,b^2}$$

input
```
int(x*cosh(a + b*x)*sinh(a + b*x),x)
```

output
```
-(sinh(2*a + 2*b*x) - 2*b*x*cosh(2*a + 2*b*x))/(8*b^2)
```

3.254 $\int \cosh(a+bx)\sinh(a+bx)\,dx$

3.254.1 Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \cosh(a+bx)\sinh(a+bx)\,dx=\frac{\sinh^2(a+bx)}{2b}$$

output
```
1/2*sinh(b*x+a)^2/b
```

3.254.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \cosh(a+bx)\sinh(a+bx)\,dx=\frac{1}{2}\left(\frac{\cosh(2a)\cosh(2bx)}{2b}+\frac{\sinh(2a)\sinh(2bx)}{2b}\right)$$

input
```
Integrate[Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
((Cosh[2*a]*Cosh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b))/2
```

3.254.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 26, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -i\sin(ia+ibx)\cos(ia+ibx)dx$$

$$\Big\downarrow \ 26$$

$$-i\int \cos(ia+ibx)\sin(ia+ibx)dx$$

$$\Big\downarrow \ 3044$$

$$-\frac{\int i\sinh(a+bx)d(i\sinh(a+bx))}{b}$$

$$\Big\downarrow \ 15$$

$$\frac{\sinh^2(a+bx)}{2b}$$

input
```
Int[Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
Sinh[a + b*x]^2/(2*b)
```

3.254.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.254.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cosh(bx+a)^2}{2b}$	14
default	$\frac{\cosh(bx+a)^2}{2b}$	14
risch	$\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}$	30

input
```
int(cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*cosh(b*x+a)^2/b
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh{(bx+a)}^2+\sinh{(bx+a)}^2}{4\,b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(cosh(b*x + a)^2 + sinh(b*x + a)^2)/b
```

3.254.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{\sinh^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sinh(a)\cosh(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a),x)
```

output
```
Piecewise((sinh(a + b*x)**2/(2*b), Ne(b, 0)), (x*sinh(a)*cosh(a), True))
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^2}{2b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*cosh(b*x + a)^2/b
```

3.254.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{e^{(2bx+2a)}}{8b} + \frac{e^{(-2bx-2a)}}{8b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b
```

3.254.9 Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)^2}{2\,b}$$

input

```
int(cosh(a + b*x)*sinh(a + b*x),x)
```

output

```
cosh(a + b*x)^2/(2*b)
```

3.255 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx$

3.255.1 Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{2}\text{Chi}(2bx)\sinh(2a) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx)$$

output
```
1/2*cosh(2*a)*Shi(2*b*x)+1/2*Chi(2*b*x)*sinh(2*a)
```

3.255.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{2}(\text{Chi}(2bx)\sinh(2a) + \cosh(2a)\text{Shi}(2bx))$$

input
```
Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x,x]
```

output
```
(CoshIntegral[2*b*x]*Sinh[2*a] + Cosh[2*a]*SinhIntegral[2*b*x])/2
```

3.255.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\cosh(a+bx)}{x}dx$$

$$\Big\downarrow \text{5971}$$

$$\int \frac{\sinh(2a+2bx)}{2x}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sinh(2a+2bx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{x}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$-\frac{1}{2}i\left(i\sinh(2a)\int \frac{\cosh(2bx)}{x}dx+\cosh(2a)\int \frac{i\sinh(2bx)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\left(i\sinh(2a)\int \frac{\cosh(2bx)}{x}dx+i\cosh(2a)\int \frac{\sinh(2bx)}{x}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}i\left(i\sinh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(2a)\int -\frac{i\sin(2ibx)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\left(i\sinh(2a)\int\frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx+\cosh(2a)\int\frac{\sin(2ibx)}{x}dx\right)$$

$$\Big\downarrow \text{3779}$$

$$-\frac{1}{2}i\left(i\sinh(2a)\int\frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(2a)\text{Shi}(2bx)\right)$$

$$\Big\downarrow \text{3782}$$

$$-\frac{1}{2}i(i\sinh(2a)\text{Chi}(2bx)+i\cosh(2a)\text{Shi}(2bx))$$

input
```
Int[(Cosh[a + b*x]*Sinh[a + b*x])/x,x]
```

output
```
(-1/2*I)*(I*CoshIntegral[2*b*x]*Sinh[2*a] + I*Cosh[2*a]*SinhIntegral[2*b*x
])
```

3.255.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.255.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{e^{-2a} \operatorname{Ei}_1(2bx)}{4} - \frac{e^{2a} \operatorname{Ei}_1(-2bx)}{4}$	26

input
```
int(cosh(b*x+a)*sinh(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(-2*a)*Ei(1,2*b*x)-1/4*exp(2*a)*Ei(1,-2*b*x)
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}\left(\operatorname{Ei}(2\,bx) - \operatorname{Ei}(-2\,bx)\right)\cosh\left(2\,a\right) + \frac{1}{4}\left(\operatorname{Ei}(2\,bx) + \operatorname{Ei}(-2\,bx)\right)\sinh\left(2\,a\right)$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x,x, algorithm="fricas")
```

output
```
1/4*(Ei(2*b*x) - Ei(-2*b*x))*cosh(2*a) + 1/4*(Ei(2*b*x) + Ei(-2*b*x))*sinh
(2*a)
```

3.255.6 Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\cosh(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x,x)
```

output
```
Integral(sinh(a + b*x)*cosh(a + b*x)/x, x)
```

3.255.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}\,\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - \frac{1}{4}\,\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x,x, algorithm="maxima")
```

output
```
1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a)
```

3.255.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}\,\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - \frac{1}{4}\,\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x,x, algorithm="giac")
```

output
```
1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)}{x}\,dx$$

input
```
int((cosh(a + b*x)*sinh(a + b*x))/x,x)
```

output
```
int((cosh(a + b*x)*sinh(a + b*x))/x, x)
```

3.256 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx$

3.256.1 Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = b\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{2x} + b\sinh(2a)\text{Shi}(2bx)$$

output
```
b*Chi(2*b*x)*cosh(2*a)+b*Shi(2*b*x)*sinh(2*a)-1/2*sinh(2*b*x+2*a)/x
```

3.256.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{1}{2}\left(2b\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2(a+bx))}{x} + 2b\sinh(2a)\text{Shi}(2bx)\right)$$

input
```
Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x^2,x]
```

output
```
(2*b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*(a + b*x)]/x + 2*b*Sinh[2*a]*S
inhIntegral[2*b*x])/2
```

3.256.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5971, 27, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\cosh(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{ 5971}$$

$$\int \frac{\sinh(2a+2bx)}{2x^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{x^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{x^2}dx$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{1}{2}i\left(2ib\int \frac{\cosh(2a+2bx)}{x}dx-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{2}i\left(2ib\int \frac{\sin\left(2ia+2ibx+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{ 3784}$$

$$-\frac{1}{2}i\left(2ib\left(\cosh(2a)\int \frac{\cosh(2bx)}{x}dx-i\sinh(2a)\int \frac{i\sinh(2bx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{1}{2}i\left(2ib\left(\sinh(2a)\int \frac{\sinh(2bx)}{x}dx+\cosh(2a)\int \frac{\cosh(2bx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}i\left(2ib\left(\sinh(2a)\int -\frac{i\sin(2ibx)}{x}dx+\cosh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\left(2ib\left(\cosh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx-i\sinh(2a)\int \frac{\sin(2ibx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3779}$$

$$-\frac{1}{2}i\left(2ib\left(\sinh(2a)\text{Shi}(2bx)+\cosh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3782}$$

$$-\frac{1}{2}i\left(2ib(\cosh(2a)\text{Chi}(2bx)+\sinh(2a)\text{Shi}(2bx))-\frac{i\sinh(2a+2bx)}{x}\right)$$

input
```
Int[(Cosh[a + b*x]*Sinh[a + b*x])/x^2,x]
```

output
```
(-1/2*I)*(((-I)*Sinh[2*a + 2*b*x])/x + (2*I)*b*(Cosh[2*a]*CoshIntegral[2*b
*x] + Sinh[2*a]*SinhIntegral[2*b*x]))
```

3.256.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.256.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

method	result	size
risch	$-\frac{2\,e^{2a}\,\mathrm{Ei}_1(-2bx)bx+2\,e^{-2a}\,\mathrm{Ei}_1(2bx)bx+e^{2bx+2a}-e^{-2bx-2a}}{4x}$	55

input
```
int(cosh(b*x+a)*sinh(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*exp(2*a)*Ei(1,-2*b*x)*b*x+2*exp(-2*a)*Ei(1,2*b*x)*b*x+exp(2*b*x+2*
a)-exp(-2*b*x-2*a))/x
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{(bx\text{Ei}(2\,bx)+bx\text{Ei}(-2\,bx))\cosh(2\,a)-2\,\cosh(bx+a)\sinh(bx+a)+(bx\text{Ei}(2\,bx)-bx\text{Ei}(-2\,bx))\sinh(}{2\,x}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
1/2*((b*x*Ei(2*b*x) + b*x*Ei(-2*b*x))*cosh(2*a) - 2*cosh(b*x + a)*sinh(b*x
 + a) + (b*x*Ei(2*b*x) - b*x*Ei(-2*b*x))*sinh(2*a))/x
```

3.256.6 Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\cosh(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x**2,x)
```

output
```
Integral(sinh(a + b*x)*cosh(a + b*x)/x**2, x)
```

3.256.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{1}{2}\,be^{(-2\,a)}\Gamma(-1,2\,bx)+\frac{1}{2}\,be^{(2\,a)}\Gamma(-1,-2\,bx)$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/2*b*e^(2*a)*gamma(-1, -2*b*x)
```

3.256.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{2\,bx\text{Ei}(2\,bx)\,e^{(2\,a)} + 2\,bx\text{Ei}(-2\,bx)\,e^{(-2\,a)} - e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)}}{4\,x}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="giac")
```

output
```
1/4*(2*b*x*Ei(2*b*x)*e^(2*a) + 2*b*x*Ei(-2*b*x)*e^(-2*a) - e^(2*b*x + 2*a)
 + e^(-2*b*x - 2*a))/x
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)}{x^2}\,dx$$

input
```
int((cosh(a + b*x)*sinh(a + b*x))/x^2,x)
```

output
```
int((cosh(a + b*x)*sinh(a + b*x))/x^2, x)
```

3.257 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx$

3.257.1 Optimal result

Integrand size = 16, antiderivative size = 60

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = -\frac{b\cosh(2a+2bx)}{2x} + b^2\mathrm{Chi}(2bx)\sinh(2a) - \frac{\sinh(2a+2bx)}{4x^2} + b^2\cosh(2a)\mathrm{Shi}(2bx)$$

output
```
-1/2*b*cosh(2*b*x+2*a)/x+b^2*cosh(2*a)*Shi(2*b*x)+b^2*Chi(2*b*x)*sinh(2*a)
-1/4*sinh(2*b*x+2*a)/x^2
```

3.257.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = \frac{1}{2}\left(2b^2\mathrm{Chi}(2bx)\sinh(2a) - \frac{2bx\cosh(2(a+bx)) + \sinh(2(a+bx))}{2x^2} + 2b^2\cosh(2a)\mathrm{Shi}(2bx)\right)$$

input
```
Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x^3,x]
```

output
```
(2*b^2*CoshIntegral[2*b*x]*Sinh[2*a] - (2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(
a + b*x)])/(2*x^2) + 2*b^2*Cosh[2*a]*SinhIntegral[2*b*x])/2
```

3.257.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.28, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5971, 27, 3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\cosh(a+bx)}{x^3}\,dx$$

$$\Big\downarrow \; 5971$$

$$\int \frac{\sinh(2a+2bx)}{2x^3}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^3}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{x^3}dx$$

$$\Big\downarrow \; 26$$

$$-\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{x^3}dx$$

$$\Big\downarrow \; 3778$$

$$-\frac{1}{2}i\left(ib\int \frac{\cosh(2a+2bx)}{x^2}dx-\frac{i\sinh(2a+2bx)}{2x^2}\right)$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{2}i\left(ib\int \frac{\sin\left(2ia+2ibx+\frac{\pi}{2}\right)}{x^2}dx-\frac{i\sinh(2a+2bx)}{2x^2}\right)$$

$$\Big\downarrow \; 3778$$

$$-\frac{1}{2}i\left(ib\left(-\frac{\cosh(2a+2bx)}{x}+2ib\int -\frac{i\sinh(2a+2bx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{2x^2}\right)$$

$$\Big\downarrow \; 26$$

$$-\frac{1}{2}i\left(ib\left(2b\int \frac{\sinh(2a+2bx)}{x}dx-\frac{\cosh(2a+2bx)}{x}\right)-\frac{i\sinh(2a+2bx)}{2x^2}\right)$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{2}i\left(ib\left(-\frac{\cosh(2a+2bx)}{x}+2b\int -\frac{i\sin(2ia+2ibx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{2x^2}\right)$$

$$\Big\downarrow \; 26$$

$$-\frac{1}{2}i\left(ib\left(-\frac{\cosh(2a+2bx)}{x}-2ib\int \frac{\sin(2ia+2ibx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{2x^2}\right)$$

$$\Big\downarrow \; 3784$$

$$-\frac{1}{2}i\left(ib\left(-\frac{\cosh(2a+2bx)}{x}-2ib\left(i\sinh(2a)\int \frac{\cosh(2bx)}{x}dx+\cosh(2a)\int \frac{i\sinh(2bx)}{x}dx\right)\right)-\frac{i\sinh(2a+2bx)}{2x^2}\right)$$

$$\Big\downarrow \; 26$$

$$-\frac{1}{2}i\left(ib\left(-\frac{\cosh(2a+2bx)}{x}-2ib\left(i\sinh(2a)\int \frac{\cosh(2bx)}{x}dx+i\cosh(2a)\int \frac{\sinh(2bx)}{x}dx\right)\right)-\frac{i\sinh(2a+2bx)}{2x^2}\right)$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{2}i\left(ib\left(-\frac{\cosh(2a+2bx)}{x}-2ib\left(i\sinh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(2a)\int -\frac{i\sin(2ibx)}{x}dx\right)\right)-\frac{i\sinh(2a+}{2x^2}$$

$$\Big\downarrow \; 26$$

$$-\frac{1}{2}i\left(ib\left(-\frac{\cosh(2a+2bx)}{x}-2ib\left(i\sinh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx+\cosh(2a)\int \frac{\sin(2ibx)}{x}dx\right)\right)-\frac{i\sinh(2a+2bx}{2x^2}$$

$$\Big\downarrow \; 3779$$

$$-\frac{1}{2}i\left(ib\left(-\frac{\cosh(2a+2bx)}{x}-2ib\left(i\sinh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(2a)\text{Shi}(2bx)\right)\right)-\frac{i\sinh(2a+2bx)}{2x^2}\right)$$

$$\Big\downarrow \; 3782$$

$$-\frac{1}{2}i\left(ib\left(-\frac{\cosh(2a+2bx)}{x}-2ib(i\sinh(2a)\text{Chi}(2bx)+i\cosh(2a)\text{Shi}(2bx))\right)-\frac{i\sinh(2a+2bx)}{2x^2}\right)$$

input
```
Int[(Cosh[a + b*x]*Sinh[a + b*x])/x^3,x]
```

3.257. $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx$

output
```
(-1/2*I)*(((-1/2*I)*Sinh[2*a + 2*b*x])/x^2 + I*b*(-(Cosh[2*a + 2*b*x]/x) -
 (2*I)*b*(I*CoshIntegral[2*b*x]*Sinh[2*a] + I*Cosh[2*a]*SinhIntegral[2*b*x
])))
```

3.257.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

3.257. $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx$

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.257.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

method	result	size
risch	$-\frac{-4\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)x^2b^2+4\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)x^2b^2+2\,\mathrm{e}^{-2bx-2a}bx+2\,\mathrm{e}^{2bx+2a}bx-\mathrm{e}^{-2bx-2a}+\mathrm{e}^{2bx+2a}}{8x^2}$	89

input
```
int(cosh(b*x+a)*sinh(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-4*exp(-2*a)*Ei(1,2*b*x)*x^2*b^2+4*exp(2*a)*Ei(1,-2*b*x)*x^2*b^2+2*e
xp(-2*b*x-2*a)*b*x+2*exp(2*b*x+2*a)*b*x-exp(-2*b*x-2*a)+exp(2*b*x+2*a))/x^
2
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.73

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx =$$
$$-\frac{bx\cosh\left(bx+a\right)^2+bx\sinh\left(bx+a\right)^2-\left(b^2x^2\mathrm{Ei}(2\,bx)-b^2x^2\mathrm{Ei}(-2\,bx)\right)\cosh\left(2\,a\right)+\cosh\left(bx+a\right)\sinh\left(\right.}{2\,x^2}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(b*x*cosh(b*x + a)^2 + b*x*sinh(b*x + a)^2 - (b^2*x^2*Ei(2*b*x) - b^2
*x^2*Ei(-2*b*x))*cosh(2*a) + cosh(b*x + a)*sinh(b*x + a) - (b^2*x^2*Ei(2*b
*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

3.257.6 Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \frac{\sinh(a+bx)\cosh(a+bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x**3,x)
```

output
```
Integral(sinh(a + b*x)*cosh(a + b*x)/x**3, x)
```

3.257.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.50

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = b^2e^{(-2\,a)}\Gamma(-2,2\,bx) - b^2e^{(2\,a)}\Gamma(-2,-2\,bx)$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
b^2*e^(-2*a)*gamma(-2, 2*b*x) - b^2*e^(2*a)*gamma(-2, -2*b*x)
```

3.257.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = \frac{4\,b^2x^2\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - 4\,b^2x^2\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - 2\,bxe^{(2\,bx+2\,a)} - 2\,bxe^{(-2\,bx-2\,a)} - e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)}}{8\,x^2}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x^3,x, algorithm="giac")
```

output
```
1/8*(4*b^2*x^2*Ei(2*b*x)*e^(2*a) - 4*b^2*x^2*Ei(-2*b*x)*e^(-2*a) - 2*b*x*e
^(2*b*x + 2*a) - 2*b*x*e^(-2*b*x - 2*a) - e^(2*b*x + 2*a) + e^(-2*b*x - 2*
a))/x^2
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)}{x^3}\,dx$$

input
```
int((cosh(a + b*x)*sinh(a + b*x))/x^3,x)
```

output
```
int((cosh(a + b*x)*sinh(a + b*x))/x^3, x)
```

3.258 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx$

3.258.1 Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx = -\frac{b\cosh(2a+2bx)}{6x^2} + \frac{2}{3}b^3\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{6x^3} - \frac{b^2\sinh(2a+2bx)}{3x} + \frac{2}{3}b^3\sinh(2a)\text{Shi}(2bx)$$

output
```
2/3*b^3*Chi(2*b*x)*cosh(2*a)-1/6*b*cosh(2*b*x+2*a)/x^2+2/3*b^3*Shi(2*b*x)*
sinh(2*a)-1/6*sinh(2*b*x+2*a)/x^3-1/3*b^2*sinh(2*b*x+2*a)/x
```

3.258.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx = -\frac{bx\cosh(2(a+bx)) - 4b^3x^3\cosh(2a)\text{Chi}(2bx) + \sinh(2(a+bx)) + 2b^2x^2\sinh(2(a+bx)) - 4b^3x^3\sinh(2}{6x^3}$$

input
```
Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x^4,x]
```

output
```
-1/6*(b*x*Cosh[2*(a + b*x)] - 4*b^3*x^3*Cosh[2*a]*CoshIntegral[2*b*x] + Si
nh[2*(a + b*x)] + 2*b^2*x^2*Sinh[2*(a + b*x)] - 4*b^3*x^3*Sinh[2*a]*SinhIn
tegral[2*b*x])/x^3
```

3.258.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {5971, 27, 3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\cosh(a+bx)}{x^4}\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \frac{\sinh(2a+2bx)}{2x^4}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{x^4}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{x^4}dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{2}i\left(\frac{2}{3}ib\int \frac{\cosh(2a+2bx)}{x^3}dx-\frac{i\sinh(2a+2bx)}{3x^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}i\left(\frac{2}{3}ib\int \frac{\sin\left(2ia+2ibx+\frac{\pi}{2}\right)}{x^3}dx-\frac{i\sinh(2a+2bx)}{3x^3}\right)$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{2}i\left(\frac{2}{3}ib\left(-\frac{\cosh(2a+2bx)}{2x^2}+ib\int -\frac{i\sinh(2a+2bx)}{x^2}dx\right)-\frac{i\sinh(2a+2bx)}{3x^3}\right)$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\left(\frac{2}{3}ib\left(b\int\frac{\sinh(2a+2bx)}{x^2}dx-\frac{\cosh(2a+2bx)}{2x^2}\right)-\frac{i\sinh(2a+2bx)}{3x^3}\right)$$

$\big\downarrow$ 3042

$$-\frac{1}{2}i\left(\frac{2}{3}ib\left(-\frac{\cosh(2a+2bx)}{2x^2}+b\int-\frac{i\sin(2ia+2ibx)}{x^2}dx\right)-\frac{i\sinh(2a+2bx)}{3x^3}\right)$$

$\big\downarrow$ 26

$$-\frac{1}{2}i\left(\frac{2}{3}ib\left(-\frac{\cosh(2a+2bx)}{2x^2}-ib\int\frac{\sin(2ia+2ibx)}{x^2}dx\right)-\frac{i\sinh(2a+2bx)}{3x^3}\right)$$

$\big\downarrow$ 3778

$$-\frac{1}{2}i\left(\frac{2}{3}ib\left(-\frac{\cosh(2a+2bx)}{2x^2}-ib\left(2ib\int\frac{\cosh(2a+2bx)}{x}dx-\frac{i\sinh(2a+2bx)}{x}\right)\right)-\frac{i\sinh(2a+2bx)}{3x^3}\right)$$

$\big\downarrow$ 3042

$$-\frac{1}{2}i\left(\frac{2}{3}ib\left(-\frac{\cosh(2a+2bx)}{2x^2}-ib\left(2ib\int\frac{\sin\left(2ia+2ibx+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh(2a+2bx)}{x}\right)\right)-\frac{i\sinh(2a+2bx)}{3x^3}\right)$$

$\big\downarrow$ 3784

$$-\frac{1}{2}i\left(\frac{2}{3}ib\left(-\frac{\cosh(2a+2bx)}{2x^2}-ib\left(2ib\left(\cosh(2a)\int\frac{\cosh(2bx)}{x}dx-i\sinh(2a)\int\frac{i\sinh(2bx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right.\right.\right.$$

$\big\downarrow$ 26

$$-\frac{1}{2}i\left(\frac{2}{3}ib\left(-\frac{\cosh(2a+2bx)}{2x^2}-ib\left(2ib\left(\sinh(2a)\int\frac{\sinh(2bx)}{x}dx+\cosh(2a)\int\frac{\cosh(2bx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right.\right.\right.$$

$\big\downarrow$ 3042

$$-\frac{1}{2}i\left(\frac{2}{3}ib\left(-\frac{\cosh(2a+2bx)}{2x^2}-ib\left(2ib\left(\sinh(2a)\int-\frac{i\sin(2ibx)}{x}dx+\cosh(2a)\int\frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx\right)-\frac{i\sinh(2a}{x}\right.\right.\right.$$

$\big\downarrow$ 26

$$-\frac{1}{2}i\left(\frac{2}{3}ib\left(-\frac{\cosh(2a+2bx)}{2x^2}-ib\left(2ib\left(\cosh(2a)\int\frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx-i\sinh(2a)\int\frac{\sin(2ibx)}{x}dx\right)-\frac{i\sinh(2a+}{x}\right.\right.\right.$$

$\big\downarrow$ 3779

3.258. $\int\frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx$

$$-\frac{1}{2}i\left(\frac{2}{3}ib\left(-\frac{\cosh(2a+2bx)}{2x^2}-ib\left(2ib\left(\sinh(2a)\text{Shi}(2bx)+\cosh(2a)\int\frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)\right.\right.$$

$\downarrow$ 3782

$$-\frac{1}{2}i\left(\frac{2}{3}ib\left(-\frac{\cosh(2a+2bx)}{2x^2}-ib\left(2ib(\cosh(2a)\text{Chi}(2bx)+\sinh(2a)\text{Shi}(2bx))-\frac{i\sinh(2a+2bx)}{x}\right)\right)-\frac{i\sinh(2a}{3x^3}\right.$$

input
```
Int[(Cosh[a + b*x]*Sinh[a + b*x])/x^4,x]
```

output
```
(-1/2*I)*(((-1/3*I)*Sinh[2*a + 2*b*x])/x^3 + ((2*I)/3)*b*(-1/2*Cosh[2*a +
2*b*x]/x^2 - I*b*(((-I)*Sinh[2*a + 2*b*x])/x + (2*I)*b*(Cosh[2*a]*CoshInte
gral[2*b*x] + Sinh[2*a]*SinhIntegral[2*b*x]))))
```

3.258.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

3.258. $\int\frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx$

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.258.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.42

method	result	si
risch	$-\frac{4\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)x^3b^3+4\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)x^3b^3+2\,\mathrm{e}^{2bx+2a}b^2x^2-2\,\mathrm{e}^{-2bx-2a}b^2x^2+\mathrm{e}^{2bx+2a}bx+\mathrm{e}^{-2bx-2a}bx+\mathrm{e}^{2bx+2a}-\mathrm{e}^{-2bx-2a}}{12x^3}$	12

input
```
int(cosh(b*x+a)*sinh(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(4*exp(2*a)*Ei(1,-2*b*x)*x^3*b^3+4*exp(-2*a)*Ei(1,2*b*x)*x^3*b^3+2*e
xp(2*b*x+2*a)*b^2*x^2-2*exp(-2*b*x-2*a)*b^2*x^2+exp(2*b*x+2*a)*b*x+exp(-2*
b*x-2*a)*b*x+exp(2*b*x+2*a)-exp(-2*b*x-2*a))/x^3
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx =$$
$$-\frac{bx\cosh{(bx+a)}^2+bx\sinh{(bx+a)}^2+2\,(2\,b^2x^2+1)\cosh{(bx+a)}\sinh{(bx+a)}-2\,(b^3x^3\mathrm{Ei}(2\,bx)+b^3}{6\,x^3}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(b*x*cosh(b*x + a)^2 + b*x*sinh(b*x + a)^2 + 2*(2*b^2*x^2 + 1)*cosh(b
*x + a)*sinh(b*x + a) - 2*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*cosh(2*
a) - 2*(b^3*x^3*Ei(2*b*x) - b^3*x^3*Ei(-2*b*x))*sinh(2*a))/x^3
```

3.258.6 Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \frac{\sinh{(a+bx)}\cosh{(a+bx)}}{x^4}\,dx$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x**4,x)
```

output
```
Integral(sinh(a + b*x)*cosh(a + b*x)/x**4, x)
```

3.258.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.36

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx = 2\,b^3e^{(-2\,a)}\Gamma(-3,2\,bx) + 2\,b^3e^{(2\,a)}\Gamma(-3,-2\,bx)$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
2*b^3*e^(-2*a)*gamma(-3, 2*b*x) + 2*b^3*e^(2*a)*gamma(-3, -2*b*x)
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.41

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx$$
$$= \frac{4\,b^3x^3\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + 4\,b^3x^3\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - 2\,b^2x^2e^{(2\,bx+2\,a)} + 2\,b^2x^2e^{(-2\,bx-2\,a)} - bxe^{(2\,bx+2\,a)} - bxe^{(-2\,bx}}{12\,x^3}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)/x^4,x, algorithm="giac")
```

output
```
1/12*(4*b^3*x^3*Ei(2*b*x)*e^(2*a) + 4*b^3*x^3*Ei(-2*b*x)*e^(-2*a) - 2*b^2*
x^2*e^(2*b*x + 2*a) + 2*b^2*x^2*e^(-2*b*x - 2*a) - b*x*e^(2*b*x + 2*a) - b
*x*e^(-2*b*x - 2*a) - e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/x^3
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)}{x^4}\,dx$$

input
```
int((cosh(a + b*x)*sinh(a + b*x))/x^4,x)
```

output
```
int((cosh(a + b*x)*sinh(a + b*x))/x^4, x)
```

3.259 $\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx$

3.259.1 Optimal result

Integrand size = 18, antiderivative size = 134

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{8b} + \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{8b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{8b} + \frac{3^{-1-m}e^{-3a}x^m(bx)^{-m}\Gamma(1+m,3bx)}{8b}$$

output
```
1/8*3^(-1-m)*exp(3*a)*x^m*GAMMA(1+m,-3*b*x)/b/((-b*x)^m)+1/8*exp(a)*x^m*GA
MMA(1+m,-b*x)/b/((-b*x)^m)+1/8*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)+1/8*3
^(-1-m)*x^m*GAMMA(1+m,3*b*x)/b/exp(3*a)/((b*x)^m)
```

3.259.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.85

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$= \frac{e^{-3a}x^m\left(3e^{2a}(e^{2a}(-bx)^{-m}\Gamma(1+m,-bx)+(bx)^{-m}\Gamma(1+m,bx))+3^{-m}(-b^2x^2)^{-m}\left(e^{6a}(bx)^m\Gamma(1+m,-3\right.\right.}{24b}$$

input
```
Integrate[x^m*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
(x^m*(3*E^(2*a)*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + Gamma[1 + m,
b*x]/(b*x)^m) + (E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] + (-(b*x))^m*Gamma[1
 + m, 3*b*x])/(3^m*(-(b^2*x^2))^m)))/(24*b*E^(3*a))
```

3.259.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{1}{4}x^m \sinh(a+bx)+\frac{1}{4}x^m \sinh(3a+3bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-3bx)}{8b}+\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{8b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{8b}+\frac{e^{-3a}3^{-m-1}x^m(bx)^{-m}\Gamma(m+1,3bx)}{8b}$$

input
```
Int[x^m*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
(3^(-1 - m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(8*b*(-(b*x))^m) + (E^a*x^m*
Gamma[1 + m, -(b*x)])/(8*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(8*b*E^a*
(b*x)^m) + (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(8*b*E^(3*a)*(b*x)^m)
```

3.259.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.259.4 Maple [F]

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)\, dx$$

input
```
int(x^m*cosh(b*x+a)^2*sinh(b*x+a),x)
```

output
```
int(x^m*cosh(b*x+a)^2*sinh(b*x+a),x)
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.21

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\, dx = \frac{\cosh(m\log(3b)+3a)\Gamma(m+1,3bx)+3\cosh(m\log(b)+a)\Gamma(m+1,bx)+3\cosh(m\log(-b)-a)\Gamma(}{}$$

input
```
integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/24*(cosh(m*log(3*b) + 3*a)*gamma(m + 1, 3*b*x) + 3*cosh(m*log(b) + a)*ga
mma(m + 1, b*x) + 3*cosh(m*log(-b) - a)*gamma(m + 1, -b*x) + cosh(m*log(-3
*b) - 3*a)*gamma(m + 1, -3*b*x) - gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*
a) - 3*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) - gamma(m + 1, -3*b*x)*sinh(
m*log(-3*b) - 3*a) - 3*gamma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

3.259.6 Sympy [F]

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx = \int x^m \sinh(a+bx)\cosh^2(a+bx)\,dx$$

input
```
integrate(x**m*cosh(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Integral(x**m*sinh(a + b*x)*cosh(a + b*x)**2, x)
```

3.259.7 Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx &= \frac{1}{8}(3\,bx)^{-m-1}x^{m+1}e^{(-3\,a)}\Gamma(m+1,3\,bx)\\ &+ \frac{1}{8}(bx)^{-m-1}x^{m+1}e^{(-a)}\Gamma(m+1,bx)\\ &- \frac{1}{8}(-bx)^{-m-1}x^{m+1}e^{a}\Gamma(m+1,-bx)\\ &- \frac{1}{8}(-3\,bx)^{-m-1}x^{m+1}e^{(3\,a)}\Gamma(m+1,-3\,bx)\end{aligned}$$

input
```
integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/8*(3*b*x)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) + 1/8*(b*x)^(-
m - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) - 1/8*(-b*x)^(-m - 1)*x^(m + 1)*
e^a*gamma(m + 1, -b*x) - 1/8*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m +
 1, -3*b*x)
```

3.259.8 Giac [F]

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx = \int x^m \cosh(bx+a)^2 \sinh(bx+a)\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^2*sinh(b*x + a), x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx = \int x^m \cosh(a+b\,x)^2 \sinh(a+b\,x)\;dx$$

input
```
int(x^m*cosh(a + b*x)^2*sinh(a + b*x),x)
```

output
```
int(x^m*cosh(a + b*x)^2*sinh(a + b*x), x)
```

3.260 $\int x^3 \cosh^2(a+bx) \sinh(a+bx)\,dx$

3.260.1 Optimal result

Integrand size = 18, antiderivative size = 117

$$\int x^3 \cosh^2(a+bx) \sinh(a+bx)\,dx = \frac{4x\cosh(a+bx)}{3b^3} + \frac{2x\cosh^3(a+bx)}{9b^3} + \frac{x^3\cosh^3(a+bx)}{3b} - \frac{14\sinh(a+bx)}{9b^4} - \frac{2x^2\sinh(a+bx)}{3b^2} - \frac{x^2\cosh^2(a+bx)\sinh(a+bx)}{3b^2} - \frac{2\sinh^3(a+bx)}{27b^4}$$

output
```
4/3*x*cosh(b*x+a)/b^3+2/9*x*cosh(b*x+a)^3/b^3+1/3*x^3*cosh(b*x+a)^3/b-14/9
*sinh(b*x+a)/b^4-2/3*x^2*sinh(b*x+a)/b^2-1/3*x^2*cosh(b*x+a)^2*sinh(b*x+a)
/b^2-2/27*sinh(b*x+a)^3/b^4
```

3.260.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int x^3 \cosh^2(a+bx) \sinh(a+bx)\,dx = \frac{27bx(6+b^2x^2)\cosh(a+bx) + (6bx+9b^3x^3)\cosh(3(a+bx)) - 2(82+45b^2x^2+(2+9b^2x^2)\cosh(2(a+b}{108b^4}$$

input
```
Integrate[x^3*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
(27*b*x*(6 + b^2*x^2)*Cosh[a + b*x] + (6*b*x + 9*b^3*x^3)*Cosh[3*(a + b*x)
] - 2*(82 + 45*b^2*x^2 + (2 + 9*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])
/(108*b^4)
```

3.260.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.30, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5896, 3042, 3792, 3042, 3113, 2009, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sinh(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 5896}$$

$$\frac{x^3\cosh^3(a+bx)}{3b}-\frac{\int x^2\cosh^3(a+bx)dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^3\cosh^3(a+bx)}{3b}-\frac{\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{x^3\cosh^3(a+bx)}{3b}-\frac{\frac{2\int\cosh^3(a+bx)dx}{9b^2}+\frac{2}{3}\int x^2\cosh(a+bx)dx-\frac{2x\cosh^3(a+bx)}{9b^2}+\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{3b}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^3\cosh^3(a+bx)}{3b}-\frac{\frac{2\int\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{9b^2}+\frac{2}{3}\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{2x\cosh^3(a+bx)}{9b^2}+\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{3b}}{b}$$

$$\Big\downarrow \text{ 3113}$$

$$\frac{x^3\cosh^3(a+bx)}{3b}-\frac{\frac{2i\int\left(\sinh^2(a+bx)+1\right)d(-i\sinh(a+bx))}{9b^3}+\frac{2}{3}\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{2x\cosh^3(a+bx)}{9b^2}+\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{3b}}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^3\cosh^3(a+bx)}{3b}-\frac{\frac{2}{3}\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{2i\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2x\cosh^3(a+bx)}{9b^2}+\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{3b}}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{x^3\cosh^3(a+bx)}{3b}-\frac{\frac{2}{3}\left(\frac{x^2\sinh(a+bx)}{b}-\frac{2i\int -ix\sinh(a+bx)dx}{b}\right)+\frac{2i\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2x\cosh^3(a+bx)}{9b^2}+\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{3b}}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{x^3\cosh^3(a+bx)}{3b}-\frac{\frac{2}{3}\left(\frac{x^2\sinh(a+bx)}{b}-\frac{2\int x\sinh(a+bx)dx}{b}\right)+\frac{2i\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2x\cosh^3(a+bx)}{9b^2}+\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{3b}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{x^3\cosh^3(a+bx)}{3b}-\frac{\frac{2}{3}\left(\frac{x^2\sinh(a+bx)}{b}-\frac{2\int -ix\sin(ia+ibx)dx}{b}\right)+\frac{2i\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2x\cosh^3(a+bx)}{9b^2}+\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{3b}}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{x^3\cosh^3(a+bx)}{3b}-\frac{\frac{2}{3}\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\int x\sin(ia+ibx)dx}{b}\right)+\frac{2i\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2x\cosh^3(a+bx)}{9b^2}+\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{3b}}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{x^3\cosh^3(a+bx)}{3b}-\frac{\frac{2}{3}\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\cosh(a+bx)dx}{b}\right)}{b}\right)+\frac{2i\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2x\cosh^3(a+bx)}{9b^2}+\frac{x^2\sinh(a+bx)\cosh^2}{3b}}{b}$$

$$\Big\downarrow \; 3042$$

3.260. $\int x^3\cosh^2(a+bx)\sinh(a+bx)\,dx$

$$\frac{x^3\cosh^3(a+bx)}{3b}-$$

$$\frac{\frac{2}{3}\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)+\frac{2i(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx))}{9b^3}-\frac{2x\cosh^3(a+bx)}{9b^2}+\frac{x^2\sinh(a+bx)}{3b}}{b}$$

$\downarrow$ 3117

$$\frac{x^3\cosh^3(a+bx)}{3b}-$$

$$\frac{\frac{2i(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx))}{9b^3}+\frac{2}{3}\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)-\frac{2x\cosh^3(a+bx)}{9b^2}+\frac{x^2\sinh(a+bx)\cosh^2(a+}{3b}}{b}$$

input
```
Int[x^3*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
(x^3*Cosh[a + b*x]^3)/(3*b) - ((-2*x*Cosh[a + b*x]^3)/(9*b^2) + (x^2*Cosh[
a + b*x]^2*Sinh[a + b*x])/(3*b) + (((2*I)/9)*((-I)*Sinh[a + b*x] - (I/3)*S
inh[a + b*x]^3))/b^3 + (2*((x^2*Sinh[a + b*x])/b + ((2*I)*((I*x*Cosh[a + b
*x])/b - (I*Sinh[a + b*x])/b^2))/b))/3)/b
```

3.260.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.260. $\int x^3\cosh^2(a+bx)\sinh(a+bx)\,dx$

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.260.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(9x^3b^3-9x^2b^2+6bx-2)\mathrm{e}^{3bx+3a}}{216b^4}+\frac{(x^3b^3-3x^2b^2+6bx-6)\mathrm{e}^{bx+a}}{8b^4}+\frac{(x^3b^3+3x^2b^2+6bx+6)\mathrm{e}^{-bx-a}}{8b^4}+\frac{(9x^3b^3+9x^2b^2}{}$
derivativedivides	$\frac{-\frac{a^3\cosh(bx+a)^3}{3}+3a^2\left(\frac{(bx+a)\cosh(bx+a)^3}{3}-\frac{2\sinh(bx+a)}{9}-\frac{\cosh(bx+a)^2\sinh(bx+a)}{9}\right)-3a\left(\frac{(bx+a)^2\cosh(bx+a)^3}{3}-\frac{4(bx+a)\sinh}{9}\right.}{}$
default	$\frac{-\frac{a^3\cosh(bx+a)^3}{3}+3a^2\left(\frac{(bx+a)\cosh(bx+a)^3}{3}-\frac{2\sinh(bx+a)}{9}-\frac{\cosh(bx+a)^2\sinh(bx+a)}{9}\right)-3a\left(\frac{(bx+a)^2\cosh(bx+a)^3}{3}-\frac{4(bx+a)\sinh}{9}\right.}{}$

input
```
int(x^3*cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/216*(9*b^3*x^3-9*b^2*x^2+6*b*x-2)/b^4*exp(3*b*x+3*a)+1/8*(b^3*x^3-3*b^2*
x^2+6*b*x-6)/b^4*exp(b*x+a)+1/8*(b^3*x^3+3*b^2*x^2+6*b*x+6)/b^4*exp(-b*x-a
)+1/216*(9*b^3*x^3+9*b^2*x^2+6*b*x+2)/b^4*exp(-3*b*x-3*a)
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.15

$$\int x^3 \cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$= \frac{3\,(3\,b^3x^3+2\,bx)\cosh\,(bx+a)^3+9\,(3\,b^3x^3+2\,bx)\cosh\,(bx+a)\sinh\,(bx+a)^2-(9\,b^2x^2+2)\sinh\,(bx+a)}{108\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/108*(3*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^3 + 9*(3*b^3*x^3 + 2*b*x)*cosh(
b*x + a)*sinh(b*x + a)^2 - (9*b^2*x^2 + 2)*sinh(b*x + a)^3 + 27*(b^3*x^3 +
 6*b*x)*cosh(b*x + a) - 3*(27*b^2*x^2 + (9*b^2*x^2 + 2)*cosh(b*x + a)^2 +
54)*sinh(b*x + a))/b^4
```

3.260.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.25

$$\int x^3 \cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$= \begin{cases} \frac{x^3\cosh^3(a+bx)}{3b}+\frac{2x^2\sinh^3(a+bx)}{3b^2}-\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{b^2}-\frac{4x\sinh^2(a+bx)\cosh(a+bx)}{3b^3}+\frac{14x\cosh^3(a+bx)}{9b^3}+\frac{40\sinh^3(a\cdots}{27b^4} \\ \frac{x^4\sinh(a)\cosh^2(a)}{4} \end{cases}$$

input
```
integrate(x**3*cosh(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Piecewise((x**3*cosh(a + b*x)**3/(3*b) + 2*x**2*sinh(a + b*x)**3/(3*b**2)
- x**2*sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 4*x*sinh(a + b*x)**2*cosh(a +
 b*x)/(3*b**3) + 14*x*cosh(a + b*x)**3/(9*b**3) + 40*sinh(a + b*x)**3/(27*
b**4) - 14*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4), Ne(b, 0)), (x**4*sinh(
a)*cosh(a)**2/4, True))
```

3.260.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.37

$$\int x^3 \cosh^2(a+bx) \sinh(a+bx)\, dx = \frac{(9\, b^3 x^3 e^{(3\, a)} - 9\, b^2 x^2 e^{(3\, a)} + 6\, bxe^{(3\, a)} - 2\, e^{(3\, a)})e^{(3\, bx)}}{216\, b^4} + \frac{(b^3 x^3 e^a - 3\, b^2 x^2 e^a + 6\, bxe^a - 6\, e^a)e^{(bx)}}{8\, b^4} + \frac{(b^3 x^3 + 3\, b^2 x^2 + 6\, bx + 6)e^{(-bx-a)}}{8\, b^4} + \frac{(9\, b^3 x^3 + 9\, b^2 x^2 + 6\, bx + 2)e^{(-3\, bx-3\, a)}}{216\, b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/216*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e^(3*a) - 2*e^(3*a))*
e^(3*b*x)/b^4 + 1/8*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*e^(b
*x)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/b^4 + 1/216*(
9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int x^3 \cosh^2(a+bx) \sinh(a+bx)\, dx = \frac{(9\, b^3 x^3 - 9\, b^2 x^2 + 6\, bx - 2)e^{(3\, bx+3\, a)}}{216\, b^4} + \frac{(b^3 x^3 - 3\, b^2 x^2 + 6\, bx - 6)e^{(bx+a)}}{8\, b^4} + \frac{(b^3 x^3 + 3\, b^2 x^2 + 6\, bx + 6)e^{(-bx-a)}}{8\, b^4} + \frac{(9\, b^3 x^3 + 9\, b^2 x^2 + 6\, bx + 2)e^{(-3\, bx-3\, a)}}{216\, b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/216*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 + 1/8*(b^3*x
^3 - 3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6
*b*x + 6)*e^(-b*x - a)/b^4 + 1/216*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(
-3*b*x - 3*a)/b^4
```

3.260.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int x^3\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\frac{2x\cosh(a+bx)^3}{9}+\frac{4x\cosh(a+bx)}{3}}{b^3} - \frac{\frac{2x^2\sinh(a+bx)}{3}+\frac{x^2\cosh(a+bx)^2\sinh(a+bx)}{3}}{b^2} - \frac{40\sinh(a+bx)}{27b^4} - \frac{2\cosh(a+bx)^2\sinh(a+bx)}{27b^4} + \frac{x^3\cosh(a+bx)^3}{3b}$$

input
```
int(x^3*cosh(a + b*x)^2*sinh(a + b*x),x)
```

output
```
((4*x*cosh(a + b*x))/3 + (2*x*cosh(a + b*x)^3)/9)/b^3 - ((2*x^2*sinh(a + b
*x))/3 + (x^2*cosh(a + b*x)^2*sinh(a + b*x))/3)/b^2 - (40*sinh(a + b*x))/(
27*b^4) - (2*cosh(a + b*x)^2*sinh(a + b*x))/(27*b^4) + (x^3*cosh(a + b*x)^
3)/(3*b)
```

3.261 $\int x^2 \cosh^2(a+bx) \sinh(a+bx) \, dx$

3.261.1 Optimal result

Integrand size = 18, antiderivative size = 83

$$\int x^2 \cosh^2(a+bx) \sinh(a+bx) \, dx = \frac{4\cosh(a+bx)}{9b^3} + \frac{2\cosh^3(a+bx)}{27b^3} + \frac{x^2\cosh^3(a+bx)}{3b} - \frac{4x\sinh(a+bx)}{9b^2} - \frac{2x\cosh^2(a+bx)\sinh(a+bx)}{9b^2}$$

output
```
4/9*cosh(b*x+a)/b^3+2/27*cosh(b*x+a)^3/b^3+1/3*x^2*cosh(b*x+a)^3/b-4/9*x*s
inh(b*x+a)/b^2-2/9*x*cosh(b*x+a)^2*sinh(b*x+a)/b^2
```

3.261.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int x^2 \cosh^2(a+bx) \sinh(a+bx) \, dx = \frac{27(2+b^2x^2)\cosh(a+bx) + (2+9b^2x^2)\cosh(3(a+bx)) - 6bx(9\sinh(a+bx) + \sinh(3(a+bx)))}{108b^3}$$

input
```
Integrate[x^2*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
(27*(2 + b^2*x^2)*Cosh[a + b*x] + (2 + 9*b^2*x^2)*Cosh[3*(a + b*x)] - 6*b*
x*(9*Sinh[a + b*x] + Sinh[3*(a + b*x)]))/(108*b^3)
```

3.261.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5896, 3042, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sinh(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{5896}$$

$$\frac{x^2\cosh^3(a+bx)}{3b}-\frac{2\int x\cosh^3(a+bx)dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^2\cosh^3(a+bx)}{3b}-\frac{2\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{x^2\cosh^3(a+bx)}{3b}-\frac{2\left(\frac{2}{3}\int x\cosh(a+bx)dx-\frac{\cosh^3(a+bx)}{9b^2}+\frac{x\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^2\cosh^3(a+bx)}{3b}-\frac{2\left(\frac{2}{3}\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{\cosh^3(a+bx)}{9b^2}+\frac{x\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{x^2\cosh^3(a+bx)}{3b}-\frac{2\left(\frac{2}{3}\left(\frac{x\sinh(a+bx)}{b}-\frac{i\int -i\sinh(a+bx)dx}{b}\right)-\frac{\cosh^3(a+bx)}{9b^2}+\frac{x\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{x^2\cosh^3(a+bx)}{3b}-\frac{2\left(\frac{2}{3}\left(\frac{x\sinh(a+bx)}{b}-\frac{\int \sinh(a+bx)dx}{b}\right)-\frac{\cosh^3(a+bx)}{9b^2}+\frac{x\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^2\cosh^3(a+bx)}{3b}-\frac{2\left(\frac{2}{3}\left(\frac{x\sinh(a+bx)}{b}-\frac{\int -i\sin(ia+ibx)dx}{b}\right)-\frac{\cosh^3(a+bx)}{9b^2}+\frac{x\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{x^2\cosh^3(a+bx)}{3b}-\frac{2\left(\frac{2}{3}\left(\frac{x\sinh(a+bx)}{b}+\frac{i\int\sin(ia+ibx)dx}{b}\right)-\frac{\cosh^3(a+bx)}{9b^2}+\frac{x\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{x^2\cosh^3(a+bx)}{3b}-\frac{2\left(-\frac{\cosh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)+\frac{x\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b}$$

input
```
Int[x^2*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
(x^2*Cosh[a + b*x]^3)/(3*b) - (2*(-1/9*Cosh[a + b*x]^3/b^2 + (x*Cosh[a + b
*x]^2*Sinh[a + b*x])/(3*b) + (2*(-(Cosh[a + b*x]/b^2) + (x*Sinh[a + b*x])/
b))/3))/(3*b)
```

3.261.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.261.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

method	result
risch	$\frac{(9x^2b^2-6bx+2)\,\mathrm{e}^{3bx+3a}}{216b^3}+\frac{(x^2b^2-2bx+2)\,\mathrm{e}^{bx+a}}{8b^3}+\frac{(x^2b^2+2bx+2)\,\mathrm{e}^{-bx-a}}{8b^3}+\frac{(9x^2b^2+6bx+2)\,\mathrm{e}^{-3bx-3a}}{216b^3}$
derivativedivides	$\frac{\frac{a^2\cosh(bx+a)^3}{3}-2a\left(\frac{(bx+a)\cosh(bx+a)^3}{3}-\frac{2\sinh(bx+a)}{9}-\frac{\cosh(bx+a)^2\sinh(bx+a)}{9}\right)+\frac{(bx+a)^2\cosh(bx+a)^3}{3}-\frac{4(bx+a)\sinh(bx+}{9}}{b^3}$
default	$\frac{\frac{a^2\cosh(bx+a)^3}{3}-2a\left(\frac{(bx+a)\cosh(bx+a)^3}{3}-\frac{2\sinh(bx+a)}{9}-\frac{\cosh(bx+a)^2\sinh(bx+a)}{9}\right)+\frac{(bx+a)^2\cosh(bx+a)^3}{3}-\frac{4(bx+a)\sinh(bx+}{9}}{b^3}$

input
```
int(x^2*cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/216*(9*b^2*x^2-6*b*x+2)/b^3*exp(3*b*x+3*a)+1/8*(b^2*x^2-2*b*x+2)/b^3*exp
(b*x+a)+1/8*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)+1/216*(9*b^2*x^2+6*b*x+2)/b^
3*exp(-3*b*x-3*a)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int x^2\cosh^2(a+bx)\sinh(a+bx)\,dx =$$
$$-\frac{6\,bx\sinh\left(bx+a\right)^3-\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)^3-3\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-27\left(b^2x^2\right.}{108\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/108*(6*b*x*sinh(b*x + a)^3 - (9*b^2*x^2 + 2)*cosh(b*x + a)^3 - 3*(9*b^2
*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^2 - 27*(b^2*x^2 + 2)*cosh(b*x + a) +
 18*(b*x*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a))/b^3
```

3.261.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int x^2\cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$=\begin{cases} \frac{x^2\cosh^3(a+bx)}{3b}+\frac{4x\sinh^3(a+bx)}{9b^2}-\frac{2x\sinh(a+bx)\cosh^2(a+bx)}{3b^2}-\frac{4\sinh^2(a+bx)\cosh(a+bx)}{9b^3}+\frac{14\cosh^3(a+bx)}{27b^3} & \text{for } b\neq 0 \\ \frac{x^3\sinh(a)\cosh^2(a)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*cosh(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Piecewise((x**2*cosh(a + b*x)**3/(3*b) + 4*x*sinh(a + b*x)**3/(9*b**2) - 2
*x*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) - 4*sinh(a + b*x)**2*cosh(a + b
*x)/(9*b**3) + 14*cosh(a + b*x)**3/(27*b**3), Ne(b, 0)), (x**3*sinh(a)*cos
h(a)**2/3, True))
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.47

$$\int x^2\cosh^2(a+bx)\sinh(a+bx)\,dx=\frac{\left(9\,b^2x^2e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^3}+\frac{\left(b^2x^2e^a-2\,bxe^a+2\,e^a\right)e^{(bx)}}{8\,b^3}+\frac{\left(b^2x^2+2\,bx+2\right)e^{(-bx-a)}}{8\,b^3}+\frac{\left(9\,b^2x^2+6\,bx+2\right)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/216*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 + 1/8*
(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*
e^(-b*x - a)/b^3 + 1/216*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

$$\int x^2 \cosh^2(a+bx) \sinh(a+bx)\, dx = \frac{(9\, b^2 x^2 - 6\, bx + 2) e^{(3\, bx + 3\, a)}}{216\, b^3} + \frac{(b^2 x^2 - 2\, bx + 2) e^{(bx+a)}}{8\, b^3} + \frac{(b^2 x^2 + 2\, bx + 2) e^{(-bx-a)}}{8\, b^3} + \frac{(9\, b^2 x^2 + 6\, bx + 2) e^{(-3\, bx - 3\, a)}}{216\, b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/216*(9*b^2*x^2 - 6*b*x + 2)*e^(3*b*x + 3*a)/b^3 + 1/8*(b^2*x^2 - 2*b*x +
 2)*e^(b*x + a)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + 1/216*(
9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

3.261.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int x^2 \cosh^2(a+bx) \sinh(a+bx)\, dx = \frac{\frac{4 \cosh(a+b\,x)}{9} - b \left(\frac{2\,x \sinh(a+b\,x) \cosh(a+b\,x)^2}{9} + \frac{4\,x \sinh(a+b\,x)}{9}\right) + \frac{2 \cosh(a+b\,x)^3}{27} + \frac{b^2\,x^2 \cosh(a+b\,x)^3}{3}}{b^3}$$

input
```
int(x^2*cosh(a + b*x)^2*sinh(a + b*x),x)
```

output
```
((4*cosh(a + b*x))/9 - b*((4*x*sinh(a + b*x))/9 + (2*x*cosh(a + b*x)^2*sin
h(a + b*x))/9) + (2*cosh(a + b*x)^3)/27 + (b^2*x^2*cosh(a + b*x)^3)/3)/b^3
```

3.262 $\int x \cosh^2(a+bx)\sinh(a+bx)\,dx$

3.262.1 Optimal result

Integrand size = 16, antiderivative size = 45

$$\int x \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{x\cosh^3(a+bx)}{3b} - \frac{\sinh(a+bx)}{3b^2} - \frac{\sinh^3(a+bx)}{9b^2}$$

output
```
1/3*x*cosh(b*x+a)^3/b-1/3*sinh(b*x+a)/b^2-1/9*sinh(b*x+a)^3/b^2
```

3.262.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int x \cosh^2(a+bx)\sinh(a+bx)\,dx = -\frac{-9bx\cosh(a+bx) - 3bx\cosh(3(a+bx)) + 9\sinh(a+bx) + \sinh(3(a+bx))}{36b^2}$$

input
```
Integrate[x*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
-1/36*(-9*b*x*Cosh[a + b*x] - 3*b*x*Cosh[3*(a + b*x)] + 9*Sinh[a + b*x] +
Sinh[3*(a + b*x)])/b^2
```

3.262.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5896, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{5896}$$

$$\frac{x\cosh^3(a+bx)}{3b}-\frac{\int \cosh^3(a+bx)dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x\cosh^3(a+bx)}{3b}-\frac{\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b}$$

$$\Big\downarrow \text{3113}$$

$$\frac{x\cosh^3(a+bx)}{3b}-\frac{i\int \left(\sinh^2(a+bx)+1\right)d(-i\sinh(a+bx))}{3b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\cosh^3(a+bx)}{3b}-\frac{i\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{3b^2}$$

input
```
Int[x*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
(x*Cosh[a + b*x]^3)/(3*b) - ((I/3)*((-I)*Sinh[a + b*x] - (I/3)*Sinh[a + b*
x]^3))/b^2
```

3.262.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.262.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$\frac{\frac{(bx+a)\cosh(bx+a)^3}{3}-\frac{2\sinh(bx+a)}{9}-\frac{\cosh(bx+a)^2\sinh(bx+a)}{9}-\frac{a\cosh(bx+a)^3}{3}}{b^2}$	56
default	$\frac{\frac{(bx+a)\cosh(bx+a)^3}{3}-\frac{2\sinh(bx+a)}{9}-\frac{\cosh(bx+a)^2\sinh(bx+a)}{9}-\frac{a\cosh(bx+a)^3}{3}}{b^2}$	56
risch	$\frac{(3bx-1)e^{3bx+3a}}{72b^2}+\frac{(bx-1)e^{bx+a}}{8b^2}+\frac{(bx+1)e^{-bx-a}}{8b^2}+\frac{(3bx+1)e^{-3bx-3a}}{72b^2}$	77

input
```
int(x*cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/3*(b*x+a)*cosh(b*x+a)^3-2/9*sinh(b*x+a)-1/9*cosh(b*x+a)^2*sinh(b*
x+a)-1/3*a*cosh(b*x+a)^3)
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.64

$$\int x\cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$=\frac{3\,bx\cosh\left(bx+a\right)^3+9\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+9\,bx\cosh\left(bx+a\right)-\sinh\left(bx+a\right)^3-3\,\left(\cosh\left(bx\right.\right.}{36\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/36*(3*b*x*cosh(b*x + a)^3 + 9*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 9*b*x*
cosh(b*x + a) - sinh(b*x + a)^3 - 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b
^2
```

3.262.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int x\cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$=\begin{cases}\frac{x\cosh^3\left(a+bx\right)}{3b}+\frac{2\sinh^3\left(a+bx\right)}{9b^2}-\frac{\sinh\left(a+bx\right)\cosh^2\left(a+bx\right)}{3b^2} & \text{for } b\neq 0\\ \frac{x^2\sinh\left(a\right)\cosh^2\left(a\right)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*cosh(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Piecewise((x*cosh(a + b*x)**3/(3*b) + 2*sinh(a + b*x)**3/(9*b**2) - sinh(a
 + b*x)*cosh(a + b*x)**2/(3*b**2), Ne(b, 0)), (x**2*sinh(a)*cosh(a)**2/2,
True))
```

3.262.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(39) = 78.

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.87

$$\int x\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{72\,b^2}+\frac{(bxe^a-e^a)e^{(bx)}}{8\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{8\,b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/72*(3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 1/8*(b*x*e^a - e^a)*e^(b*x)
/b^2 + 1/8*(b*x + 1)*e^(-b*x - a)/b^2 + 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/
b^2
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int x\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{72\,b^2}+\frac{(bx-1)e^{(bx+a)}}{8\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{8\,b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/72*(3*b*x - 1)*e^(3*b*x + 3*a)/b^2 + 1/8*(b*x - 1)*e^(b*x + a)/b^2 + 1/8
*(b*x + 1)*e^(-b*x - a)/b^2 + 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2
```

3.262.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int x\cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$= -\frac{-3\,b\,x\cosh(a+b\,x)^3+\sinh(a+b\,x)\,\cosh(a+b\,x)^2+2\sinh(a+b\,x)}{9\,b^2}$$

input
```
int(x*cosh(a + b*x)^2*sinh(a + b*x),x)
```

output
```
-(2*sinh(a + b*x) + cosh(a + b*x)^2*sinh(a + b*x) - 3*b*x*cosh(a + b*x)^3)
/(9*b^2)
```

3.263 $\int \cosh^2(a+bx)\sinh(a+bx)\,dx$

3.263.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx=\frac{\cosh^3(a+bx)}{3b}$$

output
```
1/3*cosh(b*x+a)^3/b
```

3.263.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx=\frac{\cosh^3(a+bx)}{3b}$$

input
```
Integrate[Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
Cosh[a + b*x]^3/(3*b)
```

3.263.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 26, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)\cos(ia+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \cos(ia+ibx)^2\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3045}$$

$$\frac{\int \cosh^2(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\cosh^3(a+bx)}{3b}$$

input
```
Int[Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
Cosh[a + b*x]^3/(3*b)
```

3.263.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.263.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cosh(bx+a)^3}{3b}$	14
default	$\frac{\cosh(bx+a)^3}{3b}$	14
risch	$\frac{e^{3bx+3a}}{24b}+\frac{e^{bx+a}}{8b}+\frac{e^{-bx-a}}{8b}+\frac{e^{-3bx-3a}}{24b}$	55

input
```
int(cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*cosh(b*x+a)^3/b
```

3.263.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.53

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^3+3\cosh(bx+a)\sinh(bx+a)^2+3\cosh(bx+a)}{12\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + 3*cosh(b*x + a))
/b
```

3.263.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{\cosh^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sinh(a)\cosh^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Piecewise((cosh(a + b*x)**3/(3*b), Ne(b, 0)), (x*sinh(a)*cosh(a)**2, True)
)
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^3}{3\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/3*cosh(b*x + a)^3/b
```

3.263.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(13) = 26.

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.60

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{24\,b} + \frac{e^{(bx+a)}}{8\,b} + \frac{e^{(-bx-a)}}{8\,b} + \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/24*e^(3*b*x + 3*a)/b + 1/8*e^(b*x + a)/b + 1/8*e^(-b*x - a)/b + 1/24*e^(
-3*b*x - 3*a)/b
```

3.263.9 Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)^3}{3\,b}$$

input
```
int(cosh(a + b*x)^2*sinh(a + b*x),x)
```

output
```
cosh(a + b*x)^3/(3*b)
```

3.264 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx$

3.264.1 Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}\text{Chi}(bx)\sinh(a) + \frac{1}{4}\text{Chi}(3bx)\sinh(3a) + \frac{1}{4}\cosh(a)\text{Shi}(bx) + \frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

output
```
1/4*cosh(a)*Shi(b*x)+1/4*cosh(3*a)*Shi(3*b*x)+1/4*Chi(b*x)*sinh(a)+1/4*Chi
(3*b*x)*sinh(3*a)
```

3.264.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}(\text{Chi}(bx)\sinh(a) + \text{Chi}(3bx)\sinh(3a) + \cosh(a)\text{Shi}(bx) + \cosh(3a)\text{Shi}(3bx))$$

input
```
Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x,x]
```

output
```
(CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] + Cosh[a]*SinhI
ntegral[b*x] + Cosh[3*a]*SinhIntegral[3*b*x])/4
```

3.264.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\cosh^2(a+bx)}{x}\,dx$$

$$\Big\downarrow \; 5971$$

$$\int \left(\frac{\sinh(a+bx)}{4x}+\frac{\sinh(3a+3bx)}{4x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\sinh(a)\text{Chi}(bx)+\frac{1}{4}\sinh(3a)\text{Chi}(3bx)+\frac{1}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

input
```
Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x,x]
```

output
```
(CoshIntegral[b*x]*Sinh[a])/4 + (CoshIntegral[3*b*x]*Sinh[3*a])/4 + (Cosh[
a]*SinhIntegral[b*x])/4 + (Cosh[3*a]*SinhIntegral[3*b*x])/4
```

3.264.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.264.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{e^{-3a}\,\mathrm{Ei}_1(3bx)}{8}+\frac{e^{-a}\,\mathrm{Ei}_1(bx)}{8}-\frac{e^{a}\,\mathrm{Ei}_1(-bx)}{8}-\frac{e^{3a}\,\mathrm{Ei}_1(-3bx)}{8}$	47

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/8*exp(-3*a)*Ei(1,3*b*x)+1/8*exp(-a)*Ei(1,b*x)-1/8*exp(a)*Ei(1,-b*x)-1/8*
exp(3*a)*Ei(1,-3*b*x)
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{8}\left(\mathrm{Ei}(3\,bx)-\mathrm{Ei}(-3\,bx)\right)\cosh\left(3\,a\right) + \frac{1}{8}\left(\mathrm{Ei}(bx)-\mathrm{Ei}(-bx)\right)\cosh\left(a\right) + \frac{1}{8}\left(\mathrm{Ei}(3\,bx)+\mathrm{Ei}(-3\,bx)\right)\sinh\left(3\,a\right) + \frac{1}{8}\left(\mathrm{Ei}(bx)+\mathrm{Ei}(-bx)\right)\sinh\left(a\right)$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="fricas")
```

output
```
1/8*(Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 1/8*(Ei(b*x) - Ei(-b*x))*cosh(a)
+ 1/8*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) + 1/8*(Ei(b*x) + Ei(-b*x))*sinh(a
)
```

3.264.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\cosh^2(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)/x,x)
```

output
```
Integral(sinh(a + b*x)*cosh(a + b*x)**2/x, x)
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} - \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{1}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="maxima")
```

output
```
1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 1/
8*Ei(b*x)*e^a
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} - \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{1}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="giac")
```

output
```
1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 1/
8*Ei(b*x)*e^a
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^2\sinh(a+b\,x)}{x}\,dx$$

input
```
int((cosh(a + b*x)^2*sinh(a + b*x))/x,x)
```

output
```
int((cosh(a + b*x)^2*sinh(a + b*x))/x, x)
```

3.265 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx$

3.265.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{1}{4}b\cosh(a)\text{Chi}(bx)+\frac{3}{4}b\cosh(3a)\text{Chi}(3bx) - \frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3bx)}{4x} + \frac{1}{4}b\sinh(a)\text{Shi}(bx)+\frac{3}{4}b\sinh(3a)\text{Shi}(3bx)$$

output
```
1/4*b*Chi(b*x)*cosh(a)+3/4*b*Chi(3*b*x)*cosh(3*a)+1/4*b*Shi(b*x)*sinh(a)+3
/4*b*Shi(3*b*x)*sinh(3*a)-1/4*sinh(b*x+a)/x-1/4*sinh(3*b*x+3*a)/x
```

3.265.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{bx\cosh(a)\text{Chi}(bx)+3bx\cosh(3a)\text{Chi}(3bx)-\sinh(a+bx)-\sinh(3(a+bx))+bx\sinh(a)\text{Shi}(bx)+3bx\,\text{s}}{4x}$$

input
```
Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^2,x]
```

output
```
(b*x*Cosh[a]*CoshIntegral[b*x] + 3*b*x*Cosh[3*a]*CoshIntegral[3*b*x] - Sin
h[a + b*x] - Sinh[3*(a + b*x)] + b*x*Sinh[a]*SinhIntegral[b*x] + 3*b*x*Sin
h[3*a]*SinhIntegral[3*b*x])/(4*x)
```

3.265.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\cosh^2(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{\sinh(a+bx)}{4x^2}+\frac{\sinh(3a+3bx)}{4x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}b\cosh(a)\text{Chi}(bx)+\frac{3}{4}b\cosh(3a)\text{Chi}(3bx)+\frac{1}{4}b\sinh(a)\text{Shi}(bx)+\frac{3}{4}b\sinh(3a)\text{Shi}(3bx)-\frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3bx)}{4x}$$

input
```
Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^2,x]
```

output
```
(b*Cosh[a]*CoshIntegral[b*x])/4 + (3*b*Cosh[3*a]*CoshIntegral[3*b*x])/4 -
Sinh[a + b*x]/(4*x) - Sinh[3*a + 3*b*x]/(4*x) + (b*Sinh[a]*SinhIntegral[b*
x])/4 + (3*b*Sinh[3*a]*SinhIntegral[3*b*x])/4
```

3.265.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.265.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{e^{-a} \operatorname{Ei}_1(bx) bx+3 e^{3a} \operatorname{Ei}_1(-3bx) bx+3 e^{-3a} \operatorname{Ei}_1(3bx) bx+e^{a} \operatorname{Ei}_1(-bx) bx+e^{bx+a}-e^{-bx-a}+e^{3bx+3a}-e^{-3bx-3a}}{8x}$	95

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(exp(-a)*Ei(1,b*x)*b*x+3*exp(3*a)*Ei(1,-3*b*x)*b*x+3*exp(-3*a)*Ei(1,3
*b*x)*b*x+exp(a)*Ei(1,-b*x)*b*x+exp(b*x+a)-exp(-b*x-a)+exp(3*b*x+3*a)-exp(
-3*b*x-3*a))/x
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx =$$
$$-\frac{2\sinh(bx+a)^3 - 3\left(bx\operatorname{Ei}(3bx) + bx\operatorname{Ei}(-3bx)\right)\cosh(3a) - \left(bx\operatorname{Ei}(bx) + bx\operatorname{Ei}(-bx)\right)\cosh(a) + 2\,(3\,\mathrm{co}}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sinh(b*x + a)^3 - 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*b*x))*cosh(3*a) - (
b*x*Ei(b*x) + b*x*Ei(-b*x))*cosh(a) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x +
 a) - 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))*sinh(3*a) - (b*x*Ei(b*x) - b*x*Ei
(-b*x))*sinh(a))/x
```

3.265.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\cosh^2(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)/x**2,x)
```

output
```
Integral(sinh(a + b*x)*cosh(a + b*x)**2/x**2, x)
```

3.265.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{3}{8}\,be^{(-3\,a)}\Gamma(-1,3\,bx) + \frac{1}{8}\,be^{(-a)}\Gamma(-1,bx) + \frac{1}{8}\,be^{a}\Gamma(-1,-bx) + \frac{3}{8}\,be^{(3\,a)}\Gamma(-1,-3\,bx)$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
3/8*b*e^(-3*a)*gamma(-1, 3*b*x) + 1/8*b*e^(-a)*gamma(-1, b*x) + 1/8*b*e^a*
gamma(-1, -b*x) + 3/8*b*e^(3*a)*gamma(-1, -3*b*x)
```

3.265.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{3\,bx\mathrm{Ei}(3\,bx)\,e^{(3\,a)} + bx\mathrm{Ei}(-bx)\,e^{(-a)} + 3\,bx\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + bx\mathrm{Ei}(bx)\,e^{a} - e^{(3\,bx+3\,a)} - e^{(bx+a)} + e^{(-bx-a)}}{8\,x}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="giac")
```

output
```
1/8*(3*b*x*Ei(3*b*x)*e^(3*a) + b*x*Ei(-b*x)*e^(-a) + 3*b*x*Ei(-3*b*x)*e^(-
3*a) + b*x*Ei(b*x)*e^a - e^(3*b*x + 3*a) - e^(b*x + a) + e^(-b*x - a) + e^
(-3*b*x - 3*a))/x
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^2\sinh(a+b\,x)}{x^2}\,dx$$

input
```
int((cosh(a + b*x)^2*sinh(a + b*x))/x^2,x)
```

output
```
int((cosh(a + b*x)^2*sinh(a + b*x))/x^2, x)
```

3.266 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx$

3.266.1 Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx = -\frac{b\cosh(a+bx)}{8x}-\frac{3b\cosh(3a+3bx)}{8x}+\frac{1}{8}b^2\mathrm{Chi}(bx)\sinh(a)+\frac{9}{8}b^2\mathrm{Chi}(3bx)\sinh(3a)-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{8x^2}+\frac{1}{8}b^2\cosh(a)\mathrm{Shi}(bx)+\frac{9}{8}b^2\cosh(3a)\mathrm{Shi}(3bx)$$

output
```
-1/8*b*cosh(b*x+a)/x-3/8*b*cosh(3*b*x+3*a)/x+1/8*b^2*cosh(a)*Shi(b*x)+9/8*
b^2*cosh(3*a)*Shi(3*b*x)+1/8*b^2*Chi(b*x)*sinh(a)+9/8*b^2*Chi(3*b*x)*sinh(
3*a)-1/8*sinh(b*x+a)/x^2-1/8*sinh(3*b*x+3*a)/x^2
```

3.266.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx = -\frac{bx\cosh(a+bx)+3bx\cosh(3(a+bx))-b^2x^2\mathrm{Chi}(bx)\sinh(a)-9b^2x^2\mathrm{Chi}(3bx)\sinh(3a)+\sinh(a+bx)}{8x^2}$$

input
```
Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^3,x]
```

output
```
-1/8*(b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - b^2*x^2*CoshIntegral[b
*x]*Sinh[a] - 9*b^2*x^2*CoshIntegral[3*b*x]*Sinh[3*a] + Sinh[a + b*x] + Si
nh[3*(a + b*x)] - b^2*x^2*Cosh[a]*SinhIntegral[b*x] - 9*b^2*x^2*Cosh[3*a]*
SinhIntegral[3*b*x])/x^2
```

3.266.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\cosh^2(a+bx)}{x^3}\,dx$$

$$\Big\downarrow \text{ 5971}$$

$$\int \left(\frac{\sinh(a+bx)}{4x^3}+\frac{\sinh(3a+3bx)}{4x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}b^2\sinh(a)\text{Chi}(bx)+\frac{9}{8}b^2\sinh(3a)\text{Chi}(3bx)+\frac{1}{8}b^2\cosh(a)\text{Shi}(bx)+\frac{9}{8}b^2\cosh(3a)\text{Shi}(3bx)-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{8x^2}-\frac{b\cosh(a+bx)}{8x}-\frac{3b\cosh(3a+3bx)}{8x}$$

input
```
Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^3,x]
```

output
```
-1/8*(b*Cosh[a + b*x])/x - (3*b*Cosh[3*a + 3*b*x])/(8*x) + (b^2*CoshIntegr
al[b*x]*Sinh[a])/8 + (9*b^2*CoshIntegral[3*b*x]*Sinh[3*a])/8 - Sinh[a + b*
x]/(8*x^2) - Sinh[3*a + 3*b*x]/(8*x^2) + (b^2*Cosh[a]*SinhIntegral[b*x])/8
 + (9*b^2*Cosh[3*a]*SinhIntegral[3*b*x])/8
```

3.266.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.266.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{-9\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^2b^2+9\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^2b^2-\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^2b^2+\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^2b^2+3\,\mathrm{e}^{-3bx-3a}bx+3\,\mathrm{e}^{3bx+3a}bx+\mathrm{e}^{-bx-a}bx+\mathrm{e}^{bx+a}}{16x^2}$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-9*exp(-3*a)*Ei(1,3*b*x)*x^2*b^2+9*exp(3*a)*Ei(1,-3*b*x)*x^2*b^2-ex
p(-a)*Ei(1,b*x)*x^2*b^2+exp(a)*Ei(1,-b*x)*x^2*b^2+3*exp(-3*b*x-3*a)*b*x+3*
exp(3*b*x+3*a)*b*x+exp(-b*x-a)*b*x+exp(b*x+a)*b*x-exp(-3*b*x-3*a)+exp(3*b*
x+3*a)-exp(-b*x-a)+exp(b*x+a))/x^2
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.65

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx =$$
$$-\frac{6\,bx\cosh\left(bx+a\right)^3+18\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+2\,bx\cosh\left(bx+a\right)+2\,\sinh\left(bx+a\right)^3-9\,(b^2x}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
-1/16*(6*b*x*cosh(b*x + a)^3 + 18*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 2*b*
x*cosh(b*x + a) + 2*sinh(b*x + a)^3 - 9*(b^2*x^2*Ei(3*b*x) - b^2*x^2*Ei(-3
*b*x))*cosh(3*a) - (b^2*x^2*Ei(b*x) - b^2*x^2*Ei(-b*x))*cosh(a) + 2*(3*cos
h(b*x + a)^2 + 1)*sinh(b*x + a) - 9*(b^2*x^2*Ei(3*b*x) + b^2*x^2*Ei(-3*b*x
))*sinh(3*a) - (b^2*x^2*Ei(b*x) + b^2*x^2*Ei(-b*x))*sinh(a))/x^2
```

3.266.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \frac{\sinh(a+bx)\cosh^2(a+bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)/x**3,x)
```

output
```
Integral(sinh(a + b*x)*cosh(a + b*x)**2/x**3, x)
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.49

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx = \frac{9}{8}\,b^2e^{(-3\,a)}\Gamma(-2,3\,bx) + \frac{1}{8}\,b^2e^{(-a)}\Gamma(-2,bx) - \frac{1}{8}\,b^2e^{a}\Gamma(-2,-bx) - \frac{9}{8}\,b^2e^{(3\,a)}\Gamma(-2,-3\,bx)$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
9/8*b^2*e^(-3*a)*gamma(-2, 3*b*x) + 1/8*b^2*e^(-a)*gamma(-2, b*x) - 1/8*b^
2*e^a*gamma(-2, -b*x) - 9/8*b^2*e^(3*a)*gamma(-2, -3*b*x)
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx$$
$$= \frac{9\,b^2x^2\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - b^2x^2\mathrm{Ei}(-bx)\,e^{(-a)} - 9\,b^2x^2\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + b^2x^2\mathrm{Ei}(bx)\,e^{a} - 3\,bxe^{(3\,bx+3\,a)} - bxe^{(bx}}{16\,x^2}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x, algorithm="giac")
```

output
```
1/16*(9*b^2*x^2*Ei(3*b*x)*e^(3*a) - b^2*x^2*Ei(-b*x)*e^(-a) - 9*b^2*x^2*Ei
(-3*b*x)*e^(-3*a) + b^2*x^2*Ei(b*x)*e^a - 3*b*x*e^(3*b*x + 3*a) - b*x*e^(b
*x + a) - b*x*e^(-b*x - a) - 3*b*x*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) - e^
(b*x + a) + e^(-b*x - a) + e^(-3*b*x - 3*a))/x^2
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)^2\sinh(a+b\,x)}{x^3}\,dx$$

input
```
int((cosh(a + b*x)^2*sinh(a + b*x))/x^3,x)
```

output
```
int((cosh(a + b*x)^2*sinh(a + b*x))/x^3, x)
```

3.267 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx$

3.267.1 Optimal result

Integrand size = 18, antiderivative size = 154

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx &= -\frac{b\cosh(a+bx)}{24x^2}-\frac{b\cosh(3a+3bx)}{8x^2}+\frac{1}{24}b^3\cosh(a)\text{Chi}(bx)\\ &\quad+\frac{9}{8}b^3\cosh(3a)\text{Chi}(3bx)-\frac{\sinh(a+bx)}{12x^3}-\frac{b^2\sinh(a+bx)}{24x}-\frac{\sinh(3a+3bx)}{12x^3}\\ &\quad-\frac{3b^2\sinh(3a+3bx)}{8x}+\frac{1}{24}b^3\sinh(a)\text{Shi}(bx)+\frac{9}{8}b^3\sinh(3a)\text{Shi}(3bx)\end{aligned}$$

output
```
1/24*b^3*Chi(b*x)*cosh(a)+9/8*b^3*Chi(3*b*x)*cosh(3*a)-1/24*b*cosh(b*x+a)/
x^2-1/8*b*cosh(3*b*x+3*a)/x^2+1/24*b^3*Shi(b*x)*sinh(a)+9/8*b^3*Shi(3*b*x)
*sinh(3*a)-1/12*sinh(b*x+a)/x^3-1/24*b^2*sinh(b*x+a)/x-1/12*sinh(3*b*x+3*a
)/x^3-3/8*b^2*sinh(3*b*x+3*a)/x
```

3.267.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx = -\frac{bx\cosh(a+bx)+3bx\cosh(3(a+bx))-b^3x^3\cosh(a)\text{Chi}(bx)-27b^3x^3\cosh(3a)\text{Chi}(3bx)+2\sinh(a+\cdots}{\cdots}$$

input
```
Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^4,x]
```

output
```
-1/24*(b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - b^3*x^3*Cosh[a]*CoshI
ntegral[b*x] - 27*b^3*x^3*Cosh[3*a]*CoshIntegral[3*b*x] + 2*Sinh[a + b*x]
+ b^2*x^2*Sinh[a + b*x] + 2*Sinh[3*(a + b*x)] + 9*b^2*x^2*Sinh[3*(a + b*x)
] - b^3*x^3*Sinh[a]*SinhIntegral[b*x] - 27*b^3*x^3*Sinh[3*a]*SinhIntegral[
3*b*x])/x^3
```

3.267.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\cosh^2(a+bx)}{x^4}\,dx$$
$$\Big\downarrow \text{5971}$$
$$\int \left(\frac{\sinh(a+bx)}{4x^4}+\frac{\sinh(3a+3bx)}{4x^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{24}b^3\cosh(a)\text{Chi}(bx)+\frac{9}{8}b^3\cosh(3a)\text{Chi}(3bx)+\frac{1}{24}b^3\sinh(a)\text{Shi}(bx)+\frac{9}{8}b^3\sinh(3a)\text{Shi}(3bx)-\frac{b^2\sinh(a+bx)}{24x}-\frac{3b^2\sinh(3a+3bx)}{8x}-\frac{\sinh(a+bx)}{12x^3}-\frac{\sinh(3a+3bx)}{12x^3}-\frac{b\cosh(a+bx)}{24x^2}-\frac{b\cosh(3a+3bx)}{8x^2}$$

input
```
Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^4,x]
```

output
```
-1/24*(b*Cosh[a + b*x])/x^2 - (b*Cosh[3*a + 3*b*x])/(8*x^2) + (b^3*Cosh[a]
*CoshIntegral[b*x])/24 + (9*b^3*Cosh[3*a]*CoshIntegral[3*b*x])/8 - Sinh[a
+ b*x]/(12*x^3) - (b^2*Sinh[a + b*x])/(24*x) - Sinh[3*a + 3*b*x]/(12*x^3)
- (3*b^2*Sinh[3*a + 3*b*x])/(8*x) + (b^3*Sinh[a]*SinhIntegral[b*x])/24 + (
9*b^3*Sinh[3*a]*SinhIntegral[3*b*x])/8
```

3.267.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.267.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{27\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^3b^3+27\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^3b^3+\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^3b^3+\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^3b^3+9\,\mathrm{e}^{3bx+3a}b^2x^2-9\,\mathrm{e}^{-3bx-3a}b^2x^2-\mathrm{e}^{-bx-a}b^2x^2}{48x^3}$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(27*exp(3*a)*Ei(1,-3*b*x)*x^3*b^3+27*exp(-3*a)*Ei(1,3*b*x)*x^3*b^3+e
xp(-a)*Ei(1,b*x)*x^3*b^3+exp(a)*Ei(1,-b*x)*x^3*b^3+9*exp(3*b*x+3*a)*b^2*x^
2-9*exp(-3*b*x-3*a)*b^2*x^2-exp(-b*x-a)*b^2*x^2+exp(b*x+a)*b^2*x^2+3*exp(3
*b*x+3*a)*b*x+3*exp(-3*b*x-3*a)*b*x+exp(-b*x-a)*b*x+exp(b*x+a)*b*x+2*exp(3
*b*x+3*a)-2*exp(-3*b*x-3*a)-2*exp(-b*x-a)+2*exp(b*x+a))/x^3
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.45

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx = -\frac{6\,bx\cosh\left(bx+a\right)^3+18\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+2\left(9\,b^2x^2+2\right)\sinh\left(bx+a\right)^3+2\,bx\cosh\left(bx+\right.}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
-1/48*(6*b*x*cosh(b*x + a)^3 + 18*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 2*(9
*b^2*x^2 + 2)*sinh(b*x + a)^3 + 2*b*x*cosh(b*x + a) - 27*(b^3*x^3*Ei(3*b*x
) + b^3*x^3*Ei(-3*b*x))*cosh(3*a) - (b^3*x^3*Ei(b*x) + b^3*x^3*Ei(-b*x))*c
osh(a) + 2*(b^2*x^2 + 3*(9*b^2*x^2 + 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)
 - 27*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*sinh(3*a) - (b^3*x^3*Ei(b*x
) - b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

3.267.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \frac{\sinh\left(a+bx\right)\cosh^2\left(a+bx\right)}{x^4}\,dx$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)/x**4,x)
```

output
```
Integral(sinh(a + b*x)*cosh(a + b*x)**2/x**4, x)
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.38

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx = \frac{27}{8}\,b^3e^{(-3\,a)}\Gamma(-3,3\,bx) + \frac{1}{8}\,b^3e^{(-a)}\Gamma(-3,bx) + \frac{1}{8}\,b^3e^{a}\Gamma(-3,-bx) + \frac{27}{8}\,b^3e^{(3\,a)}\Gamma(-3,-3\,bx)$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
27/8*b^3*e^(-3*a)*gamma(-3, 3*b*x) + 1/8*b^3*e^(-a)*gamma(-3, b*x) + 1/8*b
^3*e^a*gamma(-3, -b*x) + 27/8*b^3*e^(3*a)*gamma(-3, -3*b*x)
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.45

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx$$
$$= \frac{27\,b^3x^3\text{Ei}(3\,bx)\,e^{(3\,a)} + b^3x^3\text{Ei}(-bx)\,e^{(-a)} + 27\,b^3x^3\text{Ei}(-3\,bx)\,e^{(-3\,a)} + b^3x^3\text{Ei}(bx)\,e^{a} - 9\,b^2x^2e^{(3\,bx+3\,a)} - b^2}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x, algorithm="giac")
```

output
```
1/48*(27*b^3*x^3*Ei(3*b*x)*e^(3*a) + b^3*x^3*Ei(-b*x)*e^(-a) + 27*b^3*x^3*
Ei(-3*b*x)*e^(-3*a) + b^3*x^3*Ei(b*x)*e^a - 9*b^2*x^2*e^(3*b*x + 3*a) - b^
2*x^2*e^(b*x + a) + b^2*x^2*e^(-b*x - a) + 9*b^2*x^2*e^(-3*b*x - 3*a) - 3*
b*x*e^(3*b*x + 3*a) - b*x*e^(b*x + a) - b*x*e^(-b*x - a) - 3*b*x*e^(-3*b*x
 - 3*a) - 2*e^(3*b*x + 3*a) - 2*e^(b*x + a) + 2*e^(-b*x - a) + 2*e^(-3*b*x
 - 3*a))/x^3
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)^2\sinh(a+b\,x)}{x^4}\,dx$$

input
```
int((cosh(a + b*x)^2*sinh(a + b*x))/x^4,x)
```

output
```
int((cosh(a + b*x)^2*sinh(a + b*x))/x^4, x)
```

3.268 $\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx$

3.268.1 Optimal result

Integrand size = 18, antiderivative size = 139

$$\int x^m\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{2^{-2(3+m)}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b}+\frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}+\frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}+\frac{2^{-2(3+m)}e^{-4a}x^m(bx)^{-m}\Gamma(1+m,4bx)}{b}$$

output
```
exp(4*a)*x^m*GAMMA(1+m,-4*b*x)/(2^(6+2*m))/b/((-b*x)^m)+2^(-4-m)*exp(2*a)*
x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)+2^(-4-m)*x^m*GAMMA(1+m,2*b*x)/b/exp(2*a
)/((b*x)^m)+x^m*GAMMA(1+m,4*b*x)/(2^(6+2*m))/b/exp(4*a)/((b*x)^m)
```

3.268.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

$$\int x^m\cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$=\frac{4^{-3-m}e^{-4a}x^m(-b^2x^2)^{-m}\left(e^{8a}(bx)^m\Gamma(1+m,-4bx)+2^{2+m}e^{6a}(bx)^m\Gamma(1+m,-2bx)+(-bx)^m\left(2^{2+m}e^{2a}\Gamma(\right.\right.}{b}$$

input
```
Integrate[x^m*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
(4^(-3 - m)*x^m*(E^(8*a)*(b*x)^m*Gamma[1 + m, -4*b*x] + 2^(2 + m)*E^(6*a)*
(b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*(2^(2 + m)*E^(2*a)*Gamma[1 + m,
2*b*x] + Gamma[1 + m, 4*b*x])))/(b*E^(4*a)*(-(b^2*x^2))^m)
```

3.268.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{1}{4}x^m\sinh(2a+2bx)+\frac{1}{8}x^m\sinh(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\Gamma(m+1,-4bx)}{b}+\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}+\frac{e^{-4a}2^{-2(m+3)}x^m(bx)^{-m}\Gamma(m+1,4bx)}{b}$$

input
```
Int[x^m*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
(E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*b*(-(b*x))^m) + (2^(-4 -
 m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-4 - m)*x^m*Gam
ma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3
 + m))*b*E^(4*a)*(b*x)^m)
```

3.268.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.268.4 Maple [F]

$$\int x^m \cosh (bx+a)^3 \sinh (bx+a)\, dx$$

input
```
int(x^m*cosh(b*x+a)^3*sinh(b*x+a),x)
```

output
```
int(x^m*cosh(b*x+a)^3*sinh(b*x+a),x)
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.24

$$\int x^m \cosh^3(a+bx)\sinh(a+bx)\, dx = \frac{\cosh(m\log(4b)+4a)\,\Gamma(m+1,4bx)+4\cosh(m\log(2b)+2a)\,\Gamma(m+1,2bx)+4\cosh(m\log(-2b)-$$

input
```
integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/64*(cosh(m*log(4*b) + 4*a)*gamma(m + 1, 4*b*x) + 4*cosh(m*log(2*b) + 2*a
)*gamma(m + 1, 2*b*x) + 4*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + c
osh(m*log(-4*b) - 4*a)*gamma(m + 1, -4*b*x) - gamma(m + 1, 4*b*x)*sinh(m*l
og(4*b) + 4*a) - 4*gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) - 4*gamma(m
+ 1, -2*b*x)*sinh(m*log(-2*b) - 2*a) - gamma(m + 1, -4*b*x)*sinh(m*log(-4*
b) - 4*a))/b
```

3.268.6 Sympy [F]

$$\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx = \int x^m \sinh\left(a+bx\right)\cosh^3\left(a+bx\right)dx$$

input
```
integrate(x**m*cosh(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Integral(x**m*sinh(a + b*x)*cosh(a + b*x)**3, x)
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx =& \frac{1}{16}\left(4\,bx\right)^{-m-1}x^{m+1}e^{(-4\,a)}\Gamma(m+1,4\,bx)\\ &+\frac{1}{8}\left(2\,bx\right)^{-m-1}x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx)\\ &-\frac{1}{8}\left(-2\,bx\right)^{-m-1}x^{m+1}e^{(2\,a)}\Gamma(m+1,-2\,bx)\\ &-\frac{1}{16}\left(-4\,bx\right)^{-m-1}x^{m+1}e^{(4\,a)}\Gamma(m+1,-4\,bx)\end{aligned}$$

input
```
integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/16*(4*b*x)^(-m - 1)*x^(m + 1)*e^(-4*a)*gamma(m + 1, 4*b*x) + 1/8*(2*b*x)
^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/8*(-2*b*x)^(-m - 1)*x
^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/16*(-4*b*x)^(-m - 1)*x^(m + 1)*e
^(4*a)*gamma(m + 1, -4*b*x)
```

3.268.8 Giac [F]

$$\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^3\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^3*sinh(b*x + a), x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx = \int x^m \cosh(a+b\,x)^3 \sinh(a+b\,x)\;dx$$

input
```
int(x^m*cosh(a + b*x)^3*sinh(a + b*x),x)
```

output
```
int(x^m*cosh(a + b*x)^3*sinh(a + b*x), x)
```

3.269 $\int x^3 \cosh^3(a+bx) \sinh(a+bx) \, dx$

3.269.1 Optimal result

Integrand size = 18, antiderivative size = 155

$$\int x^3 \cosh^3(a+bx) \sinh(a+bx) \, dx = -\frac{45x}{256b^3} - \frac{3x^3}{32b} + \frac{9x\cosh^2(a+bx)}{32b^3} + \frac{3x\cosh^4(a+bx)}{32b^3} + \frac{x^3\cosh^4(a+bx)}{4b} - \frac{45\cosh(a+bx)\sinh(a+bx)}{256b^4} - \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{3\cosh^3(a+bx)\sinh(a+bx)}{128b^4} - \frac{3x^2\cosh^3(a+bx)\sinh(a+bx)}{16b^2}$$

output
```
-45/256*x/b^3-3/32*x^3/b+9/32*x*cosh(b*x+a)^2/b^3+3/32*x*cosh(b*x+a)^4/b^3
+1/4*x^3*cosh(b*x+a)^4/b-45/256*cosh(b*x+a)*sinh(b*x+a)/b^4-9/32*x^2*cosh(
b*x+a)*sinh(b*x+a)/b^2-3/128*cosh(b*x+a)^3*sinh(b*x+a)/b^4-3/16*x^2*cosh(b
*x+a)^3*sinh(b*x+a)/b^2
```

3.269.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.59

$$\int x^3 \cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$= \frac{32bx(3+2b^2x^2)\cosh(2(a+bx))+2bx(3+8b^2x^2)\cosh(4(a+bx))-3(16+32b^2x^2+(1+8b^2x^2)\cosh(2(}{512b^4}$$

input
```
Integrate[x^3*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
(32*b*x*(3 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + 2*b*x*(3 + 8*b^2*x^2)*Cosh[4*(
a + b*x)] - 3*(16 + 32*b^2*x^2 + (1 + 8*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[2
*(a + b*x)])/(512*b^4)
```

3.269.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.34, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {5896, 3042, 3792, 3042, 3115, 3042, 3115, 24, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sinh(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{5896}$$

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\int x^2\cosh^4(a+bx)dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx}{4b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\left(\frac{\int\cosh^4(a+bx)dx}{8b^2}+\frac{3}{4}\int x^2\cosh^2(a+bx)dx-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\left(\frac{\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx}{8b^2}+\frac{3}{4}\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{4b}$$

$\big\downarrow$ 3115

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\left(\frac{\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}}{8b^2}+\frac{3}{4}\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{4b}$$

$\big\downarrow$ 3042

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\left(\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{8b^2}+\frac{3}{4}\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{4b}$$

$\big\downarrow$ 3115

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\left(\frac{\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}}{8b^2}+\frac{3}{4}\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{x^2\sinh(a+bx)\cosh^3}{4b}\right.}{4b}$$

$\big\downarrow$ 24

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\left(\frac{3}{4}\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2\sinh(a+bx)\cosh^3(a+}{4b}\right.}{4b}$$

$\big\downarrow$ 3792

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\left(\frac{3}{4}\left(\frac{\int \cosh^2(a+bx)dx}{2b^2}+\frac{\int x^2dx}{2}-\frac{x\cosh^2(a+bx)}{2b^2}+\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+}{}\right.}{8b^2}\right.}{4b}$$

$\big\downarrow$ 15

3.269. $\int x^3\cosh^3(a+bx)\sinh(a+bx)\,dx$

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\left(\frac{3}{4}\left(\frac{\int\cosh^2(a+bx)dx}{2b^2}-\frac{x\cosh^2(a+bx)}{2b^2}+\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh}{2b}\right.}{8b^2}\right.}{4b}$$

$\downarrow$ 3042

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\left(\frac{3}{4}\left(\frac{\int\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{2b^2}-\frac{x\cosh^2(a+bx)}{2b^2}+\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+}{}\right.}{8b^2}\right.}{4b}$$

$\downarrow$ 3115

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\left(\frac{3}{4}\left(\frac{\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}-\frac{x\cosh^2(a+bx)}{2b^2}+\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\right.}{8b}\right.}{4b}$$

$\downarrow$ 24

$$\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\left(\frac{3}{4}\left(-\frac{x\cosh^2(a+bx)}{2b^2}+\frac{\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{s}{}\right.}{8b^2}\right.}{4b}$$

input
```
Int[x^3*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
(x^3*Cosh[a + b*x]^4)/(4*b) - (3*(-1/8*(x*Cosh[a + b*x]^4)/b^2 + (x^2*Cosh
[a + b*x]^3*Sinh[a + b*x])/(4*b) + ((Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b)
+ (3*(x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4)/(8*b^2) + (3*(x^3/6 -
 (x*Cosh[a + b*x]^2)/(2*b^2) + (x^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (
x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b))/(2*b^2)))/4))/(4*b)
```

3.269. $\int x^3\cosh^3(a+bx)\sinh(a+bx)\,dx$

3.269.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.269. $\int x^3 \cosh^3(a+bx)\sinh(a+bx)\,dx$

3.269.4 Maple [A] (verified)

Time = 7.58 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

method	result
risch	$\frac{(32x^3b^3-24x^2b^2+12bx-3)\,e^{4bx+4a}}{2048b^4}+\frac{(4x^3b^3-6x^2b^2+6bx-3)\,e^{2bx+2a}}{64b^4}+\frac{(4x^3b^3+6x^2b^2+6bx+3)\,e^{-2bx-2a}}{64b^4}+\frac{(32...}{}$
derivativedivides	$-\frac{a^3\cosh(bx+a)^4}{4}+3a^2\left(\frac{(bx+a)\cosh(bx+a)^4}{4}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}\right)-3a\left(\frac{(bx+a)^2\cos...}{4}\right.$
default	$-\frac{a^3\cosh(bx+a)^4}{4}+3a^2\left(\frac{(bx+a)\cosh(bx+a)^4}{4}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}\right)-3a\left(\frac{(bx+a)^2\cos...}{4}\right.$

input
```
int(x^3*cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2048*(32*b^3*x^3-24*b^2*x^2+12*b*x-3)/b^4*exp(4*b*x+4*a)+1/64*(4*b^3*x^3
-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)+1/64*(4*b^3*x^3+6*b^2*x^2+6*b*x+3)/
b^4*exp(-2*b*x-2*a)+1/2048*(32*b^3*x^3+24*b^2*x^2+12*b*x+3)/b^4*exp(-4*b*x
-4*a)
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.23

$$\int x^3\cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$=\frac{(8\,b^3x^3+3\,bx)\cosh(bx+a)^4-3\,(8\,b^2x^2+1)\cosh(bx+a)\sinh(bx+a)^3+(8\,b^3x^3+3\,bx)\sinh(bx+a)^4 ...}{}$$

input
```
integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/256*((8*b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x +
a)*sinh(b*x + a)^3 + (8*b^3*x^3 + 3*b*x)*sinh(b*x + a)^4 + 16*(2*b^3*x^3 +
 3*b*x)*cosh(b*x + a)^2 + 2*(16*b^3*x^3 + 3*(8*b^3*x^3 + 3*b*x)*cosh(b*x +
 a)^2 + 24*b*x)*sinh(b*x + a)^2 - 3*((8*b^2*x^2 + 1)*cosh(b*x + a)^3 + 16*
(2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a))/b^4
```

3.269.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.46

$$\int x^3 \cosh^3(a+bx) \sinh(a+bx)\,dx$$
$$= \begin{cases} -\frac{3x^3 \sinh^4(a+bx)}{32b} + \frac{3x^3 \sinh^2(a+bx)\cosh^2(a+bx)}{16b} + \frac{5x^3\cosh^4(a+bx)}{32b} + \frac{9x^2\sinh^3(a+bx)\cosh(a+bx)}{32b^2} - \frac{15x^2\sinh(a+bx)\cosh^3(a\cdot}{32b^2} \\ \frac{x^4\sinh(a)\cosh^3(a)}{4} \end{cases}$$

input
```
integrate(x**3*cosh(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Piecewise((-3*x**3*sinh(a + b*x)**4/(32*b) + 3*x**3*sinh(a + b*x)**2*cosh(
a + b*x)**2/(16*b) + 5*x**3*cosh(a + b*x)**4/(32*b) + 9*x**2*sinh(a + b*x)
**3*cosh(a + b*x)/(32*b**2) - 15*x**2*sinh(a + b*x)*cosh(a + b*x)**3/(32*b
**2) - 45*x*sinh(a + b*x)**4/(256*b**3) + 9*x*sinh(a + b*x)**2*cosh(a + b*
x)**2/(128*b**3) + 51*x*cosh(a + b*x)**4/(256*b**3) + 45*sinh(a + b*x)**3*
cosh(a + b*x)/(256*b**4) - 51*sinh(a + b*x)*cosh(a + b*x)**3/(256*b**4), N
e(b, 0)), (x**4*sinh(a)*cosh(a)**3/4, True))
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.10

$$\int x^3 \cosh^3(a+bx) \sinh(a+bx)\,dx$$
$$= \frac{\left(32\,b^3x^3e^{(4\,a)} - 24\,b^2x^2e^{(4\,a)} + 12\,bxe^{(4\,a)} - 3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4} + \frac{\left(4\,b^3x^3e^{(2\,a)} - 6\,b^2x^2e^{(2\,a)} + 6\,bxe^{(2\,a)} - 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{64\,b^4} + \frac{(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3)e^{(-2\,bx-2\,a)}}{64\,b^4} + \frac{(32\,b^3x^3 + 24\,b^2x^2 + 12\,bx + 3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a) - 3*e^(4*
a))*e^(4*b*x)/b^4 + 1/64*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^
(2*a) - 3*e^(2*a))*e^(2*b*x)/b^4 + 1/64*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3
)*e^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(
-4*b*x - 4*a)/b^4
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int x^3\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{(32\,b^3x^3-24\,b^2x^2+12\,bx-3)e^{(4\,bx+4\,a)}}{2048\,b^4} + \frac{(4\,b^3x^3-6\,b^2x^2+6\,bx-3)e^{(2\,bx+2\,a)}}{64\,b^4} + \frac{(4\,b^3x^3+6\,b^2x^2+6\,bx+3)e^{(-2\,bx-2\,a)}}{64\,b^4} + \frac{(32\,b^3x^3+24\,b^2x^2+12\,bx+3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b^4 + 1/64*(
4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 + 1/64*(4*b^3*x^3 +
 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2
*x^2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

3.269.9 Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.81

$$\int x^3\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\frac{x^3\cosh(2\,a+2\,b\,x)}{8}+\frac{x^3\cosh(4\,a+4\,b\,x)}{32}}{b} - \frac{\frac{3\,x^2\sinh(2\,a+2\,b\,x)}{16}+\frac{3\,x^2\sinh(4\,a+4\,b\,x)}{128}}{b^2} + \frac{\frac{3\,x\cosh(2\,a+2\,b\,x)}{16}+\frac{3\,x\cosh(4\,a+4\,b\,x)}{256}}{b^3} - \frac{3\sinh(2\,a+2\,b\,x)}{32\,b^4} - \frac{3\sinh(4\,a+4\,b\,x)}{1024\,b^4}$$

input
```
int(x^3*cosh(a + b*x)^3*sinh(a + b*x),x)
```

output
```
((x^3*cosh(2*a + 2*b*x))/8 + (x^3*cosh(4*a + 4*b*x))/32)/b - ((3*x^2*sinh(
2*a + 2*b*x))/16 + (3*x^2*sinh(4*a + 4*b*x))/128)/b^2 + ((3*x*cosh(2*a + 2
*b*x))/16 + (3*x*cosh(4*a + 4*b*x))/256)/b^3 - (3*sinh(2*a + 2*b*x))/(32*b
^4) - (3*sinh(4*a + 4*b*x))/(1024*b^4)
```

3.269. $\int x^3 \cosh^3(a+bx) \sinh(a+bx) \, dx$

3.270 $\int x^2 \cosh^3(a+bx)\sinh(a+bx)\,dx$

3.270.1 Optimal result

Integrand size = 18, antiderivative size = 101

$$\int x^2 \cosh^3(a+bx)\sinh(a+bx)\,dx = -\frac{3x^2}{32b}+\frac{3\cosh^2(a+bx)}{32b^3}+\frac{\cosh^4(a+bx)}{32b^3}+\frac{x^2\cosh^4(a+bx)}{4b}-\frac{3x\cosh(a+bx)\sinh(a+bx)}{16b^2}-\frac{x\cosh^3(a+bx)\sinh(a+bx)}{8b^2}$$

output
```
-3/32*x^2/b+3/32*cosh(b*x+a)^2/b^3+1/32*cosh(b*x+a)^4/b^3+1/4*x^2*cosh(b*x
+a)^4/b-3/16*x*cosh(b*x+a)*sinh(b*x+a)/b^2-1/8*x*cosh(b*x+a)^3*sinh(b*x+a)
/b^2
```

3.270.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int x^2 \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{16(1+2b^2x^2)\cosh(2(a+bx))+(1+8b^2x^2)\cosh(4(a+bx))-4bx(8\sinh(2(a+bx))+\sinh(4(a+bx)))}{256b^3}$$

input
```
Integrate[x^2*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
(16*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + (1 + 8*b^2*x^2)*Cosh[4*(a + b*x)]
- 4*b*x*(8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)]))/(256*b^3)
```

3.270.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5896, 3042, 3791, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sinh(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{5896}$$

$$\frac{x^2\cosh^4(a+bx)}{4b}-\frac{\int x\cosh^4(a+bx)dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^2\cosh^4(a+bx)}{4b}-\frac{\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx}{2b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{x^2\cosh^4(a+bx)}{4b}-\frac{\frac{3}{4}\int x\cosh^2(a+bx)dx-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^2\cosh^4(a+bx)}{4b}-\frac{\frac{3}{4}\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}}{2b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{x^2\cosh^4(a+bx)}{4b}-\frac{\frac{3}{4}\left(\frac{\int xdx}{2}-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}}{2b}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^2\cosh^4(a+bx)}{4b}-\frac{\frac{3}{4}\left(-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}\right)-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}}{2b}$$

input
```
Int[x^2*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
(x^2*Cosh[a + b*x]^4)/(4*b) - (-1/16*Cosh[a + b*x]^4/b^2 + (x*Cosh[a + b*x
]^3*Sinh[a + b*x])/(4*b) + (3*(x^2/4 - Cosh[a + b*x]^2/(4*b^2) + (x*Cosh[a
 + b*x]*Sinh[a + b*x])/(2*b)))/4)/(2*b)
```

3.270.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.270.4 Maple [A] (verified)

Time = 5.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.13

method	result
risch	$\frac{\left(8x^2b^2-4bx+1\right)\mathrm{e}^{4bx+4a}}{512b^3}+\frac{\left(2x^2b^2-2bx+1\right)\mathrm{e}^{2bx+2a}}{32b^3}+\frac{\left(2x^2b^2+2bx+1\right)\mathrm{e}^{-2bx-2a}}{32b^3}+\frac{\left(8x^2b^2+4bx+1\right)\mathrm{e}^{-4bx-4a}}{512b^3}$
derivativedivides	$\frac{\frac{a^2\cosh(bx+a)^4}{4}-2a\left(\frac{(bx+a)\cosh(bx+a)^4}{4}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}\right)+\frac{(bx+a)^2\cosh(bx+a\cdots}{4}\cdots}{b^3}$
default	$\frac{\frac{a^2\cosh(bx+a)^4}{4}-2a\left(\frac{(bx+a)\cosh(bx+a)^4}{4}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}\right)+\frac{(bx+a)^2\cosh(bx+a\cdots}{4}\cdots}{b^3}$

input
```
int(x^2*cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/512*(8*b^2*x^2-4*b*x+1)/b^3*exp(4*b*x+4*a)+1/32*(2*b^2*x^2-2*b*x+1)/b^3*
exp(2*b*x+2*a)+1/32*(2*b^2*x^2+2*b*x+1)/b^3*exp(-2*b*x-2*a)+1/512*(8*b^2*x
^2+4*b*x+1)/b^3*exp(-4*b*x-4*a)
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

$$\int x^2\cosh^3(a+bx)\sinh(a+bx)\,dx =$$
$$-\frac{16\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)^4-\left(8\,b^2x^2+1\right)\sinh\left(bx+a\right)^4-16\,(2\,b^2x}{}$$

input
```
integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/256*(16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 - (8*b^2*x^2 + 1)*cosh(b*x +
a)^4 - (8*b^2*x^2 + 1)*sinh(b*x + a)^4 - 16*(2*b^2*x^2 + 1)*cosh(b*x + a)^
2 - 2*(16*b^2*x^2 + 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^2
 + 16*(b*x*cosh(b*x + a)^3 + 4*b*x*cosh(b*x + a))*sinh(b*x + a))/b^3
```

3.270.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.49

$$\int x^2\cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$=\begin{cases} -\frac{3x^2\sinh^4(a+bx)}{32b}+\frac{3x^2\sinh^2(a+bx)\cosh^2(a+bx)}{16b}+\frac{5x^2\cosh^4(a+bx)}{32b}+\frac{3x\sinh^3(a+bx)\cosh(a+bx)}{16b^2}-\frac{5x\sinh(a+bx)\cosh^3(a+bx}{16b^2} \\ \frac{x^3\sinh(a)\cosh^3(a)}{3} \end{cases}$$

input
```
integrate(x**2*cosh(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Piecewise((-3*x**2*sinh(a + b*x)**4/(32*b) + 3*x**2*sinh(a + b*x)**2*cosh(
a + b*x)**2/(16*b) + 5*x**2*cosh(a + b*x)**4/(32*b) + 3*x*sinh(a + b*x)**3
*cosh(a + b*x)/(16*b**2) - 5*x*sinh(a + b*x)*cosh(a + b*x)**3/(16*b**2) -
3*sinh(a + b*x)**4/(64*b**3) + 5*cosh(a + b*x)**4/(64*b**3), Ne(b, 0)), (x
**3*sinh(a)*cosh(a)**3/3, True))
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.26

$$\int x^2 \cosh^3(a+bx) \sinh(a+bx)\,dx = \frac{\left(8\,b^2x^2e^{(4\,a)} - 4\,bxe^{(4\,a)} + e^{(4\,a)}\right)e^{(4\,bx)}}{512\,b^3} + \frac{\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^3} + \frac{(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx-2\,a)}}{32\,b^3} + \frac{(8\,b^2x^2 + 4\,bx + 1)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b^3 + 1/32*(
2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 + 1/32*(2*b^2*x
^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4
*b*x - 4*a)/b^3
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

$$\int x^2 \cosh^3(a+bx) \sinh(a+bx)\,dx = \frac{(8\,b^2x^2 - 4\,bx + 1)e^{(4\,bx+4\,a)}}{512\,b^3} + \frac{(2\,b^2x^2 - 2\,bx + 1)e^{(2\,bx+2\,a)}}{32\,b^3} + \frac{(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx-2\,a)}}{32\,b^3} + \frac{(8\,b^2x^2 + 4\,bx + 1)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 + 1/32*(2*b^2*x^2 - 2*b*
x + 1)*e^(2*b*x + 2*a)/b^3 + 1/32*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)
/b^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

3.270.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int x^2\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{3\cosh(a+b\,x)^2}{32\,b^3}-\frac{\frac{3\,x^2}{32}-\frac{x^2\cosh(a+b\,x)^4}{4}}{b}-\frac{\frac{x\sinh(a+b\,x)\cosh(a+b\,x)^3}{8}+\frac{3\,x\sinh(a+b\,x)\cosh(a+b\,x)}{16}}{b^2}+\frac{\cosh(a+b\,x)^4}{32\,b^3}$$

input
```
int(x^2*cosh(a + b*x)^3*sinh(a + b*x),x)
```

output
```
(3*cosh(a + b*x)^2)/(32*b^3) - ((3*x^2)/32 - (x^2*cosh(a + b*x)^4)/4)/b -
((x*cosh(a + b*x)^3*sinh(a + b*x))/8 + (3*x*cosh(a + b*x)*sinh(a + b*x))/1
6)/b^2 + cosh(a + b*x)^4/(32*b^3)
```

3.271 $\int x\cosh^3(a+bx)\sinh(a+bx)\,dx$

3.271.1 Optimal result

Integrand size = 16, antiderivative size = 65

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx=-\frac{3x}{32b}+\frac{x\cosh^4(a+bx)}{4b}-\frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2}-\frac{\cosh^3(a+bx)\sinh(a+bx)}{16b^2}$$

output
```
-3/32*x/b+1/4*x*cosh(b*x+a)^4/b-3/32*cosh(b*x+a)*sinh(b*x+a)/b^2-1/16*cosh
(b*x+a)^3*sinh(b*x+a)/b^2
```

3.271.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx=-\frac{-16bx\cosh(2(a+bx))-4bx\cosh(4(a+bx))+8\sinh(2(a+bx))+\sinh(4(a+bx))}{128b^2}$$

input
```
Integrate[x*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
-1/128*(-16*b*x*Cosh[2*(a + b*x)] - 4*b*x*Cosh[4*(a + b*x)] + 8*Sinh[2*(a
+ b*x)] + Sinh[4*(a + b*x)])/b^2
```

3.271.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5896, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{5896}$$

$$\frac{x\cosh^4(a+bx)}{4b}-\frac{\int \cosh^4(a+bx)dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x\cosh^4(a+bx)}{4b}-\frac{\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx}{4b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{x\cosh^4(a+bx)}{4b}-\frac{\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x\cosh^4(a+bx)}{4b}-\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{4b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{x\cosh^4(a+bx)}{4b}-\frac{\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\cosh^4(a+bx)}{4b}-\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)}{4b}$$

input
```
Int[x*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
(x*Cosh[a + b*x]^4)/(4*b) - ((Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (3*(x
/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4)/(4*b)
```

3.271.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.271.4 Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\frac{(bx+a)\cosh(bx+a)^4}{4}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}-\frac{a\cosh(bx+a)^4}{4}}{b^2}$	69
default	$\frac{\frac{(bx+a)\cosh(bx+a)^4}{4}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}-\frac{a\cosh(bx+a)^4}{4}}{b^2}$	69
risch	$\frac{(4bx-1)\mathrm{e}^{4bx+4a}}{256b^2}+\frac{(2bx-1)\mathrm{e}^{2bx+2a}}{32b^2}+\frac{(2bx+1)\mathrm{e}^{-2bx-2a}}{32b^2}+\frac{(4bx+1)\mathrm{e}^{-4bx-4a}}{256b^2}$	82

input
```
int(x*cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/4*(b*x+a)*cosh(b*x+a)^4-1/16*cosh(b*x+a)^3*sinh(b*x+a)-3/32*cosh(
b*x+a)*sinh(b*x+a)-3/32*b*x-3/32*a-1/4*a*cosh(b*x+a)^4)
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$= \frac{bx\cosh\left(bx+a\right)^4 + bx\sinh\left(bx+a\right)^4 + 4\,bx\cosh\left(bx+a\right)^2 - \cosh\left(bx+a\right)\sinh\left(bx+a\right)^3 + 2\,\big(3\,bx\cosh\left(b\right.}{32\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/32*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^2 -
cosh(b*x + a)*sinh(b*x + a)^3 + 2*(3*b*x*cosh(b*x + a)^2 + 2*b*x)*sinh(b*x
 + a)^2 - (cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a))/b^2
```

3.271.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$= \begin{cases} -\frac{3x\sinh^4(a+bx)}{32b} + \frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{16b} + \frac{5x\cosh^4(a+bx)}{32b} + \frac{3\sinh^3(a+bx)\cosh(a+bx)}{32b^2} - \frac{5\sinh(a+bx)\cosh^3(a+bx)}{32b^2} & \text{fo} \\ \frac{x^2\sinh(a)\cosh^3(a)}{2} & \text{o} \end{cases}$$

input
```
integrate(x*cosh(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Piecewise((-3*x*sinh(a + b*x)**4/(32*b) + 3*x*sinh(a + b*x)**2*cosh(a + b*
x)**2/(16*b) + 5*x*cosh(a + b*x)**4/(32*b) + 3*sinh(a + b*x)**3*cosh(a + b
*x)/(32*b**2) - 5*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2), Ne(b, 0)), (x*
*2*sinh(a)*cosh(a)**3/2, True))
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.40

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\left(4\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2}+\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^2}+\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 + 1/32*(2*b*x*e^(2*a) - e^(2
*a))*e^(2*b*x)/b^2 + 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x
+ 1)*e^(-4*b*x - 4*a)/b^2
```

3.271.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2}+\frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{32\,b^2}+\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 + 1/32*(2*b*x - 1)*e^(2*b*x + 2*a)/b
^2 + 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x + 1)*e^(-4*b*x -
 4*a)/b^2
```

3.271.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx = -\frac{\frac{3\,x}{32}-\frac{x\cosh(a+b\,x)^4}{4}}{b}-\frac{\cosh(a+b\,x)^3\sinh(a+b\,x)}{16\,b^2}-\frac{3\cosh(a+b\,x)\,\sinh(a+b\,x)}{32\,b^2}$$

input
```
int(x*cosh(a + b*x)^3*sinh(a + b*x),x)
```

output
```
- ((3*x)/32 - (x*cosh(a + b*x)^4)/4)/b - (cosh(a + b*x)^3*sinh(a + b*x))/(
16*b^2) - (3*cosh(a + b*x)*sinh(a + b*x))/(32*b^2)
```

3.271. $\int x\cosh^3(a+bx)\sinh(a+bx)\,dx$

3.272 $\int \cosh^3(a+bx)\sinh(a+bx)\,dx$

3.272.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{\cosh^4(a+bx)}{4b}$$

output
```
1/4*cosh(b*x+a)^4/b
```

3.272.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{\cosh^4(a+bx)}{4b}$$

input
```
Integrate[Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
Cosh[a + b*x]^4/(4*b)
```

3.272.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 26, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -i\sin(ia+ibx)\cos(ia+ibx)^3dx$$

$$\Big\downarrow \; 26$$

$$-i\int \cos(ia+ibx)^3\sin(ia+ibx)dx$$

$$\Big\downarrow \; 3045$$

$$\frac{\int \cosh^3(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \; 15$$

$$\frac{\cosh^4(a+bx)}{4b}$$

input
```
Int[Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
Cosh[a + b*x]^4/(4*b)
```

3.272.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.272.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cosh(bx+a)^4}{4b}$	14
default	$\frac{\cosh(bx+a)^4}{4b}$	14
risch	$\frac{e^{4bx+4a}}{64b}+\frac{e^{2bx+2a}}{16b}+\frac{e^{-2bx-2a}}{16b}+\frac{e^{-4bx-4a}}{64b}$	58

input
```
int(cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*cosh(b*x+a)^4/b
```

3.272.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.60

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^4+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2+2\right)\sinh(bx+a)^2+4\cosh(bx+a)^2}{32\,b}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/32*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 2)*sinh(b
*x + a)^2 + 4*cosh(b*x + a)^2)/b
```

3.272.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{\cosh^4(a+bx)}{4b} & \text{for } b \neq 0 \\ x\sinh(a)\cosh^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Piecewise((cosh(a + b*x)**4/(4*b), Ne(b, 0)), (x*sinh(a)*cosh(a)**3, True)
)
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^4}{4b}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/4*cosh(b*x + a)^4/b
```

3.272.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.80

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{e^{(4bx+4a)}}{64b} + \frac{e^{(2bx+2a)}}{16b} + \frac{e^{(-2bx-2a)}}{16b} + \frac{e^{(-4bx-4a)}}{64b}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/64*e^(4*b*x + 4*a)/b + 1/16*e^(2*b*x + 2*a)/b + 1/16*e^(-2*b*x - 2*a)/b
+ 1/64*e^(-4*b*x - 4*a)/b
```

3.272.9 Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)^4}{4\,b}$$

input
```
int(cosh(a + b*x)^3*sinh(a + b*x),x)
```

output
```
cosh(a + b*x)^4/(4*b)
```

3.273 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx$

3.273.1 Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}\text{Chi}(2bx)\sinh(2a) + \frac{1}{8}\text{Chi}(4bx)\sinh(4a) + \frac{1}{4}\cosh(2a)\text{Shi}(2bx) + \frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

output
```
1/4*cosh(2*a)*Shi(2*b*x)+1/8*cosh(4*a)*Shi(4*b*x)+1/4*Chi(2*b*x)*sinh(2*a)
+1/8*Chi(4*b*x)*sinh(4*a)
```

3.273.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{8}(2\text{Chi}(2bx)\sinh(2a) + \text{Chi}(4bx)\sinh(4a) + 2\cosh(2a)\text{Shi}(2bx) + \cosh(4a)\text{Shi}(4bx))$$

input
```
Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x,x]
```

output
```
(2*CoshIntegral[2*b*x]*Sinh[2*a] + CoshIntegral[4*b*x]*Sinh[4*a] + 2*Cosh[
2*a]*SinhIntegral[2*b*x] + Cosh[4*a]*SinhIntegral[4*b*x])/8
```

3.273.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x}dx$$
$$\Big\downarrow \text{5971}$$
$$\int \left(\frac{\sinh(2a+2bx)}{4x}+\frac{\sinh(4a+4bx)}{8x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}\sinh(2a)\text{Chi}(2bx)+\frac{1}{8}\sinh(4a)\text{Chi}(4bx)+\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

input
```
Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x,x]
```

output
```
(CoshIntegral[2*b*x]*Sinh[2*a])/4 + (CoshIntegral[4*b*x]*Sinh[4*a])/8 + (C
osh[2*a]*SinhIntegral[2*b*x])/4 + (Cosh[4*a]*SinhIntegral[4*b*x])/8
```

3.273.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.273.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{e^{-4a}\,\mathrm{Ei}_1(4bx)}{16}+\frac{e^{-2a}\,\mathrm{Ei}_1(2bx)}{8}-\frac{e^{2a}\,\mathrm{Ei}_1(-2bx)}{8}-\frac{e^{4a}\,\mathrm{Ei}_1(-4bx)}{16}$	50

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/16*exp(-4*a)*Ei(1,4*b*x)+1/8*exp(-2*a)*Ei(1,2*b*x)-1/8*exp(2*a)*Ei(1,-2*
b*x)-1/16*exp(4*a)*Ei(1,-4*b*x)
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{16}\left(\mathrm{Ei}(4\,bx)-\mathrm{Ei}(-4\,bx)\right)\cosh\left(4\,a\right) + \frac{1}{8}\left(\mathrm{Ei}(2\,bx)-\mathrm{Ei}(-2\,bx)\right)\cosh\left(2\,a\right) + \frac{1}{16}\left(\mathrm{Ei}(4\,bx)+\mathrm{Ei}(-4\,bx)\right)\sinh\left(4\,a\right) + \frac{1}{8}\left(\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx)\right)\sinh\left(2\,a\right)$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="fricas")
```

output
```
1/16*(Ei(4*b*x) - Ei(-4*b*x))*cosh(4*a) + 1/8*(Ei(2*b*x) - Ei(-2*b*x))*cos
h(2*a) + 1/16*(Ei(4*b*x) + Ei(-4*b*x))*sinh(4*a) + 1/8*(Ei(2*b*x) + Ei(-2*
b*x))*sinh(2*a)
```

3.273.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)/x,x)
```

output
```
Integral(sinh(a + b*x)*cosh(a + b*x)**3/x, x)
```

3.273.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + \frac{1}{8}\,\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - \frac{1}{8}\,\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - \frac{1}{16}\,\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="maxima")
```

output
```
1/16*Ei(4*b*x)*e^(4*a) + 1/8*Ei(2*b*x)*e^(2*a) - 1/8*Ei(-2*b*x)*e^(-2*a) -
 1/16*Ei(-4*b*x)*e^(-4*a)
```

3.273.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + \frac{1}{8}\,\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - \frac{1}{8}\,\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - \frac{1}{16}\,\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="giac")
```

output
```
1/16*Ei(4*b*x)*e^(4*a) + 1/8*Ei(2*b*x)*e^(2*a) - 1/8*Ei(-2*b*x)*e^(-2*a) -
 1/16*Ei(-4*b*x)*e^(-4*a)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^3\sinh(a+b\,x)}{x}\,dx$$

input
```
int((cosh(a + b*x)^3*sinh(a + b*x))/x,x)
```

output
```
int((cosh(a + b*x)^3*sinh(a + b*x))/x, x)
```

3.274 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx$

3.274.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx) - \frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x} + \frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)$$

output
```
1/2*b*Chi(2*b*x)*cosh(2*a)+1/2*b*Chi(4*b*x)*cosh(4*a)+1/2*b*Shi(2*b*x)*sin
h(2*a)+1/2*b*Shi(4*b*x)*sinh(4*a)-1/4*sinh(2*b*x+2*a)/x-1/8*sinh(4*b*x+4*a
)/x
```

3.274.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{4bx\cosh(2a)\text{Chi}(2bx)+4bx\cosh(4a)\text{Chi}(4bx)-2\sinh(2(a+bx))-\sinh(4(a+bx))+4bx\sinh(2a)\text{Shi}(2}{8x}$$

input
```
Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^2,x]
```

output
```
(4*b*x*Cosh[2*a]*CoshIntegral[2*b*x] + 4*b*x*Cosh[4*a]*CoshIntegral[4*b*x]
 - 2*Sinh[2*(a + b*x)] - Sinh[4*(a + b*x)] + 4*b*x*Sinh[2*a]*SinhIntegral[
2*b*x] + 4*b*x*Sinh[4*a]*SinhIntegral[4*b*x])/(8*x)
```

3.274.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{\sinh(2a+2bx)}{4x^2}+\frac{\sinh(4a+4bx)}{8x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)+\frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)-\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}$$

input
```
Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^2,x]
```

output
```
(b*Cosh[2*a]*CoshIntegral[2*b*x])/2 + (b*Cosh[4*a]*CoshIntegral[4*b*x])/2
- Sinh[2*a + 2*b*x]/(4*x) - Sinh[4*a + 4*b*x]/(8*x) + (b*Sinh[2*a]*SinhInt
egral[2*b*x])/2 + (b*Sinh[4*a]*SinhIntegral[4*b*x])/2
```

3.274.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.274.4 Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

method	result	si
risch	$-\frac{4\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)bx+4\,\mathrm{e}^{4a}\,\mathrm{Ei}_1(-4bx)bx+4\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)bx+4\,\mathrm{e}^{-4a}\,\mathrm{Ei}_1(4bx)bx+2\,\mathrm{e}^{2bx+2a}+\mathrm{e}^{4bx+4a}-2\,\mathrm{e}^{-2bx-2a}-\mathrm{e}^{-4bx-4a}}{16x}$	10

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(4*exp(2*a)*Ei(1,-2*b*x)*b*x+4*exp(4*a)*Ei(1,-4*b*x)*b*x+4*exp(-2*a)
*Ei(1,2*b*x)*b*x+4*exp(-4*a)*Ei(1,4*b*x)*b*x+2*exp(2*b*x+2*a)+exp(4*b*x+4*
a)-2*exp(-2*b*x-2*a)-exp(-4*b*x-4*a))/x
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.56

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx =$$
$$-\frac{2\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\left(bx\mathrm{Ei}(4\,bx)+bx\mathrm{Ei}(-4\,bx)\right)\cosh\left(4\,a\right)-\left(bx\mathrm{Ei}(2\,bx)+bx\mathrm{Ei}(-2\,bx)\right)\mathrm{co}}{}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
-1/4*(2*cosh(b*x + a)*sinh(b*x + a)^3 - (b*x*Ei(4*b*x) + b*x*Ei(-4*b*x))*c
osh(4*a) - (b*x*Ei(2*b*x) + b*x*Ei(-2*b*x))*cosh(2*a) + 2*(cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a) - (b*x*Ei(4*b*x) - b*x*Ei(-4*b*x))*sinh(4*
a) - (b*x*Ei(2*b*x) - b*x*Ei(-2*b*x))*sinh(2*a))/x
```

3.274.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)/x**2,x)
```

output
```
Integral(sinh(a + b*x)*cosh(a + b*x)**3/x**2, x)
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{1}{4}\,be^{(-4\,a)}\Gamma(-1,4\,bx) + \frac{1}{4}\,be^{(-2\,a)}\Gamma(-1,2\,bx) + \frac{1}{4}\,be^{(2\,a)}\Gamma(-1,-2\,bx) + \frac{1}{4}\,be^{(4\,a)}\Gamma(-1,-4\,bx)$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
1/4*b*e^(-4*a)*gamma(-1, 4*b*x) + 1/4*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/4*b*
e^(2*a)*gamma(-1, -2*b*x) + 1/4*b*e^(4*a)*gamma(-1, -4*b*x)
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{4\,bx\text{Ei}(4\,bx)\,e^{(4\,a)} + 4\,bx\text{Ei}(2\,bx)\,e^{(2\,a)} + 4\,bx\text{Ei}(-2\,bx)\,e^{(-2\,a)} + 4\,bx\text{Ei}(-4\,bx)\,e^{(-4\,a)} - e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+}}{16\,x}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="giac")
```

output
```
1/16*(4*b*x*Ei(4*b*x)*e^(4*a) + 4*b*x*Ei(2*b*x)*e^(2*a) + 4*b*x*Ei(-2*b*x)
*e^(-2*a) + 4*b*x*Ei(-4*b*x)*e^(-4*a) - e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a
) + 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^3\sinh(a+b\,x)}{x^2}\,dx$$

input
```
int((cosh(a + b*x)^3*sinh(a + b*x))/x^2,x)
```

output
```
int((cosh(a + b*x)^3*sinh(a + b*x))/x^2, x)
```

3.275 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx$

3.275.1 Optimal result

Integrand size = 18, antiderivative size = 125

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = -\frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} + \frac{1}{2}b^2\text{Chi}(2bx)\sinh(2a) + b^2\text{Chi}(4bx)\sinh(4a) - \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} + \frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx) + b^2\cosh(4a)\text{Shi}(4bx)$$

output
```
-1/4*b*cosh(2*b*x+2*a)/x-1/4*b*cosh(4*b*x+4*a)/x+1/2*b^2*cosh(2*a)*Shi(2*b
*x)+b^2*cosh(4*a)*Shi(4*b*x)+1/2*b^2*Chi(2*b*x)*sinh(2*a)+b^2*Chi(4*b*x)*s
inh(4*a)-1/8*sinh(2*b*x+2*a)/x^2-1/16*sinh(4*b*x+4*a)/x^2
```

3.275.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = b^2\cosh(a)\text{Chi}(2bx)\sinh(a) + b^2\text{Chi}(4bx)\sinh(4a) - \frac{2bx\cosh(2(a+bx)) + \sinh(2(a+bx))}{8x^2} - \frac{4bx\cosh(4(a+bx)) + \sinh(4(a+bx))}{16x^2} + \frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx) + b^2\cosh(4a)\text{Shi}(4bx)$$

input
```
Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^3,x]
```

output
```
b^2*Cosh[a]*CoshIntegral[2*b*x]*Sinh[a] + b^2*CoshIntegral[4*b*x]*Sinh[4*a
] - (2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])/(8*x^2) - (4*b*x*Cosh[4*
(a + b*x)] + Sinh[4*(a + b*x)])/(16*x^2) + (b^2*Cosh[2*a]*SinhIntegral[2*b
*x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

3.275.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x^3}dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{\sinh(2a+2bx)}{4x^3}+\frac{\sinh(4a+4bx)}{8x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}b^2\sinh(2a)\text{Chi}(2bx)+b^2\sinh(4a)\text{Chi}(4bx)+\frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx)+b^2\cosh(4a)\text{Shi}(4bx)-\frac{\sinh(2a+2bx)}{8x^2}-\frac{\sinh(4a+4bx)}{16x^2}-\frac{b\cosh(2a+2bx)}{4x}-\frac{b\cosh(4a+4bx)}{4x}$$

input
```
Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^3,x]
```

output
```
-1/4*(b*Cosh[2*a + 2*b*x])/x - (b*Cosh[4*a + 4*b*x])/(4*x) + (b^2*CoshInte
gral[2*b*x]*Sinh[2*a])/2 + b^2*CoshIntegral[4*b*x]*Sinh[4*a] - Sinh[2*a +
2*b*x]/(8*x^2) - Sinh[4*a + 4*b*x]/(16*x^2) + (b^2*Cosh[2*a]*SinhIntegral[
2*b*x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

3.275.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.275.4 Maple [A] (verified)

Time = 6.58 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{-16\,\mathrm{e}^{-4a}\,\mathrm{Ei}_1(4bx)x^2b^2+8\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)x^2b^2-8\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)x^2b^2+16\,\mathrm{e}^{4a}\,\mathrm{Ei}_1(-4bx)x^2b^2+4\,\mathrm{e}^{-4bx-4a}bx+4\,\mathrm{e}^{2bx+2a}bx+4\,\mathrm{e}^{-2bx}}{32x^2}$

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-16*exp(-4*a)*Ei(1,4*b*x)*x^2*b^2+8*exp(2*a)*Ei(1,-2*b*x)*x^2*b^2-8
*exp(-2*a)*Ei(1,2*b*x)*x^2*b^2+16*exp(4*a)*Ei(1,-4*b*x)*x^2*b^2+4*exp(-4*b
*x-4*a)*b*x+4*exp(2*b*x+2*a)*b*x+4*exp(-2*b*x-2*a)*b*x+4*exp(4*b*x+4*a)*b*
x-exp(-4*b*x-4*a)+2*exp(2*b*x+2*a)-2*exp(-2*b*x-2*a)+exp(4*b*x+4*a))/x^2
```

3.275.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(113) = 226.

Time = 0.25 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.82

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx =$$
$$-\frac{bx\cosh{(bx+a)}^4+bx\sinh{(bx+a)}^4+bx\cosh{(bx+a)}^2+\cosh{(bx+a)}\sinh{(bx+a)}^3+\left(6\,bx\cosh{(bx}\right.}{}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + b*x*cosh(b*x + a)^2 + co
sh(b*x + a)*sinh(b*x + a)^3 + (6*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^
2 - 2*(b^2*x^2*Ei(4*b*x) - b^2*x^2*Ei(-4*b*x))*cosh(4*a) - (b^2*x^2*Ei(2*b
*x) - b^2*x^2*Ei(-2*b*x))*cosh(2*a) + (cosh(b*x + a)^3 + cosh(b*x + a))*si
nh(b*x + a) - 2*(b^2*x^2*Ei(4*b*x) + b^2*x^2*Ei(-4*b*x))*sinh(4*a) - (b^2*
x^2*Ei(2*b*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

3.275.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \frac{\sinh\left(a+bx\right)\cosh^3\left(a+bx\right)}{x^3}\,dx$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)/x**3,x)
```

output
```
Integral(sinh(a + b*x)*cosh(a + b*x)**3/x**3, x)
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = b^2e^{(-4\,a)}\Gamma(-2,4\,bx) + \frac{1}{2}\,b^2e^{(-2\,a)}\Gamma(-2,2\,bx) - \frac{1}{2}\,b^2e^{(2\,a)}\Gamma(-2,-2\,bx) - b^2e^{(4\,a)}\Gamma(-2,-4\,bx)$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
b^2*e^(-4*a)*gamma(-2, 4*b*x) + 1/2*b^2*e^(-2*a)*gamma(-2, 2*b*x) - 1/2*b^
2*e^(2*a)*gamma(-2, -2*b*x) - b^2*e^(4*a)*gamma(-2, -4*b*x)
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.34

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx$$
$$= \frac{16\,b^2x^2\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + 8\,b^2x^2\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - 8\,b^2x^2\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - 16\,b^2x^2\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - 4\,bxe^{(4\,bx+}}{32}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x, algorithm="giac")
```

output
```
1/32*(16*b^2*x^2*Ei(4*b*x)*e^(4*a) + 8*b^2*x^2*Ei(2*b*x)*e^(2*a) - 8*b^2*x
^2*Ei(-2*b*x)*e^(-2*a) - 16*b^2*x^2*Ei(-4*b*x)*e^(-4*a) - 4*b*x*e^(4*b*x +
 4*a) - 4*b*x*e^(2*b*x + 2*a) - 4*b*x*e^(-2*b*x - 2*a) - 4*b*x*e^(-4*b*x -
 4*a) - e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 2*e^(-2*b*x - 2*a) + e^(-4*b
*x - 4*a))/x^2
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)^3\sinh(a+b\,x)}{x^3}\,dx$$

input
```
int((cosh(a + b*x)^3*sinh(a + b*x))/x^3,x)
```

output
```
int((cosh(a + b*x)^3*sinh(a + b*x))/x^3, x)
```

3.276 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx$

3.276.1 Optimal result

Integrand size = 18, antiderivative size = 169

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = &-\frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}\\ &+\frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx)\\ &-\frac{\sinh(2a+2bx)}{12x^3}-\frac{b^2\sinh(2a+2bx)}{6x}\\ &-\frac{\sinh(4a+4bx)}{24x^3}-\frac{b^2\sinh(4a+4bx)}{3x}\\ &+\frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx)\end{aligned}$$

output
```
1/3*b^3*Chi(2*b*x)*cosh(2*a)+4/3*b^3*Chi(4*b*x)*cosh(4*a)-1/12*b*cosh(2*b*
x+2*a)/x^2-1/12*b*cosh(4*b*x+4*a)/x^2+1/3*b^3*Shi(2*b*x)*sinh(2*a)+4/3*b^3
*Shi(4*b*x)*sinh(4*a)-1/12*sinh(2*b*x+2*a)/x^3-1/6*b^2*sinh(2*b*x+2*a)/x-1
/24*sinh(4*b*x+4*a)/x^3-1/3*b^2*sinh(4*b*x+4*a)/x
```

3.276.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx =$$
$$-\frac{2bx\cosh(2(a+bx))+2bx\cosh(4(a+bx))-8b^3x^3\cosh(2a)\text{Chi}(2bx)-32b^3x^3\cosh(4a)\text{Chi}(4bx)+2\,\text{si}}{}$$

input
```
Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^4,x]
```

output
```
-1/24*(2*b*x*Cosh[2*(a + b*x)] + 2*b*x*Cosh[4*(a + b*x)] - 8*b^3*x^3*Cosh[
2*a]*CoshIntegral[2*b*x] - 32*b^3*x^3*Cosh[4*a]*CoshIntegral[4*b*x] + 2*Si
nh[2*(a + b*x)] + 4*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 8*b^2*
x^2*Sinh[4*(a + b*x)] - 8*b^3*x^3*Sinh[2*a]*SinhIntegral[2*b*x] - 32*b^3*x
^3*Sinh[4*a]*SinhIntegral[4*b*x])/x^3
```

3.276.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x^4}dx$$
$$\Big\downarrow \text{5971}$$
$$\int \left(\frac{\sinh(2a+2bx)}{4x^4}+\frac{\sinh(4a+4bx)}{8x^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx)+\frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx)-\frac{b^2\sinh(2a+2bx)}{6x}-\frac{b^2\sinh(4a+4bx)}{3x}-\frac{\sinh(2a+2bx)}{12x^3}-\frac{\sinh(4a+4bx)}{24x^3}-\frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}$$

input
```
Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^4,x]
```

output
```
-1/12*(b*Cosh[2*a + 2*b*x])/x^2 - (b*Cosh[4*a + 4*b*x])/(12*x^2) + (b^3*Co
sh[2*a]*CoshIntegral[2*b*x])/3 + (4*b^3*Cosh[4*a]*CoshIntegral[4*b*x])/3 -
 Sinh[2*a + 2*b*x]/(12*x^3) - (b^2*Sinh[2*a + 2*b*x])/(6*x) - Sinh[4*a + 4
*b*x]/(24*x^3) - (b^2*Sinh[4*a + 4*b*x])/(3*x) + (b^3*Sinh[2*a]*SinhIntegr
al[2*b*x])/3 + (4*b^3*Sinh[4*a]*SinhIntegral[4*b*x])/3
```

3.276.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.276.4 Maple [A] (verified)

Time = 9.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.43

method	result
risch	$-\frac{32\,\mathrm{e}^{-4a}\,\mathrm{Ei}_1(4bx)x^3b^3+8\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)x^3b^3+32\,\mathrm{e}^{4a}\,\mathrm{Ei}_1(-4bx)x^3b^3+8\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)x^3b^3+8\,\mathrm{e}^{4bx+4a}b^2x^2-8\,\mathrm{e}^{-4bx-4a}b^2x^2+4\,\mathrm{e}^{2b}}{4}$

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(32*exp(-4*a)*Ei(1,4*b*x)*x^3*b^3+8*exp(2*a)*Ei(1,-2*b*x)*x^3*b^3+32
*exp(4*a)*Ei(1,-4*b*x)*x^3*b^3+8*exp(-2*a)*Ei(1,2*b*x)*x^3*b^3+8*exp(4*b*x
+4*a)*b^2*x^2-8*exp(-4*b*x-4*a)*b^2*x^2+4*exp(2*b*x+2*a)*b^2*x^2-4*exp(-2*
b*x-2*a)*b^2*x^2+2*exp(4*b*x+4*a)*b*x+2*exp(-4*b*x-4*a)*b*x+2*exp(2*b*x+2*
a)*b*x+2*exp(-2*b*x-2*a)*b*x+exp(4*b*x+4*a)-exp(-4*b*x-4*a)+2*exp(2*b*x+2*
a)-2*exp(-2*b*x-2*a))/x^3
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.54

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx =$$
$$-\frac{bx\cosh(bx+a)^4 + bx\sinh(bx+a)^4 + 2\,(8\,b^2x^2+1)\cosh(bx+a)\sinh(bx+a)^3 + bx\cosh(bx+a)^2 + \cdots}{\cdots}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
-1/12*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + 2*(8*b^2*x^2 + 1)*cosh(
b*x + a)*sinh(b*x + a)^3 + b*x*cosh(b*x + a)^2 + (6*b*x*cosh(b*x + a)^2 +
b*x)*sinh(b*x + a)^2 - 8*(b^3*x^3*Ei(4*b*x) + b^3*x^3*Ei(-4*b*x))*cosh(4*a
) - 2*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*cosh(2*a) + 2*((8*b^2*x^2 +
 1)*cosh(b*x + a)^3 + (2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) - 8*(b^
3*x^3*Ei(4*b*x) - b^3*x^3*Ei(-4*b*x))*sinh(4*a) - 2*(b^3*x^3*Ei(2*b*x) - b
^3*x^3*Ei(-2*b*x))*sinh(2*a))/x^3
```

3.276.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x^4}\,dx$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)/x**4,x)
```

output
```
Integral(sinh(a + b*x)*cosh(a + b*x)**3/x**4, x)
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.35

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = 4\,b^3e^{(-4\,a)}\Gamma(-3,4\,bx) + b^3e^{(-2\,a)}\Gamma(-3,2\,bx) + b^3e^{(2\,a)}\Gamma(-3,-2\,bx) + 4\,b^3e^{(4\,a)}\Gamma(-3,-4\,bx)$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
4*b^3*e^(-4*a)*gamma(-3, 4*b*x) + b^3*e^(-2*a)*gamma(-3, 2*b*x) + b^3*e^(2
*a)*gamma(-3, -2*b*x) + 4*b^3*e^(4*a)*gamma(-3, -4*b*x)
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx$$
$$= \frac{32\,b^3x^3\text{Ei}(4\,bx)\,e^{(4\,a)} + 8\,b^3x^3\text{Ei}(2\,bx)\,e^{(2\,a)} + 8\,b^3x^3\text{Ei}(-2\,bx)\,e^{(-2\,a)} + 32\,b^3x^3\text{Ei}(-4\,bx)\,e^{(-4\,a)} - 8\,b^2x^2e^{(4\,b}}{}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x, algorithm="giac")
```

output
```
1/48*(32*b^3*x^3*Ei(4*b*x)*e^(4*a) + 8*b^3*x^3*Ei(2*b*x)*e^(2*a) + 8*b^3*x
^3*Ei(-2*b*x)*e^(-2*a) + 32*b^3*x^3*Ei(-4*b*x)*e^(-4*a) - 8*b^2*x^2*e^(4*b
*x + 4*a) - 4*b^2*x^2*e^(2*b*x + 2*a) + 4*b^2*x^2*e^(-2*b*x - 2*a) + 8*b^2
*x^2*e^(-4*b*x - 4*a) - 2*b*x*e^(4*b*x + 4*a) - 2*b*x*e^(2*b*x + 2*a) - 2*
b*x*e^(-2*b*x - 2*a) - 2*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) - 2*e^(2*b
*x + 2*a) + 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x^3
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)^3\sinh(a+b\,x)}{x^4}\,dx$$

input
```
int((cosh(a + b*x)^3*sinh(a + b*x))/x^4,x)
```

output
```
int((cosh(a + b*x)^3*sinh(a + b*x))/x^4, x)
```

3.277 $\int \frac{\cosh(x)\sinh(x)}{x}\,dx$

3.277.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \frac{\text{Shi}(2x)}{2}$$

output
```
1/2*Shi(2*x)
```

3.277.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \frac{\text{Shi}(2x)}{2}$$

input
```
Integrate[(Cosh[x]*Sinh[x])/x,x]
```

output
```
SinhIntegral[2*x]/2
```

3.277.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)\cosh(x)}{x} \, dx$$

$$\Big\downarrow \; 5971$$

$$\int \frac{\sinh(2x)}{2x}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\int \frac{\sinh(2x)}{x}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int -\frac{i\sin(2ix)}{x}dx$$

$$\Big\downarrow \; 26$$

$$-\frac{1}{2}i\int \frac{\sin(2ix)}{x}dx$$

$$\Big\downarrow \; 3779$$

$$\frac{\text{Shi}(2x)}{2}$$

input
```
Int[(Cosh[x]*Sinh[x])/x,x]
```

output
```
SinhIntegral[2*x]/2
```

3.277.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.277.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\mathrm{Shi}(2x)}{2}$	7
meijerg	$-\frac{i\,\mathrm{Si}(2ix)}{2}$	9
risch	$\frac{\mathrm{Ei}_1(2x)}{4}-\frac{\mathrm{Ei}_1(-2x)}{4}$	16

input
```
int(cosh(x)*sinh(x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*Shi(2*x)
```

3.277.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(6) = 12.

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \frac{1}{4}\operatorname{Ei}(2\,x) - \frac{1}{4}\operatorname{Ei}(-2\,x)$$

input
```
integrate(cosh(x)*sinh(x)/x,x, algorithm="fricas")
```

output
```
1/4*Ei(2*x) - 1/4*Ei(-2*x)
```

3.277.6 Sympy [F]

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \int \frac{\sinh(x)\cosh(x)}{x}\,dx$$

input
```
integrate(cosh(x)*sinh(x)/x,x)
```

output
```
Integral(sinh(x)*cosh(x)/x, x)
```

3.277.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(6) = 12.

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \frac{1}{4}\operatorname{Ei}(2\,x) - \frac{1}{4}\operatorname{Ei}(-2\,x)$$

input
```
integrate(cosh(x)*sinh(x)/x,x, algorithm="maxima")
```

output
```
1/4*Ei(2*x) - 1/4*Ei(-2*x)
```

3.277.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \frac{1}{4}\,\mathrm{Ei}(2\,x) - \frac{1}{4}\,\mathrm{Ei}(-2\,x)$$

input
```
integrate(cosh(x)*sinh(x)/x,x, algorithm="giac")
```

output
```
1/4*Ei(2*x) - 1/4*Ei(-2*x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \int \frac{\cosh(x)\,\sinh(x)}{x}\,dx$$

input
```
int((cosh(x)*sinh(x))/x,x)
```

output
```
int((cosh(x)*sinh(x))/x, x)
```

3.278 $\int \frac{\cosh(x)\sinh(x)}{x^2} dx$

3.278.1 Optimal result

Integrand size = 8, antiderivative size = 16

$$\int \frac{\cosh(x)\sinh(x)}{x^2} dx = \text{Chi}(2x) - \frac{\sinh(2x)}{2x}$$

output
```
Chi(2*x)-1/2*sinh(2*x)/x
```

3.278.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)\sinh(x)}{x^2} dx = \text{Chi}(2x) - \frac{\sinh(2x)}{2x}$$

input
```
Integrate[(Cosh[x]*Sinh[x])/x^2,x]
```

output
```
CoshIntegral[2*x] - Sinh[2*x]/(2*x)
```

3.278.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5971, 27, 3042, 26, 3778, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)\cosh(x)}{x^2}dx$$

$$\Big\downarrow \text{5971}$$

$$\int \frac{\sinh(2x)}{2x^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sinh(2x)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int -\frac{i\sin(2ix)}{x^2}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\int \frac{\sin(2ix)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{2}i\left(2i\int \frac{\cosh(2x)}{x}dx-\frac{i\sinh(2x)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}i\left(2i\int \frac{\sin\left(2ix+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh(2x)}{x}\right)$$

$$\Big\downarrow \text{3782}$$

$$-\frac{1}{2}i\left(2i\text{Chi}(2x)-\frac{i\sinh(2x)}{x}\right)$$

input
```
Int[(Cosh[x]*Sinh[x])/x^2,x]
```

output
```
(-1/2*I)*((2*I)*CoshIntegral[2*x] - (I*Sinh[2*x])/x)
```

3.278.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.278. $\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx$

3.278.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\mathrm{Chi}\left(2x\right)-\frac{\sinh\left(2x\right)}{2x}$	15
risch	$\frac{-2\ \mathrm{Ei}_{1}\left(2x\right)x-2\ \mathrm{Ei}_{1}\left(-2x\right)x+\mathrm{e}^{-2x}-\mathrm{e}^{2x}}{4x}$	33
meijerg	$\frac{\sqrt{\pi}\left(\frac{4\gamma-4+4\ln\left(2\right)+4\ln\left(x\right)+2i\pi}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{2\sinh\left(2x\right)}{\sqrt{\pi}\,x}+\frac{4\ \mathrm{Chi}\left(2x\right)-4\ln\left(2x\right)-4\gamma}{\sqrt{\pi}}\right)}{4}$	65

input
```
int(cosh(x)*sinh(x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
Chi(2*x)-1/2*sinh(2*x)/x
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx = \frac{x\mathrm{Ei}(2\,x)+x\mathrm{Ei}(-2\,x)-2\,\cosh\left(x\right)\sinh\left(x\right)}{2\,x}$$

input
```
integrate(cosh(x)*sinh(x)/x^2,x, algorithm="fricas")
```

output
```
1/2*(x*Ei(2*x) + x*Ei(-2*x) - 2*cosh(x)*sinh(x))/x
```

3.278.6 Sympy [F]

$$\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx = \int \frac{\sinh\left(x\right)\cosh\left(x\right)}{x^2}\,dx$$

input
```
integrate(cosh(x)*sinh(x)/x**2,x)
```

output
```
Integral(sinh(x)*cosh(x)/x**2, x)
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx = \frac{1}{2}\,\Gamma(-1,2\,x) + \frac{1}{2}\,\Gamma(-1,-2\,x)$$

input
```
integrate(cosh(x)*sinh(x)/x^2,x, algorithm="maxima")
```

output
```
1/2*gamma(-1, 2*x) + 1/2*gamma(-1, -2*x)
```

3.278.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx = \frac{2\,x\mathrm{Ei}(2\,x) + 2\,x\mathrm{Ei}(-2\,x) - e^{(2\,x)} + e^{(-2\,x)}}{4\,x}$$

input
```
integrate(cosh(x)*sinh(x)/x^2,x, algorithm="giac")
```

output
```
1/4*(2*x*Ei(2*x) + 2*x*Ei(-2*x) - e^(2*x) + e^(-2*x))/x
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx = \int \frac{\cosh(x)\,\sinh(x)}{x^2}\,dx$$

input
```
int((cosh(x)*sinh(x))/x^2,x)
```

output
```
int((cosh(x)*sinh(x))/x^2, x)
```

3.279 $\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx$

3.279.1 Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = -\frac{\cosh(2x)}{2x} - \frac{\sinh(2x)}{4x^2} + \mathrm{Shi}(2x)$$

output
```
-1/2*cosh(2*x)/x+Shi(2*x)-1/4*sinh(2*x)/x^2
```

3.279.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = -\frac{\cosh(2x)}{2x} - \frac{\sinh(2x)}{4x^2} + \mathrm{Shi}(2x)$$

input
```
Integrate[(Cosh[x]*Sinh[x])/x^3,x]
```

output
```
-1/2*Cosh[2*x]/x - Sinh[2*x]/(4*x^2) + SinhIntegral[2*x]
```

3.279.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {5971, 27, 3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)\cosh(x)}{x^3} \, dx$$

$$\Big\downarrow \text{5971}$$

$$\int \frac{\sinh(2x)}{2x^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sinh(2x)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int -\frac{i\sin(2ix)}{x^3}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\int \frac{\sin(2ix)}{x^3}dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{2}i\left(i\int \frac{\cosh(2x)}{x^2}dx-\frac{i\sinh(2x)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}i\left(i\int \frac{\sin\left(2ix+\frac{\pi}{2}\right)}{x^2}dx-\frac{i\sinh(2x)}{2x^2}\right)$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{2}i\left(i\left(-\frac{\cosh(2x)}{x}+2i\int -\frac{i\sinh(2x)}{x}dx\right)-\frac{i\sinh(2x)}{2x^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\left(i\left(2\int \frac{\sinh(2x)}{x}dx-\frac{\cosh(2x)}{x}\right)-\frac{i\sinh(2x)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}i\left(i\left(-\frac{\cosh(2x)}{x}+2\int -\frac{i\sin(2ix)}{x}dx\right)-\frac{i\sinh(2x)}{2x^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\left(i\left(-\frac{\cosh(2x)}{x}-2i\int \frac{\sin(2ix)}{x}dx\right)-\frac{i\sinh(2x)}{2x^2}\right)$$

$$\Big\downarrow \text{3779}$$

$$-\frac{1}{2}i\left(i\left(2\text{Shi}(2x)-\frac{\cosh(2x)}{x}\right)-\frac{i\sinh(2x)}{2x^2}\right)$$

input
```
Int[(Cosh[x]*Sinh[x])/x^3,x]
```

output
```
(-1/2*I)*(((-1/2*I)*Sinh[2*x])/x^2 + I*(-(Cosh[2*x]/x) + 2*SinhIntegral[2*
x]))
```

3.279.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

3.279. $\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx$

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.279.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\cosh(2x)}{2x}+\operatorname{Shi}(2x)-\frac{\sinh(2x)}{4x^2}$	24
meijerg	$\frac{i\sqrt{\pi}\left(\frac{2i\cosh(2x)}{x\sqrt{\pi}}+\frac{i\sinh(2x)}{x^2\sqrt{\pi}}-\frac{4\operatorname{Si}(2ix)}{\sqrt{\pi}}\right)}{4}$	44
risch	$-\frac{-4\operatorname{Ei}_1(2x)x^2+4\operatorname{Ei}_1(-2x)x^2+2\,\mathrm{e}^{-2x}x+2\,\mathrm{e}^{2x}x-\mathrm{e}^{-2x}+\mathrm{e}^{2x}}{8x^2}$	51

input
```
int(cosh(x)*sinh(x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*cosh(2*x)/x+Shi(2*x)-1/4*sinh(2*x)/x^2
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = \frac{x^2\mathrm{Ei}(2\,x)-x^2\mathrm{Ei}(-2\,x)-x\cosh\left(x\right)^2-x\sinh\left(x\right)^2-\cosh\left(x\right)\sinh\left(x\right)}{2\,x^2}$$

input
```
integrate(cosh(x)*sinh(x)/x^3,x, algorithm="fricas")
```

output
```
1/2*(x^2*Ei(2*x) - x^2*Ei(-2*x) - x*cosh(x)^2 - x*sinh(x)^2 - cosh(x)*sinh
(x))/x^2
```

3.279.6 Sympy [F]

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = \int \frac{\sinh{(x)}\cosh{(x)}}{x^3}\,dx$$

input
```
integrate(cosh(x)*sinh(x)/x**3,x)
```

output
```
Integral(sinh(x)*cosh(x)/x**3, x)
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = \Gamma(-2,2\,x) - \Gamma(-2,-2\,x)$$

input
```
integrate(cosh(x)*sinh(x)/x^3,x, algorithm="maxima")
```

output
```
gamma(-2, 2*x) - gamma(-2, -2*x)
```

3.279.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = \frac{4\,x^2\mathrm{Ei}(2\,x) - 4\,x^2\mathrm{Ei}(-2\,x) - 2\,xe^{(2\,x)} - 2\,xe^{(-2\,x)} - e^{(2\,x)} + e^{(-2\,x)}}{8\,x^2}$$

input
```
integrate(cosh(x)*sinh(x)/x^3,x, algorithm="giac")
```

output
```
1/8*(4*x^2*Ei(2*x) - 4*x^2*Ei(-2*x) - 2*x*e^(2*x) - 2*x*e^(-2*x) - e^(2*x)
 + e^(-2*x))/x^2
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = \int \frac{\cosh(x)\ \sinh(x)}{x^3}\,dx$$

input
```
int((cosh(x)*sinh(x))/x^3,x)
```

output
```
int((cosh(x)*sinh(x))/x^3, x)
```

3.280 $\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx$

3.280.1 Optimal result

Integrand size = 18, antiderivative size = 134

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{8b} - \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{8b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{8b} - \frac{3^{-1-m}e^{-3a}x^m(bx)^{-m}\Gamma(1+m,3bx)}{8b}$$

output
```
1/8*3^(-1-m)*exp(3*a)*x^m*GAMMA(1+m,-3*b*x)/b/((-b*x)^m)-1/8*exp(a)*x^m*GA
MMA(1+m,-b*x)/b/((-b*x)^m)+1/8*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)-1/8*3
^(-1-m)*x^m*GAMMA(1+m,3*b*x)/b/exp(3*a)/((b*x)^m)
```

3.280.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.85

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx$$
$$= \frac{e^{-3a}x^m\left(-3e^{4a}(-bx)^{-m}\Gamma(1+m,-bx)+3e^{2a}(bx)^{-m}\Gamma(1+m,bx)+3^{-m}(-b^2x^2)^{-m}\left(e^{6a}(bx)^m\Gamma(1+m,-\right.\right.}{24b}$$

input
```
Integrate[x^m*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(x^m*((-3*E^(4*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + (3*E^(2*a)*Gamma[1 +
m, b*x])/(b*x)^m + (E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] - (-(b*x))^m*Gamm
a[1 + m, 3*b*x])/(3^m*(-(b^2*x^2))^m)))/(24*b*E^(3*a))
```

3.280.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh^2(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{1}{4}x^m\cosh(3a+3bx)-\frac{1}{4}x^m\cosh(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-3bx)}{8b}-\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{8b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{8b}-\frac{e^{-3a}3^{-m-1}x^m(bx)^{-m}\Gamma(m+1,3bx)}{8b}$$

input
```
Int[x^m*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(3^(-1 - m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(8*b*(-(b*x))^m) - (E^a*x^m*
Gamma[1 + m, -(b*x)])/(8*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(8*b*E^a*
(b*x)^m) - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(8*b*E^(3*a)*(b*x)^m)
```

3.280.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.280.4 Maple [F]

$$\int x^m \cosh(bx+a)\sinh(bx+a)^2\,dx$$

input
```
int(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

output
```
int(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.21

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx =$$

$$-\frac{\cosh(m\log(3b)+3a)\Gamma(m+1,3bx) - 3\cosh(m\log(b)+a)\Gamma(m+1,bx) + 3\cosh(m\log(-b)-a)\,\Gamma}{}$$

input
```
integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/24*(cosh(m*log(3*b) + 3*a)*gamma(m + 1, 3*b*x) - 3*cosh(m*log(b) + a)*g
amma(m + 1, b*x) + 3*cosh(m*log(-b) - a)*gamma(m + 1, -b*x) - cosh(m*log(-
3*b) - 3*a)*gamma(m + 1, -3*b*x) - gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3
*a) - 3*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) + gamma(m + 1, -3*b*x)*sinh
(m*log(-3*b) - 3*a) + 3*gamma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

3.280.6 Sympy [F]

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx = \int x^m \sinh^2(a+bx)\cosh(a+bx)\,dx$$

input
```
integrate(x**m*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**m*sinh(a + b*x)**2*cosh(a + b*x), x)
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx = & -\frac{1}{8}(3\,bx)^{-m-1}x^{m+1}e^{(-3\,a)}\Gamma(m+1,3\,bx) \\ & +\frac{1}{8}(bx)^{-m-1}x^{m+1}e^{(-a)}\Gamma(m+1,bx) \\ & +\frac{1}{8}(-bx)^{-m-1}x^{m+1}e^{a}\Gamma(m+1,-bx) \\ & -\frac{1}{8}(-3\,bx)^{-m-1}x^{m+1}e^{(3\,a)}\Gamma(m+1,-3\,bx)\end{aligned}$$

input
```
integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(3*b*x)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) + 1/8*(b*x)^(
-m - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) + 1/8*(-b*x)^(-m - 1)*x^(m + 1)
*e^a*gamma(m + 1, -b*x) - 1/8*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m
+ 1, -3*b*x)
```

3.280.8 Giac [F]

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx = \int x^m \cosh(bx+a)\sinh(bx+a)^2\,dx$$

input
```
integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)*sinh(b*x + a)^2, x)
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx = \int x^m \cosh(a+b\,x)\,\sinh(a+b\,x)^2\,dx$$

input
```
int(x^m*cosh(a + b*x)*sinh(a + b*x)^2,x)
```

output
```
int(x^m*cosh(a + b*x)*sinh(a + b*x)^2, x)
```

3.281 $\int x^3 \cosh(a+bx) \sinh^2(a+bx)\,dx$

3.281.1 Optimal result

Integrand size = 18, antiderivative size = 117

$$\int x^3 \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{14\cosh(a+bx)}{9b^4}+\frac{2x^2\cosh(a+bx)}{3b^2}-\frac{2\cosh^3(a+bx)}{27b^4}-\frac{4x\sinh(a+bx)}{3b^3}-\frac{x^2\cosh(a+bx)\sinh^2(a+bx)}{3b^2}+\frac{2x\sinh^3(a+bx)}{9b^3}+\frac{x^3\sinh^3(a+bx)}{3b}$$

output
```
14/9*cosh(b*x+a)/b^4+2/3*x^2*cosh(b*x+a)/b^2-2/27*cosh(b*x+a)^3/b^4-4/3*x*
sinh(b*x+a)/b^3-1/3*x^2*cosh(b*x+a)*sinh(b*x+a)^2/b^2+2/9*x*sinh(b*x+a)^3/
b^3+1/3*x^3*sinh(b*x+a)^3/b
```

3.281.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int x^3 \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{81(2+b^2x^2)\cosh(a+bx)-(2+9b^2x^2)\cosh(3(a+bx))+6bx(-26-3b^2x^2+(2+3b^2x^2)\cosh(2(a+bx}{108b^4}$$

input
```
Integrate[x^3*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(81*(2 + b^2*x^2)*Cosh[a + b*x] - (2 + 9*b^2*x^2)*Cosh[3*(a + b*x)] + 6*b*
x*(-26 - 3*b^2*x^2 + (2 + 3*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(10
8*b^4)
```

3.281.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.28, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {5895, 3042, 26, 3792, 26, 3042, 26, 3113, 2009, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sinh^2(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{ 5895}$$

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{\int x^2\sinh^3(a+bx)dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{\int ix^2\sin(ia+ibx)^3dx}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\int x^2\sin(ia+ibx)^3dx}{b}$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(\frac{2\int -i\sinh^3(a+bx)dx}{9b^2}+\frac{2}{3}\int ix^2\sinh(a+bx)dx+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(-\frac{2i\int \sinh^3(a+bx)dx}{9b^2}+\frac{2}{3}i\int x^2\sinh(a+bx)dx+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(-\frac{2i\int i\sin(ia+ibx)^3dx}{9b^2}+\frac{2}{3}i\int -ix^2\sin(ia+ibx)dx+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{b}$$

$\downarrow$ 26

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(\frac{2\int \sin(ia+ibx)^3dx}{9b^2}+\frac{2}{3}\int x^2\sin(ia+ibx)dx+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{b}$$

$\downarrow$ 3113

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(\frac{2i\int (1-\cosh^2(a+bx))d\cosh(a+bx)}{9b^3}+\frac{2}{3}\int x^2\sin(ia+ibx)dx+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{b}$$

$\downarrow$ 2009

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(\frac{2}{3}\int x^2\sin(ia+ibx)dx+\frac{2i\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{b}$$

$\downarrow$ 3777

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(\frac{2}{3}\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\int x\cosh(a+bx)dx}{b}\right)+\frac{2i\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{b}$$

$\downarrow$ 3042

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(\frac{2}{3}\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{2i\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{b}$$

$\downarrow$ 3777

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(\frac{2}{3}\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{i\int -i\sinh(a+bx)dx}{b}\right)}{b}\right)+\frac{2i\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)}{3b}\right.}{b}$$

$\downarrow$ 26

3.281. $\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx$

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(\frac{2}{3}\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\int\sinh(a+bx)dx}{b}\right)}{b}\right)+\frac{2i\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)\cosh}{3b}\right.}{b}$$

$\downarrow$ 3042

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(\frac{2}{3}\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\int-i\sin(ia+ibx)dx}{b}\right)}{b}\right)+\frac{2i\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)\,c}{3b}\right.}{b}$$

$\downarrow$ 26

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(\frac{2}{3}\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}+\frac{i\int\sin(ia+ibx)dx}{b}\right)}{b}\right)+\frac{2i\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)\cos}{3b}\right.}{b}$$

$\downarrow$ 3118

$$\frac{x^3\sinh^3(a+bx)}{3b}-\frac{i\left(\frac{2i\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2}{3}\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)+\frac{2ix\sinh^3(a+bx)}{9b^2}-\frac{ix^2\sinh^2(a+bx)\cosh(a+}{3b}\right.}{b}$$

input
```
Int[x^3*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(x^3*Sinh[a + b*x]^3)/(3*b) - (I*((((2*I)/9)*(Cosh[a + b*x] - Cosh[a + b*x
]^3/3))/b^3 - ((I/3)*x^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/b + (((2*I)/9)*x*S
inh[a + b*x]^3)/b^2 + (2*((I*x^2*Cosh[a + b*x])/b - ((2*I)*(-(Cosh[a + b*x
]/b^2) + (x*Sinh[a + b*x])/b))/b))/3))/b
```

3.281. $\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx$

3.281.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.281. $\int x^3 \cosh(a+bx)\sinh^2(a+bx)\,dx$

3.281.4 Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(9x^3b^3-9x^2b^2+6bx-2)\mathrm{e}^{3bx+3a}}{216b^4}-\frac{(x^3b^3-3x^2b^2+6bx-6)\mathrm{e}^{bx+a}}{8b^4}+\frac{(x^3b^3+3x^2b^2+6bx+6)\mathrm{e}^{-bx-a}}{8b^4}-\frac{(9x^3b^3+9x^2b}{}$
derivativedivides	$-\frac{a^3\sinh(bx+a)^3}{3}+3a^2\left(\frac{(bx+a)\sinh(bx+a)^3}{3}+\frac{2\cosh(bx+a)}{9}-\frac{\cosh(bx+a)\sinh(bx+a)^2}{9}\right)-3a\left(\frac{(bx+a)^2\sinh(bx+a)^3}{3}+\frac{4(bx+a)\,\mathrm{co}}{9}\right.$
default	$-\frac{a^3\sinh(bx+a)^3}{3}+3a^2\left(\frac{(bx+a)\sinh(bx+a)^3}{3}+\frac{2\cosh(bx+a)}{9}-\frac{\cosh(bx+a)\sinh(bx+a)^2}{9}\right)-3a\left(\frac{(bx+a)^2\sinh(bx+a)^3}{3}+\frac{4(bx+a)\,\mathrm{co}}{9}\right.$

input
```
int(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/216*(9*b^3*x^3-9*b^2*x^2+6*b*x-2)/b^4*exp(3*b*x+3*a)-1/8*(b^3*x^3-3*b^2*
x^2+6*b*x-6)/b^4*exp(b*x+a)+1/8*(b^3*x^3+3*b^2*x^2+6*b*x+6)/b^4*exp(-b*x-a
)-1/216*(9*b^3*x^3+9*b^2*x^2+6*b*x+2)/b^4*exp(-3*b*x-3*a)
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.15

$$\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx =$$
$$-\frac{(9\,b^2x^2+2)\cosh(bx+a)^3+3\,(9\,b^2x^2+2)\cosh(bx+a)\sinh(bx+a)^2-3\,(3\,b^3x^3+2\,bx)\sinh(bx+a)}{108\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/108*((9*b^2*x^2 + 2)*cosh(b*x + a)^3 + 3*(9*b^2*x^2 + 2)*cosh(b*x + a)*
sinh(b*x + a)^2 - 3*(3*b^3*x^3 + 2*b*x)*sinh(b*x + a)^3 - 81*(b^2*x^2 + 2)
*cosh(b*x + a) + 9*(3*b^3*x^3 - (3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^2 + 18*b
*x)*sinh(b*x + a))/b^4
```

3.281.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.25

$$\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx$$
$$=\begin{cases}\frac{x^3\sinh^3(a+bx)}{3b}-\frac{x^2\sinh^2(a+bx)\cosh(a+bx)}{b^2}+\frac{2x^2\cosh^3(a+bx)}{3b^2}+\frac{14x\sinh^3(a+bx)}{9b^3}-\frac{4x\sinh(a+bx)\cosh^2(a+bx)}{3b^3}-\frac{14\sinh^2(a\end{cases}$$
$$\frac{x^4\sinh^2(a)\cosh(a)}{4}$$

input
```
integrate(x**3*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x**3*sinh(a + b*x)**3/(3*b) - x**2*sinh(a + b*x)**2*cosh(a + b*
x)/b**2 + 2*x**2*cosh(a + b*x)**3/(3*b**2) + 14*x*sinh(a + b*x)**3/(9*b**3
) - 4*x*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**3) - 14*sinh(a + b*x)**2*cosh
(a + b*x)/(9*b**4) + 40*cosh(a + b*x)**3/(27*b**4), Ne(b, 0)), (x**4*sinh(
a)**2*cosh(a)/4, True))
```

3.281.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.37

$$\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx=\frac{\left(9\,b^3x^3e^{(3\,a)}-9\,b^2x^2e^{(3\,a)}+6\,bxe^{(3\,a)}-2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^4}-\frac{\left(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a\right)e^{(bx)}}{8\,b^4}+\frac{\left(b^3x^3+3\,b^2x^2+6\,bx+6\right)e^{(-bx-a)}}{8\,b^4}-\frac{\left(9\,b^3x^3+9\,b^2x^2+6\,bx+2\right)e^{(-3\,bx-3\,a)}}{216\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/216*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e^(3*a) - 2*e^(3*a))*
e^(3*b*x)/b^4 - 1/8*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*e^(b
*x)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/b^4 - 1/216*(
9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int x^3 \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{(9\,b^3x^3 - 9\,b^2x^2 + 6\,bx - 2)e^{(3\,bx+3\,a)}}{216\,b^4} - \frac{(b^3x^3 - 3\,b^2x^2 + 6\,bx - 6)e^{(bx+a)}}{8\,b^4} + \frac{(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)e^{(-bx-a)}}{8\,b^4} - \frac{(9\,b^3x^3 + 9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{216\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/216*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 - 1/8*(b^3*x
^3 - 3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6
*b*x + 6)*e^(-b*x - a)/b^4 - 1/216*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(
-3*b*x - 3*a)/b^4
```

3.281.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^3 \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{14\,x\,\sinh(a+b\,x)^3}{9} - \frac{4\,x\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{3}}{b^3} + \frac{\frac{2\,x^2\,\cosh(a+b\,x)^3}{3} - x^2\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b^2} + \frac{40\cosh(a+b\,x)^3}{27\,b^4} - \frac{14\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{9\,b^4} + \frac{x^3\sinh(a+b\,x)^3}{3\,b}$$

input
```
int(x^3*cosh(a + b*x)*sinh(a + b*x)^2,x)
```

output
```
((14*x*sinh(a + b*x)^3)/9 - (4*x*cosh(a + b*x)^2*sinh(a + b*x))/3)/b^3 + (
(2*x^2*cosh(a + b*x)^3)/3 - x^2*cosh(a + b*x)*sinh(a + b*x)^2)/b^2 + (40*c
osh(a + b*x)^3)/(27*b^4) - (14*cosh(a + b*x)*sinh(a + b*x)^2)/(9*b^4) + (x
^3*sinh(a + b*x)^3)/(3*b)
```

3.282 $\int x^2 \cosh(a+bx) \sinh^2(a+bx) \, dx$

3.282.1 Optimal result

Integrand size = 18, antiderivative size = 83

$$\int x^2 \cosh(a+bx) \sinh^2(a+bx) \, dx = \frac{4x \cosh(a+bx)}{9b^2} - \frac{4 \sinh(a+bx)}{9b^3} - \frac{2x \cosh(a+bx) \sinh^2(a+bx)}{9b^2} + \frac{2 \sinh^3(a+bx)}{27b^3} + \frac{x^2 \sinh^3(a+bx)}{3b}$$

output
```
4/9*x*cosh(b*x+a)/b^2-4/9*sinh(b*x+a)/b^3-2/9*x*cosh(b*x+a)*sinh(b*x+a)^2/
b^2+2/27*sinh(b*x+a)^3/b^3+1/3*x^2*sinh(b*x+a)^3/b
```

3.282.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int x^2 \cosh(a+bx) \sinh^2(a+bx) \, dx = \frac{27bx \cosh(a+bx) - 3bx \cosh(3(a+bx)) + (-26 - 9b^2x^2 + (2 + 9b^2x^2) \cosh(2(a+bx))) \sinh(a+bx)}{54b^3}$$

input
```
Integrate[x^2*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(27*b*x*Cosh[a + b*x] - 3*b*x*Cosh[3*(a + b*x)] + (-26 - 9*b^2*x^2 + (2 +
9*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(54*b^3)
```

3.282.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5895, 3042, 26, 3791, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sinh^2(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{ 5895}$$

$$\frac{x^2\sinh^3(a+bx)}{3b}-\frac{2\int x\sinh^3(a+bx)dx}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^2\sinh^3(a+bx)}{3b}-\frac{2\int ix\sin(ia+ibx)^3dx}{3b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{x^2\sinh^3(a+bx)}{3b}-\frac{2i\int x\sin(ia+ibx)^3dx}{3b}$$

$$\Big\downarrow \text{ 3791}$$

$$\frac{x^2\sinh^3(a+bx)}{3b}-\frac{2i\left(\frac{2}{3}\int ix\sinh(a+bx)dx+\frac{i\sinh^3(a+bx)}{9b^2}-\frac{ix\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{x^2\sinh^3(a+bx)}{3b}-\frac{2i\left(\frac{2}{3}i\int x\sinh(a+bx)dx+\frac{i\sinh^3(a+bx)}{9b^2}-\frac{ix\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^2\sinh^3(a+bx)}{3b}-\frac{2i\left(\frac{2}{3}i\int -ix\sin(ia+ibx)dx+\frac{i\sinh^3(a+bx)}{9b^2}-\frac{ix\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{x^2\sinh^3(a+bx)}{3b}-\frac{2i\left(\frac{2}{3}\int x\sin(ia+ibx)dx+\frac{i\sinh^3(a+bx)}{9b^2}-\frac{ix\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{x^2\sinh^3(a+bx)}{3b}-\frac{2i\left(\frac{2}{3}\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\cosh(a+bx)dx}{b}\right)+\frac{i\sinh^3(a+bx)}{9b^2}-\frac{ix\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \ 3042$$

$$\frac{x^2\sinh^3(a+bx)}{3b}-\frac{2i\left(\frac{2}{3}\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{i\sinh^3(a+bx)}{9b^2}-\frac{ix\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \ 3117$$

$$\frac{x^2\sinh^3(a+bx)}{3b}-\frac{2i\left(\frac{i\sinh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)-\frac{ix\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b}$$

input
```
Int[x^2*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(x^2*Sinh[a + b*x]^3)/(3*b) - (((2*I)/3)*(((-1/3*I)*x*Cosh[a + b*x]*Sinh[a
 + b*x]^2)/b + ((I/9)*Sinh[a + b*x]^3)/b^2 + (2*((I*x*Cosh[a + b*x])/b - (
I*Sinh[a + b*x])/b^2))/3))/b
```

3.282.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.282.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

method	result
risch	$\frac{(9x^2b^2-6bx+2)\mathrm{e}^{3bx+3a}}{216b^3}-\frac{(x^2b^2-2bx+2)\mathrm{e}^{bx+a}}{8b^3}+\frac{(x^2b^2+2bx+2)\mathrm{e}^{-bx-a}}{8b^3}-\frac{(9x^2b^2+6bx+2)\mathrm{e}^{-3bx-3a}}{216b^3}$
derivativedivides	$\frac{\frac{a^2\sinh(bx+a)^3}{3}-2a\left(\frac{(bx+a)\sinh(bx+a)^3}{3}+\frac{2\cosh(bx+a)}{9}-\frac{\cosh(bx+a)\sinh(bx+a)^2}{9}\right)+\frac{(bx+a)^2\sinh(bx+a)^3}{3}+\frac{4(bx+a)\cosh(bx+}{9}}{b^3}$
default	$\frac{\frac{a^2\sinh(bx+a)^3}{3}-2a\left(\frac{(bx+a)\sinh(bx+a)^3}{3}+\frac{2\cosh(bx+a)}{9}-\frac{\cosh(bx+a)\sinh(bx+a)^2}{9}\right)+\frac{(bx+a)^2\sinh(bx+a)^3}{3}+\frac{4(bx+a)\cosh(bx+}{9}}{b^3}$

input
```
int(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/216*(9*b^2*x^2-6*b*x+2)/b^3*exp(3*b*x+3*a)-1/8*(b^2*x^2-2*b*x+2)/b^3*exp
(b*x+a)+1/8*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)-1/216*(9*b^2*x^2+6*b*x+2)/b^
3*exp(-3*b*x-3*a)
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int x^2\cosh(a+bx)\sinh^2(a+bx)\,dx =$$
$$-\frac{6\,bx\cosh\left(bx+a\right)^3+18\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-\left(9\,b^2x^2+2\right)\sinh\left(bx+a\right)^3-54\,bx\cosh\left(bx+\right.}{108\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/108*(6*b*x*cosh(b*x + a)^3 + 18*b*x*cosh(b*x + a)*sinh(b*x + a)^2 - (9*
b^2*x^2 + 2)*sinh(b*x + a)^3 - 54*b*x*cosh(b*x + a) + 3*(9*b^2*x^2 - (9*b^
2*x^2 + 2)*cosh(b*x + a)^2 + 18)*sinh(b*x + a))/b^3
```

3.282.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int x^2\cosh(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} \frac{x^2\sinh^3(a+bx)}{3b} - \frac{2x\sinh^2(a+bx)\cosh(a+bx)}{3b^2} + \frac{4x\cosh^3(a+bx)}{9b^2} + \frac{14\sinh^3(a+bx)}{27b^3} - \frac{4\sinh(a+bx)\cosh^2(a+bx)}{9b^3} & \text{for } b \neq 0 \\ \frac{x^3\sinh^2(a)\cosh(a)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x**2*sinh(a + b*x)**3/(3*b) - 2*x*sinh(a + b*x)**2*cosh(a + b*x
)/(3*b**2) + 4*x*cosh(a + b*x)**3/(9*b**2) + 14*sinh(a + b*x)**3/(27*b**3)
 - 4*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**3), Ne(b, 0)), (x**3*sinh(a)**2*
cosh(a)/3, True))
```

3.282.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.47

$$\int x^2\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{(9\,b^2x^2e^{(3\,a)} - 6\,bxe^{(3\,a)} + 2\,e^{(3\,a)})e^{(3\,bx)}}{216\,b^3} - \frac{(b^2x^2e^a - 2\,bxe^a + 2\,e^a)e^{(bx)}}{8\,b^3} + \frac{(b^2x^2 + 2\,bx + 2)e^{(-bx-a)}}{8\,b^3} - \frac{(9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/216*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 - 1/8*
(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*
e^(-b*x - a)/b^3 - 1/216*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

$$\int x^2 \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{(9\,b^2x^2-6\,bx+2)e^{(3\,bx+3\,a)}}{216\,b^3} - \frac{(b^2x^2-2\,bx+2)e^{(bx+a)}}{8\,b^3} + \frac{(b^2x^2+2\,bx+2)e^{(-bx-a)}}{8\,b^3} - \frac{(9\,b^2x^2+6\,bx+2)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/216*(9*b^2*x^2 - 6*b*x + 2)*e^(3*b*x + 3*a)/b^3 - 1/8*(b^2*x^2 - 2*b*x +
 2)*e^(b*x + a)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - 1/216*(
9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

3.282.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int x^2 \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{4\,x\cosh(a+b\,x)^3}{9}-\frac{2\,x\cosh(a+b\,x)\sinh(a+b\,x)^2}{3}}{b^2} + \frac{14\sinh(a+b\,x)^3}{27\,b^3} - \frac{4\cosh(a+b\,x)^2\sinh(a+b\,x)}{9\,b^3} + \frac{x^2\sinh(a+b\,x)^3}{3\,b}$$

input
```
int(x^2*cosh(a + b*x)*sinh(a + b*x)^2,x)
```

output
```
((4*x*cosh(a + b*x)^3)/9 - (2*x*cosh(a + b*x)*sinh(a + b*x)^2)/3)/b^2 + (1
4*sinh(a + b*x)^3)/(27*b^3) - (4*cosh(a + b*x)^2*sinh(a + b*x))/(9*b^3) +
(x^2*sinh(a + b*x)^3)/(3*b)
```

3.283 $\int x \cosh(a+bx) \sinh^2(a+bx)\,dx$

3.283.1 Optimal result

Integrand size = 16, antiderivative size = 45

$$\int x \cosh(a+bx) \sinh^2(a+bx)\,dx = \frac{\cosh(a+bx)}{3b^2} - \frac{\cosh^3(a+bx)}{9b^2} + \frac{x\sinh^3(a+bx)}{3b}$$

output
```
1/3*cosh(b*x+a)/b^2-1/9*cosh(b*x+a)^3/b^2+1/3*x*sinh(b*x+a)^3/b
```

3.283.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int x \cosh(a+bx) \sinh^2(a+bx)\,dx = \frac{9\cosh(a+bx) - \cosh(3(a+bx)) + 12bx\sinh^3(a+bx)}{36b^2}$$

input
```
Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(9*Cosh[a + b*x] - Cosh[3*(a + b*x)] + 12*b*x*Sinh[a + b*x]^3)/(36*b^2)
```

3.283.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5895, 3042, 26, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh^2(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{5895}$$

$$\frac{x\sinh^3(a+bx)}{3b}-\frac{\int \sinh^3(a+bx)dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x\sinh^3(a+bx)}{3b}-\frac{\int i\sin(ia+ibx)^3dx}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{x\sinh^3(a+bx)}{3b}-\frac{i\int \sin(ia+ibx)^3dx}{3b}$$

$$\Big\downarrow \text{3113}$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)d\cosh(a+bx)}{3b^2}+\frac{x\sinh^3(a+bx)}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)}{3b^2}+\frac{x\sinh^3(a+bx)}{3b}$$

input
```
Int[x*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(Cosh[a + b*x] - Cosh[a + b*x]^3/3)/(3*b^2) + (x*Sinh[a + b*x]^3)/(3*b)
```

3.283.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.283.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)^3}{3}+\frac{2\cosh(bx+a)}{9}-\frac{\cosh(bx+a)\sinh(bx+a)^2}{9}-\frac{a\sinh(bx+a)^3}{3}}{b^2}$	56
default	$\frac{\frac{(bx+a)\sinh(bx+a)^3}{3}+\frac{2\cosh(bx+a)}{9}-\frac{\cosh(bx+a)\sinh(bx+a)^2}{9}-\frac{a\sinh(bx+a)^3}{3}}{b^2}$	56
risch	$\frac{(3bx-1)e^{3bx+3a}}{72b^2}-\frac{(bx-1)e^{bx+a}}{8b^2}+\frac{(bx+1)e^{-bx-a}}{8b^2}-\frac{(3bx+1)e^{-3bx-3a}}{72b^2}$	77

input
```
int(x*cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/3*(b*x+a)*sinh(b*x+a)^3+2/9*cosh(b*x+a)-1/9*cosh(b*x+a)*sinh(b*x+
a)^2-1/3*a*sinh(b*x+a)^3)
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int x\cosh(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{3\,bx\sinh\left(bx+a\right)^3-\cosh\left(bx+a\right)^3-3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+9\,\left(bx\cosh\left(bx+a\right)^2-bx\right)\sinh\left(bx\right.}{36\,b^2}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/36*(3*b*x*sinh(b*x + a)^3 - cosh(b*x + a)^3 - 3*cosh(b*x + a)*sinh(b*x +
 a)^2 + 9*(b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a) + 9*cosh(b*x + a))/b^2
```

3.283.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int x\cosh(a+bx)\sinh^2(a+bx)\,dx$$
$$=\begin{cases}\frac{x\sinh^3(a+bx)}{3b}-\frac{\sinh^2(a+bx)\cosh(a+bx)}{3b^2}+\frac{2\cosh^3(a+bx)}{9b^2} & \text{for } b\neq 0\\ \frac{x^2\sinh^2(a)\cosh(a)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x*sinh(a + b*x)**3/(3*b) - sinh(a + b*x)**2*cosh(a + b*x)/(3*b*
*2) + 2*cosh(a + b*x)**3/(9*b**2), Ne(b, 0)), (x**2*sinh(a)**2*cosh(a)/2,
True))
```

3.283.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(39) = 78.

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.87

$$\int x\cosh(a+bx)\sinh^2(a+bx)\,dx=\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{72\,b^2}-\frac{\left(bxe^a-e^a\right)e^{(bx)}}{8\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{8\,b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/72*(3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 1/8*(b*x*e^a - e^a)*e^(b*x)
/b^2 + 1/8*(b*x + 1)*e^(-b*x - a)/b^2 - 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/
b^2
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int x\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{72\,b^2} - \frac{(bx-1)e^{(bx+a)}}{8\,b^2} + \frac{(bx+1)e^{(-bx-a)}}{8\,b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/72*(3*b*x - 1)*e^(3*b*x + 3*a)/b^2 - 1/8*(b*x - 1)*e^(b*x + a)/b^2 + 1/8
*(b*x + 1)*e^(-b*x - a)/b^2 - 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2
```

3.283.9 Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int x\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{2\cosh(a+b\,x)^3 - 3\cosh(a+b\,x)\,\sinh(a+b\,x)^2 + 3\,b\,x\sinh(a+b\,x)^3}{9\,b^2}$$

input
```
int(x*cosh(a + b*x)*sinh(a + b*x)^2,x)
```

output
```
(2*cosh(a + b*x)^3 - 3*cosh(a + b*x)*sinh(a + b*x)^2 + 3*b*x*sinh(a + b*x)
^3)/(9*b^2)
```

3.284 $\int \cosh(a+bx)\sinh^2(a+bx)\,dx$

3.284.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx=\frac{\sinh^3(a+bx)}{3b}$$

output
```
1/3*sinh(b*x+a)^3/b
```

3.284.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx=\frac{\sinh^3(a+bx)}{3b}$$

input
```
Integrate[Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
Sinh[a + b*x]^3/(3*b)
```

3.284.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 25, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^2(-\cos(ia+ibx))dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(ia+ibx)\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{i\int -\sinh^2(a+bx)d(i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sinh^3(a+bx)}{3b}$$

input
```
Int[Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
Sinh[a + b*x]^3/(3*b)
```

3.284.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.284.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sinh(bx+a)^3}{3b}$	14
default	$\frac{\sinh(bx+a)^3}{3b}$	14
risch	$\frac{e^{3bx+3a}}{24b} - \frac{e^{bx+a}}{8b} + \frac{e^{-bx-a}}{8b} - \frac{e^{-3bx-3a}}{24b}$	55

input
```
int(cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*sinh(b*x+a)^3/b
```

3.284.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(bx+a)^3 + 3\left(\cosh(bx+a)^2 - 1\right)\sinh(bx+a)}{12\,b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 - 1)*sinh(b*x + a))/b
```

3.284.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} \frac{\sinh^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sinh^2(a)\cosh(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Piecewise((sinh(a + b*x)**3/(3*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a), True)
)
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(bx+a)^3}{3\,b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*sinh(b*x + a)^3/b
```

3.284.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.60

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{24\,b} - \frac{e^{(bx+a)}}{8\,b} + \frac{e^{(-bx-a)}}{8\,b} - \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/24*e^(3*b*x + 3*a)/b - 1/8*e^(b*x + a)/b + 1/8*e^(-b*x - a)/b - 1/24*e^(
-3*b*x - 3*a)/b
```

3.284.9 Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(a+b\,x)^3}{3\,b}$$

input
```
int(cosh(a + b*x)*sinh(a + b*x)^2,x)
```

output
```
sinh(a + b*x)^3/(3*b)
```

3.285 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx$

3.285.1 Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = -\frac{1}{4}\cosh(a)\text{Chi}(bx) + \frac{1}{4}\cosh(3a)\text{Chi}(3bx) - \frac{1}{4}\sinh(a)\text{Shi}(bx) + \frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

output
```
-1/4*Chi(b*x)*cosh(a)+1/4*Chi(3*b*x)*cosh(3*a)-1/4*Shi(b*x)*sinh(a)+1/4*Sh
i(3*b*x)*sinh(3*a)
```

3.285.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{4}(-\cosh(a)\text{Chi}(bx) + \cosh(3a)\text{Chi}(3bx) - \sinh(a)\text{Shi}(bx) + \sinh(3a)\text{Shi}(3bx))$$

input
```
Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x,x]
```

output
```
(-(Cosh[a]*CoshIntegral[b*x]) + Cosh[3*a]*CoshIntegral[3*b*x] - Sinh[a]*Si
nhIntegral[b*x] + Sinh[3*a]*SinhIntegral[3*b*x])/4
```

3.285.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\cosh(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{\cosh(3a+3bx)}{4x}-\frac{\cosh(a+bx)}{4x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx)-\frac{1}{4}\sinh(a)\text{Shi}(bx)+\frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

input
```
Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x,x]
```

output
```
-1/4*(Cosh[a]*CoshIntegral[b*x]) + (Cosh[3*a]*CoshIntegral[3*b*x])/4 - (Si
nh[a]*SinhIntegral[b*x])/4 + (Sinh[3*a]*SinhIntegral[3*b*x])/4
```

3.285.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.285.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{e^{-3a} \operatorname{Ei}_1(3bx)}{8}+\frac{e^{-a} \operatorname{Ei}_1(bx)}{8}+\frac{e^{a} \operatorname{Ei}_1(-bx)}{8}-\frac{e^{3a} \operatorname{Ei}_1(-3bx)}{8}$	47

input
```
int(cosh(b*x+a)*sinh(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/8*exp(-3*a)*Ei(1,3*b*x)+1/8*exp(-a)*Ei(1,b*x)+1/8*exp(a)*Ei(1,-b*x)-1/8
*exp(3*a)*Ei(1,-3*b*x)
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{8}\left(\operatorname{Ei}(3\,bx)+\operatorname{Ei}(-3\,bx)\right)\cosh\left(3\,a\right) - \frac{1}{8}\left(\operatorname{Ei}(bx)+\operatorname{Ei}(-bx)\right)\cosh\left(a\right) + \frac{1}{8}\left(\operatorname{Ei}(3\,bx)-\operatorname{Ei}(-3\,bx)\right)\sinh\left(3\,a\right) - \frac{1}{8}\left(\operatorname{Ei}(bx)-\operatorname{Ei}(-bx)\right)\sinh\left(a\right)$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
1/8*(Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 1/8*(Ei(b*x) + Ei(-b*x))*cosh(a)
+ 1/8*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 1/8*(Ei(b*x) - Ei(-b*x))*sinh(a
)
```

3.285.6 Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \frac{\sinh^2(a+bx)\cosh(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)**2/x,x)
```

output
```
Integral(sinh(a + b*x)**2*cosh(a + b*x)/x, x)
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - \frac{1}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 1/
8*Ei(b*x)*e^a
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - \frac{1}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="giac")
```

output
```
1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 1/
8*Ei(b*x)*e^a
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{x}\,dx$$

input
```
int((cosh(a + b*x)*sinh(a + b*x)^2)/x,x)
```

output
```
int((cosh(a + b*x)*sinh(a + b*x)^2)/x, x)
```

3.286 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx$

3.286.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \frac{\cosh(a+bx)}{4x} - \frac{\cosh(3a+3bx)}{4x} - \frac{1}{4}b\text{Chi}(bx)\sinh(a) + \frac{3}{4}b\text{Chi}(3bx)\sinh(3a) - \frac{1}{4}b\cosh(a)\text{Shi}(bx) + \frac{3}{4}b\cosh(3a)\text{Shi}(3bx)$$

output
```
1/4*cosh(b*x+a)/x-1/4*cosh(3*b*x+3*a)/x-1/4*b*cosh(a)*Shi(b*x)+3/4*b*cosh(
3*a)*Shi(3*b*x)-1/4*b*Chi(b*x)*sinh(a)+3/4*b*Chi(3*b*x)*sinh(3*a)
```

3.286.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{-\cosh(a+bx)+\cosh(3(a+bx))+bx\text{Chi}(bx)\sinh(a)-3bx\text{Chi}(3bx)\sinh(3a)+bx\cosh(a)\text{Shi}(bx)-}{4x}$$

input
```
Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^2,x]
```

output
```
-1/4*(-Cosh[a + b*x] + Cosh[3*(a + b*x)] + b*x*CoshIntegral[b*x]*Sinh[a] -
 3*b*x*CoshIntegral[3*b*x]*Sinh[3*a] + b*x*Cosh[a]*SinhIntegral[b*x] - 3*b
*x*Cosh[3*a]*SinhIntegral[3*b*x])/x
```

3.286.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\cosh(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{ 5971}$$

$$\int \left(\frac{\cosh(3a+3bx)}{4x^2}-\frac{\cosh(a+bx)}{4x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{4}b\sinh(a)\text{Chi}(bx)+\frac{3}{4}b\sinh(3a)\text{Chi}(3bx)-\frac{1}{4}b\cosh(a)\text{Shi}(bx)+\frac{3}{4}b\cosh(3a)\text{Shi}(3bx)+\frac{\cosh(a+bx)}{4x}-\frac{\cosh(3a+3bx)}{4x}$$

input
```
Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^2,x]
```

output
```
Cosh[a + b*x]/(4*x) - Cosh[3*a + 3*b*x]/(4*x) - (b*CoshIntegral[b*x]*Sinh[
a])/4 + (3*b*CoshIntegral[3*b*x]*Sinh[3*a])/4 - (b*Cosh[a]*SinhIntegral[b*
x])/4 + (3*b*Cosh[3*a]*SinhIntegral[3*b*x])/4
```

3.286.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.286.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{-3\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)bx+\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)bx-\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)bx+3\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)bx+\mathrm{e}^{-3bx-3a}-\mathrm{e}^{-bx-a}+\mathrm{e}^{3bx+3a}-\mathrm{e}^{bx+a}}{8x}$	96

input
```
int(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-3*exp(-3*a)*Ei(1,3*b*x)*b*x+exp(-a)*Ei(1,b*x)*b*x-exp(a)*Ei(1,-b*x)
*b*x+3*exp(3*a)*Ei(1,-3*b*x)*b*x+exp(-3*b*x-3*a)-exp(-b*x-a)+exp(3*b*x+3*a
)-exp(b*x+a))/x
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx =$$
$$-\frac{2\,\cosh\left(bx+a\right)^3+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-3\left(bx\mathrm{Ei}(3\,bx)-bx\mathrm{Ei}(-3\,bx)\right)\cosh\left(3\,a\right)+\left(bx\mathrm{Ei}(bx\right.}{}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
-1/8*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b*x*Ei(3*b*
x) - b*x*Ei(-3*b*x))*cosh(3*a) + (b*x*Ei(b*x) - b*x*Ei(-b*x))*cosh(a) - 3*
(b*x*Ei(3*b*x) + b*x*Ei(-3*b*x))*sinh(3*a) + (b*x*Ei(b*x) + b*x*Ei(-b*x))*
sinh(a) - 2*cosh(b*x + a))/x
```

3.286.6 Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh^2{(a+bx)}\cosh{(a+bx)}}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)**2/x**2,x)
```

output
```
Integral(sinh(a + b*x)**2*cosh(a + b*x)/x**2, x)
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{3}{8}\,be^{(-3\,a)}\Gamma(-1,3\,bx)+\frac{1}{8}\,be^{(-a)}\Gamma(-1,bx) - \frac{1}{8}\,be^{a}\Gamma(-1,-bx)+\frac{3}{8}\,be^{(3\,a)}\Gamma(-1,-3\,bx)$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-3/8*b*e^(-3*a)*gamma(-1, 3*b*x) + 1/8*b*e^(-a)*gamma(-1, b*x) - 1/8*b*e^a
*gamma(-1, -b*x) + 3/8*b*e^(3*a)*gamma(-1, -3*b*x)
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \frac{3\,bx\mathrm{Ei}(3\,bx)\,e^{(3\,a)}+bx\mathrm{Ei}(-bx)\,e^{(-a)}-3\,bx\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}-bx\mathrm{Ei}(bx)\,e^{a}-e^{(3\,bx+3\,a)}+e^{(bx+a)}+e^{(-bx-a)}}{8\,x}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
1/8*(3*b*x*Ei(3*b*x)*e^(3*a) + b*x*Ei(-b*x)*e^(-a) - 3*b*x*Ei(-3*b*x)*e^(-
3*a) - b*x*Ei(b*x)*e^a - e^(3*b*x + 3*a) + e^(b*x + a) + e^(-b*x - a) - e^
(-3*b*x - 3*a))/x
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{x^2}\,dx$$

input
```
int((cosh(a + b*x)*sinh(a + b*x)^2)/x^2,x)
```

output
```
int((cosh(a + b*x)*sinh(a + b*x)^2)/x^2, x)
```

3.287 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx$

3.287.1 Optimal result

Integrand size = 18, antiderivative size = 119

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx &= \frac{\cosh(a+bx)}{8x^2}-\frac{\cosh(3a+3bx)}{8x^2}\\ &\quad-\frac{1}{8}b^2\cosh(a)\text{Chi}(bx)+\frac{9}{8}b^2\cosh(3a)\text{Chi}(3bx)\\ &\quad+\frac{b\sinh(a+bx)}{8x}-\frac{3b\sinh(3a+3bx)}{8x}\\ &\quad-\frac{1}{8}b^2\sinh(a)\text{Shi}(bx)+\frac{9}{8}b^2\sinh(3a)\text{Shi}(3bx)\end{aligned}$$

output
```
-1/8*b^2*Chi(b*x)*cosh(a)+9/8*b^2*Chi(3*b*x)*cosh(3*a)+1/8*cosh(b*x+a)/x^2
-1/8*cosh(3*b*x+3*a)/x^2-1/8*b^2*Shi(b*x)*sinh(a)+9/8*b^2*Shi(3*b*x)*sinh(
3*a)+1/8*b*sinh(b*x+a)/x-3/8*b*sinh(3*b*x+3*a)/x
```

3.287.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx$$
$$= \frac{\cosh(a+bx)-\cosh(3(a+bx))-b^2x^2\cosh(a)\text{Chi}(bx)+9b^2x^2\cosh(3a)\text{Chi}(3bx)+bx\sinh(a+bx)-3b}{8x^2}$$

input
```
Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^3,x]
```

output
```
(Cosh[a + b*x] - Cosh[3*(a + b*x)] - b^2*x^2*Cosh[a]*CoshIntegral[b*x] + 9
*b^2*x^2*Cosh[3*a]*CoshIntegral[3*b*x] + b*x*Sinh[a + b*x] - 3*b*x*Sinh[3*
(a + b*x)] - b^2*x^2*Sinh[a]*SinhIntegral[b*x] + 9*b^2*x^2*Sinh[3*a]*SinhI
ntegral[3*b*x])/(8*x^2)
```

3.287.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\cosh(a+bx)}{x^3}\,dx$$

$$\Big\downarrow \text{ 5971}$$

$$\int \left(\frac{\cosh(3a+3bx)}{4x^3}-\frac{\cosh(a+bx)}{4x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{8}b^2\cosh(a)\text{Chi}(bx)+\frac{9}{8}b^2\cosh(3a)\text{Chi}(3bx)-\frac{1}{8}b^2\sinh(a)\text{Shi}(bx)+\frac{9}{8}b^2\sinh(3a)\text{Shi}(3bx)+\frac{\cosh(a+bx)}{8x^2}-\frac{\cosh(3a+3bx)}{8x^2}+\frac{b\sinh(a+bx)}{8x}-\frac{3b\sinh(3a+3bx)}{8x}$$

input
```
Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^3,x]
```

output
```
Cosh[a + b*x]/(8*x^2) - Cosh[3*a + 3*b*x]/(8*x^2) - (b^2*Cosh[a]*CoshInteg
ral[b*x])/8 + (9*b^2*Cosh[3*a]*CoshIntegral[3*b*x])/8 + (b*Sinh[a + b*x])/
(8*x) - (3*b*Sinh[3*a + 3*b*x])/(8*x) - (b^2*Sinh[a]*SinhIntegral[b*x])/8
+ (9*b^2*Sinh[3*a]*SinhIntegral[3*b*x])/8
```

3.287.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.287.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.34

method	result
risch	$\frac{-9\,e^{3a}\operatorname{Ei}_1(-3bx)x^2b^2-9\,e^{-3a}\operatorname{Ei}_1(3bx)x^2b^2+e^{-a}\operatorname{Ei}_1(bx)x^2b^2+e^{a}\operatorname{Ei}_1(-bx)x^2b^2+e^{bx+a}bx-3\,e^{3bx+3a}bx+3\,e^{-3bx-3a}bx-e^{-bx-a}bx}{16x^2}$

input
```
int(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-9*exp(3*a)*Ei(1,-3*b*x)*x^2*b^2-9*exp(-3*a)*Ei(1,3*b*x)*x^2*b^2+exp
(-a)*Ei(1,b*x)*x^2*b^2+exp(a)*Ei(1,-b*x)*x^2*b^2+exp(b*x+a)*b*x-3*exp(3*b*
x+3*a)*b*x+3*exp(-3*b*x-3*a)*b*x-exp(-b*x-a)*b*x+exp(b*x+a)-exp(3*b*x+3*a)
-exp(-3*b*x-3*a)+exp(-b*x-a))/x^2
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.64

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = -\frac{6\,bx\sinh\left(bx+a\right)^3+2\,\cosh\left(bx+a\right)^3+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-9\left(b^2x^2\operatorname{Ei}(3\,bx)+b^2x^2\operatorname{Ei}(-3\,b\right.}{}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
-1/16*(6*b*x*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*
x + a)^2 - 9*(b^2*x^2*Ei(3*b*x) + b^2*x^2*Ei(-3*b*x))*cosh(3*a) + (b^2*x^2
*Ei(b*x) + b^2*x^2*Ei(-b*x))*cosh(a) + 2*(9*b*x*cosh(b*x + a)^2 - b*x)*sin
h(b*x + a) - 9*(b^2*x^2*Ei(3*b*x) - b^2*x^2*Ei(-3*b*x))*sinh(3*a) + (b^2*x
^2*Ei(b*x) - b^2*x^2*Ei(-b*x))*sinh(a) - 2*cosh(b*x + a))/x^2
```

3.287.6 Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \int \frac{\sinh^2(a+bx)\cosh(a+bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)**2/x**3,x)
```

output
```
Integral(sinh(a + b*x)**2*cosh(a + b*x)/x**3, x)
```

3.287.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.49

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = -\frac{9}{8}\,b^2e^{(-3\,a)}\Gamma(-2,3\,bx) + \frac{1}{8}\,b^2e^{(-a)}\Gamma(-2,bx) + \frac{1}{8}\,b^2e^{a}\Gamma(-2,-bx) - \frac{9}{8}\,b^2e^{(3\,a)}\Gamma(-2,-3\,bx)$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
-9/8*b^2*e^(-3*a)*gamma(-2, 3*b*x) + 1/8*b^2*e^(-a)*gamma(-2, b*x) + 1/8*b
^2*e^a*gamma(-2, -b*x) - 9/8*b^2*e^(3*a)*gamma(-2, -3*b*x)
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \frac{9\,b^2x^2\text{Ei}(3\,bx)\,e^{(3\,a)} - b^2x^2\text{Ei}(-bx)\,e^{(-a)} + 9\,b^2x^2\text{Ei}(-3\,bx)\,e^{(-3\,a)} - b^2x^2\text{Ei}(bx)\,e^{a} - 3\,bxe^{(3\,bx+3\,a)} + bxe^{(bx}}{16\,x^2}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
1/16*(9*b^2*x^2*Ei(3*b*x)*e^(3*a) - b^2*x^2*Ei(-b*x)*e^(-a) + 9*b^2*x^2*Ei
(-3*b*x)*e^(-3*a) - b^2*x^2*Ei(b*x)*e^a - 3*b*x*e^(3*b*x + 3*a) + b*x*e^(b
*x + a) - b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) + e^
(b*x + a) + e^(-b*x - a) - e^(-3*b*x - 3*a))/x^2
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{x^3}\,dx$$

input
```
int((cosh(a + b*x)*sinh(a + b*x)^2)/x^3,x)
```

output
```
int((cosh(a + b*x)*sinh(a + b*x)^2)/x^3, x)
```

3.288 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx$

3.288.1 Optimal result

Integrand size = 18, antiderivative size = 154

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \frac{\cosh(a+bx)}{12x^3}+\frac{b^2\cosh(a+bx)}{24x} - \frac{\cosh(3a+3bx)}{12x^3}-\frac{3b^2\cosh(3a+3bx)}{8x} - \frac{1}{24}b^3\text{Chi}(bx)\sinh(a)+\frac{9}{8}b^3\text{Chi}(3bx)\sinh(3a) + \frac{b\sinh(a+bx)}{24x^2}-\frac{b\sinh(3a+3bx)}{8x^2} - \frac{1}{24}b^3\cosh(a)\text{Shi}(bx)+\frac{9}{8}b^3\cosh(3a)\text{Shi}(3bx)$$

output
```
1/12*cosh(b*x+a)/x^3+1/24*b^2*cosh(b*x+a)/x-1/12*cosh(3*b*x+3*a)/x^3-3/8*b
^2*cosh(3*b*x+3*a)/x-1/24*b^3*cosh(a)*Shi(b*x)+9/8*b^3*cosh(3*a)*Shi(3*b*x
)-1/24*b^3*Chi(b*x)*sinh(a)+9/8*b^3*Chi(3*b*x)*sinh(3*a)+1/24*b*sinh(b*x+a
)/x^2-1/8*b*sinh(3*b*x+3*a)/x^2
```

3.288.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx$$
$$= \frac{2\cosh(a+bx)+b^2x^2\cosh(a+bx)-2\cosh(3(a+bx))-9b^2x^2\cosh(3(a+bx))-b^3x^3\text{Chi}(bx)\sinh(a)+}{}$$

input
```
Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^4,x]
```

output
```
(2*Cosh[a + b*x] + b^2*x^2*Cosh[a + b*x] - 2*Cosh[3*(a + b*x)] - 9*b^2*x^2
*Cosh[3*(a + b*x)] - b^3*x^3*CoshIntegral[b*x]*Sinh[a] + 27*b^3*x^3*CoshIn
tegral[3*b*x]*Sinh[3*a] + b*x*Sinh[a + b*x] - 3*b*x*Sinh[3*(a + b*x)] - b^
3*x^3*Cosh[a]*SinhIntegral[b*x] + 27*b^3*x^3*Cosh[3*a]*SinhIntegral[3*b*x]
)/(24*x^3)
```

3.288.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\cosh(a+bx)}{x^4}\,dx$$
$$\Big\downarrow \text{ 5971}$$
$$\int \left(\frac{\cosh(3a+3bx)}{4x^4}-\frac{\cosh(a+bx)}{4x^4}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{1}{24}b^3\sinh(a)\text{Chi}(bx)+\frac{9}{8}b^3\sinh(3a)\text{Chi}(3bx)-\frac{1}{24}b^3\cosh(a)\text{Shi}(bx)+\frac{9}{8}b^3\cosh(3a)\text{Shi}(3bx)+\frac{b^2\cosh(a+bx)}{24x}-\frac{3b^2\cosh(3a+3bx)}{8x}+\frac{\cosh(a+bx)}{12x^3}-\frac{\cosh(3a+3bx)}{12x^3}+\frac{b\sinh(a+bx)}{24x^2}-\frac{b\sinh(3a+3bx)}{8x^2}$$

input
```
Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^4,x]
```

output
```
Cosh[a + b*x]/(12*x^3) + (b^2*Cosh[a + b*x])/(24*x) - Cosh[3*a + 3*b*x]/(1
2*x^3) - (3*b^2*Cosh[3*a + 3*b*x])/(8*x) - (b^3*CoshIntegral[b*x]*Sinh[a])
/24 + (9*b^3*CoshIntegral[3*b*x]*Sinh[3*a])/8 + (b*Sinh[a + b*x])/(24*x^2)
 - (b*Sinh[3*a + 3*b*x])/(8*x^2) - (b^3*Cosh[a]*SinhIntegral[b*x])/24 + (9
*b^3*Cosh[3*a]*SinhIntegral[3*b*x])/8
```

3.288.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.288.4 Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.49

method	result
risch	$-\frac{-27\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^3b^3+\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^3b^3-\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^3b^3+27\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^3b^3+9\,\mathrm{e}^{-3bx-3a}b^2x^2-\mathrm{e}^{-bx-a}b^2x^2-\mathrm{e}^{bx+a}b^2x^2+}{48x^3}$

input
```
int(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-27*exp(-3*a)*Ei(1,3*b*x)*x^3*b^3+exp(-a)*Ei(1,b*x)*x^3*b^3-exp(a)*
Ei(1,-b*x)*x^3*b^3+27*exp(3*a)*Ei(1,-3*b*x)*x^3*b^3+9*exp(-3*b*x-3*a)*b^2*
x^2-exp(-b*x-a)*b^2*x^2-exp(b*x+a)*b^2*x^2+9*exp(3*b*x+3*a)*b^2*x^2-3*exp(
-3*b*x-3*a)*b*x+exp(-b*x-a)*b*x-exp(b*x+a)*b*x+3*exp(3*b*x+3*a)*b*x+2*exp(
-3*b*x-3*a)-2*exp(-b*x-a)-2*exp(b*x+a)+2*exp(3*b*x+3*a))/x^3
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.45

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = -\frac{6\,bx\sinh\left(bx+a\right)^3+2\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)^3+6\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-2\left(b^2x^2\right.}{}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

output
```
-1/48*(6*b*x*sinh(b*x + a)^3 + 2*(9*b^2*x^2 + 2)*cosh(b*x + a)^3 + 6*(9*b^
2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^2 - 2*(b^2*x^2 + 2)*cosh(b*x + a) -
 27*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*cosh(3*a) + (b^3*x^3*Ei(b*x)
- b^3*x^3*Ei(-b*x))*cosh(a) + 2*(9*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a
) - 27*(b^3*x^3*Ei(3*b*x) + b^3*x^3*Ei(-3*b*x))*sinh(3*a) + (b^3*x^3*Ei(b*
x) + b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

3.288.6 Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \frac{\sinh^2\left(a+bx\right)\cosh\left(a+bx\right)}{x^4}\,dx$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)**2/x**4,x)
```

output
```
Integral(sinh(a + b*x)**2*cosh(a + b*x)/x**4, x)
```

3.288.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.38

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = -\frac{27}{8}\,b^3e^{(-3\,a)}\Gamma(-3,3\,bx)+\frac{1}{8}\,b^3e^{(-a)}\Gamma(-3,bx) - \frac{1}{8}\,b^3e^{a}\Gamma(-3,-bx)+\frac{27}{8}\,b^3e^{(3\,a)}\Gamma(-3,-3\,bx)$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

output
```
-27/8*b^3*e^(-3*a)*gamma(-3, 3*b*x) + 1/8*b^3*e^(-a)*gamma(-3, b*x) - 1/8*
b^3*e^a*gamma(-3, -b*x) + 27/8*b^3*e^(3*a)*gamma(-3, -3*b*x)
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.44

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \frac{27\,b^3x^3\text{Ei}(3\,bx)\,e^{(3\,a)} + b^3x^3\text{Ei}(-bx)\,e^{(-a)} - 27\,b^3x^3\text{Ei}(-3\,bx)\,e^{(-3\,a)} - b^3x^3\text{Ei}(bx)\,e^{a} - 9\,b^2x^2e^{(3\,bx+3\,a)} + b^2}{}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x, algorithm="giac")
```

output
```
1/48*(27*b^3*x^3*Ei(3*b*x)*e^(3*a) + b^3*x^3*Ei(-b*x)*e^(-a) - 27*b^3*x^3*
Ei(-3*b*x)*e^(-3*a) - b^3*x^3*Ei(b*x)*e^a - 9*b^2*x^2*e^(3*b*x + 3*a) + b^
2*x^2*e^(b*x + a) + b^2*x^2*e^(-b*x - a) - 9*b^2*x^2*e^(-3*b*x - 3*a) - 3*
b*x*e^(3*b*x + 3*a) + b*x*e^(b*x + a) - b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x
 - 3*a) - 2*e^(3*b*x + 3*a) + 2*e^(b*x + a) + 2*e^(-b*x - a) - 2*e^(-3*b*x
 - 3*a))/x^3
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{x^4}\,dx$$

input
```
int((cosh(a + b*x)*sinh(a + b*x)^2)/x^4,x)
```

output
```
int((cosh(a + b*x)*sinh(a + b*x)^2)/x^4, x)
```

3.289 $\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

3.289.1 Optimal result

Integrand size = 20, antiderivative size = 85

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{x^{1+m}}{8(1+m)} + \frac{2^{-2(3+m)}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b} - \frac{2^{-2(3+m)}e^{-4a}x^m(bx)^{-m}\Gamma(1+m,4bx)}{b}$$

output
```
-1/8*x^(1+m)/(1+m)+exp(4*a)*x^m*GAMMA(1+m,-4*b*x)/(2^(6+2*m))/b/((-b*x)^m)
-x^m*GAMMA(1+m,4*b*x)/(2^(6+2*m))/b/exp(4*a)/((b*x)^m)
```

3.289.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.25

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{2^{-2-2(2+m)}e^{-4a}x^m(-b^2x^2)^{-m}\left(2^{3+2m}be^{4a}x(-b^2x^2)^m - e^{8a}(1+m)(bx)^m\Gamma(1+m,-4bx) + (1+m)(-bx\right.}{b(1+m)}$$

input
```
Integrate[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
-((2^(-2 - 2*(2 + m))*x^m*(2^(3 + 2*m)*b*E^(4*a)*x*(-(b^2*x^2))^m - E^(8*a
)*(1 + m)*(b*x)^m*Gamma[1 + m, -4*b*x] + (1 + m)*(-(b*x))^m*Gamma[1 + m, 4
*b*x]))/(b*E^(4*a)*(1 + m)*(-(b^2*x^2))^m))
```

3.289.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh^2(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{1}{8}x^m\cosh(4a+4bx)-\frac{x^m}{8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\Gamma(m+1,-4bx)}{b}-\frac{e^{-4a}2^{-2(m+3)}x^m(bx)^{-m}\Gamma(m+1,4bx)}{b}-\frac{x^{m+1}}{8(m+1)}$$

input
```
Int[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
-1/8*x^(1 + m)/(1 + m) + (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))
*b*(-(b*x))^m) - (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 + m))*b*E^(4*a)*(b*x)^
m)
```

3.289.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.289.4 Maple [F]

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)^2\,dx$$

input
```
int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

output
```
int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx =$$
$$-\frac{8\,bx\cosh(m\log(x)) + (m+1)\cosh(m\log(4\,b)+4\,a)\,\Gamma(m+1,4\,bx) - (m+1)\cosh(m\log(-4\,b)-4}{}$$

input
```
integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/64*(8*b*x*cosh(m*log(x)) + (m + 1)*cosh(m*log(4*b) + 4*a)*gamma(m + 1,
4*b*x) - (m + 1)*cosh(m*log(-4*b) - 4*a)*gamma(m + 1, -4*b*x) - (m + 1)*ga
mma(m + 1, 4*b*x)*sinh(m*log(4*b) + 4*a) + (m + 1)*gamma(m + 1, -4*b*x)*si
nh(m*log(-4*b) - 4*a) + 8*b*x*sinh(m*log(x)))/(b*m + b)
```

3.289.6 Sympy [F]

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \int x^m \sinh^2(a+bx)\cosh^2(a+bx)\,dx$$

input
```
integrate(x**m*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Integral(x**m*sinh(a + b*x)**2*cosh(a + b*x)**2, x)
```

3.289.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{16}\,(4\,bx)^{-m-1}\,x^{m+1}e^{(-4\,a)}\Gamma(m+1,4\,bx) - \frac{1}{16}\,(-4\,bx)^{-m-1}\,x^{m+1}e^{(4\,a)}\Gamma(m+1,-4\,bx) - \frac{x^{m+1}}{8\,(m+1)}$$

input
```
integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/16*(4*b*x)^(-m - 1)*x^(m + 1)*e^(-4*a)*gamma(m + 1, 4*b*x) - 1/16*(-4*b
*x)^(-m - 1)*x^(m + 1)*e^(4*a)*gamma(m + 1, -4*b*x) - 1/8*x^(m + 1)/(m + 1
)
```

3.289.8 Giac [F]

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^2 \sinh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^2*sinh(b*x + a)^2, x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \int x^m \cosh(a+b\,x)^2 \sinh(a+b\,x)^2\,dx$$

input
```
int(x^m*cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

output
```
int(x^m*cosh(a + b*x)^2*sinh(a + b*x)^2, x)
```

3.290 $\int x^3 \cosh^2(a+bx) \sinh^2(a+bx)\,dx$

3.290.1 Optimal result

Integrand size = 20, antiderivative size = 79

$$\int x^3 \cosh^2(a+bx) \sinh^2(a+bx)\,dx = -\frac{x^4}{32} - \frac{3\cosh(4a+4bx)}{1024b^4} - \frac{3x^2\cosh(4a+4bx)}{128b^2} + \frac{3x\sinh(4a+4bx)}{256b^3} + \frac{x^3\sinh(4a+4bx)}{32b}$$

output
```
-1/32*x^4-3/1024*cosh(4*b*x+4*a)/b^4-3/128*x^2*cosh(4*b*x+4*a)/b^2+3/256*x
*sinh(4*b*x+4*a)/b^3+1/32*x^3*sinh(4*b*x+4*a)/b
```

3.290.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int x^3 \cosh^2(a+bx) \sinh^2(a+bx)\,dx = \frac{-32b^4x^4 - 3(1+8b^2x^2)\cosh(4(a+bx)) + 4bx(3+8b^2x^2)\sinh(4(a+bx))}{1024b^4}$$

input
```
Integrate[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(-32*b^4*x^4 - 3*(1 + 8*b^2*x^2)*Cosh[4*(a + b*x)] + 4*b*x*(3 + 8*b^2*x^2)
*Sinh[4*(a + b*x)])/(1024*b^4)
```

3.290.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sinh^2(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{1}{8}x^3\cosh(4a+4bx)-\frac{x^3}{8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\cosh(4a+4bx)}{1024b^4}+\frac{3x\sinh(4a+4bx)}{256b^3}-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{x^3\sinh(4a+4bx)}{32b}-\frac{x^4}{32}$$

input
```
Int[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
-1/32*x^4 - (3*Cosh[4*a + 4*b*x])/(1024*b^4) - (3*x^2*Cosh[4*a + 4*b*x])/(
128*b^2) + (3*x*Sinh[4*a + 4*b*x])/(256*b^3) + (x^3*Sinh[4*a + 4*b*x])/(32
*b)
```

3.290.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.290.4 Maple [A] (verified)

Time = 11.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{x^4}{32}+\frac{(32x^3b^3-24x^2b^2+12bx-3)\,\mathrm{e}^{4bx+4a}}{2048b^4}-\frac{(32x^3b^3+24x^2b^2+12bx+3)\,\mathrm{e}^{-4bx-4a}}{2048b^4}$
derivativedivides	$\frac{-a^3\left(\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}-\frac{(bx+a)\cosh(bx+a)\sin}{8}\right.}{}$
default	$\frac{-a^3\left(\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}-\frac{(bx+a)\cosh(bx+a)\sin}{8}\right.}{}$

input
```
int(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32*x^4+1/2048*(32*b^3*x^3-24*b^2*x^2+12*b*x-3)/b^4*exp(4*b*x+4*a)-1/204
8*(32*b^3*x^3+24*b^2*x^2+12*b*x+3)/b^4*exp(-4*b*x-4*a)
```

3.290.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(69) = 138.

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.77

$$\int x^3\cosh^2(a+bx)\sinh^2(a+bx)\,dx =$$
$$-\frac{32\,b^4x^4+3\,(8\,b^2x^2+1)\cosh{(bx+a)}^4-16\,(8\,b^3x^3+3\,bx)\cosh{(bx+a)}^3\sinh{(bx+a)}+18\,(8\,b^2x^2+1}{1}$$

input
```
integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/1024*(32*b^4*x^4 + 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^4 - 16*(8*b^3*x^3 +
3*b*x)*cosh(b*x + a)^3*sinh(b*x + a) + 18*(8*b^2*x^2 + 1)*cosh(b*x + a)^2*
sinh(b*x + a)^2 - 16*(8*b^3*x^3 + 3*b*x)*cosh(b*x + a)*sinh(b*x + a)^3 + 3
*(8*b^2*x^2 + 1)*sinh(b*x + a)^4)/b^4
```

3.290.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(76) = 152$.

Time = 0.55 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.16

$$\int x^3\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{x^4\sinh^4(a+bx)}{32} + \frac{x^4\sinh^2(a+bx)\cosh^2(a+bx)}{16} - \frac{x^4\cosh^4(a+bx)}{32} + \frac{x^3\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{8b} - \\ \frac{x^4\sinh^2(a)\cosh^2(a)}{4} \end{cases}$$

input
```
integrate(x**3*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-x**4*sinh(a + b*x)**4/32 + x**4*sinh(a + b*x)**2*cosh(a + b*x)
**2/16 - x**4*cosh(a + b*x)**4/32 + x**3*sinh(a + b*x)**3*cosh(a + b*x)/(8
*b) + x**3*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) - 3*x**2*sinh(a + b*x)**4/
(128*b**2) - 9*x**2*sinh(a + b*x)**2*cosh(a + b*x)**2/(64*b**2) - 3*x**2*c
osh(a + b*x)**4/(128*b**2) + 3*x*sinh(a + b*x)**3*cosh(a + b*x)/(64*b**3)
+ 3*x*sinh(a + b*x)*cosh(a + b*x)**3/(64*b**3) - 3*sinh(a + b*x)**4/(256*b
**4) - 3*cosh(a + b*x)**4/(256*b**4), Ne(b, 0)), (x**4*sinh(a)**2*cosh(a)*
*2/4, True))
```

3.290.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int x^3\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{32}x^4 + \frac{\left(32\,b^3x^3e^{(4\,a)} - 24\,b^2x^2e^{(4\,a)} + 12\,bxe^{(4\,a)} - 3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4} - \frac{(32\,b^3x^3 + 24\,b^2x^2 + 12\,bx + 3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/32*x^4 + 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*
a) - 3*e^(4*a))*e^(4*b*x)/b^4 - 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x +
 3)*e^(-4*b*x - 4*a)/b^4
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int x^3 \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{32}x^4 + \frac{(32\,b^3x^3 - 24\,b^2x^2 + 12\,bx - 3)e^{(4\,bx+4\,a)}}{2048\,b^4} - \frac{(32\,b^3x^3 + 24\,b^2x^2 + 12\,bx + 3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/32*x^4 + 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/
b^4 - 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

3.290.9 Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int x^3 \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{\frac{3\cosh(4\,a+4\,b\,x)}{1024} - \frac{3\,b\,x\sinh(4\,a+4\,b\,x)}{256} + \frac{3\,b^2\,x^2\cosh(4\,a+4\,b\,x)}{128} - \frac{b^3\,x^3\sinh(4\,a+4\,b\,x)}{32}}{b^4} - \frac{x^4}{32}$$

input
```
int(x^3*cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

output
```
- ((3*cosh(4*a + 4*b*x))/1024 - (3*b*x*sinh(4*a + 4*b*x))/256 + (3*b^2*x^2
*cosh(4*a + 4*b*x))/128 - (b^3*x^3*sinh(4*a + 4*b*x))/32)/b^4 - x^4/32
```

3.291 $\int x^2 \cosh^2(a+bx) \sinh^2(a+bx)\,dx$

3.291.1 Optimal result

Integrand size = 20, antiderivative size = 60

$$\int x^2 \cosh^2(a+bx) \sinh^2(a+bx)\,dx = -\frac{x^3}{24} - \frac{x\cosh(4a+4bx)}{64b^2} + \frac{\sinh(4a+4bx)}{256b^3} + \frac{x^2\sinh(4a+4bx)}{32b}$$

output
```
-1/24*x^3-1/64*x*cosh(4*b*x+4*a)/b^2+1/256*sinh(4*b*x+4*a)/b^3+1/32*x^2*si
nh(4*b*x+4*a)/b
```

3.291.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int x^2 \cosh^2(a+bx) \sinh^2(a+bx)\,dx = \frac{-32b^3x^3 - 12bx\cosh(4(a+bx)) + 3(1+8b^2x^2)\sinh(4(a+bx))}{768b^3}$$

input
```
Integrate[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(-32*b^3*x^3 - 12*b*x*Cosh[4*(a + b*x)] + 3*(1 + 8*b^2*x^2)*Sinh[4*(a + b*
x)])/(768*b^3)
```

3.291.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sinh^2(a+bx) \cosh^2(a+bx) \, dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{1}{8}x^2 \cosh(4a+4bx) - \frac{x^2}{8}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(4a+4bx)}{256b^3} - \frac{x\cosh(4a+4bx)}{64b^2} + \frac{x^2\sinh(4a+4bx)}{32b} - \frac{x^3}{24}$$

input
```
Int[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
-1/24*x^3 - (x*Cosh[4*a + 4*b*x])/(64*b^2) + Sinh[4*a + 4*b*x]/(256*b^3) +
 (x^2*Sinh[4*a + 4*b*x])/(32*b)
```

3.291.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.291.4 Maple [A] (verified)

Time = 7.81 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{x^3}{24}+\frac{(8x^2b^2-4bx+1)\,\mathrm{e}^{4bx+4a}}{512b^3}-\frac{(8x^2b^2+4bx+1)\,\mathrm{e}^{-4bx-4a}}{512b^3}$
derivativedivides	$\frac{a^2\left(\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}-\frac{(bx+a)\cosh(bx+a)\sinh(b}{8}\right.}{}$
default	$\frac{a^2\left(\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}-\frac{(bx+a)\cosh(bx+a)\sinh(b}{8}\right.}{}$

input
```
int(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24*x^3+1/512*(8*b^2*x^2-4*b*x+1)/b^3*exp(4*b*x+4*a)-1/512*(8*b^2*x^2+4*
b*x+1)/b^3*exp(-4*b*x-4*a)
```

3.291.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(52) = 104.

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.83

$$\int x^2\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{8\,b^3x^3+3\,bx\cosh(bx+a)^4+18\,bx\cosh(bx+a)^2\sinh(bx+a)^2+3\,bx\sinh(bx+a)^4-3\,(8\,b^2x^2+1)\,c}{192\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/192*(8*b^3*x^3 + 3*b*x*cosh(b*x + a)^4 + 18*b*x*cosh(b*x + a)^2*sinh(b*
x + a)^2 + 3*b*x*sinh(b*x + a)^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^3*sinh(
b*x + a) - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3)/b^3
```

3.291.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(53) = 106$.

Time = 0.44 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.40

$$\int x^2 \cosh^2(a+bx)\sinh^2(a+bx)\,dx$$
$$= \begin{cases} -\frac{x^3\sinh^4(a+bx)}{24} + \frac{x^3\sinh^2(a+bx)\cosh^2(a+bx)}{12} - \frac{x^3\cosh^4(a+bx)}{24} + \frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{8b} - \\ \frac{x^3\sinh^2(a)\cosh^2(a)}{3} \end{cases}$$

input
```
integrate(x**2*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-x**3*sinh(a + b*x)**4/24 + x**3*sinh(a + b*x)**2*cosh(a + b*x)
**2/12 - x**3*cosh(a + b*x)**4/24 + x**2*sinh(a + b*x)**3*cosh(a + b*x)/(8
*b) + x**2*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) - x*sinh(a + b*x)**4/(64*b
**2) - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/(32*b**2) - x*cosh(a + b*x)**
4/(64*b**2) + sinh(a + b*x)**3*cosh(a + b*x)/(64*b**3) + sinh(a + b*x)*cos
h(a + b*x)**3/(64*b**3), Ne(b, 0)), (x**3*sinh(a)**2*cosh(a)**2/3, True))
```

3.291.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15

$$\int x^2\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{24}x^3 + \frac{\left(8b^2x^2e^{(4a)} - 4bxe^{(4a)} + e^{(4a)}\right)e^{(4bx)}}{512b^3} - \frac{(8b^2x^2 + 4bx + 1)e^{(-4bx-4a)}}{512b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/24*x^3 + 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/
b^3 - 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int x^2\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{24}\,x^3+\frac{\left(8\,b^2x^2-4\,bx+1\right)e^{\left(4\,bx+4\,a\right)}}{512\,b^3}-\frac{\left(8\,b^2x^2+4\,bx+1\right)e^{\left(-4\,bx-4\,a\right)}}{512\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/24*x^3 + 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 - 1/512*(8*b
^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

3.291.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x^2\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{\sinh(4\,a+4\,b\,x)}{256}+\frac{b^2\,x^2\,\sinh(4\,a+4\,b\,x)}{32}-\frac{b\,x\,\cosh(4\,a+4\,b\,x)}{64}}{b^3}-\frac{x^3}{24}$$

input
```
int(x^2*cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

output
```
(sinh(4*a + 4*b*x)/256 + (b^2*x^2*sinh(4*a + 4*b*x))/32 - (b*x*cosh(4*a +
4*b*x))/64)/b^3 - x^3/24
```

3.292 $\int x \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

3.292.1 Optimal result

Integrand size = 18, antiderivative size = 41

$$\int x \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{x^2}{16} - \frac{\cosh(4a+4bx)}{128b^2} + \frac{x\sinh(4a+4bx)}{32b}$$

output
```
-1/16*x^2-1/128*cosh(4*b*x+4*a)/b^2+1/32*x*sinh(4*b*x+4*a)/b
```

3.292.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int x \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{-8a^2+8b^2x^2+\cosh(4(a+bx))-4bx\sinh(4(a+bx))}{128b^2}$$

input
```
Integrate[x*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
-1/128*(-8*a^2 + 8*b^2*x^2 + Cosh[4*(a + b*x)] - 4*b*x*Sinh[4*(a + b*x)])/
b^2
```

3.292.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh^2(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \ 5971$$

$$\int \left(\frac{1}{8}x\cosh(4a+4bx)-\frac{x}{8}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\cosh(4a+4bx)}{128b^2}+\frac{x\sinh(4a+4bx)}{32b}-\frac{x^2}{16}$$

input
```
Int[x*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
-1/16*x^2 - Cosh[4*a + 4*b*x]/(128*b^2) + (x*Sinh[4*a + 4*b*x])/(32*b)
```

3.292.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.292.4 Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{x^2}{16}+\frac{(4bx-1)\,e^{4bx+4a}}{256b^2}-\frac{(4bx+1)\,e^{-4bx-4a}}{256b^2}$
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}-\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{8}-\frac{(bx+a)^2}{16}-\frac{\cosh(bx+a)^4}{16}+\frac{\cosh(bx+a)^2}{16}-a\left(\frac{\cosh(bx+a)^3\sinh(b}{4}\right.}{b^2}$
default	$\frac{\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}-\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{8}-\frac{(bx+a)^2}{16}-\frac{\cosh(bx+a)^4}{16}+\frac{\cosh(bx+a)^2}{16}-a\left(\frac{\cosh(bx+a)^3\sinh(b}{4}\right.}{b^2}$

input
```
int(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*x^2+1/256*(4*b*x-1)/b^2*exp(4*b*x+4*a)-1/256*(4*b*x+1)/b^2*exp(-4*b*
x-4*a)
```

3.292.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(35) = 70.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.15

$$\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{16\,bx\cosh(bx+a)^3\sinh(bx+a)+16\,bx\cosh(bx+a)\sinh(bx+a)^3-8\,b^2x^2-\cosh(bx+a)^4-6\,\cosh(}{128\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/128*(16*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 16*b*x*cosh(b*x + a)*sinh(b*
x + a)^3 - 8*b^2*x^2 - cosh(b*x + a)^4 - 6*cosh(b*x + a)^2*sinh(b*x + a)^2
 - sinh(b*x + a)^4)/b^2
```

3.292.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(34) = 68$.

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.20

$$\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx$$
$$= \begin{cases} -\frac{x^2\sinh^4(a+bx)}{16} + \frac{x^2\sinh^2(a+bx)\cosh^2(a+bx)}{8} - \frac{x^2\cosh^4(a+bx)}{16} + \frac{x\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{x\sinh(a+bx)\cosh^3(a+bx)}{8b} - \\ \frac{x^2\sinh^2(a)\cosh^2(a)}{2} \end{cases}$$

input
```
integrate(x*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-x**2*sinh(a + b*x)**4/16 + x**2*sinh(a + b*x)**2*cosh(a + b*x)
**2/8 - x**2*cosh(a + b*x)**4/16 + x*sinh(a + b*x)**3*cosh(a + b*x)/(8*b)
+ x*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) - sinh(a + b*x)**4/(32*b**2) - co
sh(a + b*x)**4/(32*b**2), Ne(b, 0)), (x**2*sinh(a)**2*cosh(a)**2/2, True))
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{16}x^2 + \frac{\left(4\,bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2} - \frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/16*x^2 + 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 - 1/256*(4*b*x +
 1)*e^(-4*b*x - 4*a)/b^2
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx=-\frac{1}{16}\,x^2+\frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2}-\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/16*x^2 + 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 - 1/256*(4*b*x + 1)*e^(-
4*b*x - 4*a)/b^2
```

3.292.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx=-\frac{\frac{\cosh(4\,a+4\,b\,x)}{128}-\frac{b\,x\sinh(4\,a+4\,b\,x)}{32}}{b^2}-\frac{x^2}{16}$$

input
```
int(x*cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

output
```
- (cosh(4*a + 4*b*x)/128 - (b*x*sinh(4*a + 4*b*x))/32)/b^2 - x^2/16
```

3.293 $\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

3.293.1 Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{x}{8} - \frac{\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}$$

output
```
-1/8*x-1/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)^3*sinh(b*x+a)/b
```

3.293.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{-4(a+bx)+\sinh(4(a+bx))}{32b}$$

input
```
Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(-4*(a + b*x) + Sinh[4*(a + b*x)])/(32*b)
```

3.293.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 25, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^2\left(-\cos(ia+ibx)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(ia+ibx)^2\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{1}{4}\int \cosh^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{1}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}\left(-\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{1}{4}\left(-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}\right)$$

input
```
Int[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (-1/2*x - (Cosh[a + b*x]*Sinh[a +
b*x])/(2*b))/4
```

3.293.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.293.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{x}{8}+\frac{{\mathrm e}^{4 b x +4 a}}{64 b}-\frac{{\mathrm e}^{-4 b x -4 a}}{64 b}$	33
derivativedivides	$\frac{\frac{\cosh \left(b x +a \right)^{3} \sinh \left(b x +a \right)}{4}-\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{8}-\frac{b x}{8}-\frac{a}{8}}{b}$	43
default	$\frac{\frac{\cosh \left(b x +a \right)^{3} \sinh \left(b x +a \right)}{4}-\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{8}-\frac{b x}{8}-\frac{a}{8}}{b}$	43

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*x+1/64*exp(4*b*x+4*a)/b-1/64*exp(-4*b*x-4*a)/b
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\cosh(bx+a)^3\sinh(bx+a)+\cosh(bx+a)\sinh(bx+a)^3-bx}{8\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*x + a)^3*sinh(b*x + a) + cosh(b*x + a)*sinh(b*x + a)^3 - b*x)/
b
```

3.293.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(37) = 74$.

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{x\sinh^4(a+bx)}{8}+\frac{x\sinh^2(a+bx)\cosh^2(a+bx)}{4}-\frac{x\cosh^4(a+bx)}{8}+\frac{\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for } b \neq \\ x\sinh^2(a)\cosh^2(a) & \text{otherw} \end{cases}$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-x*sinh(a + b*x)**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 -
 x*cosh(a + b*x)**4/8 + sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*
x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**2, True))
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{bx+a}{8\,b} + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(b*x + a)/b + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

3.293.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(4\,a+4\,b\,x)}{32\,b} - \frac{x}{8}$$

input
```
int(cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

output
```
sinh(4*a + 4*b*x)/(32*b) - x/8
```

3.294 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx$

3.294.1 Optimal result

Integrand size = 20, antiderivative size = 33

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{8}\cosh(4a)\text{Chi}(4bx) - \frac{\log(x)}{8} + \frac{1}{8}\sinh(4a)\text{Shi}(4bx)$$

output
```
1/8*Chi(4*b*x)*cosh(4*a)-1/8*ln(x)+1/8*Shi(4*b*x)*sinh(4*a)
```

3.294.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{8}(\cosh(4a)\text{Chi}(4bx) - \log(2bx) + \sinh(4a)\text{Shi}(4bx))$$

input
```
Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x,x]
```

output
```
(Cosh[4*a]*CoshIntegral[4*b*x] - Log[2*b*x] + Sinh[4*a]*SinhIntegral[4*b*x
])/8
```

3.294.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{\cosh(4a+4bx)}{8x}-\frac{1}{8x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\cosh(4a)\text{Chi}(4bx)+\frac{1}{8}\sinh(4a)\text{Shi}(4bx)-\frac{\log(x)}{8}$$

input
```
Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x,x]
```

output
```
(Cosh[4*a]*CoshIntegral[4*b*x])/8 - Log[x]/8 + (Sinh[4*a]*SinhIntegral[4*b
*x])/8
```

3.294.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.294.4 Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{\ln(x)}{8}-\frac{e^{-4a}\operatorname{Ei}_1(4bx)}{16}-\frac{e^{4a}\operatorname{Ei}_1(-4bx)}{16}$	30

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(x)-1/16*exp(-4*a)*Ei(1,4*b*x)-1/16*exp(4*a)*Ei(1,-4*b*x)
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{16}\left(\operatorname{Ei}(4\,bx)+\operatorname{Ei}(-4\,bx)\right)\cosh\left(4\,a\right) + \frac{1}{16}\left(\operatorname{Ei}(4\,bx)-\operatorname{Ei}(-4\,bx)\right)\sinh\left(4\,a\right)-\frac{1}{8}\log\left(x\right)$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
1/16*(Ei(4*b*x) + Ei(-4*b*x))*cosh(4*a) + 1/16*(Ei(4*b*x) - Ei(-4*b*x))*si
nh(4*a) - 1/8*log(x)
```

3.294.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \frac{\sinh^2\left(a+bx\right)\cosh^2\left(a+bx\right)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x,x)
```

output
```
Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x, x)
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + \frac{1}{16}\,\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - \frac{1}{8}\log(x)$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
1/16*Ei(4*b*x)*e^(4*a) + 1/16*Ei(-4*b*x)*e^(-4*a) - 1/8*log(x)
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + \frac{1}{16}\,\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - \frac{1}{8}\log(x)$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="giac")
```

output
```
1/16*Ei(4*b*x)*e^(4*a) + 1/16*Ei(-4*b*x)*e^(-4*a) - 1/8*log(x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^2\sinh(a+b\,x)^2}{x}\,dx$$

input
```
int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x,x)
```

output
```
int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x, x)
```

3.295 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx$

3.295.1 Optimal result

Integrand size = 20, antiderivative size = 52

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \frac{1}{8x}-\frac{\cosh(4a+4bx)}{8x}+\frac{1}{2}b\text{Chi}(4bx)\sinh(4a)+\frac{1}{2}b\cosh(4a)\text{Shi}(4bx)$$

output
```
1/8/x-1/8*cosh(4*b*x+4*a)/x+1/2*b*cosh(4*a)*Shi(4*b*x)+1/2*b*Chi(4*b*x)*si
nh(4*a)
```

3.295.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \frac{1-\cosh(4(a+bx))+4bx\text{Chi}(4bx)\sinh(4a)+4bx\cosh(4a)\text{Shi}(4bx)}{8x}$$

input
```
Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^2,x]
```

output
```
(1 - Cosh[4*(a + b*x)] + 4*b*x*CoshIntegral[4*b*x]*Sinh[4*a] + 4*b*x*Cosh[
4*a]*SinhIntegral[4*b*x])/(8*x)
```

3.295.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{\cosh(4a+4bx)}{8x^2}-\frac{1}{8x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}b\sinh(4a)\text{Chi}(4bx)+\frac{1}{2}b\cosh(4a)\text{Shi}(4bx)-\frac{\cosh(4a+4bx)}{8x}+\frac{1}{8x}$$

input
```
Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^2,x]
```

output
```
1/(8*x) - Cosh[4*a + 4*b*x]/(8*x) + (b*CoshIntegral[4*b*x]*Sinh[4*a])/2 +
(b*Cosh[4*a]*SinhIntegral[4*b*x])/2
```

3.295.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.295.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{-4\,{\mathrm e}^{-4 a} \operatorname{Ei}_{1}\left(4 b x \right) b x +4\,{\mathrm e}^{4 a} \operatorname{Ei}_{1}\left(-4 b x \right) b x +{\mathrm e}^{-4 b x -4 a}+{\mathrm e}^{4 b x +4 a}-2}{16 x}$	54

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-4*exp(-4*a)*Ei(1,4*b*x)*b*x+4*exp(4*a)*Ei(1,-4*b*x)*b*x+exp(-4*b*x
-4*a)+exp(4*b*x+4*a)-2)/x
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.69

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{\cosh\left(bx+a\right)^{4}+6\,\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{4}-2\left(bx\mathrm{Ei}(4\,bx)-bx\mathrm{Ei}(-4\,bx)\right)\cosh(}{8\,x}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
-1/8*(cosh(b*x + a)^4 + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + sinh(b*x + a)^
4 - 2*(b*x*Ei(4*b*x) - b*x*Ei(-4*b*x))*cosh(4*a) - 2*(b*x*Ei(4*b*x) + b*x*
Ei(-4*b*x))*sinh(4*a) - 1)/x
```

3.295.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x**2,x)
```

output
```
Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x**2, x)
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{1}{4}\,be^{(-4\,a)}\Gamma(-1,4\,bx) + \frac{1}{4}\,be^{(4\,a)}\Gamma(-1,-4\,bx) + \frac{1}{8\,x}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-1/4*b*e^(-4*a)*gamma(-1, 4*b*x) + 1/4*b*e^(4*a)*gamma(-1, -4*b*x) + 1/8/x
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \frac{4\,bx\text{Ei}(4\,bx)\,e^{(4\,a)} - 4\,bx\text{Ei}(-4\,bx)\,e^{(-4\,a)} - e^{(4\,bx+4\,a)} - e^{(-4\,bx-4\,a)} + 2}{16\,x}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
1/16*(4*b*x*Ei(4*b*x)*e^(4*a) - 4*b*x*Ei(-4*b*x)*e^(-4*a) - e^(4*b*x + 4*a
) - e^(-4*b*x - 4*a) + 2)/x
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^2}{x^2}\,dx$$

input
```
int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^2,x)
```

output
```
int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^2, x)
```

3.296 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx$

3.296.1 Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}+b^2\cosh(4a)\text{Chi}(4bx) - \frac{b\sinh(4a+4bx)}{4x}+b^2\sinh(4a)\text{Shi}(4bx)$$

output
```
1/16/x^2+b^2*Chi(4*b*x)*cosh(4*a)-1/16*cosh(4*b*x+4*a)/x^2+b^2*Shi(4*b*x)*
sinh(4*a)-1/4*b*sinh(4*b*x+4*a)/x
```

3.296.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \frac{1-\cosh(4(a+bx))+16b^2x^2\cosh(4a)\text{Chi}(4bx)-4bx\sinh(4(a+bx))+16b^2x^2\sinh(4a)\text{Shi}(4bx)}{16x^2}$$

input
```
Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^3,x]
```

output
```
(1 - Cosh[4*(a + b*x)] + 16*b^2*x^2*Cosh[4*a]*CoshIntegral[4*b*x] - 4*b*x*
Sinh[4*(a + b*x)] + 16*b^2*x^2*Sinh[4*a]*SinhIntegral[4*b*x])/(16*x^2)
```

3.296.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^3}\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{\cosh(4a+4bx)}{8x^3}-\frac{1}{8x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$b^2\cosh(4a)\text{Chi}(4bx)+b^2\sinh(4a)\text{Shi}(4bx)-\frac{\cosh(4a+4bx)}{16x^2}-\frac{b\sinh(4a+4bx)}{4x}+\frac{1}{16x^2}$$

input
```
Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^3,x]
```

output
```
1/(16*x^2) - Cosh[4*a + 4*b*x]/(16*x^2) + b^2*Cosh[4*a]*CoshIntegral[4*b*x
] - (b*Sinh[4*a + 4*b*x])/(4*x) + b^2*Sinh[4*a]*SinhIntegral[4*b*x]
```

3.296.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.296.4 Maple [A] (verified)

Time = 6.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

method	result	size
risch	$\frac{-16\,\mathrm{e}^{-4a}\,\mathrm{Ei}_1(4bx)x^2b^2-16\,\mathrm{e}^{4a}\,\mathrm{Ei}_1(-4bx)x^2b^2+4\,\mathrm{e}^{-4bx-4a}bx-4\,\mathrm{e}^{4bx+4a}bx-\mathrm{e}^{-4bx-4a}-\mathrm{e}^{4bx+4a}+2}{32x^2}$	92

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(-16*exp(-4*a)*Ei(1,4*b*x)*x^2*b^2-16*exp(4*a)*Ei(1,-4*b*x)*x^2*b^2+4
*exp(-4*b*x-4*a)*b*x-4*exp(4*b*x+4*a)*b*x-exp(-4*b*x-4*a)-exp(4*b*x+4*a)+2
)/x^2
```

3.296.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(61) = 122.

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.09

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx =$$
$$-\frac{16\,bx\cosh{(bx+a)}^3\sinh{(bx+a)}+16\,bx\cosh{(bx+a)}\sinh{(bx+a)}^3+\cosh{(bx+a)}^4+6\,\cosh{(bx+a}}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
-1/16*(16*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 16*b*x*cosh(b*x + a)*sinh(b*
x + a)^3 + cosh(b*x + a)^4 + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + sinh(b*x
+ a)^4 - 8*(b^2*x^2*Ei(4*b*x) + b^2*x^2*Ei(-4*b*x))*cosh(4*a) - 8*(b^2*x^2
*Ei(4*b*x) - b^2*x^2*Ei(-4*b*x))*sinh(4*a) - 1)/x^2
```

3.296.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x**3,x)
```

output
```
Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x**3, x)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.54

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = -b^2e^{(-4a)}\Gamma(-2,4bx) - b^2e^{(4a)}\Gamma(-2,-4bx) + \frac{1}{16x^2}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
-b^2*e^(-4*a)*gamma(-2, 4*b*x) - b^2*e^(4*a)*gamma(-2, -4*b*x) + 1/16/x^2
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \frac{16b^2x^2\text{Ei}(4bx)e^{(4a)} + 16b^2x^2\text{Ei}(-4bx)e^{(-4a)} - 4bxe^{(4bx+4a)} + 4bxe^{(-4bx-4a)} - e^{(4bx+4a)} - e^{(-4bx-4a)} +}{32x^2}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
1/32*(16*b^2*x^2*Ei(4*b*x)*e^(4*a) + 16*b^2*x^2*Ei(-4*b*x)*e^(-4*a) - 4*b*
x*e^(4*b*x + 4*a) + 4*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) - e^(-4*b*x -
 4*a) + 2)/x^2
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^2}{x^3}\,dx$$

input
```
int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^3,x)
```

output
```
int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^3, x)
```

3.297 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx$

3.297.1 Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b^2\cosh(4a+4bx)}{3x}+\frac{4}{3}b^3\text{Chi}(4bx)\sinh(4a)-\frac{b\sinh(4a+4bx)}{12x^2}+\frac{4}{3}b^3\cosh(4a)\text{Shi}(4bx)$$

output
```
1/24/x^3-1/24*cosh(4*b*x+4*a)/x^3-1/3*b^2*cosh(4*b*x+4*a)/x+4/3*b^3*cosh(4
*a)*Shi(4*b*x)+4/3*b^3*Chi(4*b*x)*sinh(4*a)-1/12*b*sinh(4*b*x+4*a)/x^2
```

3.297.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx = -\frac{-1+\cosh(4(a+bx))+8b^2x^2\cosh(4(a+bx))-32b^3x^3\text{Chi}(4bx)\sinh(4a)+2bx\sinh(4(a+bx))-32b}{24x^3}$$

input
```
Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^4,x]
```

output
```
-1/24*(-1 + Cosh[4*(a + b*x)] + 8*b^2*x^2*Cosh[4*(a + b*x)] - 32*b^3*x^3*C
oshIntegral[4*b*x]*Sinh[4*a] + 2*b*x*Sinh[4*(a + b*x)] - 32*b^3*x^3*Cosh[4
*a]*SinhIntegral[4*b*x])/x^3
```

3.297.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^4}\,dx$$
$$\Big\downarrow \ 5971$$
$$\int \left(\frac{\cosh(4a+4bx)}{8x^4}-\frac{1}{8x^4}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{4}{3}b^3\sinh(4a)\text{Chi}(4bx)+\frac{4}{3}b^3\cosh(4a)\text{Shi}(4bx)-\frac{b^2\cosh(4a+4bx)}{3x}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b\sinh(4a+4bx)}{12x^2}+\frac{1}{24x^3}$$

input
```
Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^4,x]
```

output
```
1/(24*x^3) - Cosh[4*a + 4*b*x]/(24*x^3) - (b^2*Cosh[4*a + 4*b*x])/(3*x) +
(4*b^3*CoshIntegral[4*b*x]*Sinh[4*a])/3 - (b*Sinh[4*a + 4*b*x])/(12*x^2) +
 (4*b^3*Cosh[4*a]*SinhIntegral[4*b*x])/3
```

3.297.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.297.4 Maple [A] (verified)

Time = 8.79 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{-32\,\mathrm{e}^{-4a}\,\mathrm{Ei}_1(4bx)x^3b^3+32\,\mathrm{e}^{4a}\,\mathrm{Ei}_1(-4bx)x^3b^3+8\,\mathrm{e}^{-4bx-4a}b^2x^2+8\,\mathrm{e}^{4bx+4a}b^2x^2-2\,\mathrm{e}^{-4bx-4a}bx+2\,\mathrm{e}^{4bx+4a}bx+\mathrm{e}^{-4bx-4a}+\mathrm{e}^{4bx+4}}{48x^3}$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-32*exp(-4*a)*Ei(1,4*b*x)*x^3*b^3+32*exp(4*a)*Ei(1,-4*b*x)*x^3*b^3+
8*exp(-4*b*x-4*a)*b^2*x^2+8*exp(4*b*x+4*a)*b^2*x^2-2*exp(-4*b*x-4*a)*b*x+2
*exp(4*b*x+4*a)*b*x+exp(-4*b*x-4*a)+exp(4*b*x+4*a)-2)/x^3
```

3.297.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(80) = 160.

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.87

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx =$$
$$-\frac{8\,bx\cosh(bx+a)^3\sinh(bx+a)+8\,bx\cosh(bx+a)\sinh(bx+a)^3+(8\,b^2x^2+1)\cosh(bx+a)^4+6\,(8}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

output
```
-1/24*(8*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 8*b*x*cosh(b*x + a)*sinh(b*x
+ a)^3 + (8*b^2*x^2 + 1)*cosh(b*x + a)^4 + 6*(8*b^2*x^2 + 1)*cosh(b*x + a)
^2*sinh(b*x + a)^2 + (8*b^2*x^2 + 1)*sinh(b*x + a)^4 - 16*(b^3*x^3*Ei(4*b*
x) - b^3*x^3*Ei(-4*b*x))*cosh(4*a) - 16*(b^3*x^3*Ei(4*b*x) + b^3*x^3*Ei(-4
*b*x))*sinh(4*a) - 1)/x^3
```

3.297.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^4}\,dx$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x**4,x)
```

output
```
Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x**4, x)
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.39

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx = -4\,b^3e^{(-4\,a)}\Gamma(-3,4\,bx) + 4\,b^3e^{(4\,a)}\Gamma(-3,-4\,bx) + \frac{1}{24\,x^3}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

output
```
-4*b^3*e^(-4*a)*gamma(-3, 4*b*x) + 4*b^3*e^(4*a)*gamma(-3, -4*b*x) + 1/24/
x^3
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.34

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx$$
$$= \frac{32\,b^3x^3\mathrm{Ei}(4\,bx)\,e^{(4\,a)} - 32\,b^3x^3\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - 8\,b^2x^2e^{(4\,bx+4\,a)} - 8\,b^2x^2e^{(-4\,bx-4\,a)} - 2\,bxe^{(4\,bx+4\,a)} + 2\,bx}{48\,x^3}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x, algorithm="giac")
```

output
```
1/48*(32*b^3*x^3*Ei(4*b*x)*e^(4*a) - 32*b^3*x^3*Ei(-4*b*x)*e^(-4*a) - 8*b^
2*x^2*e^(4*b*x + 4*a) - 8*b^2*x^2*e^(-4*b*x - 4*a) - 2*b*x*e^(4*b*x + 4*a)
 + 2*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) - e^(-4*b*x - 4*a) + 2)/x^3
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)^2\sinh(a+b\,x)^2}{x^4}\,dx$$

input
```
int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^4,x)
```

output
```
int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^4, x)
```

3.298 $\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

3.298.1 Optimal result

Integrand size = 20, antiderivative size = 209

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{5^{-1-m}e^{5a}x^m(-bx)^{-m}\Gamma(1+m,-5bx)}{32b} + \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{32b} - \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{16b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{16b} - \frac{3^{-1-m}e^{-3a}x^m(bx)^{-m}\Gamma(1+m,3bx)}{32b} - \frac{5^{-1-m}e^{-5a}x^m(bx)^{-m}\Gamma(1+m,5bx)}{32b}$$

output
```
1/32*5^(-1-m)*exp(5*a)*x^m*GAMMA(1+m,-5*b*x)/b/((-b*x)^m)+1/32*3^(-1-m)*ex
p(3*a)*x^m*GAMMA(1+m,-3*b*x)/b/((-b*x)^m)-1/16*exp(a)*x^m*GAMMA(1+m,-b*x)/
b/((-b*x)^m)+1/16*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)-1/32*3^(-1-m)*x^m*
GAMMA(1+m,3*b*x)/b/exp(3*a)/((b*x)^m)-1/32*5^(-1-m)*x^m*GAMMA(1+m,5*b*x)/b
/exp(5*a)/((b*x)^m)
```

3.298.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.84

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{-5a}x^m\left(-30e^{6a}(-bx)^{-m}\Gamma(1+m,-bx)+30e^{4a}(bx)^{-m}\Gamma(1+m,bx)+5\,3^{-m}e^{2a}\left(-b^2x^2\right)^{-m}\left(e^{6a}(bx)^m\Gamma(1\right.\right.}{}$$

input
```
Integrate[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(x^m*((-30*E^(6*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + (30*E^(4*a)*Gamma[1
+ m, b*x])/(b*x)^m + (5*E^(2*a)*(E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] - (-
(b*x))^m*Gamma[1 + m, 3*b*x]))/(3^m*(-(b^2*x^2))^m) + (3*(E^(10*a)*(b*x)^m
*Gamma[1 + m, -5*b*x] - (-(b*x))^m*Gamma[1 + m, 5*b*x]))/(5^m*(-(b^2*x^2))
^m)))/(480*b*E^(5*a))
```

3.298.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh^2(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \; 5971$$

$$\int \left(-\frac{1}{8}x^m\cosh(a+bx)+\frac{1}{16}x^m\cosh(3a+3bx)+\frac{1}{16}x^m\cosh(5a+5bx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{e^{5a}5^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-5bx)}{32b}+\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-3bx)}{32b}-\frac{e^{a}x^m(-bx)^{-m}\Gamma(m+1,-bx)}{16b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{16b}-\frac{e^{-3a}3^{-m-1}x^m(bx)^{-m}\Gamma(m+1,3bx)}{32b}-\frac{e^{-5a}5^{-m-1}x^m(bx)^{-m}\Gamma(m+1,5bx)}{32b}$$

input
```
Int[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(5^(-1 - m)*E^(5*a)*x^m*Gamma[1 + m, -5*b*x])/(32*b*(-(b*x))^m) + (3^(-1 -
 m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(32*b*(-(b*x))^m) - (E^a*x^m*Gamma[1
 + m, -(b*x)])/(16*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(16*b*E^a*(b*x)
^m) - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(32*b*E^(3*a)*(b*x)^m) - (5^(-1
 - m)*x^m*Gamma[1 + m, 5*b*x])/(32*b*E^(5*a)*(b*x)^m)
```

3.298.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.298.4 Maple [F]

$$\int x^m \cosh(bx+a)^3 \sinh(bx+a)^2 \, dx$$

input
```
int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

output
```
int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

3.298.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.19

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{3\cosh(m\log(5b)+5a)\,\Gamma(m+1,5bx)+5\cosh(m\log(3b)+3a)\,\Gamma(m+1,3bx)-30\cosh(m\log(b)}{}$$

input
```
integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/480*(3*cosh(m*log(5*b) + 5*a)*gamma(m + 1, 5*b*x) + 5*cosh(m*log(3*b) +
 3*a)*gamma(m + 1, 3*b*x) - 30*cosh(m*log(b) + a)*gamma(m + 1, b*x) + 30*c
osh(m*log(-b) - a)*gamma(m + 1, -b*x) - 5*cosh(m*log(-3*b) - 3*a)*gamma(m
+ 1, -3*b*x) - 3*cosh(m*log(-5*b) - 5*a)*gamma(m + 1, -5*b*x) - 3*gamma(m
+ 1, 5*b*x)*sinh(m*log(5*b) + 5*a) - 5*gamma(m + 1, 3*b*x)*sinh(m*log(3*b)
 + 3*a) - 30*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) + 5*gamma(m + 1, -3*b*
x)*sinh(m*log(-3*b) - 3*a) + 3*gamma(m + 1, -5*b*x)*sinh(m*log(-5*b) - 5*a
) + 30*gamma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

3.298.6 Sympy [F]

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int x^m \sinh^2(a+bx)\cosh^3(a+bx)\,dx$$

input
```
integrate(x**m*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Integral(x**m*sinh(a + b*x)**2*cosh(a + b*x)**3, x)
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.82

$$\begin{aligned}\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = &-\frac{1}{32}\,(5\,bx)^{-m-1}\,x^{m+1}e^{(-5\,a)}\Gamma(m+1,5\,bx)\\ &-\frac{1}{32}\,(3\,bx)^{-m-1}\,x^{m+1}e^{(-3\,a)}\Gamma(m+1,3\,bx)\\ &+\frac{1}{16}\,(bx)^{-m-1}\,x^{m+1}e^{(-a)}\Gamma(m+1,bx)\\ &+\frac{1}{16}\,(-bx)^{-m-1}\,x^{m+1}e^{a}\Gamma(m+1,-bx)\\ &-\frac{1}{32}\,(-3\,bx)^{-m-1}\,x^{m+1}e^{(3\,a)}\Gamma(m+1,-3\,bx)\\ &-\frac{1}{32}\,(-5\,bx)^{-m-1}\,x^{m+1}e^{(5\,a)}\Gamma(m+1,-5\,bx)\end{aligned}$$

input
```
integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/32*(5*b*x)^(-m - 1)*x^(m + 1)*e^(-5*a)*gamma(m + 1, 5*b*x) - 1/32*(3*b*
x)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) + 1/16*(b*x)^(-m - 1)*x
^(m + 1)*e^(-a)*gamma(m + 1, b*x) + 1/16*(-b*x)^(-m - 1)*x^(m + 1)*e^a*gam
ma(m + 1, -b*x) - 1/32*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1, -3
*b*x) - 1/32*(-5*b*x)^(-m - 1)*x^(m + 1)*e^(5*a)*gamma(m + 1, -5*b*x)
```

3.298.8 Giac [F]

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int x^m \cosh(bx+a)^3 \sinh(bx+a)^2\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^3*sinh(b*x + a)^2, x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int x^m \cosh(a+b\,x)^3 \sinh(a+b\,x)^2\,dx$$

input
```
int(x^m*cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

output
```
int(x^m*cosh(a + b*x)^3*sinh(a + b*x)^2, x)
```

3.299 $\int x^3 \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

3.299.1 Optimal result

Integrand size = 20, antiderivative size = 202

$$\begin{aligned}\int x^3\cosh^3(a+bx)\sinh^2(a+bx)\,dx &= \frac{3\cosh(a+bx)}{4b^4}+\frac{3x^2\cosh(a+bx)}{8b^2}\\ &\quad-\frac{\cosh(3a+3bx)}{216b^4}-\frac{x^2\cosh(3a+3bx)}{48b^2}\\ &\quad-\frac{3\cosh(5a+5bx)}{5000b^4}-\frac{3x^2\cosh(5a+5bx)}{400b^2}\\ &\quad-\frac{3x\sinh(a+bx)}{4b^3}-\frac{x^3\sinh(a+bx)}{8b}\\ &\quad+\frac{x\sinh(3a+3bx)}{72b^3}+\frac{x^3\sinh(3a+3bx)}{48b}\\ &\quad+\frac{3x\sinh(5a+5bx)}{1000b^3}+\frac{x^3\sinh(5a+5bx)}{80b}\end{aligned}$$

output
```
3/4*cosh(b*x+a)/b^4+3/8*x^2*cosh(b*x+a)/b^2-1/216*cosh(3*b*x+3*a)/b^4-1/48
*x^2*cosh(3*b*x+3*a)/b^2-3/5000*cosh(5*b*x+5*a)/b^4-3/400*x^2*cosh(5*b*x+5
*a)/b^2-3/4*x*sinh(b*x+a)/b^3-1/8*x^3*sinh(b*x+a)/b+1/72*x*sinh(3*b*x+3*a)
/b^3+1/48*x^3*sinh(3*b*x+3*a)/b+3/1000*x*sinh(5*b*x+5*a)/b^3+1/80*x^3*sinh
(5*b*x+5*a)/b
```

3.299.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.62

$$\int x^3 \cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$= \frac{101250(2+b^2x^2)\cosh(a+bx) - 625(2+9b^2x^2)\cosh(3(a+bx)) - 81(2+25b^2x^2)\cosh(5(a+bx)) + 30b}{270000}$$

input
```
Integrate[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(101250*(2 + b^2*x^2)*Cosh[a + b*x] - 625*(2 + 9*b^2*x^2)*Cosh[3*(a + b*x)
] - 81*(2 + 25*b^2*x^2)*Cosh[5*(a + b*x)] + 30*b*x*(-6598 - 825*b^2*x^2 +
8*(38 + 75*b^2*x^2)*Cosh[2*(a + b*x)] + 9*(6 + 25*b^2*x^2)*Cosh[4*(a + b*x
)])*Sinh[a + b*x])/(270000*b^4)
```

3.299.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sinh^2(a+bx)\cosh^3(a+bx)\,dx$$
$$\Big\downarrow \text{5971}$$
$$\int \left(-\frac{1}{8}x^3\cosh(a+bx) + \frac{1}{16}x^3\cosh(3a+3bx) + \frac{1}{16}x^3\cosh(5a+5bx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3\cosh(a+bx)}{4b^4} - \frac{\cosh(3a+3bx)}{216b^4} - \frac{3\cosh(5a+5bx)}{5000b^4} - \frac{3x\sinh(a+bx)}{4b^3} + \frac{x\sinh(3a+3bx)}{72b^3} + \frac{3x\sinh(5a+5bx)}{1000b^3} + \frac{3x^2\cosh(a+bx)}{8b^2} - \frac{x^2\cosh(3a+3bx)}{48b^2} - \frac{3x^2\cosh(5a+5bx)}{400b^2} - \frac{x^3\sinh(a+bx)}{8b} + \frac{x^3\sinh(3a+3bx)}{48b} + \frac{x^3\sinh(5a+5bx)}{80b}$$

input
```
Int[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(3*Cosh[a + b*x])/(4*b^4) + (3*x^2*Cosh[a + b*x])/(8*b^2) - Cosh[3*a + 3*b
*x]/(216*b^4) - (x^2*Cosh[3*a + 3*b*x])/(48*b^2) - (3*Cosh[5*a + 5*b*x])/(
5000*b^4) - (3*x^2*Cosh[5*a + 5*b*x])/(400*b^2) - (3*x*Sinh[a + b*x])/(4*b
^3) - (x^3*Sinh[a + b*x])/(8*b) + (x*Sinh[3*a + 3*b*x])/(72*b^3) + (x^3*Si
nh[3*a + 3*b*x])/(48*b) + (3*x*Sinh[5*a + 5*b*x])/(1000*b^3) + (x^3*Sinh[5
*a + 5*b*x])/(80*b)
```

3.299.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.299.4 Maple [A] (verified)

Time = 24.68 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05

method	result
risch	$\frac{(125x^3b^3-75x^2b^2+30bx-6)\,\mathrm{e}^{5bx+5a}}{20000b^4}+\frac{(9x^3b^3-9x^2b^2+6bx-2)\,\mathrm{e}^{3bx+3a}}{864b^4}-\frac{(x^3b^3-3x^2b^2+6bx-6)\,\mathrm{e}^{bx+a}}{16b^4}+\frac{(x^3b^3+$
derivativedivides	$-a^3\left(\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}\right.$
default	$-a^3\left(\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}\right.$

input
```
int(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/20000*(125*b^3*x^3-75*b^2*x^2+30*b*x-6)/b^4*exp(5*b*x+5*a)+1/864*(9*b^3*
x^3-9*b^2*x^2+6*b*x-2)/b^4*exp(3*b*x+3*a)-1/16*(b^3*x^3-3*b^2*x^2+6*b*x-6)
/b^4*exp(b*x+a)+1/16*(b^3*x^3+3*b^2*x^2+6*b*x+6)/b^4*exp(-b*x-a)-1/864*(9*
b^3*x^3+9*b^2*x^2+6*b*x+2)/b^4*exp(-3*b*x-3*a)-1/20000*(125*b^3*x^3+75*b^2
*x^2+30*b*x+6)/b^4*exp(-5*b*x-5*a)
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.36

$$\int x^3 \cosh^3(a+bx) \sinh^2(a+bx)\,dx =$$
$$-\frac{81\,(25\,b^2x^2+2)\cosh\,(bx+a)^5+405\,(25\,b^2x^2+2)\cosh\,(bx+a)\sinh\,(bx+a)^4-135\,(25\,b^3x^3+6\,bx)\,\mathrm{si}}{}$$

input
```
integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/270000*(81*(25*b^2*x^2 + 2)*cosh(b*x + a)^5 + 405*(25*b^2*x^2 + 2)*cosh
(b*x + a)*sinh(b*x + a)^4 - 135*(25*b^3*x^3 + 6*b*x)*sinh(b*x + a)^5 + 625
*(9*b^2*x^2 + 2)*cosh(b*x + a)^3 - 75*(75*b^3*x^3 + 18*(25*b^3*x^3 + 6*b*x
)*cosh(b*x + a)^2 + 50*b*x)*sinh(b*x + a)^3 + 15*(54*(25*b^2*x^2 + 2)*cosh
(b*x + a)^3 + 125*(9*b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a)^2 - 101250*
(b^2*x^2 + 2)*cosh(b*x + a) + 225*(150*b^3*x^3 - 3*(25*b^3*x^3 + 6*b*x)*co
sh(b*x + a)^4 - 25*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^2 + 900*b*x)*sinh(b*x
 + a))/b^4
```

3.299.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.25

$$\int x^3 \cosh^3(a+bx) \sinh^2(a+bx)\,dx$$
$$=\begin{cases} -\frac{2x^3\sinh^5(a+bx)}{15b}+\frac{x^3\sinh^3(a+bx)\cosh^2(a+bx)}{3b}+\frac{2x^2\sinh^4(a+bx)\cosh(a+bx)}{5b^2}-\frac{13x^2\sinh^2(a+bx)\cosh^3(a+bx)}{15b^2}+\frac{26x^2\cosh^5(\cdots}{75b^2} \\ \frac{x^4\sinh^2(a)\cosh^3(a)}{4} \end{cases}$$

input
```
integrate(x**3*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-2*x**3*sinh(a + b*x)**5/(15*b) + x**3*sinh(a + b*x)**3*cosh(a
+ b*x)**2/(3*b) + 2*x**2*sinh(a + b*x)**4*cosh(a + b*x)/(5*b**2) - 13*x**2
*sinh(a + b*x)**2*cosh(a + b*x)**3/(15*b**2) + 26*x**2*cosh(a + b*x)**5/(7
5*b**2) - 856*x*sinh(a + b*x)**5/(1125*b**3) + 338*x*sinh(a + b*x)**3*cosh
(a + b*x)**2/(225*b**3) - 52*x*sinh(a + b*x)*cosh(a + b*x)**4/(75*b**3) +
856*sinh(a + b*x)**4*cosh(a + b*x)/(1125*b**4) - 5114*sinh(a + b*x)**2*cos
h(a + b*x)**3/(3375*b**4) + 12568*cosh(a + b*x)**5/(16875*b**4), Ne(b, 0))
, (x**4*sinh(a)**2*cosh(a)**3/4, True))
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.21

$$\int x^3\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(125\,b^3x^3e^{(5\,a)} - 75\,b^2x^2e^{(5\,a)} + 30\,bxe^{(5\,a)} - 6\,e^{(5\,a)}\right)e^{(5\,bx)}}{20000\,b^4} + \frac{\left(9\,b^3x^3e^{(3\,a)} - 9\,b^2x^2e^{(3\,a)} + 6\,bxe^{(3\,a)} - 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{864\,b^4} - \frac{(b^3x^3e^a - 3\,b^2x^2e^a + 6\,bxe^a - 6\,e^a)e^{(bx)}}{16\,b^4} + \frac{(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)e^{(-bx-a)}}{16\,b^4} - \frac{(9\,b^3x^3 + 9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{864\,b^4} - \frac{(125\,b^3x^3 + 75\,b^2x^2 + 30\,bx + 6)e^{(-5\,bx-5\,a)}}{20000\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/20000*(125*b^3*x^3*e^(5*a) - 75*b^2*x^2*e^(5*a) + 30*b*x*e^(5*a) - 6*e^(
5*a))*e^(5*b*x)/b^4 + 1/864*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x
*e^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 1/16*(b^3*x^3*e^a - 3*b^2*x^2*e^a +
6*b*x*e^a - 6*e^a)*e^(b*x)/b^4 + 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^
(-b*x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a
)/b^4 - 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5*a)/b
^4
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05

$$\int x^3\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{(125\,b^3x^3 - 75\,b^2x^2 + 30\,bx - 6)e^{(5\,bx+5\,a)}}{20000\,b^4} + \frac{(9\,b^3x^3 - 9\,b^2x^2 + 6\,bx - 2)e^{(3\,bx+3\,a)}}{864\,b^4} - \frac{(b^3x^3 - 3\,b^2x^2 + 6\,bx - 6)e^{(bx+a)}}{16\,b^4} + \frac{(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)e^{(-bx-a)}}{16\,b^4} - \frac{(9\,b^3x^3 + 9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{864\,b^4} - \frac{(125\,b^3x^3 + 75\,b^2x^2 + 30\,bx + 6)e^{(-5\,bx-5\,a)}}{20000\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/20000*(125*b^3*x^3 - 75*b^2*x^2 + 30*b*x - 6)*e^(5*b*x + 5*a)/b^4 + 1/86
4*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 - 1/16*(b^3*x^3
- 3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 + 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b
*x + 6)*e^(-b*x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3
*b*x - 3*a)/b^4 - 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*
x - 5*a)/b^4
```

3.299.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.83

$$\int x^3 \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{x\sinh(3\,a+3\,b\,x)}{72} - \frac{3\,x\sinh(a+b\,x)}{4} + \frac{3\,x\sinh(5\,a+5\,b\,x)}{1000}}{b^3} + \frac{\frac{x^3\sinh(3\,a+3\,b\,x)}{48} + \frac{x^3\sinh(5\,a+5\,b\,x)}{80} - \frac{x^3\sinh(a+b\,x)}{8}}{b} + \frac{3\cosh(a+b\,x)}{4\,b^4} - \frac{\cosh(3\,a+3\,b\,x)}{216\,b^4} - \frac{3\cosh(5\,a+5\,b\,x)}{5000\,b^4} - \frac{\frac{x^2\cosh(3\,a+3\,b\,x)}{48} - \frac{3\,x^2\cosh(a+b\,x)}{8} + \frac{3\,x^2\cosh(5\,a+5\,b\,x)}{400}}{b^2}$$

input
```
int(x^3*cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

output
```
((x*sinh(3*a + 3*b*x))/72 - (3*x*sinh(a + b*x))/4 + (3*x*sinh(5*a + 5*b*x)
)/1000)/b^3 + ((x^3*sinh(3*a + 3*b*x))/48 + (x^3*sinh(5*a + 5*b*x))/80 - (
x^3*sinh(a + b*x))/8)/b + (3*cosh(a + b*x))/(4*b^4) - cosh(3*a + 3*b*x)/(2
16*b^4) - (3*cosh(5*a + 5*b*x))/(5000*b^4) - ((x^2*cosh(3*a + 3*b*x))/48 -
 (3*x^2*cosh(a + b*x))/8 + (3*x^2*cosh(5*a + 5*b*x))/400)/b^2
```

3.300 $\int x^2 \cosh^3(a+bx) \sinh^2(a+bx)\,dx$

3.300.1 Optimal result

Integrand size = 20, antiderivative size = 148

$$\int x^2 \cosh^3(a+bx) \sinh^2(a+bx)\,dx = \frac{x\cosh(a+bx)}{4b^2} - \frac{x\cosh(3a+3bx)}{72b^2} - \frac{x\cosh(5a+5bx)}{200b^2} - \frac{\sinh(a+bx)}{4b^3} - \frac{x^2\sinh(a+bx)}{8b} + \frac{\sinh(3a+3bx)}{216b^3} + \frac{x^2\sinh(3a+3bx)}{48b} + \frac{\sinh(5a+5bx)}{1000b^3} + \frac{x^2\sinh(5a+5bx)}{80b}$$

output
```
1/4*x*cosh(b*x+a)/b^2-1/72*x*cosh(3*b*x+3*a)/b^2-1/200*x*cosh(5*b*x+5*a)/b
^2-1/4*sinh(b*x+a)/b^3-1/8*x^2*sinh(b*x+a)/b+1/216*sinh(3*b*x+3*a)/b^3+1/4
8*x^2*sinh(3*b*x+3*a)/b+1/1000*sinh(5*b*x+5*a)/b^3+1/80*x^2*sinh(5*b*x+5*a
)/b
```

3.300.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.71

$$\int x^2 \cosh^3(a+bx) \sinh^2(a+bx)\,dx = \frac{-6750(-2bx\cosh(a+bx)+(2+b^2x^2)\sinh(a+bx))+125(-6bx\cosh(3(a+bx))+(2+9b^2x^2)\sinh(3(a}{54000b^3}$$

input
```
Integrate[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(-6750*(-2*b*x*Cosh[a + b*x] + (2 + b^2*x^2)*Sinh[a + b*x]) + 125*(-6*b*x*
Cosh[3*(a + b*x)] + (2 + 9*b^2*x^2)*Sinh[3*(a + b*x)]) + 27*(-10*b*x*Cosh[
5*(a + b*x)] + (2 + 25*b^2*x^2)*Sinh[5*(a + b*x)]))/(54000*b^3)
```

3.300.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sinh^2(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \; 5971$$

$$\int \left(-\frac{1}{8}x^2\cosh(a+bx)+\frac{1}{16}x^2\cosh(3a+3bx)+\frac{1}{16}x^2\cosh(5a+5bx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sinh(a+bx)}{4b^3}+\frac{\sinh(3a+3bx)}{216b^3}+\frac{\sinh(5a+5bx)}{1000b^3}+\frac{x\cosh(a+bx)}{4b^2}-\frac{x\cosh(3a+3bx)}{72b^2}-\frac{x\cosh(5a+5bx)}{200b^2}-\frac{x^2\sinh(a+bx)}{8b}+\frac{x^2\sinh(3a+3bx)}{48b}+\frac{x^2\sinh(5a+5bx)}{80b}$$

input
```
Int[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(x*Cosh[a + b*x])/(4*b^2) - (x*Cosh[3*a + 3*b*x])/(72*b^2) - (x*Cosh[5*a +
 5*b*x])/(200*b^2) - Sinh[a + b*x]/(4*b^3) - (x^2*Sinh[a + b*x])/(8*b) + S
inh[3*a + 3*b*x]/(216*b^3) + (x^2*Sinh[3*a + 3*b*x])/(48*b) + Sinh[5*a + 5
*b*x]/(1000*b^3) + (x^2*Sinh[5*a + 5*b*x])/(80*b)
```

3.300.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.300.4 Maple [A] (verified)

Time = 18.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

method	result
risch	$\frac{(25x^2b^2-10bx+2)e^{5bx+5a}}{4000b^3}+\frac{(9x^2b^2-6bx+2)e^{3bx+3a}}{864b^3}-\frac{(x^2b^2-2bx+2)e^{bx+a}}{16b^3}+\frac{(x^2b^2+2bx+2)e^{-bx-a}}{16b^3}-\frac{(9x^2b\cdots}{}$
derivativedivides	$a^2\left(\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}-\frac{(b\cdots}{}\right.$
default	$a^2\left(\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}-\frac{(b\cdots}{}\right.$

input
```
int(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4000*(25*b^2*x^2-10*b*x+2)/b^3*exp(5*b*x+5*a)+1/864*(9*b^2*x^2-6*b*x+2)/
b^3*exp(3*b*x+3*a)-1/16*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)+1/16*(b^2*x^2+2*b
*x+2)/b^3*exp(-b*x-a)-1/864*(9*b^2*x^2+6*b*x+2)/b^3*exp(-3*b*x-3*a)-1/4000
*(25*b^2*x^2+10*b*x+2)/b^3*exp(-5*b*x-5*a)
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.41

$$\int x^2 \cosh^3(a+bx) \sinh^2(a+bx)\,dx =$$

$$-\frac{270\,bx\cosh\left(bx+a\right)^5+1350\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-27\left(25\,b^2x^2+2\right)\sinh\left(bx+a\right)^5+750\,bx\,\mathrm{c}}{}$$

input
```
integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/54000*(270*b*x*cosh(b*x + a)^5 + 1350*b*x*cosh(b*x + a)*sinh(b*x + a)^4
 - 27*(25*b^2*x^2 + 2)*sinh(b*x + a)^5 + 750*b*x*cosh(b*x + a)^3 - 5*(225*
b^2*x^2 + 54*(25*b^2*x^2 + 2)*cosh(b*x + a)^2 + 50)*sinh(b*x + a)^3 - 1350
0*b*x*cosh(b*x + a) + 450*(6*b*x*cosh(b*x + a)^3 + 5*b*x*cosh(b*x + a))*si
nh(b*x + a)^2 - 15*(9*(25*b^2*x^2 + 2)*cosh(b*x + a)^4 - 450*b^2*x^2 + 25*
(9*b^2*x^2 + 2)*cosh(b*x + a)^2 - 900)*sinh(b*x + a))/b^3
```

3.300.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.23

$$\int x^2 \cosh^3(a+bx) \sinh^2(a+bx)\,dx$$

$$= \begin{cases} -\frac{2x^2\sinh^5(a+bx)}{15b} + \frac{x^2\sinh^3(a+bx)\cosh^2(a+bx)}{3b} + \frac{4x\sinh^4(a+bx)\cosh(a+bx)}{15b^2} - \frac{26x\sinh^2(a+bx)\cosh^3(a+bx)}{45b^2} + \frac{52x\cosh^5(a+b}{225b^2} \\ \frac{x^3\sinh^2(a)\cosh^3(a)}{3} \end{cases}$$

input
```
integrate(x**2*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-2*x**2*sinh(a + b*x)**5/(15*b) + x**2*sinh(a + b*x)**3*cosh(a
+ b*x)**2/(3*b) + 4*x*sinh(a + b*x)**4*cosh(a + b*x)/(15*b**2) - 26*x*sinh
(a + b*x)**2*cosh(a + b*x)**3/(45*b**2) + 52*x*cosh(a + b*x)**5/(225*b**2)
 - 856*sinh(a + b*x)**5/(3375*b**3) + 338*sinh(a + b*x)**3*cosh(a + b*x)**
2/(675*b**3) - 52*sinh(a + b*x)*cosh(a + b*x)**4/(225*b**3), Ne(b, 0)), (x
**3*sinh(a)**2*cosh(a)**3/3, True))
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.26

$$\int x^2\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(25\,b^2x^2e^{(5\,a)} - 10\,bxe^{(5\,a)} + 2\,e^{(5\,a)}\right)e^{(5\,bx)}}{4000\,b^3} + \frac{\left(9\,b^2x^2e^{(3\,a)} - 6\,bxe^{(3\,a)} + 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{864\,b^3} - \frac{\left(b^2x^2e^a - 2\,bxe^a + 2\,e^a\right)e^{(bx)}}{16\,b^3} + \frac{\left(b^2x^2 + 2\,bx + 2\right)e^{(-bx-a)}}{16\,b^3} - \frac{\left(9\,b^2x^2 + 6\,bx + 2\right)e^{(-3\,bx-3\,a)}}{864\,b^3} - \frac{\left(25\,b^2x^2 + 10\,bx + 2\right)e^{(-5\,bx-5\,a)}}{4000\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4000*(25*b^2*x^2*e^(5*a) - 10*b*x*e^(5*a) + 2*e^(5*a))*e^(5*b*x)/b^3 + 1
/864*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 - 1/16*
(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 + 1/16*(b^2*x^2 + 2*b*x + 2)
*e^(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3 - 1
/4000*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)/b^3
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.11

$$\int x^2\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(25\,b^2x^2 - 10\,bx + 2\right)e^{(5\,bx+5\,a)}}{4000\,b^3} + \frac{\left(9\,b^2x^2 - 6\,bx + 2\right)e^{(3\,bx+3\,a)}}{864\,b^3} - \frac{\left(b^2x^2 - 2\,bx + 2\right)e^{(bx+a)}}{16\,b^3} + \frac{\left(b^2x^2 + 2\,bx + 2\right)e^{(-bx-a)}}{16\,b^3} - \frac{\left(9\,b^2x^2 + 6\,bx + 2\right)e^{(-3\,bx-3\,a)}}{864\,b^3} - \frac{\left(25\,b^2x^2 + 10\,bx + 2\right)e^{(-5\,bx-5\,a)}}{4000\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4000*(25*b^2*x^2 - 10*b*x + 2)*e^(5*b*x + 5*a)/b^3 + 1/864*(9*b^2*x^2 -
6*b*x + 2)*e^(3*b*x + 3*a)/b^3 - 1/16*(b^2*x^2 - 2*b*x + 2)*e^(b*x + a)/b^
3 + 1/16*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x
 + 2)*e^(-3*b*x - 3*a)/b^3 - 1/4000*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x -
5*a)/b^3
```

3.300.9 Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83

$$\int x^2\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{x^2\sinh(3\,a+3\,b\,x)}{48}+\frac{x^2\sinh(5\,a+5\,b\,x)}{80}-\frac{x^2\sinh(a+b\,x)}{8}}{b} - \frac{\sinh(a+b\,x)}{4\,b^3} - \frac{\frac{x\cosh(3\,a+3\,b\,x)}{72}-\frac{x\cosh(a+b\,x)}{4}+\frac{x\cosh(5\,a+5\,b\,x)}{200}}{b^2} + \frac{\sinh(3\,a+3\,b\,x)}{216\,b^3} + \frac{\sinh(5\,a+5\,b\,x)}{1000\,b^3}$$

input
```
int(x^2*cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

output
```
((x^2*sinh(3*a + 3*b*x))/48 + (x^2*sinh(5*a + 5*b*x))/80 - (x^2*sinh(a + b
*x))/8)/b - sinh(a + b*x)/(4*b^3) - ((x*cosh(3*a + 3*b*x))/72 - (x*cosh(a
+ b*x))/4 + (x*cosh(5*a + 5*b*x))/200)/b^2 + sinh(3*a + 3*b*x)/(216*b^3) +
 sinh(5*a + 5*b*x)/(1000*b^3)
```

3.301 $\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx$

3.301.1 Optimal result

Integrand size = 18, antiderivative size = 94

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx=\frac{\cosh(a+bx)}{8b^2}-\frac{\cosh(3a+3bx)}{144b^2}-\frac{\cosh(5a+5bx)}{400b^2}-\frac{x\sinh(a+bx)}{8b}+\frac{x\sinh(3a+3bx)}{48b}+\frac{x\sinh(5a+5bx)}{80b}$$

output
```
1/8*cosh(b*x+a)/b^2-1/144*cosh(3*b*x+3*a)/b^2-1/400*cosh(5*b*x+5*a)/b^2-1/
8*x*sinh(b*x+a)/b+1/48*x*sinh(3*b*x+3*a)/b+1/80*x*sinh(5*b*x+5*a)/b
```

3.301.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{450\cosh(a+bx)-25\cosh(3(a+bx))-9\cosh(5(a+bx))-450bx\sinh(a+bx)+75bx\sinh(3(a+bx))+\cdots}{3600b^2}$$

input
```
Integrate[x*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(450*Cosh[a + b*x] - 25*Cosh[3*(a + b*x)] - 9*Cosh[5*(a + b*x)] - 450*b*x*
Sinh[a + b*x] + 75*b*x*Sinh[3*(a + b*x)] + 45*b*x*Sinh[5*(a + b*x)])/(3600
*b^2)
```

3.301.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh^2(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \; 5971$$

$$\int \left(-\frac{1}{8}x\cosh(a+bx)+\frac{1}{16}x\cosh(3a+3bx)+\frac{1}{16}x\cosh(5a+5bx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\cosh(a+bx)}{8b^2}-\frac{\cosh(3a+3bx)}{144b^2}-\frac{\cosh(5a+5bx)}{400b^2}-\frac{x\sinh(a+bx)}{8b}+\frac{x\sinh(3a+3bx)}{48b}+\frac{x\sinh(5a+5bx)}{80b}$$

input
```
Int[x*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
Cosh[a + b*x]/(8*b^2) - Cosh[3*a + 3*b*x]/(144*b^2) - Cosh[5*a + 5*b*x]/(4
00*b^2) - (x*Sinh[a + b*x])/(8*b) + (x*Sinh[3*a + 3*b*x])/(48*b) + (x*Sinh
[5*a + 5*b*x])/(80*b)
```

3.301.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.301.4 Maple [A] (verified)

Time = 13.60 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

method	result
risch	$\frac{(5bx-1)\mathrm{e}^{5bx+5a}}{800b^2}+\frac{(3bx-1)\mathrm{e}^{3bx+3a}}{288b^2}-\frac{(bx-1)\mathrm{e}^{bx+a}}{16b^2}+\frac{(bx+1)\mathrm{e}^{-bx-a}}{16b^2}-\frac{(3bx+1)\mathrm{e}^{-3bx-3a}}{288b^2}-\frac{(5bx+1)\mathrm{e}^{-5bx-5a}}{800b^2}$
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}-\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^2}{15}-\frac{\cosh(bx+a)^5}{25}+\frac{2\cosh(bx+a)}{15}+\frac{\cosh(bx+a)^3}{45}}{b^2}$
default	$\frac{\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}-\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^2}{15}-\frac{\cosh(bx+a)^5}{25}+\frac{2\cosh(bx+a)}{15}+\frac{\cosh(bx+a)^3}{45}}{b^2}$

input
```
int(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/800*(5*b*x-1)/b^2*exp(5*b*x+5*a)+1/288*(3*b*x-1)/b^2*exp(3*b*x+3*a)-1/16
*(b*x-1)/b^2*exp(b*x+a)+1/16*(b*x+1)/b^2*exp(-b*x-a)-1/288*(3*b*x+1)/b^2*e
xp(-3*b*x-3*a)-1/800*(5*b*x+1)/b^2*exp(-5*b*x-5*a)
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.62

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{45\,bx\sinh\left(bx+a\right)^5-9\,\cosh\left(bx+a\right)^5-45\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+75\left(6\,bx\cosh\left(bx+a\right)^2+bx\right)\sinh\left(bx+a\right)^3-\ldots}{\ldots}$$

input
```
integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3600*(45*b*x*sinh(b*x + a)^5 - 9*cosh(b*x + a)^5 - 45*cosh(b*x + a)*sinh
(b*x + a)^4 + 75*(6*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^3 - 25*cosh(b
*x + a)^3 - 15*(6*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^2 + 225
*(b*x*cosh(b*x + a)^4 + b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a) + 450*c
osh(b*x + a))/b^2
```

3.301.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$

$$= \begin{cases} -\frac{2x\sinh^5(a+bx)}{15b} + \frac{x\sinh^3(a+bx)\cosh^2(a+bx)}{3b} + \frac{2\sinh^4(a+bx)\cosh(a+bx)}{15b^2} - \frac{13\sinh^2(a+bx)\cosh^3(a+bx)}{45b^2} + \frac{26\cosh^5(a+bx)}{225b^2} \\ \frac{x^2\sinh^2(a)\cosh^3(a)}{2} \end{cases}$$

input
```
integrate(x*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-2*x*sinh(a + b*x)**5/(15*b) + x*sinh(a + b*x)**3*cosh(a + b*x)
**2/(3*b) + 2*sinh(a + b*x)**4*cosh(a + b*x)/(15*b**2) - 13*sinh(a + b*x)*
*2*cosh(a + b*x)**3/(45*b**2) + 26*cosh(a + b*x)**5/(225*b**2), Ne(b, 0)),
 (x**2*sinh(a)**2*cosh(a)**3/2, True))
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(5\,bxe^{(5\,a)} - e^{(5\,a)}\right)e^{(5\,bx)}}{800\,b^2} + \frac{\left(3\,bxe^{(3\,a)} - e^{(3\,a)}\right)e^{(3\,bx)}}{288\,b^2} - \frac{(bxe^a - e^a)e^{(bx)}}{16\,b^2} + \frac{(bx+1)e^{(-bx-a)}}{16\,b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2} - \frac{(5\,bx+1)e^{(-5\,bx-5\,a)}}{800\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/800*(5*b*x*e^(5*a) - e^(5*a))*e^(5*b*x)/b^2 + 1/288*(3*b*x*e^(3*a) - e^(
3*a))*e^(3*b*x)/b^2 - 1/16*(b*x*e^a - e^a)*e^(b*x)/b^2 + 1/16*(b*x + 1)*e^
(-b*x - a)/b^2 - 1/288*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 - 1/800*(5*b*x + 1
)*e^(-5*b*x - 5*a)/b^2
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{(5\,bx-1)e^{(5\,bx+5\,a)}}{800\,b^2} + \frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{288\,b^2} - \frac{(bx-1)e^{(bx+a)}}{16\,b^2} + \frac{(bx+1)e^{(-bx-a)}}{16\,b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2} - \frac{(5\,bx+1)e^{(-5\,bx-5\,a)}}{800\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/800*(5*b*x - 1)*e^(5*b*x + 5*a)/b^2 + 1/288*(3*b*x - 1)*e^(3*b*x + 3*a)/
b^2 - 1/16*(b*x - 1)*e^(b*x + a)/b^2 + 1/16*(b*x + 1)*e^(-b*x - a)/b^2 - 1
/288*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 - 1/800*(5*b*x + 1)*e^(-5*b*x - 5*a)
/b^2
```

3.301.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{b\left(\frac{2\,x\sinh(a+b\,x)^5}{15}-\frac{x\cosh(a+b\,x)^2\sinh(a+b\,x)^3}{3}\right)-\frac{2\cosh(a+b\,x)\sinh(a+b\,x)^4}{15}-\frac{26\cosh(a+b\,x)^5}{225}+\frac{13\cosh(a+b\,x)^3\sinh(a+b\,x)^2}{45}}{b^2}$$

input
```
int(x*cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

output
```
-(b*((2*x*sinh(a + b*x)^5)/15 - (x*cosh(a + b*x)^2*sinh(a + b*x)^3)/3) - (
2*cosh(a + b*x)*sinh(a + b*x)^4)/15 - (26*cosh(a + b*x)^5)/225 + (13*cosh(
a + b*x)^3*sinh(a + b*x)^2)/45)/b^2
```

3.302 $\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

3.302.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx=\frac{\sinh^3(a+bx)}{3b}+\frac{\sinh^5(a+bx)}{5b}$$

output
```
1/3*sinh(b*x+a)^3/b+1/5*sinh(b*x+a)^5/b
```

3.302.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx=\frac{(7+3\cosh(2(a+bx)))\sinh^3(a+bx)}{30b}$$

input
```
Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
((7 + 3*Cosh[2*(a + b*x)])*Sinh[a + b*x]^3)/(30*b)
```

3.302.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 25, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^2\left(-\cos(ia+ibx)^3\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(ia+ibx)^3\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{i\int -\sinh^2(a+bx)\left(\sinh^2(a+bx)+1\right)d(i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{i\int \left(-\sinh^4(a+bx)-\sinh^2(a+bx)\right)d(i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{1}{5}i\sinh^5(a+bx)-\frac{1}{3}i\sinh^3(a+bx)\right)}{b}$$

input
```
Int[Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(I*((-1/3*I)*Sinh[a + b*x]^3 - (I/5)*Sinh[a + b*x]^5))/b
```

3.302.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.302.4 Maple [A] (verified)

Time = 8.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^5}{5}+\frac{\sinh(bx+a)^3}{3}}{b}$	26
default	$\frac{\frac{\sinh(bx+a)^5}{5}+\frac{\sinh(bx+a)^3}{3}}{b}$	26
risch	$\frac{e^{5bx+5a}}{160b}+\frac{e^{3bx+3a}}{96b}-\frac{e^{bx+a}}{16b}+\frac{e^{-bx-a}}{16b}-\frac{e^{-3bx-3a}}{96b}-\frac{e^{-5bx-5a}}{160b}$	83

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5*sinh(b*x+a)^5+1/3*sinh(b*x+a)^3)
```

3.302.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.06

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{3\sinh(bx+a)^5 + 5\left(6\cosh(bx+a)^2+1\right)\sinh(bx+a)^3 + 15\left(\cosh(bx+a)^4+\cosh(bx+a)^2-2\right)\sinh}{240\,b}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 15*
(cosh(b*x + a)^4 + cosh(b*x + a)^2 - 2)*sinh(b*x + a))/b
```

3.302.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{2\sinh^5(a+bx)}{15b} + \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sinh^2(a)\cosh^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-2*sinh(a + b*x)**5/(15*b) + sinh(a + b*x)**3*cosh(a + b*x)**2/
(3*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**3, True))
```

3.302.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(27) = 54$.

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.52

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(5\,e^{(-2\,bx-2\,a)} - 30\,e^{(-4\,bx-4\,a)} + 3\right)e^{(5\,bx+5\,a)}}{480\,b} + \frac{30\,e^{(-bx-a)} - 5\,e^{(-3\,bx-3\,a)} - 3\,e^{(-5\,bx-5\,a)}}{480\,b}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/480*(5*e^(-2*b*x - 2*a) - 30*e^(-4*b*x - 4*a) + 3)*e^(5*b*x + 5*a)/b + 1
/480*(30*e^(-b*x - a) - 5*e^(-3*b*x - 3*a) - 3*e^(-5*b*x - 5*a))/b
```

3.302.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.65

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{160\,b} + \frac{e^{(3\,bx+3\,a)}}{96\,b} - \frac{e^{(bx+a)}}{16\,b} + \frac{e^{(-bx-a)}}{16\,b} - \frac{e^{(-3\,bx-3\,a)}}{96\,b} - \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/160*e^(5*b*x + 5*a)/b + 1/96*e^(3*b*x + 3*a)/b - 1/16*e^(b*x + a)/b + 1/
16*e^(-b*x - a)/b - 1/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

3.302.9 Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{3\sinh(a+b\,x)^5 + 5\sinh(a+b\,x)^3}{15\,b}$$

input
```
int(cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

output
```
(5*sinh(a + b*x)^3 + 3*sinh(a + b*x)^5)/(15*b)
```

3.303 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx$

3.303.1 Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = -\frac{1}{8}\cosh(a)\text{Chi}(bx)+\frac{1}{16}\cosh(3a)\text{Chi}(3bx)+\frac{1}{16}\cosh(5a)\text{Chi}(5bx)-\frac{1}{8}\sinh(a)\text{Shi}(bx)+\frac{1}{16}\sinh(3a)\text{Shi}(3bx)+\frac{1}{16}\sinh(5a)\text{Shi}(5bx)$$

output
```
-1/8*Chi(b*x)*cosh(a)+1/16*Chi(3*b*x)*cosh(3*a)+1/16*Chi(5*b*x)*cosh(5*a)-
1/8*Shi(b*x)*sinh(a)+1/16*Shi(3*b*x)*sinh(3*a)+1/16*Shi(5*b*x)*sinh(5*a)
```

3.303.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{16}(-2\cosh(a)\text{Chi}(bx)+\cosh(3a)\text{Chi}(3bx)+\cosh(5a)\text{Chi}(5bx)-2\sinh(a)\text{Shi}(bx)+\sinh(3a)\text{Shi}(3bx)+\sinh(5a)\text{Shi}(5bx))$$

input
```
Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x,x]
```

output
```
(-2*Cosh[a]*CoshIntegral[b*x] + Cosh[3*a]*CoshIntegral[3*b*x] + Cosh[5*a]*
CoshIntegral[5*b*x] - 2*Sinh[a]*SinhIntegral[b*x] + Sinh[3*a]*SinhIntegral
[3*b*x] + Sinh[5*a]*SinhIntegral[5*b*x])/16
```

3.303.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{ 5971}$$

$$\int \left(-\frac{\cosh(a+bx)}{8x}+\frac{\cosh(3a+3bx)}{16x}+\frac{\cosh(5a+5bx)}{16x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{8}\cosh(a)\text{Chi}(bx)+\frac{1}{16}\cosh(3a)\text{Chi}(3bx)+\frac{1}{16}\cosh(5a)\text{Chi}(5bx)-\frac{1}{8}\sinh(a)\text{Shi}(bx)+\frac{1}{16}\sinh(3a)\text{Shi}(3bx)+\frac{1}{16}\sinh(5a)\text{Shi}(5bx)$$

input
```
Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x,x]
```

output
```
-1/8*(Cosh[a]*CoshIntegral[b*x]) + (Cosh[3*a]*CoshIntegral[3*b*x])/16 + (C
osh[5*a]*CoshIntegral[5*b*x])/16 - (Sinh[a]*SinhIntegral[b*x])/8 + (Sinh[3
*a]*SinhIntegral[3*b*x])/16 + (Sinh[5*a]*SinhIntegral[5*b*x])/16
```

3.303.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.303.4 Maple [A] (verified)

Time = 8.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

method	result	size
risch	$-\frac{e^{-5a}\operatorname{Ei}_1(5bx)}{32}-\frac{e^{-3a}\operatorname{Ei}_1(3bx)}{32}+\frac{e^{-a}\operatorname{Ei}_1(bx)}{16}+\frac{e^{a}\operatorname{Ei}_1(-bx)}{16}-\frac{e^{3a}\operatorname{Ei}_1(-3bx)}{32}-\frac{e^{5a}\operatorname{Ei}_1(-5bx)}{32}$	71

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/32*exp(-5*a)*Ei(1,5*b*x)-1/32*exp(-3*a)*Ei(1,3*b*x)+1/16*exp(-a)*Ei(1,b
*x)+1/16*exp(a)*Ei(1,-b*x)-1/32*exp(3*a)*Ei(1,-3*b*x)-1/32*exp(5*a)*Ei(1,-
5*b*x)
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx &= \frac{1}{32}\left(\operatorname{Ei}(5\,bx)+\operatorname{Ei}(-5\,bx)\right)\cosh(5\,a) \\ &+ \frac{1}{32}\left(\operatorname{Ei}(3\,bx)+\operatorname{Ei}(-3\,bx)\right)\cosh(3\,a) \\ &- \frac{1}{16}\left(\operatorname{Ei}(bx)+\operatorname{Ei}(-bx)\right)\cosh(a) \\ &+ \frac{1}{32}\left(\operatorname{Ei}(5\,bx)-\operatorname{Ei}(-5\,bx)\right)\sinh(5\,a) \\ &+ \frac{1}{32}\left(\operatorname{Ei}(3\,bx)-\operatorname{Ei}(-3\,bx)\right)\sinh(3\,a) \\ &- \frac{1}{16}\left(\operatorname{Ei}(bx)-\operatorname{Ei}(-bx)\right)\sinh(a)\end{aligned}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
1/32*(Ei(5*b*x) + Ei(-5*b*x))*cosh(5*a) + 1/32*(Ei(3*b*x) + Ei(-3*b*x))*co
sh(3*a) - 1/16*(Ei(b*x) + Ei(-b*x))*cosh(a) + 1/32*(Ei(5*b*x) - Ei(-5*b*x)
)*sinh(5*a) + 1/32*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 1/16*(Ei(b*x) - Ei
(-b*x))*sinh(a)
```

3.303.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x,x)
```

output
```
Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x, x)
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{32}\,\mathrm{Ei}(5\,bx)\,e^{(5\,a)} + \frac{1}{32}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{16}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{32}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{1}{32}\,\mathrm{Ei}(-5\,bx)\,e^{(-5\,a)} - \frac{1}{16}\,\mathrm{Ei}(bx)\,e^{a}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
1/32*Ei(5*b*x)*e^(5*a) + 1/32*Ei(3*b*x)*e^(3*a) - 1/16*Ei(-b*x)*e^(-a) + 1
/32*Ei(-3*b*x)*e^(-3*a) + 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{32}\operatorname{Ei}(5\,bx)\,e^{(5\,a)} + \frac{1}{32}\operatorname{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{16}\operatorname{Ei}(-bx)\,e^{(-a)} + \frac{1}{32}\operatorname{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{1}{32}\operatorname{Ei}(-5\,bx)\,e^{(-5\,a)} - \frac{1}{16}\operatorname{Ei}(bx)\,e^{a}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="giac")
```

output
```
1/32*Ei(5*b*x)*e^(5*a) + 1/32*Ei(3*b*x)*e^(3*a) - 1/16*Ei(-b*x)*e^(-a) + 1
/32*Ei(-3*b*x)*e^(-3*a) + 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^3\sinh(a+b\,x)^2}{x}\,dx$$

input
```
int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x,x)
```

output
```
int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x, x)
```

3.304 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx$

3.304.1 Optimal result

Integrand size = 20, antiderivative size = 124

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \frac{\cosh(a+bx)}{8x} - \frac{\cosh(3a+3bx)}{16x} - \frac{\cosh(5a+5bx)}{16x} - \frac{1}{8}b\text{Chi}(bx)\sinh(a) + \frac{3}{16}b\text{Chi}(3bx)\sinh(3a) + \frac{5}{16}b\text{Chi}(5bx)\sinh(5a) - \frac{1}{8}b\cosh(a)\text{Shi}(bx) + \frac{3}{16}b\cosh(3a)\text{Shi}(3bx) + \frac{5}{16}b\cosh(5a)\text{Shi}(5bx)$$

output
```
1/8*cosh(b*x+a)/x-1/16*cosh(3*b*x+3*a)/x-1/16*cosh(5*b*x+5*a)/x-1/8*b*cosh
(a)*Shi(b*x)+3/16*b*cosh(3*a)*Shi(3*b*x)+5/16*b*cosh(5*a)*Shi(5*b*x)-1/8*b
*Chi(b*x)*sinh(a)+3/16*b*Chi(3*b*x)*sinh(3*a)+5/16*b*Chi(5*b*x)*sinh(5*a)
```

3.304.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{-2\cosh(a+bx) + \cosh(3(a+bx)) + \cosh(5(a+bx)) + 2bx\text{Chi}(bx)\sinh(a) - 3bx\text{Chi}(3bx)\sinh(3a) -}{16x}$$

input
```
Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^2,x]
```

output
```
-1/16*(-2*Cosh[a + b*x] + Cosh[3*(a + b*x)] + Cosh[5*(a + b*x)] + 2*b*x*Co
shIntegral[b*x]*Sinh[a] - 3*b*x*CoshIntegral[3*b*x]*Sinh[3*a] - 5*b*x*Cosh
Integral[5*b*x]*Sinh[5*a] + 2*b*x*Cosh[a]*SinhIntegral[b*x] - 3*b*x*Cosh[3
*a]*SinhIntegral[3*b*x] - 5*b*x*Cosh[5*a]*SinhIntegral[5*b*x])/x
```

3.304.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{ 5971}$$

$$\int \left(-\frac{\cosh(a+bx)}{8x^2}+\frac{\cosh(3a+3bx)}{16x^2}+\frac{\cosh(5a+5bx)}{16x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{8}b\sinh(a)\text{Chi}(bx)+\frac{3}{16}b\sinh(3a)\text{Chi}(3bx)+\frac{5}{16}b\sinh(5a)\text{Chi}(5bx)-\frac{1}{8}b\cosh(a)\text{Shi}(bx)+\frac{3}{16}b\cosh(3a)\text{Shi}(3bx)+\frac{5}{16}b\cosh(5a)\text{Shi}(5bx)+\frac{\cosh(a+bx)}{8x}-\frac{\cosh(3a+3bx)}{16x}-\frac{\cosh(5a+5bx)}{16x}$$

input
```
Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^2,x]
```

output
```
Cosh[a + b*x]/(8*x) - Cosh[3*a + 3*b*x]/(16*x) - Cosh[5*a + 5*b*x]/(16*x)
- (b*CoshIntegral[b*x]*Sinh[a])/8 + (3*b*CoshIntegral[3*b*x]*Sinh[3*a])/16
 + (5*b*CoshIntegral[5*b*x]*Sinh[5*a])/16 - (b*Cosh[a]*SinhIntegral[b*x])/
8 + (3*b*Cosh[3*a]*SinhIntegral[3*b*x])/16 + (5*b*Cosh[5*a]*SinhIntegral[5
*b*x])/16
```

3.304.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.304.4 Maple [A] (verified)

Time = 9.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

method	result
risch	$\frac{3\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)bx-2\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)bx+2\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)bx-3\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)bx+5\,\mathrm{e}^{-5a}\,\mathrm{Ei}_1(5bx)bx-5\,\mathrm{e}^{5a}\,\mathrm{Ei}_1(-5bx)bx+2\,\mathrm{e}^{bx+a}-\mathrm{e}^{-3bx-}}{32x}$

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*(3*exp(-3*a)*Ei(1,3*b*x)*b*x-2*exp(-a)*Ei(1,b*x)*b*x+2*exp(a)*Ei(1,-b
*x)*b*x-3*exp(3*a)*Ei(1,-3*b*x)*b*x+5*exp(-5*a)*Ei(1,5*b*x)*b*x-5*exp(5*a)
*Ei(1,-5*b*x)*b*x+2*exp(b*x+a)-exp(-3*b*x-3*a)+2*exp(-b*x-a)-exp(3*b*x+3*a
)-exp(-5*b*x-5*a)-exp(5*b*x+5*a))/x
```

3.304.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(106) = 212.

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.73

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx =$$
$$-\frac{2\,\cosh\left(bx+a\right)^5+10\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+2\,\cosh\left(bx+a\right)^3+2\left(10\,\cosh\left(bx+a\right)^3+3\,\cosh\left(\right.\right.}{}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
-1/32*(2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b*x + a)^4 + 2*cosh(b*x +
 a)^3 + 2*(10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 5*(b*x*
Ei(5*b*x) - b*x*Ei(-5*b*x))*cosh(5*a) - 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))
*cosh(3*a) + 2*(b*x*Ei(b*x) - b*x*Ei(-b*x))*cosh(a) - 5*(b*x*Ei(5*b*x) + b
*x*Ei(-5*b*x))*sinh(5*a) - 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*b*x))*sinh(3*a) +
2*(b*x*Ei(b*x) + b*x*Ei(-b*x))*sinh(a) - 4*cosh(b*x + a))/x
```

3.304.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x**2,x)
```

output
```
Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x**2, x)
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{5}{32}\,be^{(-5\,a)}\Gamma(-1,5\,bx) - \frac{3}{32}\,be^{(-3\,a)}\Gamma(-1,3\,bx) + \frac{1}{16}\,be^{(-a)}\Gamma(-1,bx) - \frac{1}{16}\,be^{a}\Gamma(-1,-bx) + \frac{3}{32}\,be^{(3\,a)}\Gamma(-1,-3\,bx) + \frac{5}{32}\,be^{(5\,a)}\Gamma(-1,-5\,bx)$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-5/32*b*e^(-5*a)*gamma(-1, 5*b*x) - 3/32*b*e^(-3*a)*gamma(-1, 3*b*x) + 1/1
6*b*e^(-a)*gamma(-1, b*x) - 1/16*b*e^a*gamma(-1, -b*x) + 3/32*b*e^(3*a)*ga
mma(-1, -3*b*x) + 5/32*b*e^(5*a)*gamma(-1, -5*b*x)
```

3.304.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx$$
$$= \frac{5\,bx\mathrm{Ei}(5\,bx)\,e^{(5\,a)} + 3\,bx\mathrm{Ei}(3\,bx)\,e^{(3\,a)} + 2\,bx\mathrm{Ei}(-bx)\,e^{(-a)} - 3\,bx\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - 5\,bx\mathrm{Ei}(-5\,bx)\,e^{(-5\,a)} -}{32\,x}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
1/32*(5*b*x*Ei(5*b*x)*e^(5*a) + 3*b*x*Ei(3*b*x)*e^(3*a) + 2*b*x*Ei(-b*x)*e
^(-a) - 3*b*x*Ei(-3*b*x)*e^(-3*a) - 5*b*x*Ei(-5*b*x)*e^(-5*a) - 2*b*x*Ei(b
*x)*e^a - e^(5*b*x + 5*a) - e^(3*b*x + 3*a) + 2*e^(b*x + a) + 2*e^(-b*x -
a) - e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a))/x
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^3\,\sinh(a+b\,x)^2}{x^2}\,dx$$

input
```
int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^2,x)
```

output
```
int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^2, x)
```

3.305 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx$

3.305.1 Optimal result

Integrand size = 20, antiderivative size = 184

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx = {} & \frac{\cosh(a+bx)}{16x^2} - \frac{\cosh(3a+3bx)}{32x^2} \\ & - \frac{\cosh(5a+5bx)}{32x^2} - \frac{1}{16}b^2\cosh(a)\mathrm{Chi}(bx) \\ & + \frac{9}{32}b^2\cosh(3a)\mathrm{Chi}(3bx) + \frac{25}{32}b^2\cosh(5a)\mathrm{Chi}(5bx) \\ & + \frac{b\sinh(a+bx)}{16x} - \frac{3b\sinh(3a+3bx)}{32x} \\ & - \frac{5b\sinh(5a+5bx)}{32x} - \frac{1}{16}b^2\sinh(a)\mathrm{Shi}(bx) \\ & + \frac{9}{32}b^2\sinh(3a)\mathrm{Shi}(3bx) + \frac{25}{32}b^2\sinh(5a)\mathrm{Shi}(5bx)\end{aligned}$$

output
```
-1/16*b^2*Chi(b*x)*cosh(a)+9/32*b^2*Chi(3*b*x)*cosh(3*a)+25/32*b^2*Chi(5*b
*x)*cosh(5*a)+1/16*cosh(b*x+a)/x^2-1/32*cosh(3*b*x+3*a)/x^2-1/32*cosh(5*b*
x+5*a)/x^2-1/16*b^2*Shi(b*x)*sinh(a)+9/32*b^2*Shi(3*b*x)*sinh(3*a)+25/32*b
^2*Shi(5*b*x)*sinh(5*a)+1/16*b*sinh(b*x+a)/x-3/32*b*sinh(3*b*x+3*a)/x-5/32
*b*sinh(5*b*x+5*a)/x
```

3.305.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx =$$
$$-\frac{-2\cosh(a+bx)+\cosh(3(a+bx))+\cosh(5(a+bx))+2b^2x^2\cosh(a)\text{Chi}(bx)-9b^2x^2\cosh(3a)\text{Chi}(3b$$

input
```
Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^3,x]
```

output
```
-1/32*(-2*Cosh[a + b*x] + Cosh[3*(a + b*x)] + Cosh[5*(a + b*x)] + 2*b^2*x^
2*Cosh[a]*CoshIntegral[b*x] - 9*b^2*x^2*Cosh[3*a]*CoshIntegral[3*b*x] - 25
*b^2*x^2*Cosh[5*a]*CoshIntegral[5*b*x] - 2*b*x*Sinh[a + b*x] + 3*b*x*Sinh[
3*(a + b*x)] + 5*b*x*Sinh[5*(a + b*x)] + 2*b^2*x^2*Sinh[a]*SinhIntegral[b*
x] - 9*b^2*x^2*Sinh[3*a]*SinhIntegral[3*b*x] - 25*b^2*x^2*Sinh[5*a]*SinhIn
tegral[5*b*x])/x^2
```

3.305.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x^3}\,dx$$
$$\Big\downarrow \text{ 5971}$$
$$\int \left(-\frac{\cosh(a+bx)}{8x^3}+\frac{\cosh(3a+3bx)}{16x^3}+\frac{\cosh(5a+5bx)}{16x^3}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{1}{16}b^2\cosh(a)\text{Chi}(bx)+\frac{9}{32}b^2\cosh(3a)\text{Chi}(3bx)+\frac{25}{32}b^2\cosh(5a)\text{Chi}(5bx)-\frac{1}{16}b^2\sinh(a)\text{Shi}(bx)+\frac{9}{32}b^2\sinh(3a)\text{Shi}(3bx)+\frac{25}{32}b^2\sinh(5a)\text{Shi}(5bx)+\frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2}+\frac{b\sinh(a+bx)}{16x}-\frac{3b\sinh(3a+3bx)}{32x}-\frac{5b\sinh(5a+5bx)}{32x}$$

input
```
Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^3,x]
```

output
```
Cosh[a + b*x]/(16*x^2) - Cosh[3*a + 3*b*x]/(32*x^2) - Cosh[5*a + 5*b*x]/(3
2*x^2) - (b^2*Cosh[a]*CoshIntegral[b*x])/16 + (9*b^2*Cosh[3*a]*CoshIntegra
l[3*b*x])/32 + (25*b^2*Cosh[5*a]*CoshIntegral[5*b*x])/32 + (b*Sinh[a + b*x
])/(16*x) - (3*b*Sinh[3*a + 3*b*x])/(32*x) - (5*b*Sinh[5*a + 5*b*x])/(32*x
) - (b^2*Sinh[a]*SinhIntegral[b*x])/16 + (9*b^2*Sinh[3*a]*SinhIntegral[3*b
*x])/32 + (25*b^2*Sinh[5*a]*SinhIntegral[5*b*x])/32
```

3.305.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.305.4 Maple [A] (verified)

Time = 14.24 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.36

method	result
risch	$\frac{-9\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^2b^2+2\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^2b^2+2\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^2b^2-25\,\mathrm{e}^{-5a}\,\mathrm{Ei}_1(5bx)x^2b^2-25\,\mathrm{e}^{5a}\,\mathrm{Ei}_1(-5bx)x^2b^2-9\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^2b^2}{}$

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/64*(-9*exp(-3*a)*Ei(1,3*b*x)*x^2*b^2+2*exp(-a)*Ei(1,b*x)*x^2*b^2+2*exp(a
)*Ei(1,-b*x)*x^2*b^2-25*exp(-5*a)*Ei(1,5*b*x)*x^2*b^2-25*exp(5*a)*Ei(1,-5*
b*x)*x^2*b^2-9*exp(3*a)*Ei(1,-3*b*x)*x^2*b^2+2*exp(b*x+a)*b*x-2*exp(-b*x-a
)*b*x+3*exp(-3*b*x-3*a)*b*x+5*exp(-5*b*x-5*a)*b*x-5*exp(5*b*x+5*a)*b*x-3*e
xp(3*b*x+3*a)*b*x+2*exp(b*x+a)+2*exp(-b*x-a)-exp(-3*b*x-3*a)-exp(-5*b*x-5*
a)-exp(5*b*x+5*a)-exp(3*b*x+3*a))/x^2
```

3.305.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(160) = 320$.

Time = 0.25 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.84

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx =$$
$$-\frac{10\,bx\sinh\left(bx+a\right)^5+2\,\cosh\left(bx+a\right)^5+10\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+2\left(50\,bx\cosh\left(bx+a\right)^2+3\,b\right.}{}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
-1/64*(10*b*x*sinh(b*x + a)^5 + 2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(
b*x + a)^4 + 2*(50*b*x*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a)^3 + 2*cosh(b
*x + a)^3 + 2*(10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 25*
(b^2*x^2*Ei(5*b*x) + b^2*x^2*Ei(-5*b*x))*cosh(5*a) - 9*(b^2*x^2*Ei(3*b*x)
+ b^2*x^2*Ei(-3*b*x))*cosh(3*a) + 2*(b^2*x^2*Ei(b*x) + b^2*x^2*Ei(-b*x))*c
osh(a) + 2*(25*b*x*cosh(b*x + a)^4 + 9*b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b
*x + a) - 25*(b^2*x^2*Ei(5*b*x) - b^2*x^2*Ei(-5*b*x))*sinh(5*a) - 9*(b^2*x
^2*Ei(3*b*x) - b^2*x^2*Ei(-3*b*x))*sinh(3*a) + 2*(b^2*x^2*Ei(b*x) - b^2*x^
2*Ei(-b*x))*sinh(a) - 4*cosh(b*x + a))/x^2
```

3.305.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x**3,x)
```

output
```
Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x**3, x)
```

3.305.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.48

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx = -\frac{25}{32}\,b^2e^{(-5\,a)}\Gamma(-2,5\,bx) - \frac{9}{32}\,b^2e^{(-3\,a)}\Gamma(-2,3\,bx) + \frac{1}{16}\,b^2e^{(-a)}\Gamma(-2,bx) + \frac{1}{16}\,b^2e^{a}\Gamma(-2,-bx) - \frac{9}{32}\,b^2e^{(3\,a)}\Gamma(-2,-3\,bx) - \frac{25}{32}\,b^2e^{(5\,a)}\Gamma(-2,-5\,bx)$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
-25/32*b^2*e^(-5*a)*gamma(-2, 5*b*x) - 9/32*b^2*e^(-3*a)*gamma(-2, 3*b*x)
+ 1/16*b^2*e^(-a)*gamma(-2, b*x) + 1/16*b^2*e^a*gamma(-2, -b*x) - 9/32*b^2
*e^(3*a)*gamma(-2, -3*b*x) - 25/32*b^2*e^(5*a)*gamma(-2, -5*b*x)
```

3.305.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.32

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \frac{25\,b^2x^2\text{Ei}(5\,bx)\,e^{(5\,a)} + 9\,b^2x^2\text{Ei}(3\,bx)\,e^{(3\,a)} - 2\,b^2x^2\text{Ei}(-bx)\,e^{(-a)} + 9\,b^2x^2\text{Ei}(-3\,bx)\,e^{(-3\,a)} + 25\,b^2x^2\text{Ei}(-5}{}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
1/64*(25*b^2*x^2*Ei(5*b*x)*e^(5*a) + 9*b^2*x^2*Ei(3*b*x)*e^(3*a) - 2*b^2*x
^2*Ei(-b*x)*e^(-a) + 9*b^2*x^2*Ei(-3*b*x)*e^(-3*a) + 25*b^2*x^2*Ei(-5*b*x)
*e^(-5*a) - 2*b^2*x^2*Ei(b*x)*e^a - 5*b*x*e^(5*b*x + 5*a) - 3*b*x*e^(3*b*x
 + 3*a) + 2*b*x*e^(b*x + a) - 2*b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3*a)
+ 5*b*x*e^(-5*b*x - 5*a) - e^(5*b*x + 5*a) - e^(3*b*x + 3*a) + 2*e^(b*x +
a) + 2*e^(-b*x - a) - e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a))/x^2
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)^3\,\sinh(a+b\,x)^2}{x^3}\,dx$$

input
```
int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^3,x)
```

output
```
int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^3, x)
```

3.306 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx$

3.306.1 Optimal result

Integrand size = 20, antiderivative size = 238

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx &= \frac{\cosh(a+bx)}{24x^3}+\frac{b^2\cosh(a+bx)}{48x}-\frac{\cosh(3a+3bx)}{48x^3}\\ &-\frac{3b^2\cosh(3a+3bx)}{32x}-\frac{\cosh(5a+5bx)}{48x^3}\\ &-\frac{25b^2\cosh(5a+5bx)}{96x}-\frac{1}{48}b^3\text{Chi}(bx)\sinh(a)\\ &+\frac{9}{32}b^3\text{Chi}(3bx)\sinh(3a)+\frac{125}{96}b^3\text{Chi}(5bx)\sinh(5a)\\ &+\frac{b\sinh(a+bx)}{48x^2}-\frac{b\sinh(3a+3bx)}{32x^2}\\ &-\frac{5b\sinh(5a+5bx)}{96x^2}-\frac{1}{48}b^3\cosh(a)\text{Shi}(bx)\\ &+\frac{9}{32}b^3\cosh(3a)\text{Shi}(3bx)+\frac{125}{96}b^3\cosh(5a)\text{Shi}(5bx)\end{aligned}$$

output
```
1/24*cosh(b*x+a)/x^3+1/48*b^2*cosh(b*x+a)/x-1/48*cosh(3*b*x+3*a)/x^3-3/32*
b^2*cosh(3*b*x+3*a)/x-1/48*cosh(5*b*x+5*a)/x^3-25/96*b^2*cosh(5*b*x+5*a)/x
-1/48*b^3*cosh(a)*Shi(b*x)+9/32*b^3*cosh(3*a)*Shi(3*b*x)+125/96*b^3*cosh(5
*a)*Shi(5*b*x)-1/48*b^3*Chi(b*x)*sinh(a)+9/32*b^3*Chi(3*b*x)*sinh(3*a)+125
/96*b^3*Chi(5*b*x)*sinh(5*a)+1/48*b*sinh(b*x+a)/x^2-1/32*b*sinh(3*b*x+3*a)
/x^2-5/96*b*sinh(5*b*x+5*a)/x^2
```

3.306.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx$$
$$= \frac{4\cosh(a+bx)+2b^2x^2\cosh(a+bx)-2\cosh(3(a+bx))-9b^2x^2\cosh(3(a+bx))-2\cosh(5(a+bx))-2}{}$$

input
```
Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^4,x]
```

output
```
(4*Cosh[a + b*x] + 2*b^2*x^2*Cosh[a + b*x] - 2*Cosh[3*(a + b*x)] - 9*b^2*x
^2*Cosh[3*(a + b*x)] - 2*Cosh[5*(a + b*x)] - 25*b^2*x^2*Cosh[5*(a + b*x)]
- 2*b^3*x^3*CoshIntegral[b*x]*Sinh[a] + 27*b^3*x^3*CoshIntegral[3*b*x]*Sin
h[3*a] + 125*b^3*x^3*CoshIntegral[5*b*x]*Sinh[5*a] + 2*b*x*Sinh[a + b*x] -
 3*b*x*Sinh[3*(a + b*x)] - 5*b*x*Sinh[5*(a + b*x)] - 2*b^3*x^3*Cosh[a]*Sin
hIntegral[b*x] + 27*b^3*x^3*Cosh[3*a]*SinhIntegral[3*b*x] + 125*b^3*x^3*Co
sh[5*a]*SinhIntegral[5*b*x])/(96*x^3)
```

3.306.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x^4}\,dx$$
$$\Big\downarrow \text{5971}$$
$$\int \left(-\frac{\cosh(a+bx)}{8x^4}+\frac{\cosh(3a+3bx)}{16x^4}+\frac{\cosh(5a+5bx)}{16x^4}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{1}{48}b^3\sinh(a)\mathrm{Chi}(bx)+\frac{9}{32}b^3\sinh(3a)\mathrm{Chi}(3bx)+\frac{125}{96}b^3\sinh(5a)\mathrm{Chi}(5bx)-\frac{1}{48}b^3\cosh(a)\mathrm{Shi}(bx)+\frac{9}{32}b^3\cosh(3a)\mathrm{Shi}(3bx)+\frac{125}{96}b^3\cosh(5a)\mathrm{Shi}(5bx)+\frac{b^2\cosh(a+bx)}{48x}-\frac{3b^2\cosh(3a+3bx)}{32x}-\frac{25b^2\cosh(5a+5bx)}{96x}+\frac{\cosh(a+bx)}{24x^3}-\frac{\cosh(3a+3bx)}{48x^3}-\frac{\cosh(5a+5bx)}{48x^3}+\frac{b\sinh(a+bx)}{48x^2}-\frac{b\sinh(3a+3bx)}{32x^2}-\frac{5b\sinh(5a+5bx)}{96x^2}$$

input
```
Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^4,x]
```

output
```
Cosh[a + b*x]/(24*x^3) + (b^2*Cosh[a + b*x])/(48*x) - Cosh[3*a + 3*b*x]/(4
8*x^3) - (3*b^2*Cosh[3*a + 3*b*x])/(32*x) - Cosh[5*a + 5*b*x]/(48*x^3) - (
25*b^2*Cosh[5*a + 5*b*x])/(96*x) - (b^3*CoshIntegral[b*x]*Sinh[a])/48 + (9
*b^3*CoshIntegral[3*b*x]*Sinh[3*a])/32 + (125*b^3*CoshIntegral[5*b*x]*Sinh
[5*a])/96 + (b*Sinh[a + b*x])/(48*x^2) - (b*Sinh[3*a + 3*b*x])/(32*x^2) -
(5*b*Sinh[5*a + 5*b*x])/(96*x^2) - (b^3*Cosh[a]*SinhIntegral[b*x])/48 + (9
*b^3*Cosh[3*a]*SinhIntegral[3*b*x])/32 + (125*b^3*Cosh[5*a]*SinhIntegral[5
*b*x])/96
```

3.306.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.306.4 Maple [A] (verified)

Time = 21.04 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{-27\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^3b^3+2\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^3b^3-2\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^3b^3+27\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^3b^3-125\,\mathrm{e}^{-5a}\,\mathrm{Ei}_1(5bx)x^3b^3+125\,\mathrm{e}^{5a}\,\mathrm{Ei}_1(-5b}{}$

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-27*exp(-3*a)*Ei(1,3*b*x)*x^3*b^3+2*exp(-a)*Ei(1,b*x)*x^3*b^3-2*ex
p(a)*Ei(1,-b*x)*x^3*b^3+27*exp(3*a)*Ei(1,-3*b*x)*x^3*b^3-125*exp(-5*a)*Ei(
1,5*b*x)*x^3*b^3+125*exp(5*a)*Ei(1,-5*b*x)*x^3*b^3+9*exp(-3*b*x-3*a)*b^2*x
^2-2*exp(-b*x-a)*b^2*x^2+9*exp(3*b*x+3*a)*b^2*x^2-2*exp(b*x+a)*b^2*x^2+25*
exp(-5*b*x-5*a)*b^2*x^2+25*exp(5*b*x+5*a)*b^2*x^2-3*exp(-3*b*x-3*a)*b*x+2*
exp(-b*x-a)*b*x+3*exp(3*b*x+3*a)*b*x-2*exp(b*x+a)*b*x-5*exp(-5*b*x-5*a)*b*
x+5*exp(5*b*x+5*a)*b*x+2*exp(-3*b*x-3*a)-4*exp(-b*x-a)+2*exp(3*b*x+3*a)-4*
exp(b*x+a)+2*exp(-5*b*x-5*a)+2*exp(5*b*x+5*a))/x^3
```

3.306.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.67

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx =$$
$$-\frac{10\,bx\sinh\left(bx+a\right)^5+2\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)^5+10\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+2}{}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

output
```
-1/192*(10*b*x*sinh(b*x + a)^5 + 2*(25*b^2*x^2 + 2)*cosh(b*x + a)^5 + 10*(
25*b^2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^4 + 2*(9*b^2*x^2 + 2)*cosh(b*x
 + a)^3 + 2*(50*b*x*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a)^3 + 2*(10*(25*b
^2*x^2 + 2)*cosh(b*x + a)^3 + 3*(9*b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x +
a)^2 - 4*(b^2*x^2 + 2)*cosh(b*x + a) - 125*(b^3*x^3*Ei(5*b*x) - b^3*x^3*Ei
(-5*b*x))*cosh(5*a) - 27*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*cosh(3*a
) + 2*(b^3*x^3*Ei(b*x) - b^3*x^3*Ei(-b*x))*cosh(a) + 2*(25*b*x*cosh(b*x +
a)^4 + 9*b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a) - 125*(b^3*x^3*Ei(5*b*
x) + b^3*x^3*Ei(-5*b*x))*sinh(5*a) - 27*(b^3*x^3*Ei(3*b*x) + b^3*x^3*Ei(-3
*b*x))*sinh(3*a) + 2*(b^3*x^3*Ei(b*x) + b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

3.306.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x^4}\,dx$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x**4,x)
```

output
```
Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x**4, x)
```

3.306.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.37

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx = -\frac{125}{32}\,b^3e^{(-5\,a)}\Gamma(-3,5\,bx) - \frac{27}{32}\,b^3e^{(-3\,a)}\Gamma(-3,3\,bx) + \frac{1}{16}\,b^3e^{(-a)}\Gamma(-3,bx) - \frac{1}{16}\,b^3e^{a}\Gamma(-3,-bx) + \frac{27}{32}\,b^3e^{(3\,a)}\Gamma(-3,-3\,bx) + \frac{125}{32}\,b^3e^{(5\,a)}\Gamma(-3,-5\,bx)$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

output
```
-125/32*b^3*e^(-5*a)*gamma(-3, 5*b*x) - 27/32*b^3*e^(-3*a)*gamma(-3, 3*b*x
) + 1/16*b^3*e^(-a)*gamma(-3, b*x) - 1/16*b^3*e^a*gamma(-3, -b*x) + 27/32*
b^3*e^(3*a)*gamma(-3, -3*b*x) + 125/32*b^3*e^(5*a)*gamma(-3, -5*b*x)
```

3.306.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.44

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \frac{125\,b^3x^3\mathrm{Ei}(5\,bx)\,e^{(5\,a)} + 27\,b^3x^3\mathrm{Ei}(3\,bx)\,e^{(3\,a)} + 2\,b^3x^3\mathrm{Ei}(-bx)\,e^{(-a)} - 27\,b^3x^3\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - 125\,b^3x^3\mathrm{E}}{}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x, algorithm="giac")
```

output
```
1/192*(125*b^3*x^3*Ei(5*b*x)*e^(5*a) + 27*b^3*x^3*Ei(3*b*x)*e^(3*a) + 2*b^
3*x^3*Ei(-b*x)*e^(-a) - 27*b^3*x^3*Ei(-3*b*x)*e^(-3*a) - 125*b^3*x^3*Ei(-5
*b*x)*e^(-5*a) - 2*b^3*x^3*Ei(b*x)*e^a - 25*b^2*x^2*e^(5*b*x + 5*a) - 9*b^
2*x^2*e^(3*b*x + 3*a) + 2*b^2*x^2*e^(b*x + a) + 2*b^2*x^2*e^(-b*x - a) - 9
*b^2*x^2*e^(-3*b*x - 3*a) - 25*b^2*x^2*e^(-5*b*x - 5*a) - 5*b*x*e^(5*b*x +
 5*a) - 3*b*x*e^(3*b*x + 3*a) + 2*b*x*e^(b*x + a) - 2*b*x*e^(-b*x - a) + 3
*b*x*e^(-3*b*x - 3*a) + 5*b*x*e^(-5*b*x - 5*a) - 2*e^(5*b*x + 5*a) - 2*e^(
3*b*x + 3*a) + 4*e^(b*x + a) + 4*e^(-b*x - a) - 2*e^(-3*b*x - 3*a) - 2*e^(
-5*b*x - 5*a))/x^3
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)^3\sinh(a+b\,x)^2}{x^4}\,dx$$

input
```
int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^4,x)
```

output
```
int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^4, x)
```

3.307 $\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx$

3.307.1 Optimal result

Integrand size = 18, antiderivative size = 141

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{2^{-2(3+m)}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b} - \frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} - \frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b} + \frac{2^{-2(3+m)}e^{-4a}x^m(bx)^{-m}\Gamma(1+m,4bx)}{b}$$

output
```
exp(4*a)*x^m*GAMMA(1+m,-4*b*x)/(2^(6+2*m))/b/((-b*x)^m)-2^(-4-m)*exp(2*a)*
x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)-2^(-4-m)*x^m*GAMMA(1+m,2*b*x)/b/exp(2*a
)/((b*x)^m)+x^m*GAMMA(1+m,4*b*x)/(2^(6+2*m))/b/exp(4*a)/((b*x)^m)
```

3.307.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{4^{-3-m}e^{-4a}x^m(-b^2x^2)^{-m}\left(e^{8a}(bx)^m\Gamma(1+m,-4bx) - 2^{2+m}e^{6a}(bx)^m\Gamma(1+m,-2bx) + (-bx)^m\left(-2^{2+m}e^{2a}\right.\right.}{b}$$

input
```
Integrate[x^m*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
(4^(-3 - m)*x^m*(E^(8*a)*(b*x)^m*Gamma[1 + m, -4*b*x] - 2^(2 + m)*E^(6*a)*
(b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*(-(2^(2 + m)*E^(2*a)*Gamma[1 + m
, 2*b*x]) + Gamma[1 + m, 4*b*x])))/(b*E^(4*a)*(-(b^2*x^2))^m)
```

3.307.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh^3(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{1}{8}x^m \sinh(4a+4bx) - \frac{1}{4}x^m \sinh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\Gamma(m+1,-4bx)}{b} - \frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b} - \frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b} + \frac{e^{-4a}2^{-2(m+3)}x^m(bx)^{-m}\Gamma(m+1,4bx)}{b}$$

input
```
Int[x^m*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
(E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*b*(-(b*x))^m) - (2^(-4 -
 m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) - (2^(-4 - m)*x^m*Gam
ma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3
 + m))*b*E^(4*a)*(b*x)^m)
```

3.307.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.307.4 Maple [F]

$$\int x^m \cosh(bx+a)\sinh(bx+a)^3\,dx$$

input
```
int(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

output
```
int(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

3.307.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.22

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\cosh(m\log(4b)+4a)\Gamma(m+1,4bx) - 4\cosh(m\log(2b)+2a)\Gamma(m+1,2bx) - 4\cosh(m\log(-2b) -$$

input
```
integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/64*(cosh(m*log(4*b) + 4*a)*gamma(m + 1, 4*b*x) - 4*cosh(m*log(2*b) + 2*a
)*gamma(m + 1, 2*b*x) - 4*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + c
osh(m*log(-4*b) - 4*a)*gamma(m + 1, -4*b*x) - gamma(m + 1, 4*b*x)*sinh(m*l
og(4*b) + 4*a) + 4*gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) + 4*gamma(m
+ 1, -2*b*x)*sinh(m*log(-2*b) - 2*a) - gamma(m + 1, -4*b*x)*sinh(m*log(-4*
b) - 4*a))/b
```

3.307.6 Sympy [F]

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = \int x^m \sinh^3(a+bx)\cosh(a+bx)\,dx$$

input
```
integrate(x**m*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Integral(x**m*sinh(a + b*x)**3*cosh(a + b*x), x)
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

$$\begin{aligned}\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = &\frac{1}{16}(4\,bx)^{-m-1}x^{m+1}e^{(-4\,a)}\Gamma(m+1,4\,bx)\\ &-\frac{1}{8}(2\,bx)^{-m-1}x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx)\\ &+\frac{1}{8}(-2\,bx)^{-m-1}x^{m+1}e^{(2\,a)}\Gamma(m+1,-2\,bx)\\ &-\frac{1}{16}(-4\,bx)^{-m-1}x^{m+1}e^{(4\,a)}\Gamma(m+1,-4\,bx)\end{aligned}$$

input
```
integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/16*(4*b*x)^(-m - 1)*x^(m + 1)*e^(-4*a)*gamma(m + 1, 4*b*x) - 1/8*(2*b*x)
^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) + 1/8*(-2*b*x)^(-m - 1)*x
^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/16*(-4*b*x)^(-m - 1)*x^(m + 1)*e
^(4*a)*gamma(m + 1, -4*b*x)
```

3.307.8 Giac [F]

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = \int x^m \cosh(bx+a)\sinh(bx+a)^3\,dx$$

input
```
integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)*sinh(b*x + a)^3, x)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = \int x^m \cosh(a+b\,x)\,\sinh(a+b\,x)^3\,dx$$

input
```
int(x^m*cosh(a + b*x)*sinh(a + b*x)^3,x)
```

output
```
int(x^m*cosh(a + b*x)*sinh(a + b*x)^3, x)
```

3.308 $\int x^3 \cosh(a+bx) \sinh^3(a+bx) \, dx$

3.308.1 Optimal result

Integrand size = 18, antiderivative size = 155

$$\begin{aligned}\int x^3 \cosh(a+bx) \sinh^3(a+bx) \, dx = & -\frac{45x}{256b^3} - \frac{3x^3}{32b} + \frac{45\cosh(a+bx)\sinh(a+bx)}{256b^4} \\ & + \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} \\ & - \frac{9x\sinh^2(a+bx)}{32b^3} - \frac{3\cosh(a+bx)\sinh^3(a+bx)}{128b^4} \\ & - \frac{3x^2\cosh(a+bx)\sinh^3(a+bx)}{16b^2} \\ & + \frac{3x\sinh^4(a+bx)}{32b^3} + \frac{x^3\sinh^4(a+bx)}{4b}\end{aligned}$$

output
```
-45/256*x/b^3-3/32*x^3/b+45/256*cosh(b*x+a)*sinh(b*x+a)/b^4+9/32*x^2*cosh(
b*x+a)*sinh(b*x+a)/b^2-9/32*x*sinh(b*x+a)^2/b^3-3/128*cosh(b*x+a)*sinh(b*x
+a)^3/b^4-3/16*x^2*cosh(b*x+a)*sinh(b*x+a)^3/b^2+3/32*x*sinh(b*x+a)^4/b^3+
1/4*x^3*sinh(b*x+a)^4/b
```

3.308.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.61

$$\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{2bx(3+8b^2x^2)\cosh(4(a+bx))+48(1+2b^2x^2)\sinh(2(a+bx))-\cosh(2(a+bx))\left(32bx(3+2b^2x^2)+3(\right.}{512b^4}$$

input
```
Integrate[x^3*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
(2*b*x*(3 + 8*b^2*x^2)*Cosh[4*(a + b*x)] + 48*(1 + 2*b^2*x^2)*Sinh[2*(a +
b*x)] - Cosh[2*(a + b*x)]*(32*b*x*(3 + 2*b^2*x^2) + 3*(1 + 8*b^2*x^2)*Sinh
[2*(a + b*x)]))/(512*b^4)
```

3.308.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.34, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.056$, Rules used = {5895, 3042, 3792, 25, 3042, 25, 3115, 25, 3042, 25, 3115, 24, 3792, 15, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sinh^3(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{ 5895}$$

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\int x^2\sinh^4(a+bx)dx}{4b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\int x^2\sin(ia+ibx)^4dx}{4b}$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{\int\sinh^4(a+bx)dx}{8b^2}+\frac{3}{4}\int -x^2\sinh^2(a+bx)dx-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{4b}\right)}{4b}$$

$$\Big\downarrow 25$$

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{\int\sinh^4(a+bx)dx}{8b^2}-\frac{3}{4}\int x^2\sinh^2(a+bx)dx-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{4b}\right)}{4b}$$

$$\Big\downarrow 3042$$

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{\int\sin(ia+ibx)^4dx}{8b^2}-\frac{3}{4}\int -x^2\sin(ia+ibx)^2dx-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{4b}\right)}{4b}$$

$$\Big\downarrow 25$$

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{\int\sin(ia+ibx)^4dx}{8b^2}+\frac{3}{4}\int x^2\sin(ia+ibx)^2dx-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{4b}\right)}{4b}$$

$$\Big\downarrow 3115$$

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{\frac{3}{4}\int-\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}}{8b^2}+\frac{3}{4}\int x^2\sin(ia+ibx)^2dx-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{4b}\right)}{4b}$$

$$\Big\downarrow 25$$

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3}{4}\int\sinh^2(a+bx)dx}{8b^2}+\frac{3}{4}\int x^2\sin(ia+ibx)^2dx-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{4b}\right)}{4b}$$

$$\Big\downarrow 3042$$

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3}{4}\int-\sin(ia+ibx)^2dx}{8b^2}+\frac{3}{4}\int x^2\sin(ia+ibx)^2dx-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{4b}\right)}{4b}$$

$$\Big\downarrow 25$$

3.308. $\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx$

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\int\sin(ia+ibx)^2dx}{8b^2}+\frac{3}{4}\int x^2\sin(ia+ibx)^2dx-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{4b}\right)}{4b}$$

$\Big\downarrow$ 3115

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}}{8b^2}+\frac{3}{4}\int x^2\sin(ia+ibx)^2dx-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{4b}\right.}{4b}$$

$\Big\downarrow$ 24

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{3}{4}\int x^2\sin(ia+ibx)^2dx-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\left(\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{8b^2}+\frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{4b}\right)}{4b}$$

$\Big\downarrow$ 3792

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{3}{4}\left(\frac{\int-\sinh^2(a+bx)dx}{2b^2}+\frac{\int x^2dx}{2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\left(\frac{x}{2}-\sin\right.}{8b^2}\right.}{4b}$$

$\Big\downarrow$ 15

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{3}{4}\left(\frac{\int-\sinh^2(a+bx)dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\left(\frac{x}{2}-\sinh(a+\right.}{8b^2}\right.}{4b}$$

$\Big\downarrow$ 25

$$\frac{x^3\sinh^4(a+bx)}{4b}-\frac{3\left(\frac{3}{4}\left(-\frac{\int\sinh^2(a+bx)dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\left(\frac{x}{2}-\sinh(a+\right.}{8b^2}\right.}{4b}$$

$\Big\downarrow$ 3042

3.308. $\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx$

$$\frac{x^3\sinh^4(a+bx)}{4b}-$$

$$\frac{3\left(\frac{3}{4}\left(-\frac{\int -\sin(ia+ibx)^2dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\left(\frac{x}{2}-\sinh\right.}{8b^2}\right.}{4b}$$

$\downarrow$ 25

$$\frac{x^3\sinh^4(a+bx)}{4b}-$$

$$\frac{3\left(\frac{3}{4}\left(\frac{\int \sin(ia+ibx)^2dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\left(\frac{x}{2}-\sinh(a+b\right.}{8b^2}\right.}{4b}$$

$\downarrow$ 3115

$$\frac{x^3\sinh^4(a+bx)}{4b}-$$

$$\frac{3\left(\frac{3}{4}\left(\frac{\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\left(\right.}{8b^2}\right.}{4b}$$

$\downarrow$ 24

$$\frac{x^3\sinh^4(a+bx)}{4b}-$$

$$\frac{3\left(\frac{3}{4}\left(\frac{x\sinh^2(a+bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\sinh^4(a+bx)}{8b^2}+\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\left(\frac{x}{2}-\right.}{8b^2}\right.}{4b}$$

input
```
Int[x^3*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
(x^3*Sinh[a + b*x]^4)/(4*b) - (3*((x^2*Cosh[a + b*x]*Sinh[a + b*x]^3)/(4*b
) - (x*Sinh[a + b*x]^4)/(8*b^2) + ((Cosh[a + b*x]*Sinh[a + b*x]^3)/(4*b) +
 (3*(x/2 - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4)/(8*b^2) + (3*(x^3/6 -
(x^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (x*Sinh[a + b*x]^2)/(2*b^2) + (x
/2 - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b))/(2*b^2)))/4))/(4*b)
```

3.308. $\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx$

3.308.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.308. $\int x^3 \cosh(a+bx) \sinh^3(a+bx)\,dx$

3.308.4 Maple [A] (verified)

Time = 13.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

method	result
risch	$\frac{(32x^3b^3-24x^2b^2+12bx-3)e^{4bx+4a}}{2048b^4} - \frac{(4x^3b^3-6x^2b^2+6bx-3)e^{2bx+2a}}{64b^4} - \frac{(4x^3b^3+6x^2b^2+6bx+3)e^{-2bx-2a}}{64b^4} + \frac{(32\cdots}{}$
derivativedivides	$\frac{-\frac{a^3\sinh(bx+a)^4}{4}+3a^2\left(\frac{(bx+a)\sinh(bx+a)^4}{4}-\frac{\cosh(bx+a)\sinh(bx+a)^3}{16}+\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}\right)-3a\left(\frac{(bx+a)^2\sin\cdots}{4}\right.}{}$
default	$\frac{-\frac{a^3\sinh(bx+a)^4}{4}+3a^2\left(\frac{(bx+a)\sinh(bx+a)^4}{4}-\frac{\cosh(bx+a)\sinh(bx+a)^3}{16}+\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}\right)-3a\left(\frac{(bx+a)^2\sin\cdots}{4}\right.}{}$

input
```
int(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2048*(32*b^3*x^3-24*b^2*x^2+12*b*x-3)/b^4*exp(4*b*x+4*a)-1/64*(4*b^3*x^3
-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)-1/64*(4*b^3*x^3+6*b^2*x^2+6*b*x+3)/
b^4*exp(-2*b*x-2*a)+1/2048*(32*b^3*x^3+24*b^2*x^2+12*b*x+3)/b^4*exp(-4*b*x
-4*a)
```

3.308.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.23

$$\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{(8\,b^3x^3+3\,bx)\cosh(bx+a)^4-3\,(8\,b^2x^2+1)\cosh(bx+a)\sinh(bx+a)^3+(8\,b^3x^3+3\,bx)\sinh(bx+a)^4\cdots}{}$$

input
```
integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/256*((8*b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x +
a)*sinh(b*x + a)^3 + (8*b^3*x^3 + 3*b*x)*sinh(b*x + a)^4 - 16*(2*b^3*x^3 +
 3*b*x)*cosh(b*x + a)^2 - 2*(16*b^3*x^3 - 3*(8*b^3*x^3 + 3*b*x)*cosh(b*x +
 a)^2 + 24*b*x)*sinh(b*x + a)^2 - 3*((8*b^2*x^2 + 1)*cosh(b*x + a)^3 - 16*
(2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a))/b^4
```

3.308.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.46

$$\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases}\frac{5x^3\sinh^4(a+bx)}{32b}+\frac{3x^3\sinh^2(a+bx)\cosh^2(a+bx)}{16b}-\frac{3x^3\cosh^4(a+bx)}{32b}-\frac{15x^2\sinh^3(a+bx)\cosh(a+bx)}{32b^2}+\frac{9x^2\sinh(a+bx)\cosh^3(a+b}{32b^2}\\ \frac{x^4\sinh^3(a)\cosh(a)}{4}\end{cases}$$

input
```
integrate(x**3*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Piecewise((5*x**3*sinh(a + b*x)**4/(32*b) + 3*x**3*sinh(a + b*x)**2*cosh(a
 + b*x)**2/(16*b) - 3*x**3*cosh(a + b*x)**4/(32*b) - 15*x**2*sinh(a + b*x)
**3*cosh(a + b*x)/(32*b**2) + 9*x**2*sinh(a + b*x)*cosh(a + b*x)**3/(32*b*
*2) + 51*x*sinh(a + b*x)**4/(256*b**3) + 9*x*sinh(a + b*x)**2*cosh(a + b*x
)**2/(128*b**3) - 45*x*cosh(a + b*x)**4/(256*b**3) - 51*sinh(a + b*x)**3*c
osh(a + b*x)/(256*b**4) + 45*sinh(a + b*x)*cosh(a + b*x)**3/(256*b**4), Ne
(b, 0)), (x**4*sinh(a)**3*cosh(a)/4, True))
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.10

$$\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{\left(32\,b^3x^3e^{(4\,a)}-24\,b^2x^2e^{(4\,a)}+12\,bxe^{(4\,a)}-3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4}-\frac{\left(4\,b^3x^3e^{(2\,a)}-6\,b^2x^2e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{64\,b^4}-\frac{(4\,b^3x^3+6\,b^2x^2+6\,bx+3)e^{(-2\,bx-2\,a)}}{64\,b^4}+\frac{(32\,b^3x^3+24\,b^2x^2+12\,bx+3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a) - 3*e^(4*
a))*e^(4*b*x)/b^4 - 1/64*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^
(2*a) - 3*e^(2*a))*e^(2*b*x)/b^4 - 1/64*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3
)*e^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(
-4*b*x - 4*a)/b^4
```

3.308.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{(32\,b^3x^3-24\,b^2x^2+12\,bx-3)e^{(4\,bx+4\,a)}}{2048\,b^4} - \frac{(4\,b^3x^3-6\,b^2x^2+6\,bx-3)e^{(2\,bx+2\,a)}}{64\,b^4} - \frac{(4\,b^3x^3+6\,b^2x^2+6\,bx+3)e^{(-2\,bx-2\,a)}}{64\,b^4} + \frac{(32\,b^3x^3+24\,b^2x^2+12\,bx+3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b^4 - 1/64*(
4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 - 1/64*(4*b^3*x^3 +
 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2
*x^2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

3.308.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\frac{3\,x^2\sinh(2\,a+2\,b\,x)}{16}-\frac{3\,x^2\sinh(4\,a+4\,b\,x)}{128}}{b^2} - \frac{\frac{x^3\cosh(2\,a+2\,b\,x)}{8}-\frac{x^3\cosh(4\,a+4\,b\,x)}{32}}{b} - \frac{\frac{3\,x\cosh(2\,a+2\,b\,x)}{16}-\frac{3\,x\cosh(4\,a+4\,b\,x)}{256}}{b^3} + \frac{3\sinh(2\,a+2\,b\,x)}{32\,b^4} - \frac{3\sinh(4\,a+4\,b\,x)}{1024\,b^4}$$

input
```
int(x^3*cosh(a + b*x)*sinh(a + b*x)^3,x)
```

output
```
((3*x^2*sinh(2*a + 2*b*x))/16 - (3*x^2*sinh(4*a + 4*b*x))/128)/b^2 - ((x^3
*cosh(2*a + 2*b*x))/8 - (x^3*cosh(4*a + 4*b*x))/32)/b - ((3*x*cosh(2*a + 2
*b*x))/16 - (3*x*cosh(4*a + 4*b*x))/256)/b^3 + (3*sinh(2*a + 2*b*x))/(32*b
^4) - (3*sinh(4*a + 4*b*x))/(1024*b^4)
```

3.308. $\int x^3 \cosh(a+bx) \sinh^3(a+bx)\,dx$

3.309 $\int x^2 \cosh(a+bx) \sinh^3(a+bx)\,dx$

3.309.1 Optimal result

Integrand size = 18, antiderivative size = 101

$$\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{3x^2}{32b}+\frac{3x\cosh(a+bx)\sinh(a+bx)}{16b^2} - \frac{3\sinh^2(a+bx)}{32b^3}-\frac{x\cosh(a+bx)\sinh^3(a+bx)}{8b^2} + \frac{\sinh^4(a+bx)}{32b^3}+\frac{x^2\sinh^4(a+bx)}{4b}$$

output
```
-3/32*x^2/b+3/16*x*cosh(b*x+a)*sinh(b*x+a)/b^2-3/32*sinh(b*x+a)^2/b^3-1/8*
x*cosh(b*x+a)*sinh(b*x+a)^3/b^2+1/32*sinh(b*x+a)^4/b^3+1/4*x^2*sinh(b*x+a)
^4/b
```

3.309.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int x^2\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{-16(1+2b^2x^2)\cosh(2(a+bx))+(1+8b^2x^2)\cosh(4(a+bx))+4bx(8\sinh(2(a+bx))-\sinh(4(a+bx))}{256b^3}$$

input
```
Integrate[x^2*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
(-16*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + (1 + 8*b^2*x^2)*Cosh[4*(a + b*x)]
 + 4*b*x*(8*Sinh[2*(a + b*x)] - Sinh[4*(a + b*x)]))/(256*b^3)
```

3.309.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5895, 3042, 3791, 25, 3042, 25, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sinh^3(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{5895}$$

$$\frac{x^2\sinh^4(a+bx)}{4b}-\frac{\int x\sinh^4(a+bx)dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^2\sinh^4(a+bx)}{4b}-\frac{\int x\sin(ia+ibx)^4dx}{2b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{x^2\sinh^4(a+bx)}{4b}-\frac{\frac{3}{4}\int -x\sinh^2(a+bx)dx-\frac{\sinh^4(a+bx)}{16b^2}+\frac{x\sinh^3(a+bx)\cosh(a+bx)}{4b}}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2\sinh^4(a+bx)}{4b}-\frac{-\frac{3}{4}\int x\sinh^2(a+bx)dx-\frac{\sinh^4(a+bx)}{16b^2}+\frac{x\sinh^3(a+bx)\cosh(a+bx)}{4b}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^2\sinh^4(a+bx)}{4b}-\frac{-\frac{3}{4}\int -x\sin(ia+ibx)^2dx-\frac{\sinh^4(a+bx)}{16b^2}+\frac{x\sinh^3(a+bx)\cosh(a+bx)}{4b}}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2\sinh^4(a+bx)}{4b}-\frac{\frac{3}{4}\int x\sin(ia+ibx)^2dx-\frac{\sinh^4(a+bx)}{16b^2}+\frac{x\sinh^3(a+bx)\cosh(a+bx)}{4b}}{2b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{x^2\sinh^4(a+bx)}{4b}-\frac{\frac{3}{4}\left(\frac{\int xdx}{2}+\frac{\sinh^2(a+bx)}{4b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{\sinh^4(a+bx)}{16b^2}+\frac{x\sinh^3(a+bx)\cosh(a+bx)}{4b}}{2b}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^2\sinh^4(a+bx)}{4b}-\frac{\frac{3}{4}\left(\frac{\sinh^2(a+bx)}{4b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}\right)-\frac{\sinh^4(a+bx)}{16b^2}+\frac{x\sinh^3(a+bx)\cosh(a+bx)}{4b}}{2b}$$

input
```
Int[x^2*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
(x^2*Sinh[a + b*x]^4)/(4*b) - ((x*Cosh[a + b*x]*Sinh[a + b*x]^3)/(4*b) - S
inh[a + b*x]^4/(16*b^2) + (3*(x^2/4 - (x*Cosh[a + b*x]*Sinh[a + b*x])/(2*b
) + Sinh[a + b*x]^2/(4*b^2)))/4)/(2*b)
```

3.309.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.309.4 Maple [A] (verified)

Time = 10.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.13

method	result
risch	$\frac{(8x^2b^2-4bx+1)\,\mathrm{e}^{4bx+4a}}{512b^3} - \frac{(2x^2b^2-2bx+1)\,\mathrm{e}^{2bx+2a}}{32b^3} - \frac{(2x^2b^2+2bx+1)\,\mathrm{e}^{-2bx-2a}}{32b^3} + \frac{(8x^2b^2+4bx+1)\,\mathrm{e}^{-4bx-4a}}{512b^3}$
derivativedivides	$\frac{\frac{a^2\sinh(bx+a)^4}{4} - 2a\left(\frac{(bx+a)\sinh(bx+a)^4}{4} - \frac{\cosh(bx+a)\sinh(bx+a)^3}{16} + \frac{3\cosh(bx+a)\sinh(bx+a)}{32} - \frac{3bx}{32} - \frac{3a}{32}\right) + \frac{(bx+a)^2\sinh(bx+a}{4}}{b^3}$
default	$\frac{\frac{a^2\sinh(bx+a)^4}{4} - 2a\left(\frac{(bx+a)\sinh(bx+a)^4}{4} - \frac{\cosh(bx+a)\sinh(bx+a)^3}{16} + \frac{3\cosh(bx+a)\sinh(bx+a)}{32} - \frac{3bx}{32} - \frac{3a}{32}\right) + \frac{(bx+a)^2\sinh(bx+a}{4}}{b^3}$

input
```
int(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/512*(8*b^2*x^2-4*b*x+1)/b^3*exp(4*b*x+4*a)-1/32*(2*b^2*x^2-2*b*x+1)/b^3*
exp(2*b*x+2*a)-1/32*(2*b^2*x^2+2*b*x+1)/b^3*exp(-2*b*x-2*a)+1/512*(8*b^2*x
^2+4*b*x+1)/b^3*exp(-4*b*x-4*a)
```

3.309.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

$$\int x^2\cosh(a+bx)\sinh^3(a+bx)\,dx =$$
$$-\frac{16\,bx\cosh(bx+a)\sinh(bx+a)^3 - (8\,b^2x^2+1)\cosh(bx+a)^4 - (8\,b^2x^2+1)\sinh(bx+a)^4 + 16\,(2\,b^2x}{}$$

input
```
integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/256*(16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 - (8*b^2*x^2 + 1)*cosh(b*x +
a)^4 - (8*b^2*x^2 + 1)*sinh(b*x + a)^4 + 16*(2*b^2*x^2 + 1)*cosh(b*x + a)^
2 + 2*(16*b^2*x^2 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^2
 + 16*(b*x*cosh(b*x + a)^3 - 4*b*x*cosh(b*x + a))*sinh(b*x + a))/b^3
```

3.309.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.49

$$\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} \frac{5x^2\sinh^4(a+bx)}{32b} + \frac{3x^2\sinh^2(a+bx)\cosh^2(a+bx)}{16b} - \frac{3x^2\cosh^4(a+bx)}{32b} - \frac{5x\sinh^3(a+bx)\cosh(a+bx)}{16b^2} + \frac{3x\sinh(a+bx)\cosh^3(a+bx)}{16b^2} \\ \frac{x^3\sinh^3(a)\cosh(a)}{3} \end{cases}$$

input
```
integrate(x**2*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Piecewise((5*x**2*sinh(a + b*x)**4/(32*b) + 3*x**2*sinh(a + b*x)**2*cosh(a
 + b*x)**2/(16*b) - 3*x**2*cosh(a + b*x)**4/(32*b) - 5*x*sinh(a + b*x)**3*
cosh(a + b*x)/(16*b**2) + 3*x*sinh(a + b*x)*cosh(a + b*x)**3/(16*b**2) + 5
*sinh(a + b*x)**4/(64*b**3) - 3*cosh(a + b*x)**4/(64*b**3), Ne(b, 0)), (x*
*3*sinh(a)**3*cosh(a)/3, True))
```

3.309.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.26

$$\int x^2\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{(8\,b^2x^2e^{(4\,a)} - 4\,bxe^{(4\,a)} + e^{(4\,a)})e^{(4\,bx)}}{512\,b^3} - \frac{(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)})e^{(2\,bx)}}{32\,b^3} - \frac{(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx-2\,a)}}{32\,b^3} + \frac{(8\,b^2x^2 + 4\,bx + 1)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b^3 - 1/32*(
2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 - 1/32*(2*b^2*x
^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4
*b*x - 4*a)/b^3
```

3.309.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

$$\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{(8\,b^2x^2-4\,bx+1)e^{(4\,bx+4\,a)}}{512\,b^3} - \frac{(2\,b^2x^2-2\,bx+1)e^{(2\,bx+2\,a)}}{32\,b^3} - \frac{(2\,b^2x^2+2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^3} + \frac{(8\,b^2x^2+4\,bx+1)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 - 1/32*(2*b^2*x^2 - 2*b*
x + 1)*e^(2*b*x + 2*a)/b^3 - 1/32*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)
/b^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

3.309.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{\frac{\cosh(2\,a+2\,b\,x)}{16} - \frac{\cosh(4\,a+4\,b\,x)}{256} + b^2\left(\frac{x^2\cosh(2\,a+2\,b\,x)}{8} - \frac{x^2\cosh(4\,a+4\,b\,x)}{32}\right) - b\left(\frac{x\sinh(2\,a+2\,b\,x)}{8} - \frac{x\sinh(4\,a+4\,b\,x)}{64}\right)}{b^3}$$

input
```
int(x^2*cosh(a + b*x)*sinh(a + b*x)^3,x)
```

output
```
-(cosh(2*a + 2*b*x)/16 - cosh(4*a + 4*b*x)/256 + b^2*((x^2*cosh(2*a + 2*b*
x))/8 - (x^2*cosh(4*a + 4*b*x))/32) - b*((x*sinh(2*a + 2*b*x))/8 - (x*sinh
(4*a + 4*b*x))/64))/b^3
```

3.310 $\int x\cosh(a+bx)\sinh^3(a+bx)\,dx$

3.310.1 Optimal result

Integrand size = 16, antiderivative size = 65

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{3x}{32b}+\frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2}-\frac{\cosh(a+bx)\sinh^3(a+bx)}{16b^2}+\frac{x\sinh^4(a+bx)}{4b}$$

output
```
-3/32*x/b+3/32*cosh(b*x+a)*sinh(b*x+a)/b^2-1/16*cosh(b*x+a)*sinh(b*x+a)^3/
b^2+1/4*x*sinh(b*x+a)^4/b
```

3.310.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{16bx\cosh(2(a+bx))-4bx\cosh(4(a+bx))-8\sinh(2(a+bx))+\sinh(4(a+bx))}{128b^2}$$

input
```
Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
-1/128*(16*b*x*Cosh[2*(a + b*x)] - 4*b*x*Cosh[4*(a + b*x)] - 8*Sinh[2*(a +
 b*x)] + Sinh[4*(a + b*x)])/b^2
```

3.310.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5895, 3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sinh^3(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{5895}$$

$$\frac{x\sinh^4(a+bx)}{4b}-\frac{\int \sinh^4(a+bx)dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x\sinh^4(a+bx)}{4b}-\frac{\int \sin(ia+ibx)^4dx}{4b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{x\sinh^4(a+bx)}{4b}-\frac{\frac{3}{4}\int -\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}}{4b}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\sinh^4(a+bx)}{4b}-\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3}{4}\int \sinh^2(a+bx)dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x\sinh^4(a+bx)}{4b}-\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3}{4}\int -\sin(ia+ibx)^2dx}{4b}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\sinh^4(a+bx)}{4b}-\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\int \sin(ia+ibx)^2dx}{4b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{x\sinh^4(a+bx)}{4b}-\frac{\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\sinh^4(a+bx)}{4b}-\frac{\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\left(\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{4b}$$

input
```
Int[x*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
(x*Sinh[a + b*x]^4)/(4*b) - ((Cosh[a + b*x]*Sinh[a + b*x]^3)/(4*b) + (3*(x
/2 - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4)/(4*b)
```

3.310.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.310.4 Maple [A] (verified)

Time = 7.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)^4}{4}-\frac{\cosh(bx+a)\sinh(bx+a)^3}{16}+\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}-\frac{a\sinh(bx+a)^4}{4}}{b^2}$	69
default	$\frac{\frac{(bx+a)\sinh(bx+a)^4}{4}-\frac{\cosh(bx+a)\sinh(bx+a)^3}{16}+\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}-\frac{a\sinh(bx+a)^4}{4}}{b^2}$	69
risch	$\frac{(4bx-1)e^{4bx+4a}}{256b^2}-\frac{(2bx-1)e^{2bx+2a}}{32b^2}-\frac{(2bx+1)e^{-2bx-2a}}{32b^2}+\frac{(4bx+1)e^{-4bx-4a}}{256b^2}$	82

input
```
int(x*cosh(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/4*(b*x+a)*sinh(b*x+a)^4-1/16*cosh(b*x+a)*sinh(b*x+a)^3+3/32*cosh(
b*x+a)*sinh(b*x+a)-3/32*b*x-3/32*a-1/4*a*sinh(b*x+a)^4)
```

3.310.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{bx\cosh\left(bx+a\right)^4 + bx\sinh\left(bx+a\right)^4 - 4\,bx\cosh\left(bx+a\right)^2 - \cosh\left(bx+a\right)\sinh\left(bx+a\right)^3 + 2\left(3\,bx\cosh\left(b\right.\right.}{32\,b^2}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/32*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 - 4*b*x*cosh(b*x + a)^2 -
cosh(b*x + a)*sinh(b*x + a)^3 + 2*(3*b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x
 + a)^2 - (cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a))/b^2
```

3.310.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} \frac{5x\sinh^4(a+bx)}{32b} + \frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{16b} - \frac{3x\cosh^4(a+bx)}{32b} - \frac{5\sinh^3(a+bx)\cosh(a+bx)}{32b^2} + \frac{3\sinh(a+bx)\cosh^3(a+bx)}{32b^2} & \text{for} \\ \frac{x^2\sinh^3(a)\cosh(a)}{2} & \text{oth} \end{cases}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Piecewise((5*x*sinh(a + b*x)**4/(32*b) + 3*x*sinh(a + b*x)**2*cosh(a + b*x
)**2/(16*b) - 3*x*cosh(a + b*x)**4/(32*b) - 5*sinh(a + b*x)**3*cosh(a + b*
x)/(32*b**2) + 3*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2), Ne(b, 0)), (x**
2*sinh(a)**3*cosh(a)/2, True))
```

3.310.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.40

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\left(4\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2}-\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 - 1/32*(2*b*x*e^(2*a) - e^(2
*a))*e^(2*b*x)/b^2 - 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x
+ 1)*e^(-4*b*x - 4*a)/b^2
```

3.310.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2}-\frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{32\,b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 - 1/32*(2*b*x - 1)*e^(2*b*x + 2*a)/b
^2 - 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x + 1)*e^(-4*b*x -
 4*a)/b^2
```

3.310.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{\frac{\sinh(4\,a+4\,b\,x)}{128}-\frac{\sinh(2\,a+2\,b\,x)}{16}+b\left(\frac{x\cosh(2\,a+2\,b\,x)}{8}-\frac{x\cosh(4\,a+4\,b\,x)}{32}\right)}{b^2}$$

input
```
int(x*cosh(a + b*x)*sinh(a + b*x)^3,x)
```

output
```
-(sinh(4*a + 4*b*x)/128 - sinh(2*a + 2*b*x)/16 + b*((x*cosh(2*a + 2*b*x))/
8 - (x*cosh(4*a + 4*b*x))/32))/b^2
```

3.310. $\int x\cosh(a+bx)\sinh^3(a+bx)\,dx$

3.311 $\int \cosh(a + bx) \sinh^3(a + bx) \, dx$

3.311.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cosh(a + bx) \sinh^3(a + bx) \, dx = \frac{\sinh^4(a + bx)}{4b}$$

output
```
1/4*sinh(b*x+a)^4/b
```

3.311.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh(a + bx) \sinh^3(a + bx) \, dx = \frac{\sinh^4(a + bx)}{4b}$$

input
```
Integrate[Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
Sinh[a + b*x]^4/(4*b)
```

3.311.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 26, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\cosh(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int i\sin(ia+ibx)^3\cos(ia+ibx)dx$$
$$\Big\downarrow \text{ 26}$$
$$i\int \cos(ia+ibx)\sin(ia+ibx)^3dx$$
$$\Big\downarrow \text{ 3044}$$
$$\frac{\int -i\sinh^3(a+bx)d(i\sinh(a+bx))}{b}$$
$$\Big\downarrow \text{ 15}$$
$$\frac{\sinh^4(a+bx)}{4b}$$

input
```
Int[Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
Sinh[a + b*x]^4/(4*b)
```

3.311.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.311.4 Maple [A] (verified)

Time = 4.71 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sinh(bx+a)^4}{4b}$	14
default	$\frac{\sinh(bx+a)^4}{4b}$	14
risch	$\frac{e^{4bx+4a}}{64b} - \frac{e^{2bx+2a}}{16b} - \frac{e^{-2bx-2a}}{16b} + \frac{e^{-4bx-4a}}{64b}$	58

input
```
int(cosh(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*sinh(b*x+a)^4/b
```

3.311.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.60

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{\cosh{(bx+a)}^4 + \sinh{(bx+a)}^4 + 2\left(3\cosh{(bx+a)}^2 - 2\right)\sinh{(bx+a)}^2 - 4\cosh{(bx+a)}^2}{32\,b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/32*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 2)*sinh(b
*x + a)^2 - 4*cosh(b*x + a)^2)/b
```

3.311.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx = \begin{cases} \frac{\sinh^4(a+bx)}{4b} & \text{for } b\neq 0 \\ x\sinh^3(a)\cosh(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Piecewise((sinh(a + b*x)**4/(4*b), Ne(b, 0)), (x*sinh(a)**3*cosh(a), True)
)
```

3.311.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\sinh(bx+a)^4}{4b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*sinh(b*x + a)^4/b
```

3.311.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.80

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(4bx+4a)}}{64b} - \frac{e^{(2bx+2a)}}{16b} - \frac{e^{(-2bx-2a)}}{16b} + \frac{e^{(-4bx-4a)}}{64b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/64*e^(4*b*x + 4*a)/b - 1/16*e^(2*b*x + 2*a)/b - 1/16*e^(-2*b*x - 2*a)/b
+ 1/64*e^(-4*b*x - 4*a)/b
```

3.311.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\sinh(a+b\,x)^4}{4\,b}$$

input
```
int(cosh(a + b*x)*sinh(a + b*x)^3,x)
```

output
```
sinh(a + b*x)^4/(4*b)
```

3.312 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx$

3.312.1 Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = -\frac{1}{4}\text{Chi}(2bx)\sinh(2a)+\frac{1}{8}\text{Chi}(4bx)\sinh(4a) - \frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

output
```
-1/4*cosh(2*a)*Shi(2*b*x)+1/8*cosh(4*a)*Shi(4*b*x)-1/4*Chi(2*b*x)*sinh(2*a
)+1/8*Chi(4*b*x)*sinh(4*a)
```

3.312.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{8}(-2\text{Chi}(2bx)\sinh(2a)+\text{Chi}(4bx)\sinh(4a) - 2\cosh(2a)\text{Shi}(2bx)+\cosh(4a)\text{Shi}(4bx))$$

input
```
Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x,x]
```

output
```
(-2*CoshIntegral[2*b*x]*Sinh[2*a] + CoshIntegral[4*b*x]*Sinh[4*a] - 2*Cosh
[2*a]*SinhIntegral[2*b*x] + Cosh[4*a]*SinhIntegral[4*b*x])/8
```

3.312.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)\cosh(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{\sinh(4a+4bx)}{8x}-\frac{\sinh(2a+2bx)}{4x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}\sinh(2a)\text{Chi}(2bx)+\frac{1}{8}\sinh(4a)\text{Chi}(4bx)-\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

input
```
Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x,x]
```

output
```
-1/4*(CoshIntegral[2*b*x]*Sinh[2*a]) + (CoshIntegral[4*b*x]*Sinh[4*a])/8 -
 (Cosh[2*a]*SinhIntegral[2*b*x])/4 + (Cosh[4*a]*SinhIntegral[4*b*x])/8
```

3.312.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.312.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{e^{-4a}\operatorname{Ei}_1(4bx)}{16}-\frac{e^{-2a}\operatorname{Ei}_1(2bx)}{8}+\frac{e^{2a}\operatorname{Ei}_1(-2bx)}{8}-\frac{e^{4a}\operatorname{Ei}_1(-4bx)}{16}$	50

input
```
int(cosh(b*x+a)*sinh(b*x+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/16*exp(-4*a)*Ei(1,4*b*x)-1/8*exp(-2*a)*Ei(1,2*b*x)+1/8*exp(2*a)*Ei(1,-2*
b*x)-1/16*exp(4*a)*Ei(1,-4*b*x)
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{16}\left(\operatorname{Ei}(4\,bx)-\operatorname{Ei}(-4\,bx)\right)\cosh\left(4\,a\right) - \frac{1}{8}\left(\operatorname{Ei}(2\,bx)-\operatorname{Ei}(-2\,bx)\right)\cosh\left(2\,a\right) + \frac{1}{16}\left(\operatorname{Ei}(4\,bx)+\operatorname{Ei}(-4\,bx)\right)\sinh\left(4\,a\right) - \frac{1}{8}\left(\operatorname{Ei}(2\,bx)+\operatorname{Ei}(-2\,bx)\right)\sinh\left(2\,a\right)$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
1/16*(Ei(4*b*x) - Ei(-4*b*x))*cosh(4*a) - 1/8*(Ei(2*b*x) - Ei(-2*b*x))*cos
h(2*a) + 1/16*(Ei(4*b*x) + Ei(-4*b*x))*sinh(4*a) - 1/8*(Ei(2*b*x) + Ei(-2*
b*x))*sinh(2*a)
```

3.312.6 Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = \int \frac{\sinh^3(a+bx)\cosh(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)**3/x,x)
```

output
```
Integral(sinh(a + b*x)**3*cosh(a + b*x)/x, x)
```

3.312.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}(4\,bx)\,e^{(4\,a)} - \frac{1}{8}\,\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + \frac{1}{8}\,\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - \frac{1}{16}\,\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
1/16*Ei(4*b*x)*e^(4*a) - 1/8*Ei(2*b*x)*e^(2*a) + 1/8*Ei(-2*b*x)*e^(-2*a) -
 1/16*Ei(-4*b*x)*e^(-4*a)
```

3.312.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}(4\,bx)\,e^{(4\,a)} - \frac{1}{8}\,\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + \frac{1}{8}\,\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - \frac{1}{16}\,\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="giac")
```

output
```
1/16*Ei(4*b*x)*e^(4*a) - 1/8*Ei(2*b*x)*e^(2*a) + 1/8*Ei(-2*b*x)*e^(-2*a) -
 1/16*Ei(-4*b*x)*e^(-4*a)
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}{x}\,dx$$

input
```
int((cosh(a + b*x)*sinh(a + b*x)^3)/x,x)
```

output
```
int((cosh(a + b*x)*sinh(a + b*x)^3)/x, x)
```

3.313 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx$

3.313.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = &-\frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)\\ &+\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}\\ &-\frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)\end{aligned}$$

output
```
-1/2*b*Chi(2*b*x)*cosh(2*a)+1/2*b*Chi(4*b*x)*cosh(4*a)-1/2*b*Shi(2*b*x)*si
nh(2*a)+1/2*b*Shi(4*b*x)*sinh(4*a)+1/4*sinh(2*b*x+2*a)/x-1/8*sinh(4*b*x+4*
a)/x
```

3.313.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = -\frac{4bx\cosh(2a)\text{Chi}(2bx)-4bx\cosh(4a)\text{Chi}(4bx)-2\sinh(2(a+bx))+\sinh(4(a+bx))+4bx\sinh(2a)\text{Sh}}{8x}$$

input
```
Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^2,x]
```

output
```
-1/8*(4*b*x*Cosh[2*a]*CoshIntegral[2*b*x] - 4*b*x*Cosh[4*a]*CoshIntegral[4
*b*x] - 2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 4*b*x*Sinh[2*a]*SinhInte
gral[2*b*x] - 4*b*x*Sinh[4*a]*SinhIntegral[4*b*x])/x
```

3.313.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)\cosh(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \ 5971$$

$$\int \left(\frac{\sinh(4a+4bx)}{8x^2}-\frac{\sinh(2a+2bx)}{4x^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)-\frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)+\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}$$

input
```
Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^2,x]
```

output
```
-1/2*(b*Cosh[2*a]*CoshIntegral[2*b*x]) + (b*Cosh[4*a]*CoshIntegral[4*b*x])
/2 + Sinh[2*a + 2*b*x]/(4*x) - Sinh[4*a + 4*b*x]/(8*x) - (b*Sinh[2*a]*Sinh
Integral[2*b*x])/2 + (b*Sinh[4*a]*SinhIntegral[4*b*x])/2
```

3.313.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.313.4 Maple [A] (verified)

Time = 5.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

method	result	siz
risch	$\frac{-4\,\mathrm{e}^{-4a}\,\mathrm{Ei}_1(4bx)bx+4\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)bx-4\,\mathrm{e}^{4a}\,\mathrm{Ei}_1(-4bx)bx+4\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)bx+\mathrm{e}^{-4bx-4a}-2\,\mathrm{e}^{-2bx-2a}-\mathrm{e}^{4bx+4a}+2\,\mathrm{e}^{2bx+2a}}{16x}$	10

input
```
int(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-4*exp(-4*a)*Ei(1,4*b*x)*b*x+4*exp(-2*a)*Ei(1,2*b*x)*b*x-4*exp(4*a)*
Ei(1,-4*b*x)*b*x+4*exp(2*a)*Ei(1,-2*b*x)*b*x+exp(-4*b*x-4*a)-2*exp(-2*b*x-
2*a)-exp(4*b*x+4*a)+2*exp(2*b*x+2*a))/x
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.56

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx =$$
$$-\frac{2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\left(bx\mathrm{Ei}(4\,bx)+bx\mathrm{Ei}(-4\,bx)\right)\cosh\left(4\,a\right)+\left(bx\mathrm{Ei}(2\,bx)+bx\mathrm{Ei}(-2\,bx)\right)\mathrm{co}}{}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
-1/4*(2*cosh(b*x + a)*sinh(b*x + a)^3 - (b*x*Ei(4*b*x) + b*x*Ei(-4*b*x))*c
osh(4*a) + (b*x*Ei(2*b*x) + b*x*Ei(-2*b*x))*cosh(2*a) + 2*(cosh(b*x + a)^3
 - cosh(b*x + a))*sinh(b*x + a) - (b*x*Ei(4*b*x) - b*x*Ei(-4*b*x))*sinh(4*
a) + (b*x*Ei(2*b*x) - b*x*Ei(-2*b*x))*sinh(2*a))/x
```

3.313.6 Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \int \frac{\sinh^3(a+bx)\cosh(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)**3/x**2,x)
```

output
```
Integral(sinh(a + b*x)**3*cosh(a + b*x)/x**2, x)
```

3.313.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \frac{1}{4}\,be^{(-4\,a)}\Gamma(-1,4\,bx) - \frac{1}{4}\,be^{(-2\,a)}\Gamma(-1,2\,bx) - \frac{1}{4}\,be^{(2\,a)}\Gamma(-1,-2\,bx) + \frac{1}{4}\,be^{(4\,a)}\Gamma(-1,-4\,bx)$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/4*b*e^(-4*a)*gamma(-1, 4*b*x) - 1/4*b*e^(-2*a)*gamma(-1, 2*b*x) - 1/4*b*
e^(2*a)*gamma(-1, -2*b*x) + 1/4*b*e^(4*a)*gamma(-1, -4*b*x)
```

3.313.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \frac{4\,bx\mathrm{Ei}(4\,bx)\,e^{(4\,a)} - 4\,bx\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - 4\,bx\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} + 4\,bx\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+}}{16\,x}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
1/16*(4*b*x*Ei(4*b*x)*e^(4*a) - 4*b*x*Ei(2*b*x)*e^(2*a) - 4*b*x*Ei(-2*b*x)
*e^(-2*a) + 4*b*x*Ei(-4*b*x)*e^(-4*a) - e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a
) - 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}{x^2}\,dx$$

input
```
int((cosh(a + b*x)*sinh(a + b*x)^3)/x^2,x)
```

output
```
int((cosh(a + b*x)*sinh(a + b*x)^3)/x^2, x)
```

3.314 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx$

3.314.1 Optimal result

Integrand size = 18, antiderivative size = 125

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = {}& \frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} \\ & - \frac{1}{2}b^2\text{Chi}(2bx)\sinh(2a) + b^2\text{Chi}(4bx)\sinh(4a) \\ & + \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} \\ & - \frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx) + b^2\cosh(4a)\text{Shi}(4bx)\end{aligned}$$

output
```
1/4*b*cosh(2*b*x+2*a)/x-1/4*b*cosh(4*b*x+4*a)/x-1/2*b^2*cosh(2*a)*Shi(2*b*
x)+b^2*cosh(4*a)*Shi(4*b*x)-1/2*b^2*Chi(2*b*x)*sinh(2*a)+b^2*Chi(4*b*x)*si
nh(4*a)+1/8*sinh(2*b*x+2*a)/x^2-1/16*sinh(4*b*x+4*a)/x^2
```

3.314.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = {}& -b^2\cosh(a)\text{Chi}(2bx)\sinh(a) + b^2\text{Chi}(4bx)\sinh(4a) \\ & + \frac{2bx\cosh(2(a+bx)) + \sinh(2(a+bx))}{8x^2} \\ & - \frac{4bx\cosh(4(a+bx)) + \sinh(4(a+bx))}{16x^2} \\ & - \frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx) + b^2\cosh(4a)\text{Shi}(4bx)\end{aligned}$$

input
```
Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^3,x]
```

output
```
-(b^2*Cosh[a]*CoshIntegral[2*b*x]*Sinh[a]) + b^2*CoshIntegral[4*b*x]*Sinh[
4*a] + (2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])/(8*x^2) - (4*b*x*Cosh
[4*(a + b*x)] + Sinh[4*(a + b*x)])/(16*x^2) - (b^2*Cosh[2*a]*SinhIntegral[
2*b*x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

3.314.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)\cosh(a+bx)}{x^3}dx$$

$$\Big\downarrow \text{ 5971}$$

$$\int \left(\frac{\sinh(4a+4bx)}{8x^3}-\frac{\sinh(2a+2bx)}{4x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{2}b^2\sinh(2a)\text{Chi}(2bx)+b^2\sinh(4a)\text{Chi}(4bx)-\frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx)+b^2\cosh(4a)\text{Shi}(4bx)+\frac{\sinh(2a+2bx)}{8x^2}-\frac{\sinh(4a+4bx)}{16x^2}+\frac{b\cosh(2a+2bx)}{4x}-\frac{b\cosh(4a+4bx)}{4x}$$

input
```
Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^3,x]
```

output
```
(b*Cosh[2*a + 2*b*x])/(4*x) - (b*Cosh[4*a + 4*b*x])/(4*x) - (b^2*CoshInteg
ral[2*b*x]*Sinh[2*a])/2 + b^2*CoshIntegral[4*b*x]*Sinh[4*a] + Sinh[2*a + 2
*b*x]/(8*x^2) - Sinh[4*a + 4*b*x]/(16*x^2) - (b^2*Cosh[2*a]*SinhIntegral[2
*b*x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

3.314.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.314.4 Maple [A] (verified)

Time = 5.54 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.38

method	result
risch	$\frac{8\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)x^2b^2-16\,\mathrm{e}^{4a}\,\mathrm{Ei}_1(-4bx)x^2b^2+16\,\mathrm{e}^{-4a}\,\mathrm{Ei}_1(4bx)x^2b^2-8\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)x^2b^2+4\,\mathrm{e}^{2bx+2a}bx-4\,\mathrm{e}^{4bx+4a}bx-4\,\mathrm{e}^{-4bx-4a}bx}{32x^2}$

input
```
int(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(8*exp(2*a)*Ei(1,-2*b*x)*x^2*b^2-16*exp(4*a)*Ei(1,-4*b*x)*x^2*b^2+16*
exp(-4*a)*Ei(1,4*b*x)*x^2*b^2-8*exp(-2*a)*Ei(1,2*b*x)*x^2*b^2+4*exp(2*b*x+
2*a)*b*x-4*exp(4*b*x+4*a)*b*x-4*exp(-4*b*x-4*a)*b*x+4*exp(-2*b*x-2*a)*b*x+
2*exp(2*b*x+2*a)-exp(4*b*x+4*a)+exp(-4*b*x-4*a)-2*exp(-2*b*x-2*a))/x^2
```

3.314.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(113) = 226.

Time = 0.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.83

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx =$$
$$-\frac{bx\cosh\left(bx+a\right)^4+bx\sinh\left(bx+a\right)^4-bx\cosh\left(bx+a\right)^2+\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\left(6\,bx\cosh\left(bx\right.\right.}{}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

output
```
-1/4*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + co
sh(b*x + a)*sinh(b*x + a)^3 + (6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^
2 - 2*(b^2*x^2*Ei(4*b*x) - b^2*x^2*Ei(-4*b*x))*cosh(4*a) + (b^2*x^2*Ei(2*b
*x) - b^2*x^2*Ei(-2*b*x))*cosh(2*a) + (cosh(b*x + a)^3 - cosh(b*x + a))*si
nh(b*x + a) - 2*(b^2*x^2*Ei(4*b*x) + b^2*x^2*Ei(-4*b*x))*sinh(4*a) + (b^2*
x^2*Ei(2*b*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

3.314.6 Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \frac{\sinh^3(a+bx)\cosh(a+bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)**3/x**3,x)
```

output
```
Integral(sinh(a + b*x)**3*cosh(a + b*x)/x**3, x)
```

3.314.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = b^2e^{(-4\,a)}\Gamma(-2,4\,bx) - \frac{1}{2}\,b^2e^{(-2\,a)}\Gamma(-2,2\,bx) + \frac{1}{2}\,b^2e^{(2\,a)}\Gamma(-2,-2\,bx) - b^2e^{(4\,a)}\Gamma(-2,-4\,bx)$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

output
```
b^2*e^(-4*a)*gamma(-2, 4*b*x) - 1/2*b^2*e^(-2*a)*gamma(-2, 2*b*x) + 1/2*b^
2*e^(2*a)*gamma(-2, -2*b*x) - b^2*e^(4*a)*gamma(-2, -4*b*x)
```

3.314.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.34

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \frac{16\,b^2x^2\text{Ei}(4\,bx)\,e^{(4\,a)} - 8\,b^2x^2\text{Ei}(2\,bx)\,e^{(2\,a)} + 8\,b^2x^2\text{Ei}(-2\,bx)\,e^{(-2\,a)} - 16\,b^2x^2\text{Ei}(-4\,bx)\,e^{(-4\,a)} - 4\,bxe^{(4\,bx+}}{32}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x, algorithm="giac")
```

output
```
1/32*(16*b^2*x^2*Ei(4*b*x)*e^(4*a) - 8*b^2*x^2*Ei(2*b*x)*e^(2*a) + 8*b^2*x
^2*Ei(-2*b*x)*e^(-2*a) - 16*b^2*x^2*Ei(-4*b*x)*e^(-4*a) - 4*b*x*e^(4*b*x +
 4*a) + 4*b*x*e^(2*b*x + 2*a) + 4*b*x*e^(-2*b*x - 2*a) - 4*b*x*e^(-4*b*x -
 4*a) - e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) - 2*e^(-2*b*x - 2*a) + e^(-4*b
*x - 4*a))/x^2
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}{x^3}\,dx$$

input
```
int((cosh(a + b*x)*sinh(a + b*x)^3)/x^3,x)
```

output
```
int((cosh(a + b*x)*sinh(a + b*x)^3)/x^3, x)
```

3.315 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx$

3.315.1 Optimal result

Integrand size = 18, antiderivative size = 169

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx &= \frac{b\cosh(2a+2bx)}{12x^2} - \frac{b\cosh(4a+4bx)}{12x^2} \\ &- \frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx) + \frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx) \\ &+ \frac{\sinh(2a+2bx)}{12x^3} + \frac{b^2\sinh(2a+2bx)}{6x} \\ &- \frac{\sinh(4a+4bx)}{24x^3} - \frac{b^2\sinh(4a+4bx)}{3x} \\ &- \frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx) + \frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx)\end{aligned}$$

output
```
-1/3*b^3*Chi(2*b*x)*cosh(2*a)+4/3*b^3*Chi(4*b*x)*cosh(4*a)+1/12*b*cosh(2*b
*x+2*a)/x^2-1/12*b*cosh(4*b*x+4*a)/x^2-1/3*b^3*Shi(2*b*x)*sinh(2*a)+4/3*b^
3*Shi(4*b*x)*sinh(4*a)+1/12*sinh(2*b*x+2*a)/x^3+1/6*b^2*sinh(2*b*x+2*a)/x-
1/24*sinh(4*b*x+4*a)/x^3-1/3*b^2*sinh(4*b*x+4*a)/x
```

3.315.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx = -\frac{-2bx\cosh(2(a+bx))+2bx\cosh(4(a+bx))+8b^3x^3\cosh(2a)\mathrm{Chi}(2bx)-32b^3x^3\cosh(4a)\mathrm{Chi}(4bx)-2}{}$$

input
```
Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^4,x]
```

output
```
-1/24*(-2*b*x*Cosh[2*(a + b*x)] + 2*b*x*Cosh[4*(a + b*x)] + 8*b^3*x^3*Cosh
[2*a]*CoshIntegral[2*b*x] - 32*b^3*x^3*Cosh[4*a]*CoshIntegral[4*b*x] - 2*S
inh[2*(a + b*x)] - 4*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 8*b^2
*x^2*Sinh[4*(a + b*x)] + 8*b^3*x^3*Sinh[2*a]*SinhIntegral[2*b*x] - 32*b^3*
x^3*Sinh[4*a]*SinhIntegral[4*b*x])/x^3
```

3.315.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)\cosh(a+bx)}{x^4}\,dx$$
$$\Big\downarrow \; 5971$$
$$\int \left(\frac{\sinh(4a+4bx)}{8x^4}-\frac{\sinh(2a+2bx)}{4x^4}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{1}{3}b^3\cosh(2a)\mathrm{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\mathrm{Chi}(4bx)-\frac{1}{3}b^3\sinh(2a)\mathrm{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\mathrm{Shi}(4bx)+\frac{b^2\sinh(2a+2bx)}{6x}-\frac{b^2\sinh(4a+4bx)}{3x}+\frac{\sinh(2a+2bx)}{12x^3}-\frac{\sinh(4a+4bx)}{24x^3}+\frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}$$

input
```
Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^4,x]
```

output
```
(b*Cosh[2*a + 2*b*x])/(12*x^2) - (b*Cosh[4*a + 4*b*x])/(12*x^2) - (b^3*Cos
h[2*a]*CoshIntegral[2*b*x])/3 + (4*b^3*Cosh[4*a]*CoshIntegral[4*b*x])/3 +
Sinh[2*a + 2*b*x]/(12*x^3) + (b^2*Sinh[2*a + 2*b*x])/(6*x) - Sinh[4*a + 4*
b*x]/(24*x^3) - (b^2*Sinh[4*a + 4*b*x])/(3*x) - (b^3*Sinh[2*a]*SinhIntegra
l[2*b*x])/3 + (4*b^3*Sinh[4*a]*SinhIntegral[4*b*x])/3
```

3.315.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.315.4 Maple [A] (verified)

Time = 8.57 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.43

method	result
risch	−32 e^{−4a} Ei_1(4bx)x^3b^3+8 e^{−2a} Ei_1(2bx)x^3b^3+8 e^{2a} Ei_1(−2bx)x^3b^3−32 e^{4a} Ei_1(−4bx)x^3b^3+8 e^{−4bx−4a}b^2x^2−4 e^{−2bx−2a}b^2x^2+4 e^{2 … / 48x …

input
```
int(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/48*(-32*exp(-4*a)*Ei(1,4*b*x)*x^3*b^3+8*exp(-2*a)*Ei(1,2*b*x)*x^3*b^3+8*
exp(2*a)*Ei(1,-2*b*x)*x^3*b^3-32*exp(4*a)*Ei(1,-4*b*x)*x^3*b^3+8*exp(-4*b*
x-4*a)*b^2*x^2-4*exp(-2*b*x-2*a)*b^2*x^2+4*exp(2*b*x+2*a)*b^2*x^2-8*exp(4*
b*x+4*a)*b^2*x^2-2*exp(-4*b*x-4*a)*b*x+2*exp(-2*b*x-2*a)*b*x+2*exp(2*b*x+2
*a)*b*x-2*exp(4*b*x+4*a)*b*x+exp(-4*b*x-4*a)-2*exp(-2*b*x-2*a)+2*exp(2*b*x
+2*a)-exp(4*b*x+4*a))/x^3
```

3.315.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.56

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx =$$
$$-\frac{bx\cosh{(bx+a)}^4 + bx\sinh{(bx+a)}^4 + 2\,(8\,b^2x^2+1)\cosh{(bx+a)}\sinh{(bx+a)}^3 - bx\cosh{(bx+a)}^2 + \cdots}{\cdots}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x, algorithm="fricas")
```

output
```
-1/12*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + 2*(8*b^2*x^2 + 1)*cosh(
b*x + a)*sinh(b*x + a)^3 - b*x*cosh(b*x + a)^2 + (6*b*x*cosh(b*x + a)^2 -
b*x)*sinh(b*x + a)^2 - 8*(b^3*x^3*Ei(4*b*x) + b^3*x^3*Ei(-4*b*x))*cosh(4*a
) + 2*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*cosh(2*a) + 2*((8*b^2*x^2 +
 1)*cosh(b*x + a)^3 - (2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) - 8*(b^
3*x^3*Ei(4*b*x) - b^3*x^3*Ei(-4*b*x))*sinh(4*a) + 2*(b^3*x^3*Ei(2*b*x) - b
^3*x^3*Ei(-2*b*x))*sinh(2*a))/x^3
```

3.315.6 Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \frac{\sinh^3{(a+bx)}\cosh{(a+bx)}}{x^4}\,dx$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)**3/x**4,x)
```

output
```
Integral(sinh(a + b*x)**3*cosh(a + b*x)/x**4, x)
```

3.315.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.36

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx = 4\,b^3e^{(-4\,a)}\Gamma(-3,4\,bx) - b^3e^{(-2\,a)}\Gamma(-3,2\,bx) - b^3e^{(2\,a)}\Gamma(-3,-2\,bx) + 4\,b^3e^{(4\,a)}\Gamma(-3,-4\,bx)$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x, algorithm="maxima")
```

output
```
4*b^3*e^(-4*a)*gamma(-3, 4*b*x) - b^3*e^(-2*a)*gamma(-3, 2*b*x) - b^3*e^(2
*a)*gamma(-3, -2*b*x) + 4*b^3*e^(4*a)*gamma(-3, -4*b*x)
```

3.315.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx$$
$$= \frac{32\,b^3x^3\text{Ei}(4\,bx)\,e^{(4\,a)} - 8\,b^3x^3\text{Ei}(2\,bx)\,e^{(2\,a)} - 8\,b^3x^3\text{Ei}(-2\,bx)\,e^{(-2\,a)} + 32\,b^3x^3\text{Ei}(-4\,bx)\,e^{(-4\,a)} - 8\,b^2x^2e^{(4\,b}}{}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x, algorithm="giac")
```

output
```
1/48*(32*b^3*x^3*Ei(4*b*x)*e^(4*a) - 8*b^3*x^3*Ei(2*b*x)*e^(2*a) - 8*b^3*x
^3*Ei(-2*b*x)*e^(-2*a) + 32*b^3*x^3*Ei(-4*b*x)*e^(-4*a) - 8*b^2*x^2*e^(4*b
*x + 4*a) + 4*b^2*x^2*e^(2*b*x + 2*a) - 4*b^2*x^2*e^(-2*b*x - 2*a) + 8*b^2
*x^2*e^(-4*b*x - 4*a) - 2*b*x*e^(4*b*x + 4*a) + 2*b*x*e^(2*b*x + 2*a) + 2*
b*x*e^(-2*b*x - 2*a) - 2*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) + 2*e^(2*b
*x + 2*a) - 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x^3
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}{x^4}\,dx$$

input
```
int((cosh(a + b*x)*sinh(a + b*x)^3)/x^4,x)
```

output
```
int((cosh(a + b*x)*sinh(a + b*x)^3)/x^4, x)
```

3.316 $\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx$

3.316.1 Optimal result

Integrand size = 20, antiderivative size = 209

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{5^{-1-m}e^{5a}x^m(-bx)^{-m}\Gamma(1+m,-5bx)}{32b} - \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{32b} - \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{16b} - \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{16b} - \frac{3^{-1-m}e^{-3a}x^m(bx)^{-m}\Gamma(1+m,3bx)}{32b} + \frac{5^{-1-m}e^{-5a}x^m(bx)^{-m}\Gamma(1+m,5bx)}{32b}$$

output
```
1/32*5^(-1-m)*exp(5*a)*x^m*GAMMA(1+m,-5*b*x)/b/((-b*x)^m)-1/32*3^(-1-m)*ex
p(3*a)*x^m*GAMMA(1+m,-3*b*x)/b/((-b*x)^m)-1/16*exp(a)*x^m*GAMMA(1+m,-b*x)/
b/((-b*x)^m)-1/16*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)-1/32*3^(-1-m)*x^m*
GAMMA(1+m,3*b*x)/b/exp(3*a)/((b*x)^m)+1/32*5^(-1-m)*x^m*GAMMA(1+m,5*b*x)/b
/exp(5*a)/((b*x)^m)
```

3.316.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.83

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{e^{-5a}x^m\left(-30e^{4a}(e^{2a}(-bx)^{-m}\Gamma(1+m,-bx)+(bx)^{-m}\Gamma(1+m,bx))-5\,3^{-m}e^{2a}(-b^2x^2)^{-m}\left(e^{6a}(bx)^m\Gamma(1-\right.\right.}{}$$

input
```
Integrate[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
(x^m*(-30*E^(4*a)*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + Gamma[1 + m
, b*x]/(b*x)^m) - (5*E^(2*a)*(E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] + (-(b*
x))^m*Gamma[1 + m, 3*b*x]))/(3^m*(-(b^2*x^2))^m) + (3*(E^(10*a)*(b*x)^m*Ga
mma[1 + m, -5*b*x] + (-(b*x))^m*Gamma[1 + m, 5*b*x]))/(5^m*(-(b^2*x^2))^m)
))/(480*b*E^(5*a))
```

3.316.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh^3(a+bx)\cosh^2(a+bx)\,dx$$
$$\Big\downarrow \text{5971}$$
$$\int \left(-\frac{1}{8}x^m\sinh(a+bx)-\frac{1}{16}x^m\sinh(3a+3bx)+\frac{1}{16}x^m\sinh(5a+5bx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{e^{5a}5^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-5bx)}{32b}-\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-3bx)}{32b}-\frac{e^{a}x^m(-bx)^{-m}\Gamma(m+1,-bx)}{16b}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{16b}-\frac{e^{-3a}3^{-m-1}x^m(bx)^{-m}\Gamma(m+1,3bx)}{32b}+\frac{e^{-5a}5^{-m-1}x^m(bx)^{-m}\Gamma(m+1,5bx)}{32b}$$

input
```
Int[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
(5^(-1 - m)*E^(5*a)*x^m*Gamma[1 + m, -5*b*x])/(32*b*(-(b*x))^m) - (3^(-1 -
 m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(32*b*(-(b*x))^m) - (E^a*x^m*Gamma[1
 + m, -(b*x)])/(16*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(16*b*E^a*(b*x)
^m) - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(32*b*E^(3*a)*(b*x)^m) + (5^(-1
 - m)*x^m*Gamma[1 + m, 5*b*x])/(32*b*E^(5*a)*(b*x)^m)
```

3.316.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.316.4 Maple [F]

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)^3\,dx$$

input
```
int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

output
```
int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

3.316.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.19

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{3\cosh(m\log(5b)+5a)\Gamma(m+1,5bx) - 5\cosh(m\log(3b)+3a)\Gamma(m+1,3bx) - 30\cosh(m\log(b)+}{}$$

input
```
integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/480*(3*cosh(m*log(5*b) + 5*a)*gamma(m + 1, 5*b*x) - 5*cosh(m*log(3*b) +
3*a)*gamma(m + 1, 3*b*x) - 30*cosh(m*log(b) + a)*gamma(m + 1, b*x) - 30*co
sh(m*log(-b) - a)*gamma(m + 1, -b*x) - 5*cosh(m*log(-3*b) - 3*a)*gamma(m +
 1, -3*b*x) + 3*cosh(m*log(-5*b) - 5*a)*gamma(m + 1, -5*b*x) - 3*gamma(m +
 1, 5*b*x)*sinh(m*log(5*b) + 5*a) + 5*gamma(m + 1, 3*b*x)*sinh(m*log(3*b)
+ 3*a) + 30*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) + 5*gamma(m + 1, -3*b*x
)*sinh(m*log(-3*b) - 3*a) - 3*gamma(m + 1, -5*b*x)*sinh(m*log(-5*b) - 5*a)
 + 30*gamma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

3.316.6 Sympy [F]

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int x^m \sinh^3{(a+bx)}\cosh^2{(a+bx)}\,dx$$

input
```
integrate(x**m*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Integral(x**m*sinh(a + b*x)**3*cosh(a + b*x)**2, x)
```

3.316.7 Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.82

$$\begin{aligned}\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx &= \frac{1}{32}\,(5\,bx)^{-m-1}\,x^{m+1}e^{(-5\,a)}\Gamma(m+1,5\,bx)\\ &\quad-\frac{1}{32}\,(3\,bx)^{-m-1}\,x^{m+1}e^{(-3\,a)}\Gamma(m+1,3\,bx)\\ &\quad-\frac{1}{16}\,(bx)^{-m-1}\,x^{m+1}e^{(-a)}\Gamma(m+1,bx)\\ &\quad+\frac{1}{16}\,(-bx)^{-m-1}\,x^{m+1}e^{a}\Gamma(m+1,-bx)\\ &\quad+\frac{1}{32}\,(-3\,bx)^{-m-1}\,x^{m+1}e^{(3\,a)}\Gamma(m+1,-3\,bx)\\ &\quad-\frac{1}{32}\,(-5\,bx)^{-m-1}\,x^{m+1}e^{(5\,a)}\Gamma(m+1,-5\,bx)\end{aligned}$$

input
```
integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/32*(5*b*x)^(-m - 1)*x^(m + 1)*e^(-5*a)*gamma(m + 1, 5*b*x) - 1/32*(3*b*x
)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) - 1/16*(b*x)^(-m - 1)*x^
(m + 1)*e^(-a)*gamma(m + 1, b*x) + 1/16*(-b*x)^(-m - 1)*x^(m + 1)*e^a*gamm
a(m + 1, -b*x) + 1/32*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1, -3*
b*x) - 1/32*(-5*b*x)^(-m - 1)*x^(m + 1)*e^(5*a)*gamma(m + 1, -5*b*x)
```

3.316.8 Giac [F]

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^2 \sinh\left(bx+a\right)^3\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^2*sinh(b*x + a)^3, x)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int x^m \cosh(a+b\,x)^2 \sinh(a+b\,x)^3\,dx$$

input
```
int(x^m*cosh(a + b*x)^2*sinh(a + b*x)^3,x)
```

output
```
int(x^m*cosh(a + b*x)^2*sinh(a + b*x)^3, x)
```

3.317 $\int x^3 \cosh^2(a+bx) \sinh^3(a+bx)\,dx$

3.317.1 Optimal result

Integrand size = 20, antiderivative size = 202

$$\begin{aligned}\int x^3 \cosh^2(a+bx) \sinh^3(a+bx)\,dx = &-\frac{3x\cosh(a+bx)}{4b^3}-\frac{x^3\cosh(a+bx)}{8b} \\ &-\frac{x\cosh(3a+3bx)}{72b^3}-\frac{x^3\cosh(3a+3bx)}{48b} \\ &+\frac{3x\cosh(5a+5bx)}{1000b^3}+\frac{x^3\cosh(5a+5bx)}{80b} \\ &+\frac{3\sinh(a+bx)}{4b^4}+\frac{3x^2\sinh(a+bx)}{8b^2} \\ &+\frac{\sinh(3a+3bx)}{216b^4}+\frac{x^2\sinh(3a+3bx)}{48b^2} \\ &-\frac{3\sinh(5a+5bx)}{5000b^4}-\frac{3x^2\sinh(5a+5bx)}{400b^2}\end{aligned}$$

output
```
-3/4*x*cosh(b*x+a)/b^3-1/8*x^3*cosh(b*x+a)/b-1/72*x*cosh(3*b*x+3*a)/b^3-1/
48*x^3*cosh(3*b*x+3*a)/b+3/1000*x*cosh(5*b*x+5*a)/b^3+1/80*x^3*cosh(5*b*x+
5*a)/b+3/4*sinh(b*x+a)/b^4+3/8*x^2*sinh(b*x+a)/b^2+1/216*sinh(3*b*x+3*a)/b
^4+1/48*x^2*sinh(3*b*x+3*a)/b^2-3/5000*sinh(5*b*x+5*a)/b^4-3/400*x^2*sinh(
5*b*x+5*a)/b^2
```

3.317.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int x^3\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{-33750(bx(6+b^2x^2)\cosh(a+bx)-3(2+b^2x^2)\sinh(a+bx))-625((6bx+9b^3x^3)\cosh(3(a+bx))-(2}{270000b}$$

input
```
Integrate[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
(-33750*(b*x*(6 + b^2*x^2)*Cosh[a + b*x] - 3*(2 + b^2*x^2)*Sinh[a + b*x])
- 625*((6*b*x + 9*b^3*x^3)*Cosh[3*(a + b*x)] - (2 + 9*b^2*x^2)*Sinh[3*(a +
 b*x)]) + 27*(5*b*x*(6 + 25*b^2*x^2)*Cosh[5*(a + b*x)] - 3*(2 + 25*b^2*x^2
)*Sinh[5*(a + b*x)]))/(270000*b^4)
```

3.317.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sinh^3(a+bx)\cosh^2(a+bx)\,dx$$
$$\Big\downarrow \text{5971}$$
$$\int\left(-\frac{1}{8}x^3\sinh(a+bx)-\frac{1}{16}x^3\sinh(3a+3bx)+\frac{1}{16}x^3\sinh(5a+5bx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3\sinh(a+bx)}{4b^4}+\frac{\sinh(3a+3bx)}{216b^4}-\frac{3\sinh(5a+5bx)}{5000b^4}-\frac{3x\cosh(a+bx)}{4b^3}-\frac{x\cosh(3a+3bx)}{72b^3}+\frac{3x\cosh(5a+5bx)}{1000b^3}+\frac{3x^2\sinh(a+bx)}{8b^2}+\frac{x^2\sinh(3a+3bx)}{48b^2}-\frac{3x^2\sinh(5a+5bx)}{400b^2}-\frac{x^3\cosh(a+bx)}{8b}-\frac{x^3\cosh(3a+3bx)}{48b}+\frac{x^3\cosh(5a+5bx)}{80b}$$

input
```
Int[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
(-3*x*Cosh[a + b*x])/(4*b^3) - (x^3*Cosh[a + b*x])/(8*b) - (x*Cosh[3*a + 3
*b*x])/(72*b^3) - (x^3*Cosh[3*a + 3*b*x])/(48*b) + (3*x*Cosh[5*a + 5*b*x])
/(1000*b^3) + (x^3*Cosh[5*a + 5*b*x])/(80*b) + (3*Sinh[a + b*x])/(4*b^4) +
 (3*x^2*Sinh[a + b*x])/(8*b^2) + Sinh[3*a + 3*b*x]/(216*b^4) + (x^2*Sinh[3
*a + 3*b*x])/(48*b^2) - (3*Sinh[5*a + 5*b*x])/(5000*b^4) - (3*x^2*Sinh[5*a
 + 5*b*x])/(400*b^2)
```

3.317.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.317.4 Maple [A] (verified)

Time = 39.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05

method	result
risch	$\frac{(125x^3b^3-75x^2b^2+30bx-6)\mathrm{e}^{5bx+5a}}{20000b^4}-\frac{(9x^3b^3-9x^2b^2+6bx-2)\mathrm{e}^{3bx+3a}}{864b^4}-\frac{(x^3b^3-3x^2b^2+6bx-6)\mathrm{e}^{bx+a}}{16b^4}-\frac{(x^3b^3+$
derivativedivides	$-a^3\left(\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{5}-\frac{2(bx+a)\cosh(bx+a)^3}{15}-\frac{\sinh(bx+a)\,\mathrm{c}}{25}\right.$
default	$-a^3\left(\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{5}-\frac{2(bx+a)\cosh(bx+a)^3}{15}-\frac{\sinh(bx+a)\,\mathrm{c}}{25}\right.$

input
```
int(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/20000*(125*b^3*x^3-75*b^2*x^2+30*b*x-6)/b^4*exp(5*b*x+5*a)-1/864*(9*b^3*
x^3-9*b^2*x^2+6*b*x-2)/b^4*exp(3*b*x+3*a)-1/16*(b^3*x^3-3*b^2*x^2+6*b*x-6)
/b^4*exp(b*x+a)-1/16*(b^3*x^3+3*b^2*x^2+6*b*x+6)/b^4*exp(-b*x-a)-1/864*(9*
b^3*x^3+9*b^2*x^2+6*b*x+2)/b^4*exp(-3*b*x-3*a)+1/20000*(125*b^3*x^3+75*b^2
*x^2+30*b*x+6)/b^4*exp(-5*b*x-5*a)
```

3.317.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.36

$$\int x^3 \cosh^2(a+bx) \sinh^3(a+bx)\, dx$$
$$= \frac{135\,(25\,b^3x^3+6\,bx)\cosh{(bx+a)}^5+675\,(25\,b^3x^3+6\,bx)\cosh{(bx+a)}\sinh{(bx+a)}^4-81\,(25\,b^2x^2+2)\,\mathrm{si}}{}$$

input
```
integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/270000*(135*(25*b^3*x^3 + 6*b*x)*cosh(b*x + a)^5 + 675*(25*b^3*x^3 + 6*b
*x)*cosh(b*x + a)*sinh(b*x + a)^4 - 81*(25*b^2*x^2 + 2)*sinh(b*x + a)^5 -
1875*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^3 + 5*(1125*b^2*x^2 - 162*(25*b^2*x
^2 + 2)*cosh(b*x + a)^2 + 250)*sinh(b*x + a)^3 + 225*(6*(25*b^3*x^3 + 6*b*
x)*cosh(b*x + a)^3 - 25*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a))*sinh(b*x + a)^2
 - 33750*(b^3*x^3 + 6*b*x)*cosh(b*x + a) - 15*(27*(25*b^2*x^2 + 2)*cosh(b*
x + a)^4 - 6750*b^2*x^2 - 125*(9*b^2*x^2 + 2)*cosh(b*x + a)^2 - 13500)*sin
h(b*x + a))/b^4
```

3.317.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.25

$$\int x^3 \cosh^2(a+bx) \sinh^3(a+bx)\, dx$$
$$= \begin{cases} \frac{x^3\sinh^2{(a+bx)}\cosh^3{(a+bx)}}{3b} - \frac{2x^3\cosh^5{(a+bx)}}{15b} + \frac{26x^2\sinh^5{(a+bx)}}{75b^2} - \frac{13x^2\sinh^3{(a+bx)}\cosh^2{(a+bx)}}{15b^2} + \frac{2x^2\sinh{(a+bx)}\cosh^4{(a+}}{5b^2} \\ \frac{x^4\sinh^3{(a)}\cosh^2{(a)}}{4} \end{cases}$$

input
```
integrate(x**3*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Piecewise((x**3*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*x**3*cosh(a +
b*x)**5/(15*b) + 26*x**2*sinh(a + b*x)**5/(75*b**2) - 13*x**2*sinh(a + b*x
)**3*cosh(a + b*x)**2/(15*b**2) + 2*x**2*sinh(a + b*x)*cosh(a + b*x)**4/(5
*b**2) - 52*x*sinh(a + b*x)**4*cosh(a + b*x)/(75*b**3) + 338*x*sinh(a + b*
x)**2*cosh(a + b*x)**3/(225*b**3) - 856*x*cosh(a + b*x)**5/(1125*b**3) + 1
2568*sinh(a + b*x)**5/(16875*b**4) - 5114*sinh(a + b*x)**3*cosh(a + b*x)**
2/(3375*b**4) + 856*sinh(a + b*x)*cosh(a + b*x)**4/(1125*b**4), Ne(b, 0)),
 (x**4*sinh(a)**3*cosh(a)**2/4, True))
```

3.317.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.21

$$\int x^3\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{(125\,b^3x^3e^{(5\,a)} - 75\,b^2x^2e^{(5\,a)} + 30\,bxe^{(5\,a)} - 6\,e^{(5\,a)})e^{(5\,bx)}}{20000\,b^4} - \frac{(9\,b^3x^3e^{(3\,a)} - 9\,b^2x^2e^{(3\,a)} + 6\,bxe^{(3\,a)} - 2\,e^{(3\,a)})e^{(3\,bx)}}{864\,b^4} - \frac{(b^3x^3e^a - 3\,b^2x^2e^a + 6\,bxe^a - 6\,e^a)e^{(bx)}}{16\,b^4} - \frac{(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)e^{(-bx-a)}}{16\,b^4} - \frac{(9\,b^3x^3 + 9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{864\,b^4} + \frac{(125\,b^3x^3 + 75\,b^2x^2 + 30\,bx + 6)e^{(-5\,bx-5\,a)}}{20000\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/20000*(125*b^3*x^3*e^(5*a) - 75*b^2*x^2*e^(5*a) + 30*b*x*e^(5*a) - 6*e^(
5*a))*e^(5*b*x)/b^4 - 1/864*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x
*e^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 1/16*(b^3*x^3*e^a - 3*b^2*x^2*e^a +
6*b*x*e^a - 6*e^a)*e^(b*x)/b^4 - 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^
(-b*x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a
)/b^4 + 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5*a)/b
^4
```

3.317.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05

$$\int x^3\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{(125\,b^3x^3 - 75\,b^2x^2 + 30\,bx - 6)e^{(5\,bx+5\,a)}}{20000\,b^4} - \frac{(9\,b^3x^3 - 9\,b^2x^2 + 6\,bx - 2)e^{(3\,bx+3\,a)}}{864\,b^4} - \frac{(b^3x^3 - 3\,b^2x^2 + 6\,bx - 6)e^{(bx+a)}}{16\,b^4} - \frac{(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)e^{(-bx-a)}}{16\,b^4} - \frac{(9\,b^3x^3 + 9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{864\,b^4} + \frac{(125\,b^3x^3 + 75\,b^2x^2 + 30\,bx + 6)e^{(-5\,bx-5\,a)}}{20000\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/20000*(125*b^3*x^3 - 75*b^2*x^2 + 30*b*x - 6)*e^(5*b*x + 5*a)/b^4 - 1/86
4*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 - 1/16*(b^3*x^3
- 3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 - 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b
*x + 6)*e^(-b*x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3
*b*x - 3*a)/b^4 + 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*
x - 5*a)/b^4
```

3.317.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

$$\int x^3 \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{12568\sinh(a+b\,x)}{16875\,b^4} - \frac{\frac{x^3\cosh(a+b\,x)^3}{3} - \frac{x^3\cosh(a+b\,x)^5}{5}}{b} - \frac{-\frac{6\,x\cosh(a+b\,x)^5}{125} + \frac{26\,x\cosh(a+b\,x)^3}{225} + \frac{52\,x\cosh(a+b\,x)}{75}}{b^3} + \frac{\frac{26\,x^2\sinh(a+b\,x)}{75} + \frac{13\,x^2\cosh(a+b\,x)^2\sinh(a+b\,x)}{75} - \frac{3\,x^2\cosh(a+b\,x)^4\sinh(a+b\,x)}{25}}{b^2} + \frac{434\cosh(a+b\,x)^2\sinh(a+b\,x)}{16875\,b^4} - \frac{6\cosh(a+b\,x)^4\sinh(a+b\,x)}{625\,b^4}$$

input
```
int(x^3*cosh(a + b*x)^2*sinh(a + b*x)^3,x)
```

output
```
(12568*sinh(a + b*x))/(16875*b^4) - ((x^3*cosh(a + b*x)^3)/3 - (x^3*cosh(a
 + b*x)^5)/5)/b - ((52*x*cosh(a + b*x))/75 + (26*x*cosh(a + b*x)^3)/225 -
(6*x*cosh(a + b*x)^5)/125)/b^3 + ((26*x^2*sinh(a + b*x))/75 + (13*x^2*cosh
(a + b*x)^2*sinh(a + b*x))/75 - (3*x^2*cosh(a + b*x)^4*sinh(a + b*x))/25)/
b^2 + (434*cosh(a + b*x)^2*sinh(a + b*x))/(16875*b^4) - (6*cosh(a + b*x)^4
*sinh(a + b*x))/(625*b^4)
```

3.318 $\int x^2 \cosh^2(a+bx) \sinh^3(a+bx)\,dx$

3.318.1 Optimal result

Integrand size = 20, antiderivative size = 148

$$\int x^2 \cosh^2(a+bx) \sinh^3(a+bx)\,dx = -\frac{\cosh(a+bx)}{4b^3} - \frac{x^2\cosh(a+bx)}{8b} - \frac{\cosh(3a+3bx)}{216b^3} - \frac{x^2\cosh(3a+3bx)}{48b} + \frac{\cosh(5a+5bx)}{1000b^3} + \frac{x^2\cosh(5a+5bx)}{80b} + \frac{x\sinh(a+bx)}{4b^2} + \frac{x\sinh(3a+3bx)}{72b^2} - \frac{x\sinh(5a+5bx)}{200b^2}$$

output
```
-1/4*cosh(b*x+a)/b^3-1/8*x^2*cosh(b*x+a)/b-1/216*cosh(3*b*x+3*a)/b^3-1/48*
x^2*cosh(3*b*x+3*a)/b+1/1000*cosh(5*b*x+5*a)/b^3+1/80*x^2*cosh(5*b*x+5*a)/
b+1/4*x*sinh(b*x+a)/b^2+1/72*x*sinh(3*b*x+3*a)/b^2-1/200*x*sinh(5*b*x+5*a)
/b^2
```

3.318.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.66

$$\int x^2 \cosh^2(a+bx) \sinh^3(a+bx)\,dx = \frac{-6750(2+b^2x^2)\cosh(a+bx) - 125(2+9b^2x^2)\cosh(3(a+bx)) + 27(2+25b^2x^2)\cosh(5(a+bx)) + 30bx}{54000b^3}$$

input
```
Integrate[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
(-6750*(2 + b^2*x^2)*Cosh[a + b*x] - 125*(2 + 9*b^2*x^2)*Cosh[3*(a + b*x)]
 + 27*(2 + 25*b^2*x^2)*Cosh[5*(a + b*x)] + 30*b*x*(450*Sinh[a + b*x] + 25*
Sinh[3*(a + b*x)] - 9*Sinh[5*(a + b*x)]))/(54000*b^3)
```

3.318.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sinh^3(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(-\frac{1}{8}x^2\sinh(a+bx)-\frac{1}{16}x^2\sinh(3a+3bx)+\frac{1}{16}x^2\sinh(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cosh(a+bx)}{4b^3}-\frac{\cosh(3a+3bx)}{216b^3}+\frac{\cosh(5a+5bx)}{1000b^3}+\frac{x\sinh(a+bx)}{4b^2}+\frac{x\sinh(3a+3bx)}{72b^2}-\frac{x\sinh(5a+5bx)}{200b^2}-\frac{x^2\cosh(a+bx)}{8b}-\frac{x^2\cosh(3a+3bx)}{48b}+\frac{x^2\cosh(5a+5bx)}{80b}$$

input
```
Int[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
-1/4*Cosh[a + b*x]/b^3 - (x^2*Cosh[a + b*x])/(8*b) - Cosh[3*a + 3*b*x]/(21
6*b^3) - (x^2*Cosh[3*a + 3*b*x])/(48*b) + Cosh[5*a + 5*b*x]/(1000*b^3) + (
x^2*Cosh[5*a + 5*b*x])/(80*b) + (x*Sinh[a + b*x])/(4*b^2) + (x*Sinh[3*a +
3*b*x])/(72*b^2) - (x*Sinh[5*a + 5*b*x])/(200*b^2)
```

3.318.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.318.4 Maple [A] (verified)

Time = 26.87 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

method	result
risch	$\frac{(25x^2b^2-10bx+2)\mathrm{e}^{5bx+5a}}{4000b^3}-\frac{(9x^2b^2-6bx+2)\mathrm{e}^{3bx+3a}}{864b^3}-\frac{(x^2b^2-2bx+2)\mathrm{e}^{bx+a}}{16b^3}-\frac{(x^2b^2+2bx+2)\mathrm{e}^{-bx-a}}{16b^3}-\frac{(9x^2b}{}$
derivativedivides	$a^2\left(\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{5}-\frac{2(bx+a)\cosh(bx+a)^3}{15}-\frac{\sinh(bx+a)\cosh}{25}\right.$
default	$a^2\left(\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{5}-\frac{2(bx+a)\cosh(bx+a)^3}{15}-\frac{\sinh(bx+a)\cosh}{25}\right.$

input
```
int(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4000*(25*b^2*x^2-10*b*x+2)/b^3*exp(5*b*x+5*a)-1/864*(9*b^2*x^2-6*b*x+2)/
b^3*exp(3*b*x+3*a)-1/16*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)-1/16*(b^2*x^2+2*b
*x+2)/b^3*exp(-b*x-a)-1/864*(9*b^2*x^2+6*b*x+2)/b^3*exp(-3*b*x-3*a)+1/4000
*(25*b^2*x^2+10*b*x+2)/b^3*exp(-5*b*x-5*a)
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.45

$$\int x^2\cosh^2(a+bx)\sinh^3(a+bx)\,dx =$$
$$-\frac{270\,bx\sinh(bx+a)^5-27\,(25\,b^2x^2+2)\cosh(bx+a)^5-135\,(25\,b^2x^2+2)\cosh(bx+a)\sinh(bx+a)^4}{}$$

input
```
integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/54000*(270*b*x*sinh(b*x + a)^5 - 27*(25*b^2*x^2 + 2)*cosh(b*x + a)^5 -
135*(25*b^2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^4 + 125*(9*b^2*x^2 + 2)*c
osh(b*x + a)^3 + 150*(18*b*x*cosh(b*x + a)^2 - 5*b*x)*sinh(b*x + a)^3 - 15
*(18*(25*b^2*x^2 + 2)*cosh(b*x + a)^3 - 25*(9*b^2*x^2 + 2)*cosh(b*x + a))*
sinh(b*x + a)^2 + 6750*(b^2*x^2 + 2)*cosh(b*x + a) + 450*(3*b*x*cosh(b*x +
 a)^4 - 5*b*x*cosh(b*x + a)^2 - 30*b*x)*sinh(b*x + a))/b^3
```

3.318.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.23

$$\int x^2 \cosh^2(a+bx) \sinh^3(a+bx) \, dx$$
$$= \begin{cases} \frac{x^2 \sinh^2(a+bx) \cosh^3(a+bx)}{3b} - \frac{2x^2 \cosh^5(a+bx)}{15b} + \frac{52x \sinh^5(a+bx)}{225b^2} - \frac{26x \sinh^3(a+bx) \cosh^2(a+bx)}{45b^2} + \frac{4x \sinh(a+bx) \cosh^4(a+bx)}{15b^2} \\ \frac{x^3 \sinh^3(a) \cosh^2(a)}{3} \end{cases}$$

input
```
integrate(x**2*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Piecewise((x**2*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*x**2*cosh(a +
b*x)**5/(15*b) + 52*x*sinh(a + b*x)**5/(225*b**2) - 26*x*sinh(a + b*x)**3*
cosh(a + b*x)**2/(45*b**2) + 4*x*sinh(a + b*x)*cosh(a + b*x)**4/(15*b**2)
- 52*sinh(a + b*x)**4*cosh(a + b*x)/(225*b**3) + 338*sinh(a + b*x)**2*cosh
(a + b*x)**3/(675*b**3) - 856*cosh(a + b*x)**5/(3375*b**3), Ne(b, 0)), (x*
*3*sinh(a)**3*cosh(a)**2/3, True))
```

3.318.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.26

$$\int x^2 \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{\left(25\,b^2x^2e^{(5\,a)} - 10\,bxe^{(5\,a)} + 2\,e^{(5\,a)}\right)e^{(5\,bx)}}{4000\,b^3} - \frac{\left(9\,b^2x^2e^{(3\,a)} - 6\,bxe^{(3\,a)} + 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{864\,b^3} - \frac{(b^2x^2e^a - 2\,bxe^a + 2\,e^a)e^{(bx)}}{16\,b^3} - \frac{(b^2x^2 + 2\,bx + 2)e^{(-bx-a)}}{16\,b^3} - \frac{(9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{864\,b^3} + \frac{(25\,b^2x^2 + 10\,bx + 2)e^{(-5\,bx-5\,a)}}{4000\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4000*(25*b^2*x^2*e^(5*a) - 10*b*x*e^(5*a) + 2*e^(5*a))*e^(5*b*x)/b^3 - 1
/864*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 - 1/16*
(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 1/16*(b^2*x^2 + 2*b*x + 2)
*e^(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3 + 1
/4000*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)/b^3
```

3.318.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.11

$$\int x^2 \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{(25\,b^2x^2 - 10\,bx + 2)e^{(5\,bx+5\,a)}}{4000\,b^3} - \frac{(9\,b^2x^2 - 6\,bx + 2)e^{(3\,bx+3\,a)}}{864\,b^3} - \frac{(b^2x^2 - 2\,bx + 2)e^{(bx+a)}}{16\,b^3} - \frac{(b^2x^2 + 2\,bx + 2)e^{(-bx-a)}}{16\,b^3} - \frac{(9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{864\,b^3} + \frac{(25\,b^2x^2 + 10\,bx + 2)e^{(-5\,bx-5\,a)}}{4000\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4000*(25*b^2*x^2 - 10*b*x + 2)*e^(5*b*x + 5*a)/b^3 - 1/864*(9*b^2*x^2 -
6*b*x + 2)*e^(3*b*x + 3*a)/b^3 - 1/16*(b^2*x^2 - 2*b*x + 2)*e^(b*x + a)/b^
3 - 1/16*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x
 + 2)*e^(-3*b*x - 3*a)/b^3 + 1/4000*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x -
5*a)/b^3
```

3.318.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int x^2 \cosh^2(a+bx) \sinh^3(a+bx)\,dx =$$
$$-\frac{780\cosh(a+b\,x)+130\cosh(a+b\,x)^3-54\cosh(a+b\,x)^5-780\,b\,x\sinh(a+b\,x)+1125\,b^2\,x^2\cosh(a+$$

input
```
int(x^2*cosh(a + b*x)^2*sinh(a + b*x)^3,x)
```

output
```
-(780*cosh(a + b*x) + 130*cosh(a + b*x)^3 - 54*cosh(a + b*x)^5 - 780*b*x*s
inh(a + b*x) + 1125*b^2*x^2*cosh(a + b*x)^3 - 675*b^2*x^2*cosh(a + b*x)^5
- 390*b*x*cosh(a + b*x)^2*sinh(a + b*x) + 270*b*x*cosh(a + b*x)^4*sinh(a +
 b*x))/(3375*b^3)
```

3.319 $\int x \cosh^2(a+bx) \sinh^3(a+bx) \, dx$

3.319.1 Optimal result

Integrand size = 18, antiderivative size = 94

$$\int x \cosh^2(a+bx) \sinh^3(a+bx) \, dx = -\frac{x\cosh(a+bx)}{8b} - \frac{x\cosh(3a+3bx)}{48b} + \frac{x\cosh(5a+5bx)}{80b} + \frac{\sinh(a+bx)}{8b^2} + \frac{\sinh(3a+3bx)}{144b^2} - \frac{\sinh(5a+5bx)}{400b^2}$$

output
```
-1/8*x*cosh(b*x+a)/b-1/48*x*cosh(3*b*x+3*a)/b+1/80*x*cosh(5*b*x+5*a)/b+1/8
*sinh(b*x+a)/b^2+1/144*sinh(3*b*x+3*a)/b^2-1/400*sinh(5*b*x+5*a)/b^2
```

3.319.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int x \cosh^2(a+bx) \sinh^3(a+bx) \, dx = \frac{-450bx\cosh(a+bx) - 75bx\cosh(3(a+bx)) + 45bx\cosh(5(a+bx)) + 450\sinh(a+bx) + 25\sinh(3(a+}{3600b^2}$$

input
```
Integrate[x*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
(-450*b*x*Cosh[a + b*x] - 75*b*x*Cosh[3*(a + b*x)] + 45*b*x*Cosh[5*(a + b*
x)] + 450*Sinh[a + b*x] + 25*Sinh[3*(a + b*x)] - 9*Sinh[5*(a + b*x)])/(360
0*b^2)
```

3.319.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sinh^3(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(-\frac{1}{8}x\sinh(a+bx)-\frac{1}{16}x\sinh(3a+3bx)+\frac{1}{16}x\sinh(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh(a+bx)}{8b^2}+\frac{\sinh(3a+3bx)}{144b^2}-\frac{\sinh(5a+5bx)}{400b^2}-\frac{x\cosh(a+bx)}{8b}-\frac{x\cosh(3a+3bx)}{48b}+\frac{x\cosh(5a+5bx)}{80b}$$

input
```
Int[x*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
-1/8*(x*Cosh[a + b*x])/b - (x*Cosh[3*a + 3*b*x])/(48*b) + (x*Cosh[5*a + 5*
b*x])/(80*b) + Sinh[a + b*x]/(8*b^2) + Sinh[3*a + 3*b*x]/(144*b^2) - Sinh[
5*a + 5*b*x]/(400*b^2)
```

3.319.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.319.4 Maple [A] (verified)

Time = 19.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{5}-\frac{2(bx+a)\cosh(bx+a)^3}{15}-\frac{\sinh(bx+a)\cosh(bx+a)^4}{25}+\frac{26\sinh(bx+a)}{225}+\frac{13\cosh(bx+a)^2\sinh(bx+a)}{225}-}{b^2}$
default	$\frac{\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{5}-\frac{2(bx+a)\cosh(bx+a)^3}{15}-\frac{\sinh(bx+a)\cosh(bx+a)^4}{25}+\frac{26\sinh(bx+a)}{225}+\frac{13\cosh(bx+a)^2\sinh(bx+a)}{225}-}{b^2}$
risch	$\frac{(5bx-1)e^{5bx+5a}}{800b^2}-\frac{(3bx-1)e^{3bx+3a}}{288b^2}-\frac{(bx-1)e^{bx+a}}{16b^2}-\frac{(bx+1)e^{-bx-a}}{16b^2}-\frac{(3bx+1)e^{-3bx-3a}}{288b^2}+\frac{(5bx+1)e^{-5bx-5}}{800b^2}$

input
```
int(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/5*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)*cosh(b*x+a)^3-
1/25*sinh(b*x+a)*cosh(b*x+a)^4+26/225*sinh(b*x+a)+13/225*cosh(b*x+a)^2*sin
h(b*x+a)-a*(1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)^3))
```

3.319.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.63

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{45\,bx\cosh(bx+a)^5+225\,bx\cosh(bx+a)\sinh(bx+a)^4-75\,bx\cosh(bx+a)^3-9\,\sinh(bx+a)^5-5\,(1}{}$$

input
```
integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3600*(45*b*x*cosh(b*x + a)^5 + 225*b*x*cosh(b*x + a)*sinh(b*x + a)^4 - 7
5*b*x*cosh(b*x + a)^3 - 9*sinh(b*x + a)^5 - 5*(18*cosh(b*x + a)^2 - 5)*sin
h(b*x + a)^3 - 450*b*x*cosh(b*x + a) + 225*(2*b*x*cosh(b*x + a)^3 - b*x*co
sh(b*x + a))*sinh(b*x + a)^2 - 15*(3*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 -
 30)*sinh(b*x + a))/b^2
```

3.319.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases}\frac{x\sinh^2(a+bx)\cosh^3(a+bx)}{3b}-\frac{2x\cosh^5(a+bx)}{15b}+\frac{26\sinh^5(a+bx)}{225b^2}-\frac{13\sinh^3(a+bx)\cosh^2(a+bx)}{45b^2}+\frac{2\sinh(a+bx)\cosh^4(a+bx)}{15b^2} & \text{fo}\\ \frac{x^2\sinh^3(a)\cosh^2(a)}{2} & \text{ot}\end{cases}$$

input
```
integrate(x*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Piecewise((x*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*x*cosh(a + b*x)**
5/(15*b) + 26*sinh(a + b*x)**5/(225*b**2) - 13*sinh(a + b*x)**3*cosh(a + b
*x)**2/(45*b**2) + 2*sinh(a + b*x)*cosh(a + b*x)**4/(15*b**2), Ne(b, 0)),
(x**2*sinh(a)**3*cosh(a)**2/2, True))
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx=\frac{\left(5\,bxe^{(5\,a)}-e^{(5\,a)}\right)e^{(5\,bx)}}{800\,b^2}-\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{288\,b^2}-\frac{(bxe^a-e^a)e^{(bx)}}{16\,b^2}-\frac{(bx+1)e^{(-bx-a)}}{16\,b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2}+\frac{(5\,bx+1)e^{(-5\,bx-5\,a)}}{800\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/800*(5*b*x*e^(5*a) - e^(5*a))*e^(5*b*x)/b^2 - 1/288*(3*b*x*e^(3*a) - e^(
3*a))*e^(3*b*x)/b^2 - 1/16*(b*x*e^a - e^a)*e^(b*x)/b^2 - 1/16*(b*x + 1)*e^
(-b*x - a)/b^2 - 1/288*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 + 1/800*(5*b*x + 1
)*e^(-5*b*x - 5*a)/b^2
```

3.319.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{(5\,bx-1)e^{(5\,bx+5\,a)}}{800\,b^2} - \frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{288\,b^2} - \frac{(bx-1)e^{(bx+a)}}{16\,b^2} - \frac{(bx+1)e^{(-bx-a)}}{16\,b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2} + \frac{(5\,bx+1)e^{(-5\,bx-5\,a)}}{800\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/800*(5*b*x - 1)*e^(5*b*x + 5*a)/b^2 - 1/288*(3*b*x - 1)*e^(3*b*x + 3*a)/
b^2 - 1/16*(b*x - 1)*e^(b*x + a)/b^2 - 1/16*(b*x + 1)*e^(-b*x - a)/b^2 - 1
/288*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 + 1/800*(5*b*x + 1)*e^(-5*b*x - 5*a)
/b^2
```

3.319.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{\frac{26\sinh(a+b\,x)}{225} - b\left(\frac{x\cosh(a+b\,x)^3}{3} - \frac{x\cosh(a+b\,x)^5}{5}\right) + \frac{13\cosh(a+b\,x)^2\sinh(a+b\,x)}{225} - \frac{\cosh(a+b\,x)^4\sinh(a+b\,x)}{25}}{b^2}$$

input
```
int(x*cosh(a + b*x)^2*sinh(a + b*x)^3,x)
```

output
```
((26*sinh(a + b*x))/225 - b*((x*cosh(a + b*x)^3)/3 - (x*cosh(a + b*x)^5)/5
) + (13*cosh(a + b*x)^2*sinh(a + b*x))/225 - (cosh(a + b*x)^4*sinh(a + b*x
))/25)/b^2
```

3.320 $\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx$

3.320.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx=-\frac{\cosh^3(a+bx)}{3b}+\frac{\cosh^5(a+bx)}{5b}$$

output
```
-1/3*cosh(b*x+a)^3/b+1/5*cosh(b*x+a)^5/b
```

3.320.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx=\frac{\cosh^3(a+bx)(-7+3\cosh(2(a+bx)))}{30b}$$

input
```
Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
(Cosh[a + b*x]^3*(-7 + 3*Cosh[2*(a + b*x)]))/(30*b)
```

3.320.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 26, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int i\sin(ia+ibx)^3\cos(ia+ibx)^2dx$$

$$\Big\downarrow \; 26$$

$$i\int \cos(ia+ibx)^2\sin(ia+ibx)^3dx$$

$$\Big\downarrow \; 3045$$

$$-\frac{\int \cosh^2(a+bx)\left(1-\cosh^2(a+bx)\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$-\frac{\int \left(\cosh^2(a+bx)-\cosh^4(a+bx)\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{1}{3}\cosh^3(a+bx)-\frac{1}{5}\cosh^5(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
-((Cosh[a + b*x]^3/3 - Cosh[a + b*x]^5/5)/b)
```

3.320.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.320.4 Maple [A] (verified)

Time = 12.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^5}{5}-\frac{\cosh(bx+a)^3}{3}}{b}$	26
default	$\frac{\frac{\cosh(bx+a)^5}{5}-\frac{\cosh(bx+a)^3}{3}}{b}$	26
risch	$\frac{e^{5bx+5a}}{160b}-\frac{e^{3bx+3a}}{96b}-\frac{e^{bx+a}}{16b}-\frac{e^{-bx-a}}{16b}-\frac{e^{-3bx-3a}}{96b}+\frac{e^{-5bx-5a}}{160b}$	83

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5*cosh(b*x+a)^5-1/3*cosh(b*x+a)^3)
```

3.320.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.55

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{3\cosh(bx+a)^5 + 15\cosh(bx+a)\sinh(bx+a)^4 - 5\cosh(bx+a)^3 + 15\left(2\cosh(bx+a)^3 - \cosh(bx+\right.}{240\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/240*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 - 5*cosh(b*x +
 a)^3 + 15*(2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 30*cosh(b
*x + a))/b
```

3.320.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \begin{cases} \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{3b} - \frac{2\cosh^5(a+bx)}{15b} & \text{for } b \neq 0 \\ x\sinh^3(a)\cosh^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Piecewise((sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*cosh(a + b*x)**5/(1
5*b), Ne(b, 0)), (x*sinh(a)**3*cosh(a)**2, True))
```

3.320.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(27) = 54$.

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.52

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(5e^{(-2bx-2a)} + 30e^{(-4bx-4a)} - 3\right)e^{(5bx+5a)}}{480\,b} - \frac{30e^{(-bx-a)} + 5e^{(-3bx-3a)} - 3e^{(-5bx-5a)}}{480\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/480*(5*e^(-2*b*x - 2*a) + 30*e^(-4*b*x - 4*a) - 3)*e^(5*b*x + 5*a)/b -
1/480*(30*e^(-b*x - a) + 5*e^(-3*b*x - 3*a) - 3*e^(-5*b*x - 5*a))/b
```

3.320.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.65

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{160\,b} - \frac{e^{(3\,bx+3\,a)}}{96\,b} - \frac{e^{(bx+a)}}{16\,b} - \frac{e^{(-bx-a)}}{16\,b} - \frac{e^{(-3\,bx-3\,a)}}{96\,b} + \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/160*e^(5*b*x + 5*a)/b - 1/96*e^(3*b*x + 3*a)/b - 1/16*e^(b*x + a)/b - 1/
16*e^(-b*x - a)/b - 1/96*e^(-3*b*x - 3*a)/b + 1/160*e^(-5*b*x - 5*a)/b
```

3.320.9 Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx = -\frac{5\cosh(a+b\,x)^3 - 3\cosh(a+b\,x)^5}{15\,b}$$

input
```
int(cosh(a + b*x)^2*sinh(a + b*x)^3,x)
```

output
```
-(5*cosh(a + b*x)^3 - 3*cosh(a + b*x)^5)/(15*b)
```

3.321 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx$

3.321.1 Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = -\frac{1}{8}\text{Chi}(bx)\sinh(a) - \frac{1}{16}\text{Chi}(3bx)\sinh(3a) + \frac{1}{16}\text{Chi}(5bx)\sinh(5a) - \frac{1}{8}\cosh(a)\text{Shi}(bx) - \frac{1}{16}\cosh(3a)\text{Shi}(3bx) + \frac{1}{16}\cosh(5a)\text{Shi}(5bx)$$

output
```
-1/8*cosh(a)*Shi(b*x)-1/16*cosh(3*a)*Shi(3*b*x)+1/16*cosh(5*a)*Shi(5*b*x)-
1/8*Chi(b*x)*sinh(a)-1/16*Chi(3*b*x)*sinh(3*a)+1/16*Chi(5*b*x)*sinh(5*a)
```

3.321.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{16}(-2\text{Chi}(bx)\sinh(a) - \text{Chi}(3bx)\sinh(3a) + \text{Chi}(5bx)\sinh(5a) - 2\cosh(a)\text{Shi}(bx) - \cosh(3a)\text{Shi}(3bx) + \cosh(5a)\text{Shi}(5bx))$$

input
```
Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x,x]
```

output
```
(-2*CoshIntegral[b*x]*Sinh[a] - CoshIntegral[3*b*x]*Sinh[3*a] + CoshIntegr
al[5*b*x]*Sinh[5*a] - 2*Cosh[a]*SinhIntegral[b*x] - Cosh[3*a]*SinhIntegral
[3*b*x] + Cosh[5*a]*SinhIntegral[5*b*x])/16
```

3.321.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x}\,dx$$
$$\Big\downarrow \; 5971$$
$$\int \left(-\frac{\sinh(a+bx)}{8x}-\frac{\sinh(3a+3bx)}{16x}+\frac{\sinh(5a+5bx)}{16x}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{1}{8}\sinh(a)\text{Chi}(bx)-\frac{1}{16}\sinh(3a)\text{Chi}(3bx)+\frac{1}{16}\sinh(5a)\text{Chi}(5bx)-\frac{1}{8}\cosh(a)\text{Shi}(bx)-\frac{1}{16}\cosh(3a)\text{Shi}(3bx)+\frac{1}{16}\cosh(5a)\text{Shi}(5bx)$$

input
```
Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x,x]
```

output
```
-1/8*(CoshIntegral[b*x]*Sinh[a]) - (CoshIntegral[3*b*x]*Sinh[3*a])/16 + (C
oshIntegral[5*b*x]*Sinh[5*a])/16 - (Cosh[a]*SinhIntegral[b*x])/8 - (Cosh[3
*a]*SinhIntegral[3*b*x])/16 + (Cosh[5*a]*SinhIntegral[5*b*x])/16
```

3.321.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.321.4 Maple [A] (verified)

Time = 11.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{e^{-5a}\operatorname{Ei}_1(5bx)}{32}-\frac{e^{-3a}\operatorname{Ei}_1(3bx)}{32}-\frac{e^{-a}\operatorname{Ei}_1(bx)}{16}+\frac{e^{a}\operatorname{Ei}_1(-bx)}{16}+\frac{e^{3a}\operatorname{Ei}_1(-3bx)}{32}-\frac{e^{5a}\operatorname{Ei}_1(-5bx)}{32}$	71

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/32*exp(-5*a)*Ei(1,5*b*x)-1/32*exp(-3*a)*Ei(1,3*b*x)-1/16*exp(-a)*Ei(1,b*
x)+1/16*exp(a)*Ei(1,-b*x)+1/32*exp(3*a)*Ei(1,-3*b*x)-1/32*exp(5*a)*Ei(1,-5
*b*x)
```

3.321.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = {} & \frac{1}{32}\left(\operatorname{Ei}(5\,bx)-\operatorname{Ei}(-5\,bx)\right)\cosh\left(5\,a\right) \\ & -\frac{1}{32}\left(\operatorname{Ei}(3\,bx)-\operatorname{Ei}(-3\,bx)\right)\cosh\left(3\,a\right) \\ & -\frac{1}{16}\left(\operatorname{Ei}(bx)-\operatorname{Ei}(-bx)\right)\cosh\left(a\right) \\ & +\frac{1}{32}\left(\operatorname{Ei}(5\,bx)+\operatorname{Ei}(-5\,bx)\right)\sinh\left(5\,a\right) \\ & -\frac{1}{32}\left(\operatorname{Ei}(3\,bx)+\operatorname{Ei}(-3\,bx)\right)\sinh\left(3\,a\right) \\ & -\frac{1}{16}\left(\operatorname{Ei}(bx)+\operatorname{Ei}(-bx)\right)\sinh\left(a\right)\end{aligned}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
1/32*(Ei(5*b*x) - Ei(-5*b*x))*cosh(5*a) - 1/32*(Ei(3*b*x) - Ei(-3*b*x))*co
sh(3*a) - 1/16*(Ei(b*x) - Ei(-b*x))*cosh(a) + 1/32*(Ei(5*b*x) + Ei(-5*b*x)
)*sinh(5*a) - 1/32*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) - 1/16*(Ei(b*x) + Ei
(-b*x))*sinh(a)
```

3.321.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = \int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x,x)
```

output
```
Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x, x)
```

3.321.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{32}\,\mathrm{Ei}(5\,bx)\,e^{(5\,a)} - \frac{1}{32}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} + \frac{1}{16}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{32}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - \frac{1}{32}\,\mathrm{Ei}(-5\,bx)\,e^{(-5\,a)} - \frac{1}{16}\,\mathrm{Ei}(bx)\,e^{a}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
1/32*Ei(5*b*x)*e^(5*a) - 1/32*Ei(3*b*x)*e^(3*a) + 1/16*Ei(-b*x)*e^(-a) + 1
/32*Ei(-3*b*x)*e^(-3*a) - 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

3.321.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{32}\,\mathrm{Ei}(5\,bx)\,e^{(5\,a)} - \frac{1}{32}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} + \frac{1}{16}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{32}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - \frac{1}{32}\,\mathrm{Ei}(-5\,bx)\,e^{(-5\,a)} - \frac{1}{16}\,\mathrm{Ei}(bx)\,e^{a}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="giac")
```

output
```
1/32*Ei(5*b*x)*e^(5*a) - 1/32*Ei(3*b*x)*e^(3*a) + 1/16*Ei(-b*x)*e^(-a) + 1
/32*Ei(-3*b*x)*e^(-3*a) - 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^3}{x}\,dx$$

input
```
int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x,x)
```

output
```
int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x, x)
```

3.322 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2} dx$

3.322.1 Optimal result

Integrand size = 20, antiderivative size = 124

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2} dx = -\frac{1}{8}b\cosh(a)\text{Chi}(bx) - \frac{3}{16}b\cosh(3a)\text{Chi}(3bx) + \frac{5}{16}b\cosh(5a)\text{Chi}(5bx) + \frac{\sinh(a+bx)}{8x} + \frac{\sinh(3a+3bx)}{16x} - \frac{\sinh(5a+5bx)}{16x} - \frac{1}{8}b\sinh(a)\text{Shi}(bx) - \frac{3}{16}b\sinh(3a)\text{Shi}(3bx) + \frac{5}{16}b\sinh(5a)\text{Shi}(5bx)$$

output
```
-1/8*b*Chi(b*x)*cosh(a)-3/16*b*Chi(3*b*x)*cosh(3*a)+5/16*b*Chi(5*b*x)*cosh
(5*a)-1/8*b*Shi(b*x)*sinh(a)-3/16*b*Shi(3*b*x)*sinh(3*a)+5/16*b*Shi(5*b*x)
*sinh(5*a)+1/8*sinh(b*x+a)/x+1/16*sinh(3*b*x+3*a)/x-1/16*sinh(5*b*x+5*a)/x
```

3.322.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2} dx = \frac{-2bx\cosh(a)\text{Chi}(bx) - 3bx\cosh(3a)\text{Chi}(3bx) + 5bx\cosh(5a)\text{Chi}(5bx) + 2\sinh(a+bx) + \sinh(3(a+bx)}{16x}$$

input
```
Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^2,x]
```

output
```
(-2*b*x*Cosh[a]*CoshIntegral[b*x] - 3*b*x*Cosh[3*a]*CoshIntegral[3*b*x] +
5*b*x*Cosh[5*a]*CoshIntegral[5*b*x] + 2*Sinh[a + b*x] + Sinh[3*(a + b*x)]
- Sinh[5*(a + b*x)] - 2*b*x*Sinh[a]*SinhIntegral[b*x] - 3*b*x*Sinh[3*a]*Si
nhIntegral[3*b*x] + 5*b*x*Sinh[5*a]*SinhIntegral[5*b*x])/(16*x)
```

3.322.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{16x^2}+\frac{\sinh(5a+5bx)}{16x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}b\cosh(a)\text{Chi}(bx)-\frac{3}{16}b\cosh(3a)\text{Chi}(3bx)+\frac{5}{16}b\cosh(5a)\text{Chi}(5bx)-\frac{1}{8}b\sinh(a)\text{Shi}(bx)-\frac{3}{16}b\sinh(3a)\text{Shi}(3bx)+\frac{5}{16}b\sinh(5a)\text{Shi}(5bx)+\frac{\sinh(a+bx)}{8x}+\frac{\sinh(3a+3bx)}{16x}-\frac{\sinh(5a+5bx)}{16x}$$

input
```
Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^2,x]
```

output
```
-1/8*(b*Cosh[a]*CoshIntegral[b*x]) - (3*b*Cosh[3*a]*CoshIntegral[3*b*x])/1
6 + (5*b*Cosh[5*a]*CoshIntegral[5*b*x])/16 + Sinh[a + b*x]/(8*x) + Sinh[3*
a + 3*b*x]/(16*x) - Sinh[5*a + 5*b*x]/(16*x) - (b*Sinh[a]*SinhIntegral[b*x
])/8 - (3*b*Sinh[3*a]*SinhIntegral[3*b*x])/16 + (5*b*Sinh[5*a]*SinhIntegra
l[5*b*x])/16
```

3.322.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.322.4 Maple [A] (verified)

Time = 12.74 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{5\,e^{5a}\,\mathrm{Ei}_1(-5bx)bx-3\,e^{3a}\,\mathrm{Ei}_1(-3bx)bx+5\,e^{-5a}\,\mathrm{Ei}_1(5bx)bx-3\,e^{-3a}\,\mathrm{Ei}_1(3bx)bx-2\,e^{-a}\,\mathrm{Ei}_1(bx)bx-2\,e^{a}\,\mathrm{Ei}_1(-bx)bx+e^{5bx+5a}-2\,e^{bx}}{32x}$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(5*exp(5*a)*Ei(1,-5*b*x)*b*x-3*exp(3*a)*Ei(1,-3*b*x)*b*x+5*exp(-5*a)
*Ei(1,5*b*x)*b*x-3*exp(-3*a)*Ei(1,3*b*x)*b*x-2*exp(-a)*Ei(1,b*x)*b*x-2*exp
(a)*Ei(1,-b*x)*b*x+exp(5*b*x+5*a)-2*exp(b*x+a)-exp(3*b*x+3*a)-exp(-5*b*x-5
*a)+exp(-3*b*x-3*a)+2*exp(-b*x-a))/x
```

3.322.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.64

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx =$$
$$-\frac{2\sinh(bx+a)^5+2\left(10\cosh(bx+a)^2-1\right)\sinh(bx+a)^3-5\left(bx\mathrm{Ei}(5\,bx)+bx\mathrm{Ei}(-5\,bx)\right)\cosh(5\,a)+}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
-1/32*(2*sinh(b*x + a)^5 + 2*(10*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 5*
(b*x*Ei(5*b*x) + b*x*Ei(-5*b*x))*cosh(5*a) + 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*
b*x))*cosh(3*a) + 2*(b*x*Ei(b*x) + b*x*Ei(-b*x))*cosh(a) + 2*(5*cosh(b*x +
 a)^4 - 3*cosh(b*x + a)^2 - 2)*sinh(b*x + a) - 5*(b*x*Ei(5*b*x) - b*x*Ei(-
5*b*x))*sinh(5*a) + 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))*sinh(3*a) + 2*(b*x*
Ei(b*x) - b*x*Ei(-b*x))*sinh(a))/x
```

3.322.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x**2,x)
```

output
```
Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x**2, x)
```

3.322.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \frac{5}{32}\,be^{(-5\,a)}\Gamma(-1,5\,bx) - \frac{3}{32}\,be^{(-3\,a)}\Gamma(-1,3\,bx) - \frac{1}{16}\,be^{(-a)}\Gamma(-1,bx) - \frac{1}{16}\,be^{a}\Gamma(-1,-bx) - \frac{3}{32}\,be^{(3\,a)}\Gamma(-1,-3\,bx) + \frac{5}{32}\,be^{(5\,a)}\Gamma(-1,-5\,bx)$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
5/32*b*e^(-5*a)*gamma(-1, 5*b*x) - 3/32*b*e^(-3*a)*gamma(-1, 3*b*x) - 1/16
*b*e^(-a)*gamma(-1, b*x) - 1/16*b*e^a*gamma(-1, -b*x) - 3/32*b*e^(3*a)*gam
ma(-1, -3*b*x) + 5/32*b*e^(5*a)*gamma(-1, -5*b*x)
```

3.322.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.13

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \frac{5\,bx\mathrm{Ei}(5\,bx)\,e^{(5\,a)} - 3\,bx\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - 2\,bx\mathrm{Ei}(-bx)\,e^{(-a)} - 3\,bx\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + 5\,bx\mathrm{Ei}(-5\,bx)\,e^{(-5\,a)} -}{32\,x}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
1/32*(5*b*x*Ei(5*b*x)*e^(5*a) - 3*b*x*Ei(3*b*x)*e^(3*a) - 2*b*x*Ei(-b*x)*e
^(-a) - 3*b*x*Ei(-3*b*x)*e^(-3*a) + 5*b*x*Ei(-5*b*x)*e^(-5*a) - 2*b*x*Ei(b
*x)*e^a - e^(5*b*x + 5*a) + e^(3*b*x + 3*a) + 2*e^(b*x + a) - 2*e^(-b*x -
a) - e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a))/x
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^3}{x^2}\,dx$$

input
```
int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^2,x)
```

output
```
int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^2, x)
```

3.323 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx$

3.323.1 Optimal result

Integrand size = 20, antiderivative size = 184

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx &= \frac{b\cosh(a+bx)}{16x}+\frac{3b\cosh(3a+3bx)}{32x}\\&-\frac{5b\cosh(5a+5bx)}{32x}-\frac{1}{16}b^2\text{Chi}(bx)\sinh(a)\\&-\frac{9}{32}b^2\text{Chi}(3bx)\sinh(3a)+\frac{25}{32}b^2\text{Chi}(5bx)\sinh(5a)\\&+\frac{\sinh(a+bx)}{16x^2}+\frac{\sinh(3a+3bx)}{32x^2}\\&-\frac{\sinh(5a+5bx)}{32x^2}-\frac{1}{16}b^2\cosh(a)\text{Shi}(bx)\\&-\frac{9}{32}b^2\cosh(3a)\text{Shi}(3bx)+\frac{25}{32}b^2\cosh(5a)\text{Shi}(5bx)\end{aligned}$$

output
```
1/16*b*cosh(b*x+a)/x+3/32*b*cosh(3*b*x+3*a)/x-5/32*b*cosh(5*b*x+5*a)/x-1/1
6*b^2*cosh(a)*Shi(b*x)-9/32*b^2*cosh(3*a)*Shi(3*b*x)+25/32*b^2*cosh(5*a)*S
hi(5*b*x)-1/16*b^2*Chi(b*x)*sinh(a)-9/32*b^2*Chi(3*b*x)*sinh(3*a)+25/32*b^
2*Chi(5*b*x)*sinh(5*a)+1/16*sinh(b*x+a)/x^2+1/32*sinh(3*b*x+3*a)/x^2-1/32*
sinh(5*b*x+5*a)/x^2
```

3.323.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx$$
$$= \frac{2bx\cosh(a+bx)+3bx\cosh(3(a+bx))-5bx\cosh(5(a+bx))-2b^2x^2\text{Chi}(bx)\sinh(a)-9b^2x^2\text{Chi}(3bx)\,\text{s}}{}$$

input
```
Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^3,x]
```

output
```
(2*b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - 5*b*x*Cosh[5*(a + b*x)] -
 2*b^2*x^2*CoshIntegral[b*x]*Sinh[a] - 9*b^2*x^2*CoshIntegral[3*b*x]*Sinh[
3*a] + 25*b^2*x^2*CoshIntegral[5*b*x]*Sinh[5*a] + 2*Sinh[a + b*x] + Sinh[3
*(a + b*x)] - Sinh[5*(a + b*x)] - 2*b^2*x^2*Cosh[a]*SinhIntegral[b*x] - 9*
b^2*x^2*Cosh[3*a]*SinhIntegral[3*b*x] + 25*b^2*x^2*Cosh[5*a]*SinhIntegral[
5*b*x])/(32*x^2)
```

3.323.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x^3}\,dx$$
$$\Big\downarrow \text{5971}$$
$$\int \left(-\frac{\sinh(a+bx)}{8x^3}-\frac{\sinh(3a+3bx)}{16x^3}+\frac{\sinh(5a+5bx)}{16x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{16}b^2\sinh(a)\text{Chi}(bx)-\frac{9}{32}b^2\sinh(3a)\text{Chi}(3bx)+\frac{25}{32}b^2\sinh(5a)\text{Chi}(5bx)-\frac{1}{16}b^2\cosh(a)\text{Shi}(bx)-\frac{9}{32}b^2\cosh(3a)\text{Shi}(3bx)+\frac{25}{32}b^2\cosh(5a)\text{Shi}(5bx)+\frac{\sinh(a+bx)}{16x^2}+\frac{\sinh(3a+3bx)}{32x^2}-\frac{\sinh(5a+5bx)}{32x^2}+\frac{b\cosh(a+bx)}{16x}+\frac{3b\cosh(3a+3bx)}{32x}-\frac{5b\cosh(5a+5bx)}{32x}$$

input
```
Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^3,x]
```

output
```
(b*Cosh[a + b*x])/(16*x) + (3*b*Cosh[3*a + 3*b*x])/(32*x) - (5*b*Cosh[5*a
+ 5*b*x])/(32*x) - (b^2*CoshIntegral[b*x]*Sinh[a])/16 - (9*b^2*CoshIntegra
l[3*b*x]*Sinh[3*a])/32 + (25*b^2*CoshIntegral[5*b*x]*Sinh[5*a])/32 + Sinh[
a + b*x]/(16*x^2) + Sinh[3*a + 3*b*x]/(32*x^2) - Sinh[5*a + 5*b*x]/(32*x^2
) - (b^2*Cosh[a]*SinhIntegral[b*x])/16 - (9*b^2*Cosh[3*a]*SinhIntegral[3*b
*x])/32 + (25*b^2*Cosh[5*a]*SinhIntegral[5*b*x])/32
```

3.323.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.323.4 Maple [A] (verified)

Time = 13.45 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.34

method	result
risch	$-25\,\mathrm{e}^{5a}\,\mathrm{Ei}_1(-5bx)x^2b^2+25\,\mathrm{e}^{-5a}\,\mathrm{Ei}_1(5bx)x^2b^2-9\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^2b^2-2\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^2b^2+2\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^2b^2+9\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^2b^2$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/64*(-25*exp(5*a)*Ei(1,-5*b*x)*x^2*b^2+25*exp(-5*a)*Ei(1,5*b*x)*x^2*b^2-9
*exp(-3*a)*Ei(1,3*b*x)*x^2*b^2-2*exp(-a)*Ei(1,b*x)*x^2*b^2+2*exp(a)*Ei(1,-
b*x)*x^2*b^2+9*exp(3*a)*Ei(1,-3*b*x)*x^2*b^2+2*exp(b*x+a)*b*x-5*exp(5*b*x+
5*a)*b*x-5*exp(-5*b*x-5*a)*b*x+3*exp(-3*b*x-3*a)*b*x+2*exp(-b*x-a)*b*x+3*e
xp(3*b*x+3*a)*b*x+2*exp(b*x+a)-exp(5*b*x+5*a)+exp(-5*b*x-5*a)-exp(-3*b*x-3
*a)-2*exp(-b*x-a)+exp(3*b*x+3*a))/x^2
```

3.323.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(160) = 320$.

Time = 0.26 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.83

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx =$$
$$-\frac{10\,bx\cosh\left(bx+a\right)^5 + 50\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4 - 6\,bx\cosh\left(bx+a\right)^3 + 2\,\sinh\left(bx+a\right)^5 + 2\,(1}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

output
```
-1/64*(10*b*x*cosh(b*x + a)^5 + 50*b*x*cosh(b*x + a)*sinh(b*x + a)^4 - 6*b
*x*cosh(b*x + a)^3 + 2*sinh(b*x + a)^5 + 2*(10*cosh(b*x + a)^2 - 1)*sinh(b
*x + a)^3 - 4*b*x*cosh(b*x + a) + 2*(50*b*x*cosh(b*x + a)^3 - 9*b*x*cosh(b
*x + a))*sinh(b*x + a)^2 - 25*(b^2*x^2*Ei(5*b*x) - b^2*x^2*Ei(-5*b*x))*cos
h(5*a) + 9*(b^2*x^2*Ei(3*b*x) - b^2*x^2*Ei(-3*b*x))*cosh(3*a) + 2*(b^2*x^2
*Ei(b*x) - b^2*x^2*Ei(-b*x))*cosh(a) + 2*(5*cosh(b*x + a)^4 - 3*cosh(b*x +
 a)^2 - 2)*sinh(b*x + a) - 25*(b^2*x^2*Ei(5*b*x) + b^2*x^2*Ei(-5*b*x))*sin
h(5*a) + 9*(b^2*x^2*Ei(3*b*x) + b^2*x^2*Ei(-3*b*x))*sinh(3*a) + 2*(b^2*x^2
*Ei(b*x) + b^2*x^2*Ei(-b*x))*sinh(a))/x^2
```

3.323.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \frac{\sinh^3\left(a+bx\right)\cosh^2\left(a+bx\right)}{x^3}\,dx$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x**3,x)
```

output
```
Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x**3, x)
```

3.323.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.48

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \frac{25}{32}\,b^2e^{(-5\,a)}\Gamma(-2,5\,bx) - \frac{9}{32}\,b^2e^{(-3\,a)}\Gamma(-2,3\,bx) - \frac{1}{16}\,b^2e^{(-a)}\Gamma(-2,bx) + \frac{1}{16}\,b^2e^{a}\Gamma(-2,-bx) + \frac{9}{32}\,b^2e^{(3\,a)}\Gamma(-2,-3\,bx) - \frac{25}{32}\,b^2e^{(5\,a)}\Gamma(-2,-5\,bx)$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

output
```
25/32*b^2*e^(-5*a)*gamma(-2, 5*b*x) - 9/32*b^2*e^(-3*a)*gamma(-2, 3*b*x) -
 1/16*b^2*e^(-a)*gamma(-2, b*x) + 1/16*b^2*e^a*gamma(-2, -b*x) + 9/32*b^2*
e^(3*a)*gamma(-2, -3*b*x) - 25/32*b^2*e^(5*a)*gamma(-2, -5*b*x)
```

3.323.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \frac{25\,b^2x^2\text{Ei}(5\,bx)\,e^{(5\,a)} - 9\,b^2x^2\text{Ei}(3\,bx)\,e^{(3\,a)} + 2\,b^2x^2\text{Ei}(-bx)\,e^{(-a)} + 9\,b^2x^2\text{Ei}(-3\,bx)\,e^{(-3\,a)} - 25\,b^2x^2\text{Ei}(-5}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x, algorithm="giac")
```

output
```
1/64*(25*b^2*x^2*Ei(5*b*x)*e^(5*a) - 9*b^2*x^2*Ei(3*b*x)*e^(3*a) + 2*b^2*x
^2*Ei(-b*x)*e^(-a) + 9*b^2*x^2*Ei(-3*b*x)*e^(-3*a) - 25*b^2*x^2*Ei(-5*b*x)
*e^(-5*a) - 2*b^2*x^2*Ei(b*x)*e^a - 5*b*x*e^(5*b*x + 5*a) + 3*b*x*e^(3*b*x
 + 3*a) + 2*b*x*e^(b*x + a) + 2*b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3*a)
- 5*b*x*e^(-5*b*x - 5*a) - e^(5*b*x + 5*a) + e^(3*b*x + 3*a) + 2*e^(b*x +
a) - 2*e^(-b*x - a) - e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a))/x^2
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^3}{x^3}\,dx$$

input
```
int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^3,x)
```

output
```
int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^3, x)
```

3.324 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx$

3.324.1 Optimal result

Integrand size = 20, antiderivative size = 238

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx &= \frac{b\cosh(a+bx)}{48x^2}+\frac{b\cosh(3a+3bx)}{32x^2}-\frac{5b\cosh(5a+5bx)}{96x^2}\\ &\quad-\frac{1}{48}b^3\cosh(a)\mathrm{Chi}(bx)-\frac{9}{32}b^3\cosh(3a)\mathrm{Chi}(3bx)\\ &\quad+\frac{125}{96}b^3\cosh(5a)\mathrm{Chi}(5bx)+\frac{\sinh(a+bx)}{24x^3}\\ &\quad+\frac{b^2\sinh(a+bx)}{48x}+\frac{\sinh(3a+3bx)}{48x^3}\\ &\quad+\frac{3b^2\sinh(3a+3bx)}{32x}-\frac{\sinh(5a+5bx)}{48x^3}\\ &\quad-\frac{25b^2\sinh(5a+5bx)}{96x}-\frac{1}{48}b^3\sinh(a)\mathrm{Shi}(bx)\\ &\quad-\frac{9}{32}b^3\sinh(3a)\mathrm{Shi}(3bx)+\frac{125}{96}b^3\sinh(5a)\mathrm{Shi}(5bx)\end{aligned}$$

output
```
-1/48*b^3*Chi(b*x)*cosh(a)-9/32*b^3*Chi(3*b*x)*cosh(3*a)+125/96*b^3*Chi(5*
b*x)*cosh(5*a)+1/48*b*cosh(b*x+a)/x^2+1/32*b*cosh(3*b*x+3*a)/x^2-5/96*b*co
sh(5*b*x+5*a)/x^2-1/48*b^3*Shi(b*x)*sinh(a)-9/32*b^3*Shi(3*b*x)*sinh(3*a)+
125/96*b^3*Shi(5*b*x)*sinh(5*a)+1/24*sinh(b*x+a)/x^3+1/48*b^2*sinh(b*x+a)/
x+1/48*sinh(3*b*x+3*a)/x^3+3/32*b^2*sinh(3*b*x+3*a)/x-1/48*sinh(5*b*x+5*a)
/x^3-25/96*b^2*sinh(5*b*x+5*a)/x
```

3.324.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx$$
$$= \frac{2bx\cosh(a+bx)+3bx\cosh(3(a+bx))-5bx\cosh(5(a+bx))-2b^3x^3\cosh(a)\text{Chi}(bx)-27b^3x^3\cosh(3a)}{}$$

input
```
Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^4,x]
```

output
```
(2*b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - 5*b*x*Cosh[5*(a + b*x)] -
 2*b^3*x^3*Cosh[a]*CoshIntegral[b*x] - 27*b^3*x^3*Cosh[3*a]*CoshIntegral[3
*b*x] + 125*b^3*x^3*Cosh[5*a]*CoshIntegral[5*b*x] + 4*Sinh[a + b*x] + 2*b^
2*x^2*Sinh[a + b*x] + 2*Sinh[3*(a + b*x)] + 9*b^2*x^2*Sinh[3*(a + b*x)] -
2*Sinh[5*(a + b*x)] - 25*b^2*x^2*Sinh[5*(a + b*x)] - 2*b^3*x^3*Sinh[a]*Sin
hIntegral[b*x] - 27*b^3*x^3*Sinh[3*a]*SinhIntegral[3*b*x] + 125*b^3*x^3*Si
nh[5*a]*SinhIntegral[5*b*x])/(96*x^3)
```

3.324.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x^4}\,dx$$
$$\Big\downarrow \ 5971$$
$$\int \left(-\frac{\sinh(a+bx)}{8x^4}-\frac{\sinh(3a+3bx)}{16x^4}+\frac{\sinh(5a+5bx)}{16x^4}\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{1}{48}b^3\cosh(a)\text{Chi}(bx)-\frac{9}{32}b^3\cosh(3a)\text{Chi}(3bx)+\frac{125}{96}b^3\cosh(5a)\text{Chi}(5bx)-\frac{1}{48}b^3\sinh(a)\text{Shi}(bx)-\frac{9}{32}b^3\sinh(3a)\text{Shi}(3bx)+\frac{125}{96}b^3\sinh(5a)\text{Shi}(5bx)+\frac{b^2\sinh(a+bx)}{48x}+\frac{3b^2\sinh(3a+3bx)}{32x}-\frac{25b^2\sinh(5a+5bx)}{96x}+\frac{\sinh(a+bx)}{24x^3}+\frac{\sinh(3a+3bx)}{48x^3}-\frac{\sinh(5a+5bx)}{48x^3}+\frac{b\cosh(a+bx)}{48x^2}+\frac{b\cosh(3a+3bx)}{32x^2}-\frac{5b\cosh(5a+5bx)}{96x^2}$$

input
```
Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^4,x]
```

output
```
(b*Cosh[a + b*x])/(48*x^2) + (b*Cosh[3*a + 3*b*x])/(32*x^2) - (5*b*Cosh[5*
a + 5*b*x])/(96*x^2) - (b^3*Cosh[a]*CoshIntegral[b*x])/48 - (9*b^3*Cosh[3*
a]*CoshIntegral[3*b*x])/32 + (125*b^3*Cosh[5*a]*CoshIntegral[5*b*x])/96 +
Sinh[a + b*x]/(24*x^3) + (b^2*Sinh[a + b*x])/(48*x) + Sinh[3*a + 3*b*x]/(4
8*x^3) + (3*b^2*Sinh[3*a + 3*b*x])/(32*x) - Sinh[5*a + 5*b*x]/(48*x^3) - (
25*b^2*Sinh[5*a + 5*b*x])/(96*x) - (b^3*Sinh[a]*SinhIntegral[b*x])/48 - (9
*b^3*Sinh[3*a]*SinhIntegral[3*b*x])/32 + (125*b^3*Sinh[5*a]*SinhIntegral[5
*b*x])/96
```

3.324.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.324.4 Maple [A] (verified)

Time = 21.62 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{125\,e^{5a}\,\text{Ei}_1(-5bx)x^3b^3+125\,e^{-5a}\,\text{Ei}_1(5bx)x^3b^3-27\,e^{-3a}\,\text{Ei}_1(3bx)x^3b^3-2\,e^{-a}\,\text{Ei}_1(bx)x^3b^3-2\,e^{a}\,\text{Ei}_1(-bx)x^3b^3-27\,e^{3a}\,\text{Ei}_1(-3bx)}{}$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/192*(125*exp(5*a)*Ei(1,-5*b*x)*x^3*b^3+125*exp(-5*a)*Ei(1,5*b*x)*x^3*b^
3-27*exp(-3*a)*Ei(1,3*b*x)*x^3*b^3-2*exp(-a)*Ei(1,b*x)*x^3*b^3-2*exp(a)*Ei
(1,-b*x)*x^3*b^3-27*exp(3*a)*Ei(1,-3*b*x)*x^3*b^3+25*exp(5*b*x+5*a)*b^2*x^
2-2*exp(b*x+a)*b^2*x^2-25*exp(-5*b*x-5*a)*b^2*x^2+9*exp(-3*b*x-3*a)*b^2*x^
2+2*exp(-b*x-a)*b^2*x^2-9*exp(3*b*x+3*a)*b^2*x^2+5*exp(5*b*x+5*a)*b*x-2*ex
p(b*x+a)*b*x+5*exp(-5*b*x-5*a)*b*x-3*exp(-3*b*x-3*a)*b*x-2*exp(-b*x-a)*b*x
-3*exp(3*b*x+3*a)*b*x+2*exp(5*b*x+5*a)-4*exp(b*x+a)-2*exp(-5*b*x-5*a)+2*ex
p(-3*b*x-3*a)+4*exp(-b*x-a)-2*exp(3*b*x+3*a))/x^3
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.65

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx =$$
$$-\frac{10\,bx\cosh\left(bx+a\right)^5+50\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+2\left(25\,b^2x^2+2\right)\sinh\left(bx+a\right)^5-6\,bx\cosh\left(bx\right.}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x, algorithm="fricas")
```

output
```
-1/192*(10*b*x*cosh(b*x + a)^5 + 50*b*x*cosh(b*x + a)*sinh(b*x + a)^4 + 2*
(25*b^2*x^2 + 2)*sinh(b*x + a)^5 - 6*b*x*cosh(b*x + a)^3 - 2*(9*b^2*x^2 -
10*(25*b^2*x^2 + 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^3 - 4*b*x*cosh(b*x
+ a) + 2*(50*b*x*cosh(b*x + a)^3 - 9*b*x*cosh(b*x + a))*sinh(b*x + a)^2 -
125*(b^3*x^3*Ei(5*b*x) + b^3*x^3*Ei(-5*b*x))*cosh(5*a) + 27*(b^3*x^3*Ei(3*
b*x) + b^3*x^3*Ei(-3*b*x))*cosh(3*a) + 2*(b^3*x^3*Ei(b*x) + b^3*x^3*Ei(-b*
x))*cosh(a) + 2*(5*(25*b^2*x^2 + 2)*cosh(b*x + a)^4 - 2*b^2*x^2 - 3*(9*b^2
*x^2 + 2)*cosh(b*x + a)^2 - 4)*sinh(b*x + a) - 125*(b^3*x^3*Ei(5*b*x) - b^
3*x^3*Ei(-5*b*x))*sinh(5*a) + 27*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*
sinh(3*a) + 2*(b^3*x^3*Ei(b*x) - b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

3.324.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x^4}\,dx$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x**4,x)
```

output
```
Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x**4, x)
```

3.324.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.37

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx = {} & \frac{125}{32}\,b^3e^{(-5\,a)}\Gamma(-3,5\,bx) - \frac{27}{32}\,b^3e^{(-3\,a)}\Gamma(-3,3\,bx) \\ & - \frac{1}{16}\,b^3e^{(-a)}\Gamma(-3,bx) - \frac{1}{16}\,b^3e^{a}\Gamma(-3,-bx) \\ & - \frac{27}{32}\,b^3e^{(3\,a)}\Gamma(-3,-3\,bx) + \frac{125}{32}\,b^3e^{(5\,a)}\Gamma(-3,-5\,bx)\end{aligned}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x, algorithm="maxima")
```

output
```
125/32*b^3*e^(-5*a)*gamma(-3, 5*b*x) - 27/32*b^3*e^(-3*a)*gamma(-3, 3*b*x)
 - 1/16*b^3*e^(-a)*gamma(-3, b*x) - 1/16*b^3*e^a*gamma(-3, -b*x) - 27/32*b
^3*e^(3*a)*gamma(-3, -3*b*x) + 125/32*b^3*e^(5*a)*gamma(-3, -5*b*x)
```

3.324.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.44

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx$$

$$= \frac{125\,b^3x^3\mathrm{Ei}(5\,bx)\,e^{(5\,a)} - 27\,b^3x^3\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - 2\,b^3x^3\mathrm{Ei}(-bx)\,e^{(-a)} - 27\,b^3x^3\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + 125\,b^3x^3\mathrm{E}}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x, algorithm="giac")
```

output
```
1/192*(125*b^3*x^3*Ei(5*b*x)*e^(5*a) - 27*b^3*x^3*Ei(3*b*x)*e^(3*a) - 2*b^
3*x^3*Ei(-b*x)*e^(-a) - 27*b^3*x^3*Ei(-3*b*x)*e^(-3*a) + 125*b^3*x^3*Ei(-5
*b*x)*e^(-5*a) - 2*b^3*x^3*Ei(b*x)*e^a - 25*b^2*x^2*e^(5*b*x + 5*a) + 9*b^
2*x^2*e^(3*b*x + 3*a) + 2*b^2*x^2*e^(b*x + a) - 2*b^2*x^2*e^(-b*x - a) - 9
*b^2*x^2*e^(-3*b*x - 3*a) + 25*b^2*x^2*e^(-5*b*x - 5*a) - 5*b*x*e^(5*b*x +
 5*a) + 3*b*x*e^(3*b*x + 3*a) + 2*b*x*e^(b*x + a) + 2*b*x*e^(-b*x - a) + 3
*b*x*e^(-3*b*x - 3*a) - 5*b*x*e^(-5*b*x - 5*a) - 2*e^(5*b*x + 5*a) + 2*e^(
3*b*x + 3*a) + 4*e^(b*x + a) - 4*e^(-b*x - a) - 2*e^(-3*b*x - 3*a) + 2*e^(
-5*b*x - 5*a))/x^3
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^3}{x^4}\,dx$$

input
```
int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^4,x)
```

output
```
int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^4, x)
```

3.325 $\int x^m \cosh^3(a+bx) \sinh^3(a+bx)\,dx$

3.325.1 Optimal result

Integrand size = 20, antiderivative size = 155

$$\int x^m \cosh^3(a+bx) \sinh^3(a+bx)\,dx = \frac{2^{-7-m}3^{-1-m}e^{6a}x^m(-bx)^{-m}\Gamma(1+m,-6bx)}{b} - \frac{3\ 2^{-7-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} - \frac{3\ 2^{-7-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b} + \frac{2^{-7-m}3^{-1-m}e^{-6a}x^m(bx)^{-m}\Gamma(1+m,6bx)}{b}$$

output
```
2^(-7-m)*3^(-1-m)*exp(6*a)*x^m*GAMMA(1+m,-6*b*x)/b/((-b*x)^m)-3*2^(-7-m)*e
xp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)-3*2^(-7-m)*x^m*GAMMA(1+m,2*b*x)
/b/exp(2*a)/((b*x)^m)+2^(-7-m)*3^(-1-m)*x^m*GAMMA(1+m,6*b*x)/b/exp(6*a)/((
b*x)^m)
```

3.325.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.77

$$\int x^m \cosh^3(a+bx) \sinh^3(a+bx)\,dx$$
$$= \frac{2^{-7-m}3^{-1-m}e^{-6a}x^m(-b^2x^2)^{-m}\left(e^{12a}(bx)^m\Gamma(1+m,-6bx) - 3^{2+m}e^{8a}(bx)^m\Gamma(1+m,-2bx) + (-bx)^m(-3\right.}{b}$$

input
```
Integrate[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(2^(-7 - m)*3^(-1 - m)*x^m*(E^(12*a)*(b*x)^m*Gamma[1 + m, -6*b*x] - 3^(2 +
 m)*E^(8*a)*(b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*(-(3^(2 + m)*E^(4*a)
*Gamma[1 + m, 2*b*x]) + Gamma[1 + m, 6*b*x])))/(b*E^(6*a)*(-(b^2*x^2))^m)
```

3.325.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh^3(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{1}{32}x^m\sinh(6a+6bx)-\frac{3}{32}x^m\sinh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{6a}2^{-m-7}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-6bx)}{b}-\frac{3e^{2a}2^{-m-7}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}-\frac{3e^{-2a}2^{-m-7}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}+\frac{e^{-6a}2^{-m-7}3^{-m-1}x^m(bx)^{-m}\Gamma(m+1,6bx)}{b}$$

input
```
Int[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(2^(-7 - m)*3^(-1 - m)*E^(6*a)*x^m*Gamma[1 + m, -6*b*x])/(b*(-(b*x))^m) -
(3*2^(-7 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) - (3*2^(-7
- m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (2^(-7 - m)*3^(-1 - m)
*x^m*Gamma[1 + m, 6*b*x])/(b*E^(6*a)*(b*x)^m)
```

3.325.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.325.4 Maple [F]

$$\int x^m \cosh(bx+a)^3 \sinh(bx+a)^3 dx$$

input
```
int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

output
```
int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

3.325.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.11

$$\int x^m \cosh^3(a+bx) \sinh^3(a+bx) \, dx = \frac{\cosh(m\log(6b)+6a)\Gamma(m+1,6bx) - 9\cosh(m\log(2b)+2a)\Gamma(m+1,2bx) - 9\cosh(m\log(-2b) -$$

input
```
integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/384*(cosh(m*log(6*b) + 6*a)*gamma(m + 1, 6*b*x) - 9*cosh(m*log(2*b) + 2*
a)*gamma(m + 1, 2*b*x) - 9*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) +
cosh(m*log(-6*b) - 6*a)*gamma(m + 1, -6*b*x) - gamma(m + 1, 6*b*x)*sinh(m*
log(6*b) + 6*a) + 9*gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) + 9*gamma(m
 + 1, -2*b*x)*sinh(m*log(-2*b) - 2*a) - gamma(m + 1, -6*b*x)*sinh(m*log(-6
*b) - 6*a))/b
```

3.325.6 Sympy [F]

$$\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int x^m \sinh^3{(a+bx)}\cosh^3{(a+bx)}\,dx$$

input
```
integrate(x**m*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Integral(x**m*sinh(a + b*x)**3*cosh(a + b*x)**3, x)
```

3.325.7 Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\begin{aligned}\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx &= \frac{1}{64}\,(6\,bx)^{-m-1}\,x^{m+1}e^{(-6\,a)}\Gamma(m+1,6\,bx) \\ &\quad - \frac{3}{64}\,(2\,bx)^{-m-1}\,x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx) \\ &\quad + \frac{3}{64}\,(-2\,bx)^{-m-1}\,x^{m+1}e^{(2\,a)}\Gamma(m+1,-2\,bx) \\ &\quad - \frac{1}{64}\,(-6\,bx)^{-m-1}\,x^{m+1}e^{(6\,a)}\Gamma(m+1,-6\,bx)\end{aligned}$$

input
```
integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/64*(6*b*x)^(-m - 1)*x^(m + 1)*e^(-6*a)*gamma(m + 1, 6*b*x) - 3/64*(2*b*x
)^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) + 3/64*(-2*b*x)^(-m - 1)
*x^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/64*(-6*b*x)^(-m - 1)*x^(m + 1)
*e^(6*a)*gamma(m + 1, -6*b*x)
```

3.325.8 Giac [F]

$$\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int x^m \cosh{(bx+a)}^3 \sinh{(bx+a)}^3\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^3*sinh(b*x + a)^3, x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int x^m \cosh(a+b\,x)^3 \sinh(a+b\,x)^3\,dx$$

input
```
int(x^m*cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

output
```
int(x^m*cosh(a + b*x)^3*sinh(a + b*x)^3, x)
```

3.326 $\int x^3 \cosh^3(a+bx) \sinh^3(a+bx) \, dx$

3.326.1 Optimal result

Integrand size = 20, antiderivative size = 143

$$\int x^3 \cosh^3(a+bx) \sinh^3(a+bx) \, dx = -\frac{9x\cosh(2a+2bx)}{128b^3} - \frac{3x^3\cosh(2a+2bx)}{64b} + \frac{x\cosh(6a+6bx)}{1152b^3} + \frac{x^3\cosh(6a+6bx)}{192b} + \frac{9\sinh(2a+2bx)}{256b^4} + \frac{9x^2\sinh(2a+2bx)}{128b^2} - \frac{\sinh(6a+6bx)}{6912b^4} - \frac{x^2\sinh(6a+6bx)}{384b^2}$$

output
```
-9/128*x*cosh(2*b*x+2*a)/b^3-3/64*x^3*cosh(2*b*x+2*a)/b+1/1152*x*cosh(6*b*
x+6*a)/b^3+1/192*x^3*cosh(6*b*x+6*a)/b+9/256*sinh(2*b*x+2*a)/b^4+9/128*x^2
*sinh(2*b*x+2*a)/b^2-1/6912*sinh(6*b*x+6*a)/b^4-1/384*x^2*sinh(6*b*x+6*a)/
b^2
```

3.326.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.63

$$\int x^3 \cosh^3(a+bx) \sinh^3(a+bx) \, dx = -\frac{81bx(3+2b^2x^2)\cosh(2(a+bx)) - 3(bx+6b^3x^3)\cosh(6(a+bx)) + (-121-234b^2x^2+(1+18b^2x^2)\mathrm{co}}{3456b^4}$$

input
```
Integrate[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
-1/3456*(81*b*x*(3 + 2*b^2*x^2)*Cosh[2*(a + b*x)] - 3*(b*x + 6*b^3*x^3)*Co
sh[6*(a + b*x)] + (-121 - 234*b^2*x^2 + (1 + 18*b^2*x^2)*Cosh[4*(a + b*x)]
)*Sinh[2*(a + b*x)])/b^4
```

3.326.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sinh^3(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{1}{32}x^3\sinh(6a+6bx)-\frac{3}{32}x^3\sinh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{9\sinh(2a+2bx)}{256b^4}-\frac{\sinh(6a+6bx)}{6912b^4}-\frac{9x\cosh(2a+2bx)}{128b^3}+\frac{x\cosh(6a+6bx)}{1152b^3}+\frac{9x^2\sinh(2a+2bx)}{128b^2}-\frac{x^2\sinh(6a+6bx)}{384b^2}-\frac{3x^3\cosh(2a+2bx)}{64b}+\frac{x^3\cosh(6a+6bx)}{192b}$$

input
```
Int[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(-9*x*Cosh[2*a + 2*b*x])/(128*b^3) - (3*x^3*Cosh[2*a + 2*b*x])/(64*b) + (x
*Cosh[6*a + 6*b*x])/(1152*b^3) + (x^3*Cosh[6*a + 6*b*x])/(192*b) + (9*Sinh
[2*a + 2*b*x])/(256*b^4) + (9*x^2*Sinh[2*a + 2*b*x])/(128*b^2) - Sinh[6*a
+ 6*b*x]/(6912*b^4) - (x^2*Sinh[6*a + 6*b*x])/(384*b^2)
```

3.326.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.326.4 Maple [A] (verified)

Time = 88.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02

method	result
risch	$\frac{(36x^3b^3-18x^2b^2+6bx-1)\mathrm{e}^{6bx+6a}}{13824b^4}-\frac{3(4x^3b^3-6x^2b^2+6bx-3)\mathrm{e}^{2bx+2a}}{512b^4}-\frac{3(4x^3b^3+6x^2b^2+6bx+3)\mathrm{e}^{-2bx-2a}}{512b^4}+\frac{(3\cdots}{}$
derivativedivides	$\frac{-a^3\left(\frac{\cosh(bx+a)^4\sinh(bx+a)^2}{6}-\frac{\cosh(bx+a)^4}{12}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^4}{6}-\frac{(bx+a)\cosh(bx+a)^4}{12}-\frac{\sinh(bx+a)\cosh\cdots}{36}\right.}{}$
default	$\frac{-a^3\left(\frac{\cosh(bx+a)^4\sinh(bx+a)^2}{6}-\frac{\cosh(bx+a)^4}{12}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^4}{6}-\frac{(bx+a)\cosh(bx+a)^4}{12}-\frac{\sinh(bx+a)\cosh\cdots}{36}\right.}{}$

input
```
int(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13824*(36*b^3*x^3-18*b^2*x^2+6*b*x-1)/b^4*exp(6*b*x+6*a)-3/512*(4*b^3*x^
3-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)-3/512*(4*b^3*x^3+6*b^2*x^2+6*b*x+3
)/b^4*exp(-2*b*x-2*a)+1/13824*(36*b^3*x^3+18*b^2*x^2+6*b*x+1)/b^4*exp(-6*b
*x-6*a)
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.73

$$\int x^3\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{3\left(6\,b^3x^3+bx\right)\cosh\left(bx+a\right)^6-10\left(18\,b^2x^2+1\right)\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)^3+45\left(6\,b^3x^3+bx\right)\cosh\left(bx\cdots\right.}{}$$

input
```
integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3456*(3*(6*b^3*x^3 + b*x)*cosh(b*x + a)^6 - 10*(18*b^2*x^2 + 1)*cosh(b*x
 + a)^3*sinh(b*x + a)^3 + 45*(6*b^3*x^3 + b*x)*cosh(b*x + a)^2*sinh(b*x +
a)^4 - 3*(18*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^5 + 3*(6*b^3*x^3 + b
*x)*sinh(b*x + a)^6 - 81*(2*b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 - 9*(18*b^3*x
^3 - 5*(6*b^3*x^3 + b*x)*cosh(b*x + a)^4 + 27*b*x)*sinh(b*x + a)^2 - 3*((1
8*b^2*x^2 + 1)*cosh(b*x + a)^5 - 81*(2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*
x + a))/b^4
```

3.326.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(141) = 282$.

Time = 1.02 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.20

$$\int x^3 \cosh^3(a+bx) \sinh^3(a+bx)\,dx = \begin{cases} -\frac{x^3\sinh^6(a+bx)}{24b} + \frac{x^3\sinh^4(a+bx)\cosh^2(a+bx)}{8b} + \frac{x^3\sinh^2(a+bx)\cosh^4(a+bx)}{8b} - \frac{x^3\cosh^6(a+bx)}{24b} + \frac{x^2\sinh^5(a+bx)\cosh(a+bx)}{8b^2} \\ \frac{x^4\sinh^3(a)\cosh^3(a)}{4} \end{cases}$$

input
```
integrate(x**3*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Piecewise((-x**3*sinh(a + b*x)**6/(24*b) + x**3*sinh(a + b*x)**4*cosh(a +
b*x)**2/(8*b) + x**3*sinh(a + b*x)**2*cosh(a + b*x)**4/(8*b) - x**3*cosh(a
 + b*x)**6/(24*b) + x**2*sinh(a + b*x)**5*cosh(a + b*x)/(8*b**2) - x**2*si
nh(a + b*x)**3*cosh(a + b*x)**3/(3*b**2) + x**2*sinh(a + b*x)*cosh(a + b*x
)**5/(8*b**2) - 5*x*sinh(a + b*x)**6/(72*b**3) + x*sinh(a + b*x)**4*cosh(a
 + b*x)**2/(12*b**3) + x*sinh(a + b*x)**2*cosh(a + b*x)**4/(12*b**3) - 5*x
*cosh(a + b*x)**6/(72*b**3) + 5*sinh(a + b*x)**5*cosh(a + b*x)/(72*b**4) -
 31*sinh(a + b*x)**3*cosh(a + b*x)**3/(216*b**4) + 5*sinh(a + b*x)*cosh(a
+ b*x)**5/(72*b**4), Ne(b, 0)), (x**4*sinh(a)**3*cosh(a)**3/4, True))
```

3.326.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.20

$$\int x^3 \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{\left(36\,b^3x^3e^{(6\,a)} - 18\,b^2x^2e^{(6\,a)} + 6\,bxe^{(6\,a)} - e^{(6\,a)}\right)e^{(6\,bx)}}{13824\,b^4} - \frac{3\,\left(4\,b^3x^3e^{(2\,a)} - 6\,b^2x^2e^{(2\,a)} + 6\,bxe^{(2\,a)} - 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{512\,b^4} - \frac{3\,(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3)e^{(-2\,bx-2\,a)}}{512\,b^4} + \frac{(36\,b^3x^3 + 18\,b^2x^2 + 6\,bx + 1)e^{(-6\,bx-6\,a)}}{13824\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/13824*(36*b^3*x^3*e^(6*a) - 18*b^2*x^2*e^(6*a) + 6*b*x*e^(6*a) - e^(6*a)
)*e^(6*b*x)/b^4 - 3/512*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(
2*a) - 3*e^(2*a))*e^(2*b*x)/b^4 - 3/512*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3
)*e^(-2*b*x - 2*a)/b^4 + 1/13824*(36*b^3*x^3 + 18*b^2*x^2 + 6*b*x + 1)*e^(
-6*b*x - 6*a)/b^4
```

3.326.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\int x^3 \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{(36\,b^3x^3 - 18\,b^2x^2 + 6\,bx - 1)e^{(6\,bx+6\,a)}}{13824\,b^4} - \frac{3\,(4\,b^3x^3 - 6\,b^2x^2 + 6\,bx - 3)e^{(2\,bx+2\,a)}}{512\,b^4} - \frac{3\,(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3)e^{(-2\,bx-2\,a)}}{512\,b^4} + \frac{(36\,b^3x^3 + 18\,b^2x^2 + 6\,bx + 1)e^{(-6\,bx-6\,a)}}{13824\,b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/13824*(36*b^3*x^3 - 18*b^2*x^2 + 6*b*x - 1)*e^(6*b*x + 6*a)/b^4 - 3/512*
(4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 - 3/512*(4*b^3*x^3
 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 + 1/13824*(36*b^3*x^3 + 18*
b^2*x^2 + 6*b*x + 1)*e^(-6*b*x - 6*a)/b^4
```

3.326.9 Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int x^3\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{\frac{9\,x^2\sinh(2\,a+2\,b\,x)}{128}-\frac{x^2\sinh(6\,a+6\,b\,x)}{384}}{b^2} - \frac{\frac{3\,x^3\cosh(2\,a+2\,b\,x)}{64}-\frac{x^3\cosh(6\,a+6\,b\,x)}{192}}{b} - \frac{\frac{9\,x\cosh(2\,a+2\,b\,x)}{128}-\frac{x\cosh(6\,a+6\,b\,x)}{1152}}{b^3} + \frac{9\sinh(2\,a+2\,b\,x)}{256\,b^4} - \frac{\sinh(6\,a+6\,b\,x)}{6912\,b^4}$$

input
```
int(x^3*cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

output
```
((9*x^2*sinh(2*a + 2*b*x))/128 - (x^2*sinh(6*a + 6*b*x))/384)/b^2 - ((3*x^
3*cosh(2*a + 2*b*x))/64 - (x^3*cosh(6*a + 6*b*x))/192)/b - ((9*x*cosh(2*a
+ 2*b*x))/128 - (x*cosh(6*a + 6*b*x))/1152)/b^3 + (9*sinh(2*a + 2*b*x))/(2
56*b^4) - sinh(6*a + 6*b*x)/(6912*b^4)
```

3.327 $\int x^2 \cosh^3(a+bx) \sinh^3(a+bx) \, dx$

3.327.1 Optimal result

Integrand size = 20, antiderivative size = 105

$$\int x^2 \cosh^3(a+bx) \sinh^3(a+bx) \, dx = -\frac{3\cosh(2a+2bx)}{128b^3} - \frac{3x^2\cosh(2a+2bx)}{64b} + \frac{\cosh(6a+6bx)}{3456b^3} + \frac{x^2\cosh(6a+6bx)}{192b} + \frac{3x\sinh(2a+2bx)}{64b^2} - \frac{x\sinh(6a+6bx)}{576b^2}$$

output
```
-3/128*cosh(2*b*x+2*a)/b^3-3/64*x^2*cosh(2*b*x+2*a)/b+1/3456*cosh(6*b*x+6*
a)/b^3+1/192*x^2*cosh(6*b*x+6*a)/b+3/64*x*sinh(2*b*x+2*a)/b^2-1/576*x*sinh
(6*b*x+6*a)/b^2
```

3.327.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int x^2 \cosh^3(a+bx) \sinh^3(a+bx) \, dx = \frac{-81(1+2b^2x^2)\cosh(2(a+bx)) + (1+18b^2x^2)\cosh(6(a+bx)) + 6bx(27\sinh(2(a+bx)) - \sinh(6(a+b}{3456b^3}$$

input
```
Integrate[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(-81*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + (1 + 18*b^2*x^2)*Cosh[6*(a + b*x)
] + 6*b*x*(27*Sinh[2*(a + b*x)] - Sinh[6*(a + b*x)]))/(3456*b^3)
```

3.327.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sinh^3(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{1}{32}x^2\sinh(6a+6bx)-\frac{3}{32}x^2\sinh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\cosh(2a+2bx)}{128b^3}+\frac{\cosh(6a+6bx)}{3456b^3}+\frac{3x\sinh(2a+2bx)}{64b^2}-\frac{x\sinh(6a+6bx)}{576b^2}-\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{x^2\cosh(6a+6bx)}{192b}$$

input
```
Int[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(-3*Cosh[2*a + 2*b*x])/(128*b^3) - (3*x^2*Cosh[2*a + 2*b*x])/(64*b) + Cosh
[6*a + 6*b*x]/(3456*b^3) + (x^2*Cosh[6*a + 6*b*x])/(192*b) + (3*x*Sinh[2*a
 + 2*b*x])/(64*b^2) - (x*Sinh[6*a + 6*b*x])/(576*b^2)
```

3.327.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.327.4 Maple [A] (verified)

Time = 63.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

method	result
risch	$\frac{(18x^2b^2-6bx+1)\,\mathrm{e}^{6bx+6a}}{6912b^3}-\frac{3(2x^2b^2-2bx+1)\,\mathrm{e}^{2bx+2a}}{256b^3}-\frac{3(2x^2b^2+2bx+1)\,\mathrm{e}^{-2bx-2a}}{256b^3}+\frac{(18x^2b^2+6bx+1)\,\mathrm{e}^{-6bx-6a}}{6912b^3}$
derivativedivides	$\frac{a^2\left(\frac{\cosh(bx+a)^4\sinh(bx+a)^2}{6}-\frac{\cosh(bx+a)^4}{12}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^4}{6}-\frac{(bx+a)\cosh(bx+a)^4}{12}-\frac{\sinh(bx+a)\cosh(bx}{36}\right.}{}$
default	$\frac{a^2\left(\frac{\cosh(bx+a)^4\sinh(bx+a)^2}{6}-\frac{\cosh(bx+a)^4}{12}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^4}{6}-\frac{(bx+a)\cosh(bx+a)^4}{12}-\frac{\sinh(bx+a)\cosh(bx}{36}\right.}{}$

input
```
int(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6912*(18*b^2*x^2-6*b*x+1)/b^3*exp(6*b*x+6*a)-3/256*(2*b^2*x^2-2*b*x+1)/b
^3*exp(2*b*x+2*a)-3/256*(2*b^2*x^2+2*b*x+1)/b^3*exp(-2*b*x-2*a)+1/6912*(18
*b^2*x^2+6*b*x+1)/b^3*exp(-6*b*x-6*a)
```

3.327.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(93) = 186.

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.92

$$\int x^2\cosh^3(a+bx)\sinh^3(a+bx)\,dx =$$
$$-\frac{120\,bx\cosh(bx+a)^3\sinh(bx+a)^3+36\,bx\cosh(bx+a)\sinh(bx+a)^5-(18\,b^2x^2+1)\cosh(bx+a)^6}{}$$

input
```
integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/3456*(120*b*x*cosh(b*x + a)^3*sinh(b*x + a)^3 + 36*b*x*cosh(b*x + a)*si
nh(b*x + a)^5 - (18*b^2*x^2 + 1)*cosh(b*x + a)^6 - 15*(18*b^2*x^2 + 1)*cos
h(b*x + a)^2*sinh(b*x + a)^4 - (18*b^2*x^2 + 1)*sinh(b*x + a)^6 + 81*(2*b^
2*x^2 + 1)*cosh(b*x + a)^2 - 3*(5*(18*b^2*x^2 + 1)*cosh(b*x + a)^4 - 54*b^
2*x^2 - 27)*sinh(b*x + a)^2 + 36*(b*x*cosh(b*x + a)^5 - 9*b*x*cosh(b*x + a
))*sinh(b*x + a))/b^3
```

3.327.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(102) = 204$.

Time = 0.77 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.02

$$\int x^2\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{x^2\sinh^6(a+bx)}{24b} + \frac{x^2\sinh^4(a+bx)\cosh^2(a+bx)}{8b} + \frac{x^2\sinh^2(a+bx)\cosh^4(a+bx)}{8b} - \frac{x^2\cosh^6(a+bx)}{24b} + \frac{x\sinh^5(a+bx)\cosh(a+bx)}{12b^2} \\ \frac{x^3\sinh^3(a)\cosh^3(a)}{3} \end{cases}$$

input
```
integrate(x**2*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Piecewise((-x**2*sinh(a + b*x)**6/(24*b) + x**2*sinh(a + b*x)**4*cosh(a +
b*x)**2/(8*b) + x**2*sinh(a + b*x)**2*cosh(a + b*x)**4/(8*b) - x**2*cosh(a
 + b*x)**6/(24*b) + x*sinh(a + b*x)**5*cosh(a + b*x)/(12*b**2) - 2*x*sinh(
a + b*x)**3*cosh(a + b*x)**3/(9*b**2) + x*sinh(a + b*x)*cosh(a + b*x)**5/(
12*b**2) - sinh(a + b*x)**6/(72*b**3) + sinh(a + b*x)**2*cosh(a + b*x)**4/
(18*b**3) - 7*cosh(a + b*x)**6/(216*b**3), Ne(b, 0)), (x**3*sinh(a)**3*cos
h(a)**3/3, True))
```

3.327.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21

$$\int x^2\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{(18\,b^2x^2e^{(6\,a)} - 6\,bxe^{(6\,a)} + e^{(6\,a)})e^{(6\,bx)}}{6912\,b^3} - \frac{3\,(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)})e^{(2\,bx)}}{256\,b^3} - \frac{3\,(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx-2\,a)}}{256\,b^3} + \frac{(18\,b^2x^2 + 6\,bx + 1)e^{(-6\,bx-6\,a)}}{6912\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/6912*(18*b^2*x^2*e^(6*a) - 6*b*x*e^(6*a) + e^(6*a))*e^(6*b*x)/b^3 - 3/25
6*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 - 3/256*(2*b
^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/6912*(18*b^2*x^2 + 6*b*x + 1)
*e^(-6*b*x - 6*a)/b^3
```

3.327.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int x^2\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{(18\,b^2x^2-6\,bx+1)e^{(6\,bx+6\,a)}}{6912\,b^3} - \frac{3\,(2\,b^2x^2-2\,bx+1)e^{(2\,bx+2\,a)}}{256\,b^3} - \frac{3\,(2\,b^2x^2+2\,bx+1)e^{(-2\,bx-2\,a)}}{256\,b^3} + \frac{(18\,b^2x^2+6\,bx+1)e^{(-6\,bx-6\,a)}}{6912\,b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/6912*(18*b^2*x^2 - 6*b*x + 1)*e^(6*b*x + 6*a)/b^3 - 3/256*(2*b^2*x^2 - 2
*b*x + 1)*e^(2*b*x + 2*a)/b^3 - 3/256*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x -
2*a)/b^3 + 1/6912*(18*b^2*x^2 + 6*b*x + 1)*e^(-6*b*x - 6*a)/b^3
```

3.327.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int x^2\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{\frac{3\cosh(2\,a+2\,b\,x)}{128} - \frac{\cosh(6\,a+6\,b\,x)}{3456} + b^2\left(\frac{3\,x^2\cosh(2\,a+2\,b\,x)}{64} - \frac{x^2\cosh(6\,a+6\,b\,x)}{192}\right) - b\left(\frac{3\,x\sinh(2\,a+2\,b\,x)}{64} - \frac{x\sinh(6\,a+6\,b\,x)}{576}\right)}{b^3}$$

input
```
int(x^2*cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

output
```
-((3*cosh(2*a + 2*b*x))/128 - cosh(6*a + 6*b*x)/3456 + b^2*((3*x^2*cosh(2*
a + 2*b*x))/64 - (x^2*cosh(6*a + 6*b*x))/192) - b*((3*x*sinh(2*a + 2*b*x))
/64 - (x*sinh(6*a + 6*b*x))/576))/b^3
```

3.328 $\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx$

3.328.1 Optimal result

Integrand size = 18, antiderivative size = 67

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx=-\frac{3x\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{192b}+\frac{3\sinh(2a+2bx)}{128b^2}-\frac{\sinh(6a+6bx)}{1152b^2}$$

output
```
-3/64*x*cosh(2*b*x+2*a)/b+1/192*x*cosh(6*b*x+6*a)/b+3/128*sinh(2*b*x+2*a)/
b^2-1/1152*sinh(6*b*x+6*a)/b^2
```

3.328.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx=-\frac{54bx\cosh(2(a+bx))-6bx\cosh(6(a+bx))-27\sinh(2(a+bx))+\sinh(6(a+bx))}{1152b^2}$$

input
```
Integrate[x*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
-1/1152*(54*b*x*Cosh[2*(a + b*x)] - 6*b*x*Cosh[6*(a + b*x)] - 27*Sinh[2*(a
 + b*x)] + Sinh[6*(a + b*x)])/b^2
```

3.328.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh^3(a+bx) \cosh^3(a+bx) \, dx$$

$$\Big\downarrow \text{ 5971}$$

$$\int \left(\frac{1}{32} x \sinh(6a+6bx) - \frac{3}{32} x \sinh(2a+2bx)\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3\sinh(2a+2bx)}{128b^2} - \frac{\sinh(6a+6bx)}{1152b^2} - \frac{3x\cosh(2a+2bx)}{64b} + \frac{x\cosh(6a+6bx)}{192b}$$

input
```
Int[x*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(-3*x*Cosh[2*a + 2*b*x])/(64*b) + (x*Cosh[6*a + 6*b*x])/(192*b) + (3*Sinh[
2*a + 2*b*x])/(128*b^2) - Sinh[6*a + 6*b*x]/(1152*b^2)
```

3.328.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.328.4 Maple [A] (verified)

Time = 44.82 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

method	result
risch	$\frac{(6bx-1)e^{6bx+6a}}{2304b^2}-\frac{3(2bx-1)e^{2bx+2a}}{256b^2}-\frac{3(2bx+1)e^{-2bx-2a}}{256b^2}+\frac{(6bx+1)e^{-6bx-6a}}{2304b^2}$
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^4}{6}-\frac{(bx+a)\cosh(bx+a)^4}{12}-\frac{\sinh(bx+a)\cosh(bx+a)^5}{36}+\frac{\cosh(bx+a)^3\sinh(bx+a)}{36}+\frac{\cosh(bx+a)\sinh(bx}{24}}{b^2}$
default	$\frac{\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^4}{6}-\frac{(bx+a)\cosh(bx+a)^4}{12}-\frac{\sinh(bx+a)\cosh(bx+a)^5}{36}+\frac{\cosh(bx+a)^3\sinh(bx+a)}{36}+\frac{\cosh(bx+a)\sinh(bx}{24}}{b^2}$

input
```
int(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2304*(6*b*x-1)/b^2*exp(6*b*x+6*a)-3/256*(2*b*x-1)/b^2*exp(2*b*x+2*a)-3/2
56*(2*b*x+1)/b^2*exp(-2*b*x-2*a)+1/2304*(6*b*x+1)/b^2*exp(-6*b*x-6*a)
```

3.328.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(59) = 118.

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.21

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{3\,bx\cosh\left(bx+a\right)^6+45\,bx\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^4+3\,bx\sinh\left(bx+a\right)^6-10\,\cosh\left(bx+a\right)^3\sinh\left(bx\right.}{}$$

input
```
integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/576*(3*b*x*cosh(b*x + a)^6 + 45*b*x*cosh(b*x + a)^2*sinh(b*x + a)^4 + 3*
b*x*sinh(b*x + a)^6 - 10*cosh(b*x + a)^3*sinh(b*x + a)^3 - 3*cosh(b*x + a)
*sinh(b*x + a)^5 - 27*b*x*cosh(b*x + a)^2 + 9*(5*b*x*cosh(b*x + a)^4 - 3*b
*x)*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^5 - 9*cosh(b*x + a))*sinh(b*x + a))
/b^2
```

3.328.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(63) = 126.

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.21

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases} -\frac{x\sinh^6(a+bx)}{24b}+\frac{x\sinh^4(a+bx)\cosh^2(a+bx)}{8b}+\frac{x\sinh^2(a+bx)\cosh^4(a+bx)}{8b}-\frac{x\cosh^6(a+bx)}{24b}+\frac{\sinh^5(a+bx)\cosh(a+bx)}{24b^2}-\text{sinh} \\ \frac{x^2\sinh^3(a)\cosh^3(a)}{2} \end{cases}$$

input
```
integrate(x*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Piecewise((-x*sinh(a + b*x)**6/(24*b) + x*sinh(a + b*x)**4*cosh(a + b*x)**
2/(8*b) + x*sinh(a + b*x)**2*cosh(a + b*x)**4/(8*b) - x*cosh(a + b*x)**6/(
24*b) + sinh(a + b*x)**5*cosh(a + b*x)/(24*b**2) - sinh(a + b*x)**3*cosh(a
 + b*x)**3/(9*b**2) + sinh(a + b*x)*cosh(a + b*x)**5/(24*b**2), Ne(b, 0)),
 (x**2*sinh(a)**3*cosh(a)**3/2, True))
```

3.328.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx=\frac{\left(6\,bxe^{(6\,a)}-e^{(6\,a)}\right)e^{(6\,bx)}}{2304\,b^2}-\frac{3\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{256\,b^2}-\frac{3\,(2\,bx+1)e^{(-2\,bx-2\,a)}}{256\,b^2}+\frac{(6\,bx+1)e^{(-6\,bx-6\,a)}}{2304\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2304*(6*b*x*e^(6*a) - e^(6*a))*e^(6*b*x)/b^2 - 3/256*(2*b*x*e^(2*a) - e^
(2*a))*e^(2*b*x)/b^2 - 3/256*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/2304*(6*
b*x + 1)*e^(-6*b*x - 6*a)/b^2
```

3.328.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{(6\,bx-1)e^{(6\,bx+6\,a)}}{2304\,b^2} - \frac{3\,(2\,bx-1)e^{(2\,bx+2\,a)}}{256\,b^2} - \frac{3\,(2\,bx+1)e^{(-2\,bx-2\,a)}}{256\,b^2} + \frac{(6\,bx+1)e^{(-6\,bx-6\,a)}}{2304\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2304*(6*b*x - 1)*e^(6*b*x + 6*a)/b^2 - 3/256*(2*b*x - 1)*e^(2*b*x + 2*a)
/b^2 - 3/256*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/2304*(6*b*x + 1)*e^(-6*b
*x - 6*a)/b^2
```

3.328.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{\frac{\sinh(6\,a+6\,b\,x)}{1152} - \frac{3\sinh(2\,a+2\,b\,x)}{128} + b\left(\frac{3\,x\cosh(2\,a+2\,b\,x)}{64} - \frac{x\cosh(6\,a+6\,b\,x)}{192}\right)}{b^2}$$

input
```
int(x*cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

output
```
-(sinh(6*a + 6*b*x)/1152 - (3*sinh(2*a + 2*b*x))/128 + b*((3*x*cosh(2*a +
2*b*x))/64 - (x*cosh(6*a + 6*b*x))/192))/b^2
```

3.329 $\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx$

3.329.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx=\frac{\sinh^4(a+bx)}{4b}+\frac{\sinh^6(a+bx)}{6b}$$

output
```
1/4*sinh(b*x+a)^4/b+1/6*sinh(b*x+a)^6/b
```

3.329.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx=\frac{1}{8}\left(-\frac{3\cosh(2(a+bx))}{8b}+\frac{\cosh(6(a+bx))}{24b}\right)$$

input
```
Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
((-3*Cosh[2*(a + b*x)])/(8*b) + Cosh[6*(a + b*x)]/(24*b))/8
```

3.329.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 26, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\sin(ia+ibx)^3\cos(ia+ibx)^3dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \cos(ia+ibx)^3\sin(ia+ibx)^3dx$$

$$\Big\downarrow \text{ 3044}$$

$$\frac{\int -i\sinh^3(a+bx)\left(\sinh^2(a+bx)+1\right)d(i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left(-i\sinh^5(a+bx)-i\sinh^3(a+bx)\right)d(i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{6}\sinh^6(a+bx)+\frac{1}{4}\sinh^4(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(Sinh[a + b*x]^4/4 + Sinh[a + b*x]^6/6)/b
```

3.329.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.329.4 Maple [A] (verified)

Time = 31.92 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\frac{\left(\cosh(bx+a)^2-1\right)^3}{6}+\frac{\left(\cosh(bx+a)^2-1\right)^2}{4}}{b}$	34
default	$\frac{\frac{\left(\cosh(bx+a)^2-1\right)^3}{6}+\frac{\left(\cosh(bx+a)^2-1\right)^2}{4}}{b}$	34
risch	$\frac{e^{6bx+6a}}{384b}-\frac{3\,e^{2bx+2a}}{128b}-\frac{3\,e^{-2bx-2a}}{128b}+\frac{e^{-6bx-6a}}{384b}$	58

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/6*(cosh(b*x+a)^2-1)^3+1/4*(cosh(b*x+a)^2-1)^2)
```

3.329.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(27) = 54$.

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.32

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^6 + 15\cosh(bx+a)^2\sinh(bx+a)^4 + \sinh(bx+a)^6 + 3\left(5\cosh(bx+a)^4 - 3\right)\sinh(bx+a}{192\,b}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/192*(cosh(b*x + a)^6 + 15*cosh(b*x + a)^2*sinh(b*x + a)^4 + sinh(b*x + a
)^6 + 3*(5*cosh(b*x + a)^4 - 3)*sinh(b*x + a)^2 - 9*cosh(b*x + a)^2)/b
```

3.329.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \begin{cases} \frac{\sinh^2(a+bx)\cosh^4(a+bx)}{4b} - \frac{\cosh^6(a+bx)}{12b} & \text{for } b \neq 0 \\ x\sinh^3(a)\cosh^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Piecewise((sinh(a + b*x)**2*cosh(a + b*x)**4/(4*b) - cosh(a + b*x)**6/(12*
b), Ne(b, 0)), (x*sinh(a)**3*cosh(a)**3, True))
```

3.329.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(27) = 54$.

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(9\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{384\,b} - \frac{9\,e^{(-2\,bx-2\,a)} - e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/384*(9*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b - 1/384*(9*e^(-2*b*x - 2
*a) - e^(-6*b*x - 6*a))/b
```

3.329.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(6\,bx+6\,a)}}{384\,b} - \frac{3\,e^{(2\,bx+2\,a)}}{128\,b} - \frac{3\,e^{(-2\,bx-2\,a)}}{128\,b} + \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/384*e^(6*b*x + 6*a)/b - 3/128*e^(2*b*x + 2*a)/b - 3/128*e^(-2*b*x - 2*a)
/b + 1/384*e^(-6*b*x - 6*a)/b
```

3.329.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{2\sinh(a+b\,x)^6 + 3\sinh(a+b\,x)^4}{12\,b}$$

input
```
int(cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

output
```
(3*sinh(a + b*x)^4 + 2*sinh(a + b*x)^6)/(12*b)
```

3.330 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx$

3.330.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = -\frac{3}{32}\text{Chi}(2bx)\sinh(2a)+\frac{1}{32}\text{Chi}(6bx)\sinh(6a) -\frac{3}{32}\cosh(2a)\text{Shi}(2bx)+\frac{1}{32}\cosh(6a)\text{Shi}(6bx)$$

output
```
-3/32*cosh(2*a)*Shi(2*b*x)+1/32*cosh(6*a)*Shi(6*b*x)-3/32*Chi(2*b*x)*sinh(
2*a)+1/32*Chi(6*b*x)*sinh(6*a)
```

3.330.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{32}(-6\cosh(a)\text{Chi}(2bx)\sinh(a)+\text{Chi}(6bx)\sinh(6a) -3\cosh(2a)\text{Shi}(2bx)+\cosh(6a)\text{Shi}(6bx))$$

input
```
Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x,x]
```

output
```
(-6*Cosh[a]*CoshIntegral[2*b*x]*Sinh[a] + CoshIntegral[6*b*x]*Sinh[6*a] -
3*Cosh[2*a]*SinhIntegral[2*b*x] + Cosh[6*a]*SinhIntegral[6*b*x])/32
```

3.330.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{\sinh(6a+6bx)}{32x}-\frac{3\sinh(2a+2bx)}{32x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{32}\sinh(2a)\text{Chi}(2bx)+\frac{1}{32}\sinh(6a)\text{Chi}(6bx)-\frac{3}{32}\cosh(2a)\text{Shi}(2bx)+\frac{1}{32}\cosh(6a)\text{Shi}(6bx)$$

input
```
Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x,x]
```

output
```
(-3*CoshIntegral[2*b*x]*Sinh[2*a])/32 + (CoshIntegral[6*b*x]*Sinh[6*a])/32
 - (3*Cosh[2*a]*SinhIntegral[2*b*x])/32 + (Cosh[6*a]*SinhIntegral[6*b*x])/
32
```

3.330.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.330.4 Maple [A] (verified)

Time = 28.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{e^{-6a} \operatorname{Ei}_1(6bx)}{64} - \frac{3\,e^{-2a} \operatorname{Ei}_1(2bx)}{64} + \frac{3\,e^{2a} \operatorname{Ei}_1(-2bx)}{64} - \frac{e^{6a} \operatorname{Ei}_1(-6bx)}{64}$	50

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/64*exp(-6*a)*Ei(1,6*b*x)-3/64*exp(-2*a)*Ei(1,2*b*x)+3/64*exp(2*a)*Ei(1,-
2*b*x)-1/64*exp(6*a)*Ei(1,-6*b*x)
```

3.330.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{64}\left(\operatorname{Ei}(6\,bx) - \operatorname{Ei}(-6\,bx)\right)\cosh\left(6\,a\right) - \frac{3}{64}\left(\operatorname{Ei}(2\,bx) - \operatorname{Ei}(-2\,bx)\right)\cosh\left(2\,a\right) + \frac{1}{64}\left(\operatorname{Ei}(6\,bx) + \operatorname{Ei}(-6\,bx)\right)\sinh\left(6\,a\right) - \frac{3}{64}\left(\operatorname{Ei}(2\,bx) + \operatorname{Ei}(-2\,bx)\right)\sinh\left(2\,a\right)$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
1/64*(Ei(6*b*x) - Ei(-6*b*x))*cosh(6*a) - 3/64*(Ei(2*b*x) - Ei(-2*b*x))*co
sh(2*a) + 1/64*(Ei(6*b*x) + Ei(-6*b*x))*sinh(6*a) - 3/64*(Ei(2*b*x) + Ei(-
2*b*x))*sinh(2*a)
```

3.330.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = \int \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x,x)
```

output
```
Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x, x)
```

3.330.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{64}\,\text{Ei}(6\,bx)\,e^{(6\,a)} - \frac{3}{64}\,\text{Ei}(2\,bx)\,e^{(2\,a)} + \frac{3}{64}\,\text{Ei}(-2\,bx)\,e^{(-2\,a)} - \frac{1}{64}\,\text{Ei}(-6\,bx)\,e^{(-6\,a)}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
1/64*Ei(6*b*x)*e^(6*a) - 3/64*Ei(2*b*x)*e^(2*a) + 3/64*Ei(-2*b*x)*e^(-2*a)
 - 1/64*Ei(-6*b*x)*e^(-6*a)
```

3.330.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{64}\,\text{Ei}(6\,bx)\,e^{(6\,a)} - \frac{3}{64}\,\text{Ei}(2\,bx)\,e^{(2\,a)} + \frac{3}{64}\,\text{Ei}(-2\,bx)\,e^{(-2\,a)} - \frac{1}{64}\,\text{Ei}(-6\,bx)\,e^{(-6\,a)}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="giac")
```

output
```
1/64*Ei(6*b*x)*e^(6*a) - 3/64*Ei(2*b*x)*e^(2*a) + 3/64*Ei(-2*b*x)*e^(-2*a)
 - 1/64*Ei(-6*b*x)*e^(-6*a)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx=\int \frac{\cosh(a+b\,x)^3\,\sinh(a+b\,x)^3}{x}\,dx$$

input
```
int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x,x)
```

output
```
int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x, x)
```

3.331 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx$

3.331.1 Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = -\frac{3}{16}b\cosh(2a)\text{Chi}(2bx)+\frac{3}{16}b\cosh(6a)\text{Chi}(6bx)+\frac{3\sinh(2a+2bx)}{32x}-\frac{\sinh(6a+6bx)}{32x}-\frac{3}{16}b\sinh(2a)\text{Shi}(2bx)+\frac{3}{16}b\sinh(6a)\text{Shi}(6bx)$$

output
```
-3/16*b*Chi(2*b*x)*cosh(2*a)+3/16*b*Chi(6*b*x)*cosh(6*a)-3/16*b*Shi(2*b*x)
*sinh(2*a)+3/16*b*Shi(6*b*x)*sinh(6*a)+3/32*sinh(2*b*x+2*a)/x-1/32*sinh(6*
b*x+6*a)/x
```

3.331.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = -\frac{6bx\cosh(2a)\text{Chi}(2bx)-6bx\cosh(6a)\text{Chi}(6bx)-3\sinh(2(a+bx))+\sinh(6(a+bx))+6bx\sinh(2a)\text{Sh}}{32x}$$

input
```
Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^2,x]
```

output
```
-1/32*(6*b*x*Cosh[2*a]*CoshIntegral[2*b*x] - 6*b*x*Cosh[6*a]*CoshIntegral[
6*b*x] - 3*Sinh[2*(a + b*x)] + Sinh[6*(a + b*x)] + 6*b*x*Sinh[2*a]*SinhInt
egral[2*b*x] - 6*b*x*Sinh[6*a]*SinhIntegral[6*b*x])/x
```

3.331.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{5971}$$

$$\int \left(\frac{\sinh(6a+6bx)}{32x^2}-\frac{3\sinh(2a+2bx)}{32x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{16}b\cosh(2a)\text{Chi}(2bx)+\frac{3}{16}b\cosh(6a)\text{Chi}(6bx)-\frac{3}{16}b\sinh(2a)\text{Shi}(2bx)+\frac{3}{16}b\sinh(6a)\text{Shi}(6bx)+\frac{3\sinh(2a+2bx)}{32x}-\frac{\sinh(6a+6bx)}{32x}$$

input
```
Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^2,x]
```

output
```
(-3*b*Cosh[2*a]*CoshIntegral[2*b*x])/16 + (3*b*Cosh[6*a]*CoshIntegral[6*b*
x])/16 + (3*Sinh[2*a + 2*b*x])/(32*x) - Sinh[6*a + 6*b*x]/(32*x) - (3*b*Si
nh[2*a]*SinhIntegral[2*b*x])/16 + (3*b*Sinh[6*a]*SinhIntegral[6*b*x])/16
```

3.331.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.331.4 Maple [A] (verified)

Time = 30.96 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

method	result	size
risch	$\frac{6\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)bx-6\,\mathrm{e}^{6a}\,\mathrm{Ei}_1(-6bx)bx-6\,\mathrm{e}^{-6a}\,\mathrm{Ei}_1(6bx)bx+6\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)bx+3\,\mathrm{e}^{2bx+2a}-\mathrm{e}^{6bx+6a}+\mathrm{e}^{-6bx-6a}-3\,\mathrm{e}^{-2bx-2a}}{64x}$	105

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/64*(6*exp(2*a)*Ei(1,-2*b*x)*b*x-6*exp(6*a)*Ei(1,-6*b*x)*b*x-6*exp(-6*a)*
Ei(1,6*b*x)*b*x+6*exp(-2*a)*Ei(1,2*b*x)*b*x+3*exp(2*b*x+2*a)-exp(6*b*x+6*a
)+exp(-6*b*x-6*a)-3*exp(-2*b*x-2*a))/x
```

3.331.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(77) = 154.

Time = 0.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.79

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx =$$
$$-\frac{20\,\cosh{(bx+a)}^3\sinh{(bx+a)}^3+6\,\cosh{(bx+a)}\sinh{(bx+a)}^5-3\,(bx\mathrm{Ei}(6\,bx)+bx\mathrm{Ei}(-6\,bx))\cosh{(6}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
-1/32*(20*cosh(b*x + a)^3*sinh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^
5 - 3*(b*x*Ei(6*b*x) + b*x*Ei(-6*b*x))*cosh(6*a) + 3*(b*x*Ei(2*b*x) + b*x*
Ei(-2*b*x))*cosh(2*a) + 6*(cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)
- 3*(b*x*Ei(6*b*x) - b*x*Ei(-6*b*x))*sinh(6*a) + 3*(b*x*Ei(2*b*x) - b*x*Ei
(-2*b*x))*sinh(2*a))/x
```

3.331.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \int \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x**2,x)
```

output
```
Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x**2, x)
```

3.331.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \frac{3}{32}\,be^{(-6\,a)}\Gamma(-1,6\,bx) - \frac{3}{32}\,be^{(-2\,a)}\Gamma(-1,2\,bx) - \frac{3}{32}\,be^{(2\,a)}\Gamma(-1,-2\,bx) + \frac{3}{32}\,be^{(6\,a)}\Gamma(-1,-6\,bx)$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
3/32*b*e^(-6*a)*gamma(-1, 6*b*x) - 3/32*b*e^(-2*a)*gamma(-1, 2*b*x) - 3/32
*b*e^(2*a)*gamma(-1, -2*b*x) + 3/32*b*e^(6*a)*gamma(-1, -6*b*x)
```

3.331.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx$$
$$= \frac{6\,bx\mathrm{Ei}(6\,bx)\,e^{(6\,a)} - 6\,bx\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - 6\,bx\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} + 6\,bx\mathrm{Ei}(-6\,bx)\,e^{(-6\,a)} - e^{(6\,bx+6\,a)} + 3\,e^{(2\,bx+}}{64\,x}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
1/64*(6*b*x*Ei(6*b*x)*e^(6*a) - 6*b*x*Ei(2*b*x)*e^(2*a) - 6*b*x*Ei(-2*b*x)
*e^(-2*a) + 6*b*x*Ei(-6*b*x)*e^(-6*a) - e^(6*b*x + 6*a) + 3*e^(2*b*x + 2*a
) - 3*e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/x
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^3\sinh(a+b\,x)^3}{x^2}\,dx$$

input
```
int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^2,x)
```

output
```
int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^2, x)
```

3.332 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx$

3.332.1 Optimal result

Integrand size = 20, antiderivative size = 131

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx &= \frac{3b\cosh(2a+2bx)}{32x}-\frac{3b\cosh(6a+6bx)}{32x}\\ &\quad-\frac{3}{16}b^2\text{Chi}(2bx)\sinh(2a)+\frac{9}{16}b^2\text{Chi}(6bx)\sinh(6a)\\ &\quad+\frac{3\sinh(2a+2bx)}{64x^2}-\frac{\sinh(6a+6bx)}{64x^2}\\ &\quad-\frac{3}{16}b^2\cosh(2a)\text{Shi}(2bx)+\frac{9}{16}b^2\cosh(6a)\text{Shi}(6bx)\end{aligned}$$

output
```
3/32*b*cosh(2*b*x+2*a)/x-3/32*b*cosh(6*b*x+6*a)/x-3/16*b^2*cosh(2*a)*Shi(2
*b*x)+9/16*b^2*cosh(6*a)*Shi(6*b*x)-3/16*b^2*Chi(2*b*x)*sinh(2*a)+9/16*b^2
*Chi(6*b*x)*sinh(6*a)+3/64*sinh(2*b*x+2*a)/x^2-1/64*sinh(6*b*x+6*a)/x^2
```

3.332.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = -\frac{-6bx\cosh(2(a+bx))+6bx\cosh(6(a+bx))+12b^2x^2\text{Chi}(2bx)\sinh(2a)-36b^2x^2\text{Chi}(6bx)\sinh(6a)-3}{64x^2}$$

input
```
Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^3,x]
```

output
```
-1/64*(-6*b*x*Cosh[2*(a + b*x)] + 6*b*x*Cosh[6*(a + b*x)] + 12*b^2*x^2*Cos
hIntegral[2*b*x]*Sinh[2*a] - 36*b^2*x^2*CoshIntegral[6*b*x]*Sinh[6*a] - 3*
Sinh[2*(a + b*x)] + Sinh[6*(a + b*x)] + 12*b^2*x^2*Cosh[2*a]*SinhIntegral[
2*b*x] - 36*b^2*x^2*Cosh[6*a]*SinhIntegral[6*b*x])/x^2
```

3.332.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{x^3}\,dx$$

$$\Big\downarrow \text{ 5971}$$

$$\int \left(\frac{\sinh(6a+6bx)}{32x^3}-\frac{3\sinh(2a+2bx)}{32x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3}{16}b^2\sinh(2a)\text{Chi}(2bx)+\frac{9}{16}b^2\sinh(6a)\text{Chi}(6bx)-\frac{3}{16}b^2\cosh(2a)\text{Shi}(2bx)+\frac{9}{16}b^2\cosh(6a)\text{Shi}(6bx)+\frac{3\sinh(2a+2bx)}{64x^2}-\frac{\sinh(6a+6bx)}{64x^2}+\frac{3b\cosh(2a+2bx)}{32x}-\frac{3b\cosh(6a+6bx)}{32x}$$

input
```
Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^3,x]
```

output
```
(3*b*Cosh[2*a + 2*b*x])/(32*x) - (3*b*Cosh[6*a + 6*b*x])/(32*x) - (3*b^2*C
oshIntegral[2*b*x]*Sinh[2*a])/16 + (9*b^2*CoshIntegral[6*b*x]*Sinh[6*a])/1
6 + (3*Sinh[2*a + 2*b*x])/(64*x^2) - Sinh[6*a + 6*b*x]/(64*x^2) - (3*b^2*C
osh[2*a]*SinhIntegral[2*b*x])/16 + (9*b^2*Cosh[6*a]*SinhIntegral[6*b*x])/1
6
```

3.332.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.332.4 Maple [A] (verified)

Time = 32.46 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{-36\,\mathrm{e}^{-6a}\,\mathrm{Ei}_1(6bx)x^2b^2+12\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)x^2b^2-12\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)x^2b^2+36\,\mathrm{e}^{6a}\,\mathrm{Ei}_1(-6bx)x^2b^2+6\,\mathrm{e}^{-6bx-6a}bx-6\,\mathrm{e}^{-2bx-2a}bx-6\,\mathrm{e}^{2}}{128x^2}$

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/128*(-36*exp(-6*a)*Ei(1,6*b*x)*x^2*b^2+12*exp(-2*a)*Ei(1,2*b*x)*x^2*b^2
-12*exp(2*a)*Ei(1,-2*b*x)*x^2*b^2+36*exp(6*a)*Ei(1,-6*b*x)*x^2*b^2+6*exp(-
6*b*x-6*a)*b*x-6*exp(-2*b*x-2*a)*b*x-6*exp(2*b*x+2*a)*b*x+6*exp(6*b*x+6*a)
*b*x-exp(-6*b*x-6*a)+3*exp(-2*b*x-2*a)-3*exp(2*b*x+2*a)+exp(6*b*x+6*a))/x^
2
```

3.332.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(115) = 230.

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.09

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx =$$
$$-\frac{3\,bx\cosh\left(bx+a\right)^6+45\,bx\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^4+3\,bx\sinh\left(bx+a\right)^6+10\,\cosh\left(bx+a\right)^3\sinh\left(\right.}{}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

output
```
-1/32*(3*b*x*cosh(b*x + a)^6 + 45*b*x*cosh(b*x + a)^2*sinh(b*x + a)^4 + 3*
b*x*sinh(b*x + a)^6 + 10*cosh(b*x + a)^3*sinh(b*x + a)^3 + 3*cosh(b*x + a)
*sinh(b*x + a)^5 - 3*b*x*cosh(b*x + a)^2 + 3*(15*b*x*cosh(b*x + a)^4 - b*x
)*sinh(b*x + a)^2 - 9*(b^2*x^2*Ei(6*b*x) - b^2*x^2*Ei(-6*b*x))*cosh(6*a) +
 3*(b^2*x^2*Ei(2*b*x) - b^2*x^2*Ei(-2*b*x))*cosh(2*a) + 3*(cosh(b*x + a)^5
 - cosh(b*x + a))*sinh(b*x + a) - 9*(b^2*x^2*Ei(6*b*x) + b^2*x^2*Ei(-6*b*x
))*sinh(6*a) + 3*(b^2*x^2*Ei(2*b*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

3.332.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{x^3}\,dx$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x**3,x)
```

output
```
Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x**3, x)
```

3.332.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.47

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \frac{9}{16}\,b^2e^{(-6\,a)}\Gamma(-2,6\,bx) - \frac{3}{16}\,b^2e^{(-2\,a)}\Gamma(-2,2\,bx) + \frac{3}{16}\,b^2e^{(2\,a)}\Gamma(-2,-2\,bx) - \frac{9}{16}\,b^2e^{(6\,a)}\Gamma(-2,-6\,bx)$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

output
```
9/16*b^2*e^(-6*a)*gamma(-2, 6*b*x) - 3/16*b^2*e^(-2*a)*gamma(-2, 2*b*x) +
3/16*b^2*e^(2*a)*gamma(-2, -2*b*x) - 9/16*b^2*e^(6*a)*gamma(-2, -6*b*x)
```

3.332.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.28

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx$$
$$= \frac{36\,b^2x^2\mathrm{Ei}(6\,bx)\,e^{(6\,a)} - 12\,b^2x^2\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + 12\,b^2x^2\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - 36\,b^2x^2\mathrm{Ei}(-6\,bx)\,e^{(-6\,a)} - 6\,bxe^{(6}}{12}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x, algorithm="giac")
```

output
```
1/128*(36*b^2*x^2*Ei(6*b*x)*e^(6*a) - 12*b^2*x^2*Ei(2*b*x)*e^(2*a) + 12*b^
2*x^2*Ei(-2*b*x)*e^(-2*a) - 36*b^2*x^2*Ei(-6*b*x)*e^(-6*a) - 6*b*x*e^(6*b*
x + 6*a) + 6*b*x*e^(2*b*x + 2*a) + 6*b*x*e^(-2*b*x - 2*a) - 6*b*x*e^(-6*b*
x - 6*a) - e^(6*b*x + 6*a) + 3*e^(2*b*x + 2*a) - 3*e^(-2*b*x - 2*a) + e^(-
6*b*x - 6*a))/x^2
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)^3\sinh(a+b\,x)^3}{x^3}\,dx$$

input
```
int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^3,x)
```

output
```
int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^3, x)
```

3.333 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx$

3.333.1 Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \frac{b\cosh(2a+2bx)}{32x^2} - \frac{b\cosh(6a+6bx)}{32x^2} - \frac{1}{8}b^3\cosh(2a)\text{Chi}(2bx) + \frac{9}{8}b^3\cosh(6a)\text{Chi}(6bx) + \frac{\sinh(2a+2bx)}{32x^3} + \frac{b^2\sinh(2a+2bx)}{16x} - \frac{\sinh(6a+6bx)}{96x^3} - \frac{3b^2\sinh(6a+6bx)}{16x} - \frac{1}{8}b^3\sinh(2a)\text{Shi}(2bx) + \frac{9}{8}b^3\sinh(6a)\text{Shi}(6bx)$$

output
```
-1/8*b^3*Chi(2*b*x)*cosh(2*a)+9/8*b^3*Chi(6*b*x)*cosh(6*a)+1/32*b*cosh(2*b
*x+2*a)/x^2-1/32*b*cosh(6*b*x+6*a)/x^2-1/8*b^3*Shi(2*b*x)*sinh(2*a)+9/8*b^
3*Shi(6*b*x)*sinh(6*a)+1/32*sinh(2*b*x+2*a)/x^3+1/16*b^2*sinh(2*b*x+2*a)/x
-1/96*sinh(6*b*x+6*a)/x^3-3/16*b^2*sinh(6*b*x+6*a)/x
```

3.333.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx =$$
$$-\frac{-3bx\cosh(2(a+bx))+3bx\cosh(6(a+bx))+12b^3x^3\cosh(2a)\mathrm{Chi}(2bx)-108b^3x^3\cosh(6a)\mathrm{Chi}(6bx)-$$

input
```
Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^4,x]
```

output
```
-1/96*(-3*b*x*Cosh[2*(a + b*x)] + 3*b*x*Cosh[6*(a + b*x)] + 12*b^3*x^3*Cos
h[2*a]*CoshIntegral[2*b*x] - 108*b^3*x^3*Cosh[6*a]*CoshIntegral[6*b*x] - 3
*Sinh[2*(a + b*x)] - 6*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[6*(a + b*x)] + 18*
b^2*x^2*Sinh[6*(a + b*x)] + 12*b^3*x^3*Sinh[2*a]*SinhIntegral[2*b*x] - 108
*b^3*x^3*Sinh[6*a]*SinhIntegral[6*b*x])/x^3
```

3.333.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{x^4}\,dx$$
$$\Big\downarrow \text{5971}$$
$$\int \left(\frac{\sinh(6a+6bx)}{32x^4}-\frac{3\sinh(2a+2bx)}{32x^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{8}b^3\cosh(2a)\mathrm{Chi}(2bx)+\frac{9}{8}b^3\cosh(6a)\mathrm{Chi}(6bx)-\frac{1}{8}b^3\sinh(2a)\mathrm{Shi}(2bx)+\frac{9}{8}b^3\sinh(6a)\mathrm{Shi}(6bx)+\frac{b^2\sinh(2a+2bx)}{16x}-\frac{3b^2\sinh(6a+6bx)}{16x}+\frac{\sinh(2a+2bx)}{32x^3}-\frac{\sinh(6a+6bx)}{96x^3}+\frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}$$

input
```
Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^4,x]
```

output
```
(b*Cosh[2*a + 2*b*x])/(32*x^2) - (b*Cosh[6*a + 6*b*x])/(32*x^2) - (b^3*Cos
h[2*a]*CoshIntegral[2*b*x])/8 + (9*b^3*Cosh[6*a]*CoshIntegral[6*b*x])/8 +
Sinh[2*a + 2*b*x]/(32*x^3) + (b^2*Sinh[2*a + 2*b*x])/(16*x) - Sinh[6*a + 6
*b*x]/(96*x^3) - (3*b^2*Sinh[6*a + 6*b*x])/(16*x) - (b^3*Sinh[2*a]*SinhInt
egral[2*b*x])/8 + (9*b^3*Sinh[6*a]*SinhIntegral[6*b*x])/8
```

3.333.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.333.4 Maple [A] (verified)

Time = 49.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.43

method	result
risch	$\frac{12\,\mathrm{e}^{2a}\,\mathrm{Ei}_{1}(-2bx)x^{3}b^{3}-108\,\mathrm{e}^{-6a}\,\mathrm{Ei}_{1}(6bx)x^{3}b^{3}+12\,\mathrm{e}^{-2a}\,\mathrm{Ei}_{1}(2bx)x^{3}b^{3}-108\,\mathrm{e}^{6a}\,\mathrm{Ei}_{1}(-6bx)x^{3}b^{3}+6\,\mathrm{e}^{2bx+2a}b^{2}x^{2}+18\,\mathrm{e}^{-6bx-6a}b^{2}x^{2}-6}{1}$

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/192*(12*exp(2*a)*Ei(1,-2*b*x)*x^3*b^3-108*exp(-6*a)*Ei(1,6*b*x)*x^3*b^3+
12*exp(-2*a)*Ei(1,2*b*x)*x^3*b^3-108*exp(6*a)*Ei(1,-6*b*x)*x^3*b^3+6*exp(2
*b*x+2*a)*b^2*x^2+18*exp(-6*b*x-6*a)*b^2*x^2-6*exp(-2*b*x-2*a)*b^2*x^2-18*
exp(6*b*x+6*a)*b^2*x^2+3*exp(2*b*x+2*a)*b*x-3*exp(-6*b*x-6*a)*b*x+3*exp(-2
*b*x-2*a)*b*x-3*exp(6*b*x+6*a)*b*x+3*exp(2*b*x+2*a)+exp(-6*b*x-6*a)-3*exp(
-2*b*x-2*a)-exp(6*b*x+6*a))/x^3
```

3.333.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(149) = 298$.

Time = 0.25 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.86

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx = -\frac{3\,bx\cosh\left(bx+a\right)^6+45\,bx\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^4+3\,bx\sinh\left(bx+a\right)^6+20\left(18\,b^2x^2+1\right)\cosh\left(b\right.}{}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x, algorithm="fricas")
```

output
```
-1/96*(3*b*x*cosh(b*x + a)^6 + 45*b*x*cosh(b*x + a)^2*sinh(b*x + a)^4 + 3*
b*x*sinh(b*x + a)^6 + 20*(18*b^2*x^2 + 1)*cosh(b*x + a)^3*sinh(b*x + a)^3
+ 6*(18*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^5 - 3*b*x*cosh(b*x + a)^2
 + 3*(15*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^2 - 54*(b^3*x^3*Ei(6*b*x
) + b^3*x^3*Ei(-6*b*x))*cosh(6*a) + 6*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b
*x))*cosh(2*a) + 6*((18*b^2*x^2 + 1)*cosh(b*x + a)^5 - (2*b^2*x^2 + 1)*cos
h(b*x + a))*sinh(b*x + a) - 54*(b^3*x^3*Ei(6*b*x) - b^3*x^3*Ei(-6*b*x))*si
nh(6*a) + 6*(b^3*x^3*Ei(2*b*x) - b^3*x^3*Ei(-2*b*x))*sinh(2*a))/x^3
```

3.333.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \frac{\sinh^3\left(a+bx\right)\cosh^3\left(a+bx\right)}{x^4}\,dx$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x**4,x)
```

output
```
Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x**4, x)
```

3.333.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.36

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \frac{27}{8}\,b^3e^{(-6\,a)}\Gamma(-3,6\,bx) - \frac{3}{8}\,b^3e^{(-2\,a)}\Gamma(-3,2\,bx) - \frac{3}{8}\,b^3e^{(2\,a)}\Gamma(-3,-2\,bx) + \frac{27}{8}\,b^3e^{(6\,a)}\Gamma(-3,-6\,bx)$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x, algorithm="maxima")
```

output
```
27/8*b^3*e^(-6*a)*gamma(-3, 6*b*x) - 3/8*b^3*e^(-2*a)*gamma(-3, 2*b*x) - 3
/8*b^3*e^(2*a)*gamma(-3, -2*b*x) + 27/8*b^3*e^(6*a)*gamma(-3, -6*b*x)
```

3.333.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx$$
$$= \frac{108\,b^3x^3\mathrm{Ei}(6\,bx)\,e^{(6\,a)} - 12\,b^3x^3\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - 12\,b^3x^3\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} + 108\,b^3x^3\mathrm{Ei}(-6\,bx)\,e^{(-6\,a)} - 18\,b^2 \cdots}{\cdots}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x, algorithm="giac")
```

output
```
1/192*(108*b^3*x^3*Ei(6*b*x)*e^(6*a) - 12*b^3*x^3*Ei(2*b*x)*e^(2*a) - 12*b
^3*x^3*Ei(-2*b*x)*e^(-2*a) + 108*b^3*x^3*Ei(-6*b*x)*e^(-6*a) - 18*b^2*x^2*
e^(6*b*x + 6*a) + 6*b^2*x^2*e^(2*b*x + 2*a) - 6*b^2*x^2*e^(-2*b*x - 2*a) +
 18*b^2*x^2*e^(-6*b*x - 6*a) - 3*b*x*e^(6*b*x + 6*a) + 3*b*x*e^(2*b*x + 2*
a) + 3*b*x*e^(-2*b*x - 2*a) - 3*b*x*e^(-6*b*x - 6*a) - e^(6*b*x + 6*a) + 3
*e^(2*b*x + 2*a) - 3*e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/x^3
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx=\int \frac{{\cosh\left(a+b\,x\right)}^3\,{\sinh\left(a+b\,x\right)}^3}{x^4}\,dx$$

input
```
int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^4,x)
```

output
```
int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^4, x)
```

3.334 $\int x^m \tanh(a+bx)\,dx$

3.334.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \tanh(a+bx)\,dx = \text{Int}(x^m \tanh(a+bx),x)$$

output
```
Unintegrable(x^m*tanh(b*x+a),x)
```

3.334.2 Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(a+bx)\,dx$$

input
```
Integrate[x^m*Tanh[a + b*x],x]
```

output
```
Integrate[x^m*Tanh[a + b*x], x]
```

3.334.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \tanh(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -ix^m \tan(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int x^m \tan(ia+ibx)dx$$

$$\Big\downarrow \text{ 4222}$$

$$\int x^m \tanh(a+bx)dx$$

input
```
Int[x^m*Tanh[a + b*x],x]
```

output
```
$Aborted
```

3.334.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.334.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int x^m \operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

input
```
int(x^m*sech(b*x+a)*sinh(b*x+a),x)
```

output
```
int(x^m*sech(b*x+a)*sinh(b*x+a),x)
```

3.334.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int x^m \tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

input
```
integrate(x^m*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*sech(b*x + a)*sinh(b*x + a), x)
```

3.334.6 Sympy [N/A]

Not integrable

Time = 10.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int x^m \tanh(a+bx)\,dx = \int x^m \sinh{(a+bx)}\operatorname{sech}{(a+bx)}\,dx$$

input
```
integrate(x**m*sech(b*x+a)*sinh(b*x+a),x)
```

output
```
Integral(x**m*sinh(a + b*x)*sech(a + b*x), x)
```

3.334.7 Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 10.00

$$\int x^m \tanh(a+bx)\,dx = \int x^m \operatorname{sech}{(bx+a)}\sinh{(bx+a)}\,\,dx$$

input
```
integrate(x^m*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
x*e^(2*b*x + m*log(x) + 2*a)/((m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate
(((2*b*x*e^(2*a) + (m + 1)*e^(2*a))*e^(2*b*x) + m + 1)*x^m/((m + 1)*e^(4*b
*x + 4*a) + 2*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

3.334.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int x^m \tanh(a+bx)\,dx = \int x^m \operatorname{sech}{(bx+a)}\sinh{(bx+a)}\,\,dx$$

input
```
integrate(x^m*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*sech(b*x + a)*sinh(b*x + a), x)
```

3.334.9 Mupad [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int x^m \tanh(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)}{\cosh\left(a+b\,x\right)}\,dx$$

input
```
int((x^m*sinh(a + b*x))/cosh(a + b*x),x)
```

output
```
int((x^m*sinh(a + b*x))/cosh(a + b*x), x)
```

3.335 $\int x^3 \tanh(a+bx)\,dx$

3.335.1 Optimal result

Integrand size = 10, antiderivative size = 91

$$\int x^3 \tanh(a+bx)\,dx = -\frac{x^4}{4} + \frac{x^3 \log\left(1+e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} + \frac{3 \operatorname{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}$$

output
```
-1/4*x^4+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2
-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/4*polylog(4,-exp(2*b*x+2*a))/b^4
```

3.335.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int x^3 \tanh(a+bx)\,dx = \frac{b^4x^4 + 4b^3x^3 \log\left(1+e^{-2(a+bx)}\right) - 6b^2x^2 \operatorname{PolyLog}\left(2,-e^{-2(a+bx)}\right) - 6bx \operatorname{PolyLog}\left(3,-e^{-2(a+bx)}\right) - 3 \operatorname{PolyL}}{4b^4}$$

input
```
Integrate[x^3*Tanh[a + b*x],x]
```

output
```
(b^4*x^4 + 4*b^3*x^3*Log[1 + E^(-2*(a + b*x))] - 6*b^2*x^2*PolyLog[2, -E^(
-2*(a + b*x))] - 6*b*x*PolyLog[3, -E^(-2*(a + b*x))] - 3*PolyLog[4, -E^(-2
*(a + b*x))])/(4*b^4)
```

3.335.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 26, 4201, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \tanh(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -ix^3 \tan(ia+ibx)dx$$

$$\Big\downarrow \ 26$$

$$-i\int x^3 \tan(ia+ibx)dx$$

$$\Big\downarrow \ 4201$$

$$-i\left(2i\int \frac{e^{2(a+bx)}x^3}{1+e^{2(a+bx)}}dx-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \ 2620$$

$$-i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\int x^2\log\left(1+e^{2(a+bx)}\right)dx}{2b}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \ 3011$$

$$-i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}(2,-e^{2(a+bx)})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2(a+bx)})}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \ 7163$$

$$-i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-e^{2(a+bx)}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2(a+bx)})}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \ 2720$$

$$-i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b}-\frac{\int e^{-2(a+bx)}\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b}-\frac{\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)$$

input
```
Int[x^3*Tanh[a + b*x],x]
```

output
```
(-I)*((-1/4*I)*x^4 + (2*I)*((x^3*Log[1 + E^(2*(a + b*x))])/(2*b) - (3*(-1/
2*(x^2*PolyLog[2, -E^(2*(a + b*x))])/b + ((x*PolyLog[3, -E^(2*(a + b*x))])
/(2*b) - PolyLog[4, -E^(2*(a + b*x))]/(4*b^2))/b))/(2*b)))
```

3.335.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.335.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{x^4}{4}-\frac{3a^4}{2b^4}-\frac{2a^3x}{b^3}+\frac{x^3\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}+\frac{3x^2\operatorname{polylog}\left(2,-\mathrm{e}^{2bx+2a}\right)}{2b^2}-\frac{3x\operatorname{polylog}\left(3,-\mathrm{e}^{2bx+2a}\right)}{2b^3}+\frac{2a^3\ln\left(\mathrm{e}^{bx+a}\right)}{b^4}+\frac{3\,\mathrm{po}$

input
```
int(x^3*sech(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^4-3/2/b^4*a^4-2/b^3*a^3*x+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*x^2*polylo
g(2,-exp(2*b*x+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+2/b^4*a^3*ln
(exp(b*x+a))+3/4*polylog(4,-exp(2*b*x+2*a))/b^4
```

3.335.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.82

$$\int x^3 \tanh(a+bx)\,dx = -\frac{b^4x^4 - 12\,b^2x^2\mathrm{Li}_2(i\,\cosh\left(bx+a\right) + i\,\sinh\left(bx+a\right)) - 12\,b^2x^2\mathrm{Li}_2(-i\,\cosh\left(bx+a\right) - i\,\sinh\left(bx+a\right)) +}{}$$

input
```
integrate(x^3*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(b^4*x^4 - 12*b^2*x^2*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 12*b
^2*x^2*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*a^3*log(cosh(b*x + a)
 + sinh(b*x + a) + I) + 4*a^3*log(cosh(b*x + a) + sinh(b*x + a) - I) + 24*
b*x*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 24*b*x*polylog(3, -I*c
osh(b*x + a) - I*sinh(b*x + a)) - 4*(b^3*x^3 + a^3)*log(I*cosh(b*x + a) +
I*sinh(b*x + a) + 1) - 4*(b^3*x^3 + a^3)*log(-I*cosh(b*x + a) - I*sinh(b*x
 + a) + 1) - 24*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 24*polylog
(4, -I*cosh(b*x + a) - I*sinh(b*x + a)))/b^4
```

3.335.6 Sympy [F]

$$\int x^3 \tanh(a+bx)\,dx = \int x^3 \sinh\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

input
```
integrate(x**3*sech(b*x+a)*sinh(b*x+a),x)
```

output
```
Integral(x**3*sinh(a + b*x)*sech(a + b*x), x)
```

3.335.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int x^3 \tanh(a+bx)\,dx = -\frac{1}{4}x^4 + \frac{4\,b^3x^3\log\left(e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(-e^{(2\,bx+2\,a)})+3\,\mathrm{Li}_4(-e^{(2\,bx+2\,a)})}{3\,b^4}$$

input
```
integrate(x^3*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*x^4 + 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2
*b*x + 2*a)) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x
 + 2*a)))/b^4
```

3.335.8 Giac [F]

$$\int x^3 \tanh(a+bx)\,dx = \int x^3\,\mathrm{sech}\,(bx+a)\sinh\,(bx+a)\;dx$$

input
```
integrate(x^3*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*sech(b*x + a)*sinh(b*x + a), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \tanh(a+bx)\,dx = \int \frac{x^3\sinh(a+b\,x)}{\cosh\,(a+b\,x)}\,dx$$

input
```
int((x^3*sinh(a + b*x))/cosh(a + b*x),x)
```

output
```
int((x^3*sinh(a + b*x))/cosh(a + b*x), x)
```

3.336 $\int x^2 \tanh(a+bx)\,dx$

3.336.1 Optimal result

Integrand size = 10, antiderivative size = 65

$$\int x^2 \tanh(a+bx)\,dx = -\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}$$

output
```
-1/3*x^3+x^2*ln(1+exp(2*b*x+2*a))/b+x*polylog(2,-exp(2*b*x+2*a))/b^2-1/2*p
olylog(3,-exp(2*b*x+2*a))/b^3
```

3.336.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int x^2 \tanh(a+bx)\,dx = \frac{2b^2x^2\left(bx+3\log\left(1+e^{-2(a+bx)}\right)\right)-6bx\,\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)-3\,\text{PolyLog}\left(3,-e^{-2(a+bx)}\right)}{6b^3}$$

input
```
Integrate[x^2*Tanh[a + b*x],x]
```

output
```
(2*b^2*x^2*(b*x + 3*Log[1 + E^(-2*(a + b*x))]) - 6*b*x*PolyLog[2, -E^(-2*(
a + b*x))] - 3*PolyLog[3, -E^(-2*(a + b*x))])/(6*b^3)
```

3.336.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \tanh(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -ix^2\tan(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int x^2\tan(ia+ibx)dx$$

$$\Big\downarrow \text{ 4201}$$

$$-i\left(2i\int \frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}dx-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$-i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int x\log\left(1+e^{2(a+bx)}\right)dx}{b}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{ 3011}$$

$$-i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int \text{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{ 2720}$$

$$-i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{ 7143}$$

$$-i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x^2*Tanh[a + b*x],x]
```

output
```
(-I)*((-1/3*I)*x^3 + (2*I)*((x^2*Log[1 + E^(2*(a + b*x))])/(2*b) - (-1/2*(
x*PolyLog[2, -E^(2*(a + b*x))])/b + PolyLog[3, -E^(2*(a + b*x))]/(4*b^2))/
b))
```

3.336.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.336.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

method	result	size
risch	$-\frac{x^3}{3}-\frac{2a^2\ln(e^{bx+a})}{b^3}+\frac{2a^2x}{b^2}+\frac{4a^3}{3b^3}+\frac{x^2\ln(1+e^{2bx+2a})}{b}+\frac{x\,\mathrm{polylog}(2,-e^{2bx+2a})}{b^2}-\frac{\mathrm{polylog}(3,-e^{2bx+2a})}{2b^3}$	94

input
```
int(x^2*sech(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3-2/b^3*a^2*ln(exp(b*x+a))+2/b^2*a^2*x+4/3/b^3*a^3+x^2*ln(1+exp(2*b
*x+2*a))/b+x*polylog(2,-exp(2*b*x+2*a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))
/b^3
```

3.336.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.18

$$\int x^2\tanh(a+bx)\,dx =$$
$$-\frac{b^3x^3-6\,bx\mathrm{Li}_2(i\,\cosh{(bx+a)}+i\,\sinh{(bx+a)})-6\,bx\mathrm{Li}_2(-i\,\cosh{(bx+a)}-i\,\sinh{(bx+a)})-3\,a^2\,\mathrm{lo}}{}$$

input
```
integrate(x^2*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/3*(b^3*x^3 - 6*b*x*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*b*x*dil
og(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 3*a^2*log(cosh(b*x + a) + sinh(b*
x + a) + I) - 3*a^2*log(cosh(b*x + a) + sinh(b*x + a) - I) - 3*(b^2*x^2 -
a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 3*(b^2*x^2 - a^2)*log(-I
*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 6*polylog(3, I*cosh(b*x + a) + I*s
inh(b*x + a)) + 6*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)))/b^3
```

3.336.6 Sympy [F]

$$\int x^2 \tanh(a+bx)\,dx = \int x^2 \sinh{(a+bx)}\operatorname{sech}{(a+bx)}\,dx$$

input
```
integrate(x**2*sech(b*x+a)*sinh(b*x+a),x)
```

output
```
Integral(x**2*sinh(a + b*x)*sech(a + b*x), x)
```

3.336.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int x^2 \tanh(a+bx)\,dx = -\frac{1}{3}\,x^3 + \frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right) - \mathrm{Li}_3\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^3}$$

input
```
integrate(x^2*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-1/3*x^3 + 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x
 + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3
```

3.336.8 Giac [F]

$$\int x^2 \tanh(a + bx) \, dx = \int x^2 \operatorname{sech}(bx + a) \sinh(bx + a) \, dx$$

input
```
integrate(x^2*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*sech(b*x + a)*sinh(b*x + a), x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \tanh(a + bx) \, dx = \int \frac{x^2 \sinh(a + b\,x)}{\cosh(a + b\,x)} \, dx$$

input
```
int((x^2*sinh(a + b*x))/cosh(a + b*x),x)
```

output
```
int((x^2*sinh(a + b*x))/cosh(a + b*x), x)
```

3.337 $\int x \tanh(a+bx)\,dx$

3.337.1 Optimal result

Integrand size = 8, antiderivative size = 45

$$\int x \tanh(a+bx)\,dx = -\frac{x^2}{2} + \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}$$

output
```
-1/2*x^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^2
```

3.337.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int x \tanh(a+bx)\,dx = \frac{bx\left(bx+2\log\left(1+e^{-2(a+bx)}\right)\right) - \text{PolyLog}\left(2,-e^{-2(a+bx)}\right)}{2b^2}$$

input
```
Integrate[x*Tanh[a + b*x],x]
```

output
```
(b*x*(b*x + 2*Log[1 + E^(-2*(a + b*x))]) - PolyLog[2, -E^(-2*(a + b*x))])/
(2*b^2)
```

3.337.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix\tan(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x\tan(ia+ibx)dx$$

$$\Big\downarrow \text{4201}$$

$$-i\left(2i\int \frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$-i\left(2i\left(\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int \log\left(1+e^{2(a+bx)}\right)dx}{2b}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$-i\left(2i\left(\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int e^{-2(a+bx)}\log\left(1+e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$-i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{4b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[x*Tanh[a + b*x],x]
```

output
```
(-I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*(a + b*x))])/(2*b) + PolyLog[2
, -E^(2*(a + b*x))]/(4*b^2)))
```

3.337.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.337.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

method	result	size
risch	$-\frac{x^{2}}{2}-\frac{2 a x}{b}-\frac{a^{2}}{b^{2}}+\frac{x \ln \left(1+{\mathrm e}^{2 b x +2 a}\right)}{b}+\frac{\operatorname{polylog}\left(2,-{\mathrm e}^{2 b x +2 a}\right)}{2 b^{2}}+\frac{2 a \ln \left({\mathrm e}^{b x +a}\right)}{b^{2}}$	70

input
```
int(x*sech(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2-2/b*a*x-a^2/b^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x
+2*a))/b^2+2/b^2*a*ln(exp(b*x+a))
```

3.337.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.13

$$\int x\tanh(a+bx)\,dx = -\frac{b^2x^2 + 2\,a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+i\right) + 2\,a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-i\right) - 2\,(bx+$$

input
```
integrate(x*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^2 + 2*a*log(cosh(b*x + a) + sinh(b*x + a) + I) + 2*a*log(cosh(
b*x + a) + sinh(b*x + a) - I) - 2*(b*x + a)*log(I*cosh(b*x + a) + I*sinh(b
*x + a) + 1) - 2*(b*x + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 2
*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*dilog(-I*cosh(b*x + a) - I*s
inh(b*x + a)))/b^2
```

3.337.6 Sympy [F]

$$\int x\tanh(a+bx)\,dx = \int x\sinh\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

input
```
integrate(x*sech(b*x+a)*sinh(b*x+a),x)
```

output
```
Integral(x*sinh(a + b*x)*sech(a + b*x), x)
```

3.337.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x\tanh(a+bx)\,dx = -\frac{1}{2}x^2 + \frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}$$

input
```
integrate(x*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*x^2 + 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/
b^2
```

3.337.8 Giac [F]

$$\int x\tanh(a+bx)\,dx = \int x\,\mathrm{sech}\,(bx+a)\sinh\,(bx+a)\;dx$$

input
```
integrate(x*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sech(b*x + a)*sinh(b*x + a), x)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int x\tanh(a+bx)\,dx = \int \frac{x\sinh(a+b\,x)}{\cosh\,(a+b\,x)}\,dx$$

input
```
int((x*sinh(a + b*x))/cosh(a + b*x),x)
```

output
```
int((x*sinh(a + b*x))/cosh(a + b*x), x)
```

3.338 $\int \tanh(a+bx)\,dx$

3.338.1 Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \tanh(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b}$$

output
```
ln(cosh(b*x+a))/b
```

3.338.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \tanh(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b}$$

input
```
Integrate[Tanh[a + b*x],x]
```

output
```
Log[Cosh[a + b*x]]/b
```

3.338.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan(ia+ibx)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\log(\cosh(a+bx))}{b}$$

input
```
Int[Tanh[a + b*x],x]
```

output
```
Log[Cosh[a + b*x]]/b
```

3.338.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.338.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$-\frac{\ln(\operatorname{sech}(bx+a))}{b}$	13
default	$-\frac{\ln(\operatorname{sech}(bx+a))}{b}$	13
parallelrisch	$\frac{-bx-\ln(1-\tanh(bx+a))}{b}$	23
risch	$-x-\frac{2a}{b}+\frac{\ln(1+e^{2bx+2a})}{b}$	27

input
```
int(sech(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b*ln(sech(b*x+a))
```

3.338.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.36

$$\int \tanh(a+bx)\,dx = -\frac{bx-\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-(b*x - log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

3.338.6 Sympy [F]

$$\int \tanh(a+bx)\,dx = \int \sinh(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*sech(a + b*x), x)
```

3.338.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \tanh(a+bx)\,dx = \frac{\log\left(e^{(bx+a)} + e^{(-bx-a)}\right)}{b}$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
log(e^(b*x + a) + e^(-b*x - a))/b
```

3.338.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \tanh(a+bx)\,dx = -\frac{bx + a - \log\left(e^{(2\,bx+2\,a)} + 1\right)}{b}$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
-(b*x + a - log(e^(2*b*x + 2*a) + 1))/b
```

3.338.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \tanh(a+bx)\,dx = \frac{\ln\left(\cosh(a+b\,x)\right)}{b}$$

input
```
int(sinh(a + b*x)/cosh(a + b*x),x)
```

output
```
log(cosh(a + b*x))/b
```

3.339 $\int \frac{\tanh(a+bx)}{x} dx$

3.339.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\tanh(a+bx)}{x} dx = \operatorname{Int}\left(\frac{\tanh(a+bx)}{x}, x\right)$$

output
```
Unintegrable(tanh(b*x+a)/x,x)
```

3.339.2 Mathematica [N/A]

Not integrable

Time = 9.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\tanh(a+bx)}{x} dx = \int \frac{\tanh(a+bx)}{x} dx$$

input
```
Integrate[Tanh[a + b*x]/x,x]
```

output
```
Integrate[Tanh[a + b*x]/x, x]
```

3.339.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(a+bx)}{x} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i \tan(ia+ibx)}{x}dx$$

$$\Big\downarrow \; 26$$

$$-i \int \frac{\tan(ia+ibx)}{x}dx$$

$$\Big\downarrow \; 4222$$

$$\int \frac{\tanh(a+bx)}{x}dx$$

input
```
Int[Tanh[a + b*x]/x,x]
```

output
```
$Aborted
```

3.339.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.339.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{sech}\left(bx+a\right)\sinh\left(bx+a\right)}{x}dx$$

input
```
int(sech(b*x+a)*sinh(b*x+a)/x,x)
```

output
```
int(sech(b*x+a)*sinh(b*x+a)/x,x)
```

3.339.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)\sinh\left(bx+a\right)}{x}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)*sinh(b*x + a)/x, x)
```

3.339.6 Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\tanh(a+bx)}{x}\,dx = \int \frac{\sinh{(a+bx)}\operatorname{sech}{(a+bx)}}{x}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)/x,x)
```

output
```
Integral(sinh(a + b*x)*sech(a + b*x)/x, x)
```

3.339.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.20

$$\int \frac{\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}{(bx+a)}\sinh{(bx+a)}}{x}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)/x,x, algorithm="maxima")
```

output
```
-2*integrate(1/(x*e^(2*b*x + 2*a) + x), x) + log(x)
```

3.339.8 Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}{(bx+a)}\sinh{(bx+a)}}{x}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)*sinh(b*x + a)/x, x)
```

3.339.9 Mupad [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)}{x\cosh\left(a+b\,x\right)}\,dx$$

input
```
int(sinh(a + b*x)/(x*cosh(a + b*x)),x)
```

output
```
int(sinh(a + b*x)/(x*cosh(a + b*x)), x)
```

3.340 $\int \frac{\tanh(a+bx)}{x^2} dx$

3.340.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\tanh(a+bx)}{x^2}\, dx = \operatorname{Int}\left(\frac{\tanh(a+bx)}{x^2}, x\right)$$

output
```
Unintegrable(tanh(b*x+a)/x^2,x)
```

3.340.2 Mathematica [N/A]

Not integrable

Time = 16.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\tanh(a+bx)}{x^2}\, dx = \int \frac{\tanh(a+bx)}{x^2}\, dx$$

input
```
Integrate[Tanh[a + b*x]/x^2,x]
```

output
```
Integrate[Tanh[a + b*x]/x^2, x]
```

3.340.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(a+bx)}{x^2} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{i\tan(ia+ibx)}{x^2}dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \frac{\tan(ia+ibx)}{x^2}dx$$
$$\Big\downarrow \text{ 4222}$$
$$\int \frac{\tanh(a+bx)}{x^2}dx$$

input
```
Int[Tanh[a + b*x]/x^2,x]
```

output
```
$Aborted
```

3.340.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.340.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}dx$$

input
```
int(sech(b*x+a)*sinh(b*x+a)/x^2,x)
```

output
```
int(sech(b*x+a)*sinh(b*x+a)/x^2,x)
```

3.340.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)*sinh(b*x + a)/x^2, x)
```

3.340.6 Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)/x**2,x)
```

output
```
Integral(sinh(a + b*x)*sech(a + b*x)/x**2, x)
```

3.340.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \frac{\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
-1/x - 2*integrate(1/(x^2*e^(2*b*x + 2*a) + x^2), x)
```

3.340.8 Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)*sinh(b*x + a)/x^2, x)
```

3.340.9 Mupad [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)}{x^2\cosh\left(a+b\,x\right)}\,dx$$

input
```
int(sinh(a + b*x)/(x^2*cosh(a + b*x)),x)
```

output
```
int(sinh(a + b*x)/(x^2*cosh(a + b*x)), x)
```

3.341 $\int x^m \mathbf{sech}(a+bx)\tanh(a+bx)\,dx$

3.341.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \operatorname{Int}(x^m \operatorname{sech}(a+bx)\tanh(a+bx),x)$$

output
```
CannotIntegrate(x^m*sech(b*x+a)*tanh(b*x+a),x)
```

3.341.2 Mathematica [N/A]

Not integrable

Time = 48.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$$

input
```
Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x], x]
```

3.341.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \tanh(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^m \tanh(a+bx)\text{sech}(a+bx)dx$$

input
```
Int[x^m*Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
$Aborted
```

3.341.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.341.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)\,dx$$

input
```
int(x^m*sech(b*x+a)^2*sinh(b*x+a),x)
```

output
```
int(x^m*sech(b*x+a)^2*sinh(b*x+a),x)
```

3.341.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}\left(bx+a\right)^2 \sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^m*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*sech(b*x + a)^2*sinh(b*x + a), x)
```

3.341.6 Sympy [N/A]

Not integrable

Time = 35.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \int x^m \sinh\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

input
```
integrate(x**m*sech(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Integral(x**m*sinh(a + b*x)*sech(a + b*x)**2, x)
```

3.341.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}\left(bx+a\right)^2 \sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^m*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^m*sech(b*x + a)^2*sinh(b*x + a), x)
```

3.341.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}\left(bx+a\right)^2 \sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^m*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*sech(b*x + a)^2*sinh(b*x + a), x)
```

3.341.9 Mupad [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^2}\,dx$$

input
```
int((x^m*sinh(a + b*x))/cosh(a + b*x)^2,x)
```

output
```
int((x^m*sinh(a + b*x))/cosh(a + b*x)^2, x)
```

3.342 $\int x^3 \text{sech}(a+bx) \tanh(a+bx) \, dx$

3.342.1 Optimal result

Integrand size = 16, antiderivative size = 113

$$\int x^3 \text{sech}(a+bx) \tanh(a+bx) \, dx = \frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} - \frac{6ix \,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{6ix \,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{6i \,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4} - \frac{6i \,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4} - \frac{x^3 \text{sech}(a+bx)}{b}$$

output
```
6*x^2*arctan(exp(b*x+a))/b^2-6*I*x*polylog(2,-I*exp(b*x+a))/b^3+6*I*x*poly
log(2,I*exp(b*x+a))/b^3+6*I*polylog(3,-I*exp(b*x+a))/b^4-6*I*polylog(3,I*e
xp(b*x+a))/b^4-x^3*sech(b*x+a)/b
```

3.342.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.15

$$\int x^3 \text{sech}(a+bx) \tanh(a+bx) \, dx = \frac{3i\left(b^2x^2 \log\left(1-ie^{a+bx}\right) - b^2x^2 \log\left(1+ie^{a+bx}\right) - 2bx \,\text{PolyLog}\left(2,-ie^{a+bx}\right) + 2bx \,\text{PolyLog}\left(2,ie^{a+bx}\right) + 2\right.}{b^4} - \frac{x^3 \text{sech}(a+bx)}{b}$$

input
```
Integrate[x^3*Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
((3*I)*(b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a + b*x)] -
2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*x)] + 2*P
olyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]))/b^4 - (x^3*Sec
h[a + b*x])/b
```

3.342.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5941, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \tanh(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \; 5941$$

$$\frac{3\int x^2\text{sech}(a+bx)dx}{b}-\frac{x^3\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{x^3\text{sech}(a+bx)}{b}+\frac{3\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \; 4668$$

$$-\frac{x^3\text{sech}(a+bx)}{b}+\frac{3\left(-\frac{2i\int x\log\left(1-ie^{a+bx}\right)dx}{b}+\frac{2i\int x\log\left(1+ie^{a+bx}\right)dx}{b}+\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3011$$

$$-\frac{x^3\text{sech}(a+bx)}{b}+\frac{3\left(\frac{2i\left(\frac{\int \text{PolyLog}\left(2,-ie^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{2i\left(\frac{\int \text{PolyLog}\left(2,ie^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}+\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 2720$$

$$-\frac{x^3\text{sech}(a+bx)}{b}+\frac{3\left(\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}\left(2,-ie^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}\left(2,ie^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}+\frac{2x^2\,\text{a}}{}\right.}{b}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{x^3\text{sech}(a+bx)}{b}+\frac{3\left(\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}+\frac{2i\left(\frac{\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{2i\left(\frac{\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}\right)}{b}$$

input
```
Int[x^3*Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
(3*((2*x^2*ArcTan[E^(a + b*x)])/b + ((2*I)*(-((x*PolyLog[2, (-I)*E^(a + b*
x)])/b) + PolyLog[3, (-I)*E^(a + b*x)]/b^2))/b - ((2*I)*(-((x*PolyLog[2, I
*E^(a + b*x)])/b) + PolyLog[3, I*E^(a + b*x)]/b^2))/b))/b - (x^3*Sech[a +
b*x])/b
```

3.342.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
    Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.342.4 Maple [F]

$$\int x^3 \operatorname{sech}(bx+a)^2 \sinh(bx+a)\, dx$$

input
```
int(x^3*sech(b*x+a)^2*sinh(b*x+a),x)
```

output
```
int(x^3*sech(b*x+a)^2*sinh(b*x+a),x)
```

3.342.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(90) = 180.

Time = 0.27 (sec) , antiderivative size = 672, normalized size of antiderivative = 5.95

$$\int x^3 \text{sech}(a+bx)\tanh(a+bx)\, dx =$$

$$-\frac{2\, b^3 x^3 \cosh(bx+a) + 2\, b^3 x^3 \sinh(bx+a) + 6\left(-i\, bx \cosh(bx+a)^2 - 2i\, bx \cosh(bx+a) \sinh(bx+a)\right.}{}$$

input
```
integrate(x^3*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) + 6*(-I*b*x*cosh(b*x +
 a)^2 - 2*I*b*x*cosh(b*x + a)*sinh(b*x + a) - I*b*x*sinh(b*x + a)^2 - I*b*
x)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(I*b*x*cosh(b*x + a)^2 + 2
*I*b*x*cosh(b*x + a)*sinh(b*x + a) + I*b*x*sinh(b*x + a)^2 + I*b*x)*dilog(
-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*(-I*a^2*cosh(b*x + a)^2 - 2*I*a^2*
cosh(b*x + a)*sinh(b*x + a) - I*a^2*sinh(b*x + a)^2 - I*a^2)*log(cosh(b*x
+ a) + sinh(b*x + a) + I) + 3*(I*a^2*cosh(b*x + a)^2 + 2*I*a^2*cosh(b*x +
a)*sinh(b*x + a) + I*a^2*sinh(b*x + a)^2 + I*a^2)*log(cosh(b*x + a) + sinh
(b*x + a) - I) + 3*(I*b^2*x^2 + (I*b^2*x^2 - I*a^2)*cosh(b*x + a)^2 + 2*(I
*b^2*x^2 - I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (I*b^2*x^2 - I*a^2)*sinh(b
*x + a)^2 - I*a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 3*(-I*b^2*
x^2 + (-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^2 + 2*(-I*b^2*x^2 + I*a^2)*cosh(b
*x + a)*sinh(b*x + a) + (-I*b^2*x^2 + I*a^2)*sinh(b*x + a)^2 + I*a^2)*log(
-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 6*(I*cosh(b*x + a)^2 + 2*I*cosh(
b*x + a)*sinh(b*x + a) + I*sinh(b*x + a)^2 + I)*polylog(3, I*cosh(b*x + a)
 + I*sinh(b*x + a)) + 6*(-I*cosh(b*x + a)^2 - 2*I*cosh(b*x + a)*sinh(b*x +
 a) - I*sinh(b*x + a)^2 - I)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)
))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x
 + a)^2 + b^4)
```

3.342.6 Sympy [F]

$$\int x^3 \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \int x^3 \sinh(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(x**3*sech(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Integral(x**3*sinh(a + b*x)*sech(a + b*x)**2, x)
```

3.342.7 Maxima [F]

$$\int x^3\mathrm{sech}(a+bx)\tanh(a+bx)\,dx=\int x^3\,\mathrm{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^3*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) + 6*integrate(x^2*e^(b*x + a)/(
b*e^(2*b*x + 2*a) + b), x)
```

3.342.8 Giac [F]

$$\int x^3\mathrm{sech}(a+bx)\tanh(a+bx)\,dx=\int x^3\,\mathrm{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^3*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*sech(b*x + a)^2*sinh(b*x + a), x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int x^3\mathrm{sech}(a+bx)\tanh(a+bx)\,dx=\int \frac{x^3\sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^2}\,dx$$

input
```
int((x^3*sinh(a + b*x))/cosh(a + b*x)^2,x)
```

output
```
int((x^3*sinh(a + b*x))/cosh(a + b*x)^2, x)
```

3.343 $\int x^2 \text{sech}(a+bx)\tanh(a+bx)\,dx$

3.343.1 Optimal result

Integrand size = 16, antiderivative size = 69

$$\int x^2 \text{sech}(a+bx)\tanh(a+bx)\,dx = \frac{4x\arctan\left(e^{a+bx}\right)}{b^2} - \frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{x^2\text{sech}(a+bx)}{b}$$

output
```
4*x*arctan(exp(b*x+a))/b^2-2*I*polylog(2,-I*exp(b*x+a))/b^3+2*I*polylog(2,
I*exp(b*x+a))/b^3-x^2*sech(b*x+a)/b
```

3.343.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int x^2 \text{sech}(a+bx)\tanh(a+bx)\,dx = \frac{2i\left(bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a+bx}\right)\right)}{b^3} - \frac{x^2\text{sech}(a+bx)}{b}$$

input
```
Integrate[x^2*Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
((2*I)*(b*x*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(a + b*x)]) - PolyLog[2,
 (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*x)]))/b^3 - (x^2*Sech[a + b*x])
/b
```

3.343.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5941, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \tanh(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{5941}$$

$$\frac{2\int x\text{sech}(a+bx)dx}{b}-\frac{x^2\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x^2\text{sech}(a+bx)}{b}+\frac{2\int x\csc\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{x^2\text{sech}(a+bx)}{b}+\frac{2\left(-\frac{i\int \log\left(1-ie^{a+bx}\right)dx}{b}+\frac{i\int \log\left(1+ie^{a+bx}\right)dx}{b}+\frac{2x\arctan\left(e^{a+bx}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{x^2\text{sech}(a+bx)}{b}+\frac{2\left(-\frac{i\int e^{-a-bx}\log\left(1-ie^{a+bx}\right)de^{a+bx}}{b^2}+\frac{i\int e^{-a-bx}\log\left(1+ie^{a+bx}\right)de^{a+bx}}{b^2}+\frac{2x\arctan\left(e^{a+bx}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{x^2\text{sech}(a+bx)}{b}+\frac{2\left(\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}\right)}{b}$$

input
```
Int[x^2*Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
(2*((2*x*ArcTan[E^(a + b*x)])/b - (I*PolyLog[2, (-I)*E^(a + b*x)])/b^2 + (
I*PolyLog[2, I*E^(a + b*x)])/b^2))/b - (x^2*Sech[a + b*x])/b
```

3.343.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

3.343.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(62) = 124$.

Time = 0.75 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.23

method	result
risch	$-\frac{2x^2 e^{bx+a}}{b(1+e^{2bx+2a})}-\frac{2i\ln(1+ie^{bx+a})x}{b^2}-\frac{2i\ln(1+ie^{bx+a})a}{b^3}+\frac{2i\ln(1-ie^{bx+a})x}{b^2}+\frac{2i\ln(1-ie^{bx+a})a}{b^3}-\frac{2i\,\mathrm{dilog}(1+ie^{bx+a})}{b^3}-$

input
```
int(x^2*sech(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*x^2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))-2*I/b^2*ln(1+I*exp(b*x+a))*x-2*I/b^
3*ln(1+I*exp(b*x+a))*a+2*I/b^2*ln(1-I*exp(b*x+a))*x+2*I/b^3*ln(1-I*exp(b*x
+a))*a-2*I/b^3*dilog(1+I*exp(b*x+a))+2*I/b^3*dilog(1-I*exp(b*x+a))-4/b^3*a
*arctan(exp(b*x+a))
```

3.343.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 468 vs. 2(56) = 112.

Time = 0.25 (sec) , antiderivative size = 468, normalized size of antiderivative = 6.78

$$\int x^2 \operatorname{sech}(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{2\left(b^2x^2\cosh(bx+a)+b^2x^2\sinh(bx+a)+\left(-i\cosh(bx+a)^2-2i\cosh(bx+a)\sinh(bx+a)-i\sin\right.\right.}{}$$

input
```
integrate(x^2*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-2*(b^2*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a) + (-I*cosh(b*x + a)^2 -
2*I*cosh(b*x + a)*sinh(b*x + a) - I*sinh(b*x + a)^2 - I)*dilog(I*cosh(b*x
+ a) + I*sinh(b*x + a)) + (I*cosh(b*x + a)^2 + 2*I*cosh(b*x + a)*sinh(b*x
+ a) + I*sinh(b*x + a)^2 + I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) +
(I*a*cosh(b*x + a)^2 + 2*I*a*cosh(b*x + a)*sinh(b*x + a) + I*a*sinh(b*x +
a)^2 + I*a)*log(cosh(b*x + a) + sinh(b*x + a) + I) + (-I*a*cosh(b*x + a)^2
 - 2*I*a*cosh(b*x + a)*sinh(b*x + a) - I*a*sinh(b*x + a)^2 - I*a)*log(cosh
(b*x + a) + sinh(b*x + a) - I) + ((I*b*x + I*a)*cosh(b*x + a)^2 + 2*(I*b*x
 + I*a)*cosh(b*x + a)*sinh(b*x + a) + (I*b*x + I*a)*sinh(b*x + a)^2 + I*b*
x + I*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + ((-I*b*x - I*a)*cosh
(b*x + a)^2 + 2*(-I*b*x - I*a)*cosh(b*x + a)*sinh(b*x + a) + (-I*b*x - I*a
)*sinh(b*x + a)^2 - I*b*x - I*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) +
1))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*
x + a)^2 + b^3)
```

3.343.6 Sympy [F]

$$\int x^2 \text{sech}(a+bx)\tanh(a+bx)\,dx = \int x^2 \sinh(a+bx)\,\text{sech}^2(a+bx)\,dx$$

input
```
integrate(x**2*sech(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Integral(x**2*sinh(a + b*x)*sech(a + b*x)**2, x)
```

3.343.7 Maxima [F]

$$\int x^2 \text{sech}(a+bx)\tanh(a+bx)\,dx = \int x^2 \,\text{sech}(bx+a)^2 \sinh(bx+a)\,dx$$

input
```
integrate(x^2*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) + 4*integrate(x*e^(b*x + a)/(b*
e^(2*b*x + 2*a) + b), x)
```

3.343.8 Giac [F]

$$\int x^2 \text{sech}(a+bx)\tanh(a+bx)\,dx = \int x^2 \,\text{sech}(bx+a)^2 \sinh(bx+a)\,dx$$

input
```
integrate(x^2*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*sech(b*x + a)^2*sinh(b*x + a), x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{sech}(a+bx)\tanh(a+bx)\,dx=\int \frac{x^2\sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^2}\,dx$$

input
```
int((x^2*sinh(a + b*x))/cosh(a + b*x)^2,x)
```

output
```
int((x^2*sinh(a + b*x))/cosh(a + b*x)^2, x)
```

3.344 $\int x\text{sech}(a+bx)\tanh(a+bx)\,dx$

3.344.1 Optimal result

Integrand size = 14, antiderivative size = 24

$$\int x\text{sech}(a+bx)\tanh(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b^2}-\frac{x\text{sech}(a+bx)}{b}$$

output
```
arctan(sinh(b*x+a))/b^2-x*sech(b*x+a)/b
```

3.344.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int x\text{sech}(a+bx)\tanh(a+bx)\,dx=\frac{2\arctan\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{x\text{sech}(a+bx)}{b}$$

input
```
Integrate[x*Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
(2*ArcTan[Tanh[a/2 + (b*x)/2]])/b^2 - (x*Sech[a + b*x])/b
```

3.344.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5941, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \tanh(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \; 5941$$

$$\frac{\int \text{sech}(a+bx)dx}{b}-\frac{x\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{x\text{sech}(a+bx)}{b}+\frac{\int \csc\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \; 4257$$

$$\frac{\arctan(\sinh(a+bx))}{b^2}-\frac{x\text{sech}(a+bx)}{b}$$

input
```
Int[x*Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
ArcTan[Sinh[a + b*x]]/b^2 - (x*Sech[a + b*x])/b
```

3.344.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

3.344.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

method	result	size
risch	$-\frac{2x\,\mathrm{e}^{bx+a}}{b(1+\mathrm{e}^{2bx+2a})}+\frac{i\ln(\mathrm{e}^{bx+a}+i)}{b^2}-\frac{i\ln(\mathrm{e}^{bx+a}-i)}{b^2}$	59

input
```
int(x*sech(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*x*exp(b*x+a)/b/(1+exp(2*b*x+2*a))+I/b^2*ln(exp(b*x+a)+I)-I/b^2*ln(exp(b
*x+a)-I)
```

3.344.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.83

$$\int x\mathrm{sech}(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{2\left(bx\cosh\left(bx+a\right)+bx\sinh\left(bx+a\right)-\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)\right.}{b^2\cosh\left(bx+a\right)^2+2\,b^2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b^2\sinh\left(bx+\right.}$$

input
```
integrate(x*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-2*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a) - (cosh(b*x + a)^2 + 2*cosh(b*x
+ a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*arctan(cosh(b*x + a) + sinh(b*x
+ a)))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh
(b*x + a)^2 + b^2)
```

3.344.6 Sympy [F]

$$\int x\text{sech}(a+bx)\tanh(a+bx)\,dx = \int x\sinh(a+bx)\,\text{sech}^2(a+bx)\,dx$$

input
```
integrate(x*sech(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Integral(x*sinh(a + b*x)*sech(a + b*x)**2, x)
```

3.344.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int x\text{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}+b} + \frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^2}$$

input
```
integrate(x*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) + 2*arctan(e^(b*x + a))/b^2
```

3.344.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int x\text{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2\left(\pi + bxe^{(bx+a)} + \pi e^{(2\,bx+2\,a)} - \arctan\left(e^{(bx+a)}\right)e^{(2\,bx+2\,a)} - \arctan\left(e^{(bx+a)}\right)\right)}{b^2e^{(2\,bx+2\,a)}+b^2}$$

input
```
integrate(x*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
-2*(pi + b*x*e^(b*x + a) + pi*e^(2*b*x + 2*a) - arctan(e^(b*x + a))*e^(2*b
*x + 2*a) - arctan(e^(b*x + a)))/(b^2*e^(2*b*x + 2*a) + b^2)
```

3.344.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int x\text{sech}(a+bx)\tanh(a+bx)\,dx=\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^4}}{b^2}\right)}{\sqrt{b^4}}-\frac{2\,x\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int((x*sinh(a + b*x))/cosh(a + b*x)^2,x)
```

output
```
(2*atan((exp(b*x)*exp(a)*(b^4)^(1/2))/b^2))/(b^4)^(1/2) - (2*x*exp(a + b*x
))/(b*(exp(2*a + 2*b*x) + 1))
```

3.345 $\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$

3.345.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}(a+bx)}{b}$$

output
```
-sech(b*x+a)/b
```

3.345.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}(a+bx)}{b}$$

input
```
Integrate[Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
-(Sech[a + b*x]/b)
```

3.345.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 26, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\tan(ia+ibx)\sec(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \sec(ia+ibx)\tan(ia+ibx)dx$$

$$\Big\downarrow \text{ 3086}$$

$$-\frac{\int 1d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\text{sech}(a+bx)}{b}$$

input
```
Int[Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
-(Sech[a + b*x]/b)
```

3.345.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.345.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\operatorname{sech}(bx+a)}{b}$	12
default	$-\frac{\operatorname{sech}(bx+a)}{b}$	12
risch	$-\frac{2\,\mathrm{e}^{bx+a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)}$	25

input
```
int(sech(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-sech(b*x+a)/b
```

3.345.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(11) = 22.

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.91

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$$
$$= -\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}+b}$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

3.345.6 Sympy [F]

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \int \sinh(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*sech(a + b*x)**2, x)
```

3.345.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2}{b\left(e^{(bx+a)}+e^{(-bx-a)}\right)}$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

3.345.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2}{b\left(e^{(bx+a)}+e^{(-bx-a)}\right)}$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
-2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

3.345.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{1}{b\cosh\left(a+b\,x\right)}$$

input
```
int(sinh(a + b*x)/cosh(a + b*x)^2,x)
```

output
```
-1/(b*cosh(a + b*x))
```

3.346 $\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x} dx$

3.346.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x} dx = \text{Int}\left(\frac{\text{sech}(a+bx)\tanh(a+bx)}{x}, x\right)$$

output
```
CannotIntegrate(sech(b*x+a)*tanh(b*x+a)/x,x)
```

3.346.2 Mathematica [N/A]

Not integrable

Time = 6.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x} dx = \int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x} dx$$

input
```
Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x,x]
```

output
```
Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x, x]
```

3.346.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(a+bx)\text{sech}(a+bx)}{x} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\tanh(a+bx)\text{sech}(a+bx)}{x} dx$$

input
```
Int[(Sech[a + b*x]*Tanh[a + b*x])/x,x]
```

output
```
$Aborted
```

3.346.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.346.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{sech}\,(bx+a)^2 \sinh\,(bx+a)}{x} dx$$

input
```
int(sech(b*x+a)^2*sinh(b*x+a)/x,x)
```

output
```
int(sech(b*x+a)^2*sinh(b*x+a)/x,x)
```

3.346.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)}{x}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^2*sinh(b*x + a)/x, x)
```

3.346.6 Sympy [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

input
```
integrate(sech(b*x+a)**2*sinh(b*x+a)/x,x)
```

output
```
Integral(sinh(a + b*x)*sech(a + b*x)**2/x, x)
```

3.346.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.75

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)}{x}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="maxima")
```

output
```
-2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) + b*x) - 2*integrate(e^(b*x + a)/(b*x^
2*e^(2*b*x + 2*a) + b*x^2), x)
```

3.346.8 Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)}{x}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^2*sinh(b*x + a)/x, x)
```

3.346.9 Mupad [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)}{x\cosh(a+b\,x)^2}\,dx$$

input
```
int(sinh(a + b*x)/(x*cosh(a + b*x)^2),x)
```

output
```
int(sinh(a + b*x)/(x*cosh(a + b*x)^2), x)
```

3.347 $\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$

3.347.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx = \text{Int}\left(\frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2},x\right)$$

output
```
CannotIntegrate(sech(b*x+a)*tanh(b*x+a)/x^2,x)
```

3.347.2 Mathematica [N/A]

Not integrable

Time = 6.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x^2,x]
```

output
```
Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x^2, x]
```

3.347.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(a+bx)\text{sech}(a+bx)}{x^2} dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\tanh(a+bx)\text{sech}(a+bx)}{x^2} dx$$

input
```
Int[(Sech[a + b*x]*Tanh[a + b*x])/x^2,x]
```

output
```
$Aborted
```

3.347.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.347.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{sech}\left(bx+a\right)^2 \sinh\left(bx+a\right)}{x^2} dx$$

input
```
int(sech(b*x+a)^2*sinh(b*x+a)/x^2,x)
```

output
```
int(sech(b*x+a)^2*sinh(b*x+a)/x^2,x)
```

3.347.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^2*sinh(b*x + a)/x^2, x)
```

3.347.6 Sympy [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)**2*sinh(b*x+a)/x**2,x)
```

output
```
Integral(sinh(a + b*x)*sech(a + b*x)**2/x**2, x)
```

3.347.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.00

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
-2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) + b*x^2) - 4*integrate(e^(b*x + a)/(
b*x^3*e^(2*b*x + 2*a) + b*x^3), x)
```

3.347.8 Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^2*sinh(b*x + a)/x^2, x)
```

3.347.9 Mupad [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)}{x^2\cosh(a+b\,x)^2}\,dx$$

input
```
int(sinh(a + b*x)/(x^2*cosh(a + b*x)^2),x)
```

output
```
int(sinh(a + b*x)/(x^2*cosh(a + b*x)^2), x)
```

3.348 $\int x^m \mathbf{sech}^2(a+bx)\tanh(a+bx)\,dx$

3.348.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{sech}^2(a+bx)\tanh(a+bx),x\right)$$

output
```
CannotIntegrate(x^m*sech(b*x+a)^2*tanh(b*x+a),x)
```

3.348.2 Mathematica [N/A]

Not integrable

Time = 39.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx$$

input
```
Integrate[x^m*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
Integrate[x^m*Sech[a + b*x]^2*Tanh[a + b*x], x]
```

3.348.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \tanh(a+bx)\text{sech}^2(a+bx)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int x^m \tanh(a+bx)\text{sech}^2(a+bx)dx$$

input
```
Int[x^m*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
$Aborted
```

3.348.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.348.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \,\text{sech}\,(bx+a)^3 \sinh\,(bx+a)\,dx$$

input
```
int(x^m*sech(b*x+a)^3*sinh(b*x+a),x)
```

output
```
int(x^m*sech(b*x+a)^3*sinh(b*x+a),x)
```

3.348.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)\,dx$$

input
```
integrate(x^m*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*sech(b*x + a)^3*sinh(b*x + a), x)
```

3.348.6 Sympy [N/A]

Not integrable

Time = 110.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^m \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^m \sinh(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(x**m*sech(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Integral(x**m*sinh(a + b*x)*sech(a + b*x)**3, x)
```

3.348.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)\,dx$$

input
```
integrate(x^m*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^m*sech(b*x + a)^3*sinh(b*x + a), x)
```

3.348.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx=\int x^m \operatorname{sech}\left(bx+a\right)^3\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^m*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*sech(b*x + a)^3*sinh(b*x + a), x)
```

3.348.9 Mupad [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx=\int \frac{x^m\sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^m*sinh(a + b*x))/cosh(a + b*x)^3,x)
```

output
```
int((x^m*sinh(a + b*x))/cosh(a + b*x)^3, x)
```

3.349 $\int x^3\text{sech}^2(a+bx)\tanh(a+bx)\,dx$

3.349.1 Optimal result

Integrand size = 18, antiderivative size = 83

$$\int x^3\text{sech}^2(a+bx)\tanh(a+bx)\,dx=\frac{3x^2}{2b^2}-\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}-\frac{3\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+bx)}{2b^2}$$

output
```
3/2*x^2/b^2-3*x*ln(1+exp(2*b*x+2*a))/b^3-3/2*polylog(2,-exp(2*b*x+2*a))/b^
4-1/2*x^3*sech(b*x+a)^2/b+3/2*x^2*tanh(b*x+a)/b^2
```

3.349.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int x^3\text{sech}^2(a+bx)\tanh(a+bx)\,dx$$
$$=\frac{3\,\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)+bx\left(-\frac{6bx}{1+e^{2a}}-6\log\left(1+e^{-2(a+bx)}\right)-b^2x^2\text{sech}^2(a+bx)+3bx\text{sech}(a)\text{sech}(a+\right.}{2b^4}$$

input
```
Integrate[x^3*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
(3*PolyLog[2, -E^(-2*(a + b*x))] + b*x*((-6*b*x)/(1 + E^(2*a)) - 6*Log[1 +
 E^(-2*(a + b*x))] - b^2*x^2*Sech[a + b*x]^2 + 3*b*x*Sech[a]*Sech[a + b*x]
*Sinh[b*x]))/(2*b^4)
```

3.349.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5941, 3042, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \tanh(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{5941}$$

$$\frac{3\int x^2\text{sech}^2(a+bx)dx}{2b}-\frac{x^3\text{sech}^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{2b}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3\left(\frac{x^2\tanh(a+bx)}{b}-\frac{2i\int -ix\tanh(a+bx)dx}{b}\right)}{2b}$$

$$\Big\downarrow \text{26}$$

$$\frac{3\left(\frac{x^2\tanh(a+bx)}{b}-\frac{2\int x\tanh(a+bx)dx}{b}\right)}{2b}-\frac{x^3\text{sech}^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3\left(\frac{x^2\tanh(a+bx)}{b}-\frac{2\int -ix\tan(ia+ibx)dx}{b}\right)}{2b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3\left(\frac{x^2\tanh(a+bx)}{b}+\frac{2i\int x\tan(ia+ibx)dx}{b}\right)}{2b}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3\left(\frac{x^2\tanh(a+bx)}{b}+\frac{2i\left(2i\int\frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}dx-\frac{ix^2}{2}\right)}{b}\right)}{2b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3\left(\frac{x^2\tanh(a+bx)}{b}+\frac{2i\left(2i\left(\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int\log\left(1+e^{2(a+bx)}\right)dx}{2b}\right)-\frac{ix^2}{2}\right)}{b}\right)}{2b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3\left(\frac{x^2\tanh(a+bx)}{b}+\frac{2i\left(2i\left(\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int e^{-2(a+bx)}\log\left(1+e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}\right)-\frac{ix^2}{2}\right)}{b}\right)}{2b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3\left(\frac{x^2\tanh(a+bx)}{b}+\frac{2i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{4b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}\right)-\frac{ix^2}{2}\right)}{b}\right)}{2b}$$

input
```
Int[x^3*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-1/2*(x^3*Sech[a + b*x]^2)/b + (3*(((2*I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1
+ E^(2*(a + b*x))])/(2*b) + PolyLog[2, -E^(2*(a + b*x))]/(4*b^2))))/b + (x
^2*Tanh[a + b*x])/b))/(2*b)
```

3.349.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.349. $\int x^3\text{sech}^2(a+bx)\tanh(a+bx)\,dx$

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

3.349.4 Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.46

method	result	si
risch	$-\frac{x^{2}\left(2\,{\mathrm e}^{2 b x +2 a} b x +3\,{\mathrm e}^{2 b x +2 a}+3\right)}{b^{2} \left(1+{\mathrm e}^{2 b x +2 a}\right)^{2}}+\frac{3 x^{2}}{b^{2}}+\frac{6 a x}{b^{3}}+\frac{3 a^{2}}{b^{4}}-\frac{3 x \ln \left(1+{\mathrm e}^{2 b x +2 a}\right)}{b^{3}}-\frac{3\,\operatorname{polylog}\left(2, -{\mathrm e}^{2 b x +2 a}\right)}{2 b^{4}}-\frac{6 a \ln \left({\mathrm e}^{b x +a}\right)}{b^{4}}$	1

input
```
int(x^3*sech(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-x^2*(2*exp(2*b*x+2*a)*b*x+3*exp(2*b*x+2*a)+3)/b^2/(1+exp(2*b*x+2*a))^2+3/
b^2*x^2+6/b^3*a*x+3/b^4*a^2-3*x*ln(1+exp(2*b*x+2*a))/b^3-3/2*polylog(2,-ex
p(2*b*x+2*a))/b^4-6/b^4*a*ln(exp(b*x+a))
```

3.349.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1113, normalized size of antiderivative = 13.41

$$\int x^3 \text{sech}^2(a+bx)\tanh(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh
(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 - (2*b^3*x^3 - 3*b^2*x^2 +
 6*a^2)*cosh(b*x + a)^2 - (2*b^3*x^3 - 3*b^2*x^2 - 18*(b^2*x^2 - a^2)*cosh
(b*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 - 3*(cosh(b*x + a)^4 + 4*cosh
(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*si
nh(b*x + a) + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 3*(cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
 a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) +
3*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)
^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x + a)^2 + 4
*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a
) + sinh(b*x + a) + I) + 3*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x
 + a)^3 + a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2
 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x +
 a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 3*((b*x + a)*cosh(b*x +
a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)
^4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x +
a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cos...
```

3.349.6 Sympy [F]

$$\int x^3\text{sech}^2(a+bx)\tanh(a+bx)\,dx=\int x^3\sinh{(a+bx)}\,\text{sech}^3{(a+bx)}\,dx$$

input
```
integrate(x**3*sech(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Integral(x**3*sinh(a + b*x)*sech(a + b*x)**3, x)
```

3.349.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int x^3\text{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{3\,x^2+\left(2\,bx^3e^{(2\,a)}+3\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2}+\frac{3\,x^2}{b^2}-\frac{3\left(2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)\right)}{2\,b^4}$$

input
```
integrate(x^3*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-(3*x^2 + (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a
) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 3*x^2/b^2 - 3/2*(2*b*x*log(e^(2*b*x + 2
*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^4
```

3.349.8 Giac [F]

$$\int x^3\text{sech}^2(a+bx)\tanh(a+bx)\,dx=\int x^3\,\text{sech}\left(bx+a\right)^3\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^3*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*sech(b*x + a)^3*sinh(b*x + a), x)
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \int \frac{x^3\sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^3*sinh(a + b*x))/cosh(a + b*x)^3,x)
```

output
```
int((x^3*sinh(a + b*x))/cosh(a + b*x)^3, x)
```

3.350 $\int x^2\text{sech}^2(a+bx)\tanh(a+bx)\,dx$

3.350.1 Optimal result

Integrand size = 18, antiderivative size = 42

$$\int x^2\text{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\log(\cosh(a+bx))}{b^3}-\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{x\tanh(a+bx)}{b^2}$$

output
```
-ln(cosh(b*x+a))/b^3-1/2*x^2*sech(b*x+a)^2/b+x*tanh(b*x+a)/b^2
```

3.350.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int x^2\text{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\log(\cosh(a+bx))}{b^3}-\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{x\text{sech}(a)\text{sech}(a+bx)\sinh(bx)}{b^2}+\frac{x\tanh(a)}{b^2}$$

input
```
Integrate[x^2*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-(Log[Cosh[a + b*x]]/b^3) - (x^2*Sech[a + b*x]^2)/(2*b) + (x*Sech[a]*Sech[
a + b*x]*Sinh[b*x])/b^2 + (x*Tanh[a])/b^2
```

3.350.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5941, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \tanh(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 5941}$$

$$\frac{\int x\text{sech}^2(a+bx)dx}{b}-\frac{x^2\text{sech}^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{\int x\csc\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \text{ 4672}$$

$$-\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{\frac{x\tanh(a+bx)}{b}-\frac{i\int -i\tanh(a+bx)dx}{b}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{x\tanh(a+bx)}{b}-\frac{\int \tanh(a+bx)dx}{b}}{b}-\frac{x^2\text{sech}^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{\frac{x\tanh(a+bx)}{b}-\frac{\int -i\tan(ia+ibx)dx}{b}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{\frac{x\tanh(a+bx)}{b}+\frac{i\int \tan(ia+ibx)dx}{b}}{b}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\frac{x\tanh(a+bx)}{b}-\frac{\log(\cosh(a+bx))}{b^2}}{b}-\frac{x^2\text{sech}^2(a+bx)}{2b}$$

input
```
Int[x^2*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-1/2*(x^2*Sech[a + b*x]^2)/b + (-(Log[Cosh[a + b*x]]/b^2) + (x*Tanh[a + b*
x])/b)/b
```

3.350.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

3.350.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

method	result	size
risch	$\frac{2x}{b^2}+\frac{2a}{b^3}-\frac{2x\left(\mathrm{e}^{2bx+2a}bx+\mathrm{e}^{2bx+2a}+1\right)}{b^2\left(1+\mathrm{e}^{2bx+2a}\right)^2}-\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b^3}$	73

input
```
int(x^2*sech(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*x/b^2+2/b^3*a-2*x*(exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x
+2*a))^2-1/b^3*ln(1+exp(2*b*x+2*a))
```

3.350.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(40) = 80$.

Time = 0.26 (sec) , antiderivative size = 378, normalized size of antiderivative = 9.00

$$\int x^2\text{sech}^2(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{2\,bx\cosh\left(bx+a\right)^4+8\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+2\,bx\sinh\left(bx+a\right)^4-2\left(b^2x^2-bx\right)\cosh\left(bx+a\right)^2}{}$$

input
```
integrate(x^2*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
(2*b*x*cosh(b*x + a)^4 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 2*b*x*sinh(
b*x + a)^4 - 2*(b^2*x^2 - b*x)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 6*b*x*cosh(b
*x + a)^2 - b*x)*sinh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh
(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2
+ 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) +
1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(2*b*x*cosh(b*
x + a)^3 - (b^2*x^2 - b*x)*cosh(b*x + a))*sinh(b*x + a))/(b^3*cosh(b*x + a
)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 + 2*b^3*co
sh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a)^2 + 4*
(b^3*cosh(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a))
```

3.350.6 Sympy [F]

$$\int x^2\text{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^2\sinh\left(a+bx\right)\text{sech}^3\left(a+bx\right)dx$$

input
```
integrate(x**2*sech(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Integral(x**2*sinh(a + b*x)*sech(a + b*x)**3, x)
```

3.350.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(40) = 80.

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int x^2 \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2\left(\left(bx^2e^{(2a)}-xe^{(2a)}\right)e^{(2bx)}-xe^{(4bx+4a)}\right)}{b^2e^{(4bx+4a)}+2b^2e^{(2bx+2a)}+b^2} - \frac{\log\left(\left(e^{(2bx+2a)}+1\right)e^{(-2a)}\right)}{b^3}$$

input
```
integrate(x^2*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-2*((b*x^2*e^(2*a) - x*e^(2*a))*e^(2*b*x) - x*e^(4*b*x + 4*a))/(b^2*e^(4*b
*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) - log((e^(2*b*x + 2*a) + 1)*e^(-2
*a))/b^3
```

3.350.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(40) = 80.

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.38

$$\int x^2 \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2b^2x^2e^{(2bx+2a)}-2bxe^{(4bx+4a)}-2bxe^{(2bx+2a)}+e^{(4bx+4a)}\log\left(-e^{(2bx+2a)}-1\right)+2e^{(2bx+2a)}\log\left(-e^{(2bx \ldots\right.}{b^3e^{(4bx+4a)}+2b^3e^{(2bx+2a)}+b^3}$$

input
```
integrate(x^2*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
-(2*b^2*x^2*e^(2*b*x + 2*a) - 2*b*x*e^(4*b*x + 4*a) - 2*b*x*e^(2*b*x + 2*a
) + e^(4*b*x + 4*a)*log(-e^(2*b*x + 2*a) - 1) + 2*e^(2*b*x + 2*a)*log(-e^(
2*b*x + 2*a) - 1) + log(-e^(2*b*x + 2*a) - 1))/(b^3*e^(4*b*x + 4*a) + 2*b^
3*e^(2*b*x + 2*a) + b^3)
```

3.350.9 Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.43

$$\int x^2\mathrm{sech}^2(a+bx)\tanh(a+bx)\,dx = \frac{\frac{x^2}{b}-\frac{x^2\,e^{2\,a+2\,b\,x}}{b}}{2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1}-\frac{\ln\left(e^{2\,a}\,e^{2\,b\,x}+1\right)}{b^3}+\frac{2\,x}{b^2}-\frac{b\,x^2+2\,x}{b^2\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int((x^2*sinh(a + b*x))/cosh(a + b*x)^3,x)
```

output
```
(x^2/b - (x^2*exp(2*a + 2*b*x))/b)/(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x)
+ 1) - log(exp(2*a)*exp(2*b*x) + 1)/b^3 + (2*x)/b^2 - (2*x + b*x^2)/(b^2*(
exp(2*a + 2*b*x) + 1))
```

3.351 $\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx$

3.351.1 Optimal result

Integrand size = 16, antiderivative size = 30

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{x\text{sech}^2(a+bx)}{2b}+\frac{\tanh(a+bx)}{2b^2}$$

output
```
-1/2*x*sech(b*x+a)^2/b+1/2*tanh(b*x+a)/b^2
```

3.351.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{x\text{sech}^2(a+bx)}{2b}+\frac{\tanh(a+bx)}{2b^2}$$

input
```
Integrate[x*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-1/2*(x*Sech[a + b*x]^2)/b + Tanh[a + b*x]/(2*b^2)
```

3.351.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5941, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tanh(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 5941}$$

$$\frac{\int \text{sech}^2(a+bx)dx}{2b}-\frac{x\text{sech}^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{x\text{sech}^2(a+bx)}{2b}+\frac{\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{2b}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{x\text{sech}^2(a+bx)}{2b}+\frac{i\int 1d(-i\tanh(a+bx))}{2b^2}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\tanh(a+bx)}{2b^2}-\frac{x\text{sech}^2(a+bx)}{2b}$$

input
```
Int[x*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-1/2*(x*Sech[a + b*x]^2)/b + Tanh[a + b*x]/(2*b^2)
```

3.351.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

3.351.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{2\,{\mathrm e}^{2 b x +2 a} b x +{\mathrm e}^{2 b x +2 a}+1}{b^{2} \left(1+{\mathrm e}^{2 b x +2 a}\right)^{2}}$	43

input
```
int(x*sech(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(2*exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2
```

3.351.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.50

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{2\,\left(b x \sinh\left(b x + a\right) + \left(b x + 1\right) \cosh\left(b x + a\right)\right)}{b^{2} \cosh\left(b x + a\right)^{3} + 3\, b^{2} \cosh\left(b x + a\right) \sinh\left(b x + a\right)^{2} + b^{2} \sinh\left(b x + a\right)^{3} + 3\, b^{2} \cosh\left(b x + a\right) + \left(3\, b^{2} \cosh\right.}$$

input
```
integrate(x*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-2*(b*x*sinh(b*x + a) + (b*x + 1)*cosh(b*x + a))/(b^2*cosh(b*x + a)^3 + 3*
b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3 + 3*b^2*cosh(b*x +
 a) + (3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a))
```

3.351.6 Sympy [F]

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x\sinh(a+bx)\,\text{sech}^3(a+bx)\,dx$$

input
```
integrate(x*sech(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Integral(x*sinh(a + b*x)*sech(a + b*x)**3, x)
```

3.351.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(26) = 52$.

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.37

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2\,bxe^{(4\,bx+4\,a)} + \left(4\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)} + 1}{2\,\left(b^2e^{(4\,bx+4\,a)} + 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} + \frac{2\,bxe^{(4\,bx+4\,a)} - e^{(2\,bx+2\,a)} - 1}{2\,\left(b^2e^{(4\,bx+4\,a)} + 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)}$$

input
```
integrate(x*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*b*x*e^(4*b*x + 4*a) + (4*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) + 1)/(b^
2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 1/2*(2*b*x*e^(4*b*x + 4
*a) - e^(2*b*x + 2*a) - 1)/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) +
b^2)
```

3.351.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 6.13

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{4\,bxe^{(2\,bx+2\,a)} - e^{(4\,bx+4\,a)}\log\left(e^{(2\,bx+2\,a)}+1\right) - 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}+1\right) + e^{(4\,bx+4\,a)}\log\left(-e^{(2\,bx+2\,a)}\right.}{2\,\left(b^2e^{(4\,bx+4\,a)} + 2\right.}$$

input
```
integrate(x*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(4*b*x*e^(2*b*x + 2*a) - e^(4*b*x + 4*a)*log(e^(2*b*x + 2*a) + 1) - 2
*e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) + 1) + e^(4*b*x + 4*a)*log(-e^(2*b*x
+ 2*a) - 1) + 2*e^(2*b*x + 2*a)*log(-e^(2*b*x + 2*a) - 1) + 2*e^(2*b*x + 2
*a) - log(e^(2*b*x + 2*a) + 1) + log(-e^(2*b*x + 2*a) - 1) + 2)/(b^2*e^(4*
b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2)
```

3.351.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int x\operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{e^{2\,a+2\,b\,x}\,(2\,b\,x+1)+1}{b^2\,\left(e^{2\,a+2\,b\,x}+1\right)^2}$$

input
```
int((x*sinh(a + b*x))/cosh(a + b*x)^3,x)
```

output
```
-(exp(2*a + 2*b*x)*(2*b*x + 1) + 1)/(b^2*(exp(2*a + 2*b*x) + 1)^2)
```

3.352 $\int \text{sech}^2(a+bx)\tanh(a+bx)\,dx$

3.352.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \text{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\text{sech}^2(a+bx)}{2b}$$

output
```
-1/2*sech(b*x+a)^2/b
```

3.352.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \text{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\text{sech}^2(a+bx)}{2b}$$

input
```
Integrate[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-1/2*Sech[a + b*x]^2/b
```

3.352.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 26, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -i\tan(ia+ibx)\sec(ia+ibx)^2dx$$

$$\Big\downarrow \; 26$$

$$-i\int \sec(ia+ibx)^2\tan(ia+ibx)dx$$

$$\Big\downarrow \; 3086$$

$$-\frac{\int \text{sech}(a+bx)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \; 15$$

$$-\frac{\text{sech}^2(a+bx)}{2b}$$

input
```
Int[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-1/2*Sech[a + b*x]^2/b
```

3.352.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.352.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\operatorname{sech}(bx+a)^2}{2b}$	14
default	$-\frac{\operatorname{sech}(bx+a)^2}{2b}$	14
risch	$-\frac{2\,e^{2bx+2a}}{b\left(1+e^{2bx+2a}\right)^2}$	28

input
```
int(sech(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*sech(b*x+a)^2/b
```

3.352.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 5.60

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx =$$

$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+b\sinh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)+\left(3\,b\cosh\left(bx-\right.\right.}$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*
sinh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a))
```

3.352.6 Sympy [F]

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \int \sinh(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*sech(a + b*x)**3, x)
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2}{b\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2}$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-2/(b*(e^(b*x + a) + e^(-b*x - a))^2)
```

3.352.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2\,e^{(2\,bx+2\,a)}}{b\left(e^{(2\,bx+2\,a)}+1\right)^2}$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
-2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

3.352.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{1}{2\,b\cosh\left(a+b\,x\right)^2}$$

input
```
int(sinh(a + b*x)/cosh(a + b*x)^3,x)
```

output
```
-1/(2*b*cosh(a + b*x)^2)
```

3.353 $\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$

3.353.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x},x\right)$$

output
```
CannotIntegrate(sech(b*x+a)^2*tanh(b*x+a)/x,x)
```

3.353.2 Mathematica [N/A]

Not integrable

Time = 22.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$$

input
```
Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x,x]
```

output
```
Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x, x]
```

3.353.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(a+bx)\text{sech}^2(a+bx)}{x} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\tanh(a+bx)\text{sech}^2(a+bx)}{x}dx$$

input
```
Int[(Sech[a + b*x]^2*Tanh[a + b*x])/x,x]
```

output
```
$Aborted
```

3.353.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.353.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}\,(bx+a)^3 \sinh\,(bx+a)}{x}dx$$

input
```
int(sech(b*x+a)^3*sinh(b*x+a)/x,x)
```

output
```
int(sech(b*x+a)^3*sinh(b*x+a)/x,x)
```

3.353.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\mathrm{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\mathrm{sech}\,(bx+a)^3\sinh\,(bx+a)}{x}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^3*sinh(b*x + a)/x, x)
```

3.353.6 Sympy [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh\,(a+bx)\,\mathrm{sech}^3\,(a+bx)}{x}\,dx$$

input
```
integrate(sech(b*x+a)**3*sinh(b*x+a)/x,x)
```

output
```
Integral(sinh(a + b*x)*sech(a + b*x)**3/x, x)
```

3.353.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 5.61

$$\int \frac{\mathrm{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\mathrm{sech}\,(bx+a)^3\sinh\,(bx+a)}{x}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="maxima")
```

output
```
-((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^2*e^(4*b*x + 4*a) + 2*b^
2*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 4*integrate(1/2/(b^2*x^3*e^(2*b*x + 2*a
) + b^2*x^3), x)
```

3.353.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^3\sinh\left(bx+a\right)}{x}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^3*sinh(b*x + a)/x, x)
```

3.353.9 Mupad [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)}{x\cosh\left(a+b\,x\right)^3}\,dx$$

input
```
int(sinh(a + b*x)/(x*cosh(a + b*x)^3),x)
```

output
```
int(sinh(a + b*x)/(x*cosh(a + b*x)^3), x)
```

3.354 $\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$

3.354.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2},x\right)$$

output
```
CannotIntegrate(sech(b*x+a)^2*tanh(b*x+a)/x^2,x)
```

3.354.2 Mathematica [N/A]

Not integrable

Time = 18.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2,x]
```

output
```
Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2, x]
```

3.354.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(a+bx)\text{sech}^2(a+bx)}{x^2} \, dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\tanh(a+bx)\text{sech}^2(a+bx)}{x^2}dx$$

input
```
Int[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2,x]
```

output
```
$Aborted
```

3.354.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.354.4 Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}\left(bx+a\right)^{3}\sinh\left(bx+a\right)}{x^{2}}dx$$

input
```
int(sech(b*x+a)^3*sinh(b*x+a)/x^2,x)
```

output
```
int(sech(b*x+a)^3*sinh(b*x+a)/x^2,x)
```

3.354.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^3*sinh(b*x + a)/x^2, x)
```

3.354.6 Sympy [N/A]

Not integrable

Time = 4.62 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)**3*sinh(b*x+a)/x**2,x)
```

output
```
Integral(sinh(a + b*x)*sech(a + b*x)**3/x**2, x)
```

3.354.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.56

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
-2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^3*e^(4*b*x + 4*a) + 2*b^
2*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 12*integrate(1/2/(b^2*x^4*e^(2*b*x + 2*
a) + b^2*x^4), x)
```

3.354.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^3*sinh(b*x + a)/x^2, x)
```

3.354.9 Mupad [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)}{x^2\cosh(a+b\,x)^3}\,dx$$

input
```
int(sinh(a + b*x)/(x^2*cosh(a + b*x)^3),x)
```

output
```
int(sinh(a + b*x)/(x^2*cosh(a + b*x)^3), x)
```

3.355 $\int x^m \sinh(a+bx)\tanh(a+bx)\,dx$

3.355.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b} - \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b} - \mathrm{Int}(x^m \mathrm{sech}(a+bx),x)$$

output
```
1/2*exp(a)*x^m*GAMMA(1+m,-b*x)/b/((-b*x)^m)-1/2*x^m*GAMMA(1+m,b*x)/b/exp(a
)/((b*x)^m)-Unintegrable(x^m*sech(b*x+a),x)
```

3.355.2 Mathematica [N/A]

Not integrable

Time = 14.71 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \int x^m \sinh(a+bx)\tanh(a+bx)\,dx$$

input
```
Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x], x]
```

3.355.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5972, 3042, 3788, 26, 2612, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{5972}$$

$$\int x^m \cosh(a+bx)dx-\int x^m \text{sech}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^m \sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\int x^m \csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$-\frac{1}{2}i\int ie^{-a-bx}x^m dx+\frac{1}{2}i\int -ie^{a+bx}x^m dx-\int x^m \csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-a-bx}x^m dx+\frac{1}{2}\int e^{a+bx}x^m dx-\int x^m \csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{2612}$$

$$-\int x^m \csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}$$

$$\Big\downarrow \text{4680}$$

$$-\int x^m \text{sech}(a+bx)dx+\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}$$

input
```
Int[x^m*Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
$Aborted
```

3.355.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.355.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \operatorname{sech}(bx+a) \sinh(bx+a)^2 \, dx$$

input
```
int(x^m*sech(b*x+a)*sinh(b*x+a)^2,x)
```

output
```
int(x^m*sech(b*x+a)*sinh(b*x+a)^2,x)
```

3.355.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \sinh(a+bx)\tanh(a+bx)\, dx = \int x^m \operatorname{sech}(bx+a) \sinh(bx+a)^2 \, dx$$

input
```
integrate(x^m*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*sech(b*x + a)*sinh(b*x + a)^2, x)
```

3.355.6 Sympy [N/A]

Not integrable

Time = 38.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int x^m \sinh(a+bx)\tanh(a+bx)\, dx = \int x^m \sinh^2(a+bx) \operatorname{sech}(a+bx) \, dx$$

input
```
integrate(x**m*sech(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**m*sinh(a + b*x)**2*sech(a + b*x), x)
```

3.355.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

input
```
integrate(x^m*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*sech(b*x + a)*sinh(b*x + a)^2, x)
```

3.355.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

input
```
integrate(x^m*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*sech(b*x + a)*sinh(b*x + a)^2, x)
```

3.355.9 Mupad [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)^2}{\cosh(a+b\,x)}\,dx$$

input
```
int((x^m*sinh(a + b*x)^2)/cosh(a + b*x),x)
```

output
```
int((x^m*sinh(a + b*x)^2)/cosh(a + b*x), x)
```

3.356 $\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx$

3.356.1 Optimal result

Integrand size = 16, antiderivative size = 195

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} - \frac{6ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} + \frac{6ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} + \frac{6i\,\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4} - \frac{6i\,\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^4} + \frac{6x\sinh(a+bx)}{b^3} + \frac{x^3\sinh(a+bx)}{b}$$

output
```
-2*x^3*arctan(exp(b*x+a))/b-6*cosh(b*x+a)/b^4-3*x^2*cosh(b*x+a)/b^2+3*I*x^
2*polylog(2,-I*exp(b*x+a))/b^2-3*I*x^2*polylog(2,I*exp(b*x+a))/b^2-6*I*x*p
olylog(3,-I*exp(b*x+a))/b^3+6*I*x*polylog(3,I*exp(b*x+a))/b^3+6*I*polylog(
4,-I*exp(b*x+a))/b^4-6*I*polylog(4,I*exp(b*x+a))/b^4+6*x*sinh(b*x+a)/b^3+x
^3*sinh(b*x+a)/b
```

3.356.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.08

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{i\left(-6i\cosh(a+bx)-3ib^2x^2\cosh(a+bx)+b^3x^3\log\left(1-ie^{a+bx}\right)-b^3x^3\log\left(1+ie^{a+bx}\right)-3b^2x^2\operatorname{PolyL}\right.}{}$$

input
```
Integrate[x^3*Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
((-I)*((-6*I)*Cosh[a + b*x] - (3*I)*b^2*x^2*Cosh[a + b*x] + b^3*x^3*Log[1
- I*E^(a + b*x)] - b^3*x^3*Log[1 + I*E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, (
-I)*E^(a + b*x)] + 3*b^2*x^2*PolyLog[2, I*E^(a + b*x)] + 6*b*x*PolyLog[3,
(-I)*E^(a + b*x)] - 6*b*x*PolyLog[3, I*E^(a + b*x)] - 6*PolyLog[4, (-I)*E^
(a + b*x)] + 6*PolyLog[4, I*E^(a + b*x)] + (6*I)*b*x*Sinh[a + b*x] + I*b^3
*x^3*Sinh[a + b*x]))/b^4
```

3.356.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.16, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {5972, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{5972}$$

$$\int x^3\cosh(a+bx)dx-\int x^3\text{sech}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\int x^3\csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$-\int x^3\csc\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{3i\int -ix^2\sinh(a+bx)dx}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\int x^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{3\int x^2\sinh(a+bx)dx}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\int x^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{3\int -ix^2\sin(ia+ibx)dx}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\int x^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{3i\int x^2\sin(ia+ibx)dx}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\int x^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\int x\cosh(a+bx)dx}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\int x^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\int x^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{i\int -i\sinh(a+bx)dx}{b}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\int x^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\int\sinh(a+bx)dx}{b}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\int x^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\int -i\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

3.356. $\int x^3\sinh(a+bx)\tanh(a+bx)\,dx$

$$-\int x^3\csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}+\frac{i\int\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3118}$$

$$-\int x^3\csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{4668}$$

$$\frac{3i\int x^2\log\left(1-ie^{a+bx}\right)dx}{b}-\frac{3i\int x^2\log\left(1+ie^{a+bx}\right)dx}{b}-\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{3i\left(\frac{2\int x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}\left(2,ie^{a+bx}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}-\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{7163}$$

$$-\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{\int\text{PolyLog}\left(3,-ie^{a+bx}\right)dx}{b}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b}-\frac{\int\text{PolyLog}\left(3,ie^{a+bx}\right)dx}{b}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}-\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2720}$$

3.356. $\int x^3\sinh(a+bx)\tanh(a+bx)\,dx$

$$-\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{\int e^{-a-bx}\,\text{PolyLog}\left(3,-ie^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b}-\frac{\int e^{-a-bx}\,\text{PolyLog}\left(3,ie^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}-\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \; 7143$$

$$-\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}-\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b}-\frac{\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

input
```
Int[x^3*Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
(-2*x^3*ArcTan[E^(a + b*x)])/b - ((3*I)*(-((x^2*PolyLog[2, (-I)*E^(a + b*x
)])/b) + (2*((x*PolyLog[3, (-I)*E^(a + b*x)])/b - PolyLog[4, (-I)*E^(a + b
*x)]/b^2))/b))/b + ((3*I)*(-((x^2*PolyLog[2, I*E^(a + b*x)])/b) + (2*((x*P
olyLog[3, I*E^(a + b*x)])/b - PolyLog[4, I*E^(a + b*x)]/b^2))/b))/b + (x^3
*Sinh[a + b*x])/b + ((3*I)*((I*x^2*Cosh[a + b*x])/b - ((2*I)*(-(Cosh[a + b
*x]/b^2) + (x*Sinh[a + b*x])/b))/b))/b
```

3.356. $\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx$

3.356.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.356. $\int x^3 \sinh(a+bx) \tanh(a+bx)\,dx$

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.356.4 Maple [F]

$$\int x^3 \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

input
```
int(x^3*sech(b*x+a)*sinh(b*x+a)^2,x)
```

output
```
int(x^3*sech(b*x+a)*sinh(b*x+a)^2,x)
```

3.356.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 609 vs. 2(162) = 324.

Time = 0.27 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.12

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{b^3x^3 + 3\,b^2x^2 - (b^3x^3 - 3\,b^2x^2 + 6\,bx - 6)\cosh(bx+a)^2 - 2\,(b^3x^3 - 3\,b^2x^2 + 6\,bx - 6)\cosh(bx+a)\,\mathrm{si}}{}$$

input
```
integrate(x^3*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^3*x^3 + 3*b^2*x^2 - (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a
)^2 - 2*(b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a) - (b
^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*sinh(b*x + a)^2 + 6*b*x + 6*(I*b^2*x^2*cos
h(b*x + a) + I*b^2*x^2*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x +
 a)) + 6*(-I*b^2*x^2*cosh(b*x + a) - I*b^2*x^2*sinh(b*x + a))*dilog(-I*cos
h(b*x + a) - I*sinh(b*x + a)) + 2*(-I*a^3*cosh(b*x + a) - I*a^3*sinh(b*x +
 a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + 2*(I*a^3*cosh(b*x + a) + I*a
^3*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + 2*((-I*b^3*x^3
- I*a^3)*cosh(b*x + a) + (-I*b^3*x^3 - I*a^3)*sinh(b*x + a))*log(I*cosh(b*
x + a) + I*sinh(b*x + a) + 1) + 2*((I*b^3*x^3 + I*a^3)*cosh(b*x + a) + (I*
b^3*x^3 + I*a^3)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1
) + 12*(I*cosh(b*x + a) + I*sinh(b*x + a))*polylog(4, I*cosh(b*x + a) + I*
sinh(b*x + a)) + 12*(-I*cosh(b*x + a) - I*sinh(b*x + a))*polylog(4, -I*cos
h(b*x + a) - I*sinh(b*x + a)) + 12*(-I*b*x*cosh(b*x + a) - I*b*x*sinh(b*x
+ a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 12*(I*b*x*cosh(b*x +
 a) + I*b*x*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a))
+ 6)/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

3.356.6 Sympy [F]

$$\int x^3 \sinh(a + bx) \tanh(a + bx) \, dx = \int x^3 \sinh^2(a + bx) \operatorname{sech}(a + bx) \, dx$$

input
```
integrate(x**3*sech(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**3*sinh(a + b*x)**2*sech(a + b*x), x)
```

3.356.7 Maxima [F]

$$\int x^3 \sinh(a + bx) \tanh(a + bx) \, dx = \int x^3 \operatorname{sech}(bx + a) \sinh(bx + a)^2 \, dx$$

input
```
integrate(x^3*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b^3*x^3*e^(2*a) - 3*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a) - 6*e^(2*a))*e^(
b*x) - (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x))*e^(-a)/b^4 - 2*integrat
e(x^3*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

3.356.8 Giac [F]

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx = \int x^3 \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

input
```
integrate(x^3*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*sech(b*x + a)*sinh(b*x + a)^2, x)
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx = \int \frac{x^3 \sinh(a+b\,x)^2}{\cosh(a+b\,x)}\,dx$$

input
```
int((x^3*sinh(a + b*x)^2)/cosh(a + b*x),x)
```

output
```
int((x^3*sinh(a + b*x)^2)/cosh(a + b*x), x)
```

3.357 $\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx$

3.357.1 Optimal result

Integrand size = 16, antiderivative size = 135

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{2x\cosh(a+bx)}{b^2}+\frac{2ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{2ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{2i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{2i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}+\frac{2\sinh(a+bx)}{b^3}+\frac{x^2\sinh(a+bx)}{b}$$

output
```
-2*x^2*arctan(exp(b*x+a))/b-2*x*cosh(b*x+a)/b^2+2*I*x*polylog(2,-I*exp(b*x
+a))/b^2-2*I*x*polylog(2,I*exp(b*x+a))/b^2-2*I*polylog(3,-I*exp(b*x+a))/b^
3+2*I*polylog(3,I*exp(b*x+a))/b^3+2*sinh(b*x+a)/b^3+x^2*sinh(b*x+a)/b
```

3.357.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.13

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{i\left(-2ibx\cosh(a+bx)+b^2x^2\log\left(1-ie^{a+bx}\right)-b^2x^2\log\left(1+ie^{a+bx}\right)-2bx\,\text{PolyLog}\left(2,-ie^{a+bx}\right)+2bx\right.}{}$$

input
```
Integrate[x^2*Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
((-I)*((-2*I)*b*x*Cosh[a + b*x] + b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2
*Log[1 + I*E^(a + b*x)] - 2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyL
og[2, I*E^(a + b*x)] + 2*PolyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(
a + b*x)] + (2*I)*Sinh[a + b*x] + I*b^2*x^2*Sinh[a + b*x]))/b^3
```

3.357.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5972, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{5972}$$

$$\int x^2\cosh(a+bx)dx-\int x^2\text{sech}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$-\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{2i\int -ix\sinh(a+bx)dx}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{2\int x\sinh(a+bx)dx}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{2\int -ix\sin(ia+ibx)dx}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{2i\int x\sin(ia+ibx)dx}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\cosh(a+bx)dx}{b}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3117}$$

$$-\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{4668}$$

$$\frac{2i\int x\log\left(1-ie^{a+bx}\right)dx}{b}-\frac{2i\int x\log\left(1+ie^{a+bx}\right)dx}{b}-\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{2i\left(\frac{\int \text{PolyLog}\left(2,-ie^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{\int \text{PolyLog}\left(2,ie^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}-\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}\left(2,-ie^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}\left(2,ie^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}-\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{2i\left(\frac{\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}$$

3.357. $\int x^2\sinh(a+bx)\tanh(a+bx)\,dx$

input
```
Int[x^2*Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
(-2*x^2*ArcTan[E^(a + b*x)])/b - ((2*I)*(-((x*PolyLog[2, (-I)*E^(a + b*x)]
)/b) + PolyLog[3, (-I)*E^(a + b*x)]/b^2))/b + ((2*I)*(-((x*PolyLog[2, I*E^
(a + b*x)])/b) + PolyLog[3, I*E^(a + b*x)]/b^2))/b + (x^2*Sinh[a + b*x])/b
 + ((2*I)*((I*x*Cosh[a + b*x])/b - (I*Sinh[a + b*x])/b^2))/b
```

3.357.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.357.4 Maple [F]

$$\int x^2 \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

input
```
int(x^2*sech(b*x+a)*sinh(b*x+a)^2,x)
```

output
```
int(x^2*sech(b*x+a)*sinh(b*x+a)^2,x)
```

3.357.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(112) = 224.

Time = 0.27 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.53

$$\int x^2\sinh(a+bx)\tanh(a+bx)\,dx =$$

$$-\frac{b^2x^2-(b^2x^2-2\,bx+2)\cosh(bx+a)^2-2\,(b^2x^2-2\,bx+2)\cosh(bx+a)\sinh(bx+a)-(b^2x^2-2\,bx}{}$$

input
```
integrate(x^2*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^2 - (b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 2*b*x
 + 2)*cosh(b*x + a)*sinh(b*x + a) - (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^2
+ 2*b*x + 4*(I*b*x*cosh(b*x + a) + I*b*x*sinh(b*x + a))*dilog(I*cosh(b*x +
 a) + I*sinh(b*x + a)) + 4*(-I*b*x*cosh(b*x + a) - I*b*x*sinh(b*x + a))*di
log(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*(I*a^2*cosh(b*x + a) + I*a^2*s
inh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + 2*(-I*a^2*cosh(b*x
+ a) - I*a^2*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + 2*((-
I*b^2*x^2 + I*a^2)*cosh(b*x + a) + (-I*b^2*x^2 + I*a^2)*sinh(b*x + a))*log
(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 2*((I*b^2*x^2 - I*a^2)*cosh(b*x
+ a) + (I*b^2*x^2 - I*a^2)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*
x + a) + 1) + 4*(-I*cosh(b*x + a) - I*sinh(b*x + a))*polylog(3, I*cosh(b*x
 + a) + I*sinh(b*x + a)) + 4*(I*cosh(b*x + a) + I*sinh(b*x + a))*polylog(3
, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 2)/(b^3*cosh(b*x + a) + b^3*sinh(b
*x + a))
```

3.357.6 Sympy [F]

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx = \int x^2 \sinh^2{(a+bx)}\operatorname{sech}{(a+bx)}\,dx$$

input
```
integrate(x**2*sech(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**2*sinh(a + b*x)**2*sech(a + b*x), x)
```

3.357.7 Maxima [F]

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx = \int x^2 \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

input
```
integrate(x^2*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + 2*e^(2*a))*e^(b*x) - (b^2*x^2 + 2*
b*x + 2)*e^(-b*x))*e^(-a)/b^3 - 2*integrate(x^2*e^(b*x + a)/(e^(2*b*x + 2*
a) + 1), x)
```

3.357.8 Giac [F]

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx = \int x^2 \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

input
```
integrate(x^2*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*sech(b*x + a)*sinh(b*x + a)^2, x)
```

3.357.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx = \int \frac{x^2 \sinh(a+b\,x)^2}{\cosh(a+b\,x)}\,dx$$

input
```
int((x^2*sinh(a + b*x)^2)/cosh(a + b*x),x)
```

output
```
int((x^2*sinh(a + b*x)^2)/cosh(a + b*x), x)
```

3.358 $\int x \sinh(a+bx)\tanh(a+bx)\,dx$

3.358.1 Optimal result

Integrand size = 14, antiderivative size = 77

$$\int x \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{2x\arctan\left(e^{a+bx}\right)}{b} - \frac{\cosh(a+bx)}{b^2} + \frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{x\sinh(a+bx)}{b}$$

output
```
-2*x*arctan(exp(b*x+a))/b-cosh(b*x+a)/b^2+I*polylog(2,-I*exp(b*x+a))/b^2-I
*polylog(2,I*exp(b*x+a))/b^2+x*sinh(b*x+a)/b
```

3.358.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

$$\int x \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{i\left(-i\cosh(a+bx)+bx\log\left(1-ie^{a+bx}\right)-bx\log\left(1+ie^{a+bx}\right)-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a}\right.\right.}{b^2}$$

input
```
Integrate[x*Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
((-I)*((-I)*Cosh[a + b*x] + b*x*Log[1 - I*E^(a + b*x)] - b*x*Log[1 + I*E^(
a + b*x)] - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*x)] + I*b
*x*Sinh[a + b*x]))/b^2
```

3.358.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5972, 3042, 3777, 26, 3042, 26, 3118, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh(a+bx) \tanh(a+bx) \, dx$$

$$\Big\downarrow \text{5972}$$

$$\int x \cosh(a+bx)dx - \int x\text{sech}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x \sin\left(ia+ibx+\frac{\pi}{2}\right) dx - \int x \csc\left(ia+ibx+\frac{\pi}{2}\right) dx$$

$$\Big\downarrow \text{3777}$$

$$-\int x \csc\left(ia+ibx+\frac{\pi}{2}\right) dx - \frac{i\int -i\sinh(a+bx)dx}{b} + \frac{x\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\int x \csc\left(ia+ibx+\frac{\pi}{2}\right) dx - \frac{\int \sinh(a+bx)dx}{b} + \frac{x\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i\sin(ia+ibx)dx}{b} - \int x \csc\left(ia+ibx+\frac{\pi}{2}\right) dx + \frac{x\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \sin(ia+ibx)dx}{b} - \int x \csc\left(ia+ibx+\frac{\pi}{2}\right) dx + \frac{x\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3118}$$

$$-\int x \csc\left(ia+ibx+\frac{\pi}{2}\right) dx - \frac{\cosh(a+bx)}{b^2} + \frac{x\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{4668}$$

$$\frac{i\int \log\left(1-ie^{a+bx}\right)dx}{b}-\frac{i\int \log\left(1+ie^{a+bx}\right)dx}{b}-\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{\cosh(a+bx)}{b^2}+\frac{x\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{i\int e^{-a-bx}\log\left(1-ie^{a+bx}\right)de^{a+bx}}{b^2}-\frac{i\int e^{-a-bx}\log\left(1+ie^{a+bx}\right)de^{a+bx}}{b^2}-\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{\cosh(a+bx)}{b^2}+\frac{x\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2x\arctan\left(e^{a+bx}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\cosh(a+bx)}{b^2}+\frac{x\sinh(a+bx)}{b}$$

input
```
Int[x*Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
(-2*x*ArcTan[E^(a + b*x)])/b - Cosh[a + b*x]/b^2 + (I*PolyLog[2, (-I)*E^(a
 + b*x)])/b^2 - (I*PolyLog[2, I*E^(a + b*x)])/b^2 + (x*Sinh[a + b*x])/b
```

3.358.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.358.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(70) = 140.

Time = 0.66 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.10

method	result
risch	$\frac{(bx-1)\mathrm{e}^{bx+a}}{2b^2}-\frac{(bx+1)\mathrm{e}^{-bx-a}}{2b^2}+\frac{i\ln(1+i\mathrm{e}^{bx+a})x}{b}+\frac{i\ln(1+i\mathrm{e}^{bx+a})a}{b^2}-\frac{i\ln(1-i\mathrm{e}^{bx+a})x}{b}-\frac{i\ln(1-i\mathrm{e}^{bx+a})a}{b^2}+\frac{i\,\mathrm{dilog}(1+}{b^2}$

input
```
int(x*sech(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x-1)/b^2*exp(b*x+a)-1/2*(b*x+1)/b^2*exp(-b*x-a)+I/b*ln(1+I*exp(b*x+
a))*x+I/b^2*ln(1+I*exp(b*x+a))*a-I/b*ln(1-I*exp(b*x+a))*x-I/b^2*ln(1-I*exp
(b*x+a))*a+I/b^2*dilog(1+I*exp(b*x+a))-I/b^2*dilog(1-I*exp(b*x+a))+2/b^2*a
*arctan(exp(b*x+a))
```

3.358.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(64) = 128.

Time = 0.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 4.26

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{(bx-1)\cosh{(bx+a)}^2 + 2\,(bx-1)\cosh{(bx+a)}\sinh{(bx+a)} + (bx-1)\sinh{(bx+a)}^2 - bx - 2\,(i\cosh$$

input
```
integrate(x*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*((b*x - 1)*cosh(b*x + a)^2 + 2*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a) +
 (b*x - 1)*sinh(b*x + a)^2 - b*x - 2*(I*cosh(b*x + a) + I*sinh(b*x + a))*d
ilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*(-I*cosh(b*x + a) - I*sinh(b*x
 + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(-I*a*cosh(b*x + a) -
 I*a*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) - 2*(I*a*cosh(b
*x + a) + I*a*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) - 2*((
-I*b*x - I*a)*cosh(b*x + a) + (-I*b*x - I*a)*sinh(b*x + a))*log(I*cosh(b*x
 + a) + I*sinh(b*x + a) + 1) - 2*((I*b*x + I*a)*cosh(b*x + a) + (I*b*x + I
*a)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 1)/(b^2*c
osh(b*x + a) + b^2*sinh(b*x + a))
```

3.358.6 Sympy [F]

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx = \int x\sinh^2{(a+bx)}\,\mathrm{sech}\,{(a+bx)}\,dx$$

input
```
integrate(x*sech(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Integral(x*sinh(a + b*x)**2*sech(a + b*x), x)
```

3.358.7 Maxima [F]

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx = \int x\,\mathrm{sech}\,(bx+a)\sinh\,(bx+a)^2\,dx$$

input
```
integrate(x*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b*x*e^(2*a) - e^(2*a))*e^(b*x) - (b*x + 1)*e^(-b*x))*e^(-a)/b^2 - 2*
integrate(x*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

3.358.8 Giac [F]

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx = \int x\,\mathrm{sech}\,(bx+a)\sinh\,(bx+a)^2\,dx$$

input
```
integrate(x*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*sech(b*x + a)*sinh(b*x + a)^2, x)
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx = \int \frac{x\sinh(a+b\,x)^2}{\cosh\,(a+b\,x)}\,dx$$

input
```
int((x*sinh(a + b*x)^2)/cosh(a + b*x),x)
```

output
```
int((x*sinh(a + b*x)^2)/cosh(a + b*x), x)
```

3.359 $\int \sinh(a+bx)\tanh(a+bx)\,dx$

3.359.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \sinh(a+bx)\tanh(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}+\frac{\sinh(a+bx)}{b}$$

output
```
-arctan(sinh(b*x+a))/b+sinh(b*x+a)/b
```

3.359.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}+\frac{\sinh(a+bx)}{b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
-(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

3.359.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 25, 3072, 25, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\sin(ia+ibx)\tan(ia+ibx)dx$$

$$\Big\downarrow \; 25$$

$$-\int \sin(ia+ibx)\tan(ia+ibx)dx$$

$$\Big\downarrow \; 3072$$

$$-\frac{\int -\frac{\sinh^2(a+bx)}{\sinh^2(a+bx)+1}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sinh^2(a+bx)}{\sinh^2(a+bx)+1}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \; 262$$

$$-\frac{\int \frac{1}{\sinh^2(a+bx)+1}d\sinh(a+bx)-\sinh(a+bx)}{b}$$

$$\Big\downarrow \; 216$$

$$-\frac{\arctan(\sinh(a+bx))-\sinh(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
-((ArcTan[Sinh[a + b*x]] - Sinh[a + b*x])/b)
```

3.359.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.359.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\sinh(bx+a)-2\arctan\left(e^{bx+a}\right)}{b}$	21
default	$\frac{\sinh(bx+a)-2\arctan\left(e^{bx+a}\right)}{b}$	21
risch	$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{i\ln\left(e^{bx+a}-i\right)}{b}-\frac{i\ln\left(e^{bx+a}+i\right)}{b}$	59

input
```
int(sech(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x+a)-2*arctan(exp(b*x+a)))
```

3.359.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.74

$$\int \sinh(a+bx)\tanh(a+bx)\,dx =$$

$$-\frac{4\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-\cosh\left(bx+a\right)^{2}-2\cosh\left(bx+\right.}{2\left(b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)\right)}$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(4*(cosh(b*x + a) + sinh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x +
a)) - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2 +
1)/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

3.359.6 Sympy [F]

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \int \sinh^{2}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Integral(sinh(a + b*x)**2*sech(a + b*x), x)
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{2\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{e^{(bx+a)}}{2\,b}-\frac{e^{(-bx-a)}}{2\,b}$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
2*arctan(e^(-b*x - a))/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

3.359.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{4\arctan\left(e^{(bx+a)}\right) - e^{(bx+a)} + e^{(-bx-a)}}{2b}$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(4*arctan(e^(b*x + a)) - e^(b*x + a) + e^(-b*x - a))/b
```

3.359.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.13

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{e^{a+bx}}{2b} - \frac{2\,\mathrm{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{e^{-a-bx}}{2b}$$

input
```
int(sinh(a + b*x)^2/cosh(a + b*x),x)
```

output
```
exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
 - exp(- a - b*x)/(2*b)
```

3.360 $\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx$

3.360.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx = \cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)-\text{Int}\left(\frac{\text{sech}(a+bx)}{x},x\right)$$

output
```
Chi(b*x)*cosh(a)+Shi(b*x)*sinh(a)-Unintegrable(sech(b*x+a)/x,x)
```

3.360.2 Mathematica [N/A]

Not integrable

Time = 5.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx$$

input
```
Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x,x]
```

output
```
Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x, x]
```

3.360.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5972, 3042, 3784, 26, 3042, 26, 3779, 3782, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{5972}$$

$$\int \frac{\cosh(a+bx)}{x}dx-\int \frac{\text{sech}(a+bx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx-i\sinh(a)\int \frac{i\sinh(bx)}{x}dx+\cosh(a)\int \frac{\cosh(bx)}{x}dx$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx+\sinh(a)\int \frac{\sinh(bx)}{x}dx+\cosh(a)\int \frac{\cosh(bx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx+\sinh(a)\int -\frac{i\sin(ibx)}{x}dx+\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx-i\sinh(a)\int \frac{\sin(ibx)}{x}dx+\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3779}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx+\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx+\sinh(a)\text{Shi}(bx)$$

$$\Big\downarrow \text{3782}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx+\cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)$$

$$\Big\downarrow 4680$$

$$-\int \frac{\text{sech}(a+bx)}{x}dx + \cosh(a)\text{Chi}(bx) + \sinh(a)\text{Shi}(bx)$$

input
```
Int[(Sinh[a + b*x]*Tanh[a + b*x])/x,x]
```

output
```
$Aborted
```

3.360.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.360. $\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx$

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.360.4 Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x}dx$$

input
```
int(sech(b*x+a)*sinh(b*x+a)^2/x,x)
```

output
```
int(sech(b*x+a)*sinh(b*x+a)^2/x,x)
```

3.360.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)*sinh(b*x + a)^2/x, x)
```

3.360.6 Sympy [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh^2{(a+bx)}\operatorname{sech}{(a+bx)}}{x}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)**2/x,x)
```

output
```
Integral(sinh(a + b*x)**2*sech(a + b*x)/x, x)
```

3.360.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}{(bx+a)}\sinh{(bx+a)}^2}{x}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
integrate(sech(b*x + a)*sinh(b*x + a)^2/x, x)
```

3.360.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}{(bx+a)}\sinh{(bx+a)}^2}{x}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)*sinh(b*x + a)^2/x, x)
```

3.360.9 Mupad [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)^2}{x\cosh\left(a+b\,x\right)}\,dx$$

input
```
int(sinh(a + b*x)^2/(x*cosh(a + b*x)),x)
```

output
```
int(sinh(a + b*x)^2/(x*cosh(a + b*x)), x)
```

3.361 $\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$

3.361.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = -\frac{\cosh(a+bx)}{x} + b\text{Chi}(bx)\sinh(a) + b\cosh(a)\text{Shi}(bx) - \text{Int}\left(\frac{\text{sech}(a+bx)}{x^2},x\right)$$

output
```
-cosh(b*x+a)/x+b*cosh(a)*Shi(b*x)+b*Chi(b*x)*sinh(a)-Unintegrable(sech(b*x
+a)/x^2,x)
```

3.361.2 Mathematica [N/A]

Not integrable

Time = 4.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x^2,x]
```

output
```
Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x^2, x]
```

3.361.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5972, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{5972}$$

$$\int \frac{\cosh(a+bx)}{x^2}dx-\int \frac{\text{sech}(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx+ib\int -\frac{i\sinh(a+bx)}{x}dx-\frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx+b\int \frac{\sinh(a+bx)}{x}dx-\frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx+b\int -\frac{i\sin(ia+ibx)}{x}dx-\frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx-ib\int \frac{\sin(ia+ibx)}{x}dx-\frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{3784}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx-ib\left(i\sinh(a)\int \frac{\cosh(bx)}{x}dx+\cosh(a)\int \frac{i\sinh(bx)}{x}dx\right)-\frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx-ib\left(i\sinh(a)\int \frac{\cosh(bx)}{x}dx+i\cosh(a)\int \frac{\sinh(bx)}{x}dx\right)-\frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx - ib\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx + i\cosh(a)\int -\frac{i\sin(ibx)}{x}dx\right) - \frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx - ib\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx + \cosh(a)\int \frac{\sin(ibx)}{x}dx\right) - \frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{3779}$$

$$-ib\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx + i\cosh(a)\text{Shi}(bx)\right) - \int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx - \frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{3782}$$

$$-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx - ib(i\sinh(a)\text{Chi}(bx) + i\cosh(a)\text{Shi}(bx)) - \frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{4680}$$

$$-\int \frac{\text{sech}(a+bx)}{x^2}dx - ib(i\sinh(a)\text{Chi}(bx) + i\cosh(a)\text{Shi}(bx)) - \frac{\cosh(a+bx)}{x}$$

input
```
Int[(Sinh[a + b*x]*Tanh[a + b*x])/x^2,x]
```

output
```
$Aborted
```

3.361.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.361. $\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.361. $\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$

3.361.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x^2}dx$$

input
```
int(sech(b*x+a)*sinh(b*x+a)^2/x^2,x)
```

output
```
int(sech(b*x+a)*sinh(b*x+a)^2/x^2,x)
```

3.361.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)*sinh(b*x + a)^2/x^2, x)
```

3.361.6 Sympy [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)**2/x**2,x)
```

output
```
Integral(sinh(a + b*x)**2*sech(a + b*x)/x**2, x)
```

3.361.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(sech(b*x + a)*sinh(b*x + a)^2/x^2, x)
```

3.361.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)*sinh(b*x + a)^2/x^2, x)
```

3.361.9 Mupad [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)^2}{x^2\cosh(a+b\,x)}\,dx$$

input
```
int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)),x)
```

output
```
int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)), x)
```

3.362 $\int x^m \tanh^2(a+bx)\,dx$

3.362.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \tanh^2(a+bx)\,dx = \text{Int}\big(x^m \tanh^2(a+bx),x\big)$$

output
```
Unintegrable(x^m*tanh(b*x+a)^2,x)
```

3.362.2 Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh^2(a+bx)\,dx$$

input
```
Integrate[x^m*Tanh[a + b*x]^2,x]
```

output
```
Integrate[x^m*Tanh[a + b*x]^2, x]
```

3.362.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \tanh^2(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -x^m \tan(ia+ibx)^2dx$$
$$\Big\downarrow \text{25}$$
$$-\int x^m \tan(ia+ibx)^2dx$$
$$\Big\downarrow \text{4222}$$
$$\int x^m \tanh^2(a+bx)dx$$

input
```
Int[x^m*Tanh[a + b*x]^2,x]
```

output
```
$Aborted
```

3.362.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.362.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2 \, dx$$

input
```
int(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

output
```
int(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

3.362.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \tanh^2(a+bx) \, dx = \int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2 \, dx$$

input
```
integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

3.362.6 Sympy [N/A]

Not integrable

Time = 120.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \sinh^2{(a+bx)}\operatorname{sech}^2{(a+bx)}\,dx$$

input
```
integrate(x**m*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Integral(x**m*sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

3.362.7 Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 144, normalized size of antiderivative = 12.00

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2\,dx$$

input
```
integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
x*e^(4*b*x + m*log(x) + 4*a)/((m + 1)*e^(4*b*x + 4*a) + 2*(m + 1)*e^(2*b*x
 + 2*a) + m + 1) - integrate((2*(2*b*x*e^(4*a) + (m + 1)*e^(4*a))*e^(4*b*x
) + (m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m + 1)*e^(6*b*x + 6*a) + 3*(m +
 1)*e^(4*b*x + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

3.362.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2\,dx$$

input
```
integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

3.362.9 Mupad [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \tanh^2(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)^2}{\cosh\left(a+b\,x\right)^2}\,dx$$

input
```
int((x^m*sinh(a + b*x)^2)/cosh(a + b*x)^2,x)
```

output
```
int((x^m*sinh(a + b*x)^2)/cosh(a + b*x)^2, x)
```

3.363 $\int x^3 \tanh^2(a+bx)\,dx$

3.363.1 Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^3 \tanh^2(a+bx)\,dx = -\frac{x^3}{b}+\frac{x^4}{4}+\frac{3x^2\log\left(1+e^{2(a+bx)}\right)}{b^2}+\frac{3x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3} - \frac{3\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^4}-\frac{x^3\tanh(a+bx)}{b}$$

output
```
-x^3/b+1/4*x^4+3*x^2*ln(1+exp(2*b*x+2*a))/b^2+3*x*polylog(2,-exp(2*b*x+2*a
))/b^3-3/2*polylog(3,-exp(2*b*x+2*a))/b^4-x^3*tanh(b*x+a)/b
```

3.363.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

$$\int x^3 \tanh^2(a+bx)\,dx = \frac{1}{4}\left(-\frac{12x\,\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)}{b^3}-\frac{6\,\text{PolyLog}\left(3,-e^{-2(a+bx)}\right)}{b^4} + x^2\left(\frac{8x}{b+be^{2a}}+x^2+\frac{12\log\left(1+e^{-2(a+bx)}\right)}{b^2} - \frac{4x\text{sech}(a)\text{sech}(a+bx)\sinh(bx)}{b}\right)\right)$$

input
```
Integrate[x^3*Tanh[a + b*x]^2,x]
```

output
```
((-12*x*PolyLog[2, -E^(-2*(a + b*x))])/b^3 - (6*PolyLog[3, -E^(-2*(a + b*x
))])/b^4 + x^2*((8*x)/(b + b*E^(2*a)) + x^2 + (12*Log[1 + E^(-2*(a + b*x))
])/b^2 - (4*x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b))/4
```

3.363.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^3 \tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^3 \tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{3i\int ix^2\tanh(a+bx)dx}{b}+\int x^3dx-\frac{x^3\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3i\int ix^2\tanh(a+bx)dx}{b}-\frac{x^3\tanh(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{26}$$

$$\frac{3\int x^2\tanh(a+bx)dx}{b}-\frac{x^3\tanh(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int -ix^2\tan(ia+ibx)dx}{b}-\frac{x^3\tanh(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{26}$$

$$-\frac{3i\int x^2\tan(ia+ibx)dx}{b}-\frac{x^3\tanh(a+bx)}{b}+\frac{x^4}{4}$$

$\Big\downarrow$ 4201

$$-\frac{3i\left(2i\int\frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}dx-\frac{ix^3}{3}\right)}{b}-\frac{x^3\tanh(a+bx)}{b}+\frac{x^4}{4}$$

$\Big\downarrow$ 2620

$$-\frac{3i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int x\log\left(1+e^{2(a+bx)}\right)dx}{b}\right)-\frac{ix^3}{3}\right)}{b}-\frac{x^3\tanh(a+bx)}{b}+\frac{x^4}{4}$$

$\Big\downarrow$ 3011

$$-\frac{3i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int \text{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)}{b}-\frac{x^3\tanh(a+bx)}{b}+\frac{x^4}{4}$$

$\Big\downarrow$ 2720

$$-\frac{3i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\text{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)}{b}-\frac{x^3\tanh(a+bx)}{b}+\frac{x^4}{4}$$

$\Big\downarrow$ 7143

$$-\frac{3i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)}{b}-\frac{x^3\tanh(a+bx)}{b}+\frac{x^4}{4}$$

input
```
Int[x^3*Tanh[a + b*x]^2,x]
```

output
```
x^4/4 - ((3*I)*((-1/3*I)*x^3 + (2*I)*((x^2*Log[1 + E^(2*(a + b*x))])/(2*b)
 - (-1/2*(x*PolyLog[2, -E^(2*(a + b*x))])/b + PolyLog[3, -E^(2*(a + b*x))]
/(4*b^2))/b)))/b - (x^3*Tanh[a + b*x])/b
```

3.363. $\int x^3\tanh^2(a+bx)\,dx$

3.363.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.363. $\int x^3 \tanh^2(a + bx)\, dx$

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.363.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.40

method	result
risch	$\frac{x^4}{4}+\frac{2x^3}{b(1+e^{2bx+2a})}-\frac{6a^2\ln(e^{bx+a})}{b^4}-\frac{2x^3}{b}+\frac{6x\,a^2}{b^3}+\frac{4a^3}{b^4}+\frac{3x^2\ln(1+e^{2bx+2a})}{b^2}+\frac{3x\,\mathrm{polylog}(2,-e^{2bx+2a})}{b^3}-\frac{3\,\mathrm{polylo}}{}$

input
```
int(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+2*x^3/b/(1+exp(2*b*x+2*a))-6/b^4*a^2*ln(exp(b*x+a))-2*x^3/b+6*x/b^
3*a^2+4/b^4*a^3+3*x^2*ln(1+exp(2*b*x+2*a))/b^2+3*x*polylog(2,-exp(2*b*x+2*
a))/b^3-3/2*polylog(3,-exp(2*b*x+2*a))/b^4
```

3.363.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 721, normalized size of antiderivative = 8.10

$$\int x^3\tanh^2(a+bx)\,dx$$
$$=\frac{b^4x^4-8\,a^3+(b^4x^4-8\,b^3x^3-8\,a^3)\cosh(bx+a)^2+2\,(b^4x^4-8\,b^3x^3-8\,a^3)\cosh(bx+a)\sinh(bx+a)+}{}$$

input
```
integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(b^4*x^4 - 8*a^3 + (b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)^2 + 2*(
b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^4*x^4 - 8*b^
3*x^3 - 8*a^3)*sinh(b*x + a)^2 + 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x
+ a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*dilog(I*cosh(b*x + a) + I*
sinh(b*x + a)) + 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x +
a) + b*x*sinh(b*x + a)^2 + b*x)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a))
+ 12*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b
*x + a)^2 + a^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) + 12*(a^2*cosh(b*x
 + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2 + a^2)*l
og(cosh(b*x + a) + sinh(b*x + a) - I) + 12*(b^2*x^2 + (b^2*x^2 - a^2)*cosh
(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - a
^2)*sinh(b*x + a)^2 - a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 12
*(b^2*x^2 + (b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x +
 a)*sinh(b*x + a) + (b^2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-I*cosh(b*x
 + a) - I*sinh(b*x + a) + 1) - 24*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(
b*x + a) + sinh(b*x + a)^2 + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x +
a)) - 24*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^
2 + 1)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)))/(b^4*cosh(b*x + a)^
2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 + b^4)
```

3.363.6 Sympy [F]

$$\int x^3 \tanh^2(a+bx)\,dx = \int x^3 \sinh^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(x**3*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Integral(x**3*sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.21

$$\int x^3 \tanh^2(a+bx)\,dx = -\frac{2\,x^3}{b} + \frac{bx^4e^{(2\,bx+2\,a)} + bx^4 + 8\,x^3}{4\,(be^{(2\,bx+2\,a)} + b)} + \frac{3\,\left(2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right) + 2\,bx\mathrm{Li}_2(-e^{(2\,bx+2\,a)}) - \mathrm{Li}_3(-e^{(2\,bx+2\,a)})\right)}{2\,b^4}$$

input
```
integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*x^3/b + 1/4*(b*x^4*e^(2*b*x + 2*a) + b*x^4 + 8*x^3)/(b*e^(2*b*x + 2*a)
+ b) + 3/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2
*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^4
```

3.363.8 Giac [F]

$$\int x^3 \tanh^2(a+bx)\,dx = \int x^3 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2\,dx$$

input
```
integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \tanh^2(a+bx)\,dx = \int \frac{x^3 \sinh(a+b\,x)^2}{\cosh(a+b\,x)^2}\,dx$$

input
```
int((x^3*sinh(a + b*x)^2)/cosh(a + b*x)^2,x)
```

output
```
int((x^3*sinh(a + b*x)^2)/cosh(a + b*x)^2, x)
```

3.364 $\int x^2 \tanh^2(a+bx)\,dx$

3.364.1 Optimal result

Integrand size = 12, antiderivative size = 65

$$\int x^2 \tanh^2(a+bx)\,dx = -\frac{x^2}{b}+\frac{x^3}{3}+\frac{2x\log\left(1+e^{2(a+bx)}\right)}{b^2}+\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}-\frac{x^2\tanh(a+bx)}{b}$$

output
```
-x^2/b+1/3*x^3+2*x*ln(1+exp(2*b*x+2*a))/b^2+polylog(2,-exp(2*b*x+2*a))/b^3
-x^2*tanh(b*x+a)/b
```

3.364.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int x^2 \tanh^2(a+bx)\,dx = -\frac{\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)}{b^3}+\frac{1}{3}x\left(\frac{6x}{b+be^{2a}}+x^2+\frac{6\log\left(1+e^{-2(a+bx)}\right)}{b^2}-\frac{3x\text{sech}(a)\text{sech}(a+bx)\sinh(bx)}{b}\right)$$

input
```
Integrate[x^2*Tanh[a + b*x]^2,x]
```

output
```
-(PolyLog[2, -E^(-2*(a + b*x))]/b^3) + (x*((6*x)/(b + b*E^(2*a)) + x^2 + (
6*Log[1 + E^(-2*(a + b*x))])/b^2 - (3*x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b
))/3
```

3.364.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^2 \tan(ia+ibx)^2 dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^2 \tan(ia+ibx)^2 dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{2i\int ix\tanh(a+bx)dx}{b}+\int x^2dx-\frac{x^2\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2i\int ix\tanh(a+bx)dx}{b}-\frac{x^2\tanh(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{26}$$

$$\frac{2\int x\tanh(a+bx)dx}{b}-\frac{x^2\tanh(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int -ix\tan(ia+ibx)dx}{b}-\frac{x^2\tanh(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2i\int x\tan(ia+ibx)dx}{b}-\frac{x^2\tanh(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{2i\left(2i\int \frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}dx-\frac{ix^2}{2}\right)}{b}-\frac{x^2\tanh(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2i\left(2i\left(\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int\log\left(1+e^{2(a+bx)}\right)dx}{2b}\right)-\frac{ix^2}{2}\right)}{b}-\frac{x^2\tanh(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2i\left(2i\left(\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int e^{-2(a+bx)}\log\left(1+e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}\right)-\frac{ix^2}{2}\right)}{b}-\frac{x^2\tanh(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{4b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}\right)-\frac{ix^2}{2}\right)}{b}-\frac{x^2\tanh(a+bx)}{b}+\frac{x^3}{3}$$

input
```
Int[x^2*Tanh[a + b*x]^2,x]
```

output
```
x^3/3 - ((2*I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*(a + b*x))])/(2*b) +
 PolyLog[2, -E^(2*(a + b*x))]/(4*b^2))))/b - (x^2*Tanh[a + b*x])/b
```

3.364.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.364.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

method	result	size
risch	$\frac{x^3}{3}+\frac{2x^2}{b\left(1+e^{2bx+2a}\right)}-\frac{2x^2}{b}-\frac{4ax}{b^2}-\frac{2a^2}{b^3}+\frac{2x\ln\left(1+e^{2bx+2a}\right)}{b^2}+\frac{\operatorname{polylog}\left(2,-e^{2bx+2a}\right)}{b^3}+\frac{4a\ln\left(e^{bx+a}\right)}{b^3}$	99

input
```
int(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+2*x^2/b/(1+exp(2*b*x+2*a))-2*x^2/b-4*a*x/b^2-2/b^3*a^2+2*x*ln(1+ex
p(2*b*x+2*a))/b^2+polylog(2,-exp(2*b*x+2*a))/b^3+4/b^3*a*ln(exp(b*x+a))
```

3.364.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 515, normalized size of antiderivative = 7.92

$$\int x^2 \tanh^2(a+bx)\,dx$$
$$= \frac{b^3x^3 + (b^3x^3 - 6\,b^2x^2 + 6\,a^2)\cosh{(bx+a)}^2 + 2\,(b^3x^3 - 6\,b^2x^2 + 6\,a^2)\cosh{(bx+a)}\sinh{(bx+a)} + (b^3x^3}{}$$

input
```
integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(b^3*x^3 + (b^3*x^3 - 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)^2 + 2*(b^3*x^3
- 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 - 6*b^2*x^2 +
6*a^2)*sinh(b*x + a)^2 + 6*a^2 + 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh
(b*x + a) + sinh(b*x + a)^2 + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a))
+ 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1
)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 6*(a*cosh(b*x + a)^2 + 2*a*c
osh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2 + a)*log(cosh(b*x + a) + si
nh(b*x + a) + I) - 6*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a)
+ a*sinh(b*x + a)^2 + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) + 6*((b*x
+ a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)
*sinh(b*x + a)^2 + b*x + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 6
*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b
*x + a)*sinh(b*x + a)^2 + b*x + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a)
+ 1))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(
b*x + a)^2 + b^3)
```

3.364.6 Sympy [F]

$$\int x^2 \tanh^2(a+bx)\,dx = \int x^2 \sinh^2{(a+bx)}\operatorname{sech}^2{(a+bx)}\,dx$$

input
```
integrate(x**2*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Integral(x**2*sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int x^2 \tanh^2(a+bx)\,dx = -\frac{2\,x^2}{b} + \frac{bx^3e^{(2\,bx+2\,a)} + bx^3 + 6\,x^2}{3\,(be^{(2\,bx+2\,a)} + b)} + \frac{2\,bx\log\left(e^{(2\,bx+2\,a)} + 1\right) + \mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{b^3}$$

input
```
integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*x^2/b + 1/3*(b*x^3*e^(2*b*x + 2*a) + b*x^3 + 6*x^2)/(b*e^(2*b*x + 2*a)
+ b) + (2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^3
```

3.364.8 Giac [F]

$$\int x^2 \tanh^2(a+bx)\,dx = \int x^2\,\mathrm{sech}\,(bx+a)^2\sinh\,(bx+a)^2\,dx$$

input
```
integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \tanh^2(a+bx)\,dx = \int \frac{x^2\sinh(a+b\,x)^2}{\cosh\,(a+b\,x)^2}\,dx$$

input
```
int((x^2*sinh(a + b*x)^2)/cosh(a + b*x)^2,x)
```

output
```
int((x^2*sinh(a + b*x)^2)/cosh(a + b*x)^2, x)
```

3.365 $\int x \tanh^2(a+bx)\,dx$

3.365.1 Optimal result

Integrand size = 10, antiderivative size = 31

$$\int x \tanh^2(a+bx)\,dx = \frac{x^2}{2} + \frac{\log(\cosh(a+bx))}{b^2} - \frac{x\tanh(a+bx)}{b}$$

output
```
1/2*x^2+ln(cosh(b*x+a))/b^2-x*tanh(b*x+a)/b
```

3.365.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int x \tanh^2(a+bx)\,dx = \frac{b^2x^2 + 2\log(\cosh(a+bx)) - 2bx\text{sech}(a)\text{sech}(a+bx)\sinh(bx) - 2bx\tanh(a)}{2b^2}$$

input
```
Integrate[x*Tanh[a + b*x]^2,x]
```

output
```
(b^2*x^2 + 2*Log[Cosh[a + b*x]] - 2*b*x*Sech[a]*Sech[a + b*x]*Sinh[b*x] -
2*b*x*Tanh[a])/(2*b^2)
```

3.365.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{i\int i\tanh(a+bx)dx}{b}+\int xdx-\frac{x\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{i\int i\tanh(a+bx)dx}{b}-\frac{x\tanh(a+bx)}{b}+\frac{x^2}{2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \tanh(a+bx)dx}{b}-\frac{x\tanh(a+bx)}{b}+\frac{x^2}{2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i\tan(ia+ibx)dx}{b}-\frac{x\tanh(a+bx)}{b}+\frac{x^2}{2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \tan(ia+ibx)dx}{b}-\frac{x\tanh(a+bx)}{b}+\frac{x^2}{2}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\log(\cosh(a+bx))}{b^2}-\frac{x\tanh(a+bx)}{b}+\frac{x^2}{2}$$

input
```
Int[x*Tanh[a + b*x]^2,x]
```

output
```
x^2/2 + Log[Cosh[a + b*x]]/b^2 - (x*Tanh[a + b*x])/b
```

3.365.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.365.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

method	result	size
parallelrisch	$\frac{x^{2} b^{2}-2 \tanh \left(b x +a \right) x b -2 b x -2 \ln \left(1-\tanh \left(b x +a \right)\right)}{2 b^{2}}$	41
risch	$\frac{x^{2}}{2}-\frac{2 x}{b}-\frac{2 a}{b^{2}}+\frac{2 x}{b \left(1+{\mathrm e}^{2 b x +2 a}\right)}+\frac{\ln \left(1+{\mathrm e}^{2 b x +2 a}\right)}{b^{2}}$	54

input
```
int(x*sech(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2*b^2-2*tanh(b*x+a)*x*b-2*b*x-2*ln(1-tanh(b*x+a)))/b^2
```

3.365.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 5.97

$$\int x\tanh^2(a+bx)\,dx$$
$$= \frac{b^2x^2 + (b^2x^2 - 4\,bx)\cosh(bx+a)^2 + 2\,(b^2x^2 - 4\,bx)\cosh(bx+a)\sinh(bx+a) + (b^2x^2 - 4\,bx)\sinh(bx+}{2\left(b^2\cosh(bx+a)^2 + 2\,b^2\cosh(bx+a)\,s\right.}$$

input
```
integrate(x*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + (b^2*x^2 - 4*b*x)*cosh(b*x + a)^2 + 2*(b^2*x^2 - 4*b*x)*cos
h(b*x + a)*sinh(b*x + a) + (b^2*x^2 - 4*b*x)*sinh(b*x + a)^2 + 2*(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(2*cosh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*co
sh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 + b^2)
```

3.365.6 Sympy [F]

$$\int x\tanh^2(a+bx)\,dx = \int x\sinh^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(x*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Integral(x*sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

3.365.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(29) = 58.

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.06

$$\int x\tanh^2(a+bx)\,dx = -\frac{xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}+b} + \frac{bx^2+\left(bx^2e^{(2\,a)}-2\,xe^{(2\,a)}\right)e^{(2\,bx)}}{2\,(be^{(2\,bx+2\,a)}+b)} + \frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}$$

input
```
integrate(x*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) + 1/2*(b*x^2 + (b*x^2*e^(2*a) -
 2*x*e^(2*a))*e^(2*b*x))/(b*e^(2*b*x + 2*a) + b) + log((e^(2*b*x + 2*a) +
1)*e^(-2*a))/b^2
```

3.365.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(29) = 58.

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.06

$$\int x\tanh^2(a+bx)\,dx = \frac{b^2x^2e^{(2\,bx+2\,a)}+b^2x^2-4\,bxe^{(2\,bx+2\,a)}+2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{2\,(b^2e^{(2\,bx+2\,a)}+b^2)}$$

input
```
integrate(x*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2 - 4*b*x*e^(2*b*x + 2*a) + 2*e^(2*b*
x + 2*a)*log(e^(2*b*x + 2*a) + 1) + 2*log(e^(2*b*x + 2*a) + 1))/(b^2*e^(2*
b*x + 2*a) + b^2)
```

3.365.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int x\tanh^2(a+bx)\,dx = \frac{\frac{x^2\cosh(a+b\,x)}{2}-\frac{x\sinh(a+b\,x)}{b}}{\cosh\left(a+b\,x\right)}+\frac{\ln\left(\cosh(a+b\,x)\right)}{b^2}$$

input
```
int((x*sinh(a + b*x)^2)/cosh(a + b*x)^2,x)
```

output
```
((x^2*cosh(a + b*x))/2 - (x*sinh(a + b*x))/b)/cosh(a + b*x) + log(cosh(a +
 b*x))/b^2
```

3.366 $\int \tanh^2(a+bx)\,dx$

3.366.1 Optimal result

Integrand size = 8, antiderivative size = 13

$$\int \tanh^2(a+bx)\,dx = x - \frac{\tanh(a+bx)}{b}$$

output
```
x-tanh(b*x+a)/b
```

3.366.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \tanh^2(a+bx)\,dx = \frac{\operatorname{arctanh}(\tanh(a+bx))}{b} - \frac{\tanh(a+bx)}{b}$$

input
```
Integrate[Tanh[a + b*x]^2,x]
```

output
```
ArcTanh[Tanh[a + b*x]]/b - Tanh[a + b*x]/b
```

3.366.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx - \frac{\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$x - \frac{\tanh(a+bx)}{b}$$

input
```
Int[Tanh[a + b*x]^2,x]
```

output
```
x - Tanh[a + b*x]/b
```

3.366.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.366.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

method	result	size
parallelrisch	$\frac{bx-\tanh(bx+a)}{b}$	17
derivativedivides	$\frac{bx+a-\tanh(bx+a)}{b}$	18
default	$\frac{bx+a-\tanh(bx+a)}{b}$	18
risch	$x+\frac{2}{b\left(1+e^{2bx+2a}\right)}$	21

input
```
int(sech(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(b*x-tanh(b*x+a))/b
```

3.366.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.54

$$\int \tanh^2(a+bx)\,dx=\frac{(bx+1)\cosh(bx+a)-\sinh(bx+a)}{b\cosh(bx+a)}$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
((b*x + 1)*cosh(b*x + a) - sinh(b*x + a))/(b*cosh(b*x + a))
```

3.366.6 Sympy [F]

$$\int \tanh^2(a+bx)\,dx = \int \sinh^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Integral(sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

3.366.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \tanh^2(a+bx)\,dx = x + \frac{a}{b} - \frac{2}{b(e^{(-2\,bx-2\,a)}+1)}$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
x + a/b - 2/(b*(e^(-2*b*x - 2*a) + 1))
```

3.366.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \tanh^2(a+bx)\,dx = \frac{bx+a+\frac{2}{e^{(2\,bx+2\,a)}+1}}{b}$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
(b*x + a + 2/(e^(2*b*x + 2*a) + 1))/b
```

3.366.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \tanh^2(a+bx)\,dx = x + \frac{2}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(sinh(a + b*x)^2/cosh(a + b*x)^2,x)
```

output
```
x + 2/(b*(exp(2*a + 2*b*x) + 1))
```

3.367 $\int \frac{\tanh^2(a+bx)}{x} \, dx$

3.367.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\tanh^2(a+bx)}{x} \, dx = \text{Int}\left(\frac{\tanh^2(a+bx)}{x}, x\right)$$

output
```
Unintegrable(tanh(b*x+a)^2/x,x)
```

3.367.2 Mathematica [N/A]

Not integrable

Time = 17.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^2(a+bx)}{x} \, dx = \int \frac{\tanh^2(a+bx)}{x} \, dx$$

input
```
Integrate[Tanh[a + b*x]^2/x,x]
```

output
```
Integrate[Tanh[a + b*x]^2/x, x]
```

3.367.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(a+bx)}{x}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{\tan(ia+ibx)^2}{x}dx$$
$$\Big\downarrow \; 25$$
$$-\int \frac{\tan(ia+ibx)^2}{x}dx$$
$$\Big\downarrow \; 4222$$
$$\int \frac{\tanh^2(a+bx)}{x}dx$$

input
```
Int[Tanh[a + b*x]^2/x,x]
```

output
```
$Aborted
```

3.367.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.367.4 Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}\left(bx+a\right)^{2} \sinh\left(bx+a\right)^{2}}{x}dx$$

input
```
int(sech(b*x+a)^2*sinh(b*x+a)^2/x,x)
```

output
```
int(sech(b*x+a)^2*sinh(b*x+a)^2/x,x)
```

3.367.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^2(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^{2} \sinh\left(bx+a\right)^{2}}{x}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^2*sinh(b*x + a)^2/x, x)
```

3.367.6 Sympy [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{\tanh^2(a+bx)}{x}\,dx = \int \frac{\sinh^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

input
```
integrate(sech(b*x+a)**2*sinh(b*x+a)**2/x,x)
```

output
```
Integral(sinh(a + b*x)**2*sech(a + b*x)**2/x, x)
```

3.367.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.08

$$\int \frac{\tanh^2(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^2}{x}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
2/(b*x*e^(2*b*x + 2*a) + b*x) + 2*integrate(1/(b*x^2*e^(2*b*x + 2*a) + b*x
^2), x) + log(x)
```

3.367.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^2(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^2}{x}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^2*sinh(b*x + a)^2/x, x)
```

3.367.9 Mupad [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^2(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)^2}{x\cosh{(a+b\,x)}^2}\,dx$$

input
```
int(sinh(a + b*x)^2/(x*cosh(a + b*x)^2),x)
```

output
```
int(sinh(a + b*x)^2/(x*cosh(a + b*x)^2), x)
```

3.368 $\int \frac{\tanh^2(a+bx)}{x^2} dx$

3.368.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\tanh^2(a+bx)}{x^2} dx = \mathrm{Int}\left(\frac{\tanh^2(a+bx)}{x^2}, x\right)$$

output
```
Unintegrable(tanh(b*x+a)^2/x^2,x)
```

3.368.2 Mathematica [N/A]

Not integrable

Time = 10.98 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^2(a+bx)}{x^2} dx = \int \frac{\tanh^2(a+bx)}{x^2} dx$$

input
```
Integrate[Tanh[a + b*x]^2/x^2,x]
```

output
```
Integrate[Tanh[a + b*x]^2/x^2, x]
```

3.368.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(a+bx)}{x^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\tan(ia+ibx)^2}{x^2}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\tan(ia+ibx)^2}{x^2}dx$$
$$\Big\downarrow \text{4222}$$
$$\int \frac{\tanh^2(a+bx)}{x^2}dx$$

input
```
Int[Tanh[a + b*x]^2/x^2,x]
```

output
```
$Aborted
```

3.368.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.368.4 Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)^2}{x^2} dx$$

input
```
int(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x)
```

output
```
int(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x)
```

3.368.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^2(a+bx)}{x^2} dx = \int \frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)^2}{x^2} dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^2*sinh(b*x + a)^2/x^2, x)
```

3.368.6 Sympy [N/A]

Not integrable

Time = 3.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\tanh^2(a+bx)}{x^2} \, dx = \int \frac{\sinh^2(a+bx)\,\mathrm{sech}^2(a+bx)}{x^2} \, dx$$

input
```
integrate(sech(b*x+a)**2*sinh(b*x+a)**2/x**2,x)
```

output
```
Integral(sinh(a + b*x)**2*sech(a + b*x)**2/x**2, x)
```

3.368.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 5.67

$$\int \frac{\tanh^2(a+bx)}{x^2} \, dx = \int \frac{\mathrm{sech}\,(bx+a)^2 \sinh\,(bx+a)^2}{x^2} \, dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-(b*x*e^(2*b*x + 2*a) + b*x - 2)/(b*x^2*e^(2*b*x + 2*a) + b*x^2) + 4*integ
rate(1/(b*x^3*e^(2*b*x + 2*a) + b*x^3), x)
```

3.368.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^2(a+bx)}{x^2} \, dx = \int \frac{\mathrm{sech}\,(bx+a)^2 \sinh\,(bx+a)^2}{x^2} \, dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^2*sinh(b*x + a)^2/x^2, x)
```

3.368.9 Mupad [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)^2}{x^2\cosh\left(a+b\,x\right)^2}\,dx$$

input
```
int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)^2),x)
```

output
```
int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)^2), x)
```

3.369 $\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

3.369.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \operatorname{Int}(x^m \operatorname{sech}(a+bx),x) - \operatorname{Int}\left(x^m \operatorname{sech}^3(a+bx),x\right)$$

output
```
Unintegrable(x^m*sech(b*x+a),x)-Unintegrable(x^m*sech(b*x+a)^3,x)
```

3.369.2 Mathematica [N/A]

Not integrable

Time = 52.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$$

input
```
Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x]^2, x]
```

3.369.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5978, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \tanh^2(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{5978}$$

$$\int x^m\text{sech}(a+bx)dx-\int x^m\text{sech}^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^m \csc\left(ia+ibx+\frac{\pi}{2}\right)dx-\int x^m \csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4680}$$

$$\int x^m\text{sech}(a+bx)dx-\int x^m\text{sech}^3(a+bx)dx$$

input
```
Int[x^m*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
$Aborted
```

3.369.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5978
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

3.369.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2 \, dx$$

input
```
int(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

output
```
int(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

3.369.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \operatorname{sech}(a+bx) \tanh^2(a+bx) \, dx = \int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2 \, dx$$

input
```
integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

3.369.6 Sympy [F(-1)]

Timed out.

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**m*sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Timed out
```

3.369.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2\,dx$$

input
```
integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

3.369.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2\,dx$$

input
```
integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

3.369.9 Mupad [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)^2}{\cosh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^m*sinh(a + b*x)^2)/cosh(a + b*x)^3,x)
```

output
```
int((x^m*sinh(a + b*x)^2)/cosh(a + b*x)^3, x)
```

3.370 $\int x^3\textbf{sech}(a+bx)\tanh^2(a+bx)\,dx$

3.370.1 Optimal result

Integrand size = 18, antiderivative size = 240

$$\begin{aligned}\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx =& \frac{6x\arctan\left(e^{a+bx}\right)}{b^3}+\frac{x^3\arctan\left(e^{a+bx}\right)}{b} \\ &-\frac{3i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^4}-\frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2} \\ &+\frac{3i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^4}+\frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2} \\ &+\frac{3ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{3ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} \\ &-\frac{3i\,\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4}+\frac{3i\,\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^4} \\ &-\frac{3x^2\text{sech}(a+bx)}{2b^2}-\frac{x^3\text{sech}(a+bx)\tanh(a+bx)}{2b}\end{aligned}$$

output
```
6*x*arctan(exp(b*x+a))/b^3+x^3*arctan(exp(b*x+a))/b-3*I*polylog(2,-I*exp(b
*x+a))/b^4-3/2*I*x^2*polylog(2,-I*exp(b*x+a))/b^2+3*I*polylog(2,I*exp(b*x+
a))/b^4+3/2*I*x^2*polylog(2,I*exp(b*x+a))/b^2+3*I*x*polylog(3,-I*exp(b*x+a
))/b^3-3*I*x*polylog(3,I*exp(b*x+a))/b^3-3*I*polylog(4,-I*exp(b*x+a))/b^4+
3*I*polylog(4,I*exp(b*x+a))/b^4-3/2*x^2*sech(b*x+a)/b^2-1/2*x^3*sech(b*x+a
)*tanh(b*x+a)/b
```

3.370.2 Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.02

$$\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx =$$
$$-\frac{-i\left(6bx\log\left(1-ie^{a+bx}\right)+b^3x^3\log\left(1-ie^{a+bx}\right)-6bx\log\left(1+ie^{a+bx}\right)-b^3x^3\log\left(1+ie^{a+bx}\right)-3(2+b\right.}{}$$

input
```
Integrate[x^3*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
-1/2*((-I)*(6*b*x*Log[1 - I*E^(a + b*x)] + b^3*x^3*Log[1 - I*E^(a + b*x)]
- 6*b*x*Log[1 + I*E^(a + b*x)] - b^3*x^3*Log[1 + I*E^(a + b*x)] - 3*(2 + b
^2*x^2)*PolyLog[2, (-I)*E^(a + b*x)] + 3*(2 + b^2*x^2)*PolyLog[2, I*E^(a +
 b*x)] + 6*b*x*PolyLog[3, (-I)*E^(a + b*x)] - 6*b*x*PolyLog[3, I*E^(a + b*
x)] - 6*PolyLog[4, (-I)*E^(a + b*x)] + 6*PolyLog[4, I*E^(a + b*x)]) + b^3*
x^3*Sech[a]*Sech[a + b*x]^2*Sinh[b*x] + b^2*x^2*Sech[a + b*x]*(3 + b*x*Tan
h[a]))/b^4
```

3.370.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.75, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5978, 3042, 4668, 3011, 4674, 3042, 4668, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\tanh^2(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{5978}$$

$$\int x^3\text{sech}(a+bx)dx-\int x^3\text{sech}^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3\csc\left(ia+ibx+\frac{\pi}{2}\right)dx-\int x^3\csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4668}$$

$$-\int x^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)^3 dx - \frac{3i\int x^2 \log\left(1-ie^{a+bx}\right)dx}{b} + \frac{3i\int x^2 \log\left(1+ie^{a+bx}\right)dx}{b} + \frac{2x^3 \arctan\left(e^{a+bx}\right)}{b}$$

$\downarrow$ 3011

$$\frac{3i\left(\frac{2\int x \operatorname{PolyLog}(2,-ie^{a+bx})dx}{b} - \frac{x^2 \operatorname{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b} - \frac{3i\left(\frac{2\int x \operatorname{PolyLog}(2,ie^{a+bx})dx}{b} - \frac{x^2 \operatorname{PolyLog}(2,ie^{a+bx})}{b}\right)}{b} - \int x^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)^3 dx + \frac{2x^3 \arctan\left(e^{a+bx}\right)}{b}$$

$\downarrow$ 4674

$$\frac{3\int x\operatorname{sech}(a+bx)dx}{b^2} + \frac{3i\left(\frac{2\int x \operatorname{PolyLog}(2,-ie^{a+bx})dx}{b} - \frac{x^2 \operatorname{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b} - \frac{3i\left(\frac{2\int x \operatorname{PolyLog}(2,ie^{a+bx})dx}{b} - \frac{x^2 \operatorname{PolyLog}(2,ie^{a+bx})}{b}\right)}{b} - \frac{1}{2}\int x^3\operatorname{sech}(a+bx)dx + \frac{2x^3 \arctan\left(e^{a+bx}\right)}{b} - \frac{3x^2\operatorname{sech}(a+bx)}{2b^2} - \frac{x^3 \tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

$\downarrow$ 3042

$$\frac{3\int x \csc\left(ia+ibx+\frac{\pi}{2}\right)dx}{b^2} + \frac{3i\left(\frac{2\int x \operatorname{PolyLog}(2,-ie^{a+bx})dx}{b} - \frac{x^2 \operatorname{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b} - \frac{3i\left(\frac{2\int x \operatorname{PolyLog}(2,ie^{a+bx})dx}{b} - \frac{x^2 \operatorname{PolyLog}(2,ie^{a+bx})}{b}\right)}{b} - \frac{1}{2}\int x^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)dx + \frac{2x^3 \arctan\left(e^{a+bx}\right)}{b} - \frac{3x^2\operatorname{sech}(a+bx)}{2b^2} - \frac{x^3 \tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

$\downarrow$ 4668

$$\frac{3\left(-\frac{i\int \log(1-ie^{a+bx})dx}{b} + \frac{i\int \log(1+ie^{a+bx})dx}{b} + \frac{2x\arctan(e^{a+bx})}{b}\right)}{b^2} + \frac{1}{2}\left(\frac{3i\int x^2 \log\left(1-ie^{a+bx}\right)dx}{b} - \frac{3i\int x^2 \log\left(1+ie^{a+bx}\right)dx}{b} - \frac{2x^3 \arctan\left(e^{a+bx}\right)}{b}\right) + \frac{3i\left(\frac{2\int x \operatorname{PolyLog}(2,-ie^{a+bx})dx}{b} - \frac{x^2 \operatorname{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b} - \frac{3i\left(\frac{2\int x \operatorname{PolyLog}(2,ie^{a+bx})dx}{b} - \frac{x^2 \operatorname{PolyLog}(2,ie^{a+bx})}{b}\right)}{b} + \frac{2x^3 \arctan\left(e^{a+bx}\right)}{b} - \frac{3x^2\operatorname{sech}(a+bx)}{2b^2} - \frac{x^3 \tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

$\downarrow$ 2715

3.370. $\int x^3\operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

$$\frac{3\left(-\frac{i\int e^{-a-bx}\log(1-ie^{a+bx})de^{a+bx}}{b^2}+\frac{i\int e^{-a-bx}\log(1+ie^{a+bx})de^{a+bx}}{b^2}+\frac{2x\arctan(e^{a+bx})}{b}\right)}{b^2}+\frac{1}{2}\left(\frac{3i\int x^2\log\left(1-ie^{a+bx}\right)dx}{b}-\frac{3i\int x^2\log\left(1+ie^{a+bx}\right)dx}{b}-\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}\right)+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-ie^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,ie^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}+\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}-\frac{3x^2\text{sech}(a+bx)}{2b^2}-\frac{x^3\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$\Big\downarrow$ 2838

$$\frac{1}{2}\left(\frac{3i\int x^2\log\left(1-ie^{a+bx}\right)dx}{b}-\frac{3i\int x^2\log\left(1+ie^{a+bx}\right)dx}{b}-\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}\right)+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-ie^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,ie^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}+\frac{3\left(\frac{2x\arctan(e^{a+bx})}{b}-\frac{i\,\text{PolyLog}(2,-ie^{a+bx})}{b^2}+\frac{i\,\text{PolyLog}(2,ie^{a+bx})}{b^2}\right)}{b^2}+\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}-\frac{3x^2\text{sech}(a+bx)}{2b^2}-\frac{x^3\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$\Big\downarrow$ 3011

$$\frac{1}{2}\left(-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-ie^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,ie^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}-\frac{2x^3 a}{}\right.$$
$$\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-ie^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,ie^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}+\frac{3\left(\frac{2x\arctan(e^{a+bx})}{b}-\frac{i\,\text{PolyLog}(2,-ie^{a+bx})}{b^2}+\frac{i\,\text{PolyLog}(2,ie^{a+bx})}{b^2}\right)}{b^2}+\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}-\frac{3x^2\text{sech}(a+bx)}{2b^2}-\frac{x^3\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$\Big\downarrow$ 7163

3.370. $\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx$

$$\frac{1}{2}\left(-\frac{3i\left(\frac{2\left(\frac{x\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{\int\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)dx}{b}\right)}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b}-\frac{\int\operatorname{PolyLog}\left(3,ie^{a\cdots}\right.}{b}\right.}{b}\right.}{b}\right.$$

$$\frac{3i\left(\frac{2\left(\frac{x\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{\int\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)dx}{b}\right)}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{3i\left(\frac{2\left(\frac{x\operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b}-\frac{\int\operatorname{PolyLog}\left(3,ie^{a+bx}\right)dx}{b}\right)}{b}-\frac{x^2\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}+\frac{3\left(\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{i\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}\right)}{b^2}+\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}-\frac{3x^2\operatorname{sech}(a+bx)}{2b^2}-\frac{x^3\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{2}\left(-\frac{3i\left(\frac{2\left(\frac{x\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{\int e^{-a-bx}\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b}-\frac{\int e\cdots}{}\right.}{}\right.}{}\right.$$

$$\frac{3i\left(\frac{2\left(\frac{x\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{\int e^{-a-bx}\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{3i\left(\frac{2\left(\frac{x\operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b}-\frac{\int e^{-a-bx}\operatorname{PolyLog}\left(3,ie^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{x^2\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}+\frac{3\left(\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{i\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}\right)}{b^2}+\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}-\frac{3x^2\operatorname{sech}(a+bx)}{2b^2}-\frac{x^3\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

$$\Big\downarrow \; 7143$$

3.370. $\int x^3\operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

$$\frac{1}{2}\left(-\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}-\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,i\right.}{b}\right.}{}\right.}{}\right.$$

$$\frac{3\left(\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}\right)}{b^2}+\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b}-\frac{\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}-\frac{3x^2\text{sech}(a+bx)}{2b^2}-\frac{x^3\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

input
```
Int[x^3*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
(2*x^3*ArcTan[E^(a + b*x)])/b + (3*((2*x*ArcTan[E^(a + b*x)])/b - (I*PolyL
og[2, (-I)*E^(a + b*x)])/b^2 + (I*PolyLog[2, I*E^(a + b*x)])/b^2))/b^2 + (
(3*I)*(-((x^2*PolyLog[2, (-I)*E^(a + b*x)])/b) + (2*((x*PolyLog[3, (-I)*E^
(a + b*x)])/b - PolyLog[4, (-I)*E^(a + b*x)]/b^2))/b))/b - ((3*I)*(-((x^2*
PolyLog[2, I*E^(a + b*x)])/b) + (2*((x*PolyLog[3, I*E^(a + b*x)])/b - Poly
Log[4, I*E^(a + b*x)]/b^2))/b))/b + ((-2*x^3*ArcTan[E^(a + b*x)])/b - ((3*
I)*(-((x^2*PolyLog[2, (-I)*E^(a + b*x)])/b) + (2*((x*PolyLog[3, (-I)*E^(a
+ b*x)])/b - PolyLog[4, (-I)*E^(a + b*x)]/b^2))/b))/b + ((3*I)*(-((x^2*Pol
yLog[2, I*E^(a + b*x)])/b) + (2*((x*PolyLog[3, I*E^(a + b*x)])/b - PolyLog
[4, I*E^(a + b*x)]/b^2))/b))/b)/2 - (3*x^2*Sech[a + b*x])/(2*b^2) - (x^3*S
ech[a + b*x]*Tanh[a + b*x])/(2*b)
```

3.370. $\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx$

3.370.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.370. $\int x^3 \operatorname{sech}(a+bx) \tanh^2(a+bx) \, dx$

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5978
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.370.4 Maple [F]

$$\int x^3 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2 dx$$

input
```
int(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

output
```
int(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

3.370.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2163 vs. $2(186) = 372$.

Time = 0.29 (sec) , antiderivative size = 2163, normalized size of antiderivative = 9.01

$$\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^3 + 6*(b^3*x^3 + 3*b^2*x^2)*co
sh(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*x^3 + 3*b^2*x^2)*sinh(b*x + a)^3 - 2*
(b^3*x^3 - 3*b^2*x^2)*cosh(b*x + a) + 3*((-I*b^2*x^2 - 2*I)*cosh(b*x + a)^
4 + 4*(-I*b^2*x^2 - 2*I)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^2*x^2 - 2*I
)*sinh(b*x + a)^4 - I*b^2*x^2 + 2*(-I*b^2*x^2 - 2*I)*cosh(b*x + a)^2 + 2*(
-I*b^2*x^2 + 3*(-I*b^2*x^2 - 2*I)*cosh(b*x + a)^2 - 2*I)*sinh(b*x + a)^2 +
 4*((-I*b^2*x^2 - 2*I)*cosh(b*x + a)^3 + (-I*b^2*x^2 - 2*I)*cosh(b*x + a))
*sinh(b*x + a) - 2*I)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + 3*((I*b^2
*x^2 + 2*I)*cosh(b*x + a)^4 + 4*(I*b^2*x^2 + 2*I)*cosh(b*x + a)*sinh(b*x +
 a)^3 + (I*b^2*x^2 + 2*I)*sinh(b*x + a)^4 + I*b^2*x^2 + 2*(I*b^2*x^2 + 2*I
)*cosh(b*x + a)^2 + 2*(I*b^2*x^2 + 3*(I*b^2*x^2 + 2*I)*cosh(b*x + a)^2 + 2
*I)*sinh(b*x + a)^2 + 4*((I*b^2*x^2 + 2*I)*cosh(b*x + a)^3 + (I*b^2*x^2 +
2*I)*cosh(b*x + a))*sinh(b*x + a) + 2*I)*dilog(-I*cosh(b*x + a) - I*sinh(b
*x + a)) - ((-I*a^3 - 6*I*a)*cosh(b*x + a)^4 - 4*(I*a^3 + 6*I*a)*cosh(b*x
+ a)*sinh(b*x + a)^3 + (-I*a^3 - 6*I*a)*sinh(b*x + a)^4 - I*a^3 - 2*(I*a^3
 + 6*I*a)*cosh(b*x + a)^2 - 2*(I*a^3 + 3*(I*a^3 + 6*I*a)*cosh(b*x + a)^2 +
 6*I*a)*sinh(b*x + a)^2 - 4*((I*a^3 + 6*I*a)*cosh(b*x + a)^3 + (I*a^3 + 6*
I*a)*cosh(b*x + a))*sinh(b*x + a) - 6*I*a)*log(cosh(b*x + a) + sinh(b*x +
a) + I) - ((I*a^3 + 6*I*a)*cosh(b*x + a)^4 - 4*(-I*a^3 - 6*I*a)*cosh(b*x +
 a)*sinh(b*x + a)^3 + (I*a^3 + 6*I*a)*sinh(b*x + a)^4 + I*a^3 - 2*(-I*a...
```

3.370.6 Sympy [F]

$$\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^3\sinh^2{(a+bx)}\,\text{sech}^3{(a+bx)}\,dx$$

input
```
integrate(x**3*sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Integral(x**3*sinh(a + b*x)**2*sech(a + b*x)**3, x)
```

3.370.7 Maxima [F]

$$\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x^3\,\text{sech}\,(bx+a)^3\sinh\,(bx+a)^2\,dx$$

input
```
integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-((b*x^3*e^(3*a) + 3*x^2*e^(3*a))*e^(3*b*x) - (b*x^3*e^a - 3*x^2*e^a)*e^(b
*x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 2*integrate(1/2
*(b^2*x^3*e^a + 6*x*e^a)*e^(b*x)/(b^2*e^(2*b*x + 2*a) + b^2), x)
```

3.370.8 Giac [F]

$$\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x^3\,\text{sech}\,(bx+a)^3\sinh\,(bx+a)^2\,dx$$

input
```
integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

3.370.9 Mupad [F(-1)]

Timed out.

$$\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int \frac{x^3\sinh(a+b\,x)^2}{\cosh\,(a+b\,x)^3}\,dx$$

input
```
int((x^3*sinh(a + b*x)^2)/cosh(a + b*x)^3,x)
```

output
```
int((x^3*sinh(a + b*x)^2)/cosh(a + b*x)^3, x)
```

3.371 $\int x^2 \text{sech}(a+bx) \tanh^2(a+bx) \, dx$

3.371.1 Optimal result

Integrand size = 18, antiderivative size = 143

$$\int x^2 \text{sech}(a+bx) \tanh^2(a+bx) \, dx = \frac{x^2 \arctan\left(e^{a+bx}\right)}{b} + \frac{\arctan(\sinh(a+bx))}{b^3} - \frac{ix \,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} + \frac{ix \,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{i \,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} - \frac{i \,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{x \text{sech}(a+bx)}{b^2} - \frac{x^2 \text{sech}(a+bx) \tanh(a+bx)}{2b}$$

output
```
x^2*arctan(exp(b*x+a))/b+arctan(sinh(b*x+a))/b^3-I*x*polylog(2,-I*exp(b*x+
a))/b^2+I*x*polylog(2,I*exp(b*x+a))/b^2+I*polylog(3,-I*exp(b*x+a))/b^3-I*p
olylog(3,I*exp(b*x+a))/b^3-x*sech(b*x+a)/b^2-1/2*x^2*sech(b*x+a)*tanh(b*x+
a)/b
```

3.371.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.26

$$\int x^2 \text{sech}(a+bx) \tanh^2(a+bx) \, dx = \frac{i\left(-4i \arctan\left(e^{a+bx}\right) + b^2x^2 \log\left(1-ie^{a+bx}\right) - b^2x^2 \log\left(1+ie^{a+bx}\right) - 2bx \,\text{PolyLog}\left(2,-ie^{a+bx}\right) + 2bx \,\text{Pol}\right.}{2b^3} - \frac{x \text{sech}(a) \text{sech}(a+bx)(2\cosh(a) + bx\sinh(a))}{2b^2} - \frac{x^2 \text{sech}(a) \text{sech}^2(a+bx) \sinh(bx)}{2b}$$

input
```
Integrate[x^2*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
((I/2)*((-4*I)*ArcTan[E^(a + b*x)] + b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*
x^2*Log[1 + I*E^(a + b*x)] - 2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*Po
lyLog[2, I*E^(a + b*x)] + 2*PolyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*
E^(a + b*x)]))/b^3 - (x*Sech[a]*Sech[a + b*x]*(2*Cosh[a] + b*x*Sinh[a]))/(
2*b^2) - (x^2*Sech[a]*Sech[a + b*x]^2*Sinh[b*x])/(2*b)
```

3.371.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5978, 3042, 4668, 3011, 2720, 4674, 3042, 4257, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \tanh^2(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{5978}$$

$$\int x^2\text{sech}(a+bx)dx - \int x^2\text{sech}^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \csc\left(ia+ibx+\frac{\pi}{2}\right)dx - \int x^2 \csc\left(ia+ibx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{4668}$$

$$-\int x^2 \csc\left(ia+ibx+\frac{\pi}{2}\right)^3 dx - \frac{2i\int x\log\left(1-ie^{a+bx}\right)dx}{b} + \frac{2i\int x\log\left(1+ie^{a+bx}\right)dx}{b} + \frac{2x^2\arctan\left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2i\left(\frac{\int \text{PolyLog}(2,-ie^{a+bx})dx}{b} - \frac{x\,\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b} - \frac{2i\left(\frac{\int \text{PolyLog}(2,ie^{a+bx})dx}{b} - \frac{x\,\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b} - \int x^2 \csc\left(ia+ibx+\frac{\pi}{2}\right)^3 dx + \frac{2x^2\arctan\left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,-ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}-\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx+\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}$$

$\Big\downarrow$ 4674

$$\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,-ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}+\frac{\int\operatorname{sech}(a+bx)dx}{b^2}-\frac{1}{2}\int x^2\operatorname{sech}(a+bx)dx+\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{x\operatorname{sech}(a+bx)}{b^2}-\frac{x^2\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

$\Big\downarrow$ 3042

$$\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,-ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}+\frac{\int\csc\left(ia+ibx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{2}\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{x\operatorname{sech}(a+bx)}{b^2}-\frac{x^2\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

$\Big\downarrow$ 4257

$$\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,-ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}-\frac{1}{2}\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{\arctan(\sinh(a+bx))}{b^3}+\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{x\operatorname{sech}(a+bx)}{b^2}-\frac{x^2\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

$\Big\downarrow$ 4668

$$\frac{1}{2}\left(\frac{2i\int x\log\left(1-ie^{a+bx}\right)dx}{b}-\frac{2i\int x\log\left(1+ie^{a+bx}\right)dx}{b}-\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}\right)+\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,-ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}+\frac{\arctan(\sinh(a+bx))}{b^3}+\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{x\operatorname{sech}(a+bx)}{b^2}-\frac{x^2\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

3.371. $\int x^2\operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

$$\Big\downarrow \; 3011$$

$$\frac{1}{2}\left(-\frac{2i\left(\frac{\int \text{PolyLog}(2,-ie^{a+bx})dx}{b}-\frac{x\,\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int \text{PolyLog}(2,ie^{a+bx})dx}{b}-\frac{x\,\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}-\frac{2x^2\arctan\left(\right.}{b}\right.$$

$$\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,-ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-$$

$$\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}+\frac{\arctan(\sinh(a+bx))}{b^3}+$$

$$\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{x\text{sech}(a+bx)}{b^2}-\frac{x^2\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{2}\left(-\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,-ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,ie^{a+}}{b}\right.}{b}\right.$$

$$\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,-ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-$$

$$\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,ie^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}+\frac{\arctan(\sinh(a+bx))}{b^3}+$$

$$\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{x\text{sech}(a+bx)}{b^2}-\frac{x^2\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \; 7143$$

$$\frac{\arctan(\sinh(a+bx))}{b^3}+$$

$$\frac{1}{2}\left(-\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{2i\left(\frac{\text{PolyLog}(3,-ie^{a+bx})}{b^2}-\frac{x\,\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\text{PolyLog}(3,ie^{a+bx})}{b^2}-\frac{x\,\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}\right.$$

$$\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}+\frac{2i\left(\frac{\text{PolyLog}(3,-ie^{a+bx})}{b^2}-\frac{x\,\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-$$

$$\frac{2i\left(\frac{\text{PolyLog}(3,ie^{a+bx})}{b^2}-\frac{x\,\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}-\frac{x\text{sech}(a+bx)}{b^2}-\frac{x^2\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

input
```
Int[x^2*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

3.371. $\int x^2\text{sech}(a+bx)\tanh^2(a+bx)\,dx$

output
```
(2*x^2*ArcTan[E^(a + b*x)])/b + ArcTan[Sinh[a + b*x]]/b^3 + ((2*I)*(-((x*P
olyLog[2, (-I)*E^(a + b*x)])/b) + PolyLog[3, (-I)*E^(a + b*x)]/b^2))/b - (
(2*I)*(-((x*PolyLog[2, I*E^(a + b*x)])/b) + PolyLog[3, I*E^(a + b*x)]/b^2)
)/b + ((-2*x^2*ArcTan[E^(a + b*x)])/b - ((2*I)*(-((x*PolyLog[2, (-I)*E^(a
+ b*x)])/b) + PolyLog[3, (-I)*E^(a + b*x)]/b^2))/b + ((2*I)*(-((x*PolyLog[
2, I*E^(a + b*x)])/b) + PolyLog[3, I*E^(a + b*x)]/b^2))/b)/2 - (x*Sech[a +
 b*x])/b^2 - (x^2*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

3.371.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5978
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.371.4 Maple [F]

$$\int x^2 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2 \, dx$$

input
```
int(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

output
```
int(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

3.371.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1577 vs. 2(118) = 236.

Time = 0.29 (sec) , antiderivative size = 1577, normalized size of antiderivative = 11.03

$$\int x^2 \operatorname{sech}(a+bx) \tanh^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^3 + 6*(b^2*x^2 + 2*b*x)*cosh(b*x +
 a)*sinh(b*x + a)^2 + 2*(b^2*x^2 + 2*b*x)*sinh(b*x + a)^3 - 2*(b^2*x^2 - 2
*b*x)*cosh(b*x + a) + 2*(-I*b*x*cosh(b*x + a)^4 - 4*I*b*x*cosh(b*x + a)*si
nh(b*x + a)^3 - I*b*x*sinh(b*x + a)^4 - 2*I*b*x*cosh(b*x + a)^2 + 2*(-3*I*
b*x*cosh(b*x + a)^2 - I*b*x)*sinh(b*x + a)^2 - I*b*x + 4*(-I*b*x*cosh(b*x
+ a)^3 - I*b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sin
h(b*x + a)) + 2*(I*b*x*cosh(b*x + a)^4 + 4*I*b*x*cosh(b*x + a)*sinh(b*x +
a)^3 + I*b*x*sinh(b*x + a)^4 + 2*I*b*x*cosh(b*x + a)^2 + 2*(3*I*b*x*cosh(b
*x + a)^2 + I*b*x)*sinh(b*x + a)^2 + I*b*x + 4*(I*b*x*cosh(b*x + a)^3 + I*
b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)
) - ((I*a^2 + 2*I)*cosh(b*x + a)^4 - 4*(-I*a^2 - 2*I)*cosh(b*x + a)*sinh(b
*x + a)^3 + (I*a^2 + 2*I)*sinh(b*x + a)^4 - 2*(-I*a^2 - 2*I)*cosh(b*x + a)
^2 - 2*(3*(-I*a^2 - 2*I)*cosh(b*x + a)^2 - I*a^2 - 2*I)*sinh(b*x + a)^2 +
I*a^2 - 4*((-I*a^2 - 2*I)*cosh(b*x + a)^3 + (-I*a^2 - 2*I)*cosh(b*x + a))*
sinh(b*x + a) + 2*I)*log(cosh(b*x + a) + sinh(b*x + a) + I) - ((-I*a^2 - 2
*I)*cosh(b*x + a)^4 - 4*(I*a^2 + 2*I)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*
a^2 - 2*I)*sinh(b*x + a)^4 - 2*(I*a^2 + 2*I)*cosh(b*x + a)^2 - 2*(3*(I*a^2
 + 2*I)*cosh(b*x + a)^2 + I*a^2 + 2*I)*sinh(b*x + a)^2 - I*a^2 - 4*((I*a^2
 + 2*I)*cosh(b*x + a)^3 + (I*a^2 + 2*I)*cosh(b*x + a))*sinh(b*x + a) - 2*I
)*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((-I*b^2*x^2 + I*a^2)*cosh(b...
```

3.371.6 Sympy [F]

$$\int x^2 \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^2\sinh^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(x**2*sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Integral(x**2*sinh(a + b*x)**2*sech(a + b*x)**3, x)
```

3.371.7 Maxima [F]

$$\int x^2\mathrm{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^2\,\mathrm{sech}\,(bx+a)^3\sinh\,(bx+a)^2\;dx$$

input
```
integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
2*b^2*integrate(1/2*x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - ((b*
x^2*e^(3*a) + 2*x*e^(3*a))*e^(3*b*x) - (b*x^2*e^a - 2*x*e^a)*e^(b*x))/(b^2
*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 2*arctan(e^(b*x + a))/b^
3
```

3.371.8 Giac [F]

$$\int x^2\mathrm{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^2\,\mathrm{sech}\,(bx+a)^3\sinh\,(bx+a)^2\;dx$$

input
```
integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

3.371.9 Mupad [F(-1)]

Timed out.

$$\int x^2\mathrm{sech}(a+bx)\tanh^2(a+bx)\,dx = \int \frac{x^2\sinh(a+b\,x)^2}{\cosh\,(a+b\,x)^3}\,dx$$

input
```
int((x^2*sinh(a + b*x)^2)/cosh(a + b*x)^3,x)
```

output
```
int((x^2*sinh(a + b*x)^2)/cosh(a + b*x)^3, x)
```

3.372 $\int x \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

3.372.1 Optimal result

Integrand size = 16, antiderivative size = 91

$$\int x\operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{x\arctan\left(e^{a+bx}\right)}{b}-\frac{i\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}-\frac{\operatorname{sech}(a+bx)}{2b^2}-\frac{x\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

output
```
x*arctan(exp(b*x+a))/b-1/2*I*polylog(2,-I*exp(b*x+a))/b^2+1/2*I*polylog(2,
I*exp(b*x+a))/b^2-1/2*sech(b*x+a)/b^2-1/2*x*sech(b*x+a)*tanh(b*x+a)/b
```

3.372.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int x\operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = -\frac{-i\left(2ia\arctan\left(e^{a+bx}\right)+(a+bx)\log\left(1-ie^{a+bx}\right)-(a+bx)\log\left(1+ie^{a+bx}\right)-\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)+\right.}{2b^2}$$

input
```
Integrate[x*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
-1/2*((-I)*((2*I)*a*ArcTan[E^(a + b*x)] + (a + b*x)*Log[1 - I*E^(a + b*x)]
 - (a + b*x)*Log[1 + I*E^(a + b*x)] - PolyLog[2, (-I)*E^(a + b*x)] + PolyL
og[2, I*E^(a + b*x)]) + Sech[a + b*x] + b*x*Sech[a + b*x]*Tanh[a + b*x])/b
^2
```

3.372.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.62, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5978, 3042, 4668, 2715, 2838, 4673, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \tanh^2(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{5978}$$

$$\int x\text{sech}(a+bx)dx-\int x\text{sech}^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\csc\left(ia+ibx+\frac{\pi}{2}\right)dx-\int x\csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4668}$$

$$-\frac{i\int\log\left(1-ie^{a+bx}\right)dx}{b}+\frac{i\int\log\left(1+ie^{a+bx}\right)dx}{b}-\int x\csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx+\frac{2x\arctan\left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{i\int e^{-a-bx}\log\left(1-ie^{a+bx}\right)de^{a+bx}}{b^2}+\frac{i\int e^{-a-bx}\log\left(1+ie^{a+bx}\right)de^{a+bx}}{b^2}-\int x\csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx+\frac{2x\arctan\left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\int x\csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx+\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}$$

$$\Big\downarrow \text{4673}$$

$$-\frac{1}{2}\int x\text{sech}(a+bx)dx+\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\text{sech}(a+bx)}{2b^2}-\frac{x\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int x\csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\text{sech}(a+bx)}{2b^2}-\frac{x\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{4668}$$

$$\frac{1}{2}\left(\frac{i\int\log\left(1-ie^{a+bx}\right)dx}{b}-\frac{i\int\log\left(1+ie^{a+bx}\right)dx}{b}-\frac{2x\arctan\left(e^{a+bx}\right)}{b}\right)+\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\text{sech}(a+bx)}{2b^2}-\frac{x\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{1}{2}\left(\frac{i\int e^{-a-bx}\log\left(1-ie^{a+bx}\right)de^{a+bx}}{b^2}-\frac{i\int e^{-a-bx}\log\left(1+ie^{a+bx}\right)de^{a+bx}}{b^2}-\frac{2x\arctan\left(e^{a+bx}\right)}{b}\right)+\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\text{sech}(a+bx)}{2b^2}-\frac{x\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}\left(-\frac{2x\arctan\left(e^{a+bx}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}\right)+\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\text{sech}(a+bx)}{2b^2}-\frac{x\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

input
```
Int[x*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
(2*x*ArcTan[E^(a + b*x)])/b - (I*PolyLog[2, (-I)*E^(a + b*x)])/b^2 + (I*Po
lyLog[2, I*E^(a + b*x)])/b^2 + ((-2*x*ArcTan[E^(a + b*x)])/b + (I*PolyLog[
2, (-I)*E^(a + b*x)])/b^2 - (I*PolyLog[2, I*E^(a + b*x)])/b^2)/2 - Sech[a
+ b*x]/(2*b^2) - (x*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

3.372. $\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx$

3.372.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5978
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

3.372.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(76) = 152.

Time = 1.40 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.96

method	result
risch	$-\frac{e^{bx+a}\left(e^{2bx+2a}bx-bx+e^{2bx+2a}+1\right)}{b^{2}\left(1+e^{2bx+2a}\right)^{2}}-\frac{i\ln\left(1+ie^{bx+a}\right)x}{2b}-\frac{i\ln\left(1+ie^{bx+a}\right)a}{2b^{2}}+\frac{i\ln\left(1-ie^{bx+a}\right)x}{2b}+\frac{i\ln\left(1-ie^{bx+a}\right)a}{2b^{2}}-\frac{i\,\text{dilo}}{}$

input
```
int(x*sech(b*x+a)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-exp(b*x+a)*(exp(2*b*x+2*a)*b*x-b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a
))^2-1/2*I/b*ln(1+I*exp(b*x+a))*x-1/2*I/b^2*ln(1+I*exp(b*x+a))*a+1/2*I/b*l
n(1-I*exp(b*x+a))*x+1/2*I/b^2*ln(1-I*exp(b*x+a))*a-1/2*I/b^2*dilog(1+I*exp
(b*x+a))+1/2*I/b^2*dilog(1-I*exp(b*x+a))-1/b^2*a*arctan(exp(b*x+a))
```

3.372.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1064 vs. 2(70) = 140.

Time = 0.29 (sec) , antiderivative size = 1064, normalized size of antiderivative = 11.69

$$\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(b*x + 1)*cosh(b*x + a)^3 + 6*(b*x + 1)*cosh(b*x + a)*sinh(b*x + a
)^2 + 2*(b*x + 1)*sinh(b*x + a)^3 - 2*(b*x - 1)*cosh(b*x + a) - (I*cosh(b*
x + a)^4 + 4*I*cosh(b*x + a)*sinh(b*x + a)^3 + I*sinh(b*x + a)^4 - 2*(-3*I
*cosh(b*x + a)^2 - I)*sinh(b*x + a)^2 + 2*I*cosh(b*x + a)^2 - 4*(-I*cosh(b
*x + a)^3 - I*cosh(b*x + a))*sinh(b*x + a) + I)*dilog(I*cosh(b*x + a) + I*
sinh(b*x + a)) - (-I*cosh(b*x + a)^4 - 4*I*cosh(b*x + a)*sinh(b*x + a)^3 -
 I*sinh(b*x + a)^4 - 2*(3*I*cosh(b*x + a)^2 + I)*sinh(b*x + a)^2 - 2*I*cos
h(b*x + a)^2 - 4*(I*cosh(b*x + a)^3 + I*cosh(b*x + a))*sinh(b*x + a) - I)*
dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - (-I*a*cosh(b*x + a)^4 - 4*I*a*
cosh(b*x + a)*sinh(b*x + a)^3 - I*a*sinh(b*x + a)^4 - 2*I*a*cosh(b*x + a)^
2 - 2*(3*I*a*cosh(b*x + a)^2 + I*a)*sinh(b*x + a)^2 - 4*(I*a*cosh(b*x + a)
^3 + I*a*cosh(b*x + a))*sinh(b*x + a) - I*a)*log(cosh(b*x + a) + sinh(b*x
+ a) + I) - (I*a*cosh(b*x + a)^4 + 4*I*a*cosh(b*x + a)*sinh(b*x + a)^3 + I
*a*sinh(b*x + a)^4 + 2*I*a*cosh(b*x + a)^2 - 2*(-3*I*a*cosh(b*x + a)^2 - I
*a)*sinh(b*x + a)^2 - 4*(-I*a*cosh(b*x + a)^3 - I*a*cosh(b*x + a))*sinh(b*
x + a) + I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((-I*b*x - I*a)*cos
h(b*x + a)^4 - 4*(I*b*x + I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*x - I
*a)*sinh(b*x + a)^4 - 2*(I*b*x + I*a)*cosh(b*x + a)^2 - 2*(3*(I*b*x + I*a)
*cosh(b*x + a)^2 + I*b*x + I*a)*sinh(b*x + a)^2 - I*b*x - 4*((I*b*x + I*a)
*cosh(b*x + a)^3 + (I*b*x + I*a)*cosh(b*x + a))*sinh(b*x + a) - I*a)*lo...
```

3.372.6 Sympy [F]

$$\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x\sinh^2{(a+bx)}\operatorname{sech}^3{(a+bx)}\,dx$$

input
```
integrate(x*sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Integral(x*sinh(a + b*x)**2*sech(a + b*x)**3, x)
```

3.372.7 Maxima [F]

$$\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x\,\text{sech}\,(bx+a)^3\sinh\,(bx+a)^2\,dx$$

input
```
integrate(x*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) - (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4*b
*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 2*integrate(1/2*x*e^(b*x + a)/(
e^(2*b*x + 2*a) + 1), x)
```

3.372.8 Giac [F]

$$\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x\,\text{sech}\,(bx+a)^3\sinh\,(bx+a)^2\,dx$$

input
```
integrate(x*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int\frac{x\sinh(a+b\,x)^2}{\cosh\,(a+b\,x)^3}\,dx$$

input
```
int((x*sinh(a + b*x)^2)/cosh(a + b*x)^3,x)
```

output
```
int((x*sinh(a + b*x)^2)/cosh(a + b*x)^3, x)
```

3.373 $\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

3.373.1 Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}-\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

output
```
1/2*arctan(sinh(b*x+a))/b-1/2*sech(b*x+a)*tanh(b*x+a)/b
```

3.373.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}-\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

input
```
Integrate[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

3.373.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ia+ibx)^2(-\sec(ia+ibx))dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{2}\int \text{sech}(a+bx)dx-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}+\frac{1}{2}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\arctan(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

input
```
Int[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

3.373.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.373.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{-\frac{\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(e^{bx+a}\right)}{b}$	43
default	$\frac{-\frac{\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(e^{bx+a}\right)}{b}$	43
risch	$-\frac{e^{bx+a}\left(e^{2bx+2a}-1\right)}{b\left(1+e^{2bx+2a}\right)^2}+\frac{i\ln\left(e^{bx+a}+i\right)}{2b}-\frac{i\ln\left(e^{bx+a}-i\right)}{2b}$	69

input
```
int(sech(b*x+a)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-sinh(b*x+a)/cosh(b*x+a)^2+1/2*sech(b*x+a)*tanh(b*x+a)+arctan(exp(b*x
+a)))
```

3.373.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 269, normalized size of antiderivative = 7.91

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx =$$

$$-\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sin\right.}{b\cosh(bx+a)^4 + 4b\cosh}$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (c
osh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*
cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a
)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x +
a)) + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x +
 a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b
*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

3.373.6 Sympy [F]

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int \sinh^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Integral(sinh(a + b*x)**2*sech(a + b*x)**3, x)
```

3.373.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(30) = 60$.

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = -\frac{\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1)}$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-arctan(e^(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*
x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

3.373.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\pi - \frac{4\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4} + 2\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{4\,b}$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*(pi - 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
 4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

3.373.9 Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^a\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} + \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x} + e^{4\,a+4\,b\,x} + 1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x} + 1\right)}$$

input
```
int(sinh(a + b*x)^2/cosh(a + b*x)^3,x)
```

output
```
atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) + (2*exp(a + b*x))/(b*(2
*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*
b*x) + 1))
```

3.373. $\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

3.374 $\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx$

3.374.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx=\text{Int}\left(\frac{\text{sech}(a+bx)}{x},x\right)-\text{Int}\left(\frac{\text{sech}^3(a+bx)}{x},x\right)$$

output
```
Unintegrable(sech(b*x+a)/x,x)-Unintegrable(sech(b*x+a)^3/x,x)
```

3.374.2 Mathematica [N/A]

Not integrable

Time = 15.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx=\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx$$

input
```
Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

output
```
Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x, x]
```

3.374.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5978, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(a+bx)\text{sech}(a+bx)}{x} \, dx$$
$$\Big\downarrow \ 5978$$
$$\int \frac{\text{sech}(a+bx)}{x}dx-\int \frac{\text{sech}^3(a+bx)}{x}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)^3}{x}dx$$
$$\Big\downarrow \ 4680$$
$$\int \frac{\text{sech}(a+bx)}{x}dx-\int \frac{\text{sech}^3(a+bx)}{x}dx$$

input
```
Int[(Sech[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

output
```
$Aborted
```

3.374.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5978
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

3.374.4 Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^2}{x} dx$$

input
```
int(sech(b*x+a)^3*sinh(b*x+a)^2/x,x)
```

output
```
int(sech(b*x+a)^3*sinh(b*x+a)^2/x,x)
```

3.374.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x} dx = \int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^2}{x} dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^3*sinh(b*x + a)^2/x, x)
```

3.374.6 Sympy [N/A]

Not integrable

Time = 8.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\sinh^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

input
```
integrate(sech(b*x+a)**3*sinh(b*x+a)**2/x,x)
```

output
```
Integral(sinh(a + b*x)**2*sech(a + b*x)**3/x, x)
```

3.374.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 7.28

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^2}{x}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
-((b*x*e^(3*a) - e^(3*a))*e^(3*b*x) - (b*x*e^a + e^a)*e^(b*x))/(b^2*x^2*e^
(4*b*x + 4*a) + 2*b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 2*integrate(1/2*(b^
2*x^2*e^a + 2*e^a)*e^(b*x)/(b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3), x)
```

3.374.8 Giac [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^2}{x}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^3*sinh(b*x + a)^2/x, x)
```

3.374.9 Mupad [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx=\int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^2}{x\,{\mathrm{cosh}\left(a+b\,x\right)}^3}\,dx$$

input
```
int(sinh(a + b*x)^2/(x*cosh(a + b*x)^3),x)
```

output
```
int(sinh(a + b*x)^2/(x*cosh(a + b*x)^3), x)
```

3.375 $\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx$

3.375.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx=\text{Int}\left(\frac{\text{sech}(a+bx)}{x^2},x\right)-\text{Int}\left(\frac{\text{sech}^3(a+bx)}{x^2},x\right)$$

output
```
Unintegrable(sech(b*x+a)/x^2,x)-Unintegrable(sech(b*x+a)^3/x^2,x)
```

3.375.2 Mathematica [N/A]

Not integrable

Time = 11.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx=\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

output
```
Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x^2, x]
```

3.375.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5978, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(a+bx)\text{sech}(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{5978}$$

$$\int \frac{\text{sech}(a+bx)}{x^2}dx-\int \frac{\text{sech}^3(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx-\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)^3}{x^2}dx$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\text{sech}(a+bx)}{x^2}dx-\int \frac{\text{sech}^3(a+bx)}{x^2}dx$$

input
```
Int[(Sech[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

output
```
$Aborted
```

3.375.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5978
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

3.375.4 Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^2}{x^2}dx$$

input
```
int(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x)
```

output
```
int(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x)
```

3.375.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^2}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^3*sinh(b*x + a)^2/x^2, x)
```

3.375.6 Sympy [N/A]

Not integrable

Time = 10.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)**3*sinh(b*x+a)**2/x**2,x)
```

output
```
Integral(sinh(a + b*x)**2*sech(a + b*x)**3/x**2, x)
```

3.375.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 7.39

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^2}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-((b*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x) - (b*x*e^a + 2*e^a)*e^(b*x))/(b^2*x^
3*e^(4*b*x + 4*a) + 2*b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 2*integrate(1/2
*(b^2*x^2*e^a + 6*e^a)*e^(b*x)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4), x)
```

3.375.8 Giac [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^2}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^3*sinh(b*x + a)^2/x^2, x)
```

3.375.9 Mupad [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx=\int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^2}{x^2\,{\mathrm{cosh}\left(a+b\,x\right)}^3} \,d x$$

input
```
int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)^3),x)
```

output
```
int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)^3), x)
```

3.376 $\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx$

3.376.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} + \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b} - \text{Int}(x^m\tanh(a+bx),x)$$

output
```
2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)+2^(-3-m)*x^m*GAMMA(1+
m,2*b*x)/b/exp(2*a)/((b*x)^m)-Unintegrable(x^m*tanh(b*x+a),x)
```

3.376.2 Mathematica [N/A]

Not integrable

Time = 17.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx$$

input
```
Integrate[x^m*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
Integrate[x^m*Sinh[a + b*x]^2*Tanh[a + b*x], x]
```

3.376.3 Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5972, 3042, 26, 4222, 5971, 27, 3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{5972}$$

$$\int x^m \cosh(a+bx)\sinh(a+bx)dx-\int x^m \tanh(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^m \cosh(a+bx)\sinh(a+bx)dx-\int -ix^m \tan(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$\int x^m \cosh(a+bx)\sinh(a+bx)dx+i\int x^m \tan(ia+ibx)dx$$

$$\Big\downarrow \text{4222}$$

$$\int x^m \cosh(a+bx)\sinh(a+bx)dx-\int x^m \tanh(a+bx)dx$$

$$\Big\downarrow \text{5971}$$

$$\int \frac{1}{2}x^m \sinh(2a+2bx)dx-\int x^m \tanh(a+bx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int x^m \sinh(2a+2bx)dx-\int x^m \tanh(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$-\int x^m \tanh(a+bx)dx+\frac{1}{2}\int -ix^m \sin(2ia+2ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-\int x^m \tanh(a+bx)dx-\frac{1}{2}i\int x^m \sin(2ia+2ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-\int x^m \tanh(a+bx)dx-\frac{1}{2}i\left(\frac{1}{2}i\int e^{2(a+bx)}x^mdx-\frac{1}{2}i\int e^{-2(a+bx)}x^mdx\right)$$

$$\Big\downarrow \text{2612}$$

$$-\int x^m \tanh(a+bx)dx-\frac{1}{2}i\left(\frac{ie^{2a}2^{-m-2}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}+\frac{ie^{-2a}2^{-m-2}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}\right)$$

input
```
Int[x^m*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
$Aborted
```

3.376.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.376.4 Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{sech}(bx+a) \sinh(bx+a)^3 \, dx$$

input
```
int(x^m*sech(b*x+a)*sinh(b*x+a)^3,x)
```

output
```
int(x^m*sech(b*x+a)*sinh(b*x+a)^3,x)
```

3.376.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

input
```
integrate(x^m*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m*sech(b*x + a)*sinh(b*x + a)^3, x)
```

3.376.6 Sympy [N/A]

Not integrable

Time = 113.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^m \sinh^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(x**m*sech(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Integral(x**m*sinh(a + b*x)**3*sech(a + b*x), x)
```

3.376.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

input
```
integrate(x^m*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(x^m*sech(b*x + a)*sinh(b*x + a)^3, x)
```

3.376.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

input
```
integrate(x^m*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*sech(b*x + a)*sinh(b*x + a)^3, x)
```

3.376.9 Mupad [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)^3}{\cosh(a+b\,x)}\,dx$$

input
```
int((x^m*sinh(a + b*x)^3)/cosh(a + b*x),x)
```

output
```
int((x^m*sinh(a + b*x)^3)/cosh(a + b*x), x)
```

3.377 $\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx$

3.377.1 Optimal result

Integrand size = 18, antiderivative size = 185

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{3x}{8b^3}+\frac{x^3}{4b}+\frac{x^4}{4}-\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{3x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} + \frac{3x\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} - \frac{3\,\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4} - \frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4} - \frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{3x\sinh^2(a+bx)}{4b^3}+\frac{x^3\sinh^2(a+bx)}{2b}$$

output
```
3/8*x/b^3+1/4*x^3/b+1/4*x^4-x^3*ln(1+exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-
exp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3-3/4*polylog(4,-ex
p(2*b*x+2*a))/b^4-3/8*cosh(b*x+a)*sinh(b*x+a)/b^4-3/4*x^2*cosh(b*x+a)*sinh
(b*x+a)/b^2+3/4*x*sinh(b*x+a)^2/b^3+1/2*x^3*sinh(b*x+a)^2/b
```

3.377.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{\cosh(a)(\cosh(a)+\sinh(a))\left(4b^4x^4-6bx\cosh(2(a+bx))-4b^3x^3\cosh(2(a+bx))+16b^3x^3\log\left(1+e^{-\right.$$

input
```
Integrate[x^3*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-1/8*(Cosh[a]*(Cosh[a] + Sinh[a])*(4*b^4*x^4 - 6*b*x*Cosh[2*(a + b*x)] - 4
*b^3*x^3*Cosh[2*(a + b*x)] + 16*b^3*x^3*Log[1 + E^(-2*(a + b*x))] - 24*b^2
*x^2*PolyLog[2, -E^(-2*(a + b*x))] - 24*b*x*PolyLog[3, -E^(-2*(a + b*x))]
- 12*PolyLog[4, -E^(-2*(a + b*x))] + 3*Sinh[2*(a + b*x)] + 6*b^2*x^2*Sinh[
2*(a + b*x)]))/(b^4*(1 + E^(2*a)))
```

3.377.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.20, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.056, Rules used = {5972, 3042, 26, 4201, 2620, 3011, 5895, 3042, 25, 3792, 15, 25, 3042, 25, 3115, 24, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{ 5972}$$

$$\int x^3\cosh(a+bx)\sinh(a+bx)dx-\int x^3\tanh(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^3\cosh(a+bx)\sinh(a+bx)dx-\int -ix^3\tan(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$\int x^3\cosh(a+bx)\sinh(a+bx)dx+i\int x^3\tan(ia+ibx)dx$$

$$\Big\downarrow \text{4201}$$

$$\int x^3\cosh(a+bx)\sinh(a+bx)dx+i\left(2i\int\frac{e^{2(a+bx)}x^3}{1+e^{2(a+bx)}}dx-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{2620}$$

$$\int x^3\cosh(a+bx)\sinh(a+bx)dx+ i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\int x^2\log\left(1+e^{2(a+bx)}\right)dx}{2b}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{3011}$$

$$\int x^3\cosh(a+bx)\sinh(a+bx)dx+ i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}(2,-e^{2(a+bx)})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2(a+bx)})}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{5895}$$

$$i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}(2,-e^{2(a+bx)})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2(a+bx)})}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)- \frac{3\int x^2\sinh^2(a+bx)dx}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}(2,-e^{2(a+bx)})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2(a+bx)})}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)- \frac{3\int -x^2\sin(ia+ibx)^2dx}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}(2,-e^{2(a+bx)})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2(a+bx)})}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+ \frac{3\int x^2\sin(ia+ibx)^2dx}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3792}$$

3.377. $\int x^3\sinh^2(a+bx)\tanh(a+bx)\,dx$

$$\frac{3\left(\frac{\int -\sinh^2(a+bx)dx}{2b^2}+\frac{\int x^2dx}{2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b}+$$
$$i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+$$
$$\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow 15$$

$$\frac{3\left(\frac{\int -\sinh^2(a+bx)dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+$$
$$i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+$$
$$\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow 25$$

$$\frac{3\left(-\frac{\int \sinh^2(a+bx)dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+$$
$$i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+$$
$$\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow 3042$$

$$\frac{3\left(-\frac{\int -\sin(ia+ibx)^2dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+$$
$$i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+$$
$$\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow 25$$

3.377. $\int x^3\sinh^2(a+bx)\tanh(a+bx)\,dx$

$$\frac{3\left(\frac{\int \sin(ia+ibx)^2dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \; 3115$$

$$\frac{3\left(\frac{\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \; 24$$

$$i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+\frac{3\left(\frac{x\sinh^2(a+bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \; 7163$$

$$i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-e^{2(a+bx)}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+\frac{3\left(\frac{x\sinh^2(a+bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \; 2720$$

3.377. $\int x^3\sinh^2(a+bx)\tanh(a+bx)\,dx$

$$i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b}-\frac{\int e^{-2(a+bx)}\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right.\right.$$

$$\frac{3\left(\frac{x\sinh^2(a+bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$\downarrow$ 7143

$$i\left(2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b}-\frac{\operatorname{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^2}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+$$

$$\frac{3\left(\frac{x\sinh^2(a+bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

input
```
Int[x^3*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
I*((-1/4*I)*x^4 + (2*I)*((x^3*Log[1 + E^(2*(a + b*x))])/(2*b) - (3*(-1/2*(
x^2*PolyLog[2, -E^(2*(a + b*x))])/b + ((x*PolyLog[3, -E^(2*(a + b*x))])/(2
*b) - PolyLog[4, -E^(2*(a + b*x))]/(4*b^2))/b))/(2*b))) + (x^3*Sinh[a + b*
x]^2)/(2*b) + (3*(x^3/6 - (x^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (x*Sin
h[a + b*x]^2)/(2*b^2) + (x/2 - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b))/(2*b^2
)))/(2*b)
```

3.377.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.377. $\int x^3 \sinh^2(a+bx) \tanh(a+bx)\,dx$

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.377.4 Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.02

method	result
risch	$\frac{x^4}{4}+\frac{(4x^3b^3-6x^2b^2+6bx-3)\,\mathrm{e}^{2bx+2a}}{32b^4}+\frac{(4x^3b^3+6x^2b^2+6bx+3)\,\mathrm{e}^{-2bx-2a}}{32b^4}+\frac{3a^4}{2b^4}-\frac{3\,\mathrm{polylog}(4,-\mathrm{e}^{2bx+2a})}{4b^4}+\frac{2a^3x}{b^3}-\frac{x^3\ln}{}$

input
```
int(x^3*sech(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+1/32*(4*b^3*x^3-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)+1/32*(4*b^3*
x^3+6*b^2*x^2+6*b*x+3)/b^4*exp(-2*b*x-2*a)+3/2/b^4*a^4-3/4*polylog(4,-exp(
2*b*x+2*a))/b^4+2/b^3*a^3*x-x^3*ln(1+exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-
exp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3-2/b^4*a^3*ln(exp(
b*x+a))
```

3.377.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 966, normalized size of antiderivative = 5.22

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/32*(4*b^3*x^3 + (4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)^4 + 4*
(4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b^3
*x^3 - 6*b^2*x^2 + 6*b*x - 3)*sinh(b*x + a)^4 + 6*b^2*x^2 + 8*(b^4*x^4 - 2
*a^4)*cosh(b*x + a)^2 + 2*(4*b^4*x^4 - 8*a^4 + 3*(4*b^3*x^3 - 6*b^2*x^2 +
6*b*x - 3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x - 96*(b^2*x^2*cosh(b*x
 + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)
*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 96*(b^2*x^2*cosh(b*x + a)^2 +
2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog(-I*
cosh(b*x + a) - I*sinh(b*x + a)) + 32*(a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*
x + a)*sinh(b*x + a) + a^3*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x +
 a) + I) + 32*(a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*sinh(b*x + a) + a
^3*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 32*((b^3*x^3
+ a^3)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) + (
b^3*x^3 + a^3)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1)
 - 32*((b^3*x^3 + a^3)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*s
inh(b*x + a) + (b^3*x^3 + a^3)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*s
inh(b*x + a) + 1) - 192*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2)*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 192*(cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(4,
 -I*cosh(b*x + a) - I*sinh(b*x + a)) + 192*(b*x*cosh(b*x + a)^2 + 2*b*x...
```

3.377.6 Sympy [F]

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^3 \sinh^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(x**3*sech(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Integral(x**3*sinh(a + b*x)**3*sech(a + b*x), x)
```

3.377.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.98

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{1}{2}x^4 - \frac{\left(8b^4x^4e^{(2a)} - \left(4b^3x^3e^{(4a)} - 6b^2x^2e^{(4a)} + 6bxe^{(4a)} - 3e^{(4a)}\right)e^{(2bx)} - \left(4b^3x^3 + 6b^2x^2 + 6bx + 3\right)e^{(-2bx)}}{32b^4} - \frac{4b^3x^3\log\left(e^{(2bx+2a)}+1\right) + 6b^2x^2\mathrm{Li}_2\left(-e^{(2bx+2a)}\right) - 6bx\mathrm{Li}_3\left(-e^{(2bx+2a)}\right) + 3\mathrm{Li}_4\left(-e^{(2bx+2a)}\right)}{3b^4}$$

input
```
integrate(x^3*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*x^4 - 1/32*(8*b^4*x^4*e^(2*a) - (4*b^3*x^3*e^(4*a) - 6*b^2*x^2*e^(4*a)
 + 6*b*x*e^(4*a) - 3*e^(4*a))*e^(2*b*x) - (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x +
 3)*e^(-2*b*x))*e^(-2*a)/b^4 - 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6
*b^2*x^2*dilog(-e^(2*b*x + 2*a)) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*
polylog(4, -e^(2*b*x + 2*a)))/b^4
```

3.377.8 Giac [F]

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^3 \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

input
```
integrate(x^3*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*sech(b*x + a)*sinh(b*x + a)^3, x)
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx=\int \frac{x^3 \sinh(a+b\,x)^3}{\cosh\left(a+b\,x\right)}\,dx$$

input
```
int((x^3*sinh(a + b*x)^3)/cosh(a + b*x),x)
```

output
```
int((x^3*sinh(a + b*x)^3)/cosh(a + b*x), x)
```

3.378 $\int x^2 \sinh^2(a+bx) \tanh(a+bx)\,dx$

3.378.1 Optimal result

Integrand size = 18, antiderivative size = 130

$$\int x^2 \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{x^2}{4b}+\frac{x^3}{3}-\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} - \frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3}+\frac{x^2\sinh^2(a+bx)}{2b}$$

output
```
1/4*x^2/b+1/3*x^3-x^2*ln(1+exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/
b^2+1/2*polylog(3,-exp(2*b*x+2*a))/b^3-1/2*x*cosh(b*x+a)*sinh(b*x+a)/b^2+1
/4*sinh(b*x+a)^2/b^3+1/2*x^2*sinh(b*x+a)^2/b
```

3.378.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.94

$$\int x^2 \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{\cosh(a)(\cosh(a)+\sinh(a))\left(-8b^3x^3+3\cosh(2(a+bx))+6b^2x^2\cosh(2(a+bx))-24b^2x^2\log\left(1+e^{-2(a+\cdots}\right.\right.}{12b^3\left(1+e^{2a}\right)}$$

input
```
Integrate[x^2*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
(Cosh[a]*(Cosh[a] + Sinh[a])*(-8*b^3*x^3 + 3*Cosh[2*(a + b*x)] + 6*b^2*x^2
*Cosh[2*(a + b*x)] - 24*b^2*x^2*Log[1 + E^(-2*(a + b*x))] + 24*b*x*PolyLog
[2, -E^(-2*(a + b*x))] + 12*PolyLog[3, -E^(-2*(a + b*x))] - 6*b*x*Sinh[2*(
a + b*x)]))/(12*b^3*(1 + E^(2*a)))
```

3.378.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5972, 3042, 26, 4201, 2620, 3011, 2720, 5895, 3042, 25, 3791, 15, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sinh^2(a+bx) \tanh(a+bx) \, dx$$

$$\Big\downarrow \text{5972}$$

$$\int x^2 \cosh(a+bx) \sinh(a+bx) dx - \int x^2 \tanh(a+bx) dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \cosh(a+bx) \sinh(a+bx) dx - \int -ix^2 \tan(ia+ibx) dx$$

$$\Big\downarrow \text{26}$$

$$\int x^2 \cosh(a+bx) \sinh(a+bx) dx + i \int x^2 \tan(ia+ibx) dx$$

$$\Big\downarrow \text{4201}$$

$$\int x^2 \cosh(a+bx) \sinh(a+bx) dx + i \left(2i \int \frac{e^{2(a+bx)} x^2}{1+e^{2(a+bx)}} dx - \frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2620}$$

$$\int x^2 \cosh(a+bx) \sinh(a+bx) dx + i \left(2i \left(\frac{x^2 \log\left(e^{2(a+bx)}+1\right)}{2b} - \frac{\int x \log\left(1+e^{2(a+bx)}\right) dx}{b}\right) - \frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{3011}$$

$$\int x^2\cosh(a+bx)\sinh(a+bx)dx+$$
$$i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int \text{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2720}$$

$$\int x^2\cosh(a+bx)\sinh(a+bx)dx+$$
$$i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\text{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{5895}$$

$$i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\text{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)-$$
$$\frac{\int x\sinh^2(a+bx)dx}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\text{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)-$$
$$\frac{\int -x\sin(ia+ibx)^2dx}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\text{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)+$$
$$\frac{\int x\sin(ia+ibx)^2dx}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3791}$$

$$i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\text{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)+$$
$$\frac{\frac{\int xdx}{2}+\frac{\sinh^2(a+bx)}{4b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{15}$$

3.378. $\int x^2\sinh^2(a+bx)\tanh(a+bx)\,dx$

$$i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\text{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)+$$
$$\frac{\frac{\sinh^2(a+bx)}{4b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{7143}$$

$$i\left(2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)+$$
$$\frac{\frac{\sinh^2(a+bx)}{4b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

input
```
Int[x^2*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
I*((-1/3*I)*x^3 + (2*I)*((x^2*Log[1 + E^(2*(a + b*x))])/(2*b) - (-1/2*(x*P
olyLog[2, -E^(2*(a + b*x))])/b + PolyLog[3, -E^(2*(a + b*x))]/(4*b^2))/b))
 + (x^2*Sinh[a + b*x]^2)/(2*b) + (x^2/4 - (x*Cosh[a + b*x]*Sinh[a + b*x])/
(2*b) + Sinh[a + b*x]^2/(4*b^2))/b
```

3.378.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.378. $\int x^2 \sinh^2(a+bx) \tanh(a+bx) \, dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.378.4 Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.17

method	result
risch	$\frac{x^3}{3}+\frac{(2x^2b^2-2bx+1)\mathrm{e}^{2bx+2a}}{16b^3}+\frac{(2x^2b^2+2bx+1)\mathrm{e}^{-2bx-2a}}{16b^3}+\frac{2a^2\ln(\mathrm{e}^{bx+a})}{b^3}-\frac{2a^2x}{b^2}-\frac{4a^3}{3b^3}-\frac{x^2\ln(1+\mathrm{e}^{2bx+2a})}{b}-\frac{x\,\mathrm{poly}}{}$

input
```
int(x^2*sech(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+1/16*(2*b^2*x^2-2*b*x+1)/b^3*exp(2*b*x+2*a)+1/16*(2*b^2*x^2+2*b*x+
1)/b^3*exp(-2*b*x-2*a)+2/b^3*a^2*ln(exp(b*x+a))-2/b^2*a^2*x-4/3/b^3*a^3-x^
2*ln(1+exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2+1/2*polylog(3,-e
xp(2*b*x+2*a))/b^3
```

3.378.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 789, normalized size of antiderivative = 6.07

$$\int x^2\sinh^2(a+bx)\tanh(a+bx)\,dx=\text{Too large to display}$$

input
```
integrate(x^2*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/48*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^4 + 12*(2*b^2*x^2 - 2*b*x +
1)*cosh(b*x + a)*sinh(b*x + a)^3 + 3*(2*b^2*x^2 - 2*b*x + 1)*sinh(b*x + a)
^4 + 6*b^2*x^2 + 16*(b^3*x^3 + 2*a^3)*cosh(b*x + a)^2 + 2*(8*b^3*x^3 + 16*
a^3 + 9*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x -
 96*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*
x + a)^2)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 96*(b*x*cosh(b*x + a)
^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2)*dilog(-I*cos
h(b*x + a) - I*sinh(b*x + a)) - 48*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x +
 a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a)
 + I) - 48*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*
sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 48*((b^2*x^2 - a
^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2
*x^2 - a^2)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) -
48*((b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh
(b*x + a) + (b^2*x^2 - a^2)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh
(b*x + a) + 1) + 96*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 96*(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(3, -I*c
osh(b*x + a) - I*sinh(b*x + a)) + 4*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x +
a)^3 + 8*(b^3*x^3 + 2*a^3)*cosh(b*x + a))*sinh(b*x + a) + 3)/(b^3*cosh(...
```

3.378.6 Sympy [F]

$$\int x^2 \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^2 \sinh^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(x**2*sech(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Integral(x**2*sinh(a + b*x)**3*sech(a + b*x), x)
```

3.378.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06

$$\int x^2 \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{2}{3}x^3 - \frac{\left(16\,b^3x^3e^{(2\,a)} - 3\left(2\,b^2x^2e^{(4\,a)} - 2\,bxe^{(4\,a)} + e^{(4\,a)}\right)e^{(2\,bx)} - 3\,(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx)}\right)e^{(-2\,a)}}{48\,b^3} - \frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right) - \mathrm{Li}_3\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^3}$$

input
```
integrate(x^2*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
2/3*x^3 - 1/48*(16*b^3*x^3*e^(2*a) - 3*(2*b^2*x^2*e^(4*a) - 2*b*x*e^(4*a)
+ e^(4*a))*e^(2*b*x) - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x))*e^(-2*a)/b^3
- 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a))
- polylog(3, -e^(2*b*x + 2*a)))/b^3
```

3.378.8 Giac [F]

$$\int x^2 \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^2\operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

input
```
integrate(x^2*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*sech(b*x + a)*sinh(b*x + a)^3, x)
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh^2(a+bx)\tanh(a+bx)\,dx = \int \frac{x^2\sinh(a+b\,x)^3}{\cosh(a+b\,x)}\,dx$$

input
```
int((x^2*sinh(a + b*x)^3)/cosh(a + b*x),x)
```

output
```
int((x^2*sinh(a + b*x)^3)/cosh(a + b*x), x)
```

3.379 $\int x \sinh^2(a+bx) \tanh(a+bx) \, dx$

3.379.1 Optimal result

Integrand size = 16, antiderivative size = 89

$$\int x \sinh^2(a+bx) \tanh(a+bx) \, dx = \frac{x}{4b} + \frac{x^2}{2} - \frac{x \log\left(1+e^{2(a+bx)}\right)}{b} - \frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+bx)}{2b}$$

output
```
1/4*x/b+1/2*x^2-x*ln(1+exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b^
2-1/4*cosh(b*x+a)*sinh(b*x+a)/b^2+1/2*x*sinh(b*x+a)^2/b
```

3.379.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int x \sinh^2(a+bx) \tanh(a+bx) \, dx = -\frac{-4a^2+4b^2x^2-2bx\cosh(2(a+bx))+8bx\log\left(1+e^{-2(a+bx)}\right)-4\,\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)+\sinh(2(a-}{8b^2}$$

input
```
Integrate[x*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-1/8*(-4*a^2 + 4*b^2*x^2 - 2*b*x*Cosh[2*(a + b*x)] + 8*b*x*Log[1 + E^(-2*(
a + b*x))] - 4*PolyLog[2, -E^(-2*(a + b*x))] + Sinh[2*(a + b*x)])/b^2
```

3.379.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5972, 3042, 26, 4201, 2620, 2715, 2838, 5895, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sinh^2(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{5972}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx-\int x\tanh(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx-\int -ix\tan(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx+i\int x\tan(ia+ibx)dx$$

$$\Big\downarrow \text{4201}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx+i\left(2i\int \frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx+i\left(2i\left(\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int \log\left(1+e^{2(a+bx)}\right)dx}{2b}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx+ i\left(2i\left(\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int e^{-2(a+bx)}\log\left(1+e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx+i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{4b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{5895}$$

$$-\frac{\int \sinh^2(a+bx)dx}{2b}+i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{4b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}\right)-\frac{ix^2}{2}\right)+\frac{x\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\sin(ia+ibx)^2dx}{2b}+i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{4b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}\right)-\frac{ix^2}{2}\right)+\frac{x\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sin(ia+ibx)^2dx}{2b}+i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{4b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}\right)-\frac{ix^2}{2}\right)+\frac{x\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b}+i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{4b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}\right)-\frac{ix^2}{2}\right)+\frac{x\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{24}$$

$$i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{4b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}\right)-\frac{ix^2}{2}\right)+\frac{x\sinh^2(a+bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b}$$

input
```
Int[x*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
I*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*(a + b*x))])/(2*b) + PolyLog[2, -
E^(2*(a + b*x))]/(4*b^2))) + (x*Sinh[a + b*x]^2)/(2*b) + (x/2 - (Cosh[a +
b*x]*Sinh[a + b*x])/(2*b))/(2*b)
```

3.379. $\int x\sinh^2(a+bx)\tanh(a+bx)\,dx$

3.379.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.379. $\int x \sinh^2(a+bx) \tanh(a+bx)\, dx$

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.379.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

method	result	s
risch	$\frac{x^2}{2}+\frac{(2bx-1)e^{2bx+2a}}{16b^2}+\frac{(2bx+1)e^{-2bx-2a}}{16b^2}+\frac{2ax}{b}+\frac{a^2}{b^2}-\frac{x\ln(1+e^{2bx+2a})}{b}-\frac{\operatorname{polylog}(2,-e^{2bx+2a})}{2b^2}-\frac{2a\ln(e^{bx+a})}{b^2}$	1

input
```
int(x*sech(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/16*(2*b*x-1)/b^2*exp(2*b*x+2*a)+1/16*(2*b*x+1)/b^2*exp(-2*b*x-2*
a)+2/b*a*x+a^2/b^2-x*ln(1+exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))
/b^2-2/b^2*a*ln(exp(b*x+a))
```

3.379.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 558, normalized size of antiderivative = 6.27

$$\int x\sinh^2(a+bx)\tanh(a+bx)\,dx$$
$$=\frac{(2\,bx-1)\cosh\left(bx+a\right)^4+4\,(2\,bx-1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+(2\,bx-1)\sinh\left(bx+a\right)^4+8\,(b^2x^2-}{}$$

input
```
integrate(x*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/16*((2*b*x - 1)*cosh(b*x + a)^4 + 4*(2*b*x - 1)*cosh(b*x + a)*sinh(b*x +
 a)^3 + (2*b*x - 1)*sinh(b*x + a)^4 + 8*(b^2*x^2 - 2*a^2)*cosh(b*x + a)^2
+ 2*(4*b^2*x^2 + 3*(2*b*x - 1)*cosh(b*x + a)^2 - 8*a^2)*sinh(b*x + a)^2 +
2*b*x - 16*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a
)^2)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 16*(cosh(b*x + a)^2 + 2*co
sh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*dilog(-I*cosh(b*x + a) - I*si
nh(b*x + a)) + 16*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a
*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) + 16*(a*cosh(b*x
+ a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2)*log(cosh(b*x
 + a) + sinh(b*x + a) - I) - 16*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*c
osh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2)*log(I*cosh(b*x + a
) + I*sinh(b*x + a) + 1) - 16*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cos
h(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a)
 - I*sinh(b*x + a) + 1) + 4*((2*b*x - 1)*cosh(b*x + a)^3 + 4*(b^2*x^2 - 2*
a^2)*cosh(b*x + a))*sinh(b*x + a) + 1)/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b
*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2)
```

3.379.6 Sympy [F]

$$\int x \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x \sinh^3{(a+bx)}\operatorname{sech}{(a+bx)}\,dx$$

input
```
integrate(x*sech(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Integral(x*sinh(a + b*x)**3*sech(a + b*x), x)
```

3.379.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\begin{aligned}\int x \sinh^2(a+bx)\tanh(a+bx)\,dx &= x^2 - \frac{\left(8\,b^2x^2e^{(2\,a)} - \left(2\,bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(2\,bx)} - (2\,bx+1)e^{(-2\,bx)}\right)e^{(-2\,a)}}{16\,b^2} \\ &\quad - \frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right) + \mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}\end{aligned}$$

input
```
integrate(x*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
x^2 - 1/16*(8*b^2*x^2*e^(2*a) - (2*b*x*e^(4*a) - e^(4*a))*e^(2*b*x) - (2*b
*x + 1)*e^(-2*b*x))*e^(-2*a)/b^2 - 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + d
ilog(-e^(2*b*x + 2*a)))/b^2
```

3.379.8 Giac [F]

$$\int x \sinh^2(a+bx) \tanh(a+bx) \, dx = \int x \operatorname{sech}(bx+a) \sinh(bx+a)^3 \, dx$$

input
```
integrate(x*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x*sech(b*x + a)*sinh(b*x + a)^3, x)
```

3.379.9 Mupad [F(-1)]

Timed out.

$$\int x \sinh^2(a+bx) \tanh(a+bx) \, dx = \int \frac{x \sinh(a+b\,x)^3}{\cosh(a+b\,x)} \, dx$$

input
```
int((x*sinh(a + b*x)^3)/cosh(a + b*x),x)
```

output
```
int((x*sinh(a + b*x)^3)/cosh(a + b*x), x)
```

3.380 $\int \sinh^2(a+bx)\tanh(a+bx)\,dx$

3.380.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{\cosh^2(a+bx)}{2b} - \frac{\log(\cosh(a+bx))}{b}$$

output
```
1/2*cosh(b*x+a)^2/b-ln(cosh(b*x+a))/b
```

3.380.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{-\frac{1}{2}\cosh^2(a+bx) + \log(\cosh(a+bx))}{b}$$

input
```
Integrate[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-((-1/2*Cosh[a + b*x]^2 + Log[Cosh[a + b*x]])/b)
```

3.380.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ia+ibx)^2\tan(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin(ia+ibx)^2\tan(ia+ibx)dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int (1-\cosh^2(a+bx))\,\text{sech}(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int (\text{sech}(a+bx)-\cosh(a+bx))d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(\cosh(a+bx))-\frac{1}{2}\cosh^2(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-((-1/2*Cosh[a + b*x]^2 + Log[Cosh[a + b*x]])/b)
```

3.380.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.380.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^2}{2}-\ln(\cosh(bx+a))}{b}$	25
default	$\frac{\frac{\sinh(bx+a)^2}{2}-\ln(\cosh(bx+a))}{b}$	25
risch	$x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}+\frac{2a}{b}-\frac{\ln(1+e^{2bx+2a})}{b}$	54

input
```
int(sech(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sinh(b*x+a)^2-ln(cosh(b*x+a)))
```

3.380.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 7.04

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{8\,bx\cosh(bx+a)^2 + \cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\,(4\,bx + 3\cosh(}{}$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*(8*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x +
a)^3 + sinh(b*x + a)^4 + 2*(4*b*x + 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8
*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2
*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) +
 cosh(b*x + a)^3)*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.380.6 Sympy [F]

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = \int \sinh^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Integral(sinh(a + b*x)**3*sech(a + b*x), x)
```

3.380.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{bx+a}{b} + \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-(b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b - log(e^(-2*
b*x - 2*a) + 1)/b
```

3.380.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{8\,bx - \left(4\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 8\,a + e^{(2\,bx+2\,a)} - 8\,\log\left(e^{(2\,bx+2\,a)} + 1\right)}{8\,b}$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*(8*b*x - (4*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 8*a + e^(2*b*x + 2
*a) - 8*log(e^(2*b*x + 2*a) + 1))/b
```

3.380.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = x - \frac{\ln\left(e^{2\,a}\,e^{2\,b\,x} + 1\right)}{b} + \frac{e^{-2\,a-2\,b\,x}}{8\,b} + \frac{e^{2\,a+2\,b\,x}}{8\,b}$$

input
```
int(sinh(a + b*x)^3/cosh(a + b*x),x)
```

output
```
x - log(exp(2*a)*exp(2*b*x) + 1)/b + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a +
2*b*x)/(8*b)
```

3.381 $\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$

3.381.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \frac{1}{2}\text{Chi}(2bx)\sinh(2a) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx) - \text{Int}\left(\frac{\tanh(a+bx)}{x},x\right)$$

output
```
1/2*cosh(2*a)*Shi(2*b*x)+1/2*Chi(2*b*x)*sinh(2*a)-Unintegrable(tanh(b*x+a)
/x,x)
```

3.381.2 Mathematica [N/A]

Not integrable

Time = 12.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$$

input
```
Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x,x]
```

output
```
Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x, x]
```

3.381.3 Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5972, 3042, 26, 4222, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{ 5972}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}dx-\int \frac{\tanh(a+bx)}{x}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}dx-\int -\frac{i\tan(ia+ibx)}{x}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}dx+i\int \frac{\tan(ia+ibx)}{x}dx$$

$$\Big\downarrow \text{ 4222}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}dx-\int \frac{\tanh(a+bx)}{x}dx$$

$$\Big\downarrow \text{ 5971}$$

$$\int \frac{\sinh(2a+2bx)}{2x}dx-\int \frac{\tanh(a+bx)}{x}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{\sinh(2a+2bx)}{x}dx-\int \frac{\tanh(a+bx)}{x}dx$$

$$\Big\downarrow \text{ 3042}$$

$$-\int \frac{\tanh(a+bx)}{x}dx+\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{x}dx$$

$$\Big\downarrow \text{ 26}$$

$$-\int \frac{\tanh(a+bx)}{x}dx-\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$-\int \frac{\tanh(a+bx)}{x}dx-\frac{1}{2}i\left(i\sinh(2a)\int \frac{\cosh(2bx)}{x}dx+\cosh(2a)\int \frac{i\sinh(2bx)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\tanh(a+bx)}{x}dx-\frac{1}{2}i\left(i\sinh(2a)\int \frac{\cosh(2bx)}{x}dx+i\cosh(2a)\int \frac{\sinh(2bx)}{x}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\tanh(a+bx)}{x}dx-\frac{1}{2}i\left(i\sinh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(2a)\int -\frac{i\sin(2ibx)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\tanh(a+bx)}{x}dx-\frac{1}{2}i\left(i\sinh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx+\cosh(2a)\int \frac{\sin(2ibx)}{x}dx\right)$$

$$\Big\downarrow \text{3779}$$

$$-\int \frac{\tanh(a+bx)}{x}dx-\frac{1}{2}i\left(i\sinh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(2a)\text{Shi}(2bx)\right)$$

$$\Big\downarrow \text{3782}$$

$$-\int \frac{\tanh(a+bx)}{x}dx-\frac{1}{2}i(i\sinh(2a)\text{Chi}(2bx)+i\cosh(2a)\text{Shi}(2bx))$$

input
```
Int[(Sinh[a + b*x]^2*Tanh[a + b*x])/x,x]
```

output
```
$Aborted
```

3.381.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.381. $\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.381. $\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$

3.381.4 Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x}dx$$

input
```
int(sech(b*x+a)*sinh(b*x+a)^3/x,x)
```

output
```
int(sech(b*x+a)*sinh(b*x+a)^3/x,x)
```

3.381.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)*sinh(b*x + a)^3/x, x)
```

3.381.6 Sympy [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)**3/x,x)
```

output
```
Integral(sinh(a + b*x)**3*sech(a + b*x)/x, x)
```

3.381.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a) + 2*integrate(1/(x*e^(2*b*
x + 2*a) + x), x) - log(x)
```

3.381.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)*sinh(b*x + a)^3/x, x)
```

3.381.9 Mupad [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)^3}{x\cosh(a+b\,x)}\,dx$$

input
```
int(sinh(a + b*x)^3/(x*cosh(a + b*x)),x)
```

output
```
int(sinh(a + b*x)^3/(x*cosh(a + b*x)), x)
```

3.382 $\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx$

3.382.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx = b\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{2x} + b\sinh(2a)\text{Shi}(2bx) - \text{Int}\left(\frac{\tanh(a+bx)}{x^2},x\right)$$

output
```
b*Chi(2*b*x)*cosh(2*a)+b*Shi(2*b*x)*sinh(2*a)-1/2*sinh(2*b*x+2*a)/x-Uninte
grable(tanh(b*x+a)/x^2,x)
```

3.382.2 Mathematica [N/A]

Not integrable

Time = 10.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x^2,x]
```

output
```
Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x^2, x]
```

3.382.3 Rubi [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5972, 3042, 26, 4222, 5971, 27, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{5972}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}dx-\int \frac{\tanh(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}dx-\int -\frac{i\tan(ia+ibx)}{x^2}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}dx+i\int \frac{\tan(ia+ibx)}{x^2}dx$$

$$\Big\downarrow \text{4222}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}dx-\int \frac{\tanh(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{5971}$$

$$\int \frac{\sinh(2a+2bx)}{2x^2}dx-\int \frac{\tanh(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^2}dx-\int \frac{\tanh(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\tanh(a+bx)}{x^2}dx+\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{x^2}dx$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\tanh(a+bx)}{x^2}dx-\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$-\int \frac{\tanh(a+bx)}{x^2}dx-\frac{1}{2}i\left(2ib\int \frac{\cosh(2a+2bx)}{x}dx-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\tanh(a+bx)}{x^2}dx-\frac{1}{2}i\left(2ib\int \frac{\sin\left(2ia+2ibx+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3784}$$

$$-\int \frac{\tanh(a+bx)}{x^2}dx-$$
$$\frac{1}{2}i\left(2ib\left(\cosh(2a)\int \frac{\cosh(2bx)}{x}dx-i\sinh(2a)\int \frac{i\sinh(2bx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\tanh(a+bx)}{x^2}dx-$$
$$\frac{1}{2}i\left(2ib\left(\sinh(2a)\int \frac{\sinh(2bx)}{x}dx+\cosh(2a)\int \frac{\cosh(2bx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\tanh(a+bx)}{x^2}dx-$$
$$\frac{1}{2}i\left(2ib\left(\sinh(2a)\int -\frac{i\sin(2ibx)}{x}dx+\cosh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\tanh(a+bx)}{x^2}dx-$$
$$\frac{1}{2}i\left(2ib\left(\cosh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx-i\sinh(2a)\int \frac{\sin(2ibx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3779}$$

$$-\int \frac{\tanh(a+bx)}{x^2}dx-$$
$$\frac{1}{2}i\left(2ib\left(\sinh(2a)\text{Shi}(2bx)+\cosh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3782}$$

$$-\int \frac{\tanh(a+bx)}{x^2}dx-\frac{1}{2}i\left(2ib(\cosh(2a)\text{Chi}(2bx)+\sinh(2a)\text{Shi}(2bx))-\frac{i\sinh(2a+2bx)}{x}\right)$$

3.382. $\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx$

input
```
Int[(Sinh[a + b*x]^2*Tanh[a + b*x])/x^2,x]
```

output
```
$Aborted
```

3.382.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

3.382. $\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx$

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.382.4 Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x^2}dx$$

input
```
int(sech(b*x+a)*sinh(b*x+a)^3/x^2,x)
```

output
```
int(sech(b*x+a)*sinh(b*x+a)^3/x^2,x)
```

3.382.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)*sinh(b*x + a)^3/x^2, x)
```

3.382.6 Sympy [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\sinh^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)**3/x**2,x)
```

output
```
Integral(sinh(a + b*x)**3*sech(a + b*x)/x**2, x)
```

3.382.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/2*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/2*b*e^(2*a)*gamma(-1, -2*b*x) + 1/x +
2*integrate(1/(x^2*e^(2*b*x + 2*a) + x^2), x)
```

3.382.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)*sinh(b*x + a)^3/x^2, x)
```

3.382.9 Mupad [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)^3}{x^2\cosh(a+b\,x)}\,dx$$

input
```
int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)),x)
```

output
```
int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)), x)
```

3.383 $\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx$

3.383.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b} + \frac{e^{-a} x^m(bx)^{-m}\Gamma(1+m,bx)}{2b} - \text{Int}(x^m \text{sech}(a+bx)\tanh(a+bx),x)$$

output
```
-CannotIntegrate(x^m*sech(b*x+a)*tanh(b*x+a),x)+1/2*exp(a)*x^m*GAMMA(1+m,-
b*x)/b/((-b*x)^m)+1/2*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)
```

3.383.2 Mathematica [N/A]

Not integrable

Time = 45.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx$$

input
```
Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x]^2, x]
```

3.383.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5972, 3042, 26, 3789, 2612, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{5972}$$

$$\int x^m \sinh(a+bx)dx-\int x^m \text{sech}(a+bx)\tanh(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$-\int x^m \text{sech}(a+bx)\tanh(a+bx)dx+\int -ix^m \sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-\int x^m \text{sech}(a+bx)\tanh(a+bx)dx-i\int x^m \sin(ia+ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-\int x^m \text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{1}{2}i\int e^{a+bx}x^m dx-\frac{1}{2}i\int e^{-a-bx}x^m dx\right)$$

$$\Big\downarrow \text{2612}$$

$$-\int x^m \text{sech}(a+bx)\tanh(a+bx)dx- i\left(\frac{ie^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}+\frac{ie^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}\right)$$

$$\Big\downarrow \text{7299}$$

$$-\int x^m \text{sech}(a+bx)\tanh(a+bx)dx- i\left(\frac{ie^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}+\frac{ie^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}\right)$$

input
```
Int[x^m*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
$Aborted
```

3.383.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.383. $\int x^m \sinh(a+bx) \tanh^2(a+bx) \, dx$

3.383.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3 \, dx$$

input
```
int(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

output
```
int(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

3.383.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \sinh(a+bx) \tanh^2(a+bx) \, dx = \int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3 \, dx$$

input
```
integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

3.383.6 Sympy [F(-1)]

Timed out.

$$\int x^m \sinh(a+bx) \tanh^2(a+bx) \, dx = \text{Timed out}$$

input
```
integrate(x**m*sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Timed out
```

3.383.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3\,dx$$

input
```
integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(x^m*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

3.383.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3\,dx$$

input
```
integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

3.383.9 Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)^3}{\cosh(a+b\,x)^2}\,dx$$

input
```
int((x^m*sinh(a + b*x)^3)/cosh(a + b*x)^2,x)
```

output
```
int((x^m*sinh(a + b*x)^3)/cosh(a + b*x)^2, x)
```

3.384 $\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx$

3.384.1 Optimal result

Integrand size = 18, antiderivative size = 162

$$\begin{aligned}\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx = &-\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2}+\frac{6x\cosh(a+bx)}{b^3}\\&+\frac{x^3\cosh(a+bx)}{b}+\frac{6ix\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}\\&-\frac{6ix\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{6i\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}\\&+\frac{6i\operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}+\frac{x^3\operatorname{sech}(a+bx)}{b}\\&-\frac{6\sinh(a+bx)}{b^4}-\frac{3x^2\sinh(a+bx)}{b^2}\end{aligned}$$

output
```
-6*x^2*arctan(exp(b*x+a))/b^2+6*x*cosh(b*x+a)/b^3+x^3*cosh(b*x+a)/b+6*I*x*
polylog(2,-I*exp(b*x+a))/b^3-6*I*x*polylog(2,I*exp(b*x+a))/b^3-6*I*polylog
(3,-I*exp(b*x+a))/b^4+6*I*polylog(3,I*exp(b*x+a))/b^4+x^3*sech(b*x+a)/b-6*
sinh(b*x+a)/b^4-3*x^2*sinh(b*x+a)/b^2
```

3.384.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.21

$$\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{-3i\left(b^2x^2\log\left(1-ie^{a+bx}\right)-b^2x^2\log\left(1+ie^{a+bx}\right)-2bx\,\text{PolyLog}\left(2,-ie^{a+bx}\right)+2bx\,\text{PolyLog}\left(2,ie^{a+bx}\right)+\right.}{}$$

input
```
Integrate[x^3*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
((-3*I)*(b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a + b*x)] -
 2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*x)] + 2*
PolyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]) + b^3*x^3*Sech
[a + b*x] + Cosh[b*x]*(b*x*(6 + b^2*x^2)*Cosh[a] - 3*(2 + b^2*x^2)*Sinh[a]
) + (-3*(2 + b^2*x^2)*Cosh[a] + b*x*(6 + b^2*x^2)*Sinh[a])*Sinh[b*x])/b^4
```

3.384.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.24, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5972, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 5941, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx$$

$$\Big\downarrow \; 5972$$

$$\int x^3 \sinh(a+bx)dx - \int x^3\text{sech}(a+bx)\tanh(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$-\int x^3\text{sech}(a+bx)\tanh(a+bx)dx + \int -ix^3\sin(ia+ibx)dx$$

$$\Big\downarrow \; 26$$

$$-\int x^3\text{sech}(a+bx)\tanh(a+bx)dx - i\int x^3\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-\int x^3\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\int x^2\cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int x^3\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-\int x^3\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}-\frac{2i\int -ix\sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-\int x^3\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}-\frac{2\int x\sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int x^3\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}-\frac{2\int -ix\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-\int x^3\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\int x\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-\int x^3\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\cosh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

3.384. $\int x^3\sinh(a+bx)\tanh^2(a+bx)\,dx$

$$-\int x^3\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3117}$$

$$-\int x^3\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{5941}$$

$$-\frac{3\int x^2\text{sech}(a+bx)dx}{b}-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{x^3\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int x^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{x^3\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{3\left(-\frac{2i\int x\log\left(1-ie^{a+bx}\right)dx}{b}+\frac{2i\int x\log\left(1+ie^{a+bx}\right)dx}{b}+\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}\right)}{b}-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{x^3\text{sech}(a+bx)}{b}$$

3.384. $\int x^3\sinh(a+bx)\tanh^2(a+bx)\,dx$

$\Big\downarrow$ 3011

$$-\frac{3\left(\frac{2i\left(\frac{\int \text{PolyLog}\left(2,-ie^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{2i\left(\frac{\int \text{PolyLog}\left(2,ie^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}+\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}\right)}{b}-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{x^3\text{sech}(a+bx)}{b}$$

$\Big\downarrow$ 2720

$$-\frac{3\left(\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}\left(2,-ie^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}\left(2,ie^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}+2x\cdots\right)}{b}$$
$$-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{x^3\text{sech}(a+bx)}{b}$$

$\Big\downarrow$ 7143

$$-\frac{3\left(\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}+\frac{2i\left(\frac{\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{2i\left(\frac{\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}\right)}{b}-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{x^3\text{sech}(a+bx)}{b}$$

input
```
Int[x^3*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

3.384. $\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx$

output
```
(-3*((2*x^2*ArcTan[E^(a + b*x)])/b + ((2*I)*(-((x*PolyLog[2, (-I)*E^(a + b
*x)])/b) + PolyLog[3, (-I)*E^(a + b*x)]/b^2))/b - ((2*I)*(-((x*PolyLog[2,
I*E^(a + b*x)])/b) + PolyLog[3, I*E^(a + b*x)]/b^2))/b))/b + (x^3*Sech[a +
 b*x])/b - I*((I*x^3*Cosh[a + b*x])/b - ((3*I)*((x^2*Sinh[a + b*x])/b + ((
2*I)*((I*x*Cosh[a + b*x])/b - (I*Sinh[a + b*x])/b^2))/b))/b)
```

3.384.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.384. $\int x^3 \sinh(a+bx) \tanh^2(a+bx) \, dx$

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.384.4 Maple [F]

$$\int x^3 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3 \, dx$$

input
```
int(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

output
```
int(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

3.384.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1225 vs. $2(139) = 278$.

Time = 0.28 (sec) , antiderivative size = 1225, normalized size of antiderivative = 7.56

$$\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^4 + 4*(b^3*
x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 - 3*
b^2*x^2 + 6*b*x - 6)*sinh(b*x + a)^4 + 3*b^2*x^2 + 6*(b^3*x^3 + 2*b*x)*cos
h(b*x + a)^2 + 6*(b^3*x^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a
)^2 + 2*b*x)*sinh(b*x + a)^2 + 6*b*x - 12*(I*b*x*cosh(b*x + a)^3 + 3*I*b*x
*cosh(b*x + a)*sinh(b*x + a)^2 + I*b*x*sinh(b*x + a)^3 + I*b*x*cosh(b*x +
a) + (3*I*b*x*cosh(b*x + a)^2 + I*b*x)*sinh(b*x + a))*dilog(I*cosh(b*x + a
) + I*sinh(b*x + a)) - 12*(-I*b*x*cosh(b*x + a)^3 - 3*I*b*x*cosh(b*x + a)*
sinh(b*x + a)^2 - I*b*x*sinh(b*x + a)^3 - I*b*x*cosh(b*x + a) + (-3*I*b*x*
cosh(b*x + a)^2 - I*b*x)*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*
x + a)) - 6*(I*a^2*cosh(b*x + a)^3 + 3*I*a^2*cosh(b*x + a)*sinh(b*x + a)^2
 + I*a^2*sinh(b*x + a)^3 + I*a^2*cosh(b*x + a) + (3*I*a^2*cosh(b*x + a)^2
+ I*a^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) - 6*(-I*a^2
*cosh(b*x + a)^3 - 3*I*a^2*cosh(b*x + a)*sinh(b*x + a)^2 - I*a^2*sinh(b*x
+ a)^3 - I*a^2*cosh(b*x + a) + (-3*I*a^2*cosh(b*x + a)^2 - I*a^2)*sinh(b*x
 + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) - 6*((-I*b^2*x^2 + I*a^2)*co
sh(b*x + a)^3 + 3*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)*sinh(b*x + a)^2 + (-I
*b^2*x^2 + I*a^2)*sinh(b*x + a)^3 + (-I*b^2*x^2 + I*a^2)*cosh(b*x + a) + (
-I*b^2*x^2 + 3*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^2 + I*a^2)*sinh(b*x + a)
)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 6*((I*b^2*x^2 - I*a^2)*c...
```

3.384.6 Sympy [F]

$$\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^3 \sinh^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(x**3*sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Integral(x**3*sinh(a + b*x)**3*sech(a + b*x)**2, x)
```

3.384.7 Maxima [F]

$$\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^3 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3\,dx$$

input
```
integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*((b^3*x^3*e^(4*a) - 3*b^2*x^2*e^(4*a) + 6*b*x*e^(4*a) - 6*e^(4*a))*e^(
3*b*x) + 6*(b^3*x^3*e^(2*a) + 2*b*x*e^(2*a))*e^(b*x) + (b^3*x^3 + 3*b^2*x^
2 + 6*b*x + 6)*e^(-b*x))/(b^4*e^(2*b*x + 3*a) + b^4*e^a) - 6*integrate(x^2
*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

3.384.8 Giac [F]

$$\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^3 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3\,dx$$

input
```
integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int \frac{x^3 \sinh(a+b\,x)^3}{\cosh(a+b\,x)^2}\,dx$$

input
```
int((x^3*sinh(a + b*x)^3)/cosh(a + b*x)^2,x)
```

output
```
int((x^3*sinh(a + b*x)^3)/cosh(a + b*x)^2, x)
```

3.385 $\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx$

3.385.1 Optimal result

Integrand size = 18, antiderivative size = 104

$$\int x^2\sinh(a+bx)\tanh^2(a+bx)\,dx=-\frac{4x\arctan\left(e^{a+bx}\right)}{b^2}+\frac{2\cosh(a+bx)}{b^3}+\frac{x^2\cosh(a+bx)}{b}+\frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{x^2\text{sech}(a+bx)}{b}-\frac{2x\sinh(a+bx)}{b^2}$$

output
```
-4*x*arctan(exp(b*x+a))/b^2+2*cosh(b*x+a)/b^3+x^2*cosh(b*x+a)/b+2*I*polylo
g(2,-I*exp(b*x+a))/b^3-2*I*polylog(2,I*exp(b*x+a))/b^3+x^2*sech(b*x+a)/b-2
*x*sinh(b*x+a)/b^2
```

3.385.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.27

$$\int x^2\sinh(a+bx)\tanh^2(a+bx)\,dx$$
$$=\frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)-2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)+\frac{1}{2}\text{sech}(a+bx)\left(2+3b^2x^2+\left(2+b^2x^2\right)\cosh(2(a+bx)\right)}{b^3}$$

input
```
Integrate[x^2*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
((2*I)*PolyLog[2, (-I)*E^(a + b*x)] - (2*I)*PolyLog[2, I*E^(a + b*x)] + (S
ech[a + b*x]*(2 + 3*b^2*x^2 + (2 + b^2*x^2)*Cosh[2*(a + b*x)] - (4*I)*b*x*
Cosh[a + b*x]*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(a + b*x)]) - 2*b*x*Si
nh[2*(a + b*x)]))/2)/b^3
```

3.385.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.20, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5972, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 5941, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 5972}$$

$$\int x^2 \sinh(a+bx)dx-\int x^2\text{sech}(a+bx)\tanh(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$-\int x^2\text{sech}(a+bx)\tanh(a+bx)dx+\int -ix^2\sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$-\int x^2\text{sech}(a+bx)\tanh(a+bx)dx-i\int x^2\sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 3777}$$

$$-\int x^2\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\int x\cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-\int x^2\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{ 3777}$$

$$-\int x^2\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{i\int -i\sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-\int x^2\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\int\sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int x^2\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\int -i\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-\int x^2\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}+\frac{i\int\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3118}$$

$$-\int x^2\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{5941}$$

$$-\frac{2\int x\text{sech}(a+bx)dx}{b}-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)+\frac{x^2\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int x\csc\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)+\frac{x^2\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{2\left(-\frac{i\int\log(1-ie^{a+bx})dx}{b}+\frac{i\int\log(1+ie^{a+bx})dx}{b}+\frac{2x\arctan(e^{a+bx})}{b}\right)}{b}-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)+\frac{x^2\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2\left(-\frac{i\int e^{-a-bx}\log(1-ie^{a+bx})de^{a+bx}}{b^2}+\frac{i\int e^{-a-bx}\log(1+ie^{a+bx})de^{a+bx}}{b^2}+\frac{2x\arctan(e^{a+bx})}{b}\right)}{b}-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)+\frac{x^2\text{sech}(a+bx)}{b}$$

3.385. $\int x^2\sinh(a+bx)\tanh^2(a+bx)\,dx$

$$\Big\downarrow \text{2838}$$

$$-\frac{2\left(\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}\right)}{b}-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)+\frac{x^2\text{sech}(a+bx)}{b}$$

input
```
Int[x^2*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
(-2*((2*x*ArcTan[E^(a + b*x)])/b - (I*PolyLog[2, (-I)*E^(a + b*x)])/b^2 +
(I*PolyLog[2, I*E^(a + b*x)])/b^2))/b + (x^2*Sech[a + b*x])/b - I*((I*x^2*
Cosh[a + b*x])/b - ((2*I)*(-(Cosh[a + b*x]/b^2) + (x*Sinh[a + b*x])/b))/b)
```

3.385.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.385.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(97) = 194$.

Time = 0.89 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.97

method	result
risch	$\frac{(x^2b^2-2bx+2)\,\mathrm{e}^{bx+a}}{2b^3}+\frac{(x^2b^2+2bx+2)\,\mathrm{e}^{-bx-a}}{2b^3}+\frac{2x^2\,\mathrm{e}^{bx+a}}{b(1+\mathrm{e}^{2bx+2a})}+\frac{2i\ln(1+i\mathrm{e}^{bx+a})x}{b^2}+\frac{2i\ln(1+i\mathrm{e}^{bx+a})a}{b^3}-\frac{2i\ln(1-i\mathrm{e}^{bx+a}}{b^2}$

input
```
int(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)+1/2*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)
+2*x^2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))+2*I/b^2*ln(1+I*exp(b*x+a))*x+2*I/b^
3*ln(1+I*exp(b*x+a))*a-2*I/b^2*ln(1-I*exp(b*x+a))*x-2*I/b^3*ln(1-I*exp(b*x
+a))*a+2*I/b^3*dilog(1+I*exp(b*x+a))-2*I/b^3*dilog(1-I*exp(b*x+a))+4/b^3*a
*arctan(exp(b*x+a))
```

3.385.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(91) = 182$.

Time = 0.27 (sec) , antiderivative size = 879, normalized size of antiderivative = 8.45

$$\int x^2 \sinh(a+bx) \tanh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*b*x + 2)*cosh(
b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^4 + b^2*x^2
 + 2*(3*b^2*x^2 + 2)*cosh(b*x + a)^2 + 2*(3*b^2*x^2 + 3*(b^2*x^2 - 2*b*x +
 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 2*b*x - 4*(I*cosh(b*x + a)^3 +
3*I*cosh(b*x + a)*sinh(b*x + a)^2 + I*sinh(b*x + a)^3 + (3*I*cosh(b*x + a)
^2 + I)*sinh(b*x + a) + I*cosh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*
x + a)) - 4*(-I*cosh(b*x + a)^3 - 3*I*cosh(b*x + a)*sinh(b*x + a)^2 - I*si
nh(b*x + a)^3 + (-3*I*cosh(b*x + a)^2 - I)*sinh(b*x + a) - I*cosh(b*x + a)
)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 4*(-I*a*cosh(b*x + a)^3 - 3*
I*a*cosh(b*x + a)*sinh(b*x + a)^2 - I*a*sinh(b*x + a)^3 - I*a*cosh(b*x + a
) + (-3*I*a*cosh(b*x + a)^2 - I*a)*sinh(b*x + a))*log(cosh(b*x + a) + sinh
(b*x + a) + I) - 4*(I*a*cosh(b*x + a)^3 + 3*I*a*cosh(b*x + a)*sinh(b*x + a
)^2 + I*a*sinh(b*x + a)^3 + I*a*cosh(b*x + a) + (3*I*a*cosh(b*x + a)^2 + I
*a)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) - 4*((-I*b*x - I
*a)*cosh(b*x + a)^3 + 3*(-I*b*x - I*a)*cosh(b*x + a)*sinh(b*x + a)^2 + (-I
*b*x - I*a)*sinh(b*x + a)^3 + (-I*b*x - I*a)*cosh(b*x + a) + (3*(-I*b*x -
I*a)*cosh(b*x + a)^2 - I*b*x - I*a)*sinh(b*x + a))*log(I*cosh(b*x + a) + I
*sinh(b*x + a) + 1) - 4*((I*b*x + I*a)*cosh(b*x + a)^3 + 3*(I*b*x + I*a)*c
osh(b*x + a)*sinh(b*x + a)^2 + (I*b*x + I*a)*sinh(b*x + a)^3 + (I*b*x + I*
a)*cosh(b*x + a) + (3*(I*b*x + I*a)*cosh(b*x + a)^2 + I*b*x + I*a)*sinh...
```

3.385.6 Sympy [F]

$$\int x^2 \sinh(a+bx) \tanh^2(a+bx)\,dx = \int x^2 \sinh^3(a+bx) \operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(x**2*sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Integral(x**2*sinh(a + b*x)**3*sech(a + b*x)**2, x)
```

3.385.7 Maxima [F]

$$\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^2 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3\,dx$$

input
```
integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*((b^2*x^2*e^(4*a) - 2*b*x*e^(4*a) + 2*e^(4*a))*e^(3*b*x) + 2*(3*b^2*x^
2*e^(2*a) + 2*e^(2*a))*e^(b*x) + (b^2*x^2 + 2*b*x + 2)*e^(-b*x))/(b^3*e^(2
*b*x + 3*a) + b^3*e^a) - 4*integrate(x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b)
, x)
```

3.385.8 Giac [F]

$$\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^2 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3\,dx$$

input
```
integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int \frac{x^2 \sinh(a+b\,x)^3}{\cosh(a+b\,x)^2}\,dx$$

input
```
int((x^2*sinh(a + b*x)^3)/cosh(a + b*x)^2,x)
```

output
```
int((x^2*sinh(a + b*x)^3)/cosh(a + b*x)^2, x)
```

3.386 $\int x \sinh(a+bx) \tanh^2(a+bx) \, dx$

3.386.1 Optimal result

Integrand size = 16, antiderivative size = 46

$$\int x \sinh(a+bx) \tanh^2(a+bx) \, dx = -\frac{\arctan(\sinh(a+bx))}{b^2} + \frac{x\cosh(a+bx)}{b} + \frac{x\text{sech}(a+bx)}{b} - \frac{\sinh(a+bx)}{b^2}$$

output
```
-arctan(sinh(b*x+a))/b^2+x*cosh(b*x+a)/b+x*sech(b*x+a)/b-sinh(b*x+a)/b^2
```

3.386.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int x \sinh(a+bx) \tanh^2(a+bx) \, dx = -\frac{2\arctan\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{x\cosh(a+bx)}{b} + \frac{x\text{sech}(a+bx)}{b} - \frac{\sinh(a+bx)}{b^2}$$

input
```
Integrate[x*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
(-2*ArcTan[Tanh[(a + b*x)/2]])/b^2 + (x*Cosh[a + b*x])/b + (x*Sech[a + b*x
])/b - Sinh[a + b*x]/b^2
```

3.386.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5972, 3042, 26, 3777, 3042, 3117, 5941, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sinh(a+bx)\tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{5972}$$

$$\int x\sinh(a+bx)dx-\int x\text{sech}(a+bx)\tanh(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$-\int x\text{sech}(a+bx)\tanh(a+bx)dx+\int -ix\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-\int x\text{sech}(a+bx)\tanh(a+bx)dx-i\int x\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-\int x\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int x\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{3117}$$

$$-\int x\text{sech}(a+bx)\tanh(a+bx)dx-i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)$$

$$\Big\downarrow \text{5941}$$

$$-\frac{\int\text{sech}(a+bx)dx}{b}-i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)+\frac{x\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int\csc\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}-i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)+\frac{x\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\arctan(\sinh(a+bx))}{b^2}-i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)+\frac{x\text{sech}(a+bx)}{b}$$

input
```
Int[x*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
-(ArcTan[Sinh[a + b*x]]/b^2) + (x*Sech[a + b*x])/b - I*((I*x*Cosh[a + b*x]
)/b - (I*Sinh[a + b*x])/b^2)
```

3.386.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

3.386. $\int x\sinh(a+bx)\tanh^2(a+bx)\,dx$

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.386.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

method	result	size
risch	$\frac{(bx-1)\mathrm{e}^{bx+a}}{2b^2}+\frac{(bx+1)\mathrm{e}^{-bx-a}}{2b^2}+\frac{2x\,\mathrm{e}^{bx+a}}{b(1+\mathrm{e}^{2bx+2a})}+\frac{i\ln(\mathrm{e}^{bx+a}-i)}{b^2}-\frac{i\ln(\mathrm{e}^{bx+a}+i)}{b^2}$	94

input
```
int(x*sech(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x-1)/b^2*exp(b*x+a)+1/2*(b*x+1)/b^2*exp(-b*x-a)+2*x*exp(b*x+a)/b/(1
+exp(2*b*x+2*a))+I/b^2*ln(exp(b*x+a)-I)-I/b^2*ln(exp(b*x+a)+I)
```

3.386.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 283, normalized size of antiderivative = 6.15

$$\int x\sinh(a+bx)\tanh^2(a+bx)\,dx$$
$$= \frac{(bx-1)\cosh(bx+a)^4+4(bx-1)\cosh(bx+a)\sinh(bx+a)^3+(bx-1)\sinh(bx+a)^4+6\,bx\cosh(bx}{}$$

input
```
integrate(x*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*((b*x - 1)*cosh(b*x + a)^4 + 4*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a)^3
 + (b*x - 1)*sinh(b*x + a)^4 + 6*b*x*cosh(b*x + a)^2 + 6*((b*x - 1)*cosh(b
*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x - 4*(cosh(b*x + a)^3 + 3*cosh(b*x +
 a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x +
 a) + cosh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a)) + 4*((b*x - 1)*
cosh(b*x + a)^3 + 3*b*x*cosh(b*x + a))*sinh(b*x + a) + 1)/(b^2*cosh(b*x +
a)^3 + 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3 + b^2*cos
h(b*x + a) + (3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a))
```

3.386.6 Sympy [F]

$$\int x \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x \sinh^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(x*sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Integral(x*sinh(a + b*x)**3*sech(a + b*x)**2, x)
```

3.386.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int x \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{6\,bxe^{(bx+2\,a)} + \left(bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(3\,bx)} + (bx+1)e^{(-bx)}}{2\,\left(b^2e^{(2\,bx+3\,a)} + b^2e^{a}\right)} - \frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^2}$$

input
```
integrate(x*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(6*b*x*e^(b*x + 2*a) + (b*x*e^(4*a) - e^(4*a))*e^(3*b*x) + (b*x + 1)*e
^(-b*x))/(b^2*e^(2*b*x + 3*a) + b^2*e^a) - 2*arctan(e^(b*x + a))/b^2
```

3.386.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(46) = 92$.

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.22

$$\int x\sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{bxe^{(4\,bx+4\,a)} + 6\,bxe^{(2\,bx+2\,a)} + bx - 4\,\arctan\left(e^{(bx+a)}\right)e^{(3\,bx+3\,a)} - 4\,\arctan\left(e^{(bx+a)}\right)e^{(bx+a)} - e^{(4\,bx+4\,a)} + 1}{2\,\left(b^2e^{(3\,bx+3\,a)} + b^2e^{(bx+a)}\right)}$$

input
```
integrate(x*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*(b*x*e^(4*b*x + 4*a) + 6*b*x*e^(2*b*x + 2*a) + b*x - 4*arctan(e^(b*x +
 a))*e^(3*b*x + 3*a) - 4*arctan(e^(b*x + a))*e^(b*x + a) - e^(4*b*x + 4*a)
 + 1)/(b^2*e^(3*b*x + 3*a) + b^2*e^(b*x + a))
```

3.386.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int x\sinh(a+bx)\tanh^2(a+bx)\,dx = \mathrm{e}^{-a-b\,x}\left(\frac{x}{2\,b}+\frac{1}{2\,b^2}\right) - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^4}}{b^2}\right)}{\sqrt{b^4}} + \mathrm{e}^{a+b\,x}\left(\frac{x}{2\,b}-\frac{1}{2\,b^2}\right) + \frac{2\,x\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int((x*sinh(a + b*x)^3)/cosh(a + b*x)^2,x)
```

output
```
exp(- a - b*x)*(x/(2*b) + 1/(2*b^2)) - (2*atan((exp(b*x)*exp(a)*(b^4)^(1/2
))/b^2))/(b^4)^(1/2) + exp(a + b*x)*(x/(2*b) - 1/(2*b^2)) + (2*x*exp(a + b
*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.387 $\int \sinh(a+bx)\tanh^2(a+bx)\,dx$

3.387.1 Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{\text{sech}(a+bx)}{b}$$

output
```
cosh(b*x+a)/b+sech(b*x+a)/b
```

3.387.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{\text{sech}(a+bx)}{b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
Cosh[a + b*x]/b + Sech[a + b*x]/b
```

3.387.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \left(1-\cosh^2(a+bx)\right)\operatorname{sech}^2(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\operatorname{sech}^2(a+bx)-1\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\cosh(a+bx)-\operatorname{sech}(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
-((-Cosh[a + b*x] - Sech[a + b*x])/b)
```

3.387.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.387.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$	33
default	$\frac{\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$	33
risch	$\frac{e^{3bx+3a}+6\,e^{bx+a}+e^{-bx-a}}{2b\left(1+e^{2bx+2a}\right)}$	46

input
```
int(sech(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x+a)^2/cosh(b*x+a)+2/cosh(b*x+a))
```

3.387.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh(bx+a)^2+\sinh(bx+a)^2+3}{2\,b\cosh(bx+a)}$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 + 3)/(b*cosh(b*x + a))
```

3.387.6 Sympy [F]

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \int \sinh^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Integral(sinh(a + b*x)**3*sech(a + b*x)**2, x)
```

3.387.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{(-bx-a)}}{2\,b} + \frac{5\,e^{(-2\,bx-2\,a)}+1}{2\,b\left(e^{(-bx-a)}+e^{(-3\,bx-3\,a)}\right)}$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*e^(-b*x - a)/b + 1/2*(5*e^(-2*b*x - 2*a) + 1)/(b*(e^(-b*x - a) + e^(-3
*b*x - 3*a)))
```

3.387.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}} + e^{(bx+a)} + e^{(-bx-a)}}{2\,b}$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*(4/(e^(b*x + a) + e^(-b*x - a)) + e^(b*x + a) + e^(-b*x - a))/b
```

3.387.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh(a+b\,x)^2+1}{b\cosh(a+b\,x)}$$

input
```
int(sinh(a + b*x)^3/cosh(a + b*x)^2,x)
```

output
```
(cosh(a + b*x)^2 + 1)/(b*cosh(a + b*x))
```

3.388 $\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx$

3.388.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \text{Chi}(bx)\sinh(a)+\cosh(a)\text{Shi}(bx) - \text{Int}\left(\frac{\text{sech}(a+bx)\tanh(a+bx)}{x},x\right)$$

output
```
-CannotIntegrate(sech(b*x+a)*tanh(b*x+a)/x,x)+cosh(a)*Shi(b*x)+Chi(b*x)*si
nh(a)
```

3.388.2 Mathematica [N/A]

Not integrable

Time = 8.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx$$

input
```
Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

output
```
Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x, x]
```

3.388.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5972, 3042, 26, 3784, 26, 3042, 26, 3779, 3782, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}dx$$

$$\Big\downarrow \text{5972}$$

$$\int \frac{\sinh(a+bx)}{x}dx-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}dx+\int -\frac{i\sin(ia+ibx)}{x}dx$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}dx-i\int \frac{\sin(ia+ibx)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}dx-i\left(i\sinh(a)\int \frac{\cosh(bx)}{x}dx+\cosh(a)\int \frac{i\sinh(bx)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}dx-i\left(i\sinh(a)\int \frac{\cosh(bx)}{x}dx+i\cosh(a)\int \frac{\sinh(bx)}{x}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}dx-i\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(a)\int -\frac{i\sin(ibx)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}dx-i\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx+\cosh(a)\int \frac{\sin(ibx)}{x}dx\right)$$

$$\Big\downarrow \text{3779}$$

$$-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}dx - i\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx + i\cosh(a)\text{Shi}(bx)\right)$$

$$\Big\downarrow \text{3782}$$

$$-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}dx - i(i\sinh(a)\text{Chi}(bx) + i\cosh(a)\text{Shi}(bx))$$

$$\Big\downarrow \text{7299}$$

$$-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}dx - i(i\sinh(a)\text{Chi}(bx) + i\cosh(a)\text{Shi}(bx))$$

input
```
Int[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

output
```
$Aborted
```

3.388.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

3.388. $\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx$

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.388.4 Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)^3}{x} dx$$

input
```
int(sech(b*x+a)^2*sinh(b*x+a)^3/x,x)
```

output
```
int(sech(b*x+a)^2*sinh(b*x+a)^3/x,x)
```

3.388.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x} dx = \int \frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)^3}{x} dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^2*sinh(b*x + a)^3/x, x)
```

3.388.6 Sympy [N/A]

Not integrable

Time = 8.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\sinh^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

input
```
integrate(sech(b*x+a)**2*sinh(b*x+a)**3/x,x)
```

output
```
Integral(sinh(a + b*x)**3*sech(a + b*x)**2/x, x)
```

3.388.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.39

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^3}{x}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
-1/2*Ei(-b*x)*e^(-a) + 1/2*Ei(b*x)*e^a + 2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a
) + b*x) + 2*integrate(e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) + b*x^2), x)
```

3.388.8 Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^3}{x}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^2*sinh(b*x + a)^3/x, x)
```

3.388.9 Mupad [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)^3}{x\cosh\left(a+b\,x\right)^2}\,dx$$

input
```
int(sinh(a + b*x)^3/(x*cosh(a + b*x)^2),x)
```

output
```
int(sinh(a + b*x)^3/(x*cosh(a + b*x)^2), x)
```

3.389 $\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx$

3.389.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = b\cosh(a)\text{Chi}(bx) - \frac{\sinh(a+bx)}{x} + b\sinh(a)\text{Shi}(bx) - \text{Int}\left(\frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}, x\right)$$

output
```
-CannotIntegrate(sech(b*x+a)*tanh(b*x+a)/x^2,x)+b*Chi(b*x)*cosh(a)+b*Shi(b
*x)*sinh(a)-sinh(b*x+a)/x
```

3.389.2 Mathematica [N/A]

Not integrable

Time = 6.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

output
```
Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x^2, x]
```

3.389.3 Rubi [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5972, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{5972}$$

$$\int \frac{\sinh(a+bx)}{x^2}dx-\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}dx+\int -\frac{i\sin(ia+ibx)}{x^2}dx$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}dx-i\int \frac{\sin(ia+ibx)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$-\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}dx-i\left(ib\int \frac{\cosh(a+bx)}{x}dx-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}dx-i\left(ib\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3784}$$

$$-\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}dx-$$
$$i\left(ib\left(\cosh(a)\int \frac{\cosh(bx)}{x}dx-i\sinh(a)\int \frac{i\sinh(bx)}{x}dx\right)-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}dx-$$
$$i\left(ib\left(\sinh(a)\int \frac{\sinh(bx)}{x}dx+\cosh(a)\int \frac{\cosh(bx)}{x}dx\right)-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}dx - i\left(ib\left(\sinh(a)\int -\frac{i\sin(ibx)}{x}dx+\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx\right)-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}dx - i\left(ib\left(\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx-i\sinh(a)\int \frac{\sin(ibx)}{x}dx\right)-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3779}$$

$$-\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}dx - i\left(ib\left(\sinh(a)\text{Shi}(bx)+\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx\right)-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3782}$$

$$-\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}dx - i\left(ib(\cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx))-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{7299}$$

$$-\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}dx - i\left(ib(\cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx))-\frac{i\sinh(a+bx)}{x}\right)$$

input
```
Int[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

output
```
$Aborted
```

3.389.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.389.4 Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}\left(bx+a\right)^{2} \sinh\left(bx+a\right)^{3}}{x^{2}}dx$$

input
```
int(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x)
```

output
```
int(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x)
```

3.389. $\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx$

3.389.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^2*sinh(b*x + a)^3/x^2, x)
```

3.389.6 Sympy [N/A]

Not integrable

Time = 10.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh^3\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)**2*sinh(b*x+a)**3/x**2,x)
```

output
```
Integral(sinh(a + b*x)**3*sech(a + b*x)**2/x**2, x)
```

3.389.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.83

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/2*b*e^(-a)*gamma(-1, b*x) + 1/2*b*e^a*gamma(-1, -b*x) + 2*e^(b*x + a)/(b
*x^2*e^(2*b*x + 2*a) + b*x^2) + 4*integrate(e^(b*x + a)/(b*x^3*e^(2*b*x +
2*a) + b*x^3), x)
```

3.389.8 Giac [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^2*sinh(b*x + a)^3/x^2, x)
```

3.389.9 Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)^3}{x^2\cosh\left(a+b\,x\right)^2}\,dx$$

input
```
int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)^2),x)
```

output
```
int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)^2), x)
```

3.390 $\int x^m \tanh^3(a+bx)\,dx$

3.390.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \tanh^3(a+bx)\,dx = \mathrm{Int}\left(x^m \tanh^3(a+bx),x\right)$$

output
```
Unintegrable(x^m*tanh(b*x+a)^3,x)
```

3.390.2 Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh^3(a+bx)\,dx$$

input
```
Integrate[x^m*Tanh[a + b*x]^3,x]
```

output
```
Integrate[x^m*Tanh[a + b*x]^3, x]
```

3.390.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \tanh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix^m \tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int x^m \tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{4222}$$

$$\int x^m \tanh^3(a+bx)dx$$

input
```
Int[x^m*Tanh[a + b*x]^3,x]
```

output
```
$Aborted
```

3.390.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.390.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3 \, dx$$

input
```
int(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

output
```
int(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

3.390.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \tanh^3(a+bx) \, dx = \int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3 \, dx$$

input
```
integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

3.390.6 Sympy [F(-1)]

Timed out.

$$\int x^m \tanh^3(a + bx)\,dx = \text{Timed out}$$

input
```
integrate(x**m*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Timed out
```

3.390.7 Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 171, normalized size of antiderivative = 14.25

$$\int x^m \tanh^3(a + bx)\,dx = \int x^m \operatorname{sech}(bx + a)^3 \sinh(bx + a)^3\,dx$$

input
```
integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
x*e^(6*b*x + m*log(x) + 6*a)/((m + 1)*e^(6*b*x + 6*a) + 3*(m + 1)*e^(4*b*x
 + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate((3*(2*b*x*e^(6*a)
 + (m + 1)*e^(6*a))*e^(6*b*x) - 2*(m + 1)*e^(2*b*x + 2*a) + m + 1)*x^m/((m
 + 1)*e^(8*b*x + 8*a) + 4*(m + 1)*e^(6*b*x + 6*a) + 6*(m + 1)*e^(4*b*x + 4
*a) + 4*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

3.390.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \tanh^3(a + bx)\,dx = \int x^m \operatorname{sech}(bx + a)^3 \sinh(bx + a)^3\,dx$$

input
```
integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

3.390.9 Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \tanh^3(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)^3}{\cosh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^m*sinh(a + b*x)^3)/cosh(a + b*x)^3,x)
```

output
```
int((x^m*sinh(a + b*x)^3)/cosh(a + b*x)^3, x)
```

3.391 $\int x^3 \tanh^3(a+bx)\,dx$

3.391.1 Optimal result

Integrand size = 12, antiderivative size = 183

$$\int x^3 \tanh^3(a+bx)\,dx = -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} +\frac{3\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} -\frac{3x\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{3\,\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4} -\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{x^3\tanh^2(a+bx)}{2b}$$

output
```
-3/2*x^2/b^2+1/2*x^3/b-1/4*x^4+3*x*ln(1+exp(2*b*x+2*a))/b^3+x^3*ln(1+exp(2
*b*x+2*a))/b+3/2*polylog(2,-exp(2*b*x+2*a))/b^4+3/2*x^2*polylog(2,-exp(2*b
*x+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/4*polylog(4,-exp(2*b*x
+2*a))/b^4-3/2*x^2*tanh(b*x+a)/b^2-1/2*x^3*tanh(b*x+a)^2/b
```

3.391.2 Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.36

$$\int x^3 \tanh^3(a+bx)\,dx$$

$$= \frac{1}{4}\left(\frac{12b^2x^2+2b^4x^4+12bx\log\left(1+e^{-2(a+bx)}\right)+12be^{2a}x\log\left(1+e^{-2(a+bx)}\right)+4b^3x^3\log\left(1+e^{-2(a+bx)}\right)+4\cdots}{\cdots} +\frac{2x^3\text{sech}^2(a+bx)}{b}-\frac{6x^2\text{sech}(a)\text{sech}(a+bx)\sinh(bx)}{b^2}+x^4\tanh(a)\right)$$

input
```
Integrate[x^3*Tanh[a + b*x]^3,x]
```

output
```
((12*b^2*x^2 + 2*b^4*x^4 + 12*b*x*Log[1 + E^(-2*(a + b*x))] + 12*b*E^(2*a)
*x*Log[1 + E^(-2*(a + b*x))] + 4*b^3*x^3*Log[1 + E^(-2*(a + b*x))] + 4*b^3
*E^(2*a)*x^3*Log[1 + E^(-2*(a + b*x))] - 6*(1 + E^(2*a))*(1 + b^2*x^2)*Pol
yLog[2, -E^(-2*(a + b*x))] - 6*b*(1 + E^(2*a))*x*PolyLog[3, -E^(-2*(a + b*
x))] - 3*PolyLog[4, -E^(-2*(a + b*x))] - 3*E^(2*a)*PolyLog[4, -E^(-2*(a +
b*x))])/(b^4*(1 + E^(2*a))) + (2*x^3*Sech[a + b*x]^2)/b - (6*x^2*Sech[a]*S
ech[a + b*x]*Sinh[b*x])/b^2 + x^4*Tanh[a])/4
```

3.391.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.26, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.917$, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 4201, 2620, 3011, 4203, 15, 26, 3042, 26, 4201, 2620, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \tanh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix^3 \tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int x^3 \tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{4203}$$

$$i\left(-\int ix^3\tanh(a+bx)dx+\frac{3i\int -x^2\tanh^2(a+bx)dx}{2b}+\frac{ix^3\tanh^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\int ix^3\tanh(a+bx)dx-\frac{3i\int x^2\tanh^2(a+bx)dx}{2b}+\frac{ix^3\tanh^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-i\int x^3\tanh(a+bx)dx-\frac{3i\int x^2\tanh^2(a+bx)dx}{2b}+\frac{ix^3\tanh^2(a+bx)}{2b}\right)$$

$\big\downarrow$ 3042

$$i\left(-i\int -ix^3\tan(ia+ibx)dx-\frac{3i\int -x^2\tan(ia+ibx)^2dx}{2b}+\frac{ix^3\tanh^2(a+bx)}{2b}\right)$$

$\big\downarrow$ 25

$$i\left(-i\int -ix^3\tan(ia+ibx)dx+\frac{3i\int x^2\tan(ia+ibx)^2dx}{2b}+\frac{ix^3\tanh^2(a+bx)}{2b}\right)$$

$\big\downarrow$ 26

$$i\left(-\int x^3\tan(ia+ibx)dx+\frac{3i\int x^2\tan(ia+ibx)^2dx}{2b}+\frac{ix^3\tanh^2(a+bx)}{2b}\right)$$

$\big\downarrow$ 4201

$$i\left(-2i\int\frac{e^{2(a+bx)}x^3}{1+e^{2(a+bx)}}dx+\frac{3i\int x^2\tan(ia+ibx)^2dx}{2b}+\frac{ix^3\tanh^2(a+bx)}{2b}+\frac{ix^4}{4}\right)$$

$\big\downarrow$ 2620

$$i\left(\frac{3i\int x^2\tan(ia+ibx)^2dx}{2b}-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\int x^2\log\left(1+e^{2(a+bx)}\right)dx}{2b}\right)+\frac{ix^3\tanh^2(a+bx)}{2b}+\frac{ix^4}{4}\right.$$

$\big\downarrow$ 3011

$$i\left(-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\operatorname{PolyLog}(2,-e^{2(a+bx)})dx}{b}-\frac{x^2\operatorname{PolyLog}(2,-e^{2(a+bx)})}{2b}\right)}{2b}\right)+\frac{3i\int x^2\tan(ia+ibx)^2dx}{2b}\right.$$

$\big\downarrow$ 4203

$$i\left(-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\operatorname{PolyLog}(2,-e^{2(a+bx)})dx}{b}-\frac{x^2\operatorname{PolyLog}(2,-e^{2(a+bx)})}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\int ix\tanh(a+bx)dx}{b}-\int\right.}{2b}\right.$$

$\big\downarrow$ 15

$$i\left(-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\operatorname{PolyLog}(2,-e^{2(a+bx)})dx}{b}-\frac{x^2\operatorname{PolyLog}(2,-e^{2(a+bx)})}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\int ix\tanh(a+bx)dx}{b}+x^2\right.}{2b}\right.$$

$\big\downarrow$ 26

3.391. $\int x^3\tanh^3(a+bx)\,dx$

$$i\left(-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(-\frac{2\int x\tanh(a+bx)dx}{b}+\frac{x}{}\right.}{2b}\right.$$

$\downarrow$ 3042

$$i\left(-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(-\frac{2\int -ix\tan(ia+ibx)dx}{b}+\right.}{2b}\right.$$

$\downarrow$ 26

$$i\left(-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\int x\tan(ia+ibx)dx}{b}+\frac{x^2}{}\right.}{2b}\right.$$

$\downarrow$ 4201

$$i\left(-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\left(2i\int\frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}dx-\frac{ix^2}{2}\right.}{b}\right.}{2b}\right.$$

$\downarrow$ 2620

$$i\left(-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\left(2i\left(\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}-\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 2715

$$i\left(\frac{3i\left(\frac{2i\left(2i\left(\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int e^{-2(a+bx)}\log\left(1+e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}\right)-\frac{ix^2}{2}\right)}{b}+\frac{x^2\tanh(a+bx)}{b}-\frac{x^3}{3}\right)}{2b}-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+\right.}{2b}\right.\right.$$

3.391. $\int x^3\tanh^3(a+bx)\,dx$

$$\Big\downarrow \text{2838}$$

$$i\left(-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2(a+bx}}{4b^2}\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{7163}$$

$$i\left(-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-e^{2(a+bx)}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\left(\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2720}$$

$$i\left(-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b}-\frac{\int e^{-2(a+bx)}\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right.}{2b}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$i\left(-2i\left(\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b}-\frac{\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\left(2i\left(\right.\right.}{}\right.}{}\right.$$

input
```
Int[x^3*Tanh[a + b*x]^3,x]
```

3.391. $\int x^3\tanh^3(a+bx)\,dx$

output
```
I*((I/4)*x^4 - (2*I)*((x^3*Log[1 + E^(2*(a + b*x))])/(2*b) - (3*(-1/2*(x^2
*PolyLog[2, -E^(2*(a + b*x))])/b + ((x*PolyLog[3, -E^(2*(a + b*x))])/(2*b)
 - PolyLog[4, -E^(2*(a + b*x))]/(4*b^2))/b))/(2*b)) + ((I/2)*x^3*Tanh[a +
b*x]^2)/b + (((3*I)/2)*(-1/3*x^3 + ((2*I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1
+ E^(2*(a + b*x))])/(2*b) + PolyLog[2, -E^(2*(a + b*x))]/(4*b^2))))/b + (x
^2*Tanh[a + b*x])/b))/b)
```

3.391.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.391. $\int x^3 \tanh^3(a+bx)\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.391. $\int x^3 \tanh^3(a+bx)\,dx$

3.391.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{x^4}{4}+\frac{x^2(2\,\mathrm{e}^{2bx+2a}bx+3\,\mathrm{e}^{2bx+2a}+3)}{b^2(1+\mathrm{e}^{2bx+2a})^2}+\frac{3\,\mathrm{polylog}(2,-\mathrm{e}^{2bx+2a})}{2b^4}+\frac{3\,\mathrm{polylog}(4,-\mathrm{e}^{2bx+2a})}{4b^4}-\frac{3a^4}{2b^4}-\frac{3a^2}{b^4}-\frac{3x^2}{b^2}-\frac{6ax}{b^3}-$

input
```
int(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^4+x^2*(2*exp(2*b*x+2*a)*b*x+3*exp(2*b*x+2*a)+3)/b^2/(1+exp(2*b*x+2*
a))^2+3/2*polylog(2,-exp(2*b*x+2*a))/b^4+3/4*polylog(4,-exp(2*b*x+2*a))/b^
4-3/2/b^4*a^4-3/b^4*a^2-3/b^2*x^2-6/b^3*a*x-2/b^3*a^3*x+3*x*ln(1+exp(2*b*x
+2*a))/b^3+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b
^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+6/b^4*a*ln(exp(b*x+a))+2/b^4*a^3*l
n(exp(b*x+a))
```

3.391.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 2207, normalized size of antiderivative = 12.06

$$\int x^3\tanh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(b^4*x^4 + (b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^4 +
4*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*sinh(b*x + a)^4 - 2*a^4 + 2*(b^4*x
^4 - 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 + 2*(b^4*x^4
- 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 + 3*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2
)*cosh(b*x + a)^2 - 12*a^2)*sinh(b*x + a)^2 - 12*a^2 - 12*((b^2*x^2 + 1)*c
osh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2
+ 1)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 2*(b^2*
x^2 + 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 +
 1)*cosh(b*x + a)^3 + (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilo
g(I*cosh(b*x + a) + I*sinh(b*x + a)) - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 +
 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x +
a)^4 + b^2*x^2 + 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2
 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a
)^3 + (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x +
a) - I*sinh(b*x + a)) + 4*((a^3 + 3*a)*cosh(b*x + a)^4 + 4*(a^3 + 3*a)*cos
h(b*x + a)*sinh(b*x + a)^3 + (a^3 + 3*a)*sinh(b*x + a)^4 + a^3 + 2*(a^3 +
3*a)*cosh(b*x + a)^2 + 2*(a^3 + 3*(a^3 + 3*a)*cosh(b*x + a)^2 + 3*a)*sinh(
b*x + a)^2 + 4*((a^3 + 3*a)*cosh(b*x + a)^3 + (a^3 + 3*a)*cosh(b*x + a))*s
inh(b*x + a) + 3*a)*log(cosh(b*x + a) + sinh(b*x + a) + I) + 4*((a^3 + ...
```

3.391.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \tanh^3(a + bx)\, dx = \text{Timed out}$$

input
```
integrate(x**3*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Timed out
```

3.391.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.29

$$
\begin{aligned}
&\int x^3 \tanh^3(a+bx)\,dx \\
&= \frac{b^2x^4e^{(4\,bx+4\,a)} + b^2x^4 + 12\,x^2 + 2\left(b^2x^4e^{(2\,a)} + 4\,bx^3e^{(2\,a)} + 6\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{4\left(b^2e^{(4\,bx+4\,a)} + 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} - \frac{b^4x^4 + 6\,b^2x^2}{2\,b^4} \\
&\quad + \frac{4\,b^3x^3\log\left(e^{(2\,bx+2\,a)}+1\right) + 6\,b^2x^2\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right) - 6\,bx\mathrm{Li}_3\left(-e^{(2\,bx+2\,a)}\right) + 3\,\mathrm{Li}_4\left(-e^{(2\,bx+2\,a)}\right)}{3\,b^4} \\
&\quad + \frac{3\left(2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right) + \mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)\right)}{2\,b^4}
\end{aligned}
$$

input
```
integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(b^2*x^4*e^(4*b*x + 4*a) + b^2*x^4 + 12*x^2 + 2*(b^2*x^4*e^(2*a) + 4*b
*x^3*e^(2*a) + 6*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2
*b*x + 2*a) + b^2) - 1/2*(b^4*x^4 + 6*b^2*x^2)/b^4 + 1/3*(4*b^3*x^3*log(e^
(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)) - 6*b*x*polylog(3,
-e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^4 + 3/2*(2*b*x*log(e
^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^4
```

3.391.8 Giac [F]

$$
\int x^3 \tanh^3(a+bx)\,dx = \int x^3 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3 \,dx
$$

input
```
integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

3.391.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \tanh^3(a+bx)\,dx = \int \frac{x^3 \sinh(a+b\,x)^3}{\cosh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^3*sinh(a + b*x)^3)/cosh(a + b*x)^3,x)
```

output
```
int((x^3*sinh(a + b*x)^3)/cosh(a + b*x)^3, x)
```

3.392 $\int x^2 \tanh^3(a+bx)\,dx$

3.392.1 Optimal result

Integrand size = 12, antiderivative size = 116

$$\int x^2 \tanh^3(a+bx)\,dx = \frac{x^2}{2b} - \frac{x^3}{3} + \frac{x^2 \log\left(1+e^{2(a+bx)}\right)}{b} + \frac{\log(\cosh(a+bx))}{b^3} + \frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2} - \frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} - \frac{x\tanh(a+bx)}{b^2} - \frac{x^2\tanh^2(a+bx)}{2b}$$

output
```
1/2*x^2/b-1/3*x^3+x^2*ln(1+exp(2*b*x+2*a))/b+ln(cosh(b*x+a))/b^3+x*polylog
(2,-exp(2*b*x+2*a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b^3-x*tanh(b*x+a)/b
^2-1/2*x^2*tanh(b*x+a)^2/b
```

3.392.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.69

$$\int x^2 \tanh^3(a+bx)\,dx = \frac{e^{2a}\left(12e^{-2a}x - 12(1+e^{-2a})\,x + 4b^2e^{-2a}x^3 + 6b(1+e^{-2a})\,x^2\log\left(1+e^{-2(a+bx)}\right) + \frac{6(1+e^{-2a})\log\left(1+e^{2(a+bx)}\right)}{b} - \right.}{6b^2\left(1+e^{2a}\right)} + \frac{x^2\text{sech}^2(a+bx)}{2b} - \frac{x\,\text{sech}(a)\text{sech}(a+bx)\sinh(bx)}{b^2} + \frac{1}{3}x^3\tanh(a)$$

input
```
Integrate[x^2*Tanh[a + b*x]^3,x]
```

output
```
(E^(2*a)*((12*x)/E^(2*a) - 12*(1 + E^(-2*a))*x + (4*b^2*x^3)/E^(2*a) + 6*b
*(1 + E^(-2*a))*x^2*Log[1 + E^(-2*(a + b*x))] + (6*(1 + E^(-2*a))*Log[1 +
E^(2*(a + b*x))])/b - 6*(1 + E^(-2*a))*x*PolyLog[2, -E^(-2*(a + b*x))] - (
3*(1 + E^(-2*a))*PolyLog[3, -E^(-2*(a + b*x))])/b))/(6*b^2*(1 + E^(2*a)))
+ (x^2*Sech[a + b*x]^2)/(2*b) - (x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b^2 +
(x^3*Tanh[a])/3
```

3.392.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.26, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.583$, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 4201, 2620, 3011, 2720, 4203, 15, 26, 3042, 26, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \tanh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix^2 \tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int x^2 \tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{4203}$$

$$i\left(-\int ix^2 \tanh(a+bx)dx+\frac{i\int -x\tanh^2(a+bx)dx}{b}+\frac{ix^2\tanh^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\int ix^2 \tanh(a+bx)dx-\frac{i\int x\tanh^2(a+bx)dx}{b}+\frac{ix^2\tanh^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-i\int x^2\tanh(a+bx)dx-\frac{i\int x\tanh^2(a+bx)dx}{b}+\frac{ix^2\tanh^2(a+bx)}{2b}\right)$$

$\Big\downarrow$ 3042

$$i\left(-i\int -ix^2\tan(ia+ibx)dx-\frac{i\int -x\tan(ia+ibx)^2dx}{b}+\frac{ix^2\tanh^2(a+bx)}{2b}\right)$$

$\Big\downarrow$ 25

$$i\left(-i\int -ix^2\tan(ia+ibx)dx+\frac{i\int x\tan(ia+ibx)^2dx}{b}+\frac{ix^2\tanh^2(a+bx)}{2b}\right)$$

$\Big\downarrow$ 26

$$i\left(-\int x^2\tan(ia+ibx)dx+\frac{i\int x\tan(ia+ibx)^2dx}{b}+\frac{ix^2\tanh^2(a+bx)}{2b}\right)$$

$\Big\downarrow$ 4201

$$i\left(-2i\int \frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}dx+\frac{i\int x\tan(ia+ibx)^2dx}{b}+\frac{ix^2\tanh^2(a+bx)}{2b}+\frac{ix^3}{3}\right)$$

$\Big\downarrow$ 2620

$$i\left(-2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int x\log\left(1+e^{2(a+bx)}\right)dx}{b}\right)+\frac{i\int x\tan(ia+ibx)^2dx}{b}+\frac{ix^2\tanh^2(a+bx)}{2b}+\frac{ix^3}{3}\right)$$

$\Big\downarrow$ 3011

$$i\left(-2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int \text{PolyLog}(2,-e^{2(a+bx)})dx}{2b}-\frac{x\,\text{PolyLog}(2,-e^{2(a+bx)})}{2b}}{b}\right)+\frac{i\int x\tan(ia+ibx)^2dx}{b}+\frac{ix^2\tan}{}\right.$$

$\Big\downarrow$ 2720

$$i\left(-2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\text{PolyLog}(2,-e^{2(a+bx)})de^{2(a+bx)}}{4b^2}-\frac{x\,\text{PolyLog}(2,-e^{2(a+bx)})}{2b}}{b}\right)+\frac{i\int x\tan(ia+ibx)}{b}\right.$$

$\Big\downarrow$ 4203

$$i\left(-2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\text{PolyLog}(2,-e^{2(a+bx)})de^{2(a+bx)}}{4b^2}-\frac{x\,\text{PolyLog}(2,-e^{2(a+bx)})}{2b}}{b}\right)+\frac{i\left(\frac{i\int i\tanh(a+bx)dx}{b}\right.}{}\right.$$

$\Big\downarrow$ 15

3.392. $\int x^2\tanh^3(a+bx)\,dx$

$$i\left(-2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)+\frac{i\left(\frac{i\int i\tanh(a+bx)dx}{b}\right.}{}\right.$$

$$\Big\downarrow \textbf{26}$$

$$i\left(-2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)+\frac{i\left(-\frac{\int\tanh(a+bx)dx}{b}\right.}{}\right.$$

$$\Big\downarrow \textbf{3042}$$

$$i\left(-2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)+\frac{i\left(-\frac{\int -i\tan(ia+ibx)}{b}\right.}{}\right.$$

$$\Big\downarrow \textbf{26}$$

$$i\left(-2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)+\frac{i\left(\frac{i\int\tan(ia+ibx)dx}{b}\right.}{}\right.$$

$$\Big\downarrow \textbf{3956}$$

$$i\left(-2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\int e^{-2(a+bx)}\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)+\frac{i\left(-\frac{\log(\cosh(a+bx))}{b^2}\right.}{}\right.$$

$$\Big\downarrow \textbf{7143}$$

$$i\left(-2i\left(\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\frac{\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}}{b}\right)+\frac{i\left(-\frac{\log(\cosh(a+bx))}{b^2}+\frac{x\tanh(a+bx)}{b}-\right.}{b}\right.$$

input
```
Int[x^2*Tanh[a + b*x]^3,x]
```

output
```
I*((I/3)*x^3 - (2*I)*((x^2*Log[1 + E^(2*(a + b*x))])/(2*b) - (-1/2*(x*Poly
Log[2, -E^(2*(a + b*x))])/b + PolyLog[3, -E^(2*(a + b*x))]/(4*b^2))/b) + (
(I/2)*x^2*Tanh[a + b*x]^2)/b + (I*(-1/2*x^2 - Log[Cosh[a + b*x]]/b^2 + (x*
Tanh[a + b*x])/b))/b)
```

3.392. $\int x^2\tanh^3(a+bx)\,dx$

3.392.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.392.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{x^{3}}{3}+\frac{2 x \left({\mathrm e}^{2 b x +2 a} b x +{\mathrm e}^{2 b x +2 a}+1\right)}{b^{2} \left(1+{\mathrm e}^{2 b x +2 a}\right)^{2}}+\frac{\ln \left(1+{\mathrm e}^{2 b x +2 a}\right)}{b^{3}}-\frac{2 \ln \left({\mathrm e}^{b x +a}\right)}{b^{3}}-\frac{2 a^{2} \ln \left({\mathrm e}^{b x +a}\right)}{b^{3}}+\frac{2 a^{2} x}{b^{2}}+\frac{4 a^{3}}{3 b^{3}}+\frac{x^{2} \ln \left(1+{\mathrm e}^{2 b x +2 a}\right)}{b}$

input
```
int(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3+2*x*(exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^
2+1/b^3*ln(1+exp(2*b*x+2*a))-2/b^3*ln(exp(b*x+a))-2/b^3*a^2*ln(exp(b*x+a))
+2/b^2*a^2*x+4/3/b^3*a^3+x^2*ln(1+exp(2*b*x+2*a))/b+x*polylog(2,-exp(2*b*x
+2*a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b^3
```

3.392.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1649, normalized size of antiderivative = 14.22

$$\int x^2 \tanh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/3*(b^3*x^3 + (b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^4 + 4*(b^3*x
^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 2*a^3
 + 6*b*x + 6*a)*sinh(b*x + a)^4 + 2*a^3 + 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 +
 3*b*x + 6*a)*cosh(b*x + a)^2 + 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*(b^3*x^
3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^2 + 3*b*x + 6*a)*sinh(b*x + a)^2 -
6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*
x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*
x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a)
)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*
x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a
)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(
b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*
sinh(b*x + a)) - 3*((a^2 + 1)*cosh(b*x + a)^4 + 4*(a^2 + 1)*cosh(b*x + a)*
sinh(b*x + a)^3 + (a^2 + 1)*sinh(b*x + a)^4 + 2*(a^2 + 1)*cosh(b*x + a)^2
+ 2*(3*(a^2 + 1)*cosh(b*x + a)^2 + a^2 + 1)*sinh(b*x + a)^2 + a^2 + 4*((a^
2 + 1)*cosh(b*x + a)^3 + (a^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*log(c
osh(b*x + a) + sinh(b*x + a) + I) - 3*((a^2 + 1)*cosh(b*x + a)^4 + 4*(a^2
+ 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 + 1)*sinh(b*x + a)^4 + 2*(a^2 +
1)*cosh(b*x + a)^2 + 2*(3*(a^2 + 1)*cosh(b*x + a)^2 + a^2 + 1)*sinh(b*x +
a)^2 + a^2 + 4*((a^2 + 1)*cosh(b*x + a)^3 + (a^2 + 1)*cosh(b*x + a))*si...
```

3.392.6 Sympy [F]

$$\int x^2 \tanh^3(a+bx)\,dx = \int x^2 \sinh^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(x**2*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Integral(x**2*sinh(a + b*x)**3*sech(a + b*x)**3, x)
```

3.392.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.58

$$\int x^2 \tanh^3(a+bx)\,dx$$
$$= -\frac{2}{3}x^3 + \frac{b^2x^3e^{(4\,bx+4\,a)} + b^2x^3 + 2\left(b^2x^3e^{(2\,a)} + 3\,bx^2e^{(2\,a)} + 3\,xe^{(2\,a)}\right)e^{(2\,bx)} + 6\,x}{3\left(b^2e^{(4\,bx+4\,a)} + 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)}$$
$$- \frac{2\,x}{b^2} + \frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right) - \mathrm{Li}_3\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^3}$$
$$+ \frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b^3}$$

input
```
integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2/3*x^3 + 1/3*(b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3 + 2*(b^2*x^3*e^(2*a) + 3
*b*x^2*e^(2*a) + 3*x*e^(2*a))*e^(2*b*x) + 6*x)/(b^2*e^(4*b*x + 4*a) + 2*b^
2*e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) +
1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3 + l
og(e^(2*b*x + 2*a) + 1)/b^3
```

3.392.8 Giac [F]

$$\int x^2 \tanh^3(a+bx)\,dx = \int x^2\,\mathrm{sech}\,(bx+a)^3\sinh\,(bx+a)^3\,dx$$

input
```
integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \tanh^3(a+bx)\,dx = \int \frac{x^2 \sinh(a+b\,x)^3}{\cosh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^2*sinh(a + b*x)^3)/cosh(a + b*x)^3,x)
```

output
```
int((x^2*sinh(a + b*x)^3)/cosh(a + b*x)^3, x)
```

3.393 $\int x\tanh^3(a+bx)\,dx$

3.393.1 Optimal result

Integrand size = 10, antiderivative size = 82

$$\int x\tanh^3(a+bx)\,dx=\frac{x}{2b}-\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}$$

output
```
1/2*x/b-1/2*x^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^
2-1/2*tanh(b*x+a)/b^2-1/2*x*tanh(b*x+a)^2/b
```

3.393.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int x\tanh^3(a+bx)\,dx=\frac{bx\left(bx+2\log\left(1+e^{-2(a+bx)}\right)\right)-\operatorname{PolyLog}\left(2,-e^{-2(a+bx)}\right)+bx\operatorname{sech}^2(a+bx)-\operatorname{sech}(a)\operatorname{sech}(a+bx)\sinh(b}{2b^2}$$

input
```
Integrate[x*Tanh[a + b*x]^3,x]
```

output
```
(b*x*(b*x + 2*Log[1 + E^(-2*(a + b*x))]) - PolyLog[2, -E^(-2*(a + b*x))] +
 b*x*Sech[a + b*x]^2 - Sech[a]*Sech[a + b*x]*Sinh[b*x])/(2*b^2)
```

3.393.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 3954, 24, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tanh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int x\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{4203}$$

$$i\left(\frac{i\int -\tanh^2(a+bx)dx}{2b}-\int ix\tanh(a+bx)dx+\frac{ix\tanh^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{i\int \tanh^2(a+bx)dx}{2b}-\int ix\tanh(a+bx)dx+\frac{ix\tanh^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{i\int \tanh^2(a+bx)dx}{2b}-i\int x\tanh(a+bx)dx+\frac{ix\tanh^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-i\int -ix\tan(ia+ibx)dx-\frac{i\int -\tan(ia+ibx)^2dx}{2b}+\frac{ix\tanh^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-i\int -ix\tan(ia+ibx)dx+\frac{i\int \tan(ia+ibx)^2dx}{2b}+\frac{ix\tanh^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\int x\tan(ia+ibx)dx+\frac{i\int \tan(ia+ibx)^2dx}{2b}+\frac{ix\tanh^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3954}$$

$$i\left(-\int x\tan(ia+ibx)dx+\frac{i\left(\frac{\tanh(a+bx)}{b}-\int 1dx\right)}{2b}+\frac{ix\tanh^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{24}$$

$$i\left(-\int x\tan(ia+ibx)dx+\frac{ix\tanh^2(a+bx)}{2b}+\frac{i\left(\frac{\tanh(a+bx)}{b}-x\right)}{2b}\right)$$

$$\Big\downarrow \text{4201}$$

$$i\left(-2i\int\frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}dx+\frac{ix\tanh^2(a+bx)}{2b}+\frac{i\left(\frac{\tanh(a+bx)}{b}-x\right)}{2b}+\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$i\left(-2i\left(\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int\log\left(1+e^{2(a+bx)}\right)dx}{2b}\right)+\frac{ix\tanh^2(a+bx)}{2b}+\frac{i\left(\frac{\tanh(a+bx)}{b}-x\right)}{2b}+\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$i\left(-2i\left(\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{\int e^{-2(a+bx)}\log\left(1+e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}\right)+\frac{ix\tanh^2(a+bx)}{2b}+\frac{i\left(\frac{\tanh(a+bx)}{b}-x\right)}{2b}\right.$$

$$\Big\downarrow \text{2838}$$

$$i\left(-2i\left(\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{4b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{2b}\right)+\frac{ix\tanh^2(a+bx)}{2b}+\frac{i\left(\frac{\tanh(a+bx)}{b}-x\right)}{2b}+\frac{ix^2}{2}\right)$$

input
```
Int[x*Tanh[a + b*x]^3,x]
```

output
```
I*((I/2)*x^2 - (2*I)*((x*Log[1 + E^(2*(a + b*x))])/(2*b) + PolyLog[2, -E^(
2*(a + b*x))]/(4*b^2)) + ((I/2)*x*Tanh[a + b*x]^2)/b + ((I/2)*(-x + Tanh[a
 + b*x]/b))/b)
```

3.393. $\int x\tanh^3(a+bx)\,dx$

3.393.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.393. $\int x \tanh^3(a + bx) \, dx$

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.393.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.35

method	result	size
risch	$-\frac{x^{2}}{2}+\frac{2\,{\mathrm e}^{2 b x +2 a} b x +{\mathrm e}^{2 b x +2 a}+1}{b^{2} \left(1+{\mathrm e}^{2 b x +2 a}\right)^{2}}-\frac{2 a x}{b}-\frac{a^{2}}{b^{2}}+\frac{x \ln \left(1+{\mathrm e}^{2 b x +2 a}\right)}{b}+\frac{\operatorname{polylog}\left(2,-{\mathrm e}^{2 b x +2 a}\right)}{2 b^{2}}+\frac{2 a \ln \left({\mathrm e}^{b x +a}\right)}{b^{2}}$	111

input
```
int(x*sech(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2+(2*exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2-
2/b*a*x-a^2/b^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^
2+2/b^2*a*ln(exp(b*x+a))
```

3.393.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1106, normalized size of antiderivative = 13.49

$$\int x\tanh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x + a
)*sinh(b*x + a)^3 + (b^2*x^2 - 2*a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x
^2 - 2*a^2 - 2*b*x - 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - 2*a^2)
*cosh(b*x + a)^2 - 2*a^2 - 2*b*x - 1)*sinh(b*x + a)^2 - 2*a^2 - 2*(cosh(b*
x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b
*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 +
cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a))
 - 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
+ 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(
b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*
sinh(b*x + a)) + 2*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3
+ a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*si
nh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)
*log(cosh(b*x + a) + sinh(b*x + a) + I) + 2*(a*cosh(b*x + a)^4 + 4*a*cosh(
b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*
a*cosh(b*x + a)^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x
 + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 2*((b*x
 + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x +
 a)*sinh(b*x + a)^4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*
x + a)^2 + b*x + a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^...
```

3.393.6 Sympy [F]

$$\int x\tanh^3(a+bx)\,dx=\int x\sinh^3{(a+bx)}\operatorname{sech}^3{(a+bx)}\,dx$$

input
```
integrate(x*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Integral(x*sinh(a + b*x)**3*sech(a + b*x)**3, x)
```

3.393.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.60

$$\int x\tanh^3(a+bx)\,dx = -x^2 + \frac{b^2x^2e^{(4\,bx+4\,a)} + b^2x^2 + 2\left(b^2x^2e^{(2\,a)} + 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)} + 2}{2\left(b^2e^{(4\,bx+4\,a)} + 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} + \frac{2\,bx\log\left(e^{(2\,bx+2\,a)} + 1\right) + \mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}$$

input
```
integrate(x*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-x^2 + 1/2*(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 + 2*(b^2*x^2*e^(2*a) + 2*b*x
*e^(2*a) + e^(2*a))*e^(2*b*x) + 2)/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x +
 2*a) + b^2) + 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a
)))/b^2
```

3.393.8 Giac [F]

$$\int x\tanh^3(a+bx)\,dx = \int x\,\mathrm{sech}\left(bx+a\right)^3\sinh\left(bx+a\right)^3\,dx$$

input
```
integrate(x*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int x\tanh^3(a+bx)\,dx = \int \frac{x\sinh(a+b\,x)^3}{\cosh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x*sinh(a + b*x)^3)/cosh(a + b*x)^3,x)
```

output
```
int((x*sinh(a + b*x)^3)/cosh(a + b*x)^3, x)
```

3.394 $\int \tanh^3(a+bx)\,dx$

3.394.1 Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \tanh^3(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

output
```
ln(cosh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

3.394.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \tanh^3(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

input
```
Integrate[Tanh[a + b*x]^3,x]
```

output
```
Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

3.394.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {3042, 26, 3954, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{3954}$$

$$i\left(\frac{i\tanh^2(a+bx)}{2b}-\int i\tanh(a+bx)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\tanh^2(a+bx)}{2b}-i\int \tanh(a+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\tanh^2(a+bx)}{2b}-i\int -i\tan(ia+ibx)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\tanh^2(a+bx)}{2b}-\int \tan(ia+ibx)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$i\left(\frac{i\tanh^2(a+bx)}{2b}-\frac{i\log(\cosh(a+bx))}{b}\right)$$

input
```
Int[Tanh[a + b*x]^3,x]
```

output
```
I*(((-I)*Log[Cosh[a + b*x]])/b + ((I/2)*Tanh[a + b*x]^2)/b)
```

3.394.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.394.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\ln(\cosh(bx+a))-\frac{\tanh(bx+a)^2}{2}}{b}$	23
default	$\frac{\ln(\cosh(bx+a))-\frac{\tanh(bx+a)^2}{2}}{b}$	23
parallelrisch	$\frac{-\tanh(bx+a)^2-2bx-2\ln(1-\tanh(bx+a))}{2b}$	34
risch	$-x-\frac{2a}{b}+\frac{2\,e^{2bx+2a}}{b\left(1+e^{2bx+2a}\right)^2}+\frac{\ln\left(1+e^{2bx+2a}\right)}{b}$	54

input
```
int(sech(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(ln(cosh(b*x+a))-1/2*tanh(b*x+a)^2)
```

3.394.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 339, normalized size of antiderivative = 12.56

$$\int \tanh^3(a+bx)\,dx = -\frac{bx\cosh(bx+a)^4 + 4\,bx\cosh(bx+a)\sinh(bx+a)^3 + bx\sinh(bx+a)^4 + 2\,(bx-1)\cosh(bx+a)^2 + 2}{}$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
 + a)^4 + 2*(b*x - 1)*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x - 1
)*sinh(b*x + a)^2 + b*x - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)
^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(
b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*
cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(b*x*cosh(b*x + a)^3 +
(b*x - 1)*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x
+ a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*co
sh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a
))*sinh(b*x + a) + b)
```

3.394.6 Sympy [F]

$$\int \tanh^3(a+bx)\,dx = \int \sinh^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Integral(sinh(a + b*x)**3*sech(a + b*x)**3, x)
```

3.394.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(25) = 50$.

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \tanh^3(a+bx)\,dx = x + \frac{a}{b} + \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1)}$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
x + a/b + log(e^(-2*b*x - 2*a) + 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

3.394.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \tanh^3(a+bx)\,dx = -\frac{2\,bx+2\,a+\frac{3\,e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+3}{\left(e^{(2\,bx+2\,a)}+1\right)^2}-2\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{2\,b}$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*(2*b*x + 2*a + (3*e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 3)/(e^(2*b*x
+ 2*a) + 1)^2 - 2*log(e^(2*b*x + 2*a) + 1))/b
```

3.394.9 Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \tanh^3(a+bx)\,dx = \frac{1}{2\,b\cosh\left(a+b\,x\right)^2} + \frac{\ln\left(\cosh(a+b\,x)\right)}{b}$$

input
```
int(sinh(a + b*x)^3/cosh(a + b*x)^3,x)
```

output
```
1/(2*b*cosh(a + b*x)^2) + log(cosh(a + b*x))/b
```

3.395 $\int \frac{\tanh^3(a+bx)}{x}\,dx$

3.395.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\tanh^3(a+bx)}{x}\,dx = \text{Int}\left(\frac{\tanh^3(a+bx)}{x},x\right)$$

output
```
Unintegrable(tanh(b*x+a)^3/x,x)
```

3.395.2 Mathematica [N/A]

Not integrable

Time = 13.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^3(a+bx)}{x}\,dx = \int \frac{\tanh^3(a+bx)}{x}\,dx$$

input
```
Integrate[Tanh[a + b*x]^3/x,x]
```

output
```
Integrate[Tanh[a + b*x]^3/x, x]
```

3.395.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(a+bx)}{x} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{i\tan(ia+ibx)^3}{x}dx$$
$$\Big\downarrow \; 26$$
$$i\int \frac{\tan(ia+ibx)^3}{x}dx$$
$$\Big\downarrow \; 4222$$
$$\int \frac{\tanh^3(a+bx)}{x}dx$$

input
```
Int[Tanh[a + b*x]^3/x,x]
```

output
```
$Aborted
```

3.395.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.395.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x} dx$$

input
```
int(sech(b*x+a)^3*sinh(b*x+a)^3/x,x)
```

output
```
int(sech(b*x+a)^3*sinh(b*x+a)^3/x,x)
```

3.395.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^3(a+bx)}{x} dx = \int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x} dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^3*sinh(b*x + a)^3/x, x)
```

3.395.6 Sympy [N/A]

Not integrable

Time = 19.86 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{\tanh^3(a+bx)}{x}\,dx = \int \frac{\sinh^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

input
```
integrate(sech(b*x+a)**3*sinh(b*x+a)**3/x,x)
```

output
```
Integral(sinh(a + b*x)**3*sech(a + b*x)**3/x, x)
```

3.395.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 9.25

$$\int \frac{\tanh^3(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^3}{x}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^2*e^(4*b*x + 4*a) + 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) - integrate(2*(b^2*x^2 + 1)/(b^2*x^3*e^(2*
b*x + 2*a) + b^2*x^3), x) + log(x)
```

3.395.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^3(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^3}{x}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^3*sinh(b*x + a)^3/x, x)
```

3.395.9 Mupad [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^3(a+bx)}{x} \, dx = \int \frac{\sinh(a+b\,x)^3}{x\cosh\left(a+b\,x\right)^3} \, dx$$

input
```
int(sinh(a + b*x)^3/(x*cosh(a + b*x)^3),x)
```

output
```
int(sinh(a + b*x)^3/(x*cosh(a + b*x)^3), x)
```

3.396 $\int \frac{\tanh^3(a+bx)}{x^2}\,dx$

3.396.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \operatorname{Int}\left(\frac{\tanh^3(a+bx)}{x^2},x\right)$$

output
```
Unintegrable(tanh(b*x+a)^3/x^2,x)
```

3.396.2 Mathematica [N/A]

Not integrable

Time = 8.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \int \frac{\tanh^3(a+bx)}{x^2}\,dx$$

input
```
Integrate[Tanh[a + b*x]^3/x^2,x]
```

output
```
Integrate[Tanh[a + b*x]^3/x^2, x]
```

3.396.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(a+bx)}{x^2} dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{i\tan(ia+ibx)^3}{x^2}dx$$
$$\Big\downarrow \text{ 26}$$
$$i\int \frac{\tan(ia+ibx)^3}{x^2}dx$$
$$\Big\downarrow \text{ 4222}$$
$$\int \frac{\tanh^3(a+bx)}{x^2}dx$$

input
```
Int[Tanh[a + b*x]^3/x^2,x]
```

output
```
$Aborted
```

3.396.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.396.4 Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x^2} dx$$

input
```
int(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x)
```

output
```
int(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x)
```

3.396.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^3(a+bx)}{x^2} dx = \int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x^2} dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^3*sinh(b*x + a)^3/x^2, x)
```

3.396.6 Sympy [N/A]

Not integrable

Time = 27.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \int \frac{\sinh^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)**3*sinh(b*x+a)**3/x**2,x)
```

output
```
Integral(sinh(a + b*x)**3*sech(a + b*x)**3/x**2, x)
```

3.396.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 11.92

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^3}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
-(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 + 2*(b^2*x^2*e^(2*a) - b*x*e^(2*a) + e
^(2*a))*e^(2*b*x) + 2)/(b^2*x^3*e^(4*b*x + 4*a) + 2*b^2*x^3*e^(2*b*x + 2*a
) + b^2*x^3) - integrate(2*(b^2*x^2 + 3)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^
4), x)
```

3.396.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^3}{x^2}\,dx$$

input
```
integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^3*sinh(b*x + a)^3/x^2, x)
```

3.396.9 Mupad [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)^3}{x^2\cosh\left(a+b\,x\right)^3}\,dx$$

input
```
int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)^3),x)
```

output
```
int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)^3), x)
```

3.397 $\int x^m \coth(a+bx)\,dx$

3.397.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \coth(a+bx)\,dx = \text{Int}(x^m \coth(a+bx),x)$$

output
```
Unintegrable(x^m*coth(b*x+a),x)
```

3.397.2 Mathematica [N/A]

Not integrable

Time = 6.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \coth(a+bx)\,dx = \int x^m \coth(a+bx)\,dx$$

input
```
Integrate[x^m*Coth[a + b*x],x]
```

output
```
Integrate[x^m*Coth[a + b*x], x]
```

3.397.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^m \tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x^m \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{4222}$$

$$\int ix^m \tan\left(\frac{1}{2}(-\pi-2ia)-ibx\right)dx$$

input
```
Int[x^m*Coth[a + b*x],x]
```

output
```
$Aborted
```

3.397.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.397.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int x^m \cosh(bx+a)\operatorname{csch}(bx+a)\,dx$$

input
```
int(x^m*cosh(b*x+a)*csch(b*x+a),x)
```

output
```
int(x^m*cosh(b*x+a)*csch(b*x+a),x)
```

3.397.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int x^m \coth(a+bx)\,dx = \int x^m \cosh(bx+a)\operatorname{csch}(bx+a)\,dx$$

input
```
integrate(x^m*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*cosh(b*x + a)*csch(b*x + a), x)
```

3.397.6 Sympy [N/A]

Not integrable

Time = 108.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int x^m \coth(a+bx)\,dx = \int x^m \cosh(a+bx)\operatorname{csch}(a+bx)\,dx$$

input
```
integrate(x**m*cosh(b*x+a)*csch(b*x+a),x)
```

output
```
Integral(x**m*cosh(a + b*x)*csch(a + b*x), x)
```

3.397.7 Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 10.20

$$\int x^m \coth(a+bx)\,dx = \int x^m \cosh(bx+a)\operatorname{csch}(bx+a)\,dx$$

input
```
integrate(x^m*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

output
```
x*e^(2*b*x + m*log(x) + 2*a)/((m + 1)*e^(2*b*x + 2*a) - m - 1) + integrate
(((2*b*x*e^(2*a) + (m + 1)*e^(2*a))*e^(2*b*x) - m - 1)*x^m/((m + 1)*e^(4*b
*x + 4*a) - 2*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

3.397.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int x^m \coth(a+bx)\,dx = \int x^m \cosh(bx+a)\operatorname{csch}(bx+a)\,dx$$

input
```
integrate(x^m*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)*csch(b*x + a), x)
```

3.397.9 Mupad [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int x^m \coth(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)}{\sinh\left(a+b\,x\right)}\,dx$$

input
```
int((x^m*cosh(a + b*x))/sinh(a + b*x),x)
```

output
```
int((x^m*cosh(a + b*x))/sinh(a + b*x), x)
```

3.398 $\int x^3 \coth(a+bx)\,dx$

3.398.1 Optimal result

Integrand size = 10, antiderivative size = 87

$$\int x^3 \coth(a+bx)\,dx = -\frac{x^4}{4} + \frac{x^3 \log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3} + \frac{3 \operatorname{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}$$

output
```
-1/4*x^4+x^3*ln(1-exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,exp(2*b*x+2*a))/b^2-
3/2*x*polylog(3,exp(2*b*x+2*a))/b^3+3/4*polylog(4,exp(2*b*x+2*a))/b^4
```

3.398.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int x^3 \coth(a+bx)\,dx = -\frac{x^4}{4} + \frac{x^3 \log\left(1-e^{2a+2bx}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3} + \frac{3 \operatorname{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}$$

input
```
Integrate[x^3*Coth[a + b*x],x]
```

output
```
-1/4*x^4 + (x^3*Log[1 - E^(2*a + 2*b*x)])/b + (3*x^2*PolyLog[2, E^(2*a + 2
*b*x)])/(2*b^2) - (3*x*PolyLog[3, E^(2*a + 2*b*x)])/(2*b^3) + (3*PolyLog[4
, E^(2*a + 2*b*x)])/(4*b^4)
```

3.398.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.63, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 26, 4201, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^3 \tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x^3 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{4201}$$

$$-i\left(2i\int \frac{e^{2a+2bx-i\pi}x^3}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{2620}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\int x^2\log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{3011}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{7163}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}\right)}{2b}\right)\right)$$

$$\Big\downarrow \text{2720}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}}{}\right.}{2b}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\text{PolyLog}\left(4,-e^{2a+2bx-i\pi}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{i}{}\right.$$

input
```
Int[x^3*Coth[a + b*x],x]
```

output
```
(-I)*((-1/4*I)*x^4 + (2*I)*((x^3*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) -
(3*(-1/2*(x^2*PolyLog[2, -E^(2*a - I*Pi + 2*b*x)])/b + ((x*PolyLog[3, -E^(
2*a - I*Pi + 2*b*x)])/(2*b) - PolyLog[4, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2))
/b))/(2*b)))
```

3.398.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.398.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(79) = 158$.

Time = 0.32 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.30

method	result
risch	$-\frac{x^4}{4}+\frac{3x^2 \operatorname{polylog}(2,e^{bx+a})}{b^2}-\frac{6x \operatorname{polylog}(3,e^{bx+a})}{b^3}+\frac{\ln(e^{bx+a}+1)x^3}{b}+\frac{3x^2 \operatorname{polylog}(2,-e^{bx+a})}{b^2}-\frac{6x \operatorname{polylog}(3,-e^{bx+a})}{b^3}$

input
```
int(x^3*cosh(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^4+3*x^2*polylog(2,exp(b*x+a))/b^2-6*x*polylog(3,exp(b*x+a))/b^3+1/b
*ln(exp(b*x+a)+1)*x^3+3*x^2*polylog(2,-exp(b*x+a))/b^2-6*x*polylog(3,-exp(
b*x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^3-3/2/b^4*a^4-2/b^3*a^3*x+6*polylog(4,-
exp(b*x+a))/b^4+6*polylog(4,exp(b*x+a))/b^4+1/b^4*ln(1-exp(b*x+a))*a^3+2/b
^4*a^3*ln(exp(b*x+a))-1/b^4*a^3*ln(exp(b*x+a)-1)
```

3.398.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(78) = 156.

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.48

$$\int x^3 \coth(a+bx)\,dx =$$
$$-\frac{b^4x^4 - 4\,b^3x^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right) - 12\,b^2x^2\mathrm{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)) - 12\,b}{}$$

input
```
integrate(x^3*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(b^4*x^4 - 4*b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 12*b^2*
x^2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 12*b^2*x^2*dilog(-cosh(b*x + a)
 - sinh(b*x + a)) + 4*a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 24*b*x*
polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 24*b*x*polylog(3, -cosh(b*x +
a) - sinh(b*x + a)) - 4*(b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a)
 + 1) - 24*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 24*polylog(4, -cosh
(b*x + a) - sinh(b*x + a)))/b^4
```

3.398.6 Sympy [F]

$$\int x^3 \coth(a+bx)\,dx = \int x^3 \cosh\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

input
```
integrate(x**3*cosh(b*x+a)*csch(b*x+a),x)
```

output
```
Integral(x**3*cosh(a + b*x)*csch(a + b*x), x)
```

3.398.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.49

$$\int x^3\coth(a+bx)\,dx$$
$$= -\frac{1}{4}x^4 + \frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4} + \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(e^{(bx+a)})+6\,\mathrm{Li}_4(e^{(bx+a)})}{b^4}$$

input
```
integrate(x^3*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*x^4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) -
 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x
^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3,
 e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4
```

3.398.8 Giac [F]

$$\int x^3\coth(a+bx)\,dx = \int x^3\cosh(bx+a)\operatorname{csch}(bx+a)\,dx$$

input
```
integrate(x^3*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*cosh(b*x + a)*csch(b*x + a), x)
```

3.398.9 Mupad [F(-1)]

Timed out.

$$\int x^3\coth(a+bx)\,dx = \int \frac{x^3\cosh(a+b\,x)}{\sinh(a+b\,x)}\,dx$$

input
```
int((x^3*cosh(a + b*x))/sinh(a + b*x),x)
```

output
```
int((x^3*cosh(a + b*x))/sinh(a + b*x), x)
```

3.399 $\int x^2 \coth(a+bx)\,dx$

3.399.1 Optimal result

Integrand size = 10, antiderivative size = 63

$$\int x^2 \coth(a+bx)\,dx = -\frac{x^3}{3} + \frac{x^2 \log\left(1-e^{2(a+bx)}\right)}{b} + \frac{x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2} - \frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}$$

output
```
-1/3*x^3+x^2*ln(1-exp(2*b*x+2*a))/b+x*polylog(2,exp(2*b*x+2*a))/b^2-1/2*po
lylog(3,exp(2*b*x+2*a))/b^3
```

3.399.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int x^2 \coth(a+bx)\,dx = -\frac{x^3}{3} + \frac{x^2 \log\left(1-e^{2a+2bx}\right)}{b} + \frac{x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} - \frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}$$

input
```
Integrate[x^2*Coth[a + b*x],x]
```

output
```
-1/3*x^3 + (x^2*Log[1 - E^(2*a + 2*b*x)])/b + (x*PolyLog[2, E^(2*a + 2*b*x
)])/b^2 - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

3.399.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^2 \tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x^2 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{4201}$$

$$-i\left(2i\int \frac{e^{2a+2bx-i\pi}x^2}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2620}$$

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int x\log\left(1+e^{2a+2bx-i\pi}\right)dx}{b}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{3011}$$

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int \text{PolyLog}(2,-e^{2a+2bx-i\pi})dx}{2b}-\frac{x\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2720}$$

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{7143}$$

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\text{PolyLog}(3,-e^{2a+2bx-i\pi})}{4b^2}-\frac{x\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x^2*Coth[a + b*x],x]
```

output
```
(-I)*((-1/3*I)*x^3 + (2*I)*((x^2*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) -
(-1/2*(x*PolyLog[2, -E^(2*a - I*Pi + 2*b*x)])/b + PolyLog[3, -E^(2*a - I*P
i + 2*b*x)]/(4*b^2))/b))
```

3.399.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.399.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(59) = 118.

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.63

method	result
risch	$-\frac{x^3}{3}+\frac{a^2\ln(\mathrm{e}^{bx+a}-1)}{b^3}-\frac{2a^2\ln(\mathrm{e}^{bx+a})}{b^3}+\frac{2a^2x}{b^2}+\frac{4a^3}{3b^3}+\frac{\ln(\mathrm{e}^{bx+a}+1)x^2}{b}+\frac{2x\,\mathrm{polylog}(2,-\mathrm{e}^{bx+a})}{b^2}-\frac{2\,\mathrm{polylog}(3,-\mathrm{e}^{bx+}}{b^3}$

input
```
int(x^2*cosh(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^2*ln(exp(b*x+a))+2/b^2*a^2*x+4
/3/b^3*a^3+1/b*ln(exp(b*x+a)+1)*x^2+2*x*polylog(2,-exp(b*x+a))/b^2-2*polyl
og(3,-exp(b*x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^2-1/b^3*ln(1-exp(b*x+a))*a^2+
2*x*polylog(2,exp(b*x+a))/b^2-2*polylog(3,exp(b*x+a))/b^3
```

3.399.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(58) = 116.

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.67

$$\int x^2\coth(a+bx)\,dx = -\frac{b^3x^3-3\,b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-6\,bx\mathrm{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-6\,bx\mathrm{Li}_2}{}$$

input
```
integrate(x^2*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

output
```
-1/3*(b^3*x^3 - 3*b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 6*b*x*d
ilog(cosh(b*x + a) + sinh(b*x + a)) - 6*b*x*dilog(-cosh(b*x + a) - sinh(b*
x + a)) - 3*a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*(b^2*x^2 - a^2)
*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*polylog(3, cosh(b*x + a) + si
nh(b*x + a)) + 6*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^3
```

3.399.6 Sympy [F]

$$\int x^2 \coth(a+bx)\,dx = \int x^2 \cosh{(a+bx)}\operatorname{csch}{(a+bx)}\,dx$$

input
```
integrate(x**2*cosh(b*x+a)*csch(b*x+a),x)
```

output
```
Integral(x**2*cosh(a + b*x)*csch(a + b*x), x)
```

3.399.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.52

$$\int x^2 \coth(a+bx)\,dx = -\frac{1}{3}x^3 + \frac{b^2x^2\log\left(e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(-e^{(bx+a)})}{b^3} + \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(e^{(bx+a)})}{b^3}$$

input
```
integrate(x^2*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

output
```
-1/3*x^3 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*p
olylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilo
g(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

3.399.8 Giac [F]

$$\int x^2 \coth(a+bx)\,dx = \int x^2 \cosh(bx+a)\operatorname{csch}(bx+a)\,dx$$

input
```
integrate(x^2*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*cosh(b*x + a)*csch(b*x + a), x)
```

3.399.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)}{\sinh(a+b\,x)}\,dx$$

input
```
int((x^2*cosh(a + b*x))/sinh(a + b*x),x)
```

output
```
int((x^2*cosh(a + b*x))/sinh(a + b*x), x)
```

3.400 $\int x \coth(a+bx)\,dx$

3.400.1 Optimal result

Integrand size = 8, antiderivative size = 45

$$\int x \coth(a+bx)\,dx = -\frac{x^2}{2} + \frac{x \log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2}$$

output
```
-1/2*x^2+x*ln(1-exp(2*b*x+2*a))/b+1/2*polylog(2,exp(2*b*x+2*a))/b^2
```

3.400.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int x \coth(a+bx)\,dx = -\frac{x^2}{2} + \frac{x \log\left(1-e^{2a+2bx}\right)}{b} + \frac{\text{PolyLog}\left(2, e^{2a+2bx}\right)}{2b^2}$$

input
```
Integrate[x*Coth[a + b*x],x]
```

output
```
-1/2*x^2 + (x*Log[1 - E^(2*a + 2*b*x)])/b + PolyLog[2, E^(2*a + 2*b*x)]/(2
*b^2)
```

3.400.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{4201}$$

$$-i\left(2i\int \frac{e^{2a+2bx-i\pi}x}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$-i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int \log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$-i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\log\left(1+e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$-i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[x*Coth[a + b*x],x]
```

output
```
(-I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) + Po
lyLog[2, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2)))
```

3.400.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.400.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(41) = 82$.

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.71

method	result
risch	$-\frac{x^2}{2}-\frac{2ax}{b}-\frac{a^2}{b^2}+\frac{\ln(e^{bx+a}+1)x}{b}+\frac{\operatorname{polylog}(2,-e^{bx+a})}{b^2}+\frac{\ln(1-e^{bx+a})x}{b}+\frac{\ln(1-e^{bx+a})a}{b^2}+\frac{\operatorname{polylog}(2,e^{bx+a})}{b^2}-\frac{a\ln}{}$

input
```
int(x*cosh(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2-2/b*a*x-a^2/b^2+1/b*ln(exp(b*x+a)+1)*x+polylog(2,-exp(b*x+a))/b^2
+1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2
-1/b^2*a*ln(exp(b*x+a)-1)+2/b^2*a*ln(exp(b*x+a))
```

3.400.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(40) = 80$.

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.49

$$\int x\coth(a+bx)\,dx = -\frac{b^2x^2 - 2\,bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)-2\,(bx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^2 - 2*b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*a*log(cos
h(b*x + a) + sinh(b*x + a) - 1) - 2*(b*x + a)*log(-cosh(b*x + a) - sinh(b*
x + a) + 1) - 2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*dilog(-cosh(b*x +
 a) - sinh(b*x + a)))/b^2
```

3.400.6 Sympy [F]

$$\int x\coth(a+bx)\,dx = \int x\cosh\left(a+bx\right)\operatorname{csch}\left(a+bx\right)\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a),x)
```

output
```
Integral(x*cosh(a + b*x)*csch(a + b*x), x)
```

3.400.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int x\coth(a+bx)\,dx = -\frac{1}{2}x^2 + \frac{bx\log\left(e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*x^2 + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log
(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

3.400.8 Giac [F]

$$\int x\coth(a+bx)\,dx = \int x\cosh(bx+a)\operatorname{csch}(bx+a)\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)*csch(b*x + a), x)
```

3.400.9 Mupad [F(-1)]

Timed out.

$$\int x\coth(a+bx)\,dx = \int \frac{x\cosh(a+b\,x)}{\sinh(a+b\,x)}\,dx$$

input
```
int((x*cosh(a + b*x))/sinh(a + b*x),x)
```

output
```
int((x*cosh(a + b*x))/sinh(a + b*x), x)
```

3.401 $\int \coth(a+bx)\,dx$

3.401.1 Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \coth(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b}$$

output
```
ln(sinh(b*x+a))/b
```

3.401.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \coth(a+bx)\,dx=\frac{\log(\cosh(a+bx))+\log(\tanh(a+bx))}{b}$$

input
```
Integrate[Coth[a + b*x],x]
```

output
```
(Log[Cosh[a + b*x]] + Log[Tanh[a + b*x]])/b
```

3.401.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -i\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 26$$

$$-i\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \; 3956$$

$$\frac{\log(-i\sinh(a+bx))}{b}$$

input
```
Int[Coth[a + b*x],x]
```

output
```
Log[(-I)*Sinh[a + b*x]]/b
```

3.401.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.401.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$-\frac{\ln(\operatorname{csch}(bx+a))}{b}$	13
default	$-\frac{\ln(\operatorname{csch}(bx+a))}{b}$	13
risch	$-x-\frac{2a}{b}+\frac{\ln(e^{2bx+2a}-1)}{b}$	27
parallelrisch	$\frac{-bx+\ln(\tanh(bx+a))-\ln(1-\tanh(bx+a))}{b}$	30

input
```
int(cosh(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b*ln(csch(b*x+a))
```

3.401.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.36

$$\int \coth(a+bx)\,dx = -\frac{bx-\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

output
```
-(b*x - log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

3.401.6 Sympy [F]

$$\int \coth(a+bx)\,dx = \int \cosh(a+bx)\operatorname{csch}(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a),x)
```

output
```
Integral(cosh(a + b*x)*csch(a + b*x), x)
```

3.401.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \coth(a+bx)\,dx = \frac{\log\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{b}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

output
```
log(e^(b*x + a) - e^(-b*x - a))/b
```

3.401.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\,dx = -\frac{bx + a - \log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{b}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

output
```
-(b*x + a - log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.401.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\,dx = \frac{\ln\left(\sinh(a+b\,x)\right)}{b}$$

input
```
int(cosh(a + b*x)/sinh(a + b*x),x)
```

output
```
log(sinh(a + b*x))/b
```

3.402 $\int \frac{\coth(a+bx)}{x}\,dx$

3.402.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\coth(a+bx)}{x}\,dx = \text{Int}\left(\frac{\coth(a+bx)}{x},x\right)$$

output
```
Unintegrable(coth(b*x+a)/x,x)
```

3.402.2 Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\coth(a+bx)}{x}\,dx$$

input
```
Integrate[Coth[a + b*x]/x,x]
```

output
```
Integrate[Coth[a + b*x]/x, x]
```

3.402.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i \tan\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i \int \frac{\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)}{x}dx$$

$$\Big\downarrow \text{ 4222}$$

$$\int \frac{i \tan\left(\frac{1}{2}(-\pi-2ia)-ibx\right)}{x}dx$$

input
```
Int[Coth[a + b*x]/x,x]
```

output
```
$Aborted
```

3.402.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.402.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x}dx$$

input
```
int(cosh(b*x+a)*csch(b*x+a)/x,x)
```

output
```
int(cosh(b*x+a)*csch(b*x+a)/x,x)
```

3.402.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*csch(b*x + a)/x, x)
```

3.402.6 Sympy [N/A]

Not integrable

Time = 3.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\cosh{(a+bx)}\operatorname{csch}{(a+bx)}}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)/x,x)
```

output
```
Integral(cosh(a + b*x)*csch(a + b*x)/x, x)
```

3.402.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 3.50

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\cosh{(bx+a)}\operatorname{csch}{(bx+a)}}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)/x,x, algorithm="maxima")
```

output
```
-integrate(1/(x*e^(b*x + a) + x), x) + integrate(1/(x*e^(b*x + a) - x), x)
 + log(x)
```

3.402.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\cosh{(bx+a)}\operatorname{csch}{(bx+a)}}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)*csch(b*x + a)/x, x)
```

3.402.9 Mupad [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)}{x\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(cosh(a + b*x)/(x*sinh(a + b*x)),x)
```

output
```
int(cosh(a + b*x)/(x*sinh(a + b*x)), x)
```

3.403 $\int \frac{\coth(a+bx)}{x^2} dx$

3.403.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\coth(a+bx)}{x^2} dx = \operatorname{Int}\left(\frac{\coth(a+bx)}{x^2}, x\right)$$

output
```
Unintegrable(coth(b*x+a)/x^2,x)
```

3.403.2 Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\coth(a+bx)}{x^2} dx = \int \frac{\coth(a+bx)}{x^2} dx$$

input
```
Integrate[Coth[a + b*x]/x^2,x]
```

output
```
Integrate[Coth[a + b*x]/x^2, x]
```

3.403.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)}{x^2}dx$$

$$\Big\downarrow \text{4222}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-\pi-2ia)-ibx\right)}{x^2}dx$$

input
```
Int[Coth[a + b*x]/x^2,x]
```

output
```
$Aborted
```

3.403.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.403.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x^2}dx$$

input
```
int(cosh(b*x+a)*csch(b*x+a)/x^2,x)
```

output
```
int(cosh(b*x+a)*csch(b*x+a)/x^2,x)
```

3.403.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*csch(b*x + a)/x^2, x)
```

3.403.6 Sympy [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh{(a+bx)}\operatorname{csch}{(a+bx)}}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)/x**2,x)
```

output
```
Integral(cosh(a + b*x)*csch(a + b*x)/x**2, x)
```

3.403.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 4.60

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh{(bx+a)}\operatorname{csch}{(bx+a)}}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
-1/x - integrate(1/(x^2*e^(b*x + a) + x^2), x) + integrate(1/(x^2*e^(b*x +
 a) - x^2), x)
```

3.403.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh{(bx+a)}\operatorname{csch}{(bx+a)}}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)*csch(b*x + a)/x^2, x)
```

3.403.9 Mupad [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)}{x^2\sinh(a+b\,x)}\,dx$$

input
```
int(cosh(a + b*x)/(x^2*sinh(a + b*x)),x)
```

output
```
int(cosh(a + b*x)/(x^2*sinh(a + b*x)), x)
```

3.404 $\int x^m \cosh(a+bx)\coth(a+bx)\,dx$

3.404.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b} + \text{Int}(x^m \text{csch}(a+bx),x)$$

output
```
1/2*exp(a)*x^m*GAMMA(1+m,-b*x)/b/((-b*x)^m)+1/2*x^m*GAMMA(1+m,b*x)/b/exp(a
)/((b*x)^m)+Unintegrable(x^m*csch(b*x+a),x)
```

3.404.2 Mathematica [N/A]

Not integrable

Time = 17.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \int x^m \cosh(a+bx)\coth(a+bx)\,dx$$

input
```
Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x], x]
```

3.404.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5973, 3042, 26, 3789, 2612, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{5973}$$

$$\int x^m \sinh(a+bx)dx+\int x^m\text{csch}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^m \sin(ia+ibx)dx+\int ix^m \csc(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int x^m \csc(ia+ibx)dx-i\int x^m \sin(ia+ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$i\int x^m \csc(ia+ibx)dx-i\left(\frac{1}{2}i\int e^{a+bx}x^mdx-\frac{1}{2}i\int e^{-a-bx}x^mdx\right)$$

$$\Big\downarrow \text{2612}$$

$$i\int x^m \csc(ia+ibx)dx-i\left(\frac{ie^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}+\frac{ie^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}\right)$$

$$\Big\downarrow \text{4680}$$

$$\int x^m\text{csch}(a+bx)dx-i\left(\frac{ie^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}+\frac{ie^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}\right)$$

input
```
Int[x^m*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
$Aborted
```

3.404.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.404.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right) dx$$

input
```
int(x^m*cosh(b*x+a)^2*csch(b*x+a),x)
```

output
```
int(x^m*cosh(b*x+a)^2*csch(b*x+a),x)
```

3.404.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \cosh(a+bx) \coth(a+bx)\, dx = \int x^m \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)\, dx$$

input
```
integrate(x^m*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*cosh(b*x + a)^2*csch(b*x + a), x)
```

3.404.6 Sympy [F(-1)]

Timed out.

$$\int x^m \cosh(a+bx) \coth(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(x**m*cosh(b*x+a)**2*csch(b*x+a),x)
```

output
```
Timed out
```

3.404.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^m*cosh(b*x + a)^2*csch(b*x + a), x)
```

3.404.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^2*csch(b*x + a), x)
```

3.404.9 Mupad [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)^2}{\sinh\left(a+b\,x\right)}\,dx$$

input
```
int((x^m*cosh(a + b*x)^2)/sinh(a + b*x),x)
```

output
```
int((x^m*cosh(a + b*x)^2)/sinh(a + b*x), x)
```

3.405 $\int x^3 \cosh(a+bx) \coth(a+bx)\,dx$

3.405.1 Optimal result

Integrand size = 16, antiderivative size = 165

$$\begin{aligned}\int x^3 \cosh(a+bx) \coth(a+bx)\,dx = &-\frac{2x^3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}+\frac{6x\cosh(a+bx)}{b^3}\\ &+\frac{x^3\cosh(a+bx)}{b}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}\\ &+\frac{3x^2\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\\ &+\frac{6x\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{6x\operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^3}\\ &-\frac{6\operatorname{PolyLog}\left(4,-e^{a+bx}\right)}{b^4}+\frac{6\operatorname{PolyLog}\left(4,e^{a+bx}\right)}{b^4}\\ &-\frac{6\sinh(a+bx)}{b^4}-\frac{3x^2\sinh(a+bx)}{b^2}\end{aligned}$$

output
```
-2*x^3*arctanh(exp(b*x+a))/b+6*x*cosh(b*x+a)/b^3+x^3*cosh(b*x+a)/b-3*x^2*p
olylog(2,-exp(b*x+a))/b^2+3*x^2*polylog(2,exp(b*x+a))/b^2+6*x*polylog(3,-e
xp(b*x+a))/b^3-6*x*polylog(3,exp(b*x+a))/b^3-6*polylog(4,-exp(b*x+a))/b^4+
6*polylog(4,exp(b*x+a))/b^4-6*sinh(b*x+a)/b^4-3*x^2*sinh(b*x+a)/b^2
```

3.405.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.06

$$\int x^3 \cosh(a+bx)\coth(a+bx)\,dx$$
$$= \frac{6bx\cosh(a+bx)+b^3x^3\cosh(a+bx)+b^3x^3\log\left(1-e^{a+bx}\right)-b^3x^3\log\left(1+e^{a+bx}\right)-3b^2x^2\,\mathrm{PolyLog}\left(2,-\right.}{}$$

input
```
Integrate[x^3*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
(6*b*x*Cosh[a + b*x] + b^3*x^3*Cosh[a + b*x] + b^3*x^3*Log[1 - E^(a + b*x)
] - b^3*x^3*Log[1 + E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, -E^(a + b*x)] + 3*
b^2*x^2*PolyLog[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*x*P
olyLog[3, E^(a + b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a +
b*x)] - 6*Sinh[a + b*x] - 3*b^2*x^2*Sinh[a + b*x])/b^4
```

3.405.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.36, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$, Rules used = {5973, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cosh(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{5973}$$

$$\int x^3\sinh(a+bx)dx+\int x^3\text{csch}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^3\sin(ia+ibx)dx+\int ix^3\csc(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i \int x^3 \csc(ia+ibx)dx - i \int x^3 \sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 3777}$$

$$i \int x^3 \csc(ia+ibx)dx - i \left(\frac{ix^3 \cosh(a+bx)}{b} - \frac{3i \int x^2 \cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i \int x^3 \csc(ia+ibx)dx - i \left(\frac{ix^3 \cosh(a+bx)}{b} - \frac{3i \int x^2 \sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{ 3777}$$

$$i \int x^3 \csc(ia+ibx)dx - i \left(\frac{ix^3 \cosh(a+bx)}{b} - \frac{3i \left(\frac{x^2 \sinh(a+bx)}{b} - \frac{2i \int -ix \sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i \int x^3 \csc(ia+ibx)dx - i \left(\frac{ix^3 \cosh(a+bx)}{b} - \frac{3i \left(\frac{x^2 \sinh(a+bx)}{b} - \frac{2 \int x \sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i \int x^3 \csc(ia+ibx)dx - i \left(\frac{ix^3 \cosh(a+bx)}{b} - \frac{3i \left(\frac{x^2 \sinh(a+bx)}{b} - \frac{2 \int -ix \sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i \int x^3 \csc(ia+ibx)dx - i \left(\frac{ix^3 \cosh(a+bx)}{b} - \frac{3i \left(\frac{x^2 \sinh(a+bx)}{b} + \frac{2i \int x \sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 3777}$$

$$i \int x^3 \csc(ia+ibx)dx - i \left(\frac{ix^3 \cosh(a+bx)}{b} - \frac{3i \left(\frac{x^2 \sinh(a+bx)}{b} + \frac{2i \left(\frac{ix \cosh(a+bx)}{b} - \frac{i \int \cosh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 3042}$$

3.405. $\int x^3 \cosh(a+bx) \coth(a+bx)\, dx$

$$i\int x^3\csc(ia+ibx)dx-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3117}$$

$$i\int x^3\csc(ia+ibx)dx-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{4670}$$

$$i\left(\frac{3i\int x^2\log\left(1-e^{a+bx}\right)dx}{b}-\frac{3i\int x^2\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2ix^3\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3011}$$

$$i\left(-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^3\text{arct}\right.$$
$$i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{7163}$$

3.405. $\int x^3\cosh(a+bx)\coth(a+bx)\,dx$

$$i\left(-\frac{3i\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{\int \mathrm{PolyLog}\left(3,-e^{a+bx}\right)dx}{b}\right)}{b}-\frac{x^2\,\mathrm{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,e^{a+bx}\right)}{b}-\frac{\int \mathrm{PolyLog}\left(3,e^{a+bx}\right)\ldots}{b}\right.}{b}\right.}{b}\right.$$

$$i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)$$

$\downarrow$ 2720

$$i\left(-\frac{3i\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{\int e^{-a-bx}\,\mathrm{PolyLog}\left(3,-e^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{x^2\,\mathrm{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,e^{a+bx}\right)}{b}-\frac{\int e^{-a-\ldots}}{}\right.}{b}\right.}{}\right.$$

$$i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)$$

$\downarrow$ 7143

$$i\left(\frac{2ix^3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}-\frac{3i\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{\mathrm{PolyLog}\left(4,-e^{a+bx}\right)}{b^2}\right)}{b}-\frac{x^2\,\mathrm{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,e^{a+b\ldots}\right)}{b}\right.}{}\right.}{}\right.$$

$$i\left(\frac{ix^3\cosh(a+bx)}{b}-\frac{3i\left(\frac{x^2\sinh(a+bx)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)$$

3.405. $\int x^3\cosh(a+bx)\coth(a+bx)\,dx$

input
```
Int[x^3*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
I*(((2*I)*x^3*ArcTanh[E^(a + b*x)])/b - ((3*I)*(-((x^2*PolyLog[2, -E^(a +
b*x)])/b) + (2*((x*PolyLog[3, -E^(a + b*x)])/b - PolyLog[4, -E^(a + b*x)]/
b^2))/b))/b + ((3*I)*(-((x^2*PolyLog[2, E^(a + b*x)])/b) + (2*((x*PolyLog[
3, E^(a + b*x)])/b - PolyLog[4, E^(a + b*x)]/b^2))/b))/b) - I*((I*x^3*Cosh
[a + b*x])/b - ((3*I)*((x^2*Sinh[a + b*x])/b + ((2*I)*((I*x*Cosh[a + b*x])
/b - (I*Sinh[a + b*x])/b^2))/b))/b)
```

3.405.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.405.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.49

method	result
risch	$\frac{(x^3b^3-3x^2b^2+6bx-6)\mathrm{e}^{bx+a}}{2b^4}+\frac{(x^3b^3+3x^2b^2+6bx+6)\mathrm{e}^{-bx-a}}{2b^4}+\frac{6\,\mathrm{polylog}(4,\mathrm{e}^{bx+a})}{b^4}-\frac{6\,\mathrm{polylog}(4,-\mathrm{e}^{bx+a})}{b^4}+\frac{\ln(1-\mathrm{e}^{bx+a})x}{b}$

input
```
int(x^3*cosh(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^3*x^3-3*b^2*x^2+6*b*x-6)/b^4*exp(b*x+a)+1/2*(b^3*x^3+3*b^2*x^2+6*b*
x+6)/b^4*exp(-b*x-a)+6*polylog(4,exp(b*x+a))/b^4-6*polylog(4,-exp(b*x+a))/
b^4+1/b*ln(1-exp(b*x+a))*x^3+3*x^2*polylog(2,exp(b*x+a))/b^2-6*x*polylog(3
,exp(b*x+a))/b^3-1/b*ln(exp(b*x+a)+1)*x^3-3*x^2*polylog(2,-exp(b*x+a))/b^2
+6*x*polylog(3,-exp(b*x+a))/b^3+2/b^4*a^3*arctanh(exp(b*x+a))+1/b^4*ln(1-e
xp(b*x+a))*a^3-1/b^4*ln(exp(b*x+a)+1)*a^3
```

3.405.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(156) = 312$.

Time = 0.27 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.10

$$\int x^3 \cosh(a+bx)\coth(a+bx)\,dx$$
$$= \frac{b^3x^3 + 3\,b^2x^2 + (b^3x^3 - 3\,b^2x^2 + 6\,bx - 6)\cosh{(bx+a)}^2 + 2\,(b^3x^3 - 3\,b^2x^2 + 6\,bx - 6)\cosh{(bx+a)}\sinh}{}$$

input
```
integrate(x^3*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 + 3*b^2*x^2 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)
^2 + 2*(b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a) + (b^
3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*sinh(b*x + a)^2 + 6*b*x + 6*(b^2*x^2*cosh(b
*x + a) + b^2*x^2*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*
(b^2*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a))*dilog(-cosh(b*x + a) - sin
h(b*x + a)) - 2*(b^3*x^3*cosh(b*x + a) + b^3*x^3*sinh(b*x + a))*log(cosh(b
*x + a) + sinh(b*x + a) + 1) - 2*(a^3*cosh(b*x + a) + a^3*sinh(b*x + a))*l
og(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*((b^3*x^3 + a^3)*cosh(b*x + a) +
 (b^3*x^3 + a^3)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) +
12*(cosh(b*x + a) + sinh(b*x + a))*polylog(4, cosh(b*x + a) + sinh(b*x + a
)) - 12*(cosh(b*x + a) + sinh(b*x + a))*polylog(4, -cosh(b*x + a) - sinh(b
*x + a)) - 12*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*polylog(3, cosh(b*x
+ a) + sinh(b*x + a)) + 12*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*polylog
(3, -cosh(b*x + a) - sinh(b*x + a)) + 6)/(b^4*cosh(b*x + a) + b^4*sinh(b*x
 + a))
```

3.405.6 Sympy [F]

$$\int x^3 \cosh(a+bx)\coth(a+bx)\,dx = \int x^3 \cosh^2{(a+bx)}\operatorname{csch}{(a+bx)}\,dx$$

input
```
integrate(x**3*cosh(b*x+a)**2*csch(b*x+a),x)
```

output
```
Integral(x**3*cosh(a + b*x)**2*csch(a + b*x), x)
```

3.405.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.25

$$\int x^3\cosh(a+bx)\coth(a+bx)\,dx$$
$$= \frac{\left(\left(b^3x^3e^{(2a)} - 3b^2x^2e^{(2a)} + 6bxe^{(2a)} - 6e^{(2a)}\right)e^{(bx)} + \left(b^3x^3 + 3b^2x^2 + 6bx + 6\right)e^{(-bx)}\right)e^{(-a)}}{2b^4}$$
$$- \frac{b^3x^3\log\left(e^{(bx+a)}+1\right) + 3b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 6bx\mathrm{Li}_3(-e^{(bx+a)}) + 6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4}$$
$$+ \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right) + 3b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right) - 6bx\mathrm{Li}_3(e^{(bx+a)}) + 6\,\mathrm{Li}_4(e^{(bx+a)})}{b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

output
```
1/2*((b^3*x^3*e^(2*a) - 3*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a) - 6*e^(2*a))*e^(
b*x) + (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x))*e^(-a)/b^4 - (b^3*x^3*l
og(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^
(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) +
 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*pol
ylog(4, e^(b*x + a)))/b^4
```

3.405.8 Giac [F]

$$\int x^3\cosh(a+bx)\coth(a+bx)\,dx = \int x^3\cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)\,dx$$

input
```
integrate(x^3*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*cosh(b*x + a)^2*csch(b*x + a), x)
```

3.405.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \cosh(a+bx)\coth(a+bx)\,dx = \int \frac{x^3 \cosh(a+b\,x)^2}{\sinh(a+b\,x)}\,dx$$

input
```
int((x^3*cosh(a + b*x)^2)/sinh(a + b*x),x)
```

output
```
int((x^3*cosh(a + b*x)^2)/sinh(a + b*x), x)
```

3.406 $\int x^2 \cosh(a+bx)\coth(a+bx)\,dx$

3.406.1 Optimal result

Integrand size = 16, antiderivative size = 115

$$\int x^2\cosh(a+bx)\coth(a+bx)\,dx = -\frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{2\cosh(a+bx)}{b^3}+\frac{x^2\cosh(a+bx)}{b}-\frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{2\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{2x\sinh(a+bx)}{b^2}$$

output
```
-2*x^2*arctanh(exp(b*x+a))/b+2*cosh(b*x+a)/b^3+x^2*cosh(b*x+a)/b-2*x*polyl
og(2,-exp(b*x+a))/b^2+2*x*polylog(2,exp(b*x+a))/b^2+2*polylog(3,-exp(b*x+a
))/b^3-2*polylog(3,exp(b*x+a))/b^3-2*x*sinh(b*x+a)/b^2
```

3.406.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int x^2\cosh(a+bx)\coth(a+bx)\,dx = \frac{2\cosh(a+bx)+b^2x^2\cosh(a+bx)+b^2x^2\log\left(1-e^{a+bx}\right)-b^2x^2\log\left(1+e^{a+bx}\right)-2bx\,\text{PolyLog}\left(2,-e^{a+b}\right.}{b^3}$$

input
```
Integrate[x^2*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
(2*Cosh[a + b*x] + b^2*x^2*Cosh[a + b*x] + b^2*x^2*Log[1 - E^(a + b*x)] -
b^2*x^2*Log[1 + E^(a + b*x)] - 2*b*x*PolyLog[2, -E^(a + b*x)] + 2*b*x*Poly
Log[2, E^(a + b*x)] + 2*PolyLog[3, -E^(a + b*x)] - 2*PolyLog[3, E^(a + b*x
)] - 2*b*x*Sinh[a + b*x])/b^3
```

3.406.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.31, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5973, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cosh(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \; 5973$$

$$\int x^2 \sinh(a+bx)dx+\int x^2\text{csch}(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\int -ix^2 \sin(ia+ibx)dx+\int ix^2 \csc(ia+ibx)dx$$

$$\Big\downarrow \; 26$$

$$i\int x^2 \csc(ia+ibx)dx-i\int x^2 \sin(ia+ibx)dx$$

$$\Big\downarrow \; 3777$$

$$i\int x^2 \csc(ia+ibx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\int x\cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \; 3042$$

$$i\int x^2 \csc(ia+ibx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \; 3777$$

$$i\int x^2\csc(ia+ibx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{i\int -i\sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\int x^2\csc(ia+ibx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\int \sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\int x^2\csc(ia+ibx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\int -i\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\int x^2\csc(ia+ibx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}+\frac{i\int \sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3118}$$

$$i\int x^2\csc(ia+ibx)dx-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{4670}$$

$$i\left(\frac{2i\int x\log\left(1-e^{a+bx}\right)dx}{b}-\frac{2i\int x\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2ix^2\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{3011}$$

$$i\left(-\frac{2i\left(\frac{\int \text{PolyLog}\left(2,-e^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{\int \text{PolyLog}\left(2,e^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b}\right)}{b}+\frac{2ix^2\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{2720}$$

3.406. $\int x^2\cosh(a+bx)\coth(a+bx)\,dx$

$$i\left(-\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}\left(2,-e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}\left(2,e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b}\right)}{b}\right.$$

$$i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{7143}$$

$$i\left(\frac{2ix^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b}-\frac{2i\left(\frac{\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^2}-\frac{x\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{\operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^2}-\frac{x\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b}\right)}{b}\right)-$$

$$i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)$$

input
```
Int[x^2*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
I*(((2*I)*x^2*ArcTanh[E^(a + b*x)])/b - ((2*I)*(-((x*PolyLog[2, -E^(a + b*
x)])/b) + PolyLog[3, -E^(a + b*x)]/b^2))/b + ((2*I)*(-((x*PolyLog[2, E^(a
+ b*x)])/b) + PolyLog[3, E^(a + b*x)]/b^2))/b) - I*((I*x^2*Cosh[a + b*x])/
b - ((2*I)*(-(Cosh[a + b*x]/b^2) + (x*Sinh[a + b*x])/b))/b)
```

3.406.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.406. $\int x^2 \cosh(a+bx) \coth(a+bx) \, dx$

3.406.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.70

method	result
risch	$\frac{(x^2b^2-2bx+2)\mathrm{e}^{bx+a}}{2b^3}+\frac{(x^2b^2+2bx+2)\mathrm{e}^{-bx-a}}{2b^3}-\frac{2a^2\operatorname{arctanh}(\mathrm{e}^{bx+a})}{b^3}+\frac{\ln(1-\mathrm{e}^{bx+a})x^2}{b}-\frac{\ln(1-\mathrm{e}^{bx+a})a^2}{b^3}+\frac{2x\operatorname{polylog}(2,\mathrm{e}}{b^2}$

input
```
int(x^2*cosh(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)+1/2*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)
-2/b^3*a^2*arctanh(exp(b*x+a))+1/b*ln(1-exp(b*x+a))*x^2-1/b^3*ln(1-exp(b*x
+a))*a^2+2*x*polylog(2,exp(b*x+a))/b^2-2*polylog(3,exp(b*x+a))/b^3-1/b*ln(
exp(b*x+a)+1)*x^2+1/b^3*ln(exp(b*x+a)+1)*a^2-2*x*polylog(2,-exp(b*x+a))/b^
2+2*polylog(3,-exp(b*x+a))/b^3
```

3.406.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(108) = 216$.

Time = 0.26 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.40

$$\int x^2\cosh(a+bx)\coth(a+bx)\,dx$$

$$= \frac{b^2x^2 + (b^2x^2 - 2\,bx + 2)\cosh{(bx+a)}^2 + 2\,(b^2x^2 - 2\,bx + 2)\cosh{(bx+a)}\sinh{(bx+a)} + (b^2x^2 - 2\,bx +}{}$$

input
```
integrate(x^2*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + (b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - 2*b*x
+ 2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^2 +
 2*b*x + 4*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*dilog(cosh(b*x + a) + s
inh(b*x + a)) - 4*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*dilog(-cosh(b*x
+ a) - sinh(b*x + a)) - 2*(b^2*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a))*
log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*(a^2*cosh(b*x + a) + a^2*sinh(b
*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*((b^2*x^2 - a^2)*cosh(
b*x + a) + (b^2*x^2 - a^2)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x +
a) + 1) - 4*(cosh(b*x + a) + sinh(b*x + a))*polylog(3, cosh(b*x + a) + sin
h(b*x + a)) + 4*(cosh(b*x + a) + sinh(b*x + a))*polylog(3, -cosh(b*x + a)
- sinh(b*x + a)) + 2)/(b^3*cosh(b*x + a) + b^3*sinh(b*x + a))
```

3.406.6 Sympy [F]

$$\int x^2 \cosh(a+bx)\coth(a+bx)\,dx = \int x^2 \cosh^2(a+bx)\operatorname{csch}(a+bx)\,dx$$

input
```
integrate(x**2*cosh(b*x+a)**2*csch(b*x+a),x)
```

output
```
Integral(x**2*cosh(a + b*x)**2*csch(a + b*x), x)
```

3.406.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.32

$$\int x^2 \cosh(a+bx)\coth(a+bx)\,dx = \frac{\left((b^2x^2e^{(2a)} - 2bxe^{(2a)} + 2e^{(2a)})e^{(bx)} + (b^2x^2 + 2bx + 2)e^{(-bx)}\right)e^{(-a)}}{2b^3} - \frac{b^2x^2\log\left(e^{(bx+a)}+1\right) + 2bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(-e^{(bx+a)})}{b^3} + \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right) + 2bx\mathrm{Li}_2\left(e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(e^{(bx+a)})}{b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

output
```
1/2*((b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + 2*e^(2*a))*e^(b*x) + (b^2*x^2 + 2*
b*x + 2)*e^(-b*x))*e^(-a)/b^3 - (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilo
g(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x +
 a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

3.406.8 Giac [F]

$$\int x^2 \cosh(a+bx)\coth(a+bx)\,dx = \int x^2 \cosh(bx+a)^2\operatorname{csch}(bx+a)\,dx$$

input
```
integrate(x^2*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*cosh(b*x + a)^2*csch(b*x + a), x)
```

3.406.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh(a+bx)\coth(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)^2}{\sinh(a+b\,x)}\,dx$$

input
```
int((x^2*cosh(a + b*x)^2)/sinh(a + b*x),x)
```

output
```
int((x^2*cosh(a + b*x)^2)/sinh(a + b*x), x)
```

3.407 $\int x \cosh(a+bx)\coth(a+bx)\,dx$

3.407.1 Optimal result

Integrand size = 14, antiderivative size = 66

$$\int x\cosh(a+bx)\coth(a+bx)\,dx = -\frac{2x\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{x\cosh(a+bx)}{b}-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{\sinh(a+bx)}{b^2}$$

output
```
-2*x*arctanh(exp(b*x+a))/b+x*cosh(b*x+a)/b-polylog(2,-exp(b*x+a))/b^2+poly
log(2,exp(b*x+a))/b^2-sinh(b*x+a)/b^2
```

3.407.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int x\cosh(a+bx)\coth(a+bx)\,dx = \frac{bx\cosh(a+bx)+bx\log\left(1-e^{a+bx}\right)-bx\log\left(1+e^{a+bx}\right)-\text{PolyLog}\left(2,-e^{a+bx}\right)+\text{PolyLog}\left(2,e^{a+bx}\right)-\text{s}}{b^2}$$

input
```
Integrate[x*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
(b*x*Cosh[a + b*x] + b*x*Log[1 - E^(a + b*x)] - b*x*Log[1 + E^(a + b*x)] -
 PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)] - Sinh[a + b*x])/b^2
```

3.407.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5973, 3042, 26, 3777, 3042, 3117, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cosh(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{5973}$$

$$\int x\sinh(a+bx)dx+\int x\text{csch}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix\sin(ia+ibx)dx+\int ix\csc(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int x\csc(ia+ibx)dx-i\int x\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$i\int x\csc(ia+ibx)dx-i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\int x\csc(ia+ibx)dx-i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{3117}$$

$$i\int x\csc(ia+ibx)dx-i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)$$

$$\Big\downarrow \text{4670}$$

$$i\left(\frac{i\int\log\left(1-e^{a+bx}\right)dx}{b}-\frac{i\int\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)$$

$$\Big\downarrow \text{2715}$$

$$i\left(\frac{i\int e^{-a-bx}\log\left(1-e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{i\int e^{-a-bx}\log\left(1+e^{a+bx}\right)de^{a+bx}}{b^2}+\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)$$

$$\Big\downarrow \text{2838}$$

$$i\left(\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)-i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)$$

input
```
Int[x*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
I*(((2*I)*x*ArcTanh[E^(a + b*x)])/b + (I*PolyLog[2, -E^(a + b*x)])/b^2 - (
I*PolyLog[2, E^(a + b*x)])/b^2) - I*((I*x*Cosh[a + b*x])/b - (I*Sinh[a + b
*x])/b^2)
```

3.407.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.407.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(63) = 126.

Time = 0.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.11

method	result
risch	$\frac{(bx-1)\mathrm{e}^{bx+a}}{2b^2}+\frac{(bx+1)\mathrm{e}^{-bx-a}}{2b^2}+\frac{\ln(1-\mathrm{e}^{bx+a})x}{b}+\frac{\ln(1-\mathrm{e}^{bx+a})a}{b^2}+\frac{\operatorname{polylog}(2,\mathrm{e}^{bx+a})}{b^2}-\frac{\ln(\mathrm{e}^{bx+a}+1)x}{b}-\frac{\ln(\mathrm{e}^{bx+a}+1)a}{b^2}$

input
```
int(x*cosh(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x-1)/b^2*exp(b*x+a)+1/2*(b*x+1)/b^2*exp(-b*x-a)+1/b*ln(1-exp(b*x+a)
)*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2-1/b*ln(exp(b*x+a)+1
)*x-1/b^2*ln(exp(b*x+a)+1)*a-polylog(2,-exp(b*x+a))/b^2+2/b^2*a*arctanh(ex
p(b*x+a))
```

3.407.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(61) = 122$.

Time = 0.25 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.86

$$\int x\cosh(a+bx)\coth(a+bx)\,dx$$
$$= \frac{(bx-1)\cosh{(bx+a)}^2 + 2\,(bx-1)\cosh{(bx+a)}\sinh{(bx+a)} + (bx-1)\sinh{(bx+a)}^2 + bx + 2\,(\cosh{(}$$

input
```
integrate(x*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

output
```
1/2*((b*x - 1)*cosh(b*x + a)^2 + 2*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a) +
 (b*x - 1)*sinh(b*x + a)^2 + b*x + 2*(cosh(b*x + a) + sinh(b*x + a))*dilog
(cosh(b*x + a) + sinh(b*x + a)) - 2*(cosh(b*x + a) + sinh(b*x + a))*dilog(
-cosh(b*x + a) - sinh(b*x + a)) - 2*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a)
)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 2*(a*cosh(b*x + a) + a*sinh(b*x
 + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*((b*x + a)*cosh(b*x + a)
 + (b*x + a)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 1)/(
b^2*cosh(b*x + a) + b^2*sinh(b*x + a))
```

3.407.6 Sympy [F]

$$\int x\cosh(a+bx)\coth(a+bx)\,dx = \int x\cosh^2{(a+bx)}\operatorname{csch}{(a+bx)}\,dx$$

input
```
integrate(x*cosh(b*x+a)**2*csch(b*x+a),x)
```

output
```
Integral(x*cosh(a + b*x)**2*csch(a + b*x), x)
```

3.407.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int x\cosh(a+bx)\coth(a+bx)\,dx = \frac{\left(\left(bxe^{(2a)}-e^{(2a)}\right)e^{(bx)}+(bx+1)e^{(-bx)}\right)e^{(-a)}}{2b^2} - \frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

input
```
integrate(x*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

output
```
1/2*((b*x*e^(2*a) - e^(2*a))*e^(b*x) + (b*x + 1)*e^(-b*x))*e^(-a)/b^2 - (b
*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a)
 + 1) + dilog(e^(b*x + a)))/b^2
```

3.407.8 Giac [F]

$$\int x\cosh(a+bx)\coth(a+bx)\,dx = \int x\cosh(bx+a)^2\operatorname{csch}(bx+a)\,dx$$

input
```
integrate(x*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)^2*csch(b*x + a), x)
```

3.407.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\coth(a+bx)\,dx = \int \frac{x\cosh(a+b\,x)^2}{\sinh(a+b\,x)}\,dx$$

input
```
int((x*cosh(a + b*x)^2)/sinh(a + b*x),x)
```

output
```
int((x*cosh(a + b*x)^2)/sinh(a + b*x), x)
```

3.408 $\int \cosh(a+bx)\coth(a+bx)\,dx$

3.408.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \cosh(a+bx)\coth(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\cosh(a+bx)}{b}$$

output
```
-arctanh(cosh(b*x+a))/b+cosh(b*x+a)/b
```

3.408.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \cosh(a+bx)\coth(a+bx)\,dx=\frac{\cosh(a+bx)}{b}-\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
Cosh[a + b*x]/b - Log[Cosh[(a + b*x)/2]]/b + Log[Sinh[(a + b*x)/2]]/b
```

3.408.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin\left(ia+ibx+\frac{\pi}{2}\right)\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{\int \frac{\cosh^2(a+bx)}{1-\cosh^2(a+bx)}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\int \frac{1}{1-\cosh^2(a+bx)}d\cosh(a+bx)-\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\operatorname{arctanh}(\cosh(a+bx))-\cosh(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
-((ArcTanh[Cosh[a + b*x]] - Cosh[a + b*x])/b)
```

3.408.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.408.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\cosh(bx+a)-2\ \operatorname{arctanh}(e^{bx+a})}{b}$	21
default	$\frac{\cosh(bx+a)-2\ \operatorname{arctanh}(e^{bx+a})}{b}$	21
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{\ln(e^{bx+a}-1)}{b}-\frac{\ln(e^{bx+a}+1)}{b}$	54

input
```
int(cosh(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

3.408.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(23) = 46.

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.91

$$\int \cosh(a+bx)\coth(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^2 - 2(\cosh(bx+a)+\sinh(bx+a))\log(\cosh(bx+a)+\sinh(bx+a)+1) + 2(\cosh(bx+}{2(b\cosh(bx+}$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
 + sinh(b*x + a) + 1) + 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a
) + sinh(b*x + a) - 1) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 +
 1)/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

3.408.6 Sympy [F]

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \int \cosh^2(a+bx)\operatorname{csch}(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**2*csch(b*x+a),x)
```

output
```
Integral(cosh(a + b*x)**2*csch(a + b*x), x)
```

3.408.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(23) = 46$.

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.57

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{(bx+a)}}{2b} + \frac{e^{(-bx-a)}}{2b} - \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

output
```
1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b - log(e^(-b*x - a) + 1)/b + log(e^(
-b*x - a) - 1)/b
```

3.408.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \cosh(a+bx)\coth(a+bx)\,dx$$
$$= \frac{e^{(bx+a)} + e^{(-bx-a)} - 2\log\left(e^{(bx+a)}+1\right) + 2\log\left(\left|e^{(bx+a)}-1\right|\right)}{2b}$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

output
```
1/2*(e^(b*x + a) + e^(-b*x - a) - 2*log(e^(b*x + a) + 1) + 2*log(abs(e^(b*
x + a) - 1)))/b
```

3.408.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{a+bx}}{2b} - \frac{2\,\mathrm{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{e^{-a-bx}}{2b}$$

input
```
int(cosh(a + b*x)^2/sinh(a + b*x),x)
```

output
```
exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/
2) + exp(- a - b*x)/(2*b)
```

3.409 $\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx$

3.409.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \text{Chi}(bx)\sinh(a)+\cosh(a)\text{Shi}(bx)+\text{Int}\left(\frac{\text{csch}(a+bx)}{x},x\right)$$

output
```
cosh(a)*Shi(b*x)+Chi(b*x)*sinh(a)+Unintegrable(csch(b*x+a)/x,x)
```

3.409.2 Mathematica [N/A]

Not integrable

Time = 8.90 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx$$

input
```
Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x,x]
```

output
```
Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x, x]
```

3.409.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5973, 3042, 26, 3784, 26, 3042, 26, 3779, 3782, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}dx$$

$$\Big\downarrow \text{5973}$$

$$\int \frac{\sinh(a+bx)}{x}dx+\int \frac{\text{csch}(a+bx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ia+ibx)}{x}dx+\int \frac{i\csc(ia+ibx)}{x}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\csc(ia+ibx)}{x}dx-i\int \frac{\sin(ia+ibx)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$i\int \frac{\csc(ia+ibx)}{x}dx-i\left(i\sinh(a)\int \frac{\cosh(bx)}{x}dx+\cosh(a)\int \frac{i\sinh(bx)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\csc(ia+ibx)}{x}dx-i\left(i\sinh(a)\int \frac{\cosh(bx)}{x}dx+i\cosh(a)\int \frac{\sinh(bx)}{x}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{\csc(ia+ibx)}{x}dx-i\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(a)\int -\frac{i\sin(ibx)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\csc(ia+ibx)}{x}dx-i\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx+\cosh(a)\int \frac{\sin(ibx)}{x}dx\right)$$

$$\Big\downarrow \text{3779}$$

$$i\int \frac{\csc(ia+ibx)}{x}dx-i\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(a)\text{Shi}(bx)\right)$$

$$\Big\downarrow \text{3782}$$

$$i\int \frac{\csc(ia+ibx)}{x}dx-i(i\sinh(a)\text{Chi}(bx)+i\cosh(a)\text{Shi}(bx))$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\text{csch}(a+bx)}{x}dx-i(i\sinh(a)\text{Chi}(bx)+i\cosh(a)\text{Shi}(bx))$$

input
```
Int[(Cosh[a + b*x]*Coth[a + b*x])/x,x]
```

output
```
$Aborted
```

3.409.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.409.4 Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh\left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right)}{x}dx$$

input
```
int(cosh(b*x+a)^2*csch(b*x+a)/x,x)
```

output
```
int(cosh(b*x+a)^2*csch(b*x+a)/x,x)
```

3.409.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^2*csch(b*x + a)/x, x)
```

3.409.6 Sympy [N/A]

Not integrable

Time = 9.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh^2{(a+bx)}\operatorname{csch}{(a+bx)}}{x}\,dx$$

input
```
integrate(cosh(b*x+a)**2*csch(b*x+a)/x,x)
```

output
```
Integral(cosh(a + b*x)**2*csch(a + b*x)/x, x)
```

3.409.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh{(bx+a)}^2\operatorname{csch}{(bx+a)}}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^2*csch(b*x + a)/x, x)
```

3.409.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh{(bx+a)}^2\operatorname{csch}{(bx+a)}}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2*csch(b*x + a)/x, x)
```

3.409.9 Mupad [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^2}{x\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(cosh(a + b*x)^2/(x*sinh(a + b*x)),x)
```

output
```
int(cosh(a + b*x)^2/(x*sinh(a + b*x)), x)
```

3.410 $\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$

3.410.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx = b\cosh(a)\text{Chi}(bx) - \frac{\sinh(a+bx)}{x} + b\sinh(a)\text{Shi}(bx) + \text{Int}\left(\frac{\text{csch}(a+bx)}{x^2},x\right)$$

output
```
b*Chi(b*x)*cosh(a)+b*Shi(b*x)*sinh(a)-sinh(b*x+a)/x+Unintegrable(csch(b*x+
a)/x^2,x)
```

3.410.2 Mathematica [N/A]

Not integrable

Time = 24.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x^2,x]
```

output
```
Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x^2, x]
```

3.410.3 Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5973, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{5973}$$

$$\int \frac{\sinh(a+bx)}{x^2}dx+\int \frac{\operatorname{csch}(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ia+ibx)}{x^2}dx+\int \frac{i\csc(ia+ibx)}{x^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\csc(ia+ibx)}{x^2}dx-i\int \frac{\sin(ia+ibx)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$i\int \frac{\csc(ia+ibx)}{x^2}dx-i\left(ib\int \frac{\cosh(a+bx)}{x}dx-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{\csc(ia+ibx)}{x^2}dx-i\left(ib\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3784}$$

$$i\int \frac{\csc(ia+ibx)}{x^2}dx-i\left(ib\left(\cosh(a)\int \frac{\cosh(bx)}{x}dx-i\sinh(a)\int \frac{i\sinh(bx)}{x}dx\right)-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\csc(ia+ibx)}{x^2}dx-i\left(ib\left(\sinh(a)\int \frac{\sinh(bx)}{x}dx+\cosh(a)\int \frac{\cosh(bx)}{x}dx\right)-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{\csc(ia+ibx)}{x^2}dx-$$
$$i\left(ib\left(\sinh(a)\int -\frac{i\sin(ibx)}{x}dx+\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx\right)-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\csc(ia+ibx)}{x^2}dx-$$
$$i\left(ib\left(\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx-i\sinh(a)\int \frac{\sin(ibx)}{x}dx\right)-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3779}$$

$$i\int \frac{\csc(ia+ibx)}{x^2}dx-i\left(ib\left(\sinh(a)\text{Shi}(bx)+\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx\right)-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{3782}$$

$$i\int \frac{\csc(ia+ibx)}{x^2}dx-i\left(ib(\cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx))-\frac{i\sinh(a+bx)}{x}\right)$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\text{csch}(a+bx)}{x^2}dx-i\left(ib(\cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx))-\frac{i\sinh(a+bx)}{x}\right)$$

input
```
Int[(Cosh[a + b*x]*Coth[a + b*x])/x^2,x]
```

output
```
$Aborted
```

3.410.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.410. $\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.410. $\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$

3.410.4 Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh (bx+a)^2 \operatorname{csch}(bx+a)}{x^2} dx$$

input
```
int(cosh(b*x+a)^2*csch(b*x+a)/x^2,x)
```

output
```
int(cosh(b*x+a)^2*csch(b*x+a)/x^2,x)
```

3.410.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2} dx = \int \frac{\cosh (bx+a)^2 \operatorname{csch}(bx+a)}{x^2} dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^2*csch(b*x + a)/x^2, x)
```

3.410.6 Sympy [N/A]

Not integrable

Time = 10.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2} dx = \int \frac{\cosh^2(a+bx) \operatorname{csch}(a+bx)}{x^2} dx$$

input
```
integrate(cosh(b*x+a)**2*csch(b*x+a)/x**2,x)
```

output
```
Integral(cosh(a + b*x)**2*csch(a + b*x)/x**2, x)
```

3.410.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^2*csch(b*x + a)/x^2, x)
```

3.410.8 Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2*csch(b*x + a)/x^2, x)
```

3.410.9 Mupad [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^2}{x^2\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)),x)
```

output
```
int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)), x)
```

3.411 $\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx$

3.411.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} + \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b} + \mathrm{Int}(x^m\coth(a+bx),x)$$

output
```
2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)+2^(-3-m)*x^m*GAMMA(1+
m,2*b*x)/b/exp(2*a)/((b*x)^m)+Unintegrable(x^m*coth(b*x+a),x)
```

3.411.2 Mathematica [N/A]

Not integrable

Time = 16.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \int x^m \cosh^2(a+bx)\coth(a+bx)\,dx$$

input
```
Integrate[x^m*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
Integrate[x^m*Cosh[a + b*x]^2*Coth[a + b*x], x]
```

3.411.3 Rubi [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5973, 3042, 26, 4222, 5971, 27, 3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{5973}$$

$$\int x^m \coth(a+bx)dx+\int x^m \cosh(a+bx)\sinh(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^m \cosh(a+bx)\sinh(a+bx)dx+\int -ix^m \tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$\int x^m \cosh(a+bx)\sinh(a+bx)dx-i\int x^m \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{4222}$$

$$\int x^m \cosh(a+bx)\sinh(a+bx)dx+\int ix^m \tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)dx$$

$$\Big\downarrow \text{5971}$$

$$\int \frac{1}{2}x^m \sinh(2a+2bx)dx+\int ix^m \tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int x^m \sinh(2a+2bx)dx+\int ix^m \tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int -ix^m \sin(2ia+2ibx)dx+\int ix^m \tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)dx$$

$$\Big\downarrow \text{26}$$

$$\int ix^m \tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)dx-\frac{1}{2}i\int x^m \sin(2ia+2ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$\int ix^m \tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)dx-\frac{1}{2}i\left(\frac{1}{2}i\int e^{2(a+bx)}x^m dx-\frac{1}{2}i\int e^{-2(a+bx)}x^m dx\right)$$

$$\Big\downarrow \text{2612}$$

$$\int ix^m \tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)dx-\frac{1}{2}i\left(\frac{ie^{2a}2^{-m-2}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}+\frac{ie^{-2a}2^{-m-2}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}\right)$$

input
```
Int[x^m*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
$Aborted
```

3.411.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.411.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \cosh{(bx + a)}^3 \operatorname{csch}{(bx + a)}\, dx$$

input
```
int(x^m*cosh(b*x+a)^3*csch(b*x+a),x)
```

output
```
int(x^m*cosh(b*x+a)^3*csch(b*x+a),x)
```

3.411.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*cosh(b*x + a)^3*csch(b*x + a), x)
```

3.411.6 Sympy [F(-1)]

Timed out.

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**m*cosh(b*x+a)**3*csch(b*x+a),x)
```

output
```
Timed out
```

3.411.7 Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^m*cosh(b*x + a)^3*csch(b*x + a), x)
```

3.411.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^3*csch(b*x + a), x)
```

3.411.9 Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)^3}{\sinh(a+b\,x)}\,dx$$

input
```
int((x^m*cosh(a + b*x)^3)/sinh(a + b*x),x)
```

output
```
int((x^m*cosh(a + b*x)^3)/sinh(a + b*x), x)
```

3.412 $\int x^3 \cosh^2(a+bx)\coth(a+bx)\,dx$

3.412.1 Optimal result

Integrand size = 18, antiderivative size = 180

$$\begin{aligned}\int x^3\cosh^2(a+bx)\coth(a+bx)\,dx &= \frac{3x}{8b^3}+\frac{x^3}{4b}-\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}\\ &\quad+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}\\ &\quad-\frac{3x\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\operatorname{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}\\ &\quad-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}\\ &\quad-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}\\ &\quad+\frac{3x\sinh^2(a+bx)}{4b^3}+\frac{x^3\sinh^2(a+bx)}{2b}\end{aligned}$$

output
```
3/8*x/b^3+1/4*x^3/b-1/4*x^4+x^3*ln(1-exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,e
xp(2*b*x+2*a))/b^2-3/2*x*polylog(3,exp(2*b*x+2*a))/b^3+3/4*polylog(4,exp(2
*b*x+2*a))/b^4-3/8*cosh(b*x+a)*sinh(b*x+a)/b^4-3/4*x^2*cosh(b*x+a)*sinh(b*
x+a)/b^2+3/4*x*sinh(b*x+a)^2/b^3+1/2*x^3*sinh(b*x+a)^2/b
```

3.412.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.31

$$\int x^3 \cosh^2(a+bx)\coth(a+bx)\,dx$$
$$= \frac{\sinh(a)(\cosh(a)+\sinh(a))\left(4b^4x^4+6bx\cosh(2(a+bx))+4b^3x^3\cosh(2(a+bx))+16b^3x^3\log\left(1-e^{-a-b\right.}\right.}{}$$

input
```
Integrate[x^3*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(Sinh[a]*(Cosh[a] + Sinh[a])*(4*b^4*x^4 + 6*b*x*Cosh[2*(a + b*x)] + 4*b^3*
x^3*Cosh[2*(a + b*x)] + 16*b^3*x^3*Log[1 - E^(-a - b*x)] + 16*b^3*x^3*Log[
1 + E^(-a - b*x)] - 48*b^2*x^2*PolyLog[2, -E^(-a - b*x)] - 48*b^2*x^2*Poly
Log[2, E^(-a - b*x)] - 96*b*x*PolyLog[3, -E^(-a - b*x)] - 96*b*x*PolyLog[3
, E^(-a - b*x)] - 96*PolyLog[4, -E^(-a - b*x)] - 96*PolyLog[4, E^(-a - b*x
)] - 3*Sinh[2*(a + b*x)] - 6*b^2*x^2*Sinh[2*(a + b*x)]))/(8*b^4*(-1 + E^(2
*a)))
```

3.412.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.37, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.056$, Rules used = {5973, 3042, 26, 4201, 2620, 3011, 5895, 3042, 25, 3792, 15, 25, 3042, 25, 3115, 24, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cosh^2(a+bx)\coth(a+bx)\,dx$$
$$\Big\downarrow \; 5973$$
$$\int x^3\coth(a+bx)dx+\int x^3\cosh(a+bx)\sinh(a+bx)dx$$
$$\Big\downarrow \; 3042$$
$$\int x^3\cosh(a+bx)\sinh(a+bx)dx+\int -ix^3\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$\int x^3\cosh(a+bx)\sinh(a+bx)dx-i\int x^3\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{4201}$$

$$\int x^3\cosh(a+bx)\sinh(a+bx)dx-i\left(2i\int\frac{e^{2a+2bx-i\pi}x^3}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{2620}$$

$$\int x^3\cosh(a+bx)\sinh(a+bx)dx-$$
$$i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\int x^2\log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{3011}$$

$$\int x^3\cosh(a+bx)\sinh(a+bx)dx-$$
$$i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{5895}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)-$$
$$\frac{3\int x^2\sinh^2(a+bx)dx}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)-$$
$$\frac{3\int -x^2\sin(ia+ibx)^2dx}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+$$
$$\frac{3\int x^2\sin(ia+ibx)^2dx}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

3.412. $\int x^3\cosh^2(a+bx)\coth(a+bx)\,dx$

$$\Big\downarrow \text{3792}$$

$$\frac{3\left(\frac{\int -\sinh^2(a+bx)dx}{2b^2}+\frac{\int x^2dx}{2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b}-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\left(\frac{\int -\sinh^2(a+bx)dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(-\frac{\int \sinh^2(a+bx)dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{\int -\sin(ia+ibx)^2dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

3.412. $\int x^3\cosh^2(a+bx)\coth(a+bx)\,dx$

$$\frac{3\left(\frac{\int \sin(ia+ibx)^2dx}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3\left(\frac{\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}+\frac{x\sinh^2(a+bx)}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{24}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)+\frac{3\left(\frac{x\sinh^2(a+bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{7163}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)\right)\frac{3\left(\frac{x\sinh^2(a+bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2720}$$

3.412. $\int x^3\cosh^2(a+bx)\coth(a+bx)\,dx$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\ldots}{\ldots}\right.}{2b}\right.\right.$$

$$\frac{3\left(\frac{x\sinh^2(a+bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{7143}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\text{PolyLog}\left(4,-e^{2a+2bx-i\pi}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\frac{i\ldots}{\ldots}\right.$$

$$\frac{3\left(\frac{x\sinh^2(a+bx)}{2b^2}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}-\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)}{2b}+\frac{x^3\sinh^2(a+bx)}{2b}$$

input
```
Int[x^3*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(-I)*((-1/4*I)*x^4 + (2*I)*((x^3*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) -
(3*(-1/2*(x^2*PolyLog[2, -E^(2*a - I*Pi + 2*b*x)])/b + ((x*PolyLog[3, -E^(
2*a - I*Pi + 2*b*x)])/(2*b) - PolyLog[4, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2))
/b))/(2*b))) + (x^3*Sinh[a + b*x]^2)/(2*b) + (3*(x^3/6 - (x^2*Cosh[a + b*x
]*Sinh[a + b*x])/(2*b) + (x*Sinh[a + b*x]^2)/(2*b^2) + (x/2 - (Cosh[a + b*
x]*Sinh[a + b*x])/(2*b))/(2*b^2)))/(2*b)
```

3.412.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.412. $\int x^3 \cosh^2(a+bx)\coth(a+bx)\,dx$

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.412.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{x^4}{4}+\frac{(4x^3b^3-6x^2b^2+6bx-3)\,\mathrm{e}^{2bx+2a}}{32b^4}+\frac{(4x^3b^3+6x^2b^2+6bx+3)\,\mathrm{e}^{-2bx-2a}}{32b^4}-\frac{a^3\ln(\mathrm{e}^{bx+a}-1)}{b^4}+\frac{2a^3\ln(\mathrm{e}^{bx+a})}{b^4}-\frac{3a^4}{2b^4}+$

input
```
int(x^3*cosh(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^4+1/32*(4*b^3*x^3-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)+1/32*(4*b^3
*x^3+6*b^2*x^2+6*b*x+3)/b^4*exp(-2*b*x-2*a)-1/b^4*a^3*ln(exp(b*x+a)-1)+2/b
^4*a^3*ln(exp(b*x+a))-3/2/b^4*a^4+1/b*ln(exp(b*x+a)+1)*x^3+3*x^2*polylog(2
,-exp(b*x+a))/b^2-6*x*polylog(3,-exp(b*x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^3+
3*x^2*polylog(2,exp(b*x+a))/b^2-6*x*polylog(3,exp(b*x+a))/b^3-2/b^3*a^3*x+
6*polylog(4,-exp(b*x+a))/b^4+6*polylog(4,exp(b*x+a))/b^4+1/b^4*ln(1-exp(b*
x+a))*a^3
```

3.412.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 876 vs. $2(159) = 318$.

Time = 0.26 (sec) , antiderivative size = 876, normalized size of antiderivative = 4.87

$$\int x^3 \cosh^2(a + bx) \coth(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

output
```
1/32*(4*b^3*x^3 + (4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)^4 + 4*
(4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b^3
*x^3 - 6*b^2*x^2 + 6*b*x - 3)*sinh(b*x + a)^4 + 6*b^2*x^2 - 8*(b^4*x^4 - 2
*a^4)*cosh(b*x + a)^2 - 2*(4*b^4*x^4 - 8*a^4 - 3*(4*b^3*x^3 - 6*b^2*x^2 +
6*b*x - 3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x + 96*(b^2*x^2*cosh(b*x
 + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)
*dilog(cosh(b*x + a) + sinh(b*x + a)) + 96*(b^2*x^2*cosh(b*x + a)^2 + 2*b^
2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog(-cosh(b
*x + a) - sinh(b*x + a)) + 32*(b^3*x^3*cosh(b*x + a)^2 + 2*b^3*x^3*cosh(b*
x + a)*sinh(b*x + a) + b^3*x^3*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b
*x + a) + 1) - 32*(a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*sinh(b*x + a)
 + a^3*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 32*((b^3*
x^3 + a^3)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)
 + (b^3*x^3 + a^3)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1
) + 192*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
)*polylog(4, cosh(b*x + a) + sinh(b*x + a)) + 192*(cosh(b*x + a)^2 + 2*cos
h(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(4, -cosh(b*x + a) - si
nh(b*x + a)) - 192*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a
) + b*x*sinh(b*x + a)^2)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 192*(
b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x ...
```

3.412.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \cosh^2(a+bx) \coth(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(x**3*cosh(b*x+a)**3*csch(b*x+a),x)
```

output
```
Timed out
```

3.412.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.25

$$\int x^3 \cosh^2(a+bx) \coth(a+bx)\, dx = -\frac{1}{2}\, x^4 + \frac{\left(8\, b^4 x^4 e^{(2\, a)} + \left(4\, b^3 x^3 e^{(4\, a)} - 6\, b^2 x^2 e^{(4\, a)} + 6\, b x e^{(4\, a)} - 3\, e^{(4\, a)}\right) e^{(2\, bx)} + \left(4\, b^3 x^3 + 6\, b^2 x^2 + 6\, bx + 3\right) e^{(-2\, bx)}}{32\, b^4} + \frac{b^3 x^3 \log\left(e^{(bx+a)} + 1\right) + 3\, b^2 x^2 \text{Li}_2\left(-e^{(bx+a)}\right) - 6\, bx \text{Li}_3\left(-e^{(bx+a)}\right) + 6\, \text{Li}_4\left(-e^{(bx+a)}\right)}{b^4} + \frac{b^3 x^3 \log\left(-e^{(bx+a)} + 1\right) + 3\, b^2 x^2 \text{Li}_2\left(e^{(bx+a)}\right) - 6\, bx \text{Li}_3\left(e^{(bx+a)}\right) + 6\, \text{Li}_4\left(e^{(bx+a)}\right)}{b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*x^4 + 1/32*(8*b^4*x^4*e^(2*a) + (4*b^3*x^3*e^(4*a) - 6*b^2*x^2*e^(4*a
) + 6*b*x*e^(4*a) - 3*e^(4*a))*e^(2*b*x) + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x
+ 3)*e^(-2*b*x))*e^(-2*a)/b^4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*
dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*
x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a
)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4
```

3.412.8 Giac [F]

$$\int x^3 \cosh^2(a+bx) \coth(a+bx)\, dx = \int x^3 \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)\, dx$$

input
```
integrate(x^3*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*cosh(b*x + a)^3*csch(b*x + a), x)
```

3.412.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \cosh^2(a+bx) \coth(a+bx)\, dx = \int \frac{x^3 \cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)}\, dx$$

input
```
int((x^3*cosh(a + b*x)^3)/sinh(a + b*x),x)
```

output
```
int((x^3*cosh(a + b*x)^3)/sinh(a + b*x), x)
```

3.413 $\int x^2 \cosh^2(a+bx) \coth(a+bx) \, dx$

3.413.1 Optimal result

Integrand size = 18, antiderivative size = 126

$$\int x^2 \cosh^2(a+bx) \coth(a+bx) \, dx = \frac{x^2}{4b} - \frac{x^3}{3} + \frac{x^2 \log\left(1-e^{2(a+bx)}\right)}{b} + \frac{x \operatorname{PolyLog}\left(2, e^{2(a+bx)}\right)}{b^2} - \frac{\operatorname{PolyLog}\left(3, e^{2(a+bx)}\right)}{2b^3} - \frac{x \cosh(a+bx) \sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x^2 \sinh^2(a+bx)}{2b}$$

output
```
1/4*x^2/b-1/3*x^3+x^2*ln(1-exp(2*b*x+2*a))/b+x*polylog(2,exp(2*b*x+2*a))/b
^2-1/2*polylog(3,exp(2*b*x+2*a))/b^3-1/2*x*cosh(b*x+a)*sinh(b*x+a)/b^2+1/4
*sinh(b*x+a)^2/b^3+1/2*x^2*sinh(b*x+a)^2/b
```

3.413.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.41

$$\int x^2 \cosh^2(a+bx) \coth(a+bx) \, dx = \frac{\sinh(a)(\cosh(a)+\sinh(a))\left(8b^3x^3 + 3\cosh(2(a+bx)) + 6b^2x^2\cosh(2(a+bx)) + 24b^2x^2\log\left(1-e^{-a-bx}\right)\right.}{}$$

input
```
Integrate[x^2*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(Sinh[a]*(Cosh[a] + Sinh[a])*(8*b^3*x^3 + 3*Cosh[2*(a + b*x)] + 6*b^2*x^2*
Cosh[2*(a + b*x)] + 24*b^2*x^2*Log[1 - E^(-a - b*x)] + 24*b^2*x^2*Log[1 +
E^(-a - b*x)] - 48*b*x*PolyLog[2, -E^(-a - b*x)] - 48*b*x*PolyLog[2, E^(-a
 - b*x)] - 48*PolyLog[3, -E^(-a - b*x)] - 48*PolyLog[3, E^(-a - b*x)] - 6*
b*x*Sinh[2*(a + b*x)]))/(12*b^3*(-1 + E^(2*a)))
```

3.413.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.37, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5973, 3042, 26, 4201, 2620, 3011, 2720, 5895, 3042, 25, 3791, 15, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cosh^2(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{5973}$$

$$\int x^2\coth(a+bx)dx+\int x^2\cosh(a+bx)\sinh(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2\cosh(a+bx)\sinh(a+bx)dx+\int -ix^2\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$\int x^2\cosh(a+bx)\sinh(a+bx)dx-i\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{4201}$$

$$\int x^2\cosh(a+bx)\sinh(a+bx)dx-i\left(2i\int \frac{e^{2a+2bx-i\pi}x^2}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2620}$$

$$\int x^2\cosh(a+bx)\sinh(a+bx)dx-$$
$$i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int x\log\left(1+e^{2a+2bx-i\pi}\right)dx}{b}\right)-\frac{ix^3}{3}\right)$$

$\big\downarrow$ 3011

$$\int x^2\cosh(a+bx)\sinh(a+bx)dx-$$
$$i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int \text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

$\big\downarrow$ 2720

$$\int x^2\cosh(a+bx)\sinh(a+bx)dx-$$
$$i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

$\big\downarrow$ 5895

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$
$$\frac{\int x\sinh^2(a+bx)dx}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

$\big\downarrow$ 3042

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$
$$\frac{\int -x\sin(ia+ibx)^2dx}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

$\big\downarrow$ 25

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$
$$\frac{\int x\sin(ia+ibx)^2dx}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

$\big\downarrow$ 3791

3.413. $\int x^2\cosh^2(a+bx)\coth(a+bx)\,dx$

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\operatorname{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$
$$\frac{\frac{\int xdx}{2}+\frac{\sinh^2(a+bx)}{4b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

$\big\downarrow$ 15

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\operatorname{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$
$$\frac{\frac{\sinh^2(a+bx)}{4b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

$\big\downarrow$ 7143

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\operatorname{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)+$$
$$\frac{\frac{\sinh^2(a+bx)}{4b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}}{b}+\frac{x^2\sinh^2(a+bx)}{2b}$$

input
```
Int[x^2*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(-I)*((-1/3*I)*x^3 + (2*I)*((x^2*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) -
(-1/2*(x*PolyLog[2, -E^(2*a - I*Pi + 2*b*x)])/b + PolyLog[3, -E^(2*a - I*P
i + 2*b*x)]/(4*b^2))/b)) + (x^2*Sinh[a + b*x]^2)/(2*b) + (x^2/4 - (x*Cosh[
a + b*x]*Sinh[a + b*x])/(2*b) + Sinh[a + b*x]^2/(4*b^2))/b
```

3.413.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.413. $\int x^2 \cosh^2(a+bx) \coth(a+bx)\,dx$

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.413.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.76

method	result
risch	$-\frac{x^3}{3}+\frac{(2x^2b^2-2bx+1)\,\mathrm{e}^{2bx+2a}}{16b^3}+\frac{(2x^2b^2+2bx+1)\,\mathrm{e}^{-2bx-2a}}{16b^3}+\frac{4a^3}{3b^3}+\frac{2a^2x}{b^2}+\frac{\ln(\mathrm{e}^{bx+a}+1)x^2}{b}+\frac{2x\,\mathrm{polylog}(2,-\mathrm{e}^{bx+a})}{b^2}+$

input
```
int(x^2*cosh(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3+1/16*(2*b^2*x^2-2*b*x+1)/b^3*exp(2*b*x+2*a)+1/16*(2*b^2*x^2+2*b*x
+1)/b^3*exp(-2*b*x-2*a)+4/3/b^3*a^3+2/b^2*a^2*x+1/b*ln(exp(b*x+a)+1)*x^2+2
*x*polylog(2,-exp(b*x+a))/b^2+1/b*ln(1-exp(b*x+a))*x^2+2*x*polylog(2,exp(b
*x+a))/b^2+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^2*ln(exp(b*x+a))-1/b^3*ln(1-
exp(b*x+a))*a^2-2*polylog(3,-exp(b*x+a))/b^3-2*polylog(3,exp(b*x+a))/b^3
```

3.413.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. 2(113) = 226.

Time = 0.27 (sec) , antiderivative size = 697, normalized size of antiderivative = 5.53

$$\int x^2\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{3\left(2\,b^2x^2-2\,bx+1\right)\cosh\left(bx+a\right)^4+12\left(2\,b^2x^2-2\,bx+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+3\left(2\,b^2x^2-2\,bx\right.}{}$$

input
```
integrate(x^2*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

output
```
1/48*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^4 + 12*(2*b^2*x^2 - 2*b*x +
1)*cosh(b*x + a)*sinh(b*x + a)^3 + 3*(2*b^2*x^2 - 2*b*x + 1)*sinh(b*x + a)
^4 + 6*b^2*x^2 - 16*(b^3*x^3 + 2*a^3)*cosh(b*x + a)^2 - 2*(8*b^3*x^3 + 16*
a^3 - 9*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x +
 96*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*
x + a)^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) + 96*(b*x*cosh(b*x + a)^2 +
 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2)*dilog(-cosh(b*x
+ a) - sinh(b*x + a)) + 48*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x +
 a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x
+ a) + 1) + 48*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) +
a^2*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 48*((b^2*x^2
 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) +
(b^2*x^2 - a^2)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) -
 96*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*po
lylog(3, cosh(b*x + a) + sinh(b*x + a)) - 96*(cosh(b*x + a)^2 + 2*cosh(b*x
 + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(3, -cosh(b*x + a) - sinh(b*
x + a)) + 4*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^3 - 8*(b^3*x^3 + 2*a^
3)*cosh(b*x + a))*sinh(b*x + a) + 3)/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x
 + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2)
```

3.413.6 Sympy [F]

$$\int x^2 \cosh^2(a+bx) \coth(a+bx)\, dx = \int x^2 \cosh^3{(a+bx)} \operatorname{csch}{(a+bx)}\, dx$$

input
```
integrate(x**2*cosh(b*x+a)**3*csch(b*x+a),x)
```

output
```
Integral(x**2*cosh(a + b*x)**3*csch(a + b*x), x)
```

3.413.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.36

$$\int x^2\cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{2}{3}\,x^3 + \frac{\left(16\,b^3x^3e^{(2\,a)} + 3\,\left(2\,b^2x^2e^{(4\,a)} - 2\,bxe^{(4\,a)} + e^{(4\,a)}\right)e^{(2\,bx)} + 3\,(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx)}\right)e^{(-2\,a)}}{48\,b^3} + \frac{b^2x^2\log\left(e^{(bx+a)} + 1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)}{b^3} + \frac{b^2x^2\log\left(-e^{(bx+a)} + 1\right) + 2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right) - 2\,\mathrm{Li}_3\left(e^{(bx+a)}\right)}{b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

output
```
-2/3*x^3 + 1/48*(16*b^3*x^3*e^(2*a) + 3*(2*b^2*x^2*e^(4*a) - 2*b*x*e^(4*a)
 + e^(4*a))*e^(2*b*x) + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x))*e^(-2*a)/b^3
 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3
, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x
 + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

3.413.8 Giac [F]

$$\int x^2\cosh^2(a+bx)\coth(a+bx)\,dx = \int x^2\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)\,dx$$

input
```
integrate(x^2*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*cosh(b*x + a)^3*csch(b*x + a), x)
```

3.413.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh^2(a+bx)\coth(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)}\,dx$$

input
```
int((x^2*cosh(a + b*x)^3)/sinh(a + b*x),x)
```

output
```
int((x^2*cosh(a + b*x)^3)/sinh(a + b*x), x)
```

3.414 $\int x \cosh^2(a+bx) \coth(a+bx) \, dx$

3.414.1 Optimal result

Integrand size = 16, antiderivative size = 88

$$\int x \cosh^2(a+bx) \coth(a+bx) \, dx = \frac{x}{4b} - \frac{x^2}{2} + \frac{x \log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x \sinh^2(a+bx)}{2b}$$

output
```
1/4*x/b-1/2*x^2+x*ln(1-exp(2*b*x+2*a))/b+1/2*polylog(2,exp(2*b*x+2*a))/b^2
-1/4*cosh(b*x+a)*sinh(b*x+a)/b^2+1/2*x*sinh(b*x+a)^2/b
```

3.414.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int x \cosh^2(a+bx) \coth(a+bx) \, dx = -\frac{4a^2 - 4b^2x^2 - 2bx\cosh(2(a+bx)) - 8bx\log\left(1-e^{-2(a+bx)}\right) + 4\,\text{PolyLog}\left(2, e^{-2(a+bx)}\right) + \sinh(2(a+bx}{8b^2}$$

input
```
Integrate[x*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
-1/8*(4*a^2 - 4*b^2*x^2 - 2*b*x*Cosh[2*(a + b*x)] - 8*b*x*Log[1 - E^(-2*(a
 + b*x))] + 4*PolyLog[2, E^(-2*(a + b*x))] + Sinh[2*(a + b*x)])/b^2
```

3.414.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5973, 3042, 26, 4201, 2620, 2715, 2838, 5895, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{5973}$$

$$\int x\coth(a+bx)dx+\int x\cosh(a+bx)\sinh(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx+\int -ix\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx-i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{4201}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx-i\left(2i\int \frac{e^{2a+2bx-i\pi}x}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx-$$
$$i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int\log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx-$$
$$i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\log\left(1+e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$\int x\cosh(a+bx)\sinh(a+bx)dx - i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \; 5895$$

$$-\frac{\int\sinh^2(a+bx)dx}{2b} - i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{ix^2}{2}\right)+\frac{x\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int-\sin(ia+ibx)^2dx}{2b} - i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{ix^2}{2}\right)+\frac{x\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \; 25$$

$$\frac{\int\sin(ia+ibx)^2dx}{2b} - i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{ix^2}{2}\right)+\frac{x\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b} - i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{ix^2}{2}\right)+\frac{x\sinh^2(a+bx)}{2b}$$

$$\Big\downarrow \; 24$$

$$-i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{ix^2}{2}\right)+\frac{x\sinh^2(a+bx)}{2b}+\frac{\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b}$$

input
```
Int[x*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(-I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) + Po
lyLog[2, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2))) + (x*Sinh[a + b*x]^2)/(2*b) +
(x/2 - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b))/(2*b)
```

3.414. $\int x\cosh^2(a+bx)\coth(a+bx)\,dx$

3.414.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.414.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(78) = 156.

Time = 1.02 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.84

method	result
risch	$-\frac{x^2}{2}+\frac{(2bx-1)\mathrm{e}^{2bx+2a}}{16b^2}+\frac{(2bx+1)\mathrm{e}^{-2bx-2a}}{16b^2}-\frac{2ax}{b}-\frac{a^2}{b^2}+\frac{\ln(\mathrm{e}^{bx+a}+1)x}{b}+\frac{\mathrm{polylog}(2,-\mathrm{e}^{bx+a})}{b^2}+\frac{\ln(1-\mathrm{e}^{bx+a})x}{b}+$

input
```
int(x*cosh(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2+1/16*(2*b*x-1)/b^2*exp(2*b*x+2*a)+1/16*(2*b*x+1)/b^2*exp(-2*b*x-2
*a)-2/b*a*x-a^2/b^2+1/b*ln(exp(b*x+a)+1)*x+polylog(2,-exp(b*x+a))/b^2+1/b*
ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2-1/b^
2*a*ln(exp(b*x+a)-1)+2/b^2*a*ln(exp(b*x+a))
```

3.414.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(77) = 154.

Time = 0.27 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.55

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx$$
$$=\frac{(2\,bx-1)\cosh\left(bx+a\right)^4+4\,(2\,bx-1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+(2\,bx-1)\sinh\left(bx+a\right)^4-8\,(b^2x^2-}{}$$

input
```
integrate(x*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

output
```
1/16*((2*b*x - 1)*cosh(b*x + a)^4 + 4*(2*b*x - 1)*cosh(b*x + a)*sinh(b*x +
 a)^3 + (2*b*x - 1)*sinh(b*x + a)^4 - 8*(b^2*x^2 - 2*a^2)*cosh(b*x + a)^2
- 2*(4*b^2*x^2 - 3*(2*b*x - 1)*cosh(b*x + a)^2 - 8*a^2)*sinh(b*x + a)^2 +
2*b*x + 16*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a
)^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) + 16*(cosh(b*x + a)^2 + 2*cosh(b
*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*dilog(-cosh(b*x + a) - sinh(b*x +
 a)) + 16*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*s
inh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 16*(a*cosh(b*x +
a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2)*log(cosh(b*x +
 a) + sinh(b*x + a) - 1) + 16*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cos
h(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2)*log(-cosh(b*x + a) -
 sinh(b*x + a) + 1) + 4*((2*b*x - 1)*cosh(b*x + a)^3 - 4*(b^2*x^2 - 2*a^2)
*cosh(b*x + a))*sinh(b*x + a) + 1)/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x +
 a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2)
```

3.414.6 Sympy [F]

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx = \int x\cosh^3(a+bx)\operatorname{csch}(a+bx)\,dx$$

input
```
integrate(x*cosh(b*x+a)**3*csch(b*x+a),x)
```

output
```
Integral(x*cosh(a + b*x)**3*csch(a + b*x), x)
```

3.414.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx = -x^2 + \frac{\left(8\,b^2x^2e^{(2\,a)} + \left(2\,bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(2\,bx)} + (2\,bx+1)e^{(-2\,bx)}\right)e^{(-2\,a)}}{16\,b^2} + \frac{bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

input
```
integrate(x*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

output
```
-x^2 + 1/16*(8*b^2*x^2*e^(2*a) + (2*b*x*e^(4*a) - e^(4*a))*e^(2*b*x) + (2*
b*x + 1)*e^(-2*b*x))*e^(-2*a)/b^2 + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(
b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

3.414.8 Giac [F]

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx = \int x\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)\,dx$$

input
```
integrate(x*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)^3*csch(b*x + a), x)
```

3.414.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx = \int \frac{x\cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)}\,dx$$

input
```
int((x*cosh(a + b*x)^3)/sinh(a + b*x),x)
```

output
```
int((x*cosh(a + b*x)^3)/sinh(a + b*x), x)
```

3.415 $\int \cosh^2(a+bx)\coth(a+bx)\,dx$

3.415.1 Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{2b}$$

output
```
ln(sinh(b*x+a))/b+1/2*sinh(b*x+a)^2/b
```

3.415.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx=\frac{2\log(\sinh(a+bx))+\sinh^2(a+bx)}{2b}$$

input
```
Integrate[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(2*Log[Sinh[a + b*x]] + Sinh[a + b*x]^2)/(2*b)
```

3.415.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin\left(ia+ibx+\frac{\pi}{2}\right)^2\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int i\text{csch}(a+bx)\left(\sinh^2(a+bx)+1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int (i\text{csch}(a+bx)+i\sinh(a+bx))d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\sinh^2(a+bx)+\log(-i\sinh(a+bx))}{b}$$

input
```
Int[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(Log[(-I)*Sinh[a + b*x]] + Sinh[a + b*x]^2/2)/b
```

3.415.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.415.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	23
default	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	23
risch	$-x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}-\frac{2a}{b}+\frac{\ln(e^{2bx+2a}-1)}{b}$	55

input
```
int(cosh(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*cosh(b*x+a)^2+ln(sinh(b*x+a)))
```

3.415.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 203, normalized size of antiderivative = 7.52

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{8\,bx\cosh\left(bx+a\right)^2-\cosh\left(bx+a\right)^4-4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\sinh\left(bx+a\right)^4+2\left(4\,bx-3\cos\right.}{}$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(8*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 - 4*cosh(b*x + a)*sinh(b*x +
 a)^3 - sinh(b*x + a)^4 + 2*(4*b*x - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 -
8*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(
2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a)
- cosh(b*x + a)^3)*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x +
a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.415.6 Sympy [F]

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \int \cosh^3\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

input
```
integrate(cosh(b*x+a)**3*csch(b*x+a),x)
```

output
```
Integral(cosh(a + b*x)**3*csch(a + b*x), x)
```

3.415.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(25) = 50.

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{bx+a}{b}+\frac{e^{(2\,bx+2\,a)}}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

output
```
(b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b + log(e^(-b*x
 - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

3.415.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{8\,bx - \left(4\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)} + 8\,a - e^{(2\,bx+2\,a)} - 8\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{8\,b}$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(8*b*x - (4*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 8*a - e^(2*b*x +
2*a) - 8*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.415.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\ln\left(e^{2\,a}\,e^{2\,b\,x}-1\right)}{b} - x + \frac{e^{-2\,a-2\,b\,x}}{8\,b} + \frac{e^{2\,a+2\,b\,x}}{8\,b}$$

input
```
int(cosh(a + b*x)^3/sinh(a + b*x),x)
```

output
```
log(exp(2*a)*exp(2*b*x) - 1)/b - x + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a +
2*b*x)/(8*b)
```

3.416 $\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx$

3.416.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \frac{1}{2}\text{Chi}(2bx)\sinh(2a) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx) + \text{Int}\left(\frac{\coth(a+bx)}{x},x\right)$$

output
```
1/2*cosh(2*a)*Shi(2*b*x)+1/2*Chi(2*b*x)*sinh(2*a)+Unintegrable(coth(b*x+a)
/x,x)
```

3.416.2 Mathematica [N/A]

Not integrable

Time = 7.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx$$

input
```
Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x,x]
```

output
```
Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x, x]
```

3.416.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5973, 3042, 26, 4222, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}dx$$

$$\Big\downarrow \text{5973}$$

$$\int \frac{\coth(a+bx)}{x}dx+\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}dx+\int -\frac{i\tan\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}dx-i\int \frac{\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)}{x}dx$$

$$\Big\downarrow \text{4222}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}dx+\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x}dx$$

$$\Big\downarrow \text{5971}$$

$$\int \frac{\sinh(2a+2bx)}{2x}dx+\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sinh(2a+2bx)}{x}dx+\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{x}dx+\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x}dx-\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x}dx-\frac{1}{2}i\left(i\sinh(2a)\int\frac{\cosh(2bx)}{x}dx+\cosh(2a)\int\frac{i\sinh(2bx)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x}dx-\frac{1}{2}i\left(i\sinh(2a)\int\frac{\cosh(2bx)}{x}dx+i\cosh(2a)\int\frac{\sinh(2bx)}{x}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x}dx-$$
$$\frac{1}{2}i\left(i\sinh(2a)\int\frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(2a)\int-\frac{i\sin(2ibx)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x}dx-\frac{1}{2}i\left(i\sinh(2a)\int\frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx+\cosh(2a)\int\frac{\sin(2ibx)}{x}dx\right)$$

$$\Big\downarrow \text{3779}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x}dx-\frac{1}{2}i\left(i\sinh(2a)\int\frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx+i\cosh(2a)\text{Shi}(2bx)\right)$$

$$\Big\downarrow \text{3782}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x}dx-\frac{1}{2}i(i\sinh(2a)\text{Chi}(2bx)+i\cosh(2a)\text{Shi}(2bx))$$

input
```
Int[(Cosh[a + b*x]^2*Coth[a + b*x])/x,x]
```

output
```
$Aborted
```

3.416.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.416. $\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.416. $\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x} \, dx$

3.416.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(bx+a)^3 \operatorname{csch}(bx+a)}{x}dx$$

input
```
int(cosh(b*x+a)^3*csch(b*x+a)/x,x)
```

output
```
int(cosh(b*x+a)^3*csch(b*x+a)/x,x)
```

3.416.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^3 \operatorname{csch}(bx+a)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^3*csch(b*x + a)/x, x)
```

3.416.6 Sympy [N/A]

Not integrable

Time = 21.80 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh^3(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)**3*csch(b*x+a)/x,x)
```

output
```
Integral(cosh(a + b*x)**3*csch(a + b*x)/x, x)
```

3.416.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.17

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)/x,x, algorithm="maxima")
```

output
```
1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a) - integrate(1/(x*e^(b*x +
a) + x), x) + integrate(1/(x*e^(b*x + a) - x), x) + log(x)
```

3.416.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3*csch(b*x + a)/x, x)
```

3.416.9 Mupad [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^3}{x\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(cosh(a + b*x)^3/(x*sinh(a + b*x)),x)
```

output
```
int(cosh(a + b*x)^3/(x*sinh(a + b*x)), x)
```

3.417 $\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx$

3.417.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = b\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{2x} + b\sinh(2a)\text{Shi}(2bx) + \text{Int}\left(\frac{\coth(a+bx)}{x^2},x\right)$$

output
```
b*Chi(2*b*x)*cosh(2*a)+b*Shi(2*b*x)*sinh(2*a)-1/2*sinh(2*b*x+2*a)/x+Uninte
grable(coth(b*x+a)/x^2,x)
```

3.417.2 Mathematica [N/A]

Not integrable

Time = 7.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x^2,x]
```

output
```
Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x^2, x]
```

3.417.3 Rubi [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5973, 3042, 26, 4222, 5971, 27, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{5973}$$

$$\int \frac{\coth(a+bx)}{x^2}dx+\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}dx+\int -\frac{i\tan\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}dx-i\int \frac{\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)}{x^2}dx$$

$$\Big\downarrow \text{4222}$$

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}dx+\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx$$

$$\Big\downarrow \text{5971}$$

$$\int \frac{\sinh(2a+2bx)}{2x^2}dx+\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^2}dx+\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int -\frac{i\sin(2ia+2ibx)}{x^2}dx+\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx-\frac{1}{2}i\int \frac{\sin(2ia+2ibx)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx-\frac{1}{2}i\left(2ib\int \frac{\cosh(2a+2bx)}{x}dx-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx-\frac{1}{2}i\left(2ib\int \frac{\sin\left(2ia+2ibx+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3784}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx-\frac{1}{2}i\left(2ib\left(\cosh(2a)\int \frac{\cosh(2bx)}{x}dx-i\sinh(2a)\int \frac{i\sinh(2bx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx-\frac{1}{2}i\left(2ib\left(\sinh(2a)\int \frac{\sinh(2bx)}{x}dx+\cosh(2a)\int \frac{\cosh(2bx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx-\frac{1}{2}i\left(2ib\left(\sinh(2a)\int -\frac{i\sin(2ibx)}{x}dx+\cosh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx-\frac{1}{2}i\left(2ib\left(\cosh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx-i\sinh(2a)\int \frac{\sin(2ibx)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3779}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx-\frac{1}{2}i\left(2ib\left(\sinh(2a)\text{Shi}(2bx)+\cosh(2a)\int \frac{\sin\left(2ibx+\frac{\pi}{2}\right)}{x}dx\right)-\frac{i\sinh(2a+2bx)}{x}\right)$$

$$\Big\downarrow \text{3782}$$

3.417. $\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx$

$$\int \frac{i\tan\left(\frac{1}{2}(-2ia-\pi)-ibx\right)}{x^2}dx -$$
$$\frac{1}{2}i\left(2ib(\cosh(2a)\text{Chi}(2bx)+\sinh(2a)\text{Shi}(2bx))-\frac{i\sinh(2a+2bx)}{x}\right)$$

input
```
Int[(Cosh[a + b*x]^2*Coth[a + b*x])/x^2,x]
```

output
```
$Aborted
```

3.417.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.417.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh (bx+a)^3 \operatorname{csch}(bx+a)}{x^2} dx$$

input
```
int(cosh(b*x+a)^3*csch(b*x+a)/x^2,x)
```

output
```
int(cosh(b*x+a)^3*csch(b*x+a)/x^2,x)
```

3.417.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^3*csch(b*x + a)/x^2, x)
```

3.417.6 Sympy [N/A]

Not integrable

Time = 26.59 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh^3\left(a+bx\right)\operatorname{csch}\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)**3*csch(b*x+a)/x**2,x)
```

output
```
Integral(cosh(a + b*x)**3*csch(a + b*x)/x**2, x)
```

3.417.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 4.00

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/2*b*e^(2*a)*gamma(-1, -2*b*x) - 1/x -
integrate(1/(x^2*e^(b*x + a) + x^2), x) + integrate(1/(x^2*e^(b*x + a) - x
^2), x)
```

3.417.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3*csch(b*x + a)/x^2, x)
```

3.417.9 Mupad [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^3}{x^2\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)),x)
```

output
```
int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)), x)
```

3.418 $\int x\cosh^2(x)\coth^2(x)\,dx$

3.418.1 Optimal result

Integrand size = 10, antiderivative size = 33

$$\int x\cosh^2(x)\coth^2(x)\,dx=\frac{3x^2}{4}-\frac{\cosh^2(x)}{4}-x\coth(x)+\log(\sinh(x))+\frac{1}{2}x\cosh(x)\sinh(x)$$

output
```
3/4*x^2-1/4*cosh(x)^2-x*coth(x)+ln(sinh(x))+1/2*x*cosh(x)*sinh(x)
```

3.418.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x\cosh^2(x)\coth^2(x)\,dx=\frac{3x^2}{4}-\frac{1}{8}\cosh(2x)-x\coth(x)+\log(\sinh(x))+\frac{1}{4}x\sinh(2x)$$

input
```
Integrate[x*Cosh[x]^2*Coth[x]^2,x]
```

output
```
(3*x^2)/4 - Cosh[2*x]/8 - x*Coth[x] + Log[Sinh[x]] + (x*Sinh[2*x])/4
```

3.418.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {5973, 3042, 25, 3791, 15, 4203, 15, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh^2(x)\coth^2(x)\,dx$$

$$\Big\downarrow \text{5973}$$

$$\int x\cosh^2(x)dx+\int x\coth^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sin\left(ix+\frac{\pi}{2}\right)^2dx+\int -x\tan\left(ix+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$\int x\sin\left(ix+\frac{\pi}{2}\right)^2dx-\int x\tan\left(ix+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{\int xdx}{2}-\int x\tan\left(ix+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\cosh^2(x)+\frac{1}{2}x\sinh(x)\cosh(x)$$

$$\Big\downarrow \text{15}$$

$$-\int x\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)$$

$$\Big\downarrow \text{4203}$$

$$\int xdx-i\int i\coth(x)dx+\frac{x^2}{4}-\frac{\cosh^2(x)}{4}-x\coth(x)+\frac{1}{2}x\sinh(x)\cosh(x)$$

$$\Big\downarrow \text{15}$$

$$-i\int i\coth(x)dx+\frac{3x^2}{4}-\frac{\cosh^2(x)}{4}-x\coth(x)+\frac{1}{2}x\sinh(x)\cosh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \coth(x)dx+\frac{3x^2}{4}-\frac{\cosh^2(x)}{4}-x\coth(x)+\frac{1}{2}x\sinh(x)\cosh(x)$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan\left(ix+\frac{\pi}{2}\right)dx+\frac{3x^2}{4}-\frac{\cosh^2(x)}{4}-x\coth(x)+\frac{1}{2}x\sinh(x)\cosh(x)$$

$$\Big\downarrow \text{26}$$

$$-i\int\tan\left(ix+\frac{\pi}{2}\right)dx+\frac{3x^2}{4}-\frac{\cosh^2(x)}{4}-x\coth(x)+\frac{1}{2}x\sinh(x)\cosh(x)$$

$$\Big\downarrow \text{3956}$$

$$\frac{3x^2}{4}-\frac{\cosh^2(x)}{4}-x\coth(x)+\log(\sinh(x))+\frac{1}{2}x\sinh(x)\cosh(x)$$

input
```
Int[x*Cosh[x]^2*Coth[x]^2,x]
```

output
```
(3*x^2)/4 - Cosh[x]^2/4 - x*Coth[x] + Log[Sinh[x]] + (x*Cosh[x]*Sinh[x])/2
```

3.418.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.418. $\int x\cosh^2(x)\coth^2(x)\,dx$

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.418.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

method	result	size
risch	$\frac{3x^2}{4}+\left(-\frac{1}{16}+\frac{x}{8}\right)\mathrm{e}^{2x}+\left(-\frac{1}{16}-\frac{x}{8}\right)\mathrm{e}^{-2x}-2x-\frac{2x}{\mathrm{e}^{2x}-1}+\ln\left(\mathrm{e}^{2x}-1\right)$	48

input
```
int(x*cosh(x)^2*coth(x)^2,x,method=_RETURNVERBOSE)
```

output
```
3/4*x^2+(-1/16+1/8*x)*exp(2*x)+(-1/16-1/8*x)*exp(-2*x)-2*x-2*x/(exp(2*x)-1
)+ln(exp(2*x)-1)
```

3.418.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 336, normalized size of antiderivative = 10.18

$$\int x\cosh^2(x)\coth^2(x)\,dx$$
$$= \frac{(2\,x-1)\cosh{(x)}^6+6\,(2\,x-1)\cosh{(x)}\sinh{(x)}^5+(2\,x-1)\sinh{(x)}^6+(12\,x^2-34\,x+1)\cosh{(x)}^4+(\dots}{\dots}$$

input
```
integrate(x*cosh(x)^2*coth(x)^2,x, algorithm="fricas")
```

output
```
1/16*((2*x - 1)*cosh(x)^6 + 6*(2*x - 1)*cosh(x)*sinh(x)^5 + (2*x - 1)*sinh
(x)^6 + (12*x^2 - 34*x + 1)*cosh(x)^4 + (15*(2*x - 1)*cosh(x)^2 + 12*x^2 -
 34*x + 1)*sinh(x)^4 + 4*(5*(2*x - 1)*cosh(x)^3 + (12*x^2 - 34*x + 1)*cosh
(x))*sinh(x)^3 - (12*x^2 + 2*x + 1)*cosh(x)^2 + (15*(2*x - 1)*cosh(x)^4 +
6*(12*x^2 - 34*x + 1)*cosh(x)^2 - 12*x^2 - 2*x - 1)*sinh(x)^2 + 16*(cosh(x
)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh
(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x
))) + 2*(3*(2*x - 1)*cosh(x)^5 + 2*(12*x^2 - 34*x + 1)*cosh(x)^3 - (12*x^2
 + 2*x + 1)*cosh(x))*sinh(x) + 2*x + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 +
 sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - co
sh(x))*sinh(x))
```

3.418.6 Sympy [F]

$$\int x\cosh^2(x)\coth^2(x)\,dx = \int x\cosh^2{(x)}\coth^2{(x)}\,dx$$

input
```
integrate(x*cosh(x)**2*coth(x)**2,x)
```

output
```
Integral(x*cosh(x)**2*coth(x)**2, x)
```

3.418.7 Maxima [F(-2)]

Exception generated.

$$\int x\cosh^2(x)\coth^2(x)\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*cosh(x)^2*coth(x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.418.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.06

$$\int x\cosh^2(x)\coth^2(x)\,dx = \frac{12\,x^2e^{(4\,x)} - 12\,x^2e^{(2\,x)} + 2\,xe^{(6\,x)} - 34\,xe^{(4\,x)} - 2\,xe^{(2\,x)} + 16\,e^{(4\,x)}\log\left(e^{(2\,x)} - 1\right) - 16\,e^{(2\,x)}\log\left(e^{(2\,x)} - 1\right)}{16\left(e^{(4\,x)} - e^{(2\,x)}\right)}$$

input
```
integrate(x*cosh(x)^2*coth(x)^2,x, algorithm="giac")
```

output
```
1/16*(12*x^2*e^(4*x) - 12*x^2*e^(2*x) + 2*x*e^(6*x) - 34*x*e^(4*x) - 2*x*e
^(2*x) + 16*e^(4*x)*log(e^(2*x) - 1) - 16*e^(2*x)*log(e^(2*x) - 1) + 2*x -
 e^(6*x) + e^(4*x) - e^(2*x) + 1)/(e^(4*x) - e^(2*x))
```

3.418.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int x\cosh^2(x)\coth^2(x)\,dx = \ln\left(\mathrm{e}^{2\,x} - 1\right) - 2\,x - \mathrm{e}^{-2\,x}\left(\frac{x}{8} + \frac{1}{16}\right) + \mathrm{e}^{2\,x}\left(\frac{x}{8} - \frac{1}{16}\right) - \frac{2\,x}{\mathrm{e}^{2\,x} - 1} + \frac{3\,x^2}{4}$$

input
```
int(x*cosh(x)^2*coth(x)^2,x)
```

output
```
log(exp(2*x) - 1) - 2*x - exp(-2*x)*(x/8 + 1/16) + exp(2*x)*(x/8 - 1/16) -
 (2*x)/(exp(2*x) - 1) + (3*x^2)/4
```

3.419 $\int x^2 \cosh^2(x) \coth^2(x) \, dx$

3.419.1 Optimal result

Integrand size = 12, antiderivative size = 73

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx = \frac{x}{4} - x^2 + \frac{x^3}{2} - \frac{1}{2} x \cosh^2(x) - x^2 \coth(x) + 2x \log\left(1 - e^{2x}\right) + \operatorname{PolyLog}\left(2, e^{2x}\right) + \frac{1}{4} \cosh(x) \sinh(x) + \frac{1}{2} x^2 \cosh(x) \sinh(x)$$

output
```
1/4*x-x^2+1/2*x^3-1/2*x*cosh(x)^2-x^2*coth(x)+2*x*ln(1-exp(2*x))+polylog(2
,exp(2*x))+1/4*cosh(x)*sinh(x)+1/2*x^2*cosh(x)*sinh(x)
```

3.419.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx = \frac{1}{8} \left(8x^2 + 4x^3 - 2x \cosh(2x) - 8x^2 \coth(x) + 16x \log\left(1 - e^{-2x}\right) - 8 \operatorname{PolyLog}\left(2, e^{-2x}\right) + \sinh(2x) + 2x^2 \sinh(2x)\right)$$

input
```
Integrate[x^2*Cosh[x]^2*Coth[x]^2,x]
```

output
```
(8*x^2 + 4*x^3 - 2*x*Cosh[2*x] - 8*x^2*Coth[x] + 16*x*Log[1 - E^(-2*x)] -
8*PolyLog[2, E^(-2*x)] + Sinh[2*x] + 2*x^2*Sinh[2*x])/8
```

3.419.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.34, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {5973, 3042, 25, 3792, 15, 3042, 3115, 24, 4203, 15, 26, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx$$

$$\Big\downarrow \; 5973$$

$$\int x^2 \cosh^2(x)dx + \int x^2 \coth^2(x)dx$$

$$\Big\downarrow \; 3042$$

$$\int x^2 \sin\left(ix + \frac{\pi}{2}\right)^2 dx + \int -x^2 \tan\left(ix + \frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \; 25$$

$$\int x^2 \sin\left(ix + \frac{\pi}{2}\right)^2 dx - \int x^2 \tan\left(ix + \frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \; 3792$$

$$\frac{\int x^2 dx}{2} - \int x^2 \tan\left(ix + \frac{\pi}{2}\right)^2 dx + \frac{1}{2}\int \cosh^2(x)dx + \frac{1}{2}x^2 \sinh(x)\cosh(x) - \frac{1}{2}x\cosh^2(x)$$

$$\Big\downarrow \; 15$$

$$-\int x^2 \tan\left(ix + \frac{\pi}{2}\right)^2 dx + \frac{1}{2}\int \cosh^2(x)dx + \frac{x^3}{6} + \frac{1}{2}x^2 \sinh(x)\cosh(x) - \frac{1}{2}x\cosh^2(x)$$

$$\Big\downarrow \; 3042$$

$$-\int x^2 \tan\left(ix + \frac{\pi}{2}\right)^2 dx + \frac{1}{2}\int \sin\left(ix + \frac{\pi}{2}\right)^2 dx + \frac{x^3}{6} + \frac{1}{2}x^2 \sinh(x)\cosh(x) - \frac{1}{2}x\cosh^2(x)$$

$$\Big\downarrow \; 3115$$

$$-\int x^2 \tan\left(ix + \frac{\pi}{2}\right)^2 dx + \frac{1}{2}\left(\frac{\int 1dx}{2} + \frac{1}{2}\sinh(x)\cosh(x)\right) + \frac{x^3}{6} + \frac{1}{2}x^2 \sinh(x)\cosh(x) - \frac{1}{2}x\cosh^2(x)$$

$$\Big\downarrow \; 24$$

$$-\int x^2 \tan\left(ix + \frac{\pi}{2}\right)^2 dx + \frac{x^3}{6} + \frac{1}{2}x^2 \sinh(x)\cosh(x) - \frac{1}{2}x\cosh^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{4203}$$

$$\int x^2dx - 2i\int ix\coth(x)dx + \frac{x^3}{6} - x^2\coth(x) + \frac{1}{2}x^2\sinh(x)\cosh(x) - \frac{1}{2}x\cosh^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{15}$$

$$-2i\int ix\coth(x)dx + \frac{x^3}{2} - x^2\coth(x) + \frac{1}{2}x^2\sinh(x)\cosh(x) - \frac{1}{2}x\cosh^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{26}$$

$$2\int x\coth(x)dx + \frac{x^3}{2} - x^2\coth(x) + \frac{1}{2}x^2\sinh(x)\cosh(x) - \frac{1}{2}x\cosh^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$2\int -ix\tan\left(ix + \frac{\pi}{2}\right)dx + \frac{x^3}{2} - x^2\coth(x) + \frac{1}{2}x^2\sinh(x)\cosh(x) - \frac{1}{2}x\cosh^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-2i\int x\tan\left(ix + \frac{\pi}{2}\right)dx + \frac{x^3}{2} - x^2\coth(x) + \frac{1}{2}x^2\sinh(x)\cosh(x) - \frac{1}{2}x\cosh^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{4199}$$

$$-2i\left(2i\int -\frac{e^{2x}x}{1-e^{2x}}dx - \frac{ix^2}{2}\right) + \frac{x^3}{2} - x^2\coth(x) + \frac{1}{2}x^2\sinh(x)\cosh(x) - \frac{1}{2}x\cosh^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{25}$$

$$-2i\left(-2i\int \frac{e^{2x}x}{1-e^{2x}}dx - \frac{ix^2}{2}\right) + \frac{x^3}{2} - x^2\coth(x) + \frac{1}{2}x^2\sinh(x)\cosh(x) - \frac{1}{2}x\cosh^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{2620}$$

3.419. $\int x^2\cosh^2(x)\coth^2(x)\,dx$

$$-2i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2x}\right)dx-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)+\frac{x^3}{2}-x^2\coth(x)+\frac{1}{2}x^2\sinh(x)\cosh(x)-\frac{1}{2}x\cosh^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{2715}$$

$$-2i\left(-2i\left(\frac{1}{4}\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)+\frac{x^3}{2}-x^2\coth(x)+\frac{1}{2}x^2\sinh(x)\cosh(x)-\frac{1}{2}x\cosh^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{2838}$$

$$-2i\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)+\frac{x^3}{2}-x^2\coth(x)+\frac{1}{2}x^2\sinh(x)\cosh(x)-\frac{1}{2}x\cosh^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)$$

input
```
Int[x^2*Cosh[x]^2*Coth[x]^2,x]
```

output
```
x^3/2 - (x*Cosh[x]^2)/2 - x^2*Coth[x] - (2*I)*((-1/2*I)*x^2 - (2*I)*(-1/2*
(x*Log[1 - E^(2*x)]) - PolyLog[2, E^(2*x)]/4)) + (x^2*Cosh[x]*Sinh[x])/2 +
 (x/2 + (Cosh[x]*Sinh[x])/2)/2
```

3.419.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

3.419. $\int x^2 \cosh^2(x) \coth^2(x) \, dx$

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.419.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

method	result
risch	$\frac{x^3}{2}+\left(\frac{1}{16}-\frac{1}{8}x+\frac{1}{8}x^2\right)\mathrm{e}^{2x}+\left(-\frac{1}{16}-\frac{1}{8}x-\frac{1}{8}x^2\right)\mathrm{e}^{-2x}-\frac{2x^2}{\mathrm{e}^{2x}-1}-2x^2+2x\ln\left(1-\mathrm{e}^{x}\right)+2\,\mathrm{polylog}\left(2\right.$

input
```
int(x^2*cosh(x)^2*coth(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^3+(1/16-1/8*x+1/8*x^2)*exp(x)^2+(-1/16-1/8*x-1/8*x^2)/exp(x)^2-2*x^2
/(exp(x)^2-1)-2*x^2+2*x*ln(1-exp(x))+2*polylog(2,exp(x))+2*x*ln(exp(x)+1)+
2*polylog(2,-exp(x))
```

3.419.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(60) = 120.

Time = 0.26 (sec) , antiderivative size = 617, normalized size of antiderivative = 8.45

$$\int x^2\cosh^2(x)\coth^2(x)\,dx=\text{Too large to display}$$

input
```
integrate(x^2*cosh(x)^2*coth(x)^2,x, algorithm="fricas")
```

output
```
1/16*((2*x^2 - 2*x + 1)*cosh(x)^6 + 6*(2*x^2 - 2*x + 1)*cosh(x)*sinh(x)^5
+ (2*x^2 - 2*x + 1)*sinh(x)^6 + (8*x^3 - 34*x^2 + 2*x - 1)*cosh(x)^4 + (8*
x^3 + 15*(2*x^2 - 2*x + 1)*cosh(x)^2 - 34*x^2 + 2*x - 1)*sinh(x)^4 + 4*(5*
(2*x^2 - 2*x + 1)*cosh(x)^3 + (8*x^3 - 34*x^2 + 2*x - 1)*cosh(x))*sinh(x)^
3 - (8*x^3 + 2*x^2 + 2*x + 1)*cosh(x)^2 + (15*(2*x^2 - 2*x + 1)*cosh(x)^4
- 8*x^3 + 6*(8*x^3 - 34*x^2 + 2*x - 1)*cosh(x)^2 - 2*x^2 - 2*x - 1)*sinh(x
)^2 + 2*x^2 + 32*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)
^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*dilog(c
osh(x) + sinh(x)) + 32*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*c
osh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*d
ilog(-cosh(x) - sinh(x)) + 32*(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sin
h(x)^4 - x*cosh(x)^2 + (6*x*cosh(x)^2 - x)*sinh(x)^2 + 2*(2*x*cosh(x)^3 -
x*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 32*(x*cosh(x)^4 + 4*x*cos
h(x)*sinh(x)^3 + x*sinh(x)^4 - x*cosh(x)^2 + (6*x*cosh(x)^2 - x)*sinh(x)^2
 + 2*(2*x*cosh(x)^3 - x*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 2*
(3*(2*x^2 - 2*x + 1)*cosh(x)^5 + 2*(8*x^3 - 34*x^2 + 2*x - 1)*cosh(x)^3 -
(8*x^3 + 2*x^2 + 2*x + 1)*cosh(x))*sinh(x) + 2*x + 1)/(cosh(x)^4 + 4*cosh(
x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*
cosh(x)^3 - cosh(x))*sinh(x))
```

3.419.6 Sympy [F]

$$\int x^2 \cosh^2(x) \coth^2(x)\, dx = \int x^2 \cosh^2(x) \coth^2(x)\, dx$$

input
```
integrate(x**2*cosh(x)**2*coth(x)**2,x)
```

output
```
Integral(x**2*cosh(x)**2*coth(x)**2, x)
```

3.419.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 \cosh^2(x) \coth^2(x)\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*cosh(x)^2*coth(x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.419.8 Giac [F]

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx = \int x^2 \cosh(x)^2 \coth(x)^2 \, dx$$

input
```
integrate(x^2*cosh(x)^2*coth(x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*cosh(x)^2*coth(x)^2, x)
```

3.419.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx = \int x^2 \cosh(x)^2 \coth(x)^2 \, dx$$

input
```
int(x^2*cosh(x)^2*coth(x)^2,x)
```

output
```
int(x^2*cosh(x)^2*coth(x)^2, x)
```

3.420 $\int x^3 \cosh^2(x) \coth^2(x) \, dx$

3.420.1 Optimal result

Integrand size = 12, antiderivative size = 102

$$\int x^3 \cosh^2(x) \coth^2(x) \, dx = \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1-e^{2x}\right) + 3x\,\text{PolyLog}\left(2,e^{2x}\right) - \frac{3\,\text{PolyLog}\left(3,e^{2x}\right)}{2} + \frac{3}{4}x\cosh(x)\sinh(x) + \frac{1}{2}x^3\cosh(x)\sinh(x)$$

output
```
3/8*x^2-x^3+3/8*x^4-3/8*cosh(x)^2-3/4*x^2*cosh(x)^2-x^3*coth(x)+3*x^2*ln(1
-exp(2*x))+3*x*polylog(2,exp(2*x))-3/2*polylog(3,exp(2*x))+3/4*x*cosh(x)*s
inh(x)+1/2*x^3*cosh(x)*sinh(x)
```

3.420.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int x^3 \cosh^2(x) \coth^2(x) \, dx = \frac{3x^4}{8} - \frac{3}{16}\left(1+2x^2\right)\cosh(2x) - x^3\coth(x) + x^2\left(x+3\log\left(1-e^{-2x}\right)\right) - 3x\,\text{PolyLog}\left(2,e^{-2x}\right) - \frac{3}{2}\text{PolyLog}\left(3,e^{-2x}\right) + \frac{1}{8}x\left(3+2x^2\right)\sinh(2x)$$

input
```
Integrate[x^3*Cosh[x]^2*Coth[x]^2,x]
```

output
```
(3*x^4)/8 - (3*(1 + 2*x^2)*Cosh[2*x])/16 - x^3*Coth[x] + x^2*(x + 3*Log[1
- E^(-2*x)]) - 3*x*PolyLog[2, E^(-2*x)] - (3*PolyLog[3, E^(-2*x)])/2 + (x*
(3 + 2*x^2)*Sinh[2*x])/8
```

3.420.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.23, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.583, Rules used = {5973, 3042, 25, 3792, 15, 3042, 3791, 15, 4203, 15, 26, 3042, 26, 4199, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cosh^2(x) \coth^2(x) \, dx$$

$$\Big\downarrow \text{5973}$$

$$\int x^3 \cosh^2(x) dx + \int x^3 \coth^2(x) dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3 \sin\left(ix + \frac{\pi}{2}\right)^2 dx + \int -x^3 \tan\left(ix + \frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{25}$$

$$\int x^3 \sin\left(ix + \frac{\pi}{2}\right)^2 dx - \int x^3 \tan\left(ix + \frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{\int x^3 dx}{2} - \int x^3 \tan\left(ix + \frac{\pi}{2}\right)^2 dx + \frac{3}{2} \int x \cosh^2(x) dx + \frac{1}{2} x^3 \sinh(x) \cosh(x) - \frac{3}{4} x^2 \cosh^2(x)$$

$$\Big\downarrow \text{15}$$

$$-\int x^3 \tan\left(ix + \frac{\pi}{2}\right)^2 dx + \frac{3}{2} \int x \cosh^2(x) dx + \frac{x^4}{8} + \frac{1}{2} x^3 \sinh(x) \cosh(x) - \frac{3}{4} x^2 \cosh^2(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int x^3 \tan\left(ix + \frac{\pi}{2}\right)^2 dx + \frac{3}{2} \int x \sin\left(ix + \frac{\pi}{2}\right)^2 dx + \frac{x^4}{8} + \frac{1}{2} x^3 \sinh(x) \cosh(x) - \frac{3}{4} x^2 \cosh^2(x)$$

$$\Big\downarrow \text{3791}$$

$$-\int x^3 \tan\left(ix+\frac{\pi}{2}\right)^2 dx+\frac{3}{2}\left(\frac{\int xdx}{2}-\frac{1}{4}\cosh^2(x)+\frac{1}{2}x\sinh(x)\cosh(x)\right)+\frac{x^4}{8}+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)$$

$$\Big\downarrow \text{15}$$

$$-\int x^3 \tan\left(ix+\frac{\pi}{2}\right)^2 dx+\frac{x^4}{8}+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)+\frac{3}{2}\left(\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{4203}$$

$$\int x^3dx-3i\int ix^2\coth(x)dx+\frac{x^4}{8}-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)+\frac{3}{2}\left(\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{15}$$

$$-3i\int ix^2\coth(x)dx+\frac{3x^4}{8}-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)+\frac{3}{2}\left(\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{26}$$

$$3\int x^2\coth(x)dx+\frac{3x^4}{8}-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)+\frac{3}{2}\left(\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$3\int -ix^2\tan\left(ix+\frac{\pi}{2}\right)dx+\frac{3x^4}{8}-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)+\frac{3}{2}\left(\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-3i\int x^2\tan\left(ix+\frac{\pi}{2}\right)dx+\frac{3x^4}{8}-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)+\frac{3}{2}\left(\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{4199}$$

3.420. $\int x^3\cosh^2(x)\coth^2(x)\,dx$

$$-3i\left(2i\int -\frac{e^{2x}x^2}{1-e^{2x}}dx-\frac{ix^3}{3}\right)+\frac{3x^4}{8}-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)+\frac{3}{2}\left(\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \; 25$$

$$-3i\left(-2i\int \frac{e^{2x}x^2}{1-e^{2x}}dx-\frac{ix^3}{3}\right)+\frac{3x^4}{8}-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)+\frac{3}{2}\left(\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \; 2620$$

$$-3i\left(-2i\left(\int x\log\left(1-e^{2x}\right)dx-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)+\frac{3x^4}{8}-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)+\frac{3}{2}\left(\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \; 3011$$

$$-3i\left(-2i\left(\frac{1}{2}\int \text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)+\frac{3x^4}{8}-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)+\frac{3}{2}\left(\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \; 2720$$

$$-3i\left(-2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)+\frac{3x^4}{8}-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)+\frac{3}{2}\left(\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \; 7143$$

$$-3i\left(-2i\left(-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)+\frac{\text{PolyLog}\left(3,e^{2x}\right)}{4}-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)+\frac{3x^4}{8}-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)\cosh(x)-\frac{3}{4}x^2\cosh^2(x)+\frac{3}{2}\left(\frac{x^2}{4}-\frac{\cosh^2(x)}{4}+\frac{1}{2}x\sinh(x)\cosh(x)\right)$$

input
```
Int[x^3*Cosh[x]^2*Coth[x]^2,x]
```

output
```
(3*x^4)/8 - (3*x^2*Cosh[x]^2)/4 - x^3*Coth[x] - (3*I)*((-1/3*I)*x^3 - (2*I
)*(-1/2*(x^2*Log[1 - E^(2*x)]) - (x*PolyLog[2, E^(2*x)])/2 + PolyLog[3, E^
(2*x)]/4)) + (x^3*Cosh[x]*Sinh[x])/2 + (3*(x^2/4 - Cosh[x]^2/4 + (x*Cosh[x
]*Sinh[x])/2))/2
```

3.420. $\int x^3\cosh^2(x)\coth^2(x)\,dx$

3.420.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.420. $\int x^3 \cosh^2(x) \coth^2(x) \, dx$

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.420.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.15

method	result
risch	$\frac{3x^4}{8}+\left(-\frac{3}{32}+\frac{3}{16}x-\frac{3}{16}x^2+\frac{1}{8}x^3\right)e^{2x}+\left(-\frac{3}{32}-\frac{3}{16}x-\frac{3}{16}x^2-\frac{1}{8}x^3\right)e^{-2x}-\frac{2x^3}{e^{2x}-1}-2x^3+3x^2\ln(1$

input
```
int(x^3*cosh(x)^2*coth(x)^2,x,method=_RETURNVERBOSE)
```

output
```
3/8*x^4+(-3/32+3/16*x-3/16*x^2+1/8*x^3)*exp(x)^2+(-3/32-3/16*x-3/16*x^2-1/
8*x^3)/exp(x)^2-2*x^3/(exp(x)^2-1)-2*x^3+3*x^2*ln(1-exp(x))+6*x*polylog(2,
exp(x))-6*polylog(3,exp(x))+3*x^2*ln(exp(x)+1)+6*x*polylog(2,-exp(x))-6*po
lylog(3,-exp(x))
```

3.420.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(84) = 168$.

Time = 0.28 (sec) , antiderivative size = 875, normalized size of antiderivative = 8.58

$$\int x^3 \cosh^2(x) \coth^2(x)\, dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(x)^2*coth(x)^2,x, algorithm="fricas")
```

output
```
1/32*((4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^6 + 6*(4*x^3 - 6*x^2 + 6*x - 3)*co
sh(x)*sinh(x)^5 + (4*x^3 - 6*x^2 + 6*x - 3)*sinh(x)^6 + (12*x^4 - 68*x^3 +
 6*x^2 - 6*x + 3)*cosh(x)^4 + (12*x^4 - 68*x^3 + 15*(4*x^3 - 6*x^2 + 6*x -
 3)*cosh(x)^2 + 6*x^2 - 6*x + 3)*sinh(x)^4 + 4*(5*(4*x^3 - 6*x^2 + 6*x - 3
)*cosh(x)^3 + (12*x^4 - 68*x^3 + 6*x^2 - 6*x + 3)*cosh(x))*sinh(x)^3 + 4*x
^3 - (12*x^4 + 4*x^3 + 6*x^2 + 6*x + 3)*cosh(x)^2 + (15*(4*x^3 - 6*x^2 + 6
*x - 3)*cosh(x)^4 - 12*x^4 - 4*x^3 + 6*(12*x^4 - 68*x^3 + 6*x^2 - 6*x + 3)
*cosh(x)^2 - 6*x^2 - 6*x - 3)*sinh(x)^2 + 6*x^2 + 192*(x*cosh(x)^4 + 4*x*c
osh(x)*sinh(x)^3 + x*sinh(x)^4 - x*cosh(x)^2 + (6*x*cosh(x)^2 - x)*sinh(x)
^2 + 2*(2*x*cosh(x)^3 - x*cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 192
*(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sinh(x)^4 - x*cosh(x)^2 + (6*x*c
osh(x)^2 - x)*sinh(x)^2 + 2*(2*x*cosh(x)^3 - x*cosh(x))*sinh(x))*dilog(-co
sh(x) - sinh(x)) + 96*(x^2*cosh(x)^4 + 4*x^2*cosh(x)*sinh(x)^3 + x^2*sinh(
x)^4 - x^2*cosh(x)^2 + (6*x^2*cosh(x)^2 - x^2)*sinh(x)^2 + 2*(2*x^2*cosh(x
)^3 - x^2*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 96*(x^2*cosh(x)^4
 + 4*x^2*cosh(x)*sinh(x)^3 + x^2*sinh(x)^4 - x^2*cosh(x)^2 + (6*x^2*cosh(x
)^2 - x^2)*sinh(x)^2 + 2*(2*x^2*cosh(x)^3 - x^2*cosh(x))*sinh(x))*log(-cos
h(x) - sinh(x) + 1) - 192*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (
6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x)
)*polylog(3, cosh(x) + sinh(x)) - 192*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 ...
```

3.420.6 Sympy [F]

$$\int x^3 \cosh^2(x) \coth^2(x)\, dx = \int x^3 \cosh^2(x) \coth^2(x)\, dx$$

input
```
integrate(x**3*cosh(x)**2*coth(x)**2,x)
```

output
```
Integral(x**3*cosh(x)**2*coth(x)**2, x)
```

3.420.7 Maxima [F(-2)]

Exception generated.

$$\int x^3 \cosh^2(x) \coth^2(x)\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*cosh(x)^2*coth(x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.420.8 Giac [F]

$$\int x^3 \cosh^2(x) \coth^2(x)\, dx = \int x^3 \cosh(x)^2 \coth(x)^2\, dx$$

input
```
integrate(x^3*cosh(x)^2*coth(x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*cosh(x)^2*coth(x)^2, x)
```

3.420.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \cosh^2(x) \coth^2(x) \, dx = \int x^3 \cosh(x)^2 \coth(x)^2 \, dx$$

input
```
int(x^3*cosh(x)^2*coth(x)^2,x)
```

output
```
int(x^3*cosh(x)^2*coth(x)^2, x)
```

3.421 $\int x\cosh^2(x)\coth^3(x)\,dx$

3.421.1 Optimal result

Integrand size = 10, antiderivative size = 63

$$\int x\cosh^2(x)\coth^3(x)\,dx = \frac{3x}{4}-x^2-\frac{\coth(x)}{2}-\frac{1}{2}x\coth^2(x)+2x\log\left(1-e^{2x}\right)+\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{4}\cosh(x)\sinh(x)+\frac{1}{2}x\sinh^2(x)$$

output
```
3/4*x-x^2-1/2*coth(x)-1/2*x*coth(x)^2+2*x*ln(1-exp(2*x))+polylog(2,exp(2*x
))-1/4*cosh(x)*sinh(x)+1/2*x*sinh(x)^2
```

3.421.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x\cosh^2(x)\coth^3(x)\,dx = \frac{1}{8}\left(8x^2+2x\cosh(2x)-4\coth(x)-4x\text{csch}^2(x)+16x\log\left(1-e^{-2x}\right)-8\,\text{PolyLog}\left(2,e^{-2x}\right)-\sinh(2x)\right)$$

input
```
Integrate[x*Cosh[x]^2*Coth[x]^3,x]
```

output
```
(8*x^2 + 2*x*Cosh[2*x] - 4*Coth[x] - 4*x*Csch[x]^2 + 16*x*Log[1 - E^(-2*x)
] - 8*PolyLog[2, E^(-2*x)] - Sinh[2*x])/8
```

3.421.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.24, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.900, Rules used = {5973, 3042, 26, 4203, 25, 26, 3042, 25, 26, 3954, 24, 4199, 25, 2620, 2715, 2838, 5973, 3042, 26, 4199, 25, 2620, 2715, 2838, 5895, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh^2(x)\coth^3(x)\,dx$$

$$\Big\downarrow \text{5973}$$

$$\int x\coth^3(x)dx+\int x\cosh^2(x)\coth(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\cosh^2(x)\coth(x)dx+\int ix\tan\left(ix+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$\int x\cosh^2(x)\coth(x)dx+i\int x\tan\left(ix+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4203}$$

$$\int x\cosh^2(x)\coth(x)dx+i\left(\frac{1}{2}i\int-\coth^2(x)dx-\int ix\coth(x)dx+\frac{1}{2}ix\coth^2(x)\right)$$

$$\Big\downarrow \text{25}$$

$$\int x\cosh^2(x)\coth(x)dx+i\left(-\frac{1}{2}i\int\coth^2(x)dx-\int ix\coth(x)dx+\frac{1}{2}ix\coth^2(x)\right)$$

$$\Big\downarrow \text{26}$$

$$\int x\cosh^2(x)\coth(x)dx+i\left(-\frac{1}{2}i\int\coth^2(x)dx-i\int x\coth(x)dx+\frac{1}{2}ix\coth^2(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$\int x\cosh^2(x)\coth(x)dx+ i\left(-i\int-ix\tan\left(ix+\frac{\pi}{2}\right)dx-\frac{1}{2}i\int-\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{1}{2}ix\coth^2(x)\right)$$

$$\Big\downarrow \text{25}$$

$$\int x\cosh^2(x)\coth(x)dx+i\left(-i\int -ix\tan\left(ix+\frac{\pi}{2}\right)dx+\frac{1}{2}i\int \tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{1}{2}ix\coth^2(x)\right)$$

$$\Big\downarrow \text{26}$$

$$\int x\cosh^2(x)\coth(x)dx+i\left(-\int x\tan\left(ix+\frac{\pi}{2}\right)dx+\frac{1}{2}i\int \tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{1}{2}ix\coth^2(x)\right)$$

$$\Big\downarrow \text{3954}$$

$$\int x\cosh^2(x)\coth(x)dx+i\left(-\int x\tan\left(ix+\frac{\pi}{2}\right)dx+\frac{1}{2}i\left(\coth(x)-\int 1dx\right)+\frac{1}{2}ix\coth^2(x)\right)$$

$$\Big\downarrow \text{24}$$

$$\int x\cosh^2(x)\coth(x)dx+i\left(-\int x\tan\left(ix+\frac{\pi}{2}\right)dx+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{4199}$$

$$\int x\cosh^2(x)\coth(x)dx+i\left(-2i\int -\frac{e^{2x}x}{1-e^{2x}}dx+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{25}$$

$$\int x\cosh^2(x)\coth(x)dx+i\left(2i\int \frac{e^{2x}x}{1-e^{2x}}dx+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{2620}$$

$$\int x\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{1}{2}\int \log\left(1-e^{2x}\right)dx-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{2715}$$

$$\int x\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{2838}$$

$$\int x\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{5973}$$

3.421. $\int x\cosh^2(x)\coth^3(x)\,dx$

$$\int x\coth(x)dx+\int x\cosh(x)\sinh(x)dx+$$
$$i\left(2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{3042}$$

$$\int -ix\tan\left(ix+\frac{\pi}{2}\right)dx+\int x\cosh(x)\sinh(x)dx+$$
$$i\left(2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\int x\tan\left(ix+\frac{\pi}{2}\right)dx+\int x\cosh(x)\sinh(x)dx+$$
$$i\left(2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{4199}$$

$$-i\left(2i\int -\frac{e^{2x}x}{1-e^{2x}}dx-\frac{ix^2}{2}\right)+\int x\cosh(x)\sinh(x)dx+$$
$$i\left(2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(-2i\int \frac{e^{2x}x}{1-e^{2x}}dx-\frac{ix^2}{2}\right)+\int x\cosh(x)\sinh(x)dx+$$
$$i\left(2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{2620}$$

$$-i\left(-2i\left(\frac{1}{2}\int\log\left(1-e^{2x}\right)dx-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)+\int x\cosh(x)\sinh(x)dx+$$
$$i\left(2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{2715}$$

$$-i\left(-2i\left(\frac{1}{4}\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)+\int x\cosh(x)\sinh(x)dx+$$
$$i\left(2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{2838}$$

3.421. $\int x\cosh^2(x)\coth^3(x)\,dx$

$$\int x\cosh(x)\sinh(x)dx-i\left(-2i\left(-\frac{\text{PolyLog}(2,e^{2x})}{4}-\frac{1}{2}x\log(1-e^{2x})\right)-\frac{ix^2}{2}\right)+i\left(2i\left(-\frac{\text{PolyLog}(2,e^{2x})}{4}-\frac{1}{2}x\log(1-e^{2x})\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)$$

$$\Big\downarrow \text{ 5895}$$

$$-\frac{1}{2}\int\sinh^2(x)dx-i\left(-2i\left(-\frac{\text{PolyLog}(2,e^{2x})}{4}-\frac{1}{2}x\log(1-e^{2x})\right)-\frac{ix^2}{2}\right)+i\left(2i\left(-\frac{\text{PolyLog}(2,e^{2x})}{4}-\frac{1}{2}x\log(1-e^{2x})\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)+\frac{1}{2}x\sinh^2(x)$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{2}\int-\sin(ix)^2dx-i\left(-2i\left(-\frac{\text{PolyLog}(2,e^{2x})}{4}-\frac{1}{2}x\log(1-e^{2x})\right)-\frac{ix^2}{2}\right)+i\left(2i\left(-\frac{\text{PolyLog}(2,e^{2x})}{4}-\frac{1}{2}x\log(1-e^{2x})\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)+\frac{1}{2}x\sinh^2(x)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\int\sin(ix)^2dx-i\left(-2i\left(-\frac{\text{PolyLog}(2,e^{2x})}{4}-\frac{1}{2}x\log(1-e^{2x})\right)-\frac{ix^2}{2}\right)+i\left(2i\left(-\frac{\text{PolyLog}(2,e^{2x})}{4}-\frac{1}{2}x\log(1-e^{2x})\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)+\frac{1}{2}x\sinh^2(x)$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{1}{2}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)-i\left(-2i\left(-\frac{\text{PolyLog}(2,e^{2x})}{4}-\frac{1}{2}x\log(1-e^{2x})\right)-\frac{ix^2}{2}\right)+i\left(2i\left(-\frac{\text{PolyLog}(2,e^{2x})}{4}-\frac{1}{2}x\log(1-e^{2x})\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)+\frac{1}{2}x\sinh^2(x)$$

$$\Big\downarrow \text{ 24}$$

3.421. $\int x\cosh^2(x)\coth^3(x)\,dx$

$$-i\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)+$$
$$i\left(2i\left(-\frac{\text{PolyLog}\left(2,e^{2x}\right)}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}ix\coth^2(x)+\frac{1}{2}i(\coth(x)-x)\right)+$$
$$\frac{1}{2}x\sinh^2(x)+\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)$$

input
```
Int[x*Cosh[x]^2*Coth[x]^3,x]
```

output
```
(-I)*((-1/2*I)*x^2 - (2*I)*(-1/2*(x*Log[1 - E^(2*x)]) - PolyLog[2, E^(2*x)
]/4)) + I*((I/2)*x^2 + (I/2)*x*Coth[x]^2 + (I/2)*(-x + Coth[x]) + (2*I)*(-
1/2*(x*Log[1 - E^(2*x)]) - PolyLog[2, E^(2*x)]/4)) + (x*Sinh[x]^2)/2 + (x/
2 - (Cosh[x]*Sinh[x])/2)/2
```

3.421.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.421. $\int x\cosh^2(x)\coth^3(x)\,dx$

3.421.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

method	result
risch	$-x^2+\left(-\frac{1}{16}+\frac{x}{8}\right)\mathrm{e}^{2x}+\left(\frac{1}{16}+\frac{x}{8}\right)\mathrm{e}^{-2x}-\frac{2\,\mathrm{e}^{2x}x+\mathrm{e}^{2x}-1}{\left(\mathrm{e}^{2x}-1\right)^2}+2x\ln\left(1-\mathrm{e}^x\right)+2\,\mathrm{polylog}\left(2,\mathrm{e}^x\right)+2x\ln\left(\mathrm{e}\right.$

input
```
int(x*cosh(x)^2*coth(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-x^2+(-1/16+1/8*x)*exp(x)^2+(1/16+1/8*x)/exp(x)^2-(2*x*exp(x)^2+exp(x)^2-1
)/(exp(x)^2-1)^2+2*x*ln(1-exp(x))+2*polylog(2,exp(x))+2*x*ln(exp(x)+1)+2*p
olylog(2,-exp(x))
```

3.421.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. 2(50) = 100.

Time = 0.27 (sec) , antiderivative size = 916, normalized size of antiderivative = 14.54

$$\int x\cosh^2(x)\coth^3(x)\,dx = \text{Too large to display}$$

input
```
integrate(x*cosh(x)^2*coth(x)^3,x, algorithm="fricas")
```

output
```
1/16*((2*x - 1)*cosh(x)^8 + 8*(2*x - 1)*cosh(x)*sinh(x)^7 + (2*x - 1)*sinh
(x)^8 - 2*(8*x^2 + 2*x - 1)*cosh(x)^6 + 2*(14*(2*x - 1)*cosh(x)^2 - 8*x^2
- 2*x + 1)*sinh(x)^6 + 4*(14*(2*x - 1)*cosh(x)^3 - 3*(8*x^2 + 2*x - 1)*cos
h(x))*sinh(x)^5 + 4*(8*x^2 - 7*x - 4)*cosh(x)^4 + 2*(35*(2*x - 1)*cosh(x)^
4 - 15*(8*x^2 + 2*x - 1)*cosh(x)^2 + 16*x^2 - 14*x - 8)*sinh(x)^4 + 8*(7*(
2*x - 1)*cosh(x)^5 - 5*(8*x^2 + 2*x - 1)*cosh(x)^3 + 2*(8*x^2 - 7*x - 4)*c
osh(x))*sinh(x)^3 - 2*(8*x^2 + 2*x - 7)*cosh(x)^2 + 2*(14*(2*x - 1)*cosh(x
)^6 - 15*(8*x^2 + 2*x - 1)*cosh(x)^4 + 12*(8*x^2 - 7*x - 4)*cosh(x)^2 - 8*
x^2 - 2*x + 7)*sinh(x)^2 + 32*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6
 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x)
)*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*
(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) +
32*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(
x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4
 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3
+ cosh(x))*sinh(x))*dilog(-cosh(x) - sinh(x)) + 32*(x*cosh(x)^6 + 6*x*cosh
(x)*sinh(x)^5 + x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(
x)^4 + 4*(5*x*cosh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cos
h(x)^4 - 12*x*cosh(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3
+ x*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 32*(x*cosh(x)^6 + 6*...
```

3.421.6 Sympy [F]

$$\int x\cosh^2(x)\coth^3(x)\,dx = \int x\cosh^2(x)\coth^3(x)\,dx$$

input
```
integrate(x*cosh(x)**2*coth(x)**3,x)
```

output
```
Integral(x*cosh(x)**2*coth(x)**3, x)
```

3.421.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(50) = 100$.

Time = 0.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.32

$$\int x\cosh^2(x)\coth^3(x)\,dx = -2\,x^2 + 2\,x\log\left(e^x+1\right) + 2\,x\log\left(-e^x+1\right) + \frac{5}{8}\,x + \frac{16\,x^2 + (2\,x-1)e^{(6\,x)} + 2\left(8\,x^2 - 2\,x + 1\right)e^{(4\,x)} - \left(32\,x^2 + 8\,x + 11\right)e^{(2\,x)} + (2\,x+1)e^{(-2\,x)} - 14\,x + 9}{16\left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)} - \frac{5\left(2\,xe^{(4\,x)} + e^{(2\,x)} - 1\right)}{16\left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)} + 2\,\mathrm{Li}_2(-e^x) + 2\,\mathrm{Li}_2(e^x)$$

input
```
integrate(x*cosh(x)^2*coth(x)^3,x, algorithm="maxima")
```

output
```
-2*x^2 + 2*x*log(e^x + 1) + 2*x*log(-e^x + 1) + 5/8*x + 1/16*(16*x^2 + (2*
x - 1)*e^(6*x) + 2*(8*x^2 - 2*x + 1)*e^(4*x) - (32*x^2 + 8*x + 11)*e^(2*x)
 + (2*x + 1)*e^(-2*x) - 14*x + 9)/(e^(4*x) - 2*e^(2*x) + 1) - 5/16*(2*x*e^
(4*x) + e^(2*x) - 1)/(e^(4*x) - 2*e^(2*x) + 1) + 2*dilog(-e^x) + 2*dilog(e
^x)
```

3.421.8 Giac [F]

$$\int x\cosh^2(x)\coth^3(x)\,dx = \int x\cosh(x)^2\coth(x)^3\,dx$$

input
```
integrate(x*cosh(x)^2*coth(x)^3,x, algorithm="giac")
```

output
```
integrate(x*cosh(x)^2*coth(x)^3, x)
```

3.421.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh^2(x)\coth^3(x)\,dx = \int x\cosh(x)^2\coth(x)^3\,dx$$

input
```
int(x*cosh(x)^2*coth(x)^3,x)
```

output
```
int(x*cosh(x)^2*coth(x)^3, x)
```

3.422 $\int x^2 \cosh^2(x) \coth^3(x)\, dx$

3.422.1 Optimal result

Integrand size = 12, antiderivative size = 96

$$\int x^2 \cosh^2(x) \coth^3(x)\, dx = \frac{3x^2}{4} - \frac{2x^3}{3} - x\coth(x) - \frac{1}{2}x^2\coth^2(x) + 2x^2\log\left(1-e^{2x}\right) + \log(\sinh(x)) + 2x\,\text{PolyLog}\left(2,e^{2x}\right) - \text{PolyLog}\left(3,e^{2x}\right) - \frac{1}{2}x\cosh(x)\sinh(x) + \frac{\sinh^2(x)}{4} + \frac{1}{2}x^2\sinh^2(x)$$

output
```
3/4*x^2-2/3*x^3-x*coth(x)-1/2*x^2*coth(x)^2+2*x^2*ln(1-exp(2*x))+ln(sinh(x
))+2*x*polylog(2,exp(2*x))-polylog(3,exp(2*x))-1/2*x*cosh(x)*sinh(x)+1/4*s
inh(x)^2+1/2*x^2*sinh(x)^2
```

3.422.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.41

$$\int x^2 \cosh^2(x) \coth^3(x)\, dx = -x + \frac{2x^3}{3} + \frac{1}{8}\left(1+2x^2\right)\cosh(2x) - x\coth(x) - \frac{1}{2}x^2\text{csch}^2(x) + 2x^2\log\left(1-e^{-x}\right) + 2x^2\log\left(1+e^{-x}\right) + \log\left(1-e^{x}\right) + \log\left(1+e^{x}\right) - 4x\,\text{PolyLog}\left(2,-e^{-x}\right) - 4x\,\text{PolyLog}\left(2,e^{-x}\right) - 4\,\text{PolyLog}\left(3,-e^{-x}\right) - 4\,\text{PolyLog}\left(3,e^{-x}\right) - \frac{1}{4}x\sinh(2x)$$

input
```
Integrate[x^2*Cosh[x]^2*Coth[x]^3,x]
```

output
```
-x + (2*x^3)/3 + ((1 + 2*x^2)*Cosh[2*x])/8 - x*Coth[x] - (x^2*Csch[x]^2)/2
 + 2*x^2*Log[1 - E^(-x)] + 2*x^2*Log[1 + E^(-x)] + Log[1 - E^x] + Log[1 +
E^x] - 4*x*PolyLog[2, -E^(-x)] - 4*x*PolyLog[2, E^(-x)] - 4*PolyLog[3, -E^
(-x)] - 4*PolyLog[3, E^(-x)] - (x*Sinh[2*x])/4
```

3.422.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cosh^2(x) \coth^3(x) \, dx$$

$$\Big\downarrow \text{ 5973}$$

$$\int x^2 \coth^3(x)dx + \int x^2 \cosh^2(x) \coth(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^2 \cosh^2(x) \coth(x)dx + \int ix^2 \tan\left(ix + \frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{ 26}$$

$$\int x^2 \cosh^2(x) \coth(x)dx + i\int x^2 \tan\left(ix + \frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{ 4203}$$

$$\int x^2 \cosh^2(x) \coth(x)dx + i\left(-\int ix^2 \coth(x)dx + i\int -x \coth^2(x)dx + \frac{1}{2}ix^2 \coth^2(x)\right)$$

$$\Big\downarrow \text{ 25}$$

$$\int x^2 \cosh^2(x) \coth(x)dx + i\left(-\int ix^2 \coth(x)dx - i\int x \coth^2(x)dx + \frac{1}{2}ix^2 \coth^2(x)\right)$$

$$\Big\downarrow \text{ 26}$$

$$\int x^2 \cosh^2(x) \coth(x)dx + i\left(-i\int x^2 \coth(x)dx - i\int x \coth^2(x)dx + \frac{1}{2}ix^2 \coth^2(x)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^2\cosh^2(x)\coth(x)dx+ i\left(-i\int -ix^2\tan\left(ix+\frac{\pi}{2}\right)dx-i\int -x\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{1}{2}ix^2\coth^2(x)\right)$$

$\downarrow$ 25

$$\int x^2\cosh^2(x)\coth(x)dx+ i\left(-i\int -ix^2\tan\left(ix+\frac{\pi}{2}\right)dx+i\int x\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{1}{2}ix^2\coth^2(x)\right)$$

$\downarrow$ 26

$$\int x^2\cosh^2(x)\coth(x)dx+i\left(-\int x^2\tan\left(ix+\frac{\pi}{2}\right)dx+i\int x\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{1}{2}ix^2\coth^2(x)\right)$$

$\downarrow$ 4199

$$\int x^2\cosh^2(x)\coth(x)dx+i\left(-2i\int -\frac{e^{2x}x^2}{1-e^{2x}}dx+i\int x\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)\right)$$

$\downarrow$ 25

$$\int x^2\cosh^2(x)\coth(x)dx+i\left(2i\int \frac{e^{2x}x^2}{1-e^{2x}}dx+i\int x\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)\right)$$

$\downarrow$ 2620

$$\int x^2\cosh^2(x)\coth(x)dx+ i\left(2i\left(\int x\log\left(1-e^{2x}\right)dx-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+i\int x\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)\right)$$

$\downarrow$ 3011

$$\int x^2\cosh^2(x)\coth(x)dx+ i\left(2i\left(\frac{1}{2}\int \text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+i\int x\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{ix^3}{3}+\frac{1}{2}i\right.$$

$\downarrow$ 2720

$$\int x^2\cosh^2(x)\coth(x)dx+ i\left(2i\left(\frac{1}{4}\int e^{-2x}\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+i\int x\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{ix^3}{3}\right.$$

$\downarrow$ 4203

3.422. $\int x^2\cosh^2(x)\coth^3(x)\,dx$

$$\int x^2\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\operatorname{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\operatorname{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+i(-\int xdx+i\int i\coth(x)dx$$

$\Big\downarrow$ 15

$$\int x^2\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\operatorname{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\operatorname{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+i\left(i\int i\coth(x)dx-\frac{x^2}{2}+x\,\mathrm{c}\right.$$

$\Big\downarrow$ 26

$$\int x^2\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\operatorname{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\operatorname{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+i\left(-\int\coth(x)dx-\frac{x^2}{2}+x\,\mathrm{c}\right.$$

$\Big\downarrow$ 3042

$$\int x^2\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\operatorname{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\operatorname{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+i\left(-\int-i\tan\left(ix+\frac{\pi}{2}\right)dx-\right.$$

$\Big\downarrow$ 26

$$\int x^2\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\operatorname{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\operatorname{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+i\left(i\int\tan\left(ix+\frac{\pi}{2}\right)dx-\frac{x^2}{2}\right.$$

$\Big\downarrow$ 3956

$$\int x^2\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\operatorname{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\operatorname{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)+i\left(-\frac{x}{2}\right.$$

$\Big\downarrow$ 5973

$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\operatorname{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\operatorname{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)+i\left(-\frac{x}{2}\right.$$
$$\int x^2\coth(x)dx+\int x^2\cosh(x)\sinh(x)dx$$

$\Big\downarrow$ 3042

3.422. $\int x^2\cosh^2(x)\coth^3(x)\,dx$

$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)+i\left(-\frac{x}{2}\right.\right.$$
$$\int -ix^2\tan\left(ix+\frac{\pi}{2}\right)dx+\int x^2\cosh(x)\sinh(x)dx$$

$\downarrow$ 26

$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)+i\left(-\frac{x}{2}\right.\right.$$
$$i\int x^2\tan\left(ix+\frac{\pi}{2}\right)dx+\int x^2\cosh(x)\sinh(x)dx$$

$\downarrow$ 4199

$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)+i\left(-\frac{x}{2}\right.\right.$$
$$\int x^2\cosh(x)\sinh(x)dx-i\left(2i\int-\frac{e^{2x}x^2}{1-e^{2x}}dx-\frac{ix^3}{3}\right)$$

$\downarrow$ 25

$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)+i\left(-\frac{x}{2}\right.\right.$$
$$\int x^2\cosh(x)\sinh(x)dx-i\left(-2i\int\frac{e^{2x}x^2}{1-e^{2x}}dx-\frac{ix^3}{3}\right)$$

$\downarrow$ 2620

$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)+i\left(-\frac{x}{2}\right.\right.$$
$$\int x^2\cosh(x)\sinh(x)dx-i\left(-2i\left(\int x\log\left(1-e^{2x}\right)dx-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)$$

$\downarrow$ 3011

$$-i\left(-2i\left(\frac{1}{2}\int \text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)+$$
$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)+i\left(-\frac{x}{2}\right.\right.$$
$$\int x^2\cosh(x)\sinh(x)dx$$

$\downarrow$ 2720

3.422. $\int x^2\cosh^2(x)\coth^3(x)\,dx$

$$-i\left(-2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)+$$
$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)+i\left(-\frac{x}{2}\right.\right.$$
$$\int x^2\cosh(x)\sinh(x)dx$$

$$\Big\downarrow \; 5895$$

$$-i\left(-2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)+$$
$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)+i\left(-\frac{x}{2}\right.\right.$$
$$\int x\sinh^2(x)dx+\frac{1}{2}x^2\sinh^2(x)$$

$$\Big\downarrow \; 3042$$

$$-i\left(-2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)+$$
$$i\left(2i\left(\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)+\frac{ix^3}{3}+\frac{1}{2}ix^2\coth^2(x)+i\left(-\frac{x}{2}\right.\right.$$
$$\int -x\sin(ix)^2dx+\frac{1}{2}x^2\sinh^2(x)$$

input
```
Int[x^2*Cosh[x]^2*Coth[x]^3,x]
```

output
```
$Aborted
```

3.422.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.422. $\int x^2 \cosh^2(x) \coth^3(x) \, dx$

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.422.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{2x^3}{3}+\left(\frac{1}{16}-\frac{1}{8}x+\frac{1}{8}x^2\right)\mathrm{e}^{2x}+\left(\frac{1}{16}+\frac{1}{8}x+\frac{1}{8}x^2\right)\mathrm{e}^{-2x}-\frac{2x\left(\mathrm{e}^{2x}x+\mathrm{e}^{2x}-1\right)}{\left(\mathrm{e}^{2x}-1\right)^2}+\ln\left(\mathrm{e}^{x}-1\right)-2\ln\left(\mathrm{e}^{x}\right)+\mathrm{l}$

input
```
int(x^2*cosh(x)^2*coth(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*x^3+(1/16-1/8*x+1/8*x^2)*exp(x)^2+(1/16+1/8*x+1/8*x^2)/exp(x)^2-2*x*(
x*exp(x)^2+exp(x)^2-1)/(exp(x)^2-1)^2+ln(exp(x)-1)-2*ln(exp(x))+ln(exp(x)+
1)+2*x^2*ln(1-exp(x))+4*x*polylog(2,exp(x))-4*polylog(3,exp(x))+2*x^2*ln(e
xp(x)+1)+4*x*polylog(2,-exp(x))-4*polylog(3,-exp(x))
```

3.422.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1512 vs. $2(80) = 160$.

Time = 0.29 (sec) , antiderivative size = 1512, normalized size of antiderivative = 15.75

$$\int x^2\cosh^2(x)\coth^3(x)\,dx=\text{Too large to display}$$

input
```
integrate(x^2*cosh(x)^2*coth(x)^3,x, algorithm="fricas")
```

output
```
1/48*(3*(2*x^2 - 2*x + 1)*cosh(x)^8 + 24*(2*x^2 - 2*x + 1)*cosh(x)*sinh(x)
^7 + 3*(2*x^2 - 2*x + 1)*sinh(x)^8 - 2*(16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)
^6 - 2*(16*x^3 - 42*(2*x^2 - 2*x + 1)*cosh(x)^2 + 6*x^2 + 42*x + 3)*sinh(x
)^6 + 12*(14*(2*x^2 - 2*x + 1)*cosh(x)^3 - (16*x^3 + 6*x^2 + 42*x + 3)*cos
h(x))*sinh(x)^5 + 2*(32*x^3 - 42*x^2 + 48*x + 3)*cosh(x)^4 + 2*(105*(2*x^2
 - 2*x + 1)*cosh(x)^4 + 32*x^3 - 15*(16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)^2
- 42*x^2 + 48*x + 3)*sinh(x)^4 + 8*(21*(2*x^2 - 2*x + 1)*cosh(x)^5 - 5*(16
*x^3 + 6*x^2 + 42*x + 3)*cosh(x)^3 + (32*x^3 - 42*x^2 + 48*x + 3)*cosh(x))
*sinh(x)^3 - 2*(16*x^3 + 6*x^2 + 6*x + 3)*cosh(x)^2 + 2*(42*(2*x^2 - 2*x +
 1)*cosh(x)^6 - 15*(16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)^4 - 16*x^3 + 6*(32*
x^3 - 42*x^2 + 48*x + 3)*cosh(x)^2 - 6*x^2 - 6*x - 3)*sinh(x)^2 + 6*x^2 +
192*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^5 + x*sinh(x)^6 - 2*x*cosh(x)^4 + (
15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x*cosh(x)^3 - 2*x*cosh(x))*sinh(x)^
3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*cosh(x)^2 + x)*sinh(x)^2 + 2*(3*x
*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x))
+ 192*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^5 + x*sinh(x)^6 - 2*x*cosh(x)^4 +
 (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x*cosh(x)^3 - 2*x*cosh(x))*sinh(x
)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*cosh(x)^2 + x)*sinh(x)^2 + 2*(3
*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*sinh(x))*dilog(-cosh(x) - sinh(x
)) + 48*((2*x^2 + 1)*cosh(x)^6 + 6*(2*x^2 + 1)*cosh(x)*sinh(x)^5 + (2*x...
```

3.422.6 Sympy [F]

$$\int x^2\cosh^2(x)\coth^3(x)\,dx = \int x^2\cosh^2(x)\coth^3(x)\,dx$$

input
```
integrate(x**2*cosh(x)**2*coth(x)**3,x)
```

output
```
Integral(x**2*cosh(x)**2*coth(x)**3, x)
```

3.422.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(80) = 160.

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.81

$$\int x^2 \cosh^2(x) \coth^3(x) \, dx$$
$$= -\frac{4}{3}x^3 + 2x^2 \log(e^x + 1) + 2x^2 \log(-e^x + 1) + 4x\text{Li}_2(-e^x) + 4x\text{Li}_2(e^x) - 2x$$
$$+ \frac{32x^3 - 12x^2 + 3(2x^2 - 2x + 1)e^{(6x)} + 2(16x^3 - 6x^2 + 6x - 3)e^{(4x)} - 2(32x^3 + 42x^2 + 48x - 3)e^{(}}{48(e^{(4x)} - 2e^{(2x)} + 1)}$$
$$+ \log(e^x + 1) + \log(e^x - 1) - 4\text{Li}_3(-e^x) - 4\text{Li}_3(e^x)$$

input
```
integrate(x^2*cosh(x)^2*coth(x)^3,x, algorithm="maxima")
```

output
```
-4/3*x^3 + 2*x^2*log(e^x + 1) + 2*x^2*log(-e^x + 1) + 4*x*dilog(-e^x) + 4*
x*dilog(e^x) - 2*x + 1/48*(32*x^3 - 12*x^2 + 3*(2*x^2 - 2*x + 1)*e^(6*x) +
 2*(16*x^3 - 6*x^2 + 6*x - 3)*e^(4*x) - 2*(32*x^3 + 42*x^2 + 48*x - 3)*e^(
2*x) + 3*(2*x^2 + 2*x + 1)*e^(-2*x) + 84*x - 6)/(e^(4*x) - 2*e^(2*x) + 1)
+ log(e^x + 1) + log(e^x - 1) - 4*polylog(3, -e^x) - 4*polylog(3, e^x)
```

3.422.8 Giac [F]

$$\int x^2 \cosh^2(x) \coth^3(x) \, dx = \int x^2 \cosh(x)^2 \coth(x)^3 \, dx$$

input
```
integrate(x^2*cosh(x)^2*coth(x)^3,x, algorithm="giac")
```

output
```
integrate(x^2*cosh(x)^2*coth(x)^3, x)
```

3.422.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh^2(x) \coth^3(x) \, dx = \int x^2 \cosh(x)^2 \coth(x)^3 \, dx$$

input
```
int(x^2*cosh(x)^2*coth(x)^3,x)
```

output
```
int(x^2*cosh(x)^2*coth(x)^3, x)
```

3.423 $\int x^3 \cosh^2(x) \coth^3(x) \, dx$

3.423.1 Optimal result

Integrand size = 12, antiderivative size = 158

$$\begin{aligned}\int x^3 \cosh^2(x) \coth^3(x) \, dx = {} & \frac{3x}{8} - \frac{3x^2}{2} + \frac{3x^3}{4} - \frac{x^4}{2} - \frac{3}{2}x^2 \coth(x) - \frac{1}{2}x^3 \coth^2(x) \\ & + 3x \log\left(1 - e^{2x}\right) + 2x^3 \log\left(1 - e^{2x}\right) + \frac{3 \operatorname{PolyLog}\left(2, e^{2x}\right)}{2} \\ & + 3x^2 \operatorname{PolyLog}\left(2, e^{2x}\right) - 3x \operatorname{PolyLog}\left(3, e^{2x}\right) \\ & + \frac{3 \operatorname{PolyLog}\left(4, e^{2x}\right)}{2} - \frac{3}{8}\cosh(x)\sinh(x) \\ & - \frac{3}{4}x^2 \cosh(x)\sinh(x) + \frac{3}{4}x \sinh^2(x) + \frac{1}{2}x^3 \sinh^2(x)\end{aligned}$$

output
```
3/8*x-3/2*x^2+3/4*x^3-1/2*x^4-3/2*x^2*coth(x)-1/2*x^3*coth(x)^2+3*x*ln(1-e
xp(2*x))+2*x^3*ln(1-exp(2*x))+3/2*polylog(2,exp(2*x))+3*x^2*polylog(2,exp(
2*x))-3*x*polylog(3,exp(2*x))+3/2*polylog(4,exp(2*x))-3/8*cosh(x)*sinh(x)-
3/4*x^2*cosh(x)*sinh(x)+3/4*x*sinh(x)^2+1/2*x^3*sinh(x)^2
```

3.423.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.21

$$\int x^3 \cosh^2(x) \coth^3(x) \, dx = -3(1 + 2x^2) \operatorname{PolyLog}(2, -e^{-x}) + \frac{1}{8}(12x^2 + 4x^4 + 3x \cosh(2x) + 2x^3 \cosh(2x) - 12x^2 \coth(x) - 4x^3 \operatorname{csch}^2(x) + 24x \log(1 - e^{-x}) + 16x^3 \log(1 - e^{-x}) + 24x \log(1 + e^{-x}) + 16x^3 \log(1 + e^{-x}) - 24(1 + 2x^2) \operatorname{PolyLog}(2, e^{-x}) - 96x \operatorname{PolyLog}(3, -e^{-x}) - 96x \operatorname{PolyLog}(3, e^{-x}) - 96 \operatorname{PolyLog}(4, -e^{-x}) - 96 \operatorname{PolyLog}(4, e^{-x}) - 3 \cosh(x) \sinh(x) - 6x^2 \cosh(x) \sinh(x))$$

input
```
Integrate[x^3*Cosh[x]^2*Coth[x]^3,x]
```

output
```
-3*(1 + 2*x^2)*PolyLog[2, -E^(-x)] + (12*x^2 + 4*x^4 + 3*x*Cosh[2*x] + 2*x
^3*Cosh[2*x] - 12*x^2*Coth[x] - 4*x^3*Csch[x]^2 + 24*x*Log[1 - E^(-x)] + 1
6*x^3*Log[1 - E^(-x)] + 24*x*Log[1 + E^(-x)] + 16*x^3*Log[1 + E^(-x)] - 24
*(1 + 2*x^2)*PolyLog[2, E^(-x)] - 96*x*PolyLog[3, -E^(-x)] - 96*x*PolyLog[
3, E^(-x)] - 96*PolyLog[4, -E^(-x)] - 96*PolyLog[4, E^(-x)] - 3*Cosh[x]*Si
nh[x] - 6*x^2*Cosh[x]*Sinh[x])/8
```

3.423.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cosh^2(x) \coth^3(x) \, dx$$

$$\Big\downarrow \text{ 5973}$$

$$\int x^3 \coth^3(x) dx + \int x^3 \cosh^2(x) \coth(x) dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^3 \cosh^2(x) \coth(x) dx + \int i x^3 \tan\left(ix + \frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{ 26}$$

$$\int x^3 \cosh^2(x) \coth(x) dx + i \int x^3 \tan\left(ix + \frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{4203}$$

$$\int x^3\cosh^2(x)\coth(x)dx+i\left(-\int ix^3\coth(x)dx+\frac{3}{2}i\int -x^2\coth^2(x)dx+\frac{1}{2}ix^3\coth^2(x)\right)$$

$$\Big\downarrow \text{25}$$

$$\int x^3\cosh^2(x)\coth(x)dx+i\left(-\int ix^3\coth(x)dx-\frac{3}{2}i\int x^2\coth^2(x)dx+\frac{1}{2}ix^3\coth^2(x)\right)$$

$$\Big\downarrow \text{26}$$

$$\int x^3\cosh^2(x)\coth(x)dx+i\left(-i\int x^3\coth(x)dx-\frac{3}{2}i\int x^2\coth^2(x)dx+\frac{1}{2}ix^3\coth^2(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$\int x^3\cosh^2(x)\coth(x)dx+ \\ i\left(-i\int -ix^3\tan\left(ix+\frac{\pi}{2}\right)dx-\frac{3}{2}i\int -x^2\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{1}{2}ix^3\coth^2(x)\right)$$

$$\Big\downarrow \text{25}$$

$$\int x^3\cosh^2(x)\coth(x)dx+ \\ i\left(-i\int -ix^3\tan\left(ix+\frac{\pi}{2}\right)dx+\frac{3}{2}i\int x^2\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{1}{2}ix^3\coth^2(x)\right)$$

$$\Big\downarrow \text{26}$$

$$\int x^3\cosh^2(x)\coth(x)dx+ \\ i\left(-\int x^3\tan\left(ix+\frac{\pi}{2}\right)dx+\frac{3}{2}i\int x^2\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{1}{2}ix^3\coth^2(x)\right)$$

$$\Big\downarrow \text{4199}$$

$$\int x^3\cosh^2(x)\coth(x)dx+ \\ i\left(-2i\int -\frac{e^{2x}x^3}{1-e^{2x}}dx+\frac{3}{2}i\int x^2\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{ix^4}{4}+\frac{1}{2}ix^3\coth^2(x)\right)$$

$$\Big\downarrow \text{25}$$

$$\int x^3\cosh^2(x)\coth(x)dx+i\left(2i\int \frac{e^{2x}x^3}{1-e^{2x}}dx+\frac{3}{2}i\int x^2\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{ix^4}{4}+\frac{1}{2}ix^3\coth^2(x)\right)$$

$$\Big\downarrow \text{2620}$$

3.423. $\int x^3\cosh^2(x)\coth^3(x)\,dx$

$$\int x^3\cosh^2(x)\coth(x)dx+$$
$$i\left(\frac{3}{2}i\int x^2\tan\left(ix+\frac{\pi}{2}\right)^2dx+2i\left(\frac{3}{2}\int x^2\log\left(1-e^{2x}\right)dx-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{ix^4}{4}+\frac{1}{2}ix^3\coth^2(x)\right)$$

$$\Big\downarrow \text{3011}$$

$$\int x^3\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\int x^2\tan\left(ix+\frac{\pi}{2}\right)^2dx+\frac{i}{\cdot}\right.$$

$$\Big\downarrow \text{4203}$$

$$\int x^3\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(-\int x^2dx+2i\int ix\coth(\right.\right.$$

$$\Big\downarrow \text{15}$$

$$\int x^3\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\int ix\coth(x)dx-\frac{x^3}{3}+\right.\right.$$

$$\Big\downarrow \text{26}$$

$$\int x^3\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(-2\int x\coth(x)dx-\frac{x^3}{3}+\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\int x^3\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(-2\int -ix\tan\left(ix+\frac{\pi}{2}\right)d\right.\right.$$

$$\Big\downarrow \text{26}$$

$$\int x^3\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\int x\tan\left(ix+\frac{\pi}{2}\right)dx-\right.\right.$$

$$\Big\downarrow \text{4199}$$

$$\int x^3\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\left(2i\int -\frac{e^{2x}x}{1-e^{2x}}dx-\frac{ix}{2}\right.\right.\right.$$

3.423. $\int x^3\cosh^2(x)\coth^3(x)\,dx$

$$\Big\downarrow 25$$

$$\int x^3\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\left(-2i\int\frac{e^{2x}x}{1-e^{2x}}dx-\frac{ix}{2}\right.\right.\right.$$

$$\Big\downarrow 2620$$

$$\int x^3\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\left(-2i\left(\frac{1}{2}\int\log\left(1-e^{2x}\right.\right.\right.\right.\right.$$

$$\Big\downarrow 2715$$

$$\int x^3\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\left(-2i\left(\frac{1}{4}\int e^{-2x}\log\left(1-\right.\right.\right.\right.\right.$$

$$\Big\downarrow 2838$$

$$\int x^3\cosh^2(x)\coth(x)dx+$$
$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2}\right.}{4}\right.\right.\right.\right.$$

$$\Big\downarrow 5973$$

$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2}\right.}{4}\right.\right.\right.\right.$$
$$\int x^3\coth(x)dx+\int x^3\cosh(x)\sinh(x)dx$$

$$\Big\downarrow 3042$$

$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2}\right.}{4}\right.\right.\right.\right.$$
$$\int -ix^3\tan\left(ix+\frac{\pi}{2}\right)dx+\int x^3\cosh(x)\sinh(x)dx$$

$$\Big\downarrow 26$$

$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2}\right.}{4}\right.\right.\right.\right.$$
$$i\int x^3\tan\left(ix+\frac{\pi}{2}\right)dx+\int x^3\cosh(x)\sinh(x)dx$$

3.423. $\int x^3\cosh^2(x)\coth^3(x)\,dx$

$$\Big\downarrow \text{4199}$$

$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2}\right.}{4}\right.\right.\right.\right.$$
$$\int x^3\cosh(x)\sinh(x)dx-i\left(2i\int -\frac{e^{2x}x^3}{1-e^{2x}}dx-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2}\right.}{4}\right.\right.\right.\right.$$
$$\int x^3\cosh(x)\sinh(x)dx-i\left(-2i\int \frac{e^{2x}x^3}{1-e^{2x}}dx-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{2620}$$

$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2}\right.}{4}\right.\right.\right.\right.$$
$$\int x^3\cosh(x)\sinh(x)dx-i\left(-2i\left(\frac{3}{2}\int x^2\log\left(1-e^{2x}\right)dx-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{3011}$$

$$-i\left(-2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)-\frac{ix^4}{4}\right)+$$
$$i\left(2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)+\frac{3}{2}i\left(2i\left(-2i\left(-\frac{\text{PolyLog}\left(2,e^{2}\right.}{4}\right.\right.\right.\right.$$
$$\int x^3\cosh(x)\sinh(x)dx$$

input
```
Int[x^3*Cosh[x]^2*Coth[x]^3,x]
```

output
```
$Aborted
```

3.423. $\int x^3\cosh^2(x)\coth^3(x)\,dx$

3.423.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.423. $\int x^3 \cosh^2(x) \coth^3(x)\,dx$

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.423.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{x^4}{2}+\left(-\frac{3}{32}+\frac{3}{16}x-\frac{3}{16}x^2+\frac{1}{8}x^3\right)\mathrm{e}^{2x}+\left(\frac{3}{32}+\frac{3}{16}x+\frac{3}{16}x^2+\frac{1}{8}x^3\right)\mathrm{e}^{-2x}-\frac{x^2\left(2\,\mathrm{e}^{2x}x+3\,\mathrm{e}^{2x}-3\right)}{\left(\mathrm{e}^{2x}-1\right)^2}-3x^2+$

input
```
int(x^3*cosh(x)^2*coth(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^4+(-3/32+3/16*x-3/16*x^2+1/8*x^3)*exp(x)^2+(3/32+3/16*x+3/16*x^2+1/
8*x^3)/exp(x)^2-x^2*(2*x*exp(x)^2+3*exp(x)^2-3)/(exp(x)^2-1)^2-3*x^2+3*x*l
n(1-exp(x))+3*polylog(2,exp(x))+3*x*ln(exp(x)+1)+3*polylog(2,-exp(x))+2*x^
3*ln(1-exp(x))+6*x^2*polylog(2,exp(x))-12*x*polylog(3,exp(x))+12*polylog(4
,exp(x))+2*x^3*ln(exp(x)+1)+6*x^2*polylog(2,-exp(x))-12*x*polylog(3,-exp(x
))+12*polylog(4,-exp(x))
```

3.423.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2067 vs. $2(126) = 252$.

Time = 0.27 (sec) , antiderivative size = 2067, normalized size of antiderivative = 13.08

$$\int x^3\cosh^2(x)\coth^3(x)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(x)^2*coth(x)^3,x, algorithm="fricas")
```

output
```
1/32*((4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^8 + 8*(4*x^3 - 6*x^2 + 6*x - 3)*co
sh(x)*sinh(x)^7 + (4*x^3 - 6*x^2 + 6*x - 3)*sinh(x)^8 - 2*(8*x^4 + 4*x^3 +
 42*x^2 + 6*x - 3)*cosh(x)^6 - 2*(8*x^4 + 4*x^3 - 14*(4*x^3 - 6*x^2 + 6*x
- 3)*cosh(x)^2 + 42*x^2 + 6*x - 3)*sinh(x)^6 + 4*(14*(4*x^3 - 6*x^2 + 6*x
- 3)*cosh(x)^3 - 3*(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x))*sinh(x)^5 +
 4*(8*x^4 - 14*x^3 + 24*x^2 + 3*x)*cosh(x)^4 + 2*(35*(4*x^3 - 6*x^2 + 6*x
- 3)*cosh(x)^4 + 16*x^4 - 28*x^3 - 15*(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*c
osh(x)^2 + 48*x^2 + 6*x)*sinh(x)^4 + 8*(7*(4*x^3 - 6*x^2 + 6*x - 3)*cosh(x
)^5 - 5*(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x)^3 + 2*(8*x^4 - 14*x^3 +
 24*x^2 + 3*x)*cosh(x))*sinh(x)^3 + 4*x^3 - 2*(8*x^4 + 4*x^3 + 6*x^2 + 6*x
 + 3)*cosh(x)^2 + 2*(14*(4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^6 - 15*(8*x^4 +
4*x^3 + 42*x^2 + 6*x - 3)*cosh(x)^4 - 8*x^4 - 4*x^3 + 12*(8*x^4 - 14*x^3 +
 24*x^2 + 3*x)*cosh(x)^2 - 6*x^2 - 6*x - 3)*sinh(x)^2 + 6*x^2 + 96*((2*x^2
 + 1)*cosh(x)^6 + 6*(2*x^2 + 1)*cosh(x)*sinh(x)^5 + (2*x^2 + 1)*sinh(x)^6
- 2*(2*x^2 + 1)*cosh(x)^4 + (15*(2*x^2 + 1)*cosh(x)^2 - 4*x^2 - 2)*sinh(x)
^4 + 4*(5*(2*x^2 + 1)*cosh(x)^3 - 2*(2*x^2 + 1)*cosh(x))*sinh(x)^3 + (2*x^
2 + 1)*cosh(x)^2 + (15*(2*x^2 + 1)*cosh(x)^4 - 12*(2*x^2 + 1)*cosh(x)^2 +
2*x^2 + 1)*sinh(x)^2 + 2*(3*(2*x^2 + 1)*cosh(x)^5 - 4*(2*x^2 + 1)*cosh(x)^
3 + (2*x^2 + 1)*cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 96*((2*x^2 +
1)*cosh(x)^6 + 6*(2*x^2 + 1)*cosh(x)*sinh(x)^5 + (2*x^2 + 1)*sinh(x)^6 ...
```

3.423.6 Sympy [F]

$$\int x^3\cosh^2(x)\coth^3(x)\,dx = \int x^3\cosh^2(x)\coth^3(x)\,dx$$

input
```
integrate(x**3*cosh(x)**2*coth(x)**3,x)
```

output
```
Integral(x**3*cosh(x)**2*coth(x)**3, x)
```

3.423.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.51

$$\int x^3\cosh^2(x)\coth^3(x)\,dx = -x^4 + 2\,x^3\log\left(e^x+1\right) + 2\,x^3\log\left(-e^x+1\right) + 6\,x^2\mathrm{Li}_2(-e^x) + 6\,x^2\mathrm{Li}_2(e^x) - 3\,x^2 + 3\,x\log\left(e^x+1\right) + 3\,x\log\left(-e^x+1\right) - 12\,x\mathrm{Li}_3(-e^x) - 12\,x\mathrm{Li}_3(e^x) + \frac{16\,x^4 - 8\,x^3 + 84\,x^2 + (4\,x^3 - 6\,x^2 + 6\,x - 3)e^{(6\,x)} + 2\,(8\,x^4 - 4\,x^3 + 6\,x^2 - 6\,x + 3)e^{(4\,x)} - 4\,(8\,x^4 + 14}{32\,(e^{(4\,x)} - 2\,e^{(2\,x)} + 1)} + 3\,\mathrm{Li}_2(-e^x) + 3\,\mathrm{Li}_2(e^x) + 12\,\mathrm{Li}_4(-e^x) + 12\,\mathrm{Li}_4(e^x)$$

input
```
integrate(x^3*cosh(x)^2*coth(x)^3,x, algorithm="maxima")
```

output
```
-x^4 + 2*x^3*log(e^x + 1) + 2*x^3*log(-e^x + 1) + 6*x^2*dilog(-e^x) + 6*x^
2*dilog(e^x) - 3*x^2 + 3*x*log(e^x + 1) + 3*x*log(-e^x + 1) - 12*x*polylog
(3, -e^x) - 12*x*polylog(3, e^x) + 1/32*(16*x^4 - 8*x^3 + 84*x^2 + (4*x^3
- 6*x^2 + 6*x - 3)*e^(6*x) + 2*(8*x^4 - 4*x^3 + 6*x^2 - 6*x + 3)*e^(4*x) -
 4*(8*x^4 + 14*x^3 + 24*x^2 - 3*x)*e^(2*x) + (4*x^3 + 6*x^2 + 6*x + 3)*e^(
-2*x) - 12*x - 6)/(e^(4*x) - 2*e^(2*x) + 1) + 3*dilog(-e^x) + 3*dilog(e^x)
 + 12*polylog(4, -e^x) + 12*polylog(4, e^x)
```

3.423.8 Giac [F]

$$\int x^3\cosh^2(x)\coth^3(x)\,dx = \int x^3\cosh\left(x\right)^2\coth\left(x\right)^3\,dx$$

input
```
integrate(x^3*cosh(x)^2*coth(x)^3,x, algorithm="giac")
```

output
```
integrate(x^3*cosh(x)^2*coth(x)^3, x)
```

3.423.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \cosh^2(x) \coth^3(x)\, dx = \int x^3 \cosh(x)^2 \coth(x)^3\, dx$$

input
```
int(x^3*cosh(x)^2*coth(x)^3,x)
```

output
```
int(x^3*cosh(x)^2*coth(x)^3, x)
```

3.424 $\int x^m \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

3.424.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx = \text{Int}(x^m \coth(a+bx)\text{csch}(a+bx),x)$$

output
```
CannotIntegrate(x^m*coth(b*x+a)*csch(b*x+a),x)
```

3.424.2 Mathematica [N/A]

Not integrable

Time = 15.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx = \int x^m \coth(a+bx)\text{csch}(a+bx)\,dx$$

input
```
Integrate[x^m*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
Integrate[x^m*Coth[a + b*x]*Csch[a + b*x], x]
```

3.424.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^m \coth(a+bx)\text{csch}(a+bx)dx$$

input
```
Int[x^m*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
$Aborted
```

3.424.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.424.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \cosh(bx+a)\,\text{csch}(bx+a)^2\,dx$$

input
```
int(x^m*cosh(b*x+a)*csch(b*x+a)^2,x)
```

output
```
int(x^m*cosh(b*x+a)*csch(b*x+a)^2,x)
```

3.424.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate(x^m*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*cosh(b*x + a)*csch(b*x + a)^2, x)
```

3.424.6 Sympy [F(-1)]

Timed out.

$$\int x^m \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**m*cosh(b*x+a)*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.424.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate(x^m*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*cosh(b*x + a)*csch(b*x + a)^2, x)
```

3.424.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate(x^m*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)*csch(b*x + a)^2, x)
```

3.424.9 Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)}{\sinh(a+b\,x)^2}\,dx$$

input
```
int((x^m*cosh(a + b*x))/sinh(a + b*x)^2,x)
```

output
```
int((x^m*cosh(a + b*x))/sinh(a + b*x)^2, x)
```

3.425 $\int x^3 \coth(a+bx)\text{csch}(a+bx)\,dx$

3.425.1 Optimal result

Integrand size = 16, antiderivative size = 93

$$\begin{aligned}\int x^3 \coth(a+bx)\text{csch}(a+bx)\,dx = &-\frac{6x^2\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\text{csch}(a+bx)}{b}\\ &-\frac{6x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{6x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}\\ &+\frac{6\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4}-\frac{6\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4}\end{aligned}$$

output
```
-6*x^2*arctanh(exp(b*x+a))/b^2-x^3*csch(b*x+a)/b-6*x*polylog(2,-exp(b*x+a)
)/b^3+6*x*polylog(2,exp(b*x+a))/b^3+6*polylog(3,-exp(b*x+a))/b^4-6*polylog
(3,exp(b*x+a))/b^4
```

3.425.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)\,dx$$
$$=\frac{-b^3x^3\text{csch}(a+bx)+3b^2x^2\log\left(1-e^{a+bx}\right)-3b^2x^2\log\left(1+e^{a+bx}\right)-6bx\,\text{PolyLog}\left(2,-e^{a+bx}\right)+6bx\,\text{Poly}}{b^4}$$

input
```
Integrate[x^3*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
(-(b^3*x^3*Csch[a + b*x]) + 3*b^2*x^2*Log[1 - E^(a + b*x)] - 3*b^2*x^2*Log
[1 + E^(a + b*x)] - 6*b*x*PolyLog[2, -E^(a + b*x)] + 6*b*x*PolyLog[2, E^(a
 + b*x)] + 6*PolyLog[3, -E^(a + b*x)] - 6*PolyLog[3, E^(a + b*x)])/b^4
```

3.425.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5942, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{5942}$$

$$\frac{3\int x^2\text{csch}(a+bx)dx}{b}-\frac{x^3\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x^3\text{csch}(a+bx)}{b}+\frac{3\int ix^2\csc(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{x^3\text{csch}(a+bx)}{b}+\frac{3i\int x^2\csc(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{x^3\text{csch}(a+bx)}{b}+\frac{3i\left(\frac{2i\int x\log\left(1-e^{a+bx}\right)dx}{b}-\frac{2i\int x\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2ix^2\text{arctanh}\left(e^{a+bx}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{x^3\text{csch}(a+bx)}{b}+\frac{3i\left(-\frac{2i\left(\frac{\int \text{PolyLog}\left(2,-e^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{\int \text{PolyLog}\left(2,e^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b}\right)}{b}+\frac{2ix^2\text{arctanh}\left(e^{a+bx}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{-\frac{x^3\text{csch}(a+bx)}{b}+3i\left(-\frac{2i\left(\frac{\int e^{-a-bx}\text{PolyLog}\left(2,-e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\text{PolyLog}\left(2,e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\text{PolyLog}\left(2,e^{a+bx}\right)}{b}\right)}{b}+\frac{2ix^2}{}\right.}{b}$$

$\downarrow$ 7143

$$-\frac{x^3\text{csch}(a+bx)}{b}+\frac{3i\left(\frac{2ix^2\text{arctanh}(e^{a+bx})}{b}-\frac{2i\left(\frac{\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^2}-\frac{x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{\text{PolyLog}\left(3,e^{a+bx}\right)}{b^2}-\frac{x\text{PolyLog}\left(2,e^{a+bx}\right)}{b}\right)}{b}\right)}{b}$$

input
```
Int[x^3*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
-((x^3*Csch[a + b*x])/b) + ((3*I)*(((2*I)*x^2*ArcTanh[E^(a + b*x)])/b - ((
2*I)*(-((x*PolyLog[2, -E^(a + b*x)])/b) + PolyLog[3, -E^(a + b*x)]/b^2))/b
 + ((2*I)*(-((x*PolyLog[2, E^(a + b*x)])/b) + PolyLog[3, E^(a + b*x)]/b^2)
)/b))/b
```

3.425.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.425. $\int x^3\coth(a+bx)\text{csch}(a+bx)\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5942
```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.425.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.87

method	result
risch	$-\frac{2x^{3} {\mathrm e}^{b x +a}}{b \left({\mathrm e}^{2 b x +2 a}-1\right)}-\frac{6 a^{2} \operatorname{arctanh}\left({\mathrm e}^{b x +a}\right)}{b^{4}}+\frac{3 \ln \left(1-{\mathrm e}^{b x +a}\right) x^{2}}{b^{2}}-\frac{3 \ln \left(1-{\mathrm e}^{b x +a}\right) a^{2}}{b^{4}}+\frac{6 x \operatorname{polylog}\left(2, {\mathrm e}^{b x +a}\right)}{b^{3}}-\frac{6 \operatorname{polylog}\left(3, {\mathrm e}^{b x +a}\right)}{b^{4}}$

input
```
int(x^3*cosh(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b*x^3*exp(b*x+a)/(exp(2*b*x+2*a)-1)-6/b^4*a^2*arctanh(exp(b*x+a))+3/b^2
*ln(1-exp(b*x+a))*x^2-3/b^4*ln(1-exp(b*x+a))*a^2+6*x*polylog(2,exp(b*x+a))
/b^3-6*polylog(3,exp(b*x+a))/b^4-3/b^2*ln(exp(b*x+a)+1)*x^2+3/b^4*ln(exp(b
*x+a)+1)*a^2-6*x*polylog(2,-exp(b*x+a))/b^3+6*polylog(3,-exp(b*x+a))/b^4
```

3.425.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(86) = 172$.

Time = 0.27 (sec) , antiderivative size = 551, normalized size of antiderivative = 5.92

$$\int x^3 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\,b^3x^3\cosh(bx+a)+2\,b^3x^3\sinh(bx+a)-6\left(bx\cosh(bx+a)^2+2\,bx\cosh(bx+a)\sinh(bx+a)+bx\right.}{}$$

input
```
integrate(x^3*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) - 6*(b*x*cosh(b*x + a)
^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(
cosh(b*x + a) + sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x +
 a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(-cosh(b*x + a) - sinh
(b*x + a)) + 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x
 + a) + b^2*x^2*sinh(b*x + a)^2 - b^2*x^2)*log(cosh(b*x + a) + sinh(b*x +
a) + 1) - 3*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2
*sinh(b*x + a)^2 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(b^2*x^
2 - (b^2*x^2 - a^2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh
(b*x + a) - (b^2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - si
nh(b*x + a) + 1) + 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + si
nh(b*x + a)^2 - 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 6*(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(3,
-cosh(b*x + a) - sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a
)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 - b^4)
```

3.425.6 Sympy [F]

$$\int x^3 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^3 \cosh(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate(x**3*cosh(b*x+a)*csch(b*x+a)**2,x)
```

output
```
Integral(x**3*cosh(a + b*x)*csch(a + b*x)**2, x)
```

3.425.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.30

$$\int x^3 \coth(a+bx)\operatorname{csch}(a+bx)\,dx$$
$$= -\frac{2\,x^3e^{(bx+a)}}{be^{(2\,bx+2\,a)}-b} - \frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})\right)}{b^4}$$
$$+ \frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\,\mathrm{Li}_3(e^{(bx+a)})\right)}{b^4}$$

input
```
integrate(x^3*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 3*(b^2*x^2*log(e^(b*x + a) +
1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^4 + 3*(b^2*
x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x
 + a)))/b^4
```

3.425.8 Giac [F]

$$\int x^3 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^3 \cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)^2\,dx$$

input
```
integrate(x^3*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*cosh(b*x + a)*csch(b*x + a)^2, x)
```

3.425.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int \frac{x^3\cosh(a+b\,x)}{\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int((x^3*cosh(a + b*x))/sinh(a + b*x)^2,x)
```

output
```
int((x^3*cosh(a + b*x))/sinh(a + b*x)^2, x)
```

3.426 $\int x^2 \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

3.426.1 Optimal result

Integrand size = 16, antiderivative size = 59

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}$$

output

```
-4*x*arctanh(exp(b*x+a))/b^2-x^2*csch(b*x+a)/b-2*polylog(2,-exp(b*x+a))/b^
3+2*polylog(2,exp(b*x+a))/b^3
```

3.426.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)\,dx = \frac{-bx\left(bx\text{csch}(a+bx) - 2\log\left(1-e^{a+bx}\right) + 2\log\left(1+e^{a+bx}\right)\right) - 2\,\text{PolyLog}\left(2,-e^{a+bx}\right) + 2\,\text{PolyLog}\left(2,e^{a+}\right.}{b^3}$$

input

```
Integrate[x^2*Coth[a + b*x]*Csch[a + b*x],x]
```

output

```
(-(b*x*(b*x*Csch[a + b*x] - 2*Log[1 - E^(a + b*x)] + 2*Log[1 + E^(a + b*x)
])) - 2*PolyLog[2, -E^(a + b*x)] + 2*PolyLog[2, E^(a + b*x)])/b^3
```

3.426.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5942, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{5942}$$

$$\frac{2\int x\text{csch}(a+bx)dx}{b}-\frac{x^2\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x^2\text{csch}(a+bx)}{b}+\frac{2\int ix\csc(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{x^2\text{csch}(a+bx)}{b}+\frac{2i\int x\csc(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{x^2\text{csch}(a+bx)}{b}+\frac{2i\left(\frac{i\int \log(1-e^{a+bx})dx}{b}-\frac{i\int \log(1+e^{a+bx})dx}{b}+\frac{2ix\text{arctanh}(e^{a+bx})}{b}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{x^2\text{csch}(a+bx)}{b}+\frac{2i\left(\frac{i\int e^{-a-bx}\log(1-e^{a+bx})de^{a+bx}}{b^2}-\frac{i\int e^{-a-bx}\log(1+e^{a+bx})de^{a+bx}}{b^2}+\frac{2ix\text{arctanh}(e^{a+bx})}{b}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{x^2\text{csch}(a+bx)}{b}+\frac{2i\left(\frac{2ix\text{arctanh}(e^{a+bx})}{b}+\frac{i\,\text{PolyLog}(2,-e^{a+bx})}{b^2}-\frac{i\,\text{PolyLog}(2,e^{a+bx})}{b^2}\right)}{b}$$

input
```
Int[x^2*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
-((x^2*Csch[a + b*x])/b) + ((2*I)*(((2*I)*x*ArcTanh[E^(a + b*x)])/b + (I*P
olyLog[2, -E^(a + b*x)])/b^2 - (I*PolyLog[2, E^(a + b*x)])/b^2))/b
```

3.426.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5942
```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

3.426.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(56) = 112$.

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.27

method	result
risch	$-\frac{2x^2 e^{bx+a}}{b(e^{2bx+2a}-1)}+\frac{2\ln(1-e^{bx+a})x}{b^2}+\frac{2\ln(1-e^{bx+a})a}{b^3}+\frac{2\,\mathrm{polylog}(2,e^{bx+a})}{b^3}-\frac{2\ln(e^{bx+a}+1)x}{b^2}-\frac{2\ln(e^{bx+a}+1)a}{b^3}-\frac{2\,\mathrm{po}}{}$

input
```
int(x^2*cosh(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b*x^2*exp(b*x+a)/(exp(2*b*x+2*a)-1)+2/b^2*ln(1-exp(b*x+a))*x+2/b^3*ln(1
-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3-2/b^2*ln(exp(b*x+a)+1)*x-2/b^3*
ln(exp(b*x+a)+1)*a-2*polylog(2,-exp(b*x+a))/b^3+4/b^3*a*arctanh(exp(b*x+a)
)
```

3.426.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(54) = 108$.

Time = 0.25 (sec) , antiderivative size = 367, normalized size of antiderivative = 6.22

$$\int x^2 \coth(a+bx)\operatorname{csch}(a+bx)\,dx =$$
$$-\frac{2\left(b^2x^2\cosh\left(bx+a\right)+b^2x^2\sinh\left(bx+a\right)-\left(\cosh\left(bx+a\right)^2+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+\right.\right.}{}$$

input
```
integrate(x^2*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2*(b^2*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a) - (cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a) + si
nh(b*x + a)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
 + a)^2 - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)^2
+ 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(
b*x + a) + sinh(b*x + a) + 1) + (a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sin
h(b*x + a) + a*sinh(b*x + a)^2 - a)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 - ((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) +
(b*x + a)*sinh(b*x + a)^2 - b*x - a)*log(-cosh(b*x + a) - sinh(b*x + a) +
1))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*
x + a)^2 - b^3)
```

3.426.6 Sympy [F]

$$\int x^2 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^2 \cosh(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate(x**2*cosh(b*x+a)*csch(b*x+a)**2,x)
```

output
```
Integral(x**2*cosh(a + b*x)*csch(a + b*x)**2, x)
```

3.426.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

$$\int x^2 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\,x^2e^{(bx+a)}}{be^{(2\,bx+2\,a)}-b} - \frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3} + \frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}$$

input
```
integrate(x^2*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 2*(b*x*log(e^(b*x + a) + 1) +
 dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x +
a)))/b^3
```

3.426.8 Giac [F]

$$\int x^2 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^2 \cosh(bx+a)\operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate(x^2*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*cosh(b*x + a)*csch(b*x + a)^2, x)
```

3.426.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)}{\sinh{(a+b\,x)}^2}\,dx$$

input
```
int((x^2*cosh(a + b*x))/sinh(a + b*x)^2,x)
```

output
```
int((x^2*cosh(a + b*x))/sinh(a + b*x)^2, x)
```

3.427 $\int x \coth(a+bx)\operatorname{csch}(a+bx)\,dx$

3.427.1 Optimal result

Integrand size = 14, antiderivative size = 25

$$\int x \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b^2} - \frac{x\operatorname{csch}(a+bx)}{b}$$

output
```
-arctanh(cosh(b*x+a))/b^2-x*csch(b*x+a)/b
```

3.427.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(25) = 50.

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.56

$$\int x \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{x\operatorname{csch}(a)}{b} - \frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2} + \frac{\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2} + \frac{x\operatorname{csch}\left(\frac{a}{2}\right)\operatorname{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{2b} + \frac{x\operatorname{sech}\left(\frac{a}{2}\right)\operatorname{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{2b}$$

input
```
Integrate[x*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
-((x*Csch[a])/b) - Log[Cosh[a/2 + (b*x)/2]]/b^2 + Log[Sinh[a/2 + (b*x)/2]]
/b^2 + (x*Csch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b) + (x*Sech[a/2
]*Sech[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b)
```

3.427.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5942, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{5942}$$

$$\frac{\int \text{csch}(a+bx)dx}{b}-\frac{x\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x\text{csch}(a+bx)}{b}+\frac{\int i\csc(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{x\text{csch}(a+bx)}{b}+\frac{i\int \csc(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\text{arctanh}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}$$

input
```
Int[x*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
-(ArcTanh[Cosh[a + b*x]]/b^2) - (x*Csch[a + b*x])/b
```

3.427.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5942
```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

3.427.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(25) = 50$.

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

method	result	size
risch	$-\frac{2x\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^2}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b^2}$	54

input
```
int(x*cosh(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b*x*exp(b*x+a)/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(b*x+a)-1)-1/b^2*ln(exp(b
*x+a)+1)
```

3.427.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 169, normalized size of antiderivative = 6.76

$$\int x\coth(a+bx)\text{csch}(a+bx)\,dx =$$
$$-\frac{2\,bx\cosh\left(bx+a\right)+2\,bx\sinh\left(bx+a\right)+\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right.}{b^2\cosh\left(bx\right.}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b*x*cosh(b*x + a) + 2*b*x*sinh(b*x + a) + (cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a
)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1))/(b^2*cosh(b*x + a)^2 + 2*
b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2)
```

3.427.6 Sympy [F]

$$\int x \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int x\cosh(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)**2,x)
```

output
```
Integral(x*cosh(a + b*x)*csch(a + b*x)**2, x)
```

3.427.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.56

$$\int x \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}-b} - \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2} + \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(-a))/b
^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

3.427.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.72

$$\int x\coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\,bxe^{(bx+a)} + e^{(2\,bx+2\,a)}\log\left(e^{(bx+a)}+1\right) - e^{(2\,bx+2\,a)}\log\left(e^{(bx+a)}-1\right) - \log\left(e^{(bx+a)}+1\right) + \log\left(e^{(bx+a)}-\right.}{b^2e^{(2\,bx+2\,a)} - b^2}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-(2*b*x*e^(b*x + a) + e^(2*b*x + 2*a)*log(e^(b*x + a) + 1) - e^(2*b*x + 2*
a)*log(e^(b*x + a) - 1) - log(e^(b*x + a) + 1) + log(e^(b*x + a) - 1))/(b^
2*e^(2*b*x + 2*a) - b^2)
```

3.427.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int x\coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\,\text{atan}\left(\frac{e^{b\,x}\,e^a\,\sqrt{-b^4}}{b^2}\right)}{\sqrt{-b^4}} - \frac{2\,x\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int((x*cosh(a + b*x))/sinh(a + b*x)^2,x)
```

output
```
- (2*atan((exp(b*x)*exp(a)*(-b^4)^(1/2))/b^2))/(-b^4)^(1/2) - (2*x*exp(a +
 b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.428 $\int \coth(a+bx)\text{csch}(a+bx)\,dx$

3.428.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{\text{csch}(a+bx)}{b}$$

output
```
-csch(b*x+a)/b
```

3.428.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{\text{csch}(a+bx)}{b}$$

input
```
Integrate[Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
-(Csch[a + b*x]/b)
```

3.428.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(ia+ibx-\frac{\pi}{2}\right)\sec\left(ia+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{i\int 1d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\text{csch}(a+bx)}{b}$$

input
```
Int[Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
-(Csch[a + b*x]/b)
```

3.428.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.428.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(bx+a)}{b}$	12
default	$-\frac{\operatorname{csch}(bx+a)}{b}$	12
risch	$-\frac{2\,{\mathrm e}^{bx+a}}{b\left({\mathrm e}^{2bx+2a}-1\right)}$	25

input
```
int(cosh(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-csch(b*x+a)/b
```

3.428.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 5.09

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}-b}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

3.428.6 Sympy [F]

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int \cosh\left(a+bx\right)\operatorname{csch}^{2}\left(a+bx\right)dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)**2,x)
```

output
```
Integral(cosh(a + b*x)*csch(a + b*x)**2, x)
```

3.428.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)} - e^{(-bx-a)})}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

3.428.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)} - e^{(-bx-a)})}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

3.428.9 Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{1}{b\sinh(a+b\,x)}$$

input
```
int(cosh(a + b*x)/sinh(a + b*x)^2,x)
```

output
```
-1/(b*sinh(a + b*x))
```

3.429 $\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$

3.429.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x},x\right)$$

output
```
CannotIntegrate(coth(b*x+a)*csch(b*x+a)/x,x)
```

3.429.2 Mathematica [N/A]

Not integrable

Time = 22.78 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x}\,dx = \int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$$

input
```
Integrate[(Coth[a + b*x]*Csch[a + b*x])/x,x]
```

output
```
Integrate[(Coth[a + b*x]*Csch[a + b*x])/x, x]
```

3.429.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}dx$$

input
```
Int[(Coth[a + b*x]*Csch[a + b*x])/x,x]
```

output
```
$Aborted
```

3.429.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.429.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(bx+a)\,\text{csch}(bx+a)^2}{x}dx$$

input
```
int(cosh(b*x+a)*csch(b*x+a)^2/x,x)
```

output
```
int(cosh(b*x+a)*csch(b*x+a)^2/x,x)
```

3.429.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)\,\text{csch}(bx+a)^2}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*csch(b*x + a)^2/x, x)
```

3.429.6 Sympy [N/A]

Not integrable

Time = 9.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\,\text{csch}^2(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)**2/x,x)
```

output
```
Integral(cosh(a + b*x)*csch(a + b*x)**2/x, x)
```

3.429.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.94

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)\,\text{csch}(bx+a)^2}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
-2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) - b*x) - 2*integrate(1/2/(b*x^2*e^(b*x
 + a) + b*x^2), x) - 2*integrate(1/2/(b*x^2*e^(b*x + a) - b*x^2), x)
```

3.429.8 Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)\,\text{csch}\,(bx+a)^2}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)*csch(b*x + a)^2/x, x)
```

3.429.9 Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)}{x\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(cosh(a + b*x)/(x*sinh(a + b*x)^2),x)
```

output
```
int(cosh(a + b*x)/(x*sinh(a + b*x)^2), x)
```

3.430 $\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2} dx$

3.430.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2} dx = \text{Int}\left(\frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}, x\right)$$

output
```
CannotIntegrate(coth(b*x+a)*csch(b*x+a)/x^2,x)
```

3.430.2 Mathematica [N/A]

Not integrable

Time = 32.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2} dx = \int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2} dx$$

input
```
Integrate[(Coth[a + b*x]*Csch[a + b*x])/x^2,x]
```

output
```
Integrate[(Coth[a + b*x]*Csch[a + b*x])/x^2, x]
```

3.430.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx$$

input
```
Int[(Coth[a + b*x]*Csch[a + b*x])/x^2,x]
```

output
```
$Aborted
```

3.430.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.430.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(bx+a)\,\text{csch}(bx+a)^2}{x^2}dx$$

input
```
int(cosh(b*x+a)*csch(b*x+a)^2/x^2,x)
```

output
```
int(cosh(b*x+a)*csch(b*x+a)^2/x^2,x)
```

3.430.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*csch(b*x + a)^2/x^2, x)
```

3.430.6 Sympy [N/A]

Not integrable

Time = 11.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+bx)\operatorname{csch}^2(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)**2/x**2,x)
```

output
```
Integral(cosh(a + b*x)*csch(a + b*x)**2/x**2, x)
```

3.430.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.94

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) - 2*integrate(1/(b*x^3*e^(b
*x + a) + b*x^3), x) - 2*integrate(1/(b*x^3*e^(b*x + a) - b*x^3), x)
```

3.430.8 Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)*csch(b*x + a)^2/x^2, x)
```

3.430.9 Mupad [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)}{x^2\sinh(a+b\,x)^2}\,dx$$

input
```
int(cosh(a + b*x)/(x^2*sinh(a + b*x)^2),x)
```

output
```
int(cosh(a + b*x)/(x^2*sinh(a + b*x)^2), x)
```

3.431 $\int x^m \coth^2(a+bx)\,dx$

3.431.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \coth^2(a+bx)\,dx = \text{Int}\left(x^m \coth^2(a+bx),x\right)$$

output
```
Unintegrable(x^m*coth(b*x+a)^2,x)
```

3.431.2 Mathematica [N/A]

Not integrable

Time = 7.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \coth^2(a+bx)\,dx = \int x^m \coth^2(a+bx)\,dx$$

input
```
Integrate[x^m*Coth[a + b*x]^2,x]
```

output
```
Integrate[x^m*Coth[a + b*x]^2, x]
```

3.431.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^m \tan\left(ia+ibx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^m \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2 dx$$

$$\Big\downarrow \text{4222}$$

$$\int -x^m \tan^2\left(\frac{1}{2}(-\pi-2ia)-ibx\right) dx$$

input
```
Int[x^m*Coth[a + b*x]^2,x]
```

output
```
$Aborted
```

3.431.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.431.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)^2 \, dx$$

input
```
int(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

output
```
int(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

3.431.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \coth^2(a+bx) \, dx = \int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)^2 \, dx$$

input
```
integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

3.431.6 Sympy [F(-1)]

Timed out.

$$\int x^m \coth^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**m*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.431.7 Maxima [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 145, normalized size of antiderivative = 12.08

$$\int x^m \coth^2(a+bx)\,dx = \int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)^2 \,dx$$

input
```
integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
x*e^(4*b*x + m*log(x) + 4*a)/((m + 1)*e^(4*b*x + 4*a) - 2*(m + 1)*e^(2*b*x
 + 2*a) + m + 1) + integrate((2*(2*b*x*e^(4*a) + (m + 1)*e^(4*a))*e^(4*b*x
) - (m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m + 1)*e^(6*b*x + 6*a) - 3*(m +
 1)*e^(4*b*x + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) - m - 1), x)
```

3.431.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \coth^2(a+bx)\,dx = \int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)^2 \,dx$$

input
```
integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

3.431.9 Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \coth^2(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)^2}{\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int((x^m*cosh(a + b*x)^2)/sinh(a + b*x)^2,x)
```

output
```
int((x^m*cosh(a + b*x)^2)/sinh(a + b*x)^2, x)
```

3.432 $\int x^3 \coth^2(a+bx)\,dx$

3.432.1 Optimal result

Integrand size = 12, antiderivative size = 87

$$\int x^3 \coth^2(a+bx)\,dx = -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3\,\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^4}$$

output
```
-x^3/b+1/4*x^4-x^3*coth(b*x+a)/b+3*x^2*ln(1-exp(2*b*x+2*a))/b^2+3*x*polylo
g(2,exp(2*b*x+2*a))/b^3-3/2*polylog(3,exp(2*b*x+2*a))/b^4
```

3.432.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 222 vs. $2(87) = 174$.

Time = 0.61 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.55

$$\int x^3\coth^2(a+bx)\,dx = \frac{x^4}{4} - \frac{e^{2a}\left(2b^3e^{-2a}x^3-3b^2\left(1-e^{-2a}\right)x^2\log\left(1-e^{-a-bx}\right)-3b^2\left(1-e^{-2a}\right)x^2\log\left(1+e^{-a-bx}\right)+6b\left(1-e^{-2a}\right)x \cdots\right)}{\cdots} + \frac{x^3\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b}$$

input
```
Integrate[x^3*Coth[a + b*x]^2,x]
```

output
```
x^4/4 - (E^(2*a)*((2*b^3*x^3)/E^(2*a) - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 - E
^(-a - b*x)] - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 + E^(-a - b*x)] + 6*b*(1 - E
^(-2*a))*x*PolyLog[2, -E^(-a - b*x)] + 6*b*(1 - E^(-2*a))*x*PolyLog[2, E^(
-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[3, -E^(-a - b*x)] + 6*(1 - E^(-2*a))
*PolyLog[3, E^(-a - b*x)]))/(b^4*(-1 + E^(2*a))) + (x^3*Csch[a]*Csch[a + b
*x]*Sinh[b*x])/b
```

3.432.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^2(a+bx) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^3 \tan\left(ia+ibx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^3 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2 dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{3i\int ix^2\coth(a+bx)dx}{b}+\int x^3dx-\frac{x^3\coth(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3i\int ix^2\coth(a+bx)dx}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{26}$$

$$\frac{3\int x^2\coth(a+bx)dx}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int -ix^2\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$\Big\downarrow$ 26

$$-\frac{3i\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$\Big\downarrow$ 4201

$$-\frac{3i\left(2i\int\frac{e^{2a+2bx-i\pi}x^2}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^3}{3}\right)}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$\Big\downarrow$ 2620

$$-\frac{3i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int x\log\left(1+e^{2a+2bx-i\pi}\right)dx}{b}\right)-\frac{ix^3}{3}\right)}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$\Big\downarrow$ 3011

$$-\frac{3i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int \text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$\Big\downarrow$ 2720

$$-\frac{3i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$\Big\downarrow$ 7143

$$-\frac{3i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

input

```
Int[x^3*Coth[a + b*x]^2,x]
```

3.432. $\int x^3\coth^2(a+bx)\,dx$

output
```
x^4/4 - (x^3*Coth[a + b*x])/b - ((3*I)*((-1/3*I)*x^3 + (2*I)*((x^2*Log[1 +
 E^(2*a - I*Pi + 2*b*x)])/(2*b) - (-1/2*(x*PolyLog[2, -E^(2*a - I*Pi + 2*b
*x)])/b + PolyLog[3, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2))/b)))/b
```

3.432.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.432.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(83) = 166$.

Time = 0.99 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.28

method	result
risch	$\frac{x^4}{4}-\frac{2x^3}{b(e^{2bx+2a}-1)}+\frac{4a^3}{b^4}+\frac{6x\,a^2}{b^3}-\frac{2x^3}{b}-\frac{6\,\mathrm{polylog}(3,-e^{bx+a})}{b^4}-\frac{6\,\mathrm{polylog}(3,e^{bx+a})}{b^4}+\frac{3\ln(e^{bx+a}+1)x^2}{b^2}+\frac{6x\,\mathrm{polyl}}{}$

input
```
int(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4-2*x^3/b/(exp(2*b*x+2*a)-1)+4/b^4*a^3+6*x/b^3*a^2-2*x^3/b-6*polylog
(3,-exp(b*x+a))/b^4-6*polylog(3,exp(b*x+a))/b^4+3/b^2*ln(exp(b*x+a)+1)*x^2
+6*x*polylog(2,-exp(b*x+a))/b^3+3/b^2*ln(1-exp(b*x+a))*x^2+6*x*polylog(2,e
xp(b*x+a))/b^3+3/b^4*a^2*ln(exp(b*x+a)-1)-6/b^4*a^2*ln(exp(b*x+a))-3/b^4*l
n(1-exp(b*x+a))*a^2
```

3.432.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(82) = 164$.

Time = 0.27 (sec) , antiderivative size = 632, normalized size of antiderivative = 7.26

$$\int x^3 \coth^2(a+bx)\,dx =$$
$$-\frac{b^4x^4 - 8\,a^3 - (b^4x^4 - 8\,b^3x^3 - 8\,a^3)\cosh(bx+a)^2 - 2\,(b^4x^4 - 8\,b^3x^3 - 8\,a^3)\cosh(bx+a)\sinh(bx+a}{}$$

input
```
integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(b^4*x^4 - 8*a^3 - (b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)^2 - 2*
(b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)*sinh(b*x + a) - (b^4*x^4 - 8*b
^3*x^3 - 8*a^3)*sinh(b*x + a)^2 - 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x
 + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(cosh(b*x + a) + sin
h(b*x + a)) - 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a)
+ b*x*sinh(b*x + a)^2 - b*x)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 12*(b
^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*s
inh(b*x + a)^2 - b^2*x^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 12*(a^2
*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2
 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 12*(b^2*x^2 - (b^2*x^2 -
a^2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) - (b^
2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1
) + 24*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
- 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 24*(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(3, -cosh(b*x + a
) - sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x +
a) + b^4*sinh(b*x + a)^2 - b^4)
```

3.432.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \coth^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**3*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.432.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.68

$$\int x^3 \coth^2(a+bx)\,dx = -\frac{2\,x^3}{b} + \frac{bx^4e^{(2\,bx+2\,a)} - bx^4 - 8\,x^3}{4\,(be^{(2\,bx+2\,a)} - b)} + \frac{3\,(b^2x^2\log\left(e^{(bx+a)}+1\right) + 2\,bx\text{Li}_2\left(-e^{(bx+a)}\right) - 2\,\text{Li}_3(-e^{(bx+a)}))}{b^4} + \frac{3\,(b^2x^2\log\left(-e^{(bx+a)}+1\right) + 2\,bx\text{Li}_2\left(e^{(bx+a)}\right) - 2\,\text{Li}_3(e^{(bx+a)}))}{b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*x^3/b + 1/4*(b*x^4*e^(2*b*x + 2*a) - b*x^4 - 8*x^3)/(b*e^(2*b*x + 2*a)
- b) + 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*pol
ylog(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilo
g(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^4
```

3.432.8 Giac [F]

$$\int x^3 \coth^2(a+bx)\,dx = \int x^3 \cosh(bx+a)^2 \operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^2(a+bx)\,dx = \int \frac{x^3\cosh(a+b\,x)^2}{\sinh(a+b\,x)^2}\,dx$$

input
```
int((x^3*cosh(a + b*x)^2)/sinh(a + b*x)^2,x)
```

output
```
int((x^3*cosh(a + b*x)^2)/sinh(a + b*x)^2, x)
```

3.433 $\int x^2 \coth^2(a+bx)\,dx$

3.433.1 Optimal result

Integrand size = 12, antiderivative size = 65

$$\int x^2 \coth^2(a+bx)\,dx = -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}$$

output
```
-x^2/b+1/3*x^3-x^2*coth(b*x+a)/b+2*x*ln(1-exp(2*b*x+2*a))/b^2+polylog(2,ex
p(2*b*x+2*a))/b^3
```

3.433.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int x^2 \coth^2(a+bx)\,dx = -\frac{2\operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b^3}-\frac{2\operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b^3}+\frac{1}{3}x\left(\frac{6x}{b-be^{2a}}+x^2+\frac{6\log\left(1-e^{-a-bx}\right)}{b^2}+\frac{6\log\left(1+e^{-a-bx}\right)}{b^2}+\frac{3x\operatorname{csch}(a)\operatorname{csch}(a+bx)\sinh(bx)}{b}\right)$$

input
```
Integrate[x^2*Coth[a + b*x]^2,x]
```

output
```
(-2*PolyLog[2, -E^(-a - b*x)])/b^3 - (2*PolyLog[2, E^(-a - b*x)])/b^3 + (x
*((6*x)/(b - b*E^(2*a)) + x^2 + (6*Log[1 - E^(-a - b*x)])/b^2 + (6*Log[1 +
 E^(-a - b*x)])/b^2 + (3*x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b))/3
```

3.433.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.48, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^2(a+bx)\,dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int -x^2 \tan\left(ia+ibx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \textbf{25}$$

$$-\int x^2 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2 dx$$

$$\Big\downarrow \textbf{4203}$$

$$-\frac{2i\int ix\coth(a+bx)dx}{b}+\int x^2dx-\frac{x^2\coth(a+bx)}{b}$$

$$\Big\downarrow \textbf{15}$$

$$-\frac{2i\int ix\coth(a+bx)dx}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{2\int x\coth(a+bx)dx}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2\int -ix\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \textbf{26}$$

$$-\frac{2i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$\Big\downarrow$ 4201

$$-\frac{2i\left(2i\int\frac{e^{2a+2bx-i\pi}x}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^2}{2}\right)}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$\Big\downarrow$ 2620

$$-\frac{2i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int\log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)-\frac{ix^2}{2}\right)}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$\Big\downarrow$ 2715

$$-\frac{2i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\log\left(1+e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}\right)-\frac{ix^2}{2}\right)}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$\Big\downarrow$ 2838

$$-\frac{2i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{ix^2}{2}\right)}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

input
```
Int[x^2*Coth[a + b*x]^2,x]
```

output
```
x^3/3 - (x^2*Coth[a + b*x])/b - ((2*I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E
^(2*a - I*Pi + 2*b*x)])/(2*b) + PolyLog[2, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2
))))/b
```

3.433.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.433.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(63) = 126.

Time = 0.99 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.40

method	result
risch	$\frac{x^3}{3}-\frac{2x^2}{b\left(e^{2bx+2a}-1\right)}-\frac{2x^2}{b}-\frac{4ax}{b^2}-\frac{2a^2}{b^3}+\frac{2\ln\left(e^{bx+a}+1\right)x}{b^2}+\frac{2\operatorname{polylog}\left(2,-e^{bx+a}\right)}{b^3}+\frac{2\ln\left(1-e^{bx+a}\right)x}{b^2}+\frac{2\ln\left(1-e^{bx+a}\right)}{b^3}$

input
```
int(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3-2*x^2/b/(exp(2*b*x+2*a)-1)-2*x^2/b-4*a*x/b^2-2/b^3*a^2+2/b^2*ln(ex
p(b*x+a)+1)*x+2*polylog(2,-exp(b*x+a))/b^3+2/b^2*ln(1-exp(b*x+a))*x+2/b^3*
ln(1-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3-2/b^3*a*ln(exp(b*x+a)-1)+4/
b^3*a*ln(exp(b*x+a))
```

3.433.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(62) = 124$.

Time = 0.26 (sec) , antiderivative size = 453, normalized size of antiderivative = 6.97

$$\int x^2 \coth^2(a+bx)\,dx =$$

$$-\frac{b^3x^3 - (b^3x^3 - 6\,b^2x^2 + 6\,a^2)\cosh(bx+a)^2 - 2\,(b^3x^3 - 6\,b^2x^2 + 6\,a^2)\cosh(bx+a)\sinh(bx+a) - (b^3}{}$$

input
```
integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(b^3*x^3 - (b^3*x^3 - 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)^2 - 2*(b^3*x^3
 - 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)*sinh(b*x + a) - (b^3*x^3 - 6*b^2*x^2 +
 6*a^2)*sinh(b*x + a)^2 + 6*a^2 - 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sin
h(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6
*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*d
ilog(-cosh(b*x + a) - sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh
(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(b*x + a) + s
inh(b*x + a) + 1) + 6*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a)
 + a*sinh(b*x + a)^2 - a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*((b*x
 + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a
)*sinh(b*x + a)^2 - b*x - a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1))/(b^3
*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2
 - b^3)
```

3.433.6 Sympy [F]

$$\int x^2 \coth^2(a+bx)\,dx = \int x^2 \cosh^2(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate(x**2*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

output
```
Integral(x**2*cosh(a + b*x)**2*csch(a + b*x)**2, x)
```

3.433.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int x^2 \coth^2(a+bx)\,dx = -\frac{2x^2}{b} + \frac{bx^3e^{(2bx+2a)} - bx^3 - 6x^2}{3\left(be^{(2bx+2a)} - b\right)} + \frac{2\left(bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3} + \frac{2\left(bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*x^2/b + 1/3*(b*x^3*e^(2*b*x + 2*a) - b*x^3 - 6*x^2)/(b*e^(2*b*x + 2*a)
- b) + 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log
(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

3.433.8 Giac [F]

$$\int x^2 \coth^2(a+bx)\,dx = \int x^2 \cosh(bx+a)^2 \operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

3.433.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^2(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)^2}{\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int((x^2*cosh(a + b*x)^2)/sinh(a + b*x)^2,x)
```

output
```
int((x^2*cosh(a + b*x)^2)/sinh(a + b*x)^2, x)
```

3.434 $\int x \coth^2(a+bx)\,dx$

3.434.1 Optimal result

Integrand size = 10, antiderivative size = 31

$$\int x \coth^2(a+bx)\,dx = \frac{x^2}{2} - \frac{x\coth(a+bx)}{b} + \frac{\log(\sinh(a+bx))}{b^2}$$

output
```
1/2*x^2-x*coth(b*x+a)/b+ln(sinh(b*x+a))/b^2
```

3.434.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int x \coth^2(a+bx)\,dx = \frac{b^2x^2 - 2bx\coth(a) + 2\log(\sinh(a+bx)) + 2bx\operatorname{csch}(a)\operatorname{csch}(a+bx)\sinh(bx)}{2b^2}$$

input
```
Integrate[x*Coth[a + b*x]^2,x]
```

output
```
(b^2*x^2 - 2*b*x*Coth[a] + 2*Log[Sinh[a + b*x]] + 2*b*x*Csch[a]*Csch[a + b
*x]*Sinh[b*x])/(2*b^2)
```

3.434.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{i\int i\coth(a+bx)dx}{b}+\int xdx-\frac{x\coth(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{i\int i\coth(a+bx)dx}{b}-\frac{x\coth(a+bx)}{b}+\frac{x^2}{2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \coth(a+bx)dx}{b}-\frac{x\coth(a+bx)}{b}+\frac{x^2}{2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}-\frac{x\coth(a+bx)}{b}+\frac{x^2}{2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}-\frac{x\coth(a+bx)}{b}+\frac{x^2}{2}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\log(-i\sinh(a+bx))}{b^2}-\frac{x\coth(a+bx)}{b}+\frac{x^2}{2}$$

input
```
Int[x*Coth[a + b*x]^2,x]
```

output
```
x^2/2 - (x*Coth[a + b*x])/b + Log[(-I)*Sinh[a + b*x]]/b^2
```

3.434.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.434.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

method	result	size
risch	$\frac{x^2}{2}-\frac{2x}{b}-\frac{2a}{b^2}-\frac{2x}{b\left(e^{2bx+2a}-1\right)}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b^2}$	54
parallelrisch	$\frac{-2\ln(1-\tanh(bx+a))\tanh(bx+a)+2\ln(\tanh(bx+a))\tanh(bx+a)+xb(-2+(bx-2)\tanh(bx+a))}{2\tanh(bx+a)b^2}$	66

input
```
int(x*cosh(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-2*x/b-2*a/b^2-2*x/b/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(2*b*x+2*a)-1)
```

3.434.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 6.10

$$\int x\coth^2(a+bx)\,dx =$$

$$-\frac{b^2x^2-(b^2x^2-4\,bx)\cosh(bx+a)^2-2\,(b^2x^2-4\,bx)\cosh(bx+a)\sinh(bx+a)-(b^2x^2-4\,bx)\sinh(bx}{2\,(b^2\cosh(bx+a)^2+2\,b^2\cosh(bx+}$$

input
```
integrate(x*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^2 - (b^2*x^2 - 4*b*x)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 4*b*x)*co
sh(b*x + a)*sinh(b*x + a) - (b^2*x^2 - 4*b*x)*sinh(b*x + a)^2 - 2*(cosh(b*
x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(2*sinh
(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*c
osh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2)
```

3.434.6 Sympy [F]

$$\int x\coth^2(a+bx)\,dx = \int x\cosh^2(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate(x*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

output
```
Integral(x*cosh(a + b*x)**2*csch(a + b*x)**2, x)
```

3.434.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(29) = 58.

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.71

$$\int x\coth^2(a+bx)\,dx = -\frac{xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}-b} - \frac{bx^2-\left(bx^2e^{(2\,a)}-2\,xe^{(2\,a)}\right)e^{(2\,bx)}}{2\left(be^{(2\,bx+2\,a)}-b\right)} + \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2} + \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}$$

input
```
integrate(x*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - 1/2*(b*x^2 - (b*x^2*e^(2*a) -
 2*x*e^(2*a))*e^(2*b*x))/(b*e^(2*b*x + 2*a) - b) + log((e^(b*x + a) + 1)*e
^(-a))/b^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

3.434.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

$$\int x\coth^2(a+bx)\,dx = \frac{b^2x^2e^{(2\,bx+2\,a)}-b^2x^2-4\,bxe^{(2\,bx+2\,a)}+2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right)-2\,\log\left(e^{(2\,bx+2\,a)}-1\right)}{2\left(b^2e^{(2\,bx+2\,a)}-b^2\right)}$$

input
```
integrate(x*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^2*x^2*e^(2*b*x + 2*a) - b^2*x^2 - 4*b*x*e^(2*b*x + 2*a) + 2*e^(2*b*
x + 2*a)*log(e^(2*b*x + 2*a) - 1) - 2*log(e^(2*b*x + 2*a) - 1))/(b^2*e^(2*
b*x + 2*a) - b^2)
```

3.434.9 Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int x\coth^2(a+bx)\,dx=\frac{\frac{x^2\sinh(a+b\,x)}{2}-\frac{x\cosh(a+b\,x)}{b}}{\sinh(a+b\,x)}+\frac{\ln(\sinh(a+b\,x))}{b^2}$$

input
```
int((x*cosh(a + b*x)^2)/sinh(a + b*x)^2,x)
```

output
```
((x^2*sinh(a + b*x))/2 - (x*cosh(a + b*x))/b)/sinh(a + b*x) + log(sinh(a +
 b*x))/b^2
```

3.435 $\int \coth^2(a+bx)\,dx$

3.435.1 Optimal result

Integrand size = 8, antiderivative size = 13

$$\int \coth^2(a+bx)\,dx = x - \frac{\coth(a+bx)}{b}$$

output
```
x-coth(b*x+a)/b
```

3.435.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \coth^2(a+bx)\,dx = -\frac{\coth(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(a+bx)\right)}{b}$$

input
```
Integrate[Coth[a + b*x]^2,x]
```

output
```
-((Coth[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a + b*x]^2])/b)
```

3.435.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx-\frac{\coth(a+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$x-\frac{\coth(a+bx)}{b}$$

input
```
Int[Coth[a + b*x]^2,x]
```

output
```
x - Coth[a + b*x]/b
```

3.435.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.435.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{bx+a-\coth(bx+a)}{b}$	18
default	$\frac{bx+a-\coth(bx+a)}{b}$	18
risch	$x-\frac{2}{b\left(\mathrm{e}^{2bx+2a}-1\right)}$	21
parallelrisch	$\frac{-1+\tanh(bx+a)xb}{b\tanh(bx+a)}$	24

input
```
int(cosh(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(b*x+a-coth(b*x+a))
```

3.435.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.54

$$\int \coth^2(a+bx)\,dx=\frac{(bx+1)\sinh(bx+a)-\cosh(bx+a)}{b\sinh(bx+a)}$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
((b*x + 1)*sinh(b*x + a) - cosh(b*x + a))/(b*sinh(b*x + a))
```

3.435.6 Sympy [F]

$$\int \coth^2(a+bx)\,dx = \int \cosh^2(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

output
```
Integral(cosh(a + b*x)**2*csch(a + b*x)**2, x)
```

3.435.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \coth^2(a+bx)\,dx = x + \frac{a}{b} + \frac{2}{b(e^{(-2\,bx-2\,a)}-1)}$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
x + a/b + 2/(b*(e^(-2*b*x - 2*a) - 1))
```

3.435.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \coth^2(a+bx)\,dx = \frac{bx+a-\frac{2}{e^{(2\,bx+2\,a)}-1}}{b}$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
(b*x + a - 2/(e^(2*b*x + 2*a) - 1))/b
```

3.435.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \coth^2(a+bx)\,dx = x - \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(cosh(a + b*x)^2/sinh(a + b*x)^2,x)
```

output
```
x - 2/(b*(exp(2*a + 2*b*x) - 1))
```

3.436 $\int \frac{\coth^2(a+bx)}{x}\,dx$

3.436.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \text{Int}\left(\frac{\coth^2(a+bx)}{x},x\right)$$

output
```
Unintegrable(coth(b*x+a)^2/x,x)
```

3.436.2 Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \int \frac{\coth^2(a+bx)}{x}\,dx$$

input
```
Integrate[Coth[a + b*x]^2/x,x]
```

output
```
Integrate[Coth[a + b*x]^2/x, x]
```

3.436.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(a+bx)}{x} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\tan \left(ia+ibx+\frac{\pi}{2}\right)^2}{x}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\tan \left(\frac{1}{2}(2ia+\pi)+ibx\right)^2}{x}dx$$
$$\Big\downarrow \text{4222}$$
$$\int -\frac{\tan^2\left(\frac{1}{2}(-\pi-2ia)-ibx\right)}{x}dx$$

input
```
Int[Coth[a + b*x]^2/x,x]
```

output
```
$Aborted
```

3.436.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.436.4 Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cosh\left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right)^{2}}{x}dx$$

input
```
int(cosh(b*x+a)^2*csch(b*x+a)^2/x,x)
```

output
```
int(cosh(b*x+a)^2*csch(b*x+a)^2/x,x)
```

3.436.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right)^{2}}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^2*csch(b*x + a)^2/x, x)
```

3.436.6 Sympy [N/A]

Not integrable

Time = 21.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh^2(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)**2*csch(b*x+a)**2/x,x)
```

output
```
Integral(cosh(a + b*x)**2*csch(a + b*x)**2/x, x)
```

3.436.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 5.75

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^2\operatorname{csch}(bx+a)^2}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
-2/(b*x*e^(2*b*x + 2*a) - b*x) + integrate(1/(b*x^2*e^(b*x + a) + b*x^2),
x) - integrate(1/(b*x^2*e^(b*x + a) - b*x^2), x) + log(x)
```

3.436.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^2\operatorname{csch}(bx+a)^2}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2*csch(b*x + a)^2/x, x)
```

3.436.9 Mupad [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(a+bx)}{x}\,dx=\int \frac{\cosh(a+b\,x)^2}{x\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(cosh(a + b*x)^2/(x*sinh(a + b*x)^2),x)
```

output
```
int(cosh(a + b*x)^2/(x*sinh(a + b*x)^2), x)
```

3.437 $\int \frac{\coth^2(a+bx)}{x^2} dx$

3.437.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\coth^2(a+bx)}{x^2} dx = \text{Int}\left(\frac{\coth^2(a+bx)}{x^2}, x\right)$$

output
```
Unintegrable(coth(b*x+a)^2/x^2,x)
```

3.437.2 Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(a+bx)}{x^2} dx = \int \frac{\coth^2(a+bx)}{x^2} dx$$

input
```
Integrate[Coth[a + b*x]^2/x^2,x]
```

output
```
Integrate[Coth[a + b*x]^2/x^2, x]
```

3.437.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{\tan\left(ia+ibx+\frac{\pi}{2}\right)^2}{x^2}dx$$
$$\Big\downarrow \; 25$$
$$-\int \frac{\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2}{x^2}dx$$
$$\Big\downarrow \; 4222$$
$$\int -\frac{\tan^2\left(\frac{1}{2}(-\pi-2ia)-ibx\right)}{x^2}dx$$

input
```
Int[Coth[a + b*x]^2/x^2,x]
```

output
```
$Aborted
```

3.437.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.437.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cosh \left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right)^{2}}{x^{2}}dx$$

input
```
int(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x)
```

output
```
int(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x)
```

3.437.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx=\int \frac{\cosh \left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right)^{2}}{x^{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^2*csch(b*x + a)^2/x^2, x)
```

3.437.6 Sympy [N/A]

Not integrable

Time = 25.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh^2(a+bx)\operatorname{csch}^2(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)**2*csch(b*x+a)**2/x**2,x)
```

output
```
Integral(cosh(a + b*x)**2*csch(a + b*x)**2/x**2, x)
```

3.437.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 7.58

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)^2\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-(b*x*e^(2*b*x + 2*a) - b*x + 2)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) + 2*integ
rate(1/(b*x^3*e^(b*x + a) + b*x^3), x) - 2*integrate(1/(b*x^3*e^(b*x + a)
- b*x^3), x)
```

3.437.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)^2\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2*csch(b*x + a)^2/x^2, x)
```

3.437.9 Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^2}{x^2\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)^2),x)
```

output
```
int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)^2), x)
```

3.438 $\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx$

3.438.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b} - \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b} + \text{Int}(x^m\coth(a+bx)\text{csch}(a+bx),x)$$

output
```
CannotIntegrate(x^m*coth(b*x+a)*csch(b*x+a),x)+1/2*exp(a)*x^m*GAMMA(1+m,-b
*x)/b/((-b*x)^m)-1/2*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)
```

3.438.2 Mathematica [N/A]

Not integrable

Time = 26.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^m \cosh(a+bx)\coth^2(a+bx)\,dx$$

input
```
Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x]^2, x]
```

3.438.3 Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5973, 3042, 3788, 26, 2612, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{5973}$$

$$\int x^m \cosh(a+bx)dx+\int x^m \coth(a+bx)\text{csch}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^m \coth(a+bx)\text{csch}(a+bx)dx+\int x^m \sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$-\frac{1}{2}i\int ie^{-a-bx}x^mdx+\frac{1}{2}i\int -ie^{a+bx}x^mdx+\int x^m \coth(a+bx)\text{csch}(a+bx)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-a-bx}x^mdx+\frac{1}{2}\int e^{a+bx}x^mdx+\int x^m \coth(a+bx)\text{csch}(a+bx)dx$$

$$\Big\downarrow \text{2612}$$

$$\int x^m \coth(a+bx)\text{csch}(a+bx)dx+\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}$$

$$\Big\downarrow \text{7299}$$

$$\int x^m \coth(a+bx)\text{csch}(a+bx)dx+\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}$$

input
```
Int[x^m*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
$Aborted
```

3.438.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.438.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh (bx+a)^3 \operatorname{csch} (bx+a)^2 \, dx$$

input
```
int(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

output
```
int(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

3.438.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^m \cosh{(bx+a)}^3 \operatorname{csch}{(bx+a)}^2\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

3.438.6 Sympy [F(-1)]

Timed out.

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**m*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.438.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^m \cosh{(bx+a)}^3 \operatorname{csch}{(bx+a)}^2\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

3.438.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)^2\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

3.438.9 Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int((x^m*cosh(a + b*x)^3)/sinh(a + b*x)^2,x)
```

output
```
int((x^m*cosh(a + b*x)^3)/sinh(a + b*x)^2, x)
```

3.439 $\int x^3 \cosh(a+bx) \coth^2(a+bx) \, dx$

3.439.1 Optimal result

Integrand size = 18, antiderivative size = 143

$$\int x^3 \cosh(a+bx) \coth^2(a+bx) \, dx = -\frac{6x^2 \operatorname{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} - \frac{x^3 \operatorname{csch}(a+bx)}{b} - \frac{6x \operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{6x \operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{6 \operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^4} - \frac{6 \operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^4} + \frac{6x\sinh(a+bx)}{b^3} + \frac{x^3\sinh(a+bx)}{b}$$

output
```
-6*x^2*arctanh(exp(b*x+a))/b^2-6*cosh(b*x+a)/b^4-3*x^2*cosh(b*x+a)/b^2-x^3
*csch(b*x+a)/b-6*x*polylog(2,-exp(b*x+a))/b^3+6*x*polylog(2,exp(b*x+a))/b^
3+6*polylog(3,-exp(b*x+a))/b^4-6*polylog(3,exp(b*x+a))/b^4+6*x*sinh(b*x+a)
/b^3+x^3*sinh(b*x+a)/b
```

3.439.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.53

$$\int x^3 \cosh(a+bx)\coth^2(a+bx)\,dx$$
$$= \frac{\operatorname{csch}\left(\frac{1}{2}(a+bx)\right)\operatorname{sech}\left(\frac{1}{2}(a+bx)\right)\left(-6bx-3b^3x^3+6bx\cosh(2(a+bx))+b^3x^3\cosh(2(a+bx))+6b^2x^2\log\right.}{}$$

input
```
Integrate[x^3*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
(Csch[(a + b*x)/2]*Sech[(a + b*x)/2]*(-6*b*x - 3*b^3*x^3 + 6*b*x*Cosh[2*(a
 + b*x)] + b^3*x^3*Cosh[2*(a + b*x)] + 6*b^2*x^2*Log[1 - E^(a + b*x)]*Sinh
[a + b*x] - 6*b^2*x^2*Log[1 + E^(a + b*x)]*Sinh[a + b*x] - 12*b*x*PolyLog[
2, -E^(a + b*x)]*Sinh[a + b*x] + 12*b*x*PolyLog[2, E^(a + b*x)]*Sinh[a + b
*x] + 12*PolyLog[3, -E^(a + b*x)]*Sinh[a + b*x] - 12*PolyLog[3, E^(a + b*x
)]*Sinh[a + b*x] - 6*Sinh[2*(a + b*x)] - 3*b^2*x^2*Sinh[2*(a + b*x)]))/(4*
b^4)
```

3.439.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.29, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$, Rules used = {5973, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 5942, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cosh(a+bx)\coth^2(a+bx)\,dx$$
$$\Big\downarrow \text{5973}$$
$$\int x^3\cosh(a+bx)dx+\int x^3\coth(a+bx)\text{csch}(a+bx)dx$$
$$\Big\downarrow \text{3042}$$
$$\int x^3\coth(a+bx)\text{csch}(a+bx)dx+\int x^3\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)dx - \frac{3i\int -ix^2\sinh(a+bx)dx}{b} + \frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)dx - \frac{3\int x^2\sinh(a+bx)dx}{b} + \frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)dx - \frac{3\int -ix^2\sin(ia+ibx)dx}{b} + \frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)dx + \frac{3i\int x^2\sin(ia+ibx)dx}{b} + \frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)dx + \frac{3i\left(\frac{ix^2\cosh(a+bx)}{b} - \frac{2i\int x\cosh(a+bx)dx}{b}\right)}{b} + \frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)dx + \frac{3i\left(\frac{ix^2\cosh(a+bx)}{b} - \frac{2i\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b} + \frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)dx + \frac{3i\left(\frac{ix^2\cosh(a+bx)}{b} - \frac{2i\left(\frac{x\sinh(a+bx)}{b} - \frac{i\int -i\sinh(a+bx)dx}{b}\right)}{b}\right)}{b} + \frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)dx + \frac{3i\left(\frac{ix^2\cosh(a+bx)}{b} - \frac{2i\left(\frac{x\sinh(a+bx)}{b} - \frac{\int \sinh(a+bx)dx}{b}\right)}{b}\right)}{b} + \frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)dx + \frac{3i\left(\frac{ix^2\cosh(a+bx)}{b} - \frac{2i\left(\frac{x\sinh(a+bx)}{b} - \frac{\int -i\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b} + \frac{x^3\sinh(a+bx)}{b}$$

3.439. $\int x^3\cosh(a+bx)\coth^2(a+bx)\,dx$

$$\Big\downarrow \text{26}$$

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)dx+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}+\frac{i\int \sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)dx+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{5942}$$

$$\frac{3\int x^2\text{csch}(a+bx)dx}{b}+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}-\frac{x^3\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int ix^2\csc(ia+ibx)dx}{b}+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}-\frac{x^3\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{3i\int x^2\csc(ia+ibx)dx}{b}+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}-\frac{x^3\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{4670}$$

$$\frac{3i\left(\frac{2i\int x\log\left(1-e^{a+bx}\right)dx}{b}-\frac{2i\int x\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2ix^2\text{arctanh}\left(e^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}-\frac{x^3\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{3011}$$

3.439. $\int x^3\cosh(a+bx)\coth^2(a+bx)\,dx$

$$\frac{3i\left(-\frac{2i\left(\frac{\int \text{PolyLog}\left(2,-e^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{\int \text{PolyLog}\left(2,e^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b}\right)}{b}+\frac{2ix^2\text{arctanh}(e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}-\frac{x^3\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \; 2720$$

$$\frac{3i\left(-\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}\left(2,-e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}\left(2,e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b}\right)}{b}+\frac{2ix^2}{}\right.}{b}$$

$$\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}-\frac{x^3\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \; 7143$$

$$\frac{3i\left(\frac{2ix^2\text{arctanh}(e^{a+bx})}{b}-\frac{2i\left(\frac{\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{2i\left(\frac{\text{PolyLog}\left(3,e^{a+bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b}\right)}{b}\right)}{b}+\frac{3i\left(\frac{ix^2\cosh(a+bx)}{b}-\frac{2i\left(\frac{x\sinh(a+bx)}{b}-\frac{\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}-\frac{x^3\text{csch}(a+bx)}{b}$$

input
```
Int[x^3*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
-((x^3*Csch[a + b*x])/b) + ((3*I)*(((2*I)*x^2*ArcTanh[E^(a + b*x)])/b - ((
2*I)*(-((x*PolyLog[2, -E^(a + b*x)])/b) + PolyLog[3, -E^(a + b*x)]/b^2))/b
 + ((2*I)*(-((x*PolyLog[2, E^(a + b*x)])/b) + PolyLog[3, E^(a + b*x)]/b^2)
)/b))/b + (x^3*Sinh[a + b*x])/b + ((3*I)*((I*x^2*Cosh[a + b*x])/b - ((2*I)
*(-(Cosh[a + b*x]/b^2) + (x*Sinh[a + b*x])/b))/b))/b
```

3.439. $\int x^3 \cosh(a+bx)\coth^2(a+bx)\,dx$

3.439.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.439. $\int x^3 \cosh(a+bx) \coth^2(a+bx) \, dx$

rule 5942
```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.439.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.69

method	result
risch	$\frac{(x^3b^3-3x^2b^2+6bx-6)e^{bx+a}}{2b^4}-\frac{(x^3b^3+3x^2b^2+6bx+6)e^{-bx-a}}{2b^4}-\frac{2x^3e^{bx+a}}{b(e^{2bx+2a}-1)}-\frac{6a^2\operatorname{arctanh}(e^{bx+a})}{b^4}+\frac{3\ln(1-e^{bx+a})x^2}{b^2}-$

input
```
int(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^3*x^3-3*b^2*x^2+6*b*x-6)/b^4*exp(b*x+a)-1/2*(b^3*x^3+3*b^2*x^2+6*b*
x+6)/b^4*exp(-b*x-a)-2/b*x^3*exp(b*x+a)/(exp(2*b*x+2*a)-1)-6/b^4*a^2*arcta
nh(exp(b*x+a))+3/b^2*ln(1-exp(b*x+a))*x^2-3/b^4*ln(1-exp(b*x+a))*a^2+6*x*p
olylog(2,exp(b*x+a))/b^3-6*polylog(3,exp(b*x+a))/b^4-3/b^2*ln(exp(b*x+a)+1
)*x^2+3/b^4*ln(exp(b*x+a)+1)*a^2-6*x*polylog(2,-exp(b*x+a))/b^3+6*polylog(
3,-exp(b*x+a))/b^4
```

3.439.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(136) = 272$.

Time = 0.28 (sec) , antiderivative size = 1055, normalized size of antiderivative = 7.38

$$\int x^3 \cosh(a + bx) \coth^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^4 + 4*(b^3*
x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 - 3*
b^2*x^2 + 6*b*x - 6)*sinh(b*x + a)^4 + 3*b^2*x^2 - 6*(b^3*x^3 + 2*b*x)*cos
h(b*x + a)^2 - 6*(b^3*x^3 - (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a
)^2 + 2*b*x)*sinh(b*x + a)^2 + 6*b*x + 12*(b*x*cosh(b*x + a)^3 + 3*b*x*cos
h(b*x + a)*sinh(b*x + a)^2 + b*x*sinh(b*x + a)^3 - b*x*cosh(b*x + a) + (3*
b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x +
 a)) - 12*(b*x*cosh(b*x + a)^3 + 3*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + b*x
*sinh(b*x + a)^3 - b*x*cosh(b*x + a) + (3*b*x*cosh(b*x + a)^2 - b*x)*sinh(
b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 6*(b^2*x^2*cosh(b*x + a)
^3 + 3*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*x^2*sinh(b*x + a)^3 - b
^2*x^2*cosh(b*x + a) + (3*b^2*x^2*cosh(b*x + a)^2 - b^2*x^2)*sinh(b*x + a)
)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 6*(a^2*cosh(b*x + a)^3 + 3*a^2*
cosh(b*x + a)*sinh(b*x + a)^2 + a^2*sinh(b*x + a)^3 - a^2*cosh(b*x + a) +
(3*a^2*cosh(b*x + a)^2 - a^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x
+ a) - 1) + 6*((b^2*x^2 - a^2)*cosh(b*x + a)^3 + 3*(b^2*x^2 - a^2)*cosh(b*
x + a)*sinh(b*x + a)^2 + (b^2*x^2 - a^2)*sinh(b*x + a)^3 - (b^2*x^2 - a^2)
*cosh(b*x + a) - (b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(
b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 12*(cosh(b*x + a)^3 +
3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 ...
```

3.439.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \cosh(a + bx) \coth^2(a + bx) \, dx = \text{Timed out}$$

input
```
integrate(x**3*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.439.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.51

$$\int x^3 \cosh(a+bx)\coth^2(a+bx)\,dx$$
$$= \frac{\left(b^3x^3e^{(4a)} - 3b^2x^2e^{(4a)} + 6bxe^{(4a)} - 6e^{(4a)}\right)e^{(3bx)} - 6\left(b^3x^3e^{(2a)} + 2bxe^{(2a)}\right)e^{(bx)} + \left(b^3x^3 + 3b^2x^2 + 6bx\right.}{2\left(b^4e^{(2bx+3a)} - b^4e^a\right)}$$
$$- \frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right) + 2bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)\right)}{b^4}$$
$$+ \frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right) + 2bx\mathrm{Li}_2\left(e^{(bx+a)}\right) - 2\,\mathrm{Li}_3\left(e^{(bx+a)}\right)\right)}{b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b^3*x^3*e^(4*a) - 3*b^2*x^2*e^(4*a) + 6*b*x*e^(4*a) - 6*e^(4*a))*e^(
3*b*x) - 6*(b^3*x^3*e^(2*a) + 2*b*x*e^(2*a))*e^(b*x) + (b^3*x^3 + 3*b^2*x^
2 + 6*b*x + 6)*e^(-b*x))/(b^4*e^(2*b*x + 3*a) - b^4*e^a) - 3*(b^2*x^2*log(
e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))
/b^4 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*pol
ylog(3, e^(b*x + a)))/b^4
```

3.439.8 Giac [F]

$$\int x^3 \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^3 \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)^2\,dx$$

input
```
integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

3.439.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \cosh(a+bx) \coth^2(a+bx)\,dx = \int \frac{x^3 \cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int((x^3*cosh(a + b*x)^3)/sinh(a + b*x)^2,x)
```

output
```
int((x^3*cosh(a + b*x)^3)/sinh(a + b*x)^2, x)
```

3.440 $\int x^2 \cosh(a+bx) \coth^2(a+bx) \, dx$

3.440.1 Optimal result

Integrand size = 18, antiderivative size = 95

$$\int x^2 \cosh(a+bx) \coth^2(a+bx) \, dx = -\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{2x\cosh(a+bx)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{2\sinh(a+bx)}{b^3} + \frac{x^2\sinh(a+bx)}{b}$$

output
```
-4*x*arctanh(exp(b*x+a))/b^2-2*x*cosh(b*x+a)/b^2-x^2*csch(b*x+a)/b-2*polyl
og(2,-exp(b*x+a))/b^3+2*polylog(2,exp(b*x+a))/b^3+2*sinh(b*x+a)/b^3+x^2*si
nh(b*x+a)/b
```

3.440.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.60

$$\int x^2 \cosh(a+bx) \coth^2(a+bx) \, dx$$
$$= \frac{\text{csch}\left(\frac{1}{2}(a+bx)\right)\text{sech}\left(\frac{1}{2}(a+bx)\right)\left(-2-3b^2x^2+2\cosh(2(a+bx))+b^2x^2\cosh(2(a+bx))+4bx\log\left(1-\right.\right.}{}$$

input
```
Integrate[x^2*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
(Csch[(a + b*x)/2]*Sech[(a + b*x)/2]*(-2 - 3*b^2*x^2 + 2*Cosh[2*(a + b*x)]
 + b^2*x^2*Cosh[2*(a + b*x)] + 4*b*x*Log[1 - E^(a + b*x)]*Sinh[a + b*x] -
4*b*x*Log[1 + E^(a + b*x)]*Sinh[a + b*x] - 4*PolyLog[2, -E^(a + b*x)]*Sinh
[a + b*x] + 4*PolyLog[2, E^(a + b*x)]*Sinh[a + b*x] - 2*b*x*Sinh[2*(a + b*
x)]))/(4*b^3)
```

3.440.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.27, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5973, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 5942, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cosh(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{5973}$$

$$\int x^2 \cosh(a+bx)dx + \int x^2 \coth(a+bx)\text{csch}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)dx + \int x^2 \sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)dx - \frac{2i\int -ix\sinh(a+bx)dx}{b} + \frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)dx - \frac{2\int x\sinh(a+bx)dx}{b} + \frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)dx - \frac{2\int -ix\sin(ia+ibx)dx}{b} + \frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)dx+\frac{2i\int x\sin(ia+ibx)dx}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \textbf{3777}$$

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)dx+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int \cosh(a+bx)dx}{b}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)dx+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \textbf{3117}$$

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)dx+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}$$

$$\Big\downarrow \textbf{5942}$$

$$\frac{2\int x\text{csch}(a+bx)dx}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}-\frac{x^2\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2\int ix\csc(ia+ibx)dx}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}-\frac{x^2\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{2i\int x\csc(ia+ibx)dx}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}-\frac{x^2\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \textbf{4670}$$

$$\frac{2i\left(\frac{i\int \log(1-e^{a+bx})dx}{b}-\frac{i\int \log(1+e^{a+bx})dx}{b}+\frac{2ix\text{arctanh}(e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}-\frac{x^2\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \textbf{2715}$$

$$\frac{2i\left(\frac{i\int e^{-a-bx}\log(1-e^{a+bx})de^{a+bx}}{b^2}-\frac{i\int e^{-a-bx}\log(1+e^{a+bx})de^{a+bx}}{b^2}+\frac{2ix\text{arctanh}(e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}-\frac{x^2\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \textbf{2838}$$

3.440. $\int x^2\cosh(a+bx)\coth^2(a+bx)\,dx$

$$\frac{2i\left(\frac{2ix\text{arctanh}(e^{a+bx})}{b}+\frac{i\,\text{PolyLog}(2,-e^{a+bx})}{b^2}-\frac{i\,\text{PolyLog}(2,e^{a+bx})}{b^2}\right)}{b}+\frac{2i\left(\frac{ix\cosh(a+bx)}{b}-\frac{i\sinh(a+bx)}{b^2}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}-\frac{x^2\text{csch}(a+bx)}{b}$$

input
```
Int[x^2*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
-((x^2*Csch[a + b*x])/b) + ((2*I)*(((2*I)*x*ArcTanh[E^(a + b*x)])/b + (I*P
olyLog[2, -E^(a + b*x)])/b^2 - (I*PolyLog[2, E^(a + b*x)])/b^2))/b + (x^2*
Sinh[a + b*x])/b + ((2*I)*((I*x*Cosh[a + b*x])/b - (I*Sinh[a + b*x])/b^2))
/b
```

3.440.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5942
```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.440.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.95

method	result
risch	$\frac{(x^2b^2-2bx+2)\mathrm{e}^{bx+a}}{2b^3}-\frac{(x^2b^2+2bx+2)\mathrm{e}^{-bx-a}}{2b^3}-\frac{2x^2\mathrm{e}^{bx+a}}{b(\mathrm{e}^{2bx+2a}-1)}+\frac{2\ln(1-\mathrm{e}^{bx+a})x}{b^2}+\frac{2\ln(1-\mathrm{e}^{bx+a})a}{b^3}+\frac{2\,\mathrm{polylog}(2,\mathrm{e}^{bx+a})}{b^3}$

input
```
int(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)-1/2*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)
-2/b*x^2*exp(b*x+a)/(exp(2*b*x+2*a)-1)+2/b^2*ln(1-exp(b*x+a))*x+2/b^3*ln(1
-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3-2/b^2*ln(exp(b*x+a)+1)*x-2/b^3*
ln(exp(b*x+a)+1)*a-2*polylog(2,-exp(b*x+a))/b^3+4/b^3*a*arctanh(exp(b*x+a)
)
```

3.440.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(90) = 180$.

Time = 0.27 (sec) , antiderivative size = 731, normalized size of antiderivative = 7.69

$$\int x^2 \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{(b^2x^2-2\,bx+2)\cosh\left(bx+a\right)^4+4\,(b^2x^2-2\,bx+2)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+(b^2x^2-2\,bx+2)\sinh}{}$$

input
```
integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*b*x + 2)*cosh(
b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^4 + b^2*x^2
 - 2*(3*b^2*x^2 + 2)*cosh(b*x + a)^2 - 2*(3*b^2*x^2 - 3*(b^2*x^2 - 2*b*x +
 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 2*b*x + 4*(cosh(b*x + a)^3 + 3*
cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*
sinh(b*x + a) - cosh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 4*(c
osh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*co
sh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*dilog(-cosh(b*x + a) - s
inh(b*x + a)) - 4*(b*x*cosh(b*x + a)^3 + 3*b*x*cosh(b*x + a)*sinh(b*x + a)
^2 + b*x*sinh(b*x + a)^3 - b*x*cosh(b*x + a) + (3*b*x*cosh(b*x + a)^2 - b*
x)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 4*(a*cosh(b*x +
 a)^3 + 3*a*cosh(b*x + a)*sinh(b*x + a)^2 + a*sinh(b*x + a)^3 - a*cosh(b*x
 + a) + (3*a*cosh(b*x + a)^2 - a)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(
b*x + a) - 1) + 4*((b*x + a)*cosh(b*x + a)^3 + 3*(b*x + a)*cosh(b*x + a)*s
inh(b*x + a)^2 + (b*x + a)*sinh(b*x + a)^3 - (b*x + a)*cosh(b*x + a) + (3*
(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(b*x + a))*log(-cosh(b*x + a) - s
inh(b*x + a) + 1) + 4*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^3 - (3*b^2*x^2
+ 2)*cosh(b*x + a))*sinh(b*x + a) + 2)/(b^3*cosh(b*x + a)^3 + 3*b^3*cosh(b
*x + a)*sinh(b*x + a)^2 + b^3*sinh(b*x + a)^3 - b^3*cosh(b*x + a) + (3*b^3
*cosh(b*x + a)^2 - b^3)*sinh(b*x + a))
```

3.440.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \cosh(a+bx)\coth^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**2*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.440.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.65

$$\int x^2 \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{(b^2x^2e^{(4a)} - 2bxe^{(4a)} + 2e^{(4a)})e^{(3bx)} - 2(3b^2x^2e^{(2a)} + 2e^{(2a)})e^{(bx)} + (b^2x^2 + 2bx + 2)e^{(-bx)}}{2(b^3e^{(2bx+3a)} - b^3e^a)} - \frac{2(bx\log(e^{(bx+a)} + 1) + \text{Li}_2(-e^{(bx+a)}))}{b^3} + \frac{2(bx\log(-e^{(bx+a)} + 1) + \text{Li}_2(e^{(bx+a)}))}{b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b^2*x^2*e^(4*a) - 2*b*x*e^(4*a) + 2*e^(4*a))*e^(3*b*x) - 2*(3*b^2*x^
2*e^(2*a) + 2*e^(2*a))*e^(b*x) + (b^2*x^2 + 2*b*x + 2)*e^(-b*x))/(b^3*e^(2
*b*x + 3*a) - b^3*e^a) - 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a))
)/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

3.440.8 Giac [F]

$$\int x^2 \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^2 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh(a+bx)\coth^2(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int((x^2*cosh(a + b*x)^3)/sinh(a + b*x)^2,x)
```

output
```
int((x^2*cosh(a + b*x)^3)/sinh(a + b*x)^2, x)
```

3.441 $\int x \cosh(a+bx) \coth^2(a+bx)\, dx$

3.441.1 Optimal result

Integrand size = 16, antiderivative size = 47

$$\int x \cosh(a+bx) \coth^2(a+bx)\, dx = -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b^2} - \frac{\cosh(a+bx)}{b^2} - \frac{x\operatorname{csch}(a+bx)}{b} + \frac{x \sinh(a+bx)}{b}$$

output
```
-arctanh(cosh(b*x+a))/b^2-cosh(b*x+a)/b^2-x*csch(b*x+a)/b+x*sinh(b*x+a)/b
```

3.441.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int x \cosh(a+bx) \coth^2(a+bx)\, dx = \frac{-2\cosh(a+bx) - bx\coth\left(\frac{1}{2}(a+bx)\right) - 2\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right) + 2\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right) + 2bx\sinh(a}{2b^2}$$

input
```
Integrate[x*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
(-2*Cosh[a + b*x] - b*x*Coth[(a + b*x)/2] - 2*Log[Cosh[(a + b*x)/2]] + 2*L
og[Sinh[(a + b*x)/2]] + 2*b*x*Sinh[a + b*x] + b*x*Tanh[(a + b*x)/2])/(2*b^
2)
```

3.441.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5973, 3042, 3777, 26, 3042, 26, 3118, 5942, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{5973}$$

$$\int x\cosh(a+bx)dx+\int x\coth(a+bx)\text{csch}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\coth(a+bx)\text{csch}(a+bx)dx+\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$-\frac{i\int -i\sinh(a+bx)dx}{b}+\int x\coth(a+bx)\text{csch}(a+bx)dx+\frac{x\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\int \sinh(a+bx)dx}{b}+\int x\coth(a+bx)\text{csch}(a+bx)dx+\frac{x\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i\sin(ia+ibx)dx}{b}+\int x\coth(a+bx)\text{csch}(a+bx)dx+\frac{x\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \sin(ia+ibx)dx}{b}+\int x\coth(a+bx)\text{csch}(a+bx)dx+\frac{x\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\int x\coth(a+bx)\text{csch}(a+bx)dx-\frac{\cosh(a+bx)}{b^2}+\frac{x\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{5942}$$

$$\frac{\int \text{csch}(a+bx)dx}{b}-\frac{\cosh(a+bx)}{b^2}+\frac{x\sinh(a+bx)}{b}-\frac{x\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int i\csc(ia+ibx)dx}{b}-\frac{\cosh(a+bx)}{b^2}+\frac{x\sinh(a+bx)}{b}-\frac{x\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \csc(ia+ibx)dx}{b}-\frac{\cosh(a+bx)}{b^2}+\frac{x\sinh(a+bx)}{b}-\frac{x\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\text{arctanh}(\cosh(a+bx))}{b^2}-\frac{\cosh(a+bx)}{b^2}+\frac{x\sinh(a+bx)}{b}-\frac{x\text{csch}(a+bx)}{b}$$

input
```
Int[x*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
-(ArcTanh[Cosh[a + b*x]]/b^2) - Cosh[a + b*x]/b^2 - (x*Csch[a + b*x])/b +
(x*Sinh[a + b*x])/b
```

3.441.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5942
```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.441.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.89

method	result	size
risch	$\frac{(bx-1)\,\mathrm{e}^{bx+a}}{2b^2}-\frac{(bx+1)\,\mathrm{e}^{-bx-a}}{2b^2}-\frac{2x\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^2}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b^2}$	89

input
```
int(x*cosh(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x-1)/b^2*exp(b*x+a)-1/2*(b*x+1)/b^2*exp(-b*x-a)-2/b*x*exp(b*x+a)/(e
xp(2*b*x+2*a)-1)+1/b^2*ln(exp(b*x+a)-1)-1/b^2*ln(exp(b*x+a)+1)
```

3.441.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(47) = 94.

Time = 0.26 (sec) , antiderivative size = 367, normalized size of antiderivative = 7.81

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx$$
$$=\frac{(bx-1)\cosh{(bx+a)}^4+4\,(bx-1)\cosh{(bx+a)}\sinh{(bx+a)}^3+(bx-1)\sinh{(bx+a)}^4-6\,bx\cosh{(bx}}{}$$

input
```
integrate(x*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*((b*x - 1)*cosh(b*x + a)^4 + 4*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a)^3
 + (b*x - 1)*sinh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 + 6*((b*x - 1)*cosh(b
*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x - 2*(cosh(b*x + a)^3 + 3*cosh(b*x +
 a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x +
 a) - cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*(cosh(b*x
+ a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x +
 a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a
) - 1) + 4*((b*x - 1)*cosh(b*x + a)^3 - 3*b*x*cosh(b*x + a))*sinh(b*x + a)
 + 1)/(b^2*cosh(b*x + a)^3 + 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sin
h(b*x + a)^3 - b^2*cosh(b*x + a) + (3*b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x
+ a))
```

3.441.6 Sympy [F]

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx = \int x\cosh^3(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate(x*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

output
```
Integral(x*cosh(a + b*x)**3*csch(a + b*x)**2, x)
```

3.441.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.32

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{6\,bxe^{(bx+2\,a)} - \left(bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(3\,bx)} - (bx+1)e^{(-bx)}}{2\,\left(b^2e^{(2\,bx+3\,a)} - b^2e^a\right)} - \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2} + \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}$$

input
```
integrate(x*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(6*b*x*e^(b*x + 2*a) - (b*x*e^(4*a) - e^(4*a))*e^(3*b*x) - (b*x + 1)*
e^(-b*x))/(b^2*e^(2*b*x + 3*a) - b^2*e^a) - log((e^(b*x + a) + 1)*e^(-a))/
b^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

3.441.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(47) = 94.

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.06

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx$$
$$=\frac{bxe^{(4\,bx+4\,a)}-6\,bxe^{(2\,bx+2\,a)}+bx-2\,e^{(3\,bx+3\,a)}\log\left(e^{(bx+a)}+1\right)+2\,e^{(bx+a)}\log\left(e^{(bx+a)}+1\right)+2\,e^{(3\,bx+3\,a)}\,\mathrm{lo}}{2\left(b^2e^{(3\,bx+3\,a)}-b^2e^{(bx+a)}\right)}$$

input
```
integrate(x*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*x*e^(4*b*x + 4*a) - 6*b*x*e^(2*b*x + 2*a) + b*x - 2*e^(3*b*x + 3*a)
*log(e^(b*x + a) + 1) + 2*e^(b*x + a)*log(e^(b*x + a) + 1) + 2*e^(3*b*x +
3*a)*log(e^(b*x + a) - 1) - 2*e^(b*x + a)*log(e^(b*x + a) - 1) - e^(4*b*x
+ 4*a) + 1)/(b^2*e^(3*b*x + 3*a) - b^2*e^(b*x + a))
```

3.441.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.02

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx=\mathrm{e}^{a+b\,x}\left(\frac{x}{2\,b}-\frac{1}{2\,b^2}\right)-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^4}}{b^2}\right)}{\sqrt{-b^4}}$$
$$-\mathrm{e}^{-a-b\,x}\left(\frac{x}{2\,b}+\frac{1}{2\,b^2}\right)-\frac{2\,x\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int((x*cosh(a + b*x)^3)/sinh(a + b*x)^2,x)
```

output
```
exp(a + b*x)*(x/(2*b) - 1/(2*b^2)) - (2*atan((exp(b*x)*exp(a)*(-b^4)^(1/2)
)/b^2))/(-b^4)^(1/2) - exp(- a - b*x)*(x/(2*b) + 1/(2*b^2)) - (2*x*exp(a +
 b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.442 $\int \cosh(a+bx)\coth^2(a+bx)\,dx$

3.442.1 Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}+\frac{\sinh(a+bx)}{b}$$

output
```
-csch(b*x+a)/b+sinh(b*x+a)/b
```

3.442.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}+\frac{\sinh(a+bx)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
-(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

3.442.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(ia+ibx+\frac{\pi}{2}\right)\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{i\int -\text{csch}^2(a+bx)\left(\sinh^2(a+bx)+1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{i\int \left(-\text{csch}^2(a+bx)-1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(i\sinh(a+bx)-i\text{csch}(a+bx))}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
((-I)*((-I)*Csch[a + b*x] + I*Sinh[a + b*x]))/b
```

3.442.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.442.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	33
default	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	33
risch	$\frac{e^{3bx+3a}-6\,e^{bx+a}+e^{-bx-a}}{2b\left(e^{2bx+2a}-1\right)}$	46

input
```
int(cosh(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/sinh(b*x+a)*cosh(b*x+a)^2-2/sinh(b*x+a))
```

3.442.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{\cosh(bx+a)^2+\sinh(bx+a)^2-3}{2\,b\sinh(bx+a)}$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 - 3)/(b*sinh(b*x + a))
```

3.442.6 Sympy [F]

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \int \cosh^3(a+bx)\,\mathrm{csch}^2(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

output
```
Integral(cosh(a + b*x)**3*csch(a + b*x)**2, x)
```

3.442.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(22) = 44.

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{e^{(-bx-a)}}{2\,b} - \frac{5\,e^{(-2\,bx-2\,a)}-1}{2\,b(e^{(-bx-a)}-e^{(-3\,bx-3\,a)})}$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*e^(-b*x - a)/b - 1/2*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-b*x - a) - e^(-
3*b*x - 3*a)))
```

3.442.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{\frac{4}{e^{(bx+a)}-e^{(-bx-a)}} - e^{(bx+a)} + e^{(-bx-a)}}{2\,b}$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(4/(e^(b*x + a) - e^(-b*x - a)) - e^(b*x + a) + e^(-b*x - a))/b
```

3.442.9 Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{\sinh(a+b\,x)^2-1}{b\sinh\left(a+b\,x\right)}$$

input
```
int(cosh(a + b*x)^3/sinh(a + b*x)^2,x)
```

output
```
(sinh(a + b*x)^2 - 1)/(b*sinh(a + b*x))
```

3.443 $\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$

3.443.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx) + \text{Int}\left(\frac{\coth(a+bx)\text{csch}(a+bx)}{x},x\right)$$

output
```
CannotIntegrate(coth(b*x+a)*csch(b*x+a)/x,x)+Chi(b*x)*cosh(a)+Shi(b*x)*sin
h(a)
```

3.443.2 Mathematica [N/A]

Not integrable

Time = 18.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$$

input
```
Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x,x]
```

output
```
Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x, x]
```

3.443.3 Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5973, 3042, 3784, 26, 3042, 26, 3779, 3782, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{5973}$$

$$\int \frac{\cosh(a+bx)}{x}dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}dx+\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$-i\sinh(a)\int \frac{i\sinh(bx)}{x}dx+\cosh(a)\int \frac{\cosh(bx)}{x}dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}dx$$

$$\Big\downarrow \text{26}$$

$$\sinh(a)\int \frac{\sinh(bx)}{x}dx+\cosh(a)\int \frac{\cosh(bx)}{x}dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\sinh(a)\int -\frac{i\sin(ibx)}{x}dx+\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}dx$$

$$\Big\downarrow \text{26}$$

$$-i\sinh(a)\int \frac{\sin(ibx)}{x}dx+\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}dx$$

$$\Big\downarrow \text{3779}$$

$$\cosh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}dx+\sinh(a)\text{Shi}(bx)$$

$$\Big\downarrow \text{3782}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}dx+\cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)$$

3.443. $\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$

$$\Big\downarrow 7299$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}dx+\cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)$$

input
```
Int[(Cosh[a + b*x]*Coth[a + b*x]^2)/x,x]
```

output
```
$Aborted
```

3.443.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.443. $\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.443.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(bx+a)^{3} \operatorname{csch}(bx+a)^{2}}{x} dx$$

input
```
int(cosh(b*x+a)^3*csch(b*x+a)^2/x,x)
```

output
```
int(cosh(b*x+a)^3*csch(b*x+a)^2/x,x)
```

3.443.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x} dx = \int \frac{\cosh(bx+a)^{3} \operatorname{csch}(bx+a)^{2}}{x} dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^3*csch(b*x + a)^2/x, x)
```

3.443.6 Sympy [N/A]

Not integrable

Time = 52.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x} dx = \int \frac{\cosh^{3}(a+bx) \operatorname{csch}^{2}(a+bx)}{x} dx$$

input
```
integrate(cosh(b*x+a)**3*csch(b*x+a)**2/x,x)
```

output
```
Integral(cosh(a + b*x)**3*csch(a + b*x)**2/x, x)
```

3.443.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 5.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^2}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
1/2*Ei(-b*x)*e^(-a) + 1/2*Ei(b*x)*e^a - 2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a)
 - b*x) - integrate(1/(b*x^2*e^(b*x + a) + b*x^2), x) - integrate(1/(b*x^2
*e^(b*x + a) - b*x^2), x)
```

3.443.8 Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^2}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3*csch(b*x + a)^2/x, x)
```

3.443.9 Mupad [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^3}{x\sinh(a+b\,x)^2}\,dx$$

input
```
int(cosh(a + b*x)^3/(x*sinh(a + b*x)^2),x)
```

output
```
int(cosh(a + b*x)^3/(x*sinh(a + b*x)^2), x)
```

3.444 $\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$

3.444.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx = -\frac{\cosh(a+bx)}{x} + b\mathrm{Chi}(bx)\sinh(a) + b\cosh(a)\mathrm{Shi}(bx) + \mathrm{Int}\left(\frac{\coth(a+bx)\mathrm{csch}(a+bx)}{x^2},x\right)$$

output
```
CannotIntegrate(coth(b*x+a)*csch(b*x+a)/x^2,x)-cosh(b*x+a)/x+b*cosh(a)*Shi
(b*x)+b*Chi(b*x)*sinh(a)
```

3.444.2 Mathematica [N/A]

Not integrable

Time = 14.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x^2,x]
```

output
```
Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x^2, x]
```

3.444.3 Rubi [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5973, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{ 5973}$$

$$\int \frac{\cosh(a+bx)}{x^2}dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx+\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{ 3778}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx+ib\int -\frac{i\sinh(a+bx)}{x}dx-\frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx+b\int \frac{\sinh(a+bx)}{x}dx-\frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx+b\int -\frac{i\sin(ia+ibx)}{x}dx-\frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx-ib\int \frac{\sin(ia+ibx)}{x}dx-\frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{ 3784}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx-ib\left(i\sinh(a)\int \frac{\cosh(bx)}{x}dx+\cosh(a)\int \frac{i\sinh(bx)}{x}dx\right)-\frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx - ib\left(i\sinh(a)\int \frac{\cosh(bx)}{x}dx + i\cosh(a)\int \frac{\sinh(bx)}{x}dx\right) - \frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx - ib\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx + i\cosh(a)\int -\frac{i\sin(ibx)}{x}dx\right) - \frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx - ib\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx + \cosh(a)\int \frac{\sin(ibx)}{x}dx\right) - \frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{3779}$$

$$-ib\left(i\sinh(a)\int \frac{\sin\left(ibx+\frac{\pi}{2}\right)}{x}dx + i\cosh(a)\text{Shi}(bx)\right) + \int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx - \frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{3782}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx - ib(i\sinh(a)\text{Chi}(bx) + i\cosh(a)\text{Shi}(bx)) - \frac{\cosh(a+bx)}{x}$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}dx - ib(i\sinh(a)\text{Chi}(bx) + i\cosh(a)\text{Shi}(bx)) - \frac{\cosh(a+bx)}{x}$$

input
```
Int[(Cosh[a + b*x]*Coth[a + b*x]^2)/x^2,x]
```

output
```
$Aborted
```

3.444. $\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$

3.444.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.444. $\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$

3.444.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh \left(bx+a\right)^{3} \operatorname{csch}\left(bx+a\right)^{2}}{x^{2}}dx$$

input
```
int(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x)
```

output
```
int(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x)
```

3.444.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh \left(bx+a\right)^{3} \operatorname{csch}\left(bx+a\right)^{2}}{x^{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^3*csch(b*x + a)^2/x^2, x)
```

3.444.6 Sympy [N/A]

Not integrable

Time = 70.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh^{3}\left(a+bx\right)\operatorname{csch}^{2}\left(a+bx\right)}{x^{2}}\,dx$$

input
```
integrate(cosh(b*x+a)**3*csch(b*x+a)**2/x**2,x)
```

output
```
Integral(cosh(a + b*x)**3*csch(a + b*x)**2/x**2, x)
```

3.444.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.67

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-1/2*b*e^(-a)*gamma(-1, b*x) + 1/2*b*e^a*gamma(-1, -b*x) - 2*e^(b*x + a)/(
b*x^2*e^(2*b*x + 2*a) - b*x^2) - 2*integrate(1/(b*x^3*e^(b*x + a) + b*x^3)
, x) - 2*integrate(1/(b*x^3*e^(b*x + a) - b*x^3), x)
```

3.444.8 Giac [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3*csch(b*x + a)^2/x^2, x)
```

3.444.9 Mupad [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^3}{x^2\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)^2),x)
```

output
```
int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)^2), x)
```

3.445 $\int x^m \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

3.445.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx = \text{Int}\left(x^m \coth(a+bx)\text{csch}^2(a+bx),x\right)$$

output
```
CannotIntegrate(x^m*coth(b*x+a)*csch(b*x+a)^2,x)
```

3.445.2 Mathematica [N/A]

Not integrable

Time = 35.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

input
```
Integrate[x^m*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
Integrate[x^m*Coth[a + b*x]*Csch[a + b*x]^2, x]
```

3.445.3 Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)dx$$

input
```
Int[x^m*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
$Aborted
```

3.445.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.445.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \cosh(bx+a)\,\text{csch}(bx+a)^3\,dx$$

input
```
int(x^m*cosh(b*x+a)*csch(b*x+a)^3,x)
```

output
```
int(x^m*cosh(b*x+a)*csch(b*x+a)^3,x)
```

3.445.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int x^m \cosh(bx+a)\,\text{csch}(bx+a)^3\,dx$$

input
```
integrate(x^m*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m*cosh(b*x + a)*csch(b*x + a)^3, x)
```

3.445.6 Sympy [F(-1)]

Timed out.

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**m*cosh(b*x+a)*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.445.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int x^m \cosh(bx+a)\,\text{csch}(bx+a)^3\,dx$$

input
```
integrate(x^m*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(x^m*cosh(b*x + a)*csch(b*x + a)^3, x)
```

3.445.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)^3\,dx$$

input
```
integrate(x^m*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)*csch(b*x + a)^3, x)
```

3.445.9 Mupad [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^m*cosh(a + b*x))/sinh(a + b*x)^3,x)
```

output
```
int((x^m*cosh(a + b*x))/sinh(a + b*x)^3, x)
```

3.446 $\int x^3 \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

3.446.1 Optimal result

Integrand size = 18, antiderivative size = 83

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{3x^2}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\text{csch}^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{3\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}$$

output
```
-3/2*x^2/b^2-3/2*x^2*coth(b*x+a)/b^2-1/2*x^3*csch(b*x+a)^2/b+3*x*ln(1-exp(
2*b*x+2*a))/b^3+3/2*polylog(2,exp(2*b*x+2*a))/b^4
```

3.446.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.42

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx = \frac{-6\,\text{PolyLog}\left(2,-e^{-a-bx}\right)-6\,\text{PolyLog}\left(2,e^{-a-bx}\right)+bx\left(-b^2x^2\text{csch}^2(a+bx)+6\left(-\frac{bx}{-1+e^{2a}}+\log\left(1-e^{-a-}\right.\right.\right.}{2b^4}$$

input
```
Integrate[x^3*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
(-6*PolyLog[2, -E^(-a - b*x)] - 6*PolyLog[2, E^(-a - b*x)] + b*x*(-(b^2*x^
2*Csch[a + b*x]^2) + 6*(-((b*x)/(-1 + E^(2*a))) + Log[1 - E^(-a - b*x)] +
Log[1 + E^(-a - b*x)]) + 3*b*x*Csch[a]*Csch[a + b*x]*Sinh[b*x]))/(2*b^4)
```

3.446.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.37, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5942, 3042, 25, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \ 5942$$

$$\frac{3\int x^2\text{csch}^2(a+bx)dx}{2b}-\frac{x^3\text{csch}^2(a+bx)}{2b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{x^3\text{csch}^2(a+bx)}{2b}+\frac{3\int -x^2\csc(ia+ibx)^2dx}{2b}$$

$$\Big\downarrow \ 25$$

$$-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{3\int x^2\csc(ia+ibx)^2dx}{2b}$$

$$\Big\downarrow \ 4672$$

$$-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{3\left(\frac{x^2\coth(a+bx)}{b}-\frac{2i\int -ix\coth(a+bx)dx}{b}\right)}{2b}$$

$$\Big\downarrow \ 26$$

$$-\frac{3\left(\frac{x^2\coth(a+bx)}{b}-\frac{2\int x\coth(a+bx)dx}{b}\right)}{2b}-\frac{x^3\text{csch}^2(a+bx)}{2b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{3\left(\frac{x^2\coth(a+bx)}{b}-\frac{2\int -ix\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{2b}$$

$$\Big\downarrow \ 26$$

$$-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{3\left(\frac{x^2\coth(a+bx)}{b}+\frac{2i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}\right)}{2b}$$

$$\Big\downarrow \ 4201$$

$$-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{3\left(\frac{x^2\coth(a+bx)}{b}+\frac{2i\left(2i\int\frac{e^{2a+2bx-i\pi}x}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^2}{2}\right)}{b}\right)}{2b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{3\left(\frac{x^2\coth(a+bx)}{b}+\frac{2i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int\log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)-\frac{ix^2}{2}\right)}{b}\right)}{2b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{3\left(\frac{x^2\coth(a+bx)}{b}+\frac{2i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\log\left(1+e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}\right)-\frac{ix^2}{2}\right)}{b}\right)}{2b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{3\left(\frac{x^2\coth(a+bx)}{b}+\frac{2i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{ix^2}{2}\right)}{b}\right)}{2b}$$

input
```
Int[x^3*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
-1/2*(x^3*Csch[a + b*x]^2)/b - (3*((x^2*Coth[a + b*x])/b + ((2*I)*((-1/2*I
)*x^2 + (2*I)*((x*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) + PolyLog[2, -E^(
2*a - I*Pi + 2*b*x)]/(4*b^2)))))/b))/(2*b)
```

3.446.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5942
```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

3.446. $\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx$

3.446.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(75) = 150.

Time = 0.54 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.13

method	result
risch	$-\frac{x^2\left(2\,\mathrm{e}^{2bx+2a}bx+3\,\mathrm{e}^{2bx+2a}-3\right)}{b^2\left(\mathrm{e}^{2bx+2a}-1\right)^2}-\frac{3x^2}{b^2}-\frac{6ax}{b^3}-\frac{3a^2}{b^4}+\frac{3\ln\left(\mathrm{e}^{bx+a}+1\right)x}{b^3}+\frac{3\,\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^4}+\frac{3\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b^3}+$ 3

input
```
int(x^3*cosh(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-x^2*(2*exp(2*b*x+2*a)*b*x+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+2*a)-1)^2-3/
b^2*x^2-6/b^3*a*x-3/b^4*a^2+3/b^3*ln(exp(b*x+a)+1)*x+3*polylog(2,-exp(b*x+
a))/b^4+3/b^3*ln(1-exp(b*x+a))*x+3/b^4*ln(1-exp(b*x+a))*a+3*polylog(2,exp(
b*x+a))/b^4-3/b^4*a*ln(exp(b*x+a)-1)+6/b^4*a*ln(exp(b*x+a))
```

3.446.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 979 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 979, normalized size of antiderivative = 11.80

$$\int x^3\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sin
h(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 + (2*b^3*x^3 - 3*b^2*x^2
+ 6*a^2)*cosh(b*x + a)^2 + (2*b^3*x^3 - 3*b^2*x^2 + 18*(b^2*x^2 - a^2)*cos
h(b*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 - 3*(cosh(b*x + a)^4 + 4*cos
h(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*s
inh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*s
inh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*(cosh(b*x + a)^
4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)
^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*
x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(b*x*
cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^
4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log(c
osh(b*x + a) + sinh(b*x + a) + 1) + 3*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x +
a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh
(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))
*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*((b*x + a)*
cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*si
nh(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)
^2 - b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b...
```

3.446.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \coth(a+bx)\mathrm{csch}^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**3*cosh(b*x+a)*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.446.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.57

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx = \frac{3\,x^2 - \left(2\,bx^3e^{(2\,a)} + 3\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2} - \frac{3\,x^2}{b^2} + \frac{3\left(bx\log\left(e^{(bx+a)}+1\right) + \text{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4} + \frac{3\left(bx\log\left(-e^{(bx+a)}+1\right) + \text{Li}_2\left(e^{(bx+a)}\right)\right)}{b^4}$$

input
```
integrate(x^3*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
(3*x^2 - (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a)
 - 2*b^2*e^(2*b*x + 2*a) + b^2) - 3*x^2/b^2 + 3*(b*x*log(e^(b*x + a) + 1)
+ dilog(-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x +
 a)))/b^4
```

3.446.8 Giac [F]

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int x^3\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^3\,dx$$

input
```
integrate(x^3*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*cosh(b*x + a)*csch(b*x + a)^3, x)
```

3.446.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int \frac{x^3\cosh(a+b\,x)}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^3*cosh(a + b*x))/sinh(a + b*x)^3,x)
```

output
```
int((x^3*cosh(a + b*x))/sinh(a + b*x)^3, x)
```

3.447 $\int x^2 \coth(a+bx)\mathrm{csch}^2(a+bx)\,dx$

3.447.1 Optimal result

Integrand size = 18, antiderivative size = 42

$$\int x^2 \coth(a+bx)\mathrm{csch}^2(a+bx)\,dx=-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\mathrm{csch}^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b^3}$$

output
```
-x*coth(b*x+a)/b^2-1/2*x^2*csch(b*x+a)^2/b+ln(sinh(b*x+a))/b^3
```

3.447.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int x^2 \coth(a+bx)\mathrm{csch}^2(a+bx)\,dx=-\frac{x\coth(a)}{b^2}-\frac{x^2\mathrm{csch}^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\mathrm{csch}(a)\mathrm{csch}(a+bx)\sinh(bx)}{b^2}$$

input
```
Integrate[x^2*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
-((x*Coth[a])/b^2) - (x^2*Csch[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]/b^3
+ (x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b^2
```

3.447.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5942, 3042, 25, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{5942}$$

$$\frac{\int x\text{csch}^2(a+bx)dx}{b}-\frac{x^2\text{csch}^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x^2\text{csch}^2(a+bx)}{2b}+\frac{\int -x\csc(ia+ibx)^2dx}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x^2\text{csch}^2(a+bx)}{2b}-\frac{\int x\csc(ia+ibx)^2dx}{b}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{x^2\text{csch}^2(a+bx)}{2b}-\frac{\frac{x\coth(a+bx)}{b}-\frac{i\int -i\coth(a+bx)dx}{b}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{x\coth(a+bx)}{b}-\frac{\int \coth(a+bx)dx}{b}}{b}-\frac{x^2\text{csch}^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x^2\text{csch}^2(a+bx)}{2b}-\frac{\frac{x\coth(a+bx)}{b}-\frac{\int -i\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{x^2\text{csch}^2(a+bx)}{2b}-\frac{\frac{x\coth(a+bx)}{b}+\frac{i\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}}{b}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{x^2\text{csch}^2(a+bx)}{2b}-\frac{\frac{x\coth(a+bx)}{b}-\frac{\log(-i\sinh(a+bx))}{b^2}}{b}$$

input
```
Int[x^2*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
-1/2*(x^2*Csch[a + b*x]^2)/b - ((x*Coth[a + b*x])/b - Log[(-I)*Sinh[a + b*
x]]/b^2)/b
```

3.447.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5942
```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

3.447.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.71

method	result	size
risch	$-\frac{2x}{b^2}-\frac{2a}{b^3}-\frac{2x\left(\mathrm{e}^{2bx+2a}bx+\mathrm{e}^{2bx+2a}-1\right)}{b^2\left(\mathrm{e}^{2bx+2a}-1\right)^2}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b^3}$	72

input
```
int(x^2*cosh(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*x/b^2-2/b^3*a-2*x*(exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+
2*a)-1)^2+1/b^3*ln(exp(2*b*x+2*a)-1)
```

3.447.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(40) = 80$.

Time = 0.26 (sec) , antiderivative size = 383, normalized size of antiderivative = 9.12

$$\int x^2\coth(a+bx)\mathrm{csch}^2(a+bx)\,dx =$$

$$-\frac{2\,bx\cosh\left(bx+a\right)^4+8\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+2\,bx\sinh\left(bx+a\right)^4+2\left(b^2x^2-bx\right)\cosh\left(bx+a\right.}{}$$

input
```
integrate(x^2*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(2*b*x*cosh(b*x + a)^4 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 2*b*x*sinh
(b*x + a)^4 + 2*(b^2*x^2 - b*x)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 6*b*x*cosh(
b*x + a)^2 - b*x)*sinh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sin
h(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2
 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) +
 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(2*b*x*cosh(b
*x + a)^3 + (b^2*x^2 - b*x)*cosh(b*x + a))*sinh(b*x + a))/(b^3*cosh(b*x +
a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - 2*b^3*c
osh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2 + 4
*(b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

3.447.6 Sympy [F]

$$\int x^2 \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int x^2 \cosh(a+bx)\,\text{csch}^3(a+bx)\,dx$$

input
```
integrate(x**2*cosh(b*x+a)*csch(b*x+a)**3,x)
```

output
```
Integral(x**2*cosh(a + b*x)*csch(a + b*x)**3, x)
```

3.447.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(40) = 80.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.55

$$\int x^2 \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2\left(\left(bx^2e^{(2a)} - xe^{(2a)}\right)e^{(2bx)} + xe^{(4bx+4a)}\right)}{b^2e^{(4bx+4a)} - 2b^2e^{(2bx+2a)} + b^2} + \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^3} + \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^3}$$

input
```
integrate(x^2*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2*((b*x^2*e^(2*a) - x*e^(2*a))*e^(2*b*x) + x*e^(4*b*x + 4*a))/(b^2*e^(4*b
*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) + log((e^(b*x + a) + 1)*e^(-a))/b
^3 + log((e^(b*x + a) - 1)*e^(-a))/b^3
```

3.447.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(40) = 80.

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.31

$$\int x^2 \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2b^2x^2e^{(2bx+2a)} + 2bxe^{(4bx+4a)} - 2bxe^{(2bx+2a)} - e^{(4bx+4a)}\log\left(e^{(2bx+2a)}-1\right) + 2e^{(2bx+2a)}\log\left(e^{(2bx+2a}\right.}{b^3e^{(4bx+4a)} - 2b^3e^{(2bx+2a)} + b^3}$$

input
```
integrate(x^2*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-(2*b^2*x^2*e^(2*b*x + 2*a) + 2*b*x*e^(4*b*x + 4*a) - 2*b*x*e^(2*b*x + 2*a
) - e^(4*b*x + 4*a)*log(e^(2*b*x + 2*a) - 1) + 2*e^(2*b*x + 2*a)*log(e^(2*
b*x + 2*a) - 1) - log(e^(2*b*x + 2*a) - 1))/(b^3*e^(4*b*x + 4*a) - 2*b^3*e
^(2*b*x + 2*a) + b^3)
```

3.447.9 Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.40

$$\int x^2 \coth(a+bx)\text{csch}^2(a+bx)\,dx = \frac{\ln\left(e^{2a}\,e^{2bx}-1\right)}{b^3} - \frac{\frac{x^2}{b}+\frac{x^2\,e^{2a+2bx}}{b}}{e^{4a+4bx}-2\,e^{2a+2bx}+1} - \frac{2x}{b^2} - \frac{bx^2+2x}{b^2\,\left(e^{2a+2bx}-1\right)}$$

input
```
int((x^2*cosh(a + b*x))/sinh(a + b*x)^3,x)
```

output
```
log(exp(2*a)*exp(2*b*x) - 1)/b^3 - (x^2/b + (x^2*exp(2*a + 2*b*x))/b)/(exp
(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1) - (2*x)/b^2 - (2*x + b*x^2)/(b^2*(
exp(2*a + 2*b*x) - 1))
```

3.448 $\int x \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

3.448.1 Optimal result

Integrand size = 16, antiderivative size = 30

$$\int x\coth(a+bx)\text{csch}^2(a+bx)\,dx=-\frac{\coth(a+bx)}{2b^2}-\frac{x\text{csch}^2(a+bx)}{2b}$$

output
```
-1/2*coth(b*x+a)/b^2-1/2*x*csch(b*x+a)^2/b
```

3.448.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x\coth(a+bx)\text{csch}^2(a+bx)\,dx=-\frac{\coth(a+bx)}{2b^2}-\frac{x\text{csch}^2(a+bx)}{2b}$$

input
```
Integrate[x*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
-1/2*Coth[a + b*x]/b^2 - (x*Csch[a + b*x]^2)/(2*b)
```

3.448.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5942, 3042, 25, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{5942}$$

$$\frac{\int \text{csch}^2(a+bx)dx}{2b}-\frac{x\text{csch}^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x\text{csch}^2(a+bx)}{2b}+\frac{\int -\csc(ia+ibx)^2dx}{2b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\text{csch}^2(a+bx)}{2b}-\frac{\int \csc(ia+ibx)^2dx}{2b}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{x\text{csch}^2(a+bx)}{2b}-\frac{i\int 1d(-i\coth(a+bx))}{2b^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\coth(a+bx)}{2b^2}-\frac{x\text{csch}^2(a+bx)}{2b}$$

input
```
Int[x*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
-1/2*Coth[a + b*x]/b^2 - (x*Csch[a + b*x]^2)/(2*b)
```

3.448.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5942
```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

3.448.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}bx+\mathrm{e}^{2bx+2a}-1}{b^{2}\left(\mathrm{e}^{2bx+2a}-1\right)^{2}}$	43

input
```
int(x*cosh(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-(2*exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2
```

3.448.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.57

$$\int x \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2\,(bx\cosh(bx+a)+(bx+1)\sinh(bx+a))}{b^2\cosh(bx+a)^3+3\,b^2\cosh(bx+a)\sinh(bx+a)^2+b^2\sinh(bx+a)^3-b^2\cosh(bx+a)+3\,(b^2\cosh(\ldots$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2*(b*x*cosh(b*x + a) + (b*x + 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^3 + 3*
b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3 - b^2*cosh(b*x + a
) + 3*(b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a))
```

3.448.6 Sympy [F]

$$\int x \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int x\cosh(a+bx)\,\text{csch}^3(a+bx)\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)**3,x)
```

output
```
Integral(x*cosh(a + b*x)*csch(a + b*x)**3, x)
```

3.448.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(26) = 52.

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.33

$$\int x \coth(a+bx)\text{csch}^2(a+bx)\,dx = \frac{2\,bxe^{(4\,bx+4\,a)}-\left(4\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+1}{2\,\left(b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2\right)} - \frac{2\,bxe^{(4\,bx+4\,a)}+e^{(2\,bx+2\,a)}-1}{2\,\left(b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(2*b*x*e^(4*b*x + 4*a) - (4*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) + 1)/(b^2
*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(2*b*x*e^(4*b*x + 4*
a) + e^(2*b*x + 2*a) - 1)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b
^2)
```

3.448.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 6.13

$$\int x\coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{4\,bxe^{(2\,bx+2\,a)} - e^{(4\,bx+4\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right) + 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right) + e^{(4\,bx+4\,a)}\log\left(-e^{(2\,bx+2\,a)}\right.}{2\left(b^2e^{(4\,bx+4\,a)} - 2\right.}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*(4*b*x*e^(2*b*x + 2*a) - e^(4*b*x + 4*a)*log(e^(2*b*x + 2*a) - 1) + 2
*e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) - 1) + e^(4*b*x + 4*a)*log(-e^(2*b*x
+ 2*a) + 1) - 2*e^(2*b*x + 2*a)*log(-e^(2*b*x + 2*a) + 1) + 2*e^(2*b*x + 2
*a) - log(e^(2*b*x + 2*a) - 1) + log(-e^(2*b*x + 2*a) + 1) - 2)/(b^2*e^(4*
b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2)
```

3.448.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int x\coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{e^{2\,a+2\,b\,x}\,(2\,b\,x+1)-1}{b^2\,\left(e^{2\,a+2\,b\,x}-1\right)^2}$$

input
```
int((x*cosh(a + b*x))/sinh(a + b*x)^3,x)
```

output
```
-(exp(2*a + 2*b*x)*(2*b*x + 1) - 1)/(b^2*(exp(2*a + 2*b*x) - 1)^2)
```

3.449 $\int \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

3.449.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\text{csch}^2(a+bx)}{2b}$$

output
```
-1/2*csch(b*x+a)^2/b
```

3.449.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\text{csch}^2(a+bx)}{2b}$$

input
```
Integrate[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
-1/2*Csch[a + b*x]^2/b
```

3.449.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 26, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan\left(ia+ibx-\frac{\pi}{2}\right)\sec\left(ia+ibx-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int -i\text{csch}(a+bx)d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\text{csch}^2(a+bx)}{2b}$$

input
```
Int[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
-1/2*Csch[a + b*x]^2/b
```

3.449.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.449.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(bx+a)^{2}}{2b}$	14
default	$-\frac{\operatorname{csch}(bx+a)^{2}}{2b}$	14
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)^{2}}$	28

input
```
int(cosh(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*csch(b*x+a)^2/b
```

3.449.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 5.73

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx =$$

$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+b\sinh\left(bx+a\right)^{3}-b\cosh\left(bx+a\right)+3\left(b\cosh\left(bx+\right.\right.}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*
sinh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)
^2 - b)*sinh(b*x + a))
```

3.449.6 Sympy [F]

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int \cosh{(a+bx)}\,\text{csch}^3{(a+bx)}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)**3,x)
```

output
```
Integral(cosh(a + b*x)*csch(a + b*x)**3, x)
```

3.449.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2}{b\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2/(b*(e^(b*x + a) - e^(-b*x - a))^2)
```

3.449.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2\,e^{(2\,bx+2\,a)}}{b\left(e^{(2\,bx+2\,a)} - 1\right)^2}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

3.449.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{1}{2\,b\sinh{(a+b\,x)}^2}$$

input
```
int(cosh(a + b*x)/sinh(a + b*x)^3,x)
```

output
```
-1/(2*b*sinh(a + b*x)^2)
```

3.450 $\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx$

3.450.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx = \text{Int}\left(\frac{\coth(a+bx)\text{csch}^2(a+bx)}{x},x\right)$$

output
```
CannotIntegrate(coth(b*x+a)*csch(b*x+a)^2/x,x)
```

3.450.2 Mathematica [N/A]

Not integrable

Time = 15.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx = \int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx$$

input
```
Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x,x]
```

output
```
Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x, x]
```

3.450.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}dx$$

input
```
Int[(Coth[a + b*x]*Csch[a + b*x]^2)/x,x]
```

output
```
$Aborted
```

3.450.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.450.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(bx+a\right) \text{csch}\left(bx+a\right)^{3}}{x}dx$$

input
```
int(cosh(b*x+a)*csch(b*x+a)^3/x,x)
```

output
```
int(cosh(b*x+a)*csch(b*x+a)^3/x,x)
```

3.450.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x} \, dx = \int \frac{\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^3}{x} \, dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*csch(b*x + a)^3/x, x)
```

3.450.6 Sympy [N/A]

Not integrable

Time = 21.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x} \, dx = \int \frac{\cosh\left(a+bx\right)\text{csch}^3\left(a+bx\right)}{x} \, dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)**3/x,x)
```

output
```
Integral(cosh(a + b*x)*csch(a + b*x)**3/x, x)
```

3.450.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x} \, dx = \int \frac{\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^3}{x} \, dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
-((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^
2*x^2*e^(2*b*x + 2*a) + b^2*x^2) - 4*integrate(1/4/(b^2*x^3*e^(b*x + a) +
b^2*x^3), x) + 4*integrate(1/4/(b^2*x^3*e^(b*x + a) - b^2*x^3), x)
```

3.450.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^3}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)*csch(b*x + a)^3/x, x)
```

3.450.9 Mupad [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)}{x\sinh(a+b\,x)^3}\,dx$$

input
```
int(cosh(a + b*x)/(x*sinh(a + b*x)^3),x)
```

output
```
int(cosh(a + b*x)/(x*sinh(a + b*x)^3), x)
```

3.451 $\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x^2} dx$

3.451.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x^2} dx = \operatorname{Int}\left(\frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x^2}, x\right)$$

output
```
CannotIntegrate(coth(b*x+a)*csch(b*x+a)^2/x^2,x)
```

3.451.2 Mathematica [N/A]

Not integrable

Time = 19.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x^2} dx = \int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x^2} dx$$

input
```
Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2,x]
```

output
```
Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2, x]
```

3.451.3 Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}dx$$

input
```
Int[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2,x]
```

output
```
$Aborted
```

3.451.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.451.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(bx+a)\,\text{csch}\,(bx+a)^3}{x^2}dx$$

input
```
int(cosh(b*x+a)*csch(b*x+a)^3/x^2,x)
```

output
```
int(cosh(b*x+a)*csch(b*x+a)^3/x^2,x)
```

3.451.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)\,\text{csch}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*csch(b*x + a)^3/x^2, x)
```

3.451.6 Sympy [N/A]

Not integrable

Time = 27.85 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+bx)\,\text{csch}^3(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)**3/x**2,x)
```

output
```
Integral(cosh(a + b*x)*csch(a + b*x)**3/x**2, x)
```

3.451.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 7.06

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)\,\text{csch}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
-2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^
2*x^3*e^(2*b*x + 2*a) + b^2*x^3) - 12*integrate(1/4/(b^2*x^4*e^(b*x + a) +
 b^2*x^4), x) + 12*integrate(1/4/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

3.451.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)\,\text{csch}\,(bx+a)^3}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)*csch(b*x + a)^3/x^2, x)
```

3.451.9 Mupad [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)}{x^2\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(cosh(a + b*x)/(x^2*sinh(a + b*x)^3),x)
```

output
```
int(cosh(a + b*x)/(x^2*sinh(a + b*x)^3), x)
```

3.452 $\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx$

3.452.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx = \text{Int}(x^m\text{csch}(a+bx),x) + \text{Int}\left(x^m\text{csch}^3(a+bx),x\right)$$

output
```
Unintegrable(x^m*csch(b*x+a),x)+Unintegrable(x^m*csch(b*x+a)^3,x)
```

3.452.2 Mathematica [N/A]

Not integrable

Time = 164.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx$$

input
```
Integrate[x^m*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
Integrate[x^m*Coth[a + b*x]^2*Csch[a + b*x], x]
```

3.452.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5980, 3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{5980}$$

$$\int x^m\text{csch}^3(a+bx)dx+\int x^m\text{csch}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix^m\csc(ia+ibx)dx+\int -ix^m\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int x^m\csc(ia+ibx)dx-i\int x^m\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{4680}$$

$$\int x^m\text{csch}^3(a+bx)dx+\int x^m\text{csch}(a+bx)dx$$

input
```
Int[x^m*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
$Aborted
```

3.452.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5980
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

3.452.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^{m} \cosh\left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right)^{3} dx$$

input
```
int(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

output
```
int(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

3.452.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^{m} \cosh\left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right)^{3}\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

3.452.6 Sympy [F(-1)]

Timed out.

$$\int x^m \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**m*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.452.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)^3\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(x^m*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

3.452.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)^3\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

3.452.9 Mupad [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)^2}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^m*cosh(a + b*x)^2)/sinh(a + b*x)^3,x)
```

output
```
int((x^m*cosh(a + b*x)^2)/sinh(a + b*x)^3, x)
```

3.453 $\int x^3 \coth^2(a+bx)\text{csch}(a+bx)\,dx$

3.453.1 Optimal result

Integrand size = 18, antiderivative size = 201

$$\begin{aligned}\int x^3\coth^2(a+bx)\text{csch}(a+bx)\,dx =& -\frac{6x\text{arctanh}\left(e^{a+bx}\right)}{b^3}-\frac{x^3\text{arctanh}\left(e^{a+bx}\right)}{b} \\ &-\frac{3x^2\text{csch}(a+bx)}{2b^2}-\frac{x^3\coth(a+bx)\text{csch}(a+bx)}{2b} \\ &-\frac{3\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^4}-\frac{3x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} \\ &+\frac{3\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^4}+\frac{3x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} \\ &+\frac{3x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{3x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} \\ &-\frac{3\,\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^4}+\frac{3\,\text{PolyLog}\left(4,e^{a+bx}\right)}{b^4}\end{aligned}$$

output
```
-6*x*arctanh(exp(b*x+a))/b^3-x^3*arctanh(exp(b*x+a))/b-3/2*x^2*csch(b*x+a)
/b^2-1/2*x^3*coth(b*x+a)*csch(b*x+a)/b-3*polylog(2,-exp(b*x+a))/b^4-3/2*x^
2*polylog(2,-exp(b*x+a))/b^2+3*polylog(2,exp(b*x+a))/b^4+3/2*x^2*polylog(2
,exp(b*x+a))/b^2+3*x*polylog(3,-exp(b*x+a))/b^3-3*x*polylog(3,exp(b*x+a))/
b^3-3*polylog(4,-exp(b*x+a))/b^4+3*polylog(4,exp(b*x+a))/b^4
```

3.453.2 Mathematica [A] (verified)

Time = 3.31 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.39

$$\int x^3 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx =$$
$$-\frac{12b^2x^2\operatorname{csch}(a)+b^3x^3\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)-24bx\log\left(1-e^{a+bx}\right)-4b^3x^3\log\left(1-e^{a+bx}\right)+24bx\log\left(1+e\right.}{}$$

input
```
Integrate[x^3*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
-1/8*(12*b^2*x^2*Csch[a] + b^3*x^3*Csch[(a + b*x)/2]^2 - 24*b*x*Log[1 - E^
(a + b*x)] - 4*b^3*x^3*Log[1 - E^(a + b*x)] + 24*b*x*Log[1 + E^(a + b*x)]
+ 4*b^3*x^3*Log[1 + E^(a + b*x)] + 12*(2 + b^2*x^2)*PolyLog[2, -E^(a + b*x
)] - 12*(2 + b^2*x^2)*PolyLog[2, E^(a + b*x)] - 24*b*x*PolyLog[3, -E^(a +
b*x)] + 24*b*x*PolyLog[3, E^(a + b*x)] + 24*PolyLog[4, -E^(a + b*x)] - 24*
PolyLog[4, E^(a + b*x)] + b^3*x^3*Sech[(a + b*x)/2]^2 - 6*b^2*x^2*Csch[a/2
]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] - 6*b^2*x^2*Sech[a/2]*Sech[(a + b*x)/2]*
Sinh[(b*x)/2])/b^4
```

3.453.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.98, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {5980, 3042, 26, 4670, 3011, 4674, 26, 3042, 26, 4670, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{5980}$$

$$\int x^3\operatorname{csch}^3(a+bx)dx+\int x^3\operatorname{csch}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix^3\csc(ia+ibx)dx+\int -ix^3\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int x^3\csc(ia+ibx)dx-i\int x^3\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{4670}$$

$$i\left(\frac{3i\int x^2\log\left(1-e^{a+bx}\right)dx}{b}-\frac{3i\int x^2\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2ix^3\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-i\int x^3\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{3011}$$

$$i\left(-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^3\text{arct}}{}\right.$$
$$i\int x^3\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{4674}$$

$$i\left(-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^3\text{arct}}{}\right.$$
$$i\left(-\frac{3\int -ix\text{csch}(a+bx)dx}{b^2}+\frac{1}{2}\int -ix^3\text{csch}(a+bx)dx-\frac{3ix^2\text{csch}(a+bx)}{2b^2}-\frac{ix^3\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^3\text{arct}}{}\right.$$
$$i\left(\frac{3i\int x\text{csch}(a+bx)dx}{b^2}-\frac{1}{2}i\int x^3\text{csch}(a+bx)dx-\frac{3ix^2\text{csch}(a+bx)}{2b^2}-\frac{ix^3\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^3\text{arct}}{}\right.$$
$$i\left(\frac{3i\int ix\csc(ia+ibx)dx}{b^2}-\frac{1}{2}i\int ix^3\csc(ia+ibx)dx-\frac{3ix^2\text{csch}(a+bx)}{2b^2}-\frac{ix^3\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

3.453. $\int x^3\coth^2(a+bx)\text{csch}(a+bx)\,dx$

$$i\left(-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^3\text{arct}}{}\right.$$
$$i\left(-\frac{3\int x\csc(ia+ibx)dx}{b^2}+\frac{1}{2}\int x^3\csc(ia+ibx)dx-\frac{3ix^2\text{csch}(a+bx)}{2b^2}-\frac{ix^3\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4670}$$

$$i\left(-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^3\text{arct}}{}\right.$$
$$i\left(-\frac{3\left(\frac{i\int\log(1-e^{a+bx})dx}{b}-\frac{i\int\log(1+e^{a+bx})dx}{b}+\frac{2ix\text{arctanh}(e^{a+bx})}{b}\right)}{b^2}+\frac{1}{2}\left(\frac{3i\int x^2\log\left(1-e^{a+bx}\right)dx}{b}-\frac{3i\int x^2\log(1}{b}\right.\right.$$

$$\Big\downarrow \text{2715}$$

$$i\left(-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^3\text{arct}}{}\right.$$
$$i\left(-\frac{3\left(\frac{i\int e^{-a-bx}\log(1-e^{a+bx})de^{a+bx}}{b^2}-\frac{i\int e^{-a-bx}\log(1+e^{a+bx})de^{a+bx}}{b^2}+\frac{2ix\text{arctanh}(e^{a+bx})}{b}\right)}{b^2}+\frac{1}{2}\left(\frac{3i\int x^2\log\left(1-e^{a+bx}\right)}{b}\right.\right.$$

$$\Big\downarrow \text{2838}$$

$$i\left(-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^3\text{arct}}{}\right.$$
$$i\left(\frac{1}{2}\left(\frac{3i\int x^2\log\left(1-e^{a+bx}\right)dx}{b}-\frac{3i\int x^2\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2ix^3\text{arctanh}(e^{a+bx})}{b}\right)-\frac{3\left(\frac{2ix\text{arctanh}(e^{a+bx})}{b}+\frac{i\,\text{P}}{}\right.}{}\right.$$

$$\Big\downarrow \text{3011}$$

$$i\left(-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^3\text{arct}}{}\right.$$
$$i\left(\frac{1}{2}\left(-\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3i\left(\frac{2\int x\,\text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^3}{}\right.\right.$$

3.453. $\int x^3\coth^2(a+bx)\text{csch}(a+bx)\,dx$

$\downarrow$ 7163

$$i\left(-\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{\int \text{PolyLog}\left(3,-e^{a+bx}\right)dx}{b}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b}-\frac{\int \text{PolyLog}\left(3,e^{a+bx}\right)\cdots}{b}\right.}{b}\cdots}{b}\cdots\right.$$

$$i\left(\frac{1}{2}\left(-\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{\int \text{PolyLog}\left(3,-e^{a+bx}\right)dx}{b}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b}-\frac{\int \text{PolyLog}\left(3,e^{a}\cdots\right.}{b}\right.}{b}\cdots}{b}\cdots\right.\right.$$

$\downarrow$ 2720

$$i\left(-\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{\int e^{-a-bx}\,\text{PolyLog}\left(3,-e^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b}-\frac{\int e^{-a-}\cdots}{}\right.}{b}\cdots\right.}{}\cdots\right.$$

$$i\left(\frac{1}{2}\left(-\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{\int e^{-a-bx}\,\text{PolyLog}\left(3,-e^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b}-\frac{\int\cdots}{}\right.}{}\cdots\right.}{}\cdots\right.\right.$$

$\downarrow$ 7143

3.453. $\int x^3 \coth^2(a+bx)\text{csch}(a+bx)\,dx$

$$i\left(\frac{2ix^3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}-\frac{3i\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{\mathrm{PolyLog}\left(4,-e^{a+bx}\right)}{b^2}\right)}{b}-\frac{x^2\,\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,e^{a+b\cdots}\right.}{b}\right.}{\cdots}\right.}{\cdots}\right.$$

$$i\left(\frac{1}{2}\left(\frac{2ix^3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}-\frac{3i\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{\mathrm{PolyLog}\left(4,-e^{a+bx}\right)}{b^2}\right)}{b}-\frac{x^2\,\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{3i\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,\cdots\right.}{b}\right.}{\cdots}\right.}{\cdots}\right.\right.$$

input
```
Int[x^3*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
I*(((2*I)*x^3*ArcTanh[E^(a + b*x)])/b - ((3*I)*(-((x^2*PolyLog[2, -E^(a +
b*x)])/b) + (2*((x*PolyLog[3, -E^(a + b*x)])/b - PolyLog[4, -E^(a + b*x)]/
b^2))/b))/b + ((3*I)*(-((x^2*PolyLog[2, E^(a + b*x)])/b) + (2*((x*PolyLog[
3, E^(a + b*x)])/b - PolyLog[4, E^(a + b*x)]/b^2))/b))/b) - I*((((-3*I)/2)
*x^2*Csch[a + b*x])/b^2 - ((I/2)*x^3*Coth[a + b*x]*Csch[a + b*x])/b - (3*(
((2*I)*x*ArcTanh[E^(a + b*x)])/b + (I*PolyLog[2, -E^(a + b*x)])/b^2 - (I*P
olyLog[2, E^(a + b*x)])/b^2))/b^2 + (((2*I)*x^3*ArcTanh[E^(a + b*x)])/b -
((3*I)*(-((x^2*PolyLog[2, -E^(a + b*x)])/b) + (2*((x*PolyLog[3, -E^(a + b*
x)])/b - PolyLog[4, -E^(a + b*x)]/b^2))/b))/b + ((3*I)*(-((x^2*PolyLog[2,
E^(a + b*x)])/b) + (2*((x*PolyLog[3, E^(a + b*x)])/b - PolyLog[4, E^(a + b
*x)]/b^2))/b))/b)/2)
```

3.453.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5980
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

3.453. $\int x^3 \coth^2(a+bx)\text{csch}(a+bx)\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.453.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.69

method	result
risch	$-\frac{x^{2} \mathrm{e}^{b x+a} \left(\mathrm{e}^{2 b x+2 a} b x+b x+3 \,\mathrm{e}^{2 b x+2 a}-3\right)}{b^{2} \left(\mathrm{e}^{2 b x+2 a}-1\right)^{2}}+\frac{3 \,\mathrm{polylog}\left(2, \mathrm{e}^{b x+a}\right)}{b^{4}}+\frac{3 \,\mathrm{polylog}\left(4, \mathrm{e}^{b x+a}\right)}{b^{4}}-\frac{3 \,\mathrm{polylog}\left(2, -\mathrm{e}^{b x+a}\right)}{b^{4}}-\frac{3 \,\mathrm{polylog}\left(4,\right.}{b^{4}}$

input
```
int(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-x^2*exp(b*x+a)*(exp(2*b*x+2*a)*b*x+b*x+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x
+2*a)-1)^2+3*polylog(2,exp(b*x+a))/b^4+3*polylog(4,exp(b*x+a))/b^4-3*polyl
og(2,-exp(b*x+a))/b^4-3*polylog(4,-exp(b*x+a))/b^4+3/b^3*ln(1-exp(b*x+a))*
x-3/b^3*ln(exp(b*x+a)+1)*x+1/2/b*ln(1-exp(b*x+a))*x^3+3/2*x^2*polylog(2,ex
p(b*x+a))/b^2-3*x*polylog(3,exp(b*x+a))/b^3-1/2/b*ln(exp(b*x+a)+1)*x^3-3/2
*x^2*polylog(2,-exp(b*x+a))/b^2+3*x*polylog(3,-exp(b*x+a))/b^3+3/b^4*ln(1-
exp(b*x+a))*a-3/b^4*ln(exp(b*x+a)+1)*a+1/2/b^4*ln(1-exp(b*x+a))*a^3-1/2/b^
4*ln(exp(b*x+a)+1)*a^3+6/b^4*a*arctanh(exp(b*x+a))+1/b^4*a^3*arctanh(exp(b
*x+a))
```

3.453.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1802 vs. 2(179) = 358.

Time = 0.28 (sec) , antiderivative size = 1802, normalized size of antiderivative = 8.97

$$\int x^3 \coth^2(a+bx)\text{csch}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^3 + 6*(b^3*x^3 + 3*b^2*x^2)*co
sh(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*x^3 + 3*b^2*x^2)*sinh(b*x + a)^3 + 2*
(b^3*x^3 - 3*b^2*x^2)*cosh(b*x + a) - 3*((b^2*x^2 + 2)*cosh(b*x + a)^4 + 4
*(b^2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 2)*sinh(b*x + a)
^4 + b^2*x^2 - 2*(b^2*x^2 + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 +
 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 2)*cosh(b*x + a)^
3 - (b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a) + 2)*dilog(cosh(b*x + a) +
sinh(b*x + a)) + 3*((b^2*x^2 + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 2)*cosh(b
*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2
*x^2 + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 2)*cosh(b*x + a)^2 +
 2)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 2)*cosh(b*x + a)^3 - (b^2*x^2 + 2)*cos
h(b*x + a))*sinh(b*x + a) + 2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b^
3*x^3 + (b^3*x^3 + 6*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^3 + 6*b*x)*cosh(b*x +
 a)*sinh(b*x + a)^3 + (b^3*x^3 + 6*b*x)*sinh(b*x + a)^4 - 2*(b^3*x^3 + 6*b
*x)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 + 6*b*x)*cosh(b*x + a)^2 + 6
*b*x)*sinh(b*x + a)^2 + 6*b*x + 4*((b^3*x^3 + 6*b*x)*cosh(b*x + a)^3 - (b^
3*x^3 + 6*b*x)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x
+ a) + 1) + ((a^3 + 6*a)*cosh(b*x + a)^4 + 4*(a^3 + 6*a)*cosh(b*x + a)*sin
h(b*x + a)^3 + (a^3 + 6*a)*sinh(b*x + a)^4 + a^3 - 2*(a^3 + 6*a)*cosh(b*x
+ a)^2 - 2*(a^3 - 3*(a^3 + 6*a)*cosh(b*x + a)^2 + 6*a)*sinh(b*x + a)^2 ...
```

3.453.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \coth^2(a+bx)\text{csch}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**3*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.453.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.30

$$\int x^3\coth^2(a+bx)\text{csch}(a+bx)\,dx$$
$$= -\frac{(bx^3e^{(3a)}+3x^2e^{(3a)})e^{(3bx)}+(bx^3e^a-3x^2e^a)e^{(bx)}}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2}$$
$$-\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3b^2x^2\text{Li}_2\left(-e^{(bx+a)}\right)-6bx\text{Li}_3(-e^{(bx+a)})+6\,\text{Li}_4(-e^{(bx+a)})}{2b^4}$$
$$+\frac{b^3x^3\log\left(-e^{(bx+a)}+1\right)+3b^2x^2\text{Li}_2\left(e^{(bx+a)}\right)-6bx\text{Li}_3(e^{(bx+a)})+6\,\text{Li}_4(e^{(bx+a)})}{2b^4}$$
$$-\frac{3\left(bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4}+\frac{3\left(bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)\right)}{b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-((b*x^3*e^(3*a) + 3*x^2*e^(3*a))*e^(3*b*x) + (b*x^3*e^a - 3*x^2*e^a)*e^(b
*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b^3*x^3*lo
g(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(
b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + 1/2*(b^3*x^3*log(-e^(b*x + a
) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*
polylog(4, e^(b*x + a)))/b^4 - 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x
 + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

3.453.8 Giac [F]

$$\int x^3\coth^2(a+bx)\text{csch}(a+bx)\,dx=\int x^3\cosh(bx+a)^2\,\text{csch}(bx+a)^3\,dx$$

input
```
integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

3.453.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int \frac{x^3 \cosh(a+b\,x)^2}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^3*cosh(a + b*x)^2)/sinh(a + b*x)^3,x)
```

output
```
int((x^3*cosh(a + b*x)^2)/sinh(a + b*x)^3, x)
```

3.454 $\int x^2 \coth^2(a+bx)\text{csch}(a+bx)\,dx$

3.454.1 Optimal result

Integrand size = 18, antiderivative size = 123

$$\int x^2 \coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{x^2\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{\text{arctanh}(\cosh(a+bx))}{b^3}-\frac{x\text{csch}(a+bx)}{b^2}-\frac{x^2\coth(a+bx)\text{csch}(a+bx)}{2b}-\frac{x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}$$

output
```
-x^2*arctanh(exp(b*x+a))/b-arctanh(cosh(b*x+a))/b^3-x*csch(b*x+a)/b^2-1/2*
x^2*coth(b*x+a)*csch(b*x+a)/b-x*polylog(2,-exp(b*x+a))/b^2+x*polylog(2,exp
(b*x+a))/b^2+polylog(3,-exp(b*x+a))/b^3-polylog(3,exp(b*x+a))/b^3
```

3.454.2 Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.80

$$\int x^2 \coth^2(a+bx)\text{csch}(a+bx)\,dx =$$

$$-\frac{8bx\text{csch}(a)+b^2x^2\text{csch}^2\left(\frac{1}{2}(a+bx)\right)-8\log\left(1-e^{a+bx}\right)-4b^2x^2\log\left(1-e^{a+bx}\right)+8\log\left(1+e^{a+bx}\right)+4b}{}$$

input
```
Integrate[x^2*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
-1/8*(8*b*x*Csch[a] + b^2*x^2*Csch[(a + b*x)/2]^2 - 8*Log[1 - E^(a + b*x)]
 - 4*b^2*x^2*Log[1 - E^(a + b*x)] + 8*Log[1 + E^(a + b*x)] + 4*b^2*x^2*Log
[1 + E^(a + b*x)] + 8*b*x*PolyLog[2, -E^(a + b*x)] - 8*b*x*PolyLog[2, E^(a
 + b*x)] - 8*PolyLog[3, -E^(a + b*x)] + 8*PolyLog[3, E^(a + b*x)] + b^2*x^
2*Sech[(a + b*x)/2]^2 - 4*b*x*Csch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] -
4*b*x*Sech[a/2]*Sech[(a + b*x)/2]*Sinh[(b*x)/2])/b^3
```

3.454.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5980, 3042, 26, 4670, 3011, 2720, 4674, 26, 3042, 26, 4257, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^2(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{5980}$$

$$\int x^2\text{csch}^3(a+bx)dx+\int x^2\text{csch}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix^2\csc(ia+ibx)dx+\int -ix^2\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int x^2\csc(ia+ibx)dx-i\int x^2\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{4670}$$

$$i\left(\frac{2i\int x\log\left(1-e^{a+bx}\right)dx}{b}-\frac{2i\int x\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2ix^2\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-i\int x^2\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{3011}$$

$$i\left(-\frac{2i\left(\frac{\int \text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int \text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^2\text{arctanh}(e^{a}\right.$$

$$i\int x^2\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{2720}$$

$$i\left(-\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,e^{a+bx})}{b}\right.}{b}\right.$$

$$i\int x^2\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{4674}$$

$$i\left(-\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,e^{a+bx})}{b}\right.}{b}\right.$$

$$i\left(-\frac{\int -i\text{csch}(a+bx)dx}{b^2}+\frac{1}{2}\int -ix^2\text{csch}(a+bx)dx-\frac{ix\text{csch}(a+bx)}{b^2}-\frac{ix^2\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,e^{a+bx})}{b}\right.}{b}\right.$$

$$i\left(\frac{i\int \text{csch}(a+bx)dx}{b^2}-\frac{1}{2}i\int x^2\text{csch}(a+bx)dx-\frac{ix\text{csch}(a+bx)}{b^2}-\frac{ix^2\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,e^{a+bx})}{b}\right.}{b}\right.$$

$$i\left(\frac{i\int i\csc(ia+ibx)dx}{b^2}-\frac{1}{2}i\int ix^2\csc(ia+ibx)dx-\frac{ix\text{csch}(a+bx)}{b^2}-\frac{ix^2\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}(2,e^{a+bx})}{b}\right.}{b}\right.$$

$$i\left(-\frac{\int \csc(ia+ibx)dx}{b^2}+\frac{1}{2}\int x^2\csc(ia+ibx)dx-\frac{ix\text{csch}(a+bx)}{b^2}-\frac{ix^2\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

3.454. $\int x^2\coth^2(a+bx)\text{csch}(a+bx)\,dx$

$$\Big\downarrow \; 4257$$

$$i\left(-\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,e^{a+bx})}{b}\right.}{b}\right.$$

$$i\left(\frac{1}{2}\int x^2\csc(ia+ibx)dx-\frac{i\text{arctanh}(\cosh(a+bx))}{b^3}-\frac{ix\text{csch}(a+bx)}{b^2}-\frac{ix^2\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \; 4670$$

$$i\left(-\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,e^{a+bx})}{b}\right.}{b}\right.$$

$$i\left(\frac{1}{2}\left(\frac{2i\int x\log\left(1-e^{a+bx}\right)dx}{b}-\frac{2i\int x\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2ix^2\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-\frac{i\text{arctanh}(\cosh(a+bx))}{b^3}-\right.$$

$$\Big\downarrow \; 3011$$

$$i\left(-\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,e^{a+bx})}{b}\right.}{b}\right.$$

$$i\left(\frac{1}{2}\left(-\frac{2i\left(\frac{\int \operatorname{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{x\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int \operatorname{PolyLog}(2,e^{a+bx})dx}{b}-\frac{x\operatorname{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2ix^2\text{arctan}}{b}\right.\right.$$

$$\Big\downarrow \; 2720$$

$$i\left(-\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,e^{a+bx})}{b}\right.}{b}\right.$$

$$i\left(\frac{1}{2}\left(-\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\int e^{-a-bx}\operatorname{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\frac{x\operatorname{PolyLog}(2,e^{a}}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow \; 7143$$

$$i\left(\frac{2ix^2\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{2i\left(\frac{\operatorname{PolyLog}(3,-e^{a+bx})}{b^2}-\frac{x\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\operatorname{PolyLog}(3,e^{a+bx})}{b^2}-\frac{x\operatorname{PolyLog}(2,e^{a+bx})}{b}\right)}{b}\right)-$$

$$i\left(-\frac{i\text{arctanh}(\cosh(a+bx))}{b^3}+\frac{1}{2}\left(\frac{2ix^2\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{2i\left(\frac{\operatorname{PolyLog}(3,-e^{a+bx})}{b^2}-\frac{x\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2i\left(\frac{\operatorname{PolyLog}}{}\right.}{}\right.\right.$$

3.454. $\int x^2\coth^2(a+bx)\text{csch}(a+bx)\,dx$

input
```
Int[x^2*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
I*(((2*I)*x^2*ArcTanh[E^(a + b*x)])/b - ((2*I)*(-((x*PolyLog[2, -E^(a + b*
x)])/b) + PolyLog[3, -E^(a + b*x)]/b^2))/b + ((2*I)*(-((x*PolyLog[2, E^(a
+ b*x)])/b) + PolyLog[3, E^(a + b*x)]/b^2))/b) - I*(((-I)*ArcTanh[Cosh[a +
 b*x]])/b^3 - (I*x*Csch[a + b*x])/b^2 - ((I/2)*x^2*Coth[a + b*x]*Csch[a +
b*x])/b + (((2*I)*x^2*ArcTanh[E^(a + b*x)])/b - ((2*I)*(-((x*PolyLog[2, -E
^(a + b*x)])/b) + PolyLog[3, -E^(a + b*x)]/b^2))/b + ((2*I)*(-((x*PolyLog[
2, E^(a + b*x)])/b) + PolyLog[3, E^(a + b*x)]/b^2))/b)/2)
```

3.454.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5980
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.454.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{x\,{\mathrm e}^{b x +a} \left({\mathrm e}^{2 b x +2 a} b x +b x +2 \,{\mathrm e}^{2 b x +2 a}-2\right)}{b^{2} \left({\mathrm e}^{2 b x +2 a}-1\right)^{2}}-\frac{a^{2} \operatorname{arctanh}\left({\mathrm e}^{b x +a}\right)}{b^{3}}+\frac{\ln \left(1-{\mathrm e}^{b x +a}\right) x^{2}}{2 b}-\frac{\ln \left(1-{\mathrm e}^{b x +a}\right) a^{2}}{2 b^{3}}+\frac{x \,\operatorname{polylog}\left(2, {\mathrm e}^{b x +a}\right)}{b^{2}}-$

input
```
int(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-x*exp(b*x+a)*(exp(2*b*x+2*a)*b*x+b*x+2*exp(2*b*x+2*a)-2)/b^2/(exp(2*b*x+2
*a)-1)^2-1/b^3*a^2*arctanh(exp(b*x+a))+1/2/b*ln(1-exp(b*x+a))*x^2-1/2/b^3*
ln(1-exp(b*x+a))*a^2+x*polylog(2,exp(b*x+a))/b^2-polylog(3,exp(b*x+a))/b^3
-1/2/b*ln(exp(b*x+a)+1)*x^2+1/2/b^3*ln(exp(b*x+a)+1)*a^2-x*polylog(2,-exp(
b*x+a))/b^2+polylog(3,-exp(b*x+a))/b^3-2/b^3*arctanh(exp(b*x+a))
```

3.454.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1311 vs. 2(114) = 228.

Time = 0.28 (sec) , antiderivative size = 1311, normalized size of antiderivative = 10.66

$$\int x^2 \coth^2(a+bx)\text{csch}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^3 + 6*(b^2*x^2 + 2*b*x)*cosh(b*x +
 a)*sinh(b*x + a)^2 + 2*(b^2*x^2 + 2*b*x)*sinh(b*x + a)^3 + 2*(b^2*x^2 - 2
*b*x)*cosh(b*x + a) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*
x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x
 + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b
*x + a))*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) + 2*(b*x*cosh
(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 -
2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 +
b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-co
sh(b*x + a) - sinh(b*x + a)) + ((b^2*x^2 + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2
 + 2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 2)*sinh(b*x + a)^4 + b^2*
x^2 - 2*(b^2*x^2 + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 2)*cosh(
b*x + a)^2 + 2)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 2)*cosh(b*x + a)^3 - (b^2*
x^2 + 2)*cosh(b*x + a))*sinh(b*x + a) + 2)*log(cosh(b*x + a) + sinh(b*x +
a) + 1) - ((a^2 + 2)*cosh(b*x + a)^4 + 4*(a^2 + 2)*cosh(b*x + a)*sinh(b*x
+ a)^3 + (a^2 + 2)*sinh(b*x + a)^4 - 2*(a^2 + 2)*cosh(b*x + a)^2 + 2*(3*(a
^2 + 2)*cosh(b*x + a)^2 - a^2 - 2)*sinh(b*x + a)^2 + a^2 + 4*((a^2 + 2)*co
sh(b*x + a)^3 - (a^2 + 2)*cosh(b*x + a))*sinh(b*x + a) + 2)*log(cosh(b*x +
 a) + sinh(b*x + a) - 1) - ((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 -
 a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 +...
```

3.454.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**2*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.454.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.60

$$\begin{aligned}\int x^2 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx &= -\frac{(bx^2e^{(3a)}+2xe^{(3a)})e^{(3bx)}+(bx^2e^a-2xe^a)e^{(bx)}}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2} \\ &\quad - \frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\mathrm{Li}_3\left(-e^{(bx+a)}\right)}{2b^3} \\ &\quad + \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\mathrm{Li}_3\left(e^{(bx+a)}\right)}{2b^3} \\ &\quad - \frac{\log\left(e^{(bx+a)}+1\right)}{b^3}+\frac{\log\left(e^{(bx+a)}-1\right)}{b^3}\end{aligned}$$

input
```
integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-((b*x^2*e^(3*a) + 2*x*e^(3*a))*e^(3*b*x) + (b*x^2*e^a - 2*x*e^a)*e^(b*x))
/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b^2*x^2*log(e^
(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b
^3 + 1/2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*pol
ylog(3, e^(b*x + a)))/b^3 - log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1
)/b^3
```

3.454.8 Giac [F]

$$\int x^2 \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int x^2 \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 \,dx$$

input
```
integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)^2}{\sinh(a+b\,x)^3}\,dx$$

input
```
int((x^2*cosh(a + b*x)^2)/sinh(a + b*x)^3,x)
```

output
```
int((x^2*cosh(a + b*x)^2)/sinh(a + b*x)^3, x)
```

3.455 $\int x \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

3.455.1 Optimal result

Integrand size = 16, antiderivative size = 82

$$\int x \coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{x\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{\text{csch}(a+bx)}{2b^2}-\frac{x\coth(a+bx)\text{csch}(a+bx)}{2b}-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}$$

output
```
-x*arctanh(exp(b*x+a))/b-1/2*csch(b*x+a)/b^2-1/2*x*coth(b*x+a)*csch(b*x+a)
/b-1/2*polylog(2,-exp(b*x+a))/b^2+1/2*polylog(2,exp(b*x+a))/b^2
```

3.455.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

$$\int x \coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\coth\left(\frac{1}{2}(a+bx)\right)+bx\text{csch}^2\left(\frac{1}{2}(a+bx)\right)-4bx\log\left(1-e^{a+bx}\right)+4bx\log\left(1+e^{a+bx}\right)+4\,\text{PolyLog}\left(2,-\right.}{8b^2}$$

input
```
Integrate[x*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
-1/8*(2*Coth[(a + b*x)/2] + b*x*Csch[(a + b*x)/2]^2 - 4*b*x*Log[1 - E^(a +
 b*x)] + 4*b*x*Log[1 + E^(a + b*x)] + 4*PolyLog[2, -E^(a + b*x)] - 4*PolyL
og[2, E^(a + b*x)] + b*x*Sech[(a + b*x)/2]^2 - 2*Tanh[(a + b*x)/2])/b^2
```

3.455.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.87, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5980, 3042, 26, 4670, 2715, 2838, 4673, 26, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^2(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{5980}$$

$$\int x\text{csch}^3(a+bx)dx+\int x\text{csch}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix\csc(ia+ibx)dx+\int -ix\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int x\csc(ia+ibx)dx-i\int x\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{4670}$$

$$i\left(\frac{i\int\log\left(1-e^{a+bx}\right)dx}{b}-\frac{i\int\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-i\int x\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{2715}$$

$$i\left(\frac{i\int e^{-a-bx}\log\left(1-e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{i\int e^{-a-bx}\log\left(1+e^{a+bx}\right)de^{a+bx}}{b^2}+\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-i\int x\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{2838}$$

$$i\left(\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)-i\int x\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{4673}$$

$$i\left(\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)-i\left(\frac{1}{2}\int -ix\text{csch}(a+bx)dx-\frac{i\text{csch}(a+bx)}{2b^2}-\frac{ix\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)-i\left(-\frac{1}{2}i\int x\text{csch}(a+bx)dx-\frac{i\text{csch}(a+bx)}{2b^2}-\frac{ix\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)-i\left(-\frac{1}{2}i\int ix\csc(ia+ibx)dx-\frac{i\text{csch}(a+bx)}{2b^2}-\frac{ix\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)-i\left(\frac{1}{2}\int x\csc(ia+ibx)dx-\frac{i\text{csch}(a+bx)}{2b^2}-\frac{ix\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4670}$$

$$i\left(\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)-i\left(\frac{1}{2}\left(\frac{i\int\log\left(1-e^{a+bx}\right)dx}{b}-\frac{i\int\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-\frac{i\text{csch}(a+bx)}{2b^2}-\frac{ix\coth(a+bx)\text{cs}}{2b}\right.$$

$$\Big\downarrow \text{2715}$$

$$i\left(\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)-i\left(\frac{1}{2}\left(\frac{i\int e^{-a-bx}\log\left(1-e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{i\int e^{-a-bx}\log\left(1+e^{a+bx}\right)de^{a+bx}}{b^2}+\frac{2ix\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-\frac{i\text{csch}(a+b}{2b^2}\right.$$

$$\Big\downarrow \text{2838}$$

3.455. $\int x\coth^2(a+bx)\text{csch}(a+bx)\,dx$

$$i\left(\frac{2ix\text{arctanh}(e^{a+bx})}{b}+\frac{i\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)-$$
$$i\left(\frac{1}{2}\left(\frac{2ix\text{arctanh}(e^{a+bx})}{b}+\frac{i\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)-\frac{i\text{csch}(a+bx)}{2b^2}-\frac{ix\coth(a+bx)\text{csc}}{2b}\right.$$

input
```
Int[x*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
I*(((2*I)*x*ArcTanh[E^(a + b*x)])/b + (I*PolyLog[2, -E^(a + b*x)])/b^2 - (
I*PolyLog[2, E^(a + b*x)])/b^2) - I*(((-1/2*I)*Csch[a + b*x])/b^2 - ((I/2)
*x*Coth[a + b*x]*Csch[a + b*x])/b + (((2*I)*x*ArcTanh[E^(a + b*x)])/b + (I
*PolyLog[2, -E^(a + b*x)])/b^2 - (I*PolyLog[2, E^(a + b*x)])/b^2)/2)
```

3.455.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.455. $\int x\coth^2(a+bx)\text{csch}(a+bx)\,dx$

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5980
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

3.455.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(71) = 142$.

Time = 1.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.90

method	result
risch	$-\frac{e^{bx+a}\left(e^{2bx+2a}bx+bx+e^{2bx+2a}-1\right)}{b^{2}\left(e^{2bx+2a}-1\right)^{2}}+\frac{\ln\left(1-e^{bx+a}\right)x}{2b}+\frac{\ln\left(1-e^{bx+a}\right)a}{2b^{2}}+\frac{\operatorname{polylog}\left(2,e^{bx+a}\right)}{2b^{2}}-\frac{\ln\left(e^{bx+a}+1\right)x}{2b}-\frac{\ln\left(e^{bx+a}+\right.}{2b^{2}}$

input
```
int(x*cosh(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-exp(b*x+a)*(exp(2*b*x+2*a)*b*x+b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-
1)^2+1/2/b*ln(1-exp(b*x+a))*x+1/2/b^2*ln(1-exp(b*x+a))*a+1/2*polylog(2,exp
(b*x+a))/b^2-1/2/b*ln(exp(b*x+a)+1)*x-1/2/b^2*ln(exp(b*x+a)+1)*a-1/2*polyl
og(2,-exp(b*x+a))/b^2+1/b^2*a*arctanh(exp(b*x+a))
```

3.455.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(69) = 138$.

Time = 0.26 (sec) , antiderivative size = 842, normalized size of antiderivative = 10.27

$$\int x\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(b*x + 1)*cosh(b*x + a)^3 + 6*(b*x + 1)*cosh(b*x + a)*sinh(b*x + a
)^2 + 2*(b*x + 1)*sinh(b*x + a)^3 + 2*(b*x - 1)*cosh(b*x + a) - (cosh(b*x
+ a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
 + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - co
sh(b*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) + (co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*c
osh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)
^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a
)) + (b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh
(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh
(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x +
 a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (a*cosh(b*x + a)^4 + 4*a*cos
h(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(
3*a*cosh(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b
*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b*x
 + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x +
 a)*sinh(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*
x + a)^2 - b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 -
 (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x
 + a) + 1) + 2*(3*(b*x + 1)*cosh(b*x + a)^2 + b*x - 1)*sinh(b*x + a))/(...
```

3.455.6 Sympy [F]

$$\int x\coth^2(a+bx)\text{csch}(a+bx)\,dx=\int x\cosh^2{(a+bx)}\,\text{csch}^3{(a+bx)}\,dx$$

input
```
integrate(x*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

output
```
Integral(x*cosh(a + b*x)**2*csch(a + b*x)**3, x)
```

3.455.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.51

$$\int x\coth^2(a+bx)\mathrm{csch}(a+bx)\,dx = -\frac{(bxe^{(3a)}+e^{(3a)})e^{(3bx)}+(bxe^{a}-e^{a})e^{(bx)}}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2} - \frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{2b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)}{2b^2}$$

input
```
integrate(x*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) + (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4*b
*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b*x*log(e^(b*x + a) + 1) +
 dilog(-e^(b*x + a)))/b^2 + 1/2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x
+ a)))/b^2
```

3.455.8 Giac [F]

$$\int x\coth^2(a+bx)\mathrm{csch}(a+bx)\,dx = \int x\cosh(bx+a)^2\,\mathrm{csch}(bx+a)^3\,dx$$

input
```
integrate(x*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

3.455.9 Mupad [F(-1)]

Timed out.

$$\int x\coth^2(a+bx)\mathrm{csch}(a+bx)\,dx = \int \frac{x\cosh(a+b\,x)^2}{\sinh(a+b\,x)^3}\,dx$$

input
```
int((x*cosh(a + b*x)^2)/sinh(a + b*x)^3,x)
```

output
```
int((x*cosh(a + b*x)^2)/sinh(a + b*x)^3, x)
```

3.456 $\int \coth^2(a+bx)\text{csch}(a+bx)\,dx$

3.456.1 Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx=-\frac{\text{arctanh}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

output
```
-1/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b
```

3.456.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. $2(34)=68$.

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx=-\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

input
```
Integrate[Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
-1/8*Csch[(a + b*x)/2]^2/b - Log[Cosh[(a + b*x)/2]]/(2*b) + Log[Sinh[(a + 
b*x)/2]]/(2*b) - Sech[(a + b*x)/2]^2/(8*b)
```

3.456.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan\left(ia+ibx-\frac{\pi}{2}\right)^2\sec\left(ia+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3091}$$

$$-i\left(-\frac{1}{2}\int -i\text{csch}(a+bx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}i\int \text{csch}(a+bx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{2}i\int i\csc(ia+ibx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{1}{2}\int \csc(ia+ibx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(-\frac{i\text{arctanh}(\cosh(a+bx))}{2b}-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

input
```
Int[Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
(-I)*(((-1/2*I)*ArcTanh[Cosh[a + b*x]])/b - ((I/2)*Coth[a + b*x]*Csch[a +
b*x])/b)
```

3.456.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.456.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\operatorname{csch}(bx+a)}{2}-\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	45
default	$\frac{-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\operatorname{csch}(bx+a)}{2}-\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	45
risch	$-\frac{e^{bx+a}\left(1+e^{2bx+2a}\right)}{b\left(e^{2bx+2a}-1\right)^2}-\frac{\ln\left(e^{bx+a}+1\right)}{2b}+\frac{\ln\left(e^{bx+a}-1\right)}{2b}$	65

input
```
int(cosh(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-cosh(b*x+a)/sinh(b*x+a)^2+1/2*coth(b*x+a)*csch(b*x+a)-arctanh(exp(b*
x+a)))
```

3.456.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(30) = 60$.

Time = 0.25 (sec) , antiderivative size = 387, normalized size of antiderivative = 11.38

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\cosh(bx+a)^3 + 6\cosh(bx+a)\sinh(bx+a)^2 + 2\sinh(bx+a)^3 + \left(\cosh(bx+a)^4 + 4\cosh(bx+a\right.}{}$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a
)^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh
(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b
*x + a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b
*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
+ 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a)
 + sinh(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b
*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b
*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a
)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

3.456.6 Sympy [F]

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int \cosh^2(a+bx)\operatorname{csch}^3(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

output
```
Integral(cosh(a + b*x)**2*csch(a + b*x)**3, x)
```

3.456.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.47

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1)}$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*log(e^(-b*x - a) + 1)/b + 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a)
 + e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

3.456.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.47

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4}+\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)-\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)
+ log(e^(b*x + a) + e^(-b*x - a) + 2) - log(e^(b*x + a) + e^(-b*x - a) - 2
))/b
```

3.456.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.56

$$\int \coth^2(a+bx)\mathrm{csch}(a+bx)\,dx = -\frac{\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(cosh(a + b*x)^2/sinh(a + b*x)^3,x)
```

output
```
- atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(
b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a
+ 2*b*x) - 1))
```

3.457 $\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx$

3.457.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx = \text{Int}\left(\frac{\text{csch}(a+bx)}{x},x\right) + \text{Int}\left(\frac{\text{csch}^3(a+bx)}{x},x\right)$$

output
```
Unintegrable(csch(b*x+a)/x,x)+Unintegrable(csch(b*x+a)^3/x,x)
```

3.457.2 Mathematica [N/A]

Not integrable

Time = 47.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx$$

input
```
Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x,x]
```

output
```
Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x, x]
```

3.457.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5980, 3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx$$

$$\Big\downarrow \; 5980$$

$$\int \frac{\text{csch}^3(a+bx)}{x}dx+\int \frac{\text{csch}(a+bx)}{x}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\csc(ia+ibx)}{x}dx+\int -\frac{i\csc(ia+ibx)^3}{x}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\csc(ia+ibx)}{x}dx-i\int \frac{\csc(ia+ibx)^3}{x}dx$$

$$\Big\downarrow \; 4680$$

$$\int \frac{\text{csch}^3(a+bx)}{x}dx+\int \frac{\text{csch}(a+bx)}{x}dx$$

input
```
Int[(Coth[a + b*x]^2*Csch[a + b*x])/x,x]
```

output
```
$Aborted
```

3.457.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5980
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

3.457.4 Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh (bx+a)^2 \operatorname{csch} (bx+a)^3}{x} dx$$

input
```
int(cosh(b*x+a)^2*csch(b*x+a)^3/x,x)
```

output
```
int(cosh(b*x+a)^2*csch(b*x+a)^3/x,x)
```

3.457.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^2\,\text{csch}(bx+a)^3}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^2*csch(b*x + a)^3/x, x)
```

3.457.6 Sympy [N/A]

Not integrable

Time = 51.84 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh^2(a+bx)\,\text{csch}^3(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)**2*csch(b*x+a)**3/x,x)
```

output
```
Integral(cosh(a + b*x)**2*csch(a + b*x)**3/x, x)
```

3.457.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 8.72

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^2\,\text{csch}(bx+a)^3}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
-((b*x*e^(3*a) - e^(3*a))*e^(3*b*x) + (b*x*e^a + e^a)*e^(b*x))/(b^2*x^2*e^
(4*b*x + 4*a) - 2*b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 2*integrate(1/4*(b^
2*x^2 + 2)/(b^2*x^3*e^(b*x + a) + b^2*x^3), x) + 2*integrate(1/4*(b^2*x^2
+ 2)/(b^2*x^3*e^(b*x + a) - b^2*x^3), x)
```

3.457.8 Giac [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh{(bx+a)}^2\,\text{csch}{(bx+a)}^3}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2*csch(b*x + a)^3/x, x)
```

3.457.9 Mupad [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^2}{x\sinh{(a+b\,x)}^3}\,dx$$

input
```
int(cosh(a + b*x)^2/(x*sinh(a + b*x)^3),x)
```

output
```
int(cosh(a + b*x)^2/(x*sinh(a + b*x)^3), x)
```

3.458 $\int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx$

3.458.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)}{x^2},x\right) + \operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)}{x^2},x\right)$$

output
```
Unintegrable(csch(b*x+a)/x^2,x)+Unintegrable(csch(b*x+a)^3/x^2,x)
```

3.458.2 Mathematica [N/A]

Not integrable

Time = 47.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx = \int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x^2,x]
```

output
```
Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x^2, x]
```

3.458.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5980, 3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \text{5980}$$

$$\int \frac{\text{csch}^3(a+bx)}{x^2}dx+\int \frac{\text{csch}(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\csc(ia+ibx)}{x^2}dx+\int -\frac{i\csc(ia+ibx)^3}{x^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\csc(ia+ibx)}{x^2}dx-i\int \frac{\csc(ia+ibx)^3}{x^2}dx$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\text{csch}^3(a+bx)}{x^2}dx+\int \frac{\text{csch}(a+bx)}{x^2}dx$$

input
```
Int[(Coth[a + b*x]^2*Csch[a + b*x])/x^2,x]
```

output
```
$Aborted
```

3.458.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5980
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

3.458.4 Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)^3}{x^2} dx$$

input
```
int(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x)
```

output
```
int(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x)
```

3.458.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)^2\,\text{csch}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^2*csch(b*x + a)^3/x^2, x)
```

3.458.6 Sympy [N/A]

Not integrable

Time = 61.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh^2(a+bx)\,\text{csch}^3(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)**2*csch(b*x+a)**3/x**2,x)
```

output
```
Integral(cosh(a + b*x)**2*csch(a + b*x)**3/x**2, x)
```

3.458.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 8.83

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)^2\,\text{csch}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
-((b*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x) + (b*x*e^a + 2*e^a)*e^(b*x))/(b^2*x^
3*e^(4*b*x + 4*a) - 2*b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 2*integrate(1/4
*(b^2*x^2 + 6)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x) + 2*integrate(1/4*(b^2*
x^2 + 6)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

3.458.8 Giac [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2*csch(b*x + a)^3/x^2, x)
```

3.458.9 Mupad [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^2}{x^2\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)^3),x)
```

output
```
int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)^3), x)
```

3.459 $\int x^m \coth^3(a+bx)\,dx$

3.459.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \coth^3(a+bx)\,dx = \operatorname{Int}\left(x^m \coth^3(a+bx),x\right)$$

output
```
Unintegrable(x^m*coth(b*x+a)^3,x)
```

3.459.2 Mathematica [N/A]

Not integrable

Time = 89.98 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \coth^3(a+bx)\,dx = \int x^m \coth^3(a+bx)\,dx$$

input
```
Integrate[x^m*Coth[a + b*x]^3,x]
```

output
```
Integrate[x^m*Coth[a + b*x]^3, x]
```

3.459.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix^m \tan\left(ia+ibx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{26}$$

$$i\int x^m \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3 dx$$

$$\Big\downarrow \text{4222}$$

$$\int -ix^m \tan^3\left(\frac{1}{2}(-\pi-2ia)-ibx\right) dx$$

input
```
Int[x^m*Coth[a + b*x]^3,x]
```

output
```
$Aborted
```

3.459.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.459.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int x^m \cosh (bx+a)^3 \operatorname{csch}(bx+a)^3 dx$$

input
```
int(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

output
```
int(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

3.459.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \coth^3(a+bx)\,dx = \int x^m \cosh (bx+a)^3 \operatorname{csch}(bx+a)^3\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

3.459.6 Sympy [F(-1)]

Timed out.

$$\int x^m \coth^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**m*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.459.7 Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 173, normalized size of antiderivative = 14.42

$$\int x^m \coth^3(a+bx)\,dx = \int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
x*e^(6*b*x + m*log(x) + 6*a)/((m + 1)*e^(6*b*x + 6*a) - 3*(m + 1)*e^(4*b*x
 + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) - m - 1) + integrate((3*(2*b*x*e^(6*a)
 + (m + 1)*e^(6*a))*e^(6*b*x) - 2*(m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m
 + 1)*e^(8*b*x + 8*a) - 4*(m + 1)*e^(6*b*x + 6*a) + 6*(m + 1)*e^(4*b*x + 4
*a) - 4*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

3.459.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \coth^3(a+bx)\,dx = \int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

3.459.9 Mupad [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \coth^3(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^m*cosh(a + b*x)^3)/sinh(a + b*x)^3,x)
```

output
```
int((x^m*cosh(a + b*x)^3)/sinh(a + b*x)^3, x)
```

3.460 $\int x^3 \coth^3(a+bx)\,dx$

3.460.1 Optimal result

Integrand size = 12, antiderivative size = 179

$$\begin{aligned}\int x^3 \coth^3(a+bx)\,dx = & -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b} \\ & +\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} \\ & +\frac{3\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2} \\ & -\frac{3x\,\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\,\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}\end{aligned}$$

output
```
-3/2*x^2/b^2+1/2*x^3/b-1/4*x^4-3/2*x^2*coth(b*x+a)/b^2-1/2*x^3*coth(b*x+a)
^2/b+3*x*ln(1-exp(2*b*x+2*a))/b^3+x^3*ln(1-exp(2*b*x+2*a))/b+3/2*polylog(2
,exp(2*b*x+2*a))/b^4+3/2*x^2*polylog(2,exp(2*b*x+2*a))/b^2-3/2*x*polylog(3
,exp(2*b*x+2*a))/b^3+3/4*polylog(4,exp(2*b*x+2*a))/b^4
```

3.460.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 422 vs. $2(179) = 358$.

Time = 2.14 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.36

$$\int x^3 \coth^3(a+bx)\,dx = \frac{1}{4}\left(x^4\coth(a) - \frac{2x^3\operatorname{csch}^2(a+bx)}{b}\right.$$
$$- \frac{2e^{2a}\left(6b^2e^{-2a}x^2 + b^4e^{-2a}x^4 - 6b\left(1-e^{-2a}\right)x\log\left(1-e^{-a-bx}\right) - 2b^3e^{-2a}\left(-1+e^{2a}\right)x^3\log\left(1-e^{-a-bx}\right) - \cdots\right.}{}$$
$$\left. + \frac{6x^2\operatorname{csch}(a)\operatorname{csch}(a+bx)\sinh(bx)}{b^2}\right)$$

input
```
Integrate[x^3*Coth[a + b*x]^3,x]
```

output
```
(x^4*Coth[a] - (2*x^3*Csch[a + b*x]^2)/b - (2*E^(2*a)*((6*b^2*x^2)/E^(2*a)
 + (b^4*x^4)/E^(2*a) - 6*b*(1 - E^(-2*a))*x*Log[1 - E^(-a - b*x)] - (2*b^3
*(-1 + E^(2*a))*x^3*Log[1 - E^(-a - b*x)])/E^(2*a) - 6*b*(1 - E^(-2*a))*x*
Log[1 + E^(-a - b*x)] - (2*b^3*(-1 + E^(2*a))*x^3*Log[1 + E^(-a - b*x)])/E
^(2*a) + 6*(1 - E^(-2*a))*PolyLog[2, -E^(-a - b*x)] + 6*b^2*(1 - E^(-2*a))
*x^2*PolyLog[2, -E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[2, E^(-a - b*x)]
 + 6*b^2*(1 - E^(-2*a))*x^2*PolyLog[2, E^(-a - b*x)] + 12*b*(1 - E^(-2*a))
*x*PolyLog[3, -E^(-a - b*x)] + 12*b*(1 - E^(-2*a))*x*PolyLog[3, E^(-a - b*
x)] + 12*(1 - E^(-2*a))*PolyLog[4, -E^(-a - b*x)] + 12*(1 - E^(-2*a))*Poly
Log[4, E^(-a - b*x)]))/(b^4*(-1 + E^(2*a))) + (6*x^2*Csch[a]*Csch[a + b*x]
*Sinh[b*x])/b^2)/4
```

3.460.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.49, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.917, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 4201, 2620, 3011, 4203, 15, 26, 3042, 26, 4201, 2620, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix^3 \tan\left(ia+ibx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \; 26$$

$$i\int x^3 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3 dx$$

$$\Big\downarrow \; 4203$$

$$i\left(-\int ix^3 \coth(a+bx)dx+\frac{3i\int -x^2\coth^2(a+bx)dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \; 25$$

$$i\left(-\int ix^3 \coth(a+bx)dx-\frac{3i\int x^2\coth^2(a+bx)dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(-i\int x^3 \coth(a+bx)dx-\frac{3i\int x^2\coth^2(a+bx)dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(-i\int -ix^3 \tan\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{3i\int -x^2\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \; 25$$

$$i\left(-i\int -ix^3 \tan\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{3i\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(-\int x^3 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx+\frac{3i\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \; 4201$$

$$i\left(-2i\int \frac{e^{2a+2bx-i\pi}x^3}{1+e^{2a+2bx-i\pi}}dx+\frac{3i\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}+\frac{ix^4}{4}\right)$$

$$\Big\downarrow \; 2620$$

$$i\left(\frac{3i\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{2b}-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\int x^2\log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)+\frac{ix^3\coth^2}{2}\right.$$

$$\Big\downarrow \; 3011$$

3.460. $\int x^3 \coth^3(a+bx)\,dx$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\frac{3i\int x^2\tan\left(\frac{1}{2}(2ia\right.}{2b}\right.$$

$\Big\downarrow$ 4203

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\int ix\coth(a+bx)}{b}\right.}{}\right.$$

$\Big\downarrow$ 15

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\int ix\coth(a+bx)}{b}\right.}{}\right.$$

$\Big\downarrow$ 26

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(-\frac{2\int x\coth(a+bx}{b}\right.}{}\right.$$

$\Big\downarrow$ 3042

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(-\frac{2\int -ix\tan(ia+}{b}\right.}{}\right.$$

$\Big\downarrow$ 26

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\int x\tan\left(\frac{1}{2}(2ia+\right.}{b}\right.}{}\right.$$

$\Big\downarrow$ 4201

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\left(2i\int\frac{e^{2a+2bx-}}{1+e^{2a+2bx}}\right.}{b}\right.}{}\right.$$

$\Big\downarrow$ 2620

3.460. $\int x^3\coth^3(a+bx)\,dx$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\left(2i\left(\frac{x\log\left(1+e^{2}\right.}{}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 2715

$$i\left(\frac{3i\left(\frac{2i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\log\left(1+e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}\right)-\frac{ix^2}{2}\right)}{b}+\frac{x^2\coth(a+bx)}{b}-\frac{x^3}{3}\right)}{2b}-2i\left(\frac{x^3\log\left(1\right.}{}\right.\right.$$

$\downarrow$ 2838

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\left(2i\left(\frac{\text{PolyLog}\left(2\right.}{}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 7163

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)\right.$$

$\downarrow$ 2720

3.460. $\int x^3\coth^3(a+bx)\,dx$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}}{}\right.}{2b}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\text{PolyLog}\left(4,-e^{2a+2bx-i\pi}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\right.$$

input
```
Int[x^3*Coth[a + b*x]^3,x]
```

output
```
I*((I/4)*x^4 + ((I/2)*x^3*Coth[a + b*x]^2)/b + (((3*I)/2)*(-1/3*x^3 + (x^2
*Coth[a + b*x])/b + ((2*I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*a - I*Pi
 + 2*b*x)])/(2*b) + PolyLog[2, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2))))/b))/b -
 (2*I)*((x^3*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) - (3*(-1/2*(x^2*PolyLo
g[2, -E^(2*a - I*Pi + 2*b*x)])/b + ((x*PolyLog[3, -E^(2*a - I*Pi + 2*b*x)]
)/(2*b) - PolyLog[4, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2))/b))/(2*b)))
```

3.460.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.460. $\int x^3\coth^3(a+bx)\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.460. $\int x^3 \coth^3(a + bx)\, dx$

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.460.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(161) = 322$.

Time = 0.53 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.09

method	result
risch	$-\frac{3a^2}{b^4}+\frac{\ln(e^{bx+a}+1)x^3}{b}-\frac{x^4}{4}-\frac{3x^2}{b^2}-\frac{3a\ln(e^{bx+a}-1)}{b^4}-\frac{a^3\ln(e^{bx+a}-1)}{b^4}+\frac{3\ln(e^{bx+a}+1)x}{b^3}+\frac{3\ln(1-e^{bx+a})x}{b^3}+$ 3 ln

input
```
int(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/b^4*a^2+1/b*ln(exp(b*x+a)+1)*x^3-1/4*x^4-3/b^2*x^2-3/b^4*a*ln(exp(b*x+a
)-1)-1/b^4*a^3*ln(exp(b*x+a)-1)+3/b^3*ln(exp(b*x+a)+1)*x+3/b^3*ln(1-exp(b*
x+a))*x+3/b^4*ln(1-exp(b*x+a))*a-x^2*(2*exp(2*b*x+2*a)*b*x+3*exp(2*b*x+2*a
)-3)/b^2/(exp(2*b*x+2*a)-1)^2+6/b^4*a*ln(exp(b*x+a))-6/b^3*a*x+2/b^4*a^3*l
n(exp(b*x+a))-2/b^3*a^3*x+3*x^2*polylog(2,-exp(b*x+a))/b^2+3*x^2*polylog(2
,exp(b*x+a))/b^2-6*x*polylog(3,-exp(b*x+a))/b^3-6*x*polylog(3,exp(b*x+a))/
b^3+6*polylog(4,-exp(b*x+a))/b^4+6*polylog(4,exp(b*x+a))/b^4+1/b*ln(1-exp(
b*x+a))*x^3+1/b^4*ln(1-exp(b*x+a))*a^3-3/2/b^4*a^4+3*polylog(2,-exp(b*x+a)
)/b^4+3*polylog(2,exp(b*x+a))/b^4
```

3.460.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1985 vs. $2(159) = 318$.

Time = 0.29 (sec) , antiderivative size = 1985, normalized size of antiderivative = 11.09

$$\int x^3 \coth^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(b^4*x^4 + (b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^4 +
4*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*sinh(b*x + a)^4 - 2*a^4 - 2*(b^4*x
^4 - 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 - 2*(b^4*x^4
- 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 3*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2
)*cosh(b*x + a)^2 - 12*a^2)*sinh(b*x + a)^2 - 12*a^2 - 12*((b^2*x^2 + 1)*c
osh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2
+ 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 - 2*(b^2*
x^2 - 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 +
 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilo
g(cosh(b*x + a) + sinh(b*x + a)) - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(
b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4
 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1
)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3
- (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - s
inh(b*x + a)) - 4*(b^3*x^3 + (b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^
3 + 3*b*x)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 3*b*x)*sinh(b*x + a)
^4 - 2*(b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 + 3*b*x
)*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a)^2 + 3*b*x + 4*((b^3*x^3 + 3*b*x)*
cosh(b*x + a)^3 - (b^3*x^3 + 3*b*x)*cosh(b*x + a))*sinh(b*x + a))*log(c...
```

3.460.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \coth^3(a + bx) \, dx = \text{Timed out}$$

input
```
integrate(x**3*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.460.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.69

$$\int x^3 \coth^3(a+bx)\,dx$$
$$= \frac{b^2x^4e^{(4\,bx+4\,a)} + b^2x^4 + 12\,x^2 - 2\left(b^2x^4e^{(2\,a)} + 4\,bx^3e^{(2\,a)} + 6\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{4\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} - \frac{b^4x^4 + 6\,b^2x^2}{2\,b^4}$$
$$+ \frac{b^3x^3\log\left(e^{(bx+a)}+1\right) + 3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 6\,bx\mathrm{Li}_3(-e^{(bx+a)}) + 6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4}$$
$$+ \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right) + 3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right) - 6\,bx\mathrm{Li}_3(e^{(bx+a)}) + 6\,\mathrm{Li}_4(e^{(bx+a)})}{b^4}$$
$$+ \frac{3\left(bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4} + \frac{3\left(bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(b^2*x^4*e^(4*b*x + 4*a) + b^2*x^4 + 12*x^2 - 2*(b^2*x^4*e^(2*a) + 4*b
*x^3*e^(2*a) + 6*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2
*b*x + 2*a) + b^2) - 1/2*(b^4*x^4 + 6*b^2*x^2)/b^4 + (b^3*x^3*log(e^(b*x +
 a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a))
+ 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2
*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(
b*x + a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^4 +
3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

3.460.8 Giac [F]

$$\int x^3 \coth^3(a+bx)\,dx = \int x^3 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3\,dx$$

input
```
integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

3.460.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^3(a+bx)\,dx = \int \frac{x^3 \cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^3*cosh(a + b*x)^3)/sinh(a + b*x)^3,x)
```

output
```
int((x^3*cosh(a + b*x)^3)/sinh(a + b*x)^3, x)
```

3.461 $\int x^2 \coth^3(a+bx)\,dx$

3.461.1 Optimal result

Integrand size = 12, antiderivative size = 114

$$\int x^2 \coth^3(a+bx)\,dx = \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\log(\sinh(a+bx))}{b^3} + \frac{x\,\mathrm{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2} - \frac{\mathrm{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}$$

output
```
1/2*x^2/b-1/3*x^3-x*coth(b*x+a)/b^2-1/2*x^2*coth(b*x+a)^2/b+x^2*ln(1-exp(2
*b*x+2*a))/b+ln(sinh(b*x+a))/b^3+x*polylog(2,exp(2*b*x+2*a))/b^2-1/2*polyl
og(3,exp(2*b*x+2*a))/b^3
```

3.461.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 314 vs. 2(114) = 228.

Time = 2.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.75

$$\int x^2 \coth^3(a+bx)\,dx = \frac{1}{3}x^3\coth(a) - \frac{x^2\mathrm{csch}^2(a+bx)}{2b} - \frac{e^{2a}\left(6be^{-2a}x + 6b\left(1-e^{-2a}\right)x + 2b^3e^{-2a}x^3 - 3b^2e^{-2a}\left(-1+e^{2a}\right)x^2\log\left(1-e^{-a-bx}\right) - 3b^2e^{-2a}\left(-1+e^{2a}\right.\cdots\right.}{\cdots} + \frac{x\mathrm{csch}(a)\mathrm{csch}(a+bx)\sinh(bx)}{b^2}$$

input
```
Integrate[x^2*Coth[a + b*x]^3,x]
```

output
```
(x^3*Coth[a])/3 - (x^2*Csch[a + b*x]^2)/(2*b) - (E^(2*a)*((6*b*x)/E^(2*a)
+ 6*b*(1 - E^(-2*a))*x + (2*b^3*x^3)/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*L
og[1 - E^(-a - b*x)])/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 + E^(-a -
b*x)])/E^(2*a) - 3*(1 - E^(-2*a))*Log[1 - E^(a + b*x)] - 3*(1 - E^(-2*a))*
Log[1 + E^(a + b*x)] + 6*b*(1 - E^(-2*a))*x*PolyLog[2, -E^(-a - b*x)] + 6*
b*(1 - E^(-2*a))*x*PolyLog[2, E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[3,
-E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[3, E^(-a - b*x)]))/(3*b^3*(-1 +
E^(2*a))) + (x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b^2
```

3.461.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.47, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.583, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 4201, 2620, 3011, 2720, 4203, 15, 26, 3042, 26, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix^2 \tan\left(ia+ibx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{26}$$

$$i\int x^2 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3 dx$$

$$\Big\downarrow \text{4203}$$

$$i\left(-\int ix^2\coth(a+bx)dx+\frac{i\int -x\coth^2(a+bx)dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\int ix^2\coth(a+bx)dx-\frac{i\int x\coth^2(a+bx)dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(-i\int x^2\coth(a+bx)dx-\frac{i\int x\coth^2(a+bx)dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(-i\int -ix^2\tan\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{i\int -x\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 25}$$

$$i\left(-i\int -ix^2\tan\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(-\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx+\frac{i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 4201}$$

$$i\left(-2i\int\frac{e^{2a+2bx-i\pi}x^2}{1+e^{2a+2bx-i\pi}}dx+\frac{i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}+\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int x\log\left(1+e^{2a+2bx-i\pi}\right)dx}{b}\right)+\frac{i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{b}+\frac{ix^2\coth^2(a}{2b}\right.$$

$$\Big\downarrow \text{ 3011}$$

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int \text{PolyLog}(2,-e^{2a+2bx-i\pi})dx}{2b}-\frac{x\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}}{b}\right)+\frac{i\int x\tan\left(\frac{1}{2}(2ia+\pi)+i\right.}{b}\right.$$

$$\Big\downarrow \text{ 2720}$$

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}}{b}\right)+\frac{i\int x\,t}{}\right.$$

$$\Big\downarrow \text{ 4203}$$

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}}{b}\right)+\frac{i\left(\frac{i\int i}{}\right.}{}\right.$$

3.461. $\int x^2\coth^3(a+bx)\,dx$

$$\Big\downarrow \text{15}$$

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(\frac{i\int i\right.}{}\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(-\int\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(-\int\right.}{}\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(\frac{i\int t\right.}{}\right.$$

$$\Big\downarrow \text{3956}$$

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(-\frac{\text{lo}}{}\right.}{}\right.$$

$$\Big\downarrow \text{7143}$$

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(-\frac{\log(-i\sinh(a+bx))}{b^2}+\frac{x\coth}{}\right.}{b}\right.$$

input
```
Int[x^2*Coth[a + b*x]^3,x]
```

output
```
I*((I/3)*x^3 + ((I/2)*x^2*Coth[a + b*x]^2)/b + (I*(-1/2*x^2 + (x*Coth[a +
b*x])/b - Log[(-I)*Sinh[a + b*x]]/b^2))/b - (2*I)*((x^2*Log[1 + E^(2*a - I
*Pi + 2*b*x)])/(2*b) - (-1/2*(x*PolyLog[2, -E^(2*a - I*Pi + 2*b*x)])/b + P
olyLog[3, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2))/b))
```

3.461. $\int x^2 \coth^3(a+bx)\,dx$

3.461.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.461. $\int x^2 \coth^3(a+bx)\,dx$

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.461.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(106) = 212$.

Time = 0.52 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.16

method	result
risch	$-\frac{x^3}{3} - \frac{2x\left(e^{2bx+2a}bx+e^{2bx+2a}-1\right)}{b^2\left(e^{2bx+2a}-1\right)^2} + \frac{\ln\left(e^{bx+a}+1\right)x^2}{b} + \frac{2x\,\mathrm{polylog}\left(2,-e^{bx+a}\right)}{b^2} + \frac{\ln\left(1-e^{bx+a}\right)x^2}{b} + \frac{2x\,\mathrm{polylog}\left(2,e^{bx+a}\right)}{b^2}$

input
```
int(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3-2*x*(exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^
2+1/b*ln(exp(b*x+a)+1)*x^2+2*x*polylog(2,-exp(b*x+a))/b^2+1/b*ln(1-exp(b*x
+a))*x^2+2*x*polylog(2,exp(b*x+a))/b^2+4/3/b^3*a^3+2/b^2*a^2*x+1/b^3*ln(ex
p(b*x+a)-1)+1/b^3*ln(exp(b*x+a)+1)-2/b^3*ln(exp(b*x+a))-2*polylog(3,-exp(b
*x+a))/b^3-2*polylog(3,exp(b*x+a))/b^3+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^
2*ln(exp(b*x+a))-1/b^3*ln(1-exp(b*x+a))*a^2
```

3.461.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. $2(105) = 210$.

Time = 0.28 (sec) , antiderivative size = 1467, normalized size of antiderivative = 12.87

$$\int x^2 \coth^3(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/3*(b^3*x^3 + (b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^4 + 4*(b^3*x
^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 2*a^3
 + 6*b*x + 6*a)*sinh(b*x + a)^4 + 2*a^3 - 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 +
 3*b*x + 6*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 - 3*(b^3*x^
3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^2 + 3*b*x + 6*a)*sinh(b*x + a)^2 -
6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*
x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*
x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)
)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*co
sh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2
+ 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x
+ a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x
 + a)) - 3*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*
sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1
)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x +
a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 3*((a^2 +
1)*cosh(b*x + a)^4 + 4*(a^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 + 1)
*sinh(b*x + a)^4 - 2*(a^2 + 1)*cosh(b*x + a)^2 + 2*(3*(a^2 + 1)*cosh(b*x +
 a)^2 - a^2 - 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 + 1)*cosh(b*x + a)^3 -...
```

3.461.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \coth^3(a + bx)\, dx = \text{Timed out}$$

input
```
integrate(x**2*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.461.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(105) = 210$.

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.98

$$\int x^2 \coth^3(a+bx)\,dx$$
$$= -\frac{2}{3}x^3 + \frac{b^2x^3e^{(4\,bx+4\,a)} + b^2x^3 - 2\left(b^2x^3e^{(2\,a)} + 3\,bx^2e^{(2\,a)} + 3\,xe^{(2\,a)}\right)e^{(2\,bx)} + 6\,x}{3\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)}$$
$$- \frac{2\,x}{b^2} + \frac{b^2x^2\log\left(e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)}{b^3}$$
$$+ \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right) - 2\,\mathrm{Li}_3\left(e^{(bx+a)}\right)}{b^3}$$
$$+ \frac{\log\left(e^{(bx+a)}+1\right)}{b^3} + \frac{\log\left(e^{(bx+a)}-1\right)}{b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2/3*x^3 + 1/3*(b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3 - 2*(b^2*x^3*e^(2*a) + 3
*b*x^2*e^(2*a) + 3*x*e^(2*a))*e^(2*b*x) + 6*x)/(b^2*e^(4*b*x + 4*a) - 2*b^
2*e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x
*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(
b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
+ log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3
```

3.461.8 Giac [F]

$$\int x^2 \coth^3(a+bx)\,dx = \int x^2 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3\,dx$$

input
```
integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

3.461.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^3(a+bx)\,dx = \int \frac{x^2\cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x^2*cosh(a + b*x)^3)/sinh(a + b*x)^3,x)
```

output
```
int((x^2*cosh(a + b*x)^3)/sinh(a + b*x)^3, x)
```

3.462 $\int x \coth^3(a+bx)\,dx$

3.462.1 Optimal result

Integrand size = 10, antiderivative size = 82

$$\int x \coth^3(a+bx)\,dx = \frac{x}{2b}-\frac{x^2}{2}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}$$

output
```
1/2*x/b-1/2*x^2-1/2*coth(b*x+a)/b^2-1/2*x*coth(b*x+a)^2/b+x*ln(1-exp(2*b*x
+2*a))/b+1/2*polylog(2,exp(2*b*x+2*a))/b^2
```

3.462.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.60

$$\int x \coth^3(a+bx)\,dx = \frac{1}{2}\left(-\frac{2x^2}{-1+e^{2a}}+x^2\coth(a)-\frac{x\text{csch}^2(a+bx)}{b}+\frac{2x\log\left(1-e^{-a-bx}\right)}{b}+\frac{2x\log\left(1+e^{-a-bx}\right)}{b}-\frac{2\,\text{PolyLog}\left(2,-e^{-a-bx}\right)}{b^2}-\frac{2\,\text{PolyLog}\left(2,e^{-a-bx}\right)}{b^2}+\frac{\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b^2}\right)$$

input
```
Integrate[x*Coth[a + b*x]^3,x]
```

output
```
((-2*x^2)/(-1 + E^(2*a)) + x^2*Coth[a] - (x*Csch[a + b*x]^2)/b + (2*x*Log[
1 - E^(-a - b*x)])/b + (2*x*Log[1 + E^(-a - b*x)])/b - (2*PolyLog[2, -E^(-
a - b*x)])/b^2 - (2*PolyLog[2, E^(-a - b*x)])/b^2 + (Csch[a]*Csch[a + b*x]
*Sinh[b*x])/b^2)/2
```

3.462.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 3954, 24, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix\tan\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{4203}$$

$$i\left(\frac{i\int -\coth^2(a+bx)dx}{2b}-\int ix\coth(a+bx)dx+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{i\int \coth^2(a+bx)dx}{2b}-\int ix\coth(a+bx)dx+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{i\int \coth^2(a+bx)dx}{2b}-i\int x\coth(a+bx)dx+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-i\int -ix\tan\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{i\int -\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{2b}+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-i\int -ix\tan\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{i\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{2b}+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx+\frac{i\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{2b}+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3954}$$

$$i\left(-\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx+\frac{i\left(\frac{\coth(a+bx)}{b}-\int 1dx\right)}{2b}+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{24}$$

$$i\left(-\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx+\frac{ix\coth^2(a+bx)}{2b}+\frac{i\left(\frac{\coth(a+bx)}{b}-x\right)}{2b}\right)$$

$$\Big\downarrow \text{4201}$$

$$i\left(-2i\int \frac{e^{2a+2bx-i\pi}x}{1+e^{2a+2bx-i\pi}}dx+\frac{ix\coth^2(a+bx)}{2b}+\frac{i\left(\frac{\coth(a+bx)}{b}-x\right)}{2b}+\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$i\left(-2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int \log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)+\frac{ix\coth^2(a+bx)}{2b}+\frac{i\left(\frac{\coth(a+bx)}{b}-x\right)}{2b}+\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$i\left(-2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\log\left(1+e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}\right)+\frac{ix\coth^2(a+bx)}{2b}+\frac{i\left(\frac{\coth(a+}{b}\right.}{2}\right.$$

$$\Big\downarrow \text{2838}$$

$$i\left(-2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)+\frac{ix\coth^2(a+bx)}{2b}+\frac{i\left(\frac{\coth(a+bx)}{b}-x\right)}{2b}+\frac{ix^2}{2}\right)$$

3.462. $\int x\coth^3(a+bx)\,dx$

input
```
Int[x*Coth[a + b*x]^3,x]
```

output
```
I*((I/2)*x^2 + ((I/2)*x*Coth[a + b*x]^2)/b + ((I/2)*(-x + Coth[a + b*x]/b)
)/b - (2*I)*((x*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) + PolyLog[2, -E^(2*
a - I*Pi + 2*b*x)]/(4*b^2)))
```

3.462.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.462.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(72) = 144$.

Time = 0.52 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.00

method	result
risch	$-\frac{x^2}{2}-\frac{2\,\mathrm{e}^{2bx+2a}bx+\mathrm{e}^{2bx+2a}-1}{b^2\left(\mathrm{e}^{2bx+2a}-1\right)^2}-\frac{2ax}{b}-\frac{a^2}{b^2}+\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)x}{b}+\frac{\operatorname{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^2}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a}{b^2}$

input
```
int(x*cosh(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2-(2*exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-
2/b*a*x-a^2/b^2+1/b*ln(exp(b*x+a)+1)*x+polylog(2,-exp(b*x+a))/b^2+1/b*ln(1
-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2-1/b^2*a*
ln(exp(b*x+a)-1)+2/b^2*a*ln(exp(b*x+a))
```

3.462.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(71) = 142$.

Time = 0.28 (sec) , antiderivative size = 975, normalized size of antiderivative = 11.89

$$\int x\coth^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x + a
)*sinh(b*x + a)^3 + (b^2*x^2 - 2*a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x
^2 - 2*a^2 - 2*b*x - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 2*a^2)
*cosh(b*x + a)^2 - 2*a^2 - 2*b*x - 1)*sinh(b*x + a)^2 - 2*a^2 - 2*(cosh(b*
x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 -
cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2
*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*
(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x
+ a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x
 + a)) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*
x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x
)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh
(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*(a*cosh(b*x + a)^4 +
 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)
^2 + 2*(3*a*cosh(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 -
a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 - 2*((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^
3 + (b*x + a)*sinh(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x +
a)*cosh(b*x + a)^2 - b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh...
```

3.462.6 Sympy [F(-1)]

Timed out.

$$\int x\coth^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.462.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.82

$$\int x\coth^3(a+bx)\,dx = -x^2 + \frac{b^2x^2e^{(4\,bx+4\,a)} + b^2x^2 - 2\left(b^2x^2e^{(2\,a)} + 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)} + 2}{2\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} + \frac{bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

input
```
integrate(x*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-x^2 + 1/2*(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 - 2*(b^2*x^2*e^(2*a) + 2*b*x
*e^(2*a) + e^(2*a))*e^(2*b*x) + 2)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x +
 2*a) + b^2) + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x
*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

3.462.8 Giac [F]

$$\int x\coth^3(a+bx)\,dx = \int x\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)^3\,dx$$

input
```
integrate(x*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

3.462.9 Mupad [F(-1)]

Timed out.

$$\int x\coth^3(a+bx)\,dx = \int \frac{x\cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((x*cosh(a + b*x)^3)/sinh(a + b*x)^3,x)
```

output
```
int((x*cosh(a + b*x)^3)/sinh(a + b*x)^3, x)
```

3.463 $\int \coth^3(a+bx)\,dx$

3.463.1 Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \coth^3(a+bx)\,dx = -\frac{\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b}$$

output
```
-1/2*coth(b*x+a)^2/b+ln(sinh(b*x+a))/b
```

3.463.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \coth^3(a+bx)\,dx = -\frac{\coth^2(a+bx) - 2\log(\cosh(a+bx)) - 2\log(\tanh(a+bx))}{2b}$$

input
```
Integrate[Coth[a + b*x]^3,x]
```

output
```
-1/2*(Coth[a + b*x]^2 - 2*Log[Cosh[a + b*x]] - 2*Log[Tanh[a + b*x]])/b
```

3.463.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 26, 3954, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{3954}$$

$$i\left(\frac{i\coth^2(a+bx)}{2b}-\int i\coth(a+bx)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth^2(a+bx)}{2b}-i\int \coth(a+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\coth^2(a+bx)}{2b}-i\int -i\tan\left(ia+ibx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth^2(a+bx)}{2b}-\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$i\left(\frac{i\coth^2(a+bx)}{2b}-\frac{i\log(-i\sinh(a+bx))}{b}\right)$$

input
```
Int[Coth[a + b*x]^3,x]
```

output
```
I*(((I/2)*Coth[a + b*x]^2)/b - (I*Log[(-I)*Sinh[a + b*x]])/b)
```

3.463.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.463.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\ln(\sinh(bx+a))-\frac{\coth(bx+a)^2}{2}}{b}$	23
default	$\frac{\ln(\sinh(bx+a))-\frac{\coth(bx+a)^2}{2}}{b}$	23
parallelrisch	$\frac{-2bx+2\ln(\tanh(bx+a))-2\ln(1-\tanh(bx+a))-\coth(bx+a)^2}{2b}$	43
risch	$-x-\frac{2a}{b}-\frac{2\,\mathrm{e}^{2bx+2a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)^2}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	54

input
```
int(cosh(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(ln(sinh(b*x+a))-1/2*coth(b*x+a)^2)
```

3.463. $\int \coth^3(a+bx)\,dx$

3.463.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 346, normalized size of antiderivative = 12.81

$$\int \coth^3(a+bx)\,dx =$$

$$-\frac{bx\cosh{(bx+a)}^4 + 4\,bx\cosh{(bx+a)}\sinh{(bx+a)}^3 + bx\sinh{(bx+a)}^4 - 2\,(bx-1)\cosh{(bx+a)}^2 + 2}{}$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
 + a)^4 - 2*(b*x - 1)*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x + 1
)*sinh(b*x + a)^2 + b*x - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)
^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(
b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*
sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(b*x*cosh(b*x + a)^3 -
(b*x - 1)*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x
+ a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*co
sh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a
))*sinh(b*x + a) + b)
```

3.463.6 Sympy [F]

$$\int \coth^3(a+bx)\,dx = \int \cosh^3{(a+bx)}\operatorname{csch}^3{(a+bx)}\,dx$$

input
```
integrate(cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

output
```
Integral(cosh(a + b*x)**3*csch(a + b*x)**3, x)
```

3.463.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.93

$$\int \coth^3(a+bx)\,dx = x + \frac{a}{b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1)}$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
x + a/b + log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-2*b*x
- 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

3.463.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int \coth^3(a+bx)\,dx = -\frac{2\,bx + 2\,a + \frac{3\,e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)} + 3}{\left(e^{(2\,bx+2\,a)}-1\right)^2} - 2\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{2\,b}$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*(2*b*x + 2*a + (3*e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 3)/(e^(2*b*x
+ 2*a) - 1)^2 - 2*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.463.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \coth^3(a+bx)\,dx = \frac{\ln\left(\sinh(a+b\,x)\right)}{b} - \frac{1}{2\,b\sinh\left(a+b\,x\right)^2}$$

input
```
int(cosh(a + b*x)^3/sinh(a + b*x)^3,x)
```

output
```
log(sinh(a + b*x))/b - 1/(2*b*sinh(a + b*x)^2)
```

3.464 $\int \frac{\coth^3(a+bx)}{x} \, dx$

3.464.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\coth^3(a+bx)}{x} \, dx = \text{Int}\left(\frac{\coth^3(a+bx)}{x}, x\right)$$

output
```
Unintegrable(coth(b*x+a)^3/x,x)
```

3.464.2 Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^3(a+bx)}{x} \, dx = \int \frac{\coth^3(a+bx)}{x} \, dx$$

input
```
Integrate[Coth[a + b*x]^3/x,x]
```

output
```
Integrate[Coth[a + b*x]^3/x, x]
```

3.464.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i \tan\left(ia+ibx+\frac{\pi}{2}\right)^3}{x}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3}{x}dx$$

$$\Big\downarrow \text{4222}$$

$$\int -\frac{i\tan^3\left(\frac{1}{2}(-\pi-2ia)-ibx\right)}{x}dx$$

input
```
Int[Coth[a + b*x]^3/x,x]
```

output
```
$Aborted
```

3.464.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.464.4 Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cosh\left(bx+a\right)^{3} \operatorname{csch}\left(bx+a\right)^{3}}{x}dx$$

input
```
int(cosh(b*x+a)^3*csch(b*x+a)^3/x,x)
```

output
```
int(cosh(b*x+a)^3*csch(b*x+a)^3/x,x)
```

3.464.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^{3} \operatorname{csch}\left(bx+a\right)^{3}}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^3*csch(b*x + a)^3/x, x)
```

3.464.6 Sympy [N/A]

Not integrable

Time = 118.96 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\cosh^3(a+bx)\operatorname{csch}^3(a+bx)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)**3*csch(b*x+a)**3/x,x)
```

output
```
Integral(cosh(a + b*x)**3*csch(a + b*x)**3/x, x)
```

3.464.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 12.00

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^3}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
-((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^
2*x^2*e^(2*b*x + 2*a) + b^2*x^2) - integrate((b^2*x^2 + 1)/(b^2*x^3*e^(b*x
 + a) + b^2*x^3), x) + integrate((b^2*x^2 + 1)/(b^2*x^3*e^(b*x + a) - b^2*
x^3), x) + log(x)
```

3.464.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^3}{x}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3*csch(b*x + a)^3/x, x)
```

3.464.9 Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(a+bx)}{x}\,dx=\int \frac{\cosh(a+b\,x)^3}{x\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(cosh(a + b*x)^3/(x*sinh(a + b*x)^3),x)
```

output
```
int(cosh(a + b*x)^3/(x*sinh(a + b*x)^3), x)
```

3.465 $\int \frac{\coth^3(a+bx)}{x^2}\,dx$

3.465.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \text{Int}\left(\frac{\coth^3(a+bx)}{x^2},x\right)$$

output
```
Unintegrable(coth(b*x+a)^3/x^2,x)
```

3.465.2 Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\coth^3(a+bx)}{x^2}\,dx$$

input
```
Integrate[Coth[a + b*x]^3/x^2,x]
```

output
```
Integrate[Coth[a + b*x]^3/x^2, x]
```

3.465.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(a+bx)}{x^2} \, dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{i \tan\left(ia+ibx+\frac{\pi}{2}\right)^3}{x^2}dx$$
$$\Big\downarrow \ 26$$
$$i\int \frac{\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3}{x^2}dx$$
$$\Big\downarrow \ 4222$$
$$\int -\frac{i\tan^3\left(\frac{1}{2}(-\pi-2ia)-ibx\right)}{x^2}dx$$

input
```
Int[Coth[a + b*x]^3/x^2,x]
```

output
```
$Aborted
```

3.465.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

3.465.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cosh\left(bx+a\right)^{3} \operatorname{csch}\left(bx+a\right)^{3}}{x^{2}}dx$$

input
```
int(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x)
```

output
```
int(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x)
```

3.465.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^{3} \operatorname{csch}\left(bx+a\right)^{3}}{x^{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^3*csch(b*x + a)^3/x^2, x)
```

3.465.6 Sympy [N/A]

Not integrable

Time = 159.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\cosh^3(a+bx)\operatorname{csch}^3(a+bx)}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)**3*csch(b*x+a)**3/x**2,x)
```

output
```
Integral(cosh(a + b*x)**3*csch(a + b*x)**3/x**2, x)
```

3.465.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 175, normalized size of antiderivative = 14.58

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
-(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 - 2*(b^2*x^2*e^(2*a) - b*x*e^(2*a) + e
^(2*a))*e^(2*b*x) + 2)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^2*x^3*e^(2*b*x + 2*a
) + b^2*x^3) - integrate((b^2*x^2 + 3)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x)
 + integrate((b^2*x^2 + 3)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

3.465.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3*csch(b*x + a)^3/x^2, x)
```

3.465.9 Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^3}{x^2\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)^3),x)
```

output
```
int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)^3), x)
```

3.466 $\int x^m \mathbf{csch}(a+bx)\mathbf{sech}(a+bx)\,dx$

3.466.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx = \text{Int}(x^m \text{csch}(a+bx)\text{sech}(a+bx),x)$$

output
```
CannotIntegrate(x^m*csch(b*x+a)*sech(b*x+a),x)
```

3.466.2 Mathematica [N/A]

Not integrable

Time = 8.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx$$

input
```
Integrate[x^m*Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
Integrate[x^m*Csch[a + b*x]*Sech[a + b*x], x]
```

3.466.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int x^m \text{csch}(a+bx)\text{sech}(a+bx)dx$$

input
```
Int[x^m*Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
$Aborted
```

3.466.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.466.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)\,dx$$

input
```
int(x^m*csch(b*x+a)*sech(b*x+a),x)
```

output
```
int(x^m*csch(b*x+a)*sech(b*x+a),x)
```

3.466.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)\;dx$$

input
```
integrate(x^m*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*csch(b*x + a)*sech(b*x + a), x)
```

3.466.6 Sympy [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^m \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(x**m*csch(b*x+a)*sech(b*x+a),x)
```

output
```
Integral(x**m*csch(a + b*x)*sech(a + b*x), x)
```

3.466.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)\;dx$$

input
```
integrate(x^m*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^m*csch(b*x + a)*sech(b*x + a), x)
```

3.466.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)\,dx$$

input
```
integrate(x^m*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*csch(b*x + a)*sech(b*x + a), x)
```

3.466.9 Mupad [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int \frac{x^m}{\cosh(a+b\,x)\,\sinh(a+b\,x)}\,dx$$

input
```
int(x^m/(cosh(a + b*x)*sinh(a + b*x)),x)
```

output
```
int(x^m/(cosh(a + b*x)*sinh(a + b*x)), x)
```

3.467 $\int x^3\text{csch}(a+bx)\text{sech}(a+bx)\,dx$

3.467.1 Optimal result

Integrand size = 16, antiderivative size = 148

$$\int x^3\text{csch}(a+bx)\text{sech}(a+bx)\,dx = -\frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}+\frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{3\operatorname{PolyLog}\left(4,-e^{2a+2bx}\right)}{4b^4}+\frac{3\operatorname{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}$$

output
```
-2*x^3*arctanh(exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2+3/
2*x^2*polylog(2,exp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3-3
/2*x*polylog(3,exp(2*b*x+2*a))/b^3-3/4*polylog(4,-exp(2*b*x+2*a))/b^4+3/4*
polylog(4,exp(2*b*x+2*a))/b^4
```

3.467.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01

$$\int x^3\text{csch}(a+bx)\text{sech}(a+bx)\,dx$$
$$= \frac{4b^3x^3\log\left(1-e^{2(a+bx)}\right)-4b^3x^3\log\left(1+e^{2(a+bx)}\right)-6b^2x^2\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)+6b^2x^2\operatorname{PolyLog}\left(2,e^{2(a\cdots}\right.}{}$$

input
```
Integrate[x^3*Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
(4*b^3*x^3*Log[1 - E^(2*(a + b*x))] - 4*b^3*x^3*Log[1 + E^(2*(a + b*x))] -
 6*b^2*x^2*PolyLog[2, -E^(2*(a + b*x))] + 6*b^2*x^2*PolyLog[2, E^(2*(a + b
*x))] + 6*b*x*PolyLog[3, -E^(2*(a + b*x))] - 6*b*x*PolyLog[3, E^(2*(a + b*
x))] - 3*PolyLog[4, -E^(2*(a + b*x))] + 3*PolyLog[4, E^(2*(a + b*x))])/(4*
b^4)
```

3.467.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5984, 3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{csch}(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{5984}$$

$$2\int x^3\text{csch}(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int ix^3\csc(2ia+2ibx)dx$$

$$\Big\downarrow \text{26}$$

$$2i\int x^3\csc(2ia+2ibx)dx$$

$$\Big\downarrow \text{4670}$$

$$2i\left(\frac{3i\int x^2\log\left(1-e^{2a+2bx}\right)dx}{2b}-\frac{3i\int x^2\log\left(1+e^{2a+2bx}\right)dx}{2b}+\frac{ix^3\text{arctanh}\left(e^{2a+2bx}\right)}{b}\right)$$

$$\Big\downarrow \text{3011}$$

$$2i\left(-\frac{3i\left(\frac{\int x\,\text{PolyLog}(2,-e^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2a+2bx})}{2b}\right)}{2b}+\frac{3i\left(\frac{\int x\,\text{PolyLog}(2,e^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{2a+2bx})}{2b}\right)}{2b}+\frac{ix^3}{}\right.$$

$\Big\downarrow$ 7163

$$2i\left(-\frac{3i\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-e^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b}\right)}{2b}+\frac{3i\left(\frac{\frac{x\,\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,e^{2a+2b\right.}}{2b}}{b}\right.}{2b}\right.$$

$\Big\downarrow$ 2720

$$2i\left(-\frac{3i\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b}\right)}{2b}+\frac{3i\left(\frac{\frac{x\,\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b}-\int}{}\right.}{}\right.$$

$\Big\downarrow$ 7143

$$2i\left(\frac{ix^3\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{3i\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,-e^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b}\right)}{2b}+\frac{3i\left(\frac{x\,\text{PolyLog}\left(3,e^{2a+2\right.}}{2b}\right.}{}\right.$$

input
```
Int[x^3*Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
(2*I)*((I*x^3*ArcTanh[E^(2*a + 2*b*x)])/b - (((3*I)/2)*(-1/2*(x^2*PolyLog[
2, -E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, -E^(2*a + 2*b*x)])/(2*b) - PolyLo
g[4, -E^(2*a + 2*b*x)]/(4*b^2))/b))/b + (((3*I)/2)*(-1/2*(x^2*PolyLog[2, E
^(2*a + 2*b*x)])/b + ((x*PolyLog[3, E^(2*a + 2*b*x)])/(2*b) - PolyLog[4, E
^(2*a + 2*b*x)]/(4*b^2))/b))/b)
```

3.467. $\int x^3\text{csch}(a+bx)\text{sech}(a+bx)\,dx$

3.467.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.467. $\int x^3 \text{csch}(a+bx)\text{sech}(a+bx)\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.467.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.63

method	result
risch	$\frac{\ln(1-e^{bx+a})a^3}{b^4}+\frac{3x^2\operatorname{polylog}(2,-e^{bx+a})}{b^2}-\frac{6x\operatorname{polylog}(3,-e^{bx+a})}{b^3}-\frac{x^3\ln(1+e^{2bx+2a})}{b}-\frac{3x^2\operatorname{polylog}(2,-e^{2bx+2a})}{2b^2}+\frac{3x\,\mathrm{p}}{}$

input
```
int(x^3*csch(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*ln(1-exp(b*x+a))*a^3+3*x^2*polylog(2,-exp(b*x+a))/b^2-6*x*polylog(3,
-exp(b*x+a))/b^3-x^3*ln(1+exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-exp(2*b*x+2
*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+1/b*ln(1-exp(b*x+a))*x^3+3*x
^2*polylog(2,exp(b*x+a))/b^2-6*x*polylog(3,exp(b*x+a))/b^3+1/b*ln(exp(b*x+
a)+1)*x^3-1/b^4*a^3*ln(exp(b*x+a)-1)-3/4*polylog(4,-exp(2*b*x+2*a))/b^4+6*
polylog(4,exp(b*x+a))/b^4+6*polylog(4,-exp(b*x+a))/b^4
```

3.467.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 448, normalized size of antiderivative = 3.03

$$\int x^3\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$
$$=\frac{b^3x^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+3\,b^2x^2\operatorname{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-3\,b^2x^2\operatorname{Li}_2(i\,\cosh}{}$$

input
```
integrate(x^3*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

output
```
(b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*b^2*x^2*dilog(cosh(b*x
 + a) + sinh(b*x + a)) - 3*b^2*x^2*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)
) - 3*b^2*x^2*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*b^2*x^2*dilog(
-cosh(b*x + a) - sinh(b*x + a)) + a^3*log(cosh(b*x + a) + sinh(b*x + a) +
I) + a^3*log(cosh(b*x + a) + sinh(b*x + a) - I) - a^3*log(cosh(b*x + a) +
sinh(b*x + a) - 1) - 6*b*x*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*b
*x*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*b*x*polylog(3, -I*cos
h(b*x + a) - I*sinh(b*x + a)) - 6*b*x*polylog(3, -cosh(b*x + a) - sinh(b*x
 + a)) - (b^3*x^3 + a^3)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - (b^3
*x^3 + a^3)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b^3*x^3 + a^3)*
log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*polylog(4, cosh(b*x + a) + sin
h(b*x + a)) - 6*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*polylog(
4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*polylog(4, -cosh(b*x + a) - sin
h(b*x + a)))/b^4
```

3.467.6 Sympy [F]

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^3 \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(x**3*csch(b*x+a)*sech(b*x+a),x)
```

output
```
Integral(x**3*csch(a + b*x)*sech(a + b*x), x)
```

3.467.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.37

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{4\,b^3x^3\log\left(e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(-e^{(2\,bx+2\,a)})+3\,\mathrm{Li}_4(-e^{(2\,bx+2\,a)})}{3\,b^4} + \frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4} + \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(e^{(bx+a)})+6\,\mathrm{Li}_4(e^{(bx+a)})}{b^4}$$

input
```
integrate(x^3*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

output
```
-1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a
)) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/
b^4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*
x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*lo
g(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b
*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4
```

3.467.8 Giac [F]

$$\int x^3\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^3\,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)\;dx$$

input
```
integrate(x^3*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*csch(b*x + a)*sech(b*x + a), x)
```

3.467.9 Mupad [F(-1)]

Timed out.

$$\int x^3\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int \frac{x^3}{\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(x^3/(cosh(a + b*x)*sinh(a + b*x)),x)
```

output
```
int(x^3/(cosh(a + b*x)*sinh(a + b*x)), x)
```

3.468 $\int x^2 \text{csch}(a+bx)\text{sech}(a+bx)\,dx$

3.468.1 Optimal result

Integrand size = 16, antiderivative size = 97

$$\begin{aligned}\int x^2 \text{csch}(a+bx)\text{sech}(a+bx)\,dx &= -\frac{2x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} \\ &\quad + \frac{x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} \\ &\quad + \frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3} - \frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}\end{aligned}$$

output
```
-2*x^2*arctanh(exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2+x*polylo
g(2,exp(2*b*x+2*a))/b^2+1/2*polylog(3,-exp(2*b*x+2*a))/b^3-1/2*polylog(3,e
xp(2*b*x+2*a))/b^3
```

3.468.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int x^2 \text{csch}(a+bx)\text{sech}(a+bx)\,dx$$
$$= \frac{2b^2x^2\log\left(1-e^{2(a+bx)}\right) - 2b^2x^2\log\left(1+e^{2(a+bx)}\right) - 2bx\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right) + 2bx\,\text{PolyLog}\left(2,e^{2(a+bx)}\right.}{2b^3}$$

input
```
Integrate[x^2*Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
(2*b^2*x^2*Log[1 - E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + E^(2*(a + b*x))] -
 2*b*x*PolyLog[2, -E^(2*(a + b*x))] + 2*b*x*PolyLog[2, E^(2*(a + b*x))] +
PolyLog[3, -E^(2*(a + b*x))] - PolyLog[3, E^(2*(a + b*x))])/(2*b^3)
```

3.468.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5984, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{5984}$$

$$2\int x^2\text{csch}(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int ix^2\csc(2ia+2ibx)dx$$

$$\Big\downarrow \text{26}$$

$$2i\int x^2\csc(2ia+2ibx)dx$$

$$\Big\downarrow \text{4670}$$

$$2i\left(\frac{i\int x\log\left(1-e^{2a+2bx}\right)dx}{b}-\frac{i\int x\log\left(1+e^{2a+2bx}\right)dx}{b}+\frac{ix^2\text{arctanh}\left(e^{2a+2bx}\right)}{b}\right)$$

$$\Big\downarrow \text{3011}$$

$$2i\left(-\frac{i\left(\frac{\int \text{PolyLog}\left(2,-e^{2a+2bx}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b}\right)}{b}+\frac{i\left(\frac{\int \text{PolyLog}\left(2,e^{2a+2bx}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b}\right)}{b}+\frac{ix^2\text{arctan}}{}\right.$$

$$\Big\downarrow \text{2720}$$

$$2i\left(-\frac{i\left(\frac{\int e^{-2a-2bx}\operatorname{PolyLog}(2,-e^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}(2,-e^{2a+2bx})}{2b}\right)}{b}+\frac{i\left(\frac{\int e^{-2a-2bx}\operatorname{PolyLog}(2,e^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyL}}{}\right.}{b}\right.$$

$$\Big\downarrow \text{ 7143}$$

$$2i\left(\frac{ix^2\operatorname{arctanh}(e^{2a+2bx})}{b}-\frac{i\left(\frac{\operatorname{PolyLog}(3,-e^{2a+2bx})}{4b^2}-\frac{x\operatorname{PolyLog}(2,-e^{2a+2bx})}{2b}\right)}{b}+\frac{i\left(\frac{\operatorname{PolyLog}(3,e^{2a+2bx})}{4b^2}-\frac{x\operatorname{PolyLog}(2,e^{2a+2bx})}{2b}\right.}{b}\right.$$

input
```
Int[x^2*Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
(2*I)*((I*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - (I*(-1/2*(x*PolyLog[2, -E^(2*a
 + 2*b*x)])/b + PolyLog[3, -E^(2*a + 2*b*x)]/(4*b^2)))/b + (I*(-1/2*(x*Pol
yLog[2, E^(2*a + 2*b*x)])/b + PolyLog[3, E^(2*a + 2*b*x)]/(4*b^2)))/b)
```

3.468.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.468.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(88) = 176$.

Time = 0.67 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.92

method	result
risch	$\frac{\ln(e^{bx+a}+1)x^2}{b}+\frac{2x\,\mathrm{polylog}(2,-e^{bx+a})}{b^2}-\frac{x^2\ln(1+e^{2bx+2a})}{b}-\frac{x\,\mathrm{polylog}(2,-e^{2bx+2a})}{b^2}+\frac{\ln(1-e^{bx+a})x^2}{b}+\frac{2x\,\mathrm{polylog}(2,e$

input
```
int(x^2*csch(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*ln(exp(b*x+a)+1)*x^2+2*x*polylog(2,-exp(b*x+a))/b^2-x^2*ln(1+exp(2*b*x
+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2+1/b*ln(1-exp(b*x+a))*x^2+2*x*pol
ylog(2,exp(b*x+a))/b^2+1/b^3*a^2*ln(exp(b*x+a)-1)-1/b^3*ln(1-exp(b*x+a))*a
^2+1/2*polylog(3,-exp(2*b*x+2*a))/b^3-2*polylog(3,exp(b*x+a))/b^3-2*polylo
g(3,-exp(b*x+a))/b^3
```

3.468.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.62

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \frac{b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,bx\text{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-2\,bx\text{Li}_2(i\,\cosh\left(bx\right.}{}$$

input
```
integrate(x^2*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

output
```
(b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*b*x*dilog(cosh(b*x + a
) + sinh(b*x + a)) - 2*b*x*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*b*
x*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*b*x*dilog(-cosh(b*x + a) -
 sinh(b*x + a)) - a^2*log(cosh(b*x + a) + sinh(b*x + a) + I) - a^2*log(cos
h(b*x + a) + sinh(b*x + a) - I) + a^2*log(cosh(b*x + a) + sinh(b*x + a) -
1) - (b^2*x^2 - a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - (b^2*x^2
 - a^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b^2*x^2 - a^2)*log(
-cosh(b*x + a) - sinh(b*x + a) + 1) - 2*polylog(3, cosh(b*x + a) + sinh(b*
x + a)) + 2*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 2*polylog(3, -
I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*polylog(3, -cosh(b*x + a) - sinh(b*
x + a)))/b^3
```

3.468.6 Sympy [F]

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^2\,\text{csch}\,(a+bx)\,\text{sech}\,(a+bx)\,dx$$

input
```
integrate(x**2*csch(b*x+a)*sech(b*x+a),x)
```

output
```
Integral(x**2*csch(a + b*x)*sech(a + b*x), x)
```

3.468.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx$$
$$= -\frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\text{Li}_3\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^3}$$
$$+\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\text{Li}_2\left(-e^{(bx+a)}\right)-2\,\text{Li}_3\left(-e^{(bx+a)}\right)}{b^3}$$
$$+\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\text{Li}_2\left(e^{(bx+a)}\right)-2\,\text{Li}_3\left(e^{(bx+a)}\right)}{b^3}$$

input
```
integrate(x^2*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) -
 polylog(3, -e^(2*b*x + 2*a)))/b^3 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x
*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(
b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

3.468.8 Giac [F]

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^2\,\text{csch}\left(bx+a\right)\text{sech}\left(bx+a\right)\,dx$$

input
```
integrate(x^2*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*csch(b*x + a)*sech(b*x + a), x)
```

3.468.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int \frac{x^2}{\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(x^2/(cosh(a + b*x)*sinh(a + b*x)),x)
```

output
```
int(x^2/(cosh(a + b*x)*sinh(a + b*x)), x)
```

3.469 $\int x\text{csch}(a+bx)\text{sech}(a+bx)\,dx$

3.469.1 Optimal result

Integrand size = 14, antiderivative size = 58

$$\int x\text{csch}(a+bx)\text{sech}(a+bx)\,dx=-\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}$$

output
```
-2*x*arctanh(exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b^2+1/2*poly
log(2,exp(2*b*x+2*a))/b^2
```

3.469.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int x\text{csch}(a+bx)\text{sech}(a+bx)\,dx=\frac{2bx\left(\log\left(1-e^{2(a+bx)}\right)-\log\left(1+e^{2(a+bx)}\right)\right)-\text{PolyLog}\left(2,-e^{2(a+bx)}\right)+\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}$$

input
```
Integrate[x*Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
(2*b*x*(Log[1 - E^(2*(a + b*x))] - Log[1 + E^(2*(a + b*x))]) - PolyLog[2,
-E^(2*(a + b*x))] + PolyLog[2, E^(2*(a + b*x))])/(2*b^2)
```

3.469.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5984, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{ 5984}$$

$$2\int x\text{csch}(2a+2bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$2\int ix\csc(2ia+2ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$2i\int x\csc(2ia+2ibx)dx$$

$$\Big\downarrow \text{ 4670}$$

$$2i\left(\frac{i\int \log\left(1-e^{2a+2bx}\right)dx}{2b}-\frac{i\int \log\left(1+e^{2a+2bx}\right)dx}{2b}+\frac{ix\text{arctanh}\left(e^{2a+2bx}\right)}{b}\right)$$

$$\Big\downarrow \text{ 2715}$$

$$2i\left(\frac{i\int e^{-2a-2bx}\log\left(1-e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}-\frac{i\int e^{-2a-2bx}\log\left(1+e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}+\frac{ix\text{arctanh}\left(e^{2a+2bx}\right)}{b}\right)$$

$$\Big\downarrow \text{ 2838}$$

$$2i\left(\frac{ix\text{arctanh}\left(e^{2a+2bx}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{4b^2}-\frac{i\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{4b^2}\right)$$

input
```
Int[x*Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
(2*I)*((I*x*ArcTanh[E^(2*a + 2*b*x)])/b + ((I/4)*PolyLog[2, -E^(2*a + 2*b*
x)])/b^2 - ((I/4)*PolyLog[2, E^(2*a + 2*b*x)])/b^2)
```

3.469.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.469.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(51) = 102$.

Time = 0.53 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.16

method	result
risch	$\frac{\ln(1-e^{bx+a})x}{b} + \frac{\ln(1-e^{bx+a})a}{b^2} + \frac{\text{polylog}(2,e^{bx+a})}{b^2} + \frac{\ln(e^{bx+a}+1)x}{b} + \frac{\text{polylog}(2,-e^{bx+a})}{b^2} - \frac{x\ln(1+e^{2bx+2a})}{b} - \text{polylog}$

input
```
int(x*csch(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2+
1/b*ln(exp(b*x+a)+1)*x+polylog(2,-exp(b*x+a))/b^2-x*ln(1+exp(2*b*x+2*a))/b
-1/2*polylog(2,-exp(2*b*x+2*a))/b^2-1/b^2*a*ln(exp(b*x+a)-1)
```

3.469.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.86

$$\int x\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \frac{bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+i\right)+a\log\left(\cosh\left(bx+a\right.\right.}{}$$

input
```
integrate(x*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

output
```
(b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) + a*log(cosh(b*x + a) + sinh(b
*x + a) + I) + a*log(cosh(b*x + a) + sinh(b*x + a) - I) - a*log(cosh(b*x +
 a) + sinh(b*x + a) - 1) - (b*x + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a)
 + 1) - (b*x + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b*x + a)*
log(-cosh(b*x + a) - sinh(b*x + a) + 1) + dilog(cosh(b*x + a) + sinh(b*x +
 a)) - dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - dilog(-I*cosh(b*x + a) -
 I*sinh(b*x + a)) + dilog(-cosh(b*x + a) - sinh(b*x + a)))/b^2
```

3.469.6 Sympy [F]

$$\int x\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x\,\text{csch}\left(a+bx\right)\text{sech}\left(a+bx\right)\,dx$$

input
```
integrate(x*csch(b*x+a)*sech(b*x+a),x)
```

output
```
Integral(x*csch(a + b*x)*sech(a + b*x), x)
```

3.469.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.50

$$\int x\text{csch}(a+bx)\text{sech}(a+bx)\,dx = -\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2} + \frac{bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

input
```
integrate(x*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^2 + (b*x
*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) +
 1) + dilog(e^(b*x + a)))/b^2
```

3.469.8 Giac [F]

$$\int x\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x\,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)\;dx$$

input
```
integrate(x*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*csch(b*x + a)*sech(b*x + a), x)
```

3.469.9 Mupad [F(-1)]

Timed out.

$$\int x\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int \frac{x}{\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(x/(cosh(a + b*x)*sinh(a + b*x)),x)
```

output
```
int(x/(cosh(a + b*x)*sinh(a + b*x)), x)
```

3.470 $\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$

3.470.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\log(\tanh(a+bx))}{b}$$

output
```
ln(tanh(b*x+a))/b
```

3.470.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = 2\left(-\frac{\log(\cosh(a+bx))}{2b} + \frac{\log(\sinh(a+bx))}{2b}\right)$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
2*(-1/2*Log[Cosh[a + b*x]]/b + Log[Sinh[a + b*x]]/(2*b))
```

3.470.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 26, 3100, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)dx$$

$$\Big\downarrow \; 26$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)dx$$

$$\Big\downarrow \; 3100$$

$$\frac{\int -i\coth(a+bx)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \; 14$$

$$\frac{\log(i\tanh(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
Log[I*Tanh[a + b*x]]/b
```

3.470.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.470.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(\tanh(bx+a))}{b}$	12
default	$\frac{\ln(\tanh(bx+a))}{b}$	12
risch	$\frac{\ln(e^{2bx+2a}-1)}{b}-\frac{\ln(1+e^{2bx+2a})}{b}$	35

input
```
int(csch(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(tanh(b*x+a))/b
```

3.470.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 5.45

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

output
```
-(log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - log(2*sinh(b*x +
a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

3.470.6 Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x), x)
```

3.470.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 4.55

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

output
```
log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
 1)/b
```

3.470.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.73

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(e^{(2\,bx+2\,a)}+1\right) - \log\left(e^{(bx+a)}+1\right) - \log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

output
```
-(log(e^(2*b*x + 2*a) + 1) - log(e^(b*x + a) + 1) - log(abs(e^(b*x + a) -
1)))/b
```

3.470.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)),x)
```

output
```
-(2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

3.471 $\int \frac{\text{csch}(a+bx)\text{sech}(a+bx)}{x}\,dx$

3.471.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{csch}(a+bx)\text{sech}(a+bx)}{x}\,dx = 2\text{Int}\left(\frac{\text{csch}(2a+2bx)}{x},x\right)$$

output
```
2*Unintegrable(csch(2*b*x+2*a)/x,x)
```

3.471.2 Mathematica [N/A]

Not integrable

Time = 13.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{csch}(a+bx)\text{sech}(a+bx)}{x}\,dx = \int \frac{\text{csch}(a+bx)\text{sech}(a+bx)}{x}\,dx$$

input
```
Integrate[(Csch[a + b*x]*Sech[a + b*x])/x,x]
```

output
```
Integrate[(Csch[a + b*x]*Sech[a + b*x])/x, x]
```

3.471.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5984, 3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x} \, dx$$
$$\Big\downarrow \text{5984}$$
$$2\int \frac{\operatorname{csch}(2a+2bx)}{x}dx$$
$$\Big\downarrow \text{3042}$$
$$2\int \frac{i\csc(2ia+2ibx)}{x}dx$$
$$\Big\downarrow \text{26}$$
$$2i\int \frac{\csc(2ia+2ibx)}{x}dx$$
$$\Big\downarrow \text{4680}$$
$$2\int \frac{\operatorname{csch}(2a+2bx)}{x}dx$$

input
```
Int[(Csch[a + b*x]*Sech[a + b*x])/x,x]
```

output
```
$Aborted
```

3.471.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.471.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)}{x}dx$$

input
```
int(csch(b*x+a)*sech(b*x+a)/x,x)
```

output
```
int(csch(b*x+a)*sech(b*x+a)/x,x)
```

3.471.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)*sech(b*x + a)/x, x)
```

3.471.6 Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)/x,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)/x, x)
```

3.471.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)*sech(b*x + a)/x, x)
```

3.471.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)*sech(b*x + a)/x, x)
```

3.471.9 Mupad [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{1}{x\cosh(a+b\,x)\,\sinh(a+b\,x)}\,dx$$

input
```
int(1/(x*cosh(a + b*x)*sinh(a + b*x)),x)
```

output
```
int(1/(x*cosh(a + b*x)*sinh(a + b*x)), x)
```

3.472 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$

3.472.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = 2\mathrm{Int}\left(\frac{\operatorname{csch}(2a+2bx)}{x^2},x\right)$$

output
```
2*Unintegrable(csch(2*b*x+2*a)/x^2,x)
```

3.472.2 Mathematica [N/A]

Not integrable

Time = 13.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Csch[a + b*x]*Sech[a + b*x])/x^2,x]
```

output
```
Integrate[(Csch[a + b*x]*Sech[a + b*x])/x^2, x]
```

3.472.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5984, 3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{ 5984}$$

$$2\int \frac{\operatorname{csch}(2a+2bx)}{x^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \frac{i\csc(2ia+2ibx)}{x^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$2i\int \frac{\csc(2ia+2ibx)}{x^2}dx$$

$$\Big\downarrow \text{ 4680}$$

$$2\int \frac{\operatorname{csch}(2a+2bx)}{x^2}dx$$

input
```
Int[(Csch[a + b*x]*Sech[a + b*x])/x^2,x]
```

output
```
$Aborted
```

3.472.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.472.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2}dx$$

input
```
int(csch(b*x+a)*sech(b*x+a)/x^2,x)
```

output
```
int(csch(b*x+a)*sech(b*x+a)/x^2,x)
```

3.472.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)*sech(b*x + a)/x^2, x)
```

3.472.6 Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)/x**2,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)/x**2, x)
```

3.472.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)*sech(b*x + a)/x^2, x)
```

3.472.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)*sech(b*x + a)/x^2, x)
```

3.472.9 Mupad [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh(a+b\,x)\,\sinh(a+b\,x)}\,dx$$

input
```
int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)),x)
```

output
```
int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)), x)
```

3.473 $\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

3.473.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx),x\right)$$

output
```
CannotIntegrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x)
```

3.473.2 Mathematica [N/A]

Not integrable

Time = 102.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^2, x]
```

3.473.3 Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int x^m \text{csch}(a+bx)\text{sech}^2(a+bx)dx$$

input
```
Int[x^m*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
$Aborted
```

3.473.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.473.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\,dx$$

input
```
int(x^m*csch(b*x+a)*sech(b*x+a)^2,x)
```

output
```
int(x^m*csch(b*x+a)*sech(b*x+a)^2,x)
```

3.473.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\,dx$$

input
```
integrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*csch(b*x + a)*sech(b*x + a)^2, x)
```

3.473.6 Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(x**m*csch(b*x+a)*sech(b*x+a)**2,x)
```

output
```
Integral(x**m*csch(a + b*x)*sech(a + b*x)**2, x)
```

3.473.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\,dx$$

input
```
integrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*csch(b*x + a)*sech(b*x + a)^2, x)
```

3.473.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)\,dx = \int x^m \mathrm{csch}\,(bx+a)\,\mathrm{sech}\,(bx+a)^2\,dx$$

input
```
integrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*csch(b*x + a)*sech(b*x + a)^2, x)
```

3.473.9 Mupad [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)\,dx = \int \frac{x^m}{\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)),x)
```

output
```
int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)), x)
```

3.474 $\int x^3\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$

3.474.1 Optimal result

Integrand size = 18, antiderivative size = 226

$$\begin{aligned}\int x^3\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx =& -\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2}-\frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b}\\&-\frac{3x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}\\&+\frac{6ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{6ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}\\&+\frac{3x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{6x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}\\&-\frac{6i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}+\frac{6i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}\\&-\frac{6x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{6\,\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^4}\\&+\frac{6\,\text{PolyLog}\left(4,e^{a+bx}\right)}{b^4}+\frac{x^3\text{sech}(a+bx)}{b}\end{aligned}$$

output
```
-6*x^2*arctan(exp(b*x+a))/b^2-2*x^3*arctanh(exp(b*x+a))/b-3*x^2*polylog(2,
-exp(b*x+a))/b^2+6*I*x*polylog(2,-I*exp(b*x+a))/b^3-6*I*x*polylog(2,I*exp(
b*x+a))/b^3+3*x^2*polylog(2,exp(b*x+a))/b^2+6*x*polylog(3,-exp(b*x+a))/b^3
-6*I*polylog(3,-I*exp(b*x+a))/b^4+6*I*polylog(3,I*exp(b*x+a))/b^4-6*x*poly
log(3,exp(b*x+a))/b^3-6*polylog(4,-exp(b*x+a))/b^4+6*polylog(4,exp(b*x+a))
/b^4+x^3*sech(b*x+a)/b
```

3.474.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.16

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$
$$= \frac{b^3x^3\log\left(1-e^{a+bx}\right)-3ib^2x^2\log\left(1-ie^{a+bx}\right)+3ib^2x^2\log\left(1+ie^{a+bx}\right)-b^3x^3\log\left(1+e^{a+bx}\right)-3b^2x^2\,\mathrm{Pol}}{}$$

input
```
Integrate[x^3*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
(b^3*x^3*Log[1 - E^(a + b*x)] - (3*I)*b^2*x^2*Log[1 - I*E^(a + b*x)] + (3*
I)*b^2*x^2*Log[1 + I*E^(a + b*x)] - b^3*x^3*Log[1 + E^(a + b*x)] - 3*b^2*x
^2*PolyLog[2, -E^(a + b*x)] + (6*I)*b*x*PolyLog[2, (-I)*E^(a + b*x)] - (6*
I)*b*x*PolyLog[2, I*E^(a + b*x)] + 3*b^2*x^2*PolyLog[2, E^(a + b*x)] + 6*b
*x*PolyLog[3, -E^(a + b*x)] - (6*I)*PolyLog[3, (-I)*E^(a + b*x)] + (6*I)*P
olyLog[3, I*E^(a + b*x)] - 6*b*x*PolyLog[3, E^(a + b*x)] - 6*PolyLog[4, -E
^(a + b*x)] + 6*PolyLog[4, E^(a + b*x)] + b^3*x^3*Sech[a + b*x])/b^4
```

3.474.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5985, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{5985}$$

$$-3\int -x^2\left(\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}-\frac{\operatorname{sech}(a+bx)}{b}\right)dx-\frac{x^3\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{x^3\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$3\int x^2\left(\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}-\frac{\operatorname{sech}(a+bx)}{b}\right)dx-\frac{x^3\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{x^3\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{2010}$$

$$3\int\left(\frac{x^2\text{arctanh}(\cosh(a+bx))}{b}-\frac{x^2\text{sech}(a+bx)}{b}\right)dx-\frac{x^3\text{arctanh}(\cosh(a+bx))}{b}+\frac{x^3\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{2x^2\arctan\left(e^{a+bx}\right)}{b^2}-\frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{3b}+\frac{x^3\text{arctanh}(\cosh(a+bx))}{3b}-\frac{2i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}+\frac{2i\,\text{PolyLo}}{}\right.$$
$$\frac{x^3\text{arctanh}(\cosh(a+bx))}{b}+\frac{x^3\text{sech}(a+bx)}{b}$$

input
```
Int[x^3*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
-((x^3*ArcTanh[Cosh[a + b*x]])/b) + 3*((-2*x^2*ArcTan[E^(a + b*x)])/b^2 -
(2*x^3*ArcTanh[E^(a + b*x)])/(3*b) + (x^3*ArcTanh[Cosh[a + b*x]])/(3*b) -
(x^2*PolyLog[2, -E^(a + b*x)])/b^2 + ((2*I)*x*PolyLog[2, (-I)*E^(a + b*x)]
)/b^3 - ((2*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + (x^2*PolyLog[2, E^(a + b
*x)])/b^2 + (2*x*PolyLog[3, -E^(a + b*x)])/b^3 - ((2*I)*PolyLog[3, (-I)*E^
(a + b*x)])/b^4 + ((2*I)*PolyLog[3, I*E^(a + b*x)])/b^4 - (2*x*PolyLog[3,
E^(a + b*x)])/b^3 - (2*PolyLog[4, -E^(a + b*x)])/b^4 + (2*PolyLog[4, E^(a
+ b*x)])/b^4) + (x^3*Sech[a + b*x])/b
```

3.474.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.474.4 Maple [F]

$$\int x^3 \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\, dx$$

input
```
int(x^3*csch(b*x+a)*sech(b*x+a)^2,x)
```

output
```
int(x^3*csch(b*x+a)*sech(b*x+a)^2,x)
```

3.474.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1280 vs. $2(194) = 388$.

Time = 0.30 (sec) , antiderivative size = 1280, normalized size of antiderivative = 5.66

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate(x^3*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) + 3*(b^2*x^2*cosh(b*x +
 a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 +
b^2*x^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(I*b*x*cosh(b*x + a)^2 +
 2*I*b*x*cosh(b*x + a)*sinh(b*x + a) + I*b*x*sinh(b*x + a)^2 + I*b*x)*dilo
g(I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(-I*b*x*cosh(b*x + a)^2 - 2*I*b*x
*cosh(b*x + a)*sinh(b*x + a) - I*b*x*sinh(b*x + a)^2 - I*b*x)*dilog(-I*cos
h(b*x + a) - I*sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cos
h(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 + b^2*x^2)*dilog(-cosh(
b*x + a) - sinh(b*x + a)) - (b^3*x^3*cosh(b*x + a)^2 + 2*b^3*x^3*cosh(b*x
+ a)*sinh(b*x + a) + b^3*x^3*sinh(b*x + a)^2 + b^3*x^3)*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) - 3*(I*a^2*cosh(b*x + a)^2 + 2*I*a^2*cosh(b*x + a)*si
nh(b*x + a) + I*a^2*sinh(b*x + a)^2 + I*a^2)*log(cosh(b*x + a) + sinh(b*x
+ a) + I) - 3*(-I*a^2*cosh(b*x + a)^2 - 2*I*a^2*cosh(b*x + a)*sinh(b*x + a
) - I*a^2*sinh(b*x + a)^2 - I*a^2)*log(cosh(b*x + a) + sinh(b*x + a) - I)
- (a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*sinh(b*x + a) + a^3*sinh(b*x
+ a)^2 + a^3)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*(-I*b^2*x^2 + (-I
*b^2*x^2 + I*a^2)*cosh(b*x + a)^2 + 2*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)*s
inh(b*x + a) + (-I*b^2*x^2 + I*a^2)*sinh(b*x + a)^2 + I*a^2)*log(I*cosh(b*
x + a) + I*sinh(b*x + a) + 1) - 3*(I*b^2*x^2 + (I*b^2*x^2 - I*a^2)*cosh(b*
x + a)^2 + 2*(I*b^2*x^2 - I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (I*b^2*x...
```

3.474.6 Sympy [F]

$$\int x^3 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(x**3*csch(b*x+a)*sech(b*x+a)**2,x)
```

output
```
Integral(x**3*csch(a + b*x)*sech(a + b*x)**2, x)
```

3.474.7 Maxima [F]

$$\int x^3 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int x^3 \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\,dx$$

input
```
integrate(x^3*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) - (b^3*x^3*log(e^(b*x + a) + 1)
+ 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polyl
og(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilo
g(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a))
)/b^4 - 24*integrate(1/4*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

3.474.8 Giac [F(-1)]

Timed out.

$$\int x^3 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x^3*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.474.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int \frac{x^3}{\cosh(a+b\,x)^2\sinh(a+b\,x)}\,dx$$

input
```
int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)),x)
```

output
```
int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)), x)
```

3.475 $\int x^2\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$

3.475.1 Optimal result

Integrand size = 18, antiderivative size = 146

$$\int x^2\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2}-\frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{2\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}+\frac{x^2\text{sech}(a+bx)}{b}$$

output
```
-4*x*arctan(exp(b*x+a))/b^2-2*x^2*arctanh(exp(b*x+a))/b-2*x*polylog(2,-exp
(b*x+a))/b^2+2*I*polylog(2,-I*exp(b*x+a))/b^3-2*I*polylog(2,I*exp(b*x+a))/
b^3+2*x*polylog(2,exp(b*x+a))/b^2+2*polylog(3,-exp(b*x+a))/b^3-2*polylog(3
,exp(b*x+a))/b^3+x^2*sech(b*x+a)/b
```

3.475.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.24

$$\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$
$$= \frac{b^2x^2\log\left(1-e^{a+bx}\right)-2ibx\log\left(1-ie^{a+bx}\right)+2ibx\log\left(1+ie^{a+bx}\right)-b^2x^2\log\left(1+e^{a+bx}\right)-2bx\operatorname{PolyLog}\left(\right.}{}$$

input
```
Integrate[x^2*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
(b^2*x^2*Log[1 - E^(a + b*x)] - (2*I)*b*x*Log[1 - I*E^(a + b*x)] + (2*I)*b
*x*Log[1 + I*E^(a + b*x)] - b^2*x^2*Log[1 + E^(a + b*x)] - 2*b*x*PolyLog[2
, -E^(a + b*x)] + (2*I)*PolyLog[2, (-I)*E^(a + b*x)] - (2*I)*PolyLog[2, I*
E^(a + b*x)] + 2*b*x*PolyLog[2, E^(a + b*x)] + 2*PolyLog[3, -E^(a + b*x)]
- 2*PolyLog[3, E^(a + b*x)] + b^2*x^2*Sech[a + b*x])/b^3
```

3.475.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5985, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$
$$\Big\downarrow \; 5985$$
$$-2\int -x\left(\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}-\frac{\operatorname{sech}(a+bx)}{b}\right)dx-\frac{x^2\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{x^2\operatorname{sech}(a+bx)}{b}$$
$$\Big\downarrow \; 25$$
$$2\int x\left(\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}-\frac{\operatorname{sech}(a+bx)}{b}\right)dx-\frac{x^2\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{x^2\operatorname{sech}(a+bx)}{b}$$
$$\Big\downarrow \; 2010$$
$$2\int\left(\frac{x\operatorname{arctanh}(\cosh(a+bx))}{b}-\frac{x\operatorname{sech}(a+bx)}{b}\right)dx-\frac{x^2\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{x^2\operatorname{sech}(a+bx)}{b}$$
$$\Big\downarrow \; 2009$$

$$2\left(-\frac{2x\arctan\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{x^2\text{arctanh}(\cosh(a+bx))}{2b}+\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{i\,\text{PolyLog}\left(2\right.}{b^3}\right.$$
$$\frac{x^2\text{arctanh}(\cosh(a+bx))}{b}+\frac{x^2\text{sech}(a+bx)}{b}$$

input
```
Int[x^2*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
-((x^2*ArcTanh[Cosh[a + b*x]])/b) + 2*((-2*x*ArcTan[E^(a + b*x)])/b^2 - (x
^2*ArcTanh[E^(a + b*x)])/b + (x^2*ArcTanh[Cosh[a + b*x]])/(2*b) - (x*PolyL
og[2, -E^(a + b*x)])/b^2 + (I*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - (I*PolyL
og[2, I*E^(a + b*x)])/b^3 + (x*PolyLog[2, E^(a + b*x)])/b^2 + PolyLog[3, -
E^(a + b*x)]/b^3 - PolyLog[3, E^(a + b*x)]/b^3) + (x^2*Sech[a + b*x])/b
```

3.475.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.475.4 Maple [F]

$$\int x^2 \operatorname{csch}(bx+a) \operatorname{sech}(bx+a)^2 \, dx$$

input
```
int(x^2*csch(b*x+a)*sech(b*x+a)^2,x)
```

output
```
int(x^2*csch(b*x+a)*sech(b*x+a)^2,x)
```

3.475.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(126) = 252$.

Time = 0.28 (sec) , antiderivative size = 937, normalized size of antiderivative = 6.42

$$\int x^2 \operatorname{csch}(a+bx) \operatorname{sech}^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
(2*b^2*x^2*cosh(b*x + a) + 2*b^2*x^2*sinh(b*x + a) + 2*(b*x*cosh(b*x + a)^
2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*dilog(c
osh(b*x + a) + sinh(b*x + a)) - 2*(I*cosh(b*x + a)^2 + 2*I*cosh(b*x + a)*s
inh(b*x + a) + I*sinh(b*x + a)^2 + I)*dilog(I*cosh(b*x + a) + I*sinh(b*x +
 a)) - 2*(-I*cosh(b*x + a)^2 - 2*I*cosh(b*x + a)*sinh(b*x + a) - I*sinh(b*
x + a)^2 - I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b*x*cosh(b*x
+ a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*di
log(-cosh(b*x + a) - sinh(b*x + a)) - (b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2
*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 + b^2*x^2)*log(cosh
(b*x + a) + sinh(b*x + a) + 1) - 2*(-I*a*cosh(b*x + a)^2 - 2*I*a*cosh(b*x
+ a)*sinh(b*x + a) - I*a*sinh(b*x + a)^2 - I*a)*log(cosh(b*x + a) + sinh(b
*x + a) + I) - 2*(I*a*cosh(b*x + a)^2 + 2*I*a*cosh(b*x + a)*sinh(b*x + a)
+ I*a*sinh(b*x + a)^2 + I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (a^2
*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2
 + a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 2*((-I*b*x - I*a)*cosh(b*
x + a)^2 + 2*(-I*b*x - I*a)*cosh(b*x + a)*sinh(b*x + a) + (-I*b*x - I*a)*s
inh(b*x + a)^2 - I*b*x - I*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) -
 2*((I*b*x + I*a)*cosh(b*x + a)^2 + 2*(I*b*x + I*a)*cosh(b*x + a)*sinh(b*x
 + a) + (I*b*x + I*a)*sinh(b*x + a)^2 + I*b*x + I*a)*log(-I*cosh(b*x + a)
- I*sinh(b*x + a) + 1) + (b^2*x^2 + (b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2...
```

3.475.6 Sympy [F]

$$\int x^2 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(x**2*csch(b*x+a)*sech(b*x+a)**2,x)
```

output
```
Integral(x**2*csch(a + b*x)*sech(a + b*x)**2, x)
```

3.475.7 Maxima [F]

$$\int x^2 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int x^2 \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\,dx$$

input
```
integrate(x^2*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) - (b^2*x^2*log(e^(b*x + a) + 1)
+ 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*l
og(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)
))/b^3 - 8*integrate(1/2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

3.475.8 Giac [F]

$$\int x^2 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int x^2 \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\,dx$$

input
```
integrate(x^2*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.475.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \frac{x^2}{\cosh(a+b\,x)^2\sinh(a+b\,x)}\,dx$$

input
```
int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)),x)
```

output
```
int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)), x)
```

3.476 $\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$

3.476.1 Optimal result

Integrand size = 16, antiderivative size = 67

$$\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{\arctan(\sinh(a+bx))}{b^2} - \frac{2x\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{x\text{sech}(a+bx)}{b}$$

output
```
-arctan(sinh(b*x+a))/b^2-2*x*arctanh(exp(b*x+a))/b-polylog(2,-exp(b*x+a))/
b^2+polylog(2,exp(b*x+a))/b^2+x*sech(b*x+a)/b
```

3.476.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \frac{-2\arctan\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right) + bx\log\left(1-e^{a+bx}\right) - bx\log\left(1+e^{a+bx}\right) - \text{PolyLog}\left(2,-e^{a+bx}\right) + \text{PolyLog}}{b^2}$$

input
```
Integrate[x*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
(-2*ArcTan[Tanh[(a + b*x)/2]] + b*x*Log[1 - E^(a + b*x)] - b*x*Log[1 + E^(
a + b*x)] - PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)] + b*x*Sech[
a + b*x])/b^2
```

3.476.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5985, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{5985}$$

$$-\int \left(\frac{\text{sech}(a+bx)}{b}-\frac{\text{arctanh}(\cosh(a+bx))}{b}\right)dx-\frac{x\text{arctanh}(\cosh(a+bx))}{b}+\frac{x\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan(\sinh(a+bx))}{b^2}-\frac{2x\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{x\text{sech}(a+bx)}{b}$$

input
```
Int[x*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
-(ArcTan[Sinh[a + b*x]]/b^2) - (2*x*ArcTanh[E^(a + b*x)])/b - PolyLog[2, -
E^(a + b*x)]/b^2 + PolyLog[2, E^(a + b*x)]/b^2 + (x*Sech[a + b*x])/b
```

3.476.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.476.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.42

method	result	size
risch	$\frac{2x\,\mathrm{e}^{bx+a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)}-\frac{2\arctan\left(\mathrm{e}^{bx+a}\right)}{b^2}-\frac{\mathrm{dilog}\left(\mathrm{e}^{bx+a}+1\right)}{b^2}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)x}{b}-\frac{\mathrm{dilog}\left(\mathrm{e}^{bx+a}\right)}{b^2}-\frac{a\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^2}$	95

input
```
int(x*csch(b*x+a)*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x*exp(b*x+a)/b/(1+exp(2*b*x+2*a))-2/b^2*arctan(exp(b*x+a))-1/b^2*dilog(e
xp(b*x+a)+1)-1/b*ln(exp(b*x+a)+1)*x-1/b^2*dilog(exp(b*x+a))-1/b^2*a*ln(exp
(b*x+a)-1)
```

3.476.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(62) = 124.

Time = 0.25 (sec) , antiderivative size = 401, normalized size of antiderivative = 5.99

$$\int x\mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)\,dx$$
$$=\frac{2\,bx\cosh\left(bx+a\right)+2\,bx\sinh\left(bx+a\right)-2\,\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2\right.}{}$$

input
```
integrate(x*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
(2*b*x*cosh(b*x + a) + 2*b*x*sinh(b*x + a) - 2*(cosh(b*x + a)^2 + 2*cosh(b
*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*arctan(cosh(b*x + a) + sinh(b
*x + a)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a
)^2 + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^2 + 2*cosh(
b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*dilog(-cosh(b*x + a) - sinh(
b*x + a)) - (b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x
*sinh(b*x + a)^2 + b*x)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - (a*cosh(b
*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2 + a)*log(c
osh(b*x + a) + sinh(b*x + a) - 1) + ((b*x + a)*cosh(b*x + a)^2 + 2*(b*x +
a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2 + b*x + a)*log(
-cosh(b*x + a) - sinh(b*x + a) + 1))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x
 + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 + b^2)
```

3.476.6 Sympy [F]

$$\int x\mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)\,dx = \int x\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(x*csch(b*x+a)*sech(b*x+a)**2,x)
```

output
```
Integral(x*csch(a + b*x)*sech(a + b*x)**2, x)
```

3.476.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int x\mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)\,dx = \frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}+b} - \frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2} - \frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^2}$$

input
```
integrate(x*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) - (b*x*log(e^(b*x + a) + 1) + dilo
g(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^
2 - 2*arctan(e^(b*x + a))/b^2
```

3.476.8 Giac [F]

$$\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx=\int x\,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)^2\,dx$$

input
```
integrate(x*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*csch(b*x + a)*sech(b*x + a)^2, x)
```

3.476.9 Mupad [F(-1)]

Timed out.

$$\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx=\int \frac{x}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(x/(cosh(a + b*x)^2*sinh(a + b*x)),x)
```

output
```
int(x/(cosh(a + b*x)^2*sinh(a + b*x)), x)
```

3.477 $\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

3.477.1 Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

output
```
-arctanh(cosh(b*x+a))/b+sech(b*x+a)/b
```

3.477.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx=-\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
-(Log[Cosh[(a + b*x)/2]]/b) + Log[Sinh[(a + b*x)/2]]/b + Sech[a + b*x]/b
```

3.477.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3102, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\text{sech}^2(a+bx)}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\text{sech}^2(a+bx)}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\text{sech}(a+bx)-\int \frac{1}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{sech}(a+bx)-\text{arctanh}(\text{sech}(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
(-ArcTanh[Sech[a + b*x]] + Sech[a + b*x])/b
```

3.477.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.477.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{1}{\cosh(bx+a)}-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	23
default	$\frac{\frac{1}{\cosh(bx+a)}-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	23
risch	$\frac{2\,\mathrm{e}^{bx+a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}$	53

input
```
int(csch(b*x+a)*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

3.477.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 6.74

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\left(\cosh\left(bx+a\right)^2+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2+1\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right.}{b\cosh\left(bx\right.}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*
log(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a
)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(cosh(b*x + a) + sinh(b*x + a) -
 1) - 2*cosh(b*x + a) - 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x
 + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

3.477.6 Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**2, x)
```

3.477.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(23) = 46$.

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\,e^{(-bx-a)}}{b\left(e^{(-2\,bx-2\,a)}+1\right)}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
-log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-b*x - a)/(b*(e^
(-2*b*x - 2*a) + 1))
```

3.477.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(23) = 46$.

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}} - \log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + \log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{2\,b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(4/(e^(b*x + a) + e^(-b*x - a)) - log(e^(b*x + a) + e^(-b*x - a) + 2)
+ log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

3.477.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)),x)
```

output
```
(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(b*x)*exp(a)*(-b
^2)^(1/2))/b))/(-b^2)^(1/2)
```

3.477. $\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

3.478 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$

3.478.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x},x\right)$$

output
```
CannotIntegrate(csch(b*x+a)*sech(b*x+a)^2/x,x)
```

3.478.2 Mathematica [N/A]

Not integrable

Time = 35.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

input
```
Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x,x]
```

output
```
Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x, x]
```

3.478.3 Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{x}dx$$

input
```
Int[(Csch[a + b*x]*Sech[a + b*x]^2)/x,x]
```

output
```
$Aborted
```

3.478.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.478.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)^2}{x}dx$$

input
```
int(csch(b*x+a)*sech(b*x+a)^2/x,x)
```

output
```
int(csch(b*x+a)*sech(b*x+a)^2/x,x)
```

3.478.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)*sech(b*x + a)^2/x, x)
```

3.478.6 Sympy [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)}{x}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**2/x,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**2/x, x)
```

3.478.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 5.50

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) + b*x) + 8*integrate(1/4*e^(b*x + a)/(b
*x^2*e^(2*b*x + 2*a) + b*x^2), x) + 8*integrate(1/8/(x*e^(b*x + a) + x), x
) + 8*integrate(1/8/(x*e^(b*x + a) - x), x)
```

3.478.8 Giac [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)*sech(b*x + a)^2/x, x)
```

3.478.9 Mupad [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{1}{x\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)),x)
```

output
```
int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)), x)
```

3.479 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$

3.479.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2},x\right)$$

output
```
CannotIntegrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x)
```

3.479.2 Mathematica [N/A]

Not integrable

Time = 22.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2,x]
```

output
```
Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2, x]
```

3.479.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}dx$$

input
```
Int[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2,x]
```

output
```
$Aborted
```

3.479.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.479.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)^{2}}{x^{2}}dx$$

input
```
int(csch(b*x+a)*sech(b*x+a)^2/x^2,x)
```

output
```
int(csch(b*x+a)*sech(b*x+a)^2/x^2,x)
```

3.479.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)*sech(b*x + a)^2/x^2, x)
```

3.479.6 Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**2/x**2,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**2/x**2, x)
```

3.479.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 6.17

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) + b*x^2) + 8*integrate(1/2*e^(b*x + a
)/(b*x^3*e^(2*b*x + 2*a) + b*x^3), x) + 8*integrate(1/8/(x^2*e^(b*x + a) +
 x^2), x) + 8*integrate(1/8/(x^2*e^(b*x + a) - x^2), x)
```

3.479.8 Giac [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)*sech(b*x + a)^2/x^2, x)
```

3.479.9 Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh(a+b\,x)^2\sinh(a+b\,x)}\,dx$$

input
```
int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)),x)
```

output
```
int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)), x)
```

3.480 $\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$

3.480.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx),x\right)$$

output
```
CannotIntegrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x)
```

3.480.2 Mathematica [N/A]

Not integrable

Time = 80.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^3, x]
```

3.480.3 Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^m \text{csch}(a+bx)\text{sech}^3(a+bx)dx$$

input
```
Int[x^m*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
$Aborted
```

3.480.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.480.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3\,dx$$

input
```
int(x^m*csch(b*x+a)*sech(b*x+a)^3,x)
```

output
```
int(x^m*csch(b*x+a)*sech(b*x+a)^3,x)
```

3.480.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3\,dx$$

input
```
integrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m*csch(b*x + a)*sech(b*x + a)^3, x)
```

3.480.6 Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(x**m*csch(b*x+a)*sech(b*x+a)**3,x)
```

output
```
Integral(x**m*csch(a + b*x)*sech(a + b*x)**3, x)
```

3.480.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3\,dx$$

input
```
integrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(x^m*csch(b*x + a)*sech(b*x + a)^3, x)
```

3.480.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m \,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)^3 \,dx$$

input
```
integrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*csch(b*x + a)*sech(b*x + a)^3, x)
```

3.480.9 Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \int \frac{x^m}{\cosh\left(a+b\,x\right)^3 \sinh\left(a+b\,x\right)}\,dx$$

input
```
int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

output
```
int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)), x)
```

3.481 $\int x^3\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$

3.481.1 Optimal result

Integrand size = 18, antiderivative size = 240

$$\begin{aligned}\int x^3\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx =& -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{2x^3\text{arctanh}\left(e^{2a+2bx}\right)}{b}\\&+\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}+\frac{3\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}\\&-\frac{3x^2\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}\\&+\frac{3x\,\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\,\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}\\&-\frac{3\,\text{PolyLog}\left(4,-e^{2a+2bx}\right)}{4b^4}+\frac{3\,\text{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}\\&-\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{x^3\tanh^2(a+bx)}{2b}\end{aligned}$$

output
```
-3/2*x^2/b^2+1/2*x^3/b-2*x^3*arctanh(exp(2*b*x+2*a))/b+3*x*ln(1+exp(2*b*x+
2*a))/b^3+3/2*polylog(2,-exp(2*b*x+2*a))/b^4-3/2*x^2*polylog(2,-exp(2*b*x+
2*a))/b^2+3/2*x^2*polylog(2,exp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x
+2*a))/b^3-3/2*x*polylog(3,exp(2*b*x+2*a))/b^3-3/4*polylog(4,-exp(2*b*x+2*
a))/b^4+3/4*polylog(4,exp(2*b*x+2*a))/b^4-3/2*x^2*tanh(b*x+a)/b^2-1/2*x^3*
tanh(b*x+a)^2/b
```

3.481.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 524 vs. $2(240) = 480$.

Time = 6.40 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.18

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx =$$

$$-\frac{2e^{2a}\left(-\frac{3}{2}e^{-2a}x^2+\frac{1}{4}b^2e^{-2a}x^4-\frac{3(1+e^{-2a})x\log(1+e^{-2a-2bx})}{2b}+\frac{1}{2}b(1+e^{-2a})x^3\log\left(1+e^{-2a-2bx}\right)+\frac{3(1+e^{-2a})\ldots}{\ldots}\right.}{\ldots}$$

$$-\frac{e^{2a}\left(b^4e^{-2a}x^4-2b^3(1-e^{-2a})x^3\log\left(1-e^{-a-bx}\right)-2b^3(1-e^{-2a})x^3\log\left(1+e^{-a-bx}\right)+6b^2(1-e^{-2a})x^2\ldots\right.}{\ldots}$$

$$+\frac{1}{4}x^4\operatorname{csch}(a)\operatorname{sech}(a)+\frac{x^3\operatorname{sech}^2(a+bx)}{2b}-\frac{3x^2\operatorname{sech}(a)\operatorname{sech}(a+bx)\sinh(bx)}{2b^2}$$

input
```
Integrate[x^3*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
(-2*E^(2*a)*((-3*x^2)/(2*E^(2*a)) + (b^2*x^4)/(4*E^(2*a)) - (3*(1 + E^(-2*
a))*x*Log[1 + E^(-2*a - 2*b*x)])/(2*b) + (b*(1 + E^(-2*a))*x^3*Log[1 + E^(
-2*a - 2*b*x)])/2 + (3*(1 + E^(-2*a))*PolyLog[2, -E^(-2*a - 2*b*x)])/(4*b^
2) - (3*(1 + E^(-2*a))*x^2*PolyLog[2, -E^(-2*a - 2*b*x)])/4 - (3*(1 + E^(-
2*a))*x*PolyLog[3, -E^(-2*a - 2*b*x)])/(4*b) - (3*(1 + E^(-2*a))*PolyLog[4
, -E^(-2*a - 2*b*x)])/(8*b^2)))/(b^2*(1 + E^(2*a))) - (E^(2*a)*((b^4*x^4)/
E^(2*a) - 2*b^3*(1 - E^(-2*a))*x^3*Log[1 - E^(-a - b*x)] - 2*b^3*(1 - E^(-
2*a))*x^3*Log[1 + E^(-a - b*x)] + 6*b^2*(1 - E^(-2*a))*x^2*PolyLog[2, -E^(
-a - b*x)] + 6*b^2*(1 - E^(-2*a))*x^2*PolyLog[2, E^(-a - b*x)] + 12*b*(1 -
 E^(-2*a))*x*PolyLog[3, -E^(-a - b*x)] + 12*b*(1 - E^(-2*a))*x*PolyLog[3,
E^(-a - b*x)] + 12*(1 - E^(-2*a))*PolyLog[4, -E^(-a - b*x)] + 12*(1 - E^(-
2*a))*PolyLog[4, E^(-a - b*x)]))/(2*b^4*(-1 + E^(2*a))) + (x^4*Csch[a]*Sec
h[a])/4 + (x^3*Sech[a + b*x]^2)/(2*b) - (3*x^2*Sech[a]*Sech[a + b*x]*Sinh[
b*x])/(2*b^2)
```

3.481.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5985, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{5985}$$

$$-3\int \frac{1}{2}x^2\left(\frac{2\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{b}\right)dx-\frac{x^3\tanh^2(a+bx)}{2b}+\frac{x^3\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{2}\int x^2\left(\frac{2\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{b}\right)dx-\frac{x^3\tanh^2(a+bx)}{2b}+\frac{x^3\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2010}$$

$$-\frac{3}{2}\int \left(\frac{2x^2\log(\tanh(a+bx))}{b}-\frac{x^2\tanh^2(a+bx)}{b}\right)dx-\frac{x^3\tanh^2(a+bx)}{2b}+\frac{x^3\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}\left(\frac{4x^3\text{arctanh}\left(e^{2a+2bx}\right)}{3b}-\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^4}+\frac{\text{PolyLog}\left(4,-e^{2a+2bx}\right)}{2b^4}-\frac{\text{PolyLog}\left(4,e^{2a+2bx}\right)}{2b^4}-\frac{x\,\text{PolyL}}{}\right.$$
$$\frac{x^3\tanh^2(a+bx)}{2b}+\frac{x^3\log(\tanh(a+bx))}{b}$$

input
```
Int[x^3*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
(x^3*Log[Tanh[a + b*x]])/b - (x^3*Tanh[a + b*x]^2)/(2*b) - (3*(x^2/b^2 - x
^3/(3*b) + (4*x^3*ArcTanh[E^(2*a + 2*b*x)])/(3*b) - (2*x*Log[1 + E^(2*(a +
 b*x))])/b^3 + (2*x^3*Log[Tanh[a + b*x]])/(3*b) - PolyLog[2, -E^(2*(a + b*
x))]/b^4 + (x^2*PolyLog[2, -E^(2*a + 2*b*x)])/b^2 - (x^2*PolyLog[2, E^(2*a
 + 2*b*x)])/b^2 - (x*PolyLog[3, -E^(2*a + 2*b*x)])/b^3 + (x*PolyLog[3, E^(
2*a + 2*b*x)])/b^3 + PolyLog[4, -E^(2*a + 2*b*x)]/(2*b^4) - PolyLog[4, E^(
2*a + 2*b*x)]/(2*b^4) + (x^2*Tanh[a + b*x])/b^2))/2
```

3.481.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.481.4 Maple [A] (verified)

Time = 7.58 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.50

method	result
risch	$\frac{x^2(2\,\mathrm{e}^{2bx+2a}bx+3\,\mathrm{e}^{2bx+2a}+3)}{b^2(1+\mathrm{e}^{2bx+2a})^2}-\frac{3a^2}{b^4}-\frac{3x^2}{b^2}-\frac{6ax}{b^3}+\frac{6\,\mathrm{polylog}(4,\mathrm{e}^{bx+a})}{b^4}-\frac{3\,\mathrm{polylog}(4,-\mathrm{e}^{2bx+2a})}{4b^4}+\frac{6\,\mathrm{polylog}(4,-\mathrm{e}^{bx+a})}{b^4}$

input
```
int(x^3*csch(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
x^2*(2*exp(2*b*x+2*a)*b*x+3*exp(2*b*x+2*a)+3)/b^2/(1+exp(2*b*x+2*a))^2-3/b
^4*a^2-3/b^2*x^2-6/b^3*a*x+6*polylog(4,exp(b*x+a))/b^4-3/4*polylog(4,-exp(
2*b*x+2*a))/b^4+6*polylog(4,-exp(b*x+a))/b^4+6/b^4*a*ln(exp(b*x+a))-1/b^4*
a^3*ln(exp(b*x+a)-1)-6*x*polylog(3,exp(b*x+a))/b^3-x^3*ln(1+exp(2*b*x+2*a)
)/b-3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a)
)/b^3+1/b*ln(exp(b*x+a)+1)*x^3+3*x^2*polylog(2,-exp(b*x+a))/b^2-6*x*polylo
g(3,-exp(b*x+a))/b^3+3*x^2*polylog(2,exp(b*x+a))/b^2+1/b*ln(1-exp(b*x+a))*
x^3+3*x*ln(1+exp(2*b*x+2*a))/b^3+3/2*polylog(2,-exp(2*b*x+2*a))/b^4+1/b^4*
ln(1-exp(b*x+a))*a^3
```

3.481.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 3409, normalized size of antiderivative = 14.20

$$\int x^3 \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sin
h(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 - (2*b^3*x^3 - 3*b^2*x^2
+ 6*a^2)*cosh(b*x + a)^2 - (2*b^3*x^3 - 3*b^2*x^2 - 18*(b^2*x^2 - a^2)*cos
h(b*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 - 3*(b^2*x^2*cosh(b*x + a)^4
 + 4*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 + 2*b
^2*x^2*cosh(b*x + a)^2 + b^2*x^2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 + b^2*x^2)
*sinh(b*x + a)^2 + 4*(b^2*x^2*cosh(b*x + a)^3 + b^2*x^2*cosh(b*x + a))*sin
h(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) + 3*((b^2*x^2 - 1)*cosh(b
*x + a)^4 + 4*(b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*
sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 +
 3*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*c
osh(b*x + a)^3 + (b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(I*c
osh(b*x + a) + I*sinh(b*x + a)) + 3*((b^2*x^2 - 1)*cosh(b*x + a)^4 + 4*(b^
2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x + a)^4 +
 b^2*x^2 + 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - 1)*
cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a)^3 +
(b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(-I*cosh(b*x + a) - I
*sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*sin
h(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 + 2*b^2*x^2*cosh(b*x + a)^2 + b^2*x
^2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 + b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x...
```

3.481.6 Sympy [F]

$$\int x^3 \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(x**3*csch(b*x+a)*sech(b*x+a)**3,x)
```

output
```
Integral(x**3*csch(a + b*x)*sech(a + b*x)**3, x)
```

3.481.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.37

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= -\frac{1}{2}x^4 + \frac{3x^2 + \left(2bx^3e^{(2a)} + 3x^2e^{(2a)}\right)e^{(2bx)}}{b^2e^{(4bx+4a)} + 2b^2e^{(2bx+2a)} + b^2} + \frac{b^4x^4 - 6b^2x^2}{2b^4}$$
$$- \frac{4b^3x^3\log\left(e^{(2bx+2a)}+1\right) + 6b^2x^2\mathrm{Li}_2\left(-e^{(2bx+2a)}\right) - 6bx\mathrm{Li}_3(-e^{(2bx+2a)}) + 3\mathrm{Li}_4(-e^{(2bx+2a)})}{3b^4}$$
$$+ \frac{b^3x^3\log\left(e^{(bx+a)}+1\right) + 3b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 6bx\mathrm{Li}_3(-e^{(bx+a)}) + 6\mathrm{Li}_4(-e^{(bx+a)})}{b^4}$$
$$+ \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right) + 3b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right) - 6bx\mathrm{Li}_3(e^{(bx+a)}) + 6\mathrm{Li}_4(e^{(bx+a)})}{b^4}$$
$$+ \frac{3\left(2bx\log\left(e^{(2bx+2a)}+1\right) + \mathrm{Li}_2\left(-e^{(2bx+2a)}\right)\right)}{2b^4}$$

input
```
integrate(x^3*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*x^4 + (3*x^2 + (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4
*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 1/2*(b^4*x^4 - 6*b^2*x^2)/b^4
 - 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2
*a)) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a))
)/b^4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*
b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*
log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^
(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4 + 3/2*(2*b*x*log(e^(2*b*x + 2*
a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^4
```

3.481.8 Giac [F]

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^3 \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3\,dx$$

input
```
integrate(x^3*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*csch(b*x + a)*sech(b*x + a)^3, x)
```

3.481.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int \frac{x^3}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(x^3/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

output
```
int(x^3/(cosh(a + b*x)^3*sinh(a + b*x)), x)
```

3.482 $\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$

3.482.1 Optimal result

Integrand size = 18, antiderivative size = 148

$$\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{x^2}{2b} - \frac{2x^2 \operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x \operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} + \frac{x \operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} + \frac{\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3} - \frac{\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3} - \frac{x\tanh(a+bx)}{b^2} - \frac{x^2\tanh^2(a+bx)}{2b}$$

output
```
1/2*x^2/b-2*x^2*arctanh(exp(2*b*x+2*a))/b+ln(cosh(b*x+a))/b^3-x*polylog(2,
-exp(2*b*x+2*a))/b^2+x*polylog(2,exp(2*b*x+2*a))/b^2+1/2*polylog(3,-exp(2*
b*x+2*a))/b^3-1/2*polylog(3,exp(2*b*x+2*a))/b^3-x*tanh(b*x+a)/b^2-1/2*x^2*
tanh(b*x+a)^2/b
```

3.482.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 380 vs. $2(148) = 296$.

Time = 2.61 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.57

$$\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{1}{6}\left(-\frac{2e^{2a}\left(2b^3e^{-2a}x^3 - 3b^2(1-e^{-2a})\,x^2\log\left(1-e^{-a-bx}\right) - 3b^2(1-e^{-2a})\,x^2\log\left(1+e^{-a-bx}\right) + 6b(1-e^{-2}\right.}{}\right.$$
$$+\frac{-12be^{2a}x - 4b^3x^3 - 6b^2(1+e^{2a})\,x^2\log\left(1+e^{-2(a+bx)}\right) + 6\log\left(1+e^{2(a+bx)}\right) + 6e^{2a}\log\left(1+e^{2(a+bx)}\right) +}{b^3\left(1+e^{2a}\right)}$$
$$\left. + 2x^3\operatorname{csch}(a)\operatorname{sech}(a) + \frac{3x^2\operatorname{sech}^2(a+bx)}{b} - \frac{6x\operatorname{sech}(a)\operatorname{sech}(a+bx)\sinh(bx)}{b^2}\right)$$

input
```
Integrate[x^2*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
((-2*E^(2*a)*((2*b^3*x^3)/E^(2*a) - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 - E^(-a
 - b*x)] - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 + E^(-a - b*x)] + 6*b*(1 - E^(-2
*a))*x*PolyLog[2, -E^(-a - b*x)] + 6*b*(1 - E^(-2*a))*x*PolyLog[2, E^(-a -
 b*x)] + 6*(1 - E^(-2*a))*PolyLog[3, -E^(-a - b*x)] + 6*(1 - E^(-2*a))*Pol
yLog[3, E^(-a - b*x)]))/(b^3*(-1 + E^(2*a))) + (-12*b*E^(2*a)*x - 4*b^3*x^
3 - 6*b^2*(1 + E^(2*a))*x^2*Log[1 + E^(-2*(a + b*x))] + 6*Log[1 + E^(2*(a
+ b*x))] + 6*E^(2*a)*Log[1 + E^(2*(a + b*x))] + 6*b*(1 + E^(2*a))*x*PolyLo
g[2, -E^(-2*(a + b*x))] + 3*(1 + E^(2*a))*PolyLog[3, -E^(-2*(a + b*x))])/(
b^3*(1 + E^(2*a))) + 2*x^3*Csch[a]*Sech[a] + (3*x^2*Sech[a + b*x]^2)/b - (
6*x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b^2)/6
```

3.482.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5985, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$\Big\downarrow \text{5985}$$
$$-2\int \frac{1}{2}x\left(\frac{2\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{b}\right)dx - \frac{x^2\tanh^2(a+bx)}{2b} + \frac{x^2\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{27}$$

$$-\int x\left(\frac{2\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{b}\right)dx-\frac{x^2\tanh^2(a+bx)}{2b}+\frac{x^2\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2010}$$

$$-\int\left(\frac{2x\log(\tanh(a+bx))}{b}-\frac{x\tanh^2(a+bx)}{b}\right)dx-\frac{x^2\tanh^2(a+bx)}{2b}+\frac{x^2\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b}+\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{\log(\cosh(a+bx))}{b^3}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}+\frac{x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{x\tanh(a+bx)}{b^2}-\frac{x^2\tanh^2(a+bx)}{2b}+\frac{x^2}{2b}$$

input
```
Int[x^2*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
x^2/(2*b) - (2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b + Log[Cosh[a + b*x]]/b^3 -
(x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2 + (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2
 + PolyLog[3, -E^(2*a + 2*b*x)]/(2*b^3) - PolyLog[3, E^(2*a + 2*b*x)]/(2*b
^3) - (x*Tanh[a + b*x])/b^2 - (x^2*Tanh[a + b*x]^2)/(2*b)
```

3.482.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.482. $\int x^2\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$

3.482.4 Maple [A] (verified)

Time = 5.47 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.73

method	result
risch	$\frac{2x\left(\mathrm{e}^{2bx+2a}bx+\mathrm{e}^{2bx+2a}+1\right)}{b^{2}\left(1+\mathrm{e}^{2bx+2a}\right)^{2}}-\frac{2\ln\left(\mathrm{e}^{bx+a}\right)}{b^{3}}-\frac{2\,\mathrm{polylog}\left(3,\mathrm{e}^{bx+a}\right)}{b^{3}}+\frac{\mathrm{polylog}\left(3,-\mathrm{e}^{2bx+2a}\right)}{2b^{3}}-\frac{2\,\mathrm{polylog}\left(3,-\mathrm{e}^{bx+a}\right)}{b^{3}}+\frac{\ln\left(1-\mathrm{e}^{b}\right.}{b}$

input
```
int(x^2*csch(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*x*(exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2-2/b^3*l
n(exp(b*x+a))-2*polylog(3,exp(b*x+a))/b^3+1/2*polylog(3,-exp(2*b*x+2*a))/b
^3-2*polylog(3,-exp(b*x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^2+2*x*polylog(2,exp
(b*x+a))/b^2-x^2*ln(1+exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2+1
/b*ln(exp(b*x+a)+1)*x^2+2*x*polylog(2,-exp(b*x+a))/b^2+1/b^3*a^2*ln(exp(b*
x+a)-1)+1/b^3*ln(1+exp(2*b*x+2*a))-1/b^3*ln(1-exp(b*x+a))*a^2
```

3.482.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 2523, normalized size of antiderivative = 17.05

$$\int x^2\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(2*(b*x + a)*cosh(b*x + a)^4 + 8*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3
+ 2*(b*x + a)*sinh(b*x + a)^4 - 2*(b^2*x^2 - b*x - 2*a)*cosh(b*x + a)^2 -
2*(b^2*x^2 - 6*(b*x + a)*cosh(b*x + a)^2 - b*x - 2*a)*sinh(b*x + a)^2 - 2*
(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x
+ a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*
dilog(cosh(b*x + a) + sinh(b*x + a)) + 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh
(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 +
2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x +
a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*
x + a)) + 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b
*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*
x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sin
h(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b*x*cosh(b*x +
a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*c
osh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4
*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x
+ a) - sinh(b*x + a)) - (b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)
*sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 + 2*b^2*x^2*cosh(b*x + a)^2 + b
^2*x^2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 + b^2*x^2)*sinh(b*x + a)^2 + 4*(b...
```

3.482.6 Sympy [F]

$$\int x^2\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \int x^2\,\text{csch}\,(a+bx)\,\text{sech}^3\,(a+bx)\,dx$$

input
```
integrate(x**2*csch(b*x+a)*sech(b*x+a)**3,x)
```

output
```
Integral(x**2*csch(a + b*x)*sech(a + b*x)**3, x)
```

3.482.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.55

$$\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{2\left(\left(bx^2e^{(2a)}+xe^{(2a)}\right)e^{(2bx)}+x\right)}{b^2e^{(4bx+4a)}+2b^2e^{(2bx+2a)}+b^2} - \frac{2x}{b^2} - \frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(2bx+2a)}\right)-\mathrm{Li}_3\left(-e^{(2bx+2a)}\right)}{2b^3} + \frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\mathrm{Li}_3\left(-e^{(bx+a)}\right)}{b^3} + \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\mathrm{Li}_3\left(e^{(bx+a)}\right)}{b^3} + \frac{\log\left(e^{(2bx+2a)}+1\right)}{b^3}$$

input
```
integrate(x^2*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
2*((b*x^2*e^(2*a) + x*e^(2*a))*e^(2*b*x) + x)/(b^2*e^(4*b*x + 4*a) + 2*b^2
*e^(2*b*x + 2*a) + b^2) - 2*x/b^2 - 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1
) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3 + (b
^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^
(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)
) - 2*polylog(3, e^(b*x + a)))/b^3 + log(e^(2*b*x + 2*a) + 1)/b^3
```

3.482.8 Giac [F]

$$\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^2 \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3\,dx$$

input
```
integrate(x^2*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*csch(b*x + a)*sech(b*x + a)^3, x)
```

3.482.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \frac{x^2}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

output
```
int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)), x)
```

3.483 $\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$

3.483.1 Optimal result

Integrand size = 16, antiderivative size = 95

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\frac{x}{2b}-\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}$$

output
```
1/2*x/b-2*x*arctanh(exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b^2+1
/2*polylog(2,exp(2*b*x+2*a))/b^2-1/2*tanh(b*x+a)/b^2-1/2*x*tanh(b*x+a)^2/b
```

3.483.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\frac{2bx\log\left(1-e^{-2(a+bx)}\right)-2bx\log\left(1+e^{-2(a+bx)}\right)+\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)-\text{PolyLog}\left(2,e^{-2(a+bx)}\right)+bxs}{2b^2}$$

input
```
Integrate[x*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
(2*b*x*Log[1 - E^(-2*(a + b*x))] - 2*b*x*Log[1 + E^(-2*(a + b*x))] + PolyL
og[2, -E^(-2*(a + b*x))] - PolyLog[2, E^(-2*(a + b*x))] + b*x*Sech[a + b*x
]^2 - Tanh[a + b*x])/(2*b^2)
```

3.483.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5985, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{5985}$$

$$-\int \left(\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}\right)dx-\frac{x\tanh^2(a+bx)}{2b}+\frac{x\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}+\frac{x}{2b}$$

input
```
Int[x*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
x/(2*b) - (2*x*ArcTanh[E^(2*a + 2*b*x)])/b - PolyLog[2, -E^(2*a + 2*b*x)]/
(2*b^2) + PolyLog[2, E^(2*a + 2*b*x)]/(2*b^2) - Tanh[a + b*x]/(2*b^2) - (x
*Tanh[a + b*x]^2)/(2*b)
```

3.483.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.483.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(82) = 164.

Time = 3.82 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.75

method	result
risch	$\frac{2\,\mathrm{e}^{2bx+2a}bx+\mathrm{e}^{2bx+2a}+1}{b^{2}\left(1+\mathrm{e}^{2bx+2a}\right)^{2}}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a}{b^{2}}+\frac{\mathrm{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{b^{2}}+\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)x}{b}+\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^{2}}-$

input
```
int(x*csch(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2+1/b*ln(1-
exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2+1/b*ln(ex
p(b*x+a)+1)*x+polylog(2,-exp(b*x+a))/b^2-x*ln(1+exp(2*b*x+2*a))/b-1/2*poly
log(2,-exp(2*b*x+2*a))/b^2-1/b^2*a*ln(exp(b*x+a)-1)
```

3.483.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1543, normalized size of antiderivative = 16.24

$$\int x\mathrm{csch}(a+bx)\mathrm{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
((2*b*x + 1)*cosh(b*x + a)^2 + 2*(2*b*x + 1)*cosh(b*x + a)*sinh(b*x + a) +
 (2*b*x + 1)*sinh(b*x + a)^2 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x
 + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*
cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*d
ilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*s
inh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)
^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
 + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh
(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*si
nh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x + a
)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(
b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*
cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^
4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*log(c
osh(b*x + a) + sinh(b*x + a) + 1) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)
*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b
*x + a)^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a)...
```

3.483.6 Sympy [F]

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \int x\,\text{csch}\,(a+bx)\,\text{sech}^3\,(a+bx)\,dx$$

input
```
integrate(x*csch(b*x+a)*sech(b*x+a)**3,x)
```

output
```
Integral(x*csch(a + b*x)*sech(a + b*x)**3, x)
```

3.483.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.49

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \frac{\left(2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+1}{b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2} - \frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2} + \frac{bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

input
```
integrate(x*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
((2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) + 1)/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(
2*b*x + 2*a) + b^2) - 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*
x + 2*a)))/b^2 + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b
*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

3.483.8 Giac [F]

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \int x\,\text{csch}\left(bx+a\right)\text{sech}\left(bx+a\right)^3\,dx$$

input
```
integrate(x*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x*csch(b*x + a)*sech(b*x + a)^3, x)
```

3.483.9 Mupad [F(-1)]

Timed out.

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \int \frac{x}{\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(x/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

output
```
int(x/(cosh(a + b*x)^3*sinh(a + b*x)), x)
```

3.484 $\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$

3.484.1 Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

output
```
ln(tanh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

3.484.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=-\frac{2\log(\cosh(a+bx))-2\log(\sinh(a+bx))-\operatorname{sech}^2(a+bx)}{2b}$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
-1/2*(2*Log[Cosh[a + b*x]] - 2*Log[Sinh[a + b*x]] - Sech[a + b*x]^2)/b
```

3.484.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int -i\coth(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int (i\tanh(a+bx)-i\coth(a+bx))d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\tanh^2(a+bx)+\log(i\tanh(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
(Log[I*Tanh[a + b*x]] - Tanh[a + b*x]^2/2)/b
```

3.484.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.484.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	23
default	$\frac{\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	23
risch	$\frac{2\,\mathrm{e}^{2bx+2a}}{b(1+\mathrm{e}^{2bx+2a})^2}-\frac{\ln(1+\mathrm{e}^{2bx+2a})}{b}+\frac{\ln(\mathrm{e}^{2bx+2a}-1)}{b}$	62

input
```
int(csch(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2/cosh(b*x+a)^2+ln(tanh(b*x+a)))
```

3.484.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 371, normalized size of antiderivative = 13.74

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{2\cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2\right.\right.}{}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
(2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x +
 a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x
+ a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*
x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*
x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(
b*x + a)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(
b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*
b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x
 + a))*sinh(b*x + a) + b)
```

3.484.6 Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**3, x)
```

3.484.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1)}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

3.484.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\frac{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+6}{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2} - \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right) + \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{2\,b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
 - 2*a) + 2) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x
+ 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

3.484.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.89

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

output
```
2/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/
b))/(-b^2)^(1/2) - 2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
```

3.485 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$

3.485.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x},x\right)$$

output
```
CannotIntegrate(csch(b*x+a)*sech(b*x+a)^3/x,x)
```

3.485.2 Mathematica [N/A]

Not integrable

Time = 47.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

input
```
Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x,x]
```

output
```
Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x, x]
```

3.485.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}dx$$

input
```
Int[(Csch[a + b*x]*Sech[a + b*x]^3)/x,x]
```

output
```
$Aborted
```

3.485.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.485.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x}dx$$

input
```
int(csch(b*x+a)*sech(b*x+a)^3/x,x)
```

output
```
int(csch(b*x+a)*sech(b*x+a)^3/x,x)
```

3.485.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)*sech(b*x + a)^3/x, x)
```

3.485.6 Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**3/x,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**3/x, x)
```

3.485.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 8.17

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^2*e^(4*b*x + 4*a) + 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 16*integrate(1/8*(b^2*x^2 - 1)/(b^2*x^3*
e^(2*b*x + 2*a) + b^2*x^3), x) - 16*integrate(1/16/(x*e^(b*x + a) + x), x)
 + 16*integrate(1/16/(x*e^(b*x + a) - x), x)
```

3.485.8 Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)*sech(b*x + a)^3/x, x)
```

3.485.9 Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{1}{x\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)),x)
```

output
```
int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)), x)
```

3.486 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$

3.486.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2},x\right)$$

output
```
CannotIntegrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x)
```

3.486.2 Mathematica [N/A]

Not integrable

Time = 26.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2,x]
```

output
```
Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2, x]
```

3.486.3 Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}dx$$

input
```
Int[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2,x]
```

output
```
$Aborted
```

3.486.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.486.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x^2}dx$$

input
```
int(csch(b*x+a)*sech(b*x+a)^3/x^2,x)
```

output
```
int(csch(b*x+a)*sech(b*x+a)^3/x^2,x)
```

3.486.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)*sech(b*x + a)^3/x^2, x)
```

3.486.6 Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**3/x**2,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**3/x**2, x)
```

3.486.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 8.61

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^3*e^(4*b*x + 4*a) + 2*b^2
*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 16*integrate(1/8*(b^2*x^2 - 3)/(b^2*x^4*
e^(2*b*x + 2*a) + b^2*x^4), x) - 16*integrate(1/16/(x^2*e^(b*x + a) + x^2)
, x) + 16*integrate(1/16/(x^2*e^(b*x + a) - x^2), x)
```

3.486.8 Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)*sech(b*x + a)^3/x^2, x)
```

3.486.9 Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)}\,dx$$

input
```
int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)),x)
```

output
```
int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)), x)
```

3.487 $\int x^m \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$

3.487.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\text{Int}\left(x^m \text{csch}^2(a+bx)\text{sech}(a+bx),x\right)$$

output
```
CannotIntegrate(x^m*csch(b*x+a)^2*sech(b*x+a),x)
```

3.487.2 Mathematica [N/A]

Not integrable

Time = 45.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int x^m \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$

input
```
Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x], x]
```

3.487.3 Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int x^m \text{csch}^2(a+bx)\text{sech}(a+bx)dx$$

input
```
Int[x^m*Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
$Aborted
```

3.487.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.487.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\,dx$$

input
```
int(x^m*csch(b*x+a)^2*sech(b*x+a),x)
```

output
```
int(x^m*csch(b*x+a)^2*sech(b*x+a),x)
```

3.487.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\,dx$$

input
```
integrate(x^m*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*csch(b*x + a)^2*sech(b*x + a), x)
```

3.487.6 Sympy [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^m \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(x**m*csch(b*x+a)**2*sech(b*x+a),x)
```

output
```
Integral(x**m*csch(a + b*x)**2*sech(a + b*x), x)
```

3.487.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\,dx$$

input
```
integrate(x^m*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^m*csch(b*x + a)^2*sech(b*x + a), x)
```

3.487.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\,dx$$

input
```
integrate(x^m*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*csch(b*x + a)^2*sech(b*x + a), x)
```

3.487.9 Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \int \frac{x^m}{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}\,dx$$

input
```
int(x^m/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

output
```
int(x^m/(cosh(a + b*x)*sinh(a + b*x)^2), x)
```

3.488 $\int x^3\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$

3.488.1 Optimal result

Integrand size = 18, antiderivative size = 237

$$\begin{aligned}\int x^3\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = &-\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}-\frac{6x^2\text{arctanh}\left(e^{a+bx}\right)}{b^2}\\&-\frac{x^3\text{csch}(a+bx)}{b}-\frac{6x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}\\&+\frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}\\&-\frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{6x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}\\&+\frac{6\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4}-\frac{6ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}\\&+\frac{6ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{6\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4}\\&+\frac{6i\,\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4}-\frac{6i\,\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^4}\end{aligned}$$

output
```
-2*x^3*arctan(exp(b*x+a))/b-6*x^2*arctanh(exp(b*x+a))/b^2-x^3*csch(b*x+a)/
b-6*x*polylog(2,-exp(b*x+a))/b^3+3*I*x^2*polylog(2,-I*exp(b*x+a))/b^2-3*I*
x^2*polylog(2,I*exp(b*x+a))/b^2+6*x*polylog(2,exp(b*x+a))/b^3+6*polylog(3,
-exp(b*x+a))/b^4-6*I*x*polylog(3,-I*exp(b*x+a))/b^3+6*I*x*polylog(3,I*exp(
b*x+a))/b^3-6*polylog(3,exp(b*x+a))/b^4+6*I*polylog(4,-I*exp(b*x+a))/b^4-6
*I*polylog(4,I*exp(b*x+a))/b^4
```

3.488.2 Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.41

$$\int x^3\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \frac{-2b^3x^3\text{csch}(a)+6b^2x^2\log\left(1-e^{a+bx}\right)-2ib^3x^3\log\left(1-ie^{a+bx}\right)+2ib^3x^3\log\left(1+ie^{a+bx}\right)-6b^2x^2\log\left(1+\right.}{}$$

input
```
Integrate[x^3*Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
(-2*b^3*x^3*Csch[a] + 6*b^2*x^2*Log[1 - E^(a + b*x)] - (2*I)*b^3*x^3*Log[1
 - I*E^(a + b*x)] + (2*I)*b^3*x^3*Log[1 + I*E^(a + b*x)] - 6*b^2*x^2*Log[1
 + E^(a + b*x)] - 12*b*x*PolyLog[2, -E^(a + b*x)] + (6*I)*b^2*x^2*PolyLog[
2, (-I)*E^(a + b*x)] - (6*I)*b^2*x^2*PolyLog[2, I*E^(a + b*x)] + 12*b*x*Po
lyLog[2, E^(a + b*x)] + 12*PolyLog[3, -E^(a + b*x)] - (12*I)*b*x*PolyLog[3
, (-I)*E^(a + b*x)] + (12*I)*b*x*PolyLog[3, I*E^(a + b*x)] - 12*PolyLog[3,
 E^(a + b*x)] + (12*I)*PolyLog[4, (-I)*E^(a + b*x)] - (12*I)*PolyLog[4, I*
E^(a + b*x)] + b^3*x^3*Csch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] + b^3*x^3
*Sech[a/2]*Sech[(a + b*x)/2]*Sinh[(b*x)/2])/(2*b^4)
```

3.488.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5985, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{5985}$$

$$-3\int -x^2\left(\frac{\arctan(\sinh(a+bx))}{b}+\frac{\text{csch}(a+bx)}{b}\right)dx-\frac{x^3\arctan(\sinh(a+bx))}{b}-\frac{x^3\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$3\int x^2\left(\frac{\arctan(\sinh(a+bx))}{b}+\frac{\text{csch}(a+bx)}{b}\right)dx-\frac{x^3\arctan(\sinh(a+bx))}{b}-\frac{x^3\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{2010}$$

$$3\int \left(\frac{\arctan(\sinh(a+bx))x^2}{b}+\frac{\operatorname{csch}(a+bx)x^2}{b}\right)dx-\frac{x^3\arctan(\sinh(a+bx))}{b}-\frac{x^3\operatorname{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{2x^3\arctan\left(e^{a+bx}\right)}{3b}+\frac{x^3\arctan(\sinh(a+bx))}{3b}-\frac{2x^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b^2}+\frac{2\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^4}-\frac{2\operatorname{PolyLog}\left(3\right.}{b^4}\right.$$
$$\frac{x^3\arctan(\sinh(a+bx))}{b}-\frac{x^3\operatorname{csch}(a+bx)}{b}$$

input
```
Int[x^3*Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
-((x^3*ArcTan[Sinh[a + b*x]])/b) - (x^3*Csch[a + b*x])/b + 3*((-2*x^3*ArcT
an[E^(a + b*x)])/(3*b) + (x^3*ArcTan[Sinh[a + b*x]])/(3*b) - (2*x^2*ArcTan
h[E^(a + b*x)])/b^2 - (2*x*PolyLog[2, -E^(a + b*x)])/b^3 + (I*x^2*PolyLog[
2, (-I)*E^(a + b*x)])/b^2 - (I*x^2*PolyLog[2, I*E^(a + b*x)])/b^2 + (2*x*P
olyLog[2, E^(a + b*x)])/b^3 + (2*PolyLog[3, -E^(a + b*x)])/b^4 - ((2*I)*x*
PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((2*I)*x*PolyLog[3, I*E^(a + b*x)])/b^
3 - (2*PolyLog[3, E^(a + b*x)])/b^4 + ((2*I)*PolyLog[4, (-I)*E^(a + b*x)])
/b^4 - ((2*I)*PolyLog[4, I*E^(a + b*x)])/b^4)
```

3.488.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.488.4 Maple [F]

$$\int x^3 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\, dx$$

input
```
int(x^3*csch(b*x+a)^2*sech(b*x+a),x)
```

output
```
int(x^3*csch(b*x+a)^2*sech(b*x+a),x)
```

3.488.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1309 vs. $2(197) = 394$.

Time = 0.31 (sec) , antiderivative size = 1309, normalized size of antiderivative = 5.52

$$\int x^3 \text{csch}^2(a+bx)\text{sech}(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate(x^3*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

output
```
-(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) - 6*(b*x*cosh(b*x + a)
^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(
cosh(b*x + a) + sinh(b*x + a)) + 3*(I*b^2*x^2*cosh(b*x + a)^2 + 2*I*b^2*x^
2*cosh(b*x + a)*sinh(b*x + a) + I*b^2*x^2*sinh(b*x + a)^2 - I*b^2*x^2)*dil
og(I*cosh(b*x + a) + I*sinh(b*x + a)) + 3*(-I*b^2*x^2*cosh(b*x + a)^2 - 2*
I*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) - I*b^2*x^2*sinh(b*x + a)^2 + I*b^2*
x^2)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^2 +
2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(-cosh
(b*x + a) - sinh(b*x + a)) + 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b
*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 - b^2*x^2)*log(cosh(b*x +
a) + sinh(b*x + a) + 1) - (I*a^3*cosh(b*x + a)^2 + 2*I*a^3*cosh(b*x + a)*s
inh(b*x + a) + I*a^3*sinh(b*x + a)^2 - I*a^3)*log(cosh(b*x + a) + sinh(b*x
 + a) + I) - (-I*a^3*cosh(b*x + a)^2 - 2*I*a^3*cosh(b*x + a)*sinh(b*x + a)
 - I*a^3*sinh(b*x + a)^2 + I*a^3)*log(cosh(b*x + a) + sinh(b*x + a) - I) -
 3*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x
 + a)^2 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - (-I*b^3*x^3 - I*a^
3 + (I*b^3*x^3 + I*a^3)*cosh(b*x + a)^2 - 2*(-I*b^3*x^3 - I*a^3)*cosh(b*x
+ a)*sinh(b*x + a) + (I*b^3*x^3 + I*a^3)*sinh(b*x + a)^2)*log(I*cosh(b*x +
 a) + I*sinh(b*x + a) + 1) - (I*b^3*x^3 + I*a^3 + (-I*b^3*x^3 - I*a^3)*cos
h(b*x + a)^2 - 2*(I*b^3*x^3 + I*a^3)*cosh(b*x + a)*sinh(b*x + a) + (-I*...
```

3.488.6 Sympy [F]

$$\int x^3 \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \int x^3 \,\text{csch}^2\,(a+bx)\,\text{sech}\,(a+bx)\,dx$$

input
```
integrate(x**3*csch(b*x+a)**2*sech(b*x+a),x)
```

output
```
Integral(x**3*csch(a + b*x)**2*sech(a + b*x), x)
```

3.488.7 Maxima [F]

$$\int x^3 \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \int x^3 \,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)\;dx$$

input
```
integrate(x^3*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

output
```
-2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 3*(b^2*x^2*log(e^(b*x + a) +
1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^4 + 3*(b^2*
x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x
 + a)))/b^4 - 8*integrate(1/4*x^3*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

3.488.8 Giac [F]

$$\int x^3 \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \int x^3 \,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)\;dx$$

input
```
integrate(x^3*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*csch(b*x + a)^2*sech(b*x + a), x)
```

3.488.9 Mupad [F(-1)]

Timed out.

$$\int x^3\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int \frac{x^3}{\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^2}\,dx$$

input
```
int(x^3/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

output
```
int(x^3/(cosh(a + b*x)*sinh(a + b*x)^2), x)
```

3.489 $\int x^2 \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$

3.489.1 Optimal result

Integrand size = 18, antiderivative size = 157

$$\int x^2 \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = -\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{csch}(a+bx)}{b}-\frac{2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{2ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{2i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{2i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}$$

output
```
-2*x^2*arctan(exp(b*x+a))/b-4*x*arctanh(exp(b*x+a))/b^2-x^2*csch(b*x+a)/b-
2*polylog(2,-exp(b*x+a))/b^3+2*I*x*polylog(2,-I*exp(b*x+a))/b^2-2*I*x*poly
log(2,I*exp(b*x+a))/b^2+2*polylog(2,exp(b*x+a))/b^3-2*I*polylog(3,-I*exp(b
*x+a))/b^3+2*I*polylog(3,I*exp(b*x+a))/b^3
```

3.489.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.61

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$
$$= \frac{-2b^2x^2\text{csch}(a)+4bx\log\left(1-e^{a+bx}\right)-2ib^2x^2\log\left(1-ie^{a+bx}\right)+2ib^2x^2\log\left(1+ie^{a+bx}\right)-4bx\log\left(1+e^{a-\cdots}\right.}{}$$

input
```
Integrate[x^2*Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
(-2*b^2*x^2*Csch[a] + 4*b*x*Log[1 - E^(a + b*x)] - (2*I)*b^2*x^2*Log[1 - I
*E^(a + b*x)] + (2*I)*b^2*x^2*Log[1 + I*E^(a + b*x)] - 4*b*x*Log[1 + E^(a
+ b*x)] - 4*PolyLog[2, -E^(a + b*x)] + (4*I)*b*x*PolyLog[2, (-I)*E^(a + b*
x)] - (4*I)*b*x*PolyLog[2, I*E^(a + b*x)] + 4*PolyLog[2, E^(a + b*x)] - (4
*I)*PolyLog[3, (-I)*E^(a + b*x)] + (4*I)*PolyLog[3, I*E^(a + b*x)] + b^2*x
^2*Csch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] + b^2*x^2*Sech[a/2]*Sech[(a +
 b*x)/2]*Sinh[(b*x)/2])/(2*b^3)
```

3.489.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5985, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$
$$\Big\downarrow \text{5985}$$
$$-2\int -x\left(\frac{\arctan(\sinh(a+bx))}{b}+\frac{\text{csch}(a+bx)}{b}\right)dx-\frac{x^2\arctan(\sinh(a+bx))}{b}-\frac{x^2\text{csch}(a+bx)}{b}$$
$$\Big\downarrow \text{25}$$
$$2\int x\left(\frac{\arctan(\sinh(a+bx))}{b}+\frac{\text{csch}(a+bx)}{b}\right)dx-\frac{x^2\arctan(\sinh(a+bx))}{b}-\frac{x^2\text{csch}(a+bx)}{b}$$
$$\Big\downarrow \text{2010}$$

$$2\int\left(\frac{x\arctan(\sinh(a+bx))}{b}+\frac{x\text{csch}(a+bx)}{b}\right)dx-\frac{x^2\arctan(\sinh(a+bx))}{b}-\frac{x^2\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{x^2\arctan\left(e^{a+bx}\right)}{b}+\frac{x^2\arctan(\sinh(a+bx))}{2b}-\frac{2x\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{\text{PolyLog}\left(2,e^{a+}\right.}{b^3}\right.$$
$$\frac{x^2\arctan(\sinh(a+bx))}{b}-\frac{x^2\text{csch}(a+bx)}{b}$$

input
```
Int[x^2*Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
-((x^2*ArcTan[Sinh[a + b*x]])/b) - (x^2*Csch[a + b*x])/b + 2*(-((x^2*ArcTa
n[E^(a + b*x)])/b) + (x^2*ArcTan[Sinh[a + b*x]])/(2*b) - (2*x*ArcTanh[E^(a
 + b*x)])/b^2 - PolyLog[2, -E^(a + b*x)]/b^3 + (I*x*PolyLog[2, (-I)*E^(a +
 b*x)])/b^2 - (I*x*PolyLog[2, I*E^(a + b*x)])/b^2 + PolyLog[2, E^(a + b*x)
]/b^3 - (I*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + (I*PolyLog[3, I*E^(a + b*x)
])/b^3)
```

3.489.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.489.4 Maple [F]

$$\int x^2 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\,dx$$

input
```
int(x^2*csch(b*x+a)^2*sech(b*x+a),x)
```

output
```
int(x^2*csch(b*x+a)^2*sech(b*x+a),x)
```

3.489.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 966 vs. 2(129) = 258.

Time = 0.28 (sec) , antiderivative size = 966, normalized size of antiderivative = 6.15

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

output
```
-(2*b^2*x^2*cosh(b*x + a) + 2*b^2*x^2*sinh(b*x + a) - 2*(cosh(b*x + a)^2 +
 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a)
+ sinh(b*x + a)) + 2*(I*b*x*cosh(b*x + a)^2 + 2*I*b*x*cosh(b*x + a)*sinh(b
*x + a) + I*b*x*sinh(b*x + a)^2 - I*b*x)*dilog(I*cosh(b*x + a) + I*sinh(b*
x + a)) + 2*(-I*b*x*cosh(b*x + a)^2 - 2*I*b*x*cosh(b*x + a)*sinh(b*x + a)
- I*b*x*sinh(b*x + a)^2 + I*b*x)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a))
 + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 -
1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + 2*(b*x*cosh(b*x + a)^2 + 2*b*x*
cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(b*x + a)
 + sinh(b*x + a) + 1) - (-I*a^2*cosh(b*x + a)^2 - 2*I*a^2*cosh(b*x + a)*si
nh(b*x + a) - I*a^2*sinh(b*x + a)^2 + I*a^2)*log(cosh(b*x + a) + sinh(b*x
+ a) + I) - (I*a^2*cosh(b*x + a)^2 + 2*I*a^2*cosh(b*x + a)*sinh(b*x + a) +
 I*a^2*sinh(b*x + a)^2 - I*a^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) + 2
*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2
- a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - (-I*b^2*x^2 + (I*b^2*x^2 - I
*a^2)*cosh(b*x + a)^2 - 2*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)*sinh(b*x + a)
 + (I*b^2*x^2 - I*a^2)*sinh(b*x + a)^2 + I*a^2)*log(I*cosh(b*x + a) + I*si
nh(b*x + a) + 1) - (I*b^2*x^2 + (-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^2 - 2*(
I*b^2*x^2 - I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (-I*b^2*x^2 + I*a^2)*sinh
(b*x + a)^2 - I*a^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 2*((...
```

3.489.6 Sympy [F]

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int x^2\,\text{csch}^2\,(a+bx)\,\text{sech}\,(a+bx)\,dx$$

input
```
integrate(x**2*csch(b*x+a)**2*sech(b*x+a),x)
```

output
```
Integral(x**2*csch(a + b*x)**2*sech(a + b*x), x)
```

3.489.7 Maxima [F]

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int x^2\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)\,\,dx$$

input
```
integrate(x^2*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

output
```
-2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 2*(b*x*log(e^(b*x + a) + 1) +
 dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x +
a)))/b^3 - 8*integrate(1/4*x^2*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

3.489.8 Giac [F]

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int x^2\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)\,\,dx$$

input
```
integrate(x^2*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

output
```
sage0*x
```

3.489.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int \frac{x^2}{\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^2}\,dx$$

input
```
int(x^2/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

output
```
int(x^2/(cosh(a + b*x)*sinh(a + b*x)^2), x)
```

3.490 $\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$

3.490.1 Optimal result

Integrand size = 16, antiderivative size = 79

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = -\frac{2x\arctan\left(e^{a+bx}\right)}{b} - \frac{\text{arctanh}(\cosh(a+bx))}{b^2} - \frac{x\text{csch}(a+bx)}{b} + \frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}$$

output
```
-2*x*arctan(exp(b*x+a))/b-arctanh(cosh(b*x+a))/b^2-x*csch(b*x+a)/b+I*polyl
og(2,-I*exp(b*x+a))/b^2-I*polylog(2,I*exp(b*x+a))/b^2
```

3.490.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 184 vs. 2(79) = 158.

Time = 0.49 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.33

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$

$$= \frac{4a\arctan\left(e^{a+bx}\right) - bx\coth\left(\frac{1}{2}(a+bx)\right) - 2ia\log\left(1-ie^{a+bx}\right) - 2ibx\log\left(1-ie^{a+bx}\right) + 2ia\log\left(1+ie^{a+b\cdots}\right.}{\cdots}$$

input
```
Integrate[x*Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
(4*a*ArcTan[E^(a + b*x)] - b*x*Coth[(a + b*x)/2] - (2*I)*a*Log[1 - I*E^(a
+ b*x)] - (2*I)*b*x*Log[1 - I*E^(a + b*x)] + (2*I)*a*Log[1 + I*E^(a + b*x)
] + (2*I)*b*x*Log[1 + I*E^(a + b*x)] - 2*Log[Cosh[(a + b*x)/2]] + 2*Log[Si
nh[(a + b*x)/2]] + (2*I)*PolyLog[2, (-I)*E^(a + b*x)] - (2*I)*PolyLog[2, I
*E^(a + b*x)] + b*x*Tanh[(a + b*x)/2])/(2*b^2)
```

3.490.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5985, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{5985}$$

$$-\int \left(-\frac{\arctan(\sinh(a+bx))}{b}-\frac{\text{csch}(a+bx)}{b}\right)dx-\frac{x\arctan(\sinh(a+bx))}{b}-\frac{x\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{\text{arctanh}(\cosh(a+bx))}{b^2}+\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{x\text{csch}(a+bx)}{b}$$

input
```
Int[x*Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
(-2*x*ArcTan[E^(a + b*x)])/b - ArcTanh[Cosh[a + b*x]]/b^2 - (x*Csch[a + b*
x])/b + (I*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - (I*PolyLog[2, I*E^(a + b*x)
])/b^2
```

3.490.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.490.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(72) = 144.

Time = 0.99 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.27

method	result
risch	$-\frac{2x\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{2a\arctan\left(\mathrm{e}^{bx+a}\right)}{b^2}+\frac{i\,\mathrm{dilog}\left(1+i\mathrm{e}^{bx+a}\right)}{b^2}-\frac{i\ln\left(1-i\mathrm{e}^{bx+a}\right)x}{b}-\frac{i\ln\left(1-i\mathrm{e}^{bx+a}\right)a}{b^2}+\frac{i\ln\left(1+i\mathrm{e}^{bx+a}\right)x}{b}+\frac{i\,\mathrm{l}}{}$

input
```
int(x*csch(b*x+a)^2*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/b*x*exp(b*x+a)/(exp(2*b*x+2*a)-1)+2/b^2*a*arctan(exp(b*x+a))+I/b^2*dilo
g(1+I*exp(b*x+a))-I/b*ln(1-I*exp(b*x+a))*x-I/b^2*ln(1-I*exp(b*x+a))*a+I/b*
ln(1+I*exp(b*x+a))*x+I/b^2*ln(1+I*exp(b*x+a))*a-I/b^2*dilog(1-I*exp(b*x+a)
)+1/b^2*ln(exp(b*x+a)-1)-1/b^2*ln(exp(b*x+a)+1)
```

3.490.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(66) = 132.

Time = 0.28 (sec) , antiderivative size = 567, normalized size of antiderivative = 7.18

$$\int x\mathrm{csch}^2(a+bx)\mathrm{sech}(a+bx)\,dx =$$

$$-\frac{2\,bx\cosh\left(bx+a\right)+2\,bx\sinh\left(bx+a\right)-\left(-i\,\cosh\left(bx+a\right)^2-2i\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)-i\,\sinh\left(b\right.\right.}{}$$

input
```
integrate(x*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

output
```
-(2*b*x*cosh(b*x + a) + 2*b*x*sinh(b*x + a) - (-I*cosh(b*x + a)^2 - 2*I*co
sh(b*x + a)*sinh(b*x + a) - I*sinh(b*x + a)^2 + I)*dilog(I*cosh(b*x + a) +
 I*sinh(b*x + a)) - (I*cosh(b*x + a)^2 + 2*I*cosh(b*x + a)*sinh(b*x + a) +
 I*sinh(b*x + a)^2 - I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(
b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh
(b*x + a) + sinh(b*x + a) + 1) - (I*a*cosh(b*x + a)^2 + 2*I*a*cosh(b*x + a
)*sinh(b*x + a) + I*a*sinh(b*x + a)^2 - I*a)*log(cosh(b*x + a) + sinh(b*x
+ a) + I) - (-I*a*cosh(b*x + a)^2 - 2*I*a*cosh(b*x + a)*sinh(b*x + a) - I*
a*sinh(b*x + a)^2 + I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - (cosh(b*
x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b
*x + a) + sinh(b*x + a) - 1) - ((I*b*x + I*a)*cosh(b*x + a)^2 - 2*(-I*b*x
- I*a)*cosh(b*x + a)*sinh(b*x + a) + (I*b*x + I*a)*sinh(b*x + a)^2 - I*b*x
 - I*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((-I*b*x - I*a)*cosh(
b*x + a)^2 - 2*(I*b*x + I*a)*cosh(b*x + a)*sinh(b*x + a) + (-I*b*x - I*a)*
sinh(b*x + a)^2 + I*b*x + I*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1)
)/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x
+ a)^2 - b^2)
```

3.490.6 Sympy [F]

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \int x\,\text{csch}^2\,(a+bx)\,\text{sech}\,(a+bx)\,dx$$

input
```
integrate(x*csch(b*x+a)**2*sech(b*x+a),x)
```

output
```
Integral(x*csch(a + b*x)**2*sech(a + b*x), x)
```

3.490.7 Maxima [F]

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \int x\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)\,\,dx$$

input
```
integrate(x*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

output
```
-2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(-a))/b
^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2 - 8*integrate(1/4*x*e^(b*x + a)/(e^
(2*b*x + 2*a) + 1), x)
```

3.490.8 Giac [F]

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int x\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)\,\,dx$$

input
```
integrate(x*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*csch(b*x + a)^2*sech(b*x + a), x)
```

3.490.9 Mupad [F(-1)]

Timed out.

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int\frac{x}{\cosh\,(a+b\,x)\,\sinh\,(a+b\,x)^2}\,dx$$

input
```
int(x/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

output
```
int(x/(cosh(a + b*x)*sinh(a + b*x)^2), x)
```

3.491 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$

3.491.1 Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

output
```
-arctan(sinh(b*x+a))/b-csch(b*x+a)/b
```

3.491.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\sinh^2(a+bx)\right)}{b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
-((Csch[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[a + b*x]^2])/b)
```

3.491.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 25, 3101, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{i\int \frac{\text{csch}^2(a+bx)}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int -\frac{\text{csch}^2(a+bx)}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{i\left(-\int \frac{1}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))-i\text{csch}(a+bx)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{i(i\arctan(\text{csch}(a+bx))-i\text{csch}(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
((-I)*(I*ArcTan[Csch[a + b*x]] - I*Csch[a + b*x]))/b
```

3.491.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.491.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)}-2\arctan\left(e^{bx+a}\right)}{b}$	25
default	$\frac{-\frac{1}{\sinh(bx+a)}-2\arctan\left(e^{bx+a}\right)}{b}$	25
risch	$-\frac{2\,e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}+\frac{i\ln\left(e^{bx+a}-i\right)}{b}-\frac{i\ln\left(e^{bx+a}+i\right)}{b}$	58

input
```
int(csch(b*x+a)^2*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)-2*arctan(exp(b*x+a)))
```

3.491.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.29

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\left(\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)\arctan\left(\cosh(bx+a)+\sinh(b\right.\right.}{b\cosh(bx+a)^2+2b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

output
```
-2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1
)*arctan(cosh(b*x + a) + sinh(b*x + a)) + cosh(b*x + a) + sinh(b*x + a))/(
b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 -
b)
```

3.491.6 Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x), x)
```

3.491.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{2e^{(-bx-a)}}{b(e^{(-2bx-2a)}-1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

output
```
2*arctan(e^(-b*x - a))/b + 2*e^(-b*x - a)/(b*(e^(-2*b*x - 2*a) - 1))
```

3.491.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\pi+\frac{4}{e^{(bx+a)}-e^{(-bx-a)}}+2\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{2\,b}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(pi + 4/(e^(b*x + a) - e^(-b*x - a)) + 2*arctan(1/2*(e^(2*b*x + 2*a)
- 1)*e^(-b*x - a)))/b
```

3.491.9 Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

output
```
- (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))
/(b*(exp(2*a + 2*b*x) - 1))
```

3.492 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$

3.492.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x},x\right)$$

output
```
CannotIntegrate(csch(b*x+a)^2*sech(b*x+a)/x,x)
```

3.492.2 Mathematica [N/A]

Not integrable

Time = 31.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

input
```
Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x,x]
```

output
```
Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x, x]
```

3.492.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x} \, dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}dx$$

input
```
Int[(Csch[a + b*x]^2*Sech[a + b*x])/x,x]
```

output
```
$Aborted
```

3.492.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.492.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{2}\operatorname{sech}\left(bx+a\right)}{x}dx$$

input
```
int(csch(b*x+a)^2*sech(b*x+a)/x,x)
```

output
```
int(csch(b*x+a)^2*sech(b*x+a)/x,x)
```

3.492.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)}{x}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^2*sech(b*x + a)/x, x)
```

3.492.6 Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)/x,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)/x, x)
```

3.492.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 5.89

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)}{x}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)/x,x, algorithm="maxima")
```

output
```
-2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) - b*x) - 8*integrate(1/4*e^(b*x + a)/(
x*e^(2*b*x + 2*a) + x), x) - 8*integrate(1/8/(b*x^2*e^(b*x + a) + b*x^2),
x) - 8*integrate(1/8/(b*x^2*e^(b*x + a) - b*x^2), x)
```

3.492.8 Giac [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^2*sech(b*x + a)/x, x)
```

3.492.9 Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{1}{x\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(1/(x*cosh(a + b*x)*sinh(a + b*x)^2),x)
```

output
```
int(1/(x*cosh(a + b*x)*sinh(a + b*x)^2), x)
```

3.493 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2} dx$

3.493.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2} dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}, x\right)$$

output
```
CannotIntegrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x)
```

3.493.2 Mathematica [N/A]

Not integrable

Time = 24.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2} dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2} dx$$

input
```
Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x^2,x]
```

output
```
Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x^2, x]
```

3.493.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}dx$$

input
```
Int[(Csch[a + b*x]^2*Sech[a + b*x])/x^2,x]
```

output
```
$Aborted
```

3.493.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.493.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{2} \operatorname{sech}\left(bx+a\right)}{x^{2}}dx$$

input
```
int(csch(b*x+a)^2*sech(b*x+a)/x^2,x)
```

output
```
int(csch(b*x+a)^2*sech(b*x+a)/x^2,x)
```

3.493.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^2*sech(b*x + a)/x^2, x)
```

3.493.6 Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)/x**2,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)/x**2, x)
```

3.493.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 6.33

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
-2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) - 8*integrate(1/4*e^(b*x +
a)/(x^2*e^(2*b*x + 2*a) + x^2), x) - 8*integrate(1/4/(b*x^3*e^(b*x + a) +
b*x^3), x) - 8*integrate(1/4/(b*x^3*e^(b*x + a) - b*x^3), x)
```

3.493.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.493.9 Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh(a+b\,x)\,\sinh(a+b\,x)^2}\,dx$$

input
```
int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)^2),x)
```

output
```
int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)^2), x)
```

3.494 $\int x^m \mathbf{csch}^2(a+bx)\mathbf{sech}^2(a+bx)\,dx$

3.494.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx),x\right)$$

output
```
CannotIntegrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

3.494.2 Mathematica [N/A]

Not integrable

Time = 5.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2, x]
```

3.494.3 Rubi [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int x^m \text{csch}^2(a+bx)\text{sech}^2(a+bx)dx$$

input
```
Int[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
$Aborted
```

3.494.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.494.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \,\text{csch}\,(bx+a)^2 \,\text{sech}\,(bx+a)^2\,dx$$

input
```
int(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

output
```
int(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

3.494.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2 \,dx$$

input
```
integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

3.494.6 Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}^2(a+bx) \operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(x**m*csch(b*x+a)**2*sech(b*x+a)**2,x)
```

output
```
Integral(x**m*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

3.494.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2 \,dx$$

input
```
integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

3.494.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = \int x^m \,\text{csch}\,(bx+a)^2 \,\text{sech}\,(bx+a)^2 \,dx$$

input
```
integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

3.494.9 Mupad [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = \int \frac{x^m}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

output
```
int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

3.495 $\int x^3 \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$

3.495.1 Optimal result

Integrand size = 20, antiderivative size = 85

$$\int x^3 \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx=-\frac{2x^3}{b}-\frac{2x^3\coth(2a+2bx)}{b}+\frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2}+\frac{3x\,\text{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}-\frac{3\,\text{PolyLog}\left(3,e^{4(a+bx)}\right)}{8b^4}$$

output
```
-2*x^3/b-2*x^3*coth(2*b*x+2*a)/b+3*x^2*ln(1-exp(4*b*x+4*a))/b^2+3/2*x*poly
log(2,exp(4*b*x+4*a))/b^3-3/8*polylog(3,exp(4*b*x+4*a))/b^4
```

3.495.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 307 vs. $2(85) = 170$.

Time = 1.48 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.61

$$\int x^3 \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$$
$$=4\left(-\frac{e^{4a}\left(8b^3e^{-4a}x^3-6b^2\left(1-e^{-4a}\right)x^2\log\left(1-e^{-a-bx}\right)-6b^2\left(1-e^{-4a}\right)x^2\log\left(1+e^{-a-bx}\right)-6b^2\left(1-e^{-4a}\right.\right.}{}+\frac{x^3\text{csch}(2a)\text{csch}(2a+2bx)\sinh(2bx)}{2b}\right)$$

input
```
Integrate[x^3*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
4*(-1/8*(E^(4*a)*((8*b^3*x^3)/E^(4*a) - 6*b^2*(1 - E^(-4*a))*x^2*Log[1 - E
^(-a - b*x)] - 6*b^2*(1 - E^(-4*a))*x^2*Log[1 + E^(-a - b*x)] - 6*b^2*(1 -
 E^(-4*a))*x^2*Log[1 + E^(-2*(a + b*x))] + 12*b*(1 - E^(-4*a))*x*PolyLog[2
, -E^(-a - b*x)] + 12*b*(1 - E^(-4*a))*x*PolyLog[2, E^(-a - b*x)] + 6*b*(1
 - E^(-4*a))*x*PolyLog[2, -E^(-2*(a + b*x))] + 12*(1 - E^(-4*a))*PolyLog[3
, -E^(-a - b*x)] + 12*(1 - E^(-4*a))*PolyLog[3, E^(-a - b*x)] + 3*(1 - E^(
-4*a))*PolyLog[3, -E^(-2*(a + b*x))]))/(b^4*(-1 + E^(4*a))) + (x^3*Csch[2*
a]*Csch[2*a + 2*b*x]*Sinh[2*b*x])/(2*b))
```

3.495.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.59, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5984, 3042, 25, 4672, 26, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{5984}$$

$$4\int x^3\text{csch}^2(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int -x^3\csc(2ia+2ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-4\int x^3\csc(2ia+2ibx)^2dx$$

$$\Big\downarrow \text{4672}$$

$$-4\left(\frac{x^3\coth(2a+2bx)}{2b}-\frac{3i\int -ix^2\coth(2a+2bx)dx}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-4\left(\frac{x^3\coth(2a+2bx)}{2b}-\frac{3\int x^2\coth(2a+2bx)dx}{2b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-4\left(\frac{x^3\coth(2a+2bx)}{2b}-\frac{3\int -ix^2\tan\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{2b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-4\left(\frac{x^3\coth(2a+2bx)}{2b}+\frac{3i\int x^2\tan\left(\frac{1}{2}(4ia+\pi)+2ibx\right)dx}{2b}\right)$$

$$\Big\downarrow \text{ 4201}$$

$$-4\left(\frac{x^3\coth(2a+2bx)}{2b}+\frac{3i\left(2i\int\frac{e^{4a+4bx-i\pi}x^2}{1+e^{4a+4bx-i\pi}}dx-\frac{ix^3}{3}\right)}{2b}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$-4\left(\frac{x^3\coth(2a+2bx)}{2b}+\frac{3i\left(2i\left(\frac{x^2\log\left(1+e^{4a+4bx-i\pi}\right)}{4b}-\frac{\int x\log\left(1+e^{4a+4bx-i\pi}\right)dx}{2b}\right)-\frac{ix^3}{3}\right)}{2b}\right)$$

$$\Big\downarrow \text{ 3011}$$

$$-4\left(\frac{x^3\coth(2a+2bx)}{2b}+\frac{3i\left(2i\left(\frac{x^2\log\left(1+e^{4a+4bx-i\pi}\right)}{4b}-\frac{\frac{\int \text{PolyLog}\left(2,-e^{4a+4bx-i\pi}\right)dx}{4b}-\frac{x\,\text{PolyLog}\left(2,-e^{4a+4bx-i\pi}\right)}{4b}}{2b}\right)-\frac{ix^3}{3}\right)}{2b}\right)$$

$$\Big\downarrow \text{ 2720}$$

$$-4\left(\frac{x^3\coth(2a+2bx)}{2b}+\frac{3i\left(2i\left(\frac{x^2\log\left(1+e^{4a+4bx-i\pi}\right)}{4b}-\frac{\frac{\int e^{-4a-4bx+i\pi}\text{PolyLog}\left(2,-e^{4a+4bx-i\pi}\right)de^{4a+4bx-i\pi}}{16b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{4a+4b\cdots}\right)}{4b}}{2b}\cdots\right.\right.}{2b}\cdots\right.$$

$$\Big\downarrow \text{ 7143}$$

3.495. $\int x^3\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$

$$-4\left(\frac{x^3\coth(2a+2bx)}{2b}+\frac{3i\left(2i\left(\frac{x^2\log\left(1+e^{4a+4bx-i\pi}\right)}{4b}-\frac{\frac{\text{PolyLog}\left(3,-e^{4a+4bx-i\pi}\right)}{16b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{4a+4bx-i\pi}\right)}{4b}}{2b}\right)-\frac{ix^3}{3}\right)}{2b}\right)$$

input
```
Int[x^3*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
-4*((x^3*Coth[2*a + 2*b*x])/(2*b) + (((3*I)/2)*((-1/3*I)*x^3 + (2*I)*((x^2
*Log[1 + E^(4*a - I*Pi + 4*b*x)])/(4*b) - (-1/4*(x*PolyLog[2, -E^(4*a - I*
Pi + 4*b*x)])/b + PolyLog[3, -E^(4*a - I*Pi + 4*b*x)]/(16*b^2))/(2*b))))/b
)
```

3.495.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.495. $\int x^3\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.495.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(81) = 162$.

Time = 6.98 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.09

method	result
risch	$-\frac{4x^3}{b(1+e^{2bx+2a})(e^{2bx+2a}-1)}+\frac{8a^3}{b^4}+\frac{12x\,a^2}{b^3}-\frac{12a^2\ln(e^{bx+a})}{b^4}+\frac{3a^2\ln(e^{bx+a}-1)}{b^4}-\frac{3\ln(1-e^{bx+a})a^2}{b^4}-\frac{4x^3}{b}-$ 6 poly

input
```
int(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4*x^3/b/(1+exp(2*b*x+2*a))/(exp(2*b*x+2*a)-1)+8/b^4*a^3+12*x/b^3*a^2-12/b
^4*a^2*ln(exp(b*x+a))+3/b^4*a^2*ln(exp(b*x+a)-1)-3/b^4*ln(1-exp(b*x+a))*a^
2-4*x^3/b-6*polylog(3,-exp(b*x+a))/b^4-6*polylog(3,exp(b*x+a))/b^4-3/2*pol
ylog(3,-exp(2*b*x+2*a))/b^4+3*x^2*ln(1+exp(2*b*x+2*a))/b^2+3*x*polylog(2,-
exp(2*b*x+2*a))/b^3+3/b^2*ln(exp(b*x+a)+1)*x^2+6*x*polylog(2,-exp(b*x+a))/
b^3+3/b^2*ln(1-exp(b*x+a))*x^2+6*x*polylog(2,exp(b*x+a))/b^3
```

3.495.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1924, normalized size of antiderivative = 22.64

$$\int x^3 \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(4*(b^3*x^3 + a^3)*cosh(b*x + a)^4 + 16*(b^3*x^3 + a^3)*cosh(b*x + a)^3*s
inh(b*x + a) + 24*(b^3*x^3 + a^3)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 16*(b^
3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + 4*(b^3*x^3 + a^3)*sinh(b*x +
a)^4 - 4*a^3 - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a
) + 6*b*x*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x +
 a)^3 + b*x*sinh(b*x + a)^4 - b*x)*dilog(cosh(b*x + a) + sinh(b*x + a)) -
6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh(
b*x + a)^2*sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sin
h(b*x + a)^4 - b*x)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(b*x*cosh
(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh(b*x + a)^2*
sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^
4 - b*x)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^
4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh(b*x + a)^2*sinh(b*x +
 a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - b*x)*d
ilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*
x^2*cosh(b*x + a)^3*sinh(b*x + a) + 6*b^2*x^2*cosh(b*x + a)^2*sinh(b*x + a
)^2 + 4*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 -
b^2*x^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 3*(a^2*cosh(b*x + a)^4 +
 4*a^2*cosh(b*x + a)^3*sinh(b*x + a) + 6*a^2*cosh(b*x + a)^2*sinh(b*x + a)
^2 + 4*a^2*cosh(b*x + a)*sinh(b*x + a)^3 + a^2*sinh(b*x + a)^4 - a^2)*l...
```

3.495.6 Sympy [F]

$$\int x^3\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx=\int x^3\,\text{csch}^2\,(a+bx)\,\text{sech}^2\,(a+bx)\,dx$$

input
```
integrate(x**3*csch(b*x+a)**2*sech(b*x+a)**2,x)
```

output
```
Integral(x**3*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

3.495.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(80) = 160.

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.12

$$\begin{aligned}&\int x^3\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx\\&=-\frac{4\,x^3}{be^{(4\,bx+4\,a)}-b}-\frac{4\,x^3}{b}\\&\quad+\frac{3\,\left(2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\text{Li}_3(-e^{(2\,bx+2\,a)})\right)}{2\,b^4}\\&\quad+\frac{3\,\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\text{Li}_2\left(-e^{(bx+a)}\right)-2\,\text{Li}_3(-e^{(bx+a)})\right)}{b^4}\\&\quad+\frac{3\,\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\text{Li}_2\left(e^{(bx+a)}\right)-2\,\text{Li}_3(e^{(bx+a)})\right)}{b^4}\end{aligned}$$

input
```
integrate(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
-4*x^3/(b*e^(4*b*x + 4*a) - b) - 4*x^3/b + 3/2*(2*b^2*x^2*log(e^(2*b*x + 2
*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b
^4 + 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polyl
og(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(
e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^4
```

3.495.8 Giac [F]

$$\int x^3\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int x^3\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)^2\,dx$$

input
```
integrate(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

3.495.9 Mupad [F(-1)]

Timed out.

$$\int x^3\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int \frac{x^3}{\cosh(a+b\,x)^2\sinh(a+b\,x)^2}\,dx$$

input
```
int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

output
```
int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

3.496 $\int x^2\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$

3.496.1 Optimal result

Integrand size = 20, antiderivative size = 64

$$\int x^2\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx=-\frac{2x^2}{b}-\frac{2x^2\coth(2a+2bx)}{b}+\frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}$$

output
```
-2*x^2/b-2*x^2*coth(2*b*x+2*a)/b+2*x*ln(1-exp(4*b*x+4*a))/b^2+1/2*polylog(
2,exp(4*b*x+4*a))/b^3
```

3.496.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 216 vs. 2(64) = 128.

Time = 0.91 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.38

$$\int x^2\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$$
$$=4\left(-\frac{e^{4a}\left(4b^2e^{-4a}x^2-2b(1-e^{-4a})\,x\log\left(1-e^{-a-bx}\right)-2b(1-e^{-4a})\,x\log\left(1+e^{-a-bx}\right)-2b(1-e^{-4a})\,x\,\text{lo}\right.}{}\right.$$
$$\left.+\frac{x^2\text{csch}(2a)\text{csch}(2a+2bx)\sinh(2bx)}{2b}\right)$$

input
```
Integrate[x^2*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
4*(-1/4*(E^(4*a)*((4*b^2*x^2)/E^(4*a) - 2*b*(1 - E^(-4*a))*x*Log[1 - E^(-a
 - b*x)] - 2*b*(1 - E^(-4*a))*x*Log[1 + E^(-a - b*x)] - 2*b*(1 - E^(-4*a))
*x*Log[1 + E^(-2*(a + b*x))] + 2*(1 - E^(-4*a))*PolyLog[2, -E^(-a - b*x)]
+ 2*(1 - E^(-4*a))*PolyLog[2, E^(-a - b*x)] + (1 - E^(-4*a))*PolyLog[2, -E
^(-2*(a + b*x))]))/(b^3*(-1 + E^(4*a))) + (x^2*Csch[2*a]*Csch[2*a + 2*b*x]
*Sinh[2*b*x])/(2*b))
```

3.496.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5984, 3042, 25, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{5984}$$

$$4\int x^2\text{csch}^2(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int -x^2\csc(2ia+2ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-4\int x^2\csc(2ia+2ibx)^2dx$$

$$\Big\downarrow \text{4672}$$

$$-4\left(\frac{x^2\coth(2a+2bx)}{2b}-\frac{i\int -ix\coth(2a+2bx)dx}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-4\left(\frac{x^2\coth(2a+2bx)}{2b}-\frac{\int x\coth(2a+2bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-4\left(\frac{x^2\coth(2a+2bx)}{2b}-\frac{\int -ix\tan\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-4\left(\frac{x^2\coth(2a+2bx)}{2b}+\frac{i\int x\tan\left(\frac{1}{2}(4ia+\pi)+2ibx\right)dx}{b}\right)$$

$$\Big\downarrow \text{4201}$$

$$-4\left(\frac{x^2\coth(2a+2bx)}{2b}+\frac{i\left(2i\int\frac{e^{4a+4bx-i\pi}x}{1+e^{4a+4bx-i\pi}}dx-\frac{ix^2}{2}\right)}{b}\right)$$

$$\Big\downarrow \text{2620}$$

$$-4\left(\frac{x^2\coth(2a+2bx)}{2b}+\frac{i\left(2i\left(\frac{x\log\left(1+e^{4a+4bx-i\pi}\right)}{4b}-\frac{\int\log\left(1+e^{4a+4bx-i\pi}\right)dx}{4b}\right)-\frac{ix^2}{2}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$-4\left(\frac{x^2\coth(2a+2bx)}{2b}+\frac{i\left(2i\left(\frac{x\log\left(1+e^{4a+4bx-i\pi}\right)}{4b}-\frac{\int e^{-4a-4bx+i\pi}\log\left(1+e^{4a+4bx-i\pi}\right)de^{4a+4bx-i\pi}}{16b^2}\right)-\frac{ix^2}{2}\right)}{b}\right)$$

$$\Big\downarrow \text{2838}$$

$$-4\left(\frac{x^2\coth(2a+2bx)}{2b}+\frac{i\left(2i\left(\frac{\operatorname{PolyLog}\left(2,-e^{4a+4bx-i\pi}\right)}{16b^2}+\frac{x\log\left(1+e^{4a+4bx-i\pi}\right)}{4b}\right)-\frac{ix^2}{2}\right)}{b}\right)$$

input
```
Int[x^2*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
-4*((x^2*Coth[2*a + 2*b*x])/(2*b) + (I*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E
^(4*a - I*Pi + 4*b*x)])/(4*b) + PolyLog[2, -E^(4*a - I*Pi + 4*b*x)]/(16*b^
2))))/b)
```

3.496. $\int x^2\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$

3.496.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.496. $\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$

3.496.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(62) = 124.

Time = 4.99 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.11

method	result
risch	$-\frac{4x^2}{b(1+e^{2bx+2a})(e^{2bx+2a}-1)}-\frac{4x^2}{b}-\frac{8ax}{b^2}-\frac{4a^2}{b^3}+\frac{2\ln(e^{bx+a}+1)x}{b^2}+\frac{2\,\mathrm{polylog}(2,-e^{bx+a})}{b^3}+\frac{2\ln(1-e^{bx+a})x}{b^2}+\frac{2\ln(1$

input
```
int(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4*x^2/b/(1+exp(2*b*x+2*a))/(exp(2*b*x+2*a)-1)-4*x^2/b-8*a*x/b^2-4/b^3*a^2
+2/b^2*ln(exp(b*x+a)+1)*x+2*polylog(2,-exp(b*x+a))/b^3+2/b^2*ln(1-exp(b*x+
a))*x+2/b^3*ln(1-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3+2*x*ln(1+exp(2*
b*x+2*a))/b^2+polylog(2,-exp(2*b*x+2*a))/b^3+8/b^3*a*ln(exp(b*x+a))-2/b^3*
a*ln(exp(b*x+a)-1)
```

3.496.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1327, normalized size of antiderivative = 20.73

$$\int x^2\mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2*(2*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 8*(b^2*x^2 - a^2)*cosh(b*x + a)^3*
sinh(b*x + a) + 12*(b^2*x^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 8*(b^
2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + 2*(b^2*x^2 - a^2)*sinh(b*x +
a)^4 + 2*a^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh
(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
 a)^4 - 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cos
h(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*
x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*dilog(I*cosh(b*x + a) + I*si
nh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh
(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
 a)^4 - 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - (cosh(b*x + a)^4 +
4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*co
sh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*dilog(-cosh(b*x + a) -
sinh(b*x + a)) - (b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a
) + 6*b*x*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x +
 a)^3 + b*x*sinh(b*x + a)^4 - b*x)*log(cosh(b*x + a) + sinh(b*x + a) + 1)
+ (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)^3*sinh(b*x + a) + 6*a*cosh(b*x +
a)^2*sinh(b*x + a)^2 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)
^4 - a)*log(cosh(b*x + a) + sinh(b*x + a) + I) + (a*cosh(b*x + a)^4 + 4*a*
cosh(b*x + a)^3*sinh(b*x + a) + 6*a*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4...
```

3.496.6 Sympy [F]

$$\int x^2 \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = \int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(x**2*csch(b*x+a)**2*sech(b*x+a)**2,x)
```

output
```
Integral(x**2*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

3.496.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.84

$$\int x^2\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{4\,x^2}{be^{(4\,bx+4\,a)}-b} - \frac{4\,x^2}{b} + \frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{b^3} + \frac{2\,\left(bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3} + \frac{2\,\left(bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}$$

input
```
integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
-4*x^2/(b*e^(4*b*x + 4*a) - b) - 4*x^2/b + (2*b*x*log(e^(2*b*x + 2*a) + 1)
 + dilog(-e^(2*b*x + 2*a)))/b^3 + 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(
b*x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

3.496.8 Giac [F]

$$\int x^2\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = \int x^2\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)^2\,dx$$

input
```
integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

3.496.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = \int \frac{x^2}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

output
```
int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

3.497 $\int x\mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx$

3.497.1 Optimal result

Integrand size = 18, antiderivative size = 30

$$\int x\mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx=-\frac{2x\coth(2a+2bx)}{b}+\frac{\log(\sinh(2a+2bx))}{b^2}$$

output
```
-2*x*coth(2*b*x+2*a)/b+ln(sinh(2*b*x+2*a))/b^2
```

3.497.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x\mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx=\frac{-2bx\coth(2(a+bx))+\log(\sinh(2(a+bx)))}{b^2}$$

input
```
Integrate[x*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
(-2*b*x*Coth[2*(a + b*x)] + Log[Sinh[2*(a + b*x)]])/b^2
```

3.497.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5984, 3042, 25, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 5984}$$

$$4\int x\text{csch}^2(2a+2bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$4\int -x\csc(2ia+2ibx)^2dx$$

$$\Big\downarrow \text{ 25}$$

$$-4\int x\csc(2ia+2ibx)^2dx$$

$$\Big\downarrow \text{ 4672}$$

$$-4\left(\frac{x\coth(2a+2bx)}{2b}-\frac{i\int -i\coth(2a+2bx)dx}{2b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-4\left(\frac{x\coth(2a+2bx)}{2b}-\frac{\int \coth(2a+2bx)dx}{2b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-4\left(\frac{x\coth(2a+2bx)}{2b}-\frac{\int -i\tan\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{2b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-4\left(\frac{x\coth(2a+2bx)}{2b}+\frac{i\int \tan\left(\frac{1}{2}(4ia+\pi)+2ibx\right)dx}{2b}\right)$$

$$\Big\downarrow \text{ 3956}$$

$$-4\left(\frac{x\coth(2a+2bx)}{2b}-\frac{\log(-i\sinh(2a+2bx))}{4b^2}\right)$$

input
```
Int[x*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
-4*((x*Coth[2*a + 2*b*x])/(2*b) - Log[(-I)*Sinh[2*a + 2*b*x]]/(4*b^2))
```

3.497.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.497.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.

Time = 3.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.07

method	result	size
risch	$-\frac{4x}{b}-\frac{4a}{b^2}-\frac{4x}{b\left(1+e^{2bx+2a}\right)\left(e^{2bx+2a}-1\right)}+\frac{\ln\left(e^{4bx+4a}-1\right)}{b^2}$	62

input
```
int(x*csch(b*x+a)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4*x/b-4*a/b^2-4*x/b/(1+exp(2*b*x+2*a))/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(4*
b*x+4*a)-1)
```

3.497.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 9.73

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{4\,bx\cosh\left(bx+a\right)^4+16\,bx\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)+24\,bx\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^2+16\,bx\cos}{b^2\cosh(}$$

input
```
integrate(x*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(4*b*x*cosh(b*x + a)^4 + 16*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 24*b*x*co
sh(b*x + a)^2*sinh(b*x + a)^2 + 16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 4*b
*x*sinh(b*x + a)^4 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) +
6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh
(b*x + a)^4 - 1)*log(4*cosh(b*x + a)*sinh(b*x + a)/(cosh(b*x + a)^2 - 2*co
sh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)))/(b^2*cosh(b*x + a)^4 + 4*b^
2*cosh(b*x + a)^3*sinh(b*x + a) + 6*b^2*cosh(b*x + a)^2*sinh(b*x + a)^2 +
4*b^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - b^2)
```

3.497.6 Sympy [F]

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx=\int x\,\text{csch}^2\,(a+bx)\,\text{sech}^2\,(a+bx)\,dx$$

input
```
integrate(x*csch(b*x+a)**2*sech(b*x+a)**2,x)
```

output
```
Integral(x*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

3.497.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(30) = 60$.

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.90

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx=-\frac{4\,xe^{(4\,bx+4\,a)}}{be^{(4\,bx+4\,a)}-b}+\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}+\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}+\frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}$$

input
```
integrate(x*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
-4*x*e^(4*b*x + 4*a)/(b*e^(4*b*x + 4*a) - b) + log((e^(b*x + a) + 1)*e^(-a
))/b^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2 + log((e^(2*b*x + 2*a) + 1)*e^(
-2*a))/b^2
```

3.497.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.40

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx=-\frac{4\,bxe^{(4\,bx+4\,a)}-e^{(4\,bx+4\,a)}\log\left(e^{(4\,bx+4\,a)}-1\right)+\log\left(e^{(4\,bx+4\,a)}-1\right)}{b^2e^{(4\,bx+4\,a)}-b^2}$$

input
```
integrate(x*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
-(4*b*x*e^(4*b*x + 4*a) - e^(4*b*x + 4*a)*log(e^(4*b*x + 4*a) - 1) + log(e
^(4*b*x + 4*a) - 1))/(b^2*e^(4*b*x + 4*a) - b^2)
```

3.497.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx=\frac{\ln\left(e^{4a}\,e^{4bx}-1\right)}{b^2}-\frac{4x}{b}-\frac{4x}{b\left(e^{4a+4bx}-1\right)}$$

input
```
int(x/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

output
```
log(exp(4*a)*exp(4*b*x) - 1)/b^2 - (4*x)/b - (4*x)/(b*(exp(4*a + 4*b*x) -
1))
```

3.498 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$

3.498.1 Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=-\frac{\coth(a+bx)}{b}-\frac{\tanh(a+bx)}{b}$$

output
```
-coth(b*x+a)/b-tanh(b*x+a)/b
```

3.498.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=-\frac{2\coth(2(a+bx))}{b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
(-2*Coth[2*(a + b*x)])/b
```

3.498.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 25, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{i\int -\coth^2(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{i\int \left(1-\coth^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(i\tanh(a+bx)+i\coth(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
(I*(I*Coth[a + b*x] + I*Tanh[a + b*x]))/b
```

3.498.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.498.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)}-2\tanh(bx+a)}{b}$	32
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)}-2\tanh(bx+a)}{b}$	32
risch	$-\frac{4}{b(1+e^{2bx+2a})(e^{2bx+2a}-1)}$	32

input
```
int(csch(b*x+a)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)/cosh(b*x+a)-2*tanh(b*x+a))
```

3.498.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(23) = 46.

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.52

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx =$$

$$-\frac{4}{b\cosh(bx+a)^4+4\,b\cosh(bx+a)^3\sinh(bx+a)+6\,b\cosh(bx+a)^2\sinh(bx+a)^2+4\,b\cosh(bx+a)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-4/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x +
 a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a
)^4 - b)
```

3.498.6 Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**2, x)
```

3.498.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{4}{b(e^{(-4\,bx-4\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
4/(b*(e^(-4*b*x - 4*a) - 1))
```

3.498.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4}{b(e^{(4\,bx+4\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
-4/(b*(e^(4*b*x + 4*a) - 1))
```

3.498.9 Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

output
```
-4/(b*(exp(4*a + 4*b*x) - 1))
```

3.499 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$

3.499.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = 4\mathrm{Int}\left(\frac{\operatorname{csch}^2(2a+2bx)}{x},x\right)$$

output
```
4*Unintegrable(csch(2*b*x+2*a)^2/x,x)
```

3.499.2 Mathematica [N/A]

Not integrable

Time = 24.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

input
```
Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x,x]
```

output
```
Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x, x]
```

3.499.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5984, 3042, 25, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{5984}$$

$$4\int \frac{\operatorname{csch}^2(2a+2bx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$4\int -\frac{\csc(2ia+2ibx)^2}{x}dx$$

$$\Big\downarrow \text{25}$$

$$-4\int \frac{\csc(2ia+2ibx)^2}{x}dx$$

$$\Big\downarrow \text{4680}$$

$$4\int \frac{\operatorname{csch}^2(2a+2bx)}{x}dx$$

input
```
Int[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x,x]
```

output
```
$Aborted
```

3.499.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.499.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2}{x} dx$$

input
```
int(csch(b*x+a)^2*sech(b*x+a)^2/x,x)
```

output
```
int(csch(b*x+a)^2*sech(b*x+a)^2/x,x)
```

3.499.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^2*sech(b*x + a)^2/x, x)
```

3.499.6 Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)}{x}\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**2/x,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**2/x, x)
```

3.499.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 5.05

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
-4/(b*x*e^(4*b*x + 4*a) - b*x) + 16*integrate(1/8/(b*x^2*e^(2*b*x + 2*a) +
 b*x^2), x) + 16*integrate(1/16/(b*x^2*e^(b*x + a) + b*x^2), x) - 16*integ
rate(1/16/(b*x^2*e^(b*x + a) - b*x^2), x)
```

3.499.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^2*sech(b*x + a)^2/x, x)
```

3.499.9 Mupad [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{1}{x\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

output
```
int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

3.500 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$

3.500.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = 4\text{Int}\left(\frac{\operatorname{csch}^2(2a+2bx)}{x^2},x\right)$$

output
```
4*Unintegrable(csch(2*b*x+2*a)^2/x^2,x)
```

3.500.2 Mathematica [N/A]

Not integrable

Time = 18.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x^2,x]
```

output
```
Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x^2, x]
```

3.500.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5984, 3042, 25, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(a+bx)\text{sech}^2(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{5984}$$

$$4\int \frac{\text{csch}^2(2a+2bx)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$4\int -\frac{\csc(2ia+2ibx)^2}{x^2}dx$$

$$\Big\downarrow \text{25}$$

$$-4\int \frac{\csc(2ia+2ibx)^2}{x^2}dx$$

$$\Big\downarrow \text{4680}$$

$$4\int \frac{\text{csch}^2(2a+2bx)}{x^2}dx$$

input
```
Int[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x^2,x]
```

output
```
$Aborted
```

3.500.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.500.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2}{x^2} dx$$

input
```
int(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x)
```

output
```
int(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x)
```

3.500.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^2*sech(b*x + a)^2/x^2, x)
```

3.500.6 Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**2/x**2,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**2/x**2, x)
```

3.500.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.25

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-4/(b*x^2*e^(4*b*x + 4*a) - b*x^2) + 16*integrate(1/4/(b*x^3*e^(2*b*x + 2*
a) + b*x^3), x) + 16*integrate(1/8/(b*x^3*e^(b*x + a) + b*x^3), x) - 16*in
tegrate(1/8/(b*x^3*e^(b*x + a) - b*x^3), x)
```

3.500.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^2*sech(b*x + a)^2/x^2, x)
```

3.500.9 Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

output
```
int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

3.501 $\int x^m \mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx$

3.501.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m \mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx = \mathrm{Int}\left(x^m \mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx),x\right)$$

output
```
CannotIntegrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

3.501.2 Mathematica [N/A]

Not integrable

Time = 60.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx = \int x^m \mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx$$

input
```
Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

output
```
Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3, x]
```

3.501.3 Rubi [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int x^m \text{csch}^2(a+bx)\text{sech}^3(a+bx)dx$$

input
```
Int[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

output
```
$Aborted
```

3.501.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.501.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3\,dx$$

input
```
int(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

output
```
int(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

3.501.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3\,dx$$

input
```
integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

3.501.6 Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int x^m \operatorname{csch}^2(a+bx) \operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(x**m*csch(b*x+a)**2*sech(b*x+a)**3,x)
```

output
```
Integral(x**m*csch(a + b*x)**2*sech(a + b*x)**3, x)
```

3.501.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3\,dx$$

input
```
integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

3.501.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3\,dx$$

input
```
integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

3.501.9 Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \frac{x^m}{\cosh(a+b\,x)^3 \sinh(a+b\,x)^2}\,dx$$

input
```
int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

output
```
int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

3.502 $\int x^2\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx$

3.502.1 Optimal result

Integrand size = 20, antiderivative size = 206

$$\begin{aligned}\int x^2\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx =& -\frac{3x^2\arctan\left(e^{a+bx}\right)}{b}+\frac{\arctan(\sinh(a+bx))}{b^3} \\ &-\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{3x^2\text{csch}(a+bx)}{2b} \\ &-\frac{2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{3ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} \\ &-\frac{3ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} \\ &-\frac{3i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{3i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} \\ &-\frac{x\text{sech}(a+bx)}{b^2}+\frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}\end{aligned}$$

output
```
-3*x^2*arctan(exp(b*x+a))/b+arctan(sinh(b*x+a))/b^3-4*x*arctanh(exp(b*x+a)
)/b^2-3/2*x^2*csch(b*x+a)/b-2*polylog(2,-exp(b*x+a))/b^3+3*I*x*polylog(2,-
I*exp(b*x+a))/b^2-3*I*x*polylog(2,I*exp(b*x+a))/b^2+2*polylog(2,exp(b*x+a)
)/b^3-3*I*polylog(3,-I*exp(b*x+a))/b^3+3*I*polylog(3,I*exp(b*x+a))/b^3-x*s
ech(b*x+a)/b^2+1/2*x^2*csch(b*x+a)*sech(b*x+a)^2/b
```

3.502.2 Mathematica [A] (verified)

Time = 4.86 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.50

$$\int x^2\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx$$
$$=\frac{4\arctan\left(e^{a+bx}\right)-2b^2x^2\text{csch}(a)+4bx\log\left(1-e^{a+bx}\right)-3ib^2x^2\log\left(1-ie^{a+bx}\right)+3ib^2x^2\log\left(1+ie^{a+bx}\right)}{}$$

input
```
Integrate[x^2*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

output
```
(4*ArcTan[E^(a + b*x)] - 2*b^2*x^2*Csch[a] + 4*b*x*Log[1 - E^(a + b*x)] -
(3*I)*b^2*x^2*Log[1 - I*E^(a + b*x)] + (3*I)*b^2*x^2*Log[1 + I*E^(a + b*x)
] - 4*b*x*Log[1 + E^(a + b*x)] - 4*PolyLog[2, -E^(a + b*x)] + (6*I)*b*x*Po
lyLog[2, (-I)*E^(a + b*x)] - (6*I)*b*x*PolyLog[2, I*E^(a + b*x)] + 4*PolyL
og[2, E^(a + b*x)] - (6*I)*PolyLog[3, (-I)*E^(a + b*x)] + (6*I)*PolyLog[3,
 I*E^(a + b*x)] - 2*b*x*Sech[a + b*x] + b^2*x^2*Csch[a/2]*Csch[(a + b*x)/2
]*Sinh[(b*x)/2] + b^2*x^2*Sech[a/2]*Sech[(a + b*x)/2]*Sinh[(b*x)/2] - b^2*
x^2*Sech[a]*Sech[a + b*x]^2*Sinh[b*x] - b^2*x^2*Sech[a + b*x]*Tanh[a])/(2*
b^3)
```

3.502.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5985, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{5985}$$

$$-2\int -\frac{1}{2}x\left(-\frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{b}+\frac{3\arctan(\sinh(a+bx))}{b}+\frac{3\text{csch}(a+bx)}{b}\right)dx-\frac{3x^2\arctan(\sinh(a+bx))}{2b}-\frac{3x^2\text{csch}(a+bx)}{2b}+\frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}$$

$$\Big\downarrow \text{27}$$

$$\int x\left(-\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{b}+\frac{3\arctan(\sinh(a+bx))}{b}+\frac{3\operatorname{csch}(a+bx)}{b}\right)dx-\frac{3x^2\arctan(\sinh(a+bx))}{2b}-\frac{3x^2\operatorname{csch}(a+bx)}{2b}+\frac{x^2\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2010}$$

$$\int\left(\frac{3x(\arctan(\sinh(a+bx))+\operatorname{csch}(a+bx))}{b}-\frac{x\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{b}\right)dx-\frac{3x^2\arctan(\sinh(a+bx))}{2b}-\frac{3x^2\operatorname{csch}(a+bx)}{2b}+\frac{x^2\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan(\sinh(a+bx))}{b^3}-\frac{3x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{4x\operatorname{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{2\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{3i\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{3i\operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}+\frac{3ix\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{3ix\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{x\operatorname{sech}(a+bx)}{b^2}-\frac{3x^2\operatorname{csch}(a+bx)}{2b}+\frac{x^2\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

input
```
Int[x^2*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

output
```
(-3*x^2*ArcTan[E^(a + b*x)])/b + ArcTan[Sinh[a + b*x]]/b^3 - (4*x*ArcTanh[
E^(a + b*x)])/b^2 - (3*x^2*Csch[a + b*x])/(2*b) - (2*PolyLog[2, -E^(a + b*
x)])/b^3 + ((3*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - ((3*I)*x*PolyLog[2
, I*E^(a + b*x)])/b^2 + (2*PolyLog[2, E^(a + b*x)])/b^3 - ((3*I)*PolyLog[3
, (-I)*E^(a + b*x)])/b^3 + ((3*I)*PolyLog[3, I*E^(a + b*x)])/b^3 - (x*Sech
[a + b*x])/b^2 + (x^2*Csch[a + b*x]*Sech[a + b*x]^2)/(2*b)
```

3.502.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.502.4 Maple [F]

$$\int x^2 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3\,dx$$

input
```
int(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

output
```
int(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

3.502.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3825 vs. $2(174) = 348$.

Time = 0.33 (sec) , antiderivative size = 3825, normalized size of antiderivative = 18.57

$$\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(4*b^2*x^2*cosh(b*x + a)^3 + 2*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^5 +
10*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)*sinh(b*x + a)^4 + 2*(3*b^2*x^2 + 2*b*
x)*sinh(b*x + a)^5 + 4*(b^2*x^2 + 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^2)*s
inh(b*x + a)^3 + 4*(3*b^2*x^2*cosh(b*x + a) + 5*(3*b^2*x^2 + 2*b*x)*cosh(b
*x + a)^3)*sinh(b*x + a)^2 + 2*(3*b^2*x^2 - 2*b*x)*cosh(b*x + a) - 4*(cosh
(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3
+ cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2
 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(cosh(b*x + a) + sinh(b*
x + a)) + 6*(I*b*x*cosh(b*x + a)^6 + 6*I*b*x*cosh(b*x + a)*sinh(b*x + a)^5
 + I*b*x*sinh(b*x + a)^6 + I*b*x*cosh(b*x + a)^4 + (15*I*b*x*cosh(b*x + a)
^2 + I*b*x)*sinh(b*x + a)^4 - I*b*x*cosh(b*x + a)^2 + 4*(5*I*b*x*cosh(b*x
+ a)^3 + I*b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (15*I*b*x*cosh(b*x + a)^4
+ 6*I*b*x*cosh(b*x + a)^2 - I*b*x)*sinh(b*x + a)^2 - I*b*x + 2*(3*I*b*x*co
sh(b*x + a)^5 + 2*I*b*x*cosh(b*x + a)^3 - I*b*x*cosh(b*x + a))*sinh(b*x +
a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(-I*b*x*cosh(b*x + a)^6 -
 6*I*b*x*cosh(b*x + a)*sinh(b*x + a)^5 - I*b*x*sinh(b*x + a)^6 - I*b*x*cos
h(b*x + a)^4 + (-15*I*b*x*cosh(b*x + a)^2 - I*b*x)*sinh(b*x + a)^4 + I*b*x
*cosh(b*x + a)^2 + 4*(-5*I*b*x*cosh(b*x + a)^3 - I*b*x*cosh(b*x + a))*s...
```

3.502.6 Sympy [F]

$$\int x^2\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = \int x^2\,\text{csch}^2\,(a+bx)\,\text{sech}^3\,(a+bx)\,dx$$

input
```
integrate(x**2*csch(b*x+a)**2*sech(b*x+a)**3,x)
```

output
```
Integral(x**2*csch(a + b*x)**2*sech(a + b*x)**3, x)
```

3.502.7 Maxima [F]

$$\int x^2\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int x^2\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)^3\,dx$$

input
```
integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-96*b^2*integrate(1/32*x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - (
2*b*x^2*e^(3*b*x + 3*a) + (3*b*x^2*e^(5*a) + 2*x*e^(5*a))*e^(5*b*x) + (3*b
*x^2*e^a - 2*x*e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) + b^2*e^(4*b*x + 4*a) -
b^2*e^(2*b*x + 2*a) - b^2) - 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x +
 a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3 + 2*arc
tan(e^(b*x + a))/b^3
```

3.502.8 Giac [F]

$$\int x^2\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int x^2\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)^3\,dx$$

input
```
integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

3.502.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int\frac{x^2}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

output
```
int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

3.503 $\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx$

3.503.1 Optimal result

Integrand size = 18, antiderivative size = 120

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{3x\arctan\left(e^{a+bx}\right)}{b}-\frac{\text{arctanh}(\cosh(a+bx))}{b^2}-\frac{3x\text{csch}(a+bx)}{2b}+\frac{3i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}-\frac{3i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}-\frac{\text{sech}(a+bx)}{2b^2}+\frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}$$

output
```
-3*x*arctan(exp(b*x+a))/b-arctanh(cosh(b*x+a))/b^2-3/2*x*csch(b*x+a)/b+3/2
*I*polylog(2,-I*exp(b*x+a))/b^2-3/2*I*polylog(2,I*exp(b*x+a))/b^2-1/2*sech
(b*x+a)/b^2+1/2*x*csch(b*x+a)*sech(b*x+a)^2/b
```

3.503.2 Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.71

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{-6a\arctan\left(e^{a+bx}\right)+bx\coth\left(\frac{1}{2}(a+bx)\right)+3ia\log\left(1-ie^{a+bx}\right)+3ibx\log\left(1-ie^{a+bx}\right)-3ia\log\left(1+\right.}{}$$

input
```
Integrate[x*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

output
```
-1/2*(-6*a*ArcTan[E^(a + b*x)] + b*x*Coth[(a + b*x)/2] + (3*I)*a*Log[1 - I
*E^(a + b*x)] + (3*I)*b*x*Log[1 - I*E^(a + b*x)] - (3*I)*a*Log[1 + I*E^(a
+ b*x)] - (3*I)*b*x*Log[1 + I*E^(a + b*x)] + 2*Log[Cosh[(a + b*x)/2]] - 2*
Log[Sinh[(a + b*x)/2]] - (3*I)*PolyLog[2, (-I)*E^(a + b*x)] + (3*I)*PolyLo
g[2, I*E^(a + b*x)] + Sech[a + b*x] - b*x*Tanh[(a + b*x)/2] + b*x*Sech[a +
 b*x]*Tanh[a + b*x])/b^2
```

3.503.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5985, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{5985}$$

$$-\int \left(\frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}-\frac{3\arctan(\sinh(a+bx))}{2b}-\frac{3\text{csch}(a+bx)}{2b}\right)dx-\frac{3x\arctan(\sinh(a+bx))}{2b}-\frac{3x\text{csch}(a+bx)}{2b}+\frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3x\arctan\left(e^{a+bx}\right)}{b}-\frac{\text{arctanh}(\cosh(a+bx))}{b^2}+\frac{3i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}-\frac{3i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}-\frac{\text{sech}(a+bx)}{2b^2}-\frac{3x\text{csch}(a+bx)}{2b}+\frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}$$

input
```
Int[x*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

output
```
(-3*x*ArcTan[E^(a + b*x)])/b - ArcTanh[Cosh[a + b*x]]/b^2 - (3*x*Csch[a +
b*x])/(2*b) + (((3*I)/2)*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - (((3*I)/2)*Po
lyLog[2, I*E^(a + b*x)])/b^2 - Sech[a + b*x]/(2*b^2) + (x*Csch[a + b*x]*Se
ch[a + b*x]^2)/(2*b)
```

3.503.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.503.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(103) = 206.

Time = 8.93 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.93

method	result
risch	$-\frac{e^{bx+a}\left(3\,e^{4bx+4a}bx+2\,e^{2bx+2a}bx+e^{4bx+4a}+3bx-1\right)}{b^{2}\left(1+e^{2bx+2a}\right)^{2}\left(e^{2bx+2a}-1\right)}+\frac{\ln\left(e^{bx+a}-1\right)}{b^{2}}-\frac{\ln\left(e^{bx+a}+1\right)}{b^{2}}+\frac{3a\arctan\left(e^{bx+a}\right)}{b^{2}}+\frac{3i\,\mathrm{dilog}\left(1+ie^{bx}\right.}{2b^{2}}$

input
```
int(x*csch(b*x+a)^2*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-exp(b*x+a)*(3*exp(4*b*x+4*a)*b*x+2*exp(2*b*x+2*a)*b*x+exp(4*b*x+4*a)+3*b*
x-1)/b^2/(1+exp(2*b*x+2*a))^2/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(b*x+a)-1)-1/
b^2*ln(exp(b*x+a)+1)+3/b^2*a*arctan(exp(b*x+a))+3/2*I/b^2*dilog(1+I*exp(b*
x+a))-3/2*I/b*ln(1-I*exp(b*x+a))*x-3/2*I/b^2*ln(1-I*exp(b*x+a))*a+3/2*I/b*
ln(1+I*exp(b*x+a))*x+3/2*I/b^2*ln(1+I*exp(b*x+a))*a-3/2*I/b^2*dilog(1-I*ex
p(b*x+a))
```

3.503.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2227 vs. 2(97) = 194.

Time = 0.29 (sec) , antiderivative size = 2227, normalized size of antiderivative = 18.56

$$\int x\mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx=\text{Too large to display}$$

input
```
integrate(x*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(3*b*x + 1)*cosh(b*x + a)^5 + 10*(3*b*x + 1)*cosh(b*x + a)*sinh(b*
x + a)^4 + 2*(3*b*x + 1)*sinh(b*x + a)^5 + 4*b*x*cosh(b*x + a)^3 + 4*(5*(3
*b*x + 1)*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^3 + 4*(5*(3*b*x + 1)*cosh(b
*x + a)^3 + 3*b*x*cosh(b*x + a))*sinh(b*x + a)^2 + 2*(3*b*x - 1)*cosh(b*x
+ a) + 3*(I*cosh(b*x + a)^6 + 6*I*cosh(b*x + a)*sinh(b*x + a)^5 + I*sinh(b
*x + a)^6 + (15*I*cosh(b*x + a)^2 + I)*sinh(b*x + a)^4 + I*cosh(b*x + a)^4
 + 4*(5*I*cosh(b*x + a)^3 + I*cosh(b*x + a))*sinh(b*x + a)^3 + (15*I*cosh(
b*x + a)^4 + 6*I*cosh(b*x + a)^2 - I)*sinh(b*x + a)^2 - I*cosh(b*x + a)^2
+ 2*(3*I*cosh(b*x + a)^5 + 2*I*cosh(b*x + a)^3 - I*cosh(b*x + a))*sinh(b*x
 + a) - I)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + 3*(-I*cosh(b*x + a)^
6 - 6*I*cosh(b*x + a)*sinh(b*x + a)^5 - I*sinh(b*x + a)^6 + (-15*I*cosh(b*
x + a)^2 - I)*sinh(b*x + a)^4 - I*cosh(b*x + a)^4 + 4*(-5*I*cosh(b*x + a)^
3 - I*cosh(b*x + a))*sinh(b*x + a)^3 + (-15*I*cosh(b*x + a)^4 - 6*I*cosh(b
*x + a)^2 + I)*sinh(b*x + a)^2 + I*cosh(b*x + a)^2 + 2*(-3*I*cosh(b*x + a)
^5 - 2*I*cosh(b*x + a)^3 + I*cosh(b*x + a))*sinh(b*x + a) + I)*dilog(-I*co
sh(b*x + a) - I*sinh(b*x + a)) + 2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh
(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 +
 cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 +
 (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x +
 a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(...
```

3.503.6 Sympy [F]

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = \int x\,\text{csch}^2\,(a+bx)\,\text{sech}^3\,(a+bx)\,dx$$

input
```
integrate(x*csch(b*x+a)**2*sech(b*x+a)**3,x)
```

output
```
Integral(x*csch(a + b*x)**2*sech(a + b*x)**3, x)
```

3.503.7 Maxima [F]

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int x\,\text{csch}\left(bx+a\right)^2\text{sech}\left(bx+a\right)^3\,dx$$

input
```
integrate(x*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-(2*b*x*e^(3*b*x + 3*a) + (3*b*x*e^(5*a) + e^(5*a))*e^(5*b*x) + (3*b*x*e^a
 - e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) + b^2*e^(4*b*x + 4*a) - b^2*e^(2*b*x
 + 2*a) - b^2) - log((e^(b*x + a) + 1)*e^(-a))/b^2 + log((e^(b*x + a) - 1)
*e^(-a))/b^2 - 96*integrate(1/32*x*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

3.503.8 Giac [F]

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int x\,\text{csch}\left(bx+a\right)^2\text{sech}\left(bx+a\right)^3\,dx$$

input
```
integrate(x*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

3.503.9 Mupad [F(-1)]

Timed out.

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int \frac{x}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(x/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

output
```
int(x/(cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

3.504 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$

3.504.1 Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\arctan(\sinh(a+bx))}{2b} - \frac{3\operatorname{csch}(a+bx)}{2b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

output
```
-3/2*arctan(sinh(b*x+a))/b-3/2*csch(b*x+a)/b+1/2*csch(b*x+a)*sech(b*x+a)^2
/b
```

3.504.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\operatorname{csch}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},-\sinh^2(a+bx)\right)}{b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

output
```
-((Csch[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, -Sinh[a + b*x]^2])/b)
```

3.504.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 25, 3101, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)^3dx$$

$$\Big\downarrow \; 25$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)^3dx$$

$$\Big\downarrow \; 3101$$

$$-\frac{i\int \frac{\text{csch}^4(a+bx)}{\left(\text{csch}^2(a+bx)+1\right)^2}d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \; 252$$

$$-\frac{i\left(\frac{i\text{csch}^3(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}-\frac{3}{2}\int -\frac{\text{csch}^2(a+bx)}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))\right)}{b}$$

$$\Big\downarrow \; 262$$

$$-\frac{i\left(\frac{i\text{csch}^3(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}-\frac{3}{2}\left(\int \frac{1}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))+i\text{csch}(a+bx)\right)\right)}{b}$$

$$\Big\downarrow \; 219$$

$$-\frac{i\left(\frac{i\text{csch}^3(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}-\frac{3}{2}(i\text{csch}(a+bx)-i\arctan(\text{csch}(a+bx)))\right)}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

output
```
((-I)*((-3*((-I)*ArcTan[Csch[a + b*x]] + I*Csch[a + b*x]))/2 + ((I/2)*Csch
[a + b*x]^3)/(1 + Csch[a + b*x]^2)))/b
```

3.504.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.504.4 Maple [A] (verified)

Time = 6.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	47
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	47
risch	$-\frac{\mathrm{e}^{bx+a}\left(3\,\mathrm{e}^{4bx+4a}+2\,\mathrm{e}^{2bx+2a}+3\right)}{b\left(1+\mathrm{e}^{2bx+2a}\right)^2\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{3i\ln\left(\mathrm{e}^{bx+a}-i\right)}{2b}-\frac{3i\ln\left(\mathrm{e}^{bx+a}+i\right)}{2b}$	95

input
```
int(csch(b*x+a)^2*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^2-3/2*sech(b*x+a)*tanh(b*x+a)-3*arctan(exp
(b*x+a)))
```

3.504.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 511, normalized size of antiderivative = 10.43

$$\int \mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx =$$

$$-\frac{3\cosh(bx+a)^5+15\cosh(bx+a)\sinh(bx+a)^4+3\sinh(bx+a)^5+2\left(15\cosh(bx+a)^2+1\right)\sinh(}{}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5
 + 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*(5*c
osh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*
sinh(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*
sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b
*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(5*c
osh(b*x + a)^4 + 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b*x + a))/(
b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6
+ b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*
cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x + a)^2 + (
15*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(3*b*c
osh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) - b)
```

3.504.6 Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**3, x)
```

3.504.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(43) = 86$.

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.84

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{3\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{3\,e^{(-bx-a)} + 2\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)}}{b(e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - e^{(-6\,bx-6\,a)} + 1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
3*arctan(e^(-b*x - a))/b - (3*e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + 3*e^(-5*
b*x - 5*a))/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - e^(-6*b*x - 6*a) + 1
))
```

3.504.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.08

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+8\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+4\,e^{(bx+a)}-4\,e^{(-bx-a)}} + 6\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{4\,b}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 8)/((e^(b*x + a) - e^(-
b*x - a))^3 + 4*e^(b*x + a) - 4*e^(-b*x - a)) + 6*arctan(1/2*(e^(2*b*x + 2
*a) - 1)*e^(-b*x - a)))/b
```

3.504.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.18

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

output
```
(2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (3*atan
((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(exp(
2*a + 2*b*x) - 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

3.505 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$

3.505.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x},x\right)$$

output
```
CannotIntegrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x)
```

3.505.2 Mathematica [N/A]

Not integrable

Time = 49.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

input
```
Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x,x]
```

output
```
Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x, x]
```

3.505.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x}dx$$

input
```
Int[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x,x]
```

output
```
$Aborted
```

3.505.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.505.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{2} \operatorname{sech}\left(bx+a\right)^{3}}{x}dx$$

input
```
int(csch(b*x+a)^2*sech(b*x+a)^3/x,x)
```

output
```
int(csch(b*x+a)^2*sech(b*x+a)^3/x,x)
```

3.505.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^2*sech(b*x + a)^3/x, x)
```

3.505.6 Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)}{x}\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**3/x,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**3/x, x)
```

3.505.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 10.70

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
-(2*b*x*e^(3*b*x + 3*a) + (3*b*x*e^(5*a) - e^(5*a))*e^(5*b*x) + (3*b*x*e^a
 + e^a)*e^(b*x))/(b^2*x^2*e^(6*b*x + 6*a) + b^2*x^2*e^(4*b*x + 4*a) - b^2*
x^2*e^(2*b*x + 2*a) - b^2*x^2) - 32*integrate(1/32*(3*b^2*x^2*e^a - 2*e^a)
*e^(b*x)/(b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3), x) - 32*integrate(1/32/(b*x^
2*e^(b*x + a) + b*x^2), x) - 32*integrate(1/32/(b*x^2*e^(b*x + a) - b*x^2)
, x)
```

3.505.8 Giac [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^2*sech(b*x + a)^3/x, x)
```

3.505.9 Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{1}{x\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

output
```
int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

3.506 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$

3.506.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2},x\right)$$

output
```
CannotIntegrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x)
```

3.506.2 Mathematica [N/A]

Not integrable

Time = 32.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2,x]
```

output
```
Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2, x]
```

3.506.3 Rubi [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}dx$$

input
```
Int[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2,x]
```

output
```
$Aborted
```

3.506.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.506.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{2}\operatorname{sech}\left(bx+a\right)^{3}}{x^{2}}dx$$

input
```
int(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x)
```

output
```
int(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x)
```

3.506.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^2*sech(b*x + a)^3/x^2, x)
```

3.506.6 Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**3/x**2,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**3/x**2, x)
```

3.506.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 215, normalized size of antiderivative = 10.75

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
-(2*b*x*e^(3*b*x + 3*a) + (3*b*x*e^(5*a) - 2*e^(5*a))*e^(5*b*x) + (3*b*x*e
^a + 2*e^a)*e^(b*x))/(b^2*x^3*e^(6*b*x + 6*a) + b^2*x^3*e^(4*b*x + 4*a) -
b^2*x^3*e^(2*b*x + 2*a) - b^2*x^3) - 32*integrate(3/32*(b^2*x^2*e^a - 2*e^
a)*e^(b*x)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4), x) - 32*integrate(1/16/(b*
x^3*e^(b*x + a) + b*x^3), x) - 32*integrate(1/16/(b*x^3*e^(b*x + a) - b*x^
3), x)
```

3.506.8 Giac [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^2*sech(b*x + a)^3/x^2, x)
```

3.506.9 Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

output
```
int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

3.507 $\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$

3.507.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx),x\right)$$

output
```
CannotIntegrate(x^m*csch(b*x+a)^3*sech(b*x+a),x)
```

3.507.2 Mathematica [N/A]

Not integrable

Time = 67.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x], x]
```

3.507.3 Rubi [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int x^m \text{csch}^3(a+bx)\text{sech}(a+bx)dx$$

input
```
Int[x^m*Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
$Aborted
```

3.507.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.507.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\,dx$$

input
```
int(x^m*csch(b*x+a)^3*sech(b*x+a),x)
```

output
```
int(x^m*csch(b*x+a)^3*sech(b*x+a),x)
```

3.507.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx=\int x^m \operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)\,dx$$

input
```
integrate(x^m*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*csch(b*x + a)^3*sech(b*x + a), x)
```

3.507.6 Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx=\int x^m \operatorname{csch}^3\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

input
```
integrate(x**m*csch(b*x+a)**3*sech(b*x+a),x)
```

output
```
Integral(x**m*csch(a + b*x)**3*sech(a + b*x), x)
```

3.507.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx=\int x^m \operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)\,dx$$

input
```
integrate(x^m*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^m*csch(b*x + a)^3*sech(b*x + a), x)
```

3.507.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\,dx$$

input
```
integrate(x^m*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*csch(b*x + a)^3*sech(b*x + a), x)
```

3.507.9 Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int \frac{x^m}{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}\,dx$$

input
```
int(x^m/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

output
```
int(x^m/(cosh(a + b*x)*sinh(a + b*x)^3), x)
```

3.508 $\int x^3\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$

3.508.1 Optimal result

Integrand size = 18, antiderivative size = 240

$$\begin{aligned}\int x^3\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = &-\frac{3x^2}{2b^2}+\frac{x^3}{2b}+\frac{2x^3\text{arctanh}\left(e^{2a+2bx}\right)}{b}\\&-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}\\&+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{3\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}\\&+\frac{3x^2\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}-\frac{3x^2\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}\\&-\frac{3x\,\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{3x\,\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}\\&+\frac{3\,\text{PolyLog}\left(4,-e^{2a+2bx}\right)}{4b^4}-\frac{3\,\text{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}\end{aligned}$$

output
```
-3/2*x^2/b^2+1/2*x^3/b+2*x^3*arctanh(exp(2*b*x+2*a))/b-3/2*x^2*coth(b*x+a)
/b^2-1/2*x^3*coth(b*x+a)^2/b+3*x*ln(1-exp(2*b*x+2*a))/b^3+3/2*polylog(2,ex
p(2*b*x+2*a))/b^4+3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2-3/2*x^2*polylog(2
,exp(2*b*x+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/2*x*polylog(3,
exp(2*b*x+2*a))/b^3+3/4*polylog(4,-exp(2*b*x+2*a))/b^4-3/4*polylog(4,exp(2
*b*x+2*a))/b^4
```

3.508.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 565 vs. 2(240) = 480.

Time = 6.51 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.35

$$\int x^3\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = -\frac{x^3\text{csch}^2(a+bx)}{2b}$$
$$+\frac{e^{2a}\left(-6b^2e^{-2a}x^2+b^4e^{-2a}x^4+6b(1-e^{-2a})\,x\log\left(1-e^{-a-bx}\right)-2b^3e^{-2a}(-1+e^{2a})\,x^3\log\left(1-e^{-a-bx}\right)+\right.}{}$$
$$+\frac{e^{2a}\left(2b^4e^{-2a}x^4+4b^3(1+e^{-2a})\,x^3\log\left(1+e^{-2(a+bx)}\right)-6b^2(1+e^{-2a})\,x^2\,\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)-6b(1\right.}{4b^4\left(1+e^{2a}\right)}$$
$$-\frac{1}{4}x^4\text{csch}(a)\text{sech}(a)+\frac{3x^2\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{2b^2}$$

input
```
Integrate[x^3*Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
-1/2*(x^3*Csch[a + b*x]^2)/b + (E^(2*a)*((-6*b^2*x^2)/E^(2*a) + (b^4*x^4)/
E^(2*a) + 6*b*(1 - E^(-2*a))*x*Log[1 - E^(-a - b*x)] - (2*b^3*(-1 + E^(2*a
))*x^3*Log[1 - E^(-a - b*x)])/E^(2*a) + 6*b*(1 - E^(-2*a))*x*Log[1 + E^(-a
 - b*x)] - (2*b^3*(-1 + E^(2*a))*x^3*Log[1 + E^(-a - b*x)])/E^(2*a) - 6*(1
 - E^(-2*a))*PolyLog[2, -E^(-a - b*x)] + 6*b^2*(1 - E^(-2*a))*x^2*PolyLog[
2, -E^(-a - b*x)] - 6*(1 - E^(-2*a))*PolyLog[2, E^(-a - b*x)] + 6*b^2*(1 -
 E^(-2*a))*x^2*PolyLog[2, E^(-a - b*x)] + 12*b*(1 - E^(-2*a))*x*PolyLog[3,
 -E^(-a - b*x)] + 12*b*(1 - E^(-2*a))*x*PolyLog[3, E^(-a - b*x)] + 12*(1 -
 E^(-2*a))*PolyLog[4, -E^(-a - b*x)] + 12*(1 - E^(-2*a))*PolyLog[4, E^(-a
- b*x)]))/(2*b^4*(-1 + E^(2*a))) + (E^(2*a)*((2*b^4*x^4)/E^(2*a) + 4*b^3*(
1 + E^(-2*a))*x^3*Log[1 + E^(-2*(a + b*x))] - 6*b^2*(1 + E^(-2*a))*x^2*Pol
yLog[2, -E^(-2*(a + b*x))] - 6*b*(1 + E^(-2*a))*x*PolyLog[3, -E^(-2*(a + b
*x))] - 3*(1 + E^(-2*a))*PolyLog[4, -E^(-2*(a + b*x))]))/(4*b^4*(1 + E^(2*
a))) - (x^4*Csch[a]*Sech[a])/4 + (3*x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x])/(
2*b^2)
```

3.508.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5985, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{ 5985}$$

$$-3\int -\frac{1}{2}x^2\left(\frac{\coth^2(a+bx)}{b}+\frac{2\log(\tanh(a+bx))}{b}\right)dx-\frac{x^3\coth^2(a+bx)}{2b}-\frac{x^3\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{2}\int x^2\left(\frac{\coth^2(a+bx)}{b}+\frac{2\log(\tanh(a+bx))}{b}\right)dx-\frac{x^3\coth^2(a+bx)}{2b}-\frac{x^3\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 2010}$$

$$\frac{3}{2}\int\left(\frac{\coth^2(a+bx)x^2}{b}+\frac{2\log(\tanh(a+bx))x^2}{b}\right)dx-\frac{x^3\coth^2(a+bx)}{2b}-\frac{x^3\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{2}\left(\frac{4x^3\text{arctanh}\left(e^{2a+2bx}\right)}{3b}+\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^4}+\frac{\text{PolyLog}\left(4,-e^{2a+2bx}\right)}{2b^4}-\frac{\text{PolyLog}\left(4,e^{2a+2bx}\right)}{2b^4}-\frac{x\,\text{PolyLog}\,(}{b}\right.$$
$$\frac{x^3\coth^2(a+bx)}{2b}-\frac{x^3\log(\tanh(a+bx))}{b}$$

input
```
Int[x^3*Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
-1/2*(x^3*Coth[a + b*x]^2)/b - (x^3*Log[Tanh[a + b*x]])/b + (3*(-(x^2/b^2)
 + x^3/(3*b) + (4*x^3*ArcTanh[E^(2*a + 2*b*x)])/(3*b) - (x^2*Coth[a + b*x]
)/b^2 + (2*x*Log[1 - E^(2*(a + b*x))])/b^3 + (2*x^3*Log[Tanh[a + b*x]])/(3
*b) + PolyLog[2, E^(2*(a + b*x))]/b^4 + (x^2*PolyLog[2, -E^(2*a + 2*b*x)])
/b^2 - (x^2*PolyLog[2, E^(2*a + 2*b*x)])/b^2 - (x*PolyLog[3, -E^(2*a + 2*b
*x)])/b^3 + (x*PolyLog[3, E^(2*a + 2*b*x)])/b^3 + PolyLog[4, -E^(2*a + 2*b
*x)]/(2*b^4) - PolyLog[4, E^(2*a + 2*b*x)]/(2*b^4)))/2
```

3.508.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.508.4 Maple [A] (verified)

Time = 3.83 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.74

method	result
risch	$-\frac{3a^2}{b^4} - \frac{\ln(e^{bx+a}+1)x^3}{b} - \frac{3x^2}{b^2} - \frac{3a\ln(e^{bx+a}-1)}{b^4} + \frac{a^3\ln(e^{bx+a}-1)}{b^4} + \frac{3\ln(e^{bx+a}+1)x}{b^3} + \frac{3\ln(1-e^{bx+a})x}{b^3} + \frac{3\ln(1-e^{b}}{b^4}$

input
```
int(x^3*csch(b*x+a)^3*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-3/b^4*a^2-1/b*ln(exp(b*x+a)+1)*x^3-3/b^2*x^2-3/b^4*a*ln(exp(b*x+a)-1)+1/b
^4*a^3*ln(exp(b*x+a)-1)+3/b^3*ln(exp(b*x+a)+1)*x+3/b^3*ln(1-exp(b*x+a))*x+
3/b^4*ln(1-exp(b*x+a))*a-x^2*(2*exp(2*b*x+2*a)*b*x+3*exp(2*b*x+2*a)-3)/b^2
/(exp(2*b*x+2*a)-1)^2+6/b^4*a*ln(exp(b*x+a))-6/b^3*a*x-3*x^2*polylog(2,-ex
p(b*x+a))/b^2-3*x^2*polylog(2,exp(b*x+a))/b^2+6*x*polylog(3,-exp(b*x+a))/b
^3+6*x*polylog(3,exp(b*x+a))/b^3+3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2-3/
2*x*polylog(3,-exp(2*b*x+2*a))/b^3-6*polylog(4,-exp(b*x+a))/b^4-6*polylog(
4,exp(b*x+a))/b^4+x^3*ln(1+exp(2*b*x+2*a))/b-1/b*ln(1-exp(b*x+a))*x^3-1/b^
4*ln(1-exp(b*x+a))*a^3+3/4*polylog(4,-exp(2*b*x+2*a))/b^4+3*polylog(2,-exp
(b*x+a))/b^4+3*polylog(2,exp(b*x+a))/b^4
```

3.508.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 3394, normalized size of antiderivative = 14.14

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

output
```
-(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sin
h(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 + (2*b^3*x^3 - 3*b^2*x^2
+ 6*a^2)*cosh(b*x + a)^2 + (2*b^3*x^3 - 3*b^2*x^2 + 18*(b^2*x^2 - a^2)*cos
h(b*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 + 3*((b^2*x^2 - 1)*cosh(b*x
+ a)^4 + 4*(b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sin
h(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*
(b^2*x^2 - 1)*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh
(b*x + a)^3 - (b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(cosh(b
*x + a) + sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x
 + a)*sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 - 2*b^2*x^2*cosh(b*x + a)^
2 + b^2*x^2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 - b^2*x^2)*sinh(b*x + a)^2 + 4*
(b^2*x^2*cosh(b*x + a)^3 - b^2*x^2*cosh(b*x + a))*sinh(b*x + a))*dilog(I*c
osh(b*x + a) + I*sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*c
osh(b*x + a)*sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 - 2*b^2*x^2*cosh(b*
x + a)^2 + b^2*x^2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 - b^2*x^2)*sinh(b*x + a)
^2 + 4*(b^2*x^2*cosh(b*x + a)^3 - b^2*x^2*cosh(b*x + a))*sinh(b*x + a))*di
log(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*((b^2*x^2 - 1)*cosh(b*x + a)^4
 + 4*(b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x
+ a)^4 + b^2*x^2 - 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x
^2 - 1)*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*...
```

3.508.6 Sympy [F]

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(x**3*csch(b*x+a)**3*sech(b*x+a),x)
```

output
```
Integral(x**3*csch(a + b*x)**3*sech(a + b*x), x)
```

3.508.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.47

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$
$$= -\frac{1}{2}x^4 + \frac{3x^2 - (2bx^3e^{(2a)} + 3x^2e^{(2a)})e^{(2bx)}}{b^2e^{(4bx+4a)} - 2b^2e^{(2bx+2a)} + b^2} + \frac{b^4x^4 - 6b^2x^2}{2b^4}$$
$$+ \frac{4b^3x^3\log\left(e^{(2bx+2a)}+1\right) + 6b^2x^2\mathrm{Li}_2\left(-e^{(2bx+2a)}\right) - 6bx\mathrm{Li}_3(-e^{(2bx+2a)}) + 3\mathrm{Li}_4(-e^{(2bx+2a)})}{3b^4}$$
$$- \frac{b^3x^3\log\left(e^{(bx+a)}+1\right) + 3b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 6bx\mathrm{Li}_3(-e^{(bx+a)}) + 6\mathrm{Li}_4(-e^{(bx+a)})}{b^4}$$
$$- \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right) + 3b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right) - 6bx\mathrm{Li}_3(e^{(bx+a)}) + 6\mathrm{Li}_4(e^{(bx+a)})}{b^4}$$
$$+ \frac{3\left(bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4} + \frac{3\left(bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^4}$$

input
```
integrate(x^3*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*x^4 + (3*x^2 - (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4
*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) + 1/2*(b^4*x^4 - 6*b^2*x^2)/b^4
 + 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2
*a)) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a))
)/b^4 - (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*
b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 - (b^3*x^3*
log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^
(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1)
+ dilog(-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x +
 a)))/b^4
```

3.508.8 Giac [F]

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^3 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\,dx$$

input
```
integrate(x^3*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*csch(b*x + a)^3*sech(b*x + a), x)
```

3.508.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int \frac{x^3}{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}\,dx$$

input
```
int(x^3/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

output
```
int(x^3/(cosh(a + b*x)*sinh(a + b*x)^3), x)
```

3.509 $\int x^2\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$

3.509.1 Optimal result

Integrand size = 18, antiderivative size = 148

$$\int x^2\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx=\frac{x^2}{2b}+\frac{2x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\coth^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}$$

output
```
1/2*x^2/b+2*x^2*arctanh(exp(2*b*x+2*a))/b-x*coth(b*x+a)/b^2-1/2*x^2*coth(b
*x+a)^2/b+ln(sinh(b*x+a))/b^3+x*polylog(2,-exp(2*b*x+2*a))/b^2-x*polylog(2
,exp(2*b*x+2*a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b^3+1/2*polylog(3,exp(
2*b*x+2*a))/b^3
```

3.509.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 388 vs. 2(148) = 296.

Time = 2.95 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.62

$$\int x^2\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \frac{1}{6}\Bigg(-\frac{3x^2\text{csch}^2(a+bx)}{b}$$
$$+\frac{2e^{2a}\left(-6be^{-2a}x-6b(1-e^{-2a})\,x+2b^3e^{-2a}x^3-3b^2e^{-2a}(-1+e^{2a})\,x^2\log\left(1-e^{-a-bx}\right)-3b^2e^{-2a}(-1+\right.}{}$$
$$+\frac{2b^2x^2\left(\frac{2bx}{1+e^{2a}}+3\log\left(1+e^{-2(a+bx)}\right)\right)-6bx\,\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)-3\,\text{PolyLog}\left(3,-e^{-2(a+bx)}\right)}{b^3}$$
$$-2x^3\text{csch}(a)\text{sech}(a)+\frac{6x\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b^2}\Bigg)$$

input
```
Integrate[x^2*Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
((-3*x^2*Csch[a + b*x]^2)/b + (2*E^(2*a)*((-6*b*x)/E^(2*a) - 6*b*(1 - E^(-
2*a))*x + (2*b^3*x^3)/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 - E^(-a -
b*x)])/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 + E^(-a - b*x)])/E^(2*a)
+ 3*(1 - E^(-2*a))*Log[1 - E^(a + b*x)] + 3*(1 - E^(-2*a))*Log[1 + E^(a +
b*x)] + 6*b*(1 - E^(-2*a))*x*PolyLog[2, -E^(-a - b*x)] + 6*b*(1 - E^(-2*a)
)*x*PolyLog[2, E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[3, -E^(-a - b*x)]
+ 6*(1 - E^(-2*a))*PolyLog[3, E^(-a - b*x)]))/(b^3*(-1 + E^(2*a))) + (2*b^
2*x^2*((2*b*x)/(1 + E^(2*a)) + 3*Log[1 + E^(-2*(a + b*x))]) - 6*b*x*PolyLo
g[2, -E^(-2*(a + b*x))] - 3*PolyLog[3, -E^(-2*(a + b*x))])/b^3 - 2*x^3*Csc
h[a]*Sech[a] + (6*x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b^2)/6
```

3.509.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5985, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$$
$$\Big\downarrow \text{5985}$$
$$-2\int -\frac{1}{2}x\left(\frac{\coth^2(a+bx)}{b}+\frac{2\log(\tanh(a+bx))}{b}\right)dx-\frac{x^2\coth^2(a+bx)}{2b}-\frac{x^2\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \, 27$$

$$\int x\left(\frac{\coth^2(a+bx)}{b}+\frac{2\log(\tanh(a+bx))}{b}\right)dx-\frac{x^2\coth^2(a+bx)}{2b}-\frac{x^2\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \, 2010$$

$$\int\left(\frac{x\coth^2(a+bx)}{b}+\frac{2x\log(\tanh(a+bx))}{b}\right)dx-\frac{x^2\coth^2(a+bx)}{2b}-\frac{x^2\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \, 2009$$

$$\frac{2x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\coth^2(a+bx)}{2b}+\frac{x^2}{2b}$$

input
```
Int[x^2*Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
x^2/(2*b) + (2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - (x*Coth[a + b*x])/b^2 - (
x^2*Coth[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]/b^3 + (x*PolyLog[2, -E^(2*
a + 2*b*x)])/b^2 - (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 - PolyLog[3, -E^(2*
a + 2*b*x)]/(2*b^3) + PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

3.509.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.509. $\int x^2\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$

3.509.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.80

method	result
risch	$-\frac{2x\left(e^{2bx+2a}bx+e^{2bx+2a}-1\right)}{b^{2}\left(e^{2bx+2a}-1\right)^{2}}-\frac{a^{2}\ln\left(e^{bx+a}-1\right)}{b^{3}}-\frac{\ln\left(1-e^{bx+a}\right)x^{2}}{b}-\frac{2x\,\mathrm{polylog}\left(2,e^{bx+a}\right)}{b^{2}}+\frac{x^{2}\ln\left(1+e^{2bx+2a}\right)}{b}+\frac{x\,\mathrm{polylo}}{}$

input
```
int(x^2*csch(b*x+a)^3*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*x*(exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-1/b^3*
a^2*ln(exp(b*x+a)-1)-1/b*ln(1-exp(b*x+a))*x^2-2*x*polylog(2,exp(b*x+a))/b^
2+x^2*ln(1+exp(2*b*x+2*a))/b+x*polylog(2,-exp(2*b*x+2*a))/b^2-1/b*ln(exp(b
*x+a)+1)*x^2-2*x*polylog(2,-exp(b*x+a))/b^2+1/b^3*ln(1-exp(b*x+a))*a^2+1/b
^3*ln(exp(b*x+a)-1)+1/b^3*ln(exp(b*x+a)+1)-2/b^3*ln(exp(b*x+a))+2*polylog(
3,exp(b*x+a))/b^3-1/2*polylog(3,-exp(2*b*x+2*a))/b^3+2*polylog(3,-exp(b*x+
a))/b^3
```

3.509.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 2562, normalized size of antiderivative = 17.31

$$\int x^2\mathrm{csch}^3(a+bx)\mathrm{sech}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

output
```
-(2*(b*x + a)*cosh(b*x + a)^4 + 8*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3
+ 2*(b*x + a)*sinh(b*x + a)^4 + 2*(b^2*x^2 - b*x - 2*a)*cosh(b*x + a)^2 +
2*(b^2*x^2 + 6*(b*x + a)*cosh(b*x + a)^2 - b*x - 2*a)*sinh(b*x + a)^2 + 2*
(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x
+ a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*
dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh
(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 +
2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x +
a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*
x + a)) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b
*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*
x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sin
h(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*(b*x*cosh(b*x +
a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*c
osh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4
*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x
+ a) - sinh(b*x + a)) + ((b^2*x^2 - 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 1)*c
osh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x + a)^4 + b^2*x^2 - 2
*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 1)*cosh(b*x ...
```

3.509.6 Sympy [F]

$$\int x^2\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \int x^2\,\text{csch}^3\,(a+bx)\,\text{sech}\,(a+bx)\,dx$$

input
```
integrate(x**2*csch(b*x+a)**3*sech(b*x+a),x)
```

output
```
Integral(x**2*csch(a + b*x)**3*sech(a + b*x), x)
```

3.509.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.64

$$\int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$
$$= -\frac{2\left(\left(bx^2e^{(2a)}+xe^{(2a)}\right)e^{(2bx)}-x\right)}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2}-\frac{2x}{b^2}$$
$$+\frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(2bx+2a)}\right)-\mathrm{Li}_3\left(-e^{(2bx+2a)}\right)}{2b^3}$$
$$-\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)}{b^3}$$
$$-\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(e^{(bx+a)}\right)}{b^3}$$
$$+\frac{\log\left(e^{(bx+a)}+1\right)}{b^3}+\frac{\log\left(e^{(bx+a)}-1\right)}{b^3}$$

input
```
integrate(x^2*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

output
```
-2*((b*x^2*e^(2*a) + x*e^(2*a))*e^(2*b*x) - x)/(b^2*e^(4*b*x + 4*a) - 2*b^
2*e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) +
1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3 - (
b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e
^(b*x + a)))/b^3 - (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a
)) - 2*polylog(3, e^(b*x + a)))/b^3 + log(e^(b*x + a) + 1)/b^3 + log(e^(b*
x + a) - 1)/b^3
```

3.509.8 Giac [F]

$$\int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^2 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\,dx$$

input
```
integrate(x^2*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*csch(b*x + a)^3*sech(b*x + a), x)
```

3.509.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int \frac{x^2}{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}\,dx$$

input
```
int(x^2/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

output
```
int(x^2/(cosh(a + b*x)*sinh(a + b*x)^3), x)
```

3.510 $\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$

3.510.1 Optimal result

Integrand size = 16, antiderivative size = 95

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx=\frac{x}{2b}+\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}+\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}$$

output
```
1/2*x/b+2*x*arctanh(exp(2*b*x+2*a))/b-1/2*coth(b*x+a)/b^2-1/2*x*coth(b*x+a
)^2/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^2-1/2*polylog(2,exp(2*b*x+2*a))/b^2
```

3.510.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx=-\frac{\coth(a+bx)+bx\text{csch}^2(a+bx)+2bx\log\left(1-e^{-2(a+bx)}\right)-2bx\log\left(1+e^{-2(a+bx)}\right)+\text{PolyLog}\left(2,-e^{-2}\right.}{2b^2}$$

input
```
Integrate[x*Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
-1/2*(Coth[a + b*x] + b*x*Csch[a + b*x]^2 + 2*b*x*Log[1 - E^(-2*(a + b*x))
] - 2*b*x*Log[1 + E^(-2*(a + b*x))] + PolyLog[2, -E^(-2*(a + b*x))] - Poly
Log[2, E^(-2*(a + b*x))])/b^2
```

3.510.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5985, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{ 5985}$$

$$-\int \left(-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}\right)dx-\frac{x\coth^2(a+bx)}{2b}-\frac{x\log(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}+\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}+\frac{x}{2b}$$

input
```
Int[x*Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
x/(2*b) + (2*x*ArcTanh[E^(2*a + 2*b*x)])/b - Coth[a + b*x]/(2*b^2) - (x*Co
th[a + b*x]^2)/(2*b) + PolyLog[2, -E^(2*a + 2*b*x)]/(2*b^2) - PolyLog[2, E
^(2*a + 2*b*x)]/(2*b^2)
```

3.510.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.510.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(82) = 164$.

Time = 1.84 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.79

method	result
risch	$-\frac{2\,e^{2bx+2a}bx+e^{2bx+2a}-1}{b^2\left(e^{2bx+2a}-1\right)^2}-\frac{\ln\left(1-e^{bx+a}\right)x}{b}-\frac{\ln\left(1-e^{bx+a}\right)a}{b^2}-\frac{\operatorname{polylog}\left(2,e^{bx+a}\right)}{b^2}-\frac{\ln\left(e^{bx+a}+1\right)x}{b}-\frac{\operatorname{polylog}\left(2,-e^{bx+a}\right)}{b^2}$

input
```
int(x*csch(b*x+a)^3*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(2*exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-1/b*ln(1
-exp(b*x+a))*x-1/b^2*ln(1-exp(b*x+a))*a-polylog(2,exp(b*x+a))/b^2-1/b*ln(e
xp(b*x+a)+1)*x-polylog(2,-exp(b*x+a))/b^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*pol
ylog(2,-exp(2*b*x+2*a))/b^2+1/b^2*a*ln(exp(b*x+a)-1)
```

3.510.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1578, normalized size of antiderivative = 16.61

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

output
```
-((2*b*x + 1)*cosh(b*x + a)^2 + 2*(2*b*x + 1)*cosh(b*x + a)*sinh(b*x + a)
+ (2*b*x + 1)*sinh(b*x + a)^2 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*
x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2
*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*
dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a
)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a
) + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cos
h(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*s
inh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*s
inh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
 a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh
(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x
*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)
^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log(
cosh(b*x + a) + sinh(b*x + a) + 1) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a
)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(
b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a...
```

3.510.6 Sympy [F]

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx=\int x\,\text{csch}^3\,(a+bx)\,\text{sech}\,(a+bx)\,dx$$

input
```
integrate(x*csch(b*x+a)**3*sech(b*x+a),x)
```

output
```
Integral(x*csch(a + b*x)**3*sech(a + b*x), x)
```

3.510.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.53

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = -\frac{\left(2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}-1}{b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2} + \frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2} - \frac{bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)}{b^2} - \frac{bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

input
```
integrate(x*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

output
```
-((2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) - 1)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^
(2*b*x + 2*a) + b^2) + 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b
*x + 2*a)))/b^2 - (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 - (
b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

3.510.8 Giac [F]

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \int x\,\text{csch}\left(bx+a\right)^3\text{sech}\left(bx+a\right)\,dx$$

input
```
integrate(x*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*csch(b*x + a)^3*sech(b*x + a), x)
```

3.510.9 Mupad [F(-1)]

Timed out.

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \int \frac{x}{\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(x/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

output
```
int(x/(cosh(a + b*x)*sinh(a + b*x)^3), x)
```

3.511 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$

3.511.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}$$

output
```
-1/2*coth(b*x+a)^2/b-ln(tanh(b*x+a))/b
```

3.511.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\operatorname{csch}^2(a+bx)-2\log(\cosh(a+bx))+2\log(\sinh(a+bx))}{2b}$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
-1/2*(Csch[a + b*x]^2 - 2*Log[Cosh[a + b*x]] + 2*Log[Sinh[a + b*x]])/b
```

3.511.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)dx$$

$$\Big\downarrow \text{3100}$$

$$-\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(i\coth^3(a+bx)-i\coth(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\coth^2(a+bx)+\log(i\tanh(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
-((Coth[a + b*x]^2/2 + Log[I*Tanh[a + b*x]])/b)
```

3.511.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.511.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2}-\ln(\tanh(bx+a))}{b}$	25
default	$\frac{-\frac{1}{2\sinh(bx+a)^2}-\ln(\tanh(bx+a))}{b}$	25
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)^2}-\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}+\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}$	62

input
```
int(csch(b*x+a)^3*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2-ln(tanh(b*x+a)))
```

3.511.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 379, normalized size of antiderivative = 13.54

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx =$$

$$-\frac{2 \cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+\right.\right.}{}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

output
```
-(2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
 sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh
(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x
 + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b
*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b
*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh
(b*x + a)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh
(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3
*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*
x + a))*sinh(b*x + a) + b)
```

3.511.6 Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x), x)
```

3.511.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(26) = 52.

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.25

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1)}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

output
```
-log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + log(e^(-2*b*x - 2*a)
+ 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1)
)
```

3.511.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.32

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\frac{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-6}{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2} + \log\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)} + 2\right) - \log\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)} - 2\right)}{2\,b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

output
```
1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
 - 2*a) - 2) + log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - log(e^(2*b*x
+ 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

3.511.9 Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.79

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\,\operatorname{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{2}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

output
```
(2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 2/(b*(exp(2*
a + 2*b*x) - 1)) - 2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

3.512 $\int \frac{\text{csch}^3(a+bx)\text{sech}(a+bx)}{x}\,dx$

3.512.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\text{csch}^3(a+bx)\text{sech}(a+bx)}{x}\,dx = \text{Int}\left(\frac{\text{csch}^3(a+bx)\text{sech}(a+bx)}{x},x\right)$$

output
```
CannotIntegrate(csch(b*x+a)^3*sech(b*x+a)/x,x)
```

3.512.2 Mathematica [N/A]

Not integrable

Time = 55.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\text{csch}^3(a+bx)\text{sech}(a+bx)}{x}\,dx = \int \frac{\text{csch}^3(a+bx)\text{sech}(a+bx)}{x}\,dx$$

input
```
Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x,x]
```

output
```
Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x, x]
```

3.512.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x} \, dx$$

$$\Big\downarrow 7299$$

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}dx$$

input
```
Int[(Csch[a + b*x]^3*Sech[a + b*x])/x,x]
```

output
```
$Aborted
```

3.512.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.512.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{3} \operatorname{sech}\left(bx+a\right)}{x}dx$$

input
```
int(csch(b*x+a)^3*sech(b*x+a)/x,x)
```

output
```
int(csch(b*x+a)^3*sech(b*x+a)/x,x)
```

3.512.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^3*sech(b*x + a)/x, x)
```

3.512.6 Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^3\left(a+bx\right)\operatorname{sech}\left(a+bx\right)}{x}\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)/x,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)/x, x)
```

3.512.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 9.28

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)/x,x, algorithm="maxima")
```

output
```
-((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^
2*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 16*integrate(1/16*(b^2*x^2 - 1)/(b^2*x^
3*e^(b*x + a) + b^2*x^3), x) - 16*integrate(1/16*(b^2*x^2 - 1)/(b^2*x^3*e^
(b*x + a) - b^2*x^3), x) - 16*integrate(1/8/(x*e^(2*b*x + 2*a) + x), x)
```

3.512.8 Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)}{x}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^3*sech(b*x + a)/x, x)
```

3.512.9 Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{1}{x\cosh(a+b\,x)\,\sinh(a+b\,x)^3}\,dx$$

input
```
int(1/(x*cosh(a + b*x)*sinh(a + b*x)^3),x)
```

output
```
int(1/(x*cosh(a + b*x)*sinh(a + b*x)^3), x)
```

3.513 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$

3.513.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2},x\right)$$

output
```
CannotIntegrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x)
```

3.513.2 Mathematica [N/A]

Not integrable

Time = 23.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x^2,x]
```

output
```
Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x^2, x]
```

3.513.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}dx$$

input
```
Int[(Csch[a + b*x]^3*Sech[a + b*x])/x^2,x]
```

output
```
$Aborted
```

3.513.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.513.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{3}\operatorname{sech}\left(bx+a\right)}{x^{2}}dx$$

input
```
int(csch(b*x+a)^3*sech(b*x+a)/x^2,x)
```

output
```
int(csch(b*x+a)^3*sech(b*x+a)/x^2,x)
```

3.513.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^3*sech(b*x + a)/x^2, x)
```

3.513.6 Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)/x**2,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)/x**2, x)
```

3.513.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 9.44

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
-2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^
2*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 16*integrate(1/16*(b^2*x^2 - 3)/(b^2*x^
4*e^(b*x + a) + b^2*x^4), x) - 16*integrate(1/16*(b^2*x^2 - 3)/(b^2*x^4*e^
(b*x + a) - b^2*x^4), x) - 16*integrate(1/8/(x^2*e^(2*b*x + 2*a) + x^2), x
)
```

3.513.8 Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^3*sech(b*x + a)/x^2, x)
```

3.513.9 Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^3}\,dx$$

input
```
int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)^3),x)
```

output
```
int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)^3), x)
```

3.514 $\int x^m \mathrm{csch}^3(a+bx)\mathrm{sech}^2(a+bx)\,dx$

3.514.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m \mathrm{csch}^3(a+bx)\mathrm{sech}^2(a+bx)\,dx = \mathrm{Int}\left(x^m \mathrm{csch}^3(a+bx)\mathrm{sech}^2(a+bx),x\right)$$

output
```
CannotIntegrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

3.514.2 Mathematica [N/A]

Not integrable

Time = 60.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \mathrm{csch}^3(a+bx)\mathrm{sech}^2(a+bx)\,dx = \int x^m \mathrm{csch}^3(a+bx)\mathrm{sech}^2(a+bx)\,dx$$

input
```
Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2, x]
```

3.514.3 Rubi [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)dx$$

input
```
Int[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
$Aborted
```

3.514.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.514.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2\,dx$$

input
```
int(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

output
```
int(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

3.514.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int x^m\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^2\,dx$$

input
```
integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

3.514.6 Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int x^m\,\text{csch}^3\,(a+bx)\,\text{sech}^2\,(a+bx)\,dx$$

input
```
integrate(x**m*csch(b*x+a)**3*sech(b*x+a)**2,x)
```

output
```
Integral(x**m*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

3.514.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int x^m\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^2\,dx$$

input
```
integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

3.514.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)^2\,dx$$

input
```
integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

3.514.9 Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int \frac{x^m}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

output
```
int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

3.515 $\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$

3.515.1 Optimal result

Integrand size = 20, antiderivative size = 317

$$\begin{aligned}\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx &= \frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2}-\frac{6x\operatorname{arctanh}\left(e^{a+bx}\right)}{b^3}\\
&+\frac{3x^3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}-\frac{3x^2\operatorname{csch}(a+bx)}{2b^2}\\
&-\frac{3\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^4}+\frac{9x^2\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}\\
&-\frac{6ix\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{6ix\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}\\
&+\frac{3\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^4}-\frac{9x^2\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}\\
&-\frac{9x\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}+\frac{6i\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}\\
&-\frac{6i\operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}+\frac{9x\operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^3}\\
&+\frac{9\operatorname{PolyLog}\left(4,-e^{a+bx}\right)}{b^4}-\frac{9\operatorname{PolyLog}\left(4,e^{a+bx}\right)}{b^4}\\
&-\frac{3x^3\operatorname{sech}(a+bx)}{2b}-\frac{x^3\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}\end{aligned}$$

output
```
6*x^2*arctan(exp(b*x+a))/b^2-6*x*arctanh(exp(b*x+a))/b^3+3*x^3*arctanh(exp
(b*x+a))/b-3/2*x^2*csch(b*x+a)/b^2-3*polylog(2,-exp(b*x+a))/b^4+9/2*x^2*po
lylog(2,-exp(b*x+a))/b^2+6*I*polylog(3,-I*exp(b*x+a))/b^4-6*I*x*polylog(2,
-I*exp(b*x+a))/b^3+3*polylog(2,exp(b*x+a))/b^4-9/2*x^2*polylog(2,exp(b*x+a
))/b^2-9*x*polylog(3,-exp(b*x+a))/b^3+6*I*x*polylog(2,I*exp(b*x+a))/b^3-6*
I*polylog(3,I*exp(b*x+a))/b^4+9*x*polylog(3,exp(b*x+a))/b^3+9*polylog(4,-e
xp(b*x+a))/b^4-9*polylog(4,exp(b*x+a))/b^4-3/2*x^3*sech(b*x+a)/b-1/2*x^3*c
sch(b*x+a)^2*sech(b*x+a)/b
```

3.515.2 Mathematica [A] (verified)

Time = 6.44 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.47

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{3x^2\text{csch}(a)}{2b^2} - \frac{x^3\text{csch}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} + \frac{3i\left(b^2x^2\log\left(1-ie^{a+bx}\right)-b^2x^2\log\left(1+ie^{a+bx}\right)-2bx\,\text{PolyLog}\left(2,-ie^{a+bx}\right)+2bx\,\text{PolyLog}\left(2,ie^{a+bx}\right)+\right.}{b^4} - \frac{3\left(-\frac{x\log(1-e^{a+bx})}{b}+\frac{1}{2}bx^3\log\left(1-e^{a+bx}\right)+\frac{x\log(1+e^{a+bx})}{b}-\frac{1}{2}bx^3\log\left(1+e^{a+bx}\right)+\frac{\text{PolyLog}(2,-e^{a+bx})}{b^2}-\frac{3}{2}x^2\right.}{} - \frac{x^3\text{sech}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} - \frac{x^3\text{sech}(a+bx)}{b} + \frac{3x^2\text{csch}\left(\frac{a}{2}\right)\text{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{4b^2} + \frac{3x^2\text{sech}\left(\frac{a}{2}\right)\text{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{4b^2}$$

input
```
Integrate[x^3*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
(-3*x^2*Csch[a])/(2*b^2) - (x^3*Csch[a/2 + (b*x)/2]^2)/(8*b) + ((3*I)*(b^2
*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a + b*x)] - 2*b*x*PolyL
og[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*x)] + 2*PolyLog[3, (
-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]))/b^4 - (3*(-((x*Log[1 - E^
(a + b*x)])/b) + (b*x^3*Log[1 - E^(a + b*x)])/2 + (x*Log[1 + E^(a + b*x)])
/b - (b*x^3*Log[1 + E^(a + b*x)])/2 + PolyLog[2, -E^(a + b*x)]/b^2 - (3*x^
2*PolyLog[2, -E^(a + b*x)])/2 - PolyLog[2, E^(a + b*x)]/b^2 + (3*x^2*PolyL
og[2, E^(a + b*x)])/2 + (3*x*PolyLog[3, -E^(a + b*x)])/b - (3*x*PolyLog[3,
 E^(a + b*x)])/b - (3*PolyLog[4, -E^(a + b*x)])/b^2 + (3*PolyLog[4, E^(a +
 b*x)])/b^2))/b^2 - (x^3*Sech[a/2 + (b*x)/2]^2)/(8*b) - (x^3*Sech[a + b*x]
)/b + (3*x^2*Csch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2) + (3*x^2
*Sech[a/2]*Sech[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2)
```

3.515.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5985, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{5985}$$

$$-3\int \frac{1}{2}x^2\left(-\frac{\text{sech}(a+bx)\text{csch}^2(a+bx)}{b}+\frac{3\text{arctanh}(\cosh(a+bx))}{b}-\frac{3\text{sech}(a+bx)}{b}\right)dx+\frac{3x^3\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3x^3\text{sech}(a+bx)}{2b}-\frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{2}\int x^2\left(-\frac{\text{sech}(a+bx)\text{csch}^2(a+bx)}{b}+\frac{3\text{arctanh}(\cosh(a+bx))}{b}-\frac{3\text{sech}(a+bx)}{b}\right)dx+\frac{3x^3\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3x^3\text{sech}(a+bx)}{2b}-\frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{2010}$$

$$-\frac{3}{2}\int\left(\frac{3x^2\text{arctanh}(\cosh(a+bx))}{b}-\frac{x^2\left(\text{csch}^2(a+bx)+3\right)\text{sech}(a+bx)}{b}\right)dx+\frac{3x^3\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3x^3\text{sech}(a+bx)}{2b}-\frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}\left(-\frac{4x^2\arctan\left(e^{a+bx}\right)}{b^2}+\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^3}-\frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{x^3\text{arctanh}(\cosh(a+bx))}{b}+\frac{2\,\text{PolyLog}\left(2\right.}{b^4}\right.$$
$$\frac{3x^3\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3x^3\text{sech}(a+bx)}{2b}-\frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

input
```
Int[x^3*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
(3*x^3*ArcTanh[Cosh[a + b*x]])/(2*b) - (3*((-4*x^2*ArcTan[E^(a + b*x)])/b^
2 + (4*x*ArcTanh[E^(a + b*x)])/b^3 - (2*x^3*ArcTanh[E^(a + b*x)])/b + (x^3
*ArcTanh[Cosh[a + b*x]])/b + (x^2*Csch[a + b*x])/b^2 + (2*PolyLog[2, -E^(a
 + b*x)])/b^4 - (3*x^2*PolyLog[2, -E^(a + b*x)])/b^2 + ((4*I)*x*PolyLog[2,
 (-I)*E^(a + b*x)])/b^3 - ((4*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 - (2*Pol
yLog[2, E^(a + b*x)])/b^4 + (3*x^2*PolyLog[2, E^(a + b*x)])/b^2 + (6*x*Pol
yLog[3, -E^(a + b*x)])/b^3 - ((4*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^4 + ((
4*I)*PolyLog[3, I*E^(a + b*x)])/b^4 - (6*x*PolyLog[3, E^(a + b*x)])/b^3 -
(6*PolyLog[4, -E^(a + b*x)])/b^4 + (6*PolyLog[4, E^(a + b*x)])/b^4))/2 - (
3*x^3*Sech[a + b*x])/(2*b) - (x^3*Csch[a + b*x]^2*Sech[a + b*x])/(2*b)
```

3.515.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.515.4 Maple [F]

$$\int x^3 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2 \, dx$$

input
```
int(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

output
```
int(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

3.515.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5356 vs. 2(270) = 540.

Time = 0.33 (sec) , antiderivative size = 5356, normalized size of antiderivative = 16.90

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.515.6 Sympy [F]

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int x^3\,\text{csch}^3\,(a+bx)\,\text{sech}^2\,(a+bx)\,dx$$

input
```
integrate(x**3*csch(b*x+a)**3*sech(b*x+a)**2,x)
```

output
```
Integral(x**3*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

3.515.7 Maxima [F]

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int x^3\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^2\,dx$$

input
```
integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
(2*b*x^3*e^(3*b*x + 3*a) - 3*(b*x^3*e^(5*a) + x^2*e^(5*a))*e^(5*b*x) - 3*(
b*x^3*e^a - x^2*e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) - b^2*e^(4*b*x + 4*a) -
 b^2*e^(2*b*x + 2*a) + b^2) + 3/2*(b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^
2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(
b*x + a)))/b^4 - 3/2*(b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b
*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4
- 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(
b*x + a) + 1) + dilog(e^(b*x + a)))/b^4 + 96*integrate(1/16*x^2*e^(b*x + a
)/(b*e^(2*b*x + 2*a) + b), x)
```

3.515.8 Giac [F]

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x^3\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^2\,dx$$

input
```
integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

3.515.9 Mupad [F(-1)]

Timed out.

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int \frac{x^3}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

output
```
int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

3.516 $\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$

3.516.1 Optimal result

Integrand size = 20, antiderivative size = 197

$$\begin{aligned}\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx &= \frac{4x\arctan\left(e^{a+bx}\right)}{b^2}+\frac{3x^2\text{arctanh}\left(e^{a+bx}\right)}{b}\\&\quad-\frac{\text{arctanh}(\cosh(a+bx))}{b^3}-\frac{x\text{csch}(a+bx)}{b^2}\\&\quad+\frac{3x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}\\&\quad+\frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{3x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\\&\quad-\frac{3\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}+\frac{3\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}\\&\quad-\frac{3x^2\text{sech}(a+bx)}{2b}-\frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}\end{aligned}$$

output
```
4*x*arctan(exp(b*x+a))/b^2+3*x^2*arctanh(exp(b*x+a))/b-arctanh(cosh(b*x+a)
)/b^3-x*csch(b*x+a)/b^2+3*x*polylog(2,-exp(b*x+a))/b^2-2*I*polylog(2,-I*ex
p(b*x+a))/b^3+2*I*polylog(2,I*exp(b*x+a))/b^3-3*x*polylog(2,exp(b*x+a))/b^
2-3*polylog(3,-exp(b*x+a))/b^3+3*polylog(3,exp(b*x+a))/b^3-3/2*x^2*sech(b*
x+a)/b-1/2*x^2*csch(b*x+a)^2*sech(b*x+a)/b
```

3.516.2 Mathematica [A] (verified)

Time = 6.34 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.73

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{x\text{csch}(a)}{b^2} - \frac{x^2\text{csch}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$
$$+\frac{2i\left(bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a+bx}\right)\right)}{b^3}$$
$$+\frac{\frac{\log\left(1-e^{a+bx}\right)}{b}-\frac{3}{2}bx^2\log\left(1-e^{a+bx}\right)-\frac{\log\left(1+e^{a+bx}\right)}{b}+\frac{3}{2}bx^2\log\left(1+e^{a+bx}\right)+3x\,\text{PolyLog}\left(2,-e^{a+bx}\right)-3x\,\text{P}}{b^2}$$
$$-\frac{x^2\text{sech}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}-\frac{x^2\text{sech}(a+bx)}{b}+\frac{x\text{csch}\left(\frac{a}{2}\right)\text{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{2b^2}$$
$$+\frac{x\text{sech}\left(\frac{a}{2}\right)\text{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{2b^2}$$

input
```
Integrate[x^2*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
-((x*Csch[a])/b^2) - (x^2*Csch[a/2 + (b*x)/2]^2)/(8*b) + ((2*I)*(b*x*(Log[
1 - I*E^(a + b*x)] - Log[1 + I*E^(a + b*x)]) - PolyLog[2, (-I)*E^(a + b*x)
] + PolyLog[2, I*E^(a + b*x)]))/b^3 + (Log[1 - E^(a + b*x)]/b - (3*b*x^2*L
og[1 - E^(a + b*x)])/2 - Log[1 + E^(a + b*x)]/b + (3*b*x^2*Log[1 + E^(a +
b*x)])/2 + 3*x*PolyLog[2, -E^(a + b*x)] - 3*x*PolyLog[2, E^(a + b*x)] - (3
*PolyLog[3, -E^(a + b*x)])/b + (3*PolyLog[3, E^(a + b*x)])/b)/b^2 - (x^2*S
ech[a/2 + (b*x)/2]^2)/(8*b) - (x^2*Sech[a + b*x])/b + (x*Csch[a/2]*Csch[a/
2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b^2) + (x*Sech[a/2]*Sech[a/2 + (b*x)/2]*Sin
h[(b*x)/2])/(2*b^2)
```

3.516.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5985, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \; 5985$$

$$-2\int \frac{1}{2}x\left(-\frac{\text{sech}(a+bx)\text{csch}^2(a+bx)}{b}+\frac{3\text{arctanh}(\cosh(a+bx))}{b}-\frac{3\text{sech}(a+bx)}{b}\right)dx+\frac{3x^2\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3x^2\text{sech}(a+bx)}{2b}-\frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \textbf{27}$$

$$-\int x\left(-\frac{\text{sech}(a+bx)\text{csch}^2(a+bx)}{b}+\frac{3\text{arctanh}(\cosh(a+bx))}{b}-\frac{3\text{sech}(a+bx)}{b}\right)dx+\frac{3x^2\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3x^2\text{sech}(a+bx)}{2b}-\frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \textbf{2010}$$

$$-\int \left(\frac{3x\text{arctanh}(\cosh(a+bx))}{b}-\frac{x\left(\text{csch}^2(a+bx)+3\right)\text{sech}(a+bx)}{b}\right)dx+\frac{3x^2\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3x^2\text{sech}(a+bx)}{2b}-\frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{4x\arctan\left(e^{a+bx}\right)}{b^2}-\frac{\text{arctanh}(\cosh(a+bx))}{b^3}+\frac{3x^2\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{3\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}+\frac{3\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}+\frac{3x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{3x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{x\text{csch}(a+bx)}{b^2}-\frac{3x^2\text{sech}(a+bx)}{2b}-\frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

input

```
Int[x^2*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output

```
(4*x*ArcTan[E^(a + b*x)])/b^2 + (3*x^2*ArcTanh[E^(a + b*x)])/b - ArcTanh[C
osh[a + b*x]]/b^3 - (x*Csch[a + b*x])/b^2 + (3*x*PolyLog[2, -E^(a + b*x)])
/b^2 - ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 + ((2*I)*PolyLog[2, I*E^(a
 + b*x)])/b^3 - (3*x*PolyLog[2, E^(a + b*x)])/b^2 - (3*PolyLog[3, -E^(a +
b*x)])/b^3 + (3*PolyLog[3, E^(a + b*x)])/b^3 - (3*x^2*Sech[a + b*x])/(2*b)
 - (x^2*Csch[a + b*x]^2*Sech[a + b*x])/(2*b)
```

3.516. $\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$

3.516.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.516.4 Maple [F]

$$\int x^2 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2 \, dx$$

input
```
int(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

output
```
int(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

3.516.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3804 vs. 2(173) = 346.

Time = 0.32 (sec) , antiderivative size = 3804, normalized size of antiderivative = 19.31

$$\int x^2 \operatorname{csch}^3(a+bx) \operatorname{sech}^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(4*b^2*x^2*cosh(b*x + a)^3 - 2*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^5 - 1
0*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)*sinh(b*x + a)^4 - 2*(3*b^2*x^2 + 2*b*x
)*sinh(b*x + a)^5 + 4*(b^2*x^2 - 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^2)*si
nh(b*x + a)^3 + 4*(3*b^2*x^2*cosh(b*x + a) - 5*(3*b^2*x^2 + 2*b*x)*cosh(b*
x + a)^3)*sinh(b*x + a)^2 - 2*(3*b^2*x^2 - 2*b*x)*cosh(b*x + a) - 6*(b*x*c
osh(b*x + a)^6 + 6*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6
 - b*x*cosh(b*x + a)^4 + (15*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 -
b*x*cosh(b*x + a)^2 + 4*(5*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b
*x + a)^3 + (15*b*x*cosh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*
x + a)^2 + b*x + 2*(3*b*x*cosh(b*x + a)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*co
sh(b*x + a))*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 4*(-I*c
osh(b*x + a)^6 - 6*I*cosh(b*x + a)*sinh(b*x + a)^5 - I*sinh(b*x + a)^6 + (
-15*I*cosh(b*x + a)^2 + I)*sinh(b*x + a)^4 + I*cosh(b*x + a)^4 + 4*(-5*I*c
osh(b*x + a)^3 + I*cosh(b*x + a))*sinh(b*x + a)^3 + (-15*I*cosh(b*x + a)^4
 + 6*I*cosh(b*x + a)^2 + I)*sinh(b*x + a)^2 + I*cosh(b*x + a)^2 + 2*(-3*I*
cosh(b*x + a)^5 + 2*I*cosh(b*x + a)^3 + I*cosh(b*x + a))*sinh(b*x + a) - I
)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 4*(I*cosh(b*x + a)^6 + 6*I*co
sh(b*x + a)*sinh(b*x + a)^5 + I*sinh(b*x + a)^6 + (15*I*cosh(b*x + a)^2 -
I)*sinh(b*x + a)^4 - I*cosh(b*x + a)^4 + 4*(5*I*cosh(b*x + a)^3 - I*cosh(b
*x + a))*sinh(b*x + a)^3 + (15*I*cosh(b*x + a)^4 - 6*I*cosh(b*x + a)^2 ...
```

3.516.6 Sympy [F]

$$\int x^2 \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int x^2 \,\text{csch}^3\,(a+bx)\,\text{sech}^2\,(a+bx)\,dx$$

input
```
integrate(x**2*csch(b*x+a)**3*sech(b*x+a)**2,x)
```

output
```
Integral(x**2*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

3.516.7 Maxima [F]

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x^2\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^2\,dx$$

input
```
integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
(2*b*x^2*e^(3*b*x + 3*a) - (3*b*x^2*e^(5*a) + 2*x*e^(5*a))*e^(5*b*x) - (3*
b*x^2*e^a - 2*x*e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) - b^2*e^(4*b*x + 4*a) -
 b^2*e^(2*b*x + 2*a) + b^2) + 3/2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*di
log(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - 3/2*(b^2*x^2*log(-e^
(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
 - log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3 + 32*integrate(1/8*
x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

3.516.8 Giac [F(-2)]

Exception generated.

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\text{Exception raised: AttributeError}$$

input
```
integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.516.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int \frac{x^2}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

output
```
int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

3.517 $\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$

3.517.1 Optimal result

Integrand size = 18, antiderivative size = 109

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b^2}+\frac{3x\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{\text{csch}(a+bx)}{2b^2}+\frac{3\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{3\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{3x\text{sech}(a+bx)}{2b}-\frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

output
```
arctan(sinh(b*x+a))/b^2+3*x*arctanh(exp(b*x+a))/b-1/2*csch(b*x+a)/b^2+3/2*
polylog(2,-exp(b*x+a))/b^2-3/2*polylog(2,exp(b*x+a))/b^2-3/2*x*sech(b*x+a)
/b-1/2*x*csch(b*x+a)^2*sech(b*x+a)/b
```

3.517.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=$$

$$-\frac{-16\arctan\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)+2\coth\left(\frac{1}{2}(a+bx)\right)+bx\text{csch}^2\left(\frac{1}{2}(a+bx)\right)+12bx\log\left(1-e^{a+bx}\right)-12}{}$$

input
```
Integrate[x*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
-1/8*(-16*ArcTan[Tanh[(a + b*x)/2]] + 2*Coth[(a + b*x)/2] + b*x*Csch[(a +
b*x)/2]^2 + 12*b*x*Log[1 - E^(a + b*x)] - 12*b*x*Log[1 + E^(a + b*x)] - 12
*PolyLog[2, -E^(a + b*x)] + 12*PolyLog[2, E^(a + b*x)] + b*x*Sech[(a + b*x
)/2]^2 + 8*b*x*Sech[a + b*x] - 2*Tanh[(a + b*x)/2])/b^2
```

3.517.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5985, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{5985}$$

$$-\int \left(-\frac{\text{sech}(a+bx)\text{csch}^2(a+bx)}{2b}+\frac{3\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3\text{sech}(a+bx)}{2b}\right)dx+\frac{3x\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3x\text{sech}(a+bx)}{2b}-\frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan(\sinh(a+bx))}{b^2}+\frac{3x\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{3\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{3\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{\text{csch}(a+bx)}{2b^2}-\frac{3x\text{sech}(a+bx)}{2b}-\frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

input
```
Int[x*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
ArcTan[Sinh[a + b*x]]/b^2 + (3*x*ArcTanh[E^(a + b*x)])/b - Csch[a + b*x]/(
2*b^2) + (3*PolyLog[2, -E^(a + b*x)])/(2*b^2) - (3*PolyLog[2, E^(a + b*x)]
)/(2*b^2) - (3*x*Sech[a + b*x])/(2*b) - (x*Csch[a + b*x]^2*Sech[a + b*x])/
(2*b)
```

3.517.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.517.4 Maple [A] (verified)

Time = 5.89 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{e^{bx+a}\left(3\,e^{4bx+4a}bx-2\,e^{2bx+2a}bx+e^{4bx+4a}+3bx-1\right)}{b^{2}\left(e^{2bx+2a}-1\right)^{2}\left(1+e^{2bx+2a}\right)}+\frac{2\arctan\left(e^{bx+a}\right)}{b^{2}}+\frac{3a\ln\left(e^{bx+a}-1\right)}{2b^{2}}+\frac{3\,\mathrm{dilog}\left(e^{bx+a}\right)}{2b^{2}}+\frac{3\,\mathrm{dilog}\left(e^{bx+\right.}}{2b^{2}}$

input
```
int(x*csch(b*x+a)^3*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-exp(b*x+a)*(3*exp(4*b*x+4*a)*b*x-2*exp(2*b*x+2*a)*b*x+exp(4*b*x+4*a)+3*b*
x-1)/b^2/(exp(2*b*x+2*a)-1)^2/(1+exp(2*b*x+2*a))+2/b^2*arctan(exp(b*x+a))+
3/2/b^2*a*ln(exp(b*x+a)-1)+3/2/b^2*dilog(exp(b*x+a))+3/2/b^2*dilog(exp(b*x
+a)+1)+3/2/b*ln(exp(b*x+a)+1)*x
```

3.517.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1694 vs. $2(94) = 188$.

Time = 0.28 (sec) , antiderivative size = 1694, normalized size of antiderivative = 15.54

$$\int x\mathrm{csch}^3(a+bx)\mathrm{sech}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(3*b*x + 1)*cosh(b*x + a)^5 + 10*(3*b*x + 1)*cosh(b*x + a)*sinh(b*
x + a)^4 + 2*(3*b*x + 1)*sinh(b*x + a)^5 - 4*b*x*cosh(b*x + a)^3 + 4*(5*(3
*b*x + 1)*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^3 + 4*(5*(3*b*x + 1)*cosh(b
*x + a)^3 - 3*b*x*cosh(b*x + a))*sinh(b*x + a)^2 - 4*(cosh(b*x + a)^6 + 6*
cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))
*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x +
 a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(
b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 2*(3*
b*x - 1)*cosh(b*x + a) + 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a
)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*
x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cos
h(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 +
2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) +
1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*(cosh(b*x + a)^6 + 6*cosh(b*x
+ a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x
 + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x
 + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - c
osh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))
*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(b*x*cosh...
```

3.517.6 Sympy [F]

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x\,\text{csch}^3\,(a+bx)\,\text{sech}^2\,(a+bx)\,dx$$

input
```
integrate(x*csch(b*x+a)**3*sech(b*x+a)**2,x)
```

output
```
Integral(x*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

3.517.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.52

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$$
$$=\frac{2\,bxe^{(3\,bx+3\,a)}-\left(3\,bxe^{(5\,a)}+e^{(5\,a)}\right)e^{(5\,bx)}-\left(3\,bxe^{a}-e^{a}\right)e^{(bx)}}{b^2e^{(6\,bx+6\,a)}-b^2e^{(4\,bx+4\,a)}-b^2e^{(2\,bx+2\,a)}+b^2}$$
$$+\frac{3\left(bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)\right)}{2\,b^2}$$
$$-\frac{3\left(bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)\right)}{2\,b^2}+\frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^2}$$

input
```
integrate(x*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
(2*b*x*e^(3*b*x + 3*a) - (3*b*x*e^(5*a) + e^(5*a))*e^(5*b*x) - (3*b*x*e^a
- e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) - b^2*e^(4*b*x + 4*a) - b^2*e^(2*b*x
+ 2*a) + b^2) + 3/2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 -
 3/2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2 + 2*arctan(e^(b*
x + a))/b^2
```

3.517.8 Giac [F]

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^2\,\,dx$$

input
```
integrate(x*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

3.517.9 Mupad [F(-1)]

Timed out.

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int \frac{x}{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^3}\,dx$$

input
```
int(x/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

output
```
int(x/(cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

3.518 $\int \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$

3.518.1 Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \frac{3\text{arctanh}(\cosh(a+bx))}{2b} - \frac{3\text{sech}(a+bx)}{2b} - \frac{\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

output
```
3/2*arctanh(cosh(b*x+a))/b-3/2*sech(b*x+a)/b-1/2*csch(b*x+a)^2*sech(b*x+a)
/b
```

3.518.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\text{sech}(a+bx)}{b}$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
-1/8*Csch[(a + b*x)/2]^2/b + (3*Log[Cosh[(a + b*x)/2]])/(2*b) - (3*Log[Sin
h[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b) - Sech[a + b*x]/b
```

3.518.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 3102, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$-\frac{\int \frac{\text{sech}^4(a+bx)}{\left(1-\text{sech}^2(a+bx)\right)^2}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\text{sech}^3(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{3}{2}\int \frac{\text{sech}^2(a+bx)}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{\text{sech}^3(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{3}{2}\left(\int \frac{1}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)-\text{sech}(a+bx)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\text{sech}^3(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{3}{2}(\text{arctanh}(\text{sech}(a+bx))-\text{sech}(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
-(((-3*(ArcTanh[Sech[a + b*x]] - Sech[a + b*x]))/2 + Sech[a + b*x]^3/(2*(1
 - Sech[a + b*x]^2)))/b)
```

3.518.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.518.4 Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)}-\frac{3}{2\cosh(bx+a)}+3\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	43
default	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)}-\frac{3}{2\cosh(bx+a)}+3\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	43
risch	$-\frac{\mathrm{e}^{bx+a}\left(3\,\mathrm{e}^{4bx+4a}-2\,\mathrm{e}^{2bx+2a}+3\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^2\left(1+\mathrm{e}^{2bx+2a}\right)}-\frac{3\ln\left(\mathrm{e}^{bx+a}-1\right)}{2b}+\frac{3\ln\left(\mathrm{e}^{bx+a}+1\right)}{2b}$	91

input
```
int(csch(b*x+a)^3*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)-3/2/cosh(b*x+a)+3*arctanh(exp(b*x+a)))
```

3.518.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(43) = 86$.

Time = 0.26 (sec) , antiderivative size = 709, normalized size of antiderivative = 14.47

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx =$$

$$-\frac{6\cosh(bx+a)^5+30\cosh(bx+a)\sinh(bx+a)^4+6\sinh(bx+a)^5+4\left(15\cosh(bx+a)^2-1\right)\sinh(}{}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x +
a)^5 + 4*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 + 12
*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6
+ 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x +
 a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b
*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) +
3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (
15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x
+ a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x
 + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + si
nh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 - 2*cosh(b*x + a)^2 + 1)*sinh(b*x
+ a) + 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x +
a)^5 + b*sinh(b*x + a)^6 - b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - b)*
sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^
3 - b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 - b)*s
inh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 - b*cosh(b*x
 + a))*sinh(b*x + a) + b)
```

3.518.6 Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**2, x)
```

3.518.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(43) = 86$.

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.16

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\log\left(e^{(-bx-a)}+1\right)}{2b} - \frac{3\log\left(e^{(-bx-a)}-1\right)}{2b} + \frac{3e^{(-bx-a)} - 2e^{(-3bx-3a)} + 3e^{(-5bx-5a)}}{b\left(e^{(-2bx-2a)} + e^{(-4bx-4a)} - e^{(-6bx-6a)} - 1\right)}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
3/2*log(e^(-b*x - a) + 1)/b - 3/2*log(e^(-b*x - a) - 1)/b + (3*e^(-b*x - a
) - 2*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(e^(-2*b*x - 2*a) + e^(-4*
b*x - 4*a) - e^(-6*b*x - 6*a) - 1))
```

3.518.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(43) = 86$.

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.24

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-8\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-4e^{(bx+a)}-4e^{(-bx-a)}} - 3\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + 3\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*(4*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 8)/((e^(b*x + a) + e^(-b*x - a
))^3 - 4*e^(b*x + a) - 4*e^(-b*x - a)) - 3*log(e^(b*x + a) + e^(-b*x - a)
+ 2) + 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

3.518.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.27

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

output
```
(3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x))
/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*
a + 2*b*x) - 1)) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.519 $\int \frac{\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{x} dx$

3.519.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{x} dx = \text{Int}\left(\frac{\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{x}, x\right)$$

output
```
CannotIntegrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x)
```

3.519.2 Mathematica [N/A]

Not integrable

Time = 69.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{x} dx = \int \frac{\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{x} dx$$

input
```
Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x,x]
```

output
```
Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x, x]
```

3.519.3 Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x} \, dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}dx$$

input
```
Int[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x,x]
```

output
```
$Aborted
```

3.519.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.519.4 Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{3} \operatorname{sech}\left(bx+a\right)^{2}}{x}dx$$

input
```
int(csch(b*x+a)^3*sech(b*x+a)^2/x,x)
```

output
```
int(csch(b*x+a)^3*sech(b*x+a)^2/x,x)
```

3.519.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^3*sech(b*x + a)^2/x, x)
```

3.519.6 Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^3\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)}{x}\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**2/x,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**2/x, x)
```

3.519.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 226, normalized size of antiderivative = 11.30

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
(2*b*x*e^(3*b*x + 3*a) - (3*b*x*e^(5*a) - e^(5*a))*e^(5*b*x) - (3*b*x*e^a
+ e^a)*e^(b*x))/(b^2*x^2*e^(6*b*x + 6*a) - b^2*x^2*e^(4*b*x + 4*a) - b^2*x
^2*e^(2*b*x + 2*a) + b^2*x^2) - 32*integrate(1/64*(3*b^2*x^2 - 2)/(b^2*x^3
*e^(b*x + a) + b^2*x^3), x) - 32*integrate(1/64*(3*b^2*x^2 - 2)/(b^2*x^3*e
^(b*x + a) - b^2*x^3), x) - 32*integrate(1/16*e^(b*x + a)/(b*x^2*e^(2*b*x
+ 2*a) + b*x^2), x)
```

3.519.8 Giac [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^3*sech(b*x + a)^2/x, x)
```

3.519.9 Mupad [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{1}{x\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

output
```
int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

3.520 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$

3.520.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2},x\right)$$

output
```
CannotIntegrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x)
```

3.520.2 Mathematica [N/A]

Not integrable

Time = 43.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2,x]
```

output
```
Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2, x]
```

3.520.3 Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2} \, dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}dx$$

input
```
Int[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2,x]
```

output
```
$Aborted
```

3.520.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.520.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2}{x^2}dx$$

input
```
int(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x)
```

output
```
int(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x)
```

3.520.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^3*sech(b*x + a)^2/x^2, x)
```

3.520.6 Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^3\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**2/x**2,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**2/x**2, x)
```

3.520.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 226, normalized size of antiderivative = 11.30

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
(2*b*x*e^(3*b*x + 3*a) - (3*b*x*e^(5*a) - 2*e^(5*a))*e^(5*b*x) - (3*b*x*e^
a + 2*e^a)*e^(b*x))/(b^2*x^3*e^(6*b*x + 6*a) - b^2*x^3*e^(4*b*x + 4*a) - b
^2*x^3*e^(2*b*x + 2*a) + b^2*x^3) - 32*integrate(3/64*(b^2*x^2 - 2)/(b^2*x
^4*e^(b*x + a) + b^2*x^4), x) - 32*integrate(3/64*(b^2*x^2 - 2)/(b^2*x^4*e
^(b*x + a) - b^2*x^4), x) - 32*integrate(1/8*e^(b*x + a)/(b*x^3*e^(2*b*x +
 2*a) + b*x^3), x)
```

3.520.8 Giac [N/A]

Not integrable

Time = 20.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.520.9 Mupad [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

output
```
int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

3.521 $\int x^m \mathbf{csch}^3(a+bx)\mathbf{sech}^3(a+bx)\,dx$

3.521.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx),x\right)$$

output
```
CannotIntegrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

3.521.2 Mathematica [N/A]

Not integrable

Time = 167.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^3, x]
```

3.521.3 Rubi [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int x^m \text{csch}^3(a+bx)\text{sech}^3(a+bx)dx$$

input
```
Int[x^m*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
$Aborted
```

3.521.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.521.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3\,dx$$

input
```
int(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

output
```
int(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

3.521.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)^3\,dx$$

input
```
integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

3.521.6 Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}^3\left(a+bx\right) \operatorname{sech}^3\left(a+bx\right)dx$$

input
```
integrate(x**m*csch(b*x+a)**3*sech(b*x+a)**3,x)
```

output
```
Integral(x**m*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

3.521.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)^3\,dx$$

input
```
integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

3.521.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m \,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^3\;dx$$

input
```
integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

3.521.9 Mupad [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int \frac{x^m}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

output
```
int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

3.522 $\int x^3 \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$

3.522.1 Optimal result

Integrand size = 20, antiderivative size = 240

$$\int x^3 \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{6x\text{arctanh}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\text{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a+2bx)\text{csch}(2a+2bx)}{b} - \frac{3\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^4} + \frac{3x^2\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} + \frac{3\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^4} - \frac{3x^2\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} - \frac{3x\,\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3} + \frac{3x\,\text{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3} + \frac{3\,\text{PolyLog}\left(4,-e^{2a+2bx}\right)}{2b^4} - \frac{3\,\text{PolyLog}\left(4,e^{2a+2bx}\right)}{2b^4}$$

output
```
-6*x*arctanh(exp(2*b*x+2*a))/b^3+4*x^3*arctanh(exp(2*b*x+2*a))/b-3*x^2*csc
h(2*b*x+2*a)/b^2-2*x^3*coth(2*b*x+2*a)*csch(2*b*x+2*a)/b-3/2*polylog(2,-ex
p(2*b*x+2*a))/b^4+3*x^2*polylog(2,-exp(2*b*x+2*a))/b^2+3/2*polylog(2,exp(2
*b*x+2*a))/b^4-3*x^2*polylog(2,exp(2*b*x+2*a))/b^2-3*x*polylog(3,-exp(2*b*
x+2*a))/b^3+3*x*polylog(3,exp(2*b*x+2*a))/b^3+3/2*polylog(4,-exp(2*b*x+2*a
))/b^4-3/2*polylog(4,exp(2*b*x+2*a))/b^4
```

3.522.2 Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.14

$$\int x^3\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$$
$$= \frac{-b^3x^3\text{csch}^2(a+bx)+6bx\log\left(1-e^{2(a+bx)}\right)-4b^3x^3\log\left(1-e^{2(a+bx)}\right)-6bx\log\left(1+e^{2(a+bx)}\right)+4b^3x^3\log}{}$$

input
```
Integrate[x^3*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
(-(b^3*x^3*Csch[a + b*x]^2) + 6*b*x*Log[1 - E^(2*(a + b*x))] - 4*b^3*x^3*L
og[1 - E^(2*(a + b*x))] - 6*b*x*Log[1 + E^(2*(a + b*x))] + 4*b^3*x^3*Log[1
 + E^(2*(a + b*x))] + (-3 + 6*b^2*x^2)*PolyLog[2, -E^(2*(a + b*x))] + (3 -
 6*b^2*x^2)*PolyLog[2, E^(2*(a + b*x))] - 6*b*x*PolyLog[3, -E^(2*(a + b*x)
)] + 6*b*x*PolyLog[3, E^(2*(a + b*x))] + 3*PolyLog[4, -E^(2*(a + b*x))] -
3*PolyLog[4, E^(2*(a + b*x))] - 3*b^2*x^2*Csch[a]*Sech[a] - b^3*x^3*Sech[a
 + b*x]^2 + 3*b^2*x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x] + 3*b^2*x^2*Sech[a]*
Sech[a + b*x]*Sinh[b*x])/(2*b^4)
```

3.522.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5984, 3042, 26, 4674, 26, 3042, 26, 4670, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$$
$$\Big\downarrow \text{5984}$$
$$8\int x^3\text{csch}^3(2a+2bx)dx$$
$$\Big\downarrow \text{3042}$$
$$8\int -ix^3\csc(2ia+2ibx)^3dx$$
$$\Big\downarrow \text{26}$$

$$-8i\int x^3\csc(2ia+2ibx)^3dx$$

$\Big\downarrow$ 4674

$$-8i\left(-\frac{3\int -ix\text{csch}(2a+2bx)dx}{4b^2}+\frac{1}{2}\int -ix^3\text{csch}(2a+2bx)dx-\frac{3ix^2\text{csch}(2a+2bx)}{8b^2}-\frac{ix^3\coth(2a+2bx)\text{csch}(2\right.$$

$\Big\downarrow$ 26

$$-8i\left(\frac{3i\int x\text{csch}(2a+2bx)dx}{4b^2}-\frac{1}{2}i\int x^3\text{csch}(2a+2bx)dx-\frac{3ix^2\text{csch}(2a+2bx)}{8b^2}-\frac{ix^3\coth(2a+2bx)\text{csch}(2a+2\right.$$

$\Big\downarrow$ 3042

$$-8i\left(\frac{3i\int ix\csc(2ia+2ibx)dx}{4b^2}-\frac{1}{2}i\int ix^3\csc(2ia+2ibx)dx-\frac{3ix^2\text{csch}(2a+2bx)}{8b^2}-\frac{ix^3\coth(2a+2bx)\text{csch}(2a\right.$$

$\Big\downarrow$ 26

$$-8i\left(-\frac{3\int x\csc(2ia+2ibx)dx}{4b^2}+\frac{1}{2}\int x^3\csc(2ia+2ibx)dx-\frac{3ix^2\text{csch}(2a+2bx)}{8b^2}-\frac{ix^3\coth(2a+2bx)\text{csch}(2a+\right.$$

$\Big\downarrow$ 4670

$$-8i\left(-\frac{3\left(\frac{i\int\log\left(1-e^{2a+2bx}\right)dx}{2b}-\frac{i\int\log\left(1+e^{2a+2bx}\right)dx}{2b}+\frac{ix\text{arctanh}\left(e^{2a+2bx}\right)}{b}\right)}{4b^2}+\frac{1}{2}\left(\frac{3i\int x^2\log\left(1-e^{2a+2bx}\right)dx}{2b}-\frac{3i\int x\right.\right.$$

$\Big\downarrow$ 2715

$$-8i\left(-\frac{3\left(\frac{i\int e^{-2a-2bx}\log\left(1-e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}-\frac{i\int e^{-2a-2bx}\log\left(1+e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}+\frac{ix\text{arctanh}\left(e^{2a+2bx}\right)}{b}\right)}{4b^2}+\frac{1}{2}\left(\frac{3i\int x^2\log\right.\right.$$

$\Big\downarrow$ 2838

$$-8i\left(\frac{1}{2}\left(\frac{3i\int x^2\log\left(1-e^{2a+2bx}\right)dx}{2b}-\frac{3i\int x^2\log\left(1+e^{2a+2bx}\right)dx}{2b}+\frac{ix^3\text{arctanh}\left(e^{2a+2bx}\right)}{b}\right)-\frac{3\left(\frac{ix\text{arctanh}\left(e^{2a+2}\right.}{b}\right.}{}\right.$$

$\Big\downarrow$ 3011

3.522. $\int x^3\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$

$$-8i\left(\frac{1}{2}\left(-\frac{3i\left(\frac{\int x\,\text{PolyLog}(2,-e^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2a+2bx})}{2b}\right)}{2b}+\frac{3i\left(\frac{\int x\,\text{PolyLog}(2,e^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,e^{2a+2bx})}{2b}\right)}{2b}\right.\right.$$

$$\Big\downarrow \text{7163}$$

$$-8i\left(\frac{1}{2}\left(-\frac{3i\left(\frac{\frac{x\,\text{PolyLog}(3,-e^{2a+2bx})}{2b}-\frac{\int \text{PolyLog}(3,-e^{2a+2bx})dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2a+2bx})}{2b}\right)}{2b}+\frac{3i\left(\frac{\frac{x\,\text{PolyLog}(3,e^{2a+2bx})}{2b}-\frac{\int \text{PolyLog}(3}{}}{b}\right.}{}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$-8i\left(\frac{1}{2}\left(-\frac{3i\left(\frac{\frac{x\,\text{PolyLog}(3,-e^{2a+2bx})}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}(3,-e^{2a+2bx})de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2a+2bx})}{2b}\right)}{2b}+\frac{3i\left(\frac{\frac{x\,\text{PolyLog}(3,e^{2a+2}}{2b}}{}\right.}{}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-8i\left(\frac{1}{2}\left(\frac{ix^3\text{arctanh}(e^{2a+2bx})}{b}-\frac{3i\left(\frac{\frac{x\,\text{PolyLog}(3,-e^{2a+2bx})}{2b}-\frac{\text{PolyLog}(4,-e^{2a+2bx})}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2a+2bx})}{2b}\right)}{2b}+\frac{3i\left(\frac{x\,\text{PolyLog}(}{}\right.}{}\right.\right.$$

input
```
Int[x^3*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
(-8*I)*((((-3*I)/8)*x^2*Csch[2*a + 2*b*x])/b^2 - ((I/4)*x^3*Coth[2*a + 2*b
*x]*Csch[2*a + 2*b*x])/b - (3*((I*x*ArcTanh[E^(2*a + 2*b*x)])/b + ((I/4)*P
olyLog[2, -E^(2*a + 2*b*x)])/b^2 - ((I/4)*PolyLog[2, E^(2*a + 2*b*x)])/b^2
))/(4*b^2) + ((I*x^3*ArcTanh[E^(2*a + 2*b*x)])/b - (((3*I)/2)*(-1/2*(x^2*P
olyLog[2, -E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, -E^(2*a + 2*b*x)])/(2*b) -
 PolyLog[4, -E^(2*a + 2*b*x)]/(4*b^2))/b))/b + (((3*I)/2)*(-1/2*(x^2*PolyL
og[2, E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, E^(2*a + 2*b*x)])/(2*b) - PolyL
og[4, E^(2*a + 2*b*x)]/(4*b^2))/b))/b)/2)
```

3.522. $\int x^3\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$

3.522.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.522.4 Maple [A] (verified)

Time = 29.30 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.85

method	result
risch	$-\frac{2x^2 e^{2bx+2a}\left(2\,e^{4bx+4a}bx+3\,e^{4bx+4a}+2bx-3\right)}{b^2\left(e^{2bx+2a}-1\right)^2\left(1+e^{2bx+2a}\right)^2}-\frac{12\,\mathrm{polylog}\left(4,e^{bx+a}\right)}{b^4}+\frac{3\,\mathrm{polylog}\left(4,-e^{2bx+2a}\right)}{2b^4}-\frac{12\,\mathrm{polylog}\left(4,-e^{bx+a}\right)}{b^4}-$

input
```
int(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*x^2*exp(2*b*x+2*a)*(2*exp(4*b*x+4*a)*b*x+3*exp(4*b*x+4*a)+2*b*x-3)/b^2/
(exp(2*b*x+2*a)-1)^2/(1+exp(2*b*x+2*a))^2-12*polylog(4,exp(b*x+a))/b^4+3/2
*polylog(4,-exp(2*b*x+2*a))/b^4-12*polylog(4,-exp(b*x+a))/b^4-2/b^4*ln(1-e
xp(b*x+a))*a^3+3/b^4*ln(1-exp(b*x+a))*a+3*polylog(2,exp(b*x+a))/b^4-3/2*po
lylog(2,-exp(2*b*x+2*a))/b^4+3*polylog(2,-exp(b*x+a))/b^4-3*x*ln(1+exp(2*b
*x+2*a))/b^3+3/b^3*ln(1-exp(b*x+a))*x+3/b^3*ln(exp(b*x+a)+1)*x-2/b*ln(1-ex
p(b*x+a))*x^3-6*x^2*polylog(2,exp(b*x+a))/b^2+12*x*polylog(3,exp(b*x+a))/b
^3+2*x^3*ln(1+exp(2*b*x+2*a))/b+3*x^2*polylog(2,-exp(2*b*x+2*a))/b^2-3*x*p
olylog(3,-exp(2*b*x+2*a))/b^3-2/b*ln(exp(b*x+a)+1)*x^3-6*x^2*polylog(2,-ex
p(b*x+a))/b^2+12*x*polylog(3,-exp(b*x+a))/b^3-3/b^4*a*ln(exp(b*x+a)-1)+2/b
^4*a^3*ln(exp(b*x+a)-1)
```

3.522.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 6764, normalized size of antiderivative = 28.18

$$\int x^3 \operatorname{csch}^3(a+bx) \operatorname{sech}^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.522.6 Sympy [F]

$$\int x^3 \operatorname{csch}^3(a+bx) \operatorname{sech}^3(a+bx)\, dx = \int x^3 \operatorname{csch}^3(a+bx) \operatorname{sech}^3(a+bx)\, dx$$

input
```
integrate(x**3*csch(b*x+a)**3*sech(b*x+a)**3,x)
```

output
```
Integral(x**3*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

3.522.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.59

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= -\frac{2\left(\left(2\,bx^3e^{(6\,a)}+3\,x^2e^{(6\,a)}\right)e^{(6\,bx)}+\left(2\,bx^3e^{(2\,a)}-3\,x^2e^{(2\,a)}\right)e^{(2\,bx)}\right)}{b^2e^{(8\,bx+8\,a)}-2\,b^2e^{(4\,bx+4\,a)}+b^2}$$
$$+\frac{2\left(4\,b^3x^3\log\left(e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(-e^{(2\,bx+2\,a)})+3\,\mathrm{Li}_4(-e^{(2\,bx+2\,a)})\right)}{3\,b^4}$$
$$-\frac{2\left(b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})\right)}{b^4}$$
$$-\frac{2\left(b^3x^3\log\left(-e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(e^{(bx+a)})+6\,\mathrm{Li}_4(e^{(bx+a)})\right)}{b^4}$$
$$-\frac{3\left(2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)\right)}{2\,b^4}$$
$$+\frac{3\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4}+\frac{3\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^4}$$

input
```
integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2*((2*b*x^3*e^(6*a) + 3*x^2*e^(6*a))*e^(6*b*x) + (2*b*x^3*e^(2*a) - 3*x^2
*e^(2*a))*e^(2*b*x))/(b^2*e^(8*b*x + 8*a) - 2*b^2*e^(4*b*x + 4*a) + b^2) +
 2/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a
)) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/
b^4 - 2*(b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*
b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 - 2*(b^3*x^
3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3,
e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4 - 3/2*(2*b*x*log(e^(2*b*x +
2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + d
ilog(-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)
))/b^4
```

3.522.8 Giac [F]

$$\int x^3\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx=\int x^3\,\text{csch}\left(bx+a\right)^3\text{sech}\left(bx+a\right)^3\,dx$$

input
```
integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

3.522.9 Mupad [F(-1)]

Timed out.

$$\int x^3\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx=\int \frac{x^3}{\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(x^3/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

output
```
int(x^3/(cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

3.523 $\int x^2 \mathrm{csch}^3(a+bx)\mathrm{sech}^3(a+bx)\,dx$

3.523.1 Optimal result

Integrand size = 20, antiderivative size = 149

$$\int x^2\mathrm{csch}^3(a+bx)\mathrm{sech}^3(a+bx)\,dx = \frac{4x^2\mathrm{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{\mathrm{arctanh}(\cosh(2a+2bx))}{b^3} - \frac{2x\mathrm{csch}(2a+2bx)}{b^2} - \frac{2x^2\coth(2a+2bx)\mathrm{csch}(2a+2bx)}{b} + \frac{2x\,\mathrm{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} - \frac{2x\,\mathrm{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} - \frac{\mathrm{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3} + \frac{\mathrm{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3}$$

output
```
4*x^2*arctanh(exp(2*b*x+2*a))/b-arctanh(cosh(2*b*x+2*a))/b^3-2*x*csch(2*b*
x+2*a)/b^2-2*x^2*coth(2*b*x+2*a)*csch(2*b*x+2*a)/b+2*x*polylog(2,-exp(2*b*
x+2*a))/b^2-2*x*polylog(2,exp(2*b*x+2*a))/b^2-polylog(3,-exp(2*b*x+2*a))/b
^3+polylog(3,exp(2*b*x+2*a))/b^3
```

3.523.2 Mathematica [A] (verified)

Time = 3.06 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.29

$$\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{4\text{arctanh}\left(e^{2(a+bx)}\right)+b^2x^2\text{csch}^2(a+bx)+4b^2x^2\log\left(1-e^{2(a+bx)}\right)-4b^2x^2\log\left(1+e^{2(a+bx)}\right)-4bx\,\text{Poly}$$

input
```
Integrate[x^2*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
-1/2*(4*ArcTanh[E^(2*(a + b*x))] + b^2*x^2*Csch[a + b*x]^2 + 4*b^2*x^2*Log
[1 - E^(2*(a + b*x))] - 4*b^2*x^2*Log[1 + E^(2*(a + b*x))] - 4*b*x*PolyLog
[2, -E^(2*(a + b*x))] + 4*b*x*PolyLog[2, E^(2*(a + b*x))] + 2*PolyLog[3, -
E^(2*(a + b*x))] - 2*PolyLog[3, E^(2*(a + b*x))] + 2*b*x*Csch[a]*Sech[a] +
 b^2*x^2*Sech[a + b*x]^2 - 2*b*x*Csch[a]*Csch[a + b*x]*Sinh[b*x] - 2*b*x*S
ech[a]*Sech[a + b*x]*Sinh[b*x])/b^3
```

3.523.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.32, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5984, 3042, 26, 4674, 26, 3042, 26, 4257, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \ 5984$$

$$8\int x^2\text{csch}^3(2a+2bx)dx$$

$$\Big\downarrow \ 3042$$

$$8\int -ix^2\csc(2ia+2ibx)^3dx$$

$$\Big\downarrow \ 26$$

$$-8i\int x^2\csc(2ia+2ibx)^3dx$$

$$\Big\downarrow \text{4674}$$

$$-8i\left(-\frac{\int -i\text{csch}(2a+2bx)dx}{4b^2}+\frac{1}{2}\int -ix^2\text{csch}(2a+2bx)dx-\frac{ix\text{csch}(2a+2bx)}{4b^2}-\frac{ix^2\coth(2a+2bx)\text{csch}(2a+2}{4b}\right.$$

$$\Big\downarrow \text{26}$$

$$-8i\left(\frac{i\int \text{csch}(2a+2bx)dx}{4b^2}-\frac{1}{2}i\int x^2\text{csch}(2a+2bx)dx-\frac{ix\text{csch}(2a+2bx)}{4b^2}-\frac{ix^2\coth(2a+2bx)\text{csch}(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-8i\left(\frac{i\int i\csc(2ia+2ibx)dx}{4b^2}-\frac{1}{2}i\int ix^2\csc(2ia+2ibx)dx-\frac{ix\text{csch}(2a+2bx)}{4b^2}-\frac{ix^2\coth(2a+2bx)\text{csch}(2a+2bx}{4b}\right.$$

$$\Big\downarrow \text{26}$$

$$-8i\left(-\frac{\int \csc(2ia+2ibx)dx}{4b^2}+\frac{1}{2}\int x^2\csc(2ia+2ibx)dx-\frac{ix\text{csch}(2a+2bx)}{4b^2}-\frac{ix^2\coth(2a+2bx)\text{csch}(2a+2bx)}{4b}\right.$$

$$\Big\downarrow \text{4257}$$

$$-8i\left(\frac{1}{2}\int x^2\csc(2ia+2ibx)dx-\frac{i\text{arctanh}(\cosh(2a+2bx))}{8b^3}-\frac{ix\text{csch}(2a+2bx)}{4b^2}-\frac{ix^2\coth(2a+2bx)\text{csch}(2a+2}{4b}\right.$$

$$\Big\downarrow \text{4670}$$

$$-8i\left(\frac{1}{2}\left(\frac{i\int x\log\left(1-e^{2a+2bx}\right)dx}{b}-\frac{i\int x\log\left(1+e^{2a+2bx}\right)dx}{b}+\frac{ix^2\text{arctanh}\left(e^{2a+2bx}\right)}{b}\right)-\frac{i\text{arctanh}(\cosh(2a+2b}{8b^3}\right.$$

$$\Big\downarrow \text{3011}$$

$$-8i\left(\frac{1}{2}\left(-\frac{i\left(\frac{\int \text{PolyLog}(2,-e^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,-e^{2a+2bx})}{2b}\right)}{b}+\frac{i\left(\frac{\int \text{PolyLog}(2,e^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,e^{2a+2bx})}{2b}\right)}{b}+\frac{ix^2\text{a}}{}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$-8i\left(\frac{1}{2}\left(-\frac{i\left(\frac{\int e^{-2a-2bx}\,\text{PolyLog}(2,-e^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}(2,-e^{2a+2bx})}{2b}\right)}{b}+\frac{i\left(\frac{\int e^{-2a-2bx}\,\text{PolyLog}(2,e^{2a+2bx})de^{2a+2bx}}{4b^2}-\right.}{b}\right.\right.$$

$$\Big\downarrow \text{7143}$$

3.523. $\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$

$$-8i\left(-\frac{i\text{arctanh}(\cosh(2a+2bx))}{8b^3}+\frac{1}{2}\left(\frac{ix^2\text{arctanh}(e^{2a+2bx})}{b}-\frac{i\left(\frac{\text{PolyLog}(3,-e^{2a+2bx})}{4b^2}-\frac{x\,\text{PolyLog}(2,-e^{2a+2bx})}{2b}\right)}{b}+\frac{i(\right.\right.$$

input
```
Int[x^2*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
(-8*I)*(((-1/8*I)*ArcTanh[Cosh[2*a + 2*b*x]])/b^3 - ((I/4)*x*Csch[2*a + 2*
b*x])/b^2 - ((I/4)*x^2*Coth[2*a + 2*b*x]*Csch[2*a + 2*b*x])/b + ((I*x^2*Ar
cTanh[E^(2*a + 2*b*x)])/b - (I*(-1/2*(x*PolyLog[2, -E^(2*a + 2*b*x)])/b +
PolyLog[3, -E^(2*a + 2*b*x)]/(4*b^2)))/b + (I*(-1/2*(x*PolyLog[2, E^(2*a +
 2*b*x)])/b + PolyLog[3, E^(2*a + 2*b*x)]/(4*b^2)))/b)/2)
```

3.523.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.523. $\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.523.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(144) = 288$.

Time = 21.85 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.01

method	result
risch	$-\frac{4x\,e^{2bx+2a}\left(e^{4bx+4a}bx+e^{4bx+4a}+bx-1\right)}{b^{2}\left(e^{2bx+2a}-1\right)^{2}\left(1+e^{2bx+2a}\right)^{2}}-\frac{2a^{2}\ln\left(e^{bx+a}-1\right)}{b^{3}}-\frac{2\ln\left(1-e^{bx+a}\right)x^{2}}{b}-\frac{4x\,\mathrm{polylog}\left(2,e^{bx+a}\right)}{b^{2}}+\frac{2x^{2}\ln\left(1+e^{2bx+\right.}}{b}$

input
```
int(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-4*x*exp(2*b*x+2*a)*(exp(4*b*x+4*a)*b*x+exp(4*b*x+4*a)+b*x-1)/b^2/(exp(2*b
*x+2*a)-1)^2/(1+exp(2*b*x+2*a))^2-2/b^3*a^2*ln(exp(b*x+a)-1)-2/b*ln(1-exp(
b*x+a))*x^2-4*x*polylog(2,exp(b*x+a))/b^2+2*x^2*ln(1+exp(2*b*x+2*a))/b+2*x
*polylog(2,-exp(2*b*x+2*a))/b^2-2/b*ln(exp(b*x+a)+1)*x^2-4*x*polylog(2,-ex
p(b*x+a))/b^2+4*polylog(3,exp(b*x+a))/b^3-polylog(3,-exp(2*b*x+2*a))/b^3+4
*polylog(3,-exp(b*x+a))/b^3+2/b^3*ln(1-exp(b*x+a))*a^2+1/b^3*ln(exp(b*x+a)
-1)-1/b^3*ln(1+exp(2*b*x+2*a))+1/b^3*ln(exp(b*x+a)+1)
```

3.523.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 4779, normalized size of antiderivative = 32.07

$$\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(4*(b^2*x^2 + b*x)*cosh(b*x + a)^6 + 80*(b^2*x^2 + b*x)*cosh(b*x + a)^3*s
inh(b*x + a)^3 + 60*(b^2*x^2 + b*x)*cosh(b*x + a)^2*sinh(b*x + a)^4 + 24*(
b^2*x^2 + b*x)*cosh(b*x + a)*sinh(b*x + a)^5 + 4*(b^2*x^2 + b*x)*sinh(b*x
+ a)^6 + 4*(b^2*x^2 - b*x)*cosh(b*x + a)^2 + 4*(15*(b^2*x^2 + b*x)*cosh(b*
x + a)^4 + b^2*x^2 - b*x)*sinh(b*x + a)^2 + 4*(b*x*cosh(b*x + a)^8 + 56*b*
x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6
 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(
b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*c
osh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x +
a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)
^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x +
a)) - 4*(b*x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28
*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7
 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4
 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*si
nh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x
 + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a
))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 4*(b*x*cosh(b*x + a)^8 + 56*
b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)
^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*...
```

3.523.6 Sympy [F]

$$\int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(x**2*csch(b*x+a)**3*sech(b*x+a)**3,x)
```

output
```
Integral(x**2*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

3.523.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.83

$$\begin{aligned}
&\int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx \\
&= -\frac{4\left(\left(bx^2e^{(6a)}+xe^{(6a)}\right)e^{(6bx)}+\left(bx^2e^{(2a)}-xe^{(2a)}\right)e^{(2bx)}\right)}{b^2e^{(8bx+8a)}-2b^2e^{(4bx+4a)}+b^2} \\
&\quad+\frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right)+2bx\operatorname{Li}_2\left(-e^{(2bx+2a)}\right)-\operatorname{Li}_3\left(-e^{(2bx+2a)}\right)}{b^3} \\
&\quad-\frac{2\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2bx\operatorname{Li}_2\left(-e^{(bx+a)}\right)-2\operatorname{Li}_3\left(-e^{(bx+a)}\right)\right)}{b^3} \\
&\quad-\frac{2\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2bx\operatorname{Li}_2\left(e^{(bx+a)}\right)-2\operatorname{Li}_3\left(e^{(bx+a)}\right)\right)}{b^3} \\
&\quad-\frac{\log\left(e^{(2bx+2a)}+1\right)}{b^3}+\frac{\log\left(e^{(bx+a)}+1\right)}{b^3}+\frac{\log\left(e^{(bx+a)}-1\right)}{b^3}
\end{aligned}$$

input
```
integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-4*((b*x^2*e^(6*a) + x*e^(6*a))*e^(6*b*x) + (b*x^2*e^(2*a) - x*e^(2*a))*e^
(2*b*x))/(b^2*e^(8*b*x + 8*a) - 2*b^2*e^(4*b*x + 4*a) + b^2) + (2*b^2*x^2*
log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(
2*b*x + 2*a)))/b^3 - 2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x
 + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - 2*(b^2*x^2*log(-e^(b*x + a) + 1
) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3 - log(e^(2*b
*x + 2*a) + 1)/b^3 + log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3
```

3.523.8 Giac [F]

$$\int x^2 \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int x^2\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^3\,dx$$

input
```
integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

3.523.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int \frac{x^2}{\cosh\,(a+b\,x)^3\,\sinh\,(a+b\,x)^3}\,dx$$

input
```
int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

output
```
int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

3.524 $\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$

3.524.1 Optimal result

Integrand size = 18, antiderivative size = 91

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx=\frac{4x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{csch}(2a+2bx)}{b^2}-\frac{2x\coth(2a+2bx)\text{csch}(2a+2bx)}{b}+\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}$$

output
```
4*x*arctanh(exp(2*b*x+2*a))/b-csch(2*b*x+2*a)/b^2-2*x*coth(2*b*x+2*a)*csch
(2*b*x+2*a)/b+polylog(2,-exp(2*b*x+2*a))/b^2-polylog(2,exp(2*b*x+2*a))/b^2
```

3.524.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx=-\frac{\coth(a+bx)+bx\text{csch}^2(a+bx)+4bx\log\left(1-e^{2(a+bx)}\right)-4bx\log\left(1+e^{2(a+bx)}\right)-2\,\text{PolyLog}\left(2,-e^{2(a+\cdots\right.}}{2b^2}$$

input
```
Integrate[x*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
-1/2*(Coth[a + b*x] + b*x*Csch[a + b*x]^2 + 4*b*x*Log[1 - E^(2*(a + b*x))]
 - 4*b*x*Log[1 + E^(2*(a + b*x))] - 2*PolyLog[2, -E^(2*(a + b*x))] + 2*Pol
yLog[2, E^(2*(a + b*x))] + b*x*Sech[a + b*x]^2 - Tanh[a + b*x])/b^2
```

3.524.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5984, 3042, 26, 4673, 26, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{5984}$$

$$8\int x\text{csch}^3(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int -ix\csc(2ia+2ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$-8i\int x\csc(2ia+2ibx)^3dx$$

$$\Big\downarrow \text{4673}$$

$$-8i\left(\frac{1}{2}\int -ix\text{csch}(2a+2bx)dx-\frac{i\text{csch}(2a+2bx)}{8b^2}-\frac{ix\coth(2a+2bx)\text{csch}(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{26}$$

$$-8i\left(-\frac{1}{2}i\int x\text{csch}(2a+2bx)dx-\frac{i\text{csch}(2a+2bx)}{8b^2}-\frac{ix\coth(2a+2bx)\text{csch}(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-8i\left(-\frac{1}{2}i\int ix\csc(2ia+2ibx)dx-\frac{i\text{csch}(2a+2bx)}{8b^2}-\frac{ix\coth(2a+2bx)\text{csch}(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{26}$$

$$-8i\left(\frac{1}{2}\int x\csc(2ia+2ibx)dx-\frac{i\text{csch}(2a+2bx)}{8b^2}-\frac{ix\coth(2a+2bx)\text{csch}(2a+2bx)}{4b}\right)$$

$\Big\downarrow$ 4670

$$-8i\left(\frac{1}{2}\left(\frac{i\int\log\left(1-e^{2a+2bx}\right)dx}{2b}-\frac{i\int\log\left(1+e^{2a+2bx}\right)dx}{2b}+\frac{ix\text{arctanh}\left(e^{2a+2bx}\right)}{b}\right)-\frac{i\text{csch}(2a+2bx)}{8b^2}-\frac{ix\coth(}{}\right.$$

$\Big\downarrow$ 2715

$$-8i\left(\frac{1}{2}\left(\frac{i\int e^{-2a-2bx}\log\left(1-e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}-\frac{i\int e^{-2a-2bx}\log\left(1+e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}+\frac{ix\text{arctanh}\left(e^{2a+2bx}\right)}{b}\right)\right.$$

$\Big\downarrow$ 2838

$$-8i\left(\frac{1}{2}\left(\frac{ix\text{arctanh}\left(e^{2a+2bx}\right)}{b}+\frac{i\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(2,e^{2a+2bx}\right)}{4b^2}\right)-\frac{i\text{csch}(2a+2bx)}{8b^2}-\frac{ix\coth(2}{}\right.$$

input
```
Int[x*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
(-8*I)*(((-1/8*I)*Csch[2*a + 2*b*x])/b^2 - ((I/4)*x*Coth[2*a + 2*b*x]*Csch
[2*a + 2*b*x])/b + ((I*x*ArcTanh[E^(2*a + 2*b*x)])/b + ((I/4)*PolyLog[2, -
E^(2*a + 2*b*x)])/b^2 - ((I/4)*PolyLog[2, E^(2*a + 2*b*x)])/b^2)/2)
```

3.524.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.524.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(88) = 176$.

Time = 15.86 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.16

method	result
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}\left(2\,\mathrm{e}^{4bx+4a}bx+\mathrm{e}^{4bx+4a}+2bx-1\right)}{b^2\left(\mathrm{e}^{2bx+2a}-1\right)^2\left(1+\mathrm{e}^{2bx+2a}\right)^2}-\frac{2\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}-\frac{2\ln\left(1-\mathrm{e}^{bx+a}\right)a}{b^2}-\frac{2\,\mathrm{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{b^2}-\frac{2\ln\left(\mathrm{e}^{bx+a}+1\right)x}{b}$

input
```
int(x*csch(b*x+a)^3*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*exp(2*b*x+2*a)*(2*exp(4*b*x+4*a)*b*x+exp(4*b*x+4*a)+2*b*x-1)/b^2/(exp(2
*b*x+2*a)-1)^2/(1+exp(2*b*x+2*a))^2-2/b*ln(1-exp(b*x+a))*x-2/b^2*ln(1-exp(
b*x+a))*a-2*polylog(2,exp(b*x+a))/b^2-2/b*ln(exp(b*x+a)+1)*x-2*polylog(2,-
exp(b*x+a))/b^2+2*x*ln(1+exp(2*b*x+2*a))/b+polylog(2,-exp(2*b*x+2*a))/b^2+
2/b^2*a*ln(exp(b*x+a)-1)
```

3.524.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 3025, normalized size of antiderivative = 33.24

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2*((2*b*x + 1)*cosh(b*x + a)^6 + 20*(2*b*x + 1)*cosh(b*x + a)^3*sinh(b*x
+ a)^3 + 15*(2*b*x + 1)*cosh(b*x + a)^2*sinh(b*x + a)^4 + 6*(2*b*x + 1)*co
sh(b*x + a)*sinh(b*x + a)^5 + (2*b*x + 1)*sinh(b*x + a)^6 + (2*b*x - 1)*co
sh(b*x + a)^2 + (15*(2*b*x + 1)*cosh(b*x + a)^4 + 2*b*x - 1)*sinh(b*x + a)
^2 + (cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x +
 a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8
+ 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*co
sh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*
cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*s
inh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^8
+ 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6
+ 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^
4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b
*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*dilo
g(I*cosh(b*x + a) + I*sinh(b*x + a)) - (cosh(b*x + a)^8 + 56*cosh(b*x + a)
^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*
sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x +
a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x
+ a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*...
```

3.524.6 Sympy [F]

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int x\,\text{csch}^3\,(a+bx)\,\text{sech}^3\,(a+bx)\,dx$$

input
```
integrate(x*csch(b*x+a)**3*sech(b*x+a)**3,x)
```

output
```
Integral(x*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

3.524.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.80

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{2\left(\left(2\,bxe^{(6\,a)}+e^{(6\,a)}\right)e^{(6\,bx)}+\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}\right)}{b^2e^{(8\,bx+8\,a)}-2\,b^2e^{(4\,bx+4\,a)}+b^2} + \frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{b^2} - \frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^2} - \frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)\right)}{b^2}$$

input
```
integrate(x*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2*((2*b*x*e^(6*a) + e^(6*a))*e^(6*b*x) + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b
*x))/(b^2*e^(8*b*x + 8*a) - 2*b^2*e^(4*b*x + 4*a) + b^2) + (2*b*x*log(e^(2
*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^2 - 2*(b*x*log(e^(b*x + a) +
 1) + dilog(-e^(b*x + a)))/b^2 - 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b
*x + a)))/b^2
```

3.524.8 Giac [F]

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int x\,\text{csch}\left(bx+a\right)^3\text{sech}\left(bx+a\right)^3\,dx$$

input
```
integrate(x*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

3.524.9 Mupad [F(-1)]

Timed out.

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx=\int \frac{x}{\cosh{(a+b\,x)}^3\,\sinh{(a+b\,x)}^3}\,dx$$

input
```
int(x/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

output
```
int(x/(cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

3.525 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

3.525.1 Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}+\frac{\tanh^2(a+bx)}{2b}$$

output
```
-1/2*coth(b*x+a)^2/b-2*ln(tanh(b*x+a))/b+1/2*tanh(b*x+a)^2/b
```

3.525.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = 8\left(-\frac{\operatorname{csch}^2(a+bx)}{16b}+\frac{\log(\cosh(a+bx))}{4b}-\frac{\log(\sinh(a+bx))}{4b}-\frac{\operatorname{sech}^2(a+bx)}{16b}\right)$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
8*(-1/16*Csch[a + b*x]^2/b + Log[Cosh[a + b*x]]/(4*b) - Log[Sinh[a + b*x]]
/(4*b) - Sech[a + b*x]^2/(16*b))
```

3.525.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)^3dx$$

$$\Big\downarrow \; 26$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)^3dx$$

$$\Big\downarrow \; 3100$$

$$-\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \; 243$$

$$-\frac{\int -\coth^2(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \; 49$$

$$-\frac{\int \left(-\coth^2(a+bx)-2i\coth(a+bx)+1\right)d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\tanh^2(a+bx)+i\coth(a+bx)+2\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
-1/2*(I*Coth[a + b*x] + 2*Log[-Tanh[a + b*x]^2] - Tanh[a + b*x]^2)/b
```

3.525.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.525.4 Maple [A] (verified)

Time = 11.70 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^2}-\frac{1}{\cosh(bx+a)^2}-2\ln(\tanh(bx+a))}{b}$	43
default	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^2}-\frac{1}{\cosh(bx+a)^2}-2\ln(\tanh(bx+a))}{b}$	43
risch	$-\frac{4\,e^{2bx+2a}\left(e^{4bx+4a}+1\right)}{b\left(e^{2bx+2a}-1\right)^2\left(1+e^{2bx+2a}\right)^2}+\frac{2\ln\left(1+e^{2bx+2a}\right)}{b}-\frac{2\ln\left(e^{2bx+2a}-1\right)}{b}$	87

input
```
int(csch(b*x+a)^3*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^2-1/cosh(b*x+a)^2-2*ln(tanh(b*x+a)))
```

3.525.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 774, normalized size of antiderivative = 18.00

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2*(2*cosh(b*x + a)^6 + 40*cosh(b*x + a)^3*sinh(b*x + a)^3 + 30*cosh(b*x +
 a)^2*sinh(b*x + a)^4 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)
^6 + 2*(15*cosh(b*x + a)^4 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cos
h(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sin
h(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*c
osh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x +
a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x
+ a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x +
 a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x
+ a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x
+ a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*
x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 -
 cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2
)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) +
1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(3*cosh(b*x +
a)^5 + cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^8 + 56*b*cosh(b*x +
a)^3*sinh(b*x + a)^5 + 28*b*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*cosh(b*x
 + a)*sinh(b*x + a)^7 + b*sinh(b*x + a)^8 - 2*b*cosh(b*x + a)^4 + 2*(35*b*
cosh(b*x + a)^4 - b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - b*cosh(b*x
 + a))*sinh(b*x + a)^3 + 4*(7*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*...
```

3.525.6 Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**3, x)
```

3.525.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=-\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{4\left(e^{(-2\,bx-2\,a)}+e^{(-6\,bx-6\,a)}\right)}{b(2\,e^{(-4\,bx-4\,a)}-e^{(-8\,bx-8\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2*log(e^(-b*x - a) + 1)/b - 2*log(e^(-b*x - a) - 1)/b + 2*log(e^(-2*b*x -
 2*a) + 1)/b + 4*(e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/(b*(2*e^(-4*b*x - 4
*a) - e^(-8*b*x - 8*a) - 1))
```

3.525.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.23

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=-\frac{\frac{4\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)}{\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2-4}-\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)+\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
-(4*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/((e^(2*b*x + 2*a) + e^(-2*b*x - 2
*a))^2 - 4) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x +
 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

3.525.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.23

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=\frac{4\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{4\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}-\frac{8\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{8\,a+8\,b\,x}-2\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

output
```
(4*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (4*exp(2*a +
 2*b*x))/(b*(exp(4*a + 4*b*x) - 1)) - (8*exp(2*a + 2*b*x))/(b*(exp(8*a + 8
*b*x) - 2*exp(4*a + 4*b*x) + 1))
```

3.526 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$

3.526.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = 8\mathrm{Int}\left(\frac{\operatorname{csch}^3(2a+2bx)}{x},x\right)$$

output
```
8*Unintegrable(csch(2*b*x+2*a)^3/x,x)
```

3.526.2 Mathematica [N/A]

Not integrable

Time = 51.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

input
```
Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x,x]
```

output
```
Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x, x]
```

3.526.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5984, 3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}dx$$

$$\Big\downarrow \text{5984}$$

$$8\int \frac{\operatorname{csch}^3(2a+2bx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$8\int -\frac{i\csc(2ia+2ibx)^3}{x}dx$$

$$\Big\downarrow \text{26}$$

$$-8i\int \frac{\csc(2ia+2ibx)^3}{x}dx$$

$$\Big\downarrow \text{4680}$$

$$8\int \frac{\operatorname{csch}^3(2a+2bx)}{x}dx$$

input
```
Int[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x,x]
```

output
```
$Aborted
```

3.526.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.526.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3}{x}dx$$

input
```
int(csch(b*x+a)^3*sech(b*x+a)^3/x,x)
```

output
```
int(csch(b*x+a)^3*sech(b*x+a)^3/x,x)
```

3.526.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^3*sech(b*x + a)^3/x, x)
```

3.526.6 Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^3\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)}{x}\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**3/x,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**3/x, x)
```

3.526.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 208, normalized size of antiderivative = 10.40

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
-2*((2*b*x*e^(6*a) - e^(6*a))*e^(6*b*x) + (2*b*x*e^(2*a) + e^(2*a))*e^(2*b
*x))/(b^2*x^2*e^(8*b*x + 8*a) - 2*b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2) - 64*
integrate(1/32*(2*b^2*x^2 - 1)/(b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3), x) + 6
4*integrate(1/64*(2*b^2*x^2 - 1)/(b^2*x^3*e^(b*x + a) + b^2*x^3), x) - 64*
integrate(1/64*(2*b^2*x^2 - 1)/(b^2*x^3*e^(b*x + a) - b^2*x^3), x)
```

3.526.8 Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^3*sech(b*x + a)^3/x, x)
```

3.526.9 Mupad [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{1}{x\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

output
```
int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

3.527 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$

3.527.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = 8\mathrm{Int}\left(\frac{\operatorname{csch}^3(2a+2bx)}{x^2},x\right)$$

output
```
8*Unintegrable(csch(2*b*x+2*a)^3/x^2,x)
```

3.527.2 Mathematica [N/A]

Not integrable

Time = 38.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x^2,x]
```

output
```
Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x^2, x]
```

3.527.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5984, 3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \text{ 5984}$$

$$8 \int \frac{\operatorname{csch}^3(2a+2bx)}{x^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$8 \int -\frac{i\csc(2ia+2ibx)^3}{x^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$-8i \int \frac{\csc(2ia+2ibx)^3}{x^2}dx$$

$$\Big\downarrow \text{ 4680}$$

$$8 \int \frac{\operatorname{csch}^3(2a+2bx)}{x^2}dx$$

input
```
Int[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x^2,x]
```

output
```
$Aborted
```

3.527.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.527.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3}{x^2}dx$$

input
```
int(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x)
```

output
```
int(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x)
```

3.527.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^3*sech(b*x + a)^3/x^2, x)
```

3.527.6 Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**3/x**2,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**3/x**2, x)
```

3.527.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 206, normalized size of antiderivative = 10.30

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
-4*((b*x*e^(6*a) - e^(6*a))*e^(6*b*x) + (b*x*e^(2*a) + e^(2*a))*e^(2*b*x))
/(b^2*x^3*e^(8*b*x + 8*a) - 2*b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3) - 64*inte
grate(1/32*(2*b^2*x^2 - 3)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4), x) + 64*in
tegrate(1/64*(2*b^2*x^2 - 3)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x) - 64*inte
grate(1/64*(2*b^2*x^2 - 3)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

3.527.8 Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^3*sech(b*x + a)^3/x^2, x)
```

3.527.9 Mupad [N/A]

Not integrable

Time = 2.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{1}{x^2\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

output
```
int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

3.528 $\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx$

3.528.1 Optimal result

Integrand size = 18, antiderivative size = 87

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx=\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}+\frac{20i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{147b^2}-\frac{20\sqrt{\cosh(a+bx)}\sinh(a+bx)}{147b^2}-\frac{4\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{49b^2}$$

output
```
2/7*x*cosh(b*x+a)^(7/2)/b+20/147*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*
a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2-4/49*cosh(b*x+a)^(
5/2)*sinh(b*x+a)/b^2-20/147*sinh(b*x+a)*cosh(b*x+a)^(1/2)/b^2
```

3.528.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx=\frac{40i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+\sqrt{\cosh(a+bx)}(63bx\cosh(a+bx)+21bx\cosh(3(a+bx))-46\sinh(a+b}{294b^2}$$

input
```
Integrate[x*Cosh[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

output
```
((40*I)*EllipticF[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*(63*b*x*Cosh[a
 + b*x] + 21*b*x*Cosh[3*(a + b*x)] - 46*Sinh[a + b*x] - 6*Sinh[3*(a + b*x)
]))/(294*b^2)
```

3.528.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5896, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \ 5896$$

$$\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int \cosh^{\frac{7}{2}}(a+bx)dx}{7b}$$

$$\Big\downarrow \ 3042$$

$$\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^{7/2}dx}{7b}$$

$$\Big\downarrow \ 3115$$

$$\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{5}{7}\int \cosh^{\frac{3}{2}}(a+bx)dx+\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}\right)}{7b}$$

$$\Big\downarrow \ 3042$$

$$\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}dx\right)}{7b}$$

$$\Big\downarrow \ 3115$$

$$\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}\right)+\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}\right)}{7b}$$

$$\Big\downarrow \ 3042$$

$$\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\left(\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}+\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx\right)\right)}{7b}$$

$\big\downarrow$ 3120

$$\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\left(\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}\right)\right)}{7b}$$

input
```
Int[x*Cosh[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

output
```
(2*x*Cosh[a + b*x]^(7/2))/(7*b) - (2*((2*Cosh[a + b*x]^(5/2)*Sinh[a + b*x]
)/(7*b) + (5*((((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sqrt[Cosh[
a + b*x]]*Sinh[a + b*x])/(3*b)))/7))/(7*b)
```

3.528.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.528. $\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx$

3.528.4 Maple [F]

$$\int x\cosh\left(bx+a\right)^{\frac{5}{2}}\sinh\left(bx+a\right)dx$$

input
```
int(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x)
```

output
```
int(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x)
```

3.528.5 Fricas [F(-2)]

Exception generated.

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.528.6 Sympy [F(-1)]

Timed out.

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(b*x+a)**(5/2)*sinh(b*x+a),x)
```

output
```
Timed out
```

3.528.7 Maxima [F]

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\cosh\left(bx+a\right)^{\frac{5}{2}}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)^(5/2)*sinh(b*x + a), x)
```

3.528.8 Giac [F]

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\cosh\left(bx+a\right)^{\frac{5}{2}}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)^(5/2)*sinh(b*x + a), x)
```

3.528.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\cosh(a+b\,x)^{5/2}\sinh(a+b\,x)\,dx$$

input
```
int(x*cosh(a + b*x)^(5/2)*sinh(a + b*x),x)
```

output
```
int(x*cosh(a + b*x)^(5/2)*sinh(a + b*x), x)
```

3.529 $\int x \cosh^{\frac{3}{2}}(a+bx) \sinh(a+bx) \, dx$

3.529.1 Optimal result

Integrand size = 18, antiderivative size = 64

$$\int x \cosh^{\frac{3}{2}}(a+bx) \sinh(a+bx) \, dx = \frac{2x \cosh^{\frac{5}{2}}(a+bx)}{5b} + \frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2} - \frac{4\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{25b^2}$$

output
```
2/5*x*cosh(b*x+a)^(5/2)/b+12/25*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a
+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2-4/25*cosh(b*x+a)^(3
/2)*sinh(b*x+a)/b^2
```

3.529.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.22

$$\int x \cosh^{\frac{3}{2}}(a+bx) \sinh(a+bx) \, dx = \frac{e^{-3(a+bx)}\left(\left(1+e^{2(a+bx)}\right)\left(2+5bx+2e^{2(a+bx)}(-12+5bx)+e^{4(a+bx)}(-2+5bx)\right)+48e^{2(a+bx)}\sqrt{1+e^{2(a+bx)}}\right.}{50\sqrt{2}b^2\sqrt{e^{-a-bx}+e^{a+bx}}}$$

input
```
Integrate[x*Cosh[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

output
```
((1 + E^(2*(a + b*x)))*(2 + 5*b*x + 2*E^(2*(a + b*x))*(-12 + 5*b*x) + E^(4
*(a + b*x))*(-2 + 5*b*x)) + 48*E^(2*(a + b*x))*Sqrt[1 + E^(2*(a + b*x))]*H
ypergeometric2F1[-1/4, 1/2, 3/4, -E^(2*(a + b*x))])/(50*Sqrt[2]*b^2*E^(3*(
a + b*x))*Sqrt[E^(-a - b*x) + E^(a + b*x)])
```

3.529.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5896, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{5896}$$

$$\frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int \cosh^{\frac{5}{2}}(a+bx)dx}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^{5/2}dx}{5b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{3}{5}\int \sqrt{\cosh(a+bx)}dx+\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}+\frac{3}{5}\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx\right)}{5b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}-\frac{6iE\left(\frac{1}{2}i(a+bx)|2\right)}{5b}\right)}{5b}$$

input
```
Int[x*Cosh[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

output
```
(2*x*Cosh[a + b*x]^(5/2))/(5*b) - (2*((((-6*I)/5)*EllipticE[(I/2)*(a + b*x
), 2])/b + (2*Cosh[a + b*x]^(3/2)*Sinh[a + b*x])/(5*b)))/(5*b)
```

3.529.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.529.4 Maple [F]

$$\int x \cosh\left(bx+a\right)^{\frac{3}{2}} \sinh\left(bx+a\right) dx$$

input
```
int(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x)
```

output
```
int(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x)
```

3.529.5 Fricas [F(-2)]

Exception generated.

$$\int x \cosh^{\frac{3}{2}}(a+bx) \sinh(a+bx) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.529.6 Sympy [F(-1)]

Timed out.

$$\int x \cosh^{\frac{3}{2}}(a+bx) \sinh(a+bx) \, dx = \text{Timed out}$$

input
```
integrate(x*cosh(b*x+a)**(3/2)*sinh(b*x+a),x)
```

output
```
Timed out
```

3.529.7 Maxima [F]

$$\int x \cosh^{\frac{3}{2}}(a+bx) \sinh(a+bx) \, dx = \int x \cosh(bx+a)^{\frac{3}{2}} \sinh(bx+a) \, dx$$

input
```
integrate(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)^(3/2)*sinh(b*x + a), x)
```

3.529.8 Giac [F]

$$\int x\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\cosh\left(bx+a\right)^{\frac{3}{2}}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)^(3/2)*sinh(b*x + a), x)
```

3.529.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\cosh(a+b\,x)^{3/2}\sinh(a+b\,x)\,dx$$

input
```
int(x*cosh(a + b*x)^(3/2)*sinh(a + b*x),x)
```

output
```
int(x*cosh(a + b*x)^(3/2)*sinh(a + b*x), x)
```

3.530 $\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx$

3.530.1 Optimal result

Integrand size = 18, antiderivative size = 64

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx=\frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}+\frac{4i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{9b^2}-\frac{4\sqrt{\cosh(a+bx)}\sinh(a+bx)}{9b^2}$$

output
```
2/3*x*cosh(b*x+a)^(3/2)/b+4/9*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1
/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2-4/9*sinh(b*x+a)*cosh(
b*x+a)^(1/2)/b^2
```

3.530.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx=\frac{4i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+2\sqrt{\cosh(a+bx)}(3bx\cosh(a+bx)-2\sinh(a+bx))}{9b^2}$$

input
```
Integrate[x*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x],x]
```

output
```
((4*I)*EllipticF[(I/2)*(a + b*x), 2] + 2*Sqrt[Cosh[a + b*x]]*(3*b*x*Cosh[a
 + b*x] - 2*Sinh[a + b*x]))/(9*b^2)
```

3.530.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5896, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh(a+bx)\sqrt{\cosh(a+bx)}\,dx$$

$$\Big\downarrow \text{5896}$$

$$\frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int \cosh^{\frac{3}{2}}(a+bx)dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}dx}{3b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx\right)}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}\right)}{3b}$$

input
```
Int[x*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x],x]
```

output
```
(2*x*Cosh[a + b*x]^(3/2))/(3*b) - (2*((((-2*I)/3)*EllipticF[(I/2)*(a + b*x
), 2])/b + (2*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x])/(3*b)))/(3*b)
```

3.530.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.530.4 Maple [F]

$$\int x \sinh (bx + a) \sqrt{\cosh (bx + a)}dx$$

input
```
int(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x)
```

output
```
int(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x)
```

3.530.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

3.530.6 Sympy [F]

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx = \int x\sinh{(a+bx)}\sqrt{\cosh{(a+bx)}}\,dx$$

input
```
integrate(x*sinh(b*x+a)*cosh(b*x+a)**(1/2),x)
```

output
```
Integral(x*sinh(a + b*x)*sqrt(cosh(a + b*x)), x)
```

3.530.7 Maxima [F]

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx = \int x\sqrt{\cosh{(bx+a)}}\sinh{(bx+a)}\,dx$$

input
```
integrate(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(cosh(b*x + a))*sinh(b*x + a), x)
```

3.530.8 Giac [F]

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx = \int x\sqrt{\cosh{(bx+a)}}\sinh{(bx+a)}\,dx$$

input
```
integrate(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(cosh(b*x + a))*sinh(b*x + a), x)
```

3.530.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx=\int x\,\sqrt{\cosh\left(a+b\,x\right)}\,\mathrm{sinh}(a+b\,x)\,\,dx$$

input
```
int(x*cosh(a + b*x)^(1/2)*sinh(a + b*x),x)
```

output
```
int(x*cosh(a + b*x)^(1/2)*sinh(a + b*x), x)
```

3.531 $\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx$

3.531.1 Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \frac{2x\sqrt{\cosh(a+bx)}}{b} + \frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

output
```
4*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2
*a+1/2*b*x),2^(1/2))/b^2+2*x*cosh(b*x+a)^(1/2)/b
```

3.531.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.95

$$\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \frac{(\cosh(a+bx)-\sinh(a+bx))\left(4\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\cosh(2(a+bx))-\sinh(2(a+bx))\right)\sqrt{\ldots}\right.}{b^2\sqrt{\cosh(a+b\ldots}}$$

input
```
Integrate[(x*Sinh[a + b*x])/Sqrt[Cosh[a + b*x]],x]
```

output
```
((Cosh[a + b*x] - Sinh[a + b*x])*(4*Hypergeometric2F1[-1/4, 1/2, 3/4, -Cos
h[2*(a + b*x)] - Sinh[2*(a + b*x)]]*Sqrt[1 + Cosh[2*(a + b*x)] + Sinh[2*(a
 + b*x)]] + (-2 + b*x)*(1 + Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])))/(b^2*
Sqrt[Cosh[a + b*x]])
```

3.531.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5896, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}dx$$

$$\Big\downarrow \text{5896}$$

$$\frac{2x\sqrt{\cosh(a+bx)}}{b}-\frac{2\int \sqrt{\cosh(a+bx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sqrt{\cosh(a+bx)}}{b}-\frac{2\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2x\sqrt{\cosh(a+bx)}}{b}+\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

input
```
Int[(x*Sinh[a + b*x])/Sqrt[Cosh[a + b*x]],x]
```

output
```
(2*x*Sqrt[Cosh[a + b*x]])/b + ((4*I)*EllipticE[(I/2)*(a + b*x), 2])/b^2
```

3.531.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.531.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(61) = 122$.

Time = 0.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 6.76

method	result
risch	$\frac{(bx-2)\left(1+e^{2bx+2a}\right)\sqrt{2}\, e^{-bx-a}}{b^{2}\sqrt{\left(1+e^{2bx+2a}\right)e^{-bx-a}}}-\frac{2\left(-\frac{2\left(1+e^{2bx+2a}\right)}{\sqrt{\left(1+e^{2bx+2a}\right)e^{bx+a}}}+\frac{i\sqrt{-i\left(e^{bx+a}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{bx+a}-i\right)}\,\sqrt{i e^{bx+a}}\,\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i\left(e^{bx+a}+\right.}\right.\right.}{\sqrt{e^{3bx+3a}+e^{bx+a}}}\right.}{b^{2}\sqrt{\left(1+e^{2bx+2a}\right)e^{-bx}}}$

input
```
int(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x-2)*(exp(b*x+a)^2+1)/b^2*2^(1/2)/((exp(b*x+a)^2+1)/exp(b*x+a))^(1/2)/e
xp(b*x+a)-2/b^2*(-2*(exp(b*x+a)^2+1)/((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)+I
*(-I*(exp(b*x+a)+I))^(1/2)*2^(1/2)*(I*(exp(b*x+a)-I))^(1/2)*(I*exp(b*x+a))
^(1/2)/(exp(b*x+a)^3+exp(b*x+a))^(1/2)*(-2*I*EllipticE((-I*(exp(b*x+a)+I))
^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(b*x+a)+I))^(1/2),1/2*2^(1/2))))*2
^(1/2)/((exp(b*x+a)^2+1)/exp(b*x+a))^(1/2)*((exp(b*x+a)^2+1)*exp(b*x+a))^(
1/2)/exp(b*x+a)
```

3.531.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.531.6 Sympy [F]

$$\int \frac{x \sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{x \sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)**(1/2),x)
```

output
```
Integral(x*sinh(a + b*x)/sqrt(cosh(a + b*x)), x)
```

3.531.7 Maxima [F]

$$\int \frac{x \sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{x \sinh(bx+a)}{\sqrt{\cosh(bx+a)}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sinh(b*x + a)/sqrt(cosh(b*x + a)), x)
```

3.531.8 Giac [F]

$$\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{x\sinh(bx+a)}{\sqrt{\cosh(bx+a)}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sinh(b*x + a)/sqrt(cosh(b*x + a)), x)
```

3.531.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{x\sinh(a+b\,x)}{\sqrt{\cosh(a+b\,x)}}\,dx$$

input
```
int((x*sinh(a + b*x))/cosh(a + b*x)^(1/2),x)
```

output
```
int((x*sinh(a + b*x))/cosh(a + b*x)^(1/2), x)
```

3.532 $\int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)} dx$

3.532.1 Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)} dx = -\frac{2x}{b\sqrt{\cosh(a+bx)}} - \frac{4i \operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b^2}$$

output
```
-4*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/
2*a+1/2*b*x),2^(1/2))/b^2-2*x/b/cosh(b*x+a)^(1/2)
```

3.532.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)} dx = -\frac{2x}{b\sqrt{\cosh(a+bx)}} - \frac{4i \operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b^2}$$

input
```
Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(3/2),x]
```

output
```
(-2*x)/(b*Sqrt[Cosh[a + b*x]]) - ((4*I)*EllipticF[(I/2)*(a + b*x), 2])/b^2
```

3.532.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5896, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)} \, dx$$

$$\Big\downarrow \text{5896}$$

$$\frac{2\int \frac{1}{\sqrt{\cosh(a+bx)}}dx}{b} - \frac{2x}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{b\sqrt{\cosh(a+bx)}} + \frac{2\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2x}{b\sqrt{\cosh(a+bx)}} - \frac{4i \operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b^2}$$

input
```
Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(3/2),x]
```

output
```
(-2*x)/(b*Sqrt[Cosh[a + b*x]]) - ((4*I)*EllipticF[(I/2)*(a + b*x), 2])/b^2
```

3.532.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.532.4 Maple [F]

$$\int \frac{x \sinh \left(b x+a\right)}{\cosh \left(b x+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x)
```

output
```
int(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x)
```

3.532.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.532.6 Sympy [F]

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\, dx = \int \frac{x \sinh \left(a+bx\right)}{\cosh^{\frac{3}{2}}\left(a+bx\right)}\, dx$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)**(3/2),x)
```

output
```
Integral(x*sinh(a + b*x)/cosh(a + b*x)**(3/2), x)
```

3.532.7 Maxima [F]

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{x\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*sinh(b*x + a)/cosh(b*x + a)^(3/2), x)
```

3.532.8 Giac [F]

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{x\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*sinh(b*x + a)/cosh(b*x + a)^(3/2), x)
```

3.532.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{x\sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^{3/2}}\,dx$$

input
```
int((x*sinh(a + b*x))/cosh(a + b*x)^(3/2),x)
```

output
```
int((x*sinh(a + b*x))/cosh(a + b*x)^(3/2), x)
```

3.533 $\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

3.533.1 Optimal result

Integrand size = 18, antiderivative size = 64

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2}+\frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}}$$

output
```
-2/3*x/b/cosh(b*x+a)^(3/2)+4/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+
1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2+4/3*sinh(b*x+a)/b^2/
cosh(b*x+a)^(1/2)
```

3.533.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\left(-bx+2i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sinh(2(a+bx))\right)}{3b^2\cosh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(5/2),x]
```

output
```
(2*(-(b*x) + (2*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)*(a + b*x), 2] + Sin
h[2*(a + b*x)]))/(3*b^2*Cosh[a + b*x]^(3/2))
```

3.533.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5896, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{5896}$$

$$\frac{2\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}dx}{3b}-\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{\sin\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}}dx}{3b}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\left(\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int \sqrt{\cosh(a+bx)}dx\right)}{3b}-\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{2\left(\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx\right)}{3b}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{2\left(\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}+\frac{2iE\left(\frac{1}{2}i(a+bx)|2\right)}{b}\right)}{3b}$$

input
```
Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(5/2),x]
```

output
```
(-2*x)/(3*b*Cosh[a + b*x]^(3/2)) + (2*(((2*I)*EllipticE[(I/2)*(a + b*x), 2
])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a + b*x]])))/(3*b)
```

3.533. $\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}dx$

3.533.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.533.4 Maple [F]

$$\int \frac{x \sinh (bx+a)}{\cosh (bx+a)^{\frac{5}{2}}} dx$$

input
```
int(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x)
```

output
```
int(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x)
```

3.533.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.533.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.533.7 Maxima [F]

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x \sinh(bx+a)}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*sinh(b*x + a)/cosh(b*x + a)^(5/2), x)
```

3.533.8 Giac [F]

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x \sinh(bx+a)}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*sinh(b*x + a)/cosh(b*x + a)^(5/2), x)
```

3.533.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\sinh(a+b\,x)}{\cosh{(a+b\,x)}^{5/2}}\,dx$$

input
```
int((x*sinh(a + b*x))/cosh(a + b*x)^(5/2),x)
```

output
```
int((x*sinh(a + b*x))/cosh(a + b*x)^(5/2), x)
```

3.534 $\int \frac{x\sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx$

3.534.1 Optimal result

Integrand size = 18, antiderivative size = 64

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)} - \frac{4i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{15b^2} + \frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)}$$

output
```
-2/5*x/b/cosh(b*x+a)^(5/2)-4/15*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a
+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2+4/15*sinh(b*x+a)/b^
2/cosh(b*x+a)^(3/2)
```

3.534.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\left(-3bx-2i\cosh^{\frac{5}{2}}(a+bx)\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+\sinh(2(a+bx))\right)}{15b^2\cosh^{\frac{5}{2}}(a+bx)}$$

input
```
Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(7/2),x]
```

output
```
(2*(-3*b*x - (2*I)*Cosh[a + b*x]^(5/2)*EllipticF[(I/2)*(a + b*x), 2] + Sin
h[2*(a + b*x)]))/(15*b^2*Cosh[a + b*x]^(5/2))
```

3.534.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5896, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{5896}$$

$$\frac{2\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}dx}{5b}-\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{2\int \frac{1}{\sin\left(ia+ibx+\frac{\pi}{2}\right)^{5/2}}dx}{5b}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}\right)}{5b}-\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{2\left(\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx\right)}{5b}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{2\left(\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}\right)}{5b}$$

input
```
Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(7/2),x]
```

output
```
(-2*x)/(5*b*Cosh[a + b*x]^(5/2)) + (2*((((-2*I)/3)*EllipticF[(I/2)*(a + b*
x), 2])/b + (2*Sinh[a + b*x])/(3*b*Cosh[a + b*x]^(3/2))))/(5*b)
```

3.534.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.534.4 Maple [F]

$$\int \frac{x \sinh \left(bx+a\right)}{\cosh \left(bx+a\right)^{\frac{7}{2}}}dx$$

input
```
int(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x)
```

output
```
int(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x)
```

3.534.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

3.534.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\, dx = \text{Timed out}$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)**(7/2),x)
```

output
```
Timed out
```

3.534.7 Maxima [F]

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\, dx = \int \frac{x \sinh(bx+a)}{\cosh(bx+a)^{\frac{7}{2}}}\, dx$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(x*sinh(b*x + a)/cosh(b*x + a)^(7/2), x)
```

3.534.8 Giac [F]

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\, dx = \int \frac{x \sinh(bx+a)}{\cosh(bx+a)^{\frac{7}{2}}}\, dx$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(x*sinh(b*x + a)/cosh(b*x + a)^(7/2), x)
```

3.534.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)} \, dx = \int \frac{x \sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^{7/2}} \, dx$$

input
```
int((x*sinh(a + b*x))/cosh(a + b*x)^(7/2),x)
```

output
```
int((x*sinh(a + b*x))/cosh(a + b*x)^(7/2), x)
```

3.535 $\int \frac{x \sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)} \, dx$

3.535.1 Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)} \, dx = -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)} + \frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2} + \frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)} + \frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}}$$

output
```
-2/7*x/b/cosh(b*x+a)^(7/2)+12/35*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*
a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2+4/35*sinh(b*x+a)/b
^2/cosh(b*x+a)^(5/2)+12/35*sinh(b*x+a)/b^2/cosh(b*x+a)^(1/2)
```

3.535.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)} \, dx = \frac{-20bx + 24i\cosh^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right) + 10\sinh(2(a+bx)) + 3\sinh(4(a+bx))}{70b^2\cosh^{\frac{7}{2}}(a+bx)}$$

input
```
Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(9/2),x]
```

output
```
(-20*b*x + (24*I)*Cosh[a + b*x]^(7/2)*EllipticE[(I/2)*(a + b*x), 2] + 10*S
inh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)])/(70*b^2*Cosh[a + b*x]^(7/2))
```

3.535.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5896, 3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{5896}$$

$$\frac{2\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}dx}{7b}-\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}+\frac{2\int \frac{1}{\sin\left(ia+ibx+\frac{\pi}{2}\right)^{7/2}}dx}{7b}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\left(\frac{3}{5}\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}dx+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}+\frac{2\left(\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\int \frac{1}{\sin\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}}dx\right)}{7b}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\left(\frac{3}{5}\left(\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int \sqrt{\cosh(a+bx)}dx\right)+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}+\frac{2\left(\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\left(\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx\right)\right)}{7b}$$

$\downarrow$ 3119

$$-\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}+\frac{2\left(\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\left(\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}+\frac{2iE\left(\frac{1}{2}i(a+bx)|2\right)}{b}\right)\right)}{7b}$$

input
```
Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(9/2),x]
```

output
```
(-2*x)/(7*b*Cosh[a + b*x]^(7/2)) + (2*((2*Sinh[a + b*x])/(5*b*Cosh[a + b*x
]^(5/2)) + (3*(((2*I)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])
/(b*Sqrt[Cosh[a + b*x]])))/5))/(7*b)
```

3.535.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.535. $\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx$

3.535.4 Maple [F]

$$\int \frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{9}{2}}}dx$$

input
```
int(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x)
```

output
```
int(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x)
```

3.535.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.535.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)**(9/2),x)
```

output
```
Timed out
```

3.535.7 Maxima [F]

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x*sinh(b*x + a)/cosh(b*x + a)^(9/2), x)
```

3.535.8 Giac [F]

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
integrate(x*sinh(b*x + a)/cosh(b*x + a)^(9/2), x)
```

3.535.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\sinh(a+b\,x)}{\cosh(a+b\,x)^{9/2}}\,dx$$

input
```
int((x*sinh(a + b*x))/cosh(a + b*x)^(9/2),x)
```

output
```
int((x*sinh(a + b*x))/cosh(a + b*x)^(9/2), x)
```

3.536 $\int x\operatorname{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx$

3.536.1 Optimal result

Integrand size = 18, antiderivative size = 107

$$\int x\operatorname{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx = \frac{12i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{35b^2} - \frac{2x\operatorname{sech}^{\frac{7}{2}}(a+bx)}{7b} + \frac{12\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{35b^2} + \frac{4\operatorname{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{35b^2}$$

output
```
-2/7*x*sech(b*x+a)^(7/2)/b+4/35*sech(b*x+a)^(5/2)*sinh(b*x+a)/b^2+12/35*si
nh(b*x+a)*sech(b*x+a)^(1/2)/b^2+12/35*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh
(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)
*sech(b*x+a)^(1/2)/b^2
```

3.536.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int x\operatorname{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx = \frac{\operatorname{sech}^{\frac{7}{2}}(a+bx)\left(-20bx+24i\cosh^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)+10\sinh(2(a+bx))+3\sinh(4(a+bx))\right)}{70b^2}$$

input
```
Integrate[x*Sech[a + b*x]^(9/2)*Sinh[a + b*x],x]
```

output
```
(Sech[a + b*x]^(7/2)*(-20*b*x + (24*I)*Cosh[a + b*x]^(7/2)*EllipticE[(I/2)
*(a + b*x), 2] + 10*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)]))/(70*b^2)
```

3.536.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5967, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sinh(a+bx)\text{sech}^{\frac{9}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{5967}$$

$$\frac{2\int \text{sech}^{\frac{7}{2}}(a+bx)dx}{7b}-\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^{7/2}dx}{7b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2\left(\frac{3}{5}\int \text{sech}^{\frac{3}{2}}(a+bx)dx+\frac{2\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}-\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\left(\frac{2\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{3}{5}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}dx\right)}{7b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2\left(\frac{3}{5}\left(\frac{2\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{b}-\int \frac{1}{\sqrt{\text{sech}(a+bx)}}dx\right)+\frac{2\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}-\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\left(\frac{2\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{3}{5}\left(\frac{2\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{b}-\int\frac{1}{\sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}}dx\right)\right)}{7b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\left(\frac{3}{5}\left(\frac{2\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{b}-\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int\sqrt{\cosh(a+bx)}dx\right)+\frac{2\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}-\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\left(\frac{2\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{3}{5}\left(\frac{2\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{b}-\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx\right)\right)}{7b}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\left(\frac{2\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{3}{5}\left(\frac{2\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{b}+\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)|2\right)}{b}\right)\right)}{7b}$$

input
```
Int[x*Sech[a + b*x]^(9/2)*Sinh[a + b*x],x]
```

output
```
(-2*x*Sech[a + b*x]^(7/2))/(7*b) + (2*((2*Sech[a + b*x]^(5/2)*Sinh[a + b*x
])/(5*b) + (3*(((2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sq
rt[Sech[a + b*x]])/b + (2*Sqrt[Sech[a + b*x]]*Sinh[a + b*x])/b))/5))/(7*b)
```

3.536. $\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx$

3.536.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5967
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sech[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.536.4 Maple [F]

$$\int x \operatorname{sech}(bx+a)^{\frac{9}{2}} \sinh(bx+a)\, dx$$

input
```
int(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x)
```

output
```
int(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x)
```

3.536.5 Fricas [F(-2)]

Exception generated.

$$\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.536.6 Sympy [F(-1)]

Timed out.

$$\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*sech(b*x+a)**(9/2)*sinh(b*x+a),x)
```

output
```
Timed out
```

3.536.7 Maxima [F]

$$\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx = \int x\,\text{sech}\,(bx+a)^{\frac{9}{2}}\sinh\,(bx+a)\,\,dx$$

input
```
integrate(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sech(b*x + a)^(9/2)*sinh(b*x + a), x)
```

3.536.8 Giac [F]

$$\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\,\text{sech}\,(bx+a)^{\frac{9}{2}}\sinh\,(bx+a)\;dx$$

input
```
integrate(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sech(b*x + a)^(9/2)*sinh(b*x + a), x)
```

3.536.9 Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\sinh(a+b\,x)\,\left(\frac{1}{\cosh\,(a+b\,x)}\right)^{9/2}\,dx$$

input
```
int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(9/2),x)
```

output
```
int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(9/2), x)
```

3.537 $\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx$

3.537.1 Optimal result

Integrand size = 18, antiderivative size = 84

$$\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx = -\frac{4i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\text{sech}(a+bx)}}{15b^2} - \frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b} + \frac{4\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{15b^2}$$

output
```
-2/5*x*sech(b*x+a)^(5/2)/b+4/15*sech(b*x+a)^(3/2)*sinh(b*x+a)/b^2-4/15*I*(
cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/
2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b^2
```

3.537.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx = -\frac{2\sqrt{\text{sech}(a+bx)}\left(2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+3bx\text{sech}^2(a+bx)-2\tanh(a+bx)\right)}{15b^2}$$

input
```
Integrate[x*Sech[a + b*x]^(7/2)*Sinh[a + b*x],x]
```

output
```
(-2*Sqrt[Sech[a + b*x]]*((2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*
x), 2] + 3*b*x*Sech[a + b*x]^2 - 2*Tanh[a + b*x]))/(15*b^2)
```

3.537.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5967, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh(a+bx)\text{sech}^{\frac{7}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{5967}$$

$$\frac{2\int \text{sech}^{\frac{5}{2}}(a+bx)dx}{5b}-\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^{5/2}dx}{5b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2\left(\frac{1}{3}\int \sqrt{\text{sech}(a+bx)}dx+\frac{2\sinh(a+bx)\text{sech}^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}-\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\left(\frac{2\sinh(a+bx)\text{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{1}{3}\int \sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}dx\right)}{5b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\left(\frac{1}{3}\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)\text{sech}^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}-\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\operatorname{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\left(\frac{2\sinh(a+bx)\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{1}{3}\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\int\frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx\right)}{5b}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2x\operatorname{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\left(\frac{2\sinh(a+bx)\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}\right)}{5b}$$

input
```
Int[x*Sech[a + b*x]^(7/2)*Sinh[a + b*x],x]
```

output
```
(-2*x*Sech[a + b*x]^(5/2))/(5*b) + (2*((((-2*I)/3)*Sqrt[Cosh[a + b*x]]*Ell
ipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b + (2*Sech[a + b*x]^(3/2)
*Sinh[a + b*x])/(3*b)))/(5*b)
```

3.537.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.537. $\int x\operatorname{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx$

rule 5967
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sech[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.537.4 Maple [F]

$$\int x \operatorname{sech}(bx+a)^{\frac{7}{2}} \sinh(bx+a)\, dx$$

input
```
int(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x)
```

output
```
int(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x)
```

3.537.5 Fricas [F(-2)]

Exception generated.

$$\int x\operatorname{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.537.6 Sympy [F(-1)]

Timed out.

$$\int x\operatorname{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(x*sech(b*x+a)**(7/2)*sinh(b*x+a),x)
```

output
```
Timed out
```

3.537.7 Maxima [F]

$$\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\,\text{sech}\,(bx+a)^{\frac{7}{2}}\sinh\,(bx+a)\,dx$$

input
```
integrate(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sech(b*x + a)^(7/2)*sinh(b*x + a), x)
```

3.537.8 Giac [F]

$$\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\,\text{sech}\,(bx+a)^{\frac{7}{2}}\sinh\,(bx+a)\,dx$$

input
```
integrate(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sech(b*x + a)^(7/2)*sinh(b*x + a), x)
```

3.537.9 Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\sinh(a+b\,x)\,\left(\frac{1}{\cosh\,(a+b\,x)}\right)^{7/2}\,dx$$

input
```
int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(7/2),x)
```

output
```
int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(7/2), x)
```

3.538 $\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx$

3.538.1 Optimal result

Integrand size = 18, antiderivative size = 84

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \frac{4i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\text{sech}(a+bx)}}{3b^2} - \frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b} + \frac{4\sqrt{\text{sech}(a+bx)}\sinh(a+bx)}{3b^2}$$

output
```
-2/3*x*sech(b*x+a)^(3/2)/b+4/3*sinh(b*x+a)*sech(b*x+a)^(1/2)/b^2+4/3*I*(co
sh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*
b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b^2
```

3.538.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \frac{2\text{sech}^{\frac{3}{2}}(a+bx)\left(-bx+2i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sinh(2(a+bx))\right)}{3b^2}$$

input
```
Integrate[x*Sech[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

output
```
(2*Sech[a + b*x]^(3/2)*(-(b*x) + (2*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)
*(a + b*x), 2] + Sinh[2*(a + b*x)]))/(3*b^2)
```

3.538.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{5967}$$

$$\frac{2\int \text{sech}^{\frac{3}{2}}(a+bx)dx}{3b}-\frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}dx}{3b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2\left(\frac{2\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{b}-\int \frac{1}{\sqrt{\text{sech}(a+bx)}}dx\right)}{3b}-\frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\left(\frac{2\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{b}-\int \frac{1}{\sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}}dx\right)}{3b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\left(\frac{2\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{b}-\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \sqrt{\cosh(a+bx)}dx\right)}{3b}-\frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\left(\frac{2\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{b}-\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx\right)}{3b}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\left(\frac{2\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)|2\right)}{b}\right)}{3b}$$

input
```
Int[x*Sech[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

output
```
(-2*x*Sech[a + b*x]^(3/2))/(3*b) + (2*(((2*I)*Sqrt[Cosh[a + b*x]]*Elliptic
E[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b + (2*Sqrt[Sech[a + b*x]]*Sinh
[a + b*x])/b))/(3*b)
```

3.538.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5967
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sech[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.538. $\int x\operatorname{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx$

3.538.4 Maple [F]

$$\int x\,\mathrm{sech}\,(bx+a)^{\frac{5}{2}}\sinh\,(bx+a)\,dx$$

input
```
int(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x)
```

output
```
int(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x)
```

3.538.5 Fricas [F(-2)]

Exception generated.

$$\int x\mathrm{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.538.6 Sympy [F(-1)]

Timed out.

$$\int x\mathrm{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*sech(b*x+a)**(5/2)*sinh(b*x+a),x)
```

output
```
Timed out
```

3.538.7 Maxima [F]

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\,\text{sech}\,(bx+a)^{\frac{5}{2}}\sinh\,(bx+a)\,\,dx$$

input
```
integrate(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sech(b*x + a)^(5/2)*sinh(b*x + a), x)
```

3.538.8 Giac [F]

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\,\text{sech}\,(bx+a)^{\frac{5}{2}}\sinh\,(bx+a)\,\,dx$$

input
```
integrate(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sech(b*x + a)^(5/2)*sinh(b*x + a), x)
```

3.538.9 Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\sinh(a+b\,x)\,\left(\frac{1}{\cosh\,(a+b\,x)}\right)^{5/2}\,dx$$

input
```
int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(5/2),x)
```

output
```
int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(5/2), x)
```

3.539 $\int x\mathrm{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx$

3.539.1 Optimal result

Integrand size = 18, antiderivative size = 57

$$\int x\mathrm{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = -\frac{2x\sqrt{\mathrm{sech}(a+bx)}}{b}-\frac{4i\sqrt{\cosh(a+bx)}\,\mathrm{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\mathrm{sech}(a+bx)}}{b^2}$$

output
```
-2*x*sech(b*x+a)^(1/2)/b-4*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*
b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a
)^(1/2)/b^2
```

3.539.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int x\mathrm{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = -\frac{2\left(bx+2i\sqrt{\cosh(a+bx)}\,\mathrm{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\right)\sqrt{\mathrm{sech}(a+bx)}}{b^2}$$

input
```
Integrate[x*Sech[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

output
```
(-2*(b*x + (2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2])*Sqrt[S
ech[a + b*x]])/b^2
```

3.539.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh(a+bx)\text{sech}^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{5967}$$

$$\frac{2\int \sqrt{\text{sech}(a+bx)}dx}{b}-\frac{2x\sqrt{\text{sech}(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\sqrt{\text{sech}(a+bx)}}{b}+\frac{2\int \sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx}{b}-\frac{2x\sqrt{\text{sech}(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\sqrt{\text{sech}(a+bx)}}{b}+\frac{2\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2x\sqrt{\text{sech}(a+bx)}}{b}-\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b^2}$$

input
```
Int[x*Sech[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

output
```
(-2*x*Sqrt[Sech[a + b*x]])/b - ((4*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*
(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2
```

3.539.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5967
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sech[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.539.4 Maple [F]

$$\int x\,\mathrm{sech}\left(bx+a\right)^{\frac{3}{2}}\sinh\left(bx+a\right)dx$$

input
```
int(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x)
```

output
```
int(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x)
```

3.539.5 Fricas [F(-2)]

Exception generated.

$$\int x\mathrm{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

3.539.6 Sympy [F(-1)]

Timed out.

$$\int x\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(x*sech(b*x+a)**(3/2)*sinh(b*x+a),x)
```

output
```
Timed out
```

3.539.7 Maxima [F]

$$\int x\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\,\text{sech}\left(bx+a\right)^{\frac{3}{2}}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sech(b*x + a)^(3/2)*sinh(b*x + a), x)
```

3.539.8 Giac [F]

$$\int x\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\,\text{sech}\left(bx+a\right)^{\frac{3}{2}}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sech(b*x + a)^(3/2)*sinh(b*x + a), x)
```

3.539.9 Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\sinh(a+b\,x)\,\left(\frac{1}{\cosh\left(a+b\,x\right)}\right)^{3/2}\,dx$$

input
```
int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(3/2),x)
```

output
```
int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(3/2), x)
```

3.540 $\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx$

3.540.1 Optimal result

Integrand size = 18, antiderivative size = 57

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx=\frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}}+\frac{4i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b^2}$$

output
```
2*x/b/sech(b*x+a)^(1/2)+4*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b
*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)
^(1/2)/b^2
```

3.540.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx$$
$$=\frac{\sqrt{2}e^{-a-bx}\sqrt{\frac{e^{a+bx}}{1+e^{2(a+bx)}}}\left(\left(1+e^{2(a+bx)}\right)(-2+bx)+4\sqrt{1+e^{2(a+bx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2(a+bx)}\right)\right.}{b^2}$$

input
```
Integrate[x*Sqrt[Sech[a + b*x]]*Sinh[a + b*x],x]
```

output
```
(Sqrt[2]*E^(-a - b*x)*Sqrt[E^(a + b*x)/(1 + E^(2*(a + b*x)))]*((1 + E^(2*(
a + b*x)))*(-2 + b*x) + 4*Sqrt[1 + E^(2*(a + b*x))]*Hypergeometric2F1[-1/4
, 1/2, 3/4, -E^(2*(a + b*x))]))/b^2
```

3.540.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5967, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sinh(a+bx)\sqrt{\text{sech}(a+bx)}\,dx$$

$$\Big\downarrow \text{5967}$$

$$\frac{2x}{b\sqrt{\text{sech}(a+bx)}}-\frac{2\int \frac{1}{\sqrt{\text{sech}(a+bx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{b\sqrt{\text{sech}(a+bx)}}-\frac{2\int \frac{1}{\sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2x}{b\sqrt{\text{sech}(a+bx)}}-\frac{2\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \sqrt{\cosh(a+bx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{b\sqrt{\text{sech}(a+bx)}}-\frac{2\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2x}{b\sqrt{\text{sech}(a+bx)}}+\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

input
```
Int[x*Sqrt[Sech[a + b*x]]*Sinh[a + b*x],x]
```

output
```
(2*x)/(b*Sqrt[Sech[a + b*x]]) + ((4*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)
*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2
```

3.540.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5967
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sech[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.540.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(77) = 154$.

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 4.39

method	result
risch	$\frac{(bx-2)\left(1+\mathrm{e}^{2bx+2a}\right)\sqrt{2}\,\sqrt{\frac{\mathrm{e}^{bx+a}}{1+\mathrm{e}^{2bx+2a}}}\,\mathrm{e}^{-bx-a}}{b^{2}}-\frac{2\left(-\frac{2\left(1+\mathrm{e}^{2bx+2a}\right)}{\sqrt{\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{bx+a}}}+\frac{i\sqrt{-i\left(\mathrm{e}^{bx+a}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{bx+a}-i\right)}\,\sqrt{i\mathrm{e}^{bx+a}}\,\left(-2i\,\mathrm{EllipticE}\right.}{\sqrt{\mathrm{e}^{3bx+3}}}\right.}{}$

input
```
int(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(b*x-2)*(exp(b*x+a)^2+1)/b^2*2^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2+1))^(1/2)/e
xp(b*x+a)-2/b^2*(-2*(exp(b*x+a)^2+1)/((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)+I
*(-I*(exp(b*x+a)+I))^(1/2)*2^(1/2)*(I*(exp(b*x+a)-I))^(1/2)*(I*exp(b*x+a))
^(1/2)/(exp(b*x+a)^3+exp(b*x+a))^(1/2)*(-2*I*EllipticE((-I*(exp(b*x+a)+I))
^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(b*x+a)+I))^(1/2),1/2*2^(1/2))))*2
^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2+1))^(1/2)*((exp(b*x+a)^2+1)*exp(b*x+a))^(
1/2)/exp(b*x+a)
```

3.540.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.540.6 Sympy [F]

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx = \int x\sinh{(a+bx)}\sqrt{\operatorname{sech}{(a+bx)}}\,dx$$

input
```
integrate(x*sech(b*x+a)**(1/2)*sinh(b*x+a),x)
```

output
```
Integral(x*sinh(a + b*x)*sqrt(sech(a + b*x)), x)
```

3.540.7 Maxima [F]

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx=\int x\sqrt{\operatorname{sech}\left(bx+a\right)}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(sech(b*x + a))*sinh(b*x + a), x)
```

3.540.8 Giac [F]

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx=\int x\sqrt{\operatorname{sech}\left(bx+a\right)}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sqrt(sech(b*x + a))*sinh(b*x + a), x)
```

3.540.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx=\int x\sinh(a+b\,x)\,\sqrt{\frac{1}{\cosh\left(a+b\,x\right)}}\,dx$$

input
```
int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(1/2),x)
```

output
```
int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(1/2), x)
```

3.541 $\int \frac{x \sinh(a+bx)}{\sqrt{\text{sech}(a+bx)}}\,dx$

3.541.1 Optimal result

Integrand size = 18, antiderivative size = 84

$$\int \frac{x \sinh(a+bx)}{\sqrt{\text{sech}(a+bx)}}\,dx = \frac{2x}{3b\,\text{sech}^{\frac{3}{2}}(a+bx)} + \frac{4i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\text{sech}(a+bx)}}{9b^2} - \frac{4\sinh(a+bx)}{9b^2\sqrt{\text{sech}(a+bx)}}$$

output
```
2/3*x/b/sech(b*x+a)^(3/2)-4/9*sinh(b*x+a)/b^2/sech(b*x+a)^(1/2)+4/9*I*(cos
h(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b
*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b^2
```

3.541.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int \frac{x \sinh(a+bx)}{\sqrt{\text{sech}(a+bx)}}\,dx = \frac{\sqrt{\text{sech}(a+bx)}\left(3bx + 3bx\cosh(2(a+bx)) + 4i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right) - 2\sinh(2(a+b\right.}{9b^2}$$

input
```
Integrate[(x*Sinh[a + b*x])/Sqrt[Sech[a + b*x]],x]
```

output
```
(Sqrt[Sech[a + b*x]]*(3*b*x + 3*b*x*Cosh[2*(a + b*x)] + (4*I)*Sqrt[Cosh[a
+ b*x]]*EllipticF[(I/2)*(a + b*x), 2] - 2*Sinh[2*(a + b*x)]))/(9*b^2)
```

3.541.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5967, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sinh(a+bx)}{\sqrt{\text{sech}(a+bx)}}dx$$

$$\Big\downarrow \text{5967}$$

$$\frac{2x}{3b\text{sech}^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\text{sech}^{\frac{3}{2}}(a+bx)}dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{3b\text{sech}^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\csc\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}}dx}{3b}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2x}{3b\text{sech}^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{1}{3}\int \sqrt{\text{sech}(a+bx)}dx+\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{3b\text{sech}^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}+\frac{1}{3}\int \sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}dx\right)}{3b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2x}{3b\text{sech}^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{1}{3}\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int\frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{3b\text{sech}^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}+\frac{1}{3}\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int\frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx\right)}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x}{3b\text{sech}^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}\right)}{3b}$$

input
```
Int[(x*Sinh[a + b*x])/Sqrt[Sech[a + b*x]],x]
```

output
```
(2*x)/(3*b*Sech[a + b*x]^(3/2)) - (2*((((-2*I)/3)*Sqrt[Cosh[a + b*x]]*Elli
pticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b + (2*Sinh[a + b*x])/(3*b*
Sqrt[Sech[a + b*x]])))/(3*b)
```

3.541.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

3.541. $\int \frac{x\sinh(a+bx)}{\sqrt{\text{sech}(a+bx)}}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5967
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sech[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.541.4 Maple [F]

$$\int \frac{x \sinh (bx + a)}{\sqrt{\operatorname{sech}(bx + a)}} dx$$

input
```
int(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x)
```

output
```
int(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x)
```

3.541.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sinh(a + bx)}{\sqrt{\operatorname{sech}(a + bx)}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.541.6 Sympy [F]

$$\int \frac{x \sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{x \sinh\left(a+bx\right)}{\sqrt{\operatorname{sech}\left(a+bx\right)}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/sech(b*x+a)**(1/2),x)
```

output
```
Integral(x*sinh(a + b*x)/sqrt(sech(a + b*x)), x)
```

3.541.7 Maxima [F]

$$\int \frac{x \sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{x \sinh\left(bx+a\right)}{\sqrt{\operatorname{sech}\left(bx+a\right)}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sinh(b*x + a)/sqrt(sech(b*x + a)), x)
```

3.541.8 Giac [F]

$$\int \frac{x \sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{x \sinh\left(bx+a\right)}{\sqrt{\operatorname{sech}\left(bx+a\right)}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sinh(b*x + a)/sqrt(sech(b*x + a)), x)
```

3.541.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{x\sinh(a+b\,x)}{\sqrt{\frac{1}{\cosh(a+b\,x)}}}\,dx$$

input
```
int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(1/2),x)
```

output
```
int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(1/2), x)
```

3.542 $\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx$

3.542.1 Optimal result

Integrand size = 18, antiderivative size = 84

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)} + \frac{12i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{25b^2} - \frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)}$$

output
```
2/5*x/b/sech(b*x+a)^(5/2)-4/25*sinh(b*x+a)/b^2/sech(b*x+a)^(3/2)+12/25*I*(
cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/
2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b^2
```

3.542.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.49

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx$$
$$= \frac{e^{-3(a+bx)}\left(\left(1+e^{2(a+bx)}\right)\left(2+5bx+2e^{2(a+bx)}(-12+5bx)+e^{4(a+bx)}(-2+5bx)\right)+48e^{2(a+bx)}\sqrt{1+e^{2(a+bx)}}\right.}{100b^2}$$

input
```
Integrate[(x*Sinh[a + b*x])/Sech[a + b*x]^(3/2),x]
```

output
```
(((1 + E^(2*(a + b*x)))*(2 + 5*b*x + 2*E^(2*(a + b*x))*(-12 + 5*b*x) + E^(
4*(a + b*x))*(-2 + 5*b*x)) + 48*E^(2*(a + b*x))*Sqrt[1 + E^(2*(a + b*x))]*
Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(2*(a + b*x))])*Sqrt[Sech[a + b*x]])/
(100*b^2*E^(3*(a + b*x)))
```

3.542.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5967, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{ 5967}$$

$$\frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}-\frac{2\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}dx}{5b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}-\frac{2\int \frac{1}{\csc\left(ia+ibx+\frac{\pi}{2}\right)^{5/2}}dx}{5b}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{3}{5}\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}dx+\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}+\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}}dx\right)}{5b}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{2x}{5b\text{sech}^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{3}{5}\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int\sqrt{\cosh(a+bx)}dx+\frac{2\sinh(a+bx)}{5b\text{sech}^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{5b\text{sech}^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{2\sinh(a+bx)}{5b\text{sech}^{\frac{3}{2}}(a+bx)}+\frac{3}{5}\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx\right)}{5b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2x}{5b\text{sech}^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{2\sinh(a+bx)}{5b\text{sech}^{\frac{3}{2}}(a+bx)}-\frac{6i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)|2\right)}{5b}\right)}{5b}$$

input
```
Int[(x*Sinh[a + b*x])/Sech[a + b*x]^(3/2),x]
```

output
```
(2*x)/(5*b*Sech[a + b*x]^(5/2)) - (2*((((-6*I)/5)*Sqrt[Cosh[a + b*x]]*Elli
pticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b + (2*Sinh[a + b*x])/(5*b*
Sech[a + b*x]^(3/2))))/(5*b)
```

3.542.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
 + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.542. $\int \frac{x\sinh(a+bx)}{\text{sech}^{\frac{3}{2}}(a+bx)}\,dx$

rule 5967
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sech[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.542.4 Maple [F]

$$\int \frac{x \sinh \left(b x+a\right)}{\operatorname{sech}\left(b x+a\right)^{\frac{3}{2}}}d x$$

input
```
int(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x)
```

output
```
int(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x)
```

3.542.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.542.6 Sympy [F]

$$\int \frac{x \sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \sinh \left(a+bx\right)}{\operatorname{sech}^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

input
```
integrate(x*sinh(b*x+a)/sech(b*x+a)**(3/2),x)
```

output
```
Integral(x*sinh(a + b*x)/sech(a + b*x)**(3/2), x)
```

3.542.7 Maxima [F]

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sinh(bx+a)}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*sinh(b*x + a)/sech(b*x + a)^(3/2), x)
```

3.542.8 Giac [F]

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sinh(bx+a)}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*sinh(b*x + a)/sech(b*x + a)^(3/2), x)
```

3.542.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sinh(a+b\,x)}{\left(\frac{1}{\cosh(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(3/2),x)
```

output
```
int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(3/2), x)
```

3.543 $\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx$

3.543.1 Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)} + \frac{20i\sqrt{\cosh(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{147b^2} - \frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{20\sinh(a+bx)}{147b^2\sqrt{\operatorname{sech}(a+bx)}}$$

output
```
2/7*x/b/sech(b*x+a)^(7/2)-4/49*sinh(b*x+a)/b^2/sech(b*x+a)^(5/2)-20/147*si
nh(b*x+a)/b^2/sech(b*x+a)^(1/2)+20/147*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cos
h(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2
)*sech(b*x+a)^(1/2)/b^2
```

3.543.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.87

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \frac{\sqrt{\operatorname{sech}(a+bx)}\Big(63bx+84bx\cosh(2(a+bx))+21bx\cosh(4(a+bx))+80i\sqrt{\cosh(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}i(a\right.}{588b^2}$$

input
```
Integrate[(x*Sinh[a + b*x])/Sech[a + b*x]^(5/2),x]
```

output
```
(Sqrt[Sech[a + b*x]]*(63*b*x + 84*b*x*Cosh[2*(a + b*x)] + 21*b*x*Cosh[4*(a
 + b*x)] + (80*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2] - 52*S
inh[2*(a + b*x)] - 6*Sinh[4*(a + b*x)]))/(588*b^2)
```

3.543.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5967, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh(a+bx)}{\text{sech}^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{5967}$$

$$\frac{2x}{7b\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{2\int \frac{1}{\text{sech}^{\frac{7}{2}}(a+bx)}dx}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{7b\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{2\int \frac{1}{\csc\left(ia+ibx+\frac{\pi}{2}\right)^{7/2}}dx}{7b}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2x}{7b\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{5}{7}\int \frac{1}{\text{sech}^{\frac{3}{2}}(a+bx)}dx+\frac{2\sinh(a+bx)}{7b\text{sech}^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{7b\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{2\sinh(a+bx)}{7b\text{sech}^{\frac{5}{2}}(a+bx)}+\frac{5}{7}\int \frac{1}{\csc\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}}dx\right)}{7b}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2x}{7b\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\text{sech}(a+bx)}dx+\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}\right)+\frac{2\sinh(a+bx)}{7b\text{sech}^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{7b\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{2\sinh(a+bx)}{7b\text{sech}^{\frac{5}{2}}(a+bx)}+\frac{5}{7}\left(\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}+\frac{1}{3}\int\sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}dx\right)\right)}{7b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2x}{7b\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int\frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}\right)+\frac{2\sinh(a+bx)}{7b\text{sech}^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{7b\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{2\sinh(a+bx)}{7b\text{sech}^{\frac{5}{2}}(a+bx)}+\frac{5}{7}\left(\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}+\frac{1}{3}\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int\frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx\right)\right)}{7b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x}{7b\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{2\sinh(a+bx)}{7b\text{sech}^{\frac{5}{2}}(a+bx)}+\frac{5}{7}\left(\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}\right)\right)}{7b}$$

input
```
Int[(x*Sinh[a + b*x])/Sech[a + b*x]^(5/2),x]
```

output
```
(2*x)/(7*b*Sech[a + b*x]^(7/2)) - (2*((2*Sinh[a + b*x])/(7*b*Sech[a + b*x]
^(5/2)) + (5*((((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2
]*Sqrt[Sech[a + b*x]])/b + (2*Sinh[a + b*x])/(3*b*Sqrt[Sech[a + b*x]])))/7
))/(7*b)
```

3.543. $\int \frac{x\sinh(a+bx)}{\text{sech}^{\frac{5}{2}}(a+bx)}\,dx$

3.543.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5967
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sech[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.543.4 Maple [F]

$$\int \frac{x \sinh (bx + a)}{\operatorname{sech} (bx + a)^{\frac{5}{2}}} dx$$

input
```
int(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x)
```

output
```
int(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x)
```

3.543.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.543.6 Sympy [F]

$$\int \frac{x \sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x \sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx$$

input
```
integrate(x*sinh(b*x+a)/sech(b*x+a)**(5/2),x)
```

output
```
Integral(x*sinh(a + b*x)/sech(a + b*x)**(5/2), x)
```

3.543.7 Maxima [F]

$$\int \frac{x \sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x \sinh(bx+a)}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*sinh(b*x + a)/sech(b*x + a)^(5/2), x)
```

3.543.8 Giac [F]

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx=\int \frac{x\sinh(bx+a)}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*sinh(b*x + a)/sech(b*x + a)^(5/2), x)
```

3.543.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx=\int \frac{x\sinh(a+b\,x)}{\left(\frac{1}{\cosh(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(5/2),x)
```

output
```
int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(5/2), x)
```

3.544 $\int x \cosh(a+bx) \sinh^{\frac{5}{2}}(a+bx) \, dx$

3.544.1 Optimal result

Integrand size = 18, antiderivative size = 121

$$\int x \cosh(a+bx) \sinh^{\frac{5}{2}}(a+bx) \, dx = \frac{20i \operatorname{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{147b^2\sqrt{\sinh(a+bx)}} + \frac{20\cosh(a+bx)\sqrt{\sinh(a+bx)}}{147b^2} - \frac{4\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{49b^2} + \frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}$$

output
```
-4/49*cosh(b*x+a)*sinh(b*x+a)^(5/2)/b^2+2/7*x*sinh(b*x+a)^(7/2)/b-20/147*I
*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ell
ipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b^2/si
nh(b*x+a)^(1/2)+20/147*cosh(b*x+a)*sinh(b*x+a)^(1/2)/b^2
```

3.544.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int x \cosh(a+bx) \sinh^{\frac{5}{2}}(a+bx) \, dx = \frac{63bx - 84bx\cosh(2(a+bx)) + 21bx\cosh(4(a+bx)) - 80i \operatorname{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+}}{588b^2\sqrt{\sinh(a+bx)}}$$

input
```
Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^(5/2),x]
```

output
```
(63*b*x - 84*b*x*Cosh[2*(a + b*x)] + 21*b*x*Cosh[4*(a + b*x)] - (80*I)*Ell
ipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + 52*Sinh[2
*(a + b*x)] - 6*Sinh[4*(a + b*x)])/(588*b^2*Sqrt[Sinh[a + b*x]])
```

3.544.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5895, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \ 5895$$

$$\frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int\sinh^{\frac{7}{2}}(a+bx)dx}{7b}$$

$$\Big\downarrow \ 3042$$

$$\frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int(-i\sin(ia+ibx))^{7/2}dx}{7b}$$

$$\Big\downarrow \ 3115$$

$$\frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\int\sinh^{\frac{3}{2}}(a+bx)dx\right)}{7b}$$

$$\Big\downarrow \ 3042$$

$$\frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\int(-i\sin(ia+ibx))^{3/2}dx\right)}{7b}$$

$$\Big\downarrow \ 3115$$

$$\frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{1}{3}\int\frac{1}{\sqrt{\sinh(a+bx)}}dx\right)\right)}{7b}$$

$$\Big\downarrow \ 3042$$

$$\frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{1}{3}\int\frac{1}{\sqrt{-i\sin(ia+ibx)}}dx\right)\right)}{7b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{\sqrt{i\sinh(a+bx)}\int\frac{1}{\sqrt{i\sinh(a+bx)}}dx}{3\sqrt{\sinh(a+bx)}}\right)\right)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{\sqrt{i\sinh(a+bx)}\int\frac{1}{\sqrt{\sin(ia+ibx)}}dx}{3\sqrt{\sinh(a+bx)}}\right)\right)}{7b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{\sinh(a+bx)}}\right)\right)}{7b}$$

input
```
Int[x*Cosh[a + b*x]*Sinh[a + b*x]^(5/2),x]
```

output
```
(2*x*Sinh[a + b*x]^(7/2))/(7*b) - (2*((-5*((((2*I)/3)*EllipticF[(I*a - Pi/
2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]]) + (2*Cosh[
a + b*x]*Sqrt[Sinh[a + b*x]])/(3*b)))/7 + (2*Cosh[a + b*x]*Sinh[a + b*x]^(
5/2))/(7*b)))/(7*b)
```

3.544. $\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx$

3.544.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.544.4 Maple [F]

$$\int x \cosh(bx+a) \sinh(bx+a)^{\frac{5}{2}} dx$$

input
```
int(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x)
```

output
```
int(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x)
```

3.544.5 Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.544.6 Sympy [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.544.7 Maxima [F]

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx = \int x\cosh(bx+a)\sinh(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)*sinh(b*x + a)^(5/2), x)
```

3.544.8 Giac [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,1,0]%%%} / %%%{1,[0,0,0,2]%%%} Error: Bad Argument Value
```

3.544.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx = \int x\cosh(a+b\,x)\,\sinh(a+b\,x)^{5/2}\,dx$$

input
```
int(x*cosh(a + b*x)*sinh(a + b*x)^(5/2),x)
```

output
```
int(x*cosh(a + b*x)*sinh(a + b*x)^(5/2), x)
```

3.545 $\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx$

3.545.1 Optimal result

Integrand size = 18, antiderivative size = 98

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx=-\frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{25b^2\sqrt{i\sinh(a+bx)}}-\frac{4\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{25b^2}+\frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}$$

output
```
-4/25*cosh(b*x+a)*sinh(b*x+a)^(3/2)/b^2+2/5*x*sinh(b*x+a)^(5/2)/b+12/25*I*
(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Elli
pticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b^2/(I*sinh
(b*x+a))^(1/2)
```

3.545.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.95 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.46

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx$$
$$=\frac{e^{-3(a+bx)}\left(\left(-1+e^{2(a+bx)}\right)\left(2+5bx+e^{2(a+bx)}(24-10bx)+e^{4(a+bx)}(-2+5bx)\right)+48e^{2(a+bx)}\sqrt{1-e^{2(a+bx)}}\right.}{50\sqrt{2}b^2\sqrt{-e^{-a-bx}+e^{a+bx}}}$$

input
```
Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^(3/2),x]
```

output
```
((-1 + E^(2*(a + b*x)))*(2 + 5*b*x + E^(2*(a + b*x))*(24 - 10*b*x) + E^(4*
(a + b*x))*(-2 + 5*b*x)) + 48*E^(2*(a + b*x))*Sqrt[1 - E^(2*(a + b*x))]*Hy
pergeometric2F1[-1/4, 1/2, 3/4, E^(2*(a + b*x))])/(50*Sqrt[2]*b^2*E^(3*(a
+ b*x))*Sqrt[-E^(-a - b*x) + E^(a + b*x)])
```

3.545.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5895, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{5895}$$

$$\frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int \sinh^{\frac{5}{2}}(a+bx)dx}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int (-i\sin(ia+ibx))^{5/2}dx}{5b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}-\frac{3}{5}\int \sqrt{\sinh(a+bx)}dx\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}-\frac{3}{5}\int \sqrt{-i\sin(ia+ibx)}dx\right)}{5b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}-\frac{3\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}dx}{5\sqrt{i\sinh(a+bx)}}\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}-\frac{3\sqrt{\sinh(a+bx)}\int\sqrt{\sin(ia+ibx)}dx}{5\sqrt{i\sinh(a+bx)}}\right)}{5b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}+\frac{6i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)|2\right)}{5b\sqrt{i\sinh(a+bx)}}\right)}{5b}$$

input
```
Int[x*Cosh[a + b*x]*Sinh[a + b*x]^(3/2),x]
```

output
```
(2*x*Sinh[a + b*x]^(5/2))/(5*b) - (2*((((6*I)/5)*EllipticE[(I*a - Pi/2 + I
*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]]) + (2*Cosh[a + b
*x]*Sinh[a + b*x]^(3/2))/(5*b)))/(5*b)
```

3.545.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.545. $\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx$

3.545.4 Maple [F]

$$\int x\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{\frac{3}{2}}dx$$

input
```
int(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x)
```

output
```
int(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x)
```

3.545.5 Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.545.6 Sympy [F]

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx=\int x\sinh^{\frac{3}{2}}\left(a+bx\right)\cosh\left(a+bx\right)dx$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)**(3/2),x)
```

output
```
Integral(x*sinh(a + b*x)**(3/2)*cosh(a + b*x), x)
```

3.545.7 Maxima [F]

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx = \int x\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)*sinh(b*x + a)^(3/2), x)
```

3.545.8 Giac [F]

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx = \int x\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)*sinh(b*x + a)^(3/2), x)
```

3.545.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx = \int x\cosh(a+b\,x)\,\sinh(a+b\,x)^{3/2}\,dx$$

input
```
int(x*cosh(a + b*x)*sinh(a + b*x)^(3/2),x)
```

output
```
int(x*cosh(a + b*x)*sinh(a + b*x)^(3/2), x)
```

3.546 $\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx$

3.546.1 Optimal result

Integrand size = 18, antiderivative size = 98

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx=-\frac{4i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{9b^2\sqrt{\sinh(a+bx)}}-\frac{4\cosh(a+bx)\sqrt{\sinh(a+bx)}}{9b^2}+\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}$$

output
```
2/3*x*sinh(b*x+a)^(3/2)/b+4/9*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/si
n(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2
))*(I*sinh(b*x+a))^(1/2)/b^2/sinh(b*x+a)^(1/2)-4/9*cosh(b*x+a)*sinh(b*x+a)
^(1/2)/b^2
```

3.546.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx=\frac{2\left(2i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+bx)}+3bx\sinh^2(a+bx)-\sinh(2(a+bx))\right)}{9b^2\sqrt{\sinh(a+bx)}}$$

input
```
Integrate[x*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]],x]
```

output
```
(2*((2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]
] + 3*b*x*Sinh[a + b*x]^2 - Sinh[2*(a + b*x)]))/(9*b^2*Sqrt[Sinh[a + b*x]]
)
```

3.546.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5895, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\sinh(a+bx)}\cosh(a+bx)\,dx$$

$$\Big\downarrow \; 5895$$

$$\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int \sinh^{\frac{3}{2}}(a+bx)dx}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int (-i\sin(ia+ibx))^{3/2}dx}{3b}$$

$$\Big\downarrow \; 3115$$

$$\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+bx)}}dx\right)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{1}{3}\int \frac{1}{\sqrt{-i\sin(ia+ibx)}}dx\right)}{3b}$$

$$\Big\downarrow \; 3121$$

$$\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}dx}{3\sqrt{\sinh(a+bx)}}\right)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{\sin(ia+ibx)}}dx}{3\sqrt{\sinh(a+bx)}}\right)}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{\sinh(a+bx)}}\right)}{3b}$$

input
```
Int[x*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]],x]
```

output
```
(-2*((((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]]
)/(b*Sqrt[Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(3*b)))/
(3*b) + (2*x*Sinh[a + b*x]^(3/2))/(3*b)
```

3.546.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.546.4 Maple [F]

$$\int x\cosh\left(bx+a\right)\sqrt{\sinh\left(bx+a\right)}dx$$

input
```
int(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x)
```

output
```
int(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x)
```

3.546.5 Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.546.6 Sympy [F]

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx = \int x\sqrt{\sinh\left(a+bx\right)}\cosh\left(a+bx\right)dx$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)**(1/2),x)
```

output
```
Integral(x*sqrt(sinh(a + b*x))*cosh(a + b*x), x)
```

3.546.7 Maxima [F]

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx = \int x\cosh{(bx+a)}\sqrt{\sinh{(bx+a)}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)*sqrt(sinh(b*x + a)), x)
```

3.546.8 Giac [F]

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx = \int x\cosh{(bx+a)}\sqrt{\sinh{(bx+a)}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)*sqrt(sinh(b*x + a)), x)
```

3.546.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx = \int x\cosh(a+b\,x)\,\sqrt{\sinh{(a+b\,x)}}\,dx$$

input
```
int(x*cosh(a + b*x)*sinh(a + b*x)^(1/2),x)
```

output
```
int(x*cosh(a + b*x)*sinh(a + b*x)^(1/2), x)
```

3.547 $\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx$

3.547.1 Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2x\sqrt{\sinh(a+bx)}}{b} + \frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b^2\sqrt{i\sinh(a+bx)}}$$

output
```
2*x*sinh(b*x+a)^(1/2)/b-4*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/
2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*s
inh(b*x+a)^(1/2)/b^2/(I*sinh(b*x+a))^(1/2)
```

3.547.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.67 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.62

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx$$
$$= \frac{(-\cosh(a+bx)+\sinh(a+bx))\left(-2(-2+bx)\sinh(a+bx)(\cosh(a+bx)+\sinh(a+bx))+4\sqrt{2}\,\text{Hyperg}\right.}{b^2\sqrt{\text{sin}}}$$

input
```
Integrate[(x*Cosh[a + b*x])/Sqrt[Sinh[a + b*x]],x]
```

output
```
((-Cosh[a + b*x] + Sinh[a + b*x])*(-2*(-2 + b*x)*Sinh[a + b*x]*(Cosh[a + b
*x] + Sinh[a + b*x]) + 4*Sqrt[2]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cosh[2*
(a + b*x)] + Sinh[2*(a + b*x)]]*Sqrt[-(Sinh[a + b*x]*(Cosh[a + b*x] + Sinh
[a + b*x]))]))/(b^2*Sqrt[Sinh[a + b*x]])
```

3.547.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5895, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}dx$$

$$\Big\downarrow \text{5895}$$

$$\frac{2x\sqrt{\sinh(a+bx)}}{b}-\frac{2\int \sqrt{\sinh(a+bx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sqrt{\sinh(a+bx)}}{b}-\frac{2\int \sqrt{-i\sin(ia+ibx)}dx}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2x\sqrt{\sinh(a+bx)}}{b}-\frac{2\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}dx}{b\sqrt{i\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sqrt{\sinh(a+bx)}}{b}-\frac{2\sqrt{\sinh(a+bx)}\int \sqrt{\sin(ia+ibx)}dx}{b\sqrt{i\sinh(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2x\sqrt{\sinh(a+bx)}}{b}+\frac{4i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)|2\right)}{b^2\sqrt{i\sinh(a+bx)}}$$

input
```
Int[(x*Cosh[a + b*x])/Sqrt[Sinh[a + b*x]],x]
```

output
```
(2*x*Sqrt[Sinh[a + b*x]])/b + ((4*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*
Sqrt[Sinh[a + b*x]])/(b^2*Sqrt[I*Sinh[a + b*x]])
```

3.547.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.547.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(92) = 184$.

Time = 0.23 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.23

method	result
risch	$\frac{(bx-2)\left(\mathrm{e}^{2bx+2a}-1\right)\sqrt{2}\,\mathrm{e}^{-bx-a}}{b^2\sqrt{\left(\mathrm{e}^{2bx+2a}-1\right)\mathrm{e}^{-bx-a}}}+\frac{2\left(\frac{2\,\mathrm{e}^{2bx+2a}-2}{\sqrt{\mathrm{e}^{bx+a}\left(\mathrm{e}^{2bx+2a}-1\right)}}-\frac{\sqrt{\mathrm{e}^{bx+a}+1}\,\sqrt{-2\,\mathrm{e}^{bx+a}+2}\,\sqrt{-\mathrm{e}^{bx+a}}\left(-2\,\mathrm{EllipticE}\left(\sqrt{\mathrm{e}^{bx+a}+1},\frac{\sqrt{2}}{2}\right)+\mathrm{Ellipt}\right.}{\sqrt{\mathrm{e}^{3bx+3a}-\mathrm{e}^{bx+a}}}\right.}{b^2\sqrt{\left(\mathrm{e}^{2bx+2a}-1\right)\mathrm{e}^{-bx-a}}}$

input
```
int(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x-2)*(exp(b*x+a)^2-1)/b^2*2^(1/2)/((exp(b*x+a)^2-1)/exp(b*x+a))^(1/2)/e
xp(b*x+a)+2/b^2*(2*(exp(b*x+a)^2-1)/(exp(b*x+a)*(exp(b*x+a)^2-1))^(1/2)-(e
xp(b*x+a)+1)^(1/2)*(-2*exp(b*x+a)+2)^(1/2)*(-exp(b*x+a))^(1/2)/(exp(b*x+a)
^3-exp(b*x+a))^(1/2)*(-2*EllipticE((exp(b*x+a)+1)^(1/2),1/2*2^(1/2))+Ellip
ticF((exp(b*x+a)+1)^(1/2),1/2*2^(1/2))))*2^(1/2)/((exp(b*x+a)^2-1)/exp(b*x
+a))^(1/2)*(exp(b*x+a)*(exp(b*x+a)^2-1))^(1/2)/exp(b*x+a)
```

3.547.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.547.6 Sympy [F]

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{x\cosh\left(a+bx\right)}{\sqrt{\sinh\left(a+bx\right)}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)**(1/2),x)
```

output
```
Integral(x*cosh(a + b*x)/sqrt(sinh(a + b*x)), x)
```

3.547.7 Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{x\cosh{(bx+a)}}{\sqrt{\sinh{(bx+a)}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)/sqrt(sinh(b*x + a)), x)
```

3.547.8 Giac [F]

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{x\cosh{(bx+a)}}{\sqrt{\sinh{(bx+a)}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)/sqrt(sinh(b*x + a)), x)
```

3.547.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{x\cosh(a+b\,x)}{\sqrt{\sinh{(a+b\,x)}}}\,dx$$

input
```
int((x*cosh(a + b*x))/sinh(a + b*x)^(1/2),x)
```

output
```
int((x*cosh(a + b*x))/sinh(a + b*x)^(1/2), x)
```

3.548 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$

3.548.1 Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx=-\frac{2x}{b\sqrt{\sinh(a+bx)}}-\frac{4i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{b^2\sqrt{\sinh(a+bx)}}$$

output
```
-2*x/b/sinh(b*x+a)^(1/2)+4*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1
/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*
(I*sinh(b*x+a))^(1/2)/b^2/sinh(b*x+a)^(1/2)
```

3.548.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx=\frac{-2bx+4i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+bx)}}{b^2\sqrt{\sinh(a+bx)}}$$

input
```
Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(3/2),x]
```

output
```
(-2*b*x + (4*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a
+ b*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```

3.548.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5895, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{5895}$$

$$\frac{2\int \frac{1}{\sqrt{\sinh(a+bx)}}dx}{b}-\frac{2x}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{b\sqrt{\sinh(a+bx)}}+\frac{2\int \frac{1}{\sqrt{-i\sin(ia+ibx)}}dx}{b}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2x}{b\sqrt{\sinh(a+bx)}}+\frac{2\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}dx}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{b\sqrt{\sinh(a+bx)}}+\frac{2\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{\sin(ia+ibx)}}dx}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2x}{b\sqrt{\sinh(a+bx)}}-\frac{4i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{b^2\sqrt{\sinh(a+bx)}}$$

input
```
Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(3/2),x]
```

output
```
(-2*x)/(b*Sqrt[Sinh[a + b*x]]) - ((4*I)*EllipticF[(I*a - Pi/2 + I*b*x)/2,
2]*Sqrt[I*Sinh[a + b*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```

3.548.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.548.4 Maple [F]

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{3}{2}}}dx$$

input
```
int(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x)
```

output
```
int(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x)
```

3.548.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.548.6 Sympy [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)**(3/2),x)
```

output
```
Integral(x*cosh(a + b*x)/sinh(a + b*x)**(3/2), x)
```

3.548.7 Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)/sinh(b*x + a)^(3/2), x)
```

3.548.8 Giac [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)/sinh(b*x + a)^(3/2), x)
```

3.548.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh(a+b\,x)}{\sinh\left(a+b\,x\right)^{3/2}}\,dx$$

input
```
int((x*cosh(a + b*x))/sinh(a + b*x)^(3/2),x)
```

output
```
int((x*cosh(a + b*x))/sinh(a + b*x)^(3/2), x)
```

3.549 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

3.549.1 Optimal result

Integrand size = 18, antiderivative size = 98

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}} - \frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{3b^2\sqrt{i\sinh(a+bx)}}$$

output
```
-2/3*x/b/sinh(b*x+a)^(3/2)-4/3*cosh(b*x+a)/b^2/sinh(b*x+a)^(1/2)+4/3*I*(si
n(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellipti
cE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b^2/(I*sinh(b*
x+a))^(1/2)
```

3.549.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\left(bx+2iE\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)(i\sinh(a+bx))^{3/2}+\sinh(2(a+bx))\right)}{3b^2\sinh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(5/2),x]
```

output
```
(-2*(b*x + (2*I)*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh[a + b
*x])^(3/2) + Sinh[2*(a + b*x)]))/(3*b^2*Sinh[a + b*x]^(3/2))
```

3.549.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5895, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{5895}$$

$$\frac{2\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}dx}{3b}-\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{(-i\sin(ia+ibx))^{3/2}}dx}{3b}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\left(\int \sqrt{\sinh(a+bx)}dx-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}\right)}{3b}-\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\int \sqrt{-i\sin(ia+ibx)}dx\right)}{3b}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\frac{\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}dx}{\sqrt{i\sinh(a+bx)}}\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\frac{\sqrt{\sinh(a+bx)}\int \sqrt{\sin(ia+ibx)}dx}{\sqrt{i\sinh(a+bx)}}\right)}{3b}$$

$$\downarrow 3119$$

$$-\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)|2\right)}{b\sqrt{i\sinh(a+bx)}}\right)}{3b}$$

input
```
Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(5/2),x]
```

output
```
(2*((-2*Cosh[a + b*x])/(b*Sqrt[Sinh[a + b*x]]) - ((2*I)*EllipticE[(I*a - P
i/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])))/(3*b)
- (2*x)/(3*b*Sinh[a + b*x]^(3/2))
```

3.549.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.549. $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

3.549.4 Maple [F]

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{5}{2}}}dx$$

input
```
int(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x)
```

output
```
int(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x)
```

3.549.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.549.6 Sympy [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)**(5/2),x)
```

output
```
Integral(x*cosh(a + b*x)/sinh(a + b*x)**(5/2), x)
```

3.549.7 Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)/sinh(b*x + a)^(5/2), x)
```

3.549.8 Giac [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)/sinh(b*x + a)^(5/2), x)
```

3.549.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cosh(a+b\,x)}{\sinh(a+b\,x)^{5/2}}\,dx$$

input
```
int((x*cosh(a + b*x))/sinh(a + b*x)^(5/2),x)
```

output
```
int((x*cosh(a + b*x))/sinh(a + b*x)^(5/2), x)
```

3.550 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

3.550.1 Optimal result

Integrand size = 18, antiderivative size = 98

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)} + \frac{4i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{15b^2\sqrt{\sinh(a+bx)}}$$

output
```
-2/5*x/b/sinh(b*x+a)^(5/2)-4/15*cosh(b*x+a)/b^2/sinh(b*x+a)^(3/2)-4/15*I*(
sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellip
ticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b^2/sinh
(b*x+a)^(1/2)
```

3.550.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\left(3bx-2i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)(i\sinh(a+bx))^{5/2}+\sinh(2(a+bx))\right)}{15b^2\sinh^{\frac{5}{2}}(a+bx)}$$

input
```
Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(7/2),x]
```

output
```
(-2*(3*b*x - (2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh[a +
 b*x])^(5/2) + Sinh[2*(a + b*x)]))/(15*b^2*Sinh[a + b*x]^(5/2))
```

3.550.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5895, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{5895}$$

$$\frac{2\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}dx}{5b}-\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{2\int \frac{1}{(-i\sin(ia+ibx))^{5/2}}dx}{5b}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\left(-\frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+bx)}}dx-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}\right)}{5b}-\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{1}{3}\int \frac{1}{\sqrt{-i\sin(ia+ibx)}}dx\right)}{5b}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}dx}{3\sqrt{\sinh(a+bx)}}\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{i\sinh(a+bx)}\int\frac{1}{\sqrt{\sin(ia+ibx)}}dx}{3\sqrt{\sinh(a+bx)}}\right)}{5b}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2i\sqrt{i\sinh(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{\sinh(a+bx)}}\right)}{5b}$$

input
```
Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(7/2),x]
```

output
```
(2*((-2*Cosh[a + b*x])/(3*b*Sinh[a + b*x]^(3/2)) + (((2*I)/3)*EllipticF[(I
*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])))/
(5*b) - (2*x)/(5*b*Sinh[a + b*x]^(5/2))
```

3.550.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.550. $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

3.550.4 Maple [F]

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{7}{2}}}dx$$

input
```
int(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x)
```

output
```
int(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x)
```

3.550.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.550.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)**(7/2),x)
```

output
```
Timed out
```

3.550.7 Maxima [F]

$$\int \frac{x \cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x \cosh(bx+a)}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)/sinh(b*x + a)^(7/2), x)
```

3.550.8 Giac [F]

$$\int \frac{x \cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x \cosh(bx+a)}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)/sinh(b*x + a)^(7/2), x)
```

3.550.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x \cosh(a+b\,x)}{\sinh(a+b\,x)^{7/2}}\,dx$$

input
```
int((x*cosh(a + b*x))/sinh(a + b*x)^(7/2),x)
```

output
```
int((x*cosh(a + b*x))/sinh(a + b*x)^(7/2), x)
```

3.551 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx$

3.551.1 Optimal result

Integrand size = 18, antiderivative size = 121

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)}+\frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}}+\frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{35b^2\sqrt{i\sinh(a+bx)}}$$

output
```
-2/7*x/b/sinh(b*x+a)^(7/2)-4/35*cosh(b*x+a)/b^2/sinh(b*x+a)^(5/2)+12/35*co
sh(b*x+a)/b^2/sinh(b*x+a)^(1/2)-12/35*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^
(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x
),2^(1/2))*sinh(b*x+a)^(1/2)/b^2/(I*sinh(b*x+a))^(1/2)
```

3.551.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = -\frac{2\left(5bx-6\cosh(a+bx)\sinh^3(a+bx)+6E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}\sinh^3(a+bx)+\mathrm{s}\right.}{35b^2\sinh^{\frac{7}{2}}(a+bx)}$$

input
```
Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(9/2),x]
```

output
```
(-2*(5*b*x - 6*Cosh[a + b*x]*Sinh[a + b*x]^3 + 6*EllipticE[((-2*I)*a + Pi
- (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]*Sinh[a + b*x]^3 + Sinh[2*(a + b*x
)]))/(35*b^2*Sinh[a + b*x]^(7/2))
```

3.551.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5895, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{5895}$$

$$\frac{2\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}dx}{7b}-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}+\frac{2\int \frac{1}{(-i\sin(ia+ibx))^{7/2}}dx}{7b}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\left(-\frac{3}{5}\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}dx-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{3}{5}\int \frac{1}{(-i\sin(ia+ibx))^{3/2}}dx\right)}{7b}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\left(-\frac{3}{5}\left(\int \sqrt{\sinh(a+bx)}dx-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}\right)-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{3}{5}\left(-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\int\sqrt{-i\sin(ia+ibx)}dx\right)\right)}{7b}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{3}{5}\left(-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\frac{\sqrt{\sinh(a+bx)}\int\sqrt{i\sinh(a+bx)}dx}{\sqrt{i\sinh(a+bx)}}\right)\right)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{3}{5}\left(-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\frac{\sqrt{\sinh(a+bx)}\int\sqrt{\sin(ia+ibx)}dx}{\sqrt{i\sinh(a+bx)}}\right)\right)}{7b}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}+\frac{2\left(-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{3}{5}\left(-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)|2\right)}{b\sqrt{i\sinh(a+bx)}}\right)\right)}{7b}$$

input
```
Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(9/2),x]
```

output
```
(2*((-3*((-2*Cosh[a + b*x])/(b*Sqrt[Sinh[a + b*x]]) - ((2*I)*EllipticE[(I*
a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])))/5
 - (2*Cosh[a + b*x])/(5*b*Sinh[a + b*x]^(5/2))))/(7*b) - (2*x)/(7*b*Sinh[a
 + b*x]^(7/2))
```

3.551.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))    I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

3.551. $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx$

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.551.4 Maple [F]

$$\int \frac{x\cosh\left(bx+a\right)}{\sinh\left(bx+a\right)^{\frac{9}{2}}}dx$$

input
```
int(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x)
```

output
```
int(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x)
```

3.551.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.551.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)**(9/2),x)
```

output
```
Timed out
```

3.551.7 Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)/sinh(b*x + a)^(9/2), x)
```

3.551.8 Giac [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)/sinh(b*x + a)^(9/2), x)
```

3.551.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\cosh(a+b\,x)}{\sinh\left(a+b\,x\right)^{9/2}}\,dx$$

input
```
int((x*cosh(a + b*x))/sinh(a + b*x)^(9/2),x)
```

output
```
int((x*cosh(a + b*x))/sinh(a + b*x)^(9/2), x)
```

3.552 $\int x \cosh(a+bx)\operatorname{csch}^{\frac{9}{2}}(a+bx)\,dx$

3.552.1 Optimal result

Integrand size = 18, antiderivative size = 121

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{9}{2}}(a+bx)\,dx=\frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2}-\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\operatorname{csch}^{\frac{7}{2}}(a+bx)}{7b}+\frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{35b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

output
```
-4/35*cosh(b*x+a)*csch(b*x+a)^(5/2)/b^2-2/7*x*csch(b*x+a)^(7/2)/b+12/35*co
sh(b*x+a)*csch(b*x+a)^(1/2)/b^2-12/35*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^
(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x
),2^(1/2))/b^2/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

3.552.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{9}{2}}(a+bx)\,dx = -\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(-6\cosh(a+bx)+6E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}+\operatorname{csch}^3(a+bx)(5bx\right.}{35b^2}$$

input
```
Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(9/2),x]
```

output
```
(-2*Sqrt[Csch[a + b*x]]*(-6*Cosh[a + b*x] + 6*EllipticE[((-2*I)*a + Pi - (
2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + Csch[a + b*x]^3*(5*b*x + Sinh[2*(a
 + b*x)])))/(35*b^2)
```

3.552.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5968, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh(a+bx)\text{csch}^{\frac{9}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{5968}$$

$$\frac{2\int \text{csch}^{\frac{7}{2}}(a+bx)dx}{7b}-\frac{2x\text{csch}^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{csch}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int (i\csc(ia+ibx))^{7/2}dx}{7b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2\left(-\frac{3}{5}\int \text{csch}^{\frac{3}{2}}(a+bx)dx-\frac{2\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}-\frac{2x\text{csch}^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{csch}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\left(-\frac{2\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)}{5b}-\frac{3}{5}\int (i\csc(ia+ibx))^{3/2}dx\right)}{7b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2\left(-\frac{3}{5}\left(\int \frac{1}{\sqrt{\text{csch}(a+bx)}}dx-\frac{2\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{b}\right)-\frac{2\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}-\frac{2x\text{csch}^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{csch}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\left(-\frac{2\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)}{5b}-\frac{3}{5}\left(-\frac{2\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{b}+\int\frac{1}{\sqrt{i\csc(ia+ibx)}}dx\right)\right)}{7b}$$

$\downarrow$ 4258

$$-\frac{2x\text{csch}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\left(-\frac{2\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)}{5b}-\frac{3}{5}\left(-\frac{2\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{b}+\frac{\int\sqrt{i\sinh(a+bx)}dx}{\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}\right)\right)}{7b}$$

$\downarrow$ 3042

$$-\frac{2x\text{csch}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\left(-\frac{2\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)}{5b}-\frac{3}{5}\left(-\frac{2\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{b}+\frac{\int\sqrt{\sin(ia+ibx)}dx}{\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}\right)\right)}{7b}$$

$\downarrow$ 3119

$$-\frac{2x\text{csch}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\left(-\frac{2\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)}{5b}-\frac{3}{5}\left(-\frac{2\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}\right)\right)}{7b}$$

input
```
Int[x*Cosh[a + b*x]*Csch[a + b*x]^(9/2),x]
```

output
```
(-2*x*Csch[a + b*x]^(7/2))/(7*b) + (2*((-2*Cosh[a + b*x]*Csch[a + b*x]^(5/
2))/(5*b) - (3*((-2*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/b - ((2*I)*Elliptic
E[(I*a - Pi/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]]
)))/5))/(7*b)
```

3.552. $\int x\cosh(a+bx)\text{csch}^{\frac{9}{2}}(a+bx)\,dx$

3.552.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5968
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csch[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.552.4 Maple [F]

$$\int x\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)^{\frac{9}{2}}dx$$

input
```
int(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x)
```

output
```
int(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x)
```

3.552.5 Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\text{csch}^{\frac{9}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.552.6 Sympy [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{csch}^{\frac{9}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)**(9/2),x)
```

output
```
Timed out
```

3.552.7 Maxima [F]

$$\int x\cosh(a+bx)\text{csch}^{\frac{9}{2}}(a+bx)\,dx = \int x\cosh(bx+a)\,\text{csch}\,(bx+a)^{\frac{9}{2}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)*csch(b*x + a)^(9/2), x)
```

3.552.8 Giac [F]

$$\int x\cosh(a+bx)\text{csch}^{\frac{9}{2}}(a+bx)\,dx=\int x\cosh(bx+a)\,\text{csch}\,(bx+a)^{\frac{9}{2}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)*csch(b*x + a)^(9/2), x)
```

3.552.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{csch}^{\frac{9}{2}}(a+bx)\,dx=\int x\cosh(a+b\,x)\,\left(\frac{1}{\sinh\left(a+b\,x\right)}\right)^{9/2}\,dx$$

input
```
int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(9/2),x)
```

output
```
int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(9/2), x)
```

3.553 $\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx$

3.553.1 Optimal result

Integrand size = 18, antiderivative size = 98

$$\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx = -\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}+\frac{4i\sqrt{\text{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{15b^2}$$

output
```
-4/15*cosh(b*x+a)*csch(b*x+a)^(3/2)/b^2-2/5*x*csch(b*x+a)^(5/2)/b-4/15*I*(
sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellip
ticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+
a))^(1/2)/b^2
```

3.553.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx = -\frac{2\sqrt{\text{csch}(a+bx)}\left(2\coth(a+bx)+3bx\text{csch}^2(a+bx)+2i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+\cdots}\right.}{15b^2}$$

input
```
Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(7/2),x]
```

output
```
(-2*Sqrt[Csch[a + b*x]]*(2*Coth[a + b*x] + 3*b*x*Csch[a + b*x]^2 + (2*I)*E
llipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(15*b^2
)
```

3.553.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5968, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{5968}$$

$$\frac{2\int \text{csch}^{\frac{5}{2}}(a+bx)dx}{5b}-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int (i\csc(ia+ibx))^{5/2}dx}{5b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2\left(-\frac{1}{3}\int \sqrt{\text{csch}(a+bx)}dx-\frac{2\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\left(-\frac{2\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{1}{3}\int \sqrt{i\csc(ia+ibx)}dx\right)}{5b}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\left(-\frac{2\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{1}{3}\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}dx\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\left(-\frac{2\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{1}{3}\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}\int\frac{1}{\sqrt{\sin(ia+ibx)}}dx\right)}{5b}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\left(-\frac{2\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b}\right)}{5b}$$

input
```
Int[x*Cosh[a + b*x]*Csch[a + b*x]^(7/2),x]
```

output
```
(-2*x*Csch[a + b*x]^(5/2))/(5*b) + (2*((-2*Cosh[a + b*x]*Csch[a + b*x]^(3/
2))/(3*b) + (((2*I)/3)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/
2, 2]*Sqrt[I*Sinh[a + b*x]])/b))/(5*b)
```

3.553.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.553. $\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx$

rule 5968
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csch[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.553.4 Maple [F]

$$\int x \cosh(bx+a) \operatorname{csch}(bx+a)^{\frac{7}{2}} dx$$

input
```
int(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x)
```

output
```
int(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x)
```

3.553.5 Fricas [F(-2)]

Exception generated.

$$\int x \cosh(a+bx)\operatorname{csch}^{\frac{7}{2}}(a+bx)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.553.6 Sympy [F(-1)]

Timed out.

$$\int x \cosh(a+bx)\operatorname{csch}^{\frac{7}{2}}(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)**(7/2),x)
```

output
```
Timed out
```

3.553.7 Maxima [F]

$$\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx=\int x\cosh(bx+a)\,\text{csch}\,(bx+a)^{\frac{7}{2}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)*csch(b*x + a)^(7/2), x)
```

3.553.8 Giac [F]

$$\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx=\int x\cosh(bx+a)\,\text{csch}\,(bx+a)^{\frac{7}{2}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)*csch(b*x + a)^(7/2), x)
```

3.553.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx=\int x\cosh(a+b\,x)\,\left(\frac{1}{\sinh(a+b\,x)}\right)^{7/2}\,dx$$

input
```
int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(7/2),x)
```

output
```
int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(7/2), x)
```

3.554 $\int x\cosh(a+bx)\mathrm{csch}^{\frac{5}{2}}(a+bx)\,dx$

3.554.1 Optimal result

Integrand size = 18, antiderivative size = 98

$$\int x\cosh(a+bx)\mathrm{csch}^{\frac{5}{2}}(a+bx)\,dx=-\frac{4\cosh(a+bx)\sqrt{\mathrm{csch}(a+bx)}}{3b^2}-\frac{2x\mathrm{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{3b^2\sqrt{\mathrm{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

output
```
-2/3*x*csch(b*x+a)^(3/2)/b-4/3*cosh(b*x+a)*csch(b*x+a)^(1/2)/b^2+4/3*I*(si
n(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellipti
cE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b^2/csch(b*x+a)^(1/2)/(I*sinh(b*
x+a))^(1/2)
```

3.554.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int x\cosh(a+bx)\mathrm{csch}^{\frac{5}{2}}(a+bx)\,dx=-\frac{2\sqrt{\mathrm{csch}(a+bx)}\left(2\cosh(a+bx)+bx\mathrm{csch}(a+bx)-2E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}\right)}{3b^2}$$

input
```
Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(5/2),x]
```

output
```
(-2*Sqrt[Csch[a + b*x]]*(2*Cosh[a + b*x] + b*x*Csch[a + b*x] - 2*EllipticE
[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(3*b^2)
```

3.554.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5968, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{ 5968}$$

$$\frac{2\int \text{csch}^{\frac{3}{2}}(a+bx)dx}{3b}-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int (i\csc(ia+ibx))^{3/2}dx}{3b}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{2\left(\int \frac{1}{\sqrt{\text{csch}(a+bx)}}dx-\frac{2\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{b}\right)}{3b}-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\left(-\frac{2\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{b}+\int \frac{1}{\sqrt{i\csc(ia+ibx)}}dx\right)}{3b}$$

$$\Big\downarrow \text{ 4258}$$

$$-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\left(-\frac{2\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{b}+\frac{\int \sqrt{i\sinh(a+bx)}dx}{\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}\right)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\left(-\frac{2\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{b}+\frac{\int \sqrt{\sin(ia+ibx)}dx}{\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}\right)}{3b}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\left(-\frac{2\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}\right)}{3b}$$

input
```
Int[x*Cosh[a + b*x]*Csch[a + b*x]^(5/2),x]
```

output
```
(-2*x*Csch[a + b*x]^(3/2))/(3*b) + (2*((-2*Cosh[a + b*x]*Sqrt[Csch[a + b*x
]])/b - ((2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]
]*Sqrt[I*Sinh[a + b*x]])))/(3*b)
```

3.554.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5968
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csch[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.554. $\int x\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)\,dx$

3.554.4 Maple [F]

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)^{\frac{5}{2}}\,dx$$

input
```
int(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x)
```

output
```
int(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x)
```

3.554.5 Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.554.6 Sympy [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.554.7 Maxima [F]

$$\int x\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)\,dx=\int x\cosh(bx+a)\,\text{csch}\,(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)*csch(b*x + a)^(5/2), x)
```

3.554.8 Giac [F]

$$\int x\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)\,dx=\int x\cosh(bx+a)\,\text{csch}\,(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)*csch(b*x + a)^(5/2), x)
```

3.554.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)\,dx=\int x\cosh(a+b\,x)\,\left(\frac{1}{\sinh(a+b\,x)}\right)^{5/2}\,dx$$

input
```
int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(5/2),x)
```

output
```
int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(5/2), x)
```

3.555 $\int x \cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx$

3.555.1 Optimal result

Integrand size = 18, antiderivative size = 71

$$\int x \cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = -\frac{2x\sqrt{\operatorname{csch}(a+bx)}}{b} - \frac{4i\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{b^2}$$

output
```
-2*x*csch(b*x+a)^(1/2)/b+4*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1
/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*
csch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1/2)/b^2
```

3.555.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int x \cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = -\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(bx-2i\operatorname{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+bx)}\right)}{b^2}$$

input
```
Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(3/2),x]
```

output
```
(-2*Sqrt[Csch[a + b*x]]*(b*x - (2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)
/4, 2]*Sqrt[I*Sinh[a + b*x]]))/b^2
```

3.555.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5968, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{5968}$$

$$\frac{2\int\sqrt{\text{csch}(a+bx)}dx}{b}-\frac{2x\sqrt{\text{csch}(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\sqrt{\text{csch}(a+bx)}}{b}+\frac{2\int\sqrt{i\csc(ia+ibx)}dx}{b}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{2x\sqrt{\text{csch}(a+bx)}}{b}+\frac{2\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}\int\frac{1}{\sqrt{i\sinh(a+bx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2x\sqrt{\text{csch}(a+bx)}}{b}+\frac{2\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}\int\frac{1}{\sqrt{\sin(ia+ibx)}}dx}{b}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2x\sqrt{\text{csch}(a+bx)}}{b}-\frac{4i\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{b^2}$$

input
```
Int[x*Cosh[a + b*x]*Csch[a + b*x]^(3/2),x]
```

output
```
(-2*x*Sqrt[Csch[a + b*x]])/b - ((4*I)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a -
 Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b^2
```

3.555.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5968
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csch[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.555.4 Maple [F]

$$\int x\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)^{\frac{3}{2}}dx$$

input
```
int(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x)
```

output
```
int(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x)
```

3.555.5 Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

3.555.6 Sympy [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)**(3/2),x)
```

output
```
Timed out
```

3.555.7 Maxima [F]

$$\int x\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)\,dx = \int x\cosh(bx+a)\,\text{csch}(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)*csch(b*x + a)^(3/2), x)
```

3.555.8 Giac [F]

$$\int x\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)\,dx = \int x\cosh(bx+a)\,\text{csch}(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)*csch(b*x + a)^(3/2), x)
```

3.555.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)\,dx=\int x\cosh(a+b\,x)\left(\frac{1}{\sinh(a+b\,x)}\right)^{3/2}dx$$

input
```
int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(3/2),x)
```

output
```
int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(3/2), x)
```

3.556 $\int x \cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx$

3.556.1 Optimal result

Integrand size = 18, antiderivative size = 71

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx=\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}+\frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

output
```
2*x/b/csch(b*x+a)^(1/2)-4*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/
2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b
^2/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

3.556.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.90 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx$$
$$=\frac{\sqrt{2}e^{-a-bx}\sqrt{\frac{e^{a+bx}}{-1+e^{2(a+bx)}}}\left(\left(-1+e^{2(a+bx)}\right)(-2+bx)-4\sqrt{1-e^{2(a+bx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},e^{2(a+bx}\right.\right.}{b^2}$$

input
```
Integrate[x*Cosh[a + b*x]*Sqrt[Csch[a + b*x]],x]
```

output
```
(Sqrt[2]*E^(-a - b*x)*Sqrt[E^(a + b*x)/(-1 + E^(2*(a + b*x)))]*((-1 + E^(2
*(a + b*x)))*(-2 + b*x) - 4*Sqrt[1 - E^(2*(a + b*x))]*Hypergeometric2F1[-1
/4, 1/2, 3/4, E^(2*(a + b*x))]))/b^2
```

3.556.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5968, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx$$

$$\Big\downarrow \text{ 5968}$$

$$\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}-\frac{2\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}-\frac{2\int \frac{1}{\sqrt{i\csc(ia+ibx)}}dx}{b}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}-\frac{2\int \sqrt{i\sinh(a+bx)}dx}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}-\frac{2\int \sqrt{\sin(ia+ibx)}dx}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}+\frac{4iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

input
```
Int[x*Cosh[a + b*x]*Sqrt[Csch[a + b*x]],x]
```

output
```
(2*x)/(b*Sqrt[Csch[a + b*x]]) + ((4*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2
])/(b^2*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

3.556.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5968
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csch[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.556.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(92) = 184$.

Time = 0.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.23

method	result
risch	$\frac{(bx-2)\left(e^{2bx+2a}-1\right)\sqrt{2}\sqrt{\frac{e^{bx+a}}{e^{2bx+2a}-1}}\, e^{-bx-a}}{b^2}+\frac{2\left(\frac{2\,e^{2bx+2a}-2}{\sqrt{e^{bx+a}\left(e^{2bx+2a}-1\right)}}-\frac{\sqrt{e^{bx+a}+1}\,\sqrt{-2\,e^{bx+a}+2}\,\sqrt{-e^{bx+a}}\left(-2\,\text{EllipticE}\left(\sqrt{e^{bx+a}\dots}\right.\right.}{\sqrt{e^{3bx+3a}-e^{bx+a}}}\right.}{}$

input
```
int(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x-2)*(exp(b*x+a)^2-1)/b^2*2^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2-1))^(1/2)/e
xp(b*x+a)+2/b^2*(2*(exp(b*x+a)^2-1)/(exp(b*x+a)*(exp(b*x+a)^2-1))^(1/2)-(e
xp(b*x+a)+1)^(1/2)*(-2*exp(b*x+a)+2)^(1/2)*(-exp(b*x+a))^(1/2)/(exp(b*x+a)
^3-exp(b*x+a))^(1/2)*(-2*EllipticE((exp(b*x+a)+1)^(1/2),1/2*2^(1/2))+Ellip
ticF((exp(b*x+a)+1)^(1/2),1/2*2^(1/2))))*2^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2
-1))^(1/2)*(exp(b*x+a)*(exp(b*x+a)^2-1))^(1/2)/exp(b*x+a)
```

3.556.5 Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.556.6 Sympy [F]

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx = \int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)**(1/2),x)
```

output
```
Integral(x*cosh(a + b*x)*sqrt(csch(a + b*x)), x)
```

3.556.7 Maxima [F]

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx = \int x\cosh(bx+a)\sqrt{\operatorname{csch}(bx+a)}\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)*sqrt(csch(b*x + a)), x)
```

3.556.8 Giac [F]

$$\int x \cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx = \int x \cosh(bx+a)\sqrt{\operatorname{csch}(bx+a)}\,dx$$

input
```
integrate(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)*sqrt(csch(b*x + a)), x)
```

3.556.9 Mupad [F(-1)]

Timed out.

$$\int x \cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx = \int x \cosh(a+b\,x)\,\sqrt{\frac{1}{\sinh(a+b\,x)}}\,dx$$

input
```
int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(1/2),x)
```

output
```
int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(1/2), x)
```

3.557 $\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$

3.557.1 Optimal result

Integrand size = 18, antiderivative size = 98

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}} - \frac{4i\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{9b^2}$$

output
```
2/3*x/b/csch(b*x+a)^(3/2)-4/9*cosh(b*x+a)/b^2/csch(b*x+a)^(1/2)+4/9*I*(sin
(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Elliptic
F(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))
^(1/2)/b^2
```

3.557.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \frac{6bx-4\coth(a+bx)-\frac{4i\operatorname{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)}{(i\sinh(a+bx))^{3/2}}}{9b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[(x*Cosh[a + b*x])/Sqrt[Csch[a + b*x]],x]
```

output
```
(6*b*x - 4*Coth[a + b*x] - ((4*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4,
 2])/(I*Sinh[a + b*x])^(3/2))/(9*b^2*Csch[a + b*x]^(3/2))
```

3.557.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5968, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}dx$$

$$\Big\downarrow \text{5968}$$

$$\frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{(i\csc(ia+ibx))^{3/2}}dx}{3b}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\int \sqrt{\operatorname{csch}(a+bx)}dx\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\int \sqrt{i\csc(ia+ibx)}dx\right)}{3b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}dx\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\int \frac{1}{\sqrt{\sin(ia+ibx)}}dx\right)}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}+\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b}\right)}{3b}$$

input
```
Int[(x*Cosh[a + b*x])/Sqrt[Csch[a + b*x]],x]
```

output
```
(2*x)/(3*b*Csch[a + b*x]^(3/2)) - (2*((2*Cosh[a + b*x])/(3*b*Sqrt[Csch[a +
 b*x]]) + (((2*I)/3)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2,
 2]*Sqrt[I*Sinh[a + b*x]])/b))/(3*b)
```

3.557.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5968
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csch[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.557. $\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$

3.557.4 Maple [F]

$$\int \frac{x \cosh(bx+a)}{\sqrt{\operatorname{csch}(bx+a)}}dx$$

input
```
int(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x)
```

output
```
int(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x)
```

3.557.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.557.6 Sympy [F]

$$\int \frac{x \cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{x \cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/csch(b*x+a)**(1/2),x)
```

output
```
Integral(x*cosh(a + b*x)/sqrt(csch(a + b*x)), x)
```

3.557.7 Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{x\cosh(bx+a)}{\sqrt{\operatorname{csch}(bx+a)}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)/sqrt(csch(b*x + a)), x)
```

3.557.8 Giac [F]

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{x\cosh(bx+a)}{\sqrt{\operatorname{csch}(bx+a)}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)/sqrt(csch(b*x + a)), x)
```

3.557.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{x\cosh(a+b\,x)}{\sqrt{\frac{1}{\sinh(a+b\,x)}}}\,dx$$

input
```
int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(1/2),x)
```

output
```
int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(1/2), x)
```

3.558 $\int \frac{x \cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)} dx$

3.558.1 Optimal result

Integrand size = 18, antiderivative size = 98

$$\int \frac{x \cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)} dx = \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{25b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

output
```
2/5*x/b/csch(b*x+a)^(5/2)-4/25*cosh(b*x+a)/b^2/csch(b*x+a)^(3/2)+12/25*I*(
sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellip
ticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b^2/csch(b*x+a)^(1/2)/(I*sinh(
b*x+a))^(1/2)
```

3.558.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

$$\int \frac{x \cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)} dx = \frac{e^{-2(a+bx)}\left(2+5bx+e^{2(a+bx)}(24-10bx)+e^{4(a+bx)}(-2+5bx)-\frac{48e^{2(a+bx)}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},e^{2(a+bx)}\right)}{\sqrt{1-e^{2(a+bx)}}}\right)}{50b^2\sqrt{\operatorname{csch}(a+bx)}}$$

input
```
Integrate[(x*Cosh[a + b*x])/Csch[a + b*x]^(3/2),x]
```

output
```
(2 + 5*b*x + E^(2*(a + b*x))*(24 - 10*b*x) + E^(4*(a + b*x))*(-2 + 5*b*x)
- (48*E^(2*(a + b*x))*Hypergeometric2F1[-1/4, 1/2, 3/4, E^(2*(a + b*x))])/
Sqrt[1 - E^(2*(a + b*x))])/(50*b^2*E^(2*(a + b*x))*Sqrt[Csch[a + b*x]])
```

3.558.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5968, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)} \, dx$$

$$\Big\downarrow \text{5968}$$

$$\frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}dx}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{(i\csc(ia+ibx))^{5/2}}dx}{5b}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{3}{5}\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}dx\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{3}{5}\int \frac{1}{\sqrt{i\csc(ia+ibx)}}dx\right)}{5b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2x}{5b\text{csch}^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}-\frac{3\int\sqrt{i\sinh(a+bx)}dx}{5\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{5b\text{csch}^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}-\frac{3\int\sqrt{\sin(ia+ibx)}dx}{5\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}\right)}{5b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2x}{5b\text{csch}^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}+\frac{6iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)|2\right)}{5b\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}\right)}{5b}$$

input
```
Int[(x*Cosh[a + b*x])/Csch[a + b*x]^(3/2),x]
```

output
```
(2*x)/(5*b*Csch[a + b*x]^(5/2)) - (2*((2*Cosh[a + b*x])/(5*b*Csch[a + b*x]
^(3/2)) + (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a
+ b*x]]*Sqrt[I*Sinh[a + b*x]])))/(5*b)
```

3.558.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

3.558. $\int \frac{x\cosh(a+bx)}{\text{csch}^{\frac{3}{2}}(a+bx)}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5968
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csch[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.558.4 Maple [F]

$$\int \frac{x\cosh\left(bx+a\right)}{\operatorname{csch}\left(bx+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x)
```

output
```
int(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x)
```

3.558.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.558.6 Sympy [F]

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh\left(a+bx\right)}{\operatorname{csch}^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

input
```
integrate(x*cosh(b*x+a)/csch(b*x+a)**(3/2),x)
```

output
```
Integral(x*cosh(a + b*x)/csch(a + b*x)**(3/2), x)
```

3.558.7 Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh\left(bx+a\right)}{\operatorname{csch}\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)/csch(b*x + a)^(3/2), x)
```

3.558.8 Giac [F]

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh\left(bx+a\right)}{\operatorname{csch}\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)/csch(b*x + a)^(3/2), x)
```

3.558.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{x\cosh(a+b\,x)}{\left(\frac{1}{\sinh(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(3/2),x)
```

output
```
int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(3/2), x)
```

3.559 $\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx$

3.559.1 Optimal result

Integrand size = 18, antiderivative size = 121

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}} + \frac{20i\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{147b^2}$$

output
```
2/7*x/b/csch(b*x+a)^(7/2)-4/49*cosh(b*x+a)/b^2/csch(b*x+a)^(5/2)+20/147*co
sh(b*x+a)/b^2/csch(b*x+a)^(1/2)-20/147*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)
^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*
x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1/2)/b^2
```

3.559.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \frac{\sqrt{\operatorname{csch}(a+bx)}\Big(63bx - 84bx\cosh(2(a+bx)) + 21bx\cosh(4(a+bx)) - 80i\operatorname{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx\right.}{588b^2}$$

input
```
Integrate[(x*Cosh[a + b*x])/Csch[a + b*x]^(5/2),x]
```

output
```
(Sqrt[Csch[a + b*x]]*(63*b*x - 84*b*x*Cosh[2*(a + b*x)] + 21*b*x*Cosh[4*(a
 + b*x)] - (80*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[
a + b*x]] + 52*Sinh[2*(a + b*x)] - 6*Sinh[4*(a + b*x)]))/(588*b^2)
```

3.559.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5968, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{5968}$$

$$\frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)}-\frac{2\int \frac{1}{\operatorname{csch}^{\frac{7}{2}}(a+bx)}dx}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)}-\frac{2\int \frac{1}{(i\csc(ia+ibx))^{7/2}}dx}{7b}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{7b\operatorname{csch}^{\frac{5}{2}}(a+bx)}-\frac{5}{7}\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}dx\right)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{7b\operatorname{csch}^{\frac{5}{2}}(a+bx)}-\frac{5}{7}\int \frac{1}{(i\csc(ia+ibx))^{3/2}}dx\right)}{7b}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{7b\operatorname{csch}^{\frac{5}{2}}(a+bx)}-\frac{5}{7}\left(\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\int \sqrt{\operatorname{csch}(a+bx)}dx\right)\right)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{7b\text{csch}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{7b\text{csch}^{\frac{5}{2}}(a+bx)}-\frac{5}{7}\left(\frac{2\cosh(a+bx)}{3b\sqrt{\text{csch}(a+bx)}}-\frac{1}{3}\int\sqrt{i\csc(ia+ibx)}dx\right)\right)}{7b}$$

$\Big\downarrow$ 4258

$$\frac{2x}{7b\text{csch}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{7b\text{csch}^{\frac{5}{2}}(a+bx)}-\frac{5}{7}\left(\frac{2\cosh(a+bx)}{3b\sqrt{\text{csch}(a+bx)}}-\frac{1}{3}\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}\int\frac{1}{\sqrt{i\sinh(a+bx)}}dx\right)\right)}{7b}$$

$\Big\downarrow$ 3042

$$\frac{2x}{7b\text{csch}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{7b\text{csch}^{\frac{5}{2}}(a+bx)}-\frac{5}{7}\left(\frac{2\cosh(a+bx)}{3b\sqrt{\text{csch}(a+bx)}}-\frac{1}{3}\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}\int\frac{1}{\sqrt{\sin(ia+ibx)}}dx\right)\right)}{7b}$$

$\Big\downarrow$ 3120

$$\frac{2x}{7b\text{csch}^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{2\cosh(a+bx)}{7b\text{csch}^{\frac{5}{2}}(a+bx)}-\frac{5}{7}\left(\frac{2\cosh(a+bx)}{3b\sqrt{\text{csch}(a+bx)}}+\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b}\right)\right)}{7b}$$

input
```
Int[(x*Cosh[a + b*x])/Csch[a + b*x]^(5/2),x]
```

output
```
(2*x)/(7*b*Csch[a + b*x]^(7/2)) - (2*((2*Cosh[a + b*x])/(7*b*Csch[a + b*x]
^(5/2)) - (5*((2*Cosh[a + b*x])/(3*b*Sqrt[Csch[a + b*x]]) + (((2*I)/3)*Sqr
t[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]
])/b))/7))/(7*b)
```

3.559. $\int \frac{x\cosh(a+bx)}{\text{csch}^{\frac{5}{2}}(a+bx)}\,dx$

3.559.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 5968
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csch[a + b*x^n
]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
 NeQ[p, 1]
```

3.559.4 Maple [F]

$$\int \frac{x\cosh(bx+a)}{\operatorname{csch}(bx+a)^{\frac{5}{2}}}dx$$

input
```
int(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x)
```

output
```
int(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x)
```

3.559.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.559.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(b*x+a)/csch(b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.559.7 Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\operatorname{csch}(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*cosh(b*x + a)/csch(b*x + a)^(5/2), x)
```

3.559.8 Giac [F]

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cosh\left(bx+a\right)}{\operatorname{csch}\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*cosh(b*x + a)/csch(b*x + a)^(5/2), x)
```

3.559.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cosh(a+b\,x)}{\left(\frac{1}{\sinh(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(5/2),x)
```

output
```
int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(5/2), x)
```

3.560 $\int \sqrt{\sinh(x)\tanh(x)}\,dx$

3.560.1 Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = 2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

output
```
2*coth(x)*(sinh(x)*tanh(x))^(1/2)
```

3.560.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = 2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

input
```
Integrate[Sqrt[Sinh[x]*Tanh[x]],x]
```

output
```
2*Coth[x]*Sqrt[Sinh[x]*Tanh[x]]
```

3.560.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4898, 3042, 4900, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\sin(ix)\tan(ix)}dx$$

$$\Big\downarrow \text{4898}$$

$$\frac{\sqrt{\sinh(x)\tanh(x)}\int \sqrt{-\sinh(x)\tanh(x)}dx}{\sqrt{-\sinh(x)\tanh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sinh(x)\tanh(x)}\int \sqrt{\sin(ix)\tan(ix)}dx}{\sqrt{-\sinh(x)\tanh(x)}}$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\sinh(x)\tanh(x)}\int \sqrt{i\sinh(x)}\sqrt{i\tanh(x)}dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sinh(x)\tanh(x)}\int \sqrt{\sin(ix)}\sqrt{\tan(ix)}dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{3069}$$

$$2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

input
```
Int[Sqrt[Sinh[x]*Tanh[x]],x]
```

output
```
2*Coth[x]*Sqrt[Sinh[x]*Tanh[x]]
```

3.560.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 4898
```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Simp[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p])   In
t[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ
[v]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.560.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(11) = 22$.

Time = 0.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.23

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{\frac{\left(e^{2x}-1\right)^{2} e^{-x}}{1+e^{2x}}}\,\left(1+e^{2x}\right)}{e^{2x}-1}$	42

input
```
int((sinh(x)*tanh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*((exp(2*x)-1)^2*exp(-x)/(1+exp(2*x)))^(1/2)/(exp(2*x)-1)*(1+exp(2*
x))
```

3.560.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 4.08

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx$$
$$= \frac{2\sqrt{\frac{1}{2}}\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)}{\sqrt{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 + 1\right)\sinh(x) + \cosh(x)}}$$

input
```
integrate((sinh(x)*tanh(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/sqrt(cosh(x)^3
 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x))
```

3.560.6 Sympy [F]

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = \int \sqrt{\sinh(x)\tanh(x)}\,dx$$

input
```
integrate((sinh(x)*tanh(x))**(1/2),x)
```

output
```
Integral(sqrt(sinh(x)*tanh(x)), x)
```

3.560.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(11) = 22$.

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = -\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{e^{(-2x)} + 1}} - \frac{\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{\sqrt{e^{(-2x)} + 1}}$$

input
```
integrate((sinh(x)*tanh(x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(2)*e^(1/2*x)/sqrt(e^(-2*x) + 1) - sqrt(2)*e^(-3/2*x)/sqrt(e^(-2*x) +
 1)
```

3.560.8 Giac [F]

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = \int \sqrt{\sinh(x)\tanh(x)}\,dx$$

input
```
integrate((sinh(x)*tanh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sinh(x)*tanh(x)), x)
```

3.560.9 Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = 2\coth(x)\sqrt{-\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)\left(e^{2x}-1\right)}\sqrt{\frac{1}{e^{2x}+1}}$$

input
```
int((sinh(x)*tanh(x))^(1/2),x)
```

output
```
2*coth(x)*(-(exp(-x)/2 - exp(x)/2)*(exp(2*x) - 1))^(1/2)*(1/(exp(2*x) + 1)
)^(1/2)
```

3.561 $\int (\sinh(x)\tanh(x))^{3/2}\,dx$

3.561.1 Optimal result

Integrand size = 9, antiderivative size = 31

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx = \frac{8}{3}\operatorname{csch}(x)\sqrt{\sinh(x)\tanh(x)} + \frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)}$$

output
```
8/3*csch(x)*(sinh(x)*tanh(x))^(1/2)+2/3*sinh(x)*(sinh(x)*tanh(x))^(1/2)
```

3.561.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx = \frac{2}{3}\left(1+4\operatorname{csch}^2(x)\right)\sinh(x)\sqrt{\sinh(x)\tanh(x)}$$

input
```
Integrate[(Sinh[x]*Tanh[x])^(3/2),x]
```

output
```
(2*(1 + 4*Csch[x]^2)*Sinh[x]*Sqrt[Sinh[x]*Tanh[x]])/3
```

3.561.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.71, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {3042, 4898, 3042, 4900, 3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-\sin(ix)\tan(ix))^{3/2}dx$$

$$\Big\downarrow \text{4898}$$

$$-\frac{\sqrt{\sinh(x)\tanh(x)}\int (-\sinh(x)\tanh(x))^{3/2}dx}{\sqrt{-\sinh(x)\tanh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\sinh(x)\tanh(x)}\int (\sin(ix)\tan(ix))^{3/2}dx}{\sqrt{-\sinh(x)\tanh(x)}}$$

$$\Big\downarrow \text{4900}$$

$$-\frac{\sqrt{\sinh(x)\tanh(x)}\int (i\sinh(x))^{3/2}(i\tanh(x))^{3/2}dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\sinh(x)\tanh(x)}\int \sin(ix)^{3/2}\tan(ix)^{3/2}dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{3078}$$

$$-\frac{\sqrt{\sinh(x)\tanh(x)}\left(\frac{4}{3}\int \frac{(i\tanh(x))^{3/2}}{\sqrt{i\sinh(x)}}dx+\frac{2}{3}i(i\sinh(x))^{3/2}\sqrt{i\tanh(x)}\right)}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\sinh(x)\tanh(x)}\left(\frac{4}{3}\int \frac{\tan(ix)^{3/2}}{\sqrt{\sin(ix)}}dx+\frac{2}{3}i(i\sinh(x))^{3/2}\sqrt{i\tanh(x)}\right)}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{3069}$$

$$-\frac{\left(\frac{2}{3}i(i\sinh(x))^{3/2}\sqrt{i\tanh(x)}-\frac{8i\sqrt{i\tanh(x)}}{3\sqrt{i\sinh(x)}}\right)\sqrt{\sinh(x)\tanh(x)}}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

input
```
Int[(Sinh[x]*Tanh[x])^(3/2),x]
```

output
```
-((((((-8*I)/3)*Sqrt[I*Tanh[x]])/Sqrt[I*Sinh[x]] + ((2*I)/3)*(I*Sinh[x])^(
3/2)*Sqrt[I*Tanh[x]])*Sqrt[Sinh[x]*Tanh[x]])/(Sqrt[I*Sinh[x]]*Sqrt[I*Tanh[
x]]))
```

3.561.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4898
```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Simp[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p])   In
t[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ
[v]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.561. $\int(\sinh(x)\tanh(x))^{3/2}\,dx$

3.561.4 Maple [F]

$$\int (\sinh (x) \tanh (x))^{\frac{3}{2}} dx$$

input
```
int((sinh(x)*tanh(x))^(3/2),x)
```

output
```
int((sinh(x)*tanh(x))^(3/2),x)
```

3.561.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(23) = 46$.

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.06

$$\int (\sinh(x)\tanh(x))^{3/2}\, dx = \frac{\sqrt{\frac{1}{2}}\left(\cosh (x)^4 + 4 \cosh (x) \sinh (x)^3 + \sinh (x)^4 + 2 \left(3 \cosh (x)^2 + 7\right) \sinh (x)^2}{3 \sqrt{\cosh (x)^3 + 3 \cosh (x) \sinh (x)^2 + \sinh (x)^3 + \left(3 \cosh (x)^2 +}}$$

input
```
integrate((sinh(x)*tanh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 + 7)*sinh(x)^2 + 14*cosh(x)^2 + 4*(cosh(x)^3 + 7*cosh(x))*sinh(x) + 1)/(
sqrt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(
x) + cosh(x))*(cosh(x) + sinh(x)))
```

3.561.6 Sympy [F]

$$\int (\sinh(x)\tanh(x))^{3/2}\, dx = \int (\sinh (x) \tanh (x))^{\frac{3}{2}}\, dx$$

input
```
integrate((sinh(x)*tanh(x))**(3/2),x)
```

output
```
Integral((sinh(x)*tanh(x))**(3/2), x)
```

3.561.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(23) = 46$.

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.23

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx = -\frac{\sqrt{2}e^{\left(\frac{3}{2}\,x\right)}}{6\,\left(e^{(-2\,x)}+1\right)^{\frac{3}{2}}} - \frac{5\,\sqrt{2}e^{\left(-\frac{1}{2}\,x\right)}}{2\,\left(e^{(-2\,x)}+1\right)^{\frac{3}{2}}} - \frac{5\,\sqrt{2}e^{\left(-\frac{5}{2}\,x\right)}}{2\,\left(e^{(-2\,x)}+1\right)^{\frac{3}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{9}{2}\,x\right)}}{6\,\left(e^{(-2\,x)}+1\right)^{\frac{3}{2}}}$$

input
```
integrate((sinh(x)*tanh(x))^(3/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(2)*e^(3/2*x)/(e^(-2*x) + 1)^(3/2) - 5/2*sqrt(2)*e^(-1/2*x)/(e^(-
2*x) + 1)^(3/2) - 5/2*sqrt(2)*e^(-5/2*x)/(e^(-2*x) + 1)^(3/2) - 1/6*sqrt(2
)*e^(-9/2*x)/(e^(-2*x) + 1)^(3/2)
```

3.561.8 Giac [F]

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx = \int (\sinh(x)\tanh(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((sinh(x)*tanh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((sinh(x)*tanh(x))^(3/2), x)
```

3.561.9 Mupad [F(-1)]

Timed out.

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx = \int (\sinh(x)\,\tanh(x))^{3/2}\,dx$$

input
```
int((sinh(x)*tanh(x))^(3/2),x)
```

output
```
int((sinh(x)*tanh(x))^(3/2), x)
```

3.562 $\int (\sinh(x)\tanh(x))^{5/2}\,dx$

3.562.1 Optimal result

Integrand size = 9, antiderivative size = 50

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = -\frac{64}{15}\coth(x)\sqrt{\sinh(x)\tanh(x)} + \frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)}$$

output
```
-64/15*coth(x)*(sinh(x)*tanh(x))^(1/2)+16/15*(sinh(x)*tanh(x))^(1/2)*tanh(
x)+2/5*sinh(x)^2*(sinh(x)*tanh(x))^(1/2)*tanh(x)
```

3.562.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.58

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = -\frac{2}{15}\left(-5-3\cosh^2(x)+32\coth^2(x)\right)\tanh(x)\sqrt{\sinh(x)\tanh(x)}$$

input
```
Integrate[(Sinh[x]*Tanh[x])^(5/2),x]
```

output
```
(-2*(-5 - 3*Cosh[x]^2 + 32*Coth[x]^2)*Tanh[x]*Sqrt[Sinh[x]*Tanh[x]])/15
```

3.562.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.28, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {3042, 4898, 3042, 4900, 3042, 3078, 3042, 3074, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (-\sin(ix)\tan(ix))^{5/2}dx$$

$$\Big\downarrow \text{ 4898}$$

$$\frac{\sqrt{\sinh(x)\tanh(x)}\int(-\sinh(x)\tanh(x))^{5/2}dx}{\sqrt{-\sinh(x)\tanh(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sinh(x)\tanh(x)}\int(\sin(ix)\tan(ix))^{5/2}dx}{\sqrt{-\sinh(x)\tanh(x)}}$$

$$\Big\downarrow \text{ 4900}$$

$$\frac{\sqrt{\sinh(x)\tanh(x)}\int(i\sinh(x))^{5/2}(i\tanh(x))^{5/2}dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sinh(x)\tanh(x)}\int\sin(ix)^{5/2}\tan(ix)^{5/2}dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{ 3078}$$

$$\frac{\sqrt{\sinh(x)\tanh(x)}\left(\frac{8}{5}\int\sqrt{i\sinh(x)}(i\tanh(x))^{5/2}dx+\frac{2}{5}i(i\sinh(x))^{5/2}(i\tanh(x))^{3/2}\right)}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sinh(x)\tanh(x)}\left(\frac{8}{5}\int\sqrt{\sin(ix)}\tan(ix)^{5/2}dx+\frac{2}{5}i(i\sinh(x))^{5/2}(i\tanh(x))^{3/2}\right)}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{ 3074}$$

$$\frac{\sqrt{\sinh(x)\tanh(x)}\left(\frac{8}{5}\left(-\frac{4}{3}\int\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}dx-\frac{2}{3}i\sqrt{i\sinh(x)}(i\tanh(x))^{3/2}\right)+\frac{2}{5}i(i\sinh(x))^{5/2}(i\tanh(x))\right.}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sinh(x)\tanh(x)}\left(\frac{8}{5}\left(-\frac{4}{3}\int\sqrt{\sin(ix)}\sqrt{\tan(ix)}dx-\frac{2}{3}i\sqrt{i\sinh(x)}(i\tanh(x))^{3/2}\right)+\frac{2}{5}i(i\sinh(x))^{5/2}(i\tanh(x))^{3/2}\right)}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{\left(\frac{2}{5}i(i\sinh(x))^{5/2}(i\tanh(x))^{3/2}+\frac{8}{5}\left(-\frac{2}{3}i\sqrt{i\sinh(x)}(i\tanh(x))^{3/2}-\frac{8i\sqrt{i\sinh(x)}}{3\sqrt{i\tanh(x)}}\right)\right)\sqrt{\sinh(x)\tanh(x)}}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

input
```
Int[(Sinh[x]*Tanh[x])^(5/2),x]
```

output
```
(((8*((((-8*I)/3)*Sqrt[I*Sinh[x]])/Sqrt[I*Tanh[x]] - ((2*I)/3)*Sqrt[I*Sinh
[x]]*(I*Tanh[x])^(3/2)))/5 + ((2*I)/5)*(I*Sinh[x])^(5/2)*(I*Tanh[x])^(3/2)
)*Sqrt[Sinh[x]*Tanh[x]])/(Sqrt[I*Sinh[x]]*Sqrt[I*Tanh[x]])
```

3.562.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4898
```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Simp[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p])   In
t[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ
[v]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.562.4 Maple [F]

$$\int (\sinh (x) \tanh (x))^{\frac{5}{2}} dx$$

input
```
int((sinh(x)*tanh(x))^(5/2),x)
```

output
```
int((sinh(x)*tanh(x))^(5/2),x)
```

3.562.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(38) = 76$.

Time = 0.26 (sec) , antiderivative size = 253, normalized size of antiderivative = 5.06

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(3\cosh(x)^8 + 24\cosh(x)\sinh(x)^7 + 3\sinh(x)^8 + 12\left(7\cosh(x)^2 - 9\right)\sin\cdots\right.}{\cdots}$$

input
```
integrate((sinh(x)*tanh(x))^(5/2),x, algorithm="fricas")
```

output
```
1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*c
osh(x)^2 - 9)*sinh(x)^6 - 108*cosh(x)^6 + 24*(7*cosh(x)^3 - 27*cosh(x))*si
nh(x)^5 + 2*(105*cosh(x)^4 - 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^
4 + 8*(21*cosh(x)^5 - 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(
x)^6 - 135*cosh(x)^4 - 151*cosh(x)^2 - 9)*sinh(x)^2 - 108*cosh(x)^2 + 8*(3
*cosh(x)^7 - 81*cosh(x)^5 - 151*cosh(x)^3 - 27*cosh(x))*sinh(x) + 3)/((cos
h(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + c
osh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x))*sqrt(cosh(x)^3 + 3*cosh(x)*s
inh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x)))
```

3.562.6 Sympy [F(-1)]

Timed out.

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((sinh(x)*tanh(x))**(5/2),x)
```

output
```
Timed out
```

3.562.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(38) = 76$.

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.06

$$\begin{aligned}\int (\sinh(x)\tanh(x))^{5/2}\,dx = &-\frac{\sqrt{2}e^{\left(\frac{5}{2}x\right)}}{20\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}+\frac{7\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{4\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}\\&+\frac{41\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{6\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}+\frac{41\sqrt{2}e^{\left(-\frac{7}{2}x\right)}}{6\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}+\frac{7\sqrt{2}e^{\left(-\frac{11}{2}x\right)}}{4\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}-\frac{\sqrt{2}e^{\left(-\frac{15}{2}x\right)}}{20\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}\end{aligned}$$

input
```
integrate((sinh(x)*tanh(x))^(5/2),x, algorithm="maxima")
```

output
```
-1/20*sqrt(2)*e^(5/2*x)/(e^(-2*x) + 1)^(5/2) + 7/4*sqrt(2)*e^(1/2*x)/(e^(-
2*x) + 1)^(5/2) + 41/6*sqrt(2)*e^(-3/2*x)/(e^(-2*x) + 1)^(5/2) + 41/6*sqrt
(2)*e^(-7/2*x)/(e^(-2*x) + 1)^(5/2) + 7/4*sqrt(2)*e^(-11/2*x)/(e^(-2*x) +
1)^(5/2) - 1/20*sqrt(2)*e^(-15/2*x)/(e^(-2*x) + 1)^(5/2)
```

3.562.8 Giac [F]

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = \int (\sinh(x)\tanh(x))^{\frac{5}{2}}\,dx$$

input
```
integrate((sinh(x)*tanh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((sinh(x)*tanh(x))^(5/2), x)
```

3.562.9 Mupad [F(-1)]

Timed out.

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = \int (\sinh(x)\,\tanh(x))^{5/2}\,dx$$

input
```
int((sinh(x)*tanh(x))^(5/2),x)
```

output
```
int((sinh(x)*tanh(x))^(5/2), x)
```

3.563 $\int \sqrt{\cosh(x)\coth(x)}\,dx$

3.563.1 Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = 2\sqrt{\cosh(x)\coth(x)}\tanh(x)$$

output
```
2*(cosh(x)*coth(x))^(1/2)*tanh(x)
```

3.563.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = \frac{2\sqrt{\cosh(x)\coth(x)}\left(-1+\sqrt[4]{-\sinh^2(x)}\right)\tanh(x)}{\sqrt[4]{-\sinh^2(x)}}$$

input
```
Integrate[Sqrt[Cosh[x]*Coth[x]],x]
```

output
```
(2*Sqrt[Cosh[x]*Coth[x]]*(-1 + (-Sinh[x]^2)^(1/4))*Tanh[x])/(-Sinh[x]^2)^(
1/4)
```

3.563.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4898, 3042, 4900, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cosh(x)\coth(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{i\cos(ix)\cot(ix)}dx$$

$$\Big\downarrow \text{4898}$$

$$\frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{-i\cosh(x)\coth(x)}dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{\cos(ix)\cot(ix)}dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{\cosh(x)}\sqrt{-i\coth(x)}dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)}\sqrt{-\tan\left(ix+\frac{\pi}{2}\right)}dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3069}$$

$$2\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

input
```
Int[Sqrt[Cosh[x]*Coth[x]],x]
```

output
```
2*Sqrt[Cosh[x]*Coth[x]]*Tanh[x]
```

3.563.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 4898
```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Simp[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p])   In
t[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ
[v]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.563.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(11) = 22$.

Time = 0.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.23

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{\frac{\left(1+\mathrm{e}^{2x}\right)^{2}\mathrm{e}^{-x}}{\mathrm{e}^{2x}-1}}\,\left(\mathrm{e}^{2x}-1\right)}{1+\mathrm{e}^{2x}}$	42

input
```
int((coth(x)*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*((1+exp(2*x))^2*exp(-x)/(exp(2*x)-1))^(1/2)/(1+exp(2*x))*(exp(2*x)
-1)
```

3.563.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(11) = 22$.

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 4.23

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = \frac{2\sqrt{\frac{1}{2}}\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)}{\sqrt{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 - 1\right)\sinh(x) - \cosh(x)}}$$

input
```
integrate((cosh(x)*coth(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)/sqrt(cosh(x)^3
 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))
```

3.563.6 Sympy [F]

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = \int \sqrt{\cosh(x)\coth(x)}\,dx$$

input
```
integrate((cosh(x)*coth(x))**(1/2),x)
```

output
```
Integral(sqrt(cosh(x)*coth(x)), x)
```

3.563.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(11) = 22$.

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.15

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = \frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{e^{(-x)}+1}\sqrt{-e^{(-x)}+1}} - \frac{\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{\sqrt{e^{(-x)}+1}\sqrt{-e^{(-x)}+1}}$$

input
```
integrate((cosh(x)*coth(x))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2)*e^(1/2*x)/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1)) - sqrt(2)*e^(-3/2*x
)/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1))
```

3.563.8 Giac [F]

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = \int \sqrt{\cosh{(x)}\coth{(x)}}\,dx$$

input
```
integrate((cosh(x)*coth(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cosh(x)*coth(x)), x)
```

3.563.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = 4\,\mathrm{e}^{x}\sinh(x)\,\sqrt{\frac{\mathrm{e}^{-x}}{2\,\left(\mathrm{e}^{2\,x}-1\right)}}$$

input
```
int((cosh(x)*coth(x))^(1/2),x)
```

output
```
4*exp(x)*sinh(x)*(exp(-x)/(2*(exp(2*x) - 1)))^(1/2)
```

3.564 $\int (\cosh(x)\coth(x))^{3/2}\,dx$

3.564.1 Optimal result

Integrand size = 9, antiderivative size = 31

$$\int (\cosh(x)\coth(x))^{3/2}\,dx=\frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}-\frac{8}{3}\sqrt{\cosh(x)\coth(x)}\operatorname{sech}(x)$$

output
```
2/3*cosh(x)*(cosh(x)*coth(x))^(1/2)-8/3*sech(x)*(cosh(x)*coth(x))^(1/2)
```

3.564.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int (\cosh(x)\coth(x))^{3/2}\,dx=\frac{2}{3}\left(-4+\cosh^2(x)\right)\sqrt{\cosh(x)\coth(x)}\operatorname{sech}(x)$$

input
```
Integrate[(Cosh[x]*Coth[x])^(3/2),x]
```

output
```
(2*(-4 + Cosh[x]^2)*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

3.564.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.39, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {3042, 4898, 3042, 4900, 3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cosh(x)\coth(x))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (i\cos(ix)\cot(ix))^{3/2}dx$$

$$\Big\downarrow \text{ 4898}$$

$$\frac{i\sqrt{\cosh(x)\coth(x)}\int(-i\cosh(x)\coth(x))^{3/2}dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{i\sqrt{\cosh(x)\coth(x)}\int(\cos(ix)\cot(ix))^{3/2}dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$\Big\downarrow \text{ 4900}$$

$$\frac{i\sqrt{\cosh(x)\coth(x)}\int\cosh^{\frac{3}{2}}(x)(-i\coth(x))^{3/2}dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{i\sqrt{\cosh(x)\coth(x)}\int\sin\left(ix+\frac{\pi}{2}\right)^{3/2}\left(-\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{ 3078}$$

$$\frac{i\sqrt{\cosh(x)\coth(x)}\left(\frac{4}{3}\int\frac{(-i\coth(x))^{3/2}}{\sqrt{\cosh(x)}}dx-\frac{2}{3}i\cosh^{\frac{3}{2}}(x)\sqrt{-i\coth(x)}\right)}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{i\sqrt{\cosh(x)\coth(x)}\left(\frac{4}{3}\int\frac{\left(-\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx-\frac{2}{3}i\cosh^{\frac{3}{2}}(x)\sqrt{-i\coth(x)}\right)}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{i\left(\frac{8i\sqrt{-i\coth(x)}}{3\sqrt{\cosh(x)}}-\frac{2}{3}i\cosh^{\frac{3}{2}}(x)\sqrt{-i\coth(x)}\right)\sqrt{\cosh(x)\coth(x)}}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

input
```
Int[(Cosh[x]*Coth[x])^(3/2),x]
```

output
```
(I*((((8*I)/3)*Sqrt[(-I)*Coth[x]])/Sqrt[Cosh[x]] - ((2*I)/3)*Cosh[x]^(3/2)
*Sqrt[(-I)*Coth[x]])*Sqrt[Cosh[x]*Coth[x]])/(Sqrt[Cosh[x]]*Sqrt[(-I)*Coth[
x]])
```

3.564.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4898
```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Simp[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p])   In
t[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ
[v]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.564.4 Maple [F]

$$\int (\coth(x)\cosh(x))^{\frac{3}{2}}\, dx$$

input
```
int((coth(x)*cosh(x))^(3/2),x)
```

output
```
int((coth(x)*cosh(x))^(3/2),x)
```

3.564.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.13

$$\int (\cosh(x)\coth(x))^{3/2}\, dx = \frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 7\right)\sinh(x)^2\right.}{3\sqrt{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 -\right.}}$$

input
```
integrate((cosh(x)*coth(x))^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)/(
sqrt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(
x) - cosh(x))*(cosh(x) + sinh(x)))
```

3.564.6 Sympy [F(-1)]

Timed out.

$$\int (\cosh(x)\coth(x))^{3/2}\, dx = \text{Timed out}$$

input
```
integrate((cosh(x)*coth(x))**(3/2),x)
```

output
```
Timed out
```

3.564.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(23) = 46.

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int (\cosh(x)\coth(x))^{3/2}\,dx = \frac{\sqrt{2}e^{\left(\frac{3}{2}\,x\right)}}{6\,(e^{(-x)}+1)^{\frac{3}{2}}(-e^{(-x)}+1)^{\frac{3}{2}}} - \frac{5\,\sqrt{2}e^{\left(-\frac{1}{2}\,x\right)}}{2\,(e^{(-x)}+1)^{\frac{3}{2}}(-e^{(-x)}+1)^{\frac{3}{2}}} + \frac{5\,\sqrt{2}e^{\left(-\frac{5}{2}\,x\right)}}{2\,(e^{(-x)}+1)^{\frac{3}{2}}(-e^{(-x)}+1)^{\frac{3}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{9}{2}\,x\right)}}{6\,(e^{(-x)}+1)^{\frac{3}{2}}(-e^{(-x)}+1)^{\frac{3}{2}}}$$

input
```
integrate((cosh(x)*coth(x))^(3/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*e^(3/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 5/2*sqrt(
2)*e^(-1/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) + 5/2*sqrt(2)*e^(-5
/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 1/6*sqrt(2)*e^(-9/2*x)/((
e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2))
```

3.564.8 Giac [F]

$$\int (\cosh(x)\coth(x))^{3/2}\,dx = \int (\cosh(x)\coth(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((cosh(x)*coth(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((cosh(x)*coth(x))^(3/2), x)
```

3.564.9 Mupad [F(-1)]

Timed out.

$$\int (\cosh(x)\coth(x))^{3/2}\,dx = \int (\cosh(x)\,\coth(x))^{3/2}\,dx$$

input
```
int((cosh(x)*coth(x))^(3/2),x)
```

output
```
int((cosh(x)*coth(x))^(3/2), x)
```

3.565 $\int (\cosh(x)\coth(x))^{5/2}\,dx$

3.565.1 Optimal result

Integrand size = 9, antiderivative size = 50

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{64}{15}\sqrt{\cosh(x)\coth(x)}\tanh(x)$$

output
```
-16/15*coth(x)*(cosh(x)*coth(x))^(1/2)+2/5*cosh(x)^2*coth(x)*(cosh(x)*coth
(x))^(1/2)+64/15*(cosh(x)*coth(x))^(1/2)*tanh(x)
```

3.565.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = \frac{1}{15}\sqrt{\cosh(x)\coth(x)}\Big(-10\coth(x) + 6\cosh(x)\sinh(x) + 57\mathrm{csch}(x)\mathrm{sech}(x)\left(-\sinh^2(x)\right)^{3/4} + 64\tanh(x)\Big)$$

input
```
Integrate[(Cosh[x]*Coth[x])^(5/2),x]
```

output
```
(Sqrt[Cosh[x]*Coth[x]]*(-10*Coth[x] + 6*Cosh[x]*Sinh[x] + 57*Csch[x]*Sech[
x]*(-Sinh[x]^2)^(3/4) + 64*Tanh[x]))/15
```

3.565.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.98, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {3042, 4898, 3042, 4900, 3042, 3078, 3042, 3074, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cosh(x)\coth(x))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\cos(ix)\cot(ix))^{5/2}dx$$

$$\Big\downarrow \text{4898}$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\int(-i\cosh(x)\coth(x))^{5/2}dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\int(\cos(ix)\cot(ix))^{5/2}dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$\Big\downarrow \text{4900}$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\int\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{5/2}dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\int\sin\left(ix+\frac{\pi}{2}\right)^{5/2}\left(-\tan\left(ix+\frac{\pi}{2}\right)\right)^{5/2}dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3078}$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\left(\frac{8}{5}\int\sqrt{\cosh(x)}(-i\coth(x))^{5/2}dx-\frac{2}{5}i\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{3/2}\right)}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\left(\frac{8}{5}\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}\left(-\tan\left(ix+\frac{\pi}{2}\right)\right)^{5/2}dx-\frac{2}{5}i\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{3/2}\right)}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3074}$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\left(\frac{8}{5}\left(\frac{2}{3}i\sqrt{\cosh(x)}(-i\coth(x))^{3/2}-\frac{4}{3}\int\sqrt{\cosh(x)}\sqrt{-i\coth(x)}dx\right)-\frac{2}{5}i\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{3}\right.}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$\downarrow$ 3042

$$-\frac{\sqrt{\cosh(x)\coth(x)}\left(\frac{8}{5}\left(\frac{2}{3}i\sqrt{\cosh(x)}(-i\coth(x))^{3/2}-\frac{4}{3}\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}\sqrt{-\tan\left(ix+\frac{\pi}{2}\right)}dx\right)-\frac{2}{5}i\cosh^{\frac{5}{2}}(x)(-i\,c\right.}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$\downarrow$ 3069

$$-\frac{\left(\frac{8}{5}\left(\frac{2}{3}i\sqrt{\cosh(x)}(-i\coth(x))^{3/2}+\frac{8i\sqrt{\cosh(x)}}{3\sqrt{-i\coth(x)}}\right)-\frac{2}{5}i\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{3/2}\right)\sqrt{\cosh(x)\coth(x)}}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

input
```
Int[(Cosh[x]*Coth[x])^(5/2),x]
```

output
```
-((((8*((((8*I)/3)*Sqrt[Cosh[x]])/Sqrt[(-I)*Coth[x]] + ((2*I)/3)*Sqrt[Cosh
[x]]*((-I)*Coth[x])^(3/2)))/5 - ((2*I)/5)*Cosh[x]^(5/2)*((-I)*Coth[x])^(3/
2))*Sqrt[Cosh[x]*Coth[x]])/(Sqrt[Cosh[x]]*Sqrt[(-I)*Coth[x]]))
```

3.565.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] && !(GtQ[m, 1] && !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4898
```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Simp[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p])   In
t[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ
[v]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.565.4 Maple [F]

$$\int (\coth(x)\cosh(x))^{\frac{5}{2}}\,dx$$

input
```
int((coth(x)*cosh(x))^(5/2),x)
```

output
```
int((coth(x)*cosh(x))^(5/2),x)
```

3.565.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(38) = 76$.

Time = 0.27 (sec) , antiderivative size = 259, normalized size of antiderivative = 5.18

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(3\cosh(x)^8 + 24\cosh(x)\sinh(x)^7 + 3\sinh(x)^8 + 12\left(7\cosh(x)^2 + 9\right)\sin$$

input
```
integrate((cosh(x)*coth(x))^(5/2),x, algorithm="fricas")
```

output
```
1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*c
osh(x)^2 + 9)*sinh(x)^6 + 108*cosh(x)^6 + 24*(7*cosh(x)^3 + 27*cosh(x))*si
nh(x)^5 + 2*(105*cosh(x)^4 + 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^
4 + 8*(21*cosh(x)^5 + 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(
x)^6 + 135*cosh(x)^4 - 151*cosh(x)^2 + 9)*sinh(x)^2 + 108*cosh(x)^2 + 8*(3
*cosh(x)^7 + 81*cosh(x)^5 - 151*cosh(x)^3 + 27*cosh(x))*sinh(x) + 3)/((cos
h(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - c
osh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*sqrt(cosh(x)^3 + 3*cosh(x)*s
inh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x)))
```

3.565.6 Sympy [F(-1)]

Timed out.

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((cosh(x)*coth(x))**(5/2),x)
```

output
```
Timed out
```

3.565.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(38) = 76.

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.26

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = \frac{\sqrt{2}e^{\left(\frac{5}{2}\,x\right)}}{20\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} + \frac{7\,\sqrt{2}e^{\left(\frac{1}{2}\,x\right)}}{4\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} - \frac{41\,\sqrt{2}e^{\left(-\frac{3}{2}\,x\right)}}{6\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} + \frac{41\,\sqrt{2}e^{\left(-\frac{7}{2}\,x\right)}}{6\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} - \frac{7\,\sqrt{2}e^{\left(-\frac{11}{2}\,x\right)}}{4\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{15}{2}\,x\right)}}{20\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}}$$

input
```
integrate((cosh(x)*coth(x))^(5/2),x, algorithm="maxima")
```

output
```
1/20*sqrt(2)*e^(5/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 7/4*sqrt
(2)*e^(1/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 41/6*sqrt(2)*e^(-
3/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 41/6*sqrt(2)*e^(-7/2*x)/
((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 7/4*sqrt(2)*e^(-11/2*x)/((e^(-x
) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 1/20*sqrt(2)*e^(-15/2*x)/((e^(-x) + 1)
^(5/2)*(-e^(-x) + 1)^(5/2))
```

3.565.8 Giac [F]

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = \int (\cosh(x)\coth(x))^{\frac{5}{2}}\,dx$$

input
```
integrate((cosh(x)*coth(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((cosh(x)*coth(x))^(5/2), x)
```

3.565.9 Mupad [F(-1)]

Timed out.

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = \int (\cosh(x)\,\coth(x))^{5/2}\,dx$$

input
```
int((cosh(x)*coth(x))^(5/2),x)
```

output
```
int((cosh(x)*coth(x))^(5/2), x)
```

3.566 $\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx$

3.566.1 Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx=-\frac{2(b+c)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{\log(a+b\sinh(x))}{b}$$

output
```
ln(a+b*sinh(x))/b-2*(b+c)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+
b^2)^(1/2)
```

3.566.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx=\frac{2(b+c)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{\log(a+b\sinh(x))}{b}$$

input
```
Integrate[(b + c + Cosh[x])/(a + b*Sinh[x]),x]
```

output
```
(2*(b + c)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] +
Log[a + b*Sinh[x]]/b
```

3.566.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{b+c+\cos(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{ 4901}$$

$$\int \left(\frac{c\left(\frac{b}{c}+1\right)}{a+b\sinh(x)}+\frac{\cosh(x)}{a+b\sinh(x)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\log(a+b\sinh(x))}{b}-\frac{2(b+c)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
Int[(b + c + Cosh[x])/(a + b*Sinh[x]),x]
```

output
```
(-2*(b + c)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] +
Log[a + b*Sinh[x]]/b
```

3.566.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.566.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result
parts	$\frac{\ln(a+b\sinh(x))}{b}+\frac{2(b+c)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)-\frac{2\left(-b^2-cb\right)\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b}$
risch	$\frac{x}{b}-\frac{2x\,a^2b}{a^2b^2+b^4}-\frac{2x\,b^3}{a^2b^2+b^4}+\frac{\ln\left(\mathrm{e}^x-\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a^2b^3c+a^2b^2c^2+b^6+2b^5c+b^4c^2}}{b^2(b+c)}\right)a^2}{(a^2+b^2)b}+\frac{b\ln\left(\mathrm{e}^x-\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a^2b^3}}{b^2(b}\right.}{a^2+b^2}$

input
```
int((b+c+cosh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*sinh(x))/b+2*(b+c)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b
)/(a^2+b^2)^(1/2))
```

3.566.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(48) = 96.

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.21

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx = \frac{\sqrt{a^2+b^2}(b^2+bc)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right.}{a^2b+b^3}$$

input
```
integrate((b+c+cosh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(sqrt(a^2 + b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*co
sh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b
*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b
*cosh(x) + a)*sinh(x) - b)) - (a^2 + b^2)*x + (a^2 + b^2)*log(2*(b*sinh(x)
 + a)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

3.566.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.48 (sec) , antiderivative size = 585, normalized size of antiderivative = 11.25

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate((b+c+cosh(x))/(a+b*sinh(x)),x)
```

output
```
Piecewise((zoo*(c*log(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(x/2
))), Eq(a, 0) & Eq(b, 0)), ((b*log(tanh(x/2)) + c*log(tanh(x/2)) + x - 2*l
og(tanh(x/2) + 1) + log(tanh(x/2)))/b, Eq(a, 0)), ((c*x + sinh(x))/a, Eq(b
, 0)), (2*I*b/(b*tanh(x/2) - I*b) + 2*I*c/(b*tanh(x/2) - I*b) + x*tanh(x/2
)/(b*tanh(x/2) - I*b) - I*x/(b*tanh(x/2) - I*b) - 2*log(tanh(x/2) + 1)*tan
h(x/2)/(b*tanh(x/2) - I*b) + 2*I*log(tanh(x/2) + 1)/(b*tanh(x/2) - I*b) +
2*log(tanh(x/2) - I)*tanh(x/2)/(b*tanh(x/2) - I*b) - 2*I*log(tanh(x/2) - I
)/(b*tanh(x/2) - I*b), Eq(a, -I*b)), (-2*I*b/(b*tanh(x/2) + I*b) - 2*I*c/(
b*tanh(x/2) + I*b) + x*tanh(x/2)/(b*tanh(x/2) + I*b) + I*x/(b*tanh(x/2) +
I*b) - 2*log(tanh(x/2) + 1)*tanh(x/2)/(b*tanh(x/2) + I*b) - 2*I*log(tanh(x
/2) + 1)/(b*tanh(x/2) + I*b) + 2*log(tanh(x/2) + I)*tanh(x/2)/(b*tanh(x/2)
 + I*b) + 2*I*log(tanh(x/2) + I)/(b*tanh(x/2) + I*b), Eq(a, I*b)), (-b*log
(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + b*log(tanh(x/2
) - b/a + sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) - c*log(tanh(x/2) - b/a -
 sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + c*log(tanh(x/2) - b/a + sqrt(a**
2 + b**2)/a)/sqrt(a**2 + b**2) + x/b - 2*log(tanh(x/2) + 1)/b + log(tanh(x
/2) - b/a - sqrt(a**2 + b**2)/a)/b + log(tanh(x/2) - b/a + sqrt(a**2 + b**
2)/a)/b, True))
```

3.566.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(48) = 96.

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.35

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx = \frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{c\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{\log\left(b\sinh\left(x\right)+a\right)}{b}$$

input
```
integrate((b+c+cosh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/s
qrt(a^2 + b^2) + c*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sq
rt(a^2 + b^2)))/sqrt(a^2 + b^2) + log(b*sinh(x) + a)/b
```

3.566.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.67

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx = \frac{(b+c)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}} - \frac{x}{b} + \frac{\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{b}$$

input
```
integrate((b+c+cosh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
(b + c)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*s
qrt(a^2 + b^2)))/sqrt(a^2 + b^2) - x/b + log(abs(b*e^(2*x) + 2*a*e^x - b))
/b
```

3.566.9 Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.42

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx = \frac{\ln\left(a^2\,\mathrm{e}^x-b\,\sqrt{a^2+b^2}+b^2\,\mathrm{e}^x+a\,\mathrm{e}^x\,\sqrt{a^2+b^2}\right)\,\left(b^2\,\sqrt{a^2+b^2}+a^2+b^2+b\,c\,\sqrt{a^2+b^2}\right)}{a^2\,b+b^3} - \frac{\ln\left(b\,\sqrt{a^2+b^2}+a^2\,\mathrm{e}^x+b^2\,\mathrm{e}^x-a\,\mathrm{e}^x\,\sqrt{a^2+b^2}\right)\,\left(b^2\,\sqrt{a^2+b^2}-a^2-b^2+b\,c\,\sqrt{a^2+b^2}\right)}{a^2\,b+b^3} - \frac{x}{b}$$

input
```
int((b + c + cosh(x))/(a + b*sinh(x)),x)
```

output
```
(log(a^2*exp(x) - b*(a^2 + b^2)^(1/2) + b^2*exp(x) + a*exp(x)*(a^2 + b^2)^
(1/2))*(b^2*(a^2 + b^2)^(1/2) + a^2 + b^2 + b*c*(a^2 + b^2)^(1/2)))/(a^2*b
 + b^3) - (log(b*(a^2 + b^2)^(1/2) + a^2*exp(x) + b^2*exp(x) - a*exp(x)*(a
^2 + b^2)^(1/2))*(b^2*(a^2 + b^2)^(1/2) - a^2 - b^2 + b*c*(a^2 + b^2)^(1/2
)))/(a^2*b + b^3) - x/b
```

3.567 $\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx$

3.567.1 Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx = \frac{2(b+c)\text{arctanh}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{\log(a-b\sinh(x))}{b}$$

output
```
-ln(a-b*sinh(x))/b+2*(b+c)*arctanh((b+a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2
+b^2)^(1/2)
```

3.567.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx = -\frac{2(b+c)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \frac{\log(-a+b\sinh(x))}{b}$$

input
```
Integrate[(b + c + Cosh[x])/(a - b*Sinh[x]),x]
```

output
```
(-2*(b + c)*ArcTan[(b + a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] -
 Log[-a + b*Sinh[x]]/b
```

3.567.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{b+c+\cos(ix)}{a+ib\sin(ix)}dx$$

$$\Big\downarrow \text{ 4901}$$

$$\int \left(\frac{c\left(\frac{b}{c}+1\right)}{a-b\sinh(x)}+\frac{\cosh(x)}{a-b\sinh(x)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(b+c)\text{arctanh}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{\log(a-b\sinh(x))}{b}$$

input
```
Int[(b + c + Cosh[x])/(a - b*Sinh[x]),x]
```

output
```
(2*(b + c)*ArcTanh[(b + a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - L
og[a - b*Sinh[x]]/b
```

3.567.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.567.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result
parts	$-\frac{\ln(a-b\sinh(x))}{b}+\frac{2(b+c)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{-\ln\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)-a\right)-\frac{2\left(-b^2-cb\right)\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$
risch	$-\frac{x}{b}+\frac{2x\,a^2b}{a^2b^2+b^4}+\frac{2x\,b^3}{a^2b^2+b^4}-\frac{\ln\left(\mathrm{e}^x+\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a^2b^3c+a^2b^2c^2+b^6+2b^5c+b^4c^2}}{b^2(b+c)}\right)a^2}{(a^2+b^2)b}-\frac{b\ln\left(\mathrm{e}^x+\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a^2}}{b}\right.}{a^2+}$

input
```
int((b+c+cosh(x))/(a-b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(a-b*sinh(x))/b+2*(b+c)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)+2*
b)/(a^2+b^2)^(1/2))
```

3.567.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(47) = 94.

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.28

$$\int\frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx$$
$$=\frac{\sqrt{a^2+b^2}(b^2+bc)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2-2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)-ab)\sinh(x)+2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)-a)}{b\cosh(x)^2+b\sinh(x)^2-2\,a\cosh(x)+2\,(b\cosh(x)-a)\sinh(x)-b}\right.}{a^2b+b^3}$$

input
```
integrate((b+c+cosh(x))/(a-b*sinh(x)),x, algorithm="fricas")
```

output
```
(sqrt(a^2 + b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 - 2*a*b*co
sh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) - a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b
*cosh(x) + b*sinh(x) - a))/(b*cosh(x)^2 + b*sinh(x)^2 - 2*a*cosh(x) + 2*(b
*cosh(x) - a)*sinh(x) - b)) + (a^2 + b^2)*x - (a^2 + b^2)*log(2*(b*sinh(x)
 - a)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

3.567.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.30 (sec) , antiderivative size = 586, normalized size of antiderivative = 11.06

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate((b+c+cosh(x))/(a-b*sinh(x)),x)
```

output
```
Piecewise((zoo*(c*log(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(x/2
))), Eq(a, 0) & Eq(b, 0)), (-(b*log(tanh(x/2)) + c*log(tanh(x/2)) + x - 2*
log(tanh(x/2) + 1) + log(tanh(x/2)))/b, Eq(a, 0)), ((c*x + sinh(x))/a, Eq(
b, 0)), (2*I*b/(b*tanh(x/2) + I*b) + 2*I*c/(b*tanh(x/2) + I*b) - x*tanh(x/
2)/(b*tanh(x/2) + I*b) - I*x/(b*tanh(x/2) + I*b) + 2*log(tanh(x/2) + 1)*ta
nh(x/2)/(b*tanh(x/2) + I*b) + 2*I*log(tanh(x/2) + 1)/(b*tanh(x/2) + I*b) -
 2*log(tanh(x/2) + I)*tanh(x/2)/(b*tanh(x/2) + I*b) - 2*I*log(tanh(x/2) +
I)/(b*tanh(x/2) + I*b), Eq(a, -I*b)), (-2*I*b/(b*tanh(x/2) - I*b) - 2*I*c/
(b*tanh(x/2) - I*b) - x*tanh(x/2)/(b*tanh(x/2) - I*b) + I*x/(b*tanh(x/2) -
 I*b) + 2*log(tanh(x/2) + 1)*tanh(x/2)/(b*tanh(x/2) - I*b) - 2*I*log(tanh(
x/2) + 1)/(b*tanh(x/2) - I*b) - 2*log(tanh(x/2) - I)*tanh(x/2)/(b*tanh(x/2
) - I*b) + 2*I*log(tanh(x/2) - I)/(b*tanh(x/2) - I*b), Eq(a, I*b)), (-b*lo
g(tanh(x/2) + b/a - sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + b*log(tanh(x/
2) + b/a + sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) - c*log(tanh(x/2) + b/a
- sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + c*log(tanh(x/2) + b/a + sqrt(a*
*2 + b**2)/a)/sqrt(a**2 + b**2) - x/b + 2*log(tanh(x/2) + 1)/b - log(tanh(
x/2) + b/a - sqrt(a**2 + b**2)/a)/b - log(tanh(x/2) + b/a + sqrt(a**2 + b*
*2)/a)/b, True))
```

3.567.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(47) = 94$.

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx = -\frac{b\log\left(\frac{be^{(-x)}+a-\sqrt{a^2+b^2}}{be^{(-x)}+a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{c\log\left(\frac{be^{(-x)}+a-\sqrt{a^2+b^2}}{be^{(-x)}+a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{\log\left(b\sinh\left(x\right)-a\right)}{b}$$

input
```
integrate((b+c+cosh(x))/(a-b*sinh(x)),x, algorithm="maxima")
```

output
```
-b*log((b*e^(-x) + a - sqrt(a^2 + b^2))/(b*e^(-x) + a + sqrt(a^2 + b^2)))/
sqrt(a^2 + b^2) - c*log((b*e^(-x) + a - sqrt(a^2 + b^2))/(b*e^(-x) + a + s
qrt(a^2 + b^2)))/sqrt(a^2 + b^2) - log(b*sinh(x) - a)/b
```

3.567.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.66

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx = -\frac{(b+c)\log\left(\frac{\left|2\,be^{x}-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}} + \frac{x}{b} - \frac{\log\left(\left|be^{(2\,x)}-2\,ae^{x}-b\right|\right)}{b}$$

input
```
integrate((b+c+cosh(x))/(a-b*sinh(x)),x, algorithm="giac")
```

output
```
-(b + c)*log(abs(2*b*e^x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x - 2*a + 2*
sqrt(a^2 + b^2)))/sqrt(a^2 + b^2) + x/b - log(abs(b*e^(2*x) - 2*a*e^x - b)
)/b
```

3.567.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.34

$$\begin{aligned}\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx &= \frac{x}{b}\\ &+ \frac{\ln\left(b\sqrt{a^2+b^2}+a^2\,\mathrm{e}^x+b^2\,\mathrm{e}^x+a\,\mathrm{e}^x\sqrt{a^2+b^2}\right)\left(b^2\sqrt{a^2+b^2}-a^2-b^2+b\,c\sqrt{a^2+b^2}\right)}{a^2\,b+b^3}\\ &- \frac{\ln\left(b\sqrt{a^2+b^2}-a^2\,\mathrm{e}^x-b^2\,\mathrm{e}^x+a\,\mathrm{e}^x\sqrt{a^2+b^2}\right)\left(b^2\sqrt{a^2+b^2}+a^2+b^2+b\,c\sqrt{a^2+b^2}\right)}{a^2\,b+b^3}\end{aligned}$$

input
```
int((b + c + cosh(x))/(a - b*sinh(x)),x)
```

output
```
x/b + (log(b*(a^2 + b^2)^(1/2) + a^2*exp(x) + b^2*exp(x) + a*exp(x)*(a^2 +
 b^2)^(1/2))*(b^2*(a^2 + b^2)^(1/2) - a^2 - b^2 + b*c*(a^2 + b^2)^(1/2)))/
(a^2*b + b^3) - (log(b*(a^2 + b^2)^(1/2) - a^2*exp(x) - b^2*exp(x) + a*exp
(x)*(a^2 + b^2)^(1/2))*(b^2*(a^2 + b^2)^(1/2) + a^2 + b^2 + b*c*(a^2 + b^2
)^(1/2)))/(a^2*b + b^3)
```

3.567. $\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx$

3.568 $\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx$

3.568.1 Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx = \frac{2(b+c)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{\log(a+b\cosh(x))}{b}$$

output
```
ln(a+b*cosh(x))/b+2*(b+c)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-
b)^(1/2)/(a+b)^(1/2)
```

3.568.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx = -\frac{2(b+c)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{\log(a+b\cosh(x))}{b}$$

input
```
Integrate[(b + c + Sinh[x])/(a + b*Cosh[x]),x]
```

output
```
(-2*(b + c)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
 + Log[a + b*Cosh[x]]/b
```

3.568.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b+c-i\sin(ix)}{a+b\cos(ix)}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(\frac{b+c}{a+b\cosh(x)}+\frac{\sinh(x)}{a+b\cosh(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(b+c)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{\log(a+b\cosh(x))}{b}$$

input
```
Int[(b + c + Sinh[x])/(a + b*Cosh[x]),x]
```

output
```
(2*(b + c)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt
[a + b]) + Log[a + b*Cosh[x]]/b
```

3.568.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.568.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(47) = 94.

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.82

method	result
default	$-\frac{\ln(\tanh(\frac{x}{2})-1)}{b}-\frac{\ln(\tanh(\frac{x}{2})+1)}{b}+\frac{\ln\left(\tanh(\frac{x}{2})^2a-\tanh(\frac{x}{2})^2b-a-b\right)-\frac{2\left(-b^2-cb\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh(\frac{x}{2})}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{b}$
risch	$\frac{x}{b}+\frac{2x\,a^2b}{-a^2b^2+b^4}-\frac{2x\,b^3}{-a^2b^2+b^4}+\frac{\ln\left(\mathrm{e}^x-\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a^2b^3c+a^2b^2c^2-b^6-2b^5c-b^4c^2}}{b^2(b+c)}\right)a^2}{(a^2-b^2)b}-\frac{b\ln\left(\mathrm{e}^x-\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a}}{}\right.}{a^2-}$

input
```
int((b+c+sinh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/b*ln(tanh(1/2*x)-1)-1/b*ln(tanh(1/2*x)+1)+2/b*(1/2*ln(tanh(1/2*x)^2*a-t
anh(1/2*x)^2*b-a-b)-(-b^2-b*c)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*
x)/((a+b)*(a-b))^(1/2)))
```

3.568.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(47) = 94.

Time = 0.26 (sec) , antiderivative size = 289, normalized size of antiderivative = 5.07

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx$$
$$=\left[\frac{\sqrt{a^2-b^2}(b^2+bc)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right.}{a^2b-b^3}\right.$$
$$-\frac{2\sqrt{-a^2+b^2}(b^2+bc)\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)+(a^2-b^2)x-(a^2-b^2)\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right.}{a^2b-b^3}$$

input
```
integrate((b+c+sinh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(
b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(
b*cosh(x) + a)*sinh(x) + b)) - (a^2 - b^2)*x + (a^2 - b^2)*log(2*(b*cosh(x
) + a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*(b^2 + b*
c)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^
2 - b^2)*x - (a^2 - b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/(a^2*
b - b^3)]
```

3.568.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(49) = 98$.

Time = 16.66 (sec) , antiderivative size = 840, normalized size of antiderivative = 14.74

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate((b+c+sinh(x))/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*(2*c*atan(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(
x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (tanh(x/2) + c*tanh(x/2)/b + x/b - 2*
log(tanh(x/2) + 1)/b, Eq(a, b)), (-1/tanh(x/2) - c/(b*tanh(x/2)) + x/b - 2
*log(tanh(x/2) + 1)/b + 2*log(tanh(x/2))/b, Eq(a, -b)), ((c*x + cosh(x))/a
, Eq(b, 0)), (a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a -
 b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + a*sqrt(a/(a - b) + b/(a - b))*l
og(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a -
b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + a*sqrt(a/(a - b) + b/(a - b))*lo
g(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)
) - b**2*sqrt(a/(a - b) + b/(a - b))) - 2*a*sqrt(a/(a - b) + b/(a - b))*lo
g(tanh(x/2) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) +
b/(a - b))) - b**2*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt
(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + b**2*log(sqr
t(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b
**2*sqrt(a/(a - b) + b/(a - b))) - b*c*log(-sqrt(a/(a - b) + b/(a - b)) +
tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a -
 b))) + b*c*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a -
b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - b*x*sqrt(a/(a - b) +
 b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a
- b))) - b*sqrt(a/(a - b) + b/(a - b))*log(-sqrt(a/(a - b) + b/(a - b))...
```

3.568.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b+c+sinh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.568.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx = \frac{2\,(b+c)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - \frac{x}{b} + \frac{\log\left(be^{(2\,x)}+2\,ae^x+b\right)}{b}$$

input
```
integrate((b+c+sinh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*(b + c)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - x/b + lo
g(b*e^(2*x) + 2*a*e^x + b)/b
```

3.568.9 Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.47

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{\ln\left(b\sqrt{(a+b)\,(a-b)}+a^2\,e^x-b^2\,e^x+a\,e^x\sqrt{(a+b)\,(a-b)}\right)\left(b^2\sqrt{(a+b)\,(a-b)}+a^2-b^2+b\,c\,\sqrt{(a\right.}{a^2\,b-b^3}$$
$$-\frac{x}{b}$$
$$-\frac{\ln\left(b\sqrt{(a+b)\,(a-b)}-a^2\,e^x+b^2\,e^x+a\,e^x\sqrt{(a+b)\,(a-b)}\right)\left(b^2\sqrt{(a+b)\,(a-b)}-a^2+b^2+b\,c\,\sqrt{\right.}{a^2\,b-b^3}$$

input
```
int((b + c + sinh(x))/(a + b*cosh(x)),x)
```

output
```
(log(b*((a + b)*(a - b))^(1/2) + a^2*exp(x) - b^2*exp(x) + a*exp(x)*((a +
b)*(a - b))^(1/2))*(b^2*((a + b)*(a - b))^(1/2) + a^2 - b^2 + b*c*((a + b)
*(a - b))^(1/2)))/(a^2*b - b^3) - x/b - (log(b*((a + b)*(a - b))^(1/2) - a
^2*exp(x) + b^2*exp(x) + a*exp(x)*((a + b)*(a - b))^(1/2))*(b^2*((a + b)*(
a - b))^(1/2) - a^2 + b^2 + b*c*((a + b)*(a - b))^(1/2)))/(a^2*b - b^3)
```

3.568. $\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx$

3.569 $\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx$

3.569.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx=\frac{2(b+c)\text{arctanh}\left(\frac{\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{\log(a-b\cosh(x))}{b}$$

output
```
-ln(a-b*cosh(x))/b+2*(b+c)*arctanh((a+b)^(1/2)*tanh(1/2*x)/(a-b)^(1/2))/(a
-b)^(1/2)/(a+b)^(1/2)
```

3.569.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx=-\frac{2(b+c)\arctan\left(\frac{(a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-\frac{\log(a-b\cosh(x))}{b}$$

input
```
Integrate[(b + c + Sinh[x])/(a - b*Cosh[x]),x]
```

output
```
(-2*(b + c)*ArcTan[((a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
 - Log[a - b*Cosh[x]]/b
```

3.569.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b+c-i\sin(ix)}{a-b\cos(ix)}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(\frac{-b-c}{b\cosh(x)-a}+\frac{\sinh(x)}{a-b\cosh(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(b+c)\text{arctanh}\left(\frac{\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{\log(a-b\cosh(x))}{b}$$

input
```
Int[(b + c + Sinh[x])/(a - b*Cosh[x]),x]
```

output
```
(2*(b + c)*ArcTanh[(Sqrt[a + b]*Tanh[x/2])/Sqrt[a - b]])/(Sqrt[a - b]*Sqrt
[a + b]) - Log[a - b*Cosh[x]]/b
```

3.569.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.569.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(49) = 98.

Time = 0.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

method	result
default	$\frac{\frac{2(-a-b)\ln\left(\tanh\left(\frac{x}{2}\right)^2 a+\tanh\left(\frac{x}{2}\right)^2 b-a+b\right)}{2a+2b}-\frac{2\left(-b^2-cb\right)\operatorname{arctanh}\left(\frac{(a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$
risch	$-\frac{x}{b}-\frac{2x\,a^2 b}{-a^2b^2+b^4}+\frac{2x\,b^3}{-a^2b^2+b^4}-\frac{\ln\left(\mathrm{e}^x+\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a^2b^3c+a^2b^2c^2-b^6-2b^5c-b^4c^2}}{b^2(b+c)}\right)a^2}{(a^2-b^2)b}+\frac{b\ln\left(\mathrm{e}^x+\frac{-a\,b^2-cab+\sqrt{a^2b^4\cdots}}{\cdots}\right)}{\cdots}$

input
```
int((b+c+sinh(x))/(a-b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/b*(1/2*(-a-b)/(a+b)*ln(tanh(1/2*x)^2*a+tanh(1/2*x)^2*b-a+b)-(-b^2-b*c)/(
(a+b)*(a-b))^(1/2)*arctanh((a+b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))+1/b*ln(
tanh(1/2*x)-1)+1/b*ln(tanh(1/2*x)+1)
```

3.569.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(49) = 98.

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 5.07

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx$$
$$=\left[\frac{\sqrt{a^2-b^2}(b^2+bc)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2-2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)-ab)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)-a\cdots}{b\cosh(x)^2+b\sinh(x)^2-2\,a\cosh(x)+2\,(b\cosh(x)-a)\sinh(x)+b}\cdots}{a^2b-b^3}\cdots\right.$$

input
```
integrate((b+c+sinh(x))/(a-b*cosh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 - 2*a*b*c
osh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) - a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(
b*cosh(x) + b*sinh(x) - a))/(b*cosh(x)^2 + b*sinh(x)^2 - 2*a*cosh(x) + 2*(
b*cosh(x) - a)*sinh(x) + b)) + (a^2 - b^2)*x - (a^2 - b^2)*log(2*(b*cosh(x
) - a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), (2*sqrt(-a^2 + b^2)*(b^2 + b*c
)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) - a)/(a^2 - b^2)) + (a^2
 - b^2)*x - (a^2 - b^2)*log(2*(b*cosh(x) - a)/(cosh(x) - sinh(x))))/(a^2*b
 - b^3)]
```

3.569.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. 2(49) = 98.

Time = 15.82 (sec) , antiderivative size = 840, normalized size of antiderivative = 14.24

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate((b+c+sinh(x))/(a-b*cosh(x)),x)
```

output
```
Piecewise((zoo*(2*c*atan(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(
x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (-tanh(x/2) - c*tanh(x/2)/b - x/b + 2
*log(tanh(x/2) + 1)/b, Eq(a, -b)), (1/tanh(x/2) + c/(b*tanh(x/2)) - x/b +
2*log(tanh(x/2) + 1)/b - 2*log(tanh(x/2))/b, Eq(a, b)), ((c*x + cosh(x))/a
, Eq(b, 0)), (-a*x*sqrt(a/(a + b) - b/(a + b))/(a*b*sqrt(a/(a + b) - b/(a
+ b)) + b**2*sqrt(a/(a + b) - b/(a + b))) - a*sqrt(a/(a + b) - b/(a + b))*
log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b) - b/(a +
 b)) + b**2*sqrt(a/(a + b) - b/(a + b))) - a*sqrt(a/(a + b) - b/(a + b))*l
og(sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b) - b/(a + b
)) + b**2*sqrt(a/(a + b) - b/(a + b))) + 2*a*sqrt(a/(a + b) - b/(a + b))*l
og(tanh(x/2) + 1)/(a*b*sqrt(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) -
 b/(a + b))) - b**2*log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqr
t(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))) + b**2*log(sq
rt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b) - b/(a + b)) +
b**2*sqrt(a/(a + b) - b/(a + b))) - b*c*log(-sqrt(a/(a + b) - b/(a + b)) +
 tanh(x/2))/(a*b*sqrt(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a
+ b))) + b*c*log(sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a +
 b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))) - b*x*sqrt(a/(a + b)
- b/(a + b))/(a*b*sqrt(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a
 + b))) - b*sqrt(a/(a + b) - b/(a + b))*log(-sqrt(a/(a + b) - b/(a + b)...
```

3.569.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b+c+sinh(x))/(a-b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.569.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx = -\frac{2\,(b+c)\arctan\left(\frac{be^x-a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{x}{b} - \frac{\log\left(be^{(2\,x)} - 2\,ae^x + b\right)}{b}$$

input
```
integrate((b+c+sinh(x))/(a-b*cosh(x)),x, algorithm="giac")
```

output
```
-2*(b + c)*arctan((b*e^x - a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) + x/b - l
og(b*e^(2*x) - 2*a*e^x + b)/b
```

3.569.9 Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.37

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx = \frac{x}{b} - \frac{\ln\left(b\,\sqrt{(a+b)\,(a-b)} + a^2\,e^x - b^2\,e^x - a\,e^x\,\sqrt{(a+b)\,(a-b)}\right)\,\left(b^2\,\sqrt{(a+b)\,(a-b)} + a^2 - b^2 + b\,c\,\sqrt{\cdots}\right.}{a^2\,b - b^3} + \frac{\ln\left(b\,\sqrt{(a+b)\,(a-b)} - a^2\,e^x + b^2\,e^x - a\,e^x\,\sqrt{(a+b)\,(a-b)}\right)\,\left(b^2\,\sqrt{(a+b)\,(a-b)} - a^2 + b^2 + b\,c\,\sqrt{}\right.}{a^2\,b - b^3}$$

input
```
int((b + c + sinh(x))/(a - b*cosh(x)),x)
```

output
```
x/b - (log(b*((a + b)*(a - b))^(1/2) + a^2*exp(x) - b^2*exp(x) - a*exp(x)*
((a + b)*(a - b))^(1/2))*(b^2*((a + b)*(a - b))^(1/2) + a^2 - b^2 + b*c*((
a + b)*(a - b))^(1/2)))/(a^2*b - b^3) + (log(b*((a + b)*(a - b))^(1/2) - a
^2*exp(x) + b^2*exp(x) - a*exp(x)*((a + b)*(a - b))^(1/2))*(b^2*((a + b)*(
a - b))^(1/2) - a^2 + b^2 + b*c*((a + b)*(a - b))^(1/2)))/(a^2*b - b^3)
```

3.570 $\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx$

3.570.1 Optimal result

Integrand size = 17, antiderivative size = 25

$$\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx=\frac{\log(a+b\sinh(x))}{b}-\frac{x\cosh(x)}{a+b\sinh(x)}$$

output
```
ln(a+b*sinh(x))/b-x*cosh(x)/(a+b*sinh(x))
```

3.570.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx=\frac{\log(a+b\sinh(x))}{b}-\frac{x\cosh(x)}{a+b\sinh(x)}$$

input
```
Integrate[(x*(b - a*Sinh[x]))/(a + b*Sinh[x])^2,x]
```

output
```
Log[a + b*Sinh[x]]/b - (x*Cosh[x])/(a + b*Sinh[x])
```

3.570.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6170, 3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{6170}$$

$$\int \frac{\cosh(x)}{a+b\sinh(x)}dx-\frac{x\cosh(x)}{a+b\sinh(x)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x\cosh(x)}{a+b\sinh(x)}+\int \frac{\cos(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{1}{a+b\sinh(x)}d(b\sinh(x))}{b}-\frac{x\cosh(x)}{a+b\sinh(x)}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(a+b\sinh(x))}{b}-\frac{x\cosh(x)}{a+b\sinh(x)}$$

input
```
Int[(x*(b - a*Sinh[x]))/(a + b*Sinh[x])^2,x]
```

output
```
Log[a + b*Sinh[x]]/b - (x*Cosh[x])/(a + b*Sinh[x])
```

3.570.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 6170
```
Int[(((e_.) + (f_.)*(x_))*((A_) + (B_.)*Sinh[(c_.) + (d_.)*(x_)]))/((a_) +
(b_.)*Sinh[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[B*(e + f*x)*(Cosh[c +
d*x]/(a*d*(a + b*Sinh[c + d*x]))), x] - Simp[B*(f/(a*d))   Int[Cosh[c + d*x
]/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && Eq
Q[a*A + b*B, 0]
```

3.570.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(25) = 50$.

Time = 0.62 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

method	result	size
risch	$-\frac{2x}{b}+\frac{2x(a\,\mathrm{e}^{x}-b)}{b(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b)}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{b}$	58
parallelrisch	$\frac{(a+b\sinh(x))\ln\left(\frac{-2b\sinh(x)-2a}{\cosh(x)+1}\right)+(-2b\sinh(x)-2a)\ln(1-\coth(x)+\operatorname{csch}(x))-x(a+b\cosh(x)+b\sinh(x))}{b(a+b\sinh(x))}$	70

input
```
int(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*x/b+2*x*(a*exp(x)-b)/b/(b*exp(2*x)+2*a*exp(x)-b)+1/b*ln(exp(2*x)+2*a/b*
exp(x)-1)
```

3.570.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 5.36

$$\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx =$$
$$-\frac{2\,bx\cosh(x)^2+2\,bx\sinh(x)^2+2\,ax\cosh(x)-\left(b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)\ldots}{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)-b^2+2\,(b^2\cosh\ldots}$$

input
```
integrate(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
-(2*b*x*cosh(x)^2 + 2*b*x*sinh(x)^2 + 2*a*x*cosh(x) - (b*cosh(x)^2 + b*sin
h(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)*log(2*(b*sinh(x) + a
)/(cosh(x) - sinh(x))) + 2*(2*b*x*cosh(x) + a*x)*sinh(x))/(b^2*cosh(x)^2 +
 b^2*sinh(x)^2 + 2*a*b*cosh(x) - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

3.570.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(b-a*sinh(x))/(a+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.570.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(25) = 50.

Time = 0.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx = -\frac{2\left(bxe^{(2x)}+axe^{x}\right)}{b^2e^{(2x)}+2\,abe^{x}-b^2}+\frac{\log\left(\frac{be^{(2x)}+2\,ae^{x}-b}{b}\right)}{b}$$

input
```
integrate(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-2*(b*x*e^(2*x) + a*x*e^x)/(b^2*e^(2*x) + 2*a*b*e^x - b^2) + log((b*e^(2*x
) + 2*a*e^x - b)/b)/b
```

3.570.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(25) = 50.

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.84

$$\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx = -\frac{2\,bxe^{(2\,x)} - be^{(2\,x)}\log\left(-be^{(2\,x)} - 2\,ae^{x} + b\right) - 2\,ae^{x}\log\left(-be^{(2\,x)} - 2\,ae^{x} + b\right) + 2\,bx + b\log\left(-be^{(2\,x)} - 2\right.}{b^2e^{(2\,x)} + 2\,abe^{x} - b^2}$$

input
```
integrate(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-(2*b*x*e^(2*x) - b*e^(2*x)*log(-b*e^(2*x) - 2*a*e^x + b) - 2*a*e^x*log(-b
*e^(2*x) - 2*a*e^x + b) + 2*b*x + b*log(-b*e^(2*x) - 2*a*e^x + b))/(b^2*e^
(2*x) + 2*a*b*e^x - b^2)
```

3.570.9 Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.12

$$\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx = \frac{\ln\left(2\,a\,e^{x} - b + b\,e^{2\,x}\right)}{b} - \frac{\frac{2\,\left(x\,a^2\,b + x\,b^3\right)}{a^2\,b + b^3} - \frac{2\,e^{x}\,\left(x\,a^3\,b + x\,a\,b^3\right)}{b\,\left(a^2\,b + b^3\right)}}{2\,a\,e^{x} - b + b\,e^{2\,x}} - \frac{2\,x}{b}$$

input
```
int((x*(b - a*sinh(x)))/(a + b*sinh(x))^2,x)
```

output
```
log(2*a*exp(x) - b + b*exp(2*x))/b - ((2*(b^3*x + a^2*b*x))/(a^2*b + b^3)
- (2*exp(x)*(a*b^3*x + a^3*b*x))/(b*(a^2*b + b^3)))/(2*a*exp(x) - b + b*ex
p(2*x)) - (2*x)/b
```

3.571 $\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx$

3.571.1 Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx=-\frac{\log(a+b\cosh(x))}{b}+\frac{x\sinh(x)}{a+b\cosh(x)}$$

output
```
-ln(a+b*cosh(x))/b+x*sinh(x)/(a+b*cosh(x))
```

3.571.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx=-\frac{\log(a+b\cosh(x))}{b}+\frac{x\sinh(x)}{a+b\cosh(x)}$$

input
```
Integrate[(x*(b + a*Cosh[x]))/(a + b*Cosh[x])^2,x]
```

output
```
-(Log[a + b*Cosh[x]]/b) + (x*Sinh[x])/(a + b*Cosh[x])
```

3.571.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6171, 3042, 26, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a\cosh(x)+b)}{(a+b\cosh(x))^2}dx$$

$$\Big\downarrow \text{ 6171}$$

$$\frac{x\sinh(x)}{a+b\cosh(x)}-\int \frac{\sinh(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x\sinh(x)}{a+b\cosh(x)}-\int -\frac{i\cos\left(ix-\frac{\pi}{2}\right)}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$\frac{x\sinh(x)}{a+b\cosh(x)}+i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3147}$$

$$\frac{x\sinh(x)}{a+b\cosh(x)}-\frac{\int \frac{1}{a+b\cosh(x)}d(b\cosh(x))}{b}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{x\sinh(x)}{a+b\cosh(x)}-\frac{\log(a+b\cosh(x))}{b}$$

input
```
Int[(x*(b + a*Cosh[x]))/(a + b*Cosh[x])^2,x]
```

output
```
-(Log[a + b*Cosh[x]]/b) + (x*Sinh[x])/(a + b*Cosh[x])
```

3.571.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 6171
```
Int[((Cosh[(c_.) + (d_.)*(x_)]*(B_.) + (A_))*((e_.) + (f_.)*(x_)))/(Cosh[(c
_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[B*(e + f*x)*(Sinh[c +
d*x]/(a*d*(a + b*Cosh[c + d*x]))), x] - Simp[B*(f/(a*d))   Int[Sinh[c + d*x
]/(a + b*Cosh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && Eq
Q[a*A - b*B, 0]
```

3.571.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

method	result	size
risch	$\frac{2x}{b}-\frac{2x(a\,e^{x}+b)}{b(b\,e^{2x}+2a\,e^{x}+b)}-\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)}{b}$	55
parallelrisch	$\frac{(-b\cosh(x)-a)\ln\left(\frac{-2b\cosh(x)-2a}{\cosh(x)+1}\right)+(2b\cosh(x)+2a)\ln(1-\coth(x)+\operatorname{csch}(x))+x(a+b\cosh(x)+b\sinh(x))}{b(a+b\cosh(x))}$	72

input
```
int(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*x/b-2*x*(a*exp(x)+b)/b/(b*exp(2*x)+2*a*exp(x)+b)-1/b*ln(exp(2*x)+2*a/b*e
xp(x)+1)
```

3.571.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.16

$$\int \frac{x(b + a\cosh(x))}{(a + b\cosh(x))^2}\,dx$$
$$= \frac{2\,bx\cosh\left(x\right)^2 + 2\,bx\sinh\left(x\right)^2 + 2\,ax\cosh\left(x\right) - \left(b\cosh\left(x\right)^2 + b\sinh\left(x\right)^2 + 2\,a\cosh\left(x\right) + 2\left(b\cosh\left(x\right) + a\right.\right.}{b^2\cosh\left(x\right)^2 + b^2\sinh\left(x\right)^2 + 2\,ab\cosh\left(x\right) + b^2 + 2\left(b^2\cosh\right.}$$

input
```
integrate(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x, algorithm="fricas")
```

output
```
(2*b*x*cosh(x)^2 + 2*b*x*sinh(x)^2 + 2*a*x*cosh(x) - (b*cosh(x)^2 + b*sinh
(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*log(2*(b*cosh(x) + a)
/(cosh(x) - sinh(x))) + 2*(2*b*x*cosh(x) + a*x)*sinh(x))/(b^2*cosh(x)^2 +
b^2*sinh(x)^2 + 2*a*b*cosh(x) + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

3.571.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(b + a\cosh(x))}{(a + b\cosh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(b+a*cosh(x))/(a+b*cosh(x))**2,x)
```

output
```
Timed out
```

3.571.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.571.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.00

$$\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx$$
$$= \frac{2\,bxe^{(2\,x)} - be^{(2\,x)}\log\left(-be^{(2\,x)} - 2\,ae^{x} - b\right) - 2\,ae^{x}\log\left(-be^{(2\,x)} - 2\,ae^{x} - b\right) - 2\,bx - b\log\left(-be^{(2\,x)} - 2\,a\right.}{b^2e^{(2\,x)} + 2\,abe^{x} + b^2}$$

input
```
integrate(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x, algorithm="giac")
```

output
```
(2*b*x*e^(2*x) - b*e^(2*x)*log(-b*e^(2*x) - 2*a*e^x - b) - 2*a*e^x*log(-b*
e^(2*x) - 2*a*e^x - b) - 2*b*x - b*log(-b*e^(2*x) - 2*a*e^x - b))/(b^2*e^(
2*x) + 2*a*b*e^x + b^2)
```

3.571.9 Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 4.20

$$\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx = \frac{2\,x}{b} + \frac{\frac{2\,\left(b^3\,x-a^2\,b\,x\right)}{a^2\,b-b^3} + \frac{2\,\mathrm{e}^{x}\,\left(a\,b^3\,x-a^3\,b\,x\right)}{b\,\left(a^2\,b-b^3\right)}}{b+2\,a\,\mathrm{e}^{x}+b\,\mathrm{e}^{2\,x}} - \frac{\ln\left(b+2\,a\,\mathrm{e}^{x}+b\,\mathrm{e}^{2\,x}\right)}{b}$$

input
```
int((x*(b + a*cosh(x)))/(a + b*cosh(x))^2,x)
```

output
```
(2*x)/b + ((2*(b^3*x - a^2*b*x))/(a^2*b - b^3) + (2*exp(x)*(a*b^3*x - a^3*
b*x))/(b*(a^2*b - b^3)))/(b + 2*a*exp(x) + b*exp(2*x)) - log(b + 2*a*exp(x
) + b*exp(2*x))/b
```

3.571. $\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx$

3.572 $\int \frac{a+b\operatorname{sech}(x)}{c+d\cosh(x)}\,dx$

3.572.1 Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{a+b\operatorname{sech}(x)}{c+d\cosh(x)}\,dx=\frac{b\arctan(\sinh(x))}{c}+\frac{2(ac-bd)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}$$

output
```
b*arctan(sinh(x))/c+2*(a*c-b*d)*arctanh((c-d)^(1/2)*tanh(1/2*x)/(c+d)^(1/2
))/c/(c-d)^(1/2)/(c+d)^(1/2)
```

3.572.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{a+b\operatorname{sech}(x)}{c+d\cosh(x)}\,dx=\frac{2\left(b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{(-ac+bd)\arctan\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}}\right)}{c}$$

input
```
Integrate[(a + b*Sech[x])/(c + d*Cosh[x]),x]
```

output
```
(2*(b*ArcTan[Tanh[x/2]] + ((-(a*c) + b*d)*ArcTan[((c - d)*Tanh[x/2])/Sqrt[
-c^2 + d^2]])/Sqrt[-c^2 + d^2]))/c
```

3.572.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 3307, 3042, 3480, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}(x)}{c+d\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(\frac{\pi}{2}+ix\right)}{c+d\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3307}$$

$$\int \frac{\text{sech}(x)(a\cosh(x)+b)}{c+d\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b+a\sin\left(\frac{\pi}{2}+ix\right)}{\sin\left(\frac{\pi}{2}+ix\right)\left(c+d\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3480}$$

$$\frac{(ac-bd)\int \frac{1}{c+d\cosh(x)}dx}{c}+\frac{b\int \text{sech}(x)dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(ac-bd)\int \frac{1}{c+d\sin\left(ix+\frac{\pi}{2}\right)}dx}{c}+\frac{b\int \csc\left(ix+\frac{\pi}{2}\right)dx}{c}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2(ac-bd)\int \frac{1}{-\left((c-d)\tanh^2\left(\frac{x}{2}\right)\right)+c+d}d\tanh\left(\frac{x}{2}\right)}{c}+\frac{b\int \csc\left(ix+\frac{\pi}{2}\right)dx}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}+\frac{b\int \csc\left(ix+\frac{\pi}{2}\right)dx}{c}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}+\frac{b\arctan(\sinh(x))}{c}$$

input
```
Int[(a + b*Sech[x])/(c + d*Cosh[x]),x]
```

output
```
(b*ArcTan[Sinh[x]])/c + (2*(a*c - b*d)*ArcTanh[(Sqrt[c - d]*Tanh[x/2])/Sqr
t[c + d]])/(c*Sqrt[c - d]*Sqrt[c + d])
```

3.572.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.572.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

method	result
default	$\frac{2b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{c}-\frac{2(-ac+bd)\,\mathrm{arctanh}\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c\sqrt{(c+d)(c-d)}}$
risch	$\frac{ib\ln(\mathrm{e}^{x}+i)}{c}-\frac{ib\ln(\mathrm{e}^{x}-i)}{c}+\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{c^2-d^2}\,c-c^2+d^2}{\sqrt{c^2-d^2}\,d}\right)a}{\sqrt{c^2-d^2}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{c^2-d^2}\,c-c^2+d^2}{\sqrt{c^2-d^2}\,d}\right)bd}{\sqrt{c^2-d^2}\,c}-\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{c^2-d^2}\,c+c^2-d^2}{\sqrt{c^2-d^2}\,d}\right)a}{\sqrt{c^2-d^2}}+$ ln

input
```
int((a+b*sech(x))/(c+d*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*b/c*arctan(tanh(1/2*x))-2*(-a*c+b*d)/c/((c+d)*(c-d))^(1/2)*arctanh((c-d)
*tanh(1/2*x)/((c+d)*(c-d))^(1/2))
```

3.572.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 249, normalized size of antiderivative = 4.02

$$\int \frac{a+b\mathrm{sech}(x)}{c+d\cosh(x)}\,dx$$
$$=\left[-\frac{(ac-bd)\sqrt{c^2-d^2}\log\left(\frac{d^2\cosh(x)^2+d^2\sinh(x)^2+2\,cd\cosh(x)+2\,c^2-d^2+2\,(d^2\cosh(x)+cd)\sinh(x)+2\sqrt{c^2-d^2}(d\cosh(x)+d\sinh(x)}{d\cosh(x)^2+d\sinh(x)^2+2\,c\cosh(x)+2\,(d\cosh(x)+c)\sinh(x)+d}\right.}{c^3-cd^2}\right.$$
$$-\frac{2\left((ac-bd)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+d^2}(d\cosh(x)+d\sinh(x)+c)}{c^2-d^2}\right)-(bc^2-bd^2)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\right.}{c^3-cd^2}$$

input
```
integrate((a+b*sech(x))/(c+d*cosh(x)),x, algorithm="fricas")
```

output
```
[-((a*c - b*d)*sqrt(c^2 - d^2)*log((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*
cosh(x) + 2*c^2 - d^2 + 2*(d^2*cosh(x) + c*d)*sinh(x) + 2*sqrt(c^2 - d^2)*
(d*cosh(x) + d*sinh(x) + c))/(d*cosh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*
(d*cosh(x) + c)*sinh(x) + d)) - 2*(b*c^2 - b*d^2)*arctan(cosh(x) + sinh(x)
))/(c^3 - c*d^2), -2*((a*c - b*d)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2
)*(d*cosh(x) + d*sinh(x) + c)/(c^2 - d^2)) - (b*c^2 - b*d^2)*arctan(cosh(x
) + sinh(x)))/(c^3 - c*d^2)]
```

3.572.6 Sympy [F]

$$\int \frac{a + b\mathrm{sech}(x)}{c + d\cosh(x)}\,dx = \int \frac{a + b\,\mathrm{sech}\,(x)}{c + d\cosh\,(x)}\,dx$$

input
```
integrate((a+b*sech(x))/(c+d*cosh(x)),x)
```

output
```
Integral((a + b*sech(x))/(c + d*cosh(x)), x)
```

3.572.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\mathrm{sech}(x)}{c + d\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sech(x))/(c+d*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

3.572.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{a + b\mathrm{sech}(x)}{c + d\cosh(x)}\,dx = \frac{2\,b\arctan\,(e^x)}{c} + \frac{2\,(ac - bd)\arctan\left(\frac{de^x + c}{\sqrt{-c^2 + d^2}}\right)}{\sqrt{-c^2 + d^2}c}$$

input
```
integrate((a+b*sech(x))/(c+d*cosh(x)),x, algorithm="giac")
```

output
```
2*b*arctan(e^x)/c + 2*(a*c - b*d)*arctan((d*e^x + c)/sqrt(-c^2 + d^2))/(sq
rt(-c^2 + d^2)*c)
```

3.572.9 Mupad [B] (verification not implemented)

Time = 7.58 (sec) , antiderivative size = 636, normalized size of antiderivative = 10.26

$$\int \frac{a+b\operatorname{sech}(x)}{c+d\cosh(x)}\,dx$$

$$= \frac{\ln\left(\frac{\sqrt{(c+d)(c-d)}\,(a\,c-b\,d)\left(\frac{32\left(a^2c^2d-2abcd^2-4e^xb^2c^3-2b^2c^2d+3e^xb^2cd^2+2b^2d^3\right)}{d^5}+\frac{\sqrt{(c+d)(c-d)}\left(\frac{32c^2\left(2bd^2-4ac^2e^x+ad^2e^x-2acd+\right.}{d^5}\right.}{}\right)}{cd^2-c^3}\right)}{}$$

$$-\frac{\ln\left(-\frac{32b(ac-bd)(2bd-ade^x+4bce^x)}{d^5}-\frac{\sqrt{(c+d)(c-d)}\,(ac-bd)\left(\frac{32\left(a^2c^2d-2abcd^2-4e^xb^2c^3-2b^2c^2d+3e^xb^2cd^2+2b^2d^3\right)}{d^5}-\frac{\sqrt{(c+}{}\right.}{}\right)}{}$$

$$-\frac{b\ln\left(e^x-i\right)\,1i}{c}+\frac{b\ln\left(e^x+1i\right)\,1i}{c}$$

input
```
int((a + b/cosh(x))/(c + d*cosh(x)),x)
```

output
```
(b*log(exp(x) + 1i)*1i)/c - (b*log(exp(x) - 1i)*1i)/c + (log((((c + d)*(c
- d))^(1/2)*(a*c - b*d)*((32*(2*b^2*d^3 + a^2*c^2*d - 2*b^2*c^2*d - 4*b^2*
c^3*exp(x) + 3*b^2*c*d^2*exp(x) - 2*a*b*c*d^2))/d^5 + (((c + d)*(c - d))^(
1/2)*((32*c^2*(2*b*d^2 - 4*a*c^2*exp(x) + a*d^2*exp(x) - 2*a*c*d + 3*b*c*d
*exp(x)))/d^5 - (32*c^2*((c + d)*(c - d))^(1/2)*(a*c - b*d)*(3*c^2*d - 2*d
^3 + 4*c^3*exp(x) - 3*c*d^2*exp(x)))/(d^5*(c*d^2 - c^3)))*(a*c - b*d))/(c*
d^2 - c^3)))/(c*d^2 - c^3) - (32*b*(a*c - b*d)*(2*b*d - a*d*exp(x) + 4*b*c
*exp(x)))/d^5)*((c + d)*(c - d))^(1/2)*(a*c - b*d))/(c*d^2 - c^3) - (log(-
 (32*b*(a*c - b*d)*(2*b*d - a*d*exp(x) + 4*b*c*exp(x)))/d^5 - (((c + d)*(c
 - d))^(1/2)*(a*c - b*d)*((32*(2*b^2*d^3 + a^2*c^2*d - 2*b^2*c^2*d - 4*b^2
*c^3*exp(x) + 3*b^2*c*d^2*exp(x) - 2*a*b*c*d^2))/d^5 - (((c + d)*(c - d))^
(1/2)*((32*c^2*(2*b*d^2 - 4*a*c^2*exp(x) + a*d^2*exp(x) - 2*a*c*d + 3*b*c*
d*exp(x)))/d^5 + (32*c^2*((c + d)*(c - d))^(1/2)*(a*c - b*d)*(3*c^2*d - 2*
d^3 + 4*c^3*exp(x) - 3*c*d^2*exp(x)))/(d^5*(c*d^2 - c^3)))*(a*c - b*d))/(c
*d^2 - c^3)))/(c*d^2 - c^3))*((c + d)*(c - d))^(1/2)*(a*c - b*d))/(c*d^2 -
 c^3)
```

3.573 $\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx$

3.573.1 Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx=-\frac{b\text{arctanh}(\cosh(x))}{c}-\frac{2(ac-bd)\text{arctanh}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}$$

output
```
-b*arctanh(cosh(x))/c-2*(a*c-b*d)*arctanh((d-c*tanh(1/2*x))/(c^2+d^2)^(1/2
))/c/(c^2+d^2)^(1/2)
```

3.573.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx=\frac{\frac{2(ac-bd)\arctan\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{-c^2-d^2}}\right)}{\sqrt{-c^2-d^2}}+b\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{c}$$

input
```
Integrate[(a + b*Csch[x])/(c + d*Sinh[x]),x]
```

output
```
((2*(a*c - b*d)*ArcTan[(d - c*Tanh[x/2])/Sqrt[-c^2 - d^2]])/Sqrt[-c^2 - d^
2] + b*(-Log[Cosh[x/2]] + Log[Sinh[x/2]]))/c
```

3.573.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.933, Rules used = {3042, 3307, 26, 26, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ib\csc(ix)}{c-id\sin(ix)}dx$$

$$\Big\downarrow \text{3307}$$

$$\int -\frac{i\text{csch}(x)(ia\sinh(x)+ib)}{c+d\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{i\text{csch}(x)(b+a\sinh(x))}{c+d\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\text{csch}(x)(a\sinh(x)+b)}{c+d\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i(b-ia\sin(ix))}{\sin(ix)(c-id\sin(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{b-ia\sin(ix)}{\sin(ix)(c-id\sin(ix))}dx$$

$$\Big\downarrow \text{3480}$$

$$i\left(\frac{b\int -i\text{csch}(x)dx}{c}-\frac{i(ac-bd)\int \frac{1}{c+d\sinh(x)}dx}{c}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{i(ac-bd)\int \frac{1}{c+d\sinh(x)}dx}{c}-\frac{ib\int \operatorname{csch}(x)dx}{c}\right)$$

$\Big\downarrow$ 3042

$$i\left(-\frac{i(ac-bd)\int \frac{1}{c-id\sin(ix)}dx}{c}-\frac{ib\int i\csc(ix)dx}{c}\right)$$

$\Big\downarrow$ 26

$$i\left(\frac{b\int \csc(ix)dx}{c}-\frac{i(ac-bd)\int \frac{1}{c-id\sin(ix)}dx}{c}\right)$$

$\Big\downarrow$ 3139

$$i\left(\frac{b\int \csc(ix)dx}{c}-\frac{2i(ac-bd)\int \frac{1}{-c\tanh^2\left(\frac{x}{2}\right)+2d\tanh\left(\frac{x}{2}\right)+c}d\tanh\left(\frac{x}{2}\right)}{c}\right)$$

$\Big\downarrow$ 1083

$$i\left(\frac{4i(ac-bd)\int \frac{1}{4(c^2+d^2)-\left(2d-2c\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2d-2c\tanh\left(\frac{x}{2}\right)\right)}{c}+\frac{b\int \csc(ix)dx}{c}\right)$$

$\Big\downarrow$ 219

$$i\left(\frac{b\int \csc(ix)dx}{c}+\frac{2i(ac-bd)\operatorname{arctanh}\left(\frac{2d-2c\tanh\left(\frac{x}{2}\right)}{2\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}\right)$$

$\Big\downarrow$ 4257

$$i\left(\frac{2i(ac-bd)\operatorname{arctanh}\left(\frac{2d-2c\tanh\left(\frac{x}{2}\right)}{2\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}+\frac{ib\operatorname{arctanh}(\cosh(x))}{c}\right)$$

input
```
Int[(a + b*Csch[x])/(c + d*Sinh[x]),x]
```

output
```
I*((I*b*ArcTanh[Cosh[x]])/c + ((2*I)*(a*c - b*d)*ArcTanh[(2*d - 2*c*Tanh[x
/2])/(2*Sqrt[c^2 + d^2])])/(c*Sqrt[c^2 + d^2]))
```

3.573. $\int \frac{a+b\operatorname{csch}(x)}{c+d\sinh(x)}\,dx$

3.573.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.573. $\int \frac{a+b\operatorname{csch}(x)}{c+d\sinh(x)}\,dx$

3.573.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

method	result
default	$\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{c}-\frac{(-2ac+2bd)\,\mathrm{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}$
parts	$\frac{2a\ \mathrm{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{c}-\frac{2bd\ \mathrm{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}$
risch	$\frac{b\ln(\mathrm{e}^x-1)}{c}-\frac{b\ln(\mathrm{e}^x+1)}{c}+\frac{\ln\left(\mathrm{e}^x+\frac{\sqrt{c^2+d^2}\,c-c^2-d^2}{\sqrt{c^2+d^2}\,d}\right)a}{\sqrt{c^2+d^2}}-\frac{\ln\left(\mathrm{e}^x+\frac{\sqrt{c^2+d^2}\,c-c^2-d^2}{\sqrt{c^2+d^2}\,d}\right)bd}{\sqrt{c^2+d^2}\,c}-\frac{\ln\left(\mathrm{e}^x+\frac{\sqrt{c^2+d^2}\,c+c^2+d^2}{\sqrt{c^2+d^2}\,d}\right)a}{\sqrt{c^2+d^2}}+\frac{\ln}{}$

input
```
int((a+b*csch(x))/(c+d*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
b/c*ln(tanh(1/2*x))-(-2*a*c+2*b*d)/c/(c^2+d^2)^(1/2)*arctanh(1/2*(2*c*tanh
(1/2*x)-2*d)/(c^2+d^2)^(1/2))
```

3.573.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(54) = 108$.

Time = 0.47 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.97

$$\int \frac{a+b\operatorname{csch}(x)}{c+d\sinh(x)}\,dx = -\frac{(ac-bd)\sqrt{c^2+d^2}\log\left(\frac{d^2\cosh(x)^2+d^2\sinh(x)^2+2\,cd\cosh(x)+2\,c^2+d^2+2\,(d^2\cosh(x)+cd)\sinh(x)+2\,\sqrt{c^2+d^2}(d\cosh(x)+d\sinh(x)}{d\cosh(x)^2+d\sinh(x)^2+2\,c\cosh(x)+2\,(d\cosh(x)+c)\sinh(x)-d}\right.}{c^3+cd^2}$$

input
```
integrate((a+b*csch(x))/(c+d*sinh(x)),x, algorithm="fricas")
```

output
```
-((a*c - b*d)*sqrt(c^2 + d^2)*log((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*c
osh(x) + 2*c^2 + d^2 + 2*(d^2*cosh(x) + c*d)*sinh(x) + 2*sqrt(c^2 + d^2)*(
d*cosh(x) + d*sinh(x) + c))/(d*cosh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(
d*cosh(x) + c)*sinh(x) - d)) + (b*c^2 + b*d^2)*log(cosh(x) + sinh(x) + 1)
- (b*c^2 + b*d^2)*log(cosh(x) + sinh(x) - 1))/(c^3 + c*d^2)
```

3.573.6 Sympy [F]

$$\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx = \int \frac{a+b\,\text{csch}\,(x)}{c+d\sinh\,(x)}\,dx$$

input
```
integrate((a+b*csch(x))/(c+d*sinh(x)),x)
```

output
```
Integral((a + b*csch(x))/(c + d*sinh(x)), x)
```

3.573.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(54) = 108$.

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.43

$$\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx = -b\left(\frac{d\log\left(\frac{de^{(-x)}-c-\sqrt{c^2+d^2}}{de^{(-x)}-c+\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}c} + \frac{\log\left(e^{(-x)}+1\right)}{c} - \frac{\log\left(e^{(-x)}-1\right)}{c}\right) + \frac{a\log\left(\frac{de^{(-x)}-c-\sqrt{c^2+d^2}}{de^{(-x)}-c+\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}}$$

input
```
integrate((a+b*csch(x))/(c+d*sinh(x)),x, algorithm="maxima")
```

output
```
-b*(d*log((d*e^(-x) - c - sqrt(c^2 + d^2))/(d*e^(-x) - c + sqrt(c^2 + d^2)
))/(sqrt(c^2 + d^2)*c) + log(e^(-x) + 1)/c - log(e^(-x) - 1)/c) + a*log((d
*e^(-x) - c - sqrt(c^2 + d^2))/(d*e^(-x) - c + sqrt(c^2 + d^2)))/sqrt(c^2
+ d^2)
```

3.573.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx = -\frac{b\log\left(e^x+1\right)}{c} + \frac{b\log\left(\left|e^x-1\right|\right)}{c} + \frac{(ac-bd)\log\left(\frac{\left|2\,de^x+2\,c-2\,\sqrt{c^2+d^2}\right|}{\left|2\,de^x+2\,c+2\,\sqrt{c^2+d^2}\right|}\right)}{\sqrt{c^2+d^2}c}$$

input
```
integrate((a+b*csch(x))/(c+d*sinh(x)),x, algorithm="giac")
```

output
```
-b*log(e^x + 1)/c + b*log(abs(e^x - 1))/c + (a*c - b*d)*log(abs(2*d*e^x +
2*c - 2*sqrt(c^2 + d^2))/abs(2*d*e^x + 2*c + 2*sqrt(c^2 + d^2)))/(sqrt(c^2
 + d^2)*c)
```

3.573.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 539, normalized size of antiderivative = 9.29

$$\int \frac{a+b\operatorname{csch}(x)}{c+d\sinh(x)}\,dx = \frac{b\,\ln\left(e^x-1\right)}{c} - \frac{b\,\ln\left(e^x+1\right)}{c}$$

$$- \frac{\ln\left(\frac{\left(\frac{32\left(a^2c^2d-2abcd^2-4e^xb^2c^3+2b^2c^2d-3e^xb^2cd^2+2b^2d^3\right)}{d^5} - \frac{(ac-bd)\left(\frac{32c^2\left(2bd^2+4ac^2e^x+ad^2e^x-2acd-3bcde^x\right)}{d^5}+\frac{32c(ac-bd)(-\cdots}{\cdots}\right)}{c\sqrt{c^2+d^2}}\cdots}{c\sqrt{c^2+d^2}}\cdots\right)}{c^3+cd^2\cdots}$$

$$+ \frac{\ln\left(\frac{32b(ac-bd)(ade^x-2bd+4bce^x)}{d^5} - \frac{\left(\frac{32\left(a^2c^2d-2abcd^2-4e^xb^2c^3+2b^2c^2d-3e^xb^2cd^2+2b^2d^3\right)}{d^5}+\frac{(ac-bd)\left(\frac{32c^2\left(2bd^2+4ac^2e^x+\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{c\sqrt{c^2+d^2}\cdots}\cdots\right)}{c^3+cd^2\cdots}$$

input
```
int((a + b/sinh(x))/(c + d*sinh(x)),x)
```

output
```
(b*log(exp(x) - 1))/c - (b*log(exp(x) + 1))/c - (log((((32*(2*b^2*d^3 + a^
2*c^2*d + 2*b^2*c^2*d - 4*b^2*c^3*exp(x) - 3*b^2*c*d^2*exp(x) - 2*a*b*c*d^
2))/d^5 - ((a*c - b*d)*((32*c^2*(2*b*d^2 + 4*a*c^2*exp(x) + a*d^2*exp(x) -
 2*a*c*d - 3*b*c*d*exp(x)))/d^5 + (32*c*(a*c - b*d)*(3*c^2*d + 2*d^3 - 4*c
^3*exp(x) - 3*c*d^2*exp(x)))/(d^5*(c^2 + d^2)^(1/2))))/(c*(c^2 + d^2)^(1/2
)))*(a*c - b*d))/(c*(c^2 + d^2)^(1/2)) + (32*b*(a*c - b*d)*(a*d*exp(x) - 2
*b*d + 4*b*c*exp(x)))/d^5)*(a*c - b*d)*(c^2 + d^2)^(1/2))/(c*d^2 + c^3) +
(log((32*b*(a*c - b*d)*(a*d*exp(x) - 2*b*d + 4*b*c*exp(x)))/d^5 - (((32*(2
*b^2*d^3 + a^2*c^2*d + 2*b^2*c^2*d - 4*b^2*c^3*exp(x) - 3*b^2*c*d^2*exp(x)
 - 2*a*b*c*d^2))/d^5 + ((a*c - b*d)*((32*c^2*(2*b*d^2 + 4*a*c^2*exp(x) + a
*d^2*exp(x) - 2*a*c*d - 3*b*c*d*exp(x)))/d^5 - (32*c*(a*c - b*d)*(3*c^2*d
+ 2*d^3 - 4*c^3*exp(x) - 3*c*d^2*exp(x)))/(d^5*(c^2 + d^2)^(1/2))))/(c*(c^
2 + d^2)^(1/2)))*(a*c - b*d))/(c*(c^2 + d^2)^(1/2)))*(a*c - b*d)*(c^2 + d^
2)^(1/2))/(c*d^2 + c^3)
```

3.573. $\int \frac{a+b\,\mathrm{csch}(x)}{c+d\sinh(x)}\,dx$

3.574 $\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx$

3.574.1 Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = -x+\sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right)$$

output
```
-x+arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

3.574.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = -2\left(\frac{x}{2}-\frac{\text{arctanh}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}\right)$$

input
```
Integrate[(1 + Sinh[x]^2)/(1 - Sinh[x]^2),x]
```

output
```
-2*(x/2 - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2])
```

3.574.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3650, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)+1}{1-\sinh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-\sin(ix)^2}{1+\sin(ix)^2}dx$$

$$\Big\downarrow \text{3650}$$

$$2\int \frac{1}{1-\sinh^2(x)}dx-x$$

$$\Big\downarrow \text{3042}$$

$$-x+2\int \frac{1}{\sin(ix)^2+1}dx$$

$$\Big\downarrow \text{3660}$$

$$2\int \frac{1}{1-2\tanh^2(x)}d\tanh(x)-x$$

$$\Big\downarrow \text{219}$$

$$\sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right)-x$$

input
```
Int[(1 + Sinh[x]^2)/(1 - Sinh[x]^2),x]
```

output
```
-x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

3.574.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.574.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(15) = 30$.

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

method	result
risch	$-x+\frac{\sqrt{2}\,\ln\left(e^{2x}-3+2\sqrt{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(e^{2x}-3-2\sqrt{2}\right)}{2}$
default	$\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)+\ln\left(\tanh\left(\frac{x}{2}\right.\right.$

input
```
int((1+sinh(x)^2)/(1-sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-x+1/2*2^(1/2)*ln(exp(2*x)-3+2*2^(1/2))-1/2*2^(1/2)*ln(exp(2*x)-3-2*2^(1/2
))
```

3.574.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx$$
$$= \frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh\left(x\right)^2 - 4\left(3\sqrt{2}-4\right)\cosh\left(x\right)\sinh\left(x\right) + 3\left(2\sqrt{2}-3\right)\sinh\left(x\right)^2 - 2\sqrt{2}+3}{\cosh\left(x\right)^2 + \sinh\left(x\right)^2 - 3}\right)$$
$$- x$$

input
```
integrate((1+sinh(x)^2)/(1-sinh(x)^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*
sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x
)^2 - 3)) - x
```

3.574.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(15) = 30$.

Time = 3.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 12.53

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = -\frac{1331714x}{941664\sqrt{2}+1331714} - \frac{941664\sqrt{2}x}{941664\sqrt{2}+1331714} + \frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{941664\sqrt{2}+1331714} + \frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{941664\sqrt{2}+1331714} + \frac{941664\log\left(\tanh\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{941664\sqrt{2}+1331714} + \frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{941664\sqrt{2}+1331714} - \frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{941664\sqrt{2}+1331714} - \frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{941664\sqrt{2}+1331714} - \frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{941664\sqrt{2}+1331714} - \frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{941664\sqrt{2}+1331714}$$

input
```
integrate((1+sinh(x)**2)/(1-sinh(x)**2),x)
```

output
```
-1331714*x/(941664*sqrt(2) + 1331714) - 941664*sqrt(2)*x/(941664*sqrt(2) +
 1331714) + 941664*log(tanh(x/2) - 1 + sqrt(2))/(941664*sqrt(2) + 1331714)
 + 665857*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))/(941664*sqrt(2) + 1331714)
+ 941664*log(tanh(x/2) + 1 + sqrt(2))/(941664*sqrt(2) + 1331714) + 665857*
sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))/(941664*sqrt(2) + 1331714) - 665857*s
qrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(941664*sqrt(2) + 1331714) - 941664*lo
g(tanh(x/2) - sqrt(2) - 1)/(941664*sqrt(2) + 1331714) - 665857*sqrt(2)*log
(tanh(x/2) - sqrt(2) + 1)/(941664*sqrt(2) + 1331714) - 941664*log(tanh(x/2
) - sqrt(2) + 1)/(941664*sqrt(2) + 1331714)
```

3.574. $\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx$

3.574.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.37

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-x$$

input
```
integrate((1+sinh(x)^2)/(1-sinh(x)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/2*sqrt
(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x
```

3.574.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-x$$

input
```
integrate((1+sinh(x)^2)/(1-sinh(x)^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - x
```

3.574.9 Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.95

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = \frac{\sqrt{2}\,\ln\left(8\,e^{2\,x}+\frac{\sqrt{2}\,(12\,e^{2\,x}-4)}{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(8\,e^{2\,x}-\frac{\sqrt{2}\,(12\,e^{2\,x}-4)}{2}\right)}{2}-x$$

input
```
int(-(sinh(x)^2 + 1)/(sinh(x)^2 - 1),x)
```

output
```
(2^(1/2)*log(8*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - (2^(1/2)*log
(8*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - x
```

3.575 $\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx$

3.575.1 Optimal result

Integrand size = 17, antiderivative size = 8

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -x+2\tanh(x)$$

output
```
-x+2*tanh(x)
```

3.575.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -\frac{x}{2}-\frac{1}{2}\text{arctanh}(\tanh(x))+2\tanh(x)$$

input
```
Integrate[(1 - Sinh[x]^2)/(1 + Sinh[x]^2),x]
```

output
```
-1/2*x - ArcTanh[Tanh[x]]/2 + 2*Tanh[x]
```

3.575.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 3650, 3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\sinh^2(x)}{\sinh^2(x)+1} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1+\sin(ix)^2}{1-\sin(ix)^2}dx$$

$$\Big\downarrow \text{ 3650}$$

$$2\int \frac{1}{\sinh^2(x)+1}dx - x$$

$$\Big\downarrow \text{ 3042}$$

$$-x+2\int \frac{1}{1-\sin(ix)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$2\int \text{sech}^2(x)dx - x$$

$$\Big\downarrow \text{ 3042}$$

$$-x+2\int \csc\left(ix+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{ 4254}$$

$$-x+2i\int 1d(-i\tanh(x))$$

$$\Big\downarrow \text{ 24}$$

$$2\tanh(x) - x$$

input
```
Int[(1 - Sinh[x]^2)/(1 + Sinh[x]^2),x]
```

output
```
-x + 2*Tanh[x]
```

3.575.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.575.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
parallelrisch	$-x+2\tanh\left(x\right)$	9
risch	$-x-\frac{4}{1+{\mathrm e}^{2x}}$	15
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{4\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^{2}}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	34

input
```
int((1-sinh(x)^2)/(1+sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-x+2*tanh(x)
```

3.575.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -\frac{(x+2)\cosh(x)-2\sinh(x)}{\cosh(x)}$$

input
```
integrate((1-sinh(x)^2)/(1+sinh(x)^2),x, algorithm="fricas")
```

output
```
-((x + 2)*cosh(x) - 2*sinh(x))/cosh(x)
```

3.575.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(5) = 10$.

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 5.12

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -\frac{x\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1} - \frac{x}{\tanh^2\left(\frac{x}{2}\right)+1} + \frac{4\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

input
```
integrate((1-sinh(x)**2)/(1+sinh(x)**2),x)
```

output
```
-x*tanh(x/2)**2/(tanh(x/2)**2 + 1) - x/(tanh(x/2)**2 + 1) + 4*tanh(x/2)/(t
anh(x/2)**2 + 1)
```

3.575.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -x + \frac{4}{e^{(-2x)}+1}$$

input
```
integrate((1-sinh(x)^2)/(1+sinh(x)^2),x, algorithm="maxima")
```

output
```
-x + 4/(e^(-2*x) + 1)
```

3.575.8 Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -x - \frac{4}{e^{(2x)}+1}$$

input
```
integrate((1-sinh(x)^2)/(1+sinh(x)^2),x, algorithm="giac")
```

output
```
-x - 4/(e^(2*x) + 1)
```

3.575.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -x - \frac{4}{\mathrm{e}^{2x}+1}$$

input
```
int(-(sinh(x)^2 - 1)/(sinh(x)^2 + 1),x)
```

output
```
- x - 4/(exp(2*x) + 1)
```

3.576 $\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)} dx$

3.576.1 Optimal result

Integrand size = 17, antiderivative size = 8

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)} dx = -x + 2\coth(x)$$

output
```
-x+2*coth(x)
```

3.576.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)} dx = \coth(x) + \coth(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, \tanh^2(x)\right)$$

input
```
Integrate[(1 + Cosh[x]^2)/(1 - Cosh[x]^2),x]
```

output
```
Coth[x] + Coth[x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x]^2]
```

3.576.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 3650, 3042, 3654, 25, 3042, 25, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)+1}{1-\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1+\sin\left(\frac{\pi}{2}+ix\right)^2}{1-\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3650}$$

$$2\int \frac{1}{1-\cosh^2(x)}dx-x$$

$$\Big\downarrow \text{3042}$$

$$-x+2\int \frac{1}{1-\sin\left(ix+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$2\int -\text{csch}^2(x)dx-x$$

$$\Big\downarrow \text{25}$$

$$-2\int \text{csch}^2(x)dx-x$$

$$\Big\downarrow \text{3042}$$

$$-x-2\int -\csc(ix)^2dx$$

$$\Big\downarrow \text{25}$$

$$-x+2\int \csc(ix)^2dx$$

$$\Big\downarrow \text{4254}$$

$$-x+2i\int 1d(-i\coth(x))$$

$$\Big\downarrow \text{24}$$

$$2\coth(x)-x$$

input
```
Int[(1 + Cosh[x]^2)/(1 - Cosh[x]^2),x]
```

output
```
-x + 2*Coth[x]
```

3.576.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.576.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
parallelrisch	$-x+2\coth\left(x\right)$	9
risch	$-x+\frac{4}{{\mathrm e}^{2 x}-1}$	15
default	$\tanh\left(\frac{x}{2}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)}$	28

input
```
int((1+cosh(x)^2)/(1-cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-x+2*coth(x)
```

3.576.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx=-\frac{(x+2)\sinh\left(x\right)-2\,\cosh\left(x\right)}{\sinh\left(x\right)}$$

input
```
integrate((1+cosh(x)^2)/(1-cosh(x)^2),x, algorithm="fricas")
```

output
```
-((x + 2)*sinh(x) - 2*cosh(x))/sinh(x)
```

3.576.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(5) = 10.

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx=-x+\tanh\left(\frac{x}{2}\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)}$$

input
```
integrate((1+cosh(x)**2)/(1-cosh(x)**2),x)
```

output
```
-x + tanh(x/2) + 1/tanh(x/2)
```

3.576.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx = -x - \frac{4}{e^{(-2x)}-1}$$

input
```
integrate((1+cosh(x)^2)/(1-cosh(x)^2),x, algorithm="maxima")
```

output
```
-x - 4/(e^(-2*x) - 1)
```

3.576.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx = -x + \frac{4}{e^{(2x)}-1}$$

input
```
integrate((1+cosh(x)^2)/(1-cosh(x)^2),x, algorithm="giac")
```

output
```
-x + 4/(e^(2*x) - 1)
```

3.576.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx = \frac{4}{\mathrm{e}^{2x}-1} - x$$

input
```
int(-(cosh(x)^2 + 1)/(cosh(x)^2 - 1),x)
```

output
```
4/(exp(2*x) - 1) - x
```

3.577 $\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx$

3.577.1 Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = -x+\sqrt{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

output
```
-x+arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

3.577.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = -2\left(\frac{x}{2}-\frac{\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}\right)$$

input
```
Integrate[(1 - Cosh[x]^2)/(1 + Cosh[x]^2),x]
```

output
```
-2*(x/2 - ArcTanh[Tanh[x]/Sqrt[2]]/Sqrt[2])
```

3.577.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3650, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\cosh^2(x)}{\cosh^2(x)+1}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1-\sin\left(\frac{\pi}{2}+ix\right)^2}{1+\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \; 3650$$

$$2\int \frac{1}{\cosh^2(x)+1}dx-x$$

$$\Big\downarrow \; 3042$$

$$-x+2\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \; 3660$$

$$2\int \frac{1}{1-2\coth^2(x)}d\coth(x)-x$$

$$\Big\downarrow \; 219$$

$$\sqrt{2}\text{arctanh}\left(\sqrt{2}\coth(x)\right)-x$$

input
```
Int[(1 - Cosh[x]^2)/(1 + Cosh[x]^2),x]
```

output
```
-x + Sqrt[2]*ArcTanh[Sqrt[2]*Coth[x]]
```

3.577.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.577.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(16) = 32$.

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

method	result
risch	$-x+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3-2\sqrt{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3+2\sqrt{2}\right)}{2}$
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^{2}+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\mathrm{t}\right.\right.}{4}$

input
```
int((1-cosh(x)^2)/(1+cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-x+1/2*2^(1/2)*ln(exp(2*x)+3-2*2^(1/2))-1/2*2^(1/2)*ln(exp(2*x)+3+2*2^(1/2
))
```

3.577.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2 - 4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x) + 3\left(2\sqrt{2}-3\right)\sinh(x)^2 + 2\sqrt{2}-3}{\cosh(x)^2+\sinh(x)^2+3}\right) - x$$

input
```
integrate((1-cosh(x)^2)/(1+cosh(x)^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*
sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x
)^2 + 3)) - x
```

3.577.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(17) = 34$.

Time = 1.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = -x - \frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right) - 4\sqrt{2}\tanh\left(\frac{x}{2}\right) + 4\right)}{2} + \frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right) + 4\sqrt{2}\tanh\left(\frac{x}{2}\right) + 4\right)}{2}$$

input
```
integrate((1-cosh(x)**2)/(1+cosh(x)**2),x)
```

output
```
-x - sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/2 + sqrt(2)*log
(4*tanh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/2
```

3.577.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.37

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = \frac{3}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right) - \frac{5}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right) - 2x + \frac{1}{4}\log\left(e^{(4x)}+6e^{(2x)}+1\right) - \frac{1}{4}\log\left(6e^{(-2x)}+e^{(-4x)}+1\right)$$

input
```
integrate((1-cosh(x)^2)/(1+cosh(x)^2),x, algorithm="maxima")
```

output
```
3/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) - 5
/16*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) -
2*x + 1/4*log(e^(4*x) + 6*e^(2*x) + 1) - 1/4*log(6*e^(-2*x) + e^(-4*x) + 1
)
```

3.577.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right) - x$$

input
```
integrate((1-cosh(x)^2)/(1+cosh(x)^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) - x
```

3.577.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.95

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = \frac{\sqrt{2}\,\ln\left(-8\,e^{2\,x}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{2}\right)}{2}-x-\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{2}-8\,e^{2\,x}\right)}{2}$$

input
```
int(-(cosh(x)^2 - 1)/(cosh(x)^2 + 1),x)
```

output
```
(2^(1/2)*log(- 8*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/2))/2 - x - (2^(1/
2)*log((2^(1/2)*(12*exp(2*x) + 4))/2 - 8*exp(2*x)))/2
```

3.578 $\int \frac{a+b\operatorname{sech}^2(x)}{c+d\cosh(x)}\,dx$

3.578.1 Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{a+b\operatorname{sech}^2(x)}{c+d\cosh(x)}\,dx=-\frac{bd\arctan(\sinh(x))}{c^2}+\frac{2(ac^2+bd^2)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\sqrt{c+d}}+\frac{b\tanh(x)}{c}$$

output
```
-b*d*arctan(sinh(x))/c^2+2*(a*c^2+b*d^2)*arctanh((c-d)^(1/2)*tanh(1/2*x)/(
c+d)^(1/2))/c^2/(c-d)^(1/2)/(c+d)^(1/2)+b*tanh(x)/c
```

3.578.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

$$\int \frac{a+b\operatorname{sech}^2(x)}{c+d\cosh(x)}\,dx=$$
$$-\frac{2(b+a\cosh^2(x))\operatorname{sech}(x)\left(2\left(bd\sqrt{-c^2+d^2}\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+(ac^2+bd^2)\arctan\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{-c^2+d^2}}\right)\right)\cosh}{c^2\sqrt{-c^2+d^2}(a+2b+a\cosh(2x))}$$

input
```
Integrate[(a + b*Sech[x]^2)/(c + d*Cosh[x]),x]
```

output
```
(-2*(b + a*Cosh[x]^2)*Sech[x]*(2*(b*d*Sqrt[-c^2 + d^2]*ArcTan[Tanh[x/2]] +
 (a*c^2 + b*d^2)*ArcTan[((c - d)*Tanh[x/2])/Sqrt[-c^2 + d^2]])*Cosh[x] - b
*c*Sqrt[-c^2 + d^2]*Sinh[x]))/(c^2*Sqrt[-c^2 + d^2]*(a + 2*b + a*Cosh[2*x]
))
```

3.578.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {3042, 4722, 3042, 3535, 25, 3042, 3480, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{sech}^2(x)}{c+d\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(ix)^2}{c+d\cos(ix)}dx$$

$$\Big\downarrow \text{4722}$$

$$\int \frac{\text{sech}^2(x)\left(a\cosh^2(x)+b\right)}{c+d\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b+a\sin\left(\frac{\pi}{2}+ix\right)^2}{\sin\left(\frac{\pi}{2}+ix\right)^2\left(c+d\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3535}$$

$$\frac{\int -\frac{(bd-ac\cosh(x))\text{sech}(x)}{c+d\cosh(x)}dx}{c}+\frac{b\tanh(x)}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\tanh(x)}{c}-\frac{\int \frac{(bd-ac\cosh(x))\text{sech}(x)}{c+d\cosh(x)}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\tanh(x)}{c}-\frac{\int \frac{bd-ac\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)\left(c+d\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{c}$$

$$\Big\downarrow \text{3480}$$

$$\frac{b\tanh(x)}{c}-\frac{\frac{bd\int \text{sech}(x)dx}{c}-\frac{(ac^2+bd^2)\int \frac{1}{c+d\cosh(x)}dx}{c}}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\tanh(x)}{c}-\frac{\frac{bd\int\csc\left(ix+\frac{\pi}{2}\right)dx}{c}-\frac{\left(ac^2+bd^2\right)\int\frac{1}{c+d\sin\left(ix+\frac{\pi}{2}\right)}dx}{c}}{c}$$

$$\Big\downarrow \text{3138}$$

$$\frac{b\tanh(x)}{c}-\frac{-\frac{2\left(ac^2+bd^2\right)\int\frac{1}{-\left((c-d)\tanh^2\left(\frac{x}{2}\right)\right)+c+d}d\tanh\left(\frac{x}{2}\right)}{c}+\frac{bd\int\csc\left(ix+\frac{\pi}{2}\right)dx}{c}}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\tanh(x)}{c}-\frac{-\frac{2\left(ac^2+bd^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}+\frac{bd\int\csc\left(ix+\frac{\pi}{2}\right)dx}{c}}{c}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\tanh(x)}{c}-\frac{\frac{bd\arctan(\sinh(x))}{c}-\frac{2\left(ac^2+bd^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}}{c}$$

input
```
Int[(a + b*Sech[x]^2)/(c + d*Cosh[x]),x]
```

output
```
-(((b*d*ArcTan[Sinh[x]])/c - (2*(a*c^2 + b*d^2)*ArcTanh[(Sqrt[c - d]*Tanh[
x/2])/Sqrt[c + d]])/(c*Sqrt[c - d]*Sqrt[c + d]))/c) + (b*Tanh[x])/c
```

3.578.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.578. $\int \frac{a+b\operatorname{sech}^2(x)}{c+d\cosh(x)}\,dx$

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4722
```
Int[(u_)*((A_) + (C_.)*sec[(a_.) + (b_.)*(x_)]^2), x_Symbol] :> Int[Activat
eTrig[u]*((C + A*Cos[a + b*x]^2)/Cos[a + b*x]^2), x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

3.578.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

method	result
default	$-\frac{2\left(-a\,c^{2}-b\,d^{2}\right)\operatorname{arctanh}\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c^{2}\sqrt{(c+d)(c-d)}}-\frac{2b\left(-\frac{c\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^{2}}+d\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{c^{2}}$
risch	$-\frac{2b}{c(1+e^{2x})}+\frac{ibd\ln(e^{x}-i)}{c^{2}}-\frac{ibd\ln(e^{x}+i)}{c^{2}}+\frac{\ln\left(e^{x}+\frac{\sqrt{c^{2}-d^{2}}\,c-c^{2}+d^{2}}{\sqrt{c^{2}-d^{2}}\,d}\right)a}{\sqrt{c^{2}-d^{2}}}+\frac{\ln\left(e^{x}+\frac{\sqrt{c^{2}-d^{2}}\,c-c^{2}+d^{2}}{\sqrt{c^{2}-d^{2}}\,d}\right)b\,d^{2}}{\sqrt{c^{2}-d^{2}}\,c^{2}}-\frac{\ln\left(e^{x}+\frac{\sqrt{c^{2}-}}{\sqrt{}}\right.}{\sqrt{c^{2}}}$

input
```
int((a+sech(x)^2*b)/(c+d*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*(-a*c^2-b*d^2)/c^2/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tanh(1/2*x)/((c+d)
*(c-d))^(1/2))-2*b/c^2*(-c*tanh(1/2*x)/(1+tanh(1/2*x)^2)+d*arctan(tanh(1/2
*x)))
```

3.578.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(64) = 128$.

Time = 0.48 (sec) , antiderivative size = 598, normalized size of antiderivative = 8.08

$$\int \frac{a + b\operatorname{sech}^2(x)}{c + d\cosh(x)}\,dx$$
$$= \left[-\frac{2\,bc^3 - 2\,bcd^2 - \left(ac^2 + bd^2 + (ac^2 + bd^2)\cosh(x)^2 + 2\,(ac^2 + bd^2)\cosh(x)\sinh(x) + (ac^2 + bd^2)\sinh\right.}{}\right.$$
$$-\frac{2\left(bc^3 - bcd^2 + \left(ac^2 + bd^2 + (ac^2 + bd^2)\cosh(x)^2 + 2\,(ac^2 + bd^2)\cosh(x)\sinh(x) + (ac^2 + bd^2)\sinh(\right.\right.}{c^4 -}$$

input
```
integrate((a+b*sech(x)^2)/(c+d*cosh(x)),x, algorithm="fricas")
```

output
```
[-(2*b*c^3 - 2*b*c*d^2 - (a*c^2 + b*d^2 + (a*c^2 + b*d^2)*cosh(x)^2 + 2*(a
*c^2 + b*d^2)*cosh(x)*sinh(x) + (a*c^2 + b*d^2)*sinh(x)^2)*sqrt(c^2 - d^2)
*log((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*cosh(x) + 2*c^2 - d^2 + 2*(d^2
*cosh(x) + c*d)*sinh(x) - 2*sqrt(c^2 - d^2)*(d*cosh(x) + d*sinh(x) + c))/(
d*cosh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*cosh(x) + c)*sinh(x) + d))
 + 2*(b*c^2*d - b*d^3 + (b*c^2*d - b*d^3)*cosh(x)^2 + 2*(b*c^2*d - b*d^3)*c
osh(x)*sinh(x) + (b*c^2*d - b*d^3)*sinh(x)^2)*arctan(cosh(x) + sinh(x)))/(
c^4 - c^2*d^2 + (c^4 - c^2*d^2)*cosh(x)^2 + 2*(c^4 - c^2*d^2)*cosh(x)*sinh
(x) + (c^4 - c^2*d^2)*sinh(x)^2), -2*(b*c^3 - b*c*d^2 + (a*c^2 + b*d^2 + (
a*c^2 + b*d^2)*cosh(x)^2 + 2*(a*c^2 + b*d^2)*cosh(x)*sinh(x) + (a*c^2 + b*
d^2)*sinh(x)^2)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cosh(x) + d*s
inh(x) + c)/(c^2 - d^2)) + (b*c^2*d - b*d^3 + (b*c^2*d - b*d^3)*cosh(x)^2
 + 2*(b*c^2*d - b*d^3)*cosh(x)*sinh(x) + (b*c^2*d - b*d^3)*sinh(x)^2)*arcta
n(cosh(x) + sinh(x)))/(c^4 - c^2*d^2 + (c^4 - c^2*d^2)*cosh(x)^2 + 2*(c^4
 - c^2*d^2)*cosh(x)*sinh(x) + (c^4 - c^2*d^2)*sinh(x)^2)]
```

3.578.6 Sympy [F]

$$\int \frac{a + b\text{sech}^2(x)}{c + d\cosh(x)}\,dx = \int \frac{a + b\,\text{sech}^2(x)}{c + d\cosh(x)}\,dx$$

input
```
integrate((a+b*sech(x)**2)/(c+d*cosh(x)),x)
```

output
```
Integral((a + b*sech(x)**2)/(c + d*cosh(x)), x)
```

3.578.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^2(x)}{c + d\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sech(x)^2)/(c+d*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

3.578.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{a + b\text{sech}^2(x)}{c + d\cosh(x)}\,dx = -\frac{2\,bd\arctan(e^x)}{c^2} + \frac{2\,(ac^2 + bd^2)\arctan\left(\frac{de^x + c}{\sqrt{-c^2 + d^2}}\right)}{\sqrt{-c^2 + d^2}c^2} - \frac{2\,b}{c(e^{(2\,x)} + 1)}$$

input
```
integrate((a+b*sech(x)^2)/(c+d*cosh(x)),x, algorithm="giac")
```

output
```
-2*b*d*arctan(e^x)/c^2 + 2*(a*c^2 + b*d^2)*arctan((d*e^x + c)/sqrt(-c^2 +
d^2))/(sqrt(-c^2 + d^2)*c^2) - 2*b/(c*(e^(2*x) + 1))
```

3.578.9 Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 704, normalized size of antiderivative = 9.51

$$\int \frac{a+b\operatorname{sech}^2(x)}{c+d\cosh(x)}\,dx$$

$$= \frac{\ln\left(\frac{\sqrt{(c+d)\,(c-d)}\left(\frac{32\,\left(a^2\,c^4+2\,a\,b\,c^2\,d^2-4\,e^x\,b^2\,c^3\,d-2\,b^2\,c^2\,d^2+3\,e^x\,b^2\,c\,d^3+2\,b^2\,d^4\right)}{c^2\,d^4}-\frac{\sqrt{(c+d)\,(c-d)}\,\left(a\,c^2+b\,d^2\right)\left(\frac{32\,c\,\left(2\,b\,d^3+4\,a\,c^3\,e^x+2\,a\,c^2\,d-a\,c\,d\right.}{d^5}\right.}{}\right.}{c^2\,(c^2-d^2)}\right.}{}$$

$$-\frac{2\,b}{c\,(e^{2\,x}+1)}$$

$$-\frac{\ln\left(-\frac{32\,b\,(a\,c^2+b\,d^2)\,(2\,b\,d+a\,c\,e^x+4\,b\,c\,e^x)}{c^3\,d^3}-\frac{\sqrt{(c+d)\,(c-d)}\left(\frac{32\,\left(a^2\,c^4+2\,a\,b\,c^2\,d^2-4\,e^x\,b^2\,c^3\,d-2\,b^2\,c^2\,d^2+3\,e^x\,b^2\,c\,d^3+2\,b^2\,d^4\right)}{c^2\,d^4}+\frac{\sqrt{(c+d)\,(c-}}{}\right.}{}\right.}{}$$

$$+\frac{b\,d\,\ln\left(e^x-i\right)\,1i}{c^2}-\frac{b\,d\,\ln\left(e^x+1i\right)\,1i}{c^2}$$

input
```
int((a + b/cosh(x)^2)/(c + d*cosh(x)),x)
```

output
```
(log((((c + d)*(c - d))^(1/2)*((32*(a^2*c^4 + 2*b^2*d^4 - 2*b^2*c^2*d^2 +
3*b^2*c*d^3*exp(x) - 4*b^2*c^3*d*exp(x) + 2*a*b*c^2*d^2))/(c^2*d^4) - (((c
 + d)*(c - d))^(1/2)*(a*c^2 + b*d^2)*((32*c*(2*b*d^3 + 4*a*c^3*exp(x) + 2*
a*c^2*d - a*c*d^2*exp(x) + 3*b*c*d^2*exp(x)))/d^5 + (32*((c + d)*(c - d))^
(1/2)*(a*c^2 + b*d^2)*(3*c^2*d - 2*d^3 + 4*c^3*exp(x) - 3*c*d^2*exp(x)))/(
d^5*(c^2 - d^2))))/(c^2*(c^2 - d^2)))*(a*c^2 + b*d^2))/(c^2*(c^2 - d^2)) -
 (32*b*(a*c^2 + b*d^2)*(2*b*d + a*c*exp(x) + 4*b*c*exp(x)))/(c^3*d^3))*((c
 + d)*(c - d))^(1/2)*(a*c^2 + b*d^2))/(c^4 - c^2*d^2) - (2*b)/(c*(exp(2*x)
 + 1)) - (log(- (32*b*(a*c^2 + b*d^2)*(2*b*d + a*c*exp(x) + 4*b*c*exp(x)))
/(c^3*d^3) - (((c + d)*(c - d))^(1/2)*((32*(a^2*c^4 + 2*b^2*d^4 - 2*b^2*c^
2*d^2 + 3*b^2*c*d^3*exp(x) - 4*b^2*c^3*d*exp(x) + 2*a*b*c^2*d^2))/(c^2*d^4
) + (((c + d)*(c - d))^(1/2)*(a*c^2 + b*d^2)*((32*c*(2*b*d^3 + 4*a*c^3*exp
(x) + 2*a*c^2*d - a*c*d^2*exp(x) + 3*b*c*d^2*exp(x)))/d^5 - (32*((c + d)*(
c - d))^(1/2)*(a*c^2 + b*d^2)*(3*c^2*d - 2*d^3 + 4*c^3*exp(x) - 3*c*d^2*ex
p(x)))/(d^5*(c^2 - d^2))))/(c^2*(c^2 - d^2)))*(a*c^2 + b*d^2))/(c^2*(c^2 -
 d^2)))*((c + d)*(c - d))^(1/2)*(a*c^2 + b*d^2))/(c^4 - c^2*d^2) + (b*d*lo
g(exp(x) - 1i)*1i)/c^2 - (b*d*log(exp(x) + 1i)*1i)/c^2
```

3.578. $\int \frac{a+b\,\text{sech}^2(x)}{c+d\cosh(x)}\,dx$

3.579 $\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx$

3.579.1 Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx=\frac{bd\text{arctanh}(\cosh(x))}{c^2}-\frac{2(ac^2+bd^2)\,\text{arctanh}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c^2\sqrt{c^2+d^2}}-\frac{b\coth(x)}{c}$$

output
```
b*d*arctanh(cosh(x))/c^2-b*coth(x)/c-2*(a*c^2+b*d^2)*arctanh((d-c*tanh(1/2
*x))/(c^2+d^2)^(1/2))/c^2/(c^2+d^2)^(1/2)
```

3.579.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.54

$$\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx$$
$$=\frac{\text{csch}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\left(-bc\cosh(x)+\left(\frac{2(ac^2+bd^2)\arctan\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{-c^2-d^2}}\right)}{\sqrt{-c^2-d^2}}+bd\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\right)\sin}{2c^2}$$

input
```
Integrate[(a + b*Csch[x]^2)/(c + d*Sinh[x]),x]
```

output
```
(Csch[x/2]*Sech[x/2]*(-(b*c*Cosh[x]) + ((2*(a*c^2 + b*d^2)*ArcTan[(d - c*T
anh[x/2])/Sqrt[-c^2 - d^2]])/Sqrt[-c^2 - d^2] + b*d*(Log[Cosh[x/2]] - Log[
Sinh[x/2]]))*Sinh[x]))/(2*c^2)
```

3.579.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4721, 25, 25, 3042, 25, 3535, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a-b\csc(ix)^2}{c-id\sin(ix)}dx$$

$$\Big\downarrow \; 4721$$

$$\int -\frac{\text{csch}^2(x)\left(-a\sinh^2(x)-b\right)}{c+d\sinh(x)}dx$$

$$\Big\downarrow \; 25$$

$$-\int -\frac{\text{csch}^2(x)\left(a\sinh^2(x)+b\right)}{c+d\sinh(x)}dx$$

$$\Big\downarrow \; 25$$

$$\int \frac{\text{csch}^2(x)\left(a\sinh^2(x)+b\right)}{c+d\sinh(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{b-a\sin(ix)^2}{\sin(ix)^2(c-id\sin(ix))}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{b-a\sin(ix)^2}{\sin(ix)^2(c-id\sin(ix))}dx$$

$$\Big\downarrow \; 3535$$

$$-\frac{\int \frac{\operatorname{csch}(x)(bd-ac\sinh(x))}{c+d\sinh(x)}dx}{c}-\frac{b\coth(x)}{c}$$

$\Big\downarrow$ 3042

$$-\frac{b\coth(x)}{c}-\frac{\int \frac{i(bd+iac\sin(ix))}{\sin(ix)(c-id\sin(ix))}dx}{c}$$

$\Big\downarrow$ 26

$$-\frac{b\coth(x)}{c}-\frac{i\int \frac{bd+iac\sin(ix)}{\sin(ix)(c-id\sin(ix))}dx}{c}$$

$\Big\downarrow$ 3480

$$-\frac{b\coth(x)}{c}-\frac{i\left(\frac{i(ac^2+bd^2)\int \frac{1}{c+d\sinh(x)}dx}{c}+\frac{bd\int -i\operatorname{csch}(x)dx}{c}\right)}{c}$$

$\Big\downarrow$ 26

$$-\frac{b\coth(x)}{c}-\frac{i\left(\frac{i(ac^2+bd^2)\int \frac{1}{c+d\sinh(x)}dx}{c}-\frac{ibd\int \operatorname{csch}(x)dx}{c}\right)}{c}$$

$\Big\downarrow$ 3042

$$-\frac{b\coth(x)}{c}-\frac{i\left(\frac{i(ac^2+bd^2)\int \frac{1}{c-id\sin(ix)}dx}{c}-\frac{ibd\int i\csc(ix)dx}{c}\right)}{c}$$

$\Big\downarrow$ 26

$$-\frac{b\coth(x)}{c}-\frac{i\left(\frac{i(ac^2+bd^2)\int \frac{1}{c-id\sin(ix)}dx}{c}+\frac{bd\int \csc(ix)dx}{c}\right)}{c}$$

$\Big\downarrow$ 3139

$$-\frac{b\coth(x)}{c}-\frac{i\left(\frac{2i(ac^2+bd^2)\int \frac{1}{-c\tanh^2\left(\frac{x}{2}\right)+2d\tanh\left(\frac{x}{2}\right)+c}d\tanh\left(\frac{x}{2}\right)}{c}+\frac{bd\int \csc(ix)dx}{c}\right)}{c}$$

$\Big\downarrow$ 1083

$$-\frac{b\coth(x)}{c}-\frac{i\left(\frac{bd\int \csc(ix)dx}{c}-\frac{4i(ac^2+bd^2)\int \frac{1}{4\left(c^2+d^2\right)-\left(2d-2c\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2d-2c\tanh\left(\frac{x}{2}\right)\right)}{c}\right)}{c}$$

$\Big\downarrow$ 219

3.579. $\int \frac{a+b\operatorname{csch}^2(x)}{c+d\sinh(x)}\,dx$

$$-\frac{b\coth(x)}{c}-\frac{i\left(\frac{bd\int \csc(ix)dx}{c}-\frac{2i(ac^2+bd^2)\text{arctanh}\left(\frac{2d-2c\tanh\left(\frac{x}{2}\right)}{2\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}\right)}{c}$$

$$\Big\downarrow \; 4257$$

$$-\frac{b\coth(x)}{c}-\frac{i\left(\frac{ibd\text{arctanh}(\cosh(x))}{c}-\frac{2i(ac^2+bd^2)\text{arctanh}\left(\frac{2d-2c\tanh\left(\frac{x}{2}\right)}{2\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}\right)}{c}$$

input
```
Int[(a + b*Csch[x]^2)/(c + d*Sinh[x]),x]
```

output
```
((-I)*((I*b*d*ArcTanh[Cosh[x]])/c - ((2*I)*(a*c^2 + b*d^2)*ArcTanh[(2*d -
2*c*Tanh[x/2])/(2*Sqrt[c^2 + d^2])])/(c*Sqrt[c^2 + d^2])))/c - (b*Coth[x])
/c
```

3.579.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.579. $\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx$

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4721
```
Int[(csc[(a_.) + (b_.)*(x_)]^2*(C_.) + (A_))*(u_), x_Symbol] :> Int[Activat
eTrig[u]*((C + A*Sin[a + b*x]^2)/Sin[a + b*x]^2), x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

3.579. $\int \frac{a+b\,\mathrm{csch}^2(x)}{c+d\sinh(x)}\,dx$

3.579.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

method	result
default	$-\frac{b\tanh\left(\frac{x}{2}\right)}{2c}-\frac{\left(-4a\,c^{2}-4b\,d^{2}\right)\operatorname{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^{2}+d^{2}}}\right)}{2c^{2}\sqrt{c^{2}+d^{2}}}-\frac{b}{2c\tanh\left(\frac{x}{2}\right)}-\frac{bd\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{c^{2}}$
parts	$\frac{2a\,\operatorname{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^{2}+d^{2}}}\right)}{\sqrt{c^{2}+d^{2}}}+b\left(-\frac{\tanh\left(\frac{x}{2}\right)}{2c}-\frac{1}{2c\tanh\left(\frac{x}{2}\right)}-\frac{d\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{c^{2}}+\frac{2d^{2}\operatorname{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^{2}+d^{2}}}\right)}{c^{2}\sqrt{c^{2}+d^{2}}}\right)$
risch	$-\frac{2b}{c\left(\mathrm{e}^{2x}-1\right)}+\frac{bd\ln\left(\mathrm{e}^{x}+1\right)}{c^{2}}-\frac{bd\ln\left(\mathrm{e}^{x}-1\right)}{c^{2}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{c^{2}+d^{2}}\,c-c^{2}-d^{2}}{\sqrt{c^{2}+d^{2}}\,d}\right)a}{\sqrt{c^{2}+d^{2}}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{c^{2}+d^{2}}\,c-c^{2}-d^{2}}{\sqrt{c^{2}+d^{2}}\,d}\right)b\,d^{2}}{\sqrt{c^{2}+d^{2}}\,c^{2}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{c^{2}+}}{\sqrt{c}}\right.}{\sqrt{c^{2}-}}$

input
```
int((a+b*csch(x)^2)/(c+d*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/c*tanh(1/2*x)-1/2/c^2*(-4*a*c^2-4*b*d^2)/(c^2+d^2)^(1/2)*arctanh(1/
2*(2*c*tanh(1/2*x)-2*d)/(c^2+d^2)^(1/2))-1/2*b/c/tanh(1/2*x)-1/c^2*b*d*ln(
tanh(1/2*x))
```

3.579.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(65) = 130.

Time = 0.48 (sec) , antiderivative size = 401, normalized size of antiderivative = 5.81

$$\int \frac{a+b\operatorname{csch}^2(x)}{c+d\sinh(x)}\,dx$$
$$= \frac{2\,bc^{3}+2\,bcd^{2}+\left(ac^{2}+bd^{2}-\left(ac^{2}+bd^{2}\right)\cosh\left(x\right)^{2}-2\left(ac^{2}+bd^{2}\right)\cosh\left(x\right)\sinh\left(x\right)-\left(ac^{2}+bd^{2}\right)\sinh\left(x\right)^{2}\right.}{}$$

input
```
integrate((a+b*csch(x)^2)/(c+d*sinh(x)),x, algorithm="fricas")
```

output
```
(2*b*c^3 + 2*b*c*d^2 + (a*c^2 + b*d^2 - (a*c^2 + b*d^2)*cosh(x)^2 - 2*(a*c
^2 + b*d^2)*cosh(x)*sinh(x) - (a*c^2 + b*d^2)*sinh(x)^2)*sqrt(c^2 + d^2)*l
og((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*cosh(x) + 2*c^2 + d^2 + 2*(d^2*c
osh(x) + c*d)*sinh(x) - 2*sqrt(c^2 + d^2)*(d*cosh(x) + d*sinh(x) + c))/(d*
cosh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*cosh(x) + c)*sinh(x) - d)) +
(b*c^2*d + b*d^3 - (b*c^2*d + b*d^3)*cosh(x)^2 - 2*(b*c^2*d + b*d^3)*cosh(
x)*sinh(x) - (b*c^2*d + b*d^3)*sinh(x)^2)*log(cosh(x) + sinh(x) + 1) - (b*
c^2*d + b*d^3 - (b*c^2*d + b*d^3)*cosh(x)^2 - 2*(b*c^2*d + b*d^3)*cosh(x)*
sinh(x) - (b*c^2*d + b*d^3)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(c^4 +
c^2*d^2 - (c^4 + c^2*d^2)*cosh(x)^2 - 2*(c^4 + c^2*d^2)*cosh(x)*sinh(x) -
(c^4 + c^2*d^2)*sinh(x)^2)
```

3.579.6 Sympy [F]

$$\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx=\int \frac{a+b\operatorname{csch}^2(x)}{c+d\sinh(x)}\,dx$$

input
```
integrate((a+b*csch(x)**2)/(c+d*sinh(x)),x)
```

output
```
Integral((a + b*csch(x)**2)/(c + d*sinh(x)), x)
```

3.579.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(65)=130$.

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.29

$$\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx = b\left(\frac{d^2\log\left(\frac{de^{(-x)}-c-\sqrt{c^2+d^2}}{de^{(-x)}-c+\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}c^2}+\frac{d\log\left(e^{(-x)}+1\right)}{c^2}-\frac{d\log\left(e^{(-x)}-1\right)}{c^2}+\frac{2}{ce^{(-2x)}-c}\right)+\frac{a\log\left(\frac{de^{(-x)}-c-\sqrt{c^2+d^2}}{de^{(-x)}-c+\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}}$$

input
```
integrate((a+b*csch(x)^2)/(c+d*sinh(x)),x, algorithm="maxima")
```

output
```
b*(d^2*log((d*e^(-x) - c - sqrt(c^2 + d^2))/(d*e^(-x) - c + sqrt(c^2 + d^2
)))/(sqrt(c^2 + d^2)*c^2) + d*log(e^(-x) + 1)/c^2 - d*log(e^(-x) - 1)/c^2
+ 2/(c*e^(-2*x) - c)) + a*log((d*e^(-x) - c - sqrt(c^2 + d^2))/(d*e^(-x) -
 c + sqrt(c^2 + d^2)))/sqrt(c^2 + d^2)
```

3.579.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.58

$$\int \frac{a+b\operatorname{csch}^2(x)}{c+d\sinh(x)}\,dx = \frac{bd\log\left(e^x+1\right)}{c^2} - \frac{bd\log\left(|e^x-1|\right)}{c^2} + \frac{(ac^2+bd^2)\log\left(\frac{\left|2\,de^x+2\,c-2\,\sqrt{c^2+d^2}\right|}{\left|2\,de^x+2\,c+2\,\sqrt{c^2+d^2}\right|}\right)}{\sqrt{c^2+d^2}c^2} - \frac{2\,b}{c(e^{(2\,x)}-1)}$$

input
```
integrate((a+b*csch(x)^2)/(c+d*sinh(x)),x, algorithm="giac")
```

output
```
b*d*log(e^x + 1)/c^2 - b*d*log(abs(e^x - 1))/c^2 + (a*c^2 + b*d^2)*log(abs
(2*d*e^x + 2*c - 2*sqrt(c^2 + d^2))/abs(2*d*e^x + 2*c + 2*sqrt(c^2 + d^2))
)/(sqrt(c^2 + d^2)*c^2) - 2*b/(c*(e^(2*x) - 1))
```

3.579.9 Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 613, normalized size of antiderivative = 8.88

$$\int \frac{a+b\operatorname{csch}^2(x)}{c+d\sinh(x)}\,dx = \frac{b\,d\,\ln\left(e^x+1\right)}{c^2} - \frac{b\,d\,\ln\left(e^x-1\right)}{c^2} - \frac{2\,b}{c\,\left(e^{2\,x}-1\right)}$$

$$- \frac{\ln\left(\frac{\left(a\,c^2+b\,d^2\right)\left(\frac{32\,\left(a^2\,c^4+2\,a\,b\,c^2\,d^2-4\,e^x\,b^2\,c^3\,d+2\,b^2\,c^2\,d^2-3\,e^x\,b^2\,c\,d^3+2\,b^2\,d^4\right)}{c^2\,d^4} - \frac{\left(a\,c^2+b\,d^2\right)\left(\frac{32\,c\,\left(4\,a\,c^3\,e^x-2\,b\,d^3-2\,a\,c^2\,d+a\,c\,d^2\,e^x+3\,b\,c\,d^2\,e \cdots}{d^5} \cdots\right)}{c^2\,\sqrt{c^2+ \cdots}}\right)}{c^2\,\sqrt{c^2+d^2}} \cdots\right)}{c^4+c^2\,d^{\cdots}}$$

$$+ \frac{\ln\left(-\frac{\left(a\,c^2+b\,d^2\right)\left(\frac{32\,\left(a^2\,c^4+2\,a\,b\,c^2\,d^2-4\,e^x\,b^2\,c^3\,d+2\,b^2\,c^2\,d^2-3\,e^x\,b^2\,c\,d^3+2\,b^2\,d^4\right)}{c^2\,d^4} + \frac{\left(a\,c^2+b\,d^2\right)\left(\frac{32\,c\,\left(4\,a\,c^3\,e^x-2\,b\,d^3-2\,a\,c^2\,d+a\,c\,d^2\,e^x+3\,b\,c\,d \cdots}{d^5} \cdots\right)}{c^2\,\sqrt{c^2 \cdots}}\right)}{c^2\,\sqrt{c^2+d^2}} \cdots\right)}{c^4+c^2\,d^{\cdots}}$$

input
```
int((a + b/sinh(x)^2)/(c + d*sinh(x)),x)
```

output
```
(b*d*log(exp(x) + 1))/c^2 - (b*d*log(exp(x) - 1))/c^2 - (2*b)/(c*(exp(2*x)
 - 1)) - (log(((a*c^2 + b*d^2)*((32*(a^2*c^4 + 2*b^2*d^4 + 2*b^2*c^2*d^2 -
 3*b^2*c*d^3*exp(x) - 4*b^2*c^3*d*exp(x) + 2*a*b*c^2*d^2))/(c^2*d^4) - ((a
*c^2 + b*d^2)*((32*c*(4*a*c^3*exp(x) - 2*b*d^3 - 2*a*c^2*d + a*c*d^2*exp(x
) + 3*b*c*d^2*exp(x)))/d^5 + (32*(a*c^2 + b*d^2)*(3*c^2*d + 2*d^3 - 4*c^3*
exp(x) - 3*c*d^2*exp(x)))/(d^5*(c^2 + d^2)^(1/2))))/(c^2*(c^2 + d^2)^(1/2)
)))/(c^2*(c^2 + d^2)^(1/2)) - (32*b*(a*c^2 + b*d^2)*(2*b*d + a*c*exp(x) -
4*b*c*exp(x)))/(c^3*d^3))*(a*c^2 + b*d^2)*(c^2 + d^2)^(1/2))/(c^4 + c^2*d^
2) + (log(- ((a*c^2 + b*d^2)*((32*(a^2*c^4 + 2*b^2*d^4 + 2*b^2*c^2*d^2 - 3
*b^2*c*d^3*exp(x) - 4*b^2*c^3*d*exp(x) + 2*a*b*c^2*d^2))/(c^2*d^4) + ((a*c
^2 + b*d^2)*((32*c*(4*a*c^3*exp(x) - 2*b*d^3 - 2*a*c^2*d + a*c*d^2*exp(x)
+ 3*b*c*d^2*exp(x)))/d^5 - (32*(a*c^2 + b*d^2)*(3*c^2*d + 2*d^3 - 4*c^3*ex
p(x) - 3*c*d^2*exp(x)))/(d^5*(c^2 + d^2)^(1/2))))/(c^2*(c^2 + d^2)^(1/2)))
)/(c^2*(c^2 + d^2)^(1/2)) - (32*b*(a*c^2 + b*d^2)*(2*b*d + a*c*exp(x) - 4*
b*c*exp(x)))/(c^3*d^3))*(a*c^2 + b*d^2)*(c^2 + d^2)^(1/2))/(c^4 + c^2*d^2)
```

3.580 $\int (a\cosh(x)+b\sinh(x))\,dx$

3.580.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int (a\cosh(x)+b\sinh(x))\,dx=b\cosh(x)+a\sinh(x)$$

output
```
b*cosh(x)+a*sinh(x)
```

3.580.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (a\cosh(x)+b\sinh(x))\,dx=b\cosh(x)+a\sinh(x)$$

input
```
Integrate[a*Cosh[x] + b*Sinh[x],x]
```

output
```
b*Cosh[x] + a*Sinh[x]
```

3.580.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+b\sinh(x))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$a\sinh(x)+b\cosh(x)$$

input
```
Int[a*Cosh[x] + b*Sinh[x],x]
```

output
```
b*Cosh[x] + a*Sinh[x]
```

3.580.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.580.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$b\cosh(x)+a\sinh(x)$	10
parts	$b\cosh(x)+a\sinh(x)$	10
meijerg	$a\sinh(x)-b\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(x)}{\sqrt{\pi}}\right)$	23
risch	$\frac{(a\,e^{2x}+b\,e^{2x}-a+b)\,e^{-x}}{2}$	24

input
```
int(a*cosh(x)+b*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
b*cosh(x)+a*sinh(x)
```

3.580.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (a\cosh(x) + b\sinh(x))\,dx = b\cosh(x) + a\sinh(x)$$

input
```
integrate(a*cosh(x)+b*sinh(x),x, algorithm="fricas")
```

output
```
b*cosh(x) + a*sinh(x)
```

3.580.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int (a\cosh(x) + b\sinh(x))\,dx = a\sinh(x) + b\cosh(x)$$

input
```
integrate(a*cosh(x)+b*sinh(x),x)
```

output
```
a*sinh(x) + b*cosh(x)
```

3.580.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (a\cosh(x) + b\sinh(x))\,dx = b\cosh(x) + a\sinh(x)$$

input
```
integrate(a*cosh(x)+b*sinh(x),x, algorithm="maxima")
```

output
```
b*cosh(x) + a*sinh(x)
```

3.580.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.56

$$\int (a\cosh(x) + b\sinh(x))\,dx = \frac{1}{2}\,b\left(e^{(-x)} + e^{x}\right) - \frac{1}{2}\,a\left(e^{(-x)} - e^{x}\right)$$

input
```
integrate(a*cosh(x)+b*sinh(x),x, algorithm="giac")
```

output
```
1/2*b*(e^(-x) + e^x) - 1/2*a*(e^(-x) - e^x)
```

3.580.9 Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (a\cosh(x) + b\sinh(x))\,dx = b\cosh(x) + a\sinh(x)$$

input
```
int(a*cosh(x) + b*sinh(x),x)
```

output
```
b*cosh(x) + a*sinh(x)
```

3.581 $\int (a\cosh(x)+b\sinh(x))^2\,dx$

3.581.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int (a\cosh(x)+b\sinh(x))^2\,dx=\frac{1}{2}\left(a^2-b^2\right)x+\frac{1}{2}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))$$

output
```
1/2*(a^2-b^2)*x+1/2*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))
```

3.581.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int (a\cosh(x)+b\sinh(x))^2\,dx=\frac{1}{4}\left(2(a-b)(a+b)x+2ab\cosh(2x)+\left(a^2+b^2\right)\sinh(2x)\right)$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(2*(a - b)*(a + b)*x + 2*a*b*Cosh[2*x] + (a^2 + b^2)*Sinh[2*x])/4
```

3.581.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3552, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+b\sinh(x))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\cos(ix)-ib\sin(ix))^2dx$$

$$\Big\downarrow \text{ 3552}$$

$$\frac{1}{2}(a^2-b^2)\int 1dx+\frac{1}{2}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))$$

$$\Big\downarrow \text{ 24}$$

$$\frac{1}{2}x(a^2-b^2)+\frac{1}{2}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
((a^2 - b^2)*x)/2 + ((b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x]))/2
```

3.581.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

3.581.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

method	result	size
default	$a^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+ab\cosh\left(x\right)^2+b^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)$	37
parts	$a^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+ab\cosh\left(x\right)^2+b^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)$	37
risch	$\frac{a^2x}{2}-\frac{b^2x}{2}+\frac{a^2e^{2x}}{8}+\frac{b\,e^{2x}a}{4}+\frac{b^2e^{2x}}{8}-\frac{e^{-2x}a^2}{8}+\frac{e^{-2x}ab}{4}-\frac{e^{-2x}b^2}{8}$	66

input
```
int((a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*cosh(x)*sinh(x)+1/2*x)+a*b*cosh(x)^2+b^2*(1/2*cosh(x)*sinh(x)-1/2
*x)
```

3.581.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int (a\cosh(x)+b\sinh(x))^2\,dx=\frac{1}{2}\,ab\cosh\left(x\right)^2+\frac{1}{2}\,ab\sinh\left(x\right)^2+\frac{1}{2}\left(a^2+b^2\right)\cosh\left(x\right)\sinh\left(x\right)+\frac{1}{2}\left(a^2-b^2\right)x$$

input
```
integrate((a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
1/2*a*b*cosh(x)^2 + 1/2*a*b*sinh(x)^2 + 1/2*(a^2 + b^2)*cosh(x)*sinh(x) +
1/2*(a^2 - b^2)*x
```

3.581.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int (a\cosh(x)+b\sinh(x))^2\,dx=-\frac{a^2x\sinh^2\left(x\right)}{2}+\frac{a^2x\cosh^2\left(x\right)}{2}+\frac{a^2\sinh\left(x\right)\cosh\left(x\right)}{2}+ab\cosh^2\left(x\right)+\frac{b^2x\sinh^2\left(x\right)}{2}-\frac{b^2x\cosh^2\left(x\right)}{2}+\frac{b^2\sinh\left(x\right)\cosh\left(x\right)}{2}$$

input
```
integrate((a*cosh(x)+b*sinh(x))**2,x)
```

output
```
-a**2*x*sinh(x)**2/2 + a**2*x*cosh(x)**2/2 + a**2*sinh(x)*cosh(x)/2 + a*b*
cosh(x)**2 + b**2*x*sinh(x)**2/2 - b**2*x*cosh(x)**2/2 + b**2*sinh(x)*cosh
(x)/2
```

3.581.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int (a\cosh(x) + b\sinh(x))^2\,dx = ab\cosh{(x)}^2 + \frac{1}{8}\,a^2\big(4\,x + e^{(2\,x)} - e^{(-2\,x)}\big) - \frac{1}{8}\,b^2\big(4\,x - e^{(2\,x)} + e^{(-2\,x)}\big)$$

input
```
integrate((a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
a*b*cosh(x)^2 + 1/8*a^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*b^2*(4*x - e^(2*x
) + e^(-2*x))
```

3.581.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(33) = 66.

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.00

$$\int (a\cosh(x) + b\sinh(x))^2\,dx = \frac{1}{8}\,a^2e^{(2\,x)} + \frac{1}{4}\,abe^{(2\,x)} + \frac{1}{8}\,b^2e^{(2\,x)} + \frac{1}{2}\,(a^2 - b^2)x - \frac{1}{8}\,\big(2\,a^2e^{(2\,x)} - 2\,b^2e^{(2\,x)} + a^2 - 2\,ab + b^2\big)e^{(-2\,x)}$$

input
```
integrate((a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
1/8*a^2*e^(2*x) + 1/4*a*b*e^(2*x) + 1/8*b^2*e^(2*x) + 1/2*(a^2 - b^2)*x -
1/8*(2*a^2*e^(2*x) - 2*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-2*x)
```

3.581.9 Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int (a\cosh(x)+b\sinh(x))^2\,dx=\frac{a^2\sinh(2\,x)}{4}+\frac{b^2\sinh(2\,x)}{4}+\frac{a^2\,x}{2}-\frac{b^2\,x}{2}+\frac{a\,b\cosh(2\,x)}{2}$$

input
```
int((a*cosh(x) + b*sinh(x))^2,x)
```

output
```
(a^2*sinh(2*x))/4 + (b^2*sinh(2*x))/4 + (a^2*x)/2 - (b^2*x)/2 + (a*b*cosh(
2*x))/2
```

3.582 $\int (a\cosh(x)+b\sinh(x))^3\,dx$

3.582.1 Optimal result

Integrand size = 11, antiderivative size = 35

$$\int (a\cosh(x)+b\sinh(x))^3\,dx=\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))+\frac{1}{3}(b\cosh(x)+a\sinh(x))^3$$

output
```
(a^2-b^2)*(b*cosh(x)+a*sinh(x))+1/3*(b*cosh(x)+a*sinh(x))^3
```

3.582.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int (a\cosh(x)+b\sinh(x))^3\,dx=\frac{1}{12}\big(9b\big(a^2-b^2\big)\cosh(x)+b\big(3a^2+b^2\big)\cosh(3x)+9a\big(a^2-b^2\big)\sinh(x)+a\big(a^2+3b^2\big)\sinh(3x)\big)$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^3,x]
```

output
```
(9*b*(a^2 - b^2)*Cosh[x] + b*(3*a^2 + b^2)*Cosh[3*x] + 9*a*(a^2 - b^2)*Sin
h[x] + a*(a^2 + 3*b^2)*Sinh[3*x])/12
```

3.582.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3551, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+b\sinh(x))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(ix)-ib\sin(ix))^3dx$$

$$\Big\downarrow \text{3551}$$

$$i\int \left(a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2\right)d(-ib\cosh(x)-ia\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$i\left(\left(a^2-b^2\right)(-ia\sinh(x)-ib\cosh(x))-\frac{1}{3}(-ia\sinh(x)-ib\cosh(x))^3\right)$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^3,x]
```

output
```
I*((a^2 - b^2)*((-I)*b*Cosh[x] - I*a*Sinh[x]) - ((-I)*b*Cosh[x] - I*a*Sinh
[x])^3/3)
```

3.582.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3551
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x,
b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[(n - 1)/2, 0]
```

3.582.4 Maple [A] (verified)

Time = 5.91 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

method	result
default	$a^{3}\left(\frac{2}{3}+\frac{\cosh\left(x\right)^{2}}{3}\right)\sinh\left(x\right)+a^{2}b\cosh\left(x\right)^{3}+a\,b^{2}\sinh\left(x\right)^{3}+b^{3}\left(-\frac{2}{3}+\frac{\sinh\left(x\right)^{2}}{3}\right)\cosh\left(x\right)$
parts	$a^{3}\left(\frac{2}{3}+\frac{\cosh\left(x\right)^{2}}{3}\right)\sinh\left(x\right)+a^{2}b\cosh\left(x\right)^{3}+a\,b^{2}\sinh\left(x\right)^{3}+b^{3}\left(-\frac{2}{3}+\frac{\sinh\left(x\right)^{2}}{3}\right)\cosh\left(x\right)$
risch	$\frac{{\mathrm e}^{3x}a^{3}}{24}+\frac{{\mathrm e}^{3x}a^{2}b}{8}+\frac{{\mathrm e}^{3x}a\,b^{2}}{8}+\frac{{\mathrm e}^{3x}b^{3}}{24}+\frac{3a^{3}{\mathrm e}^{x}}{8}+\frac{3a^{2}b\,{\mathrm e}^{x}}{8}-\frac{3\,{\mathrm e}^{x}b^{2}a}{8}-\frac{3b^{3}{\mathrm e}^{x}}{8}-\frac{3\,{\mathrm e}^{-x}a^{3}}{8}+\frac{3\,{\mathrm e}^{-x}a^{2}b}{8}+\frac{3\,{\mathrm e}^{-x}a\,b^{2}}{8}-3\,{\mathrm e}$

input
```
int((a*cosh(x)+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*(2/3+1/3*cosh(x)^2)*sinh(x)+a^2*b*cosh(x)^3+a*b^2*sinh(x)^3+b^3*(-2/3+
1/3*sinh(x)^2)*cosh(x)
```

3.582.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(33) = 66$.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.77

$$\begin{aligned}\int (a\cosh(x)+b\sinh(x))^3\,dx &= \frac{1}{12}\left(3\,a^{2}b+b^{3}\right)\cosh\left(x\right)^{3}+\frac{1}{4}\left(3\,a^{2}b+b^{3}\right)\cosh\left(x\right)\sinh\left(x\right)^{2} \\ &\quad+\frac{1}{12}\left(a^{3}+3\,ab^{2}\right)\sinh\left(x\right)^{3}+\frac{3}{4}\left(a^{2}b-b^{3}\right)\cosh\left(x\right) \\ &\quad+\frac{1}{4}\left(3\,a^{3}-3\,ab^{2}+\left(a^{3}+3\,ab^{2}\right)\cosh\left(x\right)^{2}\right)\sinh\left(x\right)\end{aligned}$$

input
```
integrate((a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

output
```
1/12*(3*a^2*b + b^3)*cosh(x)^3 + 1/4*(3*a^2*b + b^3)*cosh(x)*sinh(x)^2 + 1
/12*(a^3 + 3*a*b^2)*sinh(x)^3 + 3/4*(a^2*b - b^3)*cosh(x) + 1/4*(3*a^3 - 3
*a*b^2 + (a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)
```

3.582.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(29) = 58.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int (a\cosh(x)+b\sinh(x))^3\,dx = -\frac{2a^3\sinh^3(x)}{3} + a^3\sinh(x)\cosh^2(x) + a^2b\cosh^3(x) + ab^2\sinh^3(x) + b^3\sinh^2(x)\cosh(x) - \frac{2b^3\cosh^3(x)}{3}$$

input
```
integrate((a*cosh(x)+b*sinh(x))**3,x)
```

output
```
-2*a**3*sinh(x)**3/3 + a**3*sinh(x)*cosh(x)**2 + a**2*b*cosh(x)**3 + a*b**
2*sinh(x)**3 + b**3*sinh(x)**2*cosh(x) - 2*b**3*cosh(x)**3/3
```

3.582.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(33) = 66.

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int (a\cosh(x)+b\sinh(x))^3\,dx = a^2b\cosh(x)^3 + ab^2\sinh(x)^3 + \frac{1}{24}b^3\left(e^{(3x)} - 9e^{(-x)} + e^{(-3x)} - 9e^{x}\right) + \frac{1}{24}a^3\left(e^{(3x)} - 9e^{(-x)} - e^{(-3x)} + 9e^{x}\right)$$

input
```
integrate((a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

output
```
a^2*b*cosh(x)^3 + a*b^2*sinh(x)^3 + 1/24*b^3*(e^(3*x) - 9*e^(-x) + e^(-3*x
) - 9*e^x) + 1/24*a^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x)
```

3.582.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(33) = 66.

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.83

$$\int (a\cosh(x)+b\sinh(x))^3\,dx = \frac{1}{24}\,a^3e^{(3\,x)}+\frac{1}{8}\,a^2be^{(3\,x)}+\frac{1}{8}\,ab^2e^{(3\,x)}+\frac{1}{24}\,b^3e^{(3\,x)}+\frac{3}{8}\,a^3e^x+\frac{3}{8}\,a^2be^x-\frac{3}{8}\,ab^2e^x-\frac{3}{8}\,b^3e^x - \frac{1}{24}\left(9\,a^3e^{(2\,x)}-9\,a^2be^{(2\,x)}-9\,ab^2e^{(2\,x)}+9\,b^3e^{(2\,x)}+a^3-3\,a^2b+3\,ab^2-b^3\right)e^{(-3\,x)}$$

input
```
integrate((a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

output
```
1/24*a^3*e^(3*x) + 1/8*a^2*b*e^(3*x) + 1/8*a*b^2*e^(3*x) + 1/24*b^3*e^(3*x
) + 3/8*a^3*e^x + 3/8*a^2*b*e^x - 3/8*a*b^2*e^x - 3/8*b^3*e^x - 1/24*(9*a^
3*e^(2*x) - 9*a^2*b*e^(2*x) - 9*a*b^2*e^(2*x) + 9*b^3*e^(2*x) + a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*e^(-3*x)
```

3.582.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int (a\cosh(x)+b\sinh(x))^3\,dx = \cosh(x)^3\left(a^2\,b-\frac{2\,b^3}{3}\right)+\sinh(x)^3\left(a\,b^2-\frac{2\,a^3}{3}\right) + a^3\cosh(x)^2\sinh(x)+b^3\cosh(x)\,\sinh(x)^2$$

input
```
int((a*cosh(x) + b*sinh(x))^3,x)
```

output
```
cosh(x)^3*(a^2*b - (2*b^3)/3) + sinh(x)^3*(a*b^2 - (2*a^3)/3) + a^3*cosh(x
)^2*sinh(x) + b^3*cosh(x)*sinh(x)^2
```

3.583 $\int (a\cosh(x)+b\sinh(x))^4\,dx$

3.583.1 Optimal result

Integrand size = 11, antiderivative size = 72

$$\int (a\cosh(x)+b\sinh(x))^4\,dx=\frac{3}{8}\left(a^2-b^2\right)^2x+\frac{3}{8}\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{4}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^3$$

output
```
3/8*(a^2-b^2)^2*x+3/8*(a^2-b^2)*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x)
)+1/4*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))^3
```

3.583.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int (a\cosh(x)+b\sinh(x))^4\,dx=\frac{1}{32}\left(12(a-b)^2(a+b)^2x+16ab\left(a^2-b^2\right)\cosh(2x)+4ab\left(a^2+b^2\right)\cosh(4x)+8\left(a^4-b^4\right)\sinh(2x)+\left(a^4+6a^2b^2+b^4\right)\sinh(4x)\right)$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^4,x]
```

output
```
(12*(a - b)^2*(a + b)^2*x + 16*a*b*(a^2 - b^2)*Cosh[2*x] + 4*a*b*(a^2 + b^
2)*Cosh[4*x] + 8*(a^4 - b^4)*Sinh[2*x] + (a^4 + 6*a^2*b^2 + b^4)*Sinh[4*x]
)/32
```

3.583.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3552, 3042, 3552, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+b\sinh(x))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(ix)-ib\sin(ix))^4dx$$

$$\Big\downarrow \text{3552}$$

$$\frac{3}{4}(a^2-b^2)\int (a\cosh(x)+b\sinh(x))^2dx+\frac{1}{4}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^3+\frac{3}{4}(a^2-b^2)\int (a\cos(ix)-ib\sin(ix))^2dx$$

$$\Big\downarrow \text{3552}$$

$$\frac{3}{4}(a^2-b^2)\left(\frac{1}{2}(a^2-b^2)\int 1dx+\frac{1}{2}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))\right)+\frac{1}{4}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^3$$

$$\Big\downarrow \text{24}$$

$$\frac{3}{4}(a^2-b^2)\left(\frac{1}{2}x(a^2-b^2)+\frac{1}{2}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))\right)+\frac{1}{4}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^3$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^4,x]
```

output
```
((b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x])^3)/4 + (3*(a^2 - b^2)*(((
a^2 - b^2)*x)/2 + ((b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x]))/2))/4
```

3.583.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

3.583.4 Maple [A] (verified)

Time = 24.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

method	result
default	$a^4\left(\left(\frac{\cosh(x)^3}{4}+\frac{3\cosh(x)}{8}\right)\sinh(x)+\frac{3x}{8}\right)+a^3b\cosh(x)^4+6a^2b^2\left(\frac{\cosh(x)^3\sinh(x)}{4}-\frac{\cosh(x)\sinh(x)}{8}-\frac{x}{8}\right)$
parts	$a^4\left(\left(\frac{\cosh(x)^3}{4}+\frac{3\cosh(x)}{8}\right)\sinh(x)+\frac{3x}{8}\right)+a^3b\cosh(x)^4+6a^2b^2\left(\frac{\cosh(x)^3\sinh(x)}{4}-\frac{\cosh(x)\sinh(x)}{8}-\frac{x}{8}\right)$
risch	$\frac{3x\,a^4}{8}-\frac{3a^2b^2x}{4}+\frac{3b^4x}{8}+\frac{e^{4x}a^4}{64}+\frac{e^{4x}a^3b}{16}+\frac{3\,e^{4x}a^2b^2}{32}+\frac{e^{4x}a\,b^3}{16}+\frac{e^{4x}b^4}{64}+\frac{e^{2x}a^4}{8}+\frac{e^{2x}a^3b}{4}-\frac{e^{2x}a\,b^3}{4}-\frac{e^{2x}b^4}{8}-$

input
```
int((a*cosh(x)+b*sinh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*((1/4*cosh(x)^3+3/8*cosh(x))*sinh(x)+3/8*x)+a^3*b*cosh(x)^4+6*a^2*b^2*
(1/4*cosh(x)^3*sinh(x)-1/8*cosh(x)*sinh(x)-1/8*x)+a*b^3*sinh(x)^4+b^4*((1/
4*sinh(x)^3-3/8*sinh(x))*cosh(x)+3/8*x)
```

3.583.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(66) = 132.

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.33

$$\int (a\cosh(x)+b\sinh(x))^4\,dx$$
$$=\frac{1}{8}\left(a^3b+ab^3\right)\cosh\left(x\right)^4+\frac{1}{8}\left(a^4+6\,a^2b^2+b^4\right)\cosh\left(x\right)\sinh\left(x\right)^3+\frac{1}{8}\left(a^3b+ab^3\right)\sinh\left(x\right)^4$$
$$+\frac{1}{2}\left(a^3b-ab^3\right)\cosh\left(x\right)^2+\frac{1}{4}\left(2\,a^3b-2\,ab^3+3\left(a^3b+ab^3\right)\cosh\left(x\right)^2\right)\sinh\left(x\right)^2$$
$$+\frac{3}{8}\left(a^4-2\,a^2b^2+b^4\right)x+\frac{1}{8}\left(\left(a^4+6\,a^2b^2+b^4\right)\cosh\left(x\right)^3+4\left(a^4-b^4\right)\cosh\left(x\right)\right)\sinh\left(x\right)$$

input
```
integrate((a*cosh(x)+b*sinh(x))^4,x, algorithm="fricas")
```

output
```
1/8*(a^3*b + a*b^3)*cosh(x)^4 + 1/8*(a^4 + 6*a^2*b^2 + b^4)*cosh(x)*sinh(x
)^3 + 1/8*(a^3*b + a*b^3)*sinh(x)^4 + 1/2*(a^3*b - a*b^3)*cosh(x)^2 + 1/4*
(2*a^3*b - 2*a*b^3 + 3*(a^3*b + a*b^3)*cosh(x)^2)*sinh(x)^2 + 3/8*(a^4 - 2
*a^2*b^2 + b^4)*x + 1/8*((a^4 + 6*a^2*b^2 + b^4)*cosh(x)^3 + 4*(a^4 - b^4)
*cosh(x))*sinh(x)
```

3.583.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(71) = 142.

Time = 0.19 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.68

$$\int (a\cosh(x)+b\sinh(x))^4\,dx=\frac{3a^4x\sinh^4\left(x\right)}{8}-\frac{3a^4x\sinh^2\left(x\right)\cosh^2\left(x\right)}{4}$$
$$+\frac{3a^4x\cosh^4\left(x\right)}{8}-\frac{3a^4\sinh^3\left(x\right)\cosh\left(x\right)}{8}$$
$$+\frac{5a^4\sinh\left(x\right)\cosh^3\left(x\right)}{8}+a^3b\cosh^4\left(x\right)$$
$$-\frac{3a^2b^2x\sinh^4\left(x\right)}{4}+\frac{3a^2b^2x\sinh^2\left(x\right)\cosh^2\left(x\right)}{2}$$
$$-\frac{3a^2b^2x\cosh^4\left(x\right)}{4}+\frac{3a^2b^2\sinh^3\left(x\right)\cosh\left(x\right)}{4}$$
$$+\frac{3a^2b^2\sinh\left(x\right)\cosh^3\left(x\right)}{4}+ab^3\sinh^4\left(x\right)+\frac{3b^4x\sinh^4\left(x\right)}{8}$$
$$-\frac{3b^4x\sinh^2\left(x\right)\cosh^2\left(x\right)}{4}+\frac{3b^4x\cosh^4\left(x\right)}{8}$$
$$+\frac{5b^4\sinh^3\left(x\right)\cosh\left(x\right)}{8}-\frac{3b^4\sinh\left(x\right)\cosh^3\left(x\right)}{8}$$

input
```
integrate((a*cosh(x)+b*sinh(x))**4,x)
```

output
```
3*a**4*x*sinh(x)**4/8 - 3*a**4*x*sinh(x)**2*cosh(x)**2/4 + 3*a**4*x*cosh(x
)**4/8 - 3*a**4*sinh(x)**3*cosh(x)/8 + 5*a**4*sinh(x)*cosh(x)**3/8 + a**3*
b*cosh(x)**4 - 3*a**2*b**2*x*sinh(x)**4/4 + 3*a**2*b**2*x*sinh(x)**2*cosh(
x)**2/2 - 3*a**2*b**2*x*cosh(x)**4/4 + 3*a**2*b**2*sinh(x)**3*cosh(x)/4 +
3*a**2*b**2*sinh(x)*cosh(x)**3/4 + a*b**3*sinh(x)**4 + 3*b**4*x*sinh(x)**4
/8 - 3*b**4*x*sinh(x)**2*cosh(x)**2/4 + 3*b**4*x*cosh(x)**4/8 + 5*b**4*sin
h(x)**3*cosh(x)/8 - 3*b**4*sinh(x)*cosh(x)**3/8
```

3.583.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.43

$$\int (a\cosh(x)+b\sinh(x))^4\,dx = a^3b\cosh(x)^4 + ab^3\sinh(x)^4 + \frac{1}{64}a^4\left(24x+e^{(4x)}+8e^{(2x)}-8e^{(-2x)}-e^{(-4x)}\right) + \frac{1}{64}b^4\left(24x+e^{(4x)}-8e^{(2x)}+8e^{(-2x)}-e^{(-4x)}\right) - \frac{3}{32}a^2b^2\left(8x-e^{(4x)}+e^{(-4x)}\right)$$

input
```
integrate((a*cosh(x)+b*sinh(x))^4,x, algorithm="maxima")
```

output
```
a^3*b*cosh(x)^4 + a*b^3*sinh(x)^4 + 1/64*a^4*(24*x + e^(4*x) + 8*e^(2*x) -
 8*e^(-2*x) - e^(-4*x)) + 1/64*b^4*(24*x + e^(4*x) - 8*e^(2*x) + 8*e^(-2*x
) - e^(-4*x)) - 3/32*a^2*b^2*(8*x - e^(4*x) + e^(-4*x))
```

3.583.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(66) = 132.

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.89

$$\int (a\cosh(x)+b\sinh(x))^4\,dx = \frac{1}{64}a^4e^{(4x)} + \frac{1}{16}a^3be^{(4x)} + \frac{3}{32}a^2b^2e^{(4x)} + \frac{1}{16}ab^3e^{(4x)} + \frac{1}{64}b^4e^{(4x)} + \frac{1}{8}a^4e^{(2x)} + \frac{1}{4}a^3be^{(2x)} - \frac{1}{4}ab^3e^{(2x)} - \frac{1}{8}b^4e^{(2x)} + \frac{3}{8}\left(a^4-2a^2b^2+b^4\right)x - \frac{1}{64}\left(18a^4e^{(4x)} - 36a^2b^2e^{(4x)} + 18b^4e^{(4x)} + 8a^4e^{(2x)} - 16a^3be^{(2x)} + 16ab^3e^{(2x)} - 8b^4e^{(2x)} + a^4 - 4a^3b\right.$$

input
```
integrate((a*cosh(x)+b*sinh(x))^4,x, algorithm="giac")
```

output
```
1/64*a^4*e^(4*x) + 1/16*a^3*b*e^(4*x) + 3/32*a^2*b^2*e^(4*x) + 1/16*a*b^3*
e^(4*x) + 1/64*b^4*e^(4*x) + 1/8*a^4*e^(2*x) + 1/4*a^3*b*e^(2*x) - 1/4*a*b
^3*e^(2*x) - 1/8*b^4*e^(2*x) + 3/8*(a^4 - 2*a^2*b^2 + b^4)*x - 1/64*(18*a^
4*e^(4*x) - 36*a^2*b^2*e^(4*x) + 18*b^4*e^(4*x) + 8*a^4*e^(2*x) - 16*a^3*b
*e^(2*x) + 16*a*b^3*e^(2*x) - 8*b^4*e^(2*x) + a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*e^(-4*x)
```

3.583.9 Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.99

$$\begin{aligned}\int (a\cosh(x)+b\sinh(x))^4\,dx &= \cosh(x)\,\sinh(x)^3\left(-\frac{3\,a^4}{8}+\frac{3\,a^2\,b^2}{4}+\frac{5\,b^4}{8}\right)\\ &\quad-\cosh(x)^4\left(a\,b^3-a^3\,b\right)\\ &\quad+\cosh(x)^3\sinh(x)\left(\frac{5\,a^4}{8}+\frac{3\,a^2\,b^2}{4}-\frac{3\,b^4}{8}\right)\\ &\quad+\frac{3\,x\cosh(x)^4\,(a^2-b^2)^2}{8}+\frac{3\,x\sinh(x)^4\,(a^2-b^2)^2}{8}\\ &\quad+2\,a\,b^3\cosh(x)^2\sinh(x)^2-\frac{3\,x\cosh(x)^2\sinh(x)^2\,(a^2-b^2)^2}{4}\end{aligned}$$

input
```
int((a*cosh(x) + b*sinh(x))^4,x)
```

output
```
cosh(x)*sinh(x)^3*((5*b^4)/8 - (3*a^4)/8 + (3*a^2*b^2)/4) - cosh(x)^4*(a*b
^3 - a^3*b) + cosh(x)^3*sinh(x)*((5*a^4)/8 - (3*b^4)/8 + (3*a^2*b^2)/4) +
(3*x*cosh(x)^4*(a^2 - b^2)^2)/8 + (3*x*sinh(x)^4*(a^2 - b^2)^2)/8 + 2*a*b^
3*cosh(x)^2*sinh(x)^2 - (3*x*cosh(x)^2*sinh(x)^2*(a^2 - b^2)^2)/4
```

3.584 $\int (a\cosh(x) + b\sinh(x))^5\,dx$

3.584.1 Optimal result

Integrand size = 11, antiderivative size = 61

$$\int (a\cosh(x) + b\sinh(x))^5\,dx = (a^2-b^2)^2(b\cosh(x)+a\sinh(x)) + \frac{2}{3}(a^2-b^2)(b\cosh(x)+a\sinh(x))^3 + \frac{1}{5}(b\cosh(x)+a\sinh(x))^5$$

output
```
(a^2-b^2)^2*(b*cosh(x)+a*sinh(x))+2/3*(a^2-b^2)*(b*cosh(x)+a*sinh(x))^3+1/
5*(b*cosh(x)+a*sinh(x))^5
```

3.584.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 133 vs. 2(61) = 122.

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.18

$$\int (a\cosh(x) + b\sinh(x))^5\,dx = \frac{1}{240}\Big(150b(a^2-b^2)^2\cosh(x) - 25b(-3a^4+2a^2b^2+b^4)\cosh(3x) + 3b(5a^4+10a^2b^2+b^4)\cosh(5x) + 150a(a^2-b^2)^2\sinh(x) + 25a(a^4+2a^2b^2-3b^4)\sinh(3x) + 3a(a^4+10a^2b^2+5b^4)\sinh(5x)\Big)$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^5,x]
```

output
```
(150*b*(a^2 - b^2)^2*Cosh[x] - 25*b*(-3*a^4 + 2*a^2*b^2 + b^4)*Cosh[3*x] +
 3*b*(5*a^4 + 10*a^2*b^2 + b^4)*Cosh[5*x] + 150*a*(a^2 - b^2)^2*Sinh[x] +
25*a*(a^4 + 2*a^2*b^2 - 3*b^4)*Sinh[3*x] + 3*a*(a^4 + 10*a^2*b^2 + 5*b^4)*
Sinh[5*x])/240
```

3.584.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3551, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+b\sinh(x))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(ix)-ib\sin(ix))^5dx$$

$$\Big\downarrow \text{3551}$$

$$i\int \left(a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2\right)^2 d(-ib\cosh(x)-ia\sinh(x))$$

$$\Big\downarrow \text{210}$$

$$i\int \left(\left(\frac{b^4-2a^2b^2}{a^4}+1\right)a^4-2\left(1-\frac{b^2}{a^2}\right)(-ib\cosh(x)-ia\sinh(x))^2a^2+(-ib\cosh(x)-ia\sinh(x))^4\right)d(-ib\cosh$$
$$ia\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{2}{3}\left(a^2-b^2\right)(-ia\sinh(x)-ib\cosh(x))^3+\left(a^2-b^2\right)^2(-ia\sinh(x)-ib\cosh(x))+\frac{1}{5}(-ia\sinh(x)-ib\cosh(x))\right.$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^5,x]
```

output
```
I*((a^2 - b^2)^2*((-I)*b*Cosh[x] - I*a*Sinh[x]) - (2*(a^2 - b^2)*((-I)*b*C
osh[x] - I*a*Sinh[x])^3)/3 + ((-I)*b*Cosh[x] - I*a*Sinh[x])^5/5)
```

3.584.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3551
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x,
b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[(n - 1)/2, 0]
```

3.584.4 Maple [A] (verified)

Time = 180.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

method	result
parts	$a^5\left(\frac{8}{15}+\frac{\cosh(x)^4}{5}+\frac{4\cosh(x)^2}{15}\right)\sinh(x)+b^5\left(\frac{8}{15}+\frac{\sinh(x)^4}{5}-\frac{4\sinh(x)^2}{15}\right)\cosh(x)+a^4b\cosh(x)^5+10a$
default	$a^5\left(\frac{8}{15}+\frac{\cosh(x)^4}{5}+\frac{4\cosh(x)^2}{15}\right)\sinh(x)+a^4b\cosh(x)^5+10a^3b^2\left(\frac{\cosh(x)^4\sinh(x)}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(x)^2}{3}\right)\sinh(x)}{5}\right.$
risch	$\frac{5b^5e^x}{16}+\frac{5e^xab^4}{16}+\frac{5e^xa^4b}{16}-\frac{5e^xa^3b^2}{8}-\frac{5e^xa^2b^3}{8}+\frac{5e^xa^5}{16}+\frac{e^{-5x}b^5}{160}+\frac{e^{5x}a^5}{160}+\frac{5e^{3x}a^5}{96}+\frac{5e^{-x}a^4b}{16}+\frac{5e^{-x}a^3b^2}{8}$

input
```
int((a*cosh(x)+b*sinh(x))^5,x,method=_RETURNVERBOSE)
```

output
```
a^5*(8/15+1/5*cosh(x)^4+4/15*cosh(x)^2)*sinh(x)+b^5*(8/15+1/5*sinh(x)^4-4/
15*sinh(x)^2)*cosh(x)+a^4*b*cosh(x)^5+10*a^3*b^2*(1/5*sinh(x)^5+1/3*sinh(x
)^3)+10*a^2*b^3*(1/5*cosh(x)^5-1/3*cosh(x)^3)+a*b^4*sinh(x)^5
```

3.584.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(57) = 114$.

Time = 0.25 (sec) , antiderivative size = 298, normalized size of antiderivative = 4.89

$$\int (a\cosh(x)+b\sinh(x))^5\,dx$$
$$= \frac{1}{80}\left(5\,a^4b+10\,a^2b^3+b^5\right)\cosh\left(x\right)^5+\frac{1}{16}\left(5\,a^4b+10\,a^2b^3+b^5\right)\cosh\left(x\right)\sinh\left(x\right)^4$$
$$+\frac{1}{80}\left(a^5+10\,a^3b^2+5\,ab^4\right)\sinh\left(x\right)^5+\frac{5}{48}\left(3\,a^4b-2\,a^2b^3-b^5\right)\cosh\left(x\right)^3$$
$$+\frac{1}{48}\left(5\,a^5+10\,a^3b^2-15\,ab^4+6\left(a^5+10\,a^3b^2+5\,ab^4\right)\cosh\left(x\right)^2\right)\sinh\left(x\right)^3$$
$$+\frac{1}{16}\left(2\left(5\,a^4b+10\,a^2b^3+b^5\right)\cosh\left(x\right)^3+5\left(3\,a^4b-2\,a^2b^3-b^5\right)\cosh\left(x\right)\right)\sinh\left(x\right)^2$$
$$+\frac{5}{8}\left(a^4b-2\,a^2b^3+b^5\right)\cosh\left(x\right)$$
$$+\frac{1}{16}\left(10\,a^5-20\,a^3b^2+10\,ab^4+\left(a^5+10\,a^3b^2+5\,ab^4\right)\cosh\left(x\right)^4+5\left(a^5+2\,a^3b^2-3\,ab^4\right)\cosh\left(x\right)^2\right)\sinh$$

input
```
integrate((a*cosh(x)+b*sinh(x))^5,x, algorithm="fricas")
```

output
```
1/80*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(x)^5 + 1/16*(5*a^4*b + 10*a^2*b^3 +
 b^5)*cosh(x)*sinh(x)^4 + 1/80*(a^5 + 10*a^3*b^2 + 5*a*b^4)*sinh(x)^5 + 5/
48*(3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x)^3 + 1/48*(5*a^5 + 10*a^3*b^2 - 15*a
*b^4 + 6*(a^5 + 10*a^3*b^2 + 5*a*b^4)*cosh(x)^2)*sinh(x)^3 + 1/16*(2*(5*a^
4*b + 10*a^2*b^3 + b^5)*cosh(x)^3 + 5*(3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x))
*sinh(x)^2 + 5/8*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x) + 1/16*(10*a^5 - 20*a^3
*b^2 + 10*a*b^4 + (a^5 + 10*a^3*b^2 + 5*a*b^4)*cosh(x)^4 + 5*(a^5 + 2*a^3*
b^2 - 3*a*b^4)*cosh(x)^2)*sinh(x)
```

3.584.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(56) = 112$.

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.82

$$\int (a\cosh(x)+b\sinh(x))^5\,dx = \frac{8a^5\sinh^5(x)}{15}-\frac{4a^5\sinh^3(x)\cosh^2(x)}{3}+a^5\sinh(x)\cosh^4(x)+a^4b\cosh^5(x)-\frac{4a^3b^2\sinh^5(x)}{3}+\frac{10a^3b^2\sinh^3(x)\cosh^2(x)}{3}+\frac{10a^2b^3\sinh^2(x)\cosh^3(x)}{3}-\frac{4a^2b^3\cosh^5(x)}{3}+ab^4\sinh^5(x)+b^5\sinh^4(x)\cosh(x)-\frac{4b^5\sinh^2(x)\cosh^3(x)}{3}+\frac{8b^5\cosh^5(x)}{15}$$

input
```
integrate((a*cosh(x)+b*sinh(x))**5,x)
```

output
```
8*a**5*sinh(x)**5/15 - 4*a**5*sinh(x)**3*cosh(x)**2/3 + a**5*sinh(x)*cosh(
x)**4 + a**4*b*cosh(x)**5 - 4*a**3*b**2*sinh(x)**5/3 + 10*a**3*b**2*sinh(x
)**3*cosh(x)**2/3 + 10*a**2*b**3*sinh(x)**2*cosh(x)**3/3 - 4*a**2*b**3*cos
h(x)**5/3 + a*b**4*sinh(x)**5 + b**5*sinh(x)**4*cosh(x) - 4*b**5*sinh(x)**
2*cosh(x)**3/3 + 8*b**5*cosh(x)**5/15
```

3.584.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(57) = 114$.

Time = 0.21 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.13

$$\begin{aligned}&\int (a\cosh(x)+b\sinh(x))^5\,dx\\&= a^4b\cosh(x)^5+ab^4\sinh(x)^5\\&\quad+\frac{1}{48}\left(\left(5\,e^{(-2\,x)}-30\,e^{(-4\,x)}+3\right)e^{(5\,x)}+30\,e^{(-x)}-5\,e^{(-3\,x)}-3\,e^{(-5\,x)}\right)a^3b^2\\&\quad-\frac{1}{48}\left(\left(5\,e^{(-2\,x)}+30\,e^{(-4\,x)}-3\right)e^{(5\,x)}+30\,e^{(-x)}+5\,e^{(-3\,x)}-3\,e^{(-5\,x)}\right)a^2b^3\\&\quad+\frac{1}{480}\,a^5\left(3\,e^{(5\,x)}+25\,e^{(3\,x)}-150\,e^{(-x)}-25\,e^{(-3\,x)}-3\,e^{(-5\,x)}+150\,e^{x}\right)\\&\quad+\frac{1}{480}\,b^5\left(3\,e^{(5\,x)}-25\,e^{(3\,x)}+150\,e^{(-x)}-25\,e^{(-3\,x)}+3\,e^{(-5\,x)}+150\,e^{x}\right)\end{aligned}$$

input
```
integrate((a*cosh(x)+b*sinh(x))^5,x, algorithm="maxima")
```

output
```
a^4*b*cosh(x)^5 + a*b^4*sinh(x)^5 + 1/48*((5*e^(-2*x) - 30*e^(-4*x) + 3)*e
^(5*x) + 30*e^(-x) - 5*e^(-3*x) - 3*e^(-5*x))*a^3*b^2 - 1/48*((5*e^(-2*x)
+ 30*e^(-4*x) - 3)*e^(5*x) + 30*e^(-x) + 5*e^(-3*x) - 3*e^(-5*x))*a^2*b^3
+ 1/480*a^5*(3*e^(5*x) + 25*e^(3*x) - 150*e^(-x) - 25*e^(-3*x) - 3*e^(-5*x
) + 150*e^x) + 1/480*b^5*(3*e^(5*x) - 25*e^(3*x) + 150*e^(-x) - 25*e^(-3*x
) + 3*e^(-5*x) + 150*e^x)
```

3.584.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(57) = 114$.

Time = 0.27 (sec) , antiderivative size = 344, normalized size of antiderivative = 5.64

$$\int (a\cosh(x) + b\sinh(x))^5\,dx = \frac{1}{160}\,a^5e^{(5\,x)} + \frac{1}{32}\,a^4be^{(5\,x)} + \frac{1}{16}\,a^3b^2e^{(5\,x)} + \frac{1}{16}\,a^2b^3e^{(5\,x)} + \frac{1}{32}\,ab^4e^{(5\,x)} + \frac{1}{160}\,b^5e^{(5\,x)} + \frac{5}{96}\,a^5e^{(3\,x)} + \frac{5}{32}\,a^4be^{(3\,x)} + \frac{5}{48}\,a^3b^2e^{(3\,x)} - \frac{5}{48}\,a^2b^3e^{(3\,x)} - \frac{5}{32}\,ab^4e^{(3\,x)} - \frac{5}{96}\,b^5e^{(3\,x)} + \frac{5}{16}\,a^5e^{x} + \frac{5}{16}\,a^4be^{x} - \frac{5}{8}\,a^3b^2e^{x} - \frac{5}{8}\,a^2b^3e^{x} + \frac{5}{16}\,ab^4e^{x} + \frac{5}{16}\,b^5e^{x} - \frac{1}{480}\,\big(150\,a^5e^{(4\,x)} - 150\,a^4be^{(4\,x)} - 300\,a^3b^2e^{(4\,x)} + 300\,a^2b^3e^{(4\,x)} + 150\,ab^4e^{(4\,x)} - 150\,b^5e^{(4\,x)} + 25\,a^5e^{(2\,x)}$$

input
```
integrate((a*cosh(x)+b*sinh(x))^5,x, algorithm="giac")
```

output
```
1/160*a^5*e^(5*x) + 1/32*a^4*b*e^(5*x) + 1/16*a^3*b^2*e^(5*x) + 1/16*a^2*b
^3*e^(5*x) + 1/32*a*b^4*e^(5*x) + 1/160*b^5*e^(5*x) + 5/96*a^5*e^(3*x) + 5
/32*a^4*b*e^(3*x) + 5/48*a^3*b^2*e^(3*x) - 5/48*a^2*b^3*e^(3*x) - 5/32*a*b
^4*e^(3*x) - 5/96*b^5*e^(3*x) + 5/16*a^5*e^x + 5/16*a^4*b*e^x - 5/8*a^3*b^
2*e^x - 5/8*a^2*b^3*e^x + 5/16*a*b^4*e^x + 5/16*b^5*e^x - 1/480*(150*a^5*e
^(4*x) - 150*a^4*b*e^(4*x) - 300*a^3*b^2*e^(4*x) + 300*a^2*b^3*e^(4*x) + 1
50*a*b^4*e^(4*x) - 150*b^5*e^(4*x) + 25*a^5*e^(2*x) - 75*a^4*b*e^(2*x) + 5
0*a^3*b^2*e^(2*x) + 50*a^2*b^3*e^(2*x) - 75*a*b^4*e^(2*x) + 25*b^5*e^(2*x)
 + 3*a^5 - 15*a^4*b + 30*a^3*b^2 - 30*a^2*b^3 + 15*a*b^4 - 3*b^5)*e^(-5*x)
```

3.584.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.92

$$\int (a\cosh(x)+b\sinh(x))^5\,dx = \cosh(x)^5\left(a^4\,b-\frac{4\,a^2\,b^3}{3}+\frac{8\,b^5}{15}\right) + \sinh(x)^5\left(\frac{8\,a^5}{15}-\frac{4\,a^3\,b^2}{3}+a\,b^4\right) - \cosh(x)^2\sinh(x)^3\left(\frac{4\,a^5}{3}-\frac{10\,a^3\,b^2}{3}\right)+a^5\cosh(x)^4\sinh(x) - \cosh(x)^3\sinh(x)^2\left(\frac{4\,b^5}{3}-\frac{10\,a^2\,b^3}{3}\right)+b^5\cosh(x)\,\sinh(x)^4$$

input
```
int((a*cosh(x) + b*sinh(x))^5,x)
```

output
```
cosh(x)^5*(a^4*b + (8*b^5)/15 - (4*a^2*b^3)/3) + sinh(x)^5*(a*b^4 + (8*a^5
)/15 - (4*a^3*b^2)/3) - cosh(x)^2*sinh(x)^3*((4*a^5)/3 - (10*a^3*b^2)/3) +
 a^5*cosh(x)^4*sinh(x) - cosh(x)^3*sinh(x)^2*((4*b^5)/3 - (10*a^2*b^3)/3)
+ b^5*cosh(x)*sinh(x)^4
```

3.585 $\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx$

3.585.1 Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx=\frac{\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

output
```
arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

3.585.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx=\frac{2\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^(-1),x]
```

output
```
(2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/(Sqrt[a - b]*Sqrt[
a + b])
```

3.585.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3553}$$

$$i\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))$$

$$\Big\downarrow \text{219}$$

$$\frac{i\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^(-1),x]
```

output
```
(I*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2
]
```

3.585.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.585.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2 \arctan\left(\frac{2 a \tanh\left(\frac{x}{2}\right)+2 b}{2 \sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}}$	39
risch	$-\frac{\ln\left(\mathrm{e}^{x}-\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}$	70

input
```
int(1/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

3.585.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.89

$$\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx = \left[-\frac{\sqrt{-a^{2}+b^{2}}\log\left(\frac{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}-2\,\sqrt{-a^{2}+b^{2}}(\cosh(x)+\sinh(x))-a+b}{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}+a-b}\right)}{a^{2}-b^{2}},\; -\frac{2\,\arctan\left(\frac{\sqrt{a^{2}-b^{2}}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)}{\sqrt{a^{2}-b^{2}}}\right]$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
[-sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
 + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)
*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))/(a^2
- b^2), -2*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))/sqr
t(a^2 - b^2)]
```

3.585.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(31) = 62$.

Time = 2.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.76

$$\int \frac{1}{a\cosh(x) + b\sinh(x)}\,dx = \begin{cases} \tilde{\infty}\log\left(\tanh\left(\frac{x}{2}\right)\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b} & \text{for } a = 0 \\ -\frac{1}{-b\sinh(x) + b\cosh(x)} & \text{for } a = -b \\ -\frac{1}{b\sinh(x) + b\cosh(x)} & \text{for } a = b \\ \frac{\log\left(\tanh\left(\frac{x}{2}\right) + \frac{b}{a} - \frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} - \frac{\log\left(\tanh\left(\frac{x}{2}\right) + \frac{b}{a} + \frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Piecewise((zoo*log(tanh(x/2)), Eq(a, 0) & Eq(b, 0)), (log(tanh(x/2))/b, Eq
(a, 0)), (-1/(-b*sinh(x) + b*cosh(x)), Eq(a, -b)), (-1/(b*sinh(x) + b*cosh
(x)), Eq(a, b)), (log(tanh(x/2) + b/a - sqrt(-a**2 + b**2)/a)/sqrt(-a**2 +
 b**2) - log(tanh(x/2) + b/a + sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2), T
rue))
```

3.585.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a\cosh(x) + b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.585.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{a\cosh(x) + b\sinh(x)}\,dx = \frac{2 \arctan\left(\frac{ae^x + be^x}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/sqrt(a^2 - b^2)
```

3.585.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{a\cosh(x) + b\sinh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{e^x\sqrt{a^2 - b^2}}{a - b}\right)}{\sqrt{a^2 - b^2}}$$

input
```
int(1/(a*cosh(x) + b*sinh(x)),x)
```

output
```
(2*atan((exp(x)*(a^2 - b^2)^(1/2))/(a - b)))/(a^2 - b^2)^(1/2)
```

3.586 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.586.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx=\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

output
```
sinh(x)/a/(a*cosh(x)+b*sinh(x))
```

3.586.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx=\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^(-2),x]
```

output
```
Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x]))
```

3.586.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 3554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a\cos(ix)-ib\sin(ix))^2}dx$$
$$\Big\downarrow \text{3554}$$
$$\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^(-2),x]
```

output
```
Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x]))
```

3.586.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.586.4 Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

method	result	size
risch	$-\frac{2}{(a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}+a-b)(a+b)}$	27
default	$\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a\right)}$	29

input
```
int(1/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(a*exp(2*x)+b*exp(2*x)+a-b)/(a+b)
```

3.586.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.65

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2}{(a^2+2\,ab+b^2)\cosh{(x)}^2+2\,(a^2+2\,ab+b^2)\cosh{(x)}\sinh{(x)}+(a^2+2\,ab+b^2)\sinh{(x)}^2+a^2-b^2}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
-2/((a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)
+ (a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2)
```

3.586.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. 2(14) = 28.

Time = 136.94 (sec) , antiderivative size = 604, normalized size of antiderivative = 35.53

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}\right) \\ \frac{-\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}}{b^2} \\ \frac{x\tanh^4\left(\frac{x}{2}\right)}{2b^2\sinh^2(x)\tanh^4\left(\frac{x}{2}\right)+4b^2\sinh^2(x)\tanh^2\left(\frac{x}{2}\right)+2b^2\sinh^2(x)-8b^2\sinh(x)\cosh(x)\tanh^3\left(\frac{x}{2}\right)-8b^2\sinh(x)\cosh(x)\tanh\left(\frac{x}{2}\right)+8b^2\cosh^2(x)\,\mathrm{t}} \\ \frac{2\tanh\left(\frac{x}{2}\right)}{a^2\tanh^2\left(\frac{x}{2}\right)+a^2+2ab\tanh\left(\frac{x}{2}\right)} \end{cases}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Piecewise((zoo*(-tanh(x/2)/2 - 1/(2*tanh(x/2))), Eq(a, 0) & Eq(b, 0)), ((-
tanh(x/2)/2 - 1/(2*tanh(x/2)))/b**2, Eq(a, 0)), (x*tanh(x/2)**4/(2*b**2*si
nh(x)**2*tanh(x/2)**4 + 4*b**2*sinh(x)**2*tanh(x/2)**2 + 2*b**2*sinh(x)**2
 - 8*b**2*sinh(x)*cosh(x)*tanh(x/2)**3 - 8*b**2*sinh(x)*cosh(x)*tanh(x/2)
+ 8*b**2*cosh(x)**2*tanh(x/2)**2) - 2*x*tanh(x/2)**2/(2*b**2*sinh(x)**2*ta
nh(x/2)**4 + 4*b**2*sinh(x)**2*tanh(x/2)**2 + 2*b**2*sinh(x)**2 - 8*b**2*s
inh(x)*cosh(x)*tanh(x/2)**3 - 8*b**2*sinh(x)*cosh(x)*tanh(x/2) + 8*b**2*co
sh(x)**2*tanh(x/2)**2) + x/(2*b**2*sinh(x)**2*tanh(x/2)**4 + 4*b**2*sinh(x
)**2*tanh(x/2)**2 + 2*b**2*sinh(x)**2 - 8*b**2*sinh(x)*cosh(x)*tanh(x/2)**
3 - 8*b**2*sinh(x)*cosh(x)*tanh(x/2) + 8*b**2*cosh(x)**2*tanh(x/2)**2) + 2
*tanh(x/2)**3/(2*b**2*sinh(x)**2*tanh(x/2)**4 + 4*b**2*sinh(x)**2*tanh(x/2
)**2 + 2*b**2*sinh(x)**2 - 8*b**2*sinh(x)*cosh(x)*tanh(x/2)**3 - 8*b**2*si
nh(x)*cosh(x)*tanh(x/2) + 8*b**2*cosh(x)**2*tanh(x/2)**2) + 2*tanh(x/2)/(2
*b**2*sinh(x)**2*tanh(x/2)**4 + 4*b**2*sinh(x)**2*tanh(x/2)**2 + 2*b**2*si
nh(x)**2 - 8*b**2*sinh(x)*cosh(x)*tanh(x/2)**3 - 8*b**2*sinh(x)*cosh(x)*ta
nh(x/2) + 8*b**2*cosh(x)**2*tanh(x/2)**2), Eq(a, -2*b*tanh(x/2)/(tanh(x/2)
**2 + 1))), (2*tanh(x/2)/(a**2*tanh(x/2)**2 + a**2 + 2*a*b*tanh(x/2)), Tru
e))
```

3.586. $\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.586.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2}{a^2-b^2+(a^2-2\,ab+b^2)e^{(-2\,x)}}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
2/(a^2 - b^2 + (a^2 - 2*a*b + b^2)*e^(-2*x))
```

3.586.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2}{(ae^{(2\,x)}+be^{(2\,x)}+a-b)(a+b)}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-2/((a*e^(2*x) + b*e^(2*x) + a - b)*(a + b))
```

3.586.9 Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2}{(a+b)\,\left(a-b+\mathrm{e}^{2\,x}\,(a+b)\right)}$$

input
```
int(1/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
-2/((a + b)*(a - b + exp(2*x)*(a + b)))
```

3.587 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx$

3.587.1 Optimal result

Integrand size = 11, antiderivative size = 77

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}} + \frac{b\cosh(x)+a\sinh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}$$

output
```
1/2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+1/2*(b*c
osh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^2
```

3.587.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{1}{2}\left(\frac{2\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}} + \frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))^2} + \frac{b}{a(a-b)(a+b)(a\cosh(x)+b\sinh(x))}\right)$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^(-3),x]
```

output
```
((2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*(a
 + b)^(3/2)) + Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x])^2) + b/(a*(a - b)*(a + b
)*(a*Cosh[x] + b*Sinh[x])))/2
```

3.587.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3555, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(a\cos(ix)-ib\sin(ix))^3}dx$$
$$\Big\downarrow \text{ 3555}$$
$$\frac{\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{2(a^2-b^2)}+\frac{a\sinh(x)+b\cosh(x)}{2(a^2-b^2)(a\cosh(x)+b\sinh(x))^2}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{a\sinh(x)+b\cosh(x)}{2(a^2-b^2)(a\cosh(x)+b\sinh(x))^2}+\frac{\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{2(a^2-b^2)}$$
$$\Big\downarrow \text{ 3553}$$
$$\frac{a\sinh(x)+b\cosh(x)}{2(a^2-b^2)(a\cosh(x)+b\sinh(x))^2}+\frac{i\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{2(a^2-b^2)}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{a\sinh(x)+b\cosh(x)}{2(a^2-b^2)(a\cosh(x)+b\sinh(x))^2}+\frac{i\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}}$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^(-3),x]
```

output
```
((I/2)*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2
)^(3/2) + (b*Cosh[x] + a*Sinh[x])/(2*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^2
)
```

3.587.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

3.587.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(69) = 138$.

Time = 32.79 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.90

method	result	size
risch	$\frac{e^{x}\left(a\,e^{2x}+b\,e^{2x}-a+b\right)}{(a-b)(a+b)\left(a\,e^{2x}+b\,e^{2x}+a-b\right)^{2}}-\frac{\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{2\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}+\frac{\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{2\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}$	146
default	$\frac{-\frac{\left(a^{2}-2b^{2}\right)\tanh\left(\frac{x}{2}\right)^{3}}{\left(a^{2}-b^{2}\right)a}+\frac{b\left(a^{2}+2b^{2}\right)\tanh\left(\frac{x}{2}\right)^{2}}{\left(a^{2}-b^{2}\right)a^{2}}+\frac{\left(a^{2}+2b^{2}\right)\tanh\left(\frac{x}{2}\right)}{\left(a^{2}-b^{2}\right)a}+\frac{2b}{2a^{2}-2b^{2}}}{\left(\tanh\left(\frac{x}{2}\right)^{2}a+2b\tanh\left(\frac{x}{2}\right)+a\right)^{2}}+\frac{\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{2}-b^{2}\right)^{\frac{3}{2}}}$	167

input
```
int(1/(a*cosh(x)+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
exp(x)*(a*exp(x)^2+b*exp(x)^2-a+b)/(a-b)/(a+b)/(a*exp(x)^2+b*exp(x)^2+a-b)
^2-1/2/(-a^2+b^2)^(1/2)/(a+b)/(a-b)*ln(exp(x)-(a-b)/(-a^2+b^2)^(1/2))+1/2/
(-a^2+b^2)^(1/2)/(a+b)/(a-b)*ln(exp(x)+(a-b)/(-a^2+b^2)^(1/2))
```

3.587.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(69) = 138$.

Time = 0.29 (sec) , antiderivative size = 1495, normalized size of antiderivative = 19.42

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

output
```
[1/2*(2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^3 + 6*(a^3 + a^2*b - a*b^2 - b
^3)*cosh(x)*sinh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^3 + ((a^2 +
2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3 + (a^2 +
2*a*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2
)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 2*a*b + b^2 + 4*((a^2 + 2*a*b +
 b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log(((a +
 b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^
2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(
x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) - 2*(a^3 - a^2*b - a*b^2 + b^3)*c
osh(x) - 2*(a^3 - a^2*b - a*b^2 + b^3 - 3*(a^3 + a^2*b - a*b^2 - b^3)*cosh
(x)^2)*sinh(x))/(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 +
 b^6 + (a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cos
h(x)^4 + 4*(a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)
*cosh(x)*sinh(x)^3 + (a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*
b^5 + b^6)*sinh(x)^4 + 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 2
*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 3*(a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b
^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^2)*sinh(x)^2 + 4*((a^6 + 2*a^5*b - a
^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2
 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)), ((a^3 + a^2*b - a*b^2 - b^3)*cosh(x
)^3 + 3*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)*sinh(x)^2 + (a^3 + a^2*b - ...
```

3.587.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))**3,x)
```

output
```
Timed out
```

3.587.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.587.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}} + \frac{ae^{(3x)}+be^{(3x)}-ae^x+be^x}{(a^2-b^2)\left(ae^{(2x)}+be^{(2x)}+a-b\right)^2}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

output
```
arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + (a*e^(3*x) + b
*e^(3*x) - a*e^x + b*e^x)/((a^2 - b^2)*(a*e^(2*x) + b*e^(2*x) + a - b)^2)
```

3.587.9 Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.04

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{e^x}{(a+b)\,(a-b)\,(a-b+e^{2x}\,(a+b))} - \frac{2\,e^x}{(a+b)\,\left(e^{4x}\,(a+b)^2+(a-b)^2+2\,e^{2x}\,(a+b)\,(a-b)\right)} - \frac{\operatorname{atan}\left(\frac{e^x\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{-a^3+a^2\,b+a\,b^2-b^3}\right)}{\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}$$

input
```
int(1/(a*cosh(x) + b*sinh(x))^3,x)
```

output
```
exp(x)/((a + b)*(a - b)*(a - b + exp(2*x)*(a + b))) - (2*exp(x))/((a + b)*
(exp(4*x)*(a + b)^2 + (a - b)^2 + 2*exp(2*x)*(a + b)*(a - b))) - atan((exp
(x)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a*b^2 + a^2*b - a^3 - b^3)
)/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)
```

3.588 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx$

3.588.1 Optimal result

Integrand size = 11, antiderivative size = 67

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx=\frac{b\cosh(x)+a\sinh(x)}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^3}+\frac{2\sinh(x)}{3a(a^2-b^2)(a\cosh(x)+b\sinh(x))}$$

output
```
1/3*(b*cosh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^3+2/3*sinh(x)/a/
(a^2-b^2)/(a*cosh(x)+b*sinh(x))
```

3.588.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx=\frac{ab\cosh(3x)+(2a^2-b^2+(a^2+b^2)\cosh(2x))\sinh(x)}{3a(a-b)(a+b)(a\cosh(x)+b\sinh(x))^3}$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^(-4),x]
```

output
```
(a*b*Cosh[3*x] + (2*a^2 - b^2 + (a^2 + b^2)*Cosh[2*x])*Sinh[x])/(3*a*(a -
b)*(a + b)*(a*Cosh[x] + b*Sinh[x])^3)
```

3.588.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3555, 3042, 3554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(ix)-ib\sin(ix))^4}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{2\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}dx}{3(a^2-b^2)}+\frac{a\sinh(x)+b\cosh(x)}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sinh(x)+b\cosh(x)}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^3}+\frac{2\int \frac{1}{(a\cos(ix)-ib\sin(ix))^2}dx}{3(a^2-b^2)}$$

$$\Big\downarrow \text{3554}$$

$$\frac{2\sinh(x)}{3a(a^2-b^2)(a\cosh(x)+b\sinh(x))}+\frac{a\sinh(x)+b\cosh(x)}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^3}$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^(-4),x]
```

output
```
(b*Cosh[x] + a*Sinh[x])/(3*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^3) + (2*Sin
h[x])/(3*a*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

3.588.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

3.588.4 Maple [A] (verified)

Time = 137.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{4(3a\,e^{2x}+3b\,e^{2x}+a-b)}{3(a\,e^{2x}+b\,e^{2x}+a-b)^{3}(a+b)^{2}}$	46
default	$-\frac{2\left(-\frac{\tanh\left(\frac{x}{2}\right)^{5}}{a}-\frac{2b\tanh\left(\frac{x}{2}\right)^{4}}{a^{2}}-\frac{2\left(a^{2}+2b^{2}\right)\tanh\left(\frac{x}{2}\right)^{3}}{3a^{3}}-\frac{2b\tanh\left(\frac{x}{2}\right)^{2}}{a^{2}}-\frac{\tanh\left(\frac{x}{2}\right)}{a}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}a+2b\tanh\left(\frac{x}{2}\right)+a\right)^{3}}$	87

input
```
int(1/(a*cosh(x)+b*sinh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-4/3*(3*a*exp(2*x)+3*b*exp(2*x)+a-b)/(a*exp(2*x)+b*exp(2*x)+a-b)^3/(a+b)^2
```

3.588.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(63) = 126$.

Time = 0.26 (sec) , antiderivative size = 527, normalized size of antiderivative = 7.87

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^4}\,dx =$$

$$-\frac{}{3\left((a^5 + 5\,a^4b + 10\,a^3b^2 + 10\,a^2b^3 + 5\,ab^4 + b^5)\cosh{(x)}^5 + 5\,(a^5 + 5\,a^4b + 10\,a^3b^2 + 10\,a^2b^3 + 5\,ab^4 + b\right.}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^4,x, algorithm="fricas")
```

output
```
-8/3*((2*a + b)*cosh(x) + (a + 2*b)*sinh(x))/((a^5 + 5*a^4*b + 10*a^3*b^2
+ 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^5 + 5*(a^5 + 5*a^4*b + 10*a^3*b^2 +
10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)*sinh(x)^4 + (a^5 + 5*a^4*b + 10*a^3*b^
2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sinh(x)^5 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 -
 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^3 + (3*a^5 + 9*a^4*b + 6*a^3*b^2 - 6*a
^2*b^3 - 9*a*b^4 - 3*b^5 + 10*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5
*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^3 + (10*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*
a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^3 + 9*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*
b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x)^2 + 2*(2*a^5 + a^4*b - 4*a^3*b^2 - 2
*a^2*b^3 + 2*a*b^4 + b^5)*cosh(x) + (2*a^5 + 4*a^4*b - 4*a^3*b^2 - 8*a^2*b
^3 + 2*a*b^4 + 4*b^5 + 5*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^
4 + b^5)*cosh(x)^4 + 9*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 -
b^5)*cosh(x)^2)*sinh(x))
```

3.588.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))**4,x)
```

output
```
Timed out
```

3.588.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(63) = 126$.

Time = 0.22 (sec) , antiderivative size = 498, normalized size of antiderivative = 7.43

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx = \frac{4\,(a-b)e^{(-2\,x)}}{a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5+3\,(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5)e^{(-2\,x)}+3\,(a^5-3\,a^4b+2} + \frac{4\,a}{3\,(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5+3\,(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5)e^{(-2\,x)}+3\,(a^5-3\,a^4} + \frac{4\,b}{3\,(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5+3\,(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5)e^{(-2\,x)}+3\,(a^5-3\,a^4}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^4,x, algorithm="maxima")
```

output
```
4*(a - b)*e^(-2*x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*
(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*e^(-2*x) + 3*(a^5 - 3*
a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*e^(-4*x) + (a^5 - 5*a^4*b +
 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*e^(-6*x)) + 4/3*a/(a^5 + a^4*b -
 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*
b^3 + a*b^4 - b^5)*e^(-2*x) + 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3
*a*b^4 + b^5)*e^(-4*x) + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^
4 - b^5)*e^(-6*x)) + 4/3*b/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 +
b^5 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*e^(-2*x) + 3*(
a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*e^(-4*x) + (a^5 - 5
*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*e^(-6*x))
```

3.588.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx = -\frac{4\,\left(3\,ae^{(2\,x)}+3\,be^{(2\,x)}+a-b\right)}{3\,(a^2+2\,ab+b^2)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)^3}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^4,x, algorithm="giac")
```

output
```
-4/3*(3*a*e^(2*x) + 3*b*e^(2*x) + a - b)/((a^2 + 2*a*b + b^2)*(a*e^(2*x) +
 b*e^(2*x) + a - b)^3)
```

3.588.9 Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx = -\frac{a\left(4\,e^{2\,x}+\frac{4}{3}\right)+b\left(4\,e^{2\,x}-\frac{4}{3}\right)}{(a+b)^2\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)^3}$$

input
```
int(1/(a*cosh(x) + b*sinh(x))^4,x)
```

output
```
-(a*(4*exp(2*x) + 4/3) + b*(4*exp(2*x) - 4/3))/((a + b)^2*(a - b + a*exp(2
*x) + b*exp(2*x))^3)
```

3.589 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx$

3.589.1 Optimal result

Integrand size = 11, antiderivative size = 112

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx=\frac{3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{5/2}}+\frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}+\frac{3(b\cosh(x)+a\sinh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2}$$

output
```
3/8*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+1/4*(b*c
osh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^4+3/8*(b*cosh(x)+a*sinh(
x))/(a^2-b^2)^2/(a*cosh(x)+b*sinh(x))^2
```

3.589.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx=\frac{1}{8}\left(\frac{6\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}+\frac{b(2(a-b)(a+b)+3(a\cosh(x)+b\sinh(x))^2)}{a(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))^3}+\frac{\sinh(x)\left(2+\frac{3(a\cosh(x)+b\sinh(x))^2}{(a-b)(a+b)}\right)}{a(a\cosh(x)+b\sinh(x))^4}\right)$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^(-5),x]
```

output
```
((6*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(5/2)*(a
 + b)^(5/2)) + (b*(2*(a - b)*(a + b) + 3*(a*Cosh[x] + b*Sinh[x])^2))/(a*(a
 - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])^3) + (Sinh[x]*(2 + (3*(a*Cosh[x]
 + b*Sinh[x])^2)/((a - b)*(a + b))))/(a*(a*Cosh[x] + b*Sinh[x])^4))/8
```

3.589.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 3555, 3042, 3555, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(ix)-ib\sin(ix))^5}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{3\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}dx}{4(a^2-b^2)}+\frac{a\sinh(x)+b\cosh(x)}{4(a^2-b^2)(a\cosh(x)+b\sinh(x))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sinh(x)+b\cosh(x)}{4(a^2-b^2)(a\cosh(x)+b\sinh(x))^4}+\frac{3\int \frac{1}{(a\cos(ix)-ib\sin(ix))^3}dx}{4(a^2-b^2)}$$

$$\Big\downarrow \text{3555}$$

$$\frac{3\left(\frac{\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{2(a^2-b^2)}+\frac{a\sinh(x)+b\cosh(x)}{2(a^2-b^2)(a\cosh(x)+b\sinh(x))^2}\right)}{4(a^2-b^2)}+\frac{a\sinh(x)+b\cosh(x)}{4(a^2-b^2)(a\cosh(x)+b\sinh(x))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sinh(x)+b\cosh(x)}{4(a^2-b^2)(a\cosh(x)+b\sinh(x))^4}+\frac{3\left(\frac{a\sinh(x)+b\cosh(x)}{2(a^2-b^2)(a\cosh(x)+b\sinh(x))^2}+\frac{\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{2(a^2-b^2)}\right)}{4(a^2-b^2)}$$

$$\Big\downarrow 3553$$

$$\frac{a\sinh(x)+b\cosh(x)}{4\left(a^2-b^2\right)\left(a\cosh(x)+b\sinh(x)\right)^4}+\frac{3\left(\frac{a\sinh(x)+b\cosh(x)}{2(a^2-b^2)(a\cosh(x)+b\sinh(x))^2}+\frac{i\int\frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{2(a^2-b^2)}\right)}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow 219$$

$$\frac{a\sinh(x)+b\cosh(x)}{4\left(a^2-b^2\right)\left(a\cosh(x)+b\sinh(x)\right)^4}+\frac{3\left(\frac{a\sinh(x)+b\cosh(x)}{2(a^2-b^2)(a\cosh(x)+b\sinh(x))^2}+\frac{i\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}}\right)}{4\left(a^2-b^2\right)}$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^(-5),x]
```

output
```
(b*Cosh[x] + a*Sinh[x])/(4*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^4) + (3*(((
I/2)*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^
(3/2) + (b*Cosh[x] + a*Sinh[x])/(2*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^2))
)/(4*(a^2 - b^2))
```

3.589.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

3.589.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(102) = 204$.

Time = 0.29 (sec) , antiderivative size = 462, normalized size of antiderivative = 4.12

$$\frac{-\frac{(5a^4-16a^2b^2+8b^4)\tanh\left(\frac{x}{2}\right)^7}{4a(a^4-2a^2b^2+b^4)}-\frac{3b(a^4-16a^2b^2+8b^4)\tanh\left(\frac{x}{2}\right)^6}{4(a^4-2a^2b^2+b^4)a^2}+\frac{(3a^6+36a^4b^2+56a^2b^4-32b^6)\tanh\left(\frac{x}{2}\right)^5}{4a^3(a^4-2a^2b^2+b^4)}+\frac{b(15a^6+114a^4b^2-8a^2b^4-16b^6}{4a^4(a^4-2a^2b^2+b^4)}}{\left(\tanh\left(\frac{x}{2}\right)^2a+\right.}$$

input
```
int(1/(a*cosh(x)+b*sinh(x))^5,x)
```

output
```
2*(-1/8*(5*a^4-16*a^2*b^2+8*b^4)/a/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^7-3/8*b
*(a^4-16*a^2*b^2+8*b^4)/(a^4-2*a^2*b^2+b^4)/a^2*tanh(1/2*x)^6+1/8/a^3*(3*a
^6+36*a^4*b^2+56*a^2*b^4-32*b^6)/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^5+1/8/a^4
*b*(15*a^6+114*a^4*b^2-8*a^2*b^4-16*b^6)/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^4
-1/8/a^3*(3*a^6-84*a^4*b^2-56*a^2*b^4+32*b^6)/(a^4-2*a^2*b^2+b^4)*tanh(1/2
*x)^3+1/8*b*(23*a^4+64*a^2*b^2-24*b^4)/(a^4-2*a^2*b^2+b^4)/a^2*tanh(1/2*x)
^2+1/8*(5*a^4+24*a^2*b^2-8*b^4)/a/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)+1/8*(5*a
^2-2*b^2)*b/(a^4-2*a^2*b^2+b^4))/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)^4+3/4
/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2
-b^2)^(1/2))
```

3.589.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3408 vs. $2(102) = 204$.

Time = 0.35 (sec) , antiderivative size = 6874, normalized size of antiderivative = 61.38

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^5,x, algorithm="fricas")
```

output
```
Too large to include
```

3.589.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))**5,x)
```

output
```
Timed out
```

3.589.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.589.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(102) = 204.

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.11

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \frac{3\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{4\,(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{3\,a^3e^{(7\,x)}+9\,a^2be^{(7\,x)}+9\,ab^2e^{(7\,x)}+3\,b^3e^{(7\,x)}+11\,a^3e^{(5\,x)}+11\,a^2be^{(5\,x)}-11\,ab^2e^{(5\,x)}-11\,b^3e^{(5\,x)}-11\,a}{4\,(a^4-2\,a^2b^2+b^4)(ae^{(2\,x)}+be^{(2}}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^5,x, algorithm="giac")
```

output
```
3/4*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(
a^2 - b^2)) + 1/4*(3*a^3*e^(7*x) + 9*a^2*b*e^(7*x) + 9*a*b^2*e^(7*x) + 3*b
^3*e^(7*x) + 11*a^3*e^(5*x) + 11*a^2*b*e^(5*x) - 11*a*b^2*e^(5*x) - 11*b^3
*e^(5*x) - 11*a^3*e^(3*x) + 11*a^2*b*e^(3*x) + 11*a*b^2*e^(3*x) - 11*b^3*e
^(3*x) - 3*a^3*e^x + 9*a^2*b*e^x - 9*a*b^2*e^x + 3*b^3*e^x)/((a^4 - 2*a^2*
b^2 + b^4)*(a*e^(2*x) + b*e^(2*x) + a - b)^4)
```

3.589.9 Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.16

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \frac{3\,\mathrm{atan}\left(\frac{e^x\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}{a^5-a^4\,b-2\,a^3\,b^2+2\,a^2\,b^3+a\,b^4-b^5}\right)}{4\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}$$
$$-\frac{4\,e^{3\,x}}{(a+b)\,\left(e^{8\,x}\,(a+b)^4+(a-b)^4+4\,e^{2\,x}\,(a+b)\,(a-b)^3+4\,e^{6\,x}\,(a+b)^3\,(a-b)+6\,e^{4\,x}\,(a+b)^2\,(a-b)^2\right)}$$
$$-\frac{2\,e^x}{(a+b)^2\,\left(e^{6\,x}\,(a+b)^3+(a-b)^3+3\,e^{2\,x}\,(a+b)\,(a-b)^2+3\,e^{4\,x}\,(a+b)^2\,(a-b)\right)}$$
$$+\frac{3\,e^x}{4\,(a+b)^2\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)}$$
$$+\frac{e^x}{2\,(a+b)^2\,(a-b)\,\left(e^{4\,x}\,(a+b)^2+(a-b)^2+2\,e^{2\,x}\,(a+b)\,(a-b)\right)}$$

input
```
int(1/(a*cosh(x) + b*sinh(x))^5,x)
```

output
```
(3*atan((exp(x)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8
*b^2)^(1/2))/(a*b^4 - a^4*b + a^5 - b^5 + 2*a^2*b^3 - 2*a^3*b^2)))/(4*(a^1
0 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)) - (4*ex
p(3*x))/((a + b)*(exp(8*x)*(a + b)^4 + (a - b)^4 + 4*exp(2*x)*(a + b)*(a -
 b)^3 + 4*exp(6*x)*(a + b)^3*(a - b) + 6*exp(4*x)*(a + b)^2*(a - b)^2)) -
(2*exp(x))/((a + b)^2*(exp(6*x)*(a + b)^3 + (a - b)^3 + 3*exp(2*x)*(a + b)
*(a - b)^2 + 3*exp(4*x)*(a + b)^2*(a - b))) + (3*exp(x))/(4*(a + b)^2*(a -
 b)^2*(a - b + exp(2*x)*(a + b))) + exp(x)/(2*(a + b)^2*(a - b)*(exp(4*x)*
(a + b)^2 + (a - b)^2 + 2*exp(2*x)*(a + b)*(a - b)))
```

3.590 $\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx$

3.590.1 Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx=-\frac{2iE\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|2\right)\sqrt{a\cosh(x)+b\sinh(x)}}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

output
```
-2*I*(cos(1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I
*b))*EllipticE(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*(a*cosh(x)+b*sinh(
x))^(1/2)/((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))^(1/2)
```

3.590.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.51 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.17

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx$$

$$=\frac{b(-a^2+b^2)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cosh^2\left(x+\operatorname{arctanh}\left(\frac{b}{a}\right)\right)\right)\sinh\left(x+\operatorname{arctanh}\left(\frac{b}{a}\right)\right)+\sqrt{-\sinh^2\left(x+\operatorname{arctanh}\left(\frac{b}{a}\right.\right.}}{ab\sqrt{1-\frac{b^2}{a^2}}}$$

input
```
Integrate[Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

output
```
(b*(-a^2 + b^2)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cosh[x + ArcTanh[b/
a]]^2]*Sinh[x + ArcTanh[b/a]] + Sqrt[-Sinh[x + ArcTanh[b/a]]^2]*(2*a^3*Sqr
t[1 - b^2/a^2]*Cosh[x] - 2*a*(a^2 - b^2)*Cosh[x + ArcTanh[b/a]] + 2*a^2*b*
Sqrt[1 - b^2/a^2]*Sinh[x] + a^2*b*Sinh[x + ArcTanh[b/a]] - b^3*Sinh[x + Ar
cTanh[b/a]]))/(a*b*Sqrt[1 - b^2/a^2]*Sqrt[a*Cosh[x] + b*Sinh[x]]*Sqrt[-Sin
h[x + ArcTanh[b/a]]^2])
```

3.590.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3557, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{ 3557}$$

$$\frac{\sqrt{a\cosh(x)+b\sinh(x)}\int \sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}dx}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a\cosh(x)+b\sinh(x)}\int \sqrt{\sin\left(ix-\tan^{-1}(a,-ib)+\frac{\pi}{2}\right)}dx}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

$$\Big\downarrow \text{ 3119}$$

$$-\frac{2i\sqrt{a\cosh(x)+b\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

input
```
Int[Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

output
```
((-2*I)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh[
x]])/Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
```

3.590.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.590.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.51

method	result
default	$-\frac{\sqrt{a^{2}-b^{2}}\,\cosh(x)}{\sqrt{-\sinh(x)\sqrt{a^{2}-b^{2}}}}$
risch	$\sqrt{2}\,\sqrt{\left(a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}+a-b\right)\mathrm{e}^{-x}}+\frac{(2a-2b)\left(-\frac{2\left(a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}+a-b\right)}{(a-b)\sqrt{\left(a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}+a-b\right)\mathrm{e}^{x}}}+\frac{\left(-\frac{a+b}{a-b}+\frac{2a+2b}{a-b}\right)\sqrt{-(a+b)(a-b)}\,\sqrt{\frac{\left(\mathrm{e}^{x}+\sqrt{-}\right.}{\sqrt{-}}}}{}\right.}{}$

input
```
int((a*cosh(x)+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a^2-b^2)^(1/2)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)*cosh(x)
```

3.590.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx = -2\sqrt{2}\sqrt{a+b}\text{weierstrassZeta}\left(-\frac{4(a-b)}{a+b},0,\text{weierstrassPInverse}\left(-\frac{4(a-b)}{a+b},0,\cosh(x)+\sinh(x)\right)\right)-2\sqrt{a\cosh(x)+b\sinh(x)}$$

input
```
integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(2)*sqrt(a + b)*weierstrassZeta(-4*(a - b)/(a + b), 0, weierstrassP
Inverse(-4*(a - b)/(a + b), 0, cosh(x) + sinh(x))) - 2*sqrt(a*cosh(x) + b*
sinh(x))
```

3.590.6 Sympy [F]

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx = \int \sqrt{a\cosh(x)+b\sinh(x)}\,dx$$

input
```
integrate((a*cosh(x)+b*sinh(x))**(1/2),x)
```

output
```
Integral(sqrt(a*cosh(x) + b*sinh(x)), x)
```

3.590.7 Maxima [F]

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx=\int \sqrt{a\cosh{(x)}+b\sinh{(x)}}\,dx$$

input
```
integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cosh(x) + b*sinh(x)), x)
```

3.590.8 Giac [F]

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx=\int \sqrt{a\cosh{(x)}+b\sinh{(x)}}\,dx$$

input
```
integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cosh(x) + b*sinh(x)), x)
```

3.590.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx=\int \sqrt{a\cosh{(x)}+b\sinh{(x)}}\,dx$$

input
```
int((a*cosh(x) + b*sinh(x))^(1/2),x)
```

output
```
int((a*cosh(x) + b*sinh(x))^(1/2), x)
```

3.591 $\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx$

3.591.1 Optimal result

Integrand size = 13, antiderivative size = 103

$$\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx=\frac{2}{3}(b\cosh(x)+a\sinh(x))\sqrt{a\cosh(x)+b\sinh(x)}-\frac{2i(a^2-b^2)\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib)),2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}{3\sqrt{a\cosh(x)+b\sinh(x)}}$$

output
```
2/3*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))^(1/2)-2/3*I*(a^2-b^2)*(cos
(1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*b))*Elli
pticF(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*((a*cosh(x)+b*sinh(x))/(a^2
-b^2)^(1/2))^(1/2)/(a*cosh(x)+b*sinh(x))^(1/2)
```

3.591.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

$$\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx=\frac{2}{3}\left(b\cosh(x)-\sqrt{1-\frac{a^2}{b^2}}b\sqrt{\cosh^2\left(x+\text{arctanh}\left(\frac{a}{b}\right)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\sinh^2\left(x+\text{arctanh}\left(\frac{a}{b}\right)\right)\right)\text{sech}\left(x+\text{arctanh}\left(\frac{a}{b}\right)\right)+a\sinh(x)\right)\sqrt{a\cosh(x)+b\sinh(x)}$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^(3/2),x]
```

output
```
(2*(b*Cosh[x] - Sqrt[1 - a^2/b^2]*b*Sqrt[Cosh[x + ArcTanh[a/b]]^2]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, -Sinh[x + ArcTanh[a/b]]^2]*Sech[x + ArcTanh[
a/b]] + a*Sinh[x])*Sqrt[a*Cosh[x] + b*Sinh[x]])/3
```

3.591.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3552, 3042, 3557, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(ix)-ib\sin(ix))^{3/2}dx$$

$$\Big\downarrow \text{3552}$$

$$\frac{1}{3}\left(a^2-b^2\right)\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}dx+\frac{2}{3}\sqrt{a\cosh(x)+b\sinh(x)}(a\sinh(x)+b\cosh(x))$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\sqrt{a\cosh(x)+b\sinh(x)}(a\sinh(x)+b\cosh(x))+\frac{1}{3}\left(a^2-b^2\right)\int \frac{1}{\sqrt{a\cos(ix)-ib\sin(ix)}}dx$$

$$\Big\downarrow \text{3557}$$

$$\frac{2}{3}\sqrt{a\cosh(x)+b\sinh(x)}(a\sinh(x)+b\cosh(x))+\frac{\left(a^2-b^2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\int \frac{1}{\sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}}dx}{3\sqrt{a\cosh(x)+b\sinh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\sqrt{a\cosh(x)+b\sinh(x)}(a\sinh(x)+b\cosh(x))+\frac{\left(a^2-b^2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\int \frac{1}{\sqrt{\sin\left(ix-\tan^{-1}(a,-ib)+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\cosh(x)+b\sinh(x)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2}{3}(a\sinh(x)+b\cosh(x))\sqrt{a\cosh(x)+b\sinh(x)}-\frac{2i(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\operatorname{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right),2\right)}{3\sqrt{a\cosh(x)+b\sinh(x)}}$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^(3/2),x]
```

output
```
(2*(b*Cosh[x] + a*Sinh[x])*Sqrt[a*Cosh[x] + b*Sinh[x]])/3 - (((2*I)/3)*(a^
2 - b^2)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*Sin
h[x])/Sqrt[a^2 - b^2]])/Sqrt[a*Cosh[x] + b*Sinh[x]]
```

3.591.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.591.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\sqrt{-\sqrt{a^2-b^2}\,\sinh(x)^3}\left(\cosh(x)\sqrt{-\sqrt{a^2-b^2}\,\sinh(x)^3}\,\sqrt{\sinh(x)\sqrt{a^2-b^2}}\,(a^2-b^2)+\sinh(x)(a^2-b^2)^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\sinh(x)\sqrt{a^2-b^2}}\,\cos}{\sqrt{-\sqrt{a^2-b^2}\,\sinh(x}}\right.\right.}{2\sqrt{\sinh(x)\sqrt{a^2-b^2}}\,\sinh(x)^2\sqrt{a^2-b^2}\,\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$

input
```
int((a*cosh(x)+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2)*(cosh(x)*(-(a^2-b^2)^(1/2)*sinh(x)
^3)^(1/2)*(sinh(x)*(a^2-b^2)^(1/2))^(1/2)*(a^2-b^2)+sinh(x)*(a^2-b^2)^(3/2
)*arctan((sinh(x)*(a^2-b^2)^(1/2))^(1/2)*cosh(x)/(-(a^2-b^2)^(1/2)*sinh(x)
^3)^(1/2)))/(sinh(x)*(a^2-b^2)^(1/2))^(1/2)/sinh(x)^2/(a^2-b^2)^(1/2)/(-si
nh(x)*(a^2-b^2)^(1/2))^(1/2)
```

3.591.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int (a\cosh(x) + b\sinh(x))^{3/2}\,dx = \frac{2\left(\sqrt{2}(a-b)\cosh(x) + \sqrt{2}(a-b)\sinh(x)\right)\sqrt{a+b}\,\text{weierstrassPInverse}\left(-\frac{4(a-b)}{a+b}, 0, \cosh\right.}{}$$

input
```
integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*(sqrt(2)*(a - b)*cosh(x) + sqrt(2)*(a - b)*sinh(x))*sqrt(a + b)*wei
erstrassPInverse(-4*(a - b)/(a + b), 0, cosh(x) + sinh(x)) + ((a + b)*cosh
(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)*sqrt(a*cosh
(x) + b*sinh(x)))/(cosh(x) + sinh(x))
```

3.591.6 Sympy [F]

$$\int (a\cosh(x) + b\sinh(x))^{3/2}\,dx = \int \left(a\cosh{\left(x \right)} + b\sinh{\left(x \right)}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cosh(x)+b*sinh(x))**(3/2),x)
```

output
```
Integral((a*cosh(x) + b*sinh(x))**(3/2), x)
```

3.591.7 Maxima [F]

$$\int (a\cosh(x) + b\sinh(x))^{3/2}\,dx = \int \left(a\cosh\left(x\right) + b\sinh\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x) + b*sinh(x))^(3/2), x)
```

3.591.8 Giac [F]

$$\int (a\cosh(x) + b\sinh(x))^{3/2}\,dx = \int \left(a\cosh\left(x\right) + b\sinh\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x) + b*sinh(x))^(3/2), x)
```

3.591.9 Mupad [F(-1)]

Timed out.

$$\int (a\cosh(x) + b\sinh(x))^{3/2} \, dx = \int (a\cosh(x) + b\sinh(x))^{3/2} \, dx$$

input
```
int((a*cosh(x) + b*sinh(x))^(3/2),x)
```

output
```
int((a*cosh(x) + b*sinh(x))^(3/2), x)
```

3.592 $\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx$

3.592.1 Optimal result

Integrand size = 13, antiderivative size = 103

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx = \frac{2}{5}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^{3/2} - \frac{6i(a^2-b^2)\,E\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|\,2\right)\sqrt{a\cosh(x)+b\sinh(x)}}{5\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

output
```
2/5*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))^(3/2)-6/5*I*(a^2-b^2)*(cos
(1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*b))*Elli
pticE(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*(a*cosh(x)+b*sinh(x))^(1/2)
/((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))^(1/2)
```

3.592.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.59 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.87

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx = \frac{(a\cosh(x)+b\sinh(x))\left(6a(a^2-b^2)+2ab^2\cosh(2x)+b(a^2+b^2)\sinh(2x)\right) - \frac{3(a-b)^2(a+b)\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^(5/2),x]
```

output
```
((a*Cosh[x] + b*Sinh[x])*(6*a*(a^2 - b^2) + 2*a*b^2*Cosh[2*x] + b*(a^2 + b
^2)*Sinh[2*x]) - (3*(a - b)^2*(a + b)^2*(b*HypergeometricPFQ[{-1/2, -1/4},
 {3/4}, Cosh[x + ArcTanh[b/a]]^2]*Sinh[x + ArcTanh[b/a]] + Sqrt[-Sinh[x +
ArcTanh[b/a]]^2]*(2*a*Cosh[x + ArcTanh[b/a]] - b*Sinh[x + ArcTanh[b/a]])))
/(a*Sqrt[1 - b^2/a^2]*Sqrt[-Sinh[x + ArcTanh[b/a]]^2]))/(5*b*Sqrt[a*Cosh[x
] + b*Sinh[x]])
```

3.592.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3552, 3042, 3557, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(ix)-ib\sin(ix))^{5/2}dx$$

$$\Big\downarrow \text{3552}$$

$$\frac{3}{5}(a^2-b^2)\int \sqrt{a\cosh(x)+b\sinh(x)}dx+\frac{2}{5}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^{3/2}+\frac{3}{5}(a^2-b^2)\int \sqrt{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3557}$$

$$\frac{2}{5}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^{3/2}+\frac{3(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}\int \sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}dx}{5\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^{3/2}+\frac{3(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}\int\sqrt{\sin\left(ix-\tan^{-1}(a,-ib)+\frac{\pi}{2}\right)}dx}{5\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2}{5}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^{3/2}-\frac{6i(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}E\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|2\right)}{5\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^(5/2),x]
```

output
```
(2*(b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x])^(3/2))/5 - (((6*I)/5)*(
a^2 - b^2)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Si
nh[x]])/Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
```

3.592.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.592.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{((a+b)(a-b))^{\frac{3}{2}}\left(\frac{\cosh(x)^3}{3}-\cosh(x)\right)}{\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$	42

input
```
int((a*cosh(x)+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)*((a+b)*(a-b))^(3/2)*(1/3*cosh(x)^3-cos
h(x))
```

3.592.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.66

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx =$$

$$-\frac{12\left(\sqrt{2}(a^2-b^2)\cosh(x)^2+2\sqrt{2}(a^2-b^2)\cosh(x)\sinh(x)+\sqrt{2}(a^2-b^2)\sinh(x)^2\right)\sqrt{a+b}\text{weierstrassZet}}{}$$

input
```
integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="fricas")
```

output
```
-1/10*(12*(sqrt(2)*(a^2 - b^2)*cosh(x)^2 + 2*sqrt(2)*(a^2 - b^2)*cosh(x)*s
inh(x) + sqrt(2)*(a^2 - b^2)*sinh(x)^2)*sqrt(a + b)*weierstrassZeta(-4*(a
- b)/(a + b), 0, weierstrassPInverse(-4*(a - b)/(a + b), 0, cosh(x) + sinh
(x))) - ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sin
h(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 12*(a^2 - b^2)*cosh(x)^2 + 6*((a^
2 + 2*a*b + b^2)*cosh(x)^2 - 2*a^2 + 2*b^2)*sinh(x)^2 - a^2 + 2*a*b - b^2
+ 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - 6*(a^2 - b^2)*cosh(x))*sinh(x))*sqrt(
a*cosh(x) + b*sinh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

3.592.6 Sympy [F(-1)]

Timed out.

$$\int (a\cosh(x) + b\sinh(x))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)+b*sinh(x))**(5/2),x)
```

output
```
Timed out
```

3.592.7 Maxima [F]

$$\int (a\cosh(x) + b\sinh(x))^{5/2}\,dx = \int (a\cosh(x) + b\sinh(x))^{\frac{5}{2}}\,dx$$

input
```
integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x) + b*sinh(x))^(5/2), x)
```

3.592.8 Giac [F]

$$\int (a\cosh(x) + b\sinh(x))^{5/2}\,dx = \int (a\cosh(x) + b\sinh(x))^{\frac{5}{2}}\,dx$$

input
```
integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x) + b*sinh(x))^(5/2), x)
```

3.592.9 Mupad [F(-1)]

Timed out.

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx=\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx$$

input
```
int((a*cosh(x) + b*sinh(x))^(5/2),x)
```

output
```
int((a*cosh(x) + b*sinh(x))^(5/2), x)
```

3.593 $\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx$

3.593.1 Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib)),2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

output
```
-2*I*(cos(1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I
*b))*EllipticF(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*((a*cosh(x)+b*sinh
(x))/(a^2-b^2)^(1/2))^(1/2)/(a*cosh(x)+b*sinh(x))^(1/2)
```

3.593.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx = \frac{2\sqrt{\cosh^2\left(x+\operatorname{arctanh}\left(\frac{a}{b}\right)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\sinh^2\left(x+\operatorname{arctanh}\left(\frac{a}{b}\right)\right)\right)\operatorname{sech}\left(x+\operatorname{arctanh}\left(\frac{a}{b}\right)\right)\sqrt{a\cosh(x)+b}}{\sqrt{1-\frac{a^2}{b^2}}\,b}$$

input
```
Integrate[1/Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

output
```
(2*Sqrt[Cosh[x + ArcTanh[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, -Si
nh[x + ArcTanh[a/b]]^2]*Sech[x + ArcTanh[a/b]]*Sqrt[a*Cosh[x] + b*Sinh[x]]
)/(Sqrt[1 - a^2/b^2]*b)
```

3.593.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3557, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\cos(ix)-ib\sin(ix)}}dx$$

$$\Big\downarrow \text{3557}$$

$$\frac{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\int \frac{1}{\sqrt{\cosh(x+i\tan^{-1}(a,-ib))}}dx}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\int \frac{1}{\sqrt{\sin\left(ix-\tan^{-1}(a,-ib)+\frac{\pi}{2}\right)}}dx}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2i\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right),2\right)}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

input
```
Int[1/Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

output
```
((-2*I)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*Sinh
[x])/Sqrt[a^2 - b^2]])/Sqrt[a*Cosh[x] + b*Sinh[x]]
```

3.593.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.593.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{\sqrt{-\sqrt{a^{2}-b^{2}}\, \sinh \left(x \right)^{3}}\, \arctan \left(\frac{\sqrt{\sinh \left(x \right) \sqrt{a^{2}-b^{2}}}\, \cosh \left(x \right)}{\sqrt{-\sqrt{a^{2}-b^{2}}\, \sinh \left(x \right)^{3}}}\right)}{\sqrt{\sinh \left(x \right) \sqrt{a^{2}-b^{2}}}\, \sinh \left(x \right) \sqrt{-\sinh \left(x \right) \sqrt{a^{2}-b^{2}}}}$	97

input
```
int(1/(a*cosh(x)+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2)/(sinh(x)*(a^2-b^2)^(1/2))^(1/2)*arctan(
(sinh(x)*(a^2-b^2)^(1/2))^(1/2)*cosh(x)/(-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2)
)/sinh(x)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)
```

3.593.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}\left(-\frac{4(a-b)}{a+b},0,\cosh(x)+\sinh(x)\right)}{\sqrt{a+b}}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(-4*(a - b)/(a + b), 0, cosh(x) + sinh(x))/sq
rt(a + b)
```

3.593.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))**(1/2),x)
```

output
```
Integral(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

3.593.7 Maxima [F]

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

3.593.8 Giac [F]

$$\int \frac{1}{\sqrt{a\cosh(x) + b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x) + b\sinh(x)}}\,dx$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

3.593.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cosh(x) + b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x) + b\sinh(x)}}\,dx$$

input
```
int(1/(a*cosh(x) + b*sinh(x))^(1/2),x)
```

output
```
int(1/(a*cosh(x) + b*sinh(x))^(1/2), x)
```

3.594 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx$

3.594.1 Optimal result

Integrand size = 13, antiderivative size = 112

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx=\frac{2(b\cosh(x)+a\sinh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}+\frac{2iE\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|2\right)\sqrt{a\cosh(x)+b\sinh(x)}}{(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

output
```
2*(b*cosh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^(1/2)+2*I*(cos(1/2
*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*b))*Elliptic
E(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*(a*cosh(x)+b*sinh(x))^(1/2)/(a^
2-b^2)/((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))^(1/2)
```

3.594.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.32

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx=\frac{b\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cosh^2\left(x+\operatorname{arctanh}\left(\frac{b}{a}\right)\right)\right)\sinh\left(x+\operatorname{arctanh}\left(\frac{b}{a}\right)\right)-\sqrt{-s}}{ab\sqrt{1-\frac{b^2}{a^2}}\sqrt{a\,co}}$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^(-3/2),x]
```

output
```
(b*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cosh[x + ArcTanh[b/a]]^2]*Sinh[x
 + ArcTanh[b/a]] - Sqrt[-Sinh[x + ArcTanh[b/a]]^2]*(2*a*Sqrt[1 - b^2/a^2]*
Cosh[x] - 2*a*Cosh[x + ArcTanh[b/a]] + b*Sinh[x + ArcTanh[b/a]]))/(a*b*Sqr
t[1 - b^2/a^2]*Sqrt[a*Cosh[x] + b*Sinh[x]]*Sqrt[-Sinh[x + ArcTanh[b/a]]^2]
)
```

3.594.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3555, 3042, 3557, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(ix)-ib\sin(ix))^{3/2}}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{2(a\sinh(x)+b\cosh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}-\frac{\int \sqrt{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2(a\sinh(x)+b\cosh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}-\frac{\int \sqrt{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3557}$$

$$\frac{2(a\sinh(x)+b\cosh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}-\frac{\sqrt{a\cosh(x)+b\sinh(x)}\int \sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}dx}{(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2(a\sinh(x)+b\cosh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}-\frac{\sqrt{a\cosh(x)+b\sinh(x)}\int \sqrt{\sin\left(ix-\tan^{-1}(a,-ib)+\frac{\pi}{2}\right)}dx}{(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

$$\downarrow 3119$$

$$\frac{2(a\sinh(x)+b\cosh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}+\frac{2i\sqrt{a\cosh(x)+b\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^(-3/2),x]
```

output
```
(2*(b*Cosh[x] + a*Sinh[x]))/((a^2 - b^2)*Sqrt[a*Cosh[x] + b*Sinh[x]]) + ((
2*I)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh[x]]
)/((a^2 - b^2)*Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])
```

3.594.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.594.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{\operatorname{arctanh}(\cosh(x))}{\sqrt{a^2-b^2}\,\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$	33

input
```
int(1/(a*cosh(x)+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a^2-b^2)^(1/2)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)*arctanh(cosh(x))
```

3.594.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.98

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx = \frac{2\left(\left(\sqrt{2}(a+b)\cosh(x)^2+2\sqrt{2}(a+b)\cosh(x)\sinh(x)+\sqrt{2}(a+b)\sinh\right.\right.}{}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="fricas")
```

output
```
2*((sqrt(2)*(a + b)*cosh(x)^2 + 2*sqrt(2)*(a + b)*cosh(x)*sinh(x) + sqrt(2
)*(a + b)*sinh(x)^2 + sqrt(2)*(a - b))*sqrt(a + b)*weierstrassZeta(-4*(a -
 b)/(a + b), 0, weierstrassPInverse(-4*(a - b)/(a + b), 0, cosh(x) + sinh(
x))) + 2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^
2)*sqrt(a*cosh(x) + b*sinh(x)))/(a^3 - a^2*b - a*b^2 + b^3 + (a^3 + a^2*b
- a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)*sinh(x) +
 (a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^2)
```

3.594.6 Sympy [F]

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx=\int \frac{1}{(a\cosh{(x)}+b\sinh{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))**(3/2),x)
```

output
```
Integral((a*cosh(x) + b*sinh(x))**(-3/2), x)
```

3.594.7 Maxima [F]

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx=\int \frac{1}{(a\cosh{(x)}+b\sinh{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x) + b*sinh(x))^(-3/2), x)
```

3.594.8 Giac [F]

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx=\int \frac{1}{(a\cosh{(x)}+b\sinh{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x) + b*sinh(x))^(-3/2), x)
```

3.594.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx=\int \frac{1}{\left(a\cosh\left(x\right)+b\sinh\left(x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(a*cosh(x) + b*sinh(x))^(3/2),x)
```

output
```
int(1/(a*cosh(x) + b*sinh(x))^(3/2), x)
```

3.595 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx$

3.595.1 Optimal result

Integrand size = 13, antiderivative size = 116

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx = \frac{2(b\cosh(x)+a\sinh(x))}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^{3/2}} - \frac{2i\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib)),2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}{3(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}$$

output
```
2/3*(b*cosh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^(3/2)-2/3*I*(cos
(1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*b))*Elli
pticF(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*((a*cosh(x)+b*sinh(x))/(a^2
-b^2)^(1/2))^(1/2)/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^(1/2)
```

3.595.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx = -\frac{2\left(\sqrt{1-\frac{a^2}{b^2}}b(b\cosh(x)+a\sinh(x))+\sqrt{\cosh^2\left(x+\text{arctanh}\left(\frac{a}{b}\right)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\sinh^2\left(x+\text{arctanh}\left(\frac{a}{b}\right)\right)\right)\text{s}\right.}{3\sqrt{1-\frac{a^2}{b^2}}b(-a+b)(a+b)(a\cosh(x)+b\sinh(x))^{3/2}}$$

input
```
Integrate[(a*Cosh[x] + b*Sinh[x])^(-5/2),x]
```

output
```
(-2*(Sqrt[1 - a^2/b^2]*b*(b*Cosh[x] + a*Sinh[x]) + Sqrt[Cosh[x + ArcTanh[a
/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, -Sinh[x + ArcTanh[a/b]]^2]*Se
ch[x + ArcTanh[a/b]]*(a*Cosh[x] + b*Sinh[x])^2))/(3*Sqrt[1 - a^2/b^2]*b*(-
a + b)*(a + b)*(a*Cosh[x] + b*Sinh[x])^(3/2))
```

3.595.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3555, 3042, 3557, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(ix)-ib\sin(ix))^{5/2}}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}dx}{3(a^2-b^2)}+\frac{2(a\sinh(x)+b\cosh(x))}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2(a\sinh(x)+b\cosh(x))}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^{3/2}}+\frac{\int \frac{1}{\sqrt{a\cos(ix)-ib\sin(ix)}}dx}{3(a^2-b^2)}$$

$$\Big\downarrow \text{3557}$$

$$\frac{2(a\sinh(x)+b\cosh(x))}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^{3/2}}+\frac{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\int \frac{1}{\sqrt{\cosh(x+i\tan^{-1}(a,-ib))}}dx}{3(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2(a\sinh(x)+b\cosh(x))}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^{3/2}}+\frac{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\int \frac{1}{\sqrt{\sin\left(ix-\tan^{-1}(a,-ib)+\frac{\pi}{2}\right)}}dx}{3(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2(a\sinh(x)+b\cosh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}-\frac{2i\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right),2\right)}{3\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}}$$

input
```
Int[(a*Cosh[x] + b*Sinh[x])^(-5/2),x]
```

output
```
(2*(b*Cosh[x] + a*Sinh[x]))/(3*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^(3/2))
- (((2*I)/3)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b
*Sinh[x])/Sqrt[a^2 - b^2]])/((a^2 - b^2)*Sqrt[a*Cosh[x] + b*Sinh[x]])
```

3.595.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.595. $\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx$

3.595.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.32

method	result	size
default	$-\frac{\cosh(x)}{(a^2-b^2)\sinh(x)\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$	37

input
```
int(1/(a*cosh(x)+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-cosh(x)/(a^2-b^2)/sinh(x)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)
```

3.595.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 679, normalized size of antiderivative = 5.85

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx = \frac{2\left(\left(\sqrt{2}(a^2+2\,ab+b^2)\cosh(x)^4+4\sqrt{2}(a^2+2\,ab+b^2)\cosh(x)\sinh(x)^3\right.\right.}{3\left(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5+(a^5+3\,a\right.}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="fricas")
```

output
```
2/3*((sqrt(2)*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*sqrt(2)*(a^2 + 2*a*b + b^2
)*cosh(x)*sinh(x)^3 + sqrt(2)*(a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*sqrt(2)*(a
^2 - b^2)*cosh(x)^2 + 2*(3*sqrt(2)*(a^2 + 2*a*b + b^2)*cosh(x)^2 + sqrt(2)
*(a^2 - b^2))*sinh(x)^2 + 4*(sqrt(2)*(a^2 + 2*a*b + b^2)*cosh(x)^3 + sqrt(
2)*(a^2 - b^2)*cosh(x))*sinh(x) + sqrt(2)*(a^2 - 2*a*b + b^2))*sqrt(a + b)
*weierstrassPInverse(-4*(a - b)/(a + b), 0, cosh(x) + sinh(x)) + 2*((a^2 +
 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^2 + (a^2 +
 2*a*b + b^2)*sinh(x)^3 - (a^2 - b^2)*cosh(x) + (3*(a^2 + 2*a*b + b^2)*cos
h(x)^2 - a^2 + b^2)*sinh(x))*sqrt(a*cosh(x) + b*sinh(x)))/(a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
3*a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3
- 3*a*b^4 - b^5)*sinh(x)^4 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^
4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5
+ 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2)*sin
h(x)^2 + 4*((a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x
)^3 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x))*sinh(x)
)
```

3.595.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))**(5/2),x)
```

output
```
Timed out
```

3.595.7 Maxima [F]

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh(x) + b\sinh(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x) + b*sinh(x))^(-5/2), x)
```

3.595.8 Giac [F]

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx=\int \frac{1}{(a\cosh(x)+b\sinh(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x) + b*sinh(x))^(-5/2), x)
```

3.595.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx=\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx$$

input
```
int(1/(a*cosh(x) + b*sinh(x))^(5/2),x)
```

output
```
int(1/(a*cosh(x) + b*sinh(x))^(5/2), x)
```

3.596 $\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx$

3.596.1 Optimal result

Integrand size = 17, antiderivative size = 23

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx=\frac{a\cosh(c+dx)}{d}+\frac{a\sinh(c+dx)}{d}$$

output
```
a*cosh(d*x+c)/d+a*sinh(d*x+c)/d
```

3.596.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx=\frac{a\cosh(c)\cosh(dx)}{d}+\frac{a\cosh(dx)\sinh(c)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}+\frac{a\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[a*Cosh[c + d*x] + a*Sinh[c + d*x],x]
```

output
```
(a*Cosh[c]*Cosh[d*x])/d + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x])/
d + (a*Sinh[c]*Sinh[d*x])/d
```

3.596.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sinh(c+dx)+a\cosh(c+dx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sinh(c+dx)}{d}+\frac{a\cosh(c+dx)}{d}$$

input
```
Int[a*Cosh[c + d*x] + a*Sinh[c + d*x],x]
```

output
```
(a*Cosh[c + d*x])/d + (a*Sinh[c + d*x])/d
```

3.596.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.596.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

method	result	size
risch	$\frac{e^{dx+c}a}{d}$	12
gosper	$\frac{a(\cosh(dx+c)+\sinh(dx+c))}{d}$	19
derivativedivides	$\frac{a\cosh(dx+c)+a\sinh(dx+c)}{d}$	22
default	$\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$	24
parts	$\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$	24
meijerg	$\frac{(\cosh(c)\sqrt{\pi}\,a+\sqrt{\pi}\,\sinh(c)a)\sinh(dx)}{d\sqrt{\pi}}-\frac{i(-i\cosh(c)\sqrt{\pi}\,a-i\sqrt{\pi}\,\sinh(c)a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}$	66

input
```
int(a*cosh(d*x+c)+a*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*exp(d*x+c)*a
```

3.596.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx=\frac{a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)}{d}$$

input
```
integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x, algorithm="fricas")
```

output
```
(a*cosh(d*x + c) + a*sinh(d*x + c))/d
```

3.596.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx=a\left(\begin{cases}\frac{\sinh\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\cosh\left(c\right) & \text{otherwise}\end{cases}\right)+a\left(\begin{cases}\frac{\cosh\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\sinh\left(c\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x)
```

output
```
a*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True)) + a*Piecewise(
(cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

3.596.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx=\frac{a\cosh{(dx+c)}}{d}+\frac{a\sinh{(dx+c)}}{d}$$

input
```
integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x, algorithm="maxima")
```

output
```
a*cosh(d*x + c)/d + a*sinh(d*x + c)/d
```

3.596.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx=\frac{1}{2}\,a\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}\,a\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)$$

input
```
integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x, algorithm="giac")
```

output
```
1/2*a*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(e^(d*x + c)/d - e^(-d*x -
c)/d)
```

3.596.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx=\frac{a\,\mathrm{e}^{c+d\,x}}{d}$$

input
```
int(a*cosh(c + d*x) + a*sinh(c + d*x),x)
```

output
```
(a*exp(c + d*x))/d
```

3.597 $\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx$

3.597.1 Optimal result

Integrand size = 19, antiderivative size = 26

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx=\frac{(a\cosh(c+dx)+a\sinh(c+dx))^2}{2d}$$

output
```
1/2*(a*cosh(d*x+c)+a*sinh(d*x+c))^2/d
```

3.597.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx=\frac{a^2(\cosh(c+dx)+\sinh(c+dx))^2}{2d}$$

input
```
Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2,x]
```

output
```
(a^2*(Cosh[c + d*x] + Sinh[c + d*x])^2)/(2*d)
```

3.597.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sinh(c+dx)+a\cosh(c+dx))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\cos(ic+idx)-ia\sin(ic+idx))^2dx$$

$$\Big\downarrow \; 3550$$

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^2}{2d}$$

input
```
Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2,x]
```

output
```
(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2/(2*d)
```

3.597.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.597.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{a^{2} {\mathrm e}^{2 d x +2 c}}{2 d}$	18
gosper	$\frac{a^{2} \left(\cosh \left(d x +c \right)+\sinh \left(d x +c \right)\right)^{2}}{2 d}$	24
derivativedivides	$\frac{a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+a^{2} \cosh \left(d x +c \right)^{2}+a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}-\frac{d x}{2}-\frac{c}{2}\right)}{d}$	70
default	$\frac{a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+a^{2} \cosh \left(d x +c \right)^{2}+a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}-\frac{d x}{2}-\frac{c}{2}\right)}{d}$	70
parts	$\frac{a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}+\frac{a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}-\frac{d x}{2}-\frac{c}{2}\right)}{d}+\frac{a^{2} \cosh \left(d x +c \right)^{2}}{d}$	75

input
```
int((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2/d*exp(2*d*x+2*c)
```

3.597.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx=\frac{a^2\cosh(dx+c)+a^2\sinh(dx+c)}{2\,(d\cosh(dx+c)-d\sinh(dx+c))}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(a^2*cosh(d*x + c) + a^2*sinh(d*x + c))/(d*cosh(d*x + c) - d*sinh(d*x
+ c))
```

3.597.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx = \begin{cases} \frac{a^2\sinh^2(c+dx)}{d}+\frac{a^2\sinh(c+dx)\cosh(c+dx)}{d} & \text{for } d\neq 0\\ x(a\sinh(c)+a\cosh(c))^2 & \text{otherwise}\end{cases}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**2,x)
```

output
```
Piecewise((a**2*sinh(c + d*x)**2/d + a**2*sinh(c + d*x)*cosh(c + d*x)/d, N
e(d, 0)), (x*(a*sinh(c) + a*cosh(c))**2, True))
```

3.597.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(24) = 48$.

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx = \frac{1}{8}a^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right) + \frac{a^2\cosh(dx+c)^2}{d}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*a^2*(4*x - e^
(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^2*cosh(d*x + c)^2/d
```

3.597.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx=\frac{a^2e^{(2\,dx+2\,c)}}{2\,d}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*a^2*e^(2*d*x + 2*c)/d
```

3.597.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx=\frac{a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{2\,d}$$

input
```
int((a*cosh(c + d*x) + a*sinh(c + d*x))^2,x)
```

output
```
(a^2*exp(2*c + 2*d*x))/(2*d)
```

3.598 $\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx$

3.598.1 Optimal result

Integrand size = 19, antiderivative size = 26

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx = \frac{(a\cosh(c+dx)+a\sinh(c+dx))^3}{3d}$$

output
```
1/3*(a*cosh(d*x+c)+a*sinh(d*x+c))^3/d
```

3.598.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx = \frac{a^3(\cosh(c+dx)+\sinh(c+dx))^3}{3d}$$

input
```
Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3,x]
```

output
```
(a^3*(Cosh[c + d*x] + Sinh[c + d*x])^3)/(3*d)
```

3.598.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sinh(c+dx)+a\cosh(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a\cos(ic+idx)-ia\sin(ic+idx))^3dx$$
$$\Big\downarrow \text{3550}$$
$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^3}{3d}$$

input
```
Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3,x]
```

output
```
(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3/(3*d)
```

3.598.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.598.4 Maple [A] (verified)

Time = 6.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

method	result	si
risch	$\frac{a^{3} {\mathrm e}^{3 d x +3 c}}{3 d}$	1
gosper	$\frac{a^{3} \left(\cosh \left(d x +c \right)+\sinh \left(d x +c \right)\right)^{3}}{3 d}$	2
derivativedivides	$\frac{a^{3} \left(\frac{2}{3}+\frac{\cosh \left(d x +c \right)^{2}}{3}\right) \sinh \left(d x +c \right)+a^{3} \cosh \left(d x +c \right)^{3}+a^{3} \sinh \left(d x +c \right)^{3}+a^{3} \left(-\frac{2}{3}+\frac{\sinh \left(d x +c \right)^{2}}{3}\right) \cosh \left(d x +c \right)}{d}$	7
default	$\frac{a^{3} \left(\frac{2}{3}+\frac{\cosh \left(d x +c \right)^{2}}{3}\right) \sinh \left(d x +c \right)+a^{3} \cosh \left(d x +c \right)^{3}+a^{3} \sinh \left(d x +c \right)^{3}+a^{3} \left(-\frac{2}{3}+\frac{\sinh \left(d x +c \right)^{2}}{3}\right) \cosh \left(d x +c \right)}{d}$	7
parts	$\frac{a^{3} \left(\frac{2}{3}+\frac{\cosh \left(d x +c \right)^{2}}{3}\right) \sinh \left(d x +c \right)}{d}+\frac{a^{3} \left(-\frac{2}{3}+\frac{\sinh \left(d x +c \right)^{2}}{3}\right) \cosh \left(d x +c \right)}{d}+\frac{a^{3} \cosh \left(d x +c \right)^{3}}{d}+\frac{a^{3} \sinh \left(d x +c \right)^{3}}{d}$	8

input
```
int((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/d*a^3*exp(3*d*x+3*c)
```

3.598.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(24) = 48$.

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx = \frac{a^{3} \cosh \left(d x + c\right)^{2} + 2 \, a^{3} \cosh \left(d x + c\right) \sinh \left(d x + c\right) + a^{3} \sinh \left(d x + c\right)^{2}}{3 \, {\left(d \cosh \left(d x + c\right) - d \sinh \left(d x + c\right)\right)}}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3*(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*
x + c)^2)/(d*cosh(d*x + c) - d*sinh(d*x + c))
```

3.598.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(20) = 40$.

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.19

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx$$
$$= \begin{cases} \frac{a^3\sinh^3(c+dx)}{3d}+\frac{a^3\sinh^2(c+dx)\cosh(c+dx)}{d}+\frac{a^3\sinh(c+dx)\cosh^2(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d} & \text{for } d\neq 0 \\ x(a\sinh(c)+a\cosh(c))^3 & \text{otherwise} \end{cases}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**3,x)
```

output
```
Piecewise((a**3*sinh(c + d*x)**3/(3*d) + a**3*sinh(c + d*x)**2*cosh(c + d*
x)/d + a**3*sinh(c + d*x)*cosh(c + d*x)**2/d + a**3*cosh(c + d*x)**3/(3*d)
, Ne(d, 0)), (x*(a*sinh(c) + a*cosh(c))**3, True))
```

3.598.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(24) = 48$.

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 5.62

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx$$
$$= \frac{a^3\cosh(dx+c)^3}{d}+\frac{a^3\sinh(dx+c)^3}{d} + \frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right) + \frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*cosh(d*x + c)^3/d + a^3*sinh(d*x + c)^3/d + 1/24*a^3*(e^(3*d*x + 3*c)/
d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 1/24*a^3*(e
^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d
)
```

3.598.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx=\frac{a^3e^{(3\,dx+3\,c)}}{3\,d}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
1/3*a^3*e^(3*d*x + 3*c)/d
```

3.598.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx=\frac{a^3\,\mathrm{e}^{3\,c+3\,d\,x}}{3\,d}$$

input
```
int((a*cosh(c + d*x) + a*sinh(c + d*x))^3,x)
```

output
```
(a^3*exp(3*c + 3*d*x))/(3*d)
```

3.599 $\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx$

3.599.1 Optimal result

Integrand size = 19, antiderivative size = 26

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx=\frac{(a\cosh(c+dx)+a\sinh(c+dx))^n}{dn}$$

output
```
(a*cosh(d*x+c)+a*sinh(d*x+c))^n/d/n
```

3.599.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx=\frac{(a(\cosh(c+dx)+\sinh(c+dx)))^n}{dn}$$

input
```
Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^n,x]
```

output
```
(a*(Cosh[c + d*x] + Sinh[c + d*x]))^n/(d*n)
```

3.599.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sinh(c+dx)+a\cosh(c+dx))^n\,dx$$
$$\Big\downarrow \; 3042$$
$$\int (a\cos(ic+idx)-ia\sin(ic+idx))^n dx$$
$$\Big\downarrow \; 3550$$
$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^n}{dn}$$

input
```
Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^n,x]
```

output
```
(a*Cosh[c + d*x] + a*Sinh[c + d*x])^n/(d*n)
```

3.599.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.599.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{(a\cosh(dx+c)+a\sinh(dx+c))^n}{dn}$	27
derivativedivides	$\frac{(a\cosh(dx+c)+a\sinh(dx+c))^n}{dn}$	27
default	$\frac{(a\cosh(dx+c)+a\sinh(dx+c))^n}{dn}$	27

input
```
int((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
(a*cosh(d*x+c)+a*sinh(d*x+c))^n/d/n
```

3.599.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx = \frac{\cosh(dnx+cn+n\log(a))+\sinh(dnx+cn+n\log(a))}{dn}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x, algorithm="fricas")
```

output
```
(cosh(d*n*x + c*n + n*log(a)) + sinh(d*n*x + c*n + n*log(a)))/(d*n)
```

3.599.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx = \begin{cases} x & \text{for } d=0 \wedge n=0 \\ x(a\sinh(c)+a\cosh(c))^n & \text{for } d=0 \\ x & \text{for } n=0 \\ \frac{(a\sinh(c+dx)+a\cosh(c+dx))^n}{dn} & \text{otherwise} \end{cases}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**n,x)
```

output
```
Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(a*sinh(c) + a*cosh(c))**n, Eq(d, 0
)), (x, Eq(n, 0)), ((a*sinh(c + d*x) + a*cosh(c + d*x))**n/(d*n), True))
```

3.599.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int (a\cosh(c+dx) + a\sinh(c+dx))^n \, dx = \frac{a^n e^{((dx+c)n)}}{dn}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x, algorithm="maxima")
```

output
```
a^n*e^((d*x + c)*n)/(d*n)
```

3.599.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int (a\cosh(c+dx) + a\sinh(c+dx))^n \, dx = \frac{\left(ae^{(dx+c)}\right)^n}{dn}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x, algorithm="giac")
```

output
```
(a*e^(d*x + c))^n/(d*n)
```

3.599.9 Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int (a\cosh(c+dx) + a\sinh(c+dx))^n \, dx = \frac{\left(a\,\mathrm{e}^{c+d\,x}\right)^n}{d\,n}$$

input
```
int((a*cosh(c + d*x) + a*sinh(c + d*x))^n,x)
```

output
```
(a*exp(c + d*x))^n/(d*n)
```

3.600 $\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx$

3.600.1 Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=-\frac{1}{d(a\cosh(c+dx)+a\sinh(c+dx))}$$

output
```
-1/d/(a*cosh(d*x+c)+a*sinh(d*x+c))
```

3.600.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=-\frac{1}{d(a\cosh(c+dx)+a\sinh(c+dx))}$$

input
```
Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-1),x]
```

output
```
-(1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])))
```

3.600.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sinh(c+dx)+a\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\cos(ic+idx)-ia\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3550}$$

$$-\frac{1}{d(a\sinh(c+dx)+a\cosh(c+dx))}$$

input
```
Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-1),x]
```

output
```
-(1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])))
```

3.600.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.600.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\mathrm{e}^{-dx-c}}{ad}$	18
gosper	$-\frac{1}{da(\cosh(dx+c)+\sinh(dx+c))}$	24
derivativedivides	$-\frac{1}{d(a\cosh(dx+c)+a\sinh(dx+c))}$	25
default	$-\frac{1}{d(a\cosh(dx+c)+a\sinh(dx+c))}$	25

input
```
int(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/a/d*exp(-d*x-c)
```

3.600.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = -\frac{1}{ad\cosh(dx+c)+ad\sinh(dx+c)}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/(a*d*cosh(d*x + c) + a*d*sinh(d*x + c))
```

3.600.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = \begin{cases} -\frac{1}{ad\sinh(c+dx)+ad\cosh(c+dx)} & \text{for } d\neq 0 \\ \frac{x}{a\sinh(c)+a\cosh(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x)
```

output
```
Piecewise((-1/(a*d*sinh(c + d*x) + a*d*cosh(c + d*x)), Ne(d, 0)), (x/(a*si
nh(c) + a*cosh(c)), True))
```

3.600.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = -\frac{e^{(-dx-c)}}{ad}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^(-d*x - c)/(a*d)
```

3.600.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = -\frac{e^{(-dx-c)}}{ad}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-e^(-d*x - c)/(a*d)
```

3.600.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = -\frac{\mathrm{e}^{-c-d\,x}}{a\,d}$$

input
```
int(1/(a*cosh(c + d*x) + a*sinh(c + d*x)),x)
```

output
```
-exp(- c - d*x)/(a*d)
```

3.601 $\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx$

3.601.1 Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{1}{2d(a\cosh(c+dx)+a\sinh(c+dx))^2}$$

output
```
-1/2/d/(a*cosh(d*x+c)+a*sinh(d*x+c))^2
```

3.601.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{1}{2d(a\cosh(c+dx)+a\sinh(c+dx))^2}$$

input
```
Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-2),x]
```

output
```
-1/2*1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2)
```

3.601.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sinh(c+dx)+a\cosh(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(ic+idx)-ia\sin(ic+idx))^2}dx$$

$$\Big\downarrow \text{3550}$$

$$-\frac{1}{2d(a\sinh(c+dx)+a\cosh(c+dx))^2}$$

input
```
Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-2),x]
```

output
```
-1/2*1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2)
```

3.601.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.601.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{e^{-2dx-2c}}{2a^2d}$	18
gosper	$-\frac{1}{2a^2(\cosh(dx+c)+\sinh(dx+c))^2 d}$	24
derivativedivides	$-\frac{1}{2d(a\cosh(dx+c)+a\sinh(dx+c))^2}$	25
default	$-\frac{1}{2d(a\cosh(dx+c)+a\sinh(dx+c))^2}$	25

input
```
int(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/d*exp(-2*d*x-2*c)
```

3.601.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx$$
$$= -\frac{1}{2\left(a^2d\cosh\left(dx+c\right)^2+2\,a^2d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^2d\sinh\left(dx+c\right)^2\right)}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a^2*d*
sinh(d*x + c)^2)
```

3.601.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = \begin{cases} -\frac{1}{2a^2d\sinh^2{(c+dx)}+4a^2d\sinh{(c+dx)}\cosh{(c+dx)}+2a^2d\cosh^2{(c+dx)}} & \text{for } d\neq 0 \\ \frac{x}{(a\sinh{(c)}+a\cosh{(c)})^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))**2,x)
```

output
```
Piecewise((-1/(2*a**2*d*sinh(c + d*x)**2 + 4*a**2*d*sinh(c + d*x)*cosh(c +
 d*x) + 2*a**2*d*cosh(c + d*x)**2), Ne(d, 0)), (x/(a*sinh(c) + a*cosh(c))*
*2, True))
```

3.601.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{e^{(-2\,dx-2\,c)}}{2\,a^2d}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*e^(-2*d*x - 2*c)/(a^2*d)
```

3.601.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{e^{(-2\,dx-2\,c)}}{2\,a^2d}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*e^(-2*d*x - 2*c)/(a^2*d)
```

3.601.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{e^{-2c-2dx}}{2a^2d}$$

input
```
int(1/(a*cosh(c + d*x) + a*sinh(c + d*x))^2,x)
```

output
```
-exp(- 2*c - 2*d*x)/(2*a^2*d)
```

3.602 $\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx$

3.602.1 Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx=-\frac{1}{3d(a\cosh(c+dx)+a\sinh(c+dx))^3}$$

output
```
-1/3/d/(a*cosh(d*x+c)+a*sinh(d*x+c))^3
```

3.602.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx=-\frac{1}{3d(a\cosh(c+dx)+a\sinh(c+dx))^3}$$

input
```
Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-3),x]
```

output
```
-1/3*1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3)
```

3.602.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sinh(c+dx)+a\cosh(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(ic+idx)-ia\sin(ic+idx))^3}dx$$

$$\Big\downarrow \text{3550}$$

$$-\frac{1}{3d(a\sinh(c+dx)+a\cosh(c+dx))^3}$$

input
```
Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-3),x]
```

output
```
-1/3*1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3)
```

3.602.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.602.4 Maple [A] (verified)

Time = 19.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{e^{-3dx-3c}}{3a^3d}$	18
gosper	$-\frac{1}{3a^3(\cosh(dx+c)+\sinh(dx+c))^3 d}$	24
derivativedivides	$-\frac{1}{3d(a\cosh(dx+c)+a\sinh(dx+c))^3}$	25
default	$-\frac{1}{3d(a\cosh(dx+c)+a\sinh(dx+c))^3}$	25

input
```
int(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^3/d*exp(-3*d*x-3*c)
```

3.602.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx =$$
$$-\frac{1}{3\left(a^3d\cosh\left(dx+c\right)^3+3\,a^3d\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)+3\,a^3d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+a^3d\sinh\right.}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/3/(a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^
3*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*d*sinh(d*x + c)^3)
```

3.602.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(24) = 48.

Time = 0.61 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.46

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx$$
$$= \begin{cases} -\frac{1}{3a^3d\sinh^3(c+dx)+9a^3d\sinh^2(c+dx)\cosh(c+dx)+9a^3d\sinh(c+dx)\cosh^2(c+dx)+3a^3d\cosh^3(c+dx)} & \text{for } d\neq 0 \\ \frac{x}{(a\sinh(c)+a\cosh(c))^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))**3,x)
```

output
```
Piecewise((-1/(3*a**3*d*sinh(c + d*x)**3 + 9*a**3*d*sinh(c + d*x)**2*cosh(
c + d*x) + 9*a**3*d*sinh(c + d*x)*cosh(c + d*x)**2 + 3*a**3*d*cosh(c + d*x
)**3), Ne(d, 0)), (x/(a*sinh(c) + a*cosh(c))**3, True))
```

3.602.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx = -\frac{e^{(-3\,dx-3\,c)}}{3\,a^3d}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/3*e^(-3*d*x - 3*c)/(a^3*d)
```

3.602.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx = -\frac{e^{(-3\,dx-3\,c)}}{3\,a^3d}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/3*e^(-3*d*x - 3*c)/(a^3*d)
```

3.602.9 Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx=-\frac{e^{-3c-3dx}}{3a^3d}$$

input
```
int(1/(a*cosh(c + d*x) + a*sinh(c + d*x))^3,x)
```

output
```
-exp(- 3*c - 3*d*x)/(3*a^3*d)
```

3.603 $\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx$

3.603.1 Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=\frac{2\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}{d}$$

output
```
2*(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2)/d
```

3.603.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=\frac{2\sqrt{a(\cosh(c+dx)+\sinh(c+dx))}}{d}$$

input
```
Integrate[Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
```

output
```
(2*Sqrt[a*(Cosh[c + d*x] + Sinh[c + d*x])])/d
```

3.603.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sinh(c+dx)+a\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\cos(ic+idx)-ia\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3550}$$

$$\frac{2\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}{d}$$

input
```
Int[Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
```

output
```
(2*Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]])/d
```

3.603.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.603.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{2a\,\mathrm{e}^{dx+c}}{d\sqrt{a\,\mathrm{e}^{dx+c}}}$	23
gosper	$\frac{2\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}{d}$	25
derivativedivides	$\frac{2\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}{d}$	25
default	$\frac{2\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}{d}$	25

input
```
int((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a/d*exp(d*x+c)/(a*exp(d*x+c))^(1/2)
```

3.603.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=\frac{2\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}{d}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a*cosh(d*x + c) + a*sinh(d*x + c))/d
```

3.603.6 Sympy [F]

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=\int \sqrt{a\sinh(c+dx)+a\cosh(c+dx)}\,dx$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

3.603.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=\frac{2\,\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(a)*e^(1/2*d*x + 1/2*c)/d
```

3.603.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=\frac{2\,\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

input
```
integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(a)*e^(1/2*d*x + 1/2*c)/d
```

3.603.9 Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=\frac{2\,\sqrt{a\,\mathrm{e}^{c+d\,x}}}{d}$$

input
```
int((a*cosh(c + d*x) + a*sinh(c + d*x))^(1/2),x)
```

output
```
(2*(a*exp(c + d*x))^(1/2))/d
```

3.604 $\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx$

3.604.1 Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = -\frac{2}{d\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}$$

output
```
-2/d/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2)
```

3.604.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = -\frac{2}{d\sqrt{a(\cosh(c+dx)+\sinh(c+dx))}}$$

input
```
Integrate[1/Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
```

output
```
-2/(d*Sqrt[a*(Cosh[c + d*x] + Sinh[c + d*x])])
```

3.604.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a\cos(ic+idx)-ia\sin(ic+idx)}}dx$$

$$\Big\downarrow \text{ 3550}$$

$$-\frac{2}{d\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}$$

input
```
Int[1/Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
```

output
```
-2/(d*Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]])
```

3.604.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.604.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{2}{d\sqrt{a\,e^{dx+c}}}$	16
gosper	$-\frac{2}{d\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}$	25
derivativedivides	$-\frac{2}{d\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}$	25
default	$-\frac{2}{d\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}$	25

input
```
int(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/(a*exp(d*x+c))^(1/2)
```

3.604.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = -\frac{2\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}{ad\cosh(dx+c)+ad\sinh(dx+c)}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*cosh(d*x + c) + a*sinh(d*x + c))/(a*d*cosh(d*x + c) + a*d*sinh(d
*x + c))
```

3.604.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}\,dx$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

3.604.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = -\frac{2\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2*e^(-1/2*d*x - 1/2*c)/(sqrt(a)*d)
```

3.604.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = -\frac{2\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

input
```
integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*e^(-1/2*d*x - 1/2*c)/(sqrt(a)*d)
```

3.604.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = -\frac{2}{d\,\sqrt{a\,\mathrm{e}^{c+d\,x}}}$$

input
```
int(1/(a*cosh(c + d*x) + a*sinh(c + d*x))^(1/2),x)
```

output
```
-2/(d*(a*exp(c + d*x))^(1/2))
```

3.605 $\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx$

3.605.1 Optimal result

Integrand size = 18, antiderivative size = 24

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx=-\frac{a\cosh(c+dx)}{d}+\frac{a\sinh(c+dx)}{d}$$

output
```
-a*cosh(d*x+c)/d+a*sinh(d*x+c)/d
```

3.605.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx=-\frac{a\cosh(c)\cosh(dx)}{d}+\frac{a\cosh(dx)\sinh(c)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}-\frac{a\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[a*Cosh[c + d*x] - a*Sinh[c + d*x],x]
```

output
```
-((a*Cosh[c]*Cosh[d*x])/d) + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x
])/d - (a*Sinh[c]*Sinh[d*x])/d
```

3.605.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a\sinh(c+dx)}{d}-\frac{a\cosh(c+dx)}{d}$$

input
```
Int[a*Cosh[c + d*x] - a*Sinh[c + d*x],x]
```

output
```
-((a*Cosh[c + d*x])/d) + (a*Sinh[c + d*x])/d
```

3.605.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.605.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{a\,{\mathrm e}^{-dx-c}}{d}$	16
gosper	$-\frac{a(\cosh(dx+c)-\sinh(dx+c))}{d}$	22
derivativedivides	$\frac{a\sinh(dx+c)-a\cosh(dx+c)}{d}$	23
default	$-\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$	25
parts	$-\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$	25
meijerg	$\frac{(\cosh(c)\sqrt{\pi}\,a-\sqrt{\pi}\,\sinh(c)a)\sinh(dx)}{d\sqrt{\pi}}-\frac{i(i\cosh(c)\sqrt{\pi}\,a-i\sqrt{\pi}\,\sinh(c)a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}$	67

input
```
int(a*cosh(d*x+c)-a*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-a/d*exp(-d*x-c)
```

3.605.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx=-\frac{a}{d\cosh{(dx+c)}+d\sinh{(dx+c)}}$$

input
```
integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x, algorithm="fricas")
```

output
```
-a/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

3.605.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx=a\left(\begin{cases}\frac{\sinh{(c+dx)}}{d} & \text{for}\: d\neq 0\\ x\cosh{(c)} & \text{otherwise}\end{cases}\right)-a\left(\begin{cases}\frac{\cosh{(c+dx)}}{d} & \text{for}\: d\neq 0\\ x\sinh{(c)} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x)
```

output
```
a*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True)) - a*Piecewise(
(cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

3.605.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx=-\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$$

input
```
integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x, algorithm="maxima")
```

output
```
-a*cosh(d*x + c)/d + a*sinh(d*x + c)/d
```

3.605.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(24) = 48.

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx=-\frac{1}{2}a\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}a\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)$$

input
```
integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x, algorithm="giac")
```

output
```
-1/2*a*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(e^(d*x + c)/d - e^(-d*x -
c)/d)
```

3.605.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx=-\frac{a\,\mathrm{e}^{-c-d\,x}}{d}$$

input
```
int(a*cosh(c + d*x) - a*sinh(c + d*x),x)
```

output
```
-(a*exp(- c - d*x))/d
```

3.606 $\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx$

3.606.1 Optimal result

Integrand size = 20, antiderivative size = 27

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^2}{2d}$$

output
```
-1/2*(a*cosh(d*x+c)-a*sinh(d*x+c))^2/d
```

3.606.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^2}{2d}$$

input
```
Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2,x]
```

output
```
-1/2*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2/d
```

3.606.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (ia\sin(ic+idx)+a\cos(ic+idx))^2dx$$

$$\Big\downarrow \text{3550}$$

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^2}{2d}$$

input
```
Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2,x]
```

output
```
-1/2*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2/d
```

3.606.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.606.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{a^{2} {\mathrm e}^{-2 d x -2 c}}{2 d}$	18
gosper	$-\frac{a^{2} \left(\cosh \left(d x +c \right)-\sinh \left(d x +c \right)\right)^{2}}{2 d}$	26
derivativedivides	$\frac{a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-a^{2} \cosh \left(d x +c \right)^{2}+a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}-\frac{d x}{2}-\frac{c}{2}\right)}{d}$	71
default	$\frac{a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-a^{2} \cosh \left(d x +c \right)^{2}+a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}-\frac{d x}{2}-\frac{c}{2}\right)}{d}$	71
parts	$\frac{a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}+\frac{a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}-\frac{d x}{2}-\frac{c}{2}\right)}{d}-\frac{a^{2} \cosh \left(d x +c \right)^{2}}{d}$	76

input
```
int((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/d*exp(-2*d*x-2*c)
```

3.606.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx$$
$$= -\frac{a^{2}}{2\left(d\cosh\left(dx+c\right)^{2}+2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+d\sinh\left(dx+c\right)^{2}\right)}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*a^2/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x
 + c)^2)
```

3.606.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx = \begin{cases} -\frac{a^2\sinh^2(c+dx)}{d}+\frac{a^2\sinh(c+dx)\cosh(c+dx)}{d} & \text{for } d\neq 0\\ x(-a\sinh(c)+a\cosh(c))^2 & \text{otherwise}\end{cases}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**2,x)
```

output
```
Piecewise((-a**2*sinh(c + d*x)**2/d + a**2*sinh(c + d*x)*cosh(c + d*x)/d,
Ne(d, 0)), (x*(-a*sinh(c) + a*cosh(c))**2, True))
```

3.606.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(25) = 50.

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.30

$$\begin{aligned}\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx &= \frac{1}{8}a^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)\\ &\quad-\frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)\\ &\quad-\frac{a^2\cosh(dx+c)^2}{d}\end{aligned}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*a^2*(4*x - e^
(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - a^2*cosh(d*x + c)^2/d
```

3.606.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx = -\frac{a^2e^{(-2\,dx-2\,c)}}{2\,d}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*a^2*e^(-2*d*x - 2*c)/d
```

3.606.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx = -\frac{a^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{2\,d}$$

input
```
int((a*cosh(c + d*x) - a*sinh(c + d*x))^2,x)
```

output
```
-(a^2*exp(- 2*c - 2*d*x))/(2*d)
```

3.607 $\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx$

3.607.1 Optimal result

Integrand size = 20, antiderivative size = 27

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

output
```
-1/3*(a*cosh(d*x+c)-a*sinh(d*x+c))^3/d
```

3.607.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

input
```
Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3,x]
```

output
```
-1/3*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3/d
```

3.607.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (ia\sin(ic+idx)+a\cos(ic+idx))^3dx$$

$$\Big\downarrow \text{3550}$$

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

input
```
Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3,x]
```

output
```
-1/3*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3/d
```

3.607.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.607.4 Maple [A] (verified)

Time = 6.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	si
risch	$-\frac{a^{3} {\mathrm e}^{-3 d x -3 c}}{3 d}$	1
gosper	$-\frac{a^{3} \left(\cosh \left(d x +c \right)-\sinh \left(d x +c \right)\right)^{3}}{3 d}$	2
derivativedivides	$\frac{a^{3} \left(\frac{2}{3}+\frac{\cosh \left(d x +c \right)^{2}}{3}\right) \sinh \left(d x +c \right)-a^{3} \cosh \left(d x +c \right)^{3}+a^{3} \sinh \left(d x +c \right)^{3}-a^{3} \left(-\frac{2}{3}+\frac{\sinh \left(d x +c \right)^{2}}{3}\right) \cosh \left(d x +c \right)}{d}$	7
default	$\frac{a^{3} \left(\frac{2}{3}+\frac{\cosh \left(d x +c \right)^{2}}{3}\right) \sinh \left(d x +c \right)-a^{3} \cosh \left(d x +c \right)^{3}+a^{3} \sinh \left(d x +c \right)^{3}-a^{3} \left(-\frac{2}{3}+\frac{\sinh \left(d x +c \right)^{2}}{3}\right) \cosh \left(d x +c \right)}{d}$	7
parts	$\frac{a^{3} \left(\frac{2}{3}+\frac{\cosh \left(d x +c \right)^{2}}{3}\right) \sinh \left(d x +c \right)}{d}-\frac{a^{3} \left(-\frac{2}{3}+\frac{\sinh \left(d x +c \right)^{2}}{3}\right) \cosh \left(d x +c \right)}{d}-\frac{a^{3} \cosh \left(d x +c \right)^{3}}{d}+\frac{a^{3} \sinh \left(d x +c \right)^{3}}{d}$	8

input
```
int((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/d*a^3*exp(-3*d*x-3*c)
```

3.607.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(25) = 50$.

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int (a\cosh(c+dx) - a\sinh(c+dx))^3\,dx =$$

$$-\frac{a^{3}}{3\left(d\cosh\left(dx+c\right)^{3} + 3\,d\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right) + 3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2} + d\sinh\left(dx+c\right.}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/3*a^3/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh
(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

3.607.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(22) = 44$.

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.07

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx = \begin{cases} \frac{a^3\sinh^3(c+dx)}{3d} - \frac{a^3\sinh^2(c+dx)\cosh(c+dx)}{d} + \frac{a^3\sinh(c+dx)\cosh^2(c+dx)}{d} - \frac{a^3\cosh^3(c+dx)}{3d} & \text{for } d\neq 0 \\ x(-a\sinh(c)+a\cosh(c))^3 & \text{otherwise} \end{cases}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**3,x)
```

output
```
Piecewise((a**3*sinh(c + d*x)**3/(3*d) - a**3*sinh(c + d*x)**2*cosh(c + d*
x)/d + a**3*sinh(c + d*x)*cosh(c + d*x)**2/d - a**3*cosh(c + d*x)**3/(3*d)
, Ne(d, 0)), (x*(-a*sinh(c) + a*cosh(c))**3, True))
```

3.607.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(25) = 50$.

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 5.44

$$\begin{aligned}\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx &= -\frac{a^3\cosh(dx+c)^3}{d} + \frac{a^3\sinh(dx+c)^3}{d} \\ &\quad + \frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right) \\ &\quad - \frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)\end{aligned}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-a^3*cosh(d*x + c)^3/d + a^3*sinh(d*x + c)^3/d + 1/24*a^3*(e^(3*d*x + 3*c)
/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) - 1/24*a^3*(
e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/
d)
```

3.607.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int (a\cosh(c+dx) - a\sinh(c+dx))^3\,dx = -\frac{a^3e^{(-3\,dx-3\,c)}}{3\,d}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/3*a^3*e^(-3*d*x - 3*c)/d
```

3.607.9 Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int (a\cosh(c+dx) - a\sinh(c+dx))^3\,dx = -\frac{a^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{3\,d}$$

input
```
int((a*cosh(c + d*x) - a*sinh(c + d*x))^3,x)
```

output
```
-(a^3*exp(- 3*c - 3*d*x))/(3*d)
```

3.608 $\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx$

3.608.1 Optimal result

Integrand size = 20, antiderivative size = 28

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^n}{dn}$$

output
```
-(a*cosh(d*x+c)-a*sinh(d*x+c))^n/d/n
```

3.608.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx=-\frac{(a(\cosh(c+dx)-\sinh(c+dx)))^n}{dn}$$

input
```
Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^n,x]
```

output
```
-((a*(Cosh[c + d*x] - Sinh[c + d*x]))^n/(d*n))
```

3.608.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (ia\sin(ic+idx)+a\cos(ic+idx))^n dx$$

$$\Big\downarrow \text{3550}$$

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^n}{dn}$$

input
```
Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^n,x]
```

output
```
-((a*Cosh[c + d*x] - a*Sinh[c + d*x])^n/(d*n))
```

3.608.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.608.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
gosper	$-\frac{(a\cosh(dx+c)-a\sinh(dx+c))^n}{dn}$	29
derivativedivides	$-\frac{(a\cosh(dx+c)-a\sinh(dx+c))^n}{dn}$	29
default	$-\frac{(a\cosh(dx+c)-a\sinh(dx+c))^n}{dn}$	29

input
```
int((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
-(a*cosh(d*x+c)-a*sinh(d*x+c))^n/d/n
```

3.608.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx = -\frac{\cosh\left(-dnx-cn+n\log\left(a\right)\right)+\sinh\left(-dnx-cn+n\log\left(a\right)\right)}{dn}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x, algorithm="fricas")
```

output
```
-(cosh(-d*n*x - c*n + n*log(a)) + sinh(-d*n*x - c*n + n*log(a)))/(d*n)
```

3.608.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx = \begin{cases} x & \text{for } d=0 \wedge n=0 \\ x(-a\sinh\left(c\right)+a\cosh\left(c\right))^n & \text{for } d=0 \\ x & \text{for } n=0 \\ -\frac{(-a\sinh\left(c+dx\right)+a\cosh\left(c+dx\right))^n}{dn} & \text{otherwise} \end{cases}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**n,x)
```

output
```
Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(-a*sinh(c) + a*cosh(c))**n, Eq(d,
0)), (x, Eq(n, 0)), (-(-a*sinh(c + d*x) + a*cosh(c + d*x))**n/(d*n), True)
)
```

3.608.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx=-\frac{a^n e^{(-(dx+c)n)}}{dn}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x, algorithm="maxima")
```

output
```
-a^n*e^(-(d*x + c)*n)/(d*n)
```

3.608.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx=-\frac{e^{(-dnx-cn+n\log(a))}}{dn}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x, algorithm="giac")
```

output
```
-e^(-d*n*x - c*n + n*log(a))/(d*n)
```

3.608.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx=-\frac{\left(a\,\mathrm{e}^{-c-d\,x}\right)^n}{d\,n}$$

input
```
int((a*cosh(c + d*x) - a*sinh(c + d*x))^n,x)
```

output
```
-(a*exp(- c - d*x))^n/(d*n)
```

3.608. $\int (a\cosh(c+dx) - a\sinh(c+dx))^n \, dx$

3.609 $\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx$

3.609.1 Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=\frac{1}{d(a\cosh(c+dx)-a\sinh(c+dx))}$$

output
```
1/d/(a*cosh(d*x+c)-a*sinh(d*x+c))
```

3.609.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=\frac{1}{ad\cosh(c+dx)-ad\sinh(c+dx)}$$

input
```
Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-1),x]
```

output
```
(a*d*Cosh[c + d*x] - a*d*Sinh[c + d*x])^(-1)
```

3.609.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{ia\sin(ic+idx)+a\cos(ic+idx)}dx$$
$$\Big\downarrow \; 3550$$
$$\frac{1}{d(a\cosh(c+dx)-a\sinh(c+dx))}$$

input
```
Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-1),x]
```

output
```
1/(d*(a*Cosh[c + d*x] - a*Sinh[c + d*x]))
```

3.609.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.609.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{e^{dx+c}}{ad}$	14
derivativedivides	$-\frac{2}{da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	22
default	$-\frac{2}{da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	22
gosper	$\frac{1}{da(\cosh(dx+c)-\sinh(dx+c))}$	25

input
```
int(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/a/d*exp(d*x+c)
```

3.609.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=\frac{\cosh(dx+c)+\sinh(dx+c)}{ad}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(cosh(d*x + c) + sinh(d*x + c))/(a*d)
```

3.609.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=\begin{cases}\frac{1}{-ad\sinh(c+dx)+ad\cosh(c+dx)} & \text{for } d\neq 0\\ \frac{x}{-a\sinh(c)+a\cosh(c)} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x)
```

output
```
Piecewise((1/(-a*d*sinh(c + d*x) + a*d*cosh(c + d*x)), Ne(d, 0)), (x/(-a*s
inh(c) + a*cosh(c)), True))
```

3.609.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=\frac{e^{(dx+c)}}{ad}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^(d*x + c)/(a*d)
```

3.609.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=\frac{e^{(dx+c)}}{ad}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
e^(d*x + c)/(a*d)
```

3.609.9 Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=\frac{e^{c+d\,x}}{a\,d}$$

input
```
int(1/(a*cosh(c + d*x) - a*sinh(c + d*x)),x)
```

output
```
exp(c + d*x)/(a*d)
```

3.610 $\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx$

3.610.1 Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx=\frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

output
```
1/2/d/(a*cosh(d*x+c)-a*sinh(d*x+c))^2
```

3.610.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx=\frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

input
```
Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-2),x]
```

output
```
1/(2*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2)
```

3.610.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(ia\sin(ic+idx)+a\cos(ic+idx))^2}dx$$
$$\Big\downarrow \text{3550}$$
$$\frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

input
```
Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-2),x]
```

output
```
1/(2*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2)
```

3.610.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.610.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{e^{2dx+2c}}{2a^2d}$	18
gosper	$\frac{1}{2a^2(\cosh(dx+c)-\sinh(dx+c))^2d}$	26
derivativedivides	$\frac{\frac{2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}}{d\,a^2}$	36
default	$\frac{\frac{2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}}{d\,a^2}$	36

input
```
int(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/d*exp(2*d*x+2*c)
```

3.610.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx = \frac{\cosh(dx+c)+\sinh(dx+c)}{2\left(a^2d\cosh(dx+c)-a^2d\sinh(dx+c)\right)}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(d*x + c) + sinh(d*x + c))/(a^2*d*cosh(d*x + c) - a^2*d*sinh(d*x
+ c))
```

3.610.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(22) = 44.

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx = \begin{cases} \frac{1}{2a^2d\sinh^2(c+dx)-4a^2d\sinh(c+dx)\cosh(c+dx)+2a^2d\cosh^2(c+dx)} & \text{for } d\neq 0 \\ \frac{x}{(-a\sinh(c)+a\cosh(c))^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))**2,x)
```

output
```
Piecewise((1/(2*a**2*d*sinh(c + d*x)**2 - 4*a**2*d*sinh(c + d*x)*cosh(c +
d*x) + 2*a**2*d*cosh(c + d*x)**2), Ne(d, 0)), (x/(-a*sinh(c) + a*cosh(c))*
*2, True))
```

3.610.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx = \frac{e^{(2\,dx+2\,c)}}{2\,a^2d}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*e^(2*d*x + 2*c)/(a^2*d)
```

3.610.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx = \frac{e^{(2\,dx+2\,c)}}{2\,a^2d}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*e^(2*d*x + 2*c)/(a^2*d)
```

3.610.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx=\frac{e^{2c+2dx}}{2a^2d}$$

input
```
int(1/(a*cosh(c + d*x) - a*sinh(c + d*x))^2,x)
```

output
```
exp(2*c + 2*d*x)/(2*a^2*d)
```

3.611 $\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx$

3.611.1 Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx=\frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

output
```
1/3/d/(a*cosh(d*x+c)-a*sinh(d*x+c))^3
```

3.611.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx=\frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

input
```
Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-3),x]
```

output
```
1/(3*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3)
```

3.611.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(ia\sin(ic+idx)+a\cos(ic+idx))^3}dx$$

$$\Big\downarrow \text{3550}$$

$$\frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

input
```
Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-3),x]
```

output
```
1/(3*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3)
```

3.611.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.611.4 Maple [A] (verified)

Time = 19.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{e^{3dx+3c}}{3a^3d}$	18
gosper	$\frac{1}{3a^3(\cosh(dx+c)-\sinh(dx+c))^3 d}$	26
derivativedivides	$\frac{-\frac{2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{8}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{4}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}}{a^3d}$	55
default	$\frac{-\frac{2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{8}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{4}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}}{a^3d}$	55

input
```
int(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/a^3/d*exp(3*d*x+3*c)
```

3.611.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(25) = 50.

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx = \frac{\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}{3\left(a^3d\cosh(dx+c)-a^3d\sinh(dx+c)\right)}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)/(a
^3*d*cosh(d*x + c) - a^3*d*sinh(d*x + c))
```

3.611.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(22) = 44$.

Time = 0.62 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{1}{-3a^3d\sinh^3(c+dx)+9a^3d\sinh^2(c+dx)\cosh(c+dx)-9a^3d\sinh(c+dx)\cosh^2(c+dx)+3a^3d\cosh^3(c+dx)} & \text{for } d\neq 0 \\ \frac{x}{(-a\sinh(c)+a\cosh(c))^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))**3,x)
```

output
```
Piecewise((1/(-3*a**3*d*sinh(c + d*x)**3 + 9*a**3*d*sinh(c + d*x)**2*cosh(
c + d*x) - 9*a**3*d*sinh(c + d*x)*cosh(c + d*x)**2 + 3*a**3*d*cosh(c + d*x
)**3), Ne(d, 0)), (x/(-a*sinh(c) + a*cosh(c))**3, True))
```

3.611.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx = \frac{e^{(3\,dx+3\,c)}}{3\,a^3d}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*e^(3*d*x + 3*c)/(a^3*d)
```

3.611.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx = \frac{e^{(3\,dx+3\,c)}}{3\,a^3d}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
1/3*e^(3*d*x + 3*c)/(a^3*d)
```

3.611.9 Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx=\frac{\mathrm{e}^{3\,c+3\,d\,x}}{3\,a^3\,d}$$

input
```
int(1/(a*cosh(c + d*x) - a*sinh(c + d*x))^3,x)
```

output
```
exp(3*c + 3*d*x)/(3*a^3*d)
```

3.612 $\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx$

3.612.1 Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=-\frac{2\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}{d}$$

output
```
-2*(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2)/d
```

3.612.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=-\frac{2\sqrt{a(\cosh(c+dx)-\sinh(c+dx))}}{d}$$

input
```
Integrate[Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
```

output
```
(-2*Sqrt[a*(Cosh[c + d*x] - Sinh[c + d*x])])/d
```

3.612.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{ia\sin(ic+idx)+a\cos(ic+idx)}dx$$

$$\Big\downarrow \text{3550}$$

$$-\frac{2\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}{d}$$

input
```
Int[Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
```

output
```
(-2*Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]])/d
```

3.612.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.612.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{2\sqrt{\mathrm{e}^{-dx-c}a}}{d}$	19
gosper	$-\frac{2\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}{d}$	26
derivativedivides	$-\frac{2\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}{d}$	26
default	$-\frac{2\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}{d}$	26

input
```
int((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(exp(-d*x-c)*a)^(1/2)/d
```

3.612.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=-\frac{2\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}}{d}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/d
```

3.612.6 Sympy [F]

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=\int \sqrt{-a\sinh(c+dx)+a\cosh(c+dx)}\,dx$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

3.612.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = -\frac{2\sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(a)*e^(-1/2*d*x - 1/2*c)/d
```

3.612.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = -\frac{2\sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d}$$

input
```
integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(a)*e^(-1/2*d*x - 1/2*c)/d
```

3.612.9 Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = -\frac{2\sqrt{a\,\mathrm{e}^{-c-d\,x}}}{d}$$

input
```
int((a*cosh(c + d*x) - a*sinh(c + d*x))^(1/2),x)
```

output
```
-(2*(a*exp(- c - d*x))^(1/2))/d
```

3.613 $\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx$

3.613.1 Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx = \frac{2}{d\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}$$

output
```
2/d/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2)
```

3.613.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx = \frac{2}{d\sqrt{a(\cosh(c+dx)-\sinh(c+dx))}}$$

input
```
Integrate[1/Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
```

output
```
2/(d*Sqrt[a*(Cosh[c + d*x] - Sinh[c + d*x])])
```

3.613.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{ia\sin(ic+idx)+a\cos(ic+idx)}}dx$$

$$\Big\downarrow \text{3550}$$

$$\frac{2}{d\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}$$

input
```
Int[1/Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
```

output
```
2/(d*Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]])
```

3.613.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.613.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{2}{\sqrt{e^{-dx-c}a}\,d}$	19
gosper	$\frac{2}{d\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}$	26
derivativedivides	$\frac{2}{d\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}$	26
default	$\frac{2}{d\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}$	26

input
```
int(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(exp(-d*x-c)*a)^(1/2)/d
```

3.613.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx = \frac{2\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)}{ad}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d*x + c))/
(a*d)
```

3.613.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx=\int \frac{1}{\sqrt{-a\sinh{(c+dx)}+a\cosh{(c+dx)}}}\,dx$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(-a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

3.613.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx=\frac{2\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d)
```

3.613.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx=\frac{2\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

input
```
integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d)
```

3.613.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx=\frac{2\,\mathrm{e}^{c+d\,x}\,\sqrt{a\,\mathrm{e}^{-c-d\,x}}}{a\,d}$$

input
```
int(1/(a*cosh(c + d*x) - a*sinh(c + d*x))^(1/2),x)
```

output
```
(2*exp(c + d*x)*(a*exp(- c - d*x))^(1/2))/(a*d)
```

3.614 $\int (a\text{sech}(x) + b\tanh(x))^5 \, dx$

3.614.1 Optimal result

Integrand size = 11, antiderivative size = 124

$$\begin{aligned}\int (a\text{sech}(x) + b\tanh(x))^5 \, dx = {} & \frac{1}{8}a(3a^4 + 10a^2b^2 + 15b^4)\arctan(\sinh(x)) \\ & + b^5\log(\cosh(x)) - \frac{1}{8}ab^2(3a^2 + 7b^2)\sinh(x) \\ & - \frac{1}{4}\text{sech}^4(x)(b - a\sinh(x))(a + b\sinh(x))^4 - \frac{1}{8}\text{sech}^2(x)(a \\ & + b\sinh(x))^2\left(2b(a^2 + 2b^2) - a(3a^2 + 5b^2)\sinh(x)\right)\end{aligned}$$

output
```
1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*arctan(sinh(x))+b^5*ln(cosh(x))-1/8*a*b^2*
(3*a^2+7*b^2)*sinh(x)-1/4*sech(x)^4*(b-a*sinh(x))*(a+b*sinh(x))^4-1/8*sech
(x)^2*(a+b*sinh(x))^2*(2*b*(a^2+2*b^2)-a*(3*a^2+5*b^2)*sinh(x))
```

3.614.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 355 vs. 2(124) = 248.

Time = 1.30 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.86

$$\int (a\text{sech}(x) + b\tanh(x))^5 \, dx$$

$$= \frac{4\text{sech}^4(x)(b + a\sinh(x))(a + b\sinh(x))^6 + \frac{2\text{sech}^2(x)(a+b\sinh(x))^6(6a^2b-2b^3+a(3a^2-5b^2)\sinh(x))}{a^2+b^2} + \frac{b\left(\frac{(a^2+b^2)^2((3a^5+\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(a*Sech[x] + b*Tanh[x])^5,x]
```

output
```
(4*Sech[x]^4*(b + a*Sinh[x])*(a + b*Sinh[x])^6 + (2*Sech[x]^2*(a + b*Sinh[
x])^6*(6*a^2*b - 2*b^3 + a*(3*a^2 - 5*b^2)*Sinh[x]))/(a^2 + b^2) + (b*(((a
^2 + b^2)^2*((3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 8*b^4*Sqrt[-b^2])*Log[Sqrt[-
b^2] - b*Sinh[x]] + (-3*a^5 - 10*a^3*b^2 - 15*a*b^4 + 8*(-b^2)^(5/2))*Log[
Sqrt[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] - 10*a*b*(9*a^6 + 6*a^4*b^2 + 8*a^2*b
^4 + 3*b^6)*Sinh[x] - 8*b^2*(15*a^6 - 4*a^4*b^2 + 2*a^2*b^4 + b^6)*Sinh[x]
^2 + 10*a*b^3*(-9*a^4 + 8*a^2*b^2 + b^4)*Sinh[x]^3 + 4*b^4*(-9*a^4 + 12*a^
2*b^2 + b^4)*Sinh[x]^4 + 2*a*b^5*(-3*a^2 + 5*b^2)*Sinh[x]^5))/(a^2 + b^2))
/(16*(a^2 + b^2))
```

3.614.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.48, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 4891, 3042, 3147, 25, 495, 684, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\text{sech}(x)+b\tanh(x))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sec(ix)-ib\tan(ix))^5dx$$

$$\Big\downarrow \text{4891}$$

$$\int \text{sech}^5(x)(a+b\sinh(x))^5dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a-ib\sin(ix))^5}{\cos(ix)^5}dx$$

$$\Big\downarrow \text{3147}$$

$$-b^5\int -\frac{(a+b\sinh(x))^5}{\left(\sinh^2(x)b^2+b^2\right)^3}d(b\sinh(x))$$

$$\Big\downarrow \text{25}$$

$$b^5 \int \frac{(a+b\sinh(x))^5}{\left(\sinh^2(x)b^2+b^2\right)^3}d(b\sinh(x))$$

$\Big\downarrow$ 495

$$-b^5\left(\frac{(a+b\sinh(x))^4\left(b^2-ab\sinh(x)\right)}{4b^2\left(b^2\sinh^2(x)+b^2\right)^2}-\frac{\int \frac{(a+b\sinh(x))^3(3a^2-b\sinh(x)a+4b^2)}{\left(\sinh^2(x)b^2+b^2\right)^2}d(b\sinh(x))}{4b^2}\right)$$

$\Big\downarrow$ 684

$$-b^5\left(\frac{(a+b\sinh(x))^4\left(b^2-ab\sinh(x)\right)}{4b^2\left(b^2\sinh^2(x)+b^2\right)^2}-\frac{\frac{\int \frac{(a+b\sinh(x))\left(3a^4+7b^2a^2-b\left(3a^2+7b^2\right)\sinh(x)a+8b^4\right)}{\sinh^2(x)b^2+b^2}d(b\sinh(x))}{2b^2}-\frac{(a+b\sinh(x))^2(2b^2(a^2}{2b^2(b^2}}{4b^2}\right.$$

$\Big\downarrow$ 657

$$-b^5\left(\frac{(a+b\sinh(x))^4\left(b^2-ab\sinh(x)\right)}{4b^2\left(b^2\sinh^2(x)+b^2\right)^2}-\frac{\frac{\int\left(\frac{3a^5+10b^2a^3+15b^4a+8b^5\sinh(x)}{\sinh^2(x)b^2+b^2}-a\left(3a^2+7b^2\right)\right)d(b\sinh(x))}{2b^2}-\frac{(a+b\sinh(x))^2(2b^2(a^2+2}{2b^2(b^2\sin}}{4b^2}\right.$$

$\Big\downarrow$ 2009

$$-b^5\left(\frac{(a+b\sinh(x))^4\left(b^2-ab\sinh(x)\right)}{4b^2\left(b^2\sinh^2(x)+b^2\right)^2}-\frac{\frac{-ab(3a^2+7b^2)\sinh(x)+\frac{a\left(3a^4+10a^2b^2+15b^4\right)\arctan(\sinh(x))}{b}+4b^4\log\left(b^2\sinh^2(x)+b^2\right)}{2b^2}-(}{4b^2}\right.$$

input
```
Int[(a*Sech[x] + b*Tanh[x])^5,x]
```

output
```
-(b^5*(((a + b*Sinh[x])^4*(b^2 - a*b*Sinh[x]))/(4*b^2*(b^2 + b^2*Sinh[x]^2
)^2) - (((a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Sinh[x]])/b + 4*b^4*Log[b
^2 + b^2*Sinh[x]^2] - a*b*(3*a^2 + 7*b^2)*Sinh[x])/(2*b^2) - ((a + b*Sinh[
x])^2*(2*b^2*(a^2 + 2*b^2) - a*b*(3*a^2 + 5*b^2)*Sinh[x]))/(2*b^2*(b^2 + b
^2*Sinh[x]^2)))/(4*b^2))
```

3.614. $\int (a\,\text{sech}(x)+b\tanh(x))^5\,dx$

3.614.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.614. $\int (a\text{sech}(x) + b\tanh(x))^5 \, dx$

3.614.4 Maple [A] (verified)

Time = 198.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.29

method	result
parts	$a^5\left(\left(\frac{\operatorname{sech}(x)^3}{4}+\frac{3\operatorname{sech}(x)}{8}\right)\tanh(x)+\frac{3\arctan(e^x)}{4}\right)+b^5\left(-\frac{\tanh(x)^4}{4}-\frac{\tanh(x)^2}{2}-\frac{\ln(\tanh(x)-1)}{2}-\frac{\ln(1+\tanh}{2}\right.$
default	$a^5\left(\left(\frac{\operatorname{sech}(x)^3}{4}+\frac{3\operatorname{sech}(x)}{8}\right)\tanh(x)+\frac{3\arctan(e^x)}{4}\right)-\frac{5a^4b}{4\cosh(x)^4}+10a^3b^2\left(-\frac{\sinh(x)}{3\cosh(x)^4}+\frac{\left(\frac{\operatorname{sech}(x)^3}{4}+\frac{3\operatorname{sech}(x)}{8}\right.}{3}\right.$
risch	$-b^5x+\frac{e^x(3a^5e^{6x}+10a^3b^2e^{6x}-25a\,b^4e^{6x}-80a^2b^3e^{5x}+16b^5e^{5x}+11a^5e^{4x}-70a^3b^2e^{4x}+15a\,b^4e^{4x}-80\,e^{3x}a^4b+16b^5e^{3x}-11a^5e^{2x}}{4(1+e^{2x})^4}$

input
```
int((a*sech(x)+b*tanh(x))^5,x,method=_RETURNVERBOSE)
```

output
```
a^5*((1/4*sech(x)^3+3/8*sech(x))*tanh(x)+3/4*arctan(exp(x)))+b^5*(-1/4*tan
h(x)^4-1/2*tanh(x)^2-1/2*ln(tanh(x)-1)-1/2*ln(1+tanh(x)))-5/4*a^4*b*sech(x
)^4+10*a^3*b^2*(-1/3*sinh(x)/cosh(x)^4+1/3*(1/4*sech(x)^3+3/8*sech(x))*tan
h(x)+1/4*arctan(exp(x)))+5/2*a^2*b^3*tanh(x)^4+5*a*b^4*(-sinh(x)^3/cosh(x)
^4-sinh(x)/cosh(x)^4+(1/4*sech(x)^3+3/8*sech(x))*tanh(x)+3/4*arctan(exp(x)
))
```

3.614.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2040 vs. 2(116) = 232.

Time = 0.27 (sec) , antiderivative size = 2040, normalized size of antiderivative = 16.45

$$\int (a\operatorname{sech}(x)+b\tanh(x))^5\,dx = \text{Too large to display}$$

input
```
integrate((a*sech(x)+b*tanh(x))^5,x, algorithm="fricas")
```

output
```
-1/4*(4*b^5*x*cosh(x)^8 + 4*b^5*x*sinh(x)^8 - (3*a^5 + 10*a^3*b^2 - 25*a*b
^4)*cosh(x)^7 + (32*b^5*x*cosh(x) - 3*a^5 - 10*a^3*b^2 + 25*a*b^4)*sinh(x)
^7 + 16*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^6 + (112*b^5*x*cosh(x)^2 + 16*b^
5*x + 80*a^2*b^3 - 16*b^5 - 7*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x))*sin
h(x)^6 + 4*b^5*x - (11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + (224*b^5*x
*cosh(x)^3 - 11*a^5 + 70*a^3*b^2 - 15*a*b^4 - 21*(3*a^5 + 10*a^3*b^2 - 25*
a*b^4)*cosh(x)^2 + 96*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 + 8*(3*
b^5*x + 10*a^4*b - 2*b^5)*cosh(x)^4 + (280*b^5*x*cosh(x)^4 + 24*b^5*x + 80
*a^4*b - 16*b^5 - 35*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^3 + 240*(b^5*
x + 5*a^2*b^3 - b^5)*cosh(x)^2 - 5*(11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x
))*sinh(x)^4 + (11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (224*b^5*x*cos
h(x)^5 + 11*a^5 - 70*a^3*b^2 + 15*a*b^4 - 35*(3*a^5 + 10*a^3*b^2 - 25*a*b^
4)*cosh(x)^4 + 320*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^3 - 10*(11*a^5 - 70*a
^3*b^2 + 15*a*b^4)*cosh(x)^2 + 32*(3*b^5*x + 10*a^4*b - 2*b^5)*cosh(x))*si
nh(x)^3 + 16*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^2 + (112*b^5*x*cosh(x)^6 +
16*b^5*x - 21*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^5 + 80*a^2*b^3 - 16*
b^5 + 240*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^4 - 10*(11*a^5 - 70*a^3*b^2 +
15*a*b^4)*cosh(x)^3 + 48*(3*b^5*x + 10*a^4*b - 2*b^5)*cosh(x)^2 + 3*(11*a^
5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^2 - ((3*a^5 + 10*a^3*b^2 + 15*
a*b^4)*cosh(x)^8 + 8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^7 ...
```

3.614.6 Sympy [F]

$$\int (a\text{sech}(x) + b\tanh(x))^5 \, dx = \int \left(a\,\text{sech}\,(x) + b\tanh\,(x)\right)^5 \, dx$$

input
```
integrate((a*sech(x)+b*tanh(x))**5,x)
```

output
```
Integral((a*sech(x) + b*tanh(x))**5, x)
```

3.614.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(116) = 232.

Time = 0.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.25

$$
\begin{aligned}
&\int (a\text{sech}(x) + b\tanh(x))^5 \, dx \\
&= \frac{5}{2}\,a^2b^3\tanh{(x)}^4 + b^5\left(x + \frac{4\,\left(e^{(-2\,x)} + e^{(-4\,x)} + e^{(-6\,x)}\right)}{4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1} + \log\left(e^{(-2\,x)} + 1\right)\right) \\
&\quad - \frac{5}{4}\,ab^4\left(\frac{5\,e^{(-x)} - 3\,e^{(-3\,x)} + 3\,e^{(-5\,x)} - 5\,e^{(-7\,x)}}{4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1} + 3\,\arctan\left(e^{(-x)}\right)\right) \\
&\quad + \frac{1}{4}\,a^5\left(\frac{3\,e^{(-x)} + 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} - 3\,e^{(-7\,x)}}{4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1} - 3\,\arctan\left(e^{(-x)}\right)\right) \\
&\quad + \frac{5}{2}\,a^3b^2\left(\frac{e^{(-x)} - 7\,e^{(-3\,x)} + 7\,e^{(-5\,x)} - e^{(-7\,x)}}{4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1} - \arctan\left(e^{(-x)}\right)\right) - \frac{20\,a^4b}{\left(e^{(-x)} + e^{x}\right)^4}
\end{aligned}
$$

input
```
integrate((a*sech(x)+b*tanh(x))^5,x, algorithm="maxima")
```

output
```
5/2*a^2*b^3*tanh(x)^4 + b^5*(x + 4*(e^(-2*x) + e^(-4*x) + e^(-6*x))/(4*e^(
-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + log(e^(-2*x) + 1)) - 5/4
*a*b^4*((5*e^(-x) - 3*e^(-3*x) + 3*e^(-5*x) - 5*e^(-7*x))/(4*e^(-2*x) + 6*
e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 3*arctan(e^(-x))) + 1/4*a^5*((3*e^
(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) +
4*e^(-6*x) + e^(-8*x) + 1) - 3*arctan(e^(-x))) + 5/2*a^3*b^2*((e^(-x) - 7*
e^(-3*x) + 7*e^(-5*x) - e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) +
e^(-8*x) + 1) - arctan(e^(-x))) - 20*a^4*b/(e^(-x) + e^x)^4
```

3.614.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(116) = 232.

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.94

$$
\begin{aligned}
&\int (a\text{sech}(x) + b\tanh(x))^5 \, dx = \frac{1}{2}\,b^5\log\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right) \\
&\quad + \frac{1}{16}\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(3\,a^5 + 10\,a^3b^2 + 15\,ab^4\right) \\
&\quad - \frac{3\,b^5\left(e^{(-x)} - e^{x}\right)^4 + 3\,a^5\left(e^{(-x)} - e^{x}\right)^3 + 10\,a^3b^2\left(e^{(-x)} - e^{x}\right)^3 - 25\,ab^4\left(e^{(-x)} - e^{x}\right)^3 + 80\,a^2b^3\left(e^{(-x)} - e^{x}\right.}{4\left(\left(e^{(-x)} - \right.\right.}
\end{aligned}
$$

input
```
integrate((a*sech(x)+b*tanh(x))^5,x, algorithm="giac")
```

output
```
1/2*b^5*log((e^(-x) - e^x)^2 + 4) + 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*
e^(-x)))*(3*a^5 + 10*a^3*b^2 + 15*a*b^4) - 1/4*(3*b^5*(e^(-x) - e^x)^4 + 3
*a^5*(e^(-x) - e^x)^3 + 10*a^3*b^2*(e^(-x) - e^x)^3 - 25*a*b^4*(e^(-x) - e
^x)^3 + 80*a^2*b^3*(e^(-x) - e^x)^2 + 8*b^5*(e^(-x) - e^x)^2 + 20*a^5*(e^(
-x) - e^x) - 40*a^3*b^2*(e^(-x) - e^x) - 60*a*b^4*(e^(-x) - e^x) + 80*a^4*
b + 160*a^2*b^3)/((e^(-x) - e^x)^2 + 4)^2
```

3.614.9 Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.99

$$\int (a\mathrm{sech}(x)+b\tanh(x))^5\,dx = \frac{\mathrm{e}^x\,\left(4\,a^5-40\,a^3\,b^2+20\,a\,b^4\right)-20\,a^4\,b-4\,b^5+40\,a^2\,b^3}{4\,\mathrm{e}^{2\,x}+6\,\mathrm{e}^{4\,x}+4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1} + b^5\,\ln\left(\left(\left(\frac{3\,a^5\,\mathrm{e}^x}{4}-2\,\sqrt{-\frac{9\,a^{10}}{64}-\frac{15\,a^8\,b^2}{16}-\frac{95\,a^6\,b^4}{32}-\frac{75\,a^4\,b^6}{16}-\frac{225\,a^2\,b^8}{64}}+\frac{15\,a\,b^4\,\mathrm{e}^x}{4}+\frac{5\,a^3\,b^2\,\mathrm{e}^x}{2}\right)\left(2\,\sqrt{-\frac{9\,a^{10}}{64}-\frac{15\,a^8\,b^2}{16}-\frac{95\,a^6\,b^4}{32}-\frac{75\,a^4\,b^6}{16}-\frac{225\,a^2\,b^8}{64}}+\frac{3\,a^5\,\mathrm{e}^x}{4}+\frac{15\,a\,b^4\,\mathrm{e}^x}{4}+\frac{5\,a^3\,b^2\,\mathrm{e}^x}{2}\right)\right)\right) - \frac{\mathrm{e}^x\,\left(6\,a^5-60\,a^3\,b^2+30\,a\,b^4\right)-40\,a^4\,b-8\,b^5+80\,a^2\,b^3}{3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1}-b^5\,x + \frac{\mathrm{atan}\left(\frac{4\,\mathrm{e}^x\,\left(\frac{3\,a^5}{4}+\frac{5\,a^3\,b^2}{2}+\frac{15\,a\,b^4}{4}\right)}{\sqrt{9\,a^{10}+60\,a^8\,b^2+190\,a^6\,b^4+300\,a^4\,b^6+225\,a^2\,b^8}}\right)\,\sqrt{9\,a^{10}+60\,a^8\,b^2+190\,a^6\,b^4+300\,a^4\,b^6+225\,a^2\,b^8}}{4} + \frac{\mathrm{e}^x\,\left(\frac{3\,a^5}{4}+\frac{5\,a^3\,b^2}{2}-\frac{25\,a\,b^4}{4}\right)+4\,b^5-20\,a^2\,b^3}{\mathrm{e}^{2\,x}+1} + \frac{\mathrm{e}^x\,\left(\frac{a^5}{2}-25\,a^3\,b^2+\frac{45\,a\,b^4}{2}\right)-20\,a^4\,b-8\,b^5+60\,a^2\,b^3}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}$$

input
```
int((b*tanh(x) + a/cosh(x))^5,x)
```

output
```
(exp(x)*(20*a*b^4 + 4*a^5 - 40*a^3*b^2) - 20*a^4*b - 4*b^5 + 40*a^2*b^3)/(
4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1) + b^5*log(((3*a^5*exp
(x))/4 - 2*(- (9*a^10)/64 - (225*a^2*b^8)/64 - (75*a^4*b^6)/16 - (95*a^6*b
^4)/32 - (15*a^8*b^2)/16)^(1/2) + (15*a*b^4*exp(x))/4 + (5*a^3*b^2*exp(x))
/2)*(2*(- (9*a^10)/64 - (225*a^2*b^8)/64 - (75*a^4*b^6)/16 - (95*a^6*b^4)/
32 - (15*a^8*b^2)/16)^(1/2) + (3*a^5*exp(x))/4 + (15*a*b^4*exp(x))/4 + (5*
a^3*b^2*exp(x))/2)) - (exp(x)*(30*a*b^4 + 6*a^5 - 60*a^3*b^2) - 40*a^4*b -
 8*b^5 + 80*a^2*b^3)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - b^5*x + (a
tan((4*exp(x)*((15*a*b^4)/4 + (3*a^5)/4 + (5*a^3*b^2)/2))/(9*a^10 + 225*a^
2*b^8 + 300*a^4*b^6 + 190*a^6*b^4 + 60*a^8*b^2)^(1/2))*(9*a^10 + 225*a^2*b
^8 + 300*a^4*b^6 + 190*a^6*b^4 + 60*a^8*b^2)^(1/2))/4 + (exp(x)*((3*a^5)/4
 - (25*a*b^4)/4 + (5*a^3*b^2)/2) + 4*b^5 - 20*a^2*b^3)/(exp(2*x) + 1) + (e
xp(x)*((45*a*b^4)/2 + a^5/2 - 25*a^3*b^2) - 20*a^4*b - 8*b^5 + 60*a^2*b^3)
/(2*exp(2*x) + exp(4*x) + 1)
```

3.614. $\int (a\,\mathrm{sech}(x) + b\tanh(x))^5\,dx$

3.615 $\int (a\text{sech}(x) + b\tanh(x))^4 \, dx$

3.615.1 Optimal result

Integrand size = 11, antiderivative size = 100

$$\int (a\text{sech}(x)+b\tanh(x))^4\,dx = b^4x-\frac{4}{3}ab\left(a^2+2b^2\right)\cosh(x)-\frac{1}{3}b^2\left(2a^2+3b^2\right)\cosh(x)\sinh(x) -\frac{1}{3}\text{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3 +\frac{1}{3}\text{sech}(x)(a+b\sinh(x))^2\left(ab+\left(2a^2+3b^2\right)\sinh(x)\right)$$

output
```
b^4*x-4/3*a*b*(a^2+2*b^2)*cosh(x)-1/3*b^2*(2*a^2+3*b^2)*cosh(x)*sinh(x)-1/
3*sech(x)^3*(b-a*sinh(x))*(a+b*sinh(x))^3+1/3*sech(x)*(a+b*sinh(x))^2*(a*b
+(2*a^2+3*b^2)*sinh(x))
```

3.615.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int (a\text{sech}(x)+b\tanh(x))^4\,dx = \frac{1}{3}\left(3b^4x-12ab^3\text{sech}(x)-4ab\left(a^2-b^2\right)\text{sech}^3(x) +2\left(a^4+3a^2b^2-2b^4\right)\tanh(x) +\left(a^4-6a^2b^2+b^4\right)\text{sech}^2(x)\tanh(x)\right)$$

input
```
Integrate[(a*Sech[x] + b*Tanh[x])^4,x]
```

output
```
(3*b^4*x - 12*a*b^3*Sech[x] - 4*a*b*(a^2 - b^2)*Sech[x]^3 + 2*(a^4 + 3*a^2
*b^2 - 2*b^4)*Tanh[x] + (a^4 - 6*a^2*b^2 + b^4)*Sech[x]^2*Tanh[x])/3
```

3.615.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {3042, 4891, 3042, 3170, 25, 3042, 3340, 27, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\text{sech}(x)+b\tanh(x))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sec(ix)-ib\tan(ix))^4dx$$

$$\Big\downarrow \text{4891}$$

$$\int \text{sech}^4(x)(a+b\sinh(x))^4dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a-ib\sin(ix))^4}{\cos(ix)^4}dx$$

$$\Big\downarrow \text{3170}$$

$$-\frac{1}{3}\int -\text{sech}^2(x)(a+b\sinh(x))^2\left(2a^2-b\sinh(x)a+3b^2\right)dx-\frac{1}{3}\text{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \text{sech}^2(x)(a+b\sinh(x))^2\left(2a^2-b\sinh(x)a+3b^2\right)dx-\frac{1}{3}\text{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\text{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3+\frac{1}{3}\int \frac{(a-ib\sin(ix))^2\left(2a^2+ib\sin(ix)a+3b^2\right)}{\cos(ix)^2}dx$$

$$\Big\downarrow \text{3340}$$

$$\frac{1}{3}\left(\text{sech}(x)(a+b\sinh(x))^2\left(\left(2a^2+3b^2\right)\sinh(x)+ab\right)-\int 2(a+b\sinh(x))\left(ab^2+\left(2a^2+3b^2\right)\sinh(x)b\right)dx\right)-\frac{1}{3}\text{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\text{sech}(x)(a+b\sinh(x))^2\left(\left(2a^2+3b^2\right)\sinh(x)+ab\right)-2\int (a+b\sinh(x))\left(ab^2+\left(2a^2+3b^2\right)\sinh(x)b\right)dx\right)-\frac{1}{3}\text{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{3}\text{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3+\frac{1}{3}\left(\text{sech}(x)(a+b\sinh(x))^2\left(\left(2a^2+3b^2\right)\sinh(x)+ab\right)-2\int (a-ib\sin(ix))\left(ab^2-ib\left(2a^2+3b^2\right)\sin(ix)\right)dx\right)$$

$$\Big\downarrow \; 3213$$

$$\frac{1}{3}\left(\text{sech}(x)(a+b\sinh(x))^2\left(\left(2a^2+3b^2\right)\sinh(x)+ab\right)-2\left(2ab\left(a^2+2b^2\right)\cosh(x)+\frac{1}{2}b^2\left(2a^2+3b^2\right)\sinh(x)\cosh(\right.\right.$$
$$\frac{1}{3}\text{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3$$

input
```
Int[(a*Sech[x] + b*Tanh[x])^4,x]
```

output
```
-1/3*(Sech[x]^3*(b - a*Sinh[x])*(a + b*Sinh[x])^3) + (Sech[x]*(a + b*Sinh[
x])^2*(a*b + (2*a^2 + 3*b^2)*Sinh[x]) - 2*((-3*b^4*x)/2 + 2*a*b*(a^2 + 2*b
^2)*Cosh[x] + (b^2*(2*a^2 + 3*b^2)*Cosh[x]*Sinh[x])/2))/3
```

3.615.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3340
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Simp[1/(g^2*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Si
n[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0]
&& SimplerQ[c + d*x, a + b*x])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.615.4 Maple [A] (verified)

Time = 52.87 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

method	result
parts	$a^4\left(\frac{2}{3}+\frac{\operatorname{sech}(x)^2}{3}\right)\tanh(x)+b^4\left(-\frac{\tanh(x)^3}{3}-\tanh(x)-\frac{\ln(\tanh(x)-1)}{2}+\frac{\ln(1+\tanh(x))}{2}\right)-\frac{4a^3b\operatorname{sech}(x)^3}{3}+$
default	$a^4\left(\frac{2}{3}+\frac{\operatorname{sech}(x)^2}{3}\right)\tanh(x)-\frac{4a^3b}{3\cosh(x)^3}+6a^2b^2\left(-\frac{\sinh(x)}{2\cosh(x)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(x)^2}{3}\right)\tanh(x)}{2}\right)+4a\,b^3\left(-\frac{\sinh(x)^2}{\cosh(x)^3}\right.$
risch	$b^4x-\frac{4(6a\,b^3e^{5x}+9\,e^{4x}a^2b^2-3\,e^{4x}b^4+8a^3b\,e^{3x}+4a\,b^3e^{3x}+3\,e^{2x}a^4-3\,e^{2x}b^4+6a\,b^3e^x+a^4+3a^2b^2-2b^4)}{3(1+e^{2x})^3}$

input
```
int((a*sech(x)+b*tanh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*(2/3+1/3*sech(x)^2)*tanh(x)+b^4*(-1/3*tanh(x)^3-tanh(x)-1/2*ln(tanh(x)
-1)+1/2*ln(1+tanh(x)))-4/3*a^3*b*sech(x)^3+2*a^2*b^2*tanh(x)^3+4*a*b^3*(1/
3*sech(x)^3-sech(x))
```

3.615.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(93) = 186$.

Time = 0.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.07

$$\int (a\text{sech}(x) + b\tanh(x))^4 \, dx = -\frac{24\, ab^3 \cosh{(x)}^2 + 16\, a^3 b + 8\, ab^3 - (3\, b^4 x - 2\, a^4 - 6\, a^2 b^2 + 4\, b^4) \cosh{(x)}^3 - 2\, (a^4 + 3\, a^2 b^2 - 2\, b^4) \sinh{(}$$

input
```
integrate((a*sech(x)+b*tanh(x))^4,x, algorithm="fricas")
```

output
```
-1/3*(24*a*b^3*cosh(x)^2 + 16*a^3*b + 8*a*b^3 - (3*b^4*x - 2*a^4 - 6*a^2*b
^2 + 4*b^4)*cosh(x)^3 - 2*(a^4 + 3*a^2*b^2 - 2*b^4)*sinh(x)^3 + 3*(8*a*b^3
 - (3*b^4*x - 2*a^4 - 6*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^2 - 3*(3*b^4*x -
 2*a^4 - 6*a^2*b^2 + 4*b^4)*cosh(x) - 6*(a^4 - 3*a^2*b^2 + (a^4 + 3*a^2*b^
2 - 2*b^4)*cosh(x)^2)*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 3*cosh(x
))
```

3.615.6 Sympy [F]

$$\int (a\text{sech}(x) + b\tanh(x))^4 \, dx = \int (a \operatorname{sech}{(x)} + b \tanh{(x)})^4 \, dx$$

input
```
integrate((a*sech(x)+b*tanh(x))**4,x)
```

output
```
Integral((a*sech(x) + b*tanh(x))**4, x)
```

3.615.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(93) = 186$.

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.10

$$\int (a\text{sech}(x) + b\tanh(x))^4\,dx$$
$$= 2\,a^2b^2\tanh(x)^3 + \frac{1}{3}\,b^4\left(3\,x - \frac{4\left(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + 2\right)}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1}\right)$$
$$-\frac{8}{3}\,ab^3\left(\frac{3\,e^{(-x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{2\,e^{(-3\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{3\,e^{(-5\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)}}\right.$$
$$+\frac{4}{3}\,a^4\left(\frac{3\,e^{(-2\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{1}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1}\right)$$
$$-\frac{32\,a^3b}{3\left(e^{(-x)} + e^x\right)^3}$$

input
```
integrate((a*sech(x)+b*tanh(x))^4,x, algorithm="maxima")
```

output
```
2*a^2*b^2*tanh(x)^3 + 1/3*b^4*(3*x - 4*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^
(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1)) - 8/3*a*b^3*(3*e^(-x)/(3*e^(-2*x) + 3
*e^(-4*x) + e^(-6*x) + 1) + 2*e^(-3*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x)
 + 1) + 3*e^(-5*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1)) + 4/3*a^4*(3*
e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 1/(3*e^(-2*x) + 3*e^(-
4*x) + e^(-6*x) + 1)) - 32/3*a^3*b/(e^(-x) + e^x)^3
```

3.615.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int (a\text{sech}(x) + b\tanh(x))^4\,dx = b^4x$$
$$-\frac{4\left(6\,ab^3e^{(5\,x)} + 9\,a^2b^2e^{(4\,x)} - 3\,b^4e^{(4\,x)} + 8\,a^3be^{(3\,x)} + 4\,ab^3e^{(3\,x)} + 3\,a^4e^{(2\,x)} - 3\,b^4e^{(2\,x)} + 6\,ab^3e^x + a^4 + 3\right.}{3\left(e^{(2\,x)} + 1\right)^3}$$

input
```
integrate((a*sech(x)+b*tanh(x))^4,x, algorithm="giac")
```

output
```
b^4*x - 4/3*(6*a*b^3*e^(5*x) + 9*a^2*b^2*e^(4*x) - 3*b^4*e^(4*x) + 8*a^3*b
*e^(3*x) + 4*a*b^3*e^(3*x) + 3*a^4*e^(2*x) - 3*b^4*e^(2*x) + 6*a*b^3*e^x +
 a^4 + 3*a^2*b^2 - 2*b^4)/(e^(2*x) + 1)^3
```

3.615.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.45

$$\int (a\mathrm{sech}(x)+b\tanh(x))^4\,dx=\frac{e^x\left(\frac{32\,a\,b^3}{3}-\frac{32\,a^3\,b}{3}\right)-4\,a^4-4\,b^4+24\,a^2\,b^2}{2\,e^{2\,x}+e^{4\,x}+1}-\frac{12\,a^2\,b^2+8\,e^x\,a\,b^3-4\,b^4}{e^{2\,x}+1}+b^4\,x-\frac{e^x\left(\frac{32\,a\,b^3}{3}-\frac{32\,a^3\,b}{3}\right)-\frac{8\,a^4}{3}-\frac{8\,b^4}{3}+16\,a^2\,b^2}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}$$

input
```
int((b*tanh(x) + a/cosh(x))^4,x)
```

output
```
(exp(x)*((32*a*b^3)/3 - (32*a^3*b)/3) - 4*a^4 - 4*b^4 + 24*a^2*b^2)/(2*exp
(2*x) + exp(4*x) + 1) - (12*a^2*b^2 - 4*b^4 + 8*a*b^3*exp(x))/(exp(2*x) +
1) + b^4*x - (exp(x)*((32*a*b^3)/3 - (32*a^3*b)/3) - (8*a^4)/3 - (8*b^4)/3
 + 16*a^2*b^2)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)
```

3.616 $\int (a\text{sech}(x) + b\tanh(x))^3\,dx$

3.616.1 Optimal result

Integrand size = 11, antiderivative size = 58

$$\int (a\text{sech}(x) + b\tanh(x))^3\,dx = \frac{1}{2}a(a^2+3b^2)\arctan(\sinh(x)) + b^3\log(\cosh(x)) - \frac{1}{2}ab^2\sinh(x) - \frac{1}{2}\text{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2$$

output
```
1/2*a*(a^2+3*b^2)*arctan(sinh(x))+b^3*ln(cosh(x))-1/2*a*b^2*sinh(x)-1/2*se
ch(x)^2*(b-a*sinh(x))*(a+b*sinh(x))^2
```

3.616.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 194 vs. 2(58) = 116.

Time = 1.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.34

$$\int (a\text{sech}(x) + b\tanh(x))^3\,dx = \frac{1}{4}\left(\frac{b\left(\left(a^3+3ab^2-2(-b^2)^{3/2}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-\left(a^3+3ab^2+2(-b^2)^{3/2}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)\right)}{\sqrt{-b^2}} + \frac{2a^4b\text{sech}^2(x)}{a^2+b^2} + \frac{a(2a^4-4a^2b^2-7b^4+b^4\cosh(2x))\,\text{sech}(x)\tanh(x)}{a^2+b^2} - \frac{2b(-4a^4-2a^2b^2+b^4+ab^3\sinh(x))\tanh^2(x)}{a^2+b^2}\right)$$

input
```
Integrate[(a*Sech[x] + b*Tanh[x])^3,x]
```

output
```
((b*((a^3 + 3*a*b^2 - 2*(-b^2)^(3/2))*Log[Sqrt[-b^2] - b*Sinh[x]] - (a^3 +
 3*a*b^2 + 2*(-b^2)^(3/2))*Log[Sqrt[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] + (2*a
^4*b*Sech[x]^2)/(a^2 + b^2) + (a*(2*a^4 - 4*a^2*b^2 - 7*b^4 + b^4*Cosh[2*x
])*Sech[x]*Tanh[x])/(a^2 + b^2) - (2*b*(-4*a^4 - 2*a^2*b^2 + b^4 + a*b^3*S
inh[x])*Tanh[x]^2)/(a^2 + b^2))/4
```

3.616.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.59, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 4891, 3042, 3147, 495, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\text{sech}(x) + b\tanh(x))^3 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sec(ix) - ib\tan(ix))^3 dx$$

$$\Big\downarrow \text{4891}$$

$$\int \text{sech}^3(x)(a + b\sinh(x))^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a - ib\sin(ix))^3}{\cos(ix)^3} dx$$

$$\Big\downarrow \text{3147}$$

$$b^3 \int \frac{(a + b\sinh(x))^3}{\left(\sinh^2(x)b^2 + b^2\right)^2} d(b\sinh(x))$$

$$\Big\downarrow \text{495}$$

$$b^3 \left(\frac{\int \frac{(a+b\sinh(x))\left(a^2 - b\sinh(x)a + 2b^2\right)}{\sinh^2(x)b^2 + b^2} d(b\sinh(x))}{2b^2} - \frac{(a + b\sinh(x))^2 \left(b^2 - ab\sinh(x)\right)}{2b^2 \left(b^2\sinh^2(x) + b^2\right)} \right)$$

$$\Big\downarrow \text{657}$$

$$b^3\left(\frac{\int\left(\frac{a^3+3b^2a+2b^3\sinh(x)}{\sinh^2(x)b^2+b^2}-a\right)d(b\sinh(x))}{2b^2}-\frac{(a+b\sinh(x))^2\left(b^2-ab\sinh(x)\right)}{2b^2\left(b^2\sinh^2(x)+b^2\right)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$b^3\left(\frac{\frac{a(a^2+3b^2)\arctan(\sinh(x))}{b}-ab\sinh(x)+b^2\log\left(b^2\sinh^2(x)+b^2\right)}{2b^2}-\frac{(a+b\sinh(x))^2\left(b^2-ab\sinh(x)\right)}{2b^2\left(b^2\sinh^2(x)+b^2\right)}\right)$$

input
```
Int[(a*Sech[x] + b*Tanh[x])^3,x]
```

output
```
b^3*(((a*(a^2 + 3*b^2)*ArcTan[Sinh[x]])/b + b^2*Log[b^2 + b^2*Sinh[x]^2] -
 a*b*Sinh[x])/(2*b^2) - ((a + b*Sinh[x])^2*(b^2 - a*b*Sinh[x]))/(2*b^2*(b^
2 + b^2*Sinh[x]^2)))
```

3.616.3.1 Defintions of rubi rules used

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.616. $\int (a\text{sech}(x)+b\tanh(x))^3\,dx$

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.616.4 Maple [A] (verified)

Time = 13.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

method	result
default	$a^3\left(\frac{\operatorname{sech}(x)\tanh(x)}{2}+\arctan\left(\mathrm{e}^x\right)\right)-\frac{3a^2b}{2\cosh(x)^2}+3a\,b^2\left(-\frac{\sinh(x)}{\cosh(x)^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2}+\arctan\left(\mathrm{e}^x\right)\right)+b^3\Big($
parts	$a^3\left(\frac{\operatorname{sech}(x)\tanh(x)}{2}+\arctan\left(\mathrm{e}^x\right)\right)+b^3\left(-\frac{\tanh(x)^2}{2}-\frac{\ln(\tanh(x)-1)}{2}-\frac{\ln(1+\tanh(x))}{2}\right)+3a\,b^2\Big(-\frac{\sinh(x)}{\cosh(x)^2}+$
risch	$-b^3x+\frac{\mathrm{e}^x\left(a^3\mathrm{e}^{2x}-3a\,b^2\mathrm{e}^{2x}-6a^2b\,\mathrm{e}^x+2b^3\mathrm{e}^x-a^3+3a\,b^2\right)}{\left(1+\mathrm{e}^{2x}\right)^2}+\frac{i\ln(\mathrm{e}^x+i)a^3}{2}+\frac{3i\ln(\mathrm{e}^x+i)a\,b^2}{2}+\ln\left(\mathrm{e}^x+i\right)b^3-\frac{i\ln(\mathrm{e}^x-i)}{2}$

input
```
int((a*sech(x)+b*tanh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*(1/2*sech(x)*tanh(x)+arctan(exp(x)))-3/2*a^2*b/cosh(x)^2+3*a*b^2*(-1/c
osh(x)^2*sinh(x)+1/2*sech(x)*tanh(x)+arctan(exp(x)))+b^3*(ln(cosh(x))-1/2*
tanh(x)^2)
```

3.616.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(53) = 106.

Time = 0.27 (sec) , antiderivative size = 502, normalized size of antiderivative = 8.66

$$\int (a\operatorname{sech}(x)+b\tanh(x))^3\,dx =$$

$$-\frac{b^3x\cosh(x)^4+b^3x\sinh(x)^4+b^3x-(a^3-3\,ab^2)\cosh(x)^3+(4\,b^3x\cosh(x)-a^3+3\,ab^2)\sinh(x)^3+2}{}$$

input
```
integrate((a*sech(x)+b*tanh(x))^3,x, algorithm="fricas")
```

output
```
-(b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + b^3*x - (a^3 - 3*a*b^2)*cosh(x)^3 +
(4*b^3*x*cosh(x) - a^3 + 3*a*b^2)*sinh(x)^3 + 2*(b^3*x + 3*a^2*b - b^3)*co
sh(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 6*a^2*b - 2*b^3 - 3*(a^3 - 3*a*b^
2)*cosh(x))*sinh(x)^2 - ((a^3 + 3*a*b^2)*cosh(x)^4 + 4*(a^3 + 3*a*b^2)*cos
h(x)*sinh(x)^3 + (a^3 + 3*a*b^2)*sinh(x)^4 + a^3 + 3*a*b^2 + 2*(a^3 + 3*a*
b^2)*cosh(x)^2 + 2*(a^3 + 3*a*b^2 + 3*(a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2
 + 4*((a^3 + 3*a*b^2)*cosh(x)^3 + (a^3 + 3*a*b^2)*cosh(x))*sinh(x))*arctan
(cosh(x) + sinh(x)) + (a^3 - 3*a*b^2)*cosh(x) - (b^3*cosh(x)^4 + 4*b^3*cos
h(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^
2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + (4*b^3*x*cosh(x)^3 + a^3 - 3*a*b^2 - 3*(a^3 - 3*a
*b^2)*cosh(x)^2 + 4*(b^3*x + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

3.616.6 Sympy [F]

$$\int (a\text{sech}(x) + b\tanh(x))^3\,dx = \int (a\,\text{sech}\,(x) + b\tanh\,(x))^3\,dx$$

input
```
integrate((a*sech(x)+b*tanh(x))**3,x)
```

output
```
Integral((a*sech(x) + b*tanh(x))**3, x)
```

3.616.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(53) = 106$.

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.07

$$\begin{aligned}\int (a\text{sech}(x) + b\tanh(x))^3\,dx = &\ \frac{3}{2}\,a^2 b\tanh\,(x)^2 \\ &+ b^3\left(x + \frac{2\,e^{(-2\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} + \log\left(e^{(-2\,x)} + 1\right)\right) \\ &- 3\,ab^2\left(\frac{e^{(-x)} - e^{(-3\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} + \arctan\left(e^{(-x)}\right)\right) \\ &+ a^3\left(\frac{e^{(-x)} - e^{(-3\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} - \arctan\left(e^{(-x)}\right)\right)\end{aligned}$$

input
```
integrate((a*sech(x)+b*tanh(x))^3,x, algorithm="maxima")
```

output
```
3/2*a^2*b*tanh(x)^2 + b^3*(x + 2*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + lo
g(e^(-2*x) + 1)) - 3*a*b^2*((e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1
) + arctan(e^(-x))) + a^3*((e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1)
 - arctan(e^(-x)))
```

3.616.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.02

$$\int (a\text{sech}(x) + b\tanh(x))^3 \, dx$$
$$= \frac{1}{2}b^3 \log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right) + \frac{1}{4}\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)\left(a^3 + 3ab^2\right)$$
$$- \frac{b^3\left(e^{(-x)} - e^x\right)^2 + 2a^3\left(e^{(-x)} - e^x\right) - 6ab^2\left(e^{(-x)} - e^x\right) + 12a^2b}{2\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}$$

input
```
integrate((a*sech(x)+b*tanh(x))^3,x, algorithm="giac")
```

output
```
1/2*b^3*log((e^(-x) - e^x)^2 + 4) + 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e
^(-x)))*(a^3 + 3*a*b^2) - 1/2*(b^3*(e^(-x) - e^x)^2 + 2*a^3*(e^(-x) - e^x)
 - 6*a*b^2*(e^(-x) - e^x) + 12*a^2*b)/((e^(-x) - e^x)^2 + 4)
```

3.616.9 Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.02

$$\int (a\mathrm{sech}(x) + b\tanh(x))^3\,dx = \mathrm{atan}\left(\frac{\mathrm{e}^x\,(a^3+3\,a\,b^2)}{\sqrt{a^6+6\,a^4\,b^2+9\,a^2\,b^4}}\right)\sqrt{a^6+6\,a^4\,b^2+9\,a^2\,b^4} + \frac{\mathrm{e}^x\,(6\,a\,b^2-2\,a^3)+6\,a^2\,b-2\,b^3}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1} + b^3\,\ln\left(\left(a^3\,\mathrm{e}^x-2\sqrt{-\frac{a^6}{4}-\frac{3\,a^4\,b^2}{2}-\frac{9\,a^2\,b^4}{4}} + 3\,a\,b^2\,\mathrm{e}^x\right)\left(2\sqrt{-\frac{a^6}{4}-\frac{3\,a^4\,b^2}{2}-\frac{9\,a^2\,b^4}{4}}+a^3\,\mathrm{e}^x + 3\,a\,b^2\,\mathrm{e}^x\right)\right) - b^3\,x - \frac{\mathrm{e}^x\,(3\,a\,b^2-a^3)+6\,a^2\,b-2\,b^3}{\mathrm{e}^{2\,x}+1}$$

input
```
int((b*tanh(x) + a/cosh(x))^3,x)
```

output
```
atan((exp(x)*(3*a*b^2 + a^3))/(a^6 + 9*a^2*b^4 + 6*a^4*b^2)^(1/2))*(a^6 +
9*a^2*b^4 + 6*a^4*b^2)^(1/2) + (exp(x)*(6*a*b^2 - 2*a^3) + 6*a^2*b - 2*b^3
)/(2*exp(2*x) + exp(4*x) + 1) + b^3*log((a^3*exp(x) - 2*(- a^6/4 - (9*a^2*
b^4)/4 - (3*a^4*b^2)/2)^(1/2) + 3*a*b^2*exp(x))*(2*(- a^6/4 - (9*a^2*b^4)/
4 - (3*a^4*b^2)/2)^(1/2) + a^3*exp(x) + 3*a*b^2*exp(x))) - b^3*x - (exp(x)
*(3*a*b^2 - a^3) + 6*a^2*b - 2*b^3)/(exp(2*x) + 1)
```

3.617 $\int (a\mathrm{sech}(x) + b\tanh(x))^2 \, dx$

3.617.1 Optimal result

Integrand size = 11, antiderivative size = 29

$$\int (a\mathrm{sech}(x) + b\tanh(x))^2 \, dx = b^2x - ab\cosh(x) - \mathrm{sech}(x)(b - a\sinh(x))(a + b\sinh(x))$$

output
```
b^2*x-a*b*cosh(x)-sech(x)*(b-a*sinh(x))*(a+b*sinh(x))
```

3.617.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int (a\mathrm{sech}(x) + b\tanh(x))^2 \, dx = b^2\mathrm{arctanh}(\tanh(x)) - 2ab\mathrm{sech}(x) + (a^2 - b^2)\tanh(x)$$

input
```
Integrate[(a*Sech[x] + b*Tanh[x])^2,x]
```

output
```
b^2*ArcTanh[Tanh[x]] - 2*a*b*Sech[x] + (a^2 - b^2)*Tanh[x]
```

3.617.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4891, 3042, 3170, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\text{sech}(x)+b\tanh(x))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sec(ix)-ib\tan(ix))^2dx$$

$$\Big\downarrow \text{4891}$$

$$\int \text{sech}^2(x)(a+b\sinh(x))^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a-ib\sin(ix))^2}{\cos(ix)^2}dx$$

$$\Big\downarrow \text{3170}$$

$$-\int \left(ab\sinh(x)-b^2\right)dx-\text{sech}(x)(b-a\sinh(x))(a+b\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$-ab\cosh(x)-\text{sech}(x)(b-a\sinh(x))(a+b\sinh(x))+b^2x$$

input
```
Int[(a*Sech[x] + b*Tanh[x])^2,x]
```

output
```
b^2*x - a*b*Cosh[x] - Sech[x]*(b - a*Sinh[x])*(a + b*Sinh[x])
```

3.617.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.617.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$a^2 \tanh(x) - \frac{2ab}{\cosh(x)} + b^2(x - \tanh(x))$	26
risch	$b^2 x - \frac{2(2b\,e^x a + a^2 - b^2)}{1 + e^{2x}}$	32
parts	$a^2 \tanh(x) + b^2\left(-\tanh(x) - \frac{\ln(\tanh(x)-1)}{2} + \frac{\ln(1+\tanh(x))}{2}\right) - 2ab\,\mathrm{sech}(x)$	37

input
```
int((a*sech(x)+b*tanh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*tanh(x)-2*a*b/cosh(x)+b^2*(x-tanh(x))
```

3.617.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int (a\text{sech}(x) + b\tanh(x))^2 \, dx = -\frac{2\,ab - (b^2 x - a^2 + b^2)\cosh(x) - (a^2 - b^2)\sinh(x)}{\cosh(x)}$$

input
```
integrate((a*sech(x)+b*tanh(x))^2,x, algorithm="fricas")
```

output
```
-(2*a*b - (b^2*x - a^2 + b^2)*cosh(x) - (a^2 - b^2)*sinh(x))/cosh(x)
```

3.617.6 Sympy [F]

$$\int (a\text{sech}(x) + b\tanh(x))^2 \, dx = \int (a\,\text{sech}(x) + b\tanh(x))^2 \, dx$$

input
```
integrate((a*sech(x)+b*tanh(x))**2,x)
```

output
```
Integral((a*sech(x) + b*tanh(x))**2, x)
```

3.617.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int (a\text{sech}(x) + b\tanh(x))^2 \, dx = b^2\left(x - \frac{2}{e^{(-2\,x)} + 1}\right) - \frac{4\,ab}{e^{(-x)} + e^x} + \frac{2\,a^2}{e^{(-2\,x)} + 1}$$

input
```
integrate((a*sech(x)+b*tanh(x))^2,x, algorithm="maxima")
```

output
```
b^2*(x - 2/(e^(-2*x) + 1)) - 4*a*b/(e^(-x) + e^x) + 2*a^2/(e^(-2*x) + 1)
```

3.617.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int (a\mathrm{sech}(x) + b\tanh(x))^2\,dx = b^2x - \frac{2\,(2\,abe^x + a^2 - b^2)}{e^{(2\,x)} + 1}$$

input
```
integrate((a*sech(x)+b*tanh(x))^2,x, algorithm="giac")
```

output
```
b^2*x - 2*(2*a*b*e^x + a^2 - b^2)/(e^(2*x) + 1)
```

3.617.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int (a\mathrm{sech}(x) + b\tanh(x))^2\,dx = b^2\,x - \frac{2\,a^2 + 4\,\mathrm{e}^x\,a\,b - 2\,b^2}{\mathrm{e}^{2\,x} + 1}$$

input
```
int((b*tanh(x) + a/cosh(x))^2,x)
```

output
```
b^2*x - (2*a^2 - 2*b^2 + 4*a*b*exp(x))/(exp(2*x) + 1)
```

3.618 $\int (a\text{sech}(x) + b\tanh(x))\,dx$

3.618.1 Optimal result

Integrand size = 9, antiderivative size = 11

$$\int (a\text{sech}(x) + b\tanh(x))\,dx = a\arctan(\sinh(x)) + b\log(\cosh(x))$$

output
```
a*arctan(sinh(x))+b*ln(cosh(x))
```

3.618.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (a\text{sech}(x) + b\tanh(x))\,dx = a\arctan(\sinh(x)) + b\log(\cosh(x))$$

input
```
Integrate[a*Sech[x] + b*Tanh[x],x]
```

output
```
a*ArcTan[Sinh[x]] + b*Log[Cosh[x]]
```

3.618.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\text{sech}(x) + b\tanh(x))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$a\arctan(\sinh(x)) + b\log(\cosh(x))$$

input
```
Int[a*Sech[x] + b*Tanh[x],x]
```

output
```
a*ArcTan[Sinh[x]] + b*Log[Cosh[x]]
```

3.618.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.618.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result
default	$a\arctan(\sinh(x)) + b\ln(\cosh(x))$
parts	$a\arctan(\sinh(x)) + b\ln(\cosh(x))$
risch	$ia\ln(\mathrm{e}^{x} + i) - ia\ln(\mathrm{e}^{x} - i) - bx + b\ln(1 + \mathrm{e}^{2x})$
parallelrisch	$-ia\ln(-i + \coth(x) - \text{csch}(x)) + ia\ln(i + \coth(x) - \text{csch}(x)) - b(x + \ln(1 - \tanh(x)))$

input
```
int(a*sech(x)+b*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
a*arctan(sinh(x))+b*ln(cosh(x))
```

3.618.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int (a\operatorname{sech}(x) + b\tanh(x))\,dx = -bx + 2\,a\arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right) + b\log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right) - \sinh\left(x\right)}\right)$$

input
```
integrate(a*sech(x)+b*tanh(x),x, algorithm="fricas")
```

output
```
-b*x + 2*a*arctan(cosh(x) + sinh(x)) + b*log(2*cosh(x)/(cosh(x) - sinh(x))
)
```

3.618.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (a\operatorname{sech}(x) + b\tanh(x))\,dx = 2a\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right) + b(x - \log\left(\tanh\left(x\right) + 1\right))$$

input
```
integrate(a*sech(x)+b*tanh(x),x)
```

output
```
2*a*atan(tanh(x/2)) + b*(x - log(tanh(x) + 1))
```

3.618.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (a\operatorname{sech}(x) + b\tanh(x))\,dx = a\arctan\left(\sinh\left(x\right)\right) + b\log\left(\cosh\left(x\right)\right)$$

input
```
integrate(a*sech(x)+b*tanh(x),x, algorithm="maxima")
```

output
```
a*arctan(sinh(x)) + b*log(cosh(x))
```

3.618.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int (a\text{sech}(x) + b\tanh(x))\,dx = -b\big(x - \log\left(e^{(2x)} + 1\right)\big) + 2\,a\arctan\left(e^{x}\right)$$

input
```
integrate(a*sech(x)+b*tanh(x),x, algorithm="giac")
```

output
```
-b*(x - log(e^(2*x) + 1)) + 2*a*arctan(e^x)
```

3.618.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.64

$$\int (a\text{sech}(x) + b\tanh(x))\,dx = b\,\ln\left(4\,a^2\,\mathrm{e}^{2\,x} + 4\,a^2\right) - b\,x + 2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{x}}{\sqrt{a^2}}\right)\,\sqrt{a^2}$$

input
```
int(b*tanh(x) + a/cosh(x),x)
```

output
```
b*log(4*a^2*exp(2*x) + 4*a^2) - b*x + 2*atan((a*exp(x))/(a^2)^(1/2))*(a^2)
^(1/2)
```

3.619 $\int \frac{1}{a\text{sech}(x)+b\tanh(x)}\,dx$

3.619.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{a\text{sech}(x)+b\tanh(x)}\,dx=\frac{\log(a+b\sinh(x))}{b}$$

output
```
ln(a+b*sinh(x))/b
```

3.619.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\text{sech}(x)+b\tanh(x)}\,dx=\frac{\log(a+b\sinh(x))}{b}$$

input
```
Integrate[(a*Sech[x] + b*Tanh[x])^(-1),x]
```

output
```
Log[a + b*Sinh[x]]/b
```

3.619.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3638, 3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\text{sech}(x)+b\tanh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a\sec(ix)-ib\tan(ix)}dx$$

$$\Big\downarrow \text{ 3638}$$

$$\int \frac{\cosh(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{ 3147}$$

$$\frac{\int \frac{1}{a+b\sinh(x)}d(b\sinh(x))}{b}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log(a+b\sinh(x))}{b}$$

input
```
Int[(a*Sech[x] + b*Tanh[x])^(-1),x]
```

output
```
Log[a + b*Sinh[x]]/b
```

3.619.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3638
```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.619.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

method	result	size
risch	$-\frac{x}{b}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{b}$	27
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}$	50

input
```
int(1/(a*sech(x)+b*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
-x/b+1/b*ln(exp(2*x)+2*a/b*exp(x)-1)
```

3.619.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{1}{a\text{sech}(x) + b\tanh(x)}\,dx = -\frac{x - \log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{b}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

output
```
-(x - log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))))/b
```

3.619.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.91

$$\int \frac{1}{a\text{sech}(x) + b\tanh(x)}\,dx = \begin{cases} \frac{x}{b} + \frac{\log\left(\frac{a\,\text{sech}\,(x)}{b}+\tanh\,(x)\right)}{b} - \frac{\log\left(\tanh\,(x)+1\right)}{b} & \text{for } b \neq 0 \\ \frac{\tanh\,(x)}{a\,\text{sech}\,(x)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x)),x)
```

output
```
Piecewise((x/b + log(a*sech(x)/b + tanh(x))/b - log(tanh(x) + 1)/b, Ne(b,
0)), (tanh(x)/(a*sech(x)), True))
```

3.619.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \frac{1}{a\text{sech}(x) + b\tanh(x)}\,dx = \frac{x}{b} + \frac{\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{b}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

output
```
x/b + log(-2*a*e^(-x) + b*e^(-2*x) - b)/b
```

3.619.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{a\mathrm{sech}(x) + b\tanh(x)}\,dx = \frac{\log\left(\left|-b\left(e^{(-x)} - e^{x}\right) + 2\,a\right|\right)}{b}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x)),x, algorithm="giac")
```

output
```
log(abs(-b*(e^(-x) - e^x) + 2*a))/b
```

3.619.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{1}{a\mathrm{sech}(x) + b\tanh(x)}\,dx = -\frac{x - \ln\left(2\,a\,\mathrm{e}^{x} - b + b\,\mathrm{e}^{2\,x}\right)}{b}$$

input
```
int(1/(b*tanh(x) + a/cosh(x)),x)
```

output
```
-(x - log(2*a*exp(x) - b + b*exp(2*x)))/b
```

3.620 $\int \frac{1}{(a\,\mathbf{sech}(x)+b\tanh(x))^2}\,dx$

3.620.1 Optimal result

Integrand size = 11, antiderivative size = 62

$$\int \frac{1}{(a\,\mathrm{sech}(x)+b\tanh(x))^2}\,dx=\frac{x}{b^2}+\frac{2a\,\mathrm{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}$$

output
```
x/b^2-cosh(x)/b/(a+b*sinh(x))+2*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2
))/b^2/(a^2+b^2)^(1/2)
```

3.620.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 502, normalized size of antiderivative = 8.10

$$\int \frac{1}{(a\,\mathrm{sech}(x)+b\tanh(x))^2}\,dx=$$

$$-\frac{\cosh(x)\left(-2a\sqrt{a-ib}\sqrt{a+ib}\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}(a+b\sinh(x))+2a(a-ib)\mathrm{arct}\ldots\right.}{\ldots}$$

input
```
Integrate[(a*Sech[x] + b*Tanh[x])^(-2),x]
```

output
```
-((Cosh[x]*(-2*a*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh[x
]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]]
*(a + b*Sinh[x]) + 2*a*(a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sin
h[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]
*Sqrt[1 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[
x]))/(a + I*b))]*(-2*(-1)^(1/4)*a*Sqrt[b]*(I*a + b)*ArcSin[((1/2 + I/2)*Sq
rt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] - 2*(-1)^(1/4)*
b^(3/2)*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]
))/(a - I*b))])/Sqrt[b]]*Sinh[x] + Sqrt[a - I*b]*(a^2 + b^2)*Sqrt[1 + I*Si
nh[x]]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])))/((a - I*b)^(3/2)*(a + I*b)^
(3/2)*b*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt[-((
b*(I + Sinh[x]))/(a - I*b))]*(a + b*Sinh[x])))
```

3.620.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.091$, Rules used = {3042, 4891, 3042, 3172, 26, 3042, 26, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sec(ix)-ib\tan(ix))^2}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3172}$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{i\int \frac{i\sinh(x)}{a+b\sinh(x)}dx}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\int \frac{\sinh(x)}{a+b\sinh(x)}dx}{b} - \frac{\cosh(x)}{b(a+b\sinh(x))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))} + \frac{\int -\frac{i\sin(ix)}{a-ib\sin(ix)}dx}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))} - \frac{i\int \frac{\sin(ix)}{a-ib\sin(ix)}dx}{b}$$

$$\Big\downarrow \; 3214$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))} - \frac{i\left(\frac{ix}{b} - \frac{ia\int \frac{1}{a+b\sinh(x)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))} - \frac{i\left(\frac{ix}{b} - \frac{ia\int \frac{1}{a-ib\sin(ix)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3139$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))} - \frac{i\left(\frac{ix}{b} - \frac{2ia\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 1083$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))} - \frac{i\left(\frac{4ia\int \frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b} + \frac{ix}{b}\right)}{b}$$

$$\Big\downarrow \; 219$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))} - \frac{i\left(\frac{2ia\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}} + \frac{ix}{b}\right)}{b}$$

input
```
Int[(a*Sech[x] + b*Tanh[x])^(-2),x]
```

3.620. $\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^2}\,dx$

output
```
((-I)*((I*x)/b + ((2*I)*a*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2]
)])/(b*Sqrt[a^2 + b^2])))/b - Cosh[x]/(b*(a + b*Sinh[x]))
```

3.620.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.620. $\int \frac{1}{(a\operatorname{sech}(x)+b\tanh(x))^2}\,dx$

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.620.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

method	result	size
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^{2}}+\frac{\frac{2\left(\frac{b^{2}\tanh\left(\frac{x}{2}\right)}{a}+b\right)}{\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{2a\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}}}{b^{2}}$	101
risch	$\frac{x}{b^{2}}+\frac{2a\,\mathrm{e}^{x}-2b}{b^{2}\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b\right)}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{b\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}-\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{b\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}$	140

input
```
int(1/(a*sech(x)+b*tanh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*ln(tanh(1/2*x)-1)+1/b^2*ln(tanh(1/2*x)+1)+2/b^2*((b^2/a*tanh(1/2*x)
+b)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a
*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

3.620.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(58) = 116.

Time = 0.26 (sec) , antiderivative size = 362, normalized size of antiderivative = 5.84

$$\int \frac{1}{(a\mathrm{sech}(x)+b\tanh(x))^2}\,dx =$$
$$-\frac{(a^2b+b^3)x\cosh(x)^2+(a^2b+b^3)x\sinh(x)^2-2a^2b-2b^3+\left(ab\cosh(x)^2+ab\sinh(x)^2+2a^2\cosh(x\right.}{}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))^2,x, algorithm="fricas")
```

output
```
-((a^2*b + b^3)*x*cosh(x)^2 + (a^2*b + b^3)*x*sinh(x)^2 - 2*a^2*b - 2*b^3
+ (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) +
a^2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(
b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(
b*cosh(x) + a)*sinh(x) - b)) - (a^2*b + b^3)*x + 2*(a^3 + a*b^2 + (a^3 + a
*b^2)*x)*cosh(x) + 2*(a^3 + a*b^2 + (a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2
)*x)*sinh(x))/(a^2*b^3 + b^5 - (a^2*b^3 + b^5)*cosh(x)^2 - (a^2*b^3 + b^5)
*sinh(x)^2 - 2*(a^3*b^2 + a*b^4)*cosh(x) - 2*(a^3*b^2 + a*b^4 + (a^2*b^3 +
 b^5)*cosh(x))*sinh(x))
```

3.620.6 Sympy [F]

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^2}\,dx = \int \frac{1}{\left(a\operatorname{sech}\left(x\right) + b\tanh\left(x\right)\right)^2}\,dx$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))**2,x)
```

output
```
Integral((a*sech(x) + b*tanh(x))**(-2), x)
```

3.620.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^2}\,dx = -\frac{2\left(ae^{(-x)} + b\right)}{2\,ab^2e^{(-x)} - b^3e^{(-2\,x)} + b^3} - \frac{a\log\left(\frac{be^{(-x)} - a - \sqrt{a^2+b^2}}{be^{(-x)} - a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^2} + \frac{x}{b^2}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))^2,x, algorithm="maxima")
```

output
```
-2*(a*e^(-x) + b)/(2*a*b^2*e^(-x) - b^3*e^(-2*x) + b^3) - a*log((b*e^(-x)
- a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*
b^2) + x/b^2
```

3.620.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^2}\,dx = -\frac{a\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2} + \frac{x}{b^2} + \frac{2\,(ae^x-b)}{(be^{(2\,x)}+2\,ae^x-b)b^2}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))^2,x, algorithm="giac")
```

output
```
-a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + x/b^2 + 2*(a*e^x - b)/((b*e^(2*x) + 2*
a*e^x - b)*b^2)
```

3.620.9 Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.13

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^2}\,dx = \frac{x}{b^2} - \frac{\frac{2}{b}-\frac{2\,a\,e^x}{b^2}}{2\,a\,e^x-b+b\,e^{2\,x}} - \frac{a\,\ln\left(\frac{2\,a\,e^x}{b^3}-\frac{2\,a\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}} + \frac{a\,\ln\left(\frac{2\,a\,e^x}{b^3}+\frac{2\,a\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}}$$

input
```
int(1/(b*tanh(x) + a/cosh(x))^2,x)
```

output
```
x/b^2 - (2/b - (2*a*exp(x))/b^2)/(2*a*exp(x) - b + b*exp(2*x)) - (a*log((2
*a*exp(x))/b^3 - (2*a*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2))))/(b^2*(a^2
+ b^2)^(1/2)) + (a*log((2*a*exp(x))/b^3 + (2*a*(b - a*exp(x)))/(b^3*(a^2 +
 b^2)^(1/2))))/(b^2*(a^2 + b^2)^(1/2))
```

3.621 $\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^3}\,dx$

3.621.1 Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^3}\,dx=\frac{\log(a+b\sinh(x))}{b^3}-\frac{a^2+b^2}{2b^3(a+b\sinh(x))^2}+\frac{2a}{b^3(a+b\sinh(x))}$$

output
```
ln(a+b*sinh(x))/b^3+1/2*(-a^2-b^2)/b^3/(a+b*sinh(x))^2+2*a/b^3/(a+b*sinh(x
))
```

3.621.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^3}\,dx=-\frac{-\log(a+b\sinh(x))+\frac{-3a^2+b^2-4ab\sinh(x)}{2(a+b\sinh(x))^2}}{b^3}$$

input
```
Integrate[(a*Sech[x] + b*Tanh[x])^(-3),x]
```

output
```
-((-Log[a + b*Sinh[x]] + (-3*a^2 + b^2 - 4*a*b*Sinh[x])/(2*(a + b*Sinh[x])
^2))/b^3)
```

3.621.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 4891, 3042, 3147, 25, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(a\sec(ix)-ib\tan(ix))^3}dx$$

$$\Big\downarrow \; 4891$$

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ix)^3}{(a-ib\sin(ix))^3}dx$$

$$\Big\downarrow \; 3147$$

$$-\frac{\int -\frac{\sinh^2(x)b^2+b^2}{(a+b\sinh(x))^3}d(b\sinh(x))}{b^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sinh^2(x)b^2+b^2}{(a+b\sinh(x))^3}d(b\sinh(x))}{b^3}$$

$$\Big\downarrow \; 476$$

$$\frac{\int \left(-\frac{2a}{(a+b\sinh(x))^2}+\frac{1}{a+b\sinh(x)}+\frac{a^2+b^2}{(a+b\sinh(x))^3}\right)d(b\sinh(x))}{b^3}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{a^2+b^2}{2(a+b\sinh(x))^2}-\frac{2a}{a+b\sinh(x)}-\log(a+b\sinh(x))}{b^3}$$

input
```
Int[(a*Sech[x] + b*Tanh[x])^(-3),x]
```

output
```
-((-Log[a + b*Sinh[x]] + (a^2 + b^2)/(2*(a + b*Sinh[x])^2) - (2*a)/(a + b*
Sinh[x]))/b^3)
```

3.621.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.621.4 Maple [A] (verified)

Time = 11.79 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{x}{b^3}+\frac{2\,\mathrm{e}^{x}\left(2b\,\mathrm{e}^{2x}a+3a^2\mathrm{e}^{x}-b^2\mathrm{e}^{x}-2ab\right)}{b^3\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b\right)^2}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{b^3}$
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^3}+\frac{\frac{2\left(\frac{b\left(a^2-b^2\right)\tanh\left(\frac{x}{2}\right)^3}{a}-\frac{b^2\left(3a^2-b^2\right)\tanh\left(\frac{x}{2}\right)^2}{a^2}-\frac{b\left(a^2-b^2\right)\tanh\left(\frac{x}{2}\right)}{a}\right)}{\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)^2}+\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{b^3}-\frac{\ln(}{}$

input
```
int(1/(a*sech(x)+b*tanh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-x/b^3+2/b^3*exp(x)*(2*b*exp(2*x)*a+3*a^2*exp(x)-b^2*exp(x)-2*a*b)/(b*exp(
2*x)+2*a*exp(x)-b)^2+1/b^3*ln(exp(2*x)+2*a/b*exp(x)-1)
```

3.621.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 543, normalized size of antiderivative = 11.31

$$\int \frac{1}{(a\operatorname{sech}(x)+b\tanh(x))^3}\,dx =$$
$$-\frac{b^2x\cosh(x)^4+b^2x\sinh(x)^4+4\left(abx-ab\right)\cosh(x)^3+4\left(b^2x\cosh(x)+abx-ab\right)\sinh(x)^3+b^2x-2}{}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))^3,x, algorithm="fricas")
```

output
```
-(b^2*x*cosh(x)^4 + b^2*x*sinh(x)^4 + 4*(a*b*x - a*b)*cosh(x)^3 + 4*(b^2*x
*cosh(x) + a*b*x - a*b)*sinh(x)^3 + b^2*x - 2*(3*a^2 - b^2 - (2*a^2 - b^2)
*x)*cosh(x)^2 + 2*(3*b^2*x*cosh(x)^2 - 3*a^2 + b^2 + (2*a^2 - b^2)*x + 6*(
a*b*x - a*b)*cosh(x))*sinh(x)^2 - 4*(a*b*x - a*b)*cosh(x) - (b^2*cosh(x)^4
 + b^2*sinh(x)^4 + 4*a*b*cosh(x)^3 + 4*(b^2*cosh(x) + a*b)*sinh(x)^3 - 4*a
*b*cosh(x) + 2*(2*a^2 - b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 6*a*b*cosh(x
) + 2*a^2 - b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + 3*a*b*cosh(x)^2 - a*
b + (2*a^2 - b^2)*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(
x))) + 4*(b^2*x*cosh(x)^3 - a*b*x + 3*(a*b*x - a*b)*cosh(x)^2 + a*b - (3*a
^2 - b^2 - (2*a^2 - b^2)*x)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + b^5*sinh(x)
^4 + 4*a*b^4*cosh(x)^3 - 4*a*b^4*cosh(x) + b^5 + 4*(b^5*cosh(x) + a*b^4)*s
inh(x)^3 + 2*(2*a^2*b^3 - b^5)*cosh(x)^2 + 2*(3*b^5*cosh(x)^2 + 6*a*b^4*co
sh(x) + 2*a^2*b^3 - b^5)*sinh(x)^2 + 4*(b^5*cosh(x)^3 + 3*a*b^4*cosh(x)^2
- a*b^4 + (2*a^2*b^3 - b^5)*cosh(x))*sinh(x))
```

3.621.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(48) = 96$.

Time = 1.30 (sec) , antiderivative size = 651, normalized size of antiderivative = 13.56

$$\int \frac{1}{(a\mathrm{sech}(x) + b\tanh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))**3,x)
```

output
```
Piecewise((2*a**2*x*sech(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*
sech(x) + 2*b**5*tanh(x)**2) + 2*a**2*log(a*sech(x)/b + tanh(x))*sech(x)**
2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2)
- 2*a**2*log(tanh(x) + 1)*sech(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*ta
nh(x)*sech(x) + 2*b**5*tanh(x)**2) + a**2*sech(x)**2/(2*a**2*b**3*sech(x)*
*2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) + 4*a*b*x*tanh(x)*sech(
x)/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2)
 + 4*a*b*log(a*sech(x)/b + tanh(x))*tanh(x)*sech(x)/(2*a**2*b**3*sech(x)**
2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) - 4*a*b*log(tanh(x) + 1)
*tanh(x)*sech(x)/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b*
*5*tanh(x)**2) + 2*b**2*x*tanh(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*ta
nh(x)*sech(x) + 2*b**5*tanh(x)**2) + 2*b**2*log(a*sech(x)/b + tanh(x))*tan
h(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x
)**2) - 2*b**2*log(tanh(x) + 1)*tanh(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b
**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) - b**2*tanh(x)**2/(2*a**2*b**3*se
ch(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) - b**2/(2*a**2*b*
*3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2), Ne(b, 0)),
((-2*tanh(x)**3/(3*sech(x)**3) + tanh(x)/sech(x)**3)/a**3, True))
```

3.621.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(46) = 92.

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.44

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^3}\,dx = \frac{2\left(2\,abe^{(-x)}-2\,abe^{(-3\,x)}+(3\,a^2-b^2)e^{(-2\,x)}\right)}{4\,ab^4e^{(-x)}-4\,ab^4e^{(-3\,x)}+b^5e^{(-4\,x)}+b^5+2\,(2\,a^2b^3-b^5)e^{(-2\,x)}} + \frac{x}{b^3}+\frac{\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^3}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))^3,x, algorithm="maxima")
```

output
```
2*(2*a*b*e^(-x) - 2*a*b*e^(-3*x) + (3*a^2 - b^2)*e^(-2*x))/(4*a*b^4*e^(-x)
 - 4*a*b^4*e^(-3*x) + b^5*e^(-4*x) + b^5 + 2*(2*a^2*b^3 - b^5)*e^(-2*x)) +
 x/b^3 + log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^3
```

3.621.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^3}\,dx = \frac{\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{b^3} - \frac{3\,b\left(e^{(-x)} - e^x\right)^2 - 4\,a\left(e^{(-x)} - e^x\right) + 4\,b}{2\left(b\left(e^{(-x)} - e^x\right) - 2\,a\right)^2 b^2}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))^3,x, algorithm="giac")
```

output
```
log(abs(-b*(e^(-x) - e^x) + 2*a))/b^3 - 1/2*(3*b*(e^(-x) - e^x)^2 - 4*a*(e
^(-x) - e^x) + 4*b)/((b*(e^(-x) - e^x) - 2*a)^2*b^2)
```

3.621.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^3}\,dx = \int \frac{1}{\left(b\tanh(x) + \frac{a}{\cosh(x)}\right)^3}\,dx$$

input
```
int(1/(b*tanh(x) + a/cosh(x))^3,x)
```

output
```
int(1/(b*tanh(x) + a/cosh(x))^3, x)
```

3.622 $\int \frac{1}{(a\,\mathbf{sech}(x)+b\tanh(x))^4}\,dx$

3.622.1 Optimal result

Integrand size = 11, antiderivative size = 146

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^4}\,dx=\frac{x}{b^4}+\frac{a(2a^2+3b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\,(a^2+b^2)^{3/2}}-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}+\frac{a\cosh^3(x)}{2b\,(a^2+b^2)\,(a+b\sinh(x))^2}-\frac{\cosh(x)\,(2(a^2+b^2)+ab\sinh(x))}{2b^3\,(a^2+b^2)\,(a+b\sinh(x))}$$

output
```
x/b^4+a*(2*a^2+3*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^4/(a^2+
b^2)^(3/2)-1/3*cosh(x)^3/b/(a+b*sinh(x))^3+1/2*a*cosh(x)^3/b/(a^2+b^2)/(a+
b*sinh(x))^2-1/2*cosh(x)*(2*a^2+2*b^2+a*b*sinh(x))/b^3/(a^2+b^2)/(a+b*sinh
(x))
```

3.622.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.17 (sec) , antiderivative size = 3430, normalized size of antiderivative = 23.49

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^4}\,dx=\text{Result too large to show}$$

input
```
Integrate[(a*Sech[x] + b*Tanh[x])^(-4),x]
```

output
```
((-I)*Sech[x]*(a + b*Sinh[x])^4*(((I/3)*b*(((-I)*b)/(a - I*b) - (b*Sinh[x]
)/(a - I*b))^(5/2)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((((-I
)*a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))*(
a + b*Sinh[x])^3) - (((I/2)*a*b^3*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I
*b))^(5/2)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((a^2 + b^2)*(
((-I)*a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b
))*(a + b*Sinh[x])^2) - (-(((((3*I)*a^2*b^5)/(a^2 + b^2)^2 - ((2*I)*b^5*(3
*a^2 + 2*b^2))/(a^2 + b^2)^2)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))
^(5/2)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((((-I)*a*b)/(a -
I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))*(a + b*Sinh[x
]))) - ((16*Sqrt[2]*(a - I*b)*b^6*(3*a^2 + 4*b^2)*(((-I)*b)/(a - I*b) - (b
*Sinh[x])/(a - I*b))^(5/2)*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)]*(
1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(5/2
)*((5*(1/(2*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I
*b)))/b)^2) + (1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a -
 I*b)))/b)^(-1)))/8 + (((15*I)/32)*b^3*(((-I)*(a - I*b)*(((-I)*b)/(a - I*b
) - (b*Sinh[x])/(a - I*b)))/b + ((a - I*b)^2*(((-I)*b)/(a - I*b) - (b*Sinh
[x])/(a - I*b))^2)/(3*b^2) + ((-1)^(1/4)*Sqrt[2]*Sqrt[a - I*b]*ArcSin[((-1
)^(1/4)*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(S
qrt[2]*Sqrt[b])]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqr...
```

3.622.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.22, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.636$, Rules used = {3042, 4891, 3042, 3172, 26, 3042, 26, 3343, 26, 3042, 3342, 25, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sec(ix)-ib\tan(ix))^4}dx$$

$$\Big\downarrow \text{ 4891}$$

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ix)^4}{(a-ib\sin(ix))^4}dx$$

$$\Big\downarrow \text{ 3172}$$

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\int \frac{i\cosh^2(x)\sinh(x)}{(a+b\sinh(x))^3}dx}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\int \frac{\cosh^2(x)\sinh(x)}{(a+b\sinh(x))^3}dx}{b}-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}+\frac{\int -\frac{i\cos(ix)^2\sin(ix)}{(a-ib\sin(ix))^3}dx}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\int \frac{\cos(ix)^2\sin(ix)}{(a-ib\sin(ix))^3}dx}{b}$$

$$\Big\downarrow \text{ 3343}$$

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\left(\frac{ia\cosh^3(x)}{2(a^2+b^2)(a+b\sinh(x))^2}-\frac{\int -\frac{i\cosh^2(x)(2b-a\sinh(x))}{(a+b\sinh(x))^2}dx}{2(a^2+b^2)}\right)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\left(\frac{i\int \frac{\cosh^2(x)(2b-a\sinh(x))}{(a+b\sinh(x))^2}dx}{2(a^2+b^2)}+\frac{ia\cosh^3(x)}{2(a^2+b^2)(a+b\sinh(x))^2}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\left(\frac{i\int \frac{\cos(ix)^2(2b+ia\sin(ix))}{(a-ib\sin(ix))^2}dx}{2(a^2+b^2)}+\frac{ia\cosh^3(x)}{2(a^2+b^2)(a+b\sinh(x))^2}\right)}{b}$$

$$\Big\downarrow \text{ 3342}$$

3.622. $\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^4}\,dx$

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\left(\frac{i\left(\frac{\int -\frac{ab-2\left(a^2+b^2\right)\sinh(x)}{a+b\sinh(x)}dx}{b^2}-\frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sinh(x)\right)}{b^2(a+b\sinh(x))}\right)}{2(a^2+b^2)}+\frac{ia\cosh^3(x)}{2(a^2+b^2)(a+b\sinh(x))^2}\right)}{b}$$

$\downarrow$ 25

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\left(\frac{i\left(-\frac{\int \frac{ab-2\left(a^2+b^2\right)\sinh(x)}{a+b\sinh(x)}dx}{b^2}-\frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sinh(x)\right)}{b^2(a+b\sinh(x))}\right)}{2(a^2+b^2)}+\frac{ia\cosh^3(x)}{2(a^2+b^2)(a+b\sinh(x))^2}\right)}{b}$$

$\downarrow$ 3042

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\left(\frac{i\left(-\frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sinh(x)\right)}{b^2(a+b\sinh(x))}-\frac{\int \frac{ab+2i\left(a^2+b^2\right)\sin(ix)}{a-ib\sin(ix)}dx}{b^2}\right)}{2(a^2+b^2)}+\frac{ia\cosh^3(x)}{2(a^2+b^2)(a+b\sinh(x))^2}\right)}{b}$$

$\downarrow$ 3214

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\left(\frac{i\left(-\frac{\frac{a\left(2a^2+3b^2\right)\int \frac{1}{a+b\sinh(x)}dx}{b}-\frac{2x\left(a^2+b^2\right)}{b}}{b^2}-\frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sinh(x)\right)}{b^2(a+b\sinh(x))}\right)}{2(a^2+b^2)}+\frac{ia\cosh^3(x)}{2(a^2+b^2)(a+b\sinh(x))^2}\right)}{b}$$

$\downarrow$ 3042

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\left(\frac{i\left(-\frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sinh(x)\right)}{b^2(a+b\sinh(x))}-\frac{-\frac{2x\left(a^2+b^2\right)}{b}+\frac{a\left(2a^2+3b^2\right)\int \frac{1}{a-ib\sin(ix)}dx}{b}}{b^2}\right)}{2(a^2+b^2)}+\frac{ia\cosh^3(x)}{2(a^2+b^2)(a+b\sinh(x))^2}\right)}{b}$$

$\downarrow$ 3139

3.622. $\int \frac{1}{(a\operatorname{sech}(x)+b\tanh(x))^4}\,dx$

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\left(\frac{i\left(-\frac{\frac{2a(2a^2+3b^2)\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}-\frac{2x(a^2+b^2)}{b}}{b^2}-\frac{\cosh(x)\left(2(a^2+b^2)+ab\sinh(x)\right)}{b^2(a+b\sinh(x))}\right)}{2(a^2+b^2)}+\frac{ia\cosh^3(x)}{2(a^2+b^2)(a+b\sinh(x))^2}\right)}{b}$$

$\Big\downarrow$ 1083

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\left(\frac{i\left(-\frac{-\frac{4a(2a^2+3b^2)\int\frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{2x(a^2+b^2)}{b}}{b^2}-\frac{\cosh(x)\left(2(a^2+b^2)+ab\sinh(x)\right)}{b^2(a+b\sinh(x))}\right)}{2(a^2+b^2)}+\frac{ia\cosh^3(x)}{2(a^2+b^2)(a+b\sinh(x))^2}\right)}{b}$$

$\Big\downarrow$ 219

$$-\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}-\frac{i\left(\frac{i\left(-\frac{-\frac{2a(2a^2+3b^2)\operatorname{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{2x(a^2+b^2)}{b}}{b^2}-\frac{\cosh(x)\left(2(a^2+b^2)+ab\sinh(x)\right)}{b^2(a+b\sinh(x))}\right)}{2(a^2+b^2)}+\frac{ia\cosh^3(x)}{2(a^2+b^2)(a+b\sinh(x))^2}\right)}{b}$$

input
```
Int[(a*Sech[x] + b*Tanh[x])^(-4),x]
```

output
```
-1/3*Cosh[x]^3/(b*(a + b*Sinh[x])^3) - (I*((((I/2)*a*Cosh[x]^3)/((a^2 + b^2
)*(a + b*Sinh[x])^2) + ((I/2)*(-(((-2*(a^2 + b^2)*x)/b - (2*a*(2*a^2 + 3*b
^2)*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]
))/b^2) - (Cosh[x]*(2*(a^2 + b^2) + a*b*Sinh[x]))/(b^2*(a + b*Sinh[x]))))/
(a^2 + b^2)))/b
```

3.622. $\int \frac{1}{(a\,\mathrm{sech}(x)+b\tanh(x))^4}\,dx$

3.622.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.622. $\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^4}\,dx$

rule 3342
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Simp[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.622.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(135) = 270$.

Time = 90.76 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.45

method	result
default	$\frac{\frac{2\left(\frac{b^{2}\left(a^{4}+2a^{2}b^{2}+2b^{4}\right)\tanh\left(\frac{x}{2}\right)^{5}}{2a\left(a^{2}+b^{2}\right)}+\frac{b\left(2a^{6}-3a^{4}b^{2}-4a^{2}b^{4}-4b^{6}\right)\tanh\left(\frac{x}{2}\right)^{4}}{2\left(a^{2}+b^{2}\right)a^{2}}-\frac{b^{2}\left(18a^{6}+3a^{4}b^{2}-4a^{2}b^{4}-4b^{6}\right)\tanh\left(\frac{x}{2}\right)^{3}}{3a^{3}\left(a^{2}+b^{2}\right)}-\frac{b\left(2a^{6}-8a^{4}b^{2}-7a^{2}b^{4}-2\cdots\right.}{a^{2}\left(a^{2}+b^{2}\right)\cdots}}{\left(\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a\right)^{3}}}{b^{4}}$
risch	$\frac{x}{b^{4}}+\frac{18\,\mathrm{e}^{5x}a^{3}b^{2}+15a\,b^{4}\mathrm{e}^{5x}+54a^{4}b\,\mathrm{e}^{4x}+27a^{2}b^{3}\mathrm{e}^{4x}-12b^{5}\mathrm{e}^{4x}+44\,\mathrm{e}^{3x}a^{5}-34a^{3}b^{2}\mathrm{e}^{3x}-48\,\mathrm{e}^{3x}a\,b^{4}-78a^{4}b\,\mathrm{e}^{2x}-36a^{2}b^{3}\mathrm{e}^{2x}+12b^{5}\mathrm{e}^{2x}+4\cdots}{3b^{4}(a^{2}+b^{2})(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b)^{3}}$

input
```
int(1/(a*sech(x)+b*tanh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
2/b^4*((1/2*b^2*(a^4+2*a^2*b^2+2*b^4)/a/(a^2+b^2)*tanh(1/2*x)^5+1/2*b*(2*a
^6-3*a^4*b^2-4*a^2*b^4-4*b^6)/(a^2+b^2)/a^2*tanh(1/2*x)^4-1/3/a^3*b^2*(18*
a^6+3*a^4*b^2-4*a^2*b^4-4*b^6)/(a^2+b^2)*tanh(1/2*x)^3-1/a^2*b*(2*a^6-8*a^
4*b^2-7*a^2*b^4-2*b^6)/(a^2+b^2)*tanh(1/2*x)^2+1/2/a*b^2*(11*a^4+8*a^2*b^2
+2*b^4)/(a^2+b^2)*tanh(1/2*x)+1/6*b*(6*a^4+5*a^2*b^2+2*b^4)/(a^2+b^2))/(ta
nh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)^3-1/2*a*(2*a^2+3*b^2)/(a^2+b^2)^(3/2)*arc
tanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))+1/b^4*ln(tanh(1/2*x)+1)-1
/b^4*ln(tanh(1/2*x)-1)
```

3.622.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2978 vs. $2(137) = 274$.

Time = 0.29 (sec) , antiderivative size = 2978, normalized size of antiderivative = 20.40

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))^4,x, algorithm="fricas")
```

output
```
-1/6*(6*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x)^6 + 6*(a^4*b^3 + 2*a^2*b^5 +
 b^7)*x*sinh(x)^6 - 22*a^4*b^3 - 38*a^2*b^5 - 16*b^7 + 6*(6*a^5*b^2 + 11*a
^3*b^4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^5 + 6*(6*a^5
*b^2 + 11*a^3*b^4 + 5*a*b^6 + 6*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x) + 6*
(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*sinh(x)^5 + 6*(18*a^6*b + 27*a^4*b^3 + 5*
a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x)^4 +
 6*(18*a^6*b + 27*a^4*b^3 + 5*a^2*b^5 - 4*b^7 + 15*(a^4*b^3 + 2*a^2*b^5 +
b^7)*x*cosh(x)^2 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x + 5*(6*a^5*
b^2 + 11*a^3*b^4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x))*s
inh(x)^4 + 4*(22*a^7 + 5*a^5*b^2 - 41*a^3*b^4 - 24*a*b^6 + 6*(2*a^7 + a^5*
b^2 - 4*a^3*b^4 - 3*a*b^6)*x)*cosh(x)^3 + 4*(22*a^7 + 5*a^5*b^2 - 41*a^3*b
^4 - 24*a*b^6 + 30*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x)^3 + 15*(6*a^5*b^2
 + 11*a^3*b^4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^2 + 6
*(2*a^7 + a^5*b^2 - 4*a^3*b^4 - 3*a*b^6)*x + 6*(18*a^6*b + 27*a^4*b^3 + 5*
a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x))*si
nh(x)^3 - 6*(26*a^6*b + 38*a^4*b^3 + 8*a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^
4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x)^2 - 6*(26*a^6*b + 38*a^4*b^3 + 8*a^2*b
^5 - 4*b^7 - 15*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x)^4 - 10*(6*a^5*b^2 +
11*a^3*b^4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^3 - 6*(1
8*a^6*b + 27*a^4*b^3 + 5*a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a...
```

3.622.6 Sympy [F]

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^4}\,dx = \int \frac{1}{(a\,\text{sech}\,(x) + b\tanh\,(x))^4}\,dx$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))**4,x)
```

output
```
Integral((a*sech(x) + b*tanh(x))**(-4), x)
```

3.622.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(137) = 274$.

Time = 0.32 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.57

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^4}\,dx = -\frac{(2\,a^2 + 3\,b^2)a\log\left(\frac{be^{(-x)} - a - \sqrt{a^2+b^2}}{be^{(-x)} - a + \sqrt{a^2+b^2}}\right)}{2\,(a^2b^4 + b^6)\sqrt{a^2+b^2}} - \frac{11\,a^2b^3 + 8\,b^5 + 3\,(16\,a^3b^2 + 11\,ab^4)e^{(-x)} + 6\,(13\,a^4b + 6\,a^2b^3 - 2\,b^5)e^{(-2\,x)} + 2\,(22\,a^5 - 17\,a^3b^2 - 24\,a}{3\,(a^2b^7 + b^9 + 6\,(a^3b^6 + ab^8)e^{(-x)} + 3\,(4\,a^4b^5 + 3\,a^2b^7 - b^9)e^{(-2\,x)} + 4\,(2\,a^5b^4 - a^3b^6 - 3\,ab^8)e^{(-3\,x)} - 3\,(} + \frac{x}{b^4}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))^4,x, algorithm="maxima")
```

output
```
-1/2*(2*a^2 + 3*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a
+ sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 1/3*(11*a^2*b^3 +
8*b^5 + 3*(16*a^3*b^2 + 11*a*b^4)*e^(-x) + 6*(13*a^4*b + 6*a^2*b^3 - 2*b^5
)*e^(-2*x) + 2*(22*a^5 - 17*a^3*b^2 - 24*a*b^4)*e^(-3*x) - 3*(18*a^4*b + 9
*a^2*b^3 - 4*b^5)*e^(-4*x) + 3*(6*a^3*b^2 + 5*a*b^4)*e^(-5*x))/(a^2*b^7 +
b^9 + 6*(a^3*b^6 + a*b^8)*e^(-x) + 3*(4*a^4*b^5 + 3*a^2*b^7 - b^9)*e^(-2*x
) + 4*(2*a^5*b^4 - a^3*b^6 - 3*a*b^8)*e^(-3*x) - 3*(4*a^4*b^5 + 3*a^2*b^7
- b^9)*e^(-4*x) + 6*(a^3*b^6 + a*b^8)*e^(-5*x) - (a^2*b^7 + b^9)*e^(-6*x))
 + x/b^4
```

3.622.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.83

$$\int \frac{1}{(a\operatorname{sech}(x) + b\tanh(x))^4}\,dx = -\frac{(2a^3 + 3ab^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,(a^2b^4 + b^6)\sqrt{a^2+b^2}}$$
$$+ \frac{18\,a^3b^2e^{(5\,x)} + 15\,ab^4e^{(5\,x)} + 54\,a^4be^{(4\,x)} + 27\,a^2b^3e^{(4\,x)} - 12\,b^5e^{(4\,x)} + 44\,a^5e^{(3\,x)} - 34\,a^3b^2e^{(3\,x)} - 48\,ab^4e^{(}}{3\,(a^2b^4 + b^6)(be^{(2\,x)} + 2\,ae^{x}}$$
$$+ \frac{x}{b^4}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))^4,x, algorithm="giac")
```

output
```
-1/2*(2*a^3 + 3*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*
e^x + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/3*(1
8*a^3*b^2*e^(5*x) + 15*a*b^4*e^(5*x) + 54*a^4*b*e^(4*x) + 27*a^2*b^3*e^(4*
x) - 12*b^5*e^(4*x) + 44*a^5*e^(3*x) - 34*a^3*b^2*e^(3*x) - 48*a*b^4*e^(3*
x) - 78*a^4*b*e^(2*x) - 36*a^2*b^3*e^(2*x) + 12*b^5*e^(2*x) + 48*a^3*b^2*e
^x + 33*a*b^4*e^x - 11*a^2*b^3 - 8*b^5)/((a^2*b^4 + b^6)*(b*e^(2*x) + 2*a*
e^x - b)^3) + x/b^4
```

3.622.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\operatorname{sech}(x) + b\tanh(x))^4}\,dx = \int \frac{1}{\left(b\tanh\left(x\right) + \frac{a}{\cosh(x)}\right)^4}\,dx$$

input
```
int(1/(b*tanh(x) + a/cosh(x))^4,x)
```

output
```
int(1/(b*tanh(x) + a/cosh(x))^4, x)
```

3.623 $\int \frac{1}{(a\,\text{sech}(x)+b\tanh(x))^5}\,dx$

3.623.1 Optimal result

Integrand size = 11, antiderivative size = 95

$$\int \frac{1}{(a\,\text{sech}(x)+b\tanh(x))^5}\,dx=\frac{\log(a+b\sinh(x))}{b^5}-\frac{(a^2+b^2)^2}{4b^5(a+b\sinh(x))^4}+\frac{4a(a^2+b^2)}{3b^5(a+b\sinh(x))^3}-\frac{3a^2+b^2}{b^5(a+b\sinh(x))^2}+\frac{4a}{b^5(a+b\sinh(x))}$$

output
```
ln(a+b*sinh(x))/b^5-1/4*(a^2+b^2)^2/b^5/(a+b*sinh(x))^4+4/3*a*(a^2+b^2)/b^
5/(a+b*sinh(x))^3+(-3*a^2-b^2)/b^5/(a+b*sinh(x))^2+4*a/b^5/(a+b*sinh(x))
```

3.623.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a\,\text{sech}(x)+b\tanh(x))^5}\,dx=\frac{\log(a+b\sinh(x))-\frac{(a^2+b^2)^2}{4(a+b\sinh(x))^4}+\frac{4a(a^2+b^2)}{3(a+b\sinh(x))^3}-\frac{3a^2+b^2}{(a+b\sinh(x))^2}+\frac{4a}{a+b\sinh(x)}}{b^5}$$

input
```
Integrate[(a*Sech[x] + b*Tanh[x])^(-5),x]
```

output
```
(Log[a + b*Sinh[x]] - (a^2 + b^2)^2/(4*(a + b*Sinh[x])^4) + (4*a*(a^2 + b^
2))/(3*(a + b*Sinh[x])^3) - (3*a^2 + b^2)/(a + b*Sinh[x])^2 + (4*a)/(a + b
*Sinh[x]))/b^5
```

3.623.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4891, 3042, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sec(ix)-ib\tan(ix))^5}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\cosh^5(x)}{(a+b\sinh(x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^5}{(a-ib\sin(ix))^5}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{\left(\sinh^2(x)b^2+b^2\right)^2}{(a+b\sinh(x))^5}d(b\sinh(x))}{b^5}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(\frac{\left(a^2+b^2\right)^2}{(a+b\sinh(x))^5}-\frac{4a\left(a^2+b^2\right)}{(a+b\sinh(x))^4}+\frac{1}{a+b\sinh(x)}-\frac{4a}{(a+b\sinh(x))^2}+\frac{2\left(3a^2+b^2\right)}{(a+b\sinh(x))^3}\right)d(b\sinh(x))}{b^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\left(a^2+b^2\right)^2}{4(a+b\sinh(x))^4}+\frac{4a\left(a^2+b^2\right)}{3(a+b\sinh(x))^3}-\frac{3a^2+b^2}{(a+b\sinh(x))^2}+\frac{4a}{a+b\sinh(x)}+\log(a+b\sinh(x))}{b^5}$$

input
```
Int[(a*Sech[x] + b*Tanh[x])^(-5),x]
```

output
```
(Log[a + b*Sinh[x]] - (a^2 + b^2)^2/(4*(a + b*Sinh[x])^4) + (4*a*(a^2 + b^
2))/(3*(a + b*Sinh[x])^3) - (3*a^2 + b^2)/(a + b*Sinh[x])^2 + (4*a)/(a + b
*Sinh[x]))/b^5
```

3.623.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.623. $\int \frac{1}{(a\operatorname{sech}(x)+b\tanh(x))^5}\,dx$

3.623.4 Maple [A] (verified)

Time = 289.96 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.85

method	result
risch	$-\frac{x}{b^5}+\frac{4(6a\,b^3\mathrm{e}^{6x}+27a^2b^2\mathrm{e}^{5x}-3\,\mathrm{e}^{5x}b^4+44a^3b\,\mathrm{e}^{4x}-22a\,b^3\mathrm{e}^{4x}+25a^4\mathrm{e}^{3x}-56a^2b^2\mathrm{e}^{3x}+3\,\mathrm{e}^{3x}b^4-44\,\mathrm{e}^{2x}a^3b+22\,\mathrm{e}^{2x}a\,b^3+27a^2b^2\mathrm{e}^{x}-3}{3b^5(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b)^4}$
default	$-\frac{\ln(\tanh(\frac{x}{2})+1)}{b^5}+\frac{2\left(\frac{(a^4-b^4)b\tanh(\frac{x}{2})^7}{a}-\frac{b^2(7a^4-3b^4)\tanh(\frac{x}{2})^6}{a^2}-\frac{b(9a^6-52a^4b^2-a^2b^4+12b^6)\tanh(\frac{x}{2})^5}{3a^3}+\frac{2b^2(21a^6-25a^4b^2-7a^2b}{3a^4}\right.}{\left(\tanh(\frac{x}{2})^2a-2b\tan\right.}$

input
```
int(1/(a*sech(x)+b*tanh(x))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/b^5*x+4/3*(6*a*b^3*exp(6*x)+27*a^2*b^2*exp(5*x)-3*exp(5*x)*b^4+44*a^3*b
*exp(4*x)-22*a*b^3*exp(4*x)+25*a^4*exp(3*x)-56*a^2*b^2*exp(3*x)+3*exp(3*x)
*b^4-44*exp(2*x)*a^3*b+22*exp(2*x)*a*b^3+27*a^2*b^2*exp(x)-3*exp(x)*b^4-6*
a*b^3)/b^5*exp(x)/(b*exp(2*x)+2*a*exp(x)-b)^4+1/b^5*ln(exp(2*x)+2*a/b*exp(
x)-1)
```

3.623.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2640 vs. $2(91) = 182$.

Time = 0.28 (sec) , antiderivative size = 2640, normalized size of antiderivative = 27.79

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))^5,x, algorithm="fricas")
```

output
```
-1/3*(3*b^4*x*cosh(x)^8 + 3*b^4*x*sinh(x)^8 + 24*(a*b^3*x - a*b^3)*cosh(x)
^7 + 24*(b^4*x*cosh(x) + a*b^3*x - a*b^3)*sinh(x)^7 - 12*(9*a^2*b^2 - b^4
- (6*a^2*b^2 - b^4)*x)*cosh(x)^6 + 12*(7*b^4*x*cosh(x)^2 - 9*a^2*b^2 + b^4
 + (6*a^2*b^2 - b^4)*x + 14*(a*b^3*x - a*b^3)*cosh(x))*sinh(x)^6 - 8*(22*a
^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^5 + 8*(21*b^4*x*cosh(x)
^3 - 22*a^3*b + 11*a*b^3 + 63*(a*b^3*x - a*b^3)*cosh(x)^2 + 3*(4*a^3*b - 3
*a*b^3)*x - 9*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x))*sinh(x)^5 +
 3*b^4*x - 2*(50*a^4 - 112*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2 + 3*b^4
)*x)*cosh(x)^4 + 2*(105*b^4*x*cosh(x)^4 - 50*a^4 + 112*a^2*b^2 - 6*b^4 + 4
20*(a*b^3*x - a*b^3)*cosh(x)^3 - 90*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x
)*cosh(x)^2 + 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x - 20*(22*a^3*b - 11*a*b^3 -
 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x))*sinh(x)^4 + 8*(22*a^3*b - 11*a*b^3 - 3*
(4*a^3*b - 3*a*b^3)*x)*cosh(x)^3 + 8*(21*b^4*x*cosh(x)^5 + 105*(a*b^3*x -
a*b^3)*cosh(x)^4 + 22*a^3*b - 11*a*b^3 - 30*(9*a^2*b^2 - b^4 - (6*a^2*b^2
- b^4)*x)*cosh(x)^3 - 10*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*c
osh(x)^2 - 3*(4*a^3*b - 3*a*b^3)*x - (50*a^4 - 112*a^2*b^2 + 6*b^4 - 3*(8*
a^4 - 24*a^2*b^2 + 3*b^4)*x)*cosh(x))*sinh(x)^3 - 12*(9*a^2*b^2 - b^4 - (6
*a^2*b^2 - b^4)*x)*cosh(x)^2 + 4*(21*b^4*x*cosh(x)^6 + 126*(a*b^3*x - a*b^
3)*cosh(x)^5 - 45*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x)^4 - 27*a
^2*b^2 + 3*b^4 - 20*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cos...
```

3.623.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2162 vs. $2(92) = 184$.

Time = 6.20 (sec) , antiderivative size = 2162, normalized size of antiderivative = 22.76

$$\int \frac{1}{(a\operatorname{sech}(x) + b\tanh(x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))**5,x)
```

output
```
Piecewise((36*a**4*x*sech(x)**4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*t
anh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(
x)**3*sech(x) + 36*b**9*tanh(x)**4) + 36*a**4*log(a*sech(x)/b + tanh(x))*s
ech(x)**4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 21
6*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**
9*tanh(x)**4) - 36*a**4*log(tanh(x) + 1)*sech(x)**4/(36*a**4*b**5*sech(x)*
*4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**
2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 20*a**4*sech(x)*
*4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*
b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(
x)**4) + 144*a**3*b*x*tanh(x)*sech(x)**3/(36*a**4*b**5*sech(x)**4 + 144*a*
*3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b
**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 144*a**3*b*log(a*sech(x)/b
+ tanh(x))*tanh(x)*sech(x)**3/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tan
h(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)
**3*sech(x) + 36*b**9*tanh(x)**4) - 144*a**3*b*log(tanh(x) + 1)*tanh(x)*se
ch(x)**3/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216
*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9
*tanh(x)**4) + 44*a**3*b*tanh(x)*sech(x)**3/(36*a**4*b**5*sech(x)**4 + 144
*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 1...
```

3.623.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(91) = 182$.

Time = 0.25 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.13

$$\int \frac{1}{(a\operatorname{sech}(x) + b\tanh(x))^5}\,dx$$
$$= \frac{4\left(6\,ab^3e^{(-x)} - 6\,ab^3e^{(-7\,x)} + 3\left(9\,a^2b^2 - b^4\right)e^{(-2\,x)} + 22\left(2\,a^3b - ab^3\right)e^{(-3\,x)} + (25\,a^4 - 56\,a^2b^2 +}{3\left(8\,ab^8e^{(-x)} - 8\,ab^8e^{(-7\,x)} + b^9e^{(-8\,x)} + b^9 + 4\left(6\,a^2b^7 - b^9\right)e^{(-2\,x)} + 8\left(4\,a^3b^6 - 3\,ab^8\right)e^{(-3\,x)} + 2\,(8\,a^4b^5 - 2}$$
$$+ \frac{x}{b^5} + \frac{\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{b^5}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))^5,x, algorithm="maxima")
```

output
```
4/3*(6*a*b^3*e^(-x) - 6*a*b^3*e^(-7*x) + 3*(9*a^2*b^2 - b^4)*e^(-2*x) + 22
*(2*a^3*b - a*b^3)*e^(-3*x) + (25*a^4 - 56*a^2*b^2 + 3*b^4)*e^(-4*x) - 22*
(2*a^3*b - a*b^3)*e^(-5*x) + 3*(9*a^2*b^2 - b^4)*e^(-6*x))/(8*a*b^8*e^(-x)
 - 8*a*b^8*e^(-7*x) + b^9*e^(-8*x) + b^9 + 4*(6*a^2*b^7 - b^9)*e^(-2*x) +
8*(4*a^3*b^6 - 3*a*b^8)*e^(-3*x) + 2*(8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*e^(-
4*x) - 8*(4*a^3*b^6 - 3*a*b^8)*e^(-5*x) + 4*(6*a^2*b^7 - b^9)*e^(-6*x)) +
x/b^5 + log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^5
```

3.623.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a\mathrm{sech}(x) + b\tanh(x))^5}\,dx = \frac{\log\left(\left|-b\left(e^{(-x)} - e^{x}\right) + 2\,a\right|\right)}{b^5} - \frac{25\,b^3\left(e^{(-x)} - e^{x}\right)^4 - 104\,ab^2\left(e^{(-x)} - e^{x}\right)^3 + 168\,a^2b\left(e^{(-x)} - e^{x}\right)^2 + 48\,b^3\left(e^{(-x)} - e^{x}\right)^2 - 96\,a^3\left(e^{(-x)} - e\right.}{12\,\left(b\left(e^{(-x)} - e^{x}\right) - 2\,a\right)^4 b^4}$$

input
```
integrate(1/(a*sech(x)+b*tanh(x))^5,x, algorithm="giac")
```

output
```
log(abs(-b*(e^(-x) - e^x) + 2*a))/b^5 - 1/12*(25*b^3*(e^(-x) - e^x)^4 - 10
4*a*b^2*(e^(-x) - e^x)^3 + 168*a^2*b*(e^(-x) - e^x)^2 + 48*b^3*(e^(-x) - e
^x)^2 - 96*a^3*(e^(-x) - e^x) - 64*a*b^2*(e^(-x) - e^x) + 32*a^2*b + 48*b^
3)/((b*(e^(-x) - e^x) - 2*a)^4*b^4)
```

3.623.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\mathrm{sech}(x) + b\tanh(x))^5}\,dx = \int \frac{1}{\left(b\tanh(x) + \frac{a}{\cosh(x)}\right)^5}\,dx$$

input
```
int(1/(b*tanh(x) + a/cosh(x))^5,x)
```

output
```
int(1/(b*tanh(x) + a/cosh(x))^5, x)
```

3.624 $\int (\text{sech}(x)+i\tanh(x))^5\,dx$

3.624.1 Optimal result

Integrand size = 11, antiderivative size = 40

$$\int (\text{sech}(x)+i\tanh(x))^5\,dx = i\log(i+\sinh(x)) - \frac{2i}{(1-i\sinh(x))^2} + \frac{4i}{1-i\sinh(x)}$$

output
```
I*ln(I+sinh(x))-2*I/(1-I*sinh(x))^2+4*I/(1-I*sinh(x))
```

3.624.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.55

$$\begin{aligned}\int (\text{sech}(x)+i\tanh(x))^5\,dx &= \arctan(\sinh(x)) + i\log(\cosh(x)) - \frac{5}{4}i\text{sech}^4(x) \\ &\quad + \text{sech}(x)\tanh(x) - \text{sech}^3(x)\tanh(x) \\ &\quad - \frac{1}{2}i\tanh^2(x) - 5\text{sech}(x)\tanh^3(x) - \frac{11}{4}i\tanh^4(x)\end{aligned}$$

input
```
Integrate[(Sech[x] + I*Tanh[x])^5,x]
```

output
```
ArcTan[Sinh[x]] + I*Log[Cosh[x]] - ((5*I)/4)*Sech[x]^4 + Sech[x]*Tanh[x] -
 Sech[x]^3*Tanh[x] - (I/2)*Tanh[x]^2 - 5*Sech[x]*Tanh[x]^3 - ((11*I)/4)*Ta
nh[x]^4
```

3.624.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4891, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{sech}(x)+i\tanh(x))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\tan(ix)+\sec(ix))^5dx$$

$$\Big\downarrow \text{4891}$$

$$\int (1+i\sinh(x))^5\text{sech}^5(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(1+\sin(ix))^5}{\cos(ix)^5}dx$$

$$\Big\downarrow \text{3146}$$

$$-i\int \frac{(i\sinh(x)+1)^2}{(1-i\sinh(x))^3}d(i\sinh(x))$$

$$\Big\downarrow \text{49}$$

$$-i\int \left(-\frac{4}{(i\sinh(x)-1)^2}-\frac{4}{(i\sinh(x)-1)^3}+\frac{1}{1-i\sinh(x)}\right)d(i\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{4}{1-i\sinh(x)}+\frac{2}{(1-i\sinh(x))^2}-\log(1-i\sinh(x))\right)$$

input
```
Int[(Sech[x] + I*Tanh[x])^5,x]
```

output
```
(-I)*(-Log[1 - I*Sinh[x]] + 2/(1 - I*Sinh[x])^2 - 4/(1 - I*Sinh[x]))
```

3.624.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.624.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.

Time = 0.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.95

$$\frac{8\left(\frac{\operatorname{sech}(x)^3}{4}+\frac{3\operatorname{sech}(x)}{8}\right)\tanh(x)}{3}+2\arctan(e^x)+\frac{5i}{4\cosh(x)^4}-\frac{5\sinh(x)}{3\cosh(x)^4}+\frac{5i\sinh(x)^2}{\cosh(x)^4}-\frac{5\sinh(x)^3}{\cosh(x)^4}+i\ln(\cosh$$

input
```
int((sech(x)+I*tanh(x))^5,x)
```

output
```
8/3*(1/4*sech(x)^3+3/8*sech(x))*tanh(x)+2*arctan(exp(x))+5/4*I/cosh(x)^4-5
/3*sinh(x)/cosh(x)^4+5*I*sinh(x)^2/cosh(x)^4-5*sinh(x)^3/cosh(x)^4+I*ln(co
sh(x))-1/2*I*tanh(x)^2-1/4*I*tanh(x)^4
```

3.624.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.35

$$\int (\operatorname{sech}(x) + i\tanh(x))^5 \, dx$$
$$= \frac{-i\, xe^{(4\,x)} + 4\,(x-2)e^{(3\,x)} - 2\,(-3i\,x + 4i)e^{(2\,x)} - 4\,(x-2)e^{x} - 2\left(-i\,e^{(4\,x)} + 4\,e^{(3\,x)} + 6i\,e^{(2\,x)} - 4\,e^{x} - i\right)}{e^{(4\,x)} + 4i\,e^{(3\,x)} - 6\,e^{(2\,x)} - 4i\,e^{x} + 1}$$

input
```
integrate((sech(x)+I*tanh(x))^5,x, algorithm="fricas")
```

output
```
(-I*x*e^(4*x) + 4*(x - 2)*e^(3*x) - 2*(-3*I*x + 4*I)*e^(2*x) - 4*(x - 2)*e
^x - 2*(-I*e^(4*x) + 4*e^(3*x) + 6*I*e^(2*x) - 4*e^x - I)*log(e^x + I) - I
*x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

3.624.6 Sympy [F]

$$\int (\operatorname{sech}(x) + i\tanh(x))^5 \, dx = \int (i\tanh(x) + \operatorname{sech}(x))^5 \, dx$$

input
```
integrate((sech(x)+I*tanh(x))**5,x)
```

output
```
Integral((I*tanh(x) + sech(x))**5, x)
```

3.624.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 5.88

$$\int (\operatorname{sech}(x)+i\tanh(x))^5\,dx = -\frac{5}{2}i\,\tanh(x)^4 + i\,x - \frac{5\left(5\,e^{(-x)}-3\,e^{(-3\,x)}+3\,e^{(-5\,x)}-5\,e^{(-7\,x)}\right)}{4\left(4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1\right)} + \frac{3\,e^{(-x)}+11\,e^{(-3\,x)}-11\,e^{(-5\,x)}-3\,e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1\right)} - \frac{5\left(e^{(-x)}-7\,e^{(-3\,x)}+7\,e^{(-5\,x)}-e^{(-7\,x)}\right)}{2\left(4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1\right)} + \frac{4i\left(e^{(-2\,x)}+e^{(-4\,x)}+e^{(-6\,x)}\right)}{4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1} - \frac{20i}{\left(e^{(-x)}+e^{x}\right)^4} - 2\,\arctan\left(e^{(-x)}\right) + i\,\log\left(e^{(-2\,x)}+1\right)$$

input
```
integrate((sech(x)+I*tanh(x))^5,x, algorithm="maxima")
```

output
```
-5/2*I*tanh(x)^4 + I*x - 5/4*(5*e^(-x) - 3*e^(-3*x) + 3*e^(-5*x) - 5*e^(-7
*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 1/4*(3*e^(-x)
 + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^
(-6*x) + e^(-8*x) + 1) - 5/2*(e^(-x) - 7*e^(-3*x) + 7*e^(-5*x) - e^(-7*x))
/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 4*I*(e^(-2*x) + e
^(-4*x) + e^(-6*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1)
- 20*I/(e^(-x) + e^x)^4 - 2*arctan(e^(-x)) + I*log(e^(-2*x) + 1)
```

3.624.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int (\operatorname{sech}(x)+i\tanh(x))^5\,dx = -i\,x - \frac{8\left(e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}\right)}{\left(e^{x}+i\right)^4} + 2i\,\log\left(e^{x}+i\right)$$

input
```
integrate((sech(x)+I*tanh(x))^5,x, algorithm="giac")
```

output
```
-I*x - 8*(e^(3*x) + I*e^(2*x) - e^x)/(e^x + I)^4 + 2*I*log(e^x + I)
```

3.624.9 Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int (\mathrm{sech}(x) + i\tanh(x))^5\,dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^{x} + 1\mathrm{i}\right)\,2\mathrm{i} + \frac{16\mathrm{i}}{\mathrm{e}^{2\,x} - 1 + \mathrm{e}^{x}\,2\mathrm{i}} - \frac{8\mathrm{i}}{\mathrm{e}^{4\,x} - 6\,\mathrm{e}^{2\,x} + 1 + \mathrm{e}^{3\,x}\,4\mathrm{i} - \mathrm{e}^{x}\,4\mathrm{i}} - \frac{8}{\mathrm{e}^{x} + 1\mathrm{i}} + \frac{16}{\mathrm{e}^{2\,x}\,3\mathrm{i} + \mathrm{e}^{3\,x} - 3\,\mathrm{e}^{x} - \mathrm{i}}$$

input
```
int((tanh(x)*1i + 1/cosh(x))^5,x)
```

output
```
log(exp(x) + 1i)*2i - x*1i + 16i/(exp(2*x) + exp(x)*2i - 1) - 8i/(exp(3*x)
*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) - 8/(exp(x) + 1i) + 16/(exp(2
*x)*3i + exp(3*x) - 3*exp(x) - 1i)
```

3.625 $\int (\text{sech}(x) + i\tanh(x))^4 \, dx$

3.625.1 Optimal result

Integrand size = 11, antiderivative size = 38

$$\int (\text{sech}(x) + i\tanh(x))^4 \, dx = x - \frac{2i\cosh^3(x)}{3(1 - i\sinh(x))^3} + \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

output
```
x-2/3*I*cosh(x)^3/(1-I*sinh(x))^3+2*I*cosh(x)/(1-I*sinh(x))
```

3.625.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.95

$$\int (\text{sech}(x) + i\tanh(x))^4 \, dx = \frac{3(-8i + 3x)\cosh\left(\frac{x}{2}\right) + (16i - 3x)\cosh\left(\frac{3x}{2}\right) - 6i(-4i + 2x + x\cosh(x))\sinh\left(\frac{x}{2}\right)}{6\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^3}$$

input
```
Integrate[(Sech[x] + I*Tanh[x])^4,x]
```

output
```
(3*(-8*I + 3*x)*Cosh[x/2] + (16*I - 3*x)*Cosh[(3*x)/2] - (6*I)*(-4*I + 2*x
 + x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] - I*Sinh[x/2])^3)
```

3.625.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 4891, 3042, 3149, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{sech}(x)+i\tanh(x))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\tan(ix)+\sec(ix))^4dx$$

$$\Big\downarrow \text{4891}$$

$$\int (1+i\sinh(x))^4\text{sech}^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(1+\sin(ix))^4}{\cos(ix)^4}dx$$

$$\Big\downarrow \text{3149}$$

$$\int \frac{\cosh^4(x)}{(1-i\sinh(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^4}{(1-\sin(ix))^4}dx$$

$$\Big\downarrow \text{3159}$$

$$-\int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}dx-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\cos(ix)^2}{(1-\sin(ix))^2}dx-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{24}$$

$$x - \frac{2i \cosh^3(x)}{3(1 - i\sinh(x))^3} + \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

input
```
Int[(Sech[x] + I*Tanh[x])^4,x]
```

output
```
x - (((2*I)/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3 + ((2*I)*Cosh[x])/(1 - I*Sinh[
x])
```

3.625.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.625.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$-2\left(\frac{2}{3}+\frac{\operatorname{sech}\left(x\right)^{2}}{3}\right)\tanh\left(x\right)-\frac{4i}{3\cosh\left(x\right)^{3}}+\frac{3\sinh\left(x\right)}{\cosh\left(x\right)^{3}}-4i\left(-\frac{\sinh\left(x\right)^{2}}{\cosh\left(x\right)^{3}}-\frac{2}{3\cosh\left(x\right)^{3}}\right)+x-\tanh\left(x\right)-\frac{\tanh}{3}$$

input
```
int((sech(x)+I*tanh(x))^4,x)
```

output
```
-2*(2/3+1/3*sech(x)^2)*tanh(x)-4/3*I/cosh(x)^3+3*sinh(x)/cosh(x)^3-4*I*(-s
inh(x)^2/cosh(x)^3-2/3/cosh(x)^3)+x-tanh(x)-1/3*tanh(x)^3
```

3.625.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int(\operatorname{sech}(x)+i\tanh(x))^{4}\,dx=\frac{3\,xe^{(3\,x)}-3\,(-3i\,x-8i)e^{(2\,x)}-3\,(3\,x+8)e^{x}-3i\,x-16i}{3\,\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

input
```
integrate((sech(x)+I*tanh(x))^4,x, algorithm="fricas")
```

output
```
1/3*(3*x*e^(3*x) - 3*(-3*I*x - 8*I)*e^(2*x) - 3*(3*x + 8)*e^x - 3*I*x - 16
*I)/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

3.625.6 Sympy [F]

$$\int(\operatorname{sech}(x)+i\tanh(x))^{4}\,dx=\int\left(i\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^{4}\,dx$$

input
```
integrate((sech(x)+I*tanh(x))**4,x)
```

output
```
Integral((I*tanh(x) + sech(x))**4, x)
```

3.625.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(28) = 56$.

Time = 0.19 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.76

$$\int (\mathrm{sech}(x) + i\tanh(x))^4\,dx = -2\,\tanh(x)^3 + x - \frac{4\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + 2)}{3\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1)} + \frac{8i\,e^{(-x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{4\,e^{(-2\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{16i\,e^{(-3\,x)}}{3\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1)} + \frac{8i\,e^{(-5\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{4}{3\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1)} - \frac{32i}{3\,(e^{(-x)} + e^{x})^3}$$

input
```
integrate((sech(x)+I*tanh(x))^4,x, algorithm="maxima")
```

output
```
-2*tanh(x)^3 + x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4
*x) + e^(-6*x) + 1) + 8*I*e^(-x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1)
+ 4*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 16/3*I*e^(-3*x)/(3
*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8*I*e^(-5*x)/(3*e^(-2*x) + 3*e^(-
4*x) + e^(-6*x) + 1) + 4/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 32/3
*I/(e^(-x) + e^x)^3
```

3.625.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int (\mathrm{sech}(x) + i\tanh(x))^4\,dx = x - \frac{8\,(-3i\,e^{(2\,x)} + 3\,e^{x} + 2i)}{3\,(e^{x} + i)^3}$$

input
```
integrate((sech(x)+I*tanh(x))^4,x, algorithm="giac")
```

output
```
x - 8/3*(-3*I*e^(2*x) + 3*e^x + 2*I)/(e^x + I)^3
```

3.625.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int (\operatorname{sech}(x)+i\tanh(x))^4\,dx = x+\frac{\frac{e^{2x}\,8i}{3}-\frac{8}{3}i}{e^{2x}\,3i+e^{3x}-3\,e^{x}-i}+\frac{e^{x}\,8i}{3\,\left(e^{2x}-1+e^{x}\,2i\right)}+\frac{8i}{3\,\left(e^{x}+1i\right)}$$

input
```
int((tanh(x)*1i + 1/cosh(x))^4,x)
```

output
```
x + ((exp(2*x)*8i)/3 - 8i/3)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + (e
xp(x)*8i)/(3*(exp(2*x) + exp(x)*2i - 1)) + 8i/(3*(exp(x) + 1i))
```

3.626 $\int (\mathrm{sech}(x) + i \tanh(x))^3 \, dx$

3.626.1 Optimal result

Integrand size = 11, antiderivative size = 26

$$\int (\mathrm{sech}(x) + i \tanh(x))^3 \, dx = -i \log(i + \sinh(x)) - \frac{2i}{1 - i \sinh(x)}$$

output
```
-I*ln(I+sinh(x))-2*I/(1-I*sinh(x))
```

3.626.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int (\mathrm{sech}(x) + i \tanh(x))^3 \, dx = -\arctan(\sinh(x)) - i \log(\cosh(x)) - \frac{3}{2} i \mathrm{sech}^2(x) + 2 \mathrm{sech}(x) \tanh(x) + \frac{1}{2} i \tanh^2(x)$$

input
```
Integrate[(Sech[x] + I*Tanh[x])^3,x]
```

output
```
-ArcTan[Sinh[x]] - I*Log[Cosh[x]] - ((3*I)/2)*Sech[x]^2 + 2*Sech[x]*Tanh[x
] + (I/2)*Tanh[x]^2
```

3.626.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4891, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\operatorname{sech}(x)+i\tanh(x))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (\tan(ix)+\sec(ix))^3dx$$

$$\Big\downarrow \text{ 4891}$$

$$\int (1+i\sinh(x))^3\operatorname{sech}^3(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(1+\sin(ix))^3}{\cos(ix)^3}dx$$

$$\Big\downarrow \text{ 3146}$$

$$-i\int \frac{i\sinh(x)+1}{(1-i\sinh(x))^2}d(i\sinh(x))$$

$$\Big\downarrow \text{ 49}$$

$$-i\int \left(\frac{1}{i\sinh(x)-1}+\frac{2}{(i\sinh(x)-1)^2}\right)d(i\sinh(x))$$

$$\Big\downarrow \text{ 2009}$$

$$-i\left(\frac{2}{1-i\sinh(x)}+\log(1-i\sinh(x))\right)$$

input
```
Int[(Sech[x] + I*Tanh[x])^3,x]
```

output
```
(-I)*(Log[1 - I*Sinh[x]] + 2/(1 - I*Sinh[x]))
```

3.626.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.626.4 Maple [A] (verified)

Time = 73.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result
risch	$ix + \frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}+i)^{2}} - 2i\ln\left(\mathrm{e}^{x}+i\right)$
default	$-\operatorname{sech}(x)\tanh(x) - 2\arctan\left(\mathrm{e}^{x}\right) - \frac{3i}{2\cosh(x)^{2}} + \frac{3\sinh(x)}{\cosh(x)^{2}} - i\left(\ln\left(\cosh(x)\right) - \frac{\tanh(x)^{2}}{2}\right)$
parts	$-\operatorname{sech}(x)\tanh(x) - 2\arctan\left(\mathrm{e}^{x}\right) - i\left(-\frac{\tanh(x)^{2}}{2} - \frac{\ln(\tanh(x)-1)}{2} - \frac{\ln(1+\tanh(x))}{2}\right) + \frac{3i\tanh(x)^{2}}{2} + \frac{3\sin}{\cosh}$

input
```
int((sech(x)+I*tanh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
I*x+4*exp(x)/(exp(x)+I)^2-2*I*ln(exp(x)+I)
```

3.626.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int (\operatorname{sech}(x)+i\tanh(x))^3\,dx=\frac{i\,xe^{(2\,x)}-2\,(x-2)e^{x}-2\,\left(i\,e^{(2\,x)}-2\,e^{x}-i\right)\log\left(e^{x}+i\right)-i\,x}{e^{(2\,x)}+2i\,e^{x}-1}$$

input
```
integrate((sech(x)+I*tanh(x))^3,x, algorithm="fricas")
```

output
```
(I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(I*e^(2*x) - 2*e^x - I)*log(e^x + I) - I*
x)/(e^(2*x) + 2*I*e^x - 1)
```

3.626.6 Sympy [F]

$$\int (\operatorname{sech}(x)+i\tanh(x))^3\,dx=\int \left(i\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^3\,dx$$

input
```
integrate((sech(x)+I*tanh(x))**3,x)
```

output
```
Integral((I*tanh(x) + sech(x))**3, x)
```

3.626.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(18) = 36.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.81

$$\int (\operatorname{sech}(x)+i\tanh(x))^3\,dx=\frac{3}{2}i\,\tanh\left(x\right)^2-i\,x+\frac{4\left(e^{(-x)}-e^{(-3\,x)}\right)}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}-\frac{2i\,e^{(-2\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}+2\,\arctan\left(e^{(-x)}\right)-i\,\log\left(e^{(-2\,x)}+1\right)$$

input
```
integrate((sech(x)+I*tanh(x))^3,x, algorithm="maxima")
```

output
```
3/2*I*tanh(x)^2 - I*x + 4*(e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1)
- 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + 2*arctan(e^(-x)) - I*log(e^(-
2*x) + 1)
```

3.626.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int (\operatorname{sech}(x) + i\tanh(x))^3\,dx = i\,x + \frac{4\,e^x}{(e^x + i)^2} - 2i\,\log\left(e^x + i\right)$$

input
```
integrate((sech(x)+I*tanh(x))^3,x, algorithm="giac")
```

output
```
I*x + 4*e^x/(e^x + I)^2 - 2*I*log(e^x + I)
```

3.626.9 Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int (\operatorname{sech}(x) + i\tanh(x))^3\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^x + 1\mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{\mathrm{e}^{2\,x} - 1 + \mathrm{e}^x\,2\mathrm{i}} + \frac{4}{\mathrm{e}^x + 1\mathrm{i}}$$

input
```
int((tanh(x)*1i + 1/cosh(x))^3,x)
```

output
```
x*1i - log(exp(x) + 1i)*2i - 4i/(exp(2*x) + exp(x)*2i - 1) + 4/(exp(x) + 1
i)
```

3.627 $\int (\mathrm{sech}(x)+i\tanh(x))^2\,dx$

3.627.1 Optimal result

Integrand size = 11, antiderivative size = 20

$$\int (\mathrm{sech}(x)+i\tanh(x))^2\,dx=-x-\frac{2i\cosh(x)}{1-i\sinh(x)}$$

output
```
-x-2*I*cosh(x)/(1-I*sinh(x))
```

3.627.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (\mathrm{sech}(x)+i\tanh(x))^2\,dx=-\mathrm{arctanh}(\tanh(x))-2i\mathrm{sech}(x)+2\tanh(x)$$

input
```
Integrate[(Sech[x] + I*Tanh[x])^2,x]
```

output
```
-ArcTanh[Tanh[x]] - (2*I)*Sech[x] + 2*Tanh[x]
```

3.627.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 4891, 3042, 3149, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{sech}(x)+i\tanh(x))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\tan(ix)+\sec(ix))^2dx$$

$$\Big\downarrow \text{4891}$$

$$\int (1+i\sinh(x))^2\text{sech}^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(1+\sin(ix))^2}{\cos(ix)^2}dx$$

$$\Big\downarrow \text{3149}$$

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{(1-\sin(ix))^2}dx$$

$$\Big\downarrow \text{3159}$$

$$-\int 1dx-\frac{2i\cosh(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{24}$$

$$-x-\frac{2i\cosh(x)}{1-i\sinh(x)}$$

input
```
Int[(Sech[x] + I*Tanh[x])^2,x]
```

output
```
-x - ((2*I)*Cosh[x])/(1 - I*Sinh[x])
```

3.627.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.627.4 Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
risch	$-x-\frac{4i}{e^{x}+i}$	15
default	$2\tanh\left(x\right)-\frac{2i}{\cosh\left(x\right)}-x$	16
parts	$2\tanh\left(x\right)-2i\,\operatorname{sech}\left(x\right)+\frac{\ln\left(\tanh\left(x\right)-1\right)}{2}-\frac{\ln\left(1+\tanh\left(x\right)\right)}{2}$	25

input
```
int((sech(x)+I*tanh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-x-4*I/(exp(x)+I)
```

3.627.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int (\operatorname{sech}(x) + i\tanh(x))^2 \, dx = -\frac{xe^x + i\,x + 4i}{e^x + i}$$

input
```
integrate((sech(x)+I*tanh(x))^2,x, algorithm="fricas")
```

output
```
-(x*e^x + I*x + 4*I)/(e^x + I)
```

3.627.6 Sympy [F]

$$\int (\operatorname{sech}(x) + i\tanh(x))^2 \, dx = \int (i\tanh{(x)} + \operatorname{sech}{(x)})^2 \, dx$$

input
```
integrate((sech(x)+I*tanh(x))**2,x)
```

output
```
Integral((I*tanh(x) + sech(x))**2, x)
```

3.627.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int (\operatorname{sech}(x) + i\tanh(x))^2 \, dx = -x - \frac{4i}{e^{(-x)} + e^x} + \frac{4}{e^{(-2\,x)} + 1}$$

input
```
integrate((sech(x)+I*tanh(x))^2,x, algorithm="maxima")
```

output
```
-x - 4*I/(e^(-x) + e^x) + 4/(e^(-2*x) + 1)
```

3.627.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int (\operatorname{sech}(x) + i\tanh(x))^2\,dx = -x - \frac{4i}{e^x + i}$$

input
```
integrate((sech(x)+I*tanh(x))^2,x, algorithm="giac")
```

output
```
-x - 4*I/(e^x + I)
```

3.627.9 Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int (\operatorname{sech}(x) + i\tanh(x))^2\,dx = -x - \frac{4\mathrm{i}}{\mathrm{e}^x + 1\mathrm{i}}$$

input
```
int((tanh(x)*1i + 1/cosh(x))^2,x)
```

output
```
- x - 4i/(exp(x) + 1i)
```

3.628 $\int (\mathbf{sech}(x) + i\tanh(x))\,dx$

3.628.1 Optimal result

Integrand size = 9, antiderivative size = 11

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = \arctan(\sinh(x)) + i\log(\cosh(x))$$

output
```
arctan(sinh(x))+I*ln(cosh(x))
```

3.628.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = \arctan(\sinh(x)) + i\log(\cosh(x))$$

input
```
Integrate[Sech[x] + I*Tanh[x],x]
```

output
```
ArcTan[Sinh[x]] + I*Log[Cosh[x]]
```

3.628.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{sech}(x)+i\tanh(x))\,dx$$

$$\Big\downarrow \; 2009$$

$$\arctan(\sinh(x))+i\log(\cosh(x))$$

input
```
Int[Sech[x] + I*Tanh[x],x]
```

output
```
ArcTan[Sinh[x]] + I*Log[Cosh[x]]
```

3.628.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.628.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$\arctan(\sinh(x))+i\ln(\cosh(x))$	11
parts	$\arctan(\sinh(x))+i\ln(\cosh(x))$	11
risch	$i\ln(\mathrm{e}^{x}+i)-i\ln(\mathrm{e}^{x}-i)-ix+i\ln(1+\mathrm{e}^{2x})$	34
parallelrisch	$-\frac{i(\ln(1-\tanh(x))+\ln(1+\tanh(x))+2\ln(-i+\coth(x)-\text{csch}(x))-2\ln(i+\coth(x)-\text{csch}(x)))}{2}$	41

input
```
int(sech(x)+I*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
arctan(sinh(x))+I*ln(cosh(x))
```

3.628.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = -i\,x + 2i\,\log\left(e^{x} + i\right)$$

input
```
integrate(sech(x)+I*tanh(x),x, algorithm="fricas")
```

output
```
-I*x + 2*I*log(e^x + I)
```

3.628.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = i(x - \log\left(\tanh\left(x\right) + 1\right)) + 2\,\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
integrate(sech(x)+I*tanh(x),x)
```

output
```
I*(x - log(tanh(x) + 1)) + 2*atan(tanh(x/2))
```

3.628.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = \arctan\left(\sinh\left(x\right)\right) + i\,\log\left(\cosh\left(x\right)\right)$$

input
```
integrate(sech(x)+I*tanh(x),x, algorithm="maxima")
```

output
```
arctan(sinh(x)) + I*log(cosh(x))
```

3.628.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = -i\,x + 2\,\arctan\left(e^{x}\right) + i\,\log\left(e^{(2\,x)} + 1\right)$$

input
```
integrate(sech(x)+I*tanh(x),x, algorithm="giac")
```

output
```
-I*x + 2*arctan(e^x) + I*log(e^(2*x) + 1)
```

3.628.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^{x} + 1\mathrm{i}\right)\,2\mathrm{i}$$

input
```
int(tanh(x)*1i + 1/cosh(x),x)
```

output
```
log(exp(x) + 1i)*2i - x*1i
```

3.629 $\int \frac{1}{\operatorname{sech}(x)+i\tanh(x)}\,dx$

3.629.1 Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{1}{\operatorname{sech}(x)+i\tanh(x)}\,dx=-i\log(i-\sinh(x))$$

output
```
-I*ln(I-sinh(x))
```

3.629.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{\operatorname{sech}(x)+i\tanh(x)}\,dx=2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-i\log(\cosh(x))$$

input
```
Integrate[(Sech[x] + I*Tanh[x])^(-1),x]
```

output
```
2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]]
```

3.629.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3638, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{sech}(x)+i\tanh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\tan(ix)+\sec(ix)}dx$$
$$\Big\downarrow \text{3638}$$
$$\int \frac{\cosh(x)}{1+i\sinh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)}{1+\sin(ix)}dx$$
$$\Big\downarrow \text{3146}$$
$$-i\int \frac{1}{i\sinh(x)+1}d(i\sinh(x))$$
$$\Big\downarrow \text{16}$$
$$-i\log(1+i\sinh(x))$$

input
```
Int[(Sech[x] + I*Tanh[x])^(-1),x]
```

output
```
(-I)*Log[1 + I*Sinh[x]]
```

3.629.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3638
```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.629.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
risch	$ix - 2i \ln(e^x - i)$	15
default	$-2i \ln\left(\tanh\left(\frac{x}{2}\right) - i\right) + i \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) + i \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)$	33

input
```
int(1/(sech(x)+I*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
I*x-2*I*ln(exp(x)-I)
```

3.629.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\operatorname{sech}(x) + i \tanh(x)}\, dx = i\, x - 2i\, \log\left(e^{x} - i\right)$$

input
```
integrate(1/(sech(x)+I*tanh(x)),x, algorithm="fricas")
```

output
```
I*x - 2*I*log(e^x - I)
```

3.629.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{1}{\operatorname{sech}(x) + i \tanh(x)}\, dx = -ix + i \log\left(\tanh\left(x\right) + 1\right) - i \log\left(\tanh\left(x\right) - i \operatorname{sech}\left(x\right)\right)$$

input
```
integrate(1/(sech(x)+I*tanh(x)),x)
```

output
```
-I*x + I*log(tanh(x) + 1) - I*log(tanh(x) - I*sech(x))
```

3.629.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\operatorname{sech}(x) + i \tanh(x)}\, dx = -i\, x - 2i\, \log\left(i\, e^{(-x)} - 1\right)$$

input
```
integrate(1/(sech(x)+I*tanh(x)),x, algorithm="maxima")
```

output
```
-I*x - 2*I*log(I*e^(-x) - 1)
```

3.629.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\operatorname{sech}(x) + i\tanh(x)}\,dx = i\,x - 2i\,\log\left(e^{x} - i\right)$$

input
```
integrate(1/(sech(x)+I*tanh(x)),x, algorithm="giac")
```

output
```
I*x - 2*I*log(e^x - I)
```

3.629.9 Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\operatorname{sech}(x) + i\tanh(x)}\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^{x} - \mathrm{i}\right)\,2\mathrm{i}$$

input
```
int(1/(tanh(x)*1i + 1/cosh(x)),x)
```

output
```
x*1i - log(exp(x) - 1i)*2i
```

3.630 $\int \frac{1}{(\text{sech}(x)+i\tanh(x))^2}\,dx$

3.630.1 Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{1}{(\text{sech}(x)+i\tanh(x))^2}\,dx=-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

output
```
-x+2*I*cosh(x)/(1+I*sinh(x))
```

3.630.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(\text{sech}(x)+i\tanh(x))^2}\,dx=-x+\frac{4\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

input
```
Integrate[(Sech[x] + I*Tanh[x])^(-2),x]
```

output
```
-x + (4*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

3.630.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4891, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\tan(ix)+\sec(ix))^2}dx$$

$$\Big\downarrow \text{ 4891}$$

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ix)^2}{(1+\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 3159}$$

$$-\int 1dx+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

$$\Big\downarrow \text{ 24}$$

$$-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

input
```
Int[(Sech[x] + I*Tanh[x])^(-2),x]
```

output
```
-x + ((2*I)*Cosh[x])/(1 + I*Sinh[x])
```

3.630.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.630.4 Maple [A] (verified)

Time = 7.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
risch	$-x+\frac{4i}{{\mathrm e}^{x}-i}$	15
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{4}{\tanh\left(\frac{x}{2}\right)-i}-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	29

input
```
int(1/(sech(x)+I*tanh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-x+4*I/(exp(x)-I)
```

3.630.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx=-\frac{xe^x-i\,x-4i}{e^x-i}$$

input
```
integrate(1/(sech(x)+I*tanh(x))^2,x, algorithm="fricas")
```

output
```
-(x*e^x - I*x - 4*I)/(e^x - I)
```

3.630.6 Sympy [F]

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx=\int \frac{1}{(i\tanh{(x)}+\operatorname{sech}{(x)})^2}\,dx$$

input
```
integrate(1/(sech(x)+I*tanh(x))**2,x)
```

output
```
Integral((I*tanh(x) + sech(x))**(-2), x)
```

3.630.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx=-x+\frac{4i}{e^{(-x)}+i}$$

input
```
integrate(1/(sech(x)+I*tanh(x))^2,x, algorithm="maxima")
```

output
```
-x + 4*I/(e^(-x) + I)
```

3.630.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{1}{(\operatorname{sech}(x) + i\tanh(x))^2}\,dx = -x + \frac{4i}{e^x - i}$$

input
```
integrate(1/(sech(x)+I*tanh(x))^2,x, algorithm="giac")
```

output
```
-x + 4*I/(e^x - I)
```

3.630.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{(\operatorname{sech}(x) + i\tanh(x))^2}\,dx = -x + \frac{4\mathrm{i}}{\mathrm{e}^x - \mathrm{i}}$$

input
```
int(1/(tanh(x)*1i + 1/cosh(x))^2,x)
```

output
```
4i/(exp(x) - 1i) - x
```

3.631 $\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}\,dx$

3.631.1 Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}\,dx = i\log(i-\sinh(x))+\frac{2i}{1+i\sinh(x)}$$

output
```
I*ln(I-sinh(x))+2*I/(1+I*sinh(x))
```

3.631.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}\,dx = -2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+i\log(\cosh(x)) + \frac{2i}{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^2}$$

input
```
Integrate[(Sech[x] + I*Tanh[x])^(-3),x]
```

output
```
-2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]] + (2*I)/(Cosh[x/2] + I*Sinh[x/2])^2
```

3.631.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4891, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\tan(ix)+\sec(ix))^3}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3}{(1+\sin(ix))^3}dx$$

$$\Big\downarrow \text{3146}$$

$$-i\int \frac{1-i\sinh(x)}{(i\sinh(x)+1)^2}d(i\sinh(x))$$

$$\Big\downarrow \text{49}$$

$$-i\int \left(\frac{2}{(i\sinh(x)+1)^2}+\frac{1}{-i\sinh(x)-1}\right)d(i\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{2}{1+i\sinh(x)}-\log(1+i\sinh(x))\right)$$

input
```
Int[(Sech[x] + I*Tanh[x])^(-3),x]
```

output
```
(-I)*(-Log[1 + I*Sinh[x]] - 2/(1 + I*Sinh[x]))
```

3.631.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.631.4 Maple [A] (verified)

Time = 166.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
risch	$-ix+\frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}-i)^{2}}+2i\ln\left(\mathrm{e}^{x}-i\right)$	26
default	$2i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{4}{\tanh\left(\frac{x}{2}\right)-i}-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	56

input
```
int(1/(sech(x)+I*tanh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-I*x+4/(exp(x)-I)^2*exp(x)+2*I*ln(exp(x)-I)
```

3.631.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{1}{(\operatorname{sech}(x) + i\tanh(x))^3}\,dx = \frac{-i\,xe^{(2\,x)} - 2\,(x-2)e^x - 2\left(-i\,e^{(2\,x)} - 2\,e^x + i\right)\log\left(e^x - i\right) + i\,x}{e^{(2\,x)} - 2i\,e^x - 1}$$

input
```
integrate(1/(sech(x)+I*tanh(x))^3,x, algorithm="fricas")
```

output
```
(-I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(-I*e^(2*x) - 2*e^x + I)*log(e^x - I) +
I*x)/(e^(2*x) - 2*I*e^x - 1)
```

3.631.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(17) = 34.

Time = 0.98 (sec) , antiderivative size = 432, normalized size of antiderivative = 15.43

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}\,dx = -\frac{2ix\tanh^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} - \frac{4x\tanh(x)\operatorname{sech}(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{2ix\operatorname{sech}^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{2i\log(\tanh(x)+1)\tanh^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{4\log(\tanh(x)+1)\tanh(x)\operatorname{sech}(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} - \frac{2i\log(\tanh(x)+1)\operatorname{sech}^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} - \frac{2i\log(\tanh(x)-i\operatorname{sech}(x))\tanh^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} - \frac{4\log(\tanh(x)-i\operatorname{sech}(x))\tanh(x)\operatorname{sech}(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{2i\log(\tanh(x)-i\operatorname{sech}(x))\operatorname{sech}^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{i\tanh^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{i\operatorname{sech}^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{i}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)}$$

input
```
integrate(1/(sech(x)+I*tanh(x))**3,x)
```

3.631. $\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}\,dx$

output
```
-2*I*x*tanh(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 4
*x*tanh(x)*sech(x)/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) +
2*I*x*sech(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) + 2*
I*log(tanh(x) + 1)*tanh(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sec
h(x)**2) + 4*log(tanh(x) + 1)*tanh(x)*sech(x)/(-2*tanh(x)**2 + 4*I*tanh(x)
*sech(x) + 2*sech(x)**2) - 2*I*log(tanh(x) + 1)*sech(x)**2/(-2*tanh(x)**2
+ 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 2*I*log(tanh(x) - I*sech(x))*tanh(
x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 4*log(tanh(x)
 - I*sech(x))*tanh(x)*sech(x)/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sec
h(x)**2) + 2*I*log(tanh(x) - I*sech(x))*sech(x)**2/(-2*tanh(x)**2 + 4*I*ta
nh(x)*sech(x) + 2*sech(x)**2) + I*tanh(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*
sech(x) + 2*sech(x)**2) + I*sech(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x
) + 2*sech(x)**2) + I/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2)
```

3.631.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}\,dx = i\,x - \frac{4\,e^{(-x)}}{2i\,e^{(-x)}+e^{(-2\,x)}-1} + 2i\,\log\left(e^{(-x)}+i\right)$$

input
```
integrate(1/(sech(x)+I*tanh(x))^3,x, algorithm="maxima")
```

output
```
I*x - 4*e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) + 2*I*log(e^(-x) + I)
```

3.631.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}\,dx = -i\,x + \frac{4\,e^{x}}{\left(e^{x}-i\right)^2} + 2i\,\log\left(e^{x}-i\right)$$

input
```
integrate(1/(sech(x)+I*tanh(x))^3,x, algorithm="giac")
```

output
```
-I*x + 4*e^x/(e^x - I)^2 + 2*I*log(e^x - I)
```

3.631.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}\,dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^{x}-\mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}} + \frac{4}{\mathrm{e}^{x}-\mathrm{i}}$$

input
```
int(1/(tanh(x)*1i + 1/cosh(x))^3,x)
```

output
```
log(exp(x) - 1i)*2i - x*1i - 4i/(exp(x)*2i - exp(2*x) + 1) + 4/(exp(x) - 1
i)
```

3.632 $\int \frac{1}{(\text{sech}(x)+i\tanh(x))^4}\,dx$

3.632.1 Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{1}{(\text{sech}(x)+i\tanh(x))^4}\,dx = x+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}$$

output
```
x+2/3*I*cosh(x)^3/(1+I*sinh(x))^3-2*I*cosh(x)/(1+I*sinh(x))
```

3.632.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int \frac{1}{(\text{sech}(x)+i\tanh(x))^4}\,dx = \frac{3(8i+3x)\cosh\left(\frac{x}{2}\right)-(16i+3x)\cosh\left(\frac{3x}{2}\right)+6i(4i+2x+x\cosh(x))\sinh\left(\frac{x}{2}\right)}{6\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

input
```
Integrate[(Sech[x] + I*Tanh[x])^(-4),x]
```

output
```
(3*(8*I + 3*x)*Cosh[x/2] - (16*I + 3*x)*Cosh[(3*x)/2] + (6*I)*(4*I + 2*x +
 x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] + I*Sinh[x/2])^3)
```

3.632.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 4891, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\tan(ix)+\sec(ix))^4}dx$$

$$\Big\downarrow \text{ 4891}$$

$$\int \frac{\cosh^4(x)}{(1+i\sinh(x))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ix)^4}{(1+\sin(ix))^4}dx$$

$$\Big\downarrow \text{ 3159}$$

$$\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\int \frac{\cosh^2(x)}{(i\sinh(x)+1)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\int \frac{\cos(ix)^2}{(\sin(ix)+1)^2}dx$$

$$\Big\downarrow \text{ 3159}$$

$$\int 1dx+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}$$

$$\Big\downarrow \text{ 24}$$

$$x+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}$$

input
```
Int[(Sech[x] + I*Tanh[x])^(-4),x]
```

output
```
x + (((2*I)/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3 - ((2*I)*Cosh[x])/(1 + I*Sinh[
x])
```

3.632.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.632.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^2}-\frac{16}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^3}+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

input
```
int(1/(sech(x)+I*tanh(x))^4,x)
```

output
```
8*I/(tanh(1/2*x)-I)^2-16/3/(tanh(1/2*x)-I)^3+ln(tanh(1/2*x)+1)-ln(tanh(1/2
*x)-1)
```

3.632.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx=\frac{3\,xe^{(3\,x)}-3\,(3i\,x+8i)e^{(2\,x)}-3\,(3\,x+8)e^{x}+3i\,x+16i}{3\,(e^{(3\,x)}-3i\,e^{(2\,x)}-3\,e^{x}+i)}$$

input
```
integrate(1/(sech(x)+I*tanh(x))^4,x, algorithm="fricas")
```

output
```
1/3*(3*x*e^(3*x) - 3*(3*I*x + 8*I)*e^(2*x) - 3*(3*x + 8)*e^x + 3*I*x + 16*
I)/(e^(3*x) - 3*I*e^(2*x) - 3*e^x + I)
```

3.632.6 Sympy [F]

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx=\int \frac{1}{(i\tanh{(x)}+\operatorname{sech}{(x)})^4}\,dx$$

input
```
integrate(1/(sech(x)+I*tanh(x))**4,x)
```

output
```
Integral((I*tanh(x) + sech(x))**(-4), x)
```

3.632.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx=x-\frac{8\left(3\,e^{(-x)}-3i\,e^{(-2\,x)}+2i\right)}{3\,(3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i)}$$

input
```
integrate(1/(sech(x)+I*tanh(x))^4,x, algorithm="maxima")
```

output
```
x - 8/3*(3*e^(-x) - 3*I*e^(-2*x) + 2*I)/(3*e^(-x) - 3*I*e^(-2*x) - e^(-3*x
) + I)
```

3.632.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx = x - \frac{8\left(3i\,e^{(2\,x)}+3\,e^{x}-2i\right)}{3\left(e^{x}-i\right)^{3}}$$

input
```
integrate(1/(sech(x)+I*tanh(x))^4,x, algorithm="giac")
```

output
```
x - 8/3*(3*I*e^(2*x) + 3*e^x - 2*I)/(e^x - I)^3
```

3.632.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx = x + \frac{\frac{e^{2\,x}\,8i}{3}-\frac{8}{3}i}{e^{2\,x}\,3i-e^{3\,x}+3\,e^{x}-i} - \frac{8i}{3\,\left(e^{x}-i\right)} + \frac{e^{x}\,8i}{3\,\left(1-e^{2\,x}+e^{x}\,2i\right)}$$

input
```
int(1/(tanh(x)*1i + 1/cosh(x))^4,x)
```

output
```
x + ((exp(2*x)*8i)/3 - 8i/3)/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 8i
/(3*(exp(x) - 1i)) + (exp(x)*8i)/(3*(exp(x)*2i - exp(2*x) + 1))
```

3.633 $\int \frac{1}{(\text{sech}(x)+i\tanh(x))^5}\,dx$

3.633.1 Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{(\text{sech}(x)+i\tanh(x))^5}\,dx=-i\log(i-\sinh(x))+\frac{2i}{(1+i\sinh(x))^2}-\frac{4i}{1+i\sinh(x)}$$

output
```
-I*ln(I-sinh(x))+2*I/(1+I*sinh(x))^2-4*I/(1+I*sinh(x))
```

3.633.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{(\text{sech}(x)+i\tanh(x))^5}\,dx = 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-i\log(\cosh(x))+\frac{-2i+4\sinh(x)}{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^4}$$

input
```
Integrate[(Sech[x] + I*Tanh[x])^(-5),x]
```

output
```
2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]] + (-2*I + 4*Sinh[x])/(Cosh[x/2] + I*S
inh[x/2])^4
```

3.633.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4891, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\tan(ix)+\sec(ix))^5}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\cosh^5(x)}{(1+i\sinh(x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^5}{(1+\sin(ix))^5}dx$$

$$\Big\downarrow \text{3146}$$

$$-i\int \frac{(1-i\sinh(x))^2}{(i\sinh(x)+1)^3}d(i\sinh(x))$$

$$\Big\downarrow \text{49}$$

$$-i\int \left(\frac{1}{i\sinh(x)+1}-\frac{4}{(i\sinh(x)+1)^2}+\frac{4}{(i\sinh(x)+1)^3}\right)d(i\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{4}{1+i\sinh(x)}-\frac{2}{(1+i\sinh(x))^2}+\log(1+i\sinh(x))\right)$$

input
```
Int[(Sech[x] + I*Tanh[x])^(-5),x]
```

output
```
(-I)*(Log[1 + I*Sinh[x]] - 2/(1 + I*Sinh[x])^2 + 4/(1 + I*Sinh[x]))
```

3.633.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.633.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.62

$$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{4}}-2i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}+\frac{16}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}+i\ln\left(\tan\right.$$

input
```
int(1/(sech(x)+I*tanh(x))^5,x)
```

output
```
I*ln(tanh(1/2*x)-1)+8*I/(tanh(1/2*x)-I)^4-2*I*ln(tanh(1/2*x)-I)-8*I/(tanh(
1/2*x)-I)^2+16/(tanh(1/2*x)-I)^3+I*ln(tanh(1/2*x)+1)
```

3.633.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(30) = 60.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}\,dx$$
$$= \frac{i\,xe^{(4\,x)}+4\,(x-2)e^{(3\,x)}-2\,(3i\,x-4i)e^{(2\,x)}-4\,(x-2)e^{x}-2\,\left(i\,e^{(4\,x)}+4\,e^{(3\,x)}-6i\,e^{(2\,x)}-4\,e^{x}+i\right)\log\left(e\right.}{e^{(4\,x)}-4i\,e^{(3\,x)}-6\,e^{(2\,x)}+4i\,e^{x}+1}$$

input
```
integrate(1/(sech(x)+I*tanh(x))^5,x, algorithm="fricas")
```

output
```
(I*x*e^(4*x) + 4*(x - 2)*e^(3*x) - 2*(3*I*x - 4*I)*e^(2*x) - 4*(x - 2)*e^x
 - 2*(I*e^(4*x) + 4*e^(3*x) - 6*I*e^(2*x) - 4*e^x + I)*log(e^x - I) + I*x)
/(e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)
```

3.633.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1445 vs. 2(29) = 58.

Time = 4.26 (sec) , antiderivative size = 1445, normalized size of antiderivative = 34.40

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(sech(x)+I*tanh(x))**5,x)
```

output
```
-36*I*x*tanh(x)**4/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)
**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 144*x*tanh(x)
**3*sech(x)/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sec
h(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 216*I*x*tanh(x)**2*s
ech(x)**2/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(
x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 144*x*tanh(x)*sech(x)*
*3/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 +
 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 36*I*x*sech(x)**4/(36*tanh(x)
**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)
*sech(x)**3 + 36*sech(x)**4) + 36*I*log(tanh(x) + 1)*tanh(x)**4/(36*tanh(x
)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x
)*sech(x)**3 + 36*sech(x)**4) + 144*log(tanh(x) + 1)*tanh(x)**3*sech(x)/(3
6*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*
I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 216*I*log(tanh(x) + 1)*tanh(x)**2*
sech(x)**2/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech
(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 144*log(tanh(x) + 1)*
tanh(x)*sech(x)**3/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)
**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 36*I*log(tanh
(x) + 1)*sech(x)**4/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x
)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 36*I*log(...
```

3.633.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}\,dx = -i\,x - \frac{8\left(e^{(-x)} - i\,e^{(-2\,x)} - e^{(-3\,x)}\right)}{-4i\,e^{(-x)} - 6\,e^{(-2\,x)} + 4i\,e^{(-3\,x)} + e^{(-4\,x)} + 1} - 2i\,\log\left(e^{(-x)} + i\right)$$

input
```
integrate(1/(sech(x)+I*tanh(x))^5,x, algorithm="maxima")
```

output
```
-I*x - 8*(e^(-x) - I*e^(-2*x) - e^(-3*x))/(-4*I*e^(-x) - 6*e^(-2*x) + 4*I*
e^(-3*x) + e^(-4*x) + 1) - 2*I*log(e^(-x) + I)
```

3.633.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}\,dx = i\,x - \frac{8\left(e^{(3\,x)} - i\,e^{(2\,x)} - e^{x}\right)}{\left(e^{x}-i\right)^{4}} - 2i\,\log\left(e^{x}-i\right)$$

input
```
integrate(1/(sech(x)+I*tanh(x))^5,x, algorithm="giac")
```

output
```
I*x - 8*(e^(3*x) - I*e^(2*x) - e^x)/(e^x - I)^4 - 2*I*log(e^x - I)
```

3.633.9 Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^{x}-\mathrm{i}\right)\,2\mathrm{i} - \frac{16}{\mathrm{e}^{2\,x}\,3\mathrm{i} - \mathrm{e}^{3\,x} + 3\,\mathrm{e}^{x} - \mathrm{i}} + \frac{8\mathrm{i}}{\mathrm{e}^{4\,x} - 6\,\mathrm{e}^{2\,x} + 1 - \mathrm{e}^{3\,x}\,4\mathrm{i} + \mathrm{e}^{x}\,4\mathrm{i}} + \frac{16\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^{x}\,2\mathrm{i}} - \frac{8}{\mathrm{e}^{x}-\mathrm{i}}$$

input
```
int(1/(tanh(x)*1i + 1/cosh(x))^5,x)
```

output
```
x*1i - log(exp(x) - 1i)*2i - 16/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) +
 8i/(exp(4*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1) + 16i/(exp(x)*2i
 - exp(2*x) + 1) - 8/(exp(x) - 1i)
```

3.634 $\int (\text{sech}(x) - i\tanh(x))^5 \, dx$

3.634.1 Optimal result

Integrand size = 11, antiderivative size = 42

$$\int (\text{sech}(x) - i\tanh(x))^5 \, dx = -i\log(i - \sinh(x)) + \frac{2i}{(1 + i\sinh(x))^2} - \frac{4i}{1 + i\sinh(x)}$$

output
```
-I*ln(I-sinh(x))+2*I/(1+I*sinh(x))^2-4*I/(1+I*sinh(x))
```

3.634.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\begin{aligned}\int (\text{sech}(x) - i\tanh(x))^5 \, dx = {} & \arctan(\sinh(x)) - i\log(\cosh(x)) + \frac{5}{4} i \text{sech}^4(x) \\ & + \text{sech}(x)\tanh(x) - \text{sech}^3(x)\tanh(x) \\ & + \frac{1}{2} i\tanh^2(x) - 5\text{sech}(x)\tanh^3(x) + \frac{11}{4} i\tanh^4(x)\end{aligned}$$

input
```
Integrate[(Sech[x] - I*Tanh[x])^5,x]
```

output
```
ArcTan[Sinh[x]] - I*Log[Cosh[x]] + ((5*I)/4)*Sech[x]^4 + Sech[x]*Tanh[x] -
 Sech[x]^3*Tanh[x] + (I/2)*Tanh[x]^2 - 5*Sech[x]*Tanh[x]^3 + ((11*I)/4)*Ta
nh[x]^4
```

3.634.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4891, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{sech}(x)-i\tanh(x))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sec(ix)-\tan(ix))^5dx$$

$$\Big\downarrow \text{4891}$$

$$\int (1-i\sinh(x))^5\text{sech}^5(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(1-\sin(ix))^5}{\cos(ix)^5}dx$$

$$\Big\downarrow \text{3146}$$

$$i\int \frac{(1-i\sinh(x))^2}{(i\sinh(x)+1)^3}d(-i\sinh(x))$$

$$\Big\downarrow \text{49}$$

$$i\int \left(-\frac{4}{(-i\sinh(x)-1)^2}-\frac{4}{(-i\sinh(x)-1)^3}+\frac{1}{i\sinh(x)+1}\right)d(-i\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{4}{1+i\sinh(x)}+\frac{2}{(1+i\sinh(x))^2}-\log(1+i\sinh(x))\right)$$

input
```
Int[(Sech[x] - I*Tanh[x])^5,x]
```

output
```
I*(-Log[1 + I*Sinh[x]] + 2/(1 + I*Sinh[x])^2 - 4/(1 + I*Sinh[x]))
```

3.634.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.634.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(36) = 72$.

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\frac{8\left(\frac{\operatorname{sech}(x)^3}{4}+\frac{3\operatorname{sech}(x)}{8}\right)\tanh(x)}{3}+2\arctan(e^x)-\frac{5i}{4\cosh(x)^4}-\frac{5\sinh(x)}{3\cosh(x)^4}-\frac{5i\sinh(x)^2}{\cosh(x)^4}-\frac{5\sinh(x)^3}{\cosh(x)^4}-i\ln(\cosh$$

input
```
int((sech(x)-I*tanh(x))^5,x)
```

output
```
8/3*(1/4*sech(x)^3+3/8*sech(x))*tanh(x)+2*arctan(exp(x))-5/4*I/cosh(x)^4-5
/3*sinh(x)/cosh(x)^4-5*I*sinh(x)^2/cosh(x)^4-5*sinh(x)^3/cosh(x)^4-I*ln(co
sh(x))+1/2*I*tanh(x)^2+1/4*I*tanh(x)^4
```

3.634.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int (\text{sech}(x) - i\tanh(x))^5 \, dx$$
$$= \frac{i\,xe^{(4\,x)} + 4\,(x-2)e^{(3\,x)} - 2\,(3i\,x - 4i)e^{(2\,x)} - 4\,(x-2)e^{x} - 2\left(i\,e^{(4\,x)} + 4\,e^{(3\,x)} - 6i\,e^{(2\,x)} - 4\,e^{x} + i\right)\log\left(e\right.}{e^{(4\,x)} - 4i\,e^{(3\,x)} - 6\,e^{(2\,x)} + 4i\,e^{x} + 1}$$

input
```
integrate((sech(x)-I*tanh(x))^5,x, algorithm="fricas")
```

output
```
(I*x*e^(4*x) + 4*(x - 2)*e^(3*x) - 2*(3*I*x - 4*I)*e^(2*x) - 4*(x - 2)*e^x
 - 2*(I*e^(4*x) + 4*e^(3*x) - 6*I*e^(2*x) - 4*e^x + I)*log(e^x - I) + I*x)
/(e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)
```

3.634.6 Sympy [F]

$$\int (\text{sech}(x) - i\tanh(x))^5 \, dx = \int (-i\tanh(x) + \text{sech}(x))^5 \, dx$$

input
```
integrate((sech(x)-I*tanh(x))**5,x)
```

output
```
Integral((-I*tanh(x) + sech(x))**5, x)
```

3.634.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 5.60

$$\int (\operatorname{sech}(x) - i\tanh(x))^5\,dx = \frac{5}{2}i\tanh(x)^4 - ix - \frac{5\left(5e^{(-x)} - 3e^{(-3x)} + 3e^{(-5x)} - 5e^{(-7x)}\right)}{4\left(4e^{(-2x)} + 6e^{(-4x)} + 4e^{(-6x)} + e^{(-8x)} + 1\right)} + \frac{3e^{(-x)} + 11e^{(-3x)} - 11e^{(-5x)} - 3e^{(-7x)}}{4\left(4e^{(-2x)} + 6e^{(-4x)} + 4e^{(-6x)} + e^{(-8x)} + 1\right)} - \frac{5\left(e^{(-x)} - 7e^{(-3x)} + 7e^{(-5x)} - e^{(-7x)}\right)}{2\left(4e^{(-2x)} + 6e^{(-4x)} + 4e^{(-6x)} + e^{(-8x)} + 1\right)} - \frac{4i\left(e^{(-2x)} + e^{(-4x)} + e^{(-6x)}\right)}{4e^{(-2x)} + 6e^{(-4x)} + 4e^{(-6x)} + e^{(-8x)} + 1} + \frac{20i}{\left(e^{(-x)} + e^x\right)^4} - 2\arctan\left(e^{(-x)}\right) - i\log\left(e^{(-2x)} + 1\right)$$

input
```
integrate((sech(x)-I*tanh(x))^5,x, algorithm="maxima")
```

output
```
5/2*I*tanh(x)^4 - I*x - 5/4*(5*e^(-x) - 3*e^(-3*x) + 3*e^(-5*x) - 5*e^(-7*
x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 1/4*(3*e^(-x)
+ 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(
-6*x) + e^(-8*x) + 1) - 5/2*(e^(-x) - 7*e^(-3*x) + 7*e^(-5*x) - e^(-7*x))/
(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 4*I*(e^(-2*x) + e^
(-4*x) + e^(-6*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) +
 20*I/(e^(-x) + e^x)^4 - 2*arctan(e^(-x)) - I*log(e^(-2*x) + 1)
```

3.634.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int (\operatorname{sech}(x) - i\tanh(x))^5\,dx = ix - \frac{8\left(e^{(3x)} - ie^{(2x)} - e^x\right)}{(e^x - i)^4} - 2i\log\left(e^x - i\right)$$

input
```
integrate((sech(x)-I*tanh(x))^5,x, algorithm="giac")
```

output
```
I*x - 8*(e^(3*x) - I*e^(2*x) - e^x)/(e^x - I)^4 - 2*I*log(e^x - I)
```

3.634.9 Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int (\mathrm{sech}(x)-i\tanh(x))^5\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^x-\mathrm{i}\right)\,2\mathrm{i} - \frac{16}{\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^x-\mathrm{i}} + \frac{8\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1-\mathrm{e}^{3\,x}\,4\mathrm{i}+\mathrm{e}^x\,4\mathrm{i}} + \frac{16\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,2\mathrm{i}} - \frac{8}{\mathrm{e}^x-\mathrm{i}}$$

input
```
int(-(tanh(x)*1i - 1/cosh(x))^5,x)
```

output
```
x*1i - log(exp(x) - 1i)*2i - 16/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) +
 8i/(exp(4*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1) + 16i/(exp(x)*2i
 - exp(2*x) + 1) - 8/(exp(x) - 1i)
```

3.635 $\int (\text{sech}(x) - i\tanh(x))^4 \, dx$

3.635.1 Optimal result

Integrand size = 11, antiderivative size = 38

$$\int (\text{sech}(x) - i\tanh(x))^4 \, dx = x + \frac{2i\cosh^3(x)}{3(1 + i\sinh(x))^3} - \frac{2i\cosh(x)}{1 + i\sinh(x)}$$

output
```
x+2/3*I*cosh(x)^3/(1+I*sinh(x))^3-2*I*cosh(x)/(1+I*sinh(x))
```

3.635.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int (\text{sech}(x) - i\tanh(x))^4 \, dx = \frac{3(8i + 3x)\cosh\left(\frac{x}{2}\right) - (16i + 3x)\cosh\left(\frac{3x}{2}\right) + 6i(4i + 2x + x\cosh(x))\sinh\left(\frac{x}{2}\right)}{6\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^3}$$

input
```
Integrate[(Sech[x] - I*Tanh[x])^4,x]
```

output
```
(3*(8*I + 3*x)*Cosh[x/2] - (16*I + 3*x)*Cosh[(3*x)/2] + (6*I)*(4*I + 2*x + 
 x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] + I*Sinh[x/2])^3)
```

3.635.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 4891, 3042, 3149, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{sech}(x)-i\tanh(x))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sec(ix)-\tan(ix))^4dx$$

$$\Big\downarrow \text{4891}$$

$$\int (1-i\sinh(x))^4\text{sech}^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(1-\sin(ix))^4}{\cos(ix)^4}dx$$

$$\Big\downarrow \text{3149}$$

$$\int \frac{\cosh^4(x)}{(1+i\sinh(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^4}{(1+\sin(ix))^4}dx$$

$$\Big\downarrow \text{3159}$$

$$\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\int \frac{\cosh^2(x)}{(i\sinh(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\int \frac{\cos(ix)^2}{(\sin(ix)+1)^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}$$

$$\Big\downarrow \text{24}$$

$$x+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}$$

input
```
Int[(Sech[x] - I*Tanh[x])^4,x]
```

output
```
x + (((2*I)/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3 - ((2*I)*Cosh[x])/(1 + I*Sinh[
x])
```

3.635.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.635.4 Maple [A] (verified)

Time = 278.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

method	result
risch	$x-\frac{8i\left(-3i\,\mathrm{e}^{x}+3\,\mathrm{e}^{2x}-2\right)}{3\left(\mathrm{e}^{x}-i\right)^{3}}$
parts	$\left(\frac{2}{3}+\frac{\operatorname{sech}(x)^{2}}{3}\right)\tanh\left(x\right)-\frac{7\tanh(x)^{3}}{3}-\tanh\left(x\right)-\frac{\ln(\tanh(x)-1)}{2}+\frac{\ln(1+\tanh(x))}{2}+\frac{4i\operatorname{sech}(x)^{3}}{3}+4i\left(\frac{\operatorname{sech}(x)}{3}\right.$
default	$-2\left(\frac{2}{3}+\frac{\operatorname{sech}(x)^{2}}{3}\right)\tanh\left(x\right)+\frac{4i}{3\cosh(x)^{3}}+\frac{3\sinh(x)}{\cosh(x)^{3}}+4i\left(-\frac{\sinh(x)^{2}}{\cosh(x)^{3}}-\frac{2}{3\cosh(x)^{3}}\right)+x-\tanh\left(x\right)-\frac{\tanh}{3}$

input
```
int((sech(x)-I*tanh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
x-8/3*I*(-3*I*exp(x)+3*exp(2*x)-2)/(exp(x)-I)^3
```

3.635.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int (\operatorname{sech}(x)-i\tanh(x))^4\,dx=\frac{3\,xe^{(3\,x)}-3\,(3i\,x+8i)e^{(2\,x)}-3\,(3\,x+8)e^{x}+3i\,x+16i}{3\,(e^{(3\,x)}-3i\,e^{(2\,x)}-3\,e^{x}+i)}$$

input
```
integrate((sech(x)-I*tanh(x))^4,x, algorithm="fricas")
```

output
```
1/3*(3*x*e^(3*x) - 3*(3*I*x + 8*I)*e^(2*x) - 3*(3*x + 8)*e^x + 3*I*x + 16*
I)/(e^(3*x) - 3*I*e^(2*x) - 3*e^x + I)
```

3.635.6 Sympy [F]

$$\int (\operatorname{sech}(x)-i\tanh(x))^4\,dx=\int \left(-i\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^4\,dx$$

input
```
integrate((sech(x)-I*tanh(x))**4,x)
```

output
```
Integral((-I*tanh(x) + sech(x))**4, x)
```

3.635.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(28) = 56$.

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.76

$$\int (\operatorname{sech}(x) - i\tanh(x))^4\, dx = -2\,\tanh(x)^3 + x - \frac{4\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + 2)}{3\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1)} - \frac{8i\,e^{(-x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{4\,e^{(-2\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} - \frac{16i\,e^{(-3\,x)}}{3\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1)} - \frac{8i\,e^{(-5\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{4}{3\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1)} + \frac{32i}{3\,(e^{(-x)} + e^{x})^3}$$

input
```
integrate((sech(x)-I*tanh(x))^4,x, algorithm="maxima")
```

output
```
-2*tanh(x)^3 + x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4
*x) + e^(-6*x) + 1) - 8*I*e^(-x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1)
+ 4*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 16/3*I*e^(-3*x)/(3
*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 8*I*e^(-5*x)/(3*e^(-2*x) + 3*e^(-
4*x) + e^(-6*x) + 1) + 4/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 32/3
*I/(e^(-x) + e^x)^3
```

3.635.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int (\operatorname{sech}(x) - i\tanh(x))^4\, dx = x - \frac{8\,(3i\,e^{(2\,x)} + 3\,e^{x} - 2i)}{3\,(e^{x} - i)^3}$$

input
```
integrate((sech(x)-I*tanh(x))^4,x, algorithm="giac")
```

output
```
x - 8/3*(3*I*e^(2*x) + 3*e^x - 2*I)/(e^x - I)^3
```

3.635.9 Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int (\operatorname{sech}(x) - i\tanh(x))^4 \, dx = x + \frac{\frac{e^{2x}\,8i}{3} - \frac{8}{3}i}{e^{2x}\,3i - e^{3x} + 3\,e^{x} - i} - \frac{8i}{3\,(e^{x} - i)} + \frac{e^{x}\,8i}{3\,(1 - e^{2x} + e^{x}\,2i)}$$

input
```
int((tanh(x)*1i - 1/cosh(x))^4,x)
```

output
```
x + ((exp(2*x)*8i)/3 - 8i/3)/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 8i
/(3*(exp(x) - 1i)) + (exp(x)*8i)/(3*(exp(x)*2i - exp(2*x) + 1))
```

3.636 $\int (\operatorname{sech}(x) - i\tanh(x))^3\,dx$

3.636.1 Optimal result

Integrand size = 11, antiderivative size = 28

$$\int (\operatorname{sech}(x) - i\tanh(x))^3\,dx = i\log(i-\sinh(x)) + \frac{2i}{1+i\sinh(x)}$$

output
```
I*ln(I-sinh(x))+2*I/(1+I*sinh(x))
```

3.636.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int (\operatorname{sech}(x) - i\tanh(x))^3\,dx = -\arctan(\sinh(x)) + i\log(\cosh(x)) + \frac{3}{2}i\operatorname{sech}^2(x) + 2\operatorname{sech}(x)\tanh(x) - \frac{1}{2}i\tanh^2(x)$$

input
```
Integrate[(Sech[x] - I*Tanh[x])^3,x]
```

output
```
-ArcTan[Sinh[x]] + I*Log[Cosh[x]] + ((3*I)/2)*Sech[x]^2 + 2*Sech[x]*Tanh[x
] - (I/2)*Tanh[x]^2
```

3.636.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4891, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\operatorname{sech}(x)-i\tanh(x))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (\sec(ix)-\tan(ix))^3dx$$
$$\Big\downarrow \text{4891}$$
$$\int (1-i\sinh(x))^3\operatorname{sech}^3(x)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(1-\sin(ix))^3}{\cos(ix)^3}dx$$
$$\Big\downarrow \text{3146}$$
$$i\int \frac{1-i\sinh(x)}{(i\sinh(x)+1)^2}d(-i\sinh(x))$$
$$\Big\downarrow \text{49}$$
$$i\int \left(\frac{1}{-i\sinh(x)-1}+\frac{2}{(-i\sinh(x)-1)^2}\right)d(-i\sinh(x))$$
$$\Big\downarrow \text{2009}$$
$$i\left(\frac{2}{1+i\sinh(x)}+\log(1+i\sinh(x))\right)$$

input
```
Int[(Sech[x] - I*Tanh[x])^3,x]
```

output
```
I*(Log[1 + I*Sinh[x]] + 2/(1 + I*Sinh[x]))
```

3.636.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.636.4 Maple [A] (verified)

Time = 15.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result
risch	$-ix+\frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}-i)^{2}}+2i\ln\left(\mathrm{e}^{x}-i\right)$
default	$-\operatorname{sech}\left(x\right)\tanh\left(x\right)-2\arctan\left(\mathrm{e}^{x}\right)+\frac{3i}{2\cosh(x)^{2}}+\frac{3\sinh(x)}{\cosh(x)^{2}}+i\left(\ln\left(\cosh\left(x\right)\right)-\frac{\tanh(x)^{2}}{2}\right)$
parts	$-\operatorname{sech}\left(x\right)\tanh\left(x\right)-2\arctan\left(\mathrm{e}^{x}\right)+i\left(-\frac{\tanh(x)^{2}}{2}-\frac{\ln(\tanh(x)-1)}{2}-\frac{\ln(1+\tanh(x))}{2}\right)-\frac{3i\tanh(x)^{2}}{2}+$

input
```
int((sech(x)-I*tanh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-I*x+4/(exp(x)-I)^2*exp(x)+2*I*ln(exp(x)-I)
```

3.636.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int (\operatorname{sech}(x) - i\tanh(x))^3 \, dx = \frac{-i\,xe^{(2\,x)} - 2\,(x-2)e^x - 2\left(-i\,e^{(2\,x)} - 2\,e^x + i\right)\log\left(e^x - i\right) + i\,x}{e^{(2\,x)} - 2i\,e^x - 1}$$

input
```
integrate((sech(x)-I*tanh(x))^3,x, algorithm="fricas")
```

output
```
(-I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(-I*e^(2*x) - 2*e^x + I)*log(e^x - I) +
I*x)/(e^(2*x) - 2*I*e^x - 1)
```

3.636.6 Sympy [F]

$$\int (\operatorname{sech}(x) - i\tanh(x))^3 \, dx = \int \left(-i\tanh\left(x\right) + \operatorname{sech}\left(x\right)\right)^3 \, dx$$

input
```
integrate((sech(x)-I*tanh(x))**3,x)
```

output
```
Integral((-I*tanh(x) + sech(x))**3, x)
```

3.636.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(20) = 40.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int (\operatorname{sech}(x) - i\tanh(x))^3 \, dx = -\frac{3}{2}i\,\tanh\left(x\right)^2 + i\,x + \frac{4\left(e^{(-x)} - e^{(-3\,x)}\right)}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} + \frac{2i\,e^{(-2\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} + 2\,\arctan\left(e^{(-x)}\right) + i\,\log\left(e^{(-2\,x)} + 1\right)$$

input
```
integrate((sech(x)-I*tanh(x))^3,x, algorithm="maxima")
```

output
```
-3/2*I*tanh(x)^2 + I*x + 4*(e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1)
 + 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + 2*arctan(e^(-x)) + I*log(e^(
-2*x) + 1)
```

3.636.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (\operatorname{sech}(x) - i\tanh(x))^3\, dx = -i\,x + \frac{4\,e^x}{(e^x - i)^2} + 2i\,\log(e^x - i)$$

input
```
integrate((sech(x)-I*tanh(x))^3,x, algorithm="giac")
```

output
```
-I*x + 4*e^x/(e^x - I)^2 + 2*I*log(e^x - I)
```

3.636.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int (\operatorname{sech}(x) - i\tanh(x))^3\, dx = -x\,1\mathrm{i} + \ln(\mathrm{e}^x - \mathrm{i})\,2\mathrm{i} - \frac{4\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^x\,2\mathrm{i}} + \frac{4}{\mathrm{e}^x - \mathrm{i}}$$

input
```
int(-(tanh(x)*1i - 1/cosh(x))^3,x)
```

output
```
log(exp(x) - 1i)*2i - x*1i - 4i/(exp(x)*2i - exp(2*x) + 1) + 4/(exp(x) - 1
i)
```

3.637 $\int (\text{sech}(x) - i \tanh(x))^2 \, dx$

3.637.1 Optimal result

Integrand size = 11, antiderivative size = 20

$$\int (\text{sech}(x) - i \tanh(x))^2 \, dx = -x + \frac{2i \cosh(x)}{1 + i \sinh(x)}$$

output
```
-x+2*I*cosh(x)/(1+I*sinh(x))
```

3.637.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (\text{sech}(x) - i \tanh(x))^2 \, dx = -\text{arctanh}(\tanh(x)) + 2i\text{sech}(x) + 2\tanh(x)$$

input
```
Integrate[(Sech[x] - I*Tanh[x])^2,x]
```

output
```
-ArcTanh[Tanh[x]] + (2*I)*Sech[x] + 2*Tanh[x]
```

3.637.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 4891, 3042, 3149, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{sech}(x)-i\tanh(x))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sec(ix)-\tan(ix))^2dx$$

$$\Big\downarrow \text{4891}$$

$$\int (1-i\sinh(x))^2\text{sech}^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(1-\sin(ix))^2}{\cos(ix)^2}dx$$

$$\Big\downarrow \text{3149}$$

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{(1+\sin(ix))^2}dx$$

$$\Big\downarrow \text{3159}$$

$$-\int 1dx+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

$$\Big\downarrow \text{24}$$

$$-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

input
```
Int[(Sech[x] - I*Tanh[x])^2,x]
```

output
```
-x + ((2*I)*Cosh[x])/(1 + I*Sinh[x])
```

3.637.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.637.4 Maple [A] (verified)

Time = 6.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
risch	$-x+\frac{4i}{{\mathrm e}^{x}-i}$	15
default	$2\tanh\left(x\right)+\frac{2i}{\cosh\left(x\right)}-x$	16
parts	$2\tanh\left(x\right)+2i\,\operatorname{sech}\left(x\right)+\frac{\ln\left(\tanh\left(x\right)-1\right)}{2}-\frac{\ln\left(1+\tanh\left(x\right)\right)}{2}$	25

input
```
int((sech(x)-I*tanh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-x+4*I/(exp(x)-I)
```

3.637.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int (\operatorname{sech}(x) - i\tanh(x))^2\,dx = -\frac{xe^x - i\,x - 4i}{e^x - i}$$

input
```
integrate((sech(x)-I*tanh(x))^2,x, algorithm="fricas")
```

output
```
-(x*e^x - I*x - 4*I)/(e^x - I)
```

3.637.6 Sympy [F]

$$\int (\operatorname{sech}(x) - i\tanh(x))^2\,dx = \int \left(-i\tanh\left(x\right) + \operatorname{sech}\left(x\right)\right)^2\,dx$$

input
```
integrate((sech(x)-I*tanh(x))**2,x)
```

output
```
Integral((-I*tanh(x) + sech(x))**2, x)
```

3.637.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int (\operatorname{sech}(x) - i\tanh(x))^2\,dx = -x + \frac{4i}{e^{(-x)} + e^x} + \frac{4}{e^{(-2\,x)} + 1}$$

input
```
integrate((sech(x)-I*tanh(x))^2,x, algorithm="maxima")
```

output
```
-x + 4*I/(e^(-x) + e^x) + 4/(e^(-2*x) + 1)
```

3.637.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int (\operatorname{sech}(x) - i\tanh(x))^2 \, dx = -x + \frac{4i}{e^x - i}$$

input
```
integrate((sech(x)-I*tanh(x))^2,x, algorithm="giac")
```

output
```
-x + 4*I/(e^x - I)
```

3.637.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int (\operatorname{sech}(x) - i\tanh(x))^2 \, dx = -x + \frac{4\mathrm{i}}{\mathrm{e}^x - \mathrm{i}}$$

input
```
int((tanh(x)*1i - 1/cosh(x))^2,x)
```

output
```
4i/(exp(x) - 1i) - x
```

3.638 $\int (\text{sech}(x) - i \tanh(x)) \, dx$

3.638.1 Optimal result

Integrand size = 9, antiderivative size = 11

$$\int (\text{sech}(x) - i \tanh(x)) \, dx = \arctan(\sinh(x)) - i \log(\cosh(x))$$

output
```
arctan(sinh(x))-I*ln(cosh(x))
```

3.638.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (\text{sech}(x) - i \tanh(x)) \, dx = \arctan(\sinh(x)) - i \log(\cosh(x))$$

input
```
Integrate[Sech[x] - I*Tanh[x],x]
```

output
```
ArcTan[Sinh[x]] - I*Log[Cosh[x]]
```

3.638.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{sech}(x) - i\tanh(x))\,dx$$

$$\Big\downarrow \text{2009}$$

$$\arctan(\sinh(x)) - i\log(\cosh(x))$$

input
```
Int[Sech[x] - I*Tanh[x],x]
```

output
```
ArcTan[Sinh[x]] - I*Log[Cosh[x]]
```

3.638.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.638.4 Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$\arctan(\sinh(x)) - i\ln(\cosh(x))$	11
parts	$\arctan(\sinh(x)) - i\ln(\cosh(x))$	11
risch	$i\ln(\mathrm{e}^{x}+i) - i\ln(\mathrm{e}^{x}-i) + ix - i\ln(1+\mathrm{e}^{2x})$	34
parallelrisch	$\frac{i(\ln(1-\tanh(x))+\ln(1+\tanh(x))-2\ln(-i+\coth(x)-\text{csch}(x))+2\ln(i+\coth(x)-\text{csch}(x)))}{2}$	41

input
```
int(sech(x)-I*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
arctan(sinh(x))-I*ln(cosh(x))
```

3.638.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (\operatorname{sech}(x) - i\tanh(x))\, dx = i\, x - 2i\, \log\left(e^{x} - i\right)$$

input
```
integrate(sech(x)-I*tanh(x),x, algorithm="fricas")
```

output
```
I*x - 2*I*log(e^x - I)
```

3.638.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int (\operatorname{sech}(x) - i\tanh(x))\, dx = -i(x - \log\left(\tanh\left(x\right) + 1\right)) + 2\,\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
integrate(sech(x)-I*tanh(x),x)
```

output
```
-I*(x - log(tanh(x) + 1)) + 2*atan(tanh(x/2))
```

3.638.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (\operatorname{sech}(x) - i\tanh(x))\, dx = \arctan\left(\sinh\left(x\right)\right) - i\, \log\left(\cosh\left(x\right)\right)$$

input
```
integrate(sech(x)-I*tanh(x),x, algorithm="maxima")
```

output
```
arctan(sinh(x)) - I*log(cosh(x))
```

3.638.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int (\operatorname{sech}(x) - i\tanh(x))\,dx = i\,x + 2\,\arctan\left(e^{x}\right) - i\,\log\left(e^{(2\,x)} + 1\right)$$

input
```
integrate(sech(x)-I*tanh(x),x, algorithm="giac")
```

output
```
I*x + 2*arctan(e^x) - I*log(e^(2*x) + 1)
```

3.638.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (\operatorname{sech}(x) - i\tanh(x))\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^{x} - \mathrm{i}\right)\,2\mathrm{i}$$

input
```
int(1/cosh(x) - tanh(x)*1i,x)
```

output
```
x*1i - log(exp(x) - 1i)*2i
```

3.639 $\int \frac{1}{\operatorname{sech}(x)-i\tanh(x)}\,dx$

3.639.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{\operatorname{sech}(x)-i\tanh(x)}\,dx = i\log(i+\sinh(x))$$

output
```
I*ln(I+sinh(x))
```

3.639.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{\operatorname{sech}(x)-i\tanh(x)}\,dx = 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+i\log(\cosh(x))$$

input
```
Integrate[(Sech[x] - I*Tanh[x])^(-1),x]
```

output
```
2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]]
```

3.639.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3638, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{sech}(x)-i\tanh(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(ix)-\tan(ix)}dx$$
$$\Big\downarrow \text{3638}$$
$$\int \frac{\cosh(x)}{1-i\sinh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)}{1-\sin(ix)}dx$$
$$\Big\downarrow \text{3146}$$
$$i\int \frac{1}{1-i\sinh(x)}d(-i\sinh(x))$$
$$\Big\downarrow \text{16}$$
$$i\log(1-i\sinh(x))$$

input
```
Int[(Sech[x] - I*Tanh[x])^(-1),x]
```

output
```
I*Log[1 - I*Sinh[x]]
```

3.639.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3638
```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.639.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

method	result	size
risch	$-ix+2i\ln\left(\mathrm{e}^{x}+i\right)$	15
default	$-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+2i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)$	33

input
```
int(1/(sech(x)-I*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*x+2*I*ln(exp(x)+I)
```

3.639.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\,dx = -i\,x + 2i\,\log\left(e^{x} + i\right)$$

input
```
integrate(1/(sech(x)-I*tanh(x)),x, algorithm="fricas")
```

output
```
-I*x + 2*I*log(e^x + I)
```

3.639.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(7) = 14.

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\,dx = ix - i\log\left(\tanh\left(x\right) + 1\right) + i\log\left(\tanh\left(x\right) + i\operatorname{sech}\left(x\right)\right)$$

input
```
integrate(1/(sech(x)-I*tanh(x)),x)
```

output
```
I*x - I*log(tanh(x) + 1) + I*log(tanh(x) + I*sech(x))
```

3.639.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\,dx = i\,x + 2i\,\log\left(i\,e^{(-x)} + 1\right)$$

input
```
integrate(1/(sech(x)-I*tanh(x)),x, algorithm="maxima")
```

output
```
I*x + 2*I*log(I*e^(-x) + 1)
```

3.639.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\, dx = -i\, x + 2i\, \log\left(e^{x} + i\right)$$

input
```
integrate(1/(sech(x)-I*tanh(x)),x, algorithm="giac")
```

output
```
-I*x + 2*I*log(e^x + I)
```

3.639.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\, dx = -x\, 1\mathrm{i} + \ln\left(\mathrm{e}^{x} + 1\mathrm{i}\right)\, 2\mathrm{i}$$

input
```
int(-1/(tanh(x)*1i - 1/cosh(x)),x)
```

output
```
log(exp(x) + 1i)*2i - x*1i
```

3.640 $\int \frac{1}{(\mathrm{sech}(x)-i\tanh(x))^2}\,dx$

3.640.1 Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{1}{(\mathrm{sech}(x)-i\tanh(x))^2}\,dx = -x-\frac{2i\cosh(x)}{1-i\sinh(x)}$$

output
```
-x-2*I*cosh(x)/(1-I*sinh(x))
```

3.640.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(\mathrm{sech}(x)-i\tanh(x))^2}\,dx = -x+\frac{4\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)}$$

input
```
Integrate[(Sech[x] - I*Tanh[x])^(-2),x]
```

output
```
-x + (4*Sinh[x/2])/(Cosh[x/2] - I*Sinh[x/2])
```

3.640.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4891, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\sec(ix)-\tan(ix))^2}dx$$

$$\Big\downarrow \text{ 4891}$$

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ix)^2}{(1-\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 3159}$$

$$-\int 1dx-\frac{2i\cosh(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{ 24}$$

$$-x-\frac{2i\cosh(x)}{1-i\sinh(x)}$$

input
```
Int[(Sech[x] - I*Tanh[x])^(-2),x]
```

output
```
-x - ((2*I)*Cosh[x])/(1 - I*Sinh[x])
```

3.640.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.640.4 Maple [A] (verified)

Time = 9.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
risch	$-x-\frac{4i}{e^{x}+i}$	15
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{4}{\tanh\left(\frac{x}{2}\right)+i}$	29

input
```
int(1/(sech(x)-I*tanh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-x-4*I/(exp(x)+I)
```

3.640.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^2}\,dx = -\frac{xe^x + i\,x + 4i}{e^x + i}$$

input
```
integrate(1/(sech(x)-I*tanh(x))^2,x, algorithm="fricas")
```

output
```
-(x*e^x + I*x + 4*I)/(e^x + I)
```

3.640.6 Sympy [F]

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^2}\,dx = \int \frac{1}{\left(-i\tanh{(x)} + \operatorname{sech}{(x)}\right)^2}\,dx$$

input
```
integrate(1/(sech(x)-I*tanh(x))**2,x)
```

output
```
Integral((-I*tanh(x) + sech(x))**(-2), x)
```

3.640.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^2}\,dx = -x - \frac{4i}{e^{(-x)} - i}$$

input
```
integrate(1/(sech(x)-I*tanh(x))^2,x, algorithm="maxima")
```

output
```
-x - 4*I/(e^(-x) - I)
```

3.640.8 Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^2}\,dx = -x - \frac{4i}{e^x + i}$$

input
```
integrate(1/(sech(x)-I*tanh(x))^2,x, algorithm="giac")
```

output
```
-x - 4*I/(e^x + I)
```

3.640.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^2}\,dx = -x - \frac{4\mathrm{i}}{\mathrm{e}^x + 1\mathrm{i}}$$

input
```
int(1/(tanh(x)*1i - 1/cosh(x))^2,x)
```

output
```
- x - 4i/(exp(x) + 1i)
```

3.641 $\int \frac{1}{(\text{sech}(x)-i\tanh(x))^3}\,dx$

3.641.1 Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{1}{(\text{sech}(x)-i\tanh(x))^3}\,dx=-i\log(i+\sinh(x))-\frac{2i}{1-i\sinh(x)}$$

output
```
-I*ln(I+sinh(x))-2*I/(1-I*sinh(x))
```

3.641.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{1}{(\text{sech}(x)-i\tanh(x))^3}\,dx=-2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-i\log(\cosh(x))+\frac{2}{i+\sinh(x)}$$

input
```
Integrate[(Sech[x] - I*Tanh[x])^(-3),x]
```

output
```
-2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]] + 2/(I + Sinh[x])
```

3.641.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4891, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\sec(ix)-\tan(ix))^3}dx$$
$$\Big\downarrow \text{4891}$$
$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)^3}{(1-\sin(ix))^3}dx$$
$$\Big\downarrow \text{3146}$$
$$i\int \frac{i\sinh(x)+1}{(1-i\sinh(x))^2}d(-i\sinh(x))$$
$$\Big\downarrow \text{49}$$
$$i\int \left(\frac{2}{(1-i\sinh(x))^2}+\frac{1}{i\sinh(x)-1}\right)d(-i\sinh(x))$$
$$\Big\downarrow \text{2009}$$
$$i\left(-\frac{2}{1-i\sinh(x)}-\log(1-i\sinh(x))\right)$$

input
```
Int[(Sech[x] - I*Tanh[x])^(-3),x]
```

output
```
I*(-Log[1 - I*Sinh[x]] - 2/(1 - I*Sinh[x]))
```

3.641.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.641.4 Maple [A] (verified)

Time = 44.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
risch	$ix+\frac{4\,\mathrm{e}^{x}}{\left(\mathrm{e}^{x}+i\right)^{2}}-2i\ln\left(\mathrm{e}^{x}+i\right)$	26
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-2i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{4}{\tanh\left(\frac{x}{2}\right)+i}$	56

input
```
int(1/(sech(x)-I*tanh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
I*x+4*exp(x)/(exp(x)+I)^2-2*I*ln(exp(x)+I)
```

3.641.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^3}\,dx = \frac{i\,xe^{(2\,x)} - 2\,(x-2)e^x - 2\,\left(i\,e^{(2\,x)} - 2\,e^x - i\right)\log\left(e^x + i\right) - i\,x}{e^{(2\,x)} + 2i\,e^x - 1}$$

input
```
integrate(1/(sech(x)-I*tanh(x))^3,x, algorithm="fricas")
```

output
```
(I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(I*e^(2*x) - 2*e^x - I)*log(e^x + I) - I*
x)/(e^(2*x) + 2*I*e^x - 1)
```

3.641.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(19) = 38.

Time = 0.97 (sec) , antiderivative size = 432, normalized size of antiderivative = 16.62

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^3}\,dx = \frac{2ix\tanh^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{4x\tanh(x)\operatorname{sech}(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{2ix\operatorname{sech}^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{2i\log(\tanh(x)+1)\tanh^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} + \frac{4\log(\tanh(x)+1)\tanh(x)\operatorname{sech}(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} + \frac{2i\log(\tanh(x)+1)\operatorname{sech}^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} + \frac{2i\log(\tanh(x)+i\operatorname{sech}(x))\tanh^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{4\log(\tanh(x)+i\operatorname{sech}(x))\tanh(x)\operatorname{sech}(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{2i\log(\tanh(x)+i\operatorname{sech}(x))\operatorname{sech}^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{i\tanh^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{i\operatorname{sech}^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{i}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)}$$

input
```
integrate(1/(sech(x)-I*tanh(x))**3,x)
```

3.641. $\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^3}\,dx$

output
```
2*I*x*tanh(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 4*
x*tanh(x)*sech(x)/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 2
*I*x*sech(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 2*I
*log(tanh(x) + 1)*tanh(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech
(x)**2) + 4*log(tanh(x) + 1)*tanh(x)*sech(x)/(-2*tanh(x)**2 - 4*I*tanh(x)*
sech(x) + 2*sech(x)**2) + 2*I*log(tanh(x) + 1)*sech(x)**2/(-2*tanh(x)**2 -
 4*I*tanh(x)*sech(x) + 2*sech(x)**2) + 2*I*log(tanh(x) + I*sech(x))*tanh(x
)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 4*log(tanh(x)
+ I*sech(x))*tanh(x)*sech(x)/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech
(x)**2) - 2*I*log(tanh(x) + I*sech(x))*sech(x)**2/(-2*tanh(x)**2 - 4*I*tan
h(x)*sech(x) + 2*sech(x)**2) - I*tanh(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*s
ech(x) + 2*sech(x)**2) - I*sech(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x)
 + 2*sech(x)**2) - I/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2)
```

3.641.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{1}{(\mathrm{sech}(x)-i\tanh(x))^3}\,dx = -i\,x - \frac{4\,e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1} - 2i\,\log\left(e^{(-x)}-i\right)$$

input
```
integrate(1/(sech(x)-I*tanh(x))^3,x, algorithm="maxima")
```

output
```
-I*x - 4*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - 2*I*log(e^(-x) - I)
```

3.641.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1}{(\mathrm{sech}(x)-i\tanh(x))^3}\,dx = i\,x + \frac{4\,e^{x}}{(e^{x}+i)^2} - 2i\,\log\left(e^{x}+i\right)$$

input
```
integrate(1/(sech(x)-I*tanh(x))^3,x, algorithm="giac")
```

output
```
I*x + 4*e^x/(e^x + I)^2 - 2*I*log(e^x + I)
```

3.641.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^3}\,dx = x\,1\mathrm{i}-\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,2\mathrm{i}-\frac{4\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}}+\frac{4}{\mathrm{e}^x+1\mathrm{i}}$$

input
```
int(-1/(tanh(x)*1i - 1/cosh(x))^3,x)
```

output
```
x*1i - log(exp(x) + 1i)*2i - 4i/(exp(2*x) + exp(x)*2i - 1) + 4/(exp(x) + 1
i)
```

3.642 $\int \frac{1}{(\text{sech}(x)-i\tanh(x))^4}\,dx$

3.642.1 Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{1}{(\text{sech}(x)-i\tanh(x))^4}\,dx = x-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}$$

output
```
x-2/3*I*cosh(x)^3/(1-I*sinh(x))^3+2*I*cosh(x)/(1-I*sinh(x))
```

3.642.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.95

$$\int \frac{1}{(\text{sech}(x)-i\tanh(x))^4}\,dx = \frac{3(-8i+3x)\cosh\left(\frac{x}{2}\right)+(16i-3x)\cosh\left(\frac{3x}{2}\right)-6i(-4i+2x+x\cosh(x))\sinh\left(\frac{x}{2}\right)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3}$$

input
```
Integrate[(Sech[x] - I*Tanh[x])^(-4),x]
```

output
```
(3*(-8*I + 3*x)*Cosh[x/2] + (16*I - 3*x)*Cosh[(3*x)/2] - (6*I)*(-4*I + 2*x
 + x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] - I*Sinh[x/2])^3)
```

3.642.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 4891, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\sec(ix)-\tan(ix))^4}dx$$
$$\Big\downarrow \text{4891}$$
$$\int \frac{\cosh^4(x)}{(1-i\sinh(x))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)^4}{(1-\sin(ix))^4}dx$$
$$\Big\downarrow \text{3159}$$
$$-\int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}dx-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}$$
$$\Big\downarrow \text{3042}$$
$$-\int \frac{\cos(ix)^2}{(1-\sin(ix))^2}dx-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}$$
$$\Big\downarrow \text{3159}$$
$$\int 1dx-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}$$
$$\Big\downarrow \text{24}$$
$$x-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}$$

input
```
Int[(Sech[x] - I*Tanh[x])^(-4),x]
```

output
```
x - (((2*I)/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3 + ((2*I)*Cosh[x])/(1 - I*Sinh[
x])
```

3.642.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.642.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{16}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}$$

input
```
int(1/(sech(x)-I*tanh(x))^4,x)
```

output
```
-ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)-8*I/(tanh(1/2*x)+I)^2-16/3/(tanh(1/2*
x)+I)^3
```

3.642.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^4}\,dx = \frac{3\,xe^{(3\,x)} - 3\,(-3i\,x - 8i)e^{(2\,x)} - 3\,(3\,x + 8)e^{x} - 3i\,x - 16i}{3\,(e^{(3\,x)} + 3i\,e^{(2\,x)} - 3\,e^{x} - i)}$$

input
```
integrate(1/(sech(x)-I*tanh(x))^4,x, algorithm="fricas")
```

output
```
1/3*(3*x*e^(3*x) - 3*(-3*I*x - 8*I)*e^(2*x) - 3*(3*x + 8)*e^x - 3*I*x - 16
*I)/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

3.642.6 Sympy [F]

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^4}\,dx = \int \frac{1}{(-i\tanh{(x)} + \operatorname{sech}{(x)})^4}\,dx$$

input
```
integrate(1/(sech(x)-I*tanh(x))**4,x)
```

output
```
Integral((-I*tanh(x) + sech(x))**(-4), x)
```

3.642.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^4}\,dx = x - \frac{8\left(3\,e^{(-x)} + 3i\,e^{(-2\,x)} - 2i\right)}{3\,(3\,e^{(-x)} + 3i\,e^{(-2\,x)} - e^{(-3\,x)} - i)}$$

input
```
integrate(1/(sech(x)-I*tanh(x))^4,x, algorithm="maxima")
```

output
```
x - 8/3*(3*e^(-x) + 3*I*e^(-2*x) - 2*I)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x
) - I)
```

3.642.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^4}\,dx = x - \frac{8\left(-3i\,e^{(2\,x)} + 3\,e^{x} + 2i\right)}{3\,(e^{x} + i)^3}$$

input
```
integrate(1/(sech(x)-I*tanh(x))^4,x, algorithm="giac")
```

output
```
x - 8/3*(-3*I*e^(2*x) + 3*e^x + 2*I)/(e^x + I)^3
```

3.642.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^4}\,dx = x + \frac{\frac{e^{2\,x}\,8i}{3} - \frac{8}{3}i}{e^{2\,x}\,3i + e^{3\,x} - 3\,e^{x} - i} + \frac{e^{x}\,8i}{3\,\left(e^{2\,x} - 1 + e^{x}\,2i\right)} + \frac{8i}{3\,\left(e^{x} + 1i\right)}$$

input
```
int(1/(tanh(x)*1i - 1/cosh(x))^4,x)
```

output
```
x + ((exp(2*x)*8i)/3 - 8i/3)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + (e
xp(x)*8i)/(3*(exp(2*x) + exp(x)*2i - 1)) + 8i/(3*(exp(x) + 1i))
```

3.643 $\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^5}\,dx$

3.643.1 Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^5}\,dx = i\log(i+\sinh(x)) - \frac{2i}{(1-i\sinh(x))^2} + \frac{4i}{1-i\sinh(x)}$$

output
```
I*ln(I+sinh(x))-2*I/(1-I*sinh(x))^2+4*I/(1-I*sinh(x))
```

3.643.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^5}\,dx = 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + i\log(\cosh(x)) + \frac{2i+4\sinh(x)}{\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^4}$$

input
```
Integrate[(Sech[x] - I*Tanh[x])^(-5),x]
```

output
```
2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]] + (2*I + 4*Sinh[x])/(Cosh[x/2] - I*Si
nh[x/2])^4
```

3.643.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4891, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\text{sech}(x)-i\tanh(x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sec(ix)-\tan(ix))^5}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\cosh^5(x)}{(1-i\sinh(x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^5}{(1-\sin(ix))^5}dx$$

$$\Big\downarrow \text{3146}$$

$$i\int \frac{(i\sinh(x)+1)^2}{(1-i\sinh(x))^3}d(-i\sinh(x))$$

$$\Big\downarrow \text{49}$$

$$i\int \left(\frac{1}{1-i\sinh(x)}-\frac{4}{(1-i\sinh(x))^2}+\frac{4}{(1-i\sinh(x))^3}\right)d(-i\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{4}{1-i\sinh(x)}-\frac{2}{(1-i\sinh(x))^2}+\log(1-i\sinh(x))\right)$$

input
```
Int[(Sech[x] - I*Tanh[x])^(-5),x]
```

output
```
I*(Log[1 - I*Sinh[x]] - 2/(1 - I*Sinh[x])^2 + 4/(1 - I*Sinh[x]))
```

3.643.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.643.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.70

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+2i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{}{(\mathrm{t}}$$

input
```
int(1/(sech(x)-I*tanh(x))^5,x)
```

output
```
-I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+8*I/(tanh(1/2*x)+I)^2+2*I*ln(tanh
(1/2*x)+I)-8*I/(tanh(1/2*x)+I)^4+16/(tanh(1/2*x)+I)^3
```

3.643.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.35

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^5}\,dx = \frac{-i\,xe^{(4\,x)} + 4\,(x-2)e^{(3\,x)} - 2\,(-3i\,x+4i)e^{(2\,x)} - 4\,(x-2)e^{x} - 2\,\left(-i\,e^{(4\,x)} + 4\,e^{(3\,x)} + 6i\,e^{(2\,x)} - 4\,e^{x} - i\right)}{e^{(4\,x)} + 4i\,e^{(3\,x)} - 6\,e^{(2\,x)} - 4i\,e^{x} + 1}$$

input
```
integrate(1/(sech(x)-I*tanh(x))^5,x, algorithm="fricas")
```

output
```
(-I*x*e^(4*x) + 4*(x - 2)*e^(3*x) - 2*(-3*I*x + 4*I)*e^(2*x) - 4*(x - 2)*e
^x - 2*(-I*e^(4*x) + 4*e^(3*x) + 6*I*e^(2*x) - 4*e^x - I)*log(e^x + I) - I
*x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

3.643.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1445 vs. $2(29) = 58$.

Time = 4.42 (sec) , antiderivative size = 1445, normalized size of antiderivative = 36.12

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(sech(x)-I*tanh(x))**5,x)
```

output
```
36*I*x*tanh(x)**4/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)*
*2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 144*x*tanh(x)*
*3*sech(x)/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech
(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 216*I*x*tanh(x)**2*se
ch(x)**2/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x
)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 144*x*tanh(x)*sech(x)**
3/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 -
144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 36*I*x*sech(x)**4/(36*tanh(x)*
*4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*
sech(x)**3 + 36*sech(x)**4) - 36*I*log(tanh(x) + 1)*tanh(x)**4/(36*tanh(x)
**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)
*sech(x)**3 + 36*sech(x)**4) + 144*log(tanh(x) + 1)*tanh(x)**3*sech(x)/(36
*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I
*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 216*I*log(tanh(x) + 1)*tanh(x)**2*s
ech(x)**2/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(
x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 144*log(tanh(x) + 1)*t
anh(x)*sech(x)**3/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)*
*2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 36*I*log(tanh(
x) + 1)*sech(x)**4/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)
**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 36*I*log(t...
```

3.643.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(28) = 56.

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^5}\,dx = i\,x - \frac{8\left(e^{(-x)} + i\,e^{(-2\,x)} - e^{(-3\,x)}\right)}{4i\,e^{(-x)} - 6\,e^{(-2\,x)} - 4i\,e^{(-3\,x)} + e^{(-4\,x)} + 1} + 2i\,\log\left(e^{(-x)} - i\right)$$

input
```
integrate(1/(sech(x)-I*tanh(x))^5,x, algorithm="maxima")
```

output
```
I*x - 8*(e^(-x) + I*e^(-2*x) - e^(-3*x))/(4*I*e^(-x) - 6*e^(-2*x) - 4*I*e^
(-3*x) + e^(-4*x) + 1) + 2*I*log(e^(-x) - I)
```

3.643.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^5} \, dx = -i\,x - \frac{8\left(e^{(3\,x)} + i\,e^{(2\,x)} - e^{x}\right)}{(e^{x} + i)^4} + 2i\,\log\left(e^{x} + i\right)$$

input
```
integrate(1/(sech(x)-I*tanh(x))^5,x, algorithm="giac")
```

output
```
-I*x - 8*(e^(3*x) + I*e^(2*x) - e^x)/(e^x + I)^4 + 2*I*log(e^x + I)
```

3.643.9 Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^5} \, dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^{x} + 1\mathrm{i}\right)\,2\mathrm{i} + \frac{16\mathrm{i}}{\mathrm{e}^{2\,x} - 1 + \mathrm{e}^{x}\,2\mathrm{i}} - \frac{8\mathrm{i}}{\mathrm{e}^{4\,x} - 6\,\mathrm{e}^{2\,x} + 1 + \mathrm{e}^{3\,x}\,4\mathrm{i} - \mathrm{e}^{x}\,4\mathrm{i}} - \frac{8}{\mathrm{e}^{x} + 1\mathrm{i}} + \frac{16}{\mathrm{e}^{2\,x}\,3\mathrm{i} + \mathrm{e}^{3\,x} - 3\,\mathrm{e}^{x} - \mathrm{i}}$$

input
```
int(-1/(tanh(x)*1i - 1/cosh(x))^5,x)
```

output
```
log(exp(x) + 1i)*2i - x*1i + 16i/(exp(2*x) + exp(x)*2i - 1) - 8i/(exp(3*x)
*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) - 8/(exp(x) + 1i) + 16/(exp(2
*x)*3i + exp(3*x) - 3*exp(x) - 1i)
```

3.644 $\int (a\coth(x)+b\text{csch}(x))^5\,dx$

3.644.1 Optimal result

Integrand size = 11, antiderivative size = 124

$$\int (a\coth(x)+b\text{csch}(x))^5\,dx=-\frac{1}{8}b\left(15a^4-10a^2b^2+3b^4\right)\text{arctanh}(\cosh(x))+\frac{1}{8}a^2b\left(7a^2-3b^2\right)\cosh(x)-\frac{1}{8}(b+a\cosh(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cosh(x)\right)\text{csch}^2(x)-\frac{1}{4}(b+a\cosh(x))^4(a+b\cosh(x))\text{csch}^4(x)+a^5\log(\sinh(x))$$

output
```
-1/8*b*(15*a^4-10*a^2*b^2+3*b^4)*arctanh(cosh(x))+1/8*a^2*b*(7*a^2-3*b^2)*
cosh(x)-1/8*(b+a*cosh(x))^2*(2*a*(2*a^2-b^2)+b*(5*a^2-3*b^2)*cosh(x))*csch
(x)^2-1/4*(b+a*cosh(x))^4*(a+b*cosh(x))*csch(x)^4+a^5*ln(sinh(x))
```

3.644.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15

$$\int (a\coth(x)+b\text{csch}(x))^5\,dx=\frac{1}{64}\left(-2(7a-3b)(a+b)^4\text{csch}^2\left(\frac{x}{2}\right)-(a+b)^5\text{csch}^4\left(\frac{x}{2}\right)+8(a-b)^3\left(8a^2+9ab+3b^2\right)\log\left(\cosh\left(\frac{x}{2}\right)\right)+8(a+b)^3\left(8a^2-9ab+3b^2\right)\log\left(\sinh\left(\frac{x}{2}\right)\right)+2(a-b)^4(7a+3b)\text{sech}^2\left(\frac{x}{2}\right)-(a-b)^5\text{sech}^4\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(a*Coth[x] + b*Csch[x])^5,x]
```

output
```
(-2*(7*a - 3*b)*(a + b)^4*Csch[x/2]^2 - (a + b)^5*Csch[x/2]^4 + 8*(a - b)^
3*(8*a^2 + 9*a*b + 3*b^2)*Log[Cosh[x/2]] + 8*(a + b)^3*(8*a^2 - 9*a*b + 3*
b^2)*Log[Sinh[x/2]] + 2*(a - b)^4*(7*a + 3*b)*Sech[x/2]^2 - (a - b)^5*Sech
[x/2]^4)/64
```

3.644.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.40, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3042, 4892, 26, 26, 3042, 26, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\coth(x)+b\text{csch}(x))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (ia\cot(ix)+ib\csc(ix))^5dx$$

$$\Big\downarrow \text{4892}$$

$$\int -i\text{csch}^5(x)(ia\cosh(x)+ib)^5dx$$

$$\Big\downarrow \text{26}$$

$$-i\int i(b+a\cosh(x))^5\text{csch}^5(x)dx$$

$$\Big\downarrow \text{26}$$

$$\int \text{csch}^5(x)(a\cosh(x)+b)^5dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(b-a\sin\left(-\frac{\pi}{2}+ix\right)\right)^5}{\cos\left(-\frac{\pi}{2}+ix\right)^5}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)^5}{\cos\left(ix-\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3147}$$

$$-a^5\int \frac{(b+a\cosh(x))^5}{\left(a^2-a^2\cosh^2(x)\right)^3}d(a\cosh(x))$$

$$\Big\downarrow \text{477}$$

$$-\frac{\int \left(-\frac{a^3(a-b)^5}{8(\cosh(x)a+a)^3}+\frac{a^2(7a+3b)(a-b)^4}{16(\cosh(x)a+a)^2}-\frac{a\left(8a^2+9ba+3b^2\right)(a-b)^3}{16(\cosh(x)a+a)}+\frac{a(a+b)^3\left(8a^2-9ba+3b^2\right)}{16(a-a\cosh(x))}-\frac{a^2(7a-3b)(a+b)^4}{16(a-a\cosh(x))^2}+\frac{a^3(a+b)^5}{8(a-a\cosh(x)}\right.}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{a^3(a-b)^5}{16(a\cosh(x)+a)^2}+\frac{a^3(a+b)^5}{16(a-a\cosh(x))^2}-\frac{1}{16}a\left(8a^2+9ab+3b^2\right)(a-b)^3\log(a\cosh(x)+a)-\frac{1}{16}a(a+b)^3\left(8a^2-9ab+\right.}{a}$$

input
```
Int[(a*Coth[x] + b*Csch[x])^5,x]
```

output
```
-(((a^3*(a + b)^5)/(16*(a - a*Cosh[x])^2) - (a^2*(7*a - 3*b)*(a + b)^4)/(1
6*(a - a*Cosh[x])) + (a^3*(a - b)^5)/(16*(a + a*Cosh[x])^2) - (a^2*(a - b)
^4*(7*a + 3*b))/(16*(a + a*Cosh[x])) - (a*(a + b)^3*(8*a^2 - 9*a*b + 3*b^2
)*Log[a - a*Cosh[x]])/16 - (a*(a - b)^3*(8*a^2 + 9*a*b + 3*b^2)*Log[a + a*
Cosh[x]])/16)/a)
```

3.644.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.644.4 Maple [A] (verified)

Time = 55.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.28

method	result
parts	$a^{5}\left(-\frac{\coth(x)^{4}}{4}-\frac{\coth(x)^{2}}{2}-\frac{\ln(\coth(x)-1)}{2}-\frac{\ln(1+\coth(x))}{2}\right)+b^{5}\left(\left(-\frac{\operatorname{csch}(x)^{3}}{4}+\frac{3\operatorname{csch}(x)}{8}\right)\coth\left(x\right)-\frac{3\operatorname{arcta}}{4}\right.$
default	$a^{5}\left(\ln\left(\sinh\left(x\right)\right)-\frac{\coth(x)^{2}}{2}-\frac{\coth(x)^{4}}{4}\right)+5a^{4}b\left(-\frac{\cosh(x)^{3}}{\sinh(x)^{4}}+\frac{\cosh(x)}{\sinh(x)^{4}}+\left(-\frac{\operatorname{csch}(x)^{3}}{4}+\frac{3\operatorname{csch}(x)}{8}\right)\coth\left(x\right.\right.$
risch	$-a^{5}x-\frac{e^{x}\left(25a^{4}b\,e^{6x}+10a^{2}b^{3}e^{6x}-3b^{5}e^{6x}+16\,e^{5x}a^{5}+80\,e^{5x}a^{3}b^{2}+15a^{4}b\,e^{4x}+70a^{2}b^{3}e^{4x}+11b^{5}e^{4x}-16\,e^{3x}a^{5}+80\,e^{3x}a\,b^{4}+15a^{4}b\,e\right.}{4\left(e^{2x}-1\right)^{4}}$

input
```
int((a*coth(x)+b*csch(x))^5,x,method=_RETURNVERBOSE)
```

output
```
a^5*(-1/4*coth(x)^4-1/2*coth(x)^2-1/2*ln(coth(x)-1)-1/2*ln(1+coth(x)))+b^5
*((-1/4*csch(x)^3+3/8*csch(x))*coth(x)-3/4*arctanh(exp(x)))+5*a^4*b*(-1/si
nh(x)^4*cosh(x)^3+1/sinh(x)^4*cosh(x)+(-1/4*csch(x)^3+3/8*csch(x))*coth(x)
-3/4*arctanh(exp(x)))-5/2*a^3*b^2*coth(x)^4+10*a^2*b^3*(-1/3/sinh(x)^4*cos
h(x)-1/3*(-1/4*csch(x)^3+3/8*csch(x))*coth(x)+1/4*arctanh(exp(x)))-5/4*b^4
*csch(x)^4*a
```

3.644.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2716 vs. $2(116) = 232$.

Time = 0.28 (sec) , antiderivative size = 2716, normalized size of antiderivative = 21.90

$$\int (a\coth(x) + b\text{csch}(x))^5\,dx = \text{Too large to display}$$

input
```
integrate((a*coth(x)+b*csch(x))^5,x, algorithm="fricas")
```

output
```
-1/8*(8*a^5*x*cosh(x)^8 + 8*a^5*x*sinh(x)^8 + 2*(25*a^4*b + 10*a^2*b^3 - 3
*b^5)*cosh(x)^7 + 2*(32*a^5*x*cosh(x) + 25*a^4*b + 10*a^2*b^3 - 3*b^5)*sin
h(x)^7 - 32*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^6 + 2*(112*a^5*x*cosh(x)^2 -
 16*a^5*x + 16*a^5 + 80*a^3*b^2 + 7*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x
))*sinh(x)^6 + 8*a^5*x + 2*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^5 + 2*
(224*a^5*x*cosh(x)^3 + 15*a^4*b + 70*a^2*b^3 + 11*b^5 + 21*(25*a^4*b + 10*
a^2*b^3 - 3*b^5)*cosh(x)^2 - 96*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x))*sinh(x)
^5 + 16*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x)^4 + 2*(280*a^5*x*cosh(x)^4 +
24*a^5*x - 16*a^5 + 80*a*b^4 + 35*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^
3 - 240*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^2 + 5*(15*a^4*b + 70*a^2*b^3 + 1
1*b^5)*cosh(x))*sinh(x)^4 + 2*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^3 +
 2*(224*a^5*x*cosh(x)^5 + 15*a^4*b + 70*a^2*b^3 + 11*b^5 + 35*(25*a^4*b +
10*a^2*b^3 - 3*b^5)*cosh(x)^4 - 320*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^3 +
10*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^2 + 32*(3*a^5*x - 2*a^5 + 10*a
*b^4)*cosh(x))*sinh(x)^3 - 32*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^2 + 2*(112
*a^5*x*cosh(x)^6 - 16*a^5*x + 21*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^5
 + 16*a^5 + 80*a^3*b^2 - 240*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^4 + 10*(15*
a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^3 + 48*(3*a^5*x - 2*a^5 + 10*a*b^4)*c
osh(x)^2 + 3*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^2 + 2*(25*a
^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x) - ((8*a^5 - 15*a^4*b + 10*a^2*b^3 - ...
```

3.644.6 Sympy [F]

$$\int (a\coth(x) + b\text{csch}(x))^5\,dx = \int (a\coth(x) + b\,\text{csch}(x))^5\,dx$$

input
```
integrate((a*coth(x)+b*csch(x))**5,x)
```

output
```
Integral((a*coth(x) + b*csch(x))**5, x)
```

3.644.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(116) = 232$.

Time = 0.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.66

$$
\begin{aligned}
\int (a\coth(x)+b\text{csch}(x))^5\,dx = & -\frac{5}{2}\,a^3b^2\coth\left(x\right)^4 \\
& + a^5\left(x + \frac{4\left(e^{(-2x)} - e^{(-4x)} + e^{(-6x)}\right)}{4\,e^{(-2x)} - 6\,e^{(-4x)} + 4\,e^{(-6x)} - e^{(-8x)} - 1} + \log\left(e^{(-x)}+1\right) + \log\left(e^{(-x)}-1\right)\right) \\
& + \frac{5}{8}\,a^4b\left(\frac{2\left(5\,e^{(-x)} + 3\,e^{(-3x)} + 3\,e^{(-5x)} + 5\,e^{(-7x)}\right)}{4\,e^{(-2x)} - 6\,e^{(-4x)} + 4\,e^{(-6x)} - e^{(-8x)} - 1} - 3\log\left(e^{(-x)}+1\right) + 3\log\left(e^{(-x)}-1\right)\right) \\
& - \frac{1}{8}\,b^5\left(\frac{2\left(3\,e^{(-x)} - 11\,e^{(-3x)} - 11\,e^{(-5x)} + 3\,e^{(-7x)}\right)}{4\,e^{(-2x)} - 6\,e^{(-4x)} + 4\,e^{(-6x)} - e^{(-8x)} - 1} + 3\log\left(e^{(-x)}+1\right) - 3\log\left(e^{(-x)}-1\right)\right) \\
& + \frac{5}{4}\,a^2b^3\left(\frac{2\left(e^{(-x)} + 7\,e^{(-3x)} + 7\,e^{(-5x)} + e^{(-7x)}\right)}{4\,e^{(-2x)} - 6\,e^{(-4x)} + 4\,e^{(-6x)} - e^{(-8x)} - 1} + \log\left(e^{(-x)}+1\right) - \log\left(e^{(-x)}-1\right)\right) \\
& - \frac{20\,ab^4}{\left(e^{(-x)} - e^{x}\right)^4}
\end{aligned}
$$

input
```
integrate((a*coth(x)+b*csch(x))^5,x, algorithm="maxima")
```

output
```
-5/2*a^3*b^2*coth(x)^4 + a^5*(x + 4*(e^(-2*x) - e^(-4*x) + e^(-6*x))/(4*e^
(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + log(e^(-x) + 1) + log(e
^(-x) - 1)) + 5/8*a^4*b*(2*(5*e^(-x) + 3*e^(-3*x) + 3*e^(-5*x) + 5*e^(-7*x
))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 3*log(e^(-x) +
1) + 3*log(e^(-x) - 1)) - 1/8*b^5*(2*(3*e^(-x) - 11*e^(-3*x) - 11*e^(-5*x)
 + 3*e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 3*l
og(e^(-x) + 1) - 3*log(e^(-x) - 1)) + 5/4*a^2*b^3*(2*(e^(-x) + 7*e^(-3*x)
+ 7*e^(-5*x) + e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x)
- 1) + log(e^(-x) + 1) - log(e^(-x) - 1)) - 20*a*b^4/(e^(-x) - e^x)^4
```

3.644.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(116) = 232$.

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.89

$$\int (a\coth(x) + b\text{csch}(x))^5\,dx = \frac{1}{16}\left(8\,a^5 - 15\,a^4b + 10\,a^2b^3 - 3\,b^5\right)\log\left(e^{(-x)} + e^x + 2\right) + \frac{1}{16}\left(8\,a^5 + 15\,a^4b - 10\,a^2b^3 + 3\,b^5\right)\log\left(e^{(-x)} + e^x - 2\right) - \frac{3\,a^5\left(e^{(-x)} + e^x\right)^4 + 25\,a^4b\left(e^{(-x)} + e^x\right)^3 + 10\,a^2b^3\left(e^{(-x)} + e^x\right)^3 - 3\,b^5\left(e^{(-x)} + e^x\right)^3 - 8\,a^5\left(e^{(-x)} + e^x\right)^2 +}{4\left(\left(e^{(-x)} +\right.\right.}$$

input
```
integrate((a*coth(x)+b*csch(x))^5,x, algorithm="giac")
```

output
```
1/16*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*log(e^(-x) + e^x + 2) + 1/16*
(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*log(e^(-x) + e^x - 2) - 1/4*(3*a^5
*(e^(-x) + e^x)^4 + 25*a^4*b*(e^(-x) + e^x)^3 + 10*a^2*b^3*(e^(-x) + e^x)^
3 - 3*b^5*(e^(-x) + e^x)^3 - 8*a^5*(e^(-x) + e^x)^2 + 80*a^3*b^2*(e^(-x) +
 e^x)^2 - 60*a^4*b*(e^(-x) + e^x) + 40*a^2*b^3*(e^(-x) + e^x) + 20*b^5*(e^
(-x) + e^x) - 160*a^3*b^2 + 80*a*b^4)/((e^(-x) + e^x)^2 - 4)^2
```

3.644.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.16

$$\int (a\coth(x)+b\operatorname{csch}(x))^5\,dx = \ln\left(\frac{15\,a^4\,b}{4}+\frac{3\,b^5}{4}-\frac{5\,a^2\,b^3}{2}-\frac{3\,b^5\,e^x}{4}-\frac{15\,a^4\,b\,e^x}{4}+\frac{5\,a^2\,b^3\,e^x}{2}\right)\left(a^5+\frac{15\,a^4\,b}{8}-\frac{5\,a^2\,b^3}{4}+\frac{3\,b^5}{8}\right)-\frac{e^x\,(20\,a^4\,b+40\,a^2\,b^3+4\,b^5)+20\,a\,b^4+4\,a^5+40\,a^3\,b^2}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1}-\frac{e^x\,(30\,a^4\,b+60\,a^2\,b^3+6\,b^5)+40\,a\,b^4+8\,a^5+80\,a^3\,b^2}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}-a^5\,x-\ln\left(\frac{5\,a^2\,b^3}{2}-\frac{3\,b^5}{4}-\frac{15\,a^4\,b}{4}-\frac{3\,b^5\,e^x}{4}-\frac{15\,a^4\,b\,e^x}{4}+\frac{5\,a^2\,b^3\,e^x}{2}\right)\left(-a^5+\frac{15\,a^4\,b}{8}-\frac{5\,a^2\,b^3}{4}+\frac{3\,b^5}{8}\right)-\frac{e^x\left(\frac{25\,a^4\,b}{4}+\frac{5\,a^2\,b^3}{2}-\frac{3\,b^5}{4}\right)+4\,a^5+20\,a^3\,b^2}{e^{2\,x}-1}-\frac{e^x\left(\frac{45\,a^4\,b}{2}+25\,a^2\,b^3+\frac{b^5}{2}\right)+20\,a\,b^4+8\,a^5+60\,a^3\,b^2}{e^{4\,x}-2\,e^{2\,x}+1}$$

input
```
int((b/sinh(x) + a*coth(x))^5,x)
```

output
```
log((15*a^4*b)/4 + (3*b^5)/4 - (5*a^2*b^3)/2 - (3*b^5*exp(x))/4 - (15*a^4*
b*exp(x))/4 + (5*a^2*b^3*exp(x))/2)*((15*a^4*b)/8 + a^5 + (3*b^5)/8 - (5*a
^2*b^3)/4) - (exp(x)*(20*a^4*b + 4*b^5 + 40*a^2*b^3) + 20*a*b^4 + 4*a^5 +
40*a^3*b^2)/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - (exp(x
)*(30*a^4*b + 6*b^5 + 60*a^2*b^3) + 40*a*b^4 + 8*a^5 + 80*a^3*b^2)/(3*exp(
2*x) - 3*exp(4*x) + exp(6*x) - 1) - a^5*x - log((5*a^2*b^3)/2 - (3*b^5)/4
- (15*a^4*b)/4 - (3*b^5*exp(x))/4 - (15*a^4*b*exp(x))/4 + (5*a^2*b^3*exp(x
))/2)*((15*a^4*b)/8 - a^5 + (3*b^5)/8 - (5*a^2*b^3)/4) - (exp(x)*((25*a^4*
b)/4 - (3*b^5)/4 + (5*a^2*b^3)/2) + 4*a^5 + 20*a^3*b^2)/(exp(2*x) - 1) - (
exp(x)*((45*a^4*b)/2 + b^5/2 + 25*a^2*b^3) + 20*a*b^4 + 8*a^5 + 60*a^3*b^2
)/(exp(4*x) - 2*exp(2*x) + 1)
```

3.645 $\int (a\coth(x) + b\text{csch}(x))^4\,dx$

3.645.1 Optimal result

Integrand size = 11, antiderivative size = 101

$$\begin{aligned}\int (a\coth(x)+b\text{csch}(x))^4\,dx = {} & a^4x-\frac{1}{3}(b+a\cosh(x))^2\left(ab+\left(3a^2-2b^2\right)\cosh(x)\right)\text{csch}(x)\\ & -\frac{1}{3}(b+a\cosh(x))^3(a+b\cosh(x))\text{csch}^3(x)\\ & +\frac{4}{3}ab\left(2a^2-b^2\right)\sinh(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\cosh(x)\sinh(x)\end{aligned}$$

output
```
a^4*x-1/3*(b+a*cosh(x))^2*(a*b+(3*a^2-2*b^2)*cosh(x))*csch(x)-1/3*(b+a*cos
h(x))^3*(a+b*cosh(x))*csch(x)^3+4/3*a*b*(2*a^2-b^2)*sinh(x)+1/3*a^2*(3*a^2
-2*b^2)*cosh(x)*sinh(x)
```

3.645.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\begin{aligned}\int (a\coth(x)+b\text{csch}(x))^4\,dx = {} & -\frac{1}{12}\text{csch}^3(x)\left(-8a^3b+16ab^3+6b^2\left(3a^2+b^2\right)\cosh(x)\right.\\ & +24a^3b\cosh(2x)+4a^4\cosh(3x)+6a^2b^2\cosh(3x)\\ & \left.-2b^4\cosh(3x)+9a^4x\sinh(x)-3a^4x\sinh(3x)\right)\end{aligned}$$

input
```
Integrate[(a*Coth[x] + b*Csch[x])^4,x]
```

output
```
-1/12*(Csch[x]^3*(-8*a^3*b + 16*a*b^3 + 6*b^2*(3*a^2 + b^2)*Cosh[x] + 24*a
^3*b*Cosh[2*x] + 4*a^4*Cosh[3*x] + 6*a^2*b^2*Cosh[3*x] - 2*b^4*Cosh[3*x] +
 9*a^4*x*Sinh[x] - 3*a^4*x*Sinh[3*x]))
```

3.645.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4892, 3042, 3170, 25, 3042, 25, 3340, 27, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\coth(x)+b\text{csch}(x))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (ia\cot(ix)+ib\csc(ix))^4dx$$

$$\Big\downarrow \text{4892}$$

$$\int \text{csch}^4(x)(ia\cosh(x)+ib)^4dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(ib-ia\sin\left(-\frac{\pi}{2}+ix\right)\right)^4}{\cos\left(-\frac{\pi}{2}+ix\right)^4}dx$$

$$\Big\downarrow \text{3170}$$

$$-\frac{1}{3}\int -(b+a\cosh(x))^2\left(3a^2+b\cosh(x)a-2b^2\right)\text{csch}^2(x)dx-\frac{1}{3}\text{csch}^3(x)(a\cosh(x)+b)^3(a+b\cosh(x))$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int (b+a\cosh(x))^2\left(3a^2+b\cosh(x)a-2b^2\right)\text{csch}^2(x)dx-\frac{1}{3}\text{csch}^3(x)(a\cosh(x)+b)^3(a+b\cosh(x))$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\text{csch}^3(x)(a\cosh(x)+b)^3(a+b\cosh(x))+\frac{1}{3}\int -\frac{\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)^2\left(3a^2-b\sin\left(ix-\frac{\pi}{2}\right)a-2b^2\right)}{\cos\left(ix-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{3}\text{csch}^3(x)(a\cosh(x)+b)^3(a+b\cosh(x))-\frac{1}{3}\int\frac{\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)^2\left(3a^2-b\sin\left(ix-\frac{\pi}{2}\right)a-2b^2\right)}{\cos\left(ix-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \ 3340$$

$$\frac{1}{3}\left(\int 2(b+a\cosh(x))\left(ba^2+\left(3a^2-2b^2\right)\cosh(x)a\right)dx-\text{csch}(x)(a\cosh(x)+b)^2\left(\left(3a^2-2b^2\right)\cosh(x)+ab\right)\right)-\frac{1}{3}\text{csch}^3(x)(a\cosh(x)+b)^3(a+b\cosh(x))$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(2\int(b+a\cosh(x))\left(ba^2+\left(3a^2-2b^2\right)\cosh(x)a\right)dx-\text{csch}(x)(a\cosh(x)+b)^2\left(\left(3a^2-2b^2\right)\cosh(x)+ab\right)\right)-\frac{1}{3}\text{csch}^3(x)(a\cosh(x)+b)^3(a+b\cosh(x))$$

$$\Big\downarrow \ 3042$$

$$-\frac{1}{3}\text{csch}^3(x)(a\cosh(x)+b)^3(a+b\cosh(x))+\frac{1}{3}\left(-\text{csch}(x)(a\cosh(x)+b)^2\left(\left(3a^2-2b^2\right)\cosh(x)+ab\right)+2\int\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)\left(ba^2+\left(3a^2-2b^2\right)\sin\left(ix+\right.\right.\right.$$

$$\Big\downarrow \ 3213$$

$$\frac{1}{3}\left(2\left(\frac{3a^4x}{2}+2ab\left(2a^2-b^2\right)\sinh(x)+\frac{1}{2}a^2\left(3a^2-2b^2\right)\sinh(x)\cosh(x)\right)-\text{csch}(x)(a\cosh(x)+b)^2\left(\left(3a^2-2b^2\right)\text{co}\right.\right.$$
$$\frac{1}{3}\text{csch}^3(x)(a\cosh(x)+b)^3(a+b\cosh(x))$$

input
```
Int[(a*Coth[x] + b*Csch[x])^4,x]
```

output
```
-1/3*((b + a*Cosh[x])^3*(a + b*Cosh[x])*Csch[x]^3) + (-((b + a*Cosh[x])^2*
(a*b + (3*a^2 - 2*b^2)*Cosh[x])*Csch[x]) + 2*((3*a^4*x)/2 + 2*a*b*(2*a^2 -
 b^2)*Sinh[x] + (a^2*(3*a^2 - 2*b^2)*Cosh[x]*Sinh[x])/2))/3
```

3.645. $\int(a\coth(x)+b\text{csch}(x))^4\,dx$

3.645.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3340
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Simp[1/(g^2*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Si
n[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0]
&& SimplerQ[c + d*x, a + b*x])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.645. $\int (a\coth(x)+b\text{csch}(x))^4\,dx$

3.645.4 Maple [A] (verified)

Time = 20.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

method	result
parts	$a^4\left(-\frac{\coth(x)^3}{3}-\coth(x)-\frac{\ln(\coth(x)-1)}{2}+\frac{\ln(1+\coth(x))}{2}\right)+b^4\left(\frac{2}{3}-\frac{\operatorname{csch}(x)^2}{3}\right)\coth(x)-2a^2b^2\coth(x$
default	$a^4\left(x-\coth(x)-\frac{\coth(x)^3}{3}\right)+4a^3b\left(-\frac{\cosh(x)^2}{\sinh(x)^3}+\frac{2}{3\sinh(x)^3}\right)+6a^2b^2\left(-\frac{\cosh(x)}{2\sinh(x)^3}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(x)^2}{3}\right)\coth(x}{2}\right.$
risch	$x\,a^4-\frac{4(6a^3b\,e^{5x}+3\,e^{4x}a^4+9\,e^{4x}a^2b^2-4a^3b\,e^{3x}+8a\,b^3e^{3x}-3\,e^{2x}a^4+3\,e^{2x}b^4+6a^3b\,e^{x}+2a^4+3a^2b^2-b^4)}{3(e^{2x}-1)^3}$

input
```
int((a*coth(x)+b*csch(x))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*(-1/3*coth(x)^3-coth(x)-1/2*ln(coth(x)-1)+1/2*ln(1+coth(x)))+b^4*(2/3-
1/3*csch(x)^2)*coth(x)-2*a^2*b^2*coth(x)^3-4/3*b^3*csch(x)^3*a+4*a^3*b*(-1
/3*csch(x)^3-csch(x))
```

3.645.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(93) = 186.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.07

$$\int (a\coth(x)+b\operatorname{csch}(x))^4\,dx =$$
$$-\frac{24\,a^3b\cosh(x)^2-8\,a^3b+16\,ab^3+2\,(2\,a^4+3\,a^2b^2-b^4)\cosh(x)^3-(3\,a^4x+4\,a^4+6\,a^2b^2-2\,b^4)\sinh(}{}$$

input
```
integrate((a*coth(x)+b*csch(x))^4,x, algorithm="fricas")
```

output
```
-1/3*(24*a^3*b*cosh(x)^2 - 8*a^3*b + 16*a*b^3 + 2*(2*a^4 + 3*a^2*b^2 - b^4
)*cosh(x)^3 - (3*a^4*x + 4*a^4 + 6*a^2*b^2 - 2*b^4)*sinh(x)^3 + 6*(4*a^3*b
 + (2*a^4 + 3*a^2*b^2 - b^4)*cosh(x))*sinh(x)^2 + 6*(3*a^2*b^2 + b^4)*cosh
(x) + 3*(3*a^4*x + 4*a^4 + 6*a^2*b^2 - 2*b^4 - (3*a^4*x + 4*a^4 + 6*a^2*b^
2 - 2*b^4)*cosh(x)^2)*sinh(x))/(sinh(x)^3 + 3*(cosh(x)^2 - 1)*sinh(x))
```

3.645.6 Sympy [F]

$$\int (a\coth(x)+b\text{csch}(x))^4\,dx=\int (a\coth{(x)}+b\operatorname{csch}{(x)})^4\,dx$$

input
```
integrate((a*coth(x)+b*csch(x))**4,x)
```

output
```
Integral((a*coth(x) + b*csch(x))**4, x)
```

3.645.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(93) = 186.

Time = 0.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.12

$$\begin{aligned}
&\int (a\coth(x)+b\text{csch}(x))^4\,dx \\
&= -2\,a^2b^2\coth{(x)}^3+\frac{1}{3}\,a^4\left(3\,x-\frac{4\,\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}-2\right)}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}\right) \\
&\quad+\frac{8}{3}\,a^3b\left(\frac{3\,e^{(-x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}-\frac{2\,e^{(-3\,x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}+\frac{3\,e^{(-5\,x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}}\right. \\
&\quad+\frac{4}{3}\,b^4\left(\frac{3\,e^{(-2\,x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}-\frac{1}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}\right) \\
&\quad+\frac{32\,ab^3}{3\,\left(e^{(-x)}-e^{x}\right)^3}
\end{aligned}$$

input
```
integrate((a*coth(x)+b*csch(x))^4,x, algorithm="maxima")
```

output
```
-2*a^2*b^2*coth(x)^3 + 1/3*a^4*(3*x - 4*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e
^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) + 8/3*a^3*b*(3*e^(-x)/(3*e^(-2*x) -
3*e^(-4*x) + e^(-6*x) - 1) - 2*e^(-3*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x
) - 1) + 3*e^(-5*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) + 4/3*b^4*(3
*e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 1/(3*e^(-2*x) - 3*e^(
-4*x) + e^(-6*x) - 1)) + 32/3*a*b^3/(e^(-x) - e^x)^3
```

3.645.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11

$$\int (a\coth(x)+b\operatorname{csch}(x))^4\,dx = a^4x - \frac{4\left(6\,a^3be^{(5\,x)}+3\,a^4e^{(4\,x)}+9\,a^2b^2e^{(4\,x)}-4\,a^3be^{(3\,x)}+8\,ab^3e^{(3\,x)}-3\,a^4e^{(2\,x)}+3\,b^4e^{(2\,x)}+6\,a^3be^{x}+2\,a^4+\right.}{3\left(e^{(2\,x)}-1\right)^3}$$

input
```
integrate((a*coth(x)+b*csch(x))^4,x, algorithm="giac")
```

output
```
a^4*x - 4/3*(6*a^3*b*e^(5*x) + 3*a^4*e^(4*x) + 9*a^2*b^2*e^(4*x) - 4*a^3*b
*e^(3*x) + 8*a*b^3*e^(3*x) - 3*a^4*e^(2*x) + 3*b^4*e^(2*x) + 6*a^3*b*e^x +
 2*a^4 + 3*a^2*b^2 - b^4)/(e^(2*x) - 1)^3
```

3.645.9 Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.45

$$\int (a\coth(x)+b\operatorname{csch}(x))^4\,dx = a^4\,x - \frac{4\,a^4+8\,\mathrm{e}^{x}\,a^3\,b+12\,a^2\,b^2}{\mathrm{e}^{2\,x}-1} - \frac{\mathrm{e}^{x}\left(\frac{32\,a^3\,b}{3}+\frac{32\,a\,b^3}{3}\right)+4\,a^4+4\,b^4+24\,a^2\,b^2}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1} - \frac{\mathrm{e}^{x}\left(\frac{32\,a^3\,b}{3}+\frac{32\,a\,b^3}{3}\right)+\frac{8\,a^4}{3}+\frac{8\,b^4}{3}+16\,a^2\,b^2}{3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1}$$

input
```
int((b/sinh(x) + a*coth(x))^4,x)
```

output
```
a^4*x - (4*a^4 + 12*a^2*b^2 + 8*a^3*b*exp(x))/(exp(2*x) - 1) - (exp(x)*((3
2*a*b^3)/3 + (32*a^3*b)/3) + 4*a^4 + 4*b^4 + 24*a^2*b^2)/(exp(4*x) - 2*exp
(2*x) + 1) - (exp(x)*((32*a*b^3)/3 + (32*a^3*b)/3) + (8*a^4)/3 + (8*b^4)/3
 + 16*a^2*b^2)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)
```

3.646 $\int (a\coth(x)+b\text{csch}(x))^3\,dx$

3.646.1 Optimal result

Integrand size = 11, antiderivative size = 59

$$\int (a\coth(x)+b\text{csch}(x))^3\,dx=-\frac{1}{2}b(3a^2-b^2)\,\text{arctanh}(\cosh(x))+\frac{1}{2}a^2b\cosh(x)-\frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\text{csch}^2(x)+a^3\log(\sinh(x))$$

output
```
-1/2*b*(3*a^2-b^2)*arctanh(cosh(x))+1/2*a^2*b*cosh(x)-1/2*(b+a*cosh(x))^2*
(a+b*cosh(x))*csch(x)^2+a^3*ln(sinh(x))
```

3.646.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

$$\int (a\coth(x)+b\text{csch}(x))^3\,dx=\frac{1}{8}\left(-(a+b)^3\text{csch}^2\left(\frac{x}{2}\right)+4(a-b)^2(2a+b)\log\left(\cosh\left(\frac{x}{2}\right)\right)+4(2a-b)(a+b)^2\log\left(\sinh\left(\frac{x}{2}\right)\right)+(a-b)^3\text{sech}^2\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(a*Coth[x] + b*Csch[x])^3,x]
```

output
```
(-((a + b)^3*Csch[x/2]^2) + 4*(a - b)^2*(2*a + b)*Log[Cosh[x/2]] + 4*(2*a
- b)*(a + b)^2*Log[Sinh[x/2]] + (a - b)^3*Sech[x/2]^2)/8
```

3.646.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.64, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 4892, 26, 26, 3042, 26, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\coth(x)+b\text{csch}(x))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (ia\cot(ix)+ib\csc(ix))^3dx$$

$$\Big\downarrow \text{4892}$$

$$\int i\text{csch}^3(x)(ia\cosh(x)+ib)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int -i(b+a\cosh(x))^3\text{csch}^3(x)dx$$

$$\Big\downarrow \text{26}$$

$$\int \text{csch}^3(x)(a\cosh(x)+b)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(b-a\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}{\cos\left(-\frac{\pi}{2}+ix\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)^3}{\cos\left(ix-\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3147}$$

$$a^3\int \frac{(b+a\cosh(x))^3}{\left(a^2-a^2\cosh^2(x)\right)^2}d(a\cosh(x))$$

$$\Big\downarrow \text{477}$$

$$\frac{\int \left(-\frac{a^2(a-b)^3}{4(\cosh(x)a+a)^2}+\frac{a(2a+b)(a-b)^2}{4(\cosh(x)a+a)}-\frac{a(2a-b)(a+b)^2}{4(a-a\cosh(x))}+\frac{a^2(a+b)^3}{4(a-a\cosh(x))^2}\right)d(a\cosh(x))}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2(a-b)^3}{4(a\cosh(x)+a)}+\frac{a^2(a+b)^3}{4(a-a\cosh(x))}+\frac{1}{4}a(2a+b)(a-b)^2\log(a\cosh(x)+a)+\frac{1}{4}a(2a-b)(a+b)^2\log(a-a\cosh(x))}{a}$$

input
```
Int[(a*Coth[x] + b*Csch[x])^3,x]
```

output
```
((a^2*(a + b)^3)/(4*(a - a*Cosh[x])) + (a^2*(a - b)^3)/(4*(a + a*Cosh[x]))
 + (a*(2*a - b)*(a + b)^2*Log[a - a*Cosh[x]])/4 + (a*(a - b)^2*(2*a + b)*L
og[a + a*Cosh[x]])/4)/a
```

3.646.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.646.4 Maple [A] (verified)

Time = 5.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

method	result
default	$a^{3}\left(\ln\left(\sinh\left(x\right)\right)-\frac{\coth\left(x\right)^{2}}{2}\right)+3a^{2}b\left(-\frac{\cosh\left(x\right)}{\sinh\left(x\right)^{2}}+\frac{\operatorname{csch}\left(x\right)\coth\left(x\right)}{2}-\operatorname{arctanh}\left(\mathrm{e}^{x}\right)\right)-\frac{3a\,b^{2}}{2\sinh\left(x\right)^{2}}+b^{3}\left(-\frac{\mathrm{cs}}{}\right.$
parts	$a^{3}\left(-\frac{\coth\left(x\right)^{2}}{2}-\frac{\ln\left(\coth\left(x\right)-1\right)}{2}-\frac{\ln\left(1+\coth\left(x\right)\right)}{2}\right)+b^{3}\left(-\frac{\operatorname{csch}\left(x\right)\coth\left(x\right)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{x}\right)\right)+3a^{2}b\left(-\frac{\cosh\left(x\right)}{\sinh\left(x\right)^{2}}\right.$
risch	$-a^{3}x-\frac{\mathrm{e}^{x}\left(3a^{2}b\,\mathrm{e}^{2x}+b^{3}\mathrm{e}^{2x}+2a^{3}\mathrm{e}^{x}+6\,\mathrm{e}^{x}b^{2}a+3a^{2}b+b^{3}\right)}{\left(\mathrm{e}^{2x}-1\right)^{2}}+\ln\left(\mathrm{e}^{x}-1\right)a^{3}+\frac{3\ln\left(\mathrm{e}^{x}-1\right)a^{2}b}{2}-\frac{\ln\left(\mathrm{e}^{x}-1\right)b^{3}}{2}+\ln\left(\mathrm{e}^{x}+\right.$

input
```
int((a*coth(x)+b*csch(x))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*(ln(sinh(x))-1/2*coth(x)^2)+3*a^2*b*(-1/sinh(x)^2*cosh(x)+1/2*csch(x)*
coth(x)-arctanh(exp(x)))-3/2*a*b^2/sinh(x)^2+b^3*(-1/2*csch(x)*coth(x)+arc
tanh(exp(x)))
```

3.646.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 674 vs. $2(53) = 106$.

Time = 0.28 (sec) , antiderivative size = 674, normalized size of antiderivative = 11.42

$$\int (a\coth(x)+b\operatorname{csch}(x))^3\,dx =$$
$$-\frac{2\,a^{3}x\cosh\left(x\right)^{4}+2\,a^{3}x\sinh\left(x\right)^{4}+2\,a^{3}x+2\left(3\,a^{2}b+b^{3}\right)\cosh\left(x\right)^{3}+2\left(4\,a^{3}x\cosh\left(x\right)+3\,a^{2}b+b^{3}\right)\sinh}{}$$

input
```
integrate((a*coth(x)+b*csch(x))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a^3*x*cosh(x)^4 + 2*a^3*x*sinh(x)^4 + 2*a^3*x + 2*(3*a^2*b + b^3)*
cosh(x)^3 + 2*(4*a^3*x*cosh(x) + 3*a^2*b + b^3)*sinh(x)^3 - 4*(a^3*x - a^3
 - 3*a*b^2)*cosh(x)^2 + 2*(6*a^3*x*cosh(x)^2 - 2*a^3*x + 2*a^3 + 6*a*b^2 +
 3*(3*a^2*b + b^3)*cosh(x))*sinh(x)^2 + 2*(3*a^2*b + b^3)*cosh(x) - ((2*a^
3 - 3*a^2*b + b^3)*cosh(x)^4 + 4*(2*a^3 - 3*a^2*b + b^3)*cosh(x)*sinh(x)^3
 + (2*a^3 - 3*a^2*b + b^3)*sinh(x)^4 + 2*a^3 - 3*a^2*b + b^3 - 2*(2*a^3 -
3*a^2*b + b^3)*cosh(x)^2 - 2*(2*a^3 - 3*a^2*b + b^3 - 3*(2*a^3 - 3*a^2*b +
 b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 - 3*a^2*b + b^3)*cosh(x)^3 - (2*a^3
 - 3*a^2*b + b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - ((2*a^3 +
 3*a^2*b - b^3)*cosh(x)^4 + 4*(2*a^3 + 3*a^2*b - b^3)*cosh(x)*sinh(x)^3 +
(2*a^3 + 3*a^2*b - b^3)*sinh(x)^4 + 2*a^3 + 3*a^2*b - b^3 - 2*(2*a^3 + 3*a
^2*b - b^3)*cosh(x)^2 - 2*(2*a^3 + 3*a^2*b - b^3 - 3*(2*a^3 + 3*a^2*b - b^
3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a^2*b - b^3)*cosh(x)^3 - (2*a^3 +
3*a^2*b - b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(4*a^3*x*c
osh(x)^3 + 3*a^2*b + b^3 + 3*(3*a^2*b + b^3)*cosh(x)^2 - 4*(a^3*x - a^3 -
3*a*b^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh
(x) + 1)
```

3.646.6 Sympy [F]

$$\int (a\coth(x) + b\text{csch}(x))^3 \, dx = \int (a\coth(x) + b\,\text{csch}(x))^3 \, dx$$

input
```
integrate((a*coth(x)+b*csch(x))**3,x)
```

output
```
Integral((a*coth(x) + b*csch(x))**3, x)
```

3.646.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(53) = 106.

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.58

$$\int (a\coth(x) + b\text{csch}(x))^3\,dx$$
$$= -\frac{3}{2}ab^2\coth(x)^2 + a^3\left(x + \frac{2e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)\right)$$
$$+ \frac{1}{2}b^3\left(\frac{2\left(e^{(-x)} + e^{(-3x)}\right)}{2e^{(-2x)} - e^{(-4x)} - 1} + \log\left(e^{(-x)} + 1\right) - \log\left(e^{(-x)} - 1\right)\right)$$
$$+ \frac{3}{2}a^2b\left(\frac{2\left(e^{(-x)} + e^{(-3x)}\right)}{2e^{(-2x)} - e^{(-4x)} - 1} - \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)\right)$$

input
```
integrate((a*coth(x)+b*csch(x))^3,x, algorithm="maxima")
```

output
```
-3/2*a*b^2*coth(x)^2 + a^3*(x + 2*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + l
og(e^(-x) + 1) + log(e^(-x) - 1)) + 1/2*b^3*(2*(e^(-x) + e^(-3*x))/(2*e^(-
2*x) - e^(-4*x) - 1) + log(e^(-x) + 1) - log(e^(-x) - 1)) + 3/2*a^2*b*(2*(
e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) - log(e^(-x) + 1) + log(e^(
-x) - 1))
```

3.646.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(53) = 106.

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.95

$$\int (a\coth(x) + b\text{csch}(x))^3\,dx$$
$$= \frac{1}{4}\left(2a^3 - 3a^2b + b^3\right)\log\left(e^{(-x)} + e^x + 2\right) + \frac{1}{4}\left(2a^3 + 3a^2b - b^3\right)\log\left(e^{(-x)} + e^x - 2\right)$$
$$- \frac{a^3\left(e^{(-x)} + e^x\right)^2 + 6a^2b\left(e^{(-x)} + e^x\right) + 2b^3\left(e^{(-x)} + e^x\right) + 12ab^2}{2\left(\left(e^{(-x)} + e^x\right)^2 - 4\right)}$$

input
```
integrate((a*coth(x)+b*csch(x))^3,x, algorithm="giac")
```

output
```
1/4*(2*a^3 - 3*a^2*b + b^3)*log(e^(-x) + e^x + 2) + 1/4*(2*a^3 + 3*a^2*b -
 b^3)*log(e^(-x) + e^x - 2) - 1/2*(a^3*(e^(-x) + e^x)^2 + 6*a^2*b*(e^(-x)
+ e^x) + 2*b^3*(e^(-x) + e^x) + 12*a*b^2)/((e^(-x) + e^x)^2 - 4)
```

3.646.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.86

$$\int (a\coth(x)+b\text{csch}(x))^3\,dx = \ln\left(b^3-3\,a^2\,b+b^3\,\mathrm{e}^x-3\,a^2\,b\,\mathrm{e}^x\right)\,\left(a^3-\frac{3\,a^2\,b}{2}+\frac{b^3}{2}\right) - \frac{6\,a\,b^2+2\,a^3+\mathrm{e}^x\,(3\,a^2\,b+b^3)}{\mathrm{e}^{2\,x}-1} - \frac{\mathrm{e}^x\,(6\,a^2\,b+2\,b^3)+6\,a\,b^2+2\,a^3}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1} - a^3\,x + \ln\left(3\,a^2\,b-b^3+b^3\,\mathrm{e}^x-3\,a^2\,b\,\mathrm{e}^x\right)\,\left(a^3+\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)$$

input
```
int((b/sinh(x) + a*coth(x))^3,x)
```

output
```
log(b^3 - 3*a^2*b + b^3*exp(x) - 3*a^2*b*exp(x))*(a^3 - (3*a^2*b)/2 + b^3/
2) - (6*a*b^2 + 2*a^3 + exp(x)*(3*a^2*b + b^3))/(exp(2*x) - 1) - (exp(x)*(
6*a^2*b + 2*b^3) + 6*a*b^2 + 2*a^3)/(exp(4*x) - 2*exp(2*x) + 1) - a^3*x +
log(3*a^2*b - b^3 + b^3*exp(x) - 3*a^2*b*exp(x))*((3*a^2*b)/2 + a^3 - b^3/
2)
```

3.647 $\int (a\coth(x) + b\text{csch}(x))^2 \, dx$

3.647.1 Optimal result

Integrand size = 11, antiderivative size = 27

$$\int (a\coth(x) + b\text{csch}(x))^2 \, dx = a^2x - (b + a\cosh(x))(a + b\cosh(x))\text{csch}(x) + ab\sinh(x)$$

output
```
a^2*x-(b+a*cosh(x))*(a+b*cosh(x))*csch(x)+a*b*sinh(x)
```

3.647.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int (a\coth(x) + b\text{csch}(x))^2 \, dx = -\left(\left(a^2 + b^2\right)\coth(x)\right) + a(ax - 2b\text{csch}(x))$$

input
```
Integrate[(a*Coth[x] + b*Csch[x])^2,x]
```

output
```
-((a^2 + b^2)*Coth[x]) + a*(a*x - 2*b*Csch[x])
```

3.647.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3042, 4892, 25, 25, 3042, 25, 3170, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\coth(x)+b\text{csch}(x))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (ia\cot(ix)+ib\csc(ix))^2dx$$

$$\Big\downarrow \text{4892}$$

$$\int \text{csch}^2(x)\left(-(ia\cosh(x)+ib)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int -(b+a\cosh(x))^2\text{csch}^2(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int \text{csch}^2(x)(a\cosh(x)+b)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(b-a\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}{\cos\left(-\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)^2}{\cos\left(ix-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3170}$$

$$\int \left(a^2+b\cosh(x)a\right)dx-\text{csch}(x)(a\cosh(x)+b)(a+b\cosh(x))$$

$$\Big\downarrow \text{2009}$$

$$a^2x+ab\sinh(x)-\text{csch}(x)(a\cosh(x)+b)(a+b\cosh(x))$$

input
```
Int[(a*Coth[x] + b*Csch[x])^2,x]
```

output
```
a^2*x - (b + a*Cosh[x])*(a + b*Cosh[x])*Csch[x] + a*b*Sinh[x]
```

3.647.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.647.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
default	$a^{2}\left(x-\coth \left(x \right)\right)-\frac{2 a b}{\sinh \left(x \right)}-b^{2} \coth \left(x \right)$	27
risch	$a^{2} x -\frac{2 \left(2 b \,{\mathrm e}^{x} a +a^{2}+b^{2}\right)}{{\mathrm e}^{2 x}-1}$	30
parts	$a^{2} \left(-\coth \left(x \right)-\frac{\ln \left(\coth \left(x \right)-1\right)}{2}+\frac{\ln \left(1+\coth \left(x \right)\right)}{2}\right)-b^{2} \coth \left(x \right)-2 b \,\operatorname{csch}\left(x \right) a$	38

input
```
int((a*coth(x)+b*csch(x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(x-coth(x))-2*a*b/sinh(x)-b^2*coth(x)
```

3.647.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int (a\coth(x)+b\text{csch}(x))^2\,dx=-\frac{2\,ab+(a^2+b^2)\cosh{(x)}-(a^2x+a^2+b^2)\sinh{(x)}}{\sinh{(x)}}$$

input
```
integrate((a*coth(x)+b*csch(x))^2,x, algorithm="fricas")
```

output
```
-(2*a*b + (a^2 + b^2)*cosh(x) - (a^2*x + a^2 + b^2)*sinh(x))/sinh(x)
```

3.647.6 Sympy [F]

$$\int (a\coth(x)+b\text{csch}(x))^2\,dx=\int (a\coth{(x)}+b\,\text{csch}{(x)})^2\,dx$$

input
```
integrate((a*coth(x)+b*csch(x))**2,x)
```

output
```
Integral((a*coth(x) + b*csch(x))**2, x)
```

3.647.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int (a\coth(x)+b\text{csch}(x))^2\,dx=a^2\left(x+\frac{2}{e^{(-2\,x)}-1}\right)+\frac{4\,ab}{e^{(-x)}-e^x}+\frac{2\,b^2}{e^{(-2\,x)}-1}$$

input
```
integrate((a*coth(x)+b*csch(x))^2,x, algorithm="maxima")
```

output
```
a^2*(x + 2/(e^(-2*x) - 1)) + 4*a*b/(e^(-x) - e^x) + 2*b^2/(e^(-2*x) - 1)
```

3.647.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a\coth(x) + b\mathrm{csch}(x))^2\,dx = a^2x - \frac{2\,(2\,abe^x + a^2 + b^2)}{e^{(2\,x)} - 1}$$

input
```
integrate((a*coth(x)+b*csch(x))^2,x, algorithm="giac")
```

output
```
a^2*x - 2*(2*a*b*e^x + a^2 + b^2)/(e^(2*x) - 1)
```

3.647.9 Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int (a\coth(x) + b\mathrm{csch}(x))^2\,dx = a^2\,x - \frac{2\,a^2 + 4\,\mathrm{e}^x\,a\,b + 2\,b^2}{\mathrm{e}^{2\,x} - 1}$$

input
```
int((b/sinh(x) + a*coth(x))^2,x)
```

output
```
a^2*x - (2*a^2 + 2*b^2 + 4*a*b*exp(x))/(exp(2*x) - 1)
```

3.648 $\int (a \coth(x) + b\operatorname{csch}(x))\,dx$

3.648.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int (a \coth(x) + b\operatorname{csch}(x))\,dx = -b\operatorname{arctanh}(\cosh(x)) + a\log(\sinh(x))$$

output
```
-b*arctanh(cosh(x))+a*ln(sinh(x))
```

3.648.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 30 vs. 2(12) = 24.

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int (a \coth(x) + b\operatorname{csch}(x))\,dx = -b\log\left(\cosh\left(\frac{x}{2}\right)\right) + a\log(\cosh(x)) + b\log\left(\sinh\left(\frac{x}{2}\right)\right) + a\log(\tanh(x))$$

input
```
Integrate[a*Coth[x] + b*Csch[x],x]
```

output
```
-(b*Log[Cosh[x/2]]) + a*Log[Cosh[x]] + b*Log[Sinh[x/2]] + a*Log[Tanh[x]]
```

3.648.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\coth(x)+b\text{csch}(x))\,dx$$

$$\Big\downarrow \ 2009$$

$$a\log(\sinh(x))-b\text{arctanh}(\cosh(x))$$

input
```
Int[a*Coth[x] + b*Csch[x],x]
```

output
```
-(b*ArcTanh[Cosh[x]]) + a*Log[Sinh[x]]
```

3.648.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.648.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

method	result	size
default	$a\ln(\sinh(x))+b\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	14
parts	$a\ln(\sinh(x))+b\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	14
parallelrisch	$b\ln(\coth(x)-\text{csch}(x))-a(x-\ln(\tanh(x))+\ln(1-\tanh(x)))$	29
risch	$-ax+a\ln(\mathrm{e}^{2x}-1)+b\ln(\mathrm{e}^{x}-1)-b\ln(\mathrm{e}^{x}+1)$	30

input
```
int(a*coth(x)+b*csch(x),x,method=_RETURNVERBOSE)
```

output
```
a*ln(sinh(x))+b*ln(tanh(1/2*x))
```

3.648.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int (a\coth(x) + b\text{csch}(x))\,dx = -ax + (a-b)\log(\cosh(x) + \sinh(x) + 1) + (a+b)\log(\cosh(x) + \sinh(x) - 1)$$

input
```
integrate(a*coth(x)+b*csch(x),x, algorithm="fricas")
```

output
```
-a*x + (a - b)*log(cosh(x) + sinh(x) + 1) + (a + b)*log(cosh(x) + sinh(x)
- 1)
```

3.648.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int (a\coth(x) + b\text{csch}(x))\,dx = a(x - \log(\tanh(x) + 1) + \log(\tanh(x))) + b\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
integrate(a*coth(x)+b*csch(x),x)
```

output
```
a*(x - log(tanh(x) + 1) + log(tanh(x))) + b*log(tanh(x/2))
```

3.648.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int (a\coth(x) + b\text{csch}(x))\,dx = a\log(\sinh(x)) + b\log\left(\tanh\left(\frac{1}{2}x\right)\right)$$

input
```
integrate(a*coth(x)+b*csch(x),x, algorithm="maxima")
```

output
```
a*log(sinh(x)) + b*log(tanh(1/2*x))
```

3.648.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int (a\coth(x) + b\text{csch}(x))\,dx = -a\left(x - \log\left(\left|e^{(2x)} - 1\right|\right)\right) - b(\log(e^x + 1) - \log(|e^x - 1|))$$

input
```
integrate(a*coth(x)+b*csch(x),x, algorithm="giac")
```

output
```
-a*(x - log(abs(e^(2*x) - 1))) - b*(log(e^x + 1) - log(abs(e^x - 1)))
```

3.648.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

$$\int (a\coth(x) + b\text{csch}(x))\,dx = \ln(-2b - 2b\,\mathrm{e}^x)\,(a - b) - a\,x + \ln(2b - 2b\,\mathrm{e}^x)\,(a + b)$$

input
```
int(b/sinh(x) + a*coth(x),x)
```

output
```
log(- 2*b - 2*b*exp(x))*(a - b) - a*x + log(2*b - 2*b*exp(x))*(a + b)
```

3.649 $\int \frac{1}{a\coth(x)+b\operatorname{csch}(x)}\,dx$

3.649.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{a\coth(x)+b\operatorname{csch}(x)}\,dx=\frac{\log(b+a\cosh(x))}{a}$$

output
```
ln(b+a*cosh(x))/a
```

3.649.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\coth(x)+b\operatorname{csch}(x)}\,dx=\frac{\log(b+a\cosh(x))}{a}$$

input
```
Integrate[(a*Coth[x] + b*Csch[x])^(-1),x]
```

output
```
Log[b + a*Cosh[x]]/a
```

3.649.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3042, 3639, 26, 26, 3042, 26, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\coth(x)+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{ia\cot(ix)+ib\csc(ix)}dx$$

$$\Big\downarrow \text{3639}$$

$$\int \frac{i\sinh(x)}{ia\cosh(x)+ib}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\sinh(x)}{b+a\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sinh(x)}{a\cosh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{b-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{b-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{1}{b+a\cosh(x)}d(a\cosh(x))}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(a\cosh(x)+b)}{a}$$

input
```
Int[(a*Coth[x] + b*Csch[x])^(-1),x]
```

output
```
Log[b + a*Cosh[x]]/a
```

3.649.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3639
```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.649.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

method	result	size
risch	$-\frac{x}{a}+\frac{\ln\left({\mathrm e}^{2 x}+\frac{2 b\,{\mathrm e}^{x}}{a}+1\right)}{a}$	27
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2} a-\tanh\left(\frac{x}{2}\right)^{2} b+a+b\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	51

input
```
int(1/(a*coth(x)+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-x/a+1/a*ln(exp(2*x)+2*b/a*exp(x)+1)
```

3.649.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{1}{a\coth(x)+b\text{csch}(x)}\,dx = -\frac{x-\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

input
```
integrate(1/(a*coth(x)+b*csch(x)),x, algorithm="fricas")
```

output
```
-(x - log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))))/a
```

3.649.6 Sympy [F]

$$\int \frac{1}{a\coth(x)+b\text{csch}(x)}\,dx = \int \frac{1}{a\coth(x)+b\operatorname{csch}(x)}\,dx$$

input
```
integrate(1/(a*coth(x)+b*csch(x)),x)
```

output
```
Integral(1/(a*coth(x) + b*csch(x)), x)
```

3.649.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \frac{1}{a\coth(x)+b\text{csch}(x)}\,dx = \frac{x}{a} + \frac{\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a}$$

input
```
integrate(1/(a*coth(x)+b*csch(x)),x, algorithm="maxima")
```

output
```
x/a + log(2*b*e^(-x) + a*e^(-2*x) + a)/a
```

3.649.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{1}{a\coth(x) + b\text{csch}(x)}\,dx = \frac{\log\left(\left|a\left(e^{(-x)} + e^{x}\right) + 2\,b\right|\right)}{a}$$

input
```
integrate(1/(a*coth(x)+b*csch(x)),x, algorithm="giac")
```

output
```
log(abs(a*(e^(-x) + e^x) + 2*b))/a
```

3.649.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{1}{a\coth(x) + b\text{csch}(x)}\,dx = -\frac{x - \ln\left(a + 2\,b\,e^{x} + a\,e^{2\,x}\right)}{a}$$

input
```
int(1/(b/sinh(x) + a*coth(x)),x)
```

output
```
-(x - log(a + 2*b*exp(x) + a*exp(2*x)))/a
```

3.650 $\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^2}\,dx$

3.650.1 Optimal result

Integrand size = 11, antiderivative size = 67

$$\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^2}\,dx=\frac{x}{a^2}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}-\frac{\sinh(x)}{a(b+a\cosh(x))}$$

output
```
x/a^2-sinh(x)/a/(b+a*cosh(x))-2*b*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/
2))/a^2/(a-b)^(1/2)/(a+b)^(1/2)
```

3.650.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^2}\,dx=\frac{x+\frac{2b\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{a\sinh(x)}{b+a\cosh(x)}}{a^2}$$

input
```
Integrate[(a*Coth[x] + b*Csch[x])^(-2),x]
```

output
```
(x + (2*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
(a*Sinh[x])/(b + a*Cosh[x]))/a^2
```

3.650.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.182, Rules used = {3042, 4892, 25, 25, 3042, 25, 3172, 25, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(ia\cot(ix)+ib\csc(ix))^2}dx$$

$$\Big\downarrow \text{4892}$$

$$\int -\frac{\sinh^2(x)}{(ia\cosh(x)+ib)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\sinh^2(x)}{(b+a\cosh(x))^2}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(b-a\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3172}$$

$$-\frac{\int -\frac{\cosh(x)}{b+a\cosh(x)}dx}{a}-\frac{\sinh(x)}{a(a\cosh(x)+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cosh(x)}{b+a\cosh(x)}dx}{a}-\frac{\sinh(x)}{a(a\cosh(x)+b)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)}{a(a\cosh(x)+b)}+\frac{\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{x}{a}-\frac{b\int \frac{1}{b+a\cosh(x)}dx}{a}}{a}-\frac{\sinh(x)}{a(a\cosh(x)+b)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)}{a(a\cosh(x)+b)}+\frac{\frac{x}{a}-\frac{b\int \frac{1}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{x}{a}-\frac{2b\int \frac{1}{(a-b)\tanh^2\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}}{a}-\frac{\sinh(x)}{a(a\cosh(x)+b)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}}{a}-\frac{\sinh(x)}{a(a\cosh(x)+b)}$$

input
```
Int[(a*Coth[x] + b*Csch[x])^(-2),x]
```

output
```
(x/a - (2*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sq
rt[a + b]))/a - Sinh[x]/(a*(b + a*Cosh[x]))
```

3.650.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.650. $\int \frac{1}{(a\coth(x)+b\text{csch}(x))^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.650.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{-\frac{2\tanh\left(\frac{x}{2}\right)a}{\tanh\left(\frac{x}{2}\right)^{2}a-\tanh\left(\frac{x}{2}\right)^{2}b+a+b}-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{a^{2}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^{2}}$	96
risch	$\frac{x}{a^{2}}+\frac{2\,\mathrm{e}^{x}b+2a}{a^{2}\left(a\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}b+a\right)}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}\,a^{2}}+\frac{b\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}\,a^{2}}$	148

input
```
int(1/(a*coth(x)+b*csch(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2/a^2*(-tanh(1/2*x)*a/(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b+a+b)-b/((a+b)*(a-b)
)^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))-1/a^2*ln(tanh(1/2*x
)-1)+1/a^2*ln(tanh(1/2*x)+1)
```

3.650.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(57) = 114$.

Time = 0.27 (sec) , antiderivative size = 682, normalized size of antiderivative = 10.18

$$\int \frac{1}{(a\coth(x) + b\operatorname{csch}(x))^2}\,dx$$

$$= \left[\frac{(a^3 - ab^2)x\cosh(x)^2 + (a^3 - ab^2)x\sinh(x)^2 + 2a^3 - 2ab^2 - \left(ab\cosh(x)^2 + ab\sinh(x)^2 + 2b^2\cosh(x\right.}{}\right.$$

input
```
integrate(1/(a*coth(x)+b*csch(x))^2,x, algorithm="fricas")
```

output
```
[((a^3 - a*b^2)*x*cosh(x)^2 + (a^3 - a*b^2)*x*sinh(x)^2 + 2*a^3 - 2*a*b^2
- (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*b^2*cosh(x) + a*b + 2*(a*b*cosh(x) +
b^2)*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*
cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)
*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2
*(a*cosh(x) + b)*sinh(x) + a)) + (a^3 - a*b^2)*x + 2*(a^2*b - b^3 + (a^2*b
 - b^3)*x)*cosh(x) + 2*(a^2*b - b^3 + (a^3 - a*b^2)*x*cosh(x) + (a^2*b - b
^3)*x)*sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^2 + (a^5 - a^3*b^
2)*sinh(x)^2 + 2*(a^4*b - a^2*b^3)*cosh(x) + 2*(a^4*b - a^2*b^3 + (a^5 - a
^3*b^2)*cosh(x))*sinh(x)), ((a^3 - a*b^2)*x*cosh(x)^2 + (a^3 - a*b^2)*x*si
nh(x)^2 + 2*a^3 - 2*a*b^2 + 2*(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*b^2*cosh(
x) + a*b + 2*(a*b*cosh(x) + b^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(
x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + (a^3 - a*b^2)*x + 2*(a^2*b - b^3 +
(a^2*b - b^3)*x)*cosh(x) + 2*(a^2*b - b^3 + (a^3 - a*b^2)*x*cosh(x) + (a^2
*b - b^3)*x)*sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^2 + (a^5 -
a^3*b^2)*sinh(x)^2 + 2*(a^4*b - a^2*b^3)*cosh(x) + 2*(a^4*b - a^2*b^3 + (a
^5 - a^3*b^2)*cosh(x))*sinh(x))]
```

3.650.6 Sympy [F]

$$\int \frac{1}{(a\coth(x) + b\text{csch}(x))^2}\,dx = \int \frac{1}{(a\coth{(x)} + b\,\text{csch}{(x)})^2}\,dx$$

input
```
integrate(1/(a*coth(x)+b*csch(x))**2,x)
```

output
```
Integral((a*coth(x) + b*csch(x))**(-2), x)
```

3.650.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\coth(x) + b\text{csch}(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*coth(x)+b*csch(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.650.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a\coth(x) + b\text{csch}(x))^2}\,dx = -\frac{2\,b\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^2} + \frac{x}{a^2} + \frac{2\,(be^x+a)}{(ae^{(2\,x)} + 2\,be^x + a)a^2}$$

input
```
integrate(1/(a*coth(x)+b*csch(x))^2,x, algorithm="giac")
```

output
```
-2*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^2) + x/a^2 + 2
*(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a^2)
```

3.650.9 Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^2}\,dx = \frac{x}{a^2}+\frac{\frac{2}{a}+\frac{2\,b\,e^x}{a^2}}{a+2\,b\,e^x+a\,e^{2\,x}}+\frac{b\,\ln\left(\frac{2\,b\,e^x}{a^3}-\frac{2\,b\,(a+b\,e^x)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}} - \frac{b\,\ln\left(\frac{2\,b\,e^x}{a^3}+\frac{2\,b\,(a+b\,e^x)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}$$

input
```
int(1/(b/sinh(x) + a*coth(x))^2,x)
```

output
```
x/a^2 + (2/a + (2*b*exp(x))/a^2)/(a + 2*b*exp(x) + a*exp(2*x)) + (b*log((2
*b*exp(x))/a^3 - (2*b*(a + b*exp(x)))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))))/
(a^2*(a + b)^(1/2)*(b - a)^(1/2)) - (b*log((2*b*exp(x))/a^3 + (2*b*(a + b*
exp(x)))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))))/(a^2*(a + b)^(1/2)*(b - a)^(1
/2))
```

3.651 $\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx$

3.651.1 Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx=\frac{a^2-b^2}{2a^3(b+a\cosh(x))^2}+\frac{2b}{a^3(b+a\cosh(x))}+\frac{\log(b+a\cosh(x))}{a^3}$$

output
```
1/2*(a^2-b^2)/a^3/(b+a*cosh(x))^2+2*b/a^3/(b+a*cosh(x))+ln(b+a*cosh(x))/a^
3
```

3.651.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.54

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx$$
$$=\frac{a^2+3b^2+a^2\log(b+a\cosh(x))+2b^2\log(b+a\cosh(x))+a^2\cosh(2x)\log(b+a\cosh(x))+4ab\cosh(x)(1}{2a^3(b+a\cosh(x))^2}$$

input
```
Integrate[(a*Coth[x] + b*Csch[x])^(-3),x]
```

output
```
(a^2 + 3*b^2 + a^2*Log[b + a*Cosh[x]] + 2*b^2*Log[b + a*Cosh[x]] + a^2*Cos
h[2*x]*Log[b + a*Cosh[x]] + 4*a*b*Cosh[x]*(1 + Log[b + a*Cosh[x]]))/(2*a^3
*(b + a*Cosh[x])^2)
```

3.651.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 4892, 26, 26, 3042, 26, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(ia\cot(ix)+ib\csc(ix))^3}dx$$

$$\Big\downarrow \text{ 4892}$$

$$\int -\frac{i\sinh^3(x)}{(ia\cosh(x)+ib)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{i\sinh^3(x)}{(b+a\cosh(x))^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{\left(b-a\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 3147}$$

$$-\frac{\int \frac{a^2-a^2\cosh^2(x)}{(b+a\cosh(x))^3}d(a\cosh(x))}{a^3}$$

$$\Big\downarrow \text{ 476}$$

$$-\frac{\int \left(\frac{2b}{(b+a\cosh(x))^2}+\frac{1}{-b-a\cosh(x)}+\frac{a^2-b^2}{(b+a\cosh(x))^3}\right)d(a\cosh(x))}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{a^2-b^2}{2(a\cosh(x)+b)^2}-\frac{2b}{a\cosh(x)+b}-\log(a\cosh(x)+b)}{a^3}$$

input
```
Int[(a*Coth[x] + b*Csch[x])^(-3),x]
```

output
```
-((-1/2*(a^2 - b^2)/(b + a*Cosh[x])^2 - (2*b)/(b + a*Cosh[x]) - Log[b + a*
Cosh[x]])/a^3)
```

3.651.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.651. $\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx$

3.651.4 Maple [A] (verified)

Time = 10.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{x}{a^3}+\frac{2\,e^{x}\left(2b\,e^{2x}a+a^2e^{x}+3b^2e^{x}+2ab\right)}{a^3\left(a\,e^{2x}+2\,e^{x}b+a\right)^2}+\frac{\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}+1\right)}{a^3}$
default	$\frac{\frac{2a^2(a+b)}{(a-b)\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)^2}-\frac{2a}{\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b}+\ln\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)}{a^3}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^3}-\frac{\ln(}{}$

input
```
int(1/(a*coth(x)+b*csch(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-x/a^3+2/a^3*exp(x)*(2*b*exp(2*x)*a+a^2*exp(x)+3*b^2*exp(x)+2*a*b)/(a*exp(
2*x)+2*exp(x)*b+a)^2+1/a^3*ln(exp(2*x)+2*b/a*exp(x)+1)
```

3.651.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(48) = 96.

Time = 0.25 (sec) , antiderivative size = 521, normalized size of antiderivative = 10.42

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx =$$
$$-\frac{a^2x\cosh(x)^4+a^2x\sinh(x)^4+4\,(abx-ab)\cosh(x)^3+4\,(a^2x\cosh(x)+abx-ab)\sinh(x)^3+a^2x-2}{}$$

input
```
integrate(1/(a*coth(x)+b*csch(x))^3,x, algorithm="fricas")
```

output
```
-(a^2*x*cosh(x)^4 + a^2*x*sinh(x)^4 + 4*(a*b*x - a*b)*cosh(x)^3 + 4*(a^2*x
*cosh(x) + a*b*x - a*b)*sinh(x)^3 + a^2*x - 2*(a^2 + 3*b^2 - (a^2 + 2*b^2)
*x)*cosh(x)^2 + 2*(3*a^2*x*cosh(x)^2 - a^2 - 3*b^2 + (a^2 + 2*b^2)*x + 6*(
a*b*x - a*b)*cosh(x))*sinh(x)^2 + 4*(a*b*x - a*b)*cosh(x) - (a^2*cosh(x)^4
 + a^2*sinh(x)^4 + 4*a*b*cosh(x)^3 + 4*(a^2*cosh(x) + a*b)*sinh(x)^3 + 4*a
*b*cosh(x) + 2*(a^2 + 2*b^2)*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + 6*a*b*cosh(x
) + a^2 + 2*b^2)*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + 3*a*b*cosh(x)^2 + a*
b + (a^2 + 2*b^2)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(
x))) + 4*(a^2*x*cosh(x)^3 + a*b*x + 3*(a*b*x - a*b)*cosh(x)^2 - a*b - (a^2
 + 3*b^2 - (a^2 + 2*b^2)*x)*cosh(x))*sinh(x))/(a^5*cosh(x)^4 + a^5*sinh(x)
^4 + 4*a^4*b*cosh(x)^3 + 4*a^4*b*cosh(x) + a^5 + 4*(a^5*cosh(x) + a^4*b)*s
inh(x)^3 + 2*(a^5 + 2*a^3*b^2)*cosh(x)^2 + 2*(3*a^5*cosh(x)^2 + 6*a^4*b*co
sh(x) + a^5 + 2*a^3*b^2)*sinh(x)^2 + 4*(a^5*cosh(x)^3 + 3*a^4*b*cosh(x)^2
+ a^4*b + (a^5 + 2*a^3*b^2)*cosh(x))*sinh(x))
```

3.651.6 Sympy [F]

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx=\int \frac{1}{(a\coth{(x)}+b\operatorname{csch}{(x)})^3}\,dx$$

input
```
integrate(1/(a*coth(x)+b*csch(x))**3,x)
```

output
```
Integral((a*coth(x) + b*csch(x))**(-3), x)
```

3.651.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(48) = 96$.

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.22

$$\begin{aligned}&\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx\\&=\frac{2\left(2\,abe^{(-x)}+2\,abe^{(-3\,x)}+(a^2+3\,b^2)e^{(-2\,x)}\right)}{4\,a^4be^{(-x)}+4\,a^4be^{(-3\,x)}+a^5e^{(-4\,x)}+a^5+2\,(a^5+2\,a^3b^2)e^{(-2\,x)}}\\&\quad+\frac{x}{a^3}+\frac{\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^3}\end{aligned}$$

input
```
integrate(1/(a*coth(x)+b*csch(x))^3,x, algorithm="maxima")
```

output
```
2*(2*a*b*e^(-x) + 2*a*b*e^(-3*x) + (a^2 + 3*b^2)*e^(-2*x))/(4*a^4*b*e^(-x)
 + 4*a^4*b*e^(-3*x) + a^5*e^(-4*x) + a^5 + 2*(a^5 + 2*a^3*b^2)*e^(-2*x)) +
 x/a^3 + log(2*b*e^(-x) + a*e^(-2*x) + a)/a^3
```

3.651.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx = \frac{\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{a^3} - \frac{3\,a\left(e^{(-x)}+e^{x}\right)^2+4\,b\left(e^{(-x)}+e^{x}\right)-4\,a}{2\,\left(a\left(e^{(-x)}+e^{x}\right)+2\,b\right)^2a^2}$$

input
```
integrate(1/(a*coth(x)+b*csch(x))^3,x, algorithm="giac")
```

output
```
log(abs(a*(e^(-x) + e^x) + 2*b))/a^3 - 1/2*(3*a*(e^(-x) + e^x)^2 + 4*b*(e^
(-x) + e^x) - 4*a)/((a*(e^(-x) + e^x) + 2*b)^2*a^2)
```

3.651.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx = \int \frac{1}{\left(\frac{b}{\sinh(x)}+a\coth\left(x\right)\right)^3}\,dx$$

input
```
int(1/(b/sinh(x) + a*coth(x))^3,x)
```

output
```
int(1/(b/sinh(x) + a*coth(x))^3, x)
```

3.652 $\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^4}\,dx$

3.652.1 Optimal result

Integrand size = 11, antiderivative size = 159

$$\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^4}\,dx = \frac{x}{a^4}-\frac{b(3a^2-2b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} - \frac{(2(a^2-b^2)-ab\cosh(x))\sinh(x)}{2a^3(a^2-b^2)(b+a\cosh(x))} - \frac{\sinh^3(x)}{3a(b+a\cosh(x))^3}-\frac{b\sinh^3(x)}{2a(a^2-b^2)(b+a\cosh(x))^2}$$

output
```
x/a^4-b*(3*a^2-2*b^2)*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^4/(a-b
)^(3/2)/(a+b)^(3/2)-1/2*(2*a^2-2*b^2-a*b*cosh(x))*sinh(x)/a^3/(a^2-b^2)/(b
+a*cosh(x))-1/3*sinh(x)^3/a/(b+a*cosh(x))^3-1/2*b*sinh(x)^3/a/(a^2-b^2)/(b
+a*cosh(x))^2
```

3.652.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^4}\,dx = \frac{\left(2a(a^2-b^2)+7ab(b+a\cosh(x))-\frac{a(8a^2-11b^2)(b+a\cosh(x))^2}{(a-b)(a+b)}+6x(b+a\cosh(x))^3\operatorname{csch}(x)-\frac{6b(-3a^2+2b^2)\operatorname{arct}}{}\right.}{6a^4(b+a\cosh(x))^3}$$

input
```
Integrate[(a*Coth[x] + b*Csch[x])^(-4),x]
```

output
```
((2*a*(a^2 - b^2) + 7*a*b*(b + a*Cosh[x]) - (a*(8*a^2 - 11*b^2)*(b + a*Cos
h[x])^2)/((a - b)*(a + b)) + 6*x*(b + a*Cosh[x])^3*Csch[x] - (6*b*(-3*a^2
+ 2*b^2)*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]]*(b + a*Cosh[x])^3*Cs
ch[x])/(a^2 - b^2)^(3/2))*Sinh[x])/(6*a^4*(b + a*Cosh[x])^3)
```

3.652.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.636, Rules used = {3042, 4892, 3042, 3172, 26, 3042, 25, 3343, 25, 3042, 25, 3342, 25, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(ia\cot(ix)+ib\csc(ix))^4}dx$$

$$\Big\downarrow \text{4892}$$

$$\int \frac{\sinh^4(x)}{(ia\cosh(x)+ib)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^4}{\left(ib-ia\sin\left(-\frac{\pi}{2}+ix\right)\right)^4}dx$$

$$\Big\downarrow \text{3172}$$

$$-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}-\frac{i\int \frac{i\cosh(x)\sinh^2(x)}{(b+a\cosh(x))^3}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\cosh(x)\sinh^2(x)}{(b+a\cosh(x))^3}dx}{a}-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}+\frac{\int -\frac{\cos\left(ix+\frac{\pi}{2}\right)^2\sin\left(ix+\frac{\pi}{2}\right)}{\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)^3}dx}{a}$$

$\Big\downarrow$ 25

$$-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}-\frac{\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^2\sin\left(ix+\frac{\pi}{2}\right)}{\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)^3}dx}{a}$$

$\Big\downarrow$ 3343

$$-\frac{\frac{\int -\frac{(2a+b\cosh(x))\sinh^2(x)}{(b+a\cosh(x))^2}dx}{2(a^2-b^2)}+\frac{b\sinh^3(x)}{2(a^2-b^2)(a\cosh(x)+b)^2}}{a}-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}$$

$\Big\downarrow$ 25

$$-\frac{\frac{b\sinh^3(x)}{2(a^2-b^2)(a\cosh(x)+b)^2}-\frac{\int \frac{(2a+b\cosh(x))\sinh^2(x)}{(b+a\cosh(x))^2}dx}{2(a^2-b^2)}}{a}-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}-\frac{\frac{b\sinh^3(x)}{2(a^2-b^2)(a\cosh(x)+b)^2}-\frac{\int -\frac{\cos\left(ix+\frac{\pi}{2}\right)^2\left(2a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}{\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}}{a}$$

$\Big\downarrow$ 25

$$-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}-\frac{\frac{b\sinh^3(x)}{2(a^2-b^2)(a\cosh(x)+b)^2}+\frac{\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^2\left(2a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}{\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}}{a}$$

$\Big\downarrow$ 3342

$$-\frac{\frac{\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{a^2(a\cosh(x)+b)}-\frac{\int -\frac{ab-2\left(a^2-b^2\right)\cosh(x)}{b+a\cosh(x)}dx}{a^2}}{2(a^2-b^2)}+\frac{b\sinh^3(x)}{2(a^2-b^2)(a\cosh(x)+b)^2}}{a}-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{\int \frac{ab-2\left(a^2-b^2\right)\cosh(x)}{b+a\cosh(x)}dx}{a^2}+\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{a^2(a\cosh(x)+b)}}{2(a^2-b^2)}+\frac{b\sinh^3(x)}{2(a^2-b^2)(a\cosh(x)+b)^2}}{a}-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}$$

$\Big\downarrow$ 3042

3.652. $\int \frac{1}{(a\coth(x)+b\text{csch}(x))^4}\,dx$

$$-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}-\frac{\frac{b\sinh^3(x)}{2(a^2-b^2)(a\cosh(x)+b)^2}+\frac{\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{a^2(a\cosh(x)+b)}+\frac{\int \frac{ab-2\left(a^2-b^2\right)\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}}{a}$$

$$\Big\downarrow \; 3214$$

$$-\frac{\frac{\frac{\frac{b\left(3a^2-2b^2\right)\int \frac{1}{b+a\cosh(x)}dx}{a}-\frac{2x\left(a^2-b^2\right)}{a}}{a^2}+\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{a^2(a\cosh(x)+b)}}{2(a^2-b^2)}+\frac{b\sinh^3(x)}{2(a^2-b^2)(a\cosh(x)+b)^2}}{a}-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}-\frac{\frac{b\sinh^3(x)}{2(a^2-b^2)(a\cosh(x)+b)^2}+\frac{\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{a^2(a\cosh(x)+b)}+\frac{-\frac{2x\left(a^2-b^2\right)}{a}+\frac{b\left(3a^2-2b^2\right)\int \frac{1}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}}{a^2}}{2(a^2-b^2)}}{a}$$

$$\Big\downarrow \; 3138$$

$$-\frac{\frac{\frac{\frac{2b\left(3a^2-2b^2\right)\int \frac{1}{(a-b)\tanh^2\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}-\frac{2x\left(a^2-b^2\right)}{a}}{a^2}+\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{a^2(a\cosh(x)+b)}}{2(a^2-b^2)}+\frac{b\sinh^3(x)}{2(a^2-b^2)(a\cosh(x)+b)^2}}{a}-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{\frac{\frac{2b\left(3a^2-2b^2\right)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}-\frac{2x\left(a^2-b^2\right)}{a}}{a^2}+\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{a^2(a\cosh(x)+b)}}{2(a^2-b^2)}+\frac{b\sinh^3(x)}{2(a^2-b^2)(a\cosh(x)+b)^2}}{a}-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}$$

input
```
Int[(a*Coth[x] + b*Csch[x])^(-4),x]
```

3.652. $\int \frac{1}{(a\coth(x)+b\text{csch}(x))^4}\,dx$

output
```
-1/3*Sinh[x]^3/(a*(b + a*Cosh[x])^3) - ((b*Sinh[x]^3)/(2*(a^2 - b^2)*(b +
a*Cosh[x])^2) + (((-2*(a^2 - b^2)*x)/a + (2*b*(3*a^2 - 2*b^2)*ArcTan[(Sqrt
[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]))/a^2 + ((2*(a
^2 - b^2) - a*b*Cosh[x])*Sinh[x])/(a^2*(b + a*Cosh[x])))/(2*(a^2 - b^2)))/
a
```

3.652.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.652. $\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^4}\,dx$

rule 3342
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Simp[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.652.4 Maple [A] (verified)

Time = 40.02 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

method	result
default	$\frac{\frac{2\left(-\frac{\left(2a^3-a^2b-3a\,b^2+2b^3\right)a\tanh\left(\frac{x}{2}\right)^5}{2(a+b)}-\frac{2a\left(5a^2-3b^2\right)\tanh\left(\frac{x}{2}\right)^3}{3}-\frac{\left(2a^3+a^2b-3a\,b^2-2b^3\right)a\tanh\left(\frac{x}{2}\right)}{2(a-b)}\right)}{\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)^3}-\frac{b\left(3a^2-2b^2\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right.}{\left(a^2-b^2\right)\sqrt{(a+b)(a-b)}}}{a^4}$
risch	$\frac{x}{a^4}+\frac{15a^4b\,\mathrm{e}^{5x}-18a^2b^3\mathrm{e}^{5x}+12a^5\mathrm{e}^{4x}+27a^3b^2\mathrm{e}^{4x}-54a\,b^4\mathrm{e}^{4x}+48\,\mathrm{e}^{3x}a^4b-34a^2b^3\mathrm{e}^{3x}-44b^5\mathrm{e}^{3x}+12a^5\mathrm{e}^{2x}+36a^3b^2\mathrm{e}^{2x}-78a\,b^4\mathrm{e}^{2x}+3}{3a^4\left(a\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}b+a\right)^3\left(a^2-b^2\right)}$

input
```
int(1/(a*coth(x)+b*csch(x))^4,x,method=_RETURNVERBOSE)
```

output
```
2/a^4*((-1/2*(2*a^3-a^2*b-3*a*b^2+2*b^3)*a/(a+b)*tanh(1/2*x)^5-2/3*a*(5*a^
2-3*b^2)*tanh(1/2*x)^3-1/2*(2*a^3+a^2*b-3*a*b^2-2*b^3)*a/(a-b)*tanh(1/2*x)
)/(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b+a+b)^3-1/2*b*(3*a^2-2*b^2)/(a^2-b^2)/((
a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))-1/a^4*ln(
tanh(1/2*x)-1)+1/a^4*ln(tanh(1/2*x)+1)
```

3.652.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2875 vs. 2(141) = 282.

Time = 0.35 (sec) , antiderivative size = 5830, normalized size of antiderivative = 36.67

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*coth(x)+b*csch(x))^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.652.6 Sympy [F]

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^4}\,dx = \int \frac{1}{\left(a\coth\left(x\right)+b\,\text{csch}\left(x\right)\right)^4}\,dx$$

input
```
integrate(1/(a*coth(x)+b*csch(x))**4,x)
```

output
```
Integral((a*coth(x) + b*csch(x))**(-4), x)
```

3.652.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*coth(x)+b*csch(x))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.652.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.52

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^4}\,dx = -\frac{(3\,a^2b-2\,b^3)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^6-a^4b^2)\sqrt{a^2-b^2}} + \frac{15\,a^4be^{(5\,x)}-18\,a^2b^3e^{(5\,x)}+12\,a^5e^{(4\,x)}+27\,a^3b^2e^{(4\,x)}-54\,ab^4e^{(4\,x)}+48\,a^4be^{(3\,x)}-34\,a^2b^3e^{(3\,x)}-44\,b^5e^{(}}{3\,(a^6-a^4b^2)(ae^{(2\,x)}+2\,be^{}} + \frac{x}{a^4}$$

input
```
integrate(1/(a*coth(x)+b*csch(x))^4,x, algorithm="giac")
```

output
```
-(3*a^2*b - 2*b^3)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^6 - a^4*b^2)*sq
rt(a^2 - b^2)) + 1/3*(15*a^4*b*e^(5*x) - 18*a^2*b^3*e^(5*x) + 12*a^5*e^(4*
x) + 27*a^3*b^2*e^(4*x) - 54*a*b^4*e^(4*x) + 48*a^4*b*e^(3*x) - 34*a^2*b^3
*e^(3*x) - 44*b^5*e^(3*x) + 12*a^5*e^(2*x) + 36*a^3*b^2*e^(2*x) - 78*a*b^4
*e^(2*x) + 33*a^4*b*e^x - 48*a^2*b^3*e^x + 8*a^5 - 11*a^3*b^2)/((a^6 - a^4
*b^2)*(a*e^(2*x) + 2*b*e^x + a)^3) + x/a^4
```

3.652.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^4}\,dx = \int \frac{1}{\left(\frac{b}{\sinh(x)}+a\coth\left(x\right)\right)^4}\,dx$$

input
```
int(1/(b/sinh(x) + a*coth(x))^4,x)
```

output
```
int(1/(b/sinh(x) + a*coth(x))^4, x)
```

3.653 $\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}\,dx$

3.653.1 Optimal result

Integrand size = 11, antiderivative size = 98

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}\,dx = -\frac{(a^2-b^2)^2}{4a^5(b+a\cosh(x))^4} - \frac{4b(a^2-b^2)}{3a^5(b+a\cosh(x))^3} + \frac{a^2-3b^2}{a^5(b+a\cosh(x))^2} + \frac{4b}{a^5(b+a\cosh(x))} + \frac{\log(b+a\cosh(x))}{a^5}$$

output
```
-1/4*(a^2-b^2)^2/a^5/(b+a*cosh(x))^4-4/3*b*(a^2-b^2)/a^5/(b+a*cosh(x))^3+(
a^2-3*b^2)/a^5/(b+a*cosh(x))^2+4*b/a^5/(b+a*cosh(x))+ln(b+a*cosh(x))/a^5
```

3.653.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.41

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}\,dx = \frac{-3a^4+2a^2b^2+25b^4+12b^4\log(b+a\cosh(x))+12a^4\cosh^4(x)\log(b+a\cosh(x))+48a^3b\cosh^3(x)(1+\text{lo}}{12a}$$

input
```
Integrate[(a*Coth[x] + b*Csch[x])^(-5),x]
```

output
```
(-3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*b^4*Log[b + a*Cosh[x]] + 12*a^4*Cosh[x]^
4*Log[b + a*Cosh[x]] + 48*a^3*b*Cosh[x]^3*(1 + Log[b + a*Cosh[x]]) + 12*a^
2*Cosh[x]^2*(a^2 + 9*b^2 + 6*b^2*Log[b + a*Cosh[x]]) + 8*a*b*Cosh[x]*(a^2
+ 11*b^2 + 6*b^2*Log[b + a*Cosh[x]]))/(12*a^5*(b + a*Cosh[x])^4)
```

3.653.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3042, 4892, 26, 26, 3042, 26, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(ia\cot(ix)+ib\csc(ix))^5}dx$$
$$\Big\downarrow \text{4892}$$
$$\int \frac{i\sinh^5(x)}{(ia\cosh(x)+ib)^5}dx$$
$$\Big\downarrow \text{26}$$
$$i\int -\frac{i\sinh^5(x)}{(b+a\cosh(x))^5}dx$$
$$\Big\downarrow \text{26}$$
$$\int \frac{\sinh^5(x)}{(a\cosh(x)+b)^5}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)^5}{\left(b-a\sin\left(-\frac{\pi}{2}+ix\right)\right)^5}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^5}{\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)^5}dx$$
$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{\left(a^2-a^2\cosh^2(x)\right)^2}{(b+a\cosh(x))^5}d(a\cosh(x))}{a^5}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(\frac{\left(a^2-b^2\right)^2}{(b+a\cosh(x))^5}+\frac{1}{b+a\cosh(x)}-\frac{4b}{(b+a\cosh(x))^2}-\frac{2\left(a^2-3b^2\right)}{(b+a\cosh(x))^3}-\frac{4b\left(b^2-a^2\right)}{(b+a\cosh(x))^4}\right)d(a\cosh(x))}{a^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\left(a^2-b^2\right)^2}{4(a\cosh(x)+b)^4}-\frac{4b\left(a^2-b^2\right)}{3(a\cosh(x)+b)^3}+\frac{a^2-3b^2}{(a\cosh(x)+b)^2}+\frac{4b}{a\cosh(x)+b}+\log(a\cosh(x)+b)}{a^5}$$

input
```
Int[(a*Coth[x] + b*Csch[x])^(-5),x]
```

output
```
(-1/4*(a^2 - b^2)^2/(b + a*Cosh[x])^4 - (4*b*(a^2 - b^2))/(3*(b + a*Cosh[x
])^3) + (a^2 - 3*b^2)/(b + a*Cosh[x])^2 + (4*b)/(b + a*Cosh[x]) + Log[b +
a*Cosh[x]])/a^5
```

3.653.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.653. $\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}\,dx$

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.653.4 Maple [A] (verified)

Time = 126.52 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.78

method	result
risch	$-\frac{x}{a^5}+\frac{4(6a^3b\,\mathrm{e}^{6x}+3a^4\mathrm{e}^{5x}+27a^2b^2\mathrm{e}^{5x}+22a^3b\,\mathrm{e}^{4x}+44a\,b^3\mathrm{e}^{4x}+3a^4\mathrm{e}^{3x}+56a^2b^2\mathrm{e}^{3x}+25\,\mathrm{e}^{3x}b^4+22\,\mathrm{e}^{2x}a^3b+44\,\mathrm{e}^{2x}a\,b^3+3a^4\mathrm{e}^{x}+27a^2}{3a^5(a\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}b+a)^4}$
default	$\frac{-\frac{2a^2}{\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)^2}-\frac{2a}{\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b}+\frac{8a^3\left(3a^2+2ab-b^2\right)}{3(a-b)^2\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)^3}+\ln\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)\right)}{a^5}$

input
```
int(1/(a*coth(x)+b*csch(x))^5,x,method=_RETURNVERBOSE)
```

output
```
-x/a^5+4/3*(6*a^3*b*exp(6*x)+3*a^4*exp(5*x)+27*a^2*b^2*exp(5*x)+22*a^3*b*e
xp(4*x)+44*a*b^3*exp(4*x)+3*a^4*exp(3*x)+56*a^2*b^2*exp(3*x)+25*exp(3*x)*b
^4+22*exp(2*x)*a^3*b+44*exp(2*x)*a*b^3+3*a^4*exp(x)+27*a^2*b^2*exp(x)+6*a^
3*b)/a^5*exp(x)/(a*exp(2*x)+2*exp(x)*b+a)^4+1/a^5*ln(exp(2*x)+2*b/a*exp(x)
+1)
```

3.653.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2564 vs. $2(94) = 188$.

Time = 0.28 (sec) , antiderivative size = 2564, normalized size of antiderivative = 26.16

$$\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*coth(x)+b*csch(x))^5,x, algorithm="fricas")
```

output
```
-1/3*(3*a^4*x*cosh(x)^8 + 3*a^4*x*sinh(x)^8 + 24*(a^3*b*x - a^3*b)*cosh(x)
^7 + 24*(a^4*x*cosh(x) + a^3*b*x - a^3*b)*sinh(x)^7 - 12*(a^4 + 9*a^2*b^2
- (a^4 + 6*a^2*b^2)*x)*cosh(x)^6 + 12*(7*a^4*x*cosh(x)^2 - a^4 - 9*a^2*b^2
 + (a^4 + 6*a^2*b^2)*x + 14*(a^3*b*x - a^3*b)*cosh(x))*sinh(x)^6 - 8*(11*a
^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^5 + 8*(21*a^4*x*cosh(x)
^3 - 11*a^3*b - 22*a*b^3 + 63*(a^3*b*x - a^3*b)*cosh(x)^2 + 3*(3*a^3*b + 4
*a*b^3)*x - 9*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x))*sinh(x)^5 +
 3*a^4*x - 2*(6*a^4 + 112*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4
)*x)*cosh(x)^4 + 2*(105*a^4*x*cosh(x)^4 - 6*a^4 - 112*a^2*b^2 - 50*b^4 + 4
20*(a^3*b*x - a^3*b)*cosh(x)^3 - 90*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x
)*cosh(x)^2 + 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x - 20*(11*a^3*b + 22*a*b^3 -
 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x))*sinh(x)^4 - 8*(11*a^3*b + 22*a*b^3 - 3*
(3*a^3*b + 4*a*b^3)*x)*cosh(x)^3 + 8*(21*a^4*x*cosh(x)^5 + 105*(a^3*b*x -
a^3*b)*cosh(x)^4 - 11*a^3*b - 22*a*b^3 - 30*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^
2*b^2)*x)*cosh(x)^3 - 10*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*c
osh(x)^2 + 3*(3*a^3*b + 4*a*b^3)*x - (6*a^4 + 112*a^2*b^2 + 50*b^4 - 3*(3*
a^4 + 24*a^2*b^2 + 8*b^4)*x)*cosh(x))*sinh(x)^3 - 12*(a^4 + 9*a^2*b^2 - (a
^4 + 6*a^2*b^2)*x)*cosh(x)^2 + 4*(21*a^4*x*cosh(x)^6 + 126*(a^3*b*x - a^3*
b)*cosh(x)^5 - 45*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x)^4 - 3*a^
4 - 27*a^2*b^2 - 20*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cos...
```

3.653.6 Sympy [F]

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}\,dx=\int \frac{1}{\left(a\coth\left(x\right)+b\,\text{csch}\left(x\right)\right)^5}\,dx$$

input
```
integrate(1/(a*coth(x)+b*csch(x))**5,x)
```

output
```
Integral((a*coth(x) + b*csch(x))**(-5), x)
```

3.653.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(94) = 188$.

Time = 0.22 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.91

$$\int \frac{1}{(a\coth(x) + b\text{csch}(x))^5}\,dx$$
$$= \frac{4\left(6\,a^3be^{(-x)} + 6\,a^3be^{(-7\,x)} + 3\,(a^4 + 9\,a^2b^2)e^{(-2\,x)} + 22\,(a^3b + 2\,ab^3)e^{(-3\,x)} + (3\,a^4 + 56\,a^2b^2 + 2\right.}{3\left(8\,a^8be^{(-x)} + 8\,a^8be^{(-7\,x)} + a^9e^{(-8\,x)} + a^9 + 4\,(a^9 + 6\,a^7b^2)e^{(-2\,x)} + 8\,(3\,a^8b + 4\,a^6b^3)e^{(-3\,x)} + 2\,(3\,a^9 + 24\right.}$$
$$+ \frac{x}{a^5} + \frac{\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a^5}$$

input
```
integrate(1/(a*coth(x)+b*csch(x))^5,x, algorithm="maxima")
```

output
```
4/3*(6*a^3*b*e^(-x) + 6*a^3*b*e^(-7*x) + 3*(a^4 + 9*a^2*b^2)*e^(-2*x) + 22
*(a^3*b + 2*a*b^3)*e^(-3*x) + (3*a^4 + 56*a^2*b^2 + 25*b^4)*e^(-4*x) + 22*
(a^3*b + 2*a*b^3)*e^(-5*x) + 3*(a^4 + 9*a^2*b^2)*e^(-6*x))/(8*a^8*b*e^(-x)
 + 8*a^8*b*e^(-7*x) + a^9*e^(-8*x) + a^9 + 4*(a^9 + 6*a^7*b^2)*e^(-2*x) +
8*(3*a^8*b + 4*a^6*b^3)*e^(-3*x) + 2*(3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*e^(-
4*x) + 8*(3*a^8*b + 4*a^6*b^3)*e^(-5*x) + 4*(a^9 + 6*a^7*b^2)*e^(-6*x)) +
x/a^5 + log(2*b*e^(-x) + a*e^(-2*x) + a)/a^5
```

3.653.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.38

$$\int \frac{1}{(a\coth(x) + b\text{csch}(x))^5}\,dx = \frac{\log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\,b\right|\right)}{a^5}$$
$$- \frac{25\,a^3\left(e^{(-x)} + e^x\right)^4 + 104\,a^2b\left(e^{(-x)} + e^x\right)^3 - 48\,a^3\left(e^{(-x)} + e^x\right)^2 + 168\,ab^2\left(e^{(-x)} + e^x\right)^2 - 64\,a^2b\left(e^{(-x)} + \right.}{12\left(a\left(e^{(-x)} + e^x\right) + 2\,b\right)^4a^4}$$

input
```
integrate(1/(a*coth(x)+b*csch(x))^5,x, algorithm="giac")
```

output
```
log(abs(a*(e^(-x) + e^x) + 2*b))/a^5 - 1/12*(25*a^3*(e^(-x) + e^x)^4 + 104
*a^2*b*(e^(-x) + e^x)^3 - 48*a^3*(e^(-x) + e^x)^2 + 168*a*b^2*(e^(-x) + e^
x)^2 - 64*a^2*b*(e^(-x) + e^x) + 96*b^3*(e^(-x) + e^x) + 48*a^3 - 32*a*b^2
)/((a*(e^(-x) + e^x) + 2*b)^4*a^4)
```

3.653.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}\,dx=\int \frac{1}{\left(\frac{b}{\sinh(x)}+a\coth\left(x\right)\right)^5}\,dx$$

input
```
int(1/(b/sinh(x) + a*coth(x))^5,x)
```

output
```
int(1/(b/sinh(x) + a*coth(x))^5, x)
```

3.654 $\int (\coth(x) + \operatorname{csch}(x))^5 \, dx$

3.654.1 Optimal result

Integrand size = 7, antiderivative size = 28

$$\int (\coth(x)+\operatorname{csch}(x))^5\,dx=-\frac{2}{(1-\cosh(x))^2}+\frac{4}{1-\cosh(x)}+\log(1-\cosh(x))$$

output
```
-2/(1-cosh(x))^2+4/(1-cosh(x))+ln(1-cosh(x))
```

3.654.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int (\coth(x)+\operatorname{csch}(x))^5\,dx=-2\operatorname{csch}^2\left(\frac{x}{2}\right)-\frac{1}{2}\operatorname{csch}^4\left(\frac{x}{2}\right)+2\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Coth[x] + Csch[x])^5,x]
```

output
```
-2*Csch[x/2]^2 - Csch[x/2]^4/2 + 2*Log[Sinh[x/2]]
```

3.654.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {3042, 4892, 26, 26, 3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\coth(x)+\text{csch}(x))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\cot(ix)+i\csc(ix))^5dx$$

$$\Big\downarrow \text{4892}$$

$$\int -i(i\cosh(x)+i)^5\text{csch}^5(x)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int i(\cosh(x)+1)^5\text{csch}^5(x)dx$$

$$\Big\downarrow \text{26}$$

$$\int (\cosh(x)+1)^5\text{csch}^5(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^5}{\cos\left(-\frac{\pi}{2}+ix\right)^5}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^5}{\cos\left(ix-\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{(\cosh(x)+1)^2}{(1-\cosh(x))^3}d\cosh(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(-\frac{4}{(\cosh(x)-1)^2}-\frac{4}{(\cosh(x)-1)^3}+\frac{1}{1-\cosh(x)}\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{1-\cosh(x)} - \frac{2}{(1-\cosh(x))^2} + \log(1-\cosh(x))$$

input
```
Int[(Coth[x] + Csch[x])^5,x]
```

output
```
-2/(1 - Cosh[x])^2 + 4/(1 - Cosh[x]) + Log[1 - Cosh[x]]
```

3.654.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.654.4 Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

method	result
risch	$-x-\frac{8\,\mathrm{e}^{x}\left(\mathrm{e}^{2x}-\mathrm{e}^{x}+1\right)}{\left(\mathrm{e}^{x}-1\right)^{4}}+2\ln\left(\mathrm{e}^{x}-1\right)$
default	$\ln\left(\sinh\left(x\right)\right)-\frac{\coth\left(x\right)^{2}}{2}-\frac{\coth\left(x\right)^{4}}{4}-\frac{5\cosh\left(x\right)^{3}}{\sinh\left(x\right)^{4}}+\frac{5\cosh\left(x\right)}{3\sinh\left(x\right)^{4}}+\frac{8\left(-\frac{\operatorname{csch}\left(x\right)^{3}}{4}+\frac{3\,\operatorname{csch}\left(x\right)}{8}\right)\coth\left(x\right)}{3}-2\,\operatorname{arctanh}\left(\mathrm{e}\right.$
parts	$-\frac{11\coth\left(x\right)^{4}}{4}-\frac{\coth\left(x\right)^{2}}{2}-\frac{\ln\left(\coth\left(x\right)-1\right)}{2}-\frac{\ln\left(1+\coth\left(x\right)\right)}{2}+\frac{8\left(-\frac{\operatorname{csch}\left(x\right)^{3}}{4}+\frac{3\,\operatorname{csch}\left(x\right)}{8}\right)\coth\left(x\right)}{3}-2\,\operatorname{arctanh}\left(\mathrm{e}^{x}\right)+$

input
```
int((coth(x)+csch(x))^5,x,method=_RETURNVERBOSE)
```

output
```
-x-8*exp(x)*(exp(2*x)-exp(x)+1)/(exp(x)-1)^4+2*ln(exp(x)-1)
```

3.654.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 270, normalized size of antiderivative = 9.64

$$\int(\coth(x)+\operatorname{csch}(x))^{5}\,dx =$$
$$-\frac{x\cosh\left(x\right)^{4}+x\sinh\left(x\right)^{4}-4\left(x-2\right)\cosh\left(x\right)^{3}+4\left(x\cosh\left(x\right)-x+2\right)\sinh\left(x\right)^{3}+2\left(3\,x-4\right)\cosh\left(x\right)^{2}}{}$$

input
```
integrate((coth(x)+csch(x))^5,x, algorithm="fricas")
```

output
```
-(x*cosh(x)^4 + x*sinh(x)^4 - 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) - x + 2)*
sinh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 - 6*(x - 2)*cosh(x) +
 3*x - 4)*sinh(x)^2 - 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) - 1)*s
inh(x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)
^2 + 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4
*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 4*(x*cosh(x)^3 - 3*(x - 2)*cosh
(x)^2 + (3*x - 4)*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) -
1)*sinh(x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sin
h(x)^2 + 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x)
 - 4*cosh(x) + 1)
```

3.654.6 Sympy [F]

$$\int (\coth(x) + \operatorname{csch}(x))^5 \, dx = \int (\coth(x) + \operatorname{csch}(x))^5 \, dx$$

input
```
integrate((coth(x)+csch(x))**5,x)
```

output
```
Integral((coth(x) + csch(x))**5, x)
```

3.654.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(24) = 48.

Time = 0.20 (sec) , antiderivative size = 236, normalized size of antiderivative = 8.43

$$\begin{aligned}\int (\coth(x) + \operatorname{csch}(x))^5 \, dx = & -\frac{5}{2} \coth(x)^4 + x \\ & + \frac{5 \left(5 e^{(-x)} + 3 e^{(-3x)} + 3 e^{(-5x)} + 5 e^{(-7x)}\right)}{4 \left(4 e^{(-2x)} - 6 e^{(-4x)} + 4 e^{(-6x)} - e^{(-8x)} - 1\right)} \\ & - \frac{3 e^{(-x)} - 11 e^{(-3x)} - 11 e^{(-5x)} + 3 e^{(-7x)}}{4 \left(4 e^{(-2x)} - 6 e^{(-4x)} + 4 e^{(-6x)} - e^{(-8x)} - 1\right)} \\ & + \frac{5 \left(e^{(-x)} + 7 e^{(-3x)} + 7 e^{(-5x)} + e^{(-7x)}\right)}{2 \left(4 e^{(-2x)} - 6 e^{(-4x)} + 4 e^{(-6x)} - e^{(-8x)} - 1\right)} \\ & + \frac{4 \left(e^{(-2x)} - e^{(-4x)} + e^{(-6x)}\right)}{4 e^{(-2x)} - 6 e^{(-4x)} + 4 e^{(-6x)} - e^{(-8x)} - 1} \\ & - \frac{20}{\left(e^{(-x)} - e^{x}\right)^4} + 2 \log\left(e^{(-x)} - 1\right)\end{aligned}$$

input
```
integrate((coth(x)+csch(x))^5,x, algorithm="maxima")
```

output
```
-5/2*coth(x)^4 + x + 5/4*(5*e^(-x) + 3*e^(-3*x) + 3*e^(-5*x) + 5*e^(-7*x))
/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 1/4*(3*e^(-x) - 1
1*e^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*
x) - e^(-8*x) - 1) + 5/2*(e^(-x) + 7*e^(-3*x) + 7*e^(-5*x) + e^(-7*x))/(4*
e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 4*(e^(-2*x) - e^(-4*x
) + e^(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 20/(
e^(-x) - e^x)^4 + 2*log(e^(-x) - 1)
```

3.654.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int (\coth(x) + \operatorname{csch}(x))^5 \, dx = -x - \frac{8\left(e^{(3x)} - e^{(2x)} + e^{x}\right)}{(e^{x} - 1)^4} + 2 \log\left(|e^{x} - 1|\right)$$

input
```
integrate((coth(x)+csch(x))^5,x, algorithm="giac")
```

output
```
-x - 8*(e^(3*x) - e^(2*x) + e^x)/(e^x - 1)^4 + 2*log(abs(e^x - 1))
```

3.654.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.89

$$\int (\coth(x) + \operatorname{csch}(x))^5 \, dx = 2 \ln\left(e^{x} - 1\right) - x + \frac{16}{3 e^{2x} - e^{3x} - 3 e^{x} + 1} - \frac{16}{e^{2x} - 2 e^{x} + 1} - \frac{8}{6 e^{2x} - 4 e^{3x} + e^{4x} - 4 e^{x} + 1} - \frac{8}{e^{x} - 1}$$

input
```
int((coth(x) + 1/sinh(x))^5,x)
```

output
```
2*log(exp(x) - 1) - x + 16/(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1) - 16/(ex
p(2*x) - 2*exp(x) + 1) - 8/(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x)
+ 1) - 8/(exp(x) - 1)
```

3.655 $\int (\coth(x)+\operatorname{csch}(x))^4\,dx$

3.655.1 Optimal result

Integrand size = 7, antiderivative size = 30

$$\int (\coth(x)+\operatorname{csch}(x))^4\,dx = x+\frac{2\sinh(x)}{1-\cosh(x)}+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}$$

output
```
x+2*sinh(x)/(1-cosh(x))+2/3*sinh(x)^3/(1-cosh(x))^3
```

3.655.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (\coth(x)+\operatorname{csch}(x))^4\,dx = x-\frac{8}{3}\coth\left(\frac{x}{2}\right)-\frac{2}{3}\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right)$$

input
```
Integrate[(Coth[x] + Csch[x])^4,x]
```

output
```
x - (8*Coth[x/2])/3 - (2*Coth[x/2]*Csch[x/2]^2)/3
```

3.655.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.571, Rules used = {3042, 4892, 3042, 3149, 3042, 3159, 25, 3042, 25, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\coth(x)+\operatorname{csch}(x))^4 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\cot(ix)+i\csc(ix))^4dx$$

$$\Big\downarrow \text{4892}$$

$$\int (i\cosh(x)+i)^4\operatorname{csch}^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(i-i\sin\left(-\frac{\pi}{2}+ix\right)\right)^4}{\cos\left(-\frac{\pi}{2}+ix\right)^4}dx$$

$$\Big\downarrow \text{3149}$$

$$\int \frac{\sinh^4(x)}{(1-\cosh(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^4}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^4}dx$$

$$\Big\downarrow \text{3159}$$

$$\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}-\int -\frac{\sinh^2(x)}{(1-\cosh(x))^2}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}dx+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\int -\frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\frac{2\sinh(x)}{1-\cosh(x)}$$

$$\Big\downarrow \text{24}$$

$$x+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\frac{2\sinh(x)}{1-\cosh(x)}$$

input
```
Int[(Coth[x] + Csch[x])^4,x]
```

output
```
x + (2*Sinh[x])/(1 - Cosh[x]) + (2*Sinh[x]^3)/(3*(1 - Cosh[x])^3)
```

3.655.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.655.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result
risch	$x-\frac{8(3\,\mathrm{e}^{2x}-3\,\mathrm{e}^{x}+2)}{3(\mathrm{e}^{x}-1)^{3}}$
parts	$-\frac{7\coth(x)^{3}}{3}-\coth\left(x\right)-\frac{\ln(\coth(x)-1)}{2}+\frac{\ln(1+\coth(x))}{2}+\left(\frac{2}{3}-\frac{\operatorname{csch}(x)^{2}}{3}\right)\coth\left(x\right)-\frac{8\operatorname{csch}(x)^{3}}{3}-4\operatorname{csch}(x$
default	$x-\coth\left(x\right)-\frac{\coth(x)^{3}}{3}-\frac{4\cosh(x)^{2}}{\sinh(x)^{3}}+\frac{4}{3\sinh(x)^{3}}-\frac{3\cosh(x)}{\sinh(x)^{3}}-2\left(\frac{2}{3}-\frac{\operatorname{csch}(x)^{2}}{3}\right)\coth\left(x\right)$

input
```
int((coth(x)+csch(x))^4,x,method=_RETURNVERBOSE)
```

output
```
x-8/3*(3*exp(2*x)-3*exp(x)+2)/(exp(x)-1)^3
```

3.655.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int(\coth(x)+\operatorname{csch}(x))^4\,dx$$
$$=\frac{3\,x\cosh\left(x\right)^{2}+3\,x\sinh\left(x\right)^{2}-4\left(3\,x+10\right)\cosh\left(x\right)+2\left(3\,x\cosh\left(x\right)-3\,x-4\right)\sinh\left(x\right)+9\,x+24}{3\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-4\,\cosh\left(x\right)+3\right)}$$

input
```
integrate((coth(x)+csch(x))^4,x, algorithm="fricas")
```

output
```
1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 - 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
 - 3*x - 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sin
h(x)^2 - 4*cosh(x) + 3)
```

3.655.6 Sympy [F]

$$\int (\coth(x)+\operatorname{csch}(x))^4\,dx=\int (\coth{(x)}+\operatorname{csch}{(x)})^4\,dx$$

input
```
integrate((coth(x)+csch(x))**4,x)
```

output
```
Integral((coth(x) + csch(x))**4, x)
```

3.655.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(24) = 48$.

Time = 0.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 6.10

$$\begin{aligned}\int (\coth(x)+\operatorname{csch}(x))^4\,dx &= -2\coth{(x)}^3+x-\frac{4\,(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}-2)}{3\,(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1)}\\&\quad+\frac{8\,e^{(-x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}\\&\quad+\frac{4\,e^{(-2\,x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}\\&\quad-\frac{16\,e^{(-3\,x)}}{3\,(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1)}\\&\quad+\frac{8\,e^{(-5\,x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}\\&\quad-\frac{4}{3\,(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1)}+\frac{32}{3\,(e^{(-x)}-e^x)^3}\end{aligned}$$

input
```
integrate((coth(x)+csch(x))^4,x, algorithm="maxima")
```

output
```
-2*coth(x)^3 + x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4
*x) + e^(-6*x) - 1) + 8*e^(-x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) +
4*e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 16/3*e^(-3*x)/(3*e^(
-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 8*e^(-5*x)/(3*e^(-2*x) - 3*e^(-4*x) +
 e^(-6*x) - 1) - 4/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 32/3/(e^(-
x) - e^x)^3
```

3.655.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int (\coth(x) + \operatorname{csch}(x))^4 \, dx = x - \frac{8 \left(3 \, e^{(2 \, x)} - 3 \, e^{x} + 2\right)}{3 \, {\left(e^{x} - 1\right)}^{3}}$$

input
```
integrate((coth(x)+csch(x))^4,x, algorithm="giac")
```

output
```
x - 8/3*(3*e^(2*x) - 3*e^x + 2)/(e^x - 1)^3
```

3.655.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int (\coth(x) + \operatorname{csch}(x))^4 \, dx = x - \frac{8 \, e^{x}}{3 \, \left(e^{2 \, x} - 2 \, e^{x} + 1\right)} + \frac{\frac{8 \, e^{2 \, x}}{3} + \frac{8}{3}}{3 \, e^{2 \, x} - e^{3 \, x} - 3 \, e^{x} + 1} - \frac{8}{3 \, \left(e^{x} - 1\right)}$$

input
```
int((coth(x) + 1/sinh(x))^4,x)
```

output
```
x - (8*exp(x))/(3*(exp(2*x) - 2*exp(x) + 1)) + ((8*exp(2*x))/3 + 8/3)/(3*e
xp(2*x) - exp(3*x) - 3*exp(x) + 1) - 8/(3*(exp(x) - 1))
```

3.656 $\int (\coth(x) + \operatorname{csch}(x))^3 \, dx$

3.656.1 Optimal result

Integrand size = 7, antiderivative size = 18

$$\int (\coth(x) + \operatorname{csch}(x))^3 \, dx = \frac{2}{1 - \cosh(x)} + \log(1 - \cosh(x))$$

output
```
2/(1-cosh(x))+ln(1-cosh(x))
```

3.656.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (\coth(x) + \operatorname{csch}(x))^3 \, dx = -\operatorname{csch}^2\left(\frac{x}{2}\right) + 2 \log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Coth[x] + Csch[x])^3,x]
```

output
```
-Csch[x/2]^2 + 2*Log[Sinh[x/2]]
```

3.656.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {3042, 4892, 26, 26, 3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\coth(x)+\operatorname{csch}(x))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\cot(ix)+i\csc(ix))^3dx$$

$$\Big\downarrow \text{4892}$$

$$\int i(i\cosh(x)+i)^3\operatorname{csch}^3(x)dx$$

$$\Big\downarrow \text{26}$$

$$i\int -i(\cosh(x)+1)^3\operatorname{csch}^3(x)dx$$

$$\Big\downarrow \text{26}$$

$$\int (\cosh(x)+1)^3\operatorname{csch}^3(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}{\cos\left(-\frac{\pi}{2}+ix\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^3}{\cos\left(ix-\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{\cosh(x)+1}{(1-\cosh(x))^2}d\cosh(x)$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{\cosh(x)-1}+\frac{2}{(\cosh(x)-1)^2}\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{1-\cosh(x)}+\log(1-\cosh(x))$$

input
```
Int[(Coth[x] + Csch[x])^3,x]
```

output
```
2/(1 - Cosh[x]) + Log[1 - Cosh[x]]
```

3.656.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.656.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

method	result	size
risch	$-x-\frac{4\,{\mathrm e}^{x}}{\left({\mathrm e}^{x}-1\right)^{2}}+2\ln\left({\mathrm e}^{x}-1\right)$	22
default	$\ln\left(\sinh\left(x\right)\right)-\frac{\coth\left(x\right)^{2}}{2}-\frac{3\cosh\left(x\right)}{\sinh\left(x\right)^{2}}+\operatorname{csch}\left(x\right)\coth\left(x\right)-2\,\operatorname{arctanh}\left({\mathrm e}^{x}\right)-\frac{3}{2\sinh\left(x\right)^{2}}$	35
parts	$-2\coth\left(x\right)^{2}-\frac{\ln\left(\coth\left(x\right)-1\right)}{2}-\frac{\ln\left(1+\coth\left(x\right)\right)}{2}+\operatorname{csch}\left(x\right)\coth\left(x\right)-2\,\operatorname{arctanh}\left({\mathrm e}^{x}\right)-\frac{3\cosh\left(x\right)}{\sinh\left(x\right)^{2}}$	40

input
```
int((coth(x)+csch(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-x-4*exp(x)/(exp(x)-1)^2+2*ln(exp(x)-1)
```

3.656.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(16) = 32.

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 5.06

$$\int (\coth(x)+\operatorname{csch}(x))^3\,dx = -\frac{x\cosh\left(x\right)^{2}+x\sinh\left(x\right)^{2}-2\left(x-2\right)\cosh\left(x\right)-2\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-2\cdots}{\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(\cdots\right.}$$

input
```
integrate((coth(x)+csch(x))^3,x, algorithm="fricas")
```

output
```
-(x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x
) - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2
*(x*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + s
inh(x)^2 - 2*cosh(x) + 1)
```

3.656.6 Sympy [F]

$$\int (\coth(x) + \operatorname{csch}(x))^3 \, dx = \int (\coth(x) + \operatorname{csch}(x))^3 \, dx$$

input
```
integrate((coth(x)+csch(x))**3,x)
```

output
```
Integral((coth(x) + csch(x))**3, x)
```

3.656.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(16) = 32.

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.67

$$\int (\coth(x) + \operatorname{csch}(x))^3 \, dx = -\frac{3}{2}\coth(x)^2 + x + \frac{4\left(e^{(-x)} + e^{(-3x)}\right)}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{2e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + 2\log\left(e^{(-x)} - 1\right)$$

input
```
integrate((coth(x)+csch(x))^3,x, algorithm="maxima")
```

output
```
-3/2*coth(x)^2 + x + 4*(e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 2
*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 2*log(e^(-x) - 1)
```

3.656.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int (\coth(x) + \operatorname{csch}(x))^3 \, dx = -x - \frac{4e^x}{(e^x - 1)^2} + 2\log(|e^x - 1|)$$

input
```
integrate((coth(x)+csch(x))^3,x, algorithm="giac")
```

output
```
-x - 4*e^x/(e^x - 1)^2 + 2*log(abs(e^x - 1))
```

3.656.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int (\coth(x)+\operatorname{csch}(x))^3\,dx = 2\,\ln\left(\mathrm{e}^x-1\right)-x-\frac{4}{\mathrm{e}^{2\,x}-2\,\mathrm{e}^x+1}-\frac{4}{\mathrm{e}^x-1}$$

input
```
int((coth(x) + 1/sinh(x))^3,x)
```

output
```
2*log(exp(x) - 1) - x - 4/(exp(2*x) - 2*exp(x) + 1) - 4/(exp(x) - 1)
```

3.657 $\int (\coth(x) + \mathbf{csch}(x))^2 \, dx$

3.657.1 Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (\coth(x) + \operatorname{csch}(x))^2 \, dx = x + \frac{2\sinh(x)}{1 - \cosh(x)}$$

output
```
x+2*sinh(x)/(1-cosh(x))
```

3.657.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (\coth(x) + \operatorname{csch}(x))^2 \, dx = x - 2\coth\left(\frac{x}{2}\right)$$

input
```
Integrate[(Coth[x] + Csch[x])^2,x]
```

output
```
x - 2*Coth[x/2]
```

3.657.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.714, Rules used = {3042, 4892, 25, 25, 3042, 25, 3149, 25, 3042, 25, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\coth(x)+\text{csch}(x))^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\cot(ix)+i\csc(ix))^2dx$$

$$\Big\downarrow \text{4892}$$

$$\int -(i\cosh(x)+i)^2\text{csch}^2(x)dx$$

$$\Big\downarrow \text{25}$$

$$-\int -(\cosh(x)+1)^2\text{csch}^2(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int (\cosh(x)+1)^2\text{csch}^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}{\cos\left(-\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}{\cos\left(ix-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3149}$$

$$-\int -\frac{\sinh^2(x)}{(1-\cosh(x))^2}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx+\frac{2\sinh(x)}{1-\cosh(x)}$$

$$\Big\downarrow \text{24}$$

$$x+\frac{2\sinh(x)}{1-\cosh(x)}$$

input
```
Int[(Coth[x] + Csch[x])^2,x]
```

output
```
x + (2*Sinh[x])/(1 - Cosh[x])
```

3.657.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.657.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$x-\frac{4}{{\mathrm e}^{x}-1}$	11
default	$x-2\coth\left(x\right)-\frac{2}{\sinh\left(x\right)}$	13
parts	$-2\coth\left(x\right)-\frac{\ln\left(\coth\left(x\right)-1\right)}{2}+\frac{\ln\left(1+\coth\left(x\right)\right)}{2}-2\,\operatorname{csch}\left(x\right)$	24

input
```
int((coth(x)+csch(x))^2,x,method=_RETURNVERBOSE)
```

output
```
x-4/(exp(x)-1)
```

3.657.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int (\coth(x)+\operatorname{csch}(x))^2\,dx=\frac{x\cosh\left(x\right)+x\sinh\left(x\right)-x-4}{\cosh\left(x\right)+\sinh\left(x\right)-1}$$

input
```
integrate((coth(x)+csch(x))^2,x, algorithm="fricas")
```

output
```
(x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

3.657.6 Sympy [F]

$$\int (\coth(x) + \operatorname{csch}(x))^2 \, dx = \int (\coth{(x)} + \operatorname{csch}{(x)})^2 \, dx$$

input
```
integrate((coth(x)+csch(x))**2,x)
```

output
```
Integral((coth(x) + csch(x))**2, x)
```

3.657.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int (\coth(x) + \operatorname{csch}(x))^2 \, dx = x + \frac{4}{e^{(-x)} - e^x} + \frac{4}{e^{(-2x)} - 1}$$

input
```
integrate((coth(x)+csch(x))^2,x, algorithm="maxima")
```

output
```
x + 4/(e^(-x) - e^x) + 4/(e^(-2*x) - 1)
```

3.657.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (\coth(x) + \operatorname{csch}(x))^2 \, dx = x - \frac{4}{e^x - 1}$$

input
```
integrate((coth(x)+csch(x))^2,x, algorithm="giac")
```

output
```
x - 4/(e^x - 1)
```

3.657.9 Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (\coth(x) + \operatorname{csch}(x))^2 \, dx = x - \frac{4}{e^x - 1}$$

input
```
int((coth(x) + 1/sinh(x))^2,x)
```

output
```
x - 4/(exp(x) - 1)
```

3.658 $\int (\coth(x) + \operatorname{csch}(x))\,dx$

3.658.1 Optimal result

Integrand size = 5, antiderivative size = 9

$$\int (\coth(x) + \operatorname{csch}(x))\,dx = -\operatorname{arctanh}(\cosh(x)) + \log(\sinh(x))$$

output
```
-arctanh(cosh(x))+ln(sinh(x))
```

3.658.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(9) = 18.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.56

$$\int (\coth(x)+\operatorname{csch}(x))\,dx = -\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log(\cosh(x))+\log\left(\sinh\left(\frac{x}{2}\right)\right)+\log(\tanh(x))$$

input
```
Integrate[Coth[x] + Csch[x],x]
```

output
```
-Log[Cosh[x/2]] + Log[Cosh[x]] + Log[Sinh[x/2]] + Log[Tanh[x]]
```

3.658.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\coth(x) + \operatorname{csch}(x))\,dx$$
$$\Big\downarrow \text{ 2009}$$
$$\log(\sinh(x)) - \operatorname{arctanh}(\cosh(x))$$

input
```
Int[Coth[x] + Csch[x],x]
```

output
```
-ArcTanh[Cosh[x]] + Log[Sinh[x]]
```

3.658.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.658.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\ln\left(\sinh\left(x\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	10
parts	$\ln\left(\sinh\left(x\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	10
risch	$-x+\ln\left({\rm e}^{2x}-1\right)+\ln\left({\rm e}^{x}-1\right)-\ln\left({\rm e}^{x}+1\right)$	24
parallelrisch	$-x+\ln\left(\tanh\left(x\right)\right)+\ln\left(\coth\left(x\right)-\operatorname{csch}\left(x\right)\right)-\ln\left(1-\tanh\left(x\right)\right)$	25

input
```
int(coth(x)+csch(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sinh(x))+ln(tanh(1/2*x))
```

3.658.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int (\coth(x) + \operatorname{csch}(x))\,dx = -x + 2\,\log\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right)$$

input
```
integrate(coth(x)+csch(x),x, algorithm="fricas")
```

output
```
-x + 2*log(cosh(x) + sinh(x) - 1)
```

3.658.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int (\coth(x) + \operatorname{csch}(x))\,dx = x - \log\left(\tanh\left(x\right) + 1\right) + \log\left(\tanh\left(\frac{x}{2}\right)\right) + \log\left(\tanh\left(x\right)\right)$$

input
```
integrate(coth(x)+csch(x),x)
```

output
```
x - log(tanh(x) + 1) + log(tanh(x/2)) + log(tanh(x))
```

3.658.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (\coth(x) + \operatorname{csch}(x))\,dx = \log\left(\sinh\left(x\right)\right) + \log\left(\tanh\left(\frac{1}{2}\,x\right)\right)$$

input
```
integrate(coth(x)+csch(x),x, algorithm="maxima")
```

output
```
log(sinh(x)) + log(tanh(1/2*x))
```

3.658.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(9) = 18.

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.78

$$\int (\coth(x) + \operatorname{csch}(x))\,dx = -x - \log\left(e^{x} + 1\right) + \log\left(\left|e^{(2\,x)} - 1\right|\right) + \log\left(\left|e^{x} - 1\right|\right)$$

input
```
integrate(coth(x)+csch(x),x, algorithm="giac")
```

output
```
-x - log(e^x + 1) + log(abs(e^(2*x) - 1)) + log(abs(e^x - 1))
```

3.658.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int (\coth(x) + \operatorname{csch}(x))\,dx = 2\,\ln\left(\mathrm{e}^{x} - 1\right) - x$$

input
```
int(coth(x) + 1/sinh(x),x)
```

output
```
2*log(exp(x) - 1) - x
```

3.659 $\int \frac{1}{\coth(x)+\text{csch}(x)}\,dx$

3.659.1 Optimal result

Integrand size = 7, antiderivative size = 5

$$\int \frac{1}{\coth(x)+\text{csch}(x)}\,dx = \log(1+\cosh(x))$$

output
```
ln(1+cosh(x))
```

3.659.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{1}{\coth(x)+\text{csch}(x)}\,dx = 2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Coth[x] + Csch[x])^(-1),x]
```

output
```
2*Log[Cosh[x/2]]
```

3.659.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3042, 3639, 26, 26, 3042, 26, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\coth(x)+\operatorname{csch}(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{i\cot(ix)+i\csc(ix)}dx$$
$$\Big\downarrow \; 3639$$
$$\int \frac{i\sinh(x)}{i\cosh(x)+i}dx$$
$$\Big\downarrow \; 26$$
$$i\int -\frac{i\sinh(x)}{\cosh(x)+1}dx$$
$$\Big\downarrow \; 26$$
$$\int \frac{\sinh(x)}{\cosh(x)+1}dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{1-\sin\left(-\frac{\pi}{2}+ix\right)}dx$$
$$\Big\downarrow \; 26$$
$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{1-\sin\left(ix-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \; 3146$$
$$\int \frac{1}{\cosh(x)+1}d\cosh(x)$$
$$\Big\downarrow \; 16$$
$$\log(\cosh(x)+1)$$

input
```
Int[(Coth[x] + Csch[x])^(-1),x]
```

output
```
Log[1 + Cosh[x]]
```

3.659.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3639
```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.659.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.40

method	result	size
risch	$-x + 2\ln\left(e^{x}+1\right)$	12
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	20

input
```
int(1/(coth(x)+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-x+2*ln(exp(x)+1)
```

3.659.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(5) = 10.

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.60

$$\int \frac{1}{\coth(x)+\operatorname{csch}(x)}\,dx = -x + 2\,\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)$$

input
```
integrate(1/(coth(x)+csch(x)),x, algorithm="fricas")
```

output
```
-x + 2*log(cosh(x) + sinh(x) + 1)
```

3.659.6 Sympy [F]

$$\int \frac{1}{\coth(x)+\operatorname{csch}(x)}\,dx = \int \frac{1}{\coth\left(x\right)+\operatorname{csch}\left(x\right)}\,dx$$

input
```
integrate(1/(coth(x)+csch(x)),x)
```

output
```
Integral(1/(coth(x) + csch(x)), x)
```

3.659.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \frac{1}{\coth(x)+\operatorname{csch}(x)}\,dx = x + 2\,\log\left(e^{(-x)}+1\right)$$

input
```
integrate(1/(coth(x)+csch(x)),x, algorithm="maxima")
```

output
```
x + 2*log(e^(-x) + 1)
```

3.659.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \frac{1}{\coth(x) + \operatorname{csch}(x)}\, dx = -x + 2 \log\left(e^x + 1\right)$$

input
```
integrate(1/(coth(x)+csch(x)),x, algorithm="giac")
```

output
```
-x + 2*log(e^x + 1)
```

3.659.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \frac{1}{\coth(x) + \operatorname{csch}(x)}\, dx = 2 \ln\left(\mathrm{e}^x + 1\right) - x$$

input
```
int(1/(coth(x) + 1/sinh(x)),x)
```

output
```
2*log(exp(x) + 1) - x
```

3.660 $\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2} dx$

3.660.1 Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2} dx = x - \frac{2\sinh(x)}{1+\cosh(x)}$$

output
```
x-2*sinh(x)/(1+cosh(x))
```

3.660.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2} dx = x - 2\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[(Coth[x] + Csch[x])^(-2),x]
```

output
```
x - 2*Tanh[x/2]
```

3.660.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3042, 4892, 25, 25, 3042, 25, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\coth(x)+\text{csch}(x))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(i\cot(ix)+i\csc(ix))^2}dx$$
$$\Big\downarrow \text{4892}$$
$$\int -\frac{\sinh^2(x)}{(i\cosh(x)+i)^2}dx$$
$$\Big\downarrow \text{25}$$
$$-\int -\frac{\sinh^2(x)}{(\cosh(x)+1)^2}dx$$
$$\Big\downarrow \text{25}$$
$$\int \frac{\sinh^2(x)}{(\cosh(x)+1)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{3159}$$
$$\int 1dx-\frac{2\sinh(x)}{\cosh(x)+1}$$
$$\Big\downarrow \text{24}$$
$$x-\frac{2\sinh(x)}{\cosh(x)+1}$$

input
```
Int[(Coth[x] + Csch[x])^(-2),x]
```

output
```
x - (2*Sinh[x])/(1 + Cosh[x])
```

3.660.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.660.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
risch	$x+\frac{4}{{\mathrm e}^{x}+1}$	11
default	$-2\tanh\left(\frac{x}{2}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	24

input
```
int(1/(coth(x)+csch(x))^2,x,method=_RETURNVERBOSE)
```

output
```
x+4/(exp(x)+1)
```

3.660.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2}\,dx = \frac{x\cosh(x)+x\sinh(x)+x+4}{\cosh(x)+\sinh(x)+1}$$

input
```
integrate(1/(coth(x)+csch(x))^2,x, algorithm="fricas")
```

output
```
(x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

3.660.6 Sympy [F]

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2}\,dx = \int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2}\,dx$$

input
```
integrate(1/(coth(x)+csch(x))**2,x)
```

output
```
Integral((coth(x) + csch(x))**(-2), x)
```

3.660.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2}\,dx = x-\frac{4}{e^{(-x)}+1}$$

input
```
integrate(1/(coth(x)+csch(x))^2,x, algorithm="maxima")
```

output
```
x - 4/(e^(-x) + 1)
```

3.660.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2}\,dx = x + \frac{4}{e^x+1}$$

input
```
integrate(1/(coth(x)+csch(x))^2,x, algorithm="giac")
```

output
```
x + 4/(e^x + 1)
```

3.660.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2}\,dx = x + \frac{4}{\mathrm{e}^x+1}$$

input
```
int(1/(coth(x) + 1/sinh(x))^2,x)
```

output
```
x + 4/(exp(x) + 1)
```

3.661 $\int \frac{1}{(\coth(x)+\text{csch}(x))^3} \, dx$

3.661.1 Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(\coth(x)+\text{csch}(x))^3} \, dx = \frac{2}{1+\cosh(x)} + \log(1+\cosh(x))$$

output
```
2/(1+cosh(x))+ln(1+cosh(x))
```

3.661.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{(\coth(x)+\text{csch}(x))^3} \, dx = 2\log\left(\cosh\left(\frac{x}{2}\right)\right) + \text{sech}^2\left(\frac{x}{2}\right)$$

input
```
Integrate[(Coth[x] + Csch[x])^(-3),x]
```

output
```
2*Log[Cosh[x/2]] + Sech[x/2]^2
```

3.661.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {3042, 4892, 26, 26, 3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(i\cot(ix)+i\csc(ix))^3}dx$$
$$\Big\downarrow \text{4892}$$
$$\int -\frac{i\sinh^3(x)}{(i\cosh(x)+i)^3}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{i\sinh^3(x)}{(\cosh(x)+1)^3}dx$$
$$\Big\downarrow \text{26}$$
$$\int \frac{\sinh^3(x)}{(\cosh(x)+1)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{3146}$$
$$-\int \frac{1-\cosh(x)}{(\cosh(x)+1)^2}d\cosh(x)$$
$$\Big\downarrow \text{49}$$
$$-\int \left(\frac{2}{(\cosh(x)+1)^2}+\frac{1}{-\cosh(x)-1}\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

input
```
Int[(Coth[x] + Csch[x])^(-3),x]
```

output
```
2/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

3.661.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.661. $\int \frac{1}{(\coth(x)+\text{csch}(x))^3}\,dx$

3.661.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

method	result	size
risch	$-x+\frac{4\,{\mathrm e}^{x}}{\left({\mathrm e}^{x}+1\right)^{2}}+2\ln\left({\mathrm e}^{x}+1\right)$	22
default	$-\tanh\left(\frac{x}{2}\right)^{2}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	28

input
```
int(1/(coth(x)+csch(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-x+4*exp(x)/(exp(x)+1)^2+2*ln(exp(x)+1)
```

3.661.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 6.36

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx = -\frac{x\cosh(x)^2+x\sinh(x)^2+2(x-2)\cosh(x)-2\left(\cosh(x)^2+2(\cosh(x)+1)\sinh(x)+\sinh(x)^2+2\right.}{\cosh(x)^2+2(\cosh(x)+1)\sinh(x)+\sinh(}$$

input
```
integrate(1/(coth(x)+csch(x))^3,x, algorithm="fricas")
```

output
```
-(x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x
) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2
*(x*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + s
inh(x)^2 + 2*cosh(x) + 1)
```

3.661.6 Sympy [F]

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx = \int \frac{1}{(\coth{(x)}+\operatorname{csch}{(x)})^3}\,dx$$

input
```
integrate(1/(coth(x)+csch(x))**3,x)
```

output
```
Integral((coth(x) + csch(x))**(-3), x)
```

3.661.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx = x + \frac{4\,e^{(-x)}}{2\,e^{(-x)}+e^{(-2\,x)}+1} + 2\,\log\left(e^{(-x)}+1\right)$$

input
```
integrate(1/(coth(x)+csch(x))^3,x, algorithm="maxima")
```

output
```
x + 4*e^(-x)/(2*e^(-x) + e^(-2*x) + 1) + 2*log(e^(-x) + 1)
```

3.661.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx = -x + \frac{4\,e^{x}}{(e^{x}+1)^2} + 2\,\log\left(e^{x}+1\right)$$

input
```
integrate(1/(coth(x)+csch(x))^3,x, algorithm="giac")
```

output
```
-x + 4*e^x/(e^x + 1)^2 + 2*log(e^x + 1)
```

3.661.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx = 2\,\ln\left(e^x+1\right)-x-\frac{4}{e^{2\,x}+2\,e^x+1}+\frac{4}{e^x+1}$$

input
```
int(1/(coth(x) + 1/sinh(x))^3,x)
```

output
```
2*log(exp(x) + 1) - x - 4/(exp(2*x) + 2*exp(x) + 1) + 4/(exp(x) + 1)
```

3.662 $\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4} dx$

3.662.1 Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4} dx = x - \frac{2\sinh(x)}{1+\cosh(x)} - \frac{2\sinh^3(x)}{3(1+\cosh(x))^3}$$

output
```
x-2*sinh(x)/(1+cosh(x))-2/3*sinh(x)^3/(1+cosh(x))^3
```

3.662.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4} dx = x - \frac{8}{3}\tanh\left(\frac{x}{2}\right) + \frac{2}{3}\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[(Coth[x] + Csch[x])^(-4),x]
```

output
```
x - (8*Tanh[x/2])/3 + (2*Sech[x/2]^2*Tanh[x/2])/3
```

3.662.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3042, 4892, 3042, 3159, 3042, 25, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i\cot(ix)+i\csc(ix))^4}dx$$

$$\Big\downarrow \text{4892}$$

$$\int \frac{\sinh^4(x)}{(i\cosh(x)+i)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^4}{\left(i-i\sin\left(-\frac{\pi}{2}+ix\right)\right)^4}dx$$

$$\Big\downarrow \text{3159}$$

$$\int \frac{\sinh^2(x)}{(\cosh(x)+1)^2}dx-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}+\int -\frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}-\frac{2\sinh(x)}{\cosh(x)+1}$$

$$\Big\downarrow \text{24}$$

$$x-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}-\frac{2\sinh(x)}{\cosh(x)+1}$$

input
```
Int[(Coth[x] + Csch[x])^(-4),x]
```

output
```
x - (2*Sinh[x])/(1 + Cosh[x]) - (2*Sinh[x]^3)/(3*(1 + Cosh[x])^3)
```

3.662.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.662.4 Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.08

method	result	size
parallelrisch	0	2
risch	$x+\frac{8\,{\mathrm e}^{2 x}+8\,{\mathrm e}^{x}+\frac{16}{3}}{\left({\mathrm e}^{x}+1\right)^{3}}$	23
default	$-\frac{2 \tanh \left(\frac{x}{2}\right)^{3}}{3}-2 \tanh \left(\frac{x}{2}\right)-\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)+\ln \left(\tanh \left(\frac{x}{2}\right)+1\right)$	32

input
```
int(1/(coth(x)+csch(x))^4,x,method=_RETURNVERBOSE)
```

output
```
0
```

3.662.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx = \frac{3\,x\cosh\left(x\right)^2+3\,x\sinh\left(x\right)^2+4\,(3\,x+10)\cosh\left(x\right)+2\,(3\,x\cosh\left(x\right)+3\,x+4)\sinh\left(x\right)+9\,x+24}{3\,\left(\cosh\left(x\right)^2+2\,(\cosh\left(x\right)+1)\sinh\left(x\right)+\sinh\left(x\right)^2+4\,\cosh\left(x\right)+3\right)}$$

input
```
integrate(1/(coth(x)+csch(x))^4,x, algorithm="fricas")
```

output
```
1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 + 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
 + 3*x + 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sin
h(x)^2 + 4*cosh(x) + 3)
```

3.662.6 Sympy [F]

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx = \int \frac{1}{\left(\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^4}\,dx$$

input
```
integrate(1/(coth(x)+csch(x))**4,x)
```

output
```
Integral((coth(x) + csch(x))**(-4), x)
```

3.662.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx = x - \frac{8\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+2\right)}{3\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1\right)}$$

input
```
integrate(1/(coth(x)+csch(x))^4,x, algorithm="maxima")
```

output
```
x - 8/3*(3*e^(-x) + 3*e^(-2*x) + 2)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1)
```

3.662.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx = x + \frac{8\left(3\,e^{(2\,x)}+3\,e^{x}+2\right)}{3\left(e^{x}+1\right)^3}$$

input
```
integrate(1/(coth(x)+csch(x))^4,x, algorithm="giac")
```

output
```
x + 8/3*(3*e^(2*x) + 3*e^x + 2)/(e^x + 1)^3
```

3.662.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx = x + \frac{\frac{8\,\mathrm{e}^{2\,x}}{3}+\frac{8}{3}}{3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}+1} + \frac{8\,\mathrm{e}^{x}}{3\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^{x}+1\right)} + \frac{8}{3\,\left(\mathrm{e}^{x}+1\right)}$$

input
```
int(1/(coth(x) + 1/sinh(x))^4,x)
```

output
```
x + ((8*exp(2*x))/3 + 8/3)/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) + (8*exp
(x))/(3*(exp(2*x) + 2*exp(x) + 1)) + 8/(3*(exp(x) + 1))
```

3.663 $\int \frac{1}{(\coth(x)+\text{csch}(x))^5} dx$

3.663.1 Optimal result

Integrand size = 7, antiderivative size = 22

$$\int \frac{1}{(\coth(x)+\text{csch}(x))^5} dx = -\frac{2}{(1+\cosh(x))^2}+\frac{4}{1+\cosh(x)}+\log(1+\cosh(x))$$

output
```
-2/(1+cosh(x))^2+4/(1+cosh(x))+ln(1+cosh(x))
```

3.663.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{1}{(\coth(x)+\text{csch}(x))^5} dx = 2\log\left(\cosh\left(\frac{x}{2}\right)\right)+2\text{sech}^2\left(\frac{x}{2}\right)-\frac{1}{2}\text{sech}^4\left(\frac{x}{2}\right)$$

input
```
Integrate[(Coth[x] + Csch[x])^(-5),x]
```

output
```
2*Log[Cosh[x/2]] + 2*Sech[x/2]^2 - Sech[x/2]^4/2
```

3.663.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {3042, 4892, 26, 26, 3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(i\cot(ix)+i\csc(ix))^5}dx$$
$$\Big\downarrow \text{ 4892}$$
$$\int \frac{i\sinh^5(x)}{(i\cosh(x)+i)^5}dx$$
$$\Big\downarrow \text{ 26}$$
$$i\int -\frac{i\sinh^5(x)}{(\cosh(x)+1)^5}dx$$
$$\Big\downarrow \text{ 26}$$
$$\int \frac{\sinh^5(x)}{(\cosh(x)+1)^5}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)^5}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^5}dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^5}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^5}dx$$
$$\Big\downarrow \text{ 3146}$$
$$\int \frac{(1-\cosh(x))^2}{(\cosh(x)+1)^3}d\cosh(x)$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{1}{\cosh(x)+1}-\frac{4}{(\cosh(x)+1)^2}+\frac{4}{(\cosh(x)+1)^3}\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{\cosh(x)+1} - \frac{2}{(\cosh(x)+1)^2} + \log(\cosh(x)+1)$$

input
```
Int[(Coth[x] + Csch[x])^(-5),x]
```

output
```
-2/(1 + Cosh[x])^2 + 4/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

3.663.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.663. $\int \frac{1}{(\coth(x)+\text{csch}(x))^5}\,dx$

3.663.4 Maple [A] (verified)

Time = 10.66 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.09

method	result	size
parallelrisch	0	2
risch	$-x+\frac{8\,\mathrm{e}^{x}\left(1+\mathrm{e}^{x}+\mathrm{e}^{2x}\right)}{\left(\mathrm{e}^{x}+1\right)^{4}}+2\ln\left(\mathrm{e}^{x}+1\right)$	30
default	$-\frac{\tanh\left(\frac{x}{2}\right)^{4}}{2}-\tanh\left(\frac{x}{2}\right)^{2}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	36

input
```
int(1/(coth(x)+csch(x))^5,x,method=_RETURNVERBOSE)
```

output
```
0
```

3.663.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(22) = 44.

Time = 0.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 12.09

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx = -\frac{x\cosh(x)^4+x\sinh(x)^4+4(x-2)\cosh(x)^3+4(x\cosh(x)+x-2)\sinh(x)^3+2(3x-4)\cosh(x)^2}{}$$

input
```
integrate(1/(coth(x)+csch(x))^5,x, algorithm="fricas")
```

output
```
-(x*cosh(x)^4 + x*sinh(x)^4 + 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) + x - 2)*
sinh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 + 6*(x - 2)*cosh(x) +
 3*x - 4)*sinh(x)^2 + 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) + 1)*s
inh(x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)
^2 + 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4
*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 4*(x*cosh(x)^3 + 3*(x - 2)*cosh
(x)^2 + (3*x - 4)*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) +
1)*sinh(x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sin
h(x)^2 + 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)
 + 4*cosh(x) + 1)
```

3.663.6 Sympy [F]

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx = \int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx$$

input
```
integrate(1/(coth(x)+csch(x))**5,x)
```

output
```
Integral((coth(x) + csch(x))**(-5), x)
```

3.663.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(22) = 44.

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx = x + \frac{8\left(e^{(-x)} + e^{(-2x)} + e^{(-3x)}\right)}{4\,e^{(-x)} + 6\,e^{(-2x)} + 4\,e^{(-3x)} + e^{(-4x)} + 1} + 2\,\log\left(e^{(-x)} + 1\right)$$

input
```
integrate(1/(coth(x)+csch(x))^5,x, algorithm="maxima")
```

output
```
x + 8*(e^(-x) + e^(-2*x) + e^(-3*x))/(4*e^(-x) + 6*e^(-2*x) + 4*e^(-3*x) +
 e^(-4*x) + 1) + 2*log(e^(-x) + 1)
```

3.663.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx = -x + \frac{8\left(e^{(3x)} + e^{(2x)} + e^{x}\right)}{(e^x+1)^4} + 2\,\log\left(e^x + 1\right)$$

input
```
integrate(1/(coth(x)+csch(x))^5,x, algorithm="giac")
```

output
```
-x + 8*(e^(3*x) + e^(2*x) + e^x)/(e^x + 1)^4 + 2*log(e^x + 1)
```

3.663.9 Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.59

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx = 2\,\ln\left(e^x+1\right)-x-\frac{16}{e^{2\,x}+2\,e^x+1}-\frac{8}{6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^x+1}+\frac{8}{e^x+1}+\frac{16}{3\,e^{2\,x}+e^{3\,x}+3\,e^x+1}$$

input
```
int(1/(coth(x) + 1/sinh(x))^5,x)
```

output
```
2*log(exp(x) + 1) - x - 16/(exp(2*x) + 2*exp(x) + 1) - 8/(6*exp(2*x) + 4*e
xp(3*x) + exp(4*x) + 4*exp(x) + 1) + 8/(exp(x) + 1) + 16/(3*exp(2*x) + exp
(3*x) + 3*exp(x) + 1)
```

3.664 $\int (-\coth(x)+\mathbf{csch}(x))^5\,dx$

3.664.1 Optimal result

Integrand size = 9, antiderivative size = 24

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx=\frac{2}{(1+\cosh(x))^2}-\frac{4}{1+\cosh(x)}-\log(1+\cosh(x))$$

output
```
2/(1+cosh(x))^2-4/(1+cosh(x))-ln(1+cosh(x))
```

3.664.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx=-2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\tanh^2\left(\frac{x}{2}\right)+\frac{1}{2}\tanh^4\left(\frac{x}{2}\right)$$

input
```
Integrate[(-Coth[x] + Csch[x])^5,x]
```

output
```
-2*Log[Cosh[x/2]] + Tanh[x/2]^2 + Tanh[x/2]^4/2
```

3.664.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4892, 26, 26, 3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\operatorname{csch}(x)-\coth(x))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\csc(ix)-i\cot(ix))^5dx$$

$$\Big\downarrow \text{4892}$$

$$\int -i(i-i\cosh(x))^5\operatorname{csch}^5(x)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int i(1-\cosh(x))^5\operatorname{csch}^5(x)dx$$

$$\Big\downarrow \text{26}$$

$$\int (1-\cosh(x))^5\operatorname{csch}^5(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^5}{\cos\left(-\frac{\pi}{2}+ix\right)^5}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^5}{\cos\left(ix-\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{(1-\cosh(x))^2}{(\cosh(x)+1)^3}d(-\cosh(x))$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{4}{(-\cosh(x)-1)^2}-\frac{4}{(-\cosh(x)-1)^3}+\frac{1}{\cosh(x)+1}\right)d(-\cosh(x))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4}{\cosh(x)+1}+\frac{2}{(\cosh(x)+1)^2}-\log(\cosh(x)+1)$$

input
```
Int[(-Coth[x] + Csch[x])^5,x]
```

output
```
2/(1 + Cosh[x])^2 - 4/(1 + Cosh[x]) - Log[1 + Cosh[x]]
```

3.664.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.664.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

method	result
risch	$x-\frac{8\,{\mathrm e}^{x} \left(1+{\mathrm e}^{x}+{\mathrm e}^{2 x}\right)}{\left({\mathrm e}^{x}+1\right)^{4}}-2 \ln \left({\mathrm e}^{x}+1\right)$
parts	$\frac{8 \left(-\frac{\operatorname{csch}\left(x \right)^{3}}{4}+\frac{3 \operatorname{csch}\left(x \right)}{8}\right) \coth \left(x \right)}{3}-2 \,\operatorname{arctanh}\left({\mathrm e}^{x}\right)+\frac{11 \coth \left(x \right)^{4}}{4}+\frac{\coth \left(x \right)^{2}}{2}+\frac{\ln \left(\coth \left(x \right)-1\right)}{2}+\frac{\ln \left(1+\coth \left(x \right)\right)}{2}+\frac{5}{3 s}$
default	$-\ln \left(\sinh \left(x \right)\right)+\frac{\coth \left(x \right)^{2}}{2}+\frac{\coth \left(x \right)^{4}}{4}-\frac{5 \cosh \left(x \right)^{3}}{\sinh \left(x \right)^{4}}+\frac{5 \cosh \left(x \right)}{3 \sinh \left(x \right)^{4}}+\frac{8 \left(-\frac{\operatorname{csch}\left(x \right)^{3}}{4}+\frac{3 \operatorname{csch}\left(x \right)}{8}\right) \coth \left(x \right)}{3}-2 \,\operatorname{arctanh}$

input
```
int((-coth(x)+csch(x))^5,x,method=_RETURNVERBOSE)
```

output
```
x-8*exp(x)*(1+exp(x)+exp(2*x))/(exp(x)+1)^4-2*ln(exp(x)+1)
```

3.664.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 265, normalized size of antiderivative = 11.04

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx$$
$$=\frac{x\cosh\left(x\right)^{4}+x\sinh\left(x\right)^{4}+4\,{\left(x-2\right)}\cosh\left(x\right)^{3}+4\,{\left(x\cosh\left(x\right)+x-2\right)}\sinh\left(x\right)^{3}+2\,{\left(3\,x-4\right)}\cosh\left(x\right)^{2}+2}{}$$

input
```
integrate((-coth(x)+csch(x))^5,x, algorithm="fricas")
```

output
```
(x*cosh(x)^4 + x*sinh(x)^4 + 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) + x - 2)*s
inh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 + 6*(x - 2)*cosh(x) +
3*x - 4)*sinh(x)^2 + 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) + 1)*si
nh(x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^
2 + 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*
cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 4*(x*cosh(x)^3 + 3*(x - 2)*cosh(
x)^2 + (3*x - 4)*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) + 1
)*sinh(x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh
(x)^2 + 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)
+ 4*cosh(x) + 1)
```

3.664.6 Sympy [F]

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx = -\int 5\coth(x)\operatorname{csch}^4(x)\,dx - \int \left(-10\coth^2(x)\operatorname{csch}^3(x)\right)\,dx - \int 10\coth^3(x)\operatorname{csch}^2(x)\,dx - \int \left(-5\coth^4(x)\operatorname{csch}(x)\right)\,dx - \int \coth^5(x)\,dx - \int \left(-\operatorname{csch}^5(x)\right)\,dx$$

input
```
integrate((-coth(x)+csch(x))**5,x)
```

output
```
-Integral(5*coth(x)*csch(x)**4, x) - Integral(-10*coth(x)**2*csch(x)**3, x
) - Integral(10*coth(x)**3*csch(x)**2, x) - Integral(-5*coth(x)**4*csch(x)
, x) - Integral(coth(x)**5, x) - Integral(-csch(x)**5, x)
```

3.664.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(24) = 48$.

Time = 0.21 (sec) , antiderivative size = 238, normalized size of antiderivative = 9.92

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx = \frac{5}{2}\coth(x)^4 - x + \frac{5\left(5e^{(-x)}+3e^{(-3x)}+3e^{(-5x)}+5e^{(-7x)}\right)}{4\left(4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1\right)} - \frac{3e^{(-x)}-11e^{(-3x)}-11e^{(-5x)}+3e^{(-7x)}}{4\left(4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1\right)} + \frac{5\left(e^{(-x)}+7e^{(-3x)}+7e^{(-5x)}+e^{(-7x)}\right)}{2\left(4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1\right)} - \frac{4\left(e^{(-2x)}-e^{(-4x)}+e^{(-6x)}\right)}{4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1} + \frac{20}{\left(e^{(-x)}-e^{x}\right)^4} - 2\log\left(e^{(-x)}+1\right)$$

input
```
integrate((-coth(x)+csch(x))^5,x, algorithm="maxima")
```

output
```
5/2*coth(x)^4 - x + 5/4*(5*e^(-x) + 3*e^(-3*x) + 3*e^(-5*x) + 5*e^(-7*x))/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 1/4*(3*e^(-x) - 11
*e^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x
) - e^(-8*x) - 1) + 5/2*(e^(-x) + 7*e^(-3*x) + 7*e^(-5*x) + e^(-7*x))/(4*e
^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 4*(e^(-2*x) - e^(-4*x)
 + e^(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 20/(e
^(-x) - e^x)^4 - 2*log(e^(-x) + 1)
```

3.664.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx = x - \frac{8\left(e^{(3x)} + e^{(2x)} + e^{x}\right)}{(e^{x}+1)^4} - 2\log\left(e^{x}+1\right)$$

input
```
integrate((-coth(x)+csch(x))^5,x, algorithm="giac")
```

output
```
x - 8*(e^(3*x) + e^(2*x) + e^x)/(e^x + 1)^4 - 2*log(e^x + 1)
```

3.664.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.21

$$\begin{aligned}\int (-\coth(x)+\operatorname{csch}(x))^5\,dx &= x - 2\ln\left(e^{x}+1\right) + \frac{16}{e^{2x}+2e^{x}+1} \\ &+ \frac{8}{6e^{2x}+4e^{3x}+e^{4x}+4e^{x}+1} \\ &- \frac{8}{e^{x}+1} - \frac{16}{3e^{2x}+e^{3x}+3e^{x}+1}\end{aligned}$$

input
```
int(-(coth(x) - 1/sinh(x))^5,x)
```

output
```
x - 2*log(exp(x) + 1) + 16/(exp(2*x) + 2*exp(x) + 1) + 8/(6*exp(2*x) + 4*e
xp(3*x) + exp(4*x) + 4*exp(x) + 1) - 8/(exp(x) + 1) - 16/(3*exp(2*x) + exp
(3*x) + 3*exp(x) + 1)
```

3.665 $\int (-\coth(x)+\text{csch}(x))^4\,dx$

3.665.1 Optimal result

Integrand size = 9, antiderivative size = 26

$$\int (-\coth(x)+\text{csch}(x))^4\,dx = x-\frac{2\sinh(x)}{1+\cosh(x)}-\frac{2\sinh^3(x)}{3(1+\cosh(x))^3}$$

output
```
x-2*sinh(x)/(1+cosh(x))-2/3*sinh(x)^3/(1+cosh(x))^3
```

3.665.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int (-\coth(x)+\text{csch}(x))^4\,dx = 2\text{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)-\frac{2}{3}\tanh^3\left(\frac{x}{2}\right)$$

input
```
Integrate[(-Coth[x] + Csch[x])^4,x]
```

output
```
2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2] - (2*Tanh[x/2]^3)/3
```

3.665.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.222, Rules used = {3042, 4892, 3042, 3149, 3042, 3159, 25, 3042, 25, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\operatorname{csch}(x)-\coth(x))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\csc(ix)-i\cot(ix))^4dx$$

$$\Big\downarrow \text{4892}$$

$$\int (i-i\cosh(x))^4\operatorname{csch}^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(i\sin\left(-\frac{\pi}{2}+ix\right)+i\right)^4}{\cos\left(-\frac{\pi}{2}+ix\right)^4}dx$$

$$\Big\downarrow \text{3149}$$

$$\int \frac{\sinh^4(x)}{(\cosh(x)+1)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^4}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^4}dx$$

$$\Big\downarrow \text{3159}$$

$$-\int -\frac{\sinh^2(x)}{(\cosh(x)+1)^2}dx-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh^2(x)}{(\cosh(x)+1)^2}dx-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}+\int -\frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}-\int\frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}-\frac{2\sinh(x)}{\cosh(x)+1}$$

$$\Big\downarrow \text{24}$$

$$x-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}-\frac{2\sinh(x)}{\cosh(x)+1}$$

input
```
Int[(-Coth[x] + Csch[x])^4,x]
```

output
```
x - (2*Sinh[x])/(1 + Cosh[x]) - (2*Sinh[x]^3)/(3*(1 + Cosh[x])^3)
```

3.665.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.665.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result
risch	$x+\frac{8\,{\mathrm e}^{2 x}+8\,{\mathrm e}^{x}+\frac{16}{3}}{\left({\mathrm e}^{x}+1\right)^{3}}$
parts	$-\frac{7 \coth\left(x\right)^{3}}{3}-\coth\left(x\right)-\frac{\ln\left(\coth\left(x\right)-1\right)}{2}+\frac{\ln\left(1+\coth\left(x\right)\right)}{2}+\left(\frac{2}{3}-\frac{\operatorname{csch}\left(x\right)^{2}}{3}\right)\coth\left(x\right)+\frac{8\operatorname{csch}\left(x\right)^{3}}{3}+4\operatorname{csch}\left(x\right.$
default	$x-\coth\left(x\right)-\frac{\coth\left(x\right)^{3}}{3}+\frac{4\cosh\left(x\right)^{2}}{\sinh\left(x\right)^{3}}-\frac{4}{3\sinh\left(x\right)^{3}}-\frac{3\cosh\left(x\right)}{\sinh\left(x\right)^{3}}-2\left(\frac{2}{3}-\frac{\operatorname{csch}\left(x\right)^{2}}{3}\right)\coth\left(x\right)$

input
```
int((-coth(x)+csch(x))^4,x,method=_RETURNVERBOSE)
```

output
```
x+8/3*(3*exp(2*x)+3*exp(x)+2)/(exp(x)+1)^3
```

3.665.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

$$\int(-\coth(x)+\operatorname{csch}(x))^4\,dx$$
$$=\frac{3\,x\cosh\left(x\right)^{2}+3\,x\sinh\left(x\right)^{2}+4\,\left(3\,x+10\right)\cosh\left(x\right)+2\,\left(3\,x\cosh\left(x\right)+3\,x+4\right)\sinh\left(x\right)+9\,x+24}{3\,\left(\cosh\left(x\right)^{2}+2\,\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+4\,\cosh\left(x\right)+3\right)}$$

input
```
integrate((-coth(x)+csch(x))^4,x, algorithm="fricas")
```

output
```
1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 + 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
 + 3*x + 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sin
h(x)^2 + 4*cosh(x) + 3)
```

3.665.6 Sympy [F]

$$\int (-\coth(x) + \operatorname{csch}(x))^4\,dx = \int (-\coth(x) + \operatorname{csch}(x))^4\,dx$$

input
```
integrate((-coth(x)+csch(x))**4,x)
```

output
```
Integral((-coth(x) + csch(x))**4, x)
```

3.665.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(24) = 48$.

Time = 0.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 7.04

$$\begin{aligned}\int (-\coth(x) + \operatorname{csch}(x))^4\,dx = & -2\coth(x)^3 + x - \frac{4\left(3e^{(-2x)} - 3e^{(-4x)} - 2\right)}{3\left(3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1\right)} \\ & - \frac{8e^{(-x)}}{3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1} \\ & + \frac{4e^{(-2x)}}{3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1} \\ & + \frac{16e^{(-3x)}}{3\left(3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1\right)} \\ & - \frac{8e^{(-5x)}}{3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1} \\ & - \frac{4}{3\left(3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1\right)} - \frac{32}{3\left(e^{(-x)} - e^{x}\right)^3}\end{aligned}$$

input
```
integrate((-coth(x)+csch(x))^4,x, algorithm="maxima")
```

output
```
-2*coth(x)^3 + x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4
*x) + e^(-6*x) - 1) - 8*e^(-x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) +
4*e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 16/3*e^(-3*x)/(3*e^(
-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 8*e^(-5*x)/(3*e^(-2*x) - 3*e^(-4*x) +
 e^(-6*x) - 1) - 4/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 32/3/(e^(-
x) - e^x)^3
```

3.665.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int (-\coth(x)+\operatorname{csch}(x))^4\,dx = x+\frac{8\,\left(3\,e^{(2\,x)}+3\,e^{x}+2\right)}{3\,\left(e^{x}+1\right)^{3}}$$

input
```
integrate((-coth(x)+csch(x))^4,x, algorithm="giac")
```

output
```
x + 8/3*(3*e^(2*x) + 3*e^x + 2)/(e^x + 1)^3
```

3.665.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int (-\coth(x)+\operatorname{csch}(x))^4\,dx = x+\frac{\frac{8\,\mathrm{e}^{2\,x}}{3}+\frac{8}{3}}{3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}+1}+\frac{8\,\mathrm{e}^{x}}{3\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^{x}+1\right)}+\frac{8}{3\,\left(\mathrm{e}^{x}+1\right)}$$

input
```
int((coth(x) - 1/sinh(x))^4,x)
```

output
```
x + ((8*exp(2*x))/3 + 8/3)/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) + (8*exp
(x))/(3*(exp(2*x) + 2*exp(x) + 1)) + 8/(3*(exp(x) + 1))
```

3.666 $\int (-\coth(x) + \operatorname{csch}(x))^3 \, dx$

3.666.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (-\coth(x) + \operatorname{csch}(x))^3 \, dx = -\frac{2}{1+\cosh(x)} - \log(1+\cosh(x))$$

output
```
-2/(1+cosh(x))-ln(1+cosh(x))
```

3.666.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (-\coth(x) + \operatorname{csch}(x))^3 \, dx = -2\log\left(\cosh\left(\frac{x}{2}\right)\right) + \tanh^2\left(\frac{x}{2}\right)$$

input
```
Integrate[(-Coth[x] + Csch[x])^3,x]
```

output
```
-2*Log[Cosh[x/2]] + Tanh[x/2]^2
```

3.666.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4892, 26, 26, 3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{csch}(x)-\coth(x))^3 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i \csc(ix)-i \cot(ix))^3 dx$$

$$\Big\downarrow \text{4892}$$

$$\int i(i-i\cosh(x))^3\text{csch}^3(x)dx$$

$$\Big\downarrow \text{26}$$

$$i\int -i(1-\cosh(x))^3\text{csch}^3(x)dx$$

$$\Big\downarrow \text{26}$$

$$\int (1-\cosh(x))^3\text{csch}^3(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}{\cos\left(-\frac{\pi}{2}+ix\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^3}{\cos\left(ix-\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1-\cosh(x)}{(\cosh(x)+1)^2}d(-\cosh(x))$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{1}{-\cosh(x)-1}+\frac{2}{(-\cosh(x)-1)^2}\right)d(-\cosh(x))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{\cosh(x)+1}-\log(\cosh(x)+1)$$

input
```
Int[(-Coth[x] + Csch[x])^3,x]
```

output
```
-2/(1 + Cosh[x]) - Log[1 + Cosh[x]]
```

3.666.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.666. $\int(-\coth(x)+\operatorname{csch}(x))^3\,dx$

3.666.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

method	result	size
risch	$x-\frac{4\,{\mathrm e}^{x}}{\left({\mathrm e}^{x}+1\right)^{2}}-2\ln\left({\mathrm e}^{x}+1\right)$	20
default	$-\ln\left(\sinh\left(x\right)\right)+\frac{\coth\left(x\right)^{2}}{2}-\frac{3\cosh\left(x\right)}{\sinh\left(x\right)^{2}}+\operatorname{csch}\left(x\right)\coth\left(x\right)-2\,\operatorname{arctanh}\left({\mathrm e}^{x}\right)+\frac{3}{2\sinh\left(x\right)^{2}}$	37
parts	$\operatorname{csch}\left(x\right)\coth\left(x\right)-2\,\operatorname{arctanh}\left({\mathrm e}^{x}\right)+2\coth\left(x\right)^{2}+\frac{\ln\left(\coth\left(x\right)-1\right)}{2}+\frac{\ln\left(1+\coth\left(x\right)\right)}{2}-\frac{3\cosh\left(x\right)}{\sinh\left(x\right)^{2}}$	40

input
```
int((-coth(x)+csch(x))^3,x,method=_RETURNVERBOSE)
```

output
```
x-4*exp(x)/(exp(x)+1)^2-2*ln(exp(x)+1)
```

3.666.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.50

$$\int\left(-\coth(x)+\operatorname{csch}(x)\right)^{3}\,dx$$
$$=\frac{x\cosh\left(x\right)^{2}+x\sinh\left(x\right)^{2}+2\left(x-2\right)\cosh\left(x\right)-2\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+2\cos\right.}{\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^{}}$$

input
```
integrate((-coth(x)+csch(x))^3,x, algorithm="fricas")
```

output
```
(x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
 + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2*
(x*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + si
nh(x)^2 + 2*cosh(x) + 1)
```

3.666.6 Sympy [F]

$$\int (-\coth(x)+\operatorname{csch}(x))^3\,dx = -\int 3\coth(x)\operatorname{csch}^2(x)\,dx - \int \left(-3\coth^2(x)\operatorname{csch}(x)\right)\,dx - \int \coth^3(x)\,dx - \int \left(-\operatorname{csch}^3(x)\right)\,dx$$

input
```
integrate((-coth(x)+csch(x))**3,x)
```

output
```
-Integral(3*coth(x)*csch(x)**2, x) - Integral(-3*coth(x)**2*csch(x), x) -
Integral(coth(x)**3, x) - Integral(-csch(x)**3, x)
```

3.666.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(16) = 32$.

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int (-\coth(x)+\operatorname{csch}(x))^3\,dx = \frac{3}{2}\coth(x)^2 - x + \frac{4\left(e^{(-x)}+e^{(-3x)}\right)}{2e^{(-2x)}-e^{(-4x)}-1} - \frac{2e^{(-2x)}}{2e^{(-2x)}-e^{(-4x)}-1} - 2\log\left(e^{(-x)}+1\right)$$

input
```
integrate((-coth(x)+csch(x))^3,x, algorithm="maxima")
```

output
```
3/2*coth(x)^2 - x + 4*(e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) - 2*
e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2*log(e^(-x) + 1)
```

3.666.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int (-\coth(x)+\operatorname{csch}(x))^3\,dx = x - \frac{4e^x}{(e^x+1)^2} - 2\log(e^x+1)$$

input
```
integrate((-coth(x)+csch(x))^3,x, algorithm="giac")
```

output
```
x - 4*e^x/(e^x + 1)^2 - 2*log(e^x + 1)
```

3.666.9 Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int (-\coth(x)+\operatorname{csch}(x))^3\,dx = x-2\,\ln\left(e^x+1\right)+\frac{4}{e^{2\,x}+2\,e^x+1}-\frac{4}{e^x+1}$$

input
```
int(-(coth(x) - 1/sinh(x))^3,x)
```

output
```
x - 2*log(exp(x) + 1) + 4/(exp(2*x) + 2*exp(x) + 1) - 4/(exp(x) + 1)
```

3.667 $\int (-\coth(x) + \mathbf{csch}(x))^2 \, dx$

3.667.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int (-\coth(x) + \operatorname{csch}(x))^2 \, dx = x - \frac{2\sinh(x)}{1 + \cosh(x)}$$

output
```
x-2*sinh(x)/(1+cosh(x))
```

3.667.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int (-\coth(x) + \operatorname{csch}(x))^2 \, dx = 2\text{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right) - 2\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[(-Coth[x] + Csch[x])^2,x]
```

output
```
2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2]
```

3.667.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {3042, 4892, 25, 25, 3042, 25, 3149, 25, 3042, 25, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{csch}(x)-\coth(x))^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\csc(ix)-i\cot(ix))^2dx$$

$$\Big\downarrow \text{4892}$$

$$\int -(i-i\cosh(x))^2\text{csch}^2(x)dx$$

$$\Big\downarrow \text{25}$$

$$-\int -(1-\cosh(x))^2\text{csch}^2(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int (1-\cosh(x))^2\text{csch}^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}{\cos\left(-\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}{\cos\left(ix-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3149}$$

$$-\int -\frac{\sinh^2(x)}{(\cosh(x)+1)^2}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh^2(x)}{(\cosh(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx - \frac{2\sinh(x)}{\cosh(x)+1}$$

$$\Big\downarrow \text{24}$$

$$x - \frac{2\sinh(x)}{\cosh(x)+1}$$

input
```
Int[(-Coth[x] + Csch[x])^2,x]
```

output
```
x - (2*Sinh[x])/(1 + Cosh[x])
```

3.667.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

3.667. $\int (-\coth(x)+\text{csch}(x))^2\,dx$

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.667.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
risch	$x+\frac{4}{{\mathrm e}^{x}+1}$	11
default	$x-2\coth\left(x\right)+\frac{2}{\sinh\left(x\right)}$	13
parts	$-2\coth\left(x\right)-\frac{\ln\left(\coth\left(x\right)-1\right)}{2}+\frac{\ln\left(1+\coth\left(x\right)\right)}{2}+2\,\operatorname{csch}\left(x\right)$	24

input
```
int((-coth(x)+csch(x))^2,x,method=_RETURNVERBOSE)
```

output
```
x+4/(exp(x)+1)
```

3.667.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int(-\coth(x)+\operatorname{csch}(x))^2\,dx=\frac{x\cosh\left(x\right)+x\sinh\left(x\right)+x+4}{\cosh\left(x\right)+\sinh\left(x\right)+1}$$

input
```
integrate((-coth(x)+csch(x))^2,x, algorithm="fricas")
```

output
```
(x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

3.667.6 Sympy [F]

$$\int (-\coth(x)+\operatorname{csch}(x))^2\,dx=\int (-\coth{(x)}+\operatorname{csch}{(x)})^2\,dx$$

input
```
integrate((-coth(x)+csch(x))**2,x)
```

output
```
Integral((-coth(x) + csch(x))**2, x)
```

3.667.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int (-\coth(x)+\operatorname{csch}(x))^2\,dx=x-\frac{4}{e^{(-x)}-e^x}+\frac{4}{e^{(-2x)}-1}$$

input
```
integrate((-coth(x)+csch(x))^2,x, algorithm="maxima")
```

output
```
x - 4/(e^(-x) - e^x) + 4/(e^(-2*x) - 1)
```

3.667.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (-\coth(x)+\operatorname{csch}(x))^2\,dx=x+\frac{4}{e^x+1}$$

input
```
integrate((-coth(x)+csch(x))^2,x, algorithm="giac")
```

output
```
x + 4/(e^x + 1)
```

3.667.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (-\coth(x) + \operatorname{csch}(x))^2 \, dx = x + \frac{4}{e^x + 1}$$

input
```
int((coth(x) - 1/sinh(x))^2,x)
```

output
```
x + 4/(exp(x) + 1)
```

3.668 $\int (-\coth(x) + \operatorname{csch}(x))\,dx$

3.668.1 Optimal result

Integrand size = 7, antiderivative size = 11

$$\int (-\coth(x) + \operatorname{csch}(x))\,dx = -\operatorname{arctanh}(\cosh(x)) - \log(\sinh(x))$$

output
```
-arctanh(cosh(x))-ln(sinh(x))
```

3.668.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int (-\coth(x) + \operatorname{csch}(x))\,dx = -\log\left(\cosh\left(\frac{x}{2}\right)\right) - \log(\cosh(x)) + \log\left(\sinh\left(\frac{x}{2}\right)\right) - \log(\tanh(x))$$

input
```
Integrate[-Coth[x] + Csch[x],x]
```

output
```
-Log[Cosh[x/2]] - Log[Cosh[x]] + Log[Sinh[x/2]] - Log[Tanh[x]]
```

3.668.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\operatorname{csch}(x)-\coth(x))\,dx$$

$$\Big\downarrow \ 2009$$

$$-\operatorname{arctanh}(\cosh(x))-\log(\sinh(x))$$

input
```
Int[-Coth[x] + Csch[x],x]
```

output
```
-ArcTanh[Cosh[x]] - Log[Sinh[x]]
```

3.668.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.668.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$-\ln\left(\sinh\left(x\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	12
parts	$-\ln\left(\sinh\left(x\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	12
parallelrisch	$x-\ln\left(\tanh\left(x\right)\right)+\ln\left(\coth\left(x\right)-\operatorname{csch}\left(x\right)\right)+\ln\left(1-\tanh\left(x\right)\right)$	23
risch	$x-\ln\left(e^{2x}-1\right)+\ln\left(e^{x}-1\right)-\ln\left(e^{x}+1\right)$	24

input
```
int(-coth(x)+csch(x),x,method=_RETURNVERBOSE)
```

output
```
-ln(sinh(x))+ln(tanh(1/2*x))
```

3.668.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (-\coth(x) + \operatorname{csch}(x))\,dx = x - 2\log(\cosh(x) + \sinh(x) + 1)$$

input
```
integrate(-coth(x)+csch(x),x, algorithm="fricas")
```

output
```
x - 2*log(cosh(x) + sinh(x) + 1)
```

3.668.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-\coth(x) + \operatorname{csch}(x))\,dx = -x + \log(\tanh(x) + 1) + \log\left(\tanh\left(\frac{x}{2}\right)\right) - \log(\tanh(x))$$

input
```
integrate(-coth(x)+csch(x),x)
```

output
```
-x + log(tanh(x) + 1) + log(tanh(x/2)) - log(tanh(x))
```

3.668.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (-\coth(x) + \operatorname{csch}(x))\,dx = -\log(\sinh(x)) + \log\left(\tanh\left(\frac{1}{2}x\right)\right)$$

input
```
integrate(-coth(x)+csch(x),x, algorithm="maxima")
```

output
```
-log(sinh(x)) + log(tanh(1/2*x))
```

3.668.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int (-\coth(x) + \operatorname{csch}(x))\,dx = x - \log\left(e^x + 1\right) - \log\left(\left|e^{(2x)} - 1\right|\right) + \log\left(\left|e^x - 1\right|\right)$$

input
```
integrate(-coth(x)+csch(x),x, algorithm="giac")
```

output
```
x - log(e^x + 1) - log(abs(e^(2*x) - 1)) + log(abs(e^x - 1))
```

3.668.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-\coth(x) + \operatorname{csch}(x))\,dx = x - 2\,\ln\left(\mathrm{e}^x + 1\right)$$

input
```
int(1/sinh(x) - coth(x),x)
```

output
```
x - 2*log(exp(x) + 1)
```

3.669 $\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx$

3.669.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = -\log(1-\cosh(x))$$

output
```
-ln(1-cosh(x))
```

3.669.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = -2\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(-Coth[x] + Csch[x])^(-1),x]
```

output
```
-2*Log[Sinh[x/2]]
```

3.669.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {3042, 3639, 26, 26, 3042, 26, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{csch}(x)-\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{i\csc(ix)-i\cot(ix)}dx$$

$$\Big\downarrow \text{3639}$$

$$\int \frac{i\sinh(x)}{i-i\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\sinh(x)}{1-\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sinh(x)}{1-\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{1+\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{\sin\left(ix-\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1}{1-\cosh(x)}d(-\cosh(x))$$

$$\Big\downarrow \text{16}$$

$$-\log(1-\cosh(x))$$

input
```
Int[(-Coth[x] + Csch[x])^(-1),x]
```

output
```
-Log[1 - Cosh[x]]
```

3.669.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3639
```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.669.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
risch	$x-2\ln\left(\mathrm{e}^{x}-1\right)$	10
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-2\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	23

input
```
int(1/(-coth(x)+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
x-2*ln(exp(x)-1)
```

3.669.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = x - 2\,\log\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right)$$

input
```
integrate(1/(-coth(x)+csch(x)),x, algorithm="fricas")
```

output
```
x - 2*log(cosh(x) + sinh(x) - 1)
```

3.669.6 Sympy [F]

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = -\int \frac{1}{\coth\left(x\right) - \operatorname{csch}\left(x\right)}\,dx$$

input
```
integrate(1/(-coth(x)+csch(x)),x)
```

output
```
-Integral(1/(coth(x) - csch(x)), x)
```

3.669.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = -x - 2\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(1/(-coth(x)+csch(x)),x, algorithm="maxima")
```

output
```
-x - 2*log(e^(-x) - 1)
```

3.669.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = x - 2\,\log\left(|e^{x}-1|\right)$$

input
```
integrate(1/(-coth(x)+csch(x)),x, algorithm="giac")
```

output
```
x - 2*log(abs(e^x - 1))
```

3.669.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = x - 2\,\ln\left(\mathrm{e}^{x}-1\right)$$

input
```
int(-1/(coth(x) - 1/sinh(x)),x)
```

output
```
x - 2*log(exp(x) - 1)
```

3.670 $\int \frac{1}{(-\coth(x)+\mathrm{csch}(x))^2}\,dx$

3.670.1 Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{(-\coth(x)+\mathrm{csch}(x))^2}\,dx = x+\frac{2\sinh(x)}{1-\cosh(x)}$$

output
```
x+2*sinh(x)/(1-cosh(x))
```

3.670.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{(-\coth(x)+\mathrm{csch}(x))^2}\,dx = -2\coth\left(\frac{x}{2}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(-Coth[x] + Csch[x])^(-2),x]
```

output
```
-2*Coth[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x/2]^2]
```

3.670.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {3042, 4892, 25, 25, 3042, 25, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\operatorname{csch}(x)-\coth(x))^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(i\csc(ix)-i\cot(ix))^2}dx$$
$$\Big\downarrow \text{ 4892}$$
$$\int -\frac{\sinh^2(x)}{(i-i\cosh(x))^2}dx$$
$$\Big\downarrow \text{ 25}$$
$$-\int -\frac{\sinh^2(x)}{(1-\cosh(x))^2}dx$$
$$\Big\downarrow \text{ 25}$$
$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$
$$\Big\downarrow \text{ 25}$$
$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}dx$$
$$\Big\downarrow \text{ 3159}$$
$$\int 1dx+\frac{2\sinh(x)}{1-\cosh(x)}$$
$$\Big\downarrow \text{ 24}$$
$$x+\frac{2\sinh(x)}{1-\cosh(x)}$$

input
```
Int[(-Coth[x] + Csch[x])^(-2),x]
```

output
```
x + (2*Sinh[x])/(1 - Cosh[x])
```

3.670.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.670.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$x - \frac{4}{e^{x}-1}$	11
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)}$	26

input
```
int(1/(-coth(x)+csch(x))^2,x,method=_RETURNVERBOSE)
```

output
```
x-4/(exp(x)-1)
```

3.670.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx=\frac{x\cosh{(x)}+x\sinh{(x)}-x-4}{\cosh{(x)}+\sinh{(x)}-1}$$

input
```
integrate(1/(-coth(x)+csch(x))^2,x, algorithm="fricas")
```

output
```
(x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

3.670.6 Sympy [F]

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx=\int \frac{1}{\left(-\coth{(x)}+\operatorname{csch}{(x)}\right)^2}\,dx$$

input
```
integrate(1/(-coth(x)+csch(x))**2,x)
```

output
```
Integral((-coth(x) + csch(x))**(-2), x)
```

3.670.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx=x+\frac{4}{e^{(-x)}-1}$$

input
```
integrate(1/(-coth(x)+csch(x))^2,x, algorithm="maxima")
```

output
```
x + 4/(e^(-x) - 1)
```

3.670.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx = x - \frac{4}{e^x - 1}$$

input
```
integrate(1/(-coth(x)+csch(x))^2,x, algorithm="giac")
```

output
```
x - 4/(e^x - 1)
```

3.670.9 Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx = x - \frac{4}{\mathrm{e}^x - 1}$$

input
```
int(1/(coth(x) - 1/sinh(x))^2,x)
```

output
```
x - 4/(exp(x) - 1)
```

3.671 $\int \frac{1}{(-\coth(x)+\text{csch}(x))^3} dx$

3.671.1 Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \frac{1}{(-\coth(x)+\text{csch}(x))^3} dx = -\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

output
```
-2/(1-cosh(x))-ln(1-cosh(x))
```

3.671.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(-\coth(x)+\text{csch}(x))^3} dx = \coth^2\left(\frac{x}{2}\right) - 2\log\left(\cosh\left(\frac{x}{2}\right)\right) - 2\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(-Coth[x] + Csch[x])^(-3),x]
```

output
```
Coth[x/2]^2 - 2*Log[Cosh[x/2]] - 2*Log[Tanh[x/2]]
```

3.671.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4892, 26, 26, 3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\operatorname{csch}(x)-\coth(x))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(i\csc(ix)-i\cot(ix))^3}dx$$
$$\Big\downarrow \text{4892}$$
$$\int -\frac{i\sinh^3(x)}{(i-i\cosh(x))^3}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{i\sinh^3(x)}{(1-\cosh(x))^3}dx$$
$$\Big\downarrow \text{26}$$
$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^3}dx$$
$$\Big\downarrow \text{3146}$$
$$\int \frac{\cosh(x)+1}{(1-\cosh(x))^2}d(-\cosh(x))$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{2}{(1-\cosh(x))^2}+\frac{1}{\cosh(x)-1}\right)d(-\cosh(x))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{1-\cosh(x)}-\log(1-\cosh(x))$$

input
```
Int[(-Coth[x] + Csch[x])^(-3),x]
```

output
```
-2/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

3.671.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.671. $\int \frac{1}{(-\coth(x)+\text{csch}(x))^3}\,dx$

3.671.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
risch	$x+\frac{4\,\mathrm{e}^{x}}{\left(\mathrm{e}^{x}-1\right)^{2}}-2\ln\left(\mathrm{e}^{x}-1\right)$	20
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)^{2}}-2\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	29

input
```
int(1/(-coth(x)+csch(x))^3,x,method=_RETURNVERBOSE)
```

output
```
x+4*exp(x)/(exp(x)-1)^2-2*ln(exp(x)-1)
```

3.671.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.50

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3}\,dx = \frac{x\cosh\left(x\right)^{2}+x\sinh\left(x\right)^{2}-2\left(x-2\right)\cosh\left(x\right)-2\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-2\cos}{\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)}$$

input
```
integrate(1/(-coth(x)+csch(x))^3,x, algorithm="fricas")
```

output
```
(x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
 - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*
(x*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + si
nh(x)^2 - 2*cosh(x) + 1)
```

3.671.6 Sympy [F]

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3}\,dx = -\int \frac{1}{\coth^3(x) - 3\coth^2(x)\operatorname{csch}(x) + 3\coth(x)\operatorname{csch}^2(x) - \operatorname{csch}^3(x)}\,dx$$

input
```
integrate(1/(-coth(x)+csch(x))**3,x)
```

output
```
-Integral(1/(coth(x)**3 - 3*coth(x)**2*csch(x) + 3*coth(x)*csch(x)**2 - cs
ch(x)**3), x)
```

3.671.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3}\,dx = -x - \frac{4\,e^{(-x)}}{2\,e^{(-x)} - e^{(-2\,x)} - 1} - 2\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(1/(-coth(x)+csch(x))^3,x, algorithm="maxima")
```

output
```
-x - 4*e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 2*log(e^(-x) - 1)
```

3.671.8 Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3}\,dx = x + \frac{4\,e^x}{(e^x-1)^2} - 2\,\log\left(|e^x - 1|\right)$$

input
```
integrate(1/(-coth(x)+csch(x))^3,x, algorithm="giac")
```

output
```
x + 4*e^x/(e^x - 1)^2 - 2*log(abs(e^x - 1))
```

3.671.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3}\,dx = x - 2\,\ln\left(e^x - 1\right) + \frac{4}{e^{2x} - 2\,e^x + 1} + \frac{4}{e^x - 1}$$

input
```
int(-1/(coth(x) - 1/sinh(x))^3,x)
```

output
```
x - 2*log(exp(x) - 1) + 4/(exp(2*x) - 2*exp(x) + 1) + 4/(exp(x) - 1)
```

3.672 $\int \frac{1}{(-\coth(x)+\mathrm{csch}(x))^4}\,dx$

3.672.1 Optimal result

Integrand size = 9, antiderivative size = 30

$$\int \frac{1}{(-\coth(x)+\mathrm{csch}(x))^4}\,dx = x+\frac{2\sinh(x)}{1-\cosh(x)}+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}$$

output
```
x+2*sinh(x)/(1-cosh(x))+2/3*sinh(x)^3/(1-cosh(x))^3
```

3.672.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(-\coth(x)+\mathrm{csch}(x))^4}\,dx = -\frac{2}{3}\coth^3\left(\frac{x}{2}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(-Coth[x] + Csch[x])^(-4),x]
```

output
```
(-2*Coth[x/2]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[x/2]^2])/3
```

3.672.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {3042, 4892, 3042, 3159, 3042, 25, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\operatorname{csch}(x)-\coth(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i\csc(ix)-i\cot(ix))^4}dx$$

$$\Big\downarrow \text{4892}$$

$$\int \frac{\sinh^4(x)}{(i-i\cosh(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^4}{\left(i\sin\left(-\frac{\pi}{2}+ix\right)+i\right)^4}dx$$

$$\Big\downarrow \text{3159}$$

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}dx+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\int -\frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\frac{2\sinh(x)}{1-\cosh(x)}$$

$$\Big\downarrow \text{24}$$

$$x+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\frac{2\sinh(x)}{1-\cosh(x)}$$

input
```
Int[(-Coth[x] + Csch[x])^(-4),x]
```

output
```
x + (2*Sinh[x])/(1 - Cosh[x]) + (2*Sinh[x]^3)/(3*(1 - Cosh[x])^3)
```

3.672.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.672.4 Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
risch	$x-\frac{8\left(3\,{\mathrm e}^{2 x}-3\,{\mathrm e}^{x}+2\right)}{3\left({\mathrm e}^{x}-1\right)^{3}}$	23
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{2}{3\tanh\left(\frac{x}{2}\right)^{3}}-\frac{2}{\tanh\left(\frac{x}{2}\right)}+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	34

input
```
int(1/(-coth(x)+csch(x))^4,x,method=_RETURNVERBOSE)
```

output
```
x-8/3*(3*exp(2*x)-3*exp(x)+2)/(exp(x)-1)^3
```

3.672.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx$$
$$= \frac{3\,x\cosh\left(x\right)^{2} + 3\,x\sinh\left(x\right)^{2} - 4\,(3\,x + 10)\cosh\left(x\right) + 2\,(3\,x\cosh\left(x\right) - 3\,x - 4)\sinh\left(x\right) + 9\,x + 24}{3\,\left(\cosh\left(x\right)^{2} + 2\,(\cosh\left(x\right) - 1)\sinh\left(x\right) + \sinh\left(x\right)^{2} - 4\,\cosh\left(x\right) + 3\right)}$$

input
```
integrate(1/(-coth(x)+csch(x))^4,x, algorithm="fricas")
```

output
```
1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 - 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
 - 3*x - 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sin
h(x)^2 - 4*cosh(x) + 3)
```

3.672.6 Sympy [F]

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx = \int \frac{1}{\left(-\coth\left(x\right) + \operatorname{csch}\left(x\right)\right)^{4}}\,dx$$

input
```
integrate(1/(-coth(x)+csch(x))**4,x)
```

output
```
Integral((-coth(x) + csch(x))**(-4), x)
```

3.672.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx = x - \frac{8\left(3\,e^{(-x)} - 3\,e^{(-2\,x)} - 2\right)}{3\left(3\,e^{(-x)} - 3\,e^{(-2\,x)} + e^{(-3\,x)} - 1\right)}$$

input
```
integrate(1/(-coth(x)+csch(x))^4,x, algorithm="maxima")
```

output
```
x - 8/3*(3*e^(-x) - 3*e^(-2*x) - 2)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1)
```

3.672.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx = x - \frac{8\left(3\,e^{(2\,x)} - 3\,e^{x} + 2\right)}{3\,(e^{x}-1)^3}$$

input
```
integrate(1/(-coth(x)+csch(x))^4,x, algorithm="giac")
```

output
```
x - 8/3*(3*e^(2*x) - 3*e^x + 2)/(e^x - 1)^3
```

3.672.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx = x - \frac{8\,e^{x}}{3\,\left(e^{2\,x} - 2\,e^{x} + 1\right)} + \frac{\frac{8\,e^{2\,x}}{3} + \frac{8}{3}}{3\,e^{2\,x} - e^{3\,x} - 3\,e^{x} + 1} - \frac{8}{3\,\left(e^{x} - 1\right)}$$

input
```
int(1/(coth(x) - 1/sinh(x))^4,x)
```

output
```
x - (8*exp(x))/(3*(exp(2*x) - 2*exp(x) + 1)) + ((8*exp(2*x))/3 + 8/3)/(3*e
xp(2*x) - exp(3*x) - 3*exp(x) + 1) - 8/(3*(exp(x) - 1))
```

3.673 $\int \frac{1}{(-\coth(x)+\mathbf{csch}(x))^5}\,dx$

3.673.1 Optimal result

Integrand size = 9, antiderivative size = 30

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = \frac{2}{(1-\cosh(x))^2} - \frac{4}{1-\cosh(x)} - \log(1-\cosh(x))$$

output
```
2/(1-cosh(x))^2-4/(1-cosh(x))-ln(1-cosh(x))
```

3.673.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = \coth^2\left(\frac{x}{2}\right) + \frac{1}{2}\coth^4\left(\frac{x}{2}\right) - 2\log\left(\cosh\left(\frac{x}{2}\right)\right) - 2\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(-Coth[x] + Csch[x])^(-5),x]
```

output
```
Coth[x/2]^2 + Coth[x/2]^4/2 - 2*Log[Cosh[x/2]] - 2*Log[Tanh[x/2]]
```

3.673.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4892, 26, 26, 3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\operatorname{csch}(x)-\coth(x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i\csc(ix)-i\cot(ix))^5}dx$$

$$\Big\downarrow \text{4892}$$

$$\int \frac{i\sinh^5(x)}{(i-i\cosh(x))^5}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\sinh^5(x)}{(1-\cosh(x))^5}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sinh^5(x)}{(1-\cosh(x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)^5}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^5}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^5}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^5}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{(\cosh(x)+1)^2}{(1-\cosh(x))^3}d(-\cosh(x))$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{1}{1-\cosh(x)}-\frac{4}{(1-\cosh(x))^2}+\frac{4}{(1-\cosh(x))^3}\right)d(-\cosh(x))$$

$$\downarrow \text{2009}$$

$$-\frac{4}{1-\cosh(x)}+\frac{2}{(1-\cosh(x))^2}-\log(1-\cosh(x))$$

input
```
Int[(-Coth[x] + Csch[x])^(-5),x]
```

output
```
2/(1 - Cosh[x])^2 - 4/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

3.673.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.673. $\int \frac{1}{(-\coth(x)+\text{csch}(x))^5}\,dx$

3.673.4 Maple [A] (verified)

Time = 10.36 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.07

method	result	size
parallelrisch	0	2
risch	$x+\frac{8\,{\mathrm e}^{x} \left({\mathrm e}^{2 x}-{\mathrm e}^{x}+1\right)}{\left({\mathrm e}^{x}-1\right)^{4}}-2 \ln \left({\mathrm e}^{x}-1\right)$	30
default	$\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)+\frac{1}{2 \tanh \left(\frac{x}{2}\right)^{4}}+\frac{1}{\tanh \left(\frac{x}{2}\right)^{2}}-2 \ln \left(\tanh \left(\frac{x}{2}\right)\right)+\ln \left(\tanh \left(\frac{x}{2}\right)+1\right)$	37

input
```
int(1/(-coth(x)+csch(x))^5,x,method=_RETURNVERBOSE)
```

output
```
0
```

3.673.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 8.97

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx$$
$$=\frac{x\cosh\left(x\right)^{4}+x\sinh\left(x\right)^{4}-4\left(x-2\right)\cosh\left(x\right)^{3}+4\left(x\cosh\left(x\right)-x+2\right)\sinh\left(x\right)^{3}+2\left(3\,x-4\right)\cosh\left(x\right)^{2}+\cdots}{\cdots}$$

input
```
integrate(1/(-coth(x)+csch(x))^5,x, algorithm="fricas")
```

output
```
(x*cosh(x)^4 + x*sinh(x)^4 - 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) - x + 2)*s
inh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 - 6*(x - 2)*cosh(x) +
3*x - 4)*sinh(x)^2 - 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) - 1)*si
nh(x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^
2 + 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*
cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 4*(x*cosh(x)^3 - 3*(x - 2)*cosh(
x)^2 + (3*x - 4)*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) - 1
)*sinh(x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh
(x)^2 + 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x)
- 4*cosh(x) + 1)
```

3.673.6 Sympy [F]

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = -\int \frac{1}{\coth^5(x) - 5\coth^4(x)\operatorname{csch}(x) + 10\coth^3(x)\operatorname{csch}^2(x) - 10\coth^2(x)\operatorname{csch}^3(x) + 5\coth(x)\operatorname{csch}^4(x)$$

input
```
integrate(1/(-coth(x)+csch(x))**5,x)
```

output
```
-Integral(1/(coth(x)**5 - 5*coth(x)**4*csch(x) + 10*coth(x)**3*csch(x)**2
- 10*coth(x)**2*csch(x)**3 + 5*coth(x)*csch(x)**4 - csch(x)**5), x)
```

3.673.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = -x - \frac{8\left(e^{(-x)} - e^{(-2x)} + e^{(-3x)}\right)}{4e^{(-x)} - 6e^{(-2x)} + 4e^{(-3x)} - e^{(-4x)} - 1} - 2\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(1/(-coth(x)+csch(x))^5,x, algorithm="maxima")
```

output
```
-x - 8*(e^(-x) - e^(-2*x) + e^(-3*x))/(4*e^(-x) - 6*e^(-2*x) + 4*e^(-3*x)
- e^(-4*x) - 1) - 2*log(e^(-x) - 1)
```

3.673.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = x + \frac{8\left(e^{(3x)} - e^{(2x)} + e^{x}\right)}{(e^{x} - 1)^4} - 2\log\left(|e^{x} - 1|\right)$$

input
```
integrate(1/(-coth(x)+csch(x))^5,x, algorithm="giac")
```

output
```
x + 8*(e^(3*x) - e^(2*x) + e^x)/(e^x - 1)^4 - 2*log(abs(e^x - 1))
```

3.673.9 Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.63

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = x-2\,\ln\left(e^x-1\right)-\frac{16}{3\,e^{2\,x}-e^{3\,x}-3\,e^x+1}+\frac{16}{e^{2\,x}-2\,e^x+1}+\frac{8}{6\,e^{2\,x}-4\,e^{3\,x}+e^{4\,x}-4\,e^x+1}+\frac{8}{e^x-1}$$

input
```
int(-1/(coth(x) - 1/sinh(x))^5,x)
```

output
```
x - 2*log(exp(x) - 1) - 16/(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1) + 16/(ex
p(2*x) - 2*exp(x) + 1) + 8/(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x)
+ 1) + 8/(exp(x) - 1)
```

3.674 $\int (\operatorname{csch}(x)+\sinh(x))\,dx$

3.674.1 Optimal result

Integrand size = 5, antiderivative size = 8

$$\int (\operatorname{csch}(x)+\sinh(x))\,dx=-\operatorname{arctanh}(\cosh(x))+\cosh(x)$$

output
```
-arctanh(cosh(x))+cosh(x)
```

3.674.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int (\operatorname{csch}(x)+\sinh(x))\,dx=\cosh(x)-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Csch[x] + Sinh[x],x]
```

output
```
Cosh[x] - Log[Cosh[x/2]] + Log[Sinh[x/2]]
```

3.674.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sinh(x) + \operatorname{csch}(x))\,dx$$
$$\Big\downarrow \text{2009}$$
$$\cosh(x) - \operatorname{arctanh}(\cosh(x))$$

input
```
Int[Csch[x] + Sinh[x],x]
```

output
```
-ArcTanh[Cosh[x]] + Cosh[x]
```

3.674.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.674.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\cosh\left(x\right)$	9
parts	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\cosh\left(x\right)$	9
parallelrisch	$\cosh\left(x\right)+\ln\left(\coth\left(x\right)-\operatorname{csch}\left(x\right)\right)+1$	13
risch	$\ln\left({\mathrm e}^{x}-1\right)-\ln\left({\mathrm e}^{x}+1\right)+\frac{{\mathrm e}^{x}}{2}+\frac{{\mathrm e}^{-x}}{2}$	24

input
```
int(csch(x)+sinh(x),x,method=_RETURNVERBOSE)
```

output
```
ln(tanh(1/2*x))+cosh(x)
```

3.674.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 6.62

$$\int (\operatorname{csch}(x)+\sinh(x))\,dx$$
$$= \frac{\cosh(x)^2 - 2(\cosh(x)+\sinh(x))\log(\cosh(x)+\sinh(x)+1) + 2(\cosh(x)+\sinh(x))\log(\cosh(x)+\text{si}}{2(\cosh(x)+\sinh(x))}$$

input
```
integrate(csch(x)+sinh(x),x, algorithm="fricas")
```

output
```
1/2*(cosh(x)^2 - 2*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) + 2*(cos
h(x) + sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*cosh(x)*sinh(x) + sinh(x)^2
 + 1)/(cosh(x) + sinh(x))
```

3.674.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (\operatorname{csch}(x)+\sinh(x))\,dx = \log\left(\tanh\left(\frac{x}{2}\right)\right) + \cosh(x)$$

input
```
integrate(csch(x)+sinh(x),x)
```

output
```
log(tanh(x/2)) + cosh(x)
```

3.674.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (\operatorname{csch}(x)+\sinh(x))\,dx = \cosh(x) + \log\left(\tanh\left(\frac{1}{2}\,x\right)\right)$$

input
```
integrate(csch(x)+sinh(x),x, algorithm="maxima")
```

output
```
cosh(x) + log(tanh(1/2*x))
```

3.674.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 3.00

$$\int (\operatorname{csch}(x) + \sinh(x))\, dx = \frac{1}{2}\, e^{(-x)} + \frac{1}{2}\, e^{x} - \log\left(e^{x} + 1\right) + \log\left(\left|e^{x} - 1\right|\right)$$

input
```
integrate(csch(x)+sinh(x),x, algorithm="giac")
```

output
```
1/2*e^(-x) + 1/2*e^x - log(e^x + 1) + log(abs(e^x - 1))
```

3.674.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int (\operatorname{csch}(x) + \sinh(x))\, dx = \ln\left(2 - 2\,\mathrm{e}^{x}\right) - \ln\left(-2\,\mathrm{e}^{x} - 2\right) + \frac{\mathrm{e}^{-x}}{2} + \frac{\mathrm{e}^{x}}{2}$$

input
```
int(sinh(x) + 1/sinh(x),x)
```

output
```
log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + exp(-x)/2 + exp(x)/2
```

3.675 $\int (\operatorname{csch}(x)+\sinh(x))^2\,dx$

3.675.1 Optimal result

Integrand size = 7, antiderivative size = 22

$$\int (\operatorname{csch}(x)+\sinh(x))^2\,dx=\frac{3x}{2}-\frac{3\coth(x)}{2}+\frac{1}{2}\cosh^2(x)\coth(x)$$

output
```
3/2*x-3/2*coth(x)+1/2*cosh(x)^2*coth(x)
```

3.675.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (\operatorname{csch}(x)+\sinh(x))^2\,dx=\frac{3x}{2}-\coth(x)+\frac{1}{4}\sinh(2x)$$

input
```
Integrate[(Csch[x] + Sinh[x])^2,x]
```

output
```
(3*x)/2 - Coth[x] + Sinh[2*x]/4
```

3.675.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4889, 253, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sinh(x)+\text{csch}(x))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (i\csc(ix)-i\sin(ix))^2dx$$

$$\Big\downarrow \ 4889$$

$$\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)^2}d\tanh(x)$$

$$\Big\downarrow \ 253$$

$$\frac{3}{2}\int \frac{\coth^2(x)}{1-\tanh^2(x)}d\tanh(x)+\frac{\coth(x)}{2\left(1-\tanh^2(x)\right)}$$

$$\Big\downarrow \ 264$$

$$\frac{3}{2}\left(\int \frac{1}{1-\tanh^2(x)}d\tanh(x)-\coth(x)\right)+\frac{\coth(x)}{2\left(1-\tanh^2(x)\right)}$$

$$\Big\downarrow \ 219$$

$$\frac{3}{2}(\text{arctanh}(\tanh(x))-\coth(x))+\frac{\coth(x)}{2\left(1-\tanh^2(x)\right)}$$

input
```
Int[(Csch[x] + Sinh[x])^2,x]
```

output
```
(3*(ArcTanh[Tanh[x]] - Coth[x]))/2 + Coth[x]/(2*(1 - Tanh[x]^2))
```

3.675.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.675.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

method	result	size
default	$-\coth\left(x\right)+\frac{3x}{2}+\frac{\cosh\left(x\right)\sinh\left(x\right)}{2}$	15
parts	$-\coth\left(x\right)+\frac{3x}{2}+\frac{\cosh\left(x\right)\sinh\left(x\right)}{2}$	15
parallelrisch	$\frac{\coth\left(x\right)\cosh\left(2x\right)}{4}-\frac{5\coth\left(x\right)}{4}+\frac{3x}{2}$	17
risch	$\frac{3x}{2}+\frac{\mathrm{e}^{2x}}{8}-\frac{\mathrm{e}^{-2x}}{8}-\frac{2}{\mathrm{e}^{2x}-1}$	27

input
```
int((csch(x)+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-coth(x)+3/2*x+1/2*cosh(x)*sinh(x)
```

3.675.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int (\operatorname{csch}(x)+\sinh(x))^2\,dx = \frac{\cosh{(x)}^3 + 3\,\cosh{(x)}\sinh{(x)}^2 + 4\,(3\,x+2)\sinh{(x)} - 9\,\cosh{(x)}}{8\,\sinh{(x)}}$$

input
```
integrate((csch(x)+sinh(x))^2,x, algorithm="fricas")
```

output
```
1/8*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 4*(3*x + 2)*sinh(x) - 9*cosh(x))/si
nh(x)
```

3.675.6 Sympy [F]

$$\int (\operatorname{csch}(x)+\sinh(x))^2\,dx = \int \left(\sinh{(x)} + \operatorname{csch}{(x)}\right)^2\,dx$$

input
```
integrate((csch(x)+sinh(x))**2,x)
```

output
```
Integral((sinh(x) + csch(x))**2, x)
```

3.675.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (\operatorname{csch}(x)+\sinh(x))^2\,dx = \frac{3}{2}\,x + \frac{2}{e^{(-2\,x)} - 1} + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{8}\,e^{(-2\,x)}$$

input
```
integrate((csch(x)+sinh(x))^2,x, algorithm="maxima")
```

output
```
3/2*x + 2/(e^(-2*x) - 1) + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

3.675.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int (\operatorname{csch}(x)+\sinh(x))^2\,dx = \frac{3}{2}\,x - \frac{3\,e^{(4\,x)} + 14\,e^{(2\,x)} - 1}{8\,\left(e^{(4\,x)} - e^{(2\,x)}\right)} + \frac{1}{8}\,e^{(2\,x)}$$

input
```
integrate((csch(x)+sinh(x))^2,x, algorithm="giac")
```

output
```
3/2*x - 1/8*(3*e^(4*x) + 14*e^(2*x) - 1)/(e^(4*x) - e^(2*x)) + 1/8*e^(2*x)
```

3.675.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (\operatorname{csch}(x)+\sinh(x))^2\,dx = \frac{3\,x}{2} - \frac{\mathrm{e}^{-2\,x}}{8} + \frac{\mathrm{e}^{2\,x}}{8} - \frac{2}{\mathrm{e}^{2\,x}-1}$$

input
```
int((sinh(x) + 1/sinh(x))^2,x)
```

output
```
(3*x)/2 - exp(-2*x)/8 + exp(2*x)/8 - 2/(exp(2*x) - 1)
```

3.676 $\int (\operatorname{csch}(x)+\sinh(x))^3\,dx$

3.676.1 Optimal result

Integrand size = 7, antiderivative size = 34

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx = -\frac{5}{2}\operatorname{arctanh}(\cosh(x))+\frac{5\cosh(x)}{2}+\frac{5\cosh^3(x)}{6}-\frac{1}{2}\cosh^3(x)\coth^2(x)$$

output
```
-5/2*arctanh(cosh(x))+5/2*cosh(x)+5/6*cosh(x)^3-1/2*cosh(x)^3*coth(x)^2
```

3.676.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx = \frac{1}{48}\operatorname{csch}^2(x)\left(-50\cosh(x)+25\cosh(3x)+\cosh(5x)+60\log\left(\cosh\left(\frac{x}{2}\right)\right)-60\cosh(2x)\log\left(\cosh\left(\frac{x}{2}\right)\right)-60\log\left(\sinh\left(\frac{x}{2}\right)\right)+60\cosh(2x)\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[(Csch[x] + Sinh[x])^3,x]
```

output
```
(Csch[x]^2*(-50*Cosh[x] + 25*Cosh[3*x] + Cosh[5*x] + 60*Log[Cosh[x/2]] - 6
0*Cosh[2*x]*Log[Cosh[x/2]] - 60*Log[Sinh[x/2]] + 60*Cosh[2*x]*Log[Sinh[x/2
]]))/48
```

3.676.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3042, 4897, 3042, 26, 3072, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sinh(x)+\operatorname{csch}(x))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\csc(ix)-i\sin(ix))^3dx$$

$$\Big\downarrow \text{4897}$$

$$\int \cosh^3(x)\coth^3(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin\left(\frac{\pi}{2}+ix\right)^3\tan\left(\frac{\pi}{2}+ix\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin\left(ix+\frac{\pi}{2}\right)^3\tan\left(ix+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3072}$$

$$\int \frac{\cosh^6(x)}{(1-\cosh^2(x))^2}d\cosh(x)$$

$$\Big\downarrow \text{252}$$

$$\frac{\cosh^5(x)}{2(1-\cosh^2(x))}-\frac{5}{2}\int \frac{\cosh^4(x)}{1-\cosh^2(x)}d\cosh(x)$$

$$\Big\downarrow \text{254}$$

$$\frac{\cosh^5(x)}{2(1-\cosh^2(x))}-\frac{5}{2}\int \left(-\cosh^2(x)+\frac{1}{1-\cosh^2(x)}-1\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh^5(x)}{2(1-\cosh^2(x))}-\frac{5}{2}\left(\operatorname{arctanh}(\cosh(x))-\frac{1}{3}\cosh^3(x)-\cosh(x)\right)$$

input
```
Int[(Csch[x] + Sinh[x])^3,x]
```

output
```
Cosh[x]^5/(2*(1 - Cosh[x]^2)) - (5*(ArcTanh[Cosh[x]] - Cosh[x] - Cosh[x]^3
/3))/2
```

3.676.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.676.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\operatorname{csch}(x)\coth(x)}{2}-5\,\operatorname{arctanh}\left(\mathrm{e}^{x}\right)+3\cosh\left(x\right)+\left(-\frac{2}{3}+\frac{\sinh(x)^{2}}{3}\right)\cosh\left(x\right)$	28
parts	$-\frac{\operatorname{csch}(x)\coth(x)}{2}-5\,\operatorname{arctanh}\left(\mathrm{e}^{x}\right)+3\cosh\left(x\right)+\left(-\frac{2}{3}+\frac{\sinh(x)^{2}}{3}\right)\cosh\left(x\right)$	28
parallelrisch	$\frac{5}{12}+\frac{5\ln(\coth(x)-\operatorname{csch}(x))}{2}+\frac{\cosh(3x)}{12}+\frac{9\cosh(x)}{4}-\frac{\operatorname{csch}(x)\coth(x)}{2}$	29
risch	$\frac{\mathrm{e}^{3x}}{24}+\frac{9\,\mathrm{e}^{x}}{8}+\frac{9\,\mathrm{e}^{-x}}{8}+\frac{\mathrm{e}^{-3x}}{24}-\frac{\mathrm{e}^{x}\left(1+\mathrm{e}^{2x}\right)}{\left(\mathrm{e}^{2x}-1\right)^{2}}-\frac{5\ln(\mathrm{e}^{x}+1)}{2}+\frac{5\ln(\mathrm{e}^{x}-1)}{2}$	56

input
```
int((csch(x)+sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*csch(x)*coth(x)-5*arctanh(exp(x))+3*cosh(x)+(-2/3+1/3*sinh(x)^2)*cosh
(x)
```

3.676.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 616, normalized size of antiderivative = 18.12

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx = \text{Too large to display}$$

input
```
integrate((csch(x)+sinh(x))^3,x, algorithm="fricas")
```

output
```
1/24*(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 + 5)
*sinh(x)^8 + 25*cosh(x)^8 + 40*(3*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 10*(2
1*cosh(x)^4 + 70*cosh(x)^2 - 5)*sinh(x)^6 - 50*cosh(x)^6 + 4*(63*cosh(x)^5
 + 350*cosh(x)^3 - 75*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 175*cosh(x)^
4 - 75*cosh(x)^2 - 5)*sinh(x)^4 - 50*cosh(x)^4 + 40*(3*cosh(x)^7 + 35*cosh
(x)^5 - 25*cosh(x)^3 - 5*cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 140*cosh(x)
^6 - 150*cosh(x)^4 - 60*cosh(x)^2 + 5)*sinh(x)^2 + 25*cosh(x)^2 - 60*(cosh
(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(x)^2 - 2)*sinh(x)^5 - 2
*cosh(x)^5 + 5*(7*cosh(x)^3 - 2*cosh(x))*sinh(x)^4 + (35*cosh(x)^4 - 20*co
sh(x)^2 + 1)*sinh(x)^3 + cosh(x)^3 + (21*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh
(x))*sinh(x)^2 + (7*cosh(x)^6 - 10*cosh(x)^4 + 3*cosh(x)^2)*sinh(x))*log(c
osh(x) + sinh(x) + 1) + 60*(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 +
(21*cosh(x)^2 - 2)*sinh(x)^5 - 2*cosh(x)^5 + 5*(7*cosh(x)^3 - 2*cosh(x))*s
inh(x)^4 + (35*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^3 + cosh(x)^3 + (21*c
osh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 - 10*cosh(x)
^4 + 3*cosh(x)^2)*sinh(x))*log(cosh(x) + sinh(x) - 1) + 10*(cosh(x)^9 + 20
*cosh(x)^7 - 30*cosh(x)^5 - 20*cosh(x)^3 + 5*cosh(x))*sinh(x) + 1)/(cosh(x
)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(x)^2 - 2)*sinh(x)^5 - 2*c
osh(x)^5 + 5*(7*cosh(x)^3 - 2*cosh(x))*sinh(x)^4 + (35*cosh(x)^4 - 20*cosh
(x)^2 + 1)*sinh(x)^3 + cosh(x)^3 + (21*cosh(x)^5 - 20*cosh(x)^3 + 3*cos...
```

3.676.6 Sympy [F]

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx=\int (\sinh{(x)}+\operatorname{csch}{(x)})^3\,dx$$

input
```
integrate((csch(x)+sinh(x))**3,x)
```

output
```
Integral((sinh(x) + csch(x))**3, x)
```

3.676.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(26) = 52$.

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx = \frac{e^{(-x)} + e^{(-3\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{1}{24}\,e^{(3\,x)} + \frac{9}{8}\,e^{(-x)} + \frac{1}{24}\,e^{(-3\,x)} + \frac{9}{8}\,e^{x} - \frac{5}{2}\,\log\left(e^{(-x)} + 1\right) + \frac{5}{2}\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate((csch(x)+sinh(x))^3,x, algorithm="maxima")
```

output
```
(e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/24*e^(3*x) + 9/8*e^(-x
) + 1/24*e^(-3*x) + 9/8*e^x - 5/2*log(e^(-x) + 1) + 5/2*log(e^(-x) - 1)
```

3.676.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.82

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx = \frac{1}{24}\left(e^{(-x)} + e^{x}\right)^3 - \frac{e^{(-x)} + e^{x}}{\left(e^{(-x)} + e^{x}\right)^2 - 4} + e^{(-x)} + e^{x} - \frac{5}{4}\,\log\left(e^{(-x)} + e^{x} + 2\right) + \frac{5}{4}\,\log\left(e^{(-x)} + e^{x} - 2\right)$$

input
```
integrate((csch(x)+sinh(x))^3,x, algorithm="giac")
```

output
```
1/24*(e^(-x) + e^x)^3 - (e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) + e^(-x) + e
^x - 5/4*log(e^(-x) + e^x + 2) + 5/4*log(e^(-x) + e^x - 2)
```

3.676.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx=\frac{5\,\ln\left(5-5\,{\mathrm{e}}^x\right)}{2}-\frac{5\,\ln\left(-5\,{\mathrm{e}}^x-5\right)}{2}+\frac{9\,{\mathrm{e}}^{-x}}{8}+\frac{{\mathrm{e}}^{-3\,x}}{24}+\frac{{\mathrm{e}}^{3\,x}}{24}+\frac{9\,{\mathrm{e}}^x}{8}-\frac{{\mathrm{e}}^x}{{\mathrm{e}}^{2\,x}-1}-\frac{2\,{\mathrm{e}}^x}{{\mathrm{e}}^{4\,x}-2\,{\mathrm{e}}^{2\,x}+1}$$

input
```
int((sinh(x) + 1/sinh(x))^3,x)
```

output
```
(5*log(5 - 5*exp(x)))/2 - (5*log(- 5*exp(x) - 5))/2 + (9*exp(-x))/8 + exp(
-3*x)/24 + exp(3*x)/24 + (9*exp(x))/8 - exp(x)/(exp(2*x) - 1) - (2*exp(x))
/(exp(4*x) - 2*exp(2*x) + 1)
```

3.677 $\int \sqrt{\text{csch}(x)+\sinh(x)}\,dx$

3.677.1 Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{\text{csch}(x)+\sinh(x)}\,dx = 2\sqrt{\cosh(x)\coth(x)}\tanh(x)$$

output
```
2*(cosh(x)*coth(x))^(1/2)*tanh(x)
```

3.677.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int \sqrt{\text{csch}(x)+\sinh(x)}\,dx = \frac{2\sqrt{\cosh(x)\coth(x)}\left(-1+\sqrt[4]{-\sinh^2(x)}\right)\tanh(x)}{\sqrt[4]{-\sinh^2(x)}}$$

input
```
Integrate[Sqrt[Csch[x] + Sinh[x]],x]
```

output
```
(2*Sqrt[Cosh[x]*Coth[x]]*(-1 + (-Sinh[x]^2)^(1/4))*Tanh[x])/(-Sinh[x]^2)^(
1/4)
```

3.677.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {3042, 4897, 3042, 4898, 3042, 4900, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sinh(x)+\operatorname{csch}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{i\csc(ix)-i\sin(ix)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sqrt{\cosh(x)\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{i\cos(ix)\cot(ix)}dx$$

$$\Big\downarrow \text{4898}$$

$$\frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{-i\cosh(x)\coth(x)}dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{\cos(ix)\cot(ix)}dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{\cosh(x)}\sqrt{-i\coth(x)}dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)}\sqrt{-\tan\left(ix+\frac{\pi}{2}\right)}dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3069}$$

$$2\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

input
```
Int[Sqrt[Csch[x] + Sinh[x]],x]
```

output
```
2*Sqrt[Cosh[x]*Coth[x]]*Tanh[x]
```

3.677.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4898
```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Simp[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p])   In
t[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ
[v]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.677.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(11) = 22$.

Time = 0.93 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.23

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{\frac{\left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-x}}{{\mathrm e}^{2 x}-1}}\,\left({\mathrm e}^{2 x}-1\right)}{1+{\mathrm e}^{2 x}}$	42

input
```
int((csch(x)+sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*((1+exp(2*x))^2*exp(-x)/(exp(2*x)-1))^(1/2)/(1+exp(2*x))*(exp(2*x)
-1)
```

3.677.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 4.23

$$\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx$$
$$= \frac{2\sqrt{\frac{1}{2}}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)}{\sqrt{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(3\cosh(x)^2-1\right)\sinh(x)-\cosh(x)}}$$

input
```
integrate((csch(x)+sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)/sqrt(cosh(x)^3
 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))
```

3.677.6 Sympy [F]

$$\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx = \int \sqrt{\sinh(x)+\operatorname{csch}(x)}\,dx$$

input
```
integrate((csch(x)+sinh(x))**(1/2),x)
```

output
```
Integral(sqrt(sinh(x) + csch(x)), x)
```

3.677.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.15

$$\int \sqrt{\operatorname{csch}(x) + \sinh(x)}\, dx = \frac{\sqrt{2}e^{\left(\frac{1}{2}\,x\right)}}{\sqrt{e^{(-x)} + 1}\sqrt{-e^{(-x)} + 1}} - \frac{\sqrt{2}e^{\left(-\frac{3}{2}\,x\right)}}{\sqrt{e^{(-x)} + 1}\sqrt{-e^{(-x)} + 1}}$$

input
```
integrate((csch(x)+sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2)*e^(1/2*x)/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1)) - sqrt(2)*e^(-3/2*x
)/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1))
```

3.677.8 Giac [F]

$$\int \sqrt{\operatorname{csch}(x) + \sinh(x)}\, dx = \int \sqrt{\operatorname{csch}\left(x\right) + \sinh\left(x\right)}\, dx$$

input
```
integrate((csch(x)+sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(csch(x) + sinh(x)), x)
```

3.677.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{\operatorname{csch}(x) + \sinh(x)}\, dx = 2\tanh(x)\,\sqrt{\sinh\left(x\right) + \frac{1}{\sinh\left(x\right)}}$$

input
```
int((sinh(x) + 1/sinh(x))^(1/2),x)
```

output
```
2*tanh(x)*(sinh(x) + 1/sinh(x))^(1/2)
```

3.678 $\int (\operatorname{csch}(x)+\sinh(x))^{3/2}\,dx$

3.678.1 Optimal result

Integrand size = 9, antiderivative size = 31

$$\int (\operatorname{csch}(x)+\sinh(x))^{3/2}\,dx=\frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}-\frac{8}{3}\sqrt{\cosh(x)\coth(x)}\operatorname{sech}(x)$$

output
```
2/3*cosh(x)*(cosh(x)*coth(x))^(1/2)-8/3*sech(x)*(cosh(x)*coth(x))^(1/2)
```

3.678.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int (\operatorname{csch}(x)+\sinh(x))^{3/2}\,dx=\frac{2}{3}\left(-4+\cosh^2(x)\right)\sqrt{\cosh(x)\coth(x)}\operatorname{sech}(x)$$

input
```
Integrate[(Csch[x] + Sinh[x])^(3/2),x]
```

output
```
(2*(-4 + Cosh[x]^2)*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

3.678.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.39, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {3042, 4897, 3042, 4898, 3042, 4900, 3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sinh(x)+\operatorname{csch}(x))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\csc(ix)-i\sin(ix))^{3/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int (\cosh(x)\coth(x))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\cos(ix)\cot(ix))^{3/2}dx$$

$$\Big\downarrow \text{4898}$$

$$\frac{i\sqrt{\cosh(x)\coth(x)}\int (-i\cosh(x)\coth(x))^{3/2}dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\sqrt{\cosh(x)\coth(x)}\int (\cos(ix)\cot(ix))^{3/2}dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$\Big\downarrow \text{4900}$$

$$\frac{i\sqrt{\cosh(x)\coth(x)}\int \cosh^{\frac{3}{2}}(x)(-i\coth(x))^{3/2}dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\sqrt{\cosh(x)\coth(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^{3/2}\left(-\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{i\sqrt{\cosh(x)\coth(x)}\left(\frac{4}{3}\int\frac{(-i\coth(x))^{3/2}}{\sqrt{\cosh(x)}}dx-\frac{2}{3}i\cosh^{\frac{3}{2}}(x)\sqrt{-i\coth(x)}\right)}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\sqrt{\cosh(x)\coth(x)}\left(\frac{4}{3}\int\frac{\left(-\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx-\frac{2}{3}i\cosh^{\frac{3}{2}}(x)\sqrt{-i\coth(x)}\right)}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{i\left(\frac{8i\sqrt{-i\coth(x)}}{3\sqrt{\cosh(x)}}-\frac{2}{3}i\cosh^{\frac{3}{2}}(x)\sqrt{-i\coth(x)}\right)\sqrt{\cosh(x)\coth(x)}}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

input
```
Int[(Csch[x] + Sinh[x])^(3/2),x]
```

output
```
(I*((((8*I)/3)*Sqrt[(-I)*Coth[x]])/Sqrt[Cosh[x]] - ((2*I)/3)*Cosh[x]^(3/2)
*Sqrt[(-I)*Coth[x]])*Sqrt[Cosh[x]*Coth[x]])/(Sqrt[Cosh[x]]*Sqrt[(-I)*Coth[
x]])
```

3.678.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.678. $\int(\operatorname{csch}(x)+\sinh(x))^{3/2}\,dx$

rule 4898
```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Simp[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p])   In
t[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ
[v]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.678.4 Maple [F]

$$\int (\operatorname{csch}(x) + \sinh(x))^{\frac{3}{2}} dx$$

input
```
int((csch(x)+sinh(x))^(3/2),x)
```

output
```
int((csch(x)+sinh(x))^(3/2),x)
```

3.678.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.13

$$\int (\operatorname{csch}(x) + \sinh(x))^{3/2} dx = \frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 7\right)\sinh(x)^2 - 14\cosh(x)^2 + 4\left(\cosh(x)^3 - 7\cosh(x)\right)\sinh(x) + 1\right)}{3\sqrt{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 - 1\right)\sinh(x) - \cosh(x)}\left(\cosh(x) + \sinh(x)\right)}$$

input
```
integrate((csch(x)+sinh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)/(
sqrt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(
x) - cosh(x))*(cosh(x) + sinh(x)))
```

3.678.6 Sympy [F(-1)]

Timed out.

$$\int (\mathrm{csch}(x) + \sinh(x))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((csch(x)+sinh(x))**(3/2),x)
```

output
```
Timed out
```

3.678.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int (\mathrm{csch}(x)+\sinh(x))^{3/2}\,dx = \frac{\sqrt{2}e^{\left(\frac{3}{2}\,x\right)}}{6\,(e^{(-x)}+1)^{\frac{3}{2}}(-e^{(-x)}+1)^{\frac{3}{2}}} - \frac{5\,\sqrt{2}e^{\left(-\frac{1}{2}\,x\right)}}{2\,(e^{(-x)}+1)^{\frac{3}{2}}(-e^{(-x)}+1)^{\frac{3}{2}}} + \frac{5\,\sqrt{2}e^{\left(-\frac{5}{2}\,x\right)}}{2\,(e^{(-x)}+1)^{\frac{3}{2}}(-e^{(-x)}+1)^{\frac{3}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{9}{2}\,x\right)}}{6\,(e^{(-x)}+1)^{\frac{3}{2}}(-e^{(-x)}+1)^{\frac{3}{2}}}$$

input
```
integrate((csch(x)+sinh(x))^(3/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*e^(3/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 5/2*sqrt(
2)*e^(-1/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) + 5/2*sqrt(2)*e^(-5
/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 1/6*sqrt(2)*e^(-9/2*x)/((
e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2))
```

3.678.8 Giac [F]

$$\int (\mathrm{csch}(x) + \sinh(x))^{3/2}\,dx = \int (\mathrm{csch}\,(x) + \sinh\,(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((csch(x)+sinh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((csch(x) + sinh(x))^(3/2), x)
```

3.678.9 Mupad [F(-1)]

Timed out.

$$\int (\operatorname{csch}(x)+\sinh(x))^{3/2}\,dx=\int \left(\sinh(x)+\frac{1}{\sinh(x)}\right)^{3/2}\,dx$$

input
```
int((sinh(x) + 1/sinh(x))^(3/2),x)
```

output
```
int((sinh(x) + 1/sinh(x))^(3/2), x)
```

3.679 $\int (\operatorname{csch}(x)+\sinh(x))^{5/2}\,dx$

3.679.1 Optimal result

Integrand size = 9, antiderivative size = 50

$$\int (\operatorname{csch}(x)+\sinh(x))^{5/2}\,dx=-\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)}+\frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)}+\frac{64}{15}\sqrt{\cosh(x)\coth(x)}\tanh(x)$$

output
```
-16/15*coth(x)*(cosh(x)*coth(x))^(1/2)+2/5*cosh(x)^2*coth(x)*(cosh(x)*coth
(x))^(1/2)+64/15*(cosh(x)*coth(x))^(1/2)*tanh(x)
```

3.679.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int (\operatorname{csch}(x)+\sinh(x))^{5/2}\,dx=\frac{1}{15}\sqrt{\cosh(x)\coth(x)}\Big(-10\coth(x)+6\cosh(x)\sinh(x)+57\operatorname{csch}(x)\operatorname{sech}(x)\left(-\sinh^2(x)\right)^{3/4}+64\tanh(x)\Big)$$

input
```
Integrate[(Csch[x] + Sinh[x])^(5/2),x]
```

output
```
(Sqrt[Cosh[x]*Coth[x]]*(-10*Coth[x] + 6*Cosh[x]*Sinh[x] + 57*Csch[x]*Sech[
x]*(-Sinh[x]^2)^(3/4) + 64*Tanh[x]))/15
```

3.679.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.98, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {3042, 4897, 3042, 4898, 3042, 4900, 3042, 3078, 3042, 3074, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sinh(x)+\text{csch}(x))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\csc(ix)-i\sin(ix))^{5/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int (\cosh(x)\coth(x))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\cos(ix)\cot(ix))^{5/2}dx$$

$$\Big\downarrow \text{4898}$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\int (-i\cosh(x)\coth(x))^{5/2}dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\int (\cos(ix)\cot(ix))^{5/2}dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$\Big\downarrow \text{4900}$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\int \cosh^{\frac{5}{2}}(x)(-i\coth(x))^{5/2}dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^{5/2}\left(-\tan\left(ix+\frac{\pi}{2}\right)\right)^{5/2}dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \text{3078}$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\left(\frac{8}{5}\int\sqrt{\cosh(x)}(-i\coth(x))^{5/2}dx-\frac{2}{5}i\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{3/2}\right)}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\left(\frac{8}{5}\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}\left(-\tan\left(ix+\frac{\pi}{2}\right)\right)^{5/2}dx-\frac{2}{5}i\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{3/2}\right)}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \ 3074$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\left(\frac{8}{5}\left(\frac{2}{3}i\sqrt{\cosh(x)}(-i\coth(x))^{3/2}-\frac{4}{3}\int\sqrt{\cosh(x)}\sqrt{-i\coth(x)}dx\right)-\frac{2}{5}i\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{3}\right.}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\sqrt{\cosh(x)\coth(x)}\left(\frac{8}{5}\left(\frac{2}{3}i\sqrt{\cosh(x)}(-i\coth(x))^{3/2}-\frac{4}{3}\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}\sqrt{-\tan\left(ix+\frac{\pi}{2}\right)}dx\right)-\frac{2}{5}i\cosh^{\frac{5}{2}}(x)(-i\,c\right.}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$\Big\downarrow \ 3069$$

$$-\frac{\left(\frac{8}{5}\left(\frac{2}{3}i\sqrt{\cosh(x)}(-i\coth(x))^{3/2}+\frac{8i\sqrt{\cosh(x)}}{3\sqrt{-i\coth(x)}}\right)-\frac{2}{5}i\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{3/2}\right)\sqrt{\cosh(x)\coth(x)}}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

input
```
Int[(Csch[x] + Sinh[x])^(5/2),x]
```

output
```
-((((8*((((8*I)/3)*Sqrt[Cosh[x]])/Sqrt[(-I)*Coth[x]] + ((2*I)/3)*Sqrt[Cosh
[x]]*((-I)*Coth[x])^(3/2)))/5 - ((2*I)/5)*Cosh[x]^(5/2)*((-I)*Coth[x])^(3/
2))*Sqrt[Cosh[x]*Coth[x]])/(Sqrt[Cosh[x]]*Sqrt[(-I)*Coth[x]]))
```

3.679.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4898
```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Simp[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p])   In
t[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ
[v]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.679.4 Maple [F]

$$\int (\operatorname{csch}(x) + \sinh(x))^{\frac{5}{2}} \, dx$$

input
```
int((csch(x)+sinh(x))^(5/2),x)
```

output
```
int((csch(x)+sinh(x))^(5/2),x)
```

3.679.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(38) = 76.

Time = 0.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 5.18

$$\int (\operatorname{csch}(x) + \sinh(x))^{5/2} dx = \frac{\sqrt{\frac{1}{2}}\left(3 \cosh(x)^8 + 24 \cosh(x) \sinh(x)^7 + 3 \sinh(x)^8 + 12 \left(7 \cosh(x)^2 + 9\right) \sinh(x)^6 + 10\right.}{}$$

input
```
integrate((csch(x)+sinh(x))^(5/2),x, algorithm="fricas")
```

output
```
1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*c
osh(x)^2 + 9)*sinh(x)^6 + 108*cosh(x)^6 + 24*(7*cosh(x)^3 + 27*cosh(x))*si
nh(x)^5 + 2*(105*cosh(x)^4 + 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^
4 + 8*(21*cosh(x)^5 + 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(
x)^6 + 135*cosh(x)^4 - 151*cosh(x)^2 + 9)*sinh(x)^2 + 108*cosh(x)^2 + 8*(3
*cosh(x)^7 + 81*cosh(x)^5 - 151*cosh(x)^3 + 27*cosh(x))*sinh(x) + 3)/((cos
h(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - c
osh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*sqrt(cosh(x)^3 + 3*cosh(x)*s
inh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x)))
```

3.679.6 Sympy [F(-1)]

Timed out.

$$\int (\operatorname{csch}(x) + \sinh(x))^{5/2} dx = \text{Timed out}$$

input
```
integrate((csch(x)+sinh(x))**(5/2),x)
```

output
```
Timed out
```

3.679.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(38) = 76$.

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.26

$$\int (\operatorname{csch}(x)+\sinh(x))^{5/2}\,dx = \frac{\sqrt{2}e^{\left(\frac{5}{2}x\right)}}{20\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}} + \frac{7\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{4\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}} - \frac{41\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}} + \frac{41\sqrt{2}e^{\left(-\frac{7}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}} - \frac{7\sqrt{2}e^{\left(-\frac{11}{2}x\right)}}{4\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{15}{2}x\right)}}{20\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}}$$

input
```
integrate((csch(x)+sinh(x))^(5/2),x, algorithm="maxima")
```

output
```
1/20*sqrt(2)*e^(5/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 7/4*sqrt
(2)*e^(1/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 41/6*sqrt(2)*e^(-
3/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 41/6*sqrt(2)*e^(-7/2*x)/
((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 7/4*sqrt(2)*e^(-11/2*x)/((e^(-x
) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 1/20*sqrt(2)*e^(-15/2*x)/((e^(-x) + 1)
^(5/2)*(-e^(-x) + 1)^(5/2))
```

3.679.8 Giac [F]

$$\int (\operatorname{csch}(x)+\sinh(x))^{5/2}\,dx = \int (\operatorname{csch}(x)+\sinh(x))^{\frac{5}{2}}\,dx$$

input
```
integrate((csch(x)+sinh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((csch(x) + sinh(x))^(5/2), x)
```

3.679.9 Mupad [F(-1)]

Timed out.

$$\int (\operatorname{csch}(x)+\sinh(x))^{5/2}\,dx=\int \left(\sinh(x)+\frac{1}{\sinh(x)}\right)^{5/2}\,dx$$

input
```
int((sinh(x) + 1/sinh(x))^(5/2),x)
```

output
```
int((sinh(x) + 1/sinh(x))^(5/2), x)
```

3.680 $\int (-\cosh(x) + \mathbf{sech}(x)) \, dx$

3.680.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int (-\cosh(x) + \operatorname{sech}(x)) \, dx = \arctan(\sinh(x)) - \sinh(x)$$

output
```
arctan(sinh(x))-sinh(x)
```

3.680.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (-\cosh(x) + \operatorname{sech}(x)) \, dx = \arctan(\sinh(x)) - \sinh(x)$$

input
```
Integrate[-Cosh[x] + Sech[x],x]
```

output
```
ArcTan[Sinh[x]] - Sinh[x]
```

3.680.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\operatorname{sech}(x) - \cosh(x))\,dx$$

$$\Big\downarrow \; 2009$$

$$\arctan(\sinh(x)) - \sinh(x)$$

input
```
Int[-Cosh[x] + Sech[x],x]
```

output
```
ArcTan[Sinh[x]] - Sinh[x]
```

3.680.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.680.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\arctan(\sinh(x)) - \sinh(x)$	9
parts	$\arctan(\sinh(x)) - \sinh(x)$	9
risch	$-\frac{e^{x}}{2} + \frac{e^{-x}}{2} + i\ln(e^{x} + i) - i\ln(e^{x} - i)$	30
parallelrisch	$-i\ln(-i + \coth(x) - \operatorname{csch}(x)) + i\ln(i + \coth(x) - \operatorname{csch}(x)) - \sinh(x)$	32

input
```
int(-cosh(x)+sech(x),x,method=_RETURNVERBOSE)
```

output
```
arctan(sinh(x))-sinh(x)
```

3.680.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(8) = 16$.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 5.25

$$\int (-\cosh(x) + \operatorname{sech}(x))\, dx$$
$$= \frac{4 (\cosh(x) + \sinh(x)) \arctan(\cosh(x) + \sinh(x)) - \cosh(x)^2 - 2 \cosh(x) \sinh(x) - \sinh(x)^2 + 1}{2 (\cosh(x) + \sinh(x))}$$

input
```
integrate(-cosh(x)+sech(x),x, algorithm="fricas")
```

output
```
1/2*(4*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - cosh(x)^2 - 2*cosh(
x)*sinh(x) - sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

3.680.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int (-\cosh(x) + \operatorname{sech}(x))\, dx = -\sinh(x) + 2 \operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
integrate(-cosh(x)+sech(x),x)
```

output
```
-sinh(x) + 2*atan(tanh(x/2))
```

3.680.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (-\cosh(x) + \operatorname{sech}(x))\, dx = \arctan(\sinh(x)) - \sinh(x)$$

input
```
integrate(-cosh(x)+sech(x),x, algorithm="maxima")
```

output
```
arctan(sinh(x)) - sinh(x)
```

3.680.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int (-\cosh(x) + \mathrm{sech}(x))\,dx = 2\arctan(e^x) + \frac{1}{2}e^{(-x)} - \frac{1}{2}e^x$$

input
```
integrate(-cosh(x)+sech(x),x, algorithm="giac")
```

output
```
2*arctan(e^x) + 1/2*e^(-x) - 1/2*e^x
```

3.680.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int (-\cosh(x) + \mathrm{sech}(x))\,dx = \frac{e^{-x}}{2} + 2\,\mathrm{atan}(e^x) - \frac{e^x}{2}$$

input
```
int(1/cosh(x) - cosh(x),x)
```

output
```
exp(-x)/2 + 2*atan(exp(x)) - exp(x)/2
```

3.681 $\int (-\cosh(x)+\operatorname{sech}(x))^2\,dx$

3.681.1 Optimal result

Integrand size = 9, antiderivative size = 22

$$\int (-\cosh(x)+\operatorname{sech}(x))^2\,dx=-\frac{3x}{2}+\frac{3\tanh(x)}{2}+\frac{1}{2}\sinh^2(x)\tanh(x)$$

output
```
-3/2*x+3/2*tanh(x)+1/2*sinh(x)^2*tanh(x)
```

3.681.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int (-\cosh(x)+\operatorname{sech}(x))^2\,dx=-\frac{3x}{2}+\frac{1}{4}\sinh(2x)+\tanh(x)$$

input
```
Integrate[(-Cosh[x] + Sech[x])^2,x]
```

output
```
(-3*x)/2 + Sinh[2*x]/4 + Tanh[x]
```

3.681.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4889, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\operatorname{sech}(x)-\cosh(x))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sec(ix)-\cos(ix))^2dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tanh^4(x)}{\left(1-\tanh^2(x)\right)^2}d\tanh(x)$$

$$\Big\downarrow \text{252}$$

$$\frac{\tanh^3(x)}{2\left(1-\tanh^2(x)\right)}-\frac{3}{2}\int \frac{\tanh^2(x)}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{262}$$

$$\frac{\tanh^3(x)}{2\left(1-\tanh^2(x)\right)}-\frac{3}{2}\left(\int \frac{1}{1-\tanh^2(x)}d\tanh(x)-\tanh(x)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{\tanh^3(x)}{2\left(1-\tanh^2(x)\right)}-\frac{3}{2}(\operatorname{arctanh}(\tanh(x))-\tanh(x))$$

input
```
Int[(-Cosh[x] + Sech[x])^2,x]
```

output
```
(-3*(ArcTanh[Tanh[x]] - Tanh[x]))/2 + Tanh[x]^3/(2*(1 - Tanh[x]^2))
```

3.681.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.681. $\int (-\cosh(x) + \text{sech}(x))^2 \, dx$

3.681.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\cosh(x)\sinh(x)}{2}-\frac{3x}{2}+\tanh(x)$	13
parallelrisch	$-\frac{3x}{2}+\frac{\sinh(2x)}{4}+\tanh(x)$	13
parts	$\frac{\cosh(x)\sinh(x)}{2}-\frac{3x}{2}+\tanh(x)$	13
risch	$-\frac{3x}{2}+\frac{e^{2x}}{8}-\frac{e^{-2x}}{8}-\frac{2}{1+e^{2x}}$	27

input
```
int((-cosh(x)+sech(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*cosh(x)*sinh(x)-3/2*x+tanh(x)
```

3.681.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int (-\cosh(x)+\operatorname{sech}(x))^2\,dx = \frac{\sinh(x)^3-4(3x+2)\cosh(x)+3\left(\cosh(x)^2+3\right)\sinh(x)}{8\cosh(x)}$$

input
```
integrate((-cosh(x)+sech(x))^2,x, algorithm="fricas")
```

output
```
1/8*(sinh(x)^3 - 4*(3*x + 2)*cosh(x) + 3*(cosh(x)^2 + 3)*sinh(x))/cosh(x)
```

3.681.6 Sympy [F]

$$\int (-\cosh(x)+\operatorname{sech}(x))^2\,dx = \int (-\cosh(x)+\operatorname{sech}(x))^2\,dx$$

input
```
integrate((-cosh(x)+sech(x))**2,x)
```

output
```
Integral((-cosh(x) + sech(x))**2, x)
```

3.681.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (-\cosh(x)+\text{sech}(x))^2\,dx = -\frac{3}{2}\,x+\frac{2}{e^{(-2\,x)}+1}+\frac{1}{8}\,e^{(2\,x)}-\frac{1}{8}\,e^{(-2\,x)}$$

input
```
integrate((-cosh(x)+sech(x))^2,x, algorithm="maxima")
```

output
```
-3/2*x + 2/(e^(-2*x) + 1) + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

3.681.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int (-\cosh(x)+\text{sech}(x))^2\,dx = -\frac{3}{2}\,x+\frac{3\,e^{(4\,x)}-14\,e^{(2\,x)}-1}{8\,(e^{(4\,x)}+e^{(2\,x)})}+\frac{1}{8}\,e^{(2\,x)}$$

input
```
integrate((-cosh(x)+sech(x))^2,x, algorithm="giac")
```

output
```
-3/2*x + 1/8*(3*e^(4*x) - 14*e^(2*x) - 1)/(e^(4*x) + e^(2*x)) + 1/8*e^(2*x
)
```

3.681.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (-\cosh(x)+\text{sech}(x))^2\,dx = \frac{e^{2\,x}}{8}-\frac{e^{-2\,x}}{8}-\frac{3\,x}{2}-\frac{2}{e^{2\,x}+1}$$

input
```
int((cosh(x) - 1/cosh(x))^2,x)
```

output
```
exp(2*x)/8 - exp(-2*x)/8 - (3*x)/2 - 2/(exp(2*x) + 1)
```

3.682 $\int (-\cosh(x) + \operatorname{sech}(x))^3 \, dx$

3.682.1 Optimal result

Integrand size = 9, antiderivative size = 34

$$\int (-\cosh(x) + \operatorname{sech}(x))^3 \, dx = -\frac{5}{2}\arctan(\sinh(x)) + \frac{5\sinh(x)}{2} - \frac{5\sinh^3(x)}{6} + \frac{1}{2}\sinh^3(x)\tanh^2(x)$$

output
```
-5/2*arctan(sinh(x))+5/2*sinh(x)-5/6*sinh(x)^3+1/2*sinh(x)^3*tanh(x)^2
```

3.682.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int (-\cosh(x) + \operatorname{sech}(x))^3 \, dx = -\frac{1}{48}\operatorname{sech}^2(x)(60\arctan(\sinh(x)) + 60\arctan(\sinh(x))\cosh(2x) - 50\sinh(x) - 25\sinh(3x) + \sinh(5x))$$

input
```
Integrate[(-Cosh[x] + Sech[x])^3,x]
```

output
```
-1/48*(Sech[x]^2*(60*ArcTan[Sinh[x]] + 60*ArcTan[Sinh[x]]*Cosh[2*x] - 50*S
inh[x] - 25*Sinh[3*x] + Sinh[5*x]))
```

3.682.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {3042, 4897, 25, 3042, 25, 3072, 25, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\operatorname{sech}(x)-\cosh(x))^3 \, dx$$

$$\Big\downarrow \ 3042$$

$$\int (\sec(ix)-\cos(ix))^3dx$$

$$\Big\downarrow \ 4897$$

$$\int -\sinh^3(x)\tanh^3(x)dx$$

$$\Big\downarrow \ 25$$

$$-\int \sinh^3(x)\tanh^3(x)dx$$

$$\Big\downarrow \ 3042$$

$$-\int -\sin(ix)^3\tan(ix)^3dx$$

$$\Big\downarrow \ 25$$

$$\int \sin(ix)^3\tan(ix)^3dx$$

$$\Big\downarrow \ 3072$$

$$\int -\frac{\sinh^6(x)}{\left(\sinh^2(x)+1\right)^2}d\sinh(x)$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\sinh^6(x)}{\left(\sinh^2(x)+1\right)^2}d\sinh(x)$$

$$\Big\downarrow \ 252$$

$$\frac{\sinh^5(x)}{2\left(\sinh^2(x)+1\right)}-\frac{5}{2}\int \frac{\sinh^4(x)}{\sinh^2(x)+1}d\sinh(x)$$

$$\Big\downarrow \ 254$$

$$\frac{\sinh^5(x)}{2\left(\sinh^2(x)+1\right)}-\frac{5}{2}\int\left(\sinh^2(x)+\frac{1}{\sinh^2(x)+1}-1\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sinh^5(x)}{2\left(\sinh^2(x)+1\right)}-\frac{5}{2}\left(\arctan(\sinh(x))+\frac{\sinh^3(x)}{3}-\sinh(x)\right)$$

input
```
Int[(-Cosh[x] + Sech[x])^3,x]
```

output
```
Sinh[x]^5/(2*(1 + Sinh[x]^2)) - (5*(ArcTan[Sinh[x]] - Sinh[x] + Sinh[x]^3/
3))/2
```

3.682.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.682.4 Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
default	$-\left(\frac{2}{3}+\frac{\cosh(x)^2}{3}\right)\sinh(x)+3\sinh(x)-5\arctan(\mathrm{e}^x)+\frac{\operatorname{sech}(x)\tanh(x)}{2}$	29
parts	$-\left(\frac{2}{3}+\frac{\cosh(x)^2}{3}\right)\sinh(x)+3\sinh(x)-5\arctan(\mathrm{e}^x)+\frac{\operatorname{sech}(x)\tanh(x)}{2}$	29
parallelrisch	$\frac{\operatorname{sech}(x)\tanh(x)}{2}+\frac{5i\ln(-i+\coth(x)-\operatorname{csch}(x))}{2}-\frac{5i\ln(i+\coth(x)-\operatorname{csch}(x))}{2}+\frac{9\sinh(x)}{4}-\frac{\sinh(3x)}{12}$	44
risch	$-\frac{\mathrm{e}^{3x}}{24}+\frac{9\,\mathrm{e}^x}{8}-\frac{9\,\mathrm{e}^{-x}}{8}+\frac{\mathrm{e}^{-3x}}{24}+\frac{(\mathrm{e}^{2x}-1)\mathrm{e}^x}{(1+\mathrm{e}^{2x})^2}+\frac{5i\ln(\mathrm{e}^x-i)}{2}-\frac{5i\ln(\mathrm{e}^x+i)}{2}$	59

input
```
int((-cosh(x)+sech(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-(2/3+1/3*cosh(x)^2)*sinh(x)+3*sinh(x)-5*arctan(exp(x))+1/2*sech(x)*tanh(x
)
```

3.682.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 486, normalized size of antiderivative = 14.29

$$\int(-\cosh(x)+\operatorname{sech}(x))^3\,dx =$$
$$-\frac{\cosh(x)^{10}+10\cosh(x)\sinh(x)^9+\sinh(x)^{10}+5\left(9\cosh(x)^2-5\right)\sinh(x)^8-25\cosh(x)^8+40\,(3\ co$$

input
```
integrate((-cosh(x)+sech(x))^3,x, algorithm="fricas")
```

output
```
-1/24*(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 - 5
)*sinh(x)^8 - 25*cosh(x)^8 + 40*(3*cosh(x)^3 - 5*cosh(x))*sinh(x)^7 + 10*(
21*cosh(x)^4 - 70*cosh(x)^2 - 5)*sinh(x)^6 - 50*cosh(x)^6 + 4*(63*cosh(x)^
5 - 350*cosh(x)^3 - 75*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 - 175*cosh(x)
^4 - 75*cosh(x)^2 + 5)*sinh(x)^4 + 50*cosh(x)^4 + 40*(3*cosh(x)^7 - 35*cos
h(x)^5 - 25*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 - 140*cosh(x
)^6 - 150*cosh(x)^4 + 60*cosh(x)^2 + 5)*sinh(x)^2 + 120*(cosh(x)^7 + 7*cos
h(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(x)^2 + 2)*sinh(x)^5 + 2*cosh(x)^5 +
5*(7*cosh(x)^3 + 2*cosh(x))*sinh(x)^4 + (35*cosh(x)^4 + 20*cosh(x)^2 + 1)*
sinh(x)^3 + cosh(x)^3 + (21*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^
2 + (7*cosh(x)^6 + 10*cosh(x)^4 + 3*cosh(x)^2)*sinh(x))*arctan(cosh(x) + s
inh(x)) + 25*cosh(x)^2 + 10*(cosh(x)^9 - 20*cosh(x)^7 - 30*cosh(x)^5 + 20*
cosh(x)^3 + 5*cosh(x))*sinh(x) - 1)/(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sin
h(x)^7 + (21*cosh(x)^2 + 2)*sinh(x)^5 + 2*cosh(x)^5 + 5*(7*cosh(x)^3 + 2*c
osh(x))*sinh(x)^4 + (35*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^3 + cosh(x)^
3 + (21*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 + 1
0*cosh(x)^4 + 3*cosh(x)^2)*sinh(x))
```

3.682.6 Sympy [F]

$$\int (-\cosh(x)+\operatorname{sech}(x))^3\,dx = -\int 3\cosh(x)\operatorname{sech}^2(x)\,dx - \int \left(-3\cosh^2(x)\operatorname{sech}(x)\right)\,dx - \int \cosh^3(x)\,dx - \int \left(-\operatorname{sech}^3(x)\right)\,dx$$

input
```
integrate((-cosh(x)+sech(x))**3,x)
```

output
```
-Integral(3*cosh(x)*sech(x)**2, x) - Integral(-3*cosh(x)**2*sech(x), x) -
Integral(cosh(x)**3, x) - Integral(-sech(x)**3, x)
```

3.682.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int (-\cosh(x) + \operatorname{sech}(x))^3\,dx = \frac{e^{(-x)} - e^{(-3\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} + 5\,\arctan\left(e^{(-x)}\right) - \frac{1}{24}\,e^{(3\,x)} - \frac{9}{8}\,e^{(-x)} + \frac{1}{24}\,e^{(-3\,x)} + \frac{9}{8}\,e^{x}$$

input
```
integrate((-cosh(x)+sech(x))^3,x, algorithm="maxima")
```

output
```
(e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) + 5*arctan(e^(-x)) - 1/24*
e^(3*x) - 9/8*e^(-x) + 1/24*e^(-3*x) + 9/8*e^x
```

3.682.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int (-\cosh(x) + \operatorname{sech}(x))^3\,dx = -\frac{5}{4}\,\pi + \frac{1}{24}\,\left(e^{(-x)} - e^{x}\right)^3 - \frac{e^{(-x)} - e^{x}}{\left(e^{(-x)} - e^{x}\right)^2 + 4} - \frac{5}{2}\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,x)} - 1\right)e^{(-x)}\right) - e^{(-x)} + e^{x}$$

input
```
integrate((-cosh(x)+sech(x))^3,x, algorithm="giac")
```

output
```
-5/4*pi + 1/24*(e^(-x) - e^x)^3 - (e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) -
5/2*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - e^(-x) + e^x
```

3.682.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int (-\cosh(x)+\operatorname{sech}(x))^3\,dx = \frac{e^{-3x}}{24} - \frac{9\,e^{-x}}{8} - \frac{e^{3x}}{24} - 5\,\mathrm{atan}(e^x) + \frac{9\,e^x}{8} + \frac{e^x}{e^{2x}+1} - \frac{2\,e^x}{2\,e^{2x}+e^{4x}+1}$$

input
```
int(-(cosh(x) - 1/cosh(x))^3,x)
```

output
```
exp(-3*x)/24 - (9*exp(-x))/8 - exp(3*x)/24 - 5*atan(exp(x)) + (9*exp(x))/8
 + exp(x)/(exp(2*x) + 1) - (2*exp(x))/(2*exp(2*x) + exp(4*x) + 1)
```

3.683 $\int \sqrt{-\cosh(x)+\text{sech}(x)}\,dx$

3.683.1 Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \sqrt{-\cosh(x)+\text{sech}(x)}\,dx = 2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

output
```
2*coth(x)*(-sinh(x)*tanh(x))^(1/2)
```

3.683.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{-\cosh(x)+\text{sech}(x)}\,dx = 2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

input
```
Integrate[Sqrt[-Cosh[x] + Sech[x]],x]
```

output
```
2*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])]
```

3.683.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4897, 3042, 4900, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\text{sech}(x) - \cosh(x)}\, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\sec(ix) - \cos(ix)}dx$$

$$\Big\downarrow \text{ 4897}$$

$$\int \sqrt{-\sinh(x)\tanh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\sin(ix)\tan(ix)}dx$$

$$\Big\downarrow \text{ 4900}$$

$$\frac{\sqrt{-\sinh(x)\tanh(x)}\int \sqrt{i\sinh(x)}\sqrt{i\tanh(x)}dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{-\sinh(x)\tanh(x)}\int \sqrt{\sin(ix)}\sqrt{\tan(ix)}dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{ 3069}$$

$$2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

input
```
Int[Sqrt[-Cosh[x] + Sech[x]],x]
```

output
```
2*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])]
```

3.683.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.683.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.60 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{-\frac{\left(e^{2x}-1\right)^{2} e^{-x}}{1+e^{2x}}}\,\left(1+e^{2x}\right)}{e^{2x}-1}$	43

input
```
int((-cosh(x)+sech(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*(-(exp(2*x)-1)^2*exp(-x)/(1+exp(2*x)))^(1/2)/(exp(2*x)-1)*(1+exp(2
*x))
```

3.683.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx$$
$$=2\sqrt{\frac{1}{2}}\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)\sqrt{-\frac{1}{\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+(}}$$

input
```
integrate((-cosh(x)+sech(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-1/(cosh(
x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(
x)))
```

3.683.6 Sympy [F]

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx=\int \sqrt{-\cosh\left(x\right)+\operatorname{sech}\left(x\right)}\,dx$$

input
```
integrate((-cosh(x)+sech(x))**(1/2),x)
```

output
```
Integral(sqrt(-cosh(x) + sech(x)), x)
```

3.683.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx=-\frac{\sqrt{2}e^{\left(\frac{1}{2}\,x\right)}}{\sqrt{-e^{\left(-2\,x\right)}-1}}-\frac{\sqrt{2}e^{\left(-\frac{3}{2}\,x\right)}}{\sqrt{-e^{\left(-2\,x\right)}-1}}$$

input
```
integrate((-cosh(x)+sech(x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(2)*e^(1/2*x)/sqrt(-e^(-2*x) - 1) - sqrt(2)*e^(-3/2*x)/sqrt(-e^(-2*x)
 - 1)
```

3.683.8 Giac [F]

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx = \int \sqrt{-\cosh{(x)} + \operatorname{sech}{(x)}}\,dx$$

input
```
integrate((-cosh(x)+sech(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-cosh(x) + sech(x)), x)
```

3.683.9 Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx = 2\coth(x)\,\sqrt{\frac{1}{\cosh{(x)}} - \cosh{(x)}}$$

input
```
int((1/cosh(x) - cosh(x))^(1/2),x)
```

output
```
2*coth(x)*(1/cosh(x) - cosh(x))^(1/2)
```

3.684 $\int (-\cosh(x)+\text{sech}(x))^{3/2}\,dx$

3.684.1 Optimal result

Integrand size = 11, antiderivative size = 33

$$\int (-\cosh(x)+\text{sech}(x))^{3/2}\,dx = -\frac{8}{3}\text{csch}(x)\sqrt{-\sinh(x)\tanh(x)}-\frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)}$$

output
```
-8/3*csch(x)*(-sinh(x)*tanh(x))^(1/2)-2/3*sinh(x)*(-sinh(x)*tanh(x))^(1/2)
```

3.684.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int (-\cosh(x)+\text{sech}(x))^{3/2}\,dx = \frac{2}{3}\coth(x)\left(1+4\text{csch}^2(x)\right)(-\sinh(x)\tanh(x))^{3/2}$$

input
```
Integrate[(-Cosh[x] + Sech[x])^(3/2),x]
```

output
```
(2*Coth[x]*(1 + 4*Csch[x]^2)*(-(Sinh[x]*Tanh[x]))^(3/2))/3
```

3.684.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.55, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3042, 4897, 3042, 4900, 3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{sech}(x)-\cosh(x))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (\sec(ix)-\cos(ix))^{3/2}dx$$

$$\Big\downarrow \text{ 4897}$$

$$\int (-\sinh(x)\tanh(x))^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (\sin(ix)\tan(ix))^{3/2}dx$$

$$\Big\downarrow \text{ 4900}$$

$$\frac{\sqrt{-\sinh(x)\tanh(x)}\int (i\sinh(x))^{3/2}(i\tanh(x))^{3/2}dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{-\sinh(x)\tanh(x)}\int \sin(ix)^{3/2}\tan(ix)^{3/2}dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{ 3078}$$

$$\frac{\sqrt{-\sinh(x)\tanh(x)}\left(\frac{4}{3}\int \frac{(i\tanh(x))^{3/2}}{\sqrt{i\sinh(x)}}dx+\frac{2}{3}i(i\sinh(x))^{3/2}\sqrt{i\tanh(x)}\right)}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{-\sinh(x)\tanh(x)}\left(\frac{4}{3}\int \frac{\tan(ix)^{3/2}}{\sqrt{\sin(ix)}}dx+\frac{2}{3}i(i\sinh(x))^{3/2}\sqrt{i\tanh(x)}\right)}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{ 3069}$$

$$\frac{\left(\frac{2}{3}i(i\sinh(x))^{3/2}\sqrt{i\tanh(x)}-\frac{8i\sqrt{i\tanh(x)}}{3\sqrt{i\sinh(x)}}\right)\sqrt{-\sinh(x)\tanh(x)}}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

input
```
Int[(-Cosh[x] + Sech[x])^(3/2),x]
```

output
```
(((((-8*I)/3)*Sqrt[I*Tanh[x]])/Sqrt[I*Sinh[x]] + ((2*I)/3)*(I*Sinh[x])^(3/
2)*Sqrt[I*Tanh[x]])*Sqrt[-(Sinh[x]*Tanh[x])])/(Sqrt[I*Sinh[x]]*Sqrt[I*Tanh
[x]])
```

3.684.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.684.4 Maple [F]

$$\int (-\cosh(x)+\operatorname{sech}(x))^{\frac{3}{2}}\,dx$$

input
```
int((-cosh(x)+sech(x))^(3/2),x)
```

output
```
int((-cosh(x)+sech(x))^(3/2),x)
```

3.684.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.00

$$\int (-\cosh(x)+\operatorname{sech}(x))^{3/2}\,dx =$$
$$-\frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2+7\right)\sinh(x)^2+14\cosh(x)^2+4\left(\cosh(x)\right.\right.}{3\left(\cosh(x)+\sinh(\right.}$$

input
```
integrate((-cosh(x)+sech(x))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)
^2 + 7)*sinh(x)^2 + 14*cosh(x)^2 + 4*(cosh(x)^3 + 7*cosh(x))*sinh(x) + 1)*
sqrt(-1/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*s
inh(x) + cosh(x)))/(cosh(x) + sinh(x))
```

3.684.6 Sympy [F]

$$\int (-\cosh(x)+\operatorname{sech}(x))^{3/2}\,dx = \int (-\cosh(x)+\operatorname{sech}(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((-cosh(x)+sech(x))**(3/2),x)
```

output
```
Integral((-cosh(x) + sech(x))**(3/2), x)
```

3.684.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(25) = 50.

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int (-\cosh(x)+\operatorname{sech}(x))^{3/2}\,dx = -\frac{\sqrt{2}e^{\left(\frac{3}{2}\,x\right)}}{6\,(-e^{(-2\,x)}-1)^{\frac{3}{2}}} - \frac{5\,\sqrt{2}e^{\left(-\frac{1}{2}\,x\right)}}{2\,(-e^{(-2\,x)}-1)^{\frac{3}{2}}} - \frac{5\,\sqrt{2}e^{\left(-\frac{5}{2}\,x\right)}}{2\,(-e^{(-2\,x)}-1)^{\frac{3}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{9}{2}\,x\right)}}{6\,(-e^{(-2\,x)}-1)^{\frac{3}{2}}}$$

input
```
integrate((-cosh(x)+sech(x))^(3/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(2)*e^(3/2*x)/(-e^(-2*x) - 1)^(3/2) - 5/2*sqrt(2)*e^(-1/2*x)/(-e^
(-2*x) - 1)^(3/2) - 5/2*sqrt(2)*e^(-5/2*x)/(-e^(-2*x) - 1)^(3/2) - 1/6*sqr
t(2)*e^(-9/2*x)/(-e^(-2*x) - 1)^(3/2)
```

3.684.8 Giac [F]

$$\int (-\cosh(x)+\operatorname{sech}(x))^{3/2}\,dx = \int (-\cosh(x)+\operatorname{sech}(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((-cosh(x)+sech(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((-cosh(x) + sech(x))^(3/2), x)
```

3.684.9 Mupad [F(-1)]

Timed out.

$$\int (-\cosh(x)+\operatorname{sech}(x))^{3/2}\,dx = \int \left(\frac{1}{\cosh(x)}-\cosh(x)\right)^{3/2}\,dx$$

input
```
int((1/cosh(x) - cosh(x))^(3/2),x)
```

output
```
int((1/cosh(x) - cosh(x))^(3/2), x)
```

3.685 $\int (-\cosh(x) + \text{sech}(x))^{5/2} \, dx$

3.685.1 Optimal result

Integrand size = 11, antiderivative size = 53

$$\int (-\cosh(x) + \text{sech}(x))^{5/2} \, dx = -\frac{64}{15} \coth(x) \sqrt{-\sinh(x) \tanh(x)} + \frac{16}{15} \tanh(x) \sqrt{-\sinh(x) \tanh(x)} + \frac{2}{5} \sinh^2(x) \tanh(x) \sqrt{-\sinh(x) \tanh(x)}$$

output
```
-64/15*coth(x)*(-sinh(x)*tanh(x))^(1/2)+16/15*(-sinh(x)*tanh(x))^(1/2)*tan
h(x)+2/5*sinh(x)^2*(-sinh(x)*tanh(x))^(1/2)*tanh(x)
```

3.685.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$\int (-\cosh(x) + \text{sech}(x))^{5/2} \, dx = \frac{2}{15} \left(-5 - 3\cosh^2(x) + 32\coth^2(x)\right) \text{csch}(x) (-\sinh(x) \tanh(x))^{3/2}$$

input
```
Integrate[(-Cosh[x] + Sech[x])^(5/2),x]
```

output
```
(2*(-5 - 3*Cosh[x]^2 + 32*Coth[x]^2)*Csch[x]*(-(Sinh[x]*Tanh[x]))^(3/2))/1
5
```

3.685.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {3042, 4897, 3042, 4900, 3042, 3078, 3042, 3074, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\operatorname{sech}(x)-\cosh(x))^{5/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (\sec(ix)-\cos(ix))^{5/2}dx$$

$$\Big\downarrow \ 4897$$

$$\int (-\sinh(x)\tanh(x))^{5/2}dx$$

$$\Big\downarrow \ 3042$$

$$\int (\sin(ix)\tan(ix))^{5/2}dx$$

$$\Big\downarrow \ 4900$$

$$\frac{\sqrt{-\sinh(x)\tanh(x)}\int (i\sinh(x))^{5/2}(i\tanh(x))^{5/2}dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{-\sinh(x)\tanh(x)}\int \sin(ix)^{5/2}\tan(ix)^{5/2}dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \ 3078$$

$$\frac{\sqrt{-\sinh(x)\tanh(x)}\left(\frac{8}{5}\int \sqrt{i\sinh(x)}(i\tanh(x))^{5/2}dx+\frac{2}{5}i(i\sinh(x))^{5/2}(i\tanh(x))^{3/2}\right)}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{-\sinh(x)\tanh(x)}\left(\frac{8}{5}\int \sqrt{\sin(ix)}\tan(ix)^{5/2}dx+\frac{2}{5}i(i\sinh(x))^{5/2}(i\tanh(x))^{3/2}\right)}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \ 3074$$

$$\frac{\sqrt{-\sinh(x)\tanh(x)}\left(\frac{8}{5}\left(-\frac{4}{3}\int\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}dx-\frac{2}{3}i\sqrt{i\sinh(x)}(i\tanh(x))^{3/2}\right)+\frac{2}{5}i(i\sinh(x))^{5/2}(i\tanh(x}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{-\sinh(x)\tanh(x)}\left(\frac{8}{5}\left(-\frac{4}{3}\int\sqrt{\sin(ix)}\sqrt{\tan(ix)}dx-\frac{2}{3}i\sqrt{i\sinh(x)}(i\tanh(x))^{3/2}\right)+\frac{2}{5}i(i\sinh(x))^{5/2}(i\tanh(x))^{3}}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{\left(\frac{2}{5}i(i\sinh(x))^{5/2}(i\tanh(x))^{3/2}+\frac{8}{5}\left(-\frac{2}{3}i\sqrt{i\sinh(x)}(i\tanh(x))^{3/2}-\frac{8i\sqrt{i\sinh(x)}}{3\sqrt{i\tanh(x)}}\right)\right)\sqrt{-\sinh(x)\tanh(x)}}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

input
```
Int[(-Cosh[x] + Sech[x])^(5/2),x]
```

output
```
(((8*((((-8*I)/3)*Sqrt[I*Sinh[x]])/Sqrt[I*Tanh[x]] - ((2*I)/3)*Sqrt[I*Sinh
[x]]*(I*Tanh[x])^(3/2)))/5 + ((2*I)/5)*(I*Sinh[x])^(5/2)*(I*Tanh[x])^(3/2)
)*Sqrt[-(Sinh[x]*Tanh[x])])/(Sqrt[I*Sinh[x]]*Sqrt[I*Tanh[x]])
```

3.685.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

3.685. $\int(-\cosh(x)+\operatorname{sech}(x))^{5/2}\,dx$

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.685.4 Maple [F]

$$\int \left(-\cosh \left(x \right)+\operatorname{sech}\left(x \right)\right)^{\frac{5}{2}}d x$$

input
```
int((-cosh(x)+sech(x))^(5/2),x)
```

output
```
int((-cosh(x)+sech(x))^(5/2),x)
```

3.685.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(41) = 82$.

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 4.85

$$\int (-\cosh(x) +\text{sech}(x))^{5/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(3\,\cosh\left(x\right)^{8} + 24\,\cosh\left(x\right)\sinh\left(x\right)^{7} + 3\,\sinh\left(x\right)^{8} + 12\,\left(7\,\cosh\left(x\right)^{2} - 9\right)\sinh\left(x\right)^{6} - 1}{}$$

input
```
integrate((-cosh(x)+sech(x))^(5/2),x, algorithm="fricas")
```

output
```
1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*c
osh(x)^2 - 9)*sinh(x)^6 - 108*cosh(x)^6 + 24*(7*cosh(x)^3 - 27*cosh(x))*si
nh(x)^5 + 2*(105*cosh(x)^4 - 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^
4 + 8*(21*cosh(x)^5 - 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(
x)^6 - 135*cosh(x)^4 - 151*cosh(x)^2 - 9)*sinh(x)^2 - 108*cosh(x)^2 + 8*(3
*cosh(x)^7 - 81*cosh(x)^5 - 151*cosh(x)^3 - 27*cosh(x))*sinh(x) + 3)*sqrt(
-1/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x
) + cosh(x)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2
+ 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x))
```

3.685.6 Sympy [F(-1)]

Timed out.

$$\int (-\cosh(x) + \operatorname{sech}(x))^{5/2}\, dx = \text{Timed out}$$

input
```
integrate((-cosh(x)+sech(x))**(5/2),x)
```

output
```
Timed out
```

3.685.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(41) = 82$.

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.17

$$\int (-\cosh(x) + \operatorname{sech}(x))^{5/2}\, dx = -\frac{\sqrt{2}e^{\left(\frac{5}{2}\,x\right)}}{20\,(-e^{(-2\,x)} - 1)^{\frac{5}{2}}} + \frac{7\,\sqrt{2}e^{\left(\frac{1}{2}\,x\right)}}{4\,(-e^{(-2\,x)} - 1)^{\frac{5}{2}}} + \frac{41\,\sqrt{2}e^{\left(-\frac{3}{2}\,x\right)}}{6\,(-e^{(-2\,x)} - 1)^{\frac{5}{2}}} + \frac{41\,\sqrt{2}e^{\left(-\frac{7}{2}\,x\right)}}{6\,(-e^{(-2\,x)} - 1)^{\frac{5}{2}}} + \frac{7\,\sqrt{2}e^{\left(-\frac{11}{2}\,x\right)}}{4\,(-e^{(-2\,x)} - 1)^{\frac{5}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{15}{2}\,x\right)}}{20\,(-e^{(-2\,x)} - 1)^{\frac{5}{2}}}$$

input
```
integrate((-cosh(x)+sech(x))^(5/2),x, algorithm="maxima")
```

output
```
-1/20*sqrt(2)*e^(5/2*x)/(-e^(-2*x) - 1)^(5/2) + 7/4*sqrt(2)*e^(1/2*x)/(-e^
(-2*x) - 1)^(5/2) + 41/6*sqrt(2)*e^(-3/2*x)/(-e^(-2*x) - 1)^(5/2) + 41/6*s
qrt(2)*e^(-7/2*x)/(-e^(-2*x) - 1)^(5/2) + 7/4*sqrt(2)*e^(-11/2*x)/(-e^(-2*
x) - 1)^(5/2) - 1/20*sqrt(2)*e^(-15/2*x)/(-e^(-2*x) - 1)^(5/2)
```

3.685.8 Giac [F]

$$\int (-\cosh(x) + \operatorname{sech}(x))^{5/2} \, dx = \int (-\cosh(x) + \operatorname{sech}(x))^{\frac{5}{2}} \, dx$$

input
```
integrate((-cosh(x)+sech(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((-cosh(x) + sech(x))^(5/2), x)
```

3.685.9 Mupad [F(-1)]

Timed out.

$$\int (-\cosh(x) + \operatorname{sech}(x))^{5/2} \, dx = \int \left(\frac{1}{\cosh(x)} - \cosh(x) \right)^{5/2} \, dx$$

input
```
int((1/cosh(x) - cosh(x))^(5/2),x)
```

output
```
int((1/cosh(x) - cosh(x))^(5/2), x)
```

3.686 $\int \frac{1}{\sinh(x)+\tanh(x)} dx$

3.686.1 Optimal result

Integrand size = 7, antiderivative size = 18

$$\int \frac{1}{\sinh(x)+\tanh(x)}\,dx = -\frac{1}{2}\text{arctanh}(\cosh(x)) - \frac{1}{2(1+\cosh(x))}$$

output
```
-1/2*arctanh(cosh(x))-1/2/(1+cosh(x))
```

3.686.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sinh(x)+\tanh(x)}\,dx = -\frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right) + \frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{1}{4}\text{sech}^2\left(\frac{x}{2}\right)$$

input
```
Integrate[(Sinh[x] + Tanh[x])^(-1),x]
```

output
```
-1/2*Log[Cosh[x/2]] + Log[Sinh[x/2]]/2 - Sech[x/2]^2/4
```

3.686.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.429, Rules used = {3042, 4897, 26, 26, 3042, 26, 3185, 26, 3042, 26, 3086, 15, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sinh(x)+\tanh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{-i\sin(ix)-i\tan(ix)}dx$$
$$\Big\downarrow \text{4897}$$
$$\int -\frac{i\coth(x)}{-i\cosh(x)-i}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{i\coth(x)}{\cosh(x)+1}dx$$
$$\Big\downarrow \text{26}$$
$$\int \frac{\coth(x)}{\cosh(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\tan\left(-\frac{\pi}{2}+ix\right)}{1-\sin\left(-\frac{\pi}{2}+ix\right)}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{\tan\left(ix-\frac{\pi}{2}\right)}{1-\sin\left(ix-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3185}$$
$$-i\left(\int i\coth^2(x)\operatorname{csch}(x)dx+\int -i\coth(x)\operatorname{csch}^2(x)dx\right)$$
$$\Big\downarrow \text{26}$$
$$-i\left(i\int \coth^2(x)\operatorname{csch}(x)dx-i\int \coth(x)\operatorname{csch}^2(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(i\int -i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx-i\int i\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)dx+\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \text{3086}$$

$$-i\left(\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx-i\int -i\text{csch}(x)d(-i\text{csch}(x))\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx+\frac{1}{2}i\text{csch}^2(x)\right)$$

$$\Big\downarrow \text{3091}$$

$$-i\left(-\frac{1}{2}\int -i\text{csch}(x)dx+\frac{1}{2}i\text{csch}^2(x)-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}i\int \text{csch}(x)dx+\frac{1}{2}i\text{csch}^2(x)-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{2}i\int i\csc(ix)dx+\frac{1}{2}i\text{csch}^2(x)-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{1}{2}\int \csc(ix)dx+\frac{1}{2}i\text{csch}^2(x)-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(-\frac{1}{2}i\text{arctanh}(\cosh(x))+\frac{1}{2}i\text{csch}^2(x)-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

input
```
Int[(Sinh[x] + Tanh[x])^(-1),x]
```

output
```
(-I)*((-1/2*I)*ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csch[x] + (I/2)*Csch[x]^2)
```

3.686. $\int \frac{1}{\sinh(x)+\tanh(x)}\,dx$

3.686.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.686. $\int \frac{1}{\sinh(x)+\tanh(x)}\,dx$

3.686.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)^{2}}{4}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$	17
risch	$-\frac{e^{x}}{\left(e^{x}+1\right)^{2}}+\frac{\ln\left(e^{x}-1\right)}{2}-\frac{\ln\left(e^{x}+1\right)}{2}$	26

input
```
int(1/(sinh(x)+tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*tanh(1/2*x)^2+1/2*ln(tanh(1/2*x))
```

3.686.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(14) = 28.

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 5.33

$$\int \frac{1}{\sinh(x)+\tanh(x)}\,dx = -\frac{\left(\cosh(x)^2+2\left(\cosh(x)+1\right)\sinh(x)+\sinh(x)^2+2\cosh(x)+1\right)\log\left(\cosh(x)+\sinh(x)+1\right)-(\cos\cdots}{2\left(\cosh(x)^2+2\left(\cosh(x)+\cdots\right.\right.}$$

input
```
integrate(1/(sinh(x)+tanh(x)),x, algorithm="fricas")
```

output
```
-1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^
2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(co
sh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)
```

3.686.6 Sympy [F]

$$\int \frac{1}{\sinh(x) + \tanh(x)}\,dx = \int \frac{1}{\sinh(x) + \tanh(x)}\,dx$$

input
```
integrate(1/(sinh(x)+tanh(x)),x)
```

output
```
Integral(1/(sinh(x) + tanh(x)), x)
```

3.686.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sinh(x) + \tanh(x)}\,dx = -\frac{e^{(-x)}}{2\,e^{(-x)} + e^{(-2\,x)} + 1} - \frac{1}{2}\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(1/(sinh(x)+tanh(x)),x, algorithm="maxima")
```

output
```
-e^(-x)/(2*e^(-x) + e^(-2*x) + 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) -
 1)
```

3.686.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \frac{1}{\sinh(x) + \tanh(x)}\,dx = \frac{e^{(-x)} + e^{x} - 2}{4\left(e^{(-x)} + e^{x} + 2\right)} - \frac{1}{4}\log\left(e^{(-x)} + e^{x} + 2\right) + \frac{1}{4}\log\left(e^{(-x)} + e^{x} - 2\right)$$

input
```
integrate(1/(sinh(x)+tanh(x)),x, algorithm="giac")
```

output
```
1/4*(e^(-x) + e^x - 2)/(e^(-x) + e^x + 2) - 1/4*log(e^(-x) + e^x + 2) + 1/
4*log(e^(-x) + e^x - 2)
```

3.686.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sinh(x)+\tanh(x)}\,dx = \frac{\ln(1-e^x)}{2} - \frac{\ln(-e^x-1)}{2} + \frac{1}{e^{2x}+2e^x+1} - \frac{1}{e^x+1}$$

input
```
int(1/(sinh(x) + tanh(x)),x)
```

output
```
log(1 - exp(x))/2 - log(- exp(x) - 1)/2 + 1/(exp(2*x) + 2*exp(x) + 1) - 1/
(exp(x) + 1)
```

3.687 $\int \frac{1}{\sinh(x)-\tanh(x)}\,dx$

3.687.1 Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \frac{1}{\sinh(x)-\tanh(x)}\,dx=-\frac{1}{2}\text{arctanh}(\cosh(x))+\frac{1}{2(1-\cosh(x))}$$

output
```
-1/2*arctanh(cosh(x))+1/2/(1-cosh(x))
```

3.687.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(20) = 40.

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sinh(x)-\tanh(x)}\,dx=-\frac{1}{4}\text{csch}^2\left(\frac{x}{2}\right)\left(1-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\cosh(x)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[(Sinh[x] - Tanh[x])^(-1),x]
```

output
```
-1/4*(Csch[x/2]^2*(1 - Log[Cosh[x/2]] + Cosh[x]*(Log[Cosh[x/2]] - Log[Sinh
[x/2]]) + Log[Sinh[x/2]]))
```

3.687.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.889, Rules used = {3042, 4897, 26, 26, 3042, 26, 3185, 26, 3042, 26, 3086, 15, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sinh(x)-\tanh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{i\tan(ix)-i\sin(ix)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int -\frac{i\coth(x)}{i-i\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int -\frac{i\coth(x)}{1-\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\coth(x)}{1-\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int -\frac{i\tan\left(ix-\frac{\pi}{2}\right)}{\sin\left(ix-\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan\left(ix-\frac{\pi}{2}\right)}{\sin\left(ix-\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3185}$$

$$i\left(\int -i\coth(x)\text{csch}^2(x)dx-\int i\coth^2(x)\text{csch}(x)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-i\int \coth^2(x)\text{csch}(x)dx-i\int \coth(x)\text{csch}^2(x)dx\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(-i\int i\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)dx-i\int -i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)dx-\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \; 3086$$

$$i\left(-i\int -i\text{csch}(x)d(-i\text{csch}(x))-\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \; 15$$

$$i\left(\frac{1}{2}i\text{csch}^2(x)-\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \; 3091$$

$$i\left(\frac{1}{2}\int -i\text{csch}(x)dx+\frac{1}{2}i\text{csch}^2(x)+\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \; 26$$

$$i\left(-\frac{1}{2}i\int \text{csch}(x)dx+\frac{1}{2}i\text{csch}^2(x)+\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(-\frac{1}{2}i\int i\csc(ix)dx+\frac{1}{2}i\text{csch}^2(x)+\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{1}{2}\int \csc(ix)dx+\frac{1}{2}i\text{csch}^2(x)+\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \; 4257$$

$$i\left(\frac{1}{2}i\text{arctanh}(\cosh(x))+\frac{1}{2}i\text{csch}^2(x)+\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

input
```
Int[(Sinh[x] - Tanh[x])^(-1),x]
```

output
```
I*((I/2)*ArcTanh[Cosh[x]] + (I/2)*Coth[x]*Csch[x] + (I/2)*Csch[x]^2)
```

3.687. $\int \frac{1}{\sinh(x)-\tanh(x)}\,dx$

3.687.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.687. $\int \frac{1}{\sinh(x)-\tanh(x)}\,dx$

3.687.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{1}{4\tanh\left(\frac{x}{2}\right)^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$	17
risch	$-\frac{e^{x}}{\left(e^{x}-1\right)^{2}}+\frac{\ln\left(e^{x}-1\right)}{2}-\frac{\ln\left(e^{x}+1\right)}{2}$	26

input
```
int(1/(sinh(x)-tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/tanh(1/2*x)^2+1/2*ln(tanh(1/2*x))
```

3.687.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.80

$$\int \frac{1}{\sinh(x)-\tanh(x)}\,dx = -\frac{\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-2\,\cosh\left(x\right)+1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-(\cos}{2\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)-\right.\right.}$$

input
```
integrate(1/(sinh(x)-tanh(x)),x, algorithm="fricas")
```

output
```
-1/2*((cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^
2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(co
sh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)
```

3.687.6 Sympy [F]

$$\int \frac{1}{\sinh(x) - \tanh(x)}\, dx = \int \frac{1}{\sinh(x) - \tanh(x)}\, dx$$

input
```
integrate(1/(sinh(x)-tanh(x)),x)
```

output
```
Integral(1/(sinh(x) - tanh(x)), x)
```

3.687.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sinh(x) - \tanh(x)}\, dx = \frac{e^{(-x)}}{2\,e^{(-x)} - e^{(-2\,x)} - 1} - \frac{1}{2}\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(1/(sinh(x)-tanh(x)),x, algorithm="maxima")
```

output
```
e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) -
1)
```

3.687.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{1}{\sinh(x) - \tanh(x)}\, dx = -\frac{e^{(-x)} + e^{x} + 2}{4\left(e^{(-x)} + e^{x} - 2\right)} - \frac{1}{4}\log\left(e^{(-x)} + e^{x} + 2\right) + \frac{1}{4}\log\left(e^{(-x)} + e^{x} - 2\right)$$

input
```
integrate(1/(sinh(x)-tanh(x)),x, algorithm="giac")
```

output
```
-1/4*(e^(-x) + e^x + 2)/(e^(-x) + e^x - 2) - 1/4*log(e^(-x) + e^x + 2) + 1
/4*log(e^(-x) + e^x - 2)
```

3.687.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sinh(x) - \tanh(x)}\,dx = \frac{\ln\left(1 - e^x\right)}{2} - \frac{\ln\left(-e^x - 1\right)}{2} - \frac{1}{e^{2x} - 2\,e^x + 1} - \frac{1}{e^x - 1}$$

input
```
int(1/(sinh(x) - tanh(x)),x)
```

output
```
log(1 - exp(x))/2 - log(- exp(x) - 1)/2 - 1/(exp(2*x) - 2*exp(x) + 1) - 1/
(exp(x) - 1)
```

3.688 $\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.688.1 Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx=-\frac{bx}{a^2-b^2}+\frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

output
```
-b*x/(a^2-b^2)+a*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
```

3.688.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{-bx+a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

input
```
Integrate[Sinh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(-(b*x) + a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

3.688.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 26, 3576, 26, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh (x)}{a \cosh (x)+b \sinh (x)} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i \sin (i x)}{a \cos (i x)-i b \sin (i x)}dx$$

$$\Big\downarrow \; 26$$

$$-i \int \frac{\sin (i x)}{a \cos (i x)-i b \sin (i x)}dx$$

$$\Big\downarrow \; 3576$$

$$-i \left(-\frac{a \int -\frac{i (b \cosh (x)+a \sinh (x))}{a \cosh (x)+b \sinh (x)}dx}{a^2-b^2}-\frac{i b x}{a^2-b^2}\right)$$

$$\Big\downarrow \; 26$$

$$-i \left(\frac{i a \int \frac{b \cosh (x)+a \sinh (x)}{a \cosh (x)+b \sinh (x)}dx}{a^2-b^2}-\frac{i b x}{a^2-b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$-i \left(\frac{i a \int \frac{b \cos (i x)-i a \sin (i x)}{a \cos (i x)-i b \sin (i x)}dx}{a^2-b^2}-\frac{i b x}{a^2-b^2}\right)$$

$$\Big\downarrow \; 3612$$

$$-i \left(\frac{i a \log (a \cosh (x)+b \sinh (x))}{a^2-b^2}-\frac{i b x}{a^2-b^2}\right)$$

input
```
Int[Sinh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(-I)*(((-I)*b*x)/(a^2 - b^2) + (I*a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2
))
```

3.688.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.688.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{a\ln(a+b\tanh(x))-a\ln(1-\tanh(x))-(a+b)x}{a^2-b^2}$	39
risch	$\frac{x}{a+b}-\frac{2ax}{a^2-b^2}+\frac{a\ln\left(\mathrm{e}^{2x}+\frac{a-b}{a+b}\right)}{a^2-b^2}$	54
default	$-\frac{4\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4a+4b}-\frac{4\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{4a-4b}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a\right)}{(a+b)(a-b)}$	70

input
```
int(sinh(x)/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
(a*ln(a+b*tanh(x))-a*ln(1-tanh(x))-(a+b)*x)/(a^2-b^2)
```

3.688.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{(a+b)x - a\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

input
```
integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
-((a + b)*x - a*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
 b^2)
```

3.688.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.67

$$\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \\ \frac{x}{b} & \text{for } a=0 \\ -\frac{x\sinh(x)}{-2b\sinh(x)+2b\cosh(x)} + \frac{x\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} - \frac{\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} & \text{for } a=-b \\ \frac{x\sinh(x)}{2b\sinh(x)+2b\cosh(x)} + \frac{x\cosh(x)}{2b\sinh(x)+2b\cosh(x)} + \frac{\cosh(x)}{2b\sinh(x)+2b\cosh(x)} & \text{for } a=b \\ \frac{a\log\left(\cosh(x)+\frac{b\sinh(x)}{a}\right)}{a^2-b^2} - \frac{bx}{a^2-b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (-x*sinh(x)/(-2*b
*sinh(x) + 2*b*cosh(x)) + x*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) - cosh(x)
/(-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, -b)), (x*sinh(x)/(2*b*sinh(x) + 2*b*c
osh(x)) + x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + cosh(x)/(2*b*sinh(x) + 2
*b*cosh(x)), Eq(a, b)), (a*log(cosh(x) + b*sinh(x)/a)/(a**2 - b**2) - b*x/
(a**2 - b**2), True))
```

3.688.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(x)}{a\cosh(x) + b\sinh(x)}\,dx = \frac{a\log\left(-(a-b)e^{(-2x)} - a - b\right)}{a^2 - b^2} + \frac{x}{a+b}$$

input
```
integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
a*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

3.688.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\sinh(x)}{a\cosh(x) + b\sinh(x)}\,dx = \frac{a\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^2 - b^2} - \frac{x}{a-b}$$

input
```
integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
a*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) - x/(a - b)
```

3.688.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{\sinh(x)}{a\cosh(x) + b\sinh(x)}\,dx = -\frac{b\,x - a\,\ln\left(a\cosh(x) + b\sinh(x)\right)}{a^2 - b^2}$$

input
```
int(sinh(x)/(a*cosh(x) + b*sinh(x)),x)
```

output
```
-(b*x - a*log(a*cosh(x) + b*sinh(x)))/(a^2 - b^2)
```

3.689 $\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.689.1 Optimal result

Integrand size = 16, antiderivative size = 74

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

output
```
-a^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-b*cosh(
x)/(a^2-b^2)+a*sinh(x)/(a^2-b^2)
```

3.689.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{-\sqrt{a-b}b(a+b)\cosh(x)+a\left(-2a\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)+\sqrt{a-b}(a+b)\sinh(x)\right)}{(a-b)^{3/2}(a+b)^2}$$

input
```
Integrate[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(-(Sqrt[a - b]*b*(a + b)*Cosh[x]) + a*(-2*a*Sqrt[a + b]*ArcTan[(b + a*Tanh
[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + Sqrt[a - b]*(a + b)*Sinh[x]))/((a - b)
^(3/2)*(a + b)^2)
```

3.689.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {3042, 25, 3578, 26, 3042, 26, 3118, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3578}$$

$$\frac{ib\int i\sinh(x)dx}{a^2-b^2}-\frac{a^2\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \sinh(x)dx}{a^2-b^2}-\frac{a^2\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int -i\sin(ix)dx}{a^2-b^2}-\frac{a^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{ib\int \sin(ix)dx}{a^2-b^2}-\frac{a^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{a^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{ia^2\int\frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}$$

input
```
Int[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
((-I)*a^2*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 -
b^2)^(3/2) - (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

3.689.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

3.689.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{8}{(8a-8b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{8}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{2a^2\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a+b)(a-b)\sqrt{a^2-b^2}}$	93
risch	$\frac{e^x}{2a+2b}-\frac{e^{-x}}{2(a-b)}-\frac{a^2\ln\left(e^x+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}+\frac{a^2\ln\left(e^x-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	122

input
```
int(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-8/(8*a-8*b)/(tanh(1/2*x)+1)-8/(8*a+8*b)/(tanh(1/2*x)-1)-2*a^2/(a+b)/(a-b)
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

3.689.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(70) = 140.

Time = 0.26 (sec) , antiderivative size = 435, normalized size of antiderivative = 5.88

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$=\left[-\frac{a^3+a^2b-ab^2-b^3-(a^3-a^2b-ab^2+b^3)\cosh(x)^2-2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)-(a^3}{2((a^4-}\right.$$
$$-\frac{a^3+a^2b-ab^2-b^3-(a^3-a^2b-ab^2+b^3)\cosh(x)^2-2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)-(a^3}{2((a^4-2a^2b^2+b^4)\cosh(x)+}$$

input
```
integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
[-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^
3)*sinh(x)^2 - 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)
*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 +
 b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*
sinh(x) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) +
(a^4 - 2*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 -
 a^2*b - a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*si
nh(x) - (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(a^2*cosh(x) + a^2*sinh(
x))*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh
(x))))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))
]
```

3.689.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(58) = 116$.

Time = 127.11 (sec) , antiderivative size = 685, normalized size of antiderivative = 9.26

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \begin{cases} \tilde{\infty}\cosh(x) \\ \frac{\cosh(x)}{b} \\ -\frac{\sinh^2(x)}{-3b\sinh(x)+3b\cosh(x)} - \frac{2\sinh(x)\cosh(x)}{-3b\sinh(x)+3b\cosh(x)} + \frac{2\cosh^2(x)}{-3b\sinh(x)+3b\cosh(x)} \\ -\frac{\sinh^2(x)}{3b\sinh(x)+3b\cosh(x)} + \frac{2\sinh(x)\cosh(x)}{3b\sinh(x)+3b\cosh(x)} + \frac{2\cosh^2(x)}{3b\sinh(x)+3b\cosh(x)} \\ -\frac{a^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)\tanh^2\left(\frac{x}{2}\right)}{a^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)-a^2\sqrt{-a^2+b^2}-b^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)+b^2\sqrt{-a^2+b^2}} + \frac{a^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)-a^2\sqrt{-a^2+b^2}-b^2\sqrt{-a^2+b^2}\tanh^2} \end{cases}$$

input
```
integrate(sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Piecewise((zoo*cosh(x), Eq(a, 0) & Eq(b, 0)), (cosh(x)/b, Eq(a, 0)), (-sin
h(x)**2/(-3*b*sinh(x) + 3*b*cosh(x)) - 2*sinh(x)*cosh(x)/(-3*b*sinh(x) + 3
*b*cosh(x)) + 2*cosh(x)**2/(-3*b*sinh(x) + 3*b*cosh(x)), Eq(a, -b)), (-sin
h(x)**2/(3*b*sinh(x) + 3*b*cosh(x)) + 2*sinh(x)*cosh(x)/(3*b*sinh(x) + 3*b
*cosh(x)) + 2*cosh(x)**2/(3*b*sinh(x) + 3*b*cosh(x)), Eq(a, b)), (-a**2*lo
g(tanh(x/2) + b/a - sqrt(-a**2 + b**2)/a)*tanh(x/2)**2/(a**2*sqrt(-a**2 +
b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tan
h(x/2)**2 + b**2*sqrt(-a**2 + b**2)) + a**2*log(tanh(x/2) + b/a - sqrt(-a*
*2 + b**2)/a)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b*
*2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) + a*
*2*log(tanh(x/2) + b/a + sqrt(-a**2 + b**2)/a)*tanh(x/2)**2/(a**2*sqrt(-a*
*2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2
)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) - a**2*log(tanh(x/2) + b/a + sqr
t(-a**2 + b**2)/a)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2
 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2))
 - 2*a*sqrt(-a**2 + b**2)*tanh(x/2)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2
- a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sq
rt(-a**2 + b**2)) + 2*b*sqrt(-a**2 + b**2)/(a**2*sqrt(-a**2 + b**2)*tanh(x
/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 +
b**2*sqrt(-a**2 + b**2)), True))
```

3.689.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(x)}{a\cosh(x) + b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.689.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{2\,a^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}} - \frac{e^{(-x)}}{2\,(a-b)} + \frac{e^x}{2\,(a+b)}$$

input
```
integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
-2*a^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 1/2*e^(
-x)/(a - b) + 1/2*e^x/(a + b)
```

3.689.9 Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^x}{2\,a+2\,b} - \frac{e^{-x}}{2\,a-2\,b} - \frac{a^2\,\ln\left(-\frac{2\,a^2}{(a+b)^{5/2}\,\sqrt{b-a}} - \frac{2\,a^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}} + \frac{a^2\,\ln\left(\frac{2\,a^2}{(a+b)^{5/2}\,\sqrt{b-a}} - \frac{2\,a^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}}$$

input
```
int(sinh(x)^2/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(x)/(2*a + 2*b) - exp(-x)/(2*a - 2*b) - (a^2*log(- (2*a^2)/((a + b)^(5/
2)*(b - a)^(1/2)) - (2*a^2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^
(3/2)*(b - a)^(3/2)) + (a^2*log((2*a^2)/((a + b)^(5/2)*(b - a)^(1/2)) - (2
*a^2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3/2)*(b - a)^(3/2))
```

3.690 $\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.690.1 Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^2bx}{(a^2-b^2)^2}+\frac{bx}{2(a^2-b^2)}-\frac{a^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}-\frac{b\cosh(x)\sinh(x)}{2(a^2-b^2)}+\frac{a\sinh^2(x)}{2(a^2-b^2)}$$

output
```
a^2*b*x/(a^2-b^2)^2+1/2*b*x/(a^2-b^2)-a^3*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2
)^2-1/2*b*cosh(x)*sinh(x)/(a^2-b^2)+1/2*a*sinh(x)^2/(a^2-b^2)
```

3.690.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{6a^2bx-2b^3x+a(a^2-b^2)\cosh(2x)-4a^3\log(a\cosh(x)+b\sinh(x))+(-a^2b+b^3)\sinh(2x)}{4(a-b)^2(a+b)^2}$$

input
```
Integrate[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(6*a^2*b*x - 2*b^3*x + a*(a^2 - b^2)*Cosh[2*x] - 4*a^3*Log[a*Cosh[x] + b*S
inh[x]] + (-(a^2*b) + b^3)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

3.690.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 26, 3578, 25, 26, 3042, 25, 26, 3115, 24, 3576, 26, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3578}$$

$$i\left(-\frac{ib\int -\sinh^2(x)dx}{a^2-b^2}+\frac{a^2\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(\frac{ib\int \sinh^2(x)dx}{a^2-b^2}+\frac{a^2\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{ib\int \sinh^2(x)dx}{a^2-b^2}+\frac{ia^2\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{ib\int -\sin(ix)^2dx}{a^2-b^2}+\frac{ia^2\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{ib\int \sin(ix)^2dx}{a^2-b^2}+\frac{ia^2\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(-\frac{ib\int \sin(ix)^2dx}{a^2-b^2}+\frac{a^2\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)$$

$$\Big\downarrow \; 3115$$

$$i\left(\frac{a^2\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)$$

$$\Big\downarrow \; 24$$

$$i\left(\frac{a^2\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 3576$$

$$i\left(\frac{a^2\left(-\frac{a\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{a^2\left(\frac{ia\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{a^2\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 3612$$

$$i\left(-\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}+\frac{a^2\left(\frac{ia\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}\right)$$

input
```
Int[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

3.690. $\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

output
```
I*((a^2*(((-I)*b*x)/(a^2 - b^2) + (I*a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 -
b^2)))/(a^2 - b^2) - ((I/2)*a*Sinh[x]^2)/(a^2 - b^2) - (I*b*(x/2 - (Cosh[x
]*Sinh[x])/2))/(a^2 - b^2))
```

3.690.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

3.690. $\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.690.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{ax}{(a+b)^2}-\frac{xb}{2(a+b)^2}+\frac{e^{2x}}{8a+8b}+\frac{e^{-2x}}{8a-8b}+\frac{2a^3x}{a^4-2a^2b^2+b^4}-\frac{a^3\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$\frac{8}{(16a+16b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{16}{(32a+32b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{(2a+b)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2(a+b)^2}-\frac{16}{(32a-32b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{8}{(16a-16b)(\tan}$

input
```
int(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-a*x/(a+b)^2-1/2*x/(a+b)^2*b+1/8/(a+b)*exp(2*x)+1/8/(a-b)*exp(-2*x)+2*a^3/
(a^4-2*a^2*b^2+b^4)*x-a^3/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)+(a-b)/(a+b))
```

3.690.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(95) = 190.

Time = 0.25 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.34

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$=\frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x}{}$$

input
```
integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)
*cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(2*a^3 + 3*
a^2*b - b^3)*x*cosh(x)^2 + a^3 + a^2*b - a*b^2 - b^3 + 2*(3*(a^3 - a^2*b -
 a*b^2 + b^3)*cosh(x)^2 + 2*(2*a^3 + 3*a^2*b - b^3)*x)*sinh(x)^2 - 8*(a^3*
cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*log(2*(a*cosh(x) + b*si
nh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2
*(2*a^3 + 3*a^2*b - b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh
(x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4
)*sinh(x)^2)
```

3.690.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Timed out
```

3.690.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^3\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-2a^2b^2+b^4} - \frac{(2a+b)x}{2(a^2+2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)} + \frac{e^{(-2x)}}{8(a-b)}$$

input
```
integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
-a^3*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*(2*a + b
)*x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) + 1/8*e^(-2*x)/(a - b)
```

3.690.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^3\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4}+\frac{(2a-b)x}{2(a^2-2ab+b^2)} - \frac{\left(4ae^{(2x)}-2be^{(2x)}-a+b\right)e^{(-2x)}}{8(a^2-2ab+b^2)}+\frac{e^{(2x)}}{8(a+b)}$$

input
```
integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
-a^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/2
*(2*a - b)*x/(a^2 - 2*a*b + b^2) - 1/8*(4*a*e^(2*x) - 2*b*e^(2*x) - a + b)
*e^(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

3.690.9 Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{-2x}}{8a-8b}+\frac{e^{2x}}{8a+8b}-\frac{a^3\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^4-2a^2b^2+b^4}+\frac{x(2a-b)}{2(a-b)^2}$$

input
```
int(sinh(x)^3/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(-2*x)/(8*a - 8*b) + exp(2*x)/(8*a + 8*b) - (a^3*log(a - b + a*exp(2*x)
 + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2) + (x*(2*a - b))/(2*(a - b)^2)
```

3.691 $\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.691.1 Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

output
```
a*x/(a^2-b^2)-b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
```

3.691.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{ax-b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

input
```
Integrate[Cosh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(a*x - b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

3.691.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3577, 26, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3577}$$

$$\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3612}$$

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

input
```
Int[Cosh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

3.691.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.691.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
parallelrisch	$\frac{-b\ln(a+b\tanh(x))+\ln(1-\tanh(x))b+(a+b)x}{a^2-b^2}$	38
risch	$\frac{x}{a+b}+\frac{2xb}{a^2-b^2}-\frac{b\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^2-b^2}$	55
default	$-\frac{2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a+2b}+\frac{2\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a-2b}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a\right)}{(a-b)(a+b)}$	71

input
```
int(cosh(x)/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
(-b*ln(a+b*tanh(x))+ln(1-tanh(x))*b+(a+b)*x)/(a^2-b^2)
```

3.691.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{(a+b)x-b\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

input
```
integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

3.691.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(29) = 58.

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.85

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$=\begin{cases}\tilde{\infty}\log(\sinh(x)) & \text{for } a=0\wedge b=0\\ \frac{\log(\sinh(x))}{b} & \text{for } a=0\\ \frac{x\sinh(x)}{-2b\sinh(x)+2b\cosh(x)}-\frac{x\cosh(x)}{-2b\sinh(x)+2b\cosh(x)}-\frac{\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} & \text{for } a=-b\\ \frac{x\sinh(x)}{2b\sinh(x)+2b\cosh(x)}+\frac{x\cosh(x)}{2b\sinh(x)+2b\cosh(x)}-\frac{\cosh(x)}{2b\sinh(x)+2b\cosh(x)} & \text{for } a=b\\ \frac{ax}{a^2-b^2}-\frac{b\log\left(\cosh(x)+\frac{b\sinh(x)}{a}\right)}{a^2-b^2} & \text{otherwise}\end{cases}$$

input
```
integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Piecewise((zoo*log(sinh(x)), Eq(a, 0) & Eq(b, 0)), (log(sinh(x))/b, Eq(a,
0)), (x*sinh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) - x*cosh(x)/(-2*b*sinh(x) + 2
*b*cosh(x)) - cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, -b)), (x*sinh(x)
/(2*b*sinh(x) + 2*b*cosh(x)) + x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) - cos
h(x)/(2*b*sinh(x) + 2*b*cosh(x)), Eq(a, b)), (a*x/(a**2 - b**2) - b*log(co
sh(x) + b*sinh(x)/a)/(a**2 - b**2), True))
```

3.691.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

input
```
integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
-b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

3.691.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{b\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^2-b^2}+\frac{x}{a-b}$$

input
```
integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
-b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

3.691.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\,x-b\,\ln\left(a\cosh(x)+b\sinh(x)\right)}{a^2-b^2}$$

input
```
int(cosh(x)/(a*cosh(x) + b*sinh(x)),x)
```

output
```
(a*x - b*log(a*cosh(x) + b*sinh(x)))/(a^2 - b^2)
```

3.692 $\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.692.1 Optimal result

Integrand size = 16, antiderivative size = 74

$$\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{b^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

output
```
-b^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-b*cosh(
x)/(a^2-b^2)+a*sinh(x)/(a^2-b^2)
```

3.692.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{2b^2\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{b\cosh(x)}{-a^2+b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

input
```
Integrate[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(-2*b^2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2
)*(a + b)^(3/2)) + (b*Cosh[x])/(-a^2 + b^2) + (a*Sinh[x])/(a^2 - b^2)
```

3.692.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3579, 3042, 3117, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3579}$$

$$\frac{a\int\cosh(x)dx}{a^2-b^2}-\frac{b^2\int\frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int\sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}-\frac{b^2\int\frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{b^2\int\frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{ib^2\int\frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}$$

input
```
Int[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
((-I)*b^2*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 -
b^2)^(3/2) - (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

3.692.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

3.692.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+1\right)(2a-2b)}-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)-1\right)(2a+2b)}-\frac{2b^{2}\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{(a-b)(a+b)\sqrt{a^{2}-b^{2}}}$	93
risch	$\frac{e^{x}}{2a+2b}-\frac{e^{-x}}{2(a-b)}-\frac{b^{2}\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}+\frac{b^{2}\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}$	122

input
```
int(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(tanh(1/2*x)+1)/(2*a-2*b)-2/(tanh(1/2*x)-1)/(2*a+2*b)-2*b^2/(a-b)/(a+b)
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

3.692.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 435, normalized size of antiderivative = 5.88

$$\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$= \left[-\frac{a^3+a^2b-ab^2-b^3-(a^3-a^2b-ab^2+b^3)\cosh(x)^2-2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)-(a^3}{2((a^4-}\right.$$
$$-\frac{a^3+a^2b-ab^2-b^3-(a^3-a^2b-ab^2+b^3)\cosh(x)^2-2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)-(a^3}{2((a^4-2a^2b^2+b^4)\cosh(x)+}$$

input
```
integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
[-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^
3)*sinh(x)^2 - 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)
*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 +
 b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*
sinh(x) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) +
(a^4 - 2*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 -
 a^2*b - a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*si
nh(x) - (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(b^2*cosh(x) + b^2*sinh(
x))*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh
(x))))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))
]
```

3.692.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(58) = 116$.

Time = 123.84 (sec) , antiderivative size = 774, normalized size of antiderivative = 10.46

$$\int \frac{\cosh^2(x)}{a\cosh(x) + b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Piecewise((zoo*(log(tanh(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(
x/2))/(tanh(x/2)**2 - 1) - 2/(tanh(x/2)**2 - 1)), Eq(a, 0) & Eq(b, 0)), ((
log(tanh(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(x/2))/(tanh(x/2)
**2 - 1) - 2/(tanh(x/2)**2 - 1))/b, Eq(a, 0)), (2*sinh(x)**2/(-3*b*sinh(x)
 + 3*b*cosh(x)) - 2*sinh(x)*cosh(x)/(-3*b*sinh(x) + 3*b*cosh(x)) - cosh(x)
**2/(-3*b*sinh(x) + 3*b*cosh(x)), Eq(a, -b)), (2*sinh(x)**2/(3*b*sinh(x) +
 3*b*cosh(x)) + 2*sinh(x)*cosh(x)/(3*b*sinh(x) + 3*b*cosh(x)) - cosh(x)**2
/(3*b*sinh(x) + 3*b*cosh(x)), Eq(a, b)), (-2*a*sqrt(-a**2 + b**2)*tanh(x/2
)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*s
qrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) - b**2*log(tanh(
x/2) + b/a - sqrt(-a**2 + b**2)/a)*tanh(x/2)**2/(a**2*sqrt(-a**2 + b**2)*t
anh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)*
*2 + b**2*sqrt(-a**2 + b**2)) + b**2*log(tanh(x/2) + b/a - sqrt(-a**2 + b*
*2)/a)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b
**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) + b**2*log(
tanh(x/2) + b/a + sqrt(-a**2 + b**2)/a)*tanh(x/2)**2/(a**2*sqrt(-a**2 + b*
*2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(
x/2)**2 + b**2*sqrt(-a**2 + b**2)) - b**2*log(tanh(x/2) + b/a + sqrt(-a**2
 + b**2)/a)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2
) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) + 2...
```

3.692.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)}{a\cosh(x) + b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.692.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{2\,b^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}} - \frac{e^{(-x)}}{2\,(a-b)} + \frac{e^x}{2\,(a+b)}$$

input
```
integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
-2*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 1/2*e^(
-x)/(a - b) + 1/2*e^x/(a + b)
```

3.692.9 Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^x}{2\,a+2\,b} - \frac{e^{-x}}{2\,a-2\,b} - \frac{b^2\,\ln\left(-\frac{2\,b^2}{(a+b)^{5/2}\,\sqrt{b-a}} - \frac{2\,b^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}} + \frac{b^2\,\ln\left(\frac{2\,b^2}{(a+b)^{5/2}\,\sqrt{b-a}} - \frac{2\,b^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}}$$

input
```
int(cosh(x)^2/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(x)/(2*a + 2*b) - exp(-x)/(2*a - 2*b) - (b^2*log(- (2*b^2)/((a + b)^(5/
2)*(b - a)^(1/2)) - (2*b^2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^
(3/2)*(b - a)^(3/2)) + (b^2*log((2*b^2)/((a + b)^(5/2)*(b - a)^(1/2)) - (2
*b^2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3/2)*(b - a)^(3/2))
```

3.693 $\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.693.1 Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{ab^2x}{(a^2-b^2)^2}+\frac{ax}{2(a^2-b^2)}-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{b^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}+\frac{a\cosh(x)\sinh(x)}{2(a^2-b^2)}$$

output
```
-a*b^2*x/(a^2-b^2)^2+1/2*a*x/(a^2-b^2)-1/2*b*cosh(x)^2/(a^2-b^2)+b^3*ln(a*
cosh(x)+b*sinh(x))/(a^2-b^2)^2+1/2*a*cosh(x)*sinh(x)/(a^2-b^2)
```

3.693.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{2a^3x-6ab^2x+(-a^2b+b^3)\cosh(2x)+4b^3\log(a\cosh(x)+b\sinh(x))+a(a^2-b^2)\sinh(2x)}{4(a-b)^2(a+b)^2}$$

input
```
Integrate[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(2*a^3*x - 6*a*b^2*x + (-(a^2*b) + b^3)*Cosh[2*x] + 4*b^3*Log[a*Cosh[x] +
b*Sinh[x]] + a*(a^2 - b^2)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

3.693.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 3579, 3042, 3115, 24, 3577, 26, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3579}$$

$$\frac{a\int \cosh^2(x)dx}{a^2-b^2}-\frac{b^2\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b^2\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{b^2\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{24}$$

$$-\frac{b^2\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3577}$$

$$-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\int\frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3612}$$

$$-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\right)}{a^2-b^2}$$

input
```
Int[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
-1/2*(b*Cosh[x]^2)/(a^2 - b^2) - (b^2*((a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x
] + b*Sinh[x]])/(a^2 - b^2)))/(a^2 - b^2) + (a*(x/2 + (Cosh[x]*Sinh[x])/2)
)/(a^2 - b^2)
```

3.693.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.693. $\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.693.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

method	result
risch	$\frac{ax}{2(a+b)^2}+\frac{xb}{(a+b)^2}+\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8(a-b)}-\frac{2b^3x}{a^4-2a^2b^2+b^4}+\frac{b^3\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$\frac{1}{(2a+2b)(\tanh(\frac{x}{2})-1)^2}+\frac{2}{(4a+4b)(\tanh(\frac{x}{2})-1)}+\frac{(-a-2b)\ln(\tanh(\frac{x}{2})-1)}{2(a+b)^2}-\frac{1}{(2a-2b)(\tanh(\frac{x}{2})+1)^2}+\frac{2}{(4a-4b)(\tanh(\frac{x}{2})+}$

input
```
int(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x/(a+b)^2+x/(a+b)^2*b+1/8/(a+b)*exp(2*x)-1/8/(a-b)*exp(-2*x)-2*b^3/(
a^4-2*a^2*b^2+b^4)*x+b^3/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)+(a-b)/(a+b))
```

3.693.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(95) = 190.

Time = 0.27 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.28

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$=\frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x}{}$$

input
```
integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)
*cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^3 - 3*a*
b^2 - 2*b^3)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b -
 a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 - 3*a*b^2 - 2*b^3)*x)*sinh(x)^2 + 8*(b^3*
cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*log(2*(a*cosh(x) + b*si
nh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2
*(a^3 - 3*a*b^2 - 2*b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh
(x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4
)*sinh(x)^2)
```

3.693.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)}{a\cosh(x) + b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Timed out
```

3.693.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(x)}{a\cosh(x) + b\sinh(x)}\,dx = \frac{b^3\log\left(-(a-b)e^{(-2x)} - a - b\right)}{a^4 - 2a^2b^2 + b^4} + \frac{(a+2b)x}{2(a^2 + 2ab + b^2)} + \frac{e^{(2x)}}{8(a+b)} - \frac{e^{(-2x)}}{8(a-b)}$$

input
```
integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
b^3*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(a + 2*b)
*x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

3.693.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{b^3\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4}+\frac{(a-2b)x}{2(a^2-2ab+b^2)} - \frac{\left(2ae^{(2x)}-4be^{(2x)}+a-b\right)e^{(-2x)}}{8(a^2-2ab+b^2)}+\frac{e^{(2x)}}{8(a+b)}$$

input
```
integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
b^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/2*
(a - 2*b)*x/(a^2 - 2*a*b + b^2) - 1/8*(2*a*e^(2*x) - 4*b*e^(2*x) + a - b)*
e^(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

3.693.9 Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8a-8b}+\frac{b^3\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^4-2a^2b^2+b^4}+\frac{x(a-2b)}{2(a-b)^2}$$

input
```
int(cosh(x)^3/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) + (b^3*log(a - b + a*exp(2*x)
 + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2) + (x*(a - 2*b))/(2*(a - b)^2)
```

3.694 $\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx$

3.694.1 Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx=\frac{\arctan(\sinh(x))}{a}+\frac{b\operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

output
```
arctan(sinh(x))/a+b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/a/(a^2-
b^2)^(1/2)
```

3.694.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx=\frac{2\left(\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{b\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{a}$$

input
```
Integrate[Tanh[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

output
```
(2*(ArcTan[Tanh[x/2]] - (b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a +
 b])])/(Sqrt[-a + b]*Sqrt[a + b])))/a
```

3.694.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 3589, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{a\sinh(x)+b\cosh(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\sin(ix)}{\cos(ix)(b\cos(ix)-ia\sin(ix))}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\sin(ix)}{\cos(ix)(b\cos(ix)-ia\sin(ix))}dx$$

$$\Big\downarrow \; 3589$$

$$-i\int \left(\frac{i\text{sech}(x)}{a}-\frac{ib}{a(b\cosh(x)+a\sinh(x))}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-i\left(\frac{ib\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{i\arctan(\sinh(x))}{a}\right)$$

input
```
Int[Tanh[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

output
```
(-I)*((I*ArcTan[Sinh[x]])/a + (I*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^
2 - b^2]])/(a*Sqrt[a^2 - b^2]))
```

3.694.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3589
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

3.694.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{2b\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	54
risch	$\frac{b\ln\left(\mathrm{e}^{x}+\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a}-\frac{b\ln\left(\mathrm{e}^{x}-\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a}+\frac{i\ln(\mathrm{e}^{x}+i)}{a}-\frac{i\ln(\mathrm{e}^{x}-i)}{a}$	102

input
```
int(tanh(x)/(b*cosh(x)+a*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*b/a/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*a)/(-a^2+b^2)^(1/2))
+2/a*arctan(tanh(1/2*x))
```

3.694.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 4.00

$$\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}b\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+2\,\sqrt{a^2-b^2}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)+2\,(a^2-b^2)\,\text{arcta}}{a^3-ab^2}\right.$$
$$\left. -\frac{2\left(\sqrt{-a^2+b^2}b\arctan\left(\frac{\sqrt{-a^2+b^2}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)-(a^2-b^2)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\right)}{a^3-ab^2}\right]$$

input
```
integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*b*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (
a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)
*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + 2*(
a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), -2*(sqrt(-a^2 + b^2)*
b*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2 - b^
2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

3.694.6 Sympy [F]

$$\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx = \int \frac{\tanh\left(x\right)}{a\sinh\left(x\right)+b\cosh\left(x\right)}\,dx$$

input
```
integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x)
```

output
```
Integral(tanh(x)/(a*sinh(x) + b*cosh(x)), x)
```

3.694.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tanh(x)}{b\cosh(x) + a\sinh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.694.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{\tanh(x)}{b\cosh(x) + a\sinh(x)}\, dx = -\frac{2\, b \arctan\left(\frac{ae^x + be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2 + b^2} a} + \frac{2\, \arctan\left(e^x\right)}{a}$$

input
```
integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="giac")
```

output
```
-2*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arc
tan(e^x)/a
```

3.694.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.28

$$\begin{aligned}\int \frac{\tanh(x)}{b\cosh(x) + a\sinh(x)}\, dx = & \frac{b\, \ln\left(32\, a\, b^2\, e^x + 32\, a^2\, b\, e^x + 32\, a\, b\, \sqrt{a^2 - b^2}\right)}{a\, \sqrt{a^2 - b^2}} \\ & - \frac{b\, \ln\left(32\, a\, b^2\, e^x + 32\, a^2\, b\, e^x - 32\, a\, b\, \sqrt{a^2 - b^2}\right)}{a\, \sqrt{a^2 - b^2}} \\ & + \frac{\ln\left(32\, a\, b\, e^x - 32\, a^2\, e^x + a\, b\, 32\mathrm{i} - a^2\, 32\mathrm{i}\right)\, 1\mathrm{i}}{a} \\ & - \frac{\ln\left(32\, a^2\, e^x - 32\, a\, b\, e^x + a\, b\, 32\mathrm{i} - a^2\, 32\mathrm{i}\right)\, 1\mathrm{i}}{a}\end{aligned}$$

input
```
int(tanh(x)/(b*cosh(x) + a*sinh(x)),x)
```

output
```
(log(a*b*32i - a^2*32i - 32*a^2*exp(x) + 32*a*b*exp(x))*1i)/a - (log(a*b*3
2i - a^2*32i + 32*a^2*exp(x) - 32*a*b*exp(x))*1i)/a - (b*log(32*a*b^2*exp(
x) + 32*a^2*b*exp(x) - 32*a*b*(a^2 - b^2)^(1/2)))/(a*(a^2 - b^2)^(1/2)) +
(b*log(32*a*b^2*exp(x) + 32*a^2*b*exp(x) + 32*a*b*(a^2 - b^2)^(1/2)))/(a*(
a^2 - b^2)^(1/2))
```

3.694. $\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx$

3.695 $\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx$

3.695.1 Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx=-\frac{\text{arctanh}(\cosh(x))}{b}+\frac{a\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$$

output
```
-arctanh(cosh(x))/b+a*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/b/(a^
2-b^2)^(1/2)
```

3.695.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx=\frac{-\frac{2a\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{\sqrt{-a+b}\sqrt{a+b}}-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)}{b}$$

input
```
Integrate[Coth[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

output
```
((-2*a*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/(Sqrt[-a + b]
*Sqrt[a + b]) - Log[Cosh[x/2]] + Log[Sinh[x/2]])/b
```

3.695.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 3589, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{a\sinh(x)+b\cosh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\cos(ix)}{\sin(ix)(b\cos(ix)-ia\sin(ix))}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\cos(ix)}{\sin(ix)(b\cos(ix)-ia\sin(ix))}dx$$
$$\Big\downarrow \text{3589}$$
$$i\int \left(-\frac{a}{b(ib\cosh(x)+ia\sinh(x))}-\frac{i\text{csch}(x)}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$i\left(\frac{i\text{arctanh}(\cosh(x))}{b}-\frac{ia\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}\right)$$

input
```
Int[Coth[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

output
```
I*((I*ArcTanh[Cosh[x]])/b - (I*a*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2
- b^2]])/(b*Sqrt[a^2 - b^2]))
```

3.695.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3589
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

3.695.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{2a\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{b\sqrt{-a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b}$	53
risch	$\frac{\ln(\mathrm{e}^x-1)}{b}-\frac{\ln(\mathrm{e}^x+1)}{b}+\frac{a\ln\left(\mathrm{e}^x+\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b}-\frac{a\ln\left(\mathrm{e}^x-\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b}$	97

input
```
int(coth(x)/(b*cosh(x)+a*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*a/b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*a)/(-a^2+b^2)^(1/2))
+1/b*ln(tanh(1/2*x))
```

3.695.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 4.69

$$\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}a\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+2\,\sqrt{a^2-b^2}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)-(a^2-b^2)\log(\mathrm{co}}{a^2b-b^3}\right.$$
$$-\frac{2\sqrt{-a^2+b^2}a\arctan\left(\frac{\sqrt{-a^2+b^2}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)+(a^2-b^2)\log(\cosh(x)+\sinh(x)+1)-(a^2-b^2)\log(}{a^2b-b^3}$$

input
```
integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*a*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (
a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)
*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) - (a^
2 - b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 - b^2)*log(cosh(x) + sinh(x) -
1))/(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*a*arctan(sqrt(-a^2 + b^2)/((a + b)
*cosh(x) + (a + b)*sinh(x))) + (a^2 - b^2)*log(cosh(x) + sinh(x) + 1) - (a
^2 - b^2)*log(cosh(x) + sinh(x) - 1))/(a^2*b - b^3)]
```

3.695.6 Sympy [F]

$$\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx = \int \frac{\coth(x)}{a\sinh(x)+b\cosh(x)}\,dx$$

input
```
integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x)
```

output
```
Integral(coth(x)/(a*sinh(x) + b*cosh(x)), x)
```

3.695.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\coth(x)}{b\cosh(x) + a\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.695.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{\coth(x)}{b\cosh(x) + a\sinh(x)}\,dx = -\frac{2\,a\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b} - \frac{\log\left(e^x+1\right)}{b} + \frac{\log\left(|e^x-1|\right)}{b}$$

input
```
integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="giac")
```

output
```
-2*a*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b) - log(e
^x + 1)/b + log(abs(e^x - 1))/b
```

3.695.9 Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.47

$$\begin{aligned}\int \frac{\coth(x)}{b\cosh(x) + a\sinh(x)}\,dx = {} & \frac{\ln\left(32\,a\,b - 32\,b^2 + 32\,b^2\,\mathrm{e}^x - 32\,a\,b\,\mathrm{e}^x\right)}{b} \\ & - \frac{\ln\left(32\,a\,b - 32\,b^2 - 32\,b^2\,\mathrm{e}^x + 32\,a\,b\,\mathrm{e}^x\right)}{b} \\ & - \frac{a\,\ln\left(32\,a\,b^2\,\mathrm{e}^x + 32\,a^2\,b\,\mathrm{e}^x - 32\,a\,b\,\sqrt{a^2-b^2}\right)\,\sqrt{a^2-b^2}}{a^2\,b - b^3} \\ & + \frac{a\,\ln\left(32\,a\,b^2\,\mathrm{e}^x + 32\,a^2\,b\,\mathrm{e}^x + 32\,a\,b\,\sqrt{a^2-b^2}\right)\,\sqrt{a^2-b^2}}{a^2\,b - b^3}\end{aligned}$$

input
```
int(coth(x)/(b*cosh(x) + a*sinh(x)),x)
```

output
```
log(32*a*b - 32*b^2 + 32*b^2*exp(x) - 32*a*b*exp(x))/b - log(32*a*b - 32*b
^2 - 32*b^2*exp(x) + 32*a*b*exp(x))/b - (a*log(32*a*b^2*exp(x) + 32*a^2*b*
exp(x) - 32*a*b*(a^2 - b^2)^(1/2))*(a^2 - b^2)^(1/2))/(a^2*b - b^3) + (a*l
og(32*a*b^2*exp(x) + 32*a^2*b*exp(x) + 32*a*b*(a^2 - b^2)^(1/2))*(a^2 - b^
2)^(1/2))/(a^2*b - b^3)
```

3.695. $\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx$

3.696 $\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.696.1 Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}$$

output
```
-b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-a/(a^2-b^
2)/(a*cosh(x)+b*sinh(x))
```

3.696.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.89

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{a\sqrt{a-b}(a+b)+2ab\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\cosh(x)+2b^2\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sinh(x)}{(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))}$$

input
```
Integrate[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
-((a*Sqrt[a - b]*(a + b) + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqr
t[a - b]*Sqrt[a + b])]*Cosh[x] + 2*b^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2]
)/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[x]
+ b*Sinh[x])))
```

3.696.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 26, 3633, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3633}$$

$$-i\left(-\frac{ib\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ia}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{ib\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3553}$$

$$-i\left(\frac{b\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}-\frac{ia}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(\frac{b\operatorname{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{ia}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}\right)$$

input
```
Int[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(-I)*((b*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b
^2)^(3/2) - (I*a)/((a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])))
```

3.696.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3633
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - c*C)/(a^2 - b^2 - c^2)   Int[1/(
a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

3.696. $\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.696.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{-8b\tanh\left(\frac{x}{2}\right)-8a}{\left(4a^2-4b^2\right)\left(\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a\right)}-\frac{8b\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(4a^2-4b^2\right)\sqrt{a^2-b^2}}$	99
risch	$-\frac{2a\,e^{x}}{(a-b)(a+b)\left(a\,e^{2x}+b\,e^{2x}+a-b\right)}-\frac{b\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}+\frac{b\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	132

input
```
int(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
4*(-2*b*tanh(1/2*x)-2*a)/(4*a^2-4*b^2)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)
-8*b/(4*a^2-4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b
^2)^(1/2))
```

3.696.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(62) = 124$.

Time = 0.26 (sec) , antiderivative size = 594, normalized size of antiderivative = 9.00

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \left[\frac{\left(\left(ab+b^2\right)\cosh(x)^2+2\left(ab+b^2\right)\cosh(x)\sinh(x)+\left(ab+b^2\right)\sinh(x)^2+ab-b^2\right)\sqrt{-a^2+b^2}\log\left(\frac{(a+b)c}{}\right.}{a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+\left(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5\right)\cosh(x)^2+2\left(a^5\right.}\right.$$

input
```
integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
[(((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sin
h(x)^2 + a*b - b^2)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*co
sh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x))
 - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)
^2 + a - b)) - 2*(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2)*sinh(x))/(a^5 - a
^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*
a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3
+ a*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*
b^4 + b^5)*sinh(x)^2), 2*(((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*s
inh(x) + (a*b + b^2)*sinh(x)^2 + a*b - b^2)*sqrt(a^2 - b^2)*arctan(sqrt(a^
2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^3 - a*b^2)*cosh(x) - (a
^3 - a*b^2)*sinh(x))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 +
(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a
^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b
 - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2)]
```

3.696.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx=\text{Timed out}$$

input
```
integrate(sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.696.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.696.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,b\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}} - \frac{2\,ae^x}{(a^2-b^2)(ae^{(2\,x)}+be^{(2\,x)}+a-b)}$$

input
```
integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-2*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 2*a*e^x/(
(a^2 - b^2)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

3.696.9 Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.77

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{e^x\left(b^2\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+a\,b\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}\right)}{a^4\sqrt{b^2}-2\,a^2\,(b^2)^{3/2}+b^4\sqrt{b^2}+a\,b\,(b^2)^{3/2}-a\,b^3\sqrt{b^2}}\right)\sqrt{b^2}}{\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}} - \frac{2\,a\,e^x}{(a+b)\,(a-b)\,(a-b+e^{2\,x}\,(a+b))}$$

input
```
int(sinh(x)/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
- (2*atan((exp(x)*(b^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + a*b*(a^
6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)))/(a^4*(b^2)^(1/2) - 2*a^2*(b^2)^(3
/2) + b^4*(b^2)^(1/2) + a*b*(b^2)^(3/2) - a*b^3*(b^2)^(1/2)))*(b^2)^(1/2))
/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - (2*a*exp(x))/((a + b)*(a - b)
*(a - b + exp(2*x)*(a + b)))
```

3.697 $\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.697.1 Optimal result

Integrand size = 16, antiderivative size = 68

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} - \frac{a}{(a^2-b^2)\,(b+a\coth(x))} - \frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

output
```
(a^2+b^2)*x/(a^2-b^2)^2-a/(a^2-b^2)/(b+a*coth(x))-2*a*b*ln(a*cosh(x)+b*sin
h(x))/(a^2-b^2)^2
```

3.697.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{(a^2+b^2)\,x-2ab\log(a\cosh(x)+b\sinh(x))-\frac{a(a-b)(a+b)\sinh(x)}{a\cosh(x)+b\sinh(x)}}{(a-b)^2(a+b)^2}$$

input
```
Integrate[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
((a^2 + b^2)*x - 2*a*b*Log[a*Cosh[x] + b*Sinh[x]] - (a*(a - b)*(a + b)*Sin
h[x])/(a*Cosh[x] + b*Sinh[x]))/((a - b)^2*(a + b)^2)
```

3.697.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 25, 3564, 3042, 3964, 3042, 4014, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3564}$$

$$-\int \frac{1}{(-ib-ia\coth(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3964}$$

$$-\frac{\int \frac{b-a\coth(x)}{b+a\coth(x)}dx}{a^2-b^2}-\frac{a}{(a^2-b^2)(a\coth(x)+b)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a}{(a^2-b^2)(a\coth(x)+b)}-\frac{\int \frac{b+ia\tan\left(ix+\frac{\pi}{2}\right)}{b-ia\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{4014}$$

$$-\frac{a}{(a^2-b^2)(a\coth(x)+b)}-\frac{-\frac{x(a^2+b^2)}{a^2-b^2}+\frac{2iab\int -\frac{i(a+b\coth(x))}{b+a\coth(x)}dx}{a^2-b^2}}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{2ab\int \frac{a+b\coth(x)}{b+a\coth(x)}dx}{a^2-b^2}-\frac{x(a^2+b^2)}{a^2-b^2}}{a^2-b^2}-\frac{a}{(a^2-b^2)(a\coth(x)+b)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a}{(a^2-b^2)(a\coth(x)+b)}-\frac{-\frac{x(a^2+b^2)}{a^2-b^2}+\frac{2ab\int \frac{a-ib\tan\left(ix+\frac{\pi}{2}\right)}{b-ia\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{a^2-b^2}$$

$$\Big\downarrow \text{4013}$$

$$-\frac{a}{(a^2-b^2)(a\coth(x)+b)}-\frac{\frac{2ab\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{x(a^2+b^2)}{a^2-b^2}}{a^2-b^2}$$

input
```
Int[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
-(a/((a^2 - b^2)*(b + a*Coth[x]))) - (-(((a^2 + b^2)*x)/(a^2 - b^2)) + (2*
a*b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2))/(a^2 - b^2)
```

3.697.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3564
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.697.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.49

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{(a+b)^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{(a-b)^2}+\frac{2a\left(\frac{\left(-a^2+b^2\right)\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a}-b\ln\left(\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a\right)\right)}{(a-b)^2(a+b)^2}$
parallelrisch	$\frac{\left(-2\tanh(x)a\,b^3-2a^2b^2\right)\ln(a+b\tanh(x))+\left(2\tanh(x)a\,b^3+2a^2b^2\right)\ln(1-\tanh(x))+\left(x\,b^2(a+b)\tanh(x)+\left(b^2x+a(-1+x)b+a^2\right)\right.}{(a-b)^2(a+b)^2(a+b\tanh(x))b}$
risch	$\frac{x}{a^2+2ab+b^2}+\frac{4abx}{a^4-2a^2b^2+b^4}+\frac{2a^2}{(a-b)(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}-\frac{2ab\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$

input
```
int(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a+b)^2*ln(tanh(1/2*x)-1)+1/(a-b)^2*ln(tanh(1/2*x)+1)+2*a/(a-b)^2/(a+b)
^2*((-a^2+b^2)*tanh(1/2*x)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)-b*ln(tanh(1
/2*x)^2*a+2*b*tanh(1/2*x)+a))
```

3.697.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(68) = 136$.

Time = 0.26 (sec) , antiderivative size = 348, normalized size of antiderivative = 5.12

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \frac{(a^3+3\,a^2b+3\,ab^2+b^3)x\cosh{(x)}^2+2\,(a^3+3\,a^2b+3\,ab^2+b^3)x\cosh{(x)}\sinh{(x)}+(a^3+3\,a^2b+3\,ab^2+}{a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5+(a^5+a^4b-2\,a^3b^2-2\,a^2}$$

input
```
integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*cosh(x)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*x*cosh(x)*sinh(x) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*sinh(x)^2 + 2
*a^3 - 2*a^2*b + (a^3 + a^2*b - a*b^2 - b^3)*x - 2*(a^2*b - a*b^2 + (a^2*b
 + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^2)*
sinh(x)^2)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^5 - a^4*
b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2
*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a
*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4
 + b^5)*sinh(x)^2)
```

3.697.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(56) = 112$.

Time = 0.82 (sec) , antiderivative size = 983, normalized size of antiderivative = 14.46

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b**2, Eq(a, 0)), (2*x*sinh(x)**
2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) - 4*x*
sinh(x)*cosh(x)/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh
(x)**2) + 2*x*cosh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*
b**2*cosh(x)**2) + 3*sinh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(
x) + 8*b**2*cosh(x)**2) - cosh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*
cosh(x) + 8*b**2*cosh(x)**2), Eq(a, -b)), (2*x*sinh(x)**2/(8*b**2*sinh(x)*
*2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 4*x*sinh(x)*cosh(x)/(8
*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 2*x*cosh
(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) -
 3*sinh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x
)**2) + cosh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*c
osh(x)**2), Eq(a, b)), ((x - sinh(x)/cosh(x))/a**2, Eq(b, 0)), (a**4*cosh(
x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4
*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)) + a**3*b*x*cosh(x)/(a**5*b*cosh(
x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**
5*cosh(x) + b**6*sinh(x)) + a**2*b**2*x*sinh(x)/(a**5*b*cosh(x) + a**4*b**
2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b
**6*sinh(x)) - 2*a**2*b**2*log(cosh(x) + b*sinh(x)/a)*cosh(x)/(a**5*b*cosh
(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a...
```

3.697.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,ab\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4} - \frac{2\,a^2}{a^4-2\,a^2b^2+b^4+(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,x)}} + \frac{x}{a^2+2\,ab+b^2}$$

input
```
integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-2*a*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 2*a^2/(a^4
 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x)) + x/(a^2 +
2*a*b + b^2)
```

3.697.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.66

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,ab\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{x}{a^2-2\,ab+b^2}+\frac{2\left(abe^{(2\,x)}+a^2-ab\right)}{(a^3-a^2b-ab^2+b^3)(ae^{(2\,x)}+be^{(2\,x)}+a-b)}$$

input
```
integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-2*a*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + x
/(a^2 - 2*a*b + b^2) + 2*(a*b*e^(2*x) + a^2 - a*b)/((a^3 - a^2*b - a*b^2 +
 b^3)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

3.697.9 Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.59

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{\frac{a^2\cosh(x)}{b\,(a^2-b^2)}+\frac{a\,x\cosh(x)\,(a^2+b^2)}{(a^2-b^2)^2}+\frac{b\,x\sinh(x)\,(a^2+b^2)}{(a^2-b^2)^2}}{a\cosh\left(x\right)+b\sinh\left(x\right)}+\ln\left(a\cosh(x)+b\sinh(x)\right)\left(\frac{1}{2\left(a+b\right)^2}-\frac{1}{2\left(a-b\right)^2}\right)$$

input
```
int(sinh(x)^2/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
((a^2*cosh(x))/(b*(a^2 - b^2)) + (a*x*cosh(x)*(a^2 + b^2))/(a^2 - b^2)^2 +
 (b*x*sinh(x)*(a^2 + b^2))/(a^2 - b^2)^2)/(a*cosh(x) + b*sinh(x)) + log(a*
cosh(x) + b*sinh(x))*(1/(2*(a + b)^2) - 1/(2*(a - b)^2))
```

3.698 $\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.698.1 Optimal result

Integrand size = 16, antiderivative size = 195

$$\begin{aligned}\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= \frac{3a^2b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} \\ &\quad+\frac{(2a^2+b^2)\cosh(x)}{-a^2b^2+b^4}+\frac{a(a^2+2b^2)\sinh(x)}{b^3(a^2-b^2)} \\ &\quad-\frac{a^3}{b^3(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{a^3}{(a-b)^2b^3\left(1+\tanh\left(\frac{x}{2}\right)\right)} \\ &\quad+\frac{2a^2\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{(a^2-b^2)^2\left(a+2b\tanh\left(\frac{x}{2}\right)+a\tanh^2\left(\frac{x}{2}\right)\right)}\end{aligned}$$

output
```
3*a^2*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+(2*a
^2+b^2)*cosh(x)/(-a^2*b^2+b^4)+a*(a^2+2*b^2)*sinh(x)/b^3/(a^2-b^2)-a^3/b^3
/(a+b)^2/(1-tanh(1/2*x))+a^3/(a-b)^2/b^3/(1+tanh(1/2*x))+2*a^2*(a+b*tanh(1
/2*x))/(a^2-b^2)^2/(a+2*b*tanh(1/2*x)+a*tanh(1/2*x)^2)
```

3.698.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.05

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \frac{a\sqrt{a-b}(a^3+a^2b+ab^2+b^3)\cosh^2(x) - b\cosh(x)\left(-6a^3\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)+(a-b)^{3/2}(a+b)^{2}\cdots\right.}{(a-b)^{5/2}(a+b)^3(a\cdots}$$

input
```
Integrate[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(a*Sqrt[a - b]*(a^3 + a^2*b + a*b^2 + b^3)*Cosh[x]^2 - b*Cosh[x]*(-6*a^3*S
qrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + (a - b)^(
3/2)*(a + b)^2*Sinh[x]) + a*(a^2*Sqrt[a - b]*(a + b) + 6*a*b^2*Sqrt[a + b]
*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x] - 2*Sqrt[a -
b]*b^2*(a + b)*Sinh[x]^2))/((a - b)^(5/2)*(a + b)^3*(a*Cosh[x] + b*Sinh[x]
))
```

3.698.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 26, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{i\sin(ix)^3}{(a\cos(ix)-ib\sin(ix))^2}dx$$
$$\Big\downarrow \text{ 26}$$
$$i\int \frac{\sin(ix)^3}{(a\cos(ix)-ib\sin(ix))^2}dx$$
$$\Big\downarrow \text{ 4901}$$

$$i\int\left(-\frac{ia^3\cosh^3(x)}{b^3(ia\cosh(x)+ib\sinh(x))^2}-\frac{3ia^2\cosh^2(x)}{b^3(a\cosh(x)+b\sinh(x))}+\frac{2ia\cosh(x)}{b^3}-\frac{i\sinh(x)}{b^2}\right)dx$$

$$\downarrow \text{2009}$$

$$i\left(\frac{ia^3}{b^3(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{ia^3}{b^3(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{2ia^2\left(3a^2-b^2\right)\arctan\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}}-\frac{2ia^2b\arctan\left(\right.}{(a^2-b}\right.$$

input
```
Int[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
I*(((3*I)*a^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(b*(a^2 - b
^2)^(3/2)) - ((2*I)*a^2*b*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a^2
- b^2)^(5/2) - ((2*I)*a^2*(3*a^2 - b^2)*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2
- b^2]])/(b*(a^2 - b^2)^(5/2)) - (I*Cosh[x])/b^2 + ((3*I)*a^2*Cosh[x])/(b^
2*(a^2 - b^2)) + ((2*I)*a*Sinh[x])/b^3 - ((3*I)*a^3*Sinh[x])/(b^3*(a^2 - b
^2)) + (I*a^3)/(b^3*(a + b)^2*(1 - Tanh[x/2])) - (I*a^3)/((a - b)^2*b^3*(1
 + Tanh[x/2])) - ((2*I)*a^2*(a + b*Tanh[x/2]))/((a^2 - b^2)^2*(a + 2*b*Tan
h[x/2] + a*Tanh[x/2]^2)))
```

3.698.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.698.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.62

method	result
default	$-\frac{1}{(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{1}{(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{4a^2\left(\frac{-\frac{b\tanh\left(\frac{x}{2}\right)}{2}-\frac{a}{2}}{\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a}-\frac{3b\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{(a+b)^2(a-b)^2}$
risch	$\frac{e^x}{2a^2+4ab+2b^2}+\frac{e^{-x}}{2a^2-4ab+2b^2}+\frac{2a^3e^x}{(a-b)^2(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}-\frac{3b\,a^2\ln\left(e^x-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}+\frac{3b\,a^2\ln\left(e^x+\frac{a-b}{\sqrt{-a^2+\cdots}}\right.}{\sqrt{-a^2+b^2}\,(a+b)^2(a\cdots}$

input
```
int(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a+b)^2/(tanh(1/2*x)-1)+1/(a-b)^2/(tanh(1/2*x)+1)-4*a^2/(a+b)^2/(a-b)^2
*((-1/2*b*tanh(1/2*x)-1/2*a)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)-3/2*b/(a^
2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

3.698.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(180) = 360$.

Time = 0.29 (sec) , antiderivative size = 1633, normalized size of antiderivative = 8.37

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
[1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2
 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^5 - a*b^4)*cosh(x)^2 + 6*(a^5 -
 a*b^4 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*si
nh(x)^2 - 6*((a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^3*b + a^2*b^2)*cosh(x)*sin
h(x)^2 + (a^3*b + a^2*b^2)*sinh(x)^3 + (a^3*b - a^2*b^2)*cosh(x) + (a^3*b
- a^2*b^2 + 3*(a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log((
(a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt
(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*c
osh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 4*((a^5 - a^4*b - 2*a^3*b^2
 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^5 - a*b^4)*cosh(x))*sinh(x))/
((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^
7)*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*
b^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^
3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*
b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^
6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7
 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*co
sh(x)^2)*sinh(x)), 1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b...
```

3.698.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{(a\cosh(x) + b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.698.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(x)}{(a\cosh(x) + b\sinh(x))^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.698.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.89

$$\int \frac{\sinh^3(x)}{(a\cosh(x) + b\sinh(x))^2} \, dx = \frac{6\,a^2 b \arctan\left(\frac{ae^x + be^x}{\sqrt{a^2 - b^2}}\right)}{(a^4 - 2\,a^2b^2 + b^4)\sqrt{a^2 - b^2}} + \frac{e^x}{2\,(a^2 + 2\,ab + b^2)} + \frac{5\,a^3 e^{(2\,x)} + 3\,a^2 b e^{(2\,x)} + 3\,ab^2 e^{(2\,x)} + b^3 e^{(2\,x)} + a^3 + a^2 b - ab^2 - b^3}{2\,(a^4 - 2\,a^2b^2 + b^4)(ae^{(3\,x)} + be^{(3\,x)} + ae^x - be^x)}$$

input
```
integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
6*a^2*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) + 1/2*(5*a^3*e^(2*x) + 3*a^2
*b*e^(2*x) + 3*a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b^3)/((
a^4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

3.698.9 Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{-x}}{2(a-b)^2}+\frac{e^x}{2(a+b)^2}+\frac{6\,\mathrm{atan}\left(\frac{a^2\,b\,e^x\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}{a^5\,\sqrt{a^4\,b^2}-b^5\,\sqrt{a^4\,b^2}+2\,a^2\,b^3\,\sqrt{a^4\,b^2}-2\,a^3\,b^2\,\sqrt{a^4\,b^2}+a\,b^4\,\sqrt{a^4\,b^2}-a^4\,b\,\sqrt{a^4\,b^2}}\right)\,\sqrt{a^4\,b^2}}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}+\frac{2\,a^3\,e^x}{(a+b)^2\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)}$$

input
```
int(sinh(x)^3/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
exp(-x)/(2*(a - b)^2) + exp(x)/(2*(a + b)^2) + (6*atan((a^2*b*exp(x)*(a^10
 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^
4*b^2)^(1/2) - b^5*(a^4*b^2)^(1/2) + 2*a^2*b^3*(a^4*b^2)^(1/2) - 2*a^3*b^2
*(a^4*b^2)^(1/2) + a*b^4*(a^4*b^2)^(1/2) - a^4*b*(a^4*b^2)^(1/2)))*(a^4*b^
2)^(1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^
(1/2) + (2*a^3*exp(x))/((a + b)^2*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

3.699 $\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.699.1 Optimal result

Integrand size = 14, antiderivative size = 64

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{b}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}$$

output
```
a*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+b/(a^2-b^2
)/(a*cosh(x)+b*sinh(x))
```

3.699.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.94

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{\sqrt{a-b}b(a+b)+2a^2\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\cosh(x)+2ab\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sinh(x)}{(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))}$$

input
```
Integrate[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(Sqrt[a - b]*b*(a + b) + 2*a^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[
a - b]*Sqrt[a + b])]*Cosh[x] + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/
(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[x] +
b*Sinh[x]))
```

3.699.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3634, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 3634}$$

$$\frac{a\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{b}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}+\frac{a\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{ 3553}$$

$$\frac{b}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}+\frac{ia\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{b}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}+\frac{ia\,\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}$$

input
```
Int[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(I*a*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^
(3/2) + b/((a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

3.699.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3634
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - b*B)/(a^2 - b^2 - c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

3.699.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

method	result	size
default	$\frac{\frac{2b^2 \tanh\left(\frac{x}{2}\right)}{a\left(a^2-b^2\right)}+\frac{2b}{a^2-b^2}}{\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}$	98
risch	$\frac{2b\,\mathrm{e}^{x}}{(a-b)(a+b)(a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}+a-b)}-\frac{a\ln\left(\mathrm{e}^{x}-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	132

input
```
int(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(b^2/a/(a^2-b^2)*tanh(1/2*x)+b/(a^2-b^2))/(tanh(1/2*x)^2*a+2*b*tanh(1/2*
x)+a)+2/(a^2-b^2)^(3/2)*a*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)
)
```

3.699.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(60) = 120$.

Time = 0.26 (sec) , antiderivative size = 596, normalized size of antiderivative = 9.31

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \left[\frac{\left((a^2+ab)\cosh(x)^2+2(a^2+ab)\cosh(x)\sinh(x)+(a^2+ab)\sinh(x)^2+a^2-ab\right)\sqrt{-a^2+b^2}\log\left(\frac{(a+b)}{\cdots}\right.}{a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5)\cosh(x)^2+2(a^5\cdots}\right.$$
$$-\frac{2\left(\left((a^2+ab)\cosh(x)^2+2(a^2+ab)\cosh(x)\sinh(x)+(a^2+ab)\sinh(x)^2+a^2-ab\right)\sqrt{a\cdots}\right.}{a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5)\cosh(x)^2+2(a^5+a^4b\cdots}$$

input
```
integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
[(((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sin
h(x)^2 + a^2 - a*b)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*co
sh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x))
 - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)
^2 + a - b)) + 2*(a^2*b - b^3)*cosh(x) + 2*(a^2*b - b^3)*sinh(x))/(a^5 - a
^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*
a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3
+ a*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*
b^4 + b^5)*sinh(x)^2), -2*(((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*
sinh(x) + (a^2 + a*b)*sinh(x)^2 + a^2 - a*b)*sqrt(a^2 - b^2)*arctan(sqrt(a
^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2*b - b^3)*cosh(x) - (
a^2*b - b^3)*sinh(x))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 +
 (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 +
a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*
b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2)]
```

3.699.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{(a\cosh(x) + b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.699.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)}{(a\cosh(x) + b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.699.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(x)}{(a\cosh(x) + b\sinh(x))^2}\,dx = \frac{2\,a\arctan\left(\frac{ae^x + be^x}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}} + \frac{2\,be^x}{(a^2-b^2)(ae^{(2\,x)} + be^{(2\,x)} + a - b)}$$

input
```
integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
2*a*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 2*b*e^x/((
a^2 - b^2)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

3.699.9 Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.86

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\left(a^2\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+a\,b\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}\right)}{a^4\,\sqrt{a^2}-2\,b^2\,{\left(a^2\right)}^{3/2}+b^4\,\sqrt{a^2}+a\,b\,{\left(a^2\right)}^{3/2}-a^3\,b\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}} + \frac{2\,b\,\mathrm{e}^x}{(a+b)\,(a-b)\,\left(a-b+\mathrm{e}^{2\,x}\,(a+b)\right)}$$

input
```
int(cosh(x)/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
(2*atan((exp(x)*(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + a*b*(a^6
- b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)))/(a^4*(a^2)^(1/2) - 2*b^2*(a^2)^(3/2
) + b^4*(a^2)^(1/2) + a*b*(a^2)^(3/2) - a^3*b*(a^2)^(1/2)))*(a^2)^(1/2))/(
a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + (2*b*exp(x))/((a + b)*(a - b)*(
a - b + exp(2*x)*(a + b)))
```

3.700 $\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.700.1 Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} - \frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} + \frac{b}{(a^2-b^2)\,(a+b\tanh(x))}$$

output
```
(a^2+b^2)*x/(a^2-b^2)^2-2*a*b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^2+b/(a^2-b
^2)/(a+b*tanh(x))
```

3.700.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{(a^2+b^2)\,x-2ab\log(a\cosh(x)+b\sinh(x))+\frac{b^2(-a^2+b^2)\sinh(x)}{a(a\cosh(x)+b\sinh(x))}}{(a-b)^2(a+b)^2}$$

input
```
Integrate[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
((a^2 + b^2)*x - 2*a*b*Log[a*Cosh[x] + b*Sinh[x]] + (b^2*(-a^2 + b^2)*Sinh
[x])/(a*(a*Cosh[x] + b*Sinh[x])))/((a - b)^2*(a + b)^2)
```

3.700.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {3042, 3565, 3042, 3964, 3042, 4014, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3565}$$

$$\int \frac{1}{(a+b\tanh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-ib\tan(ix))^2}dx$$

$$\Big\downarrow \text{3964}$$

$$\frac{\int \frac{a-b\tanh(x)}{a+b\tanh(x)}dx}{a^2-b^2}+\frac{b}{(a^2-b^2)(a+b\tanh(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b}{(a^2-b^2)(a+b\tanh(x))}+\frac{\int \frac{a+ib\tan(ix)}{a-ib\tan(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{4014}$$

$$\frac{b}{(a^2-b^2)(a+b\tanh(x))}+\frac{\frac{x(a^2+b^2)}{a^2-b^2}-\frac{2iab\int -\frac{i(b+a\tanh(x))}{a+b\tanh(x)}dx}{a^2-b^2}}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{x(a^2+b^2)}{a^2-b^2}-\frac{2ab\int \frac{b+a\tanh(x)}{a+b\tanh(x)}dx}{a^2-b^2}}{a^2-b^2}+\frac{b}{(a^2-b^2)(a+b\tanh(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b}{(a^2-b^2)(a+b\tanh(x))}+\frac{\frac{x(a^2+b^2)}{a^2-b^2}-\frac{2ab\int\frac{b-ia\tan(ix)}{a-ib\tan(ix)}dx}{a^2-b^2}}{a^2-b^2}$$

$$\Big\downarrow \text{4013}$$

$$\frac{b}{(a^2-b^2)(a+b\tanh(x))}+\frac{\frac{x(a^2+b^2)}{a^2-b^2}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}}{a^2-b^2}$$

input
```
Int[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(((a^2 + b^2)*x)/(a^2 - b^2) - (2*a*b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b
^2))/(a^2 - b^2) + b/((a^2 - b^2)*(a + b*Tanh[x]))
```

3.700.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3565
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

3.700. $\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.700.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{(a+b)^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{(a-b)^2}-\frac{2b\left(\frac{b\left(a^2-b^2\right)\tanh\left(\frac{x}{2}\right)}{a\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a\right)}+a\ln\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a\right)\right)}{(a+b)^2(a-b)^2}$
parallelrisch	$\frac{\left(-2b^2\tanh(x)a^2-2a^3b\right)\ln(a+b\tanh(x))+\left(2b^2\tanh(x)a^2+2a^3b\right)\ln(1-\tanh(x))+\left(b\left(a^2x+a(-1+x)b+b^2\right)\tanh(x)+a^2x(a+b\right.}{(a-b)^2(a+b)^2(a+b\tanh(x))a}$
risch	$\frac{x}{a^2+2ab+b^2}+\frac{4abx}{a^4-2a^2b^2+b^4}+\frac{2b^2}{(a-b)(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}-\frac{2ab\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$

input
```
int(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a+b)^2*ln(tanh(1/2*x)-1)+1/(a-b)^2*ln(tanh(1/2*x)+1)-2*b/(a+b)^2/(a-b)
^2*(b*(a^2-b^2)/a*tanh(1/2*x)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+a*ln(tan
h(1/2*x)^2*a+2*b*tanh(1/2*x)+a))
```

3.700.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(67) = 134.

Time = 0.26 (sec) , antiderivative size = 348, normalized size of antiderivative = 5.19

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$=\frac{(a^3+3\,a^2b+3\,ab^2+b^3)x\cosh(x)^2+2\,(a^3+3\,a^2b+3\,ab^2+b^3)x\cosh(x)\sinh(x)+(a^3+3\,a^2b+3\,ab^2+}{a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5+(a^5+a^4b-2\,a^3b^2-2\,a^2b}$$

input
```
integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*cosh(x)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*x*cosh(x)*sinh(x) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*sinh(x)^2 + 2
*a*b^2 - 2*b^3 + (a^3 + a^2*b - a*b^2 - b^3)*x - 2*(a^2*b - a*b^2 + (a^2*b
 + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^2)*
sinh(x)^2)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^5 - a^4*
b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2
*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a
*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4
 + b^5)*sinh(x)^2)
```

3.700.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(56) = 112$.

Time = 0.76 (sec) , antiderivative size = 952, normalized size of antiderivative = 14.21

$$\int \frac{\cosh^2(x)}{(a\cosh(x) + b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Piecewise((zoo*(x - cosh(x)/sinh(x)), Eq(a, 0) & Eq(b, 0)), ((x - cosh(x)/
sinh(x))/b**2, Eq(a, 0)), (2*x*sinh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sin
h(x)*cosh(x) + 8*b**2*cosh(x)**2) - 4*x*sinh(x)*cosh(x)/(8*b**2*sinh(x)**2
 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 2*x*cosh(x)**2/(8*b**2*s
inh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) - sinh(x)**2/(8*b
**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 3*cosh(x)*
*2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2), Eq(a
, -b)), (2*x*sinh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b
**2*cosh(x)**2) + 4*x*sinh(x)*cosh(x)/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)
*cosh(x) + 8*b**2*cosh(x)**2) + 2*x*cosh(x)**2/(8*b**2*sinh(x)**2 + 16*b**
2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + sinh(x)**2/(8*b**2*sinh(x)**2 + 1
6*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) - 3*cosh(x)**2/(8*b**2*sinh(x)
**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2), Eq(a, b)), (a**3*x*cos
h(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*si
nh(x) + a*b**4*cosh(x) + b**5*sinh(x)) + a**2*b*x*sinh(x)/(a**5*cosh(x) +
a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x
) + b**5*sinh(x)) - 2*a**2*b*log(cosh(x) + b*sinh(x)/a)*cosh(x)/(a**5*cosh
(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*
cosh(x) + b**5*sinh(x)) + a**2*b*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) -
2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(...
```

3.700.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,ab\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4} - \frac{2\,b^2}{a^4-2\,a^2b^2+b^4+(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,x)}} + \frac{x}{a^2+2\,ab+b^2}$$

input
```
integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-2*a*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 2*b^2/(a^4
 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x)) + x/(a^2 +
2*a*b + b^2)
```

3.700.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,ab\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4} + \frac{x}{a^2-2\,ab+b^2} + \frac{2\left(abe^{(2\,x)}+ab-b^2\right)}{(a^3-a^2b-ab^2+b^3)(ae^{(2\,x)}+be^{(2\,x)}+a-b)}$$

input
```
integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-2*a*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + x
/(a^2 - 2*a*b + b^2) + 2*(a*b*e^(2*x) + a*b - b^2)/((a^3 - a^2*b - a*b^2 +
 b^3)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

3.700.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{\frac{b\cosh(x)}{a^2-b^2}+\frac{x\sinh(x)\,(a^2\,b+b^3)}{(a^2-b^2)^2}+\frac{a\,x\cosh(x)\,(a^2+b^2)}{(a^2-b^2)^2}}{a\cosh\left(x\right)+b\sinh\left(x\right)} + \ln\left(a\cosh(x)+b\sinh(x)\right)\left(\frac{1}{2\left(a+b\right)^2}-\frac{1}{2\left(a-b\right)^2}\right)$$

input
```
int(cosh(x)^2/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
((b*cosh(x))/(a^2 - b^2) + (x*sinh(x)*(a^2*b + b^3))/(a^2 - b^2)^2 + (a*x*
cosh(x)*(a^2 + b^2))/(a^2 - b^2)^2)/(a*cosh(x) + b*sinh(x)) + log(a*cosh(x
) + b*sinh(x))*(1/(2*(a + b)^2) - 1/(2*(a - b)^2))
```

3.701 $\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.701.1 Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{3ab^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)} - \frac{1}{(a-b)^2\left(1+\tanh\left(\frac{x}{2}\right)\right)} - \frac{2b^3\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2-b^2\right)^2\left(a+2b\tanh\left(\frac{x}{2}\right)+a\tanh^2\left(\frac{x}{2}\right)\right)}$$

output
```
-3*a*b^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+1/(
a+b)^2/(1-tanh(1/2*x))-1/(a-b)^2/(1+tanh(1/2*x))-2*b^3*(a+b*tanh(1/2*x))/a
/(a^2-b^2)^2/(a+2*b*tanh(1/2*x)+a*tanh(1/2*x)^2)
```

3.701.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.53

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \frac{-\sqrt{a-b}bb^3(a+b) - 2a^2\sqrt{a-b}b b(a+b)\cosh^2(x) - 6ab^3\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sinh(x) + \sqrt{a-b}b b(a^3 \ldots}{(a-b)^{5/2}(a+b)^3(a \ldots}$$

input
```
Integrate[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(-(Sqrt[a - b]*b^3*(a + b)) - 2*a^2*Sqrt[a - b]*b*(a + b)*Cosh[x]^2 - 6*a*
b^3*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x
] + Sqrt[a - b]*b*(a^3 + a^2*b + a*b^2 + b^3)*Sinh[x]^2 + a*Cosh[x]*(-6*a*
b^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + (a -
 b)^(3/2)*(a + b)^2*Sinh[x]))/((a - b)^(5/2)*(a + b)^3*(a*Cosh[x] + b*Sinh
[x]))
```

3.701.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.50, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4902, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{4902}$$

$$2\int \frac{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}{\left(1-\tanh^2\left(\frac{x}{2}\right)\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a\right)^2}d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{7293}$$

$$2\int \left(-\frac{2\tanh\left(\frac{x}{2}\right)b^3}{a\left(b^2-a^2\right)\left(a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a\right)^2}+\frac{b^4-3a^2b^2}{a\left(a^2-b^2\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a\right)}+\frac{}{2(a+b)^2\,(}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{b^2\left(3a^2-b^2\right)\arctan\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}-\frac{b^4\arctan\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}-\frac{b^3\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2-b^2\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)\right)}\right.$$

input
```
Int[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
2*(-((b^4*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5/2))
) - (b^2*(3*a^2 - b^2)*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a*(a^2
- b^2)^(5/2)) + 1/(2*(a + b)^2*(1 - Tanh[x/2])) - 1/(2*(a - b)^2*(1 + Tanh
[x/2])) - (b^3*(a + b*Tanh[x/2]))/(a*(a^2 - b^2)^2*(a + 2*b*Tanh[x/2] + a*
Tanh[x/2]^2)))
```

3.701.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.701. $\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.701.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

method	result
default	$-\frac{1}{(a+b)^2(\tanh(\frac{x}{2})-1)}-\frac{1}{(a-b)^2(\tanh(\frac{x}{2})+1)}-\frac{2b^2\left(\frac{\frac{b^2\tanh(\frac{x}{2})}{a}+b}{\tanh(\frac{x}{2})^2a+2b\tanh(\frac{x}{2})+a}+\frac{3a\arctan\left(\frac{2a\tanh(\frac{x}{2})+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a+b)^2(a-b)^2}$
risch	$\frac{e^x}{2a^2+4ab+2b^2}-\frac{e^{-x}}{2(a^2-2ab+b^2)}-\frac{2b^3e^x}{(a-b)^2(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}-\frac{3b^2a\ln\left(e^x+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}+\frac{3b^2a\ln\left(e^x-\frac{a-b}{\sqrt{-a^2-}}\right.}{\sqrt{-a^2+b^2}\,(a+b)^2(a}$

input
```
int(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a+b)^2/(tanh(1/2*x)-1)-1/(a-b)^2/(tanh(1/2*x)+1)-2*b^2/(a+b)^2/(a-b)^2
*((b^2/a*tanh(1/2*x)+b)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+3*a/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

3.701.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(118) = 236$.

Time = 0.29 (sec) , antiderivative size = 1645, normalized size of antiderivative = 12.37

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
[-1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^
2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2 + 6*(a^4*
b - b^5 - (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*s
inh(x)^2 + 6*((a^2*b^2 + a*b^3)*cosh(x)^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)*si
nh(x)^2 + (a^2*b^2 + a*b^3)*sinh(x)^3 + (a^2*b^2 - a*b^3)*cosh(x) + (a^2*b
^2 - a*b^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(
((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqr
t(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*
cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) - 4*((a^5 - a^4*b - 2*a^3*b^
2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^4*b - b^5)*cosh(x))*sinh(x))
/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b
^7)*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2
*b^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b
^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a
^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^
7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*c
osh(x)^2)*sinh(x)), -1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 +...
```

3.701.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx=\text{Timed out}$$

input
```
integrate(cosh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.701.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.701.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.31

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{6\,ab^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{e^x}{2\,(a^2+2\,ab+b^2)} - \frac{a^3e^{(2\,x)}+3\,a^2be^{(2\,x)}+3\,ab^2e^{(2\,x)}+5\,b^3e^{(2\,x)}+a^3+a^2b-ab^2-b^3}{2\,(a^4-2\,a^2b^2+b^4)(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x)}$$

input
```
integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-6*a*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*
sqrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) - 1/2*(a^3*e^(2*x) + 3*a^2*
b*e^(2*x) + 3*a*b^2*e^(2*x) + 5*b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b^3)/(
(a^4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

3.701.9 Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.92

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^x}{2(a+b)^2}-\frac{e^{-x}}{2(a-b)^2} - \frac{6\,\mathrm{atan}\left(\frac{a\,b^2\,e^x\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}{a^5\sqrt{a^2\,b^4}-b^5\sqrt{a^2\,b^4}+2\,a^2\,b^3\sqrt{a^2\,b^4}-2\,a^3\,b^2\sqrt{a^2\,b^4}+a\,b^4\sqrt{a^2\,b^4}-a^4\,b\sqrt{a^2\,b^4}}\right)\sqrt{a^2\,b^4}}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}} - \frac{2\,b^3\,e^x}{(a+b)^2\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)}$$

input
```
int(cosh(x)^3/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
exp(x)/(2*(a + b)^2) - exp(-x)/(2*(a - b)^2) - (6*atan((a*b^2*exp(x)*(a^10
 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^
2*b^4)^(1/2) - b^5*(a^2*b^4)^(1/2) + 2*a^2*b^3*(a^2*b^4)^(1/2) - 2*a^3*b^2
*(a^2*b^4)^(1/2) + a*b^4*(a^2*b^4)^(1/2) - a^4*b*(a^2*b^4)^(1/2)))*(a^2*b^
4)^(1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^
(1/2) - (2*b^3*exp(x))/((a + b)^2*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

3.702 $\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

3.702.1 Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{\tanh^2(x)}{2a(a+b\tanh(x))^2}$$

output
```
1/2*tanh(x)^2/a/(a+b*tanh(x))^2
```

3.702.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(19) = 38.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.84

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{a^2-b^2+b^2\cosh(2x)+ab\sinh(2x)}{2a(a-b)(a+b)(a\cosh(x)+b\sinh(x))^2}$$

input
```
Integrate[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

output
```
-1/2*(a^2 - b^2 + b^2*Cosh[2*x] + a*b*Sinh[2*x])/(a*(a - b)*(a + b)*(a*Cos
h[x] + b*Sinh[x])^2)
```

3.702.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 3566, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{(a\cos(ix)-ib\sin(ix))^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^3}dx$$

$$\Big\downarrow \text{3566}$$

$$-\int \frac{i\tanh(x)}{(a+b\tanh(x))^3}d(i\tanh(x))$$

$$\Big\downarrow \text{48}$$

$$\frac{\tanh^2(x)}{2a(a+b\tanh(x))^2}$$

input
```
Int[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

output
```
Tanh[x]^2/(2*a*(a + b*Tanh[x])^2)
```

3.702.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3566
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[1/d   Subst[Int[x^m*((a + b*
x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0
] && GtQ[m, 1])
```

3.702.4 Maple [A] (verified)

Time = 12.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

method	result	size
default	$\frac{2\tanh\left(\frac{x}{2}\right)^{2}}{a\left(\tanh\left(\frac{x}{2}\right)^{2}a+2b\tanh\left(\frac{x}{2}\right)+a\right)^{2}}$	31
risch	$-\frac{2\left(a\,e^{2x}+b\,e^{2x}-b\right)}{\left(a\,e^{2x}+b\,e^{2x}+a-b\right)^{2}\left(a+b\right)^{2}}$	43

input
```
int(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2/a*tanh(1/2*x)^2/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)^2
```

3.702.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(17) = 34.

Time = 0.24 (sec) , antiderivative size = 216, normalized size of antiderivative = 11.37

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx =$$

$$-\frac{}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\cosh{(x)}^3+3\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\cosh{(x)}\sinh{(x)}^2+(a}$$

input
```
integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

output
```
-2*(a*cosh(x) + (a + 2*b)*sinh(x))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
 b^4)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)*si
nh(x)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(x)^3 + (3*a^4 +
 4*a^3*b - 2*a^2*b^2 - 4*a*b^3 - b^4)*cosh(x) + (a^4 + 4*a^3*b + 2*a^2*b^2
 - 4*a*b^3 - 3*b^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)
^2)*sinh(x))
```

3.702.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)/(a*cosh(x)+b*sinh(x))**3,x)
```

output
```
Timed out
```

3.702.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(17) = 34$.

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 8.79

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{2\,(a-b)e^{(-2\,x)}}{a^4-2\,a^2b^2+b^4+2\,(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,x)}+(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)e^{(-4\,x)}} - \frac{2\,b}{a^4-2\,a^2b^2+b^4+2\,(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,x)}+(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)e^{(-4\,x)}}$$

input
```
integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

output
```
-2*(a - b)*e^(-2*x)/(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 -
b^4)*e^(-2*x) + (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x)) - 2*
b/(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x) + (a
^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x))
```

3.702.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.63

$$\int \frac{\sinh(x)}{(a\cosh(x) + b\sinh(x))^3}\,dx = -\frac{2\left(ae^{(2x)} + be^{(2x)} - b\right)}{(a^2 + 2\,ab + b^2)(ae^{(2x)} + be^{(2x)} + a - b)^2}$$

input
```
integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

output
```
-2*(a*e^(2*x) + b*e^(2*x) - b)/((a^2 + 2*a*b + b^2)*(a*e^(2*x) + b*e^(2*x)
 + a - b)^2)
```

3.702.9 Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{\sinh(x)}{(a\cosh(x) + b\sinh(x))^3}\,dx = \frac{2\,b - \mathrm{e}^{2\,x}\,(2\,a + 2\,b)}{(a + b)^2\,(a - b + a\,\mathrm{e}^{2\,x} + b\,\mathrm{e}^{2\,x})^2}$$

input
```
int(sinh(x)/(a*cosh(x) + b*sinh(x))^3,x)
```

output
```
(2*b - exp(2*x)*(2*a + 2*b))/((a + b)^2*(a - b + a*exp(2*x) + b*exp(2*x))^
2)
```

3.703 $\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

3.703.1 Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{b(3a^2+b^2)\,x}{(a^2-b^2)^3} - \frac{a}{2(a^2-b^2)(b+a\coth(x))^2} + \frac{2ab}{(a^2-b^2)^2(b+a\coth(x))} + \frac{a(a^2+3b^2)\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}$$

output
```
-b*(3*a^2+b^2)*x/(a^2-b^2)^3-1/2*a/(a^2-b^2)/(b+a*coth(x))^2+2*a*b/(a^2-b^
2)^2/(b+a*coth(x))+a*(a^2+3*b^2)*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3
```

3.703.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{b(3a^2+b^2)\,x}{(a-b)^3(a+b)^3} + \frac{(a^3+3ab^2)\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} + \frac{a^3}{2(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))^2} + \frac{3ab\sinh(x)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))}$$

input
```
Integrate[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

output
```
-((b*(3*a^2 + b^2)*x)/((a - b)^3*(a + b)^3)) + ((a^3 + 3*a*b^2)*Log[a*Cosh
[x] + b*Sinh[x]])/(a^2 - b^2)^3 + a^3/(2*(a - b)^2*(a + b)^2*(a*Cosh[x] +
b*Sinh[x])^2) + (3*a*b*Sinh[x])/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x
]))
```

3.703.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.35, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {3042, 26, 3564, 3042, 3964, 26, 3042, 4012, 3042, 4014, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ix)^3}{(a\cos(ix)-ib\sin(ix))^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ix)^3}{(a\cos(ix)-ib\sin(ix))^3}dx$$

$$\Big\downarrow \text{3564}$$

$$i\int \frac{1}{(-ib-ia\coth(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3964}$$

$$i\left(\frac{\int \frac{i(b-a\coth(x))}{(b+a\coth(x))^2}dx}{a^2-b^2}+\frac{ia}{2(a^2-b^2)(a\coth(x)+b)^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\int \frac{b-a\coth(x)}{(b+a\coth(x))^2}dx}{a^2-b^2}+\frac{ia}{2(a^2-b^2)(a\coth(x)+b)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\int \frac{b+ia\tan\left(ix+\frac{\pi}{2}\right)}{\left(b-ia\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}+\frac{ia}{2(a^2-b^2)(a\coth(x)+b)^2}\right)$$

$$\Big\downarrow \text{4012}$$

$$i\left(\frac{i\left(-\frac{\int \frac{a^2-2b\coth(x)a+b^2}{b+a\coth(x)}dx}{a^2-b^2}-\frac{2ab}{(a^2-b^2)(a\coth(x)+b)}\right)}{a^2-b^2}+\frac{ia}{2(a^2-b^2)(a\coth(x)+b)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\left(-\frac{2ab}{(a^2-b^2)(a\coth(x)+b)}-\frac{\int \frac{a^2+2ib\tan\left(ix+\frac{\pi}{2}\right)a+b^2}{b-ia\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia}{2(a^2-b^2)(a\coth(x)+b)^2}\right)$$

$$\Big\downarrow \text{4014}$$

$$i\left(\frac{i\left(-\frac{2ab}{(a^2-b^2)(a\coth(x)+b)}-\frac{-\frac{bx\left(3a^2+b^2\right)}{a^2-b^2}+\frac{ia\left(a^2+3b^2\right)\int -\frac{i(a+b\coth(x))}{b+a\coth(x)}dx}{a^2-b^2}}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia}{2(a^2-b^2)(a\coth(x)+b)^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\left(-\frac{\frac{a\left(a^2+3b^2\right)\int \frac{a+b\coth(x)}{b+a\coth(x)}dx}{a^2-b^2}-\frac{bx\left(3a^2+b^2\right)}{a^2-b^2}}{a^2-b^2}-\frac{2ab}{(a^2-b^2)(a\coth(x)+b)}\right)}{a^2-b^2}+\frac{ia}{2(a^2-b^2)(a\coth(x)+b)^2}\right)$$

$$\Big\downarrow \text{3042}$$

3.703. $\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

$$i\left(\frac{i\left(-\frac{2ab}{(a^2-b^2)(a\coth(x)+b)}-\frac{-\frac{bx(3a^2+b^2)}{a^2-b^2}+\frac{a(a^2+3b^2)\int\frac{a-ib\tan\left(ix+\frac{\pi}{2}\right)}{b-ia\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia}{2\left(a^2-b^2\right)\left(a\coth(x)+b\right)^2}\right)$$

$\Big\downarrow$ 4013

$$i\left(\frac{ia}{2\left(a^2-b^2\right)\left(a\coth(x)+b\right)^2}+\frac{i\left(-\frac{2ab}{(a^2-b^2)(a\coth(x)+b)}-\frac{\frac{a(a^2+3b^2)\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{bx(3a^2+b^2)}{a^2-b^2}}{a^2-b^2}\right)}{a^2-b^2}\right)$$

input
```
Int[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

output
```
I*(((I/2)*a)/((a^2 - b^2)*(b + a*Coth[x])^2) + (I*((-2*a*b)/((a^2 - b^2)*(
b + a*Coth[x])) - (-((b*(3*a^2 + b^2)*x)/(a^2 - b^2)) + (a*(a^2 + 3*b^2)*L
og[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2))/(a^2 - b^2)))/(a^2 - b^2))
```

3.703.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3564
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

3.703. $\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.703.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{(a+b)^{3}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{(a-b)^{3}}+\frac{2a\left(\frac{2ba\left(a^{2}-b^{2}\right)\tanh\left(\frac{x}{2}\right)^{3}+\left(-a^{4}+6a^{2}b^{2}-5b^{4}\right)\tanh\left(\frac{x}{2}\right)^{2}+2ba\left(a^{2}-b^{2}\right)\tanh\left(\frac{x}{2}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}a+2b\tanh\left(\frac{x}{2}\right)+a\right)^{2}}+\frac{(a^{2}}{}\right.}{(a+b)^{3}(a-b)^{3}}$
parallelrisch	$\frac{2b^{2}a\left(a^{2}+3b^{2}\right)\left(a+b\tanh(x)\right)^{2}\ln\left(a+b\tanh(x)\right)-2b^{2}a\left(a^{2}+3b^{2}\right)\left(a+b\tanh(x)\right)^{2}\ln\left(1-\tanh(x)\right)+\left(-2x\,b^{4}(a+b)^{2}\tanh(x)^{2}+2(-\right.}{2(a-b)^{3}(a+b)^{3}b^{2}(a+b}$
risch	$\frac{x}{a^{3}+3a^{2}b+3a\,b^{2}+b^{3}}-\frac{2a^{3}x}{a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}}-\frac{6ax\,b^{2}}{a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}}+\frac{2a^{2}\left(a^{2}\mathrm{e}^{2x}-2b\,\mathrm{e}^{2x}a-3b^{2}\mathrm{e}^{2x}-3ab+3b^{2}\right)}{(a-b)^{2}\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)\left(a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}+a-b\right)^{2}}$

input
```
int(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/(a+b)^3*ln(tanh(1/2*x)-1)-1/(a-b)^3*ln(tanh(1/2*x)+1)+2*a/(a+b)^3/(a-b)
^3*((2*b*a*(a^2-b^2)*tanh(1/2*x)^3+(-a^4+6*a^2*b^2-5*b^4)*tanh(1/2*x)^2+2*
b*a*(a^2-b^2)*tanh(1/2*x))/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)^2+1/2*(a^2+
3*b^2)*ln(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a))
```

3.703.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(102) = 204$.

Time = 0.29 (sec) , antiderivative size = 1268, normalized size of antiderivative = 12.19

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

output
```
-((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^4 +
4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)*sinh
(x)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*sinh(x
)^4 + 6*a^4*b - 12*a^3*b^2 + 6*a^2*b^3 - 2*(a^5 - 3*a^4*b - a^3*b^2 + 3*a^
2*b^3 - (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*cosh(x)
^2 - 2*(a^5 - 3*a^4*b - a^3*b^2 + 3*a^2*b^3 - 3*(a^5 + 5*a^4*b + 10*a^3*b^
2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^2 - (a^5 + 3*a^4*b + 2*a^3*b^2 -
 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*sinh(x)^2 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^
2*b^3 + a*b^4 + b^5)*x - (a^5 - 2*a^4*b + 4*a^3*b^2 - 6*a^2*b^3 + 3*a*b^4
+ (a^5 + 2*a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 3*a*b^4)*cosh(x)^4 + 4*(a^5 + 2
*a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 3*a*b^4)*cosh(x)*sinh(x)^3 + (a^5 + 2*a^4
*b + 4*a^3*b^2 + 6*a^2*b^3 + 3*a*b^4)*sinh(x)^4 + 2*(a^5 + 2*a^3*b^2 - 3*a
*b^4)*cosh(x)^2 + 2*(a^5 + 2*a^3*b^2 - 3*a*b^4 + 3*(a^5 + 2*a^4*b + 4*a^3*
b^2 + 6*a^2*b^3 + 3*a*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^4*b + 4*a^
3*b^2 + 6*a^2*b^3 + 3*a*b^4)*cosh(x)^3 + (a^5 + 2*a^3*b^2 - 3*a*b^4)*cosh(
x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 4*((a^5
+ 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^3 - (a^5 -
3*a^4*b - a^3*b^2 + 3*a^2*b^3 - (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3
*a*b^4 - b^5)*x)*cosh(x))*sinh(x))/(a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3
- 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8 + (a^8 + 2*a^7*b - 2*a^6*b^2 - ...
```

3.703.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3813 vs. $2(88) = 176$.

Time = 1.86 (sec) , antiderivative size = 3813, normalized size of antiderivative = 36.66

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)**3/(a*cosh(x)+b*sinh(x))**3,x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b**3, Eq(a, 0)), (-3*x*sinh(x)*
*3/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cos
h(x)**2 + 24*b**3*cosh(x)**3) + 9*x*sinh(x)**2*cosh(x)/(-24*b**3*sinh(x)**
3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh
(x)**3) - 9*x*sinh(x)*cosh(x)**2/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2
*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 3*x*cosh(x)*
*3/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cos
h(x)**2 + 24*b**3*cosh(x)**3) - 7*sinh(x)**3/(-24*b**3*sinh(x)**3 + 72*b**
3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) +
6*sinh(x)*cosh(x)**2/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 7
2*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 3*cosh(x)**3/(-24*b**3*s
inh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b
**3*cosh(x)**3), Eq(a, -b)), (3*x*sinh(x)**3/(24*b**3*sinh(x)**3 + 72*b**3
*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 9
*x*sinh(x)**2*cosh(x)/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 7
2*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 9*x*sinh(x)*cosh(x)**2/(
24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)*
*2 + 24*b**3*cosh(x)**3) + 3*x*cosh(x)**3/(24*b**3*sinh(x)**3 + 72*b**3*si
nh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 7*si
nh(x)**3/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sin...
```

3.703.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(102) = 204$.

Time = 0.22 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.78

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{(a^3+3\,ab^2)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}$$
$$+\frac{2\left(3\,a^3b+3\,a^2b^2+(a^4+2\,a^3b-3\right.}{a^7+a^6b-3\,a^5b^2-3\,a^4b^3+3\,a^3b^4+3\,a^2b^5-ab^6-b^7+2\left(a^7-a^6b-3\,a^5b^2+3\,a^4b^3+3\,a^3b^4-3\,a^2b^5\right.}$$
$$+\frac{x}{a^3+3\,a^2b+3\,ab^2+b^3}$$

input
```
integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

output
```
(a^3 + 3*a*b^2)*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) + 2*(3*a^3*b + 3*a^2*b^2 + (a^4 + 2*a^3*b - 3*a^2*b^2)*e^(-2*x))/
(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7
 + 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
+ b^7)*e^(-2*x) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b
^5 + 3*a*b^6 - b^7)*e^(-4*x)) + x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.703.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(102) = 204$.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.41

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{(a^3+3\,ab^2)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{x}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{3\,a^4e^{(4\,x)}+3\,a^3be^{(4\,x)}+9\,a^2b^2e^{(4\,x)}+9\,ab^3e^{(4\,x)}+2\,a^4e^{(2\,x)}+10\,a^3be^{(2\,x)}+6\,a^2b^2e^{(2\,x)}-18\,ab^3e^{(2\,x)}+3}{2\,(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5)(ae^{(2\,x)}+be^{(2\,x)}+a-b)^2}$$

input
```
integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

output
```
(a^3 + 3*a*b^2)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6) - x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/2*(3*a^4*e^(4*x)
 + 3*a^3*b*e^(4*x) + 9*a^2*b^2*e^(4*x) + 9*a*b^3*e^(4*x) + 2*a^4*e^(2*x) +
 10*a^3*b*e^(2*x) + 6*a^2*b^2*e^(2*x) - 18*a*b^3*e^(2*x) + 3*a^4 + 3*a^3*b
 - 15*a^2*b^2 + 9*a*b^3)/((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b
^5)*(a*e^(2*x) + b*e^(2*x) + a - b)^2)
```

3.703.9 Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)\,\left(a^3+3\,a\,b^2\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{x}{{\left(a-b\right)}^3}-\frac{2\,\left(3\,a^2\,b-a^3\right)}{{\left(a+b\right)}^3\,{\left(a-b\right)}^2\,\left(a-b+e^{2\,x}\,\left(a+b\right)\right)}-\frac{2\,a^3}{{\left(a+b\right)}^3\,\left(a-b\right)\,\left(e^{4\,x}\,{\left(a+b\right)}^2+{\left(a-b\right)}^2+2\,e^{2\,x}\,\left(a+b\right)\,\left(a-b\right)\right)}$$

input
```
int(sinh(x)^3/(a*cosh(x) + b*sinh(x))^3,x)
```

output
```
(log(a - b + a*exp(2*x) + b*exp(2*x))*(3*a*b^2 + a^3))/(a^6 - b^6 + 3*a^2*
b^4 - 3*a^4*b^2) - x/(a - b)^3 - (2*(3*a^2*b - a^3))/((a + b)^3*(a - b)^2*
(a - b + exp(2*x)*(a + b))) - (2*a^3)/((a + b)^3*(a - b)*(exp(4*x)*(a + b)
^2 + (a - b)^2 + 2*exp(2*x)*(a + b)*(a - b)))
```

3.704 $\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

3.704.1 Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{\coth^2(x)}{2b(b+a\coth(x))^2}$$

output
```
-1/2*coth(x)^2/b/(b+a*coth(x))^2
```

3.704.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{b\cosh(2x)+a\sinh(2x)}{2(a-b)(a+b)(a\cosh(x)+b\sinh(x))^2}$$

input
```
Integrate[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

output
```
(b*Cosh[2*x] + a*Sinh[2*x])/(2*(a - b)*(a + b)*(a*Cosh[x] + b*Sinh[x])^2)
```

3.704.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3567, 25, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^3}dx$$
$$\Big\downarrow \text{ 3567}$$
$$i\int \frac{i\coth(x)}{(ib+ia\coth(x))^3}d(-i\coth(x))$$
$$\Big\downarrow \text{ 25}$$
$$-i\int -\frac{i\coth(x)}{(ib+ia\coth(x))^3}d(-i\coth(x))$$
$$\Big\downarrow \text{ 48}$$
$$\frac{\coth^2(x)}{2b(ia\coth(x)+ib)^2}$$

input
```
Int[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

output
```
Coth[x]^2/(2*b*(I*b + I*a*Coth[x])^2)
```

3.704.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.704.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.

Time = 5.86 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

method	result	size
risch	$-\frac{2\left(a\,{\rm e}^{2x}+b\,{\rm e}^{2x}+a\right)}{\left(a\,{\rm e}^{2x}+b\,{\rm e}^{2x}+a-b\right)^{2}\left(a+b\right)^{2}}$	41
default	$-\frac{2\left(-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{a}-\frac{b\tanh\left(\frac{x}{2}\right)^{2}}{a^{2}}-\frac{\tanh\left(\frac{x}{2}\right)}{a}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}a+2b\tanh\left(\frac{x}{2}\right)+a\right)^{2}}$	55

input
```
int(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(a*exp(2*x)+b*exp(2*x)+a)/(a*exp(2*x)+b*exp(2*x)+a-b)^2/(a+b)^2
```

3.704.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 11.37

$$\int \frac{\cosh(x)}{(a\cosh(x) + b\sinh(x))^3}\,dx =$$

$$-\frac{}{(a^4 + 4\,a^3b + 6\,a^2b^2 + 4\,ab^3 + b^4)\cosh{(x)}^3 + 3\,(a^4 + 4\,a^3b + 6\,a^2b^2 + 4\,ab^3 + b^4)\cosh{(x)}\sinh{(x)}^2 + (a}$$

input
```
integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

output
```
-2*((2*a + b)*cosh(x) + b*sinh(x))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
 b^4)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)*si
nh(x)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(x)^3 + (3*a^4 +
 4*a^3*b - 2*a^2*b^2 - 4*a*b^3 - b^4)*cosh(x) + (a^4 + 4*a^3*b + 2*a^2*b^2
 - 4*a*b^3 - 3*b^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)
^2)*sinh(x))
```

3.704.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{(a\cosh(x) + b\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)/(a*cosh(x)+b*sinh(x))**3,x)
```

output
```
Timed out
```

3.704.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(17) = 34$.

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 8.79

$$\int \frac{\cosh(x)}{(a\cosh(x) + b\sinh(x))^3}\,dx$$

$$= \frac{2\,(a-b)e^{(-2\,x)}}{a^4 - 2\,a^2b^2 + b^4 + 2\,(a^4 - 2\,a^3b + 2\,ab^3 - b^4)e^{(-2\,x)} + (a^4 - 4\,a^3b + 6\,a^2b^2 - 4\,ab^3 + b^4)e^{(-4\,x)}}$$
$$+ \frac{2\,a}{a^4 - 2\,a^2b^2 + b^4 + 2\,(a^4 - 2\,a^3b + 2\,ab^3 - b^4)e^{(-2\,x)} + (a^4 - 4\,a^3b + 6\,a^2b^2 - 4\,ab^3 + b^4)e^{(-4\,x)}}$$

input
```
integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

output
```
2*(a - b)*e^(-2*x)/(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b
^4)*e^(-2*x) + (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x)) + 2*a
/(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x) + (a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x))
```

3.704.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{2\left(ae^{(2x)}+be^{(2x)}+a\right)}{(a^2+2\,ab+b^2)\left(ae^{(2x)}+be^{(2x)}+a-b\right)^2}$$

input
```
integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

output
```
-2*(a*e^(2*x) + b*e^(2*x) + a)/((a^2 + 2*a*b + b^2)*(a*e^(2*x) + b*e^(2*x)
 + a - b)^2)
```

3.704.9 Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{2\,a+e^{2\,x}\,(2\,a+2\,b)}{(a+b)^2\,\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)^2}$$

input
```
int(cosh(x)/(a*cosh(x) + b*sinh(x))^3,x)
```

output
```
-(2*a + exp(2*x)*(2*a + 2*b))/((a + b)^2*(a - b + a*exp(2*x) + b*exp(2*x))
^2)
```

3.705 $\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

3.705.1 Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3} - \frac{b(3a^2+b^2)\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} + \frac{b}{2\,(a^2-b^2)\,(a+b\tanh(x))^2} + \frac{2ab}{(a^2-b^2)^2\,(a+b\tanh(x))}$$

output
```
a*(a^2+3*b^2)*x/(a^2-b^2)^3-b*(3*a^2+b^2)*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2
)^3+1/2*b/(a^2-b^2)/(a+b*tanh(x))^2+2*a*b/(a^2-b^2)^2/(a+b*tanh(x))
```

3.705.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{a(a^2+3b^2)\,x}{(a-b)^3(a+b)^3} + \frac{(-3a^2b-b^3)\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} - \frac{b^3}{2(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))^2} - \frac{3b^2\sinh(x)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))}$$

input
```
Integrate[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

output
```
(a*(a^2 + 3*b^2)*x)/((a - b)^3*(a + b)^3) + ((-3*a^2*b - b^3)*Log[a*Cosh[x
] + b*Sinh[x]])/(a^2 - b^2)^3 - b^3/(2*(a - b)^2*(a + b)^2*(a*Cosh[x] + b*
Sinh[x])^2) - (3*b^2*Sinh[x])/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])
)
```

3.705.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 3565, 3042, 3964, 3042, 4012, 3042, 4014, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ix)^3}{(a\cos(ix)-ib\sin(ix))^3}dx$$

$$\Big\downarrow \; 3565$$

$$\int \frac{1}{(a+b\tanh(x))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(a-ib\tan(ix))^3}dx$$

$$\Big\downarrow \; 3964$$

$$\frac{\int \frac{a-b\tanh(x)}{(a+b\tanh(x))^2}dx}{a^2-b^2}+\frac{b}{2(a^2-b^2)(a+b\tanh(x))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{b}{2(a^2-b^2)(a+b\tanh(x))^2}+\frac{\int \frac{a+ib\tan(ix)}{(a-ib\tan(ix))^2}dx}{a^2-b^2}$$

$$\Big\downarrow \; 4012$$

$$\frac{\frac{\int \frac{a^2-2b\tanh(x)a+b^2}{a+b\tanh(x)}dx}{a^2-b^2}+\frac{2ab}{(a^2-b^2)(a+b\tanh(x))}}{a^2-b^2}+\frac{b}{2(a^2-b^2)(a+b\tanh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b}{2(a^2-b^2)(a+b\tanh(x))^2}+\frac{\frac{2ab}{(a^2-b^2)(a+b\tanh(x))}+\frac{\int \frac{a^2+2ib\tan(ix)a+b^2}{a-ib\tan(ix)}dx}{a^2-b^2}}{a^2-b^2}$$

$$\Big\downarrow \text{4014}$$

$$\frac{b}{2(a^2-b^2)(a+b\tanh(x))^2}+\frac{\frac{2ab}{(a^2-b^2)(a+b\tanh(x))}+\frac{\frac{ax(a^2+3b^2)}{a^2-b^2}-\frac{ib(3a^2+b^2)\int -\frac{i(b+a\tanh(x))}{a+b\tanh(x)}dx}{a^2-b^2}}{a^2-b^2}}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{\frac{ax(a^2+3b^2)}{a^2-b^2}-\frac{b(3a^2+b^2)\int \frac{b+a\tanh(x)}{a+b\tanh(x)}dx}{a^2-b^2}}{a^2-b^2}+\frac{2ab}{(a^2-b^2)(a+b\tanh(x))}}{a^2-b^2}+\frac{b}{2(a^2-b^2)(a+b\tanh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b}{2(a^2-b^2)(a+b\tanh(x))^2}+\frac{\frac{2ab}{(a^2-b^2)(a+b\tanh(x))}+\frac{\frac{ax(a^2+3b^2)}{a^2-b^2}-\frac{b(3a^2+b^2)\int \frac{b-ia\tan(ix)}{a-ib\tan(ix)}dx}{a^2-b^2}}{a^2-b^2}}{a^2-b^2}$$

$$\Big\downarrow \text{4013}$$

$$\frac{b}{2(a^2-b^2)(a+b\tanh(x))^2}+\frac{\frac{2ab}{(a^2-b^2)(a+b\tanh(x))}+\frac{\frac{ax(a^2+3b^2)}{a^2-b^2}-\frac{b(3a^2+b^2)\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}}{a^2-b^2}}{a^2-b^2}$$

input
```
Int[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

output
```
b/(2*(a^2 - b^2)*(a + b*Tanh[x])^2) + (((a*(a^2 + 3*b^2)*x)/(a^2 - b^2) -
(b*(3*a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2))/(a^2 - b^2) + (2
*a*b)/((a^2 - b^2)*(a + b*Tanh[x])))/(a^2 - b^2)
```

3.705.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.705. $\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

rule 3565
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.705. $\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

3.705.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.52

method	result
parallelrisch	$\frac{-3(a+b\tanh(x))^2 b\left(a^2+\frac{b^2}{3}\right)\ln(a+b\tanh(x))+3(a+b\tanh(x))^2 b\left(a^2+\frac{b^2}{3}\right)\ln(1-\tanh(x))+(a+b)\left(x\,b^2(a+b)^2\tanh(x)^2+2(b^2\right.}{(a-b)^3(a+b)^3(a+b\tanh(x))^2}$
default	$-\frac{\ln(\tanh(\frac{x}{2})-1)}{(a+b)^3}+\frac{\ln(\tanh(\frac{x}{2})+1)}{(a-b)^3}-\frac{2b\left(\frac{\frac{b(3a^4-4a^2b^2+b^4)\tanh(\frac{x}{2})^3}{a}+\frac{b^2(5a^4-6a^2b^2+b^4)\tanh(\frac{x}{2})^2}{a^2}+\frac{b(3a^4-4a^2b^2+b^4)\,t}{a}}{\left(\tanh(\frac{x}{2})^2 a+2b\tanh(\frac{x}{2})+a\right)^2}\right.}{(a+b)^3(a-b)^3}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}+\frac{6b\,a^2x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{2b^3x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{2b^2(3a^2e^{2x}+2b\,e^{2x}a-b^2e^{2x}+3a^2-3ab)}{(a-b)^2(a^3+3a^2b+3a\,b^2+b^3)(a\,e^{2x}+b\,e^{2x}+a-b)^2}$

input
```
int(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
(-3*(a+b*tanh(x))^2*b*(a^2+1/3*b^2)*ln(a+b*tanh(x))+3*(a+b*tanh(x))^2*b*(a
^2+1/3*b^2)*ln(1-tanh(x))+(a+b)*(x*b^2*(a+b)^2*tanh(x)^2+2*(b^2*(-1+x)+a*(
1+2*x)*b+a^2*x)*b*a*tanh(x)+1/2*b^4-1/2*a*b^3+a^2*(x-5/2)*b^2+2*a^3*(x+5/4
)*b+x*a^4))/(a-b)^3/(a+b)^3/(a+b*tanh(x))^2
```

3.705.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1269 vs. 2(102) = 204.

Time = 0.26 (sec) , antiderivative size = 1269, normalized size of antiderivative = 12.20

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

output
```
((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^4 + 4
*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)*sinh(
x)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*sinh(x)
^4 + 6*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 + 2*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 +
 b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*cosh(x)^
2 + 2*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5 + 3*(a^5 + 5*a^4*b + 10*a^3*b^2
 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 -
2*a^2*b^3 - 3*a*b^4 - b^5)*x)*sinh(x)^2 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2
*b^3 + a*b^4 + b^5)*x - (3*a^4*b - 6*a^3*b^2 + 4*a^2*b^3 - 2*a*b^4 + b^5 +
 (3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)^4 + 4*(3*a^4*b
+ 6*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)*sinh(x)^3 + (3*a^4*b + 6*
a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*sinh(x)^4 + 2*(3*a^4*b - 2*a^2*b^3 -
b^5)*cosh(x)^2 + 2*(3*a^4*b - 2*a^2*b^3 - b^5 + 3*(3*a^4*b + 6*a^3*b^2 + 4
*a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 + 4*((3*a^4*b + 6*a^3*b^2 +
 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)^3 + (3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x
))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 4*((a^5 +
 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^3 + (3*a^3*b
^2 - a^2*b^3 - 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*
a*b^4 - b^5)*x)*cosh(x))*sinh(x))/(a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 -
 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6...
```

3.705.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3840 vs. $2(88) = 176$.

Time = 1.77 (sec) , antiderivative size = 3840, normalized size of antiderivative = 36.92

$$\int \frac{\cosh^3(x)}{(a\cosh(x) + b\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)**3/(a*cosh(x)+b*sinh(x))**3,x)
```

output
```
Piecewise((zoo*(log(sinh(x)) - cosh(x)**2/(2*sinh(x)**2)), Eq(a, 0) & Eq(b
, 0)), ((log(sinh(x)) - cosh(x)**2/(2*sinh(x)**2))/b**3, Eq(a, 0)), (3*x*s
inh(x)**3/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh
(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 9*x*sinh(x)**2*cosh(x)/(-24*b**3*si
nh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b*
*3*cosh(x)**3) + 9*x*sinh(x)*cosh(x)**2/(-24*b**3*sinh(x)**3 + 72*b**3*sin
h(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 3*x*c
osh(x)**3/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh
(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - sinh(x)**3/(-24*b**3*sinh(x)**3 + 7
2*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**
3) + 6*sinh(x)*cosh(x)**2/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x
) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 9*cosh(x)**3/(-24*b
**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 +
 24*b**3*cosh(x)**3), Eq(a, -b)), (3*x*sinh(x)**3/(24*b**3*sinh(x)**3 + 72
*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3
) + 9*x*sinh(x)**2*cosh(x)/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x
) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 9*x*sinh(x)*cosh(x)
**2/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cos
h(x)**2 + 24*b**3*cosh(x)**3) + 3*x*cosh(x)**3/(24*b**3*sinh(x)**3 + 72*b*
*3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3...
```

3.705.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(102) = 204$.

Time = 0.21 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.81

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{(3\,a^2b+b^3)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{2\left(3\,a^2b^2+3\,ab^3+\left(3\,a^2b^2-2\,ab^3\right.\right.}{a^7+a^6b-3\,a^5b^2-3\,a^4b^3+3\,a^3b^4+3\,a^2b^5-ab^6-b^7+2\left(a^7-a^6b-3\,a^5b^2+3\,a^4b^3+3\,a^3b^4-3\,a^2b^5\right.} + \frac{x}{a^3+3\,a^2b+3\,ab^2+b^3}$$

input
```
integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

output
```
-(3*a^2*b + b^3)*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6) - 2*(3*a^2*b^2 + 3*a*b^3 + (3*a^2*b^2 - 2*a*b^3 - b^4)*e^(-2*x))
/(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^
7 + 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
 + b^7)*e^(-2*x) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*
b^5 + 3*a*b^6 - b^7)*e^(-4*x)) + x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.705.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(102) = 204$.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.41

$$\int \frac{\cosh^3(x)}{(a\cosh(x) + b\sinh(x))^3}\,dx$$
$$= -\frac{(3\,a^2b + b^3)\log\left(\left|ae^{(2\,x)} + be^{(2\,x)} + a - b\right|\right)}{a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6} + \frac{x}{a^3 - 3\,a^2b + 3\,ab^2 - b^3}$$
$$+ \frac{9\,a^3be^{(4\,x)} + 9\,a^2b^2e^{(4\,x)} + 3\,ab^3e^{(4\,x)} + 3\,b^4e^{(4\,x)} + 18\,a^3be^{(2\,x)} - 6\,a^2b^2e^{(2\,x)} - 10\,ab^3e^{(2\,x)} - 2\,b^4e^{(2\,x)} + 9\,a}{2\,(a^5 - a^4b - 2\,a^3b^2 + 2\,a^2b^3 + ab^4 - b^5)(ae^{(2\,x)} + be^{(2\,x)} + a - b)^2}$$

input
```
integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

output
```
-(3*a^2*b + b^3)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2
+ 3*a^2*b^4 - b^6) + x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/2*(9*a^3*b*e^(4
*x) + 9*a^2*b^2*e^(4*x) + 3*a*b^3*e^(4*x) + 3*b^4*e^(4*x) + 18*a^3*b*e^(2*
x) - 6*a^2*b^2*e^(2*x) - 10*a*b^3*e^(2*x) - 2*b^4*e^(2*x) + 9*a^3*b - 15*a
^2*b^2 + 3*a*b^3 + 3*b^4)/((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5)*(a*e^(2*x) + b*e^(2*x) + a - b)^2)
```

3.705.9 Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.53

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$$
$$= \frac{x}{(a-b)^3} - \frac{\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)\,\left(3\,a^2\,b+b^3\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}$$
$$+ \frac{2\,\left(3\,a\,b^2-b^3\right)}{(a+b)^3\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)}$$
$$+ \frac{2\,b^3}{(a+b)^3\,(a-b)\,\left(e^{4\,x}\,(a+b)^2+(a-b)^2+2\,e^{2\,x}\,(a+b)\,(a-b)\right)}$$

input
```
int(cosh(x)^3/(a*cosh(x) + b*sinh(x))^3,x)
```

output
```
x/(a - b)^3 - (log(a - b + a*exp(2*x) + b*exp(2*x))*(3*a^2*b + b^3))/(a^6
- b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (2*(3*a*b^2 - b^3))/((a + b)^3*(a - b)^2*
(a - b + exp(2*x)*(a + b))) + (2*b^3)/((a + b)^3*(a - b)*(exp(4*x)*(a + b)
^2 + (a - b)^2 + 2*exp(2*x)*(a + b)*(a - b)))
```

3.706 $\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.706.1 Optimal result

Integrand size = 16, antiderivative size = 72

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{ab\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}$$

output
```
a*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+a*cosh(x
)/(a^2-b^2)-b*sinh(x)/(a^2-b^2)
```

3.706.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{2ab\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{a\cosh(x)}{a^2-b^2}+\frac{b\sinh(x)}{-a^2+b^2}$$

input
```
Integrate[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(2*a*b*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)
*(a + b)^(3/2)) + (a*Cosh[x])/(a^2 - b^2) + (b*Sinh[x])/(-a^2 + b^2)
```

3.706.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 26, 3588, 26, 3042, 26, 3117, 3118, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)\cosh(x)}{a\cosh(x)+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3588}$$

$$-i\left(\frac{a\int i\sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ia\int \sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{ia\int -i\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a\int \sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3117}$$

$$-i\left(\frac{a\int \sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3118}$$

$$-i\left(\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3553}$$

$$-i\left(-\frac{ab\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(-\frac{a\text{barctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)$$

input
```
Int[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(-I)*(-((a*b*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(3/2)) + (I*a*Cosh[x])/(a^2 - b^2) - (I*b*Sinh[x])/(a^2 - b^2))
```

3.706.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.706. $\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

3.706.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{2ab\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a+b)(a-b)\sqrt{a^2-b^2}}+\frac{4}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{4}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	92
risch	$\frac{e^{x}}{2a+2b}+\frac{e^{-x}}{2a-2b}-\frac{ba\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}+\frac{ba\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	120

input
```
int(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*a*b/(a+b)/(a-b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^
2)^(1/2))+4/(4*a-4*b)/(tanh(1/2*x)+1)-4/(4*a+4*b)/(tanh(1/2*x)-1)
```

3.706.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(68) = 136.

Time = 0.28 (sec) , antiderivative size = 427, normalized size of antiderivative = 5.93

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$$

$$= \left[\frac{a^3+a^2b-ab^2-b^3+\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)^2+2\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)\sinh(x)+\left(a^3-\right.}{2\left(\left(a^4-2\right.\right.}\right.$$

input
```
integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
[1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 +
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3 - a^2*b - a*b^2 + b^3
)*sinh(x)^2 + 2*(a*b*cosh(x) + a*b*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*
cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 +
b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*s
inh(x) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (
a^4 - 2*a^2*b^2 + b^4)*sinh(x)), 1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a
^2*b - a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh
(x) + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(a*b*cosh(x) + a*b*sinh(x)
)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x
))))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

3.706.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(58) = 116$.

Time = 124.53 (sec) , antiderivative size = 678, normalized size of antiderivative = 9.42

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\sinh(x) \\ \frac{\sinh(x)}{b} \\ -\frac{\sinh^2(x)}{-3b\sinh(x)+3b\cosh(x)} + \frac{\sinh(x)\cosh(x)}{-3b\sinh(x)+3b\cosh(x)} - \frac{\cosh^2(x)}{-3b\sinh(x)+3b\cosh(x)} \\ \frac{\sinh^2(x)}{3b\sinh(x)+3b\cosh(x)} + \frac{\sinh(x)\cosh(x)}{3b\sinh(x)+3b\cosh(x)} + \frac{\cosh^2(x)}{3b\sinh(x)+3b\cosh(x)} \\ \frac{ab\log\left(\tanh\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)\tanh^2\left(\frac{x}{2}\right)}{a^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)-a^2\sqrt{-a^2+b^2}-b^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)+b^2\sqrt{-a^2+b^2}} - \frac{ab\log\left(\tanh\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)-a^2\sqrt{-a^2+b^2}-b^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right.} \end{cases}$$

input
```
integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Piecewise((zoo*sinh(x), Eq(a, 0) & Eq(b, 0)), (sinh(x)/b, Eq(a, 0)), (-sin
h(x)**2/(-3*b*sinh(x) + 3*b*cosh(x)) + sinh(x)*cosh(x)/(-3*b*sinh(x) + 3*b
*cosh(x)) - cosh(x)**2/(-3*b*sinh(x) + 3*b*cosh(x)), Eq(a, -b)), (sinh(x)*
*2/(3*b*sinh(x) + 3*b*cosh(x)) + sinh(x)*cosh(x)/(3*b*sinh(x) + 3*b*cosh(x
)) + cosh(x)**2/(3*b*sinh(x) + 3*b*cosh(x)), Eq(a, b)), (a*b*log(tanh(x/2)
 + b/a - sqrt(-a**2 + b**2)/a)*tanh(x/2)**2/(a**2*sqrt(-a**2 + b**2)*tanh(
x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 +
 b**2*sqrt(-a**2 + b**2)) - a*b*log(tanh(x/2) + b/a - sqrt(-a**2 + b**2)/a
)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*s
qrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) - a*b*log(tanh(x
/2) + b/a + sqrt(-a**2 + b**2)/a)*tanh(x/2)**2/(a**2*sqrt(-a**2 + b**2)*ta
nh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**
2 + b**2*sqrt(-a**2 + b**2)) + a*b*log(tanh(x/2) + b/a + sqrt(-a**2 + b**2
)/a)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**
2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) - 2*a*sqrt(-a
**2 + b**2)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2
) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) + 2*b*
sqrt(-a**2 + b**2)*tanh(x/2)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*
sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**
2 + b**2)), True))
```

3.706.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x) + b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.706.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{2\,ab\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}} + \frac{e^{(-x)}}{2\,(a-b)} + \frac{e^x}{2\,(a+b)}$$

input
```
integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
2*a*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 1/2*e^(-
x)/(a - b) + 1/2*e^x/(a + b)
```

3.706.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.18

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^x}{2\,a+2\,b} + \frac{e^{-x}}{2\,a-2\,b} + \frac{2\,\mathrm{atan}\left(\frac{a\,b\,e^x\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{a^3\,\sqrt{a^2\,b^2}+b^3\,\sqrt{a^2\,b^2}-a\,b^2\,\sqrt{a^2\,b^2}-a^2\,b\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}$$

input
```
int((cosh(x)*sinh(x))/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(x)/(2*a + 2*b) + exp(-x)/(2*a - 2*b) + (2*atan((a*b*exp(x)*(a^6 - b^6
+ 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a^3*(a^2*b^2)^(1/2) + b^3*(a^2*b^2)^(1/2)
 - a*b^2*(a^2*b^2)^(1/2) - a^2*b*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(a^6 -
 b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)
```

3.707 $\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.707.1 Optimal result

Integrand size = 18, antiderivative size = 102

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{ab^2x}{(a^2-b^2)^2}-\frac{ax}{2(a^2-b^2)}+\frac{a^2b\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}+\frac{a\cosh(x)\sinh(x)}{2(a^2-b^2)}-\frac{b\sinh^2(x)}{2(a^2-b^2)}$$

output
```
-a*b^2*x/(a^2-b^2)^2-1/2*a*x/(a^2-b^2)+a^2*b*ln(a*cosh(x)+b*sinh(x))/(a^2-
b^2)^2+1/2*a*cosh(x)*sinh(x)/(a^2-b^2)-1/2*b*sinh(x)^2/(a^2-b^2)
```

3.707.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{(-a^2b+b^3)\cosh(2x)+a(-2(a^2+b^2)x+4ab\log(a\cosh(x)+b\sinh(x))+(a^2-b^2)\sinh(2x))}{4(a-b)^2(a+b)^2}$$

input
```
Integrate[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
((-(a^2*b) + b^3)*Cosh[2*x] + a*(-2*(a^2 + b^2)*x + 4*a*b*Log[a*Cosh[x] +
b*Sinh[x]] + (a^2 - b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

3.707.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {3042, 25, 3588, 25, 26, 3042, 25, 26, 3044, 15, 3115, 24, 3576, 26, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ix)^2\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3588}$$

$$-\frac{a\int -\sinh^2(x)dx}{a^2-b^2}+\frac{ib\int i\cosh(x)\sinh(x)dx}{a^2-b^2}-\frac{iab\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\int \sinh^2(x)dx}{a^2-b^2}+\frac{ib\int i\cosh(x)\sinh(x)dx}{a^2-b^2}-\frac{iab\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{a\int \sinh^2(x)dx}{a^2-b^2}-\frac{b\int \cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{ab\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int -\sin(ix)^2dx}{a^2-b^2}-\frac{b\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{ab\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int \sin(ix)^2dx}{a^2-b^2}-\frac{b\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{ab\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \sin(ix)^2dx}{a^2-b^2}+\frac{ib\int \cos(ix)\sin(ix)dx}{a^2-b^2}-\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3044}$$

$$-\frac{a\int \sin(ix)^2dx}{a^2-b^2}+\frac{b\int i\sinh(x)d(i\sinh(x))}{a^2-b^2}-\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{a\int \sin(ix)^2dx}{a^2-b^2}-\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\sinh^2(x)}{2(a^2-b^2)}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}-\frac{b\sinh^2(x)}{2(a^2-b^2)}$$

$$\Big\downarrow \text{24}$$

$$-\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\sinh^2(x)}{2(a^2-b^2)}-\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3576}$$

$$-\frac{iab\left(-\frac{a\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\sinh^2(x)}{2(a^2-b^2)}-\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{iab\left(\frac{ia\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\sinh^2(x)}{2(a^2-b^2)}-\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{iab\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\sinh^2(x)}{2(a^2-b^2)}-\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3612}$$

$$-\frac{b\sinh^2(x)}{2(a^2-b^2)}-\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}-\frac{iab\left(\frac{ia\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}$$

input
```
Int[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

3.707. $\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

output
```
((-I)*a*b*(((-I)*b*x)/(a^2 - b^2) + (I*a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2
 - b^2)))/(a^2 - b^2) - (b*Sinh[x]^2)/(2*(a^2 - b^2)) - (a*(x/2 - (Cosh[x]*
Sinh[x])/2))/(a^2 - b^2)
```

3.707.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.707. $\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.707.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{ax}{2(a+b)^2}+\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8(a-b)}-\frac{2a^2bx}{a^4-2a^2b^2+b^4}+\frac{a^2b\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$\frac{8}{(16a+16b)(\tanh(\frac{x}{2})-1)}+\frac{4}{(\tanh(\frac{x}{2})-1)^2(8a+8b)}+\frac{a\ln(\tanh(\frac{x}{2})-1)}{2(a+b)^2}-\frac{4}{(\tanh(\frac{x}{2})+1)^2(8a-8b)}+\frac{8}{(16a-16b)(\tanh(\frac{x}{2})+1)}$

input
```
int(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*x/(a+b)^2+1/8/(a+b)*exp(2*x)-1/8/(a-b)*exp(-2*x)-2*a^2*b/(a^4-2*a^2
*b^2+b^4)*x+a^2*b/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)+(a-b)/(a+b))
```

3.707.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(96) = 192.

Time = 0.25 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.27

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$= \frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x}{}$$

input
```
integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)
*cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 - 4*(a^3 + 2*a^
2*b + a*b^2)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b -
 a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 + 2*a^2*b + a*b^2)*x)*sinh(x)^2 + 8*(a^2*
b*cosh(x)^2 + 2*a^2*b*cosh(x)*sinh(x) + a^2*b*sinh(x)^2)*log(2*(a*cosh(x)
+ b*sinh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)
^3 - 2*(a^3 + 2*a^2*b + a*b^2)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4
)*cosh(x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2
 + b^4)*sinh(x)^2)
```

3.707.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)*sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Timed out
```

3.707.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^2b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4} - \frac{ax}{2\,(a^2+2\,ab+b^2)} + \frac{e^{(2\,x)}}{8\,(a+b)} - \frac{e^{(-2\,x)}}{8\,(a-b)}$$

input
```
integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
a^2*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*a*x/(a^
2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

3.707.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^2b\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4} - \frac{ax}{2\,(a^2-2\,ab+b^2)} + \frac{\left(2\,ae^{(2\,x)}-a+b\right)e^{(-2\,x)}}{8\,(a^2-2\,ab+b^2)} + \frac{e^{(2\,x)}}{8\,(a+b)}$$

input
```
integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
a^2*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) - 1/
2*a*x/(a^2 - 2*a*b + b^2) + 1/8*(2*a*e^(2*x) - a + b)*e^(-2*x)/(a^2 - 2*a*
b + b^2) + 1/8*e^(2*x)/(a + b)
```

3.707.9 Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{\mathrm{e}^{2\,x}}{8\,a+8\,b} - \frac{\mathrm{e}^{-2\,x}}{8\,a-8\,b} - \frac{a\,x}{2\,{(a-b)}^2} + \frac{a^2\,b\,\ln\left(a-b+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{a^4-2\,a^2\,b^2+b^4}$$

input
```
int((cosh(x)*sinh(x)^2)/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) - (a*x)/(2*(a - b)^2) + (a^2*
b*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2)
```

3.707. $\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.708 $\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.708.1 Optimal result

Integrand size = 18, antiderivative size = 137

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx=-\frac{a^3b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{ab^2\cosh(x)}{(a^2-b^2)^2}-\frac{a\cosh(x)}{a^2-b^2}+\frac{a\cosh^3(x)}{3(a^2-b^2)}+\frac{a^2b\sinh(x)}{(a^2-b^2)^2}-\frac{b\sinh^3(x)}{3(a^2-b^2)}$$

output
```
-a^3*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-a*b^2
*cosh(x)/(a^2-b^2)^2-a*cosh(x)/(a^2-b^2)+1/3*a*cosh(x)^3/(a^2-b^2)+a^2*b*s
inh(x)/(a^2-b^2)^2-1/3*b*sinh(x)^3/(a^2-b^2)
```

3.708.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$=\frac{-3a\sqrt{a-b}\sqrt{a+b}(3a^2+b^2)\cosh(x)+a\sqrt{a-b}\sqrt{a+b}(a^2-b^2)\cosh(3x)+b\left(-24a^3\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\right.}{12(a-b)^{5/2}(a+b)^{5/2}}$$

input
```
Integrate[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(-3*a*Sqrt[a - b]*Sqrt[a + b]*(3*a^2 + b^2)*Cosh[x] + a*Sqrt[a - b]*Sqrt[a
 + b]*(a^2 - b^2)*Cosh[3*x] + b*(-24*a^3*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a
- b]*Sqrt[a + b])] + 3*Sqrt[a - b]*Sqrt[a + b]*(5*a^2 - b^2)*Sinh[x] - Sqr
t[a - b]*Sqrt[a + b]*(a^2 - b^2)*Sinh[3*x]))/(12*(a - b)^(5/2)*(a + b)^(5/
2))
```

3.708.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.056, Rules used = {3042, 26, 3588, 25, 26, 3042, 25, 26, 3044, 15, 3113, 2009, 3578, 26, 3042, 26, 3118, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)\cosh(x)}{a\cosh(x)+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ix)^3\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos(ix)\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3588}$$

$$i\left(\frac{a\int -i\sinh^3(x)dx}{a^2-b^2}-\frac{ib\int -\cosh(x)\sinh^2(x)dx}{a^2-b^2}+\frac{iab\int -\frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(\frac{a\int -i\sinh^3(x)dx}{a^2-b^2}+\frac{ib\int \cosh(x)\sinh^2(x)dx}{a^2-b^2}-\frac{iab\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{ia\int \sinh^3(x)dx}{a^2-b^2}+\frac{ib\int \cosh(x)\sinh^2(x)dx}{a^2-b^2}-\frac{iab\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{ia\int i\sin(ix)^3dx}{a^2-b^2}+\frac{ib\int -\cos(ix)\sin(ix)^2dx}{a^2-b^2}-\frac{iab\int -\frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{ia\int i\sin(ix)^3dx}{a^2-b^2}-\frac{ib\int \cos(ix)\sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{a\int \sin(ix)^3dx}{a^2-b^2}-\frac{ib\int \cos(ix)\sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3044}$$

$$i\left(\frac{a\int \sin(ix)^3dx}{a^2-b^2}-\frac{b\int -\sinh^2(x)d(i\sinh(x))}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{15}$$

$$i\left(\frac{a\int \sin(ix)^3dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ib\sinh^3(x)}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{3113}$$

$$i\left(\frac{ia\int \left(1-\cosh^2(x)\right)d\cosh(x)}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ib\sinh^3(x)}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{iab\int \frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ib\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3578}$$

$$i\left(\frac{iab\left(-\frac{ib\int i\sinh(x)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

3.708. $\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$$i\left(\frac{iab\left(\frac{b\int \sinh(x)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{iab\left(\frac{b\int -i\sin(ix)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{iab\left(-\frac{ib\int \sin(ix)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 3118$$

$$i\left(\frac{iab\left(\frac{a^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 3553$$

$$i\left(\frac{iab\left(\frac{ia^2\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 219$$

$$i\left(\frac{iab\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)}{a^2-b^2}\right)$$

input
```
Int[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

3.708. $\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

output
```
I*((I*a*(Cosh[x] - Cosh[x]^3/3))/(a^2 - b^2) + ((I/3)*b*Sinh[x]^3)/(a^2 -
b^2) + (I*a*b*((I*a^2*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^
2]])/(a^2 - b^2)^(3/2) + (b*Cosh[x])/(a^2 - b^2) - (a*Sinh[x])/(a^2 - b^2)
))/(a^2 - b^2))
```

3.708.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

3.708.4 Maple [A] (verified)

Time = 3.86 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.21

method	result
default	$-\frac{2 a^{3} b \arctan\left(\frac{2 a \tanh\left(\frac{x}{2}\right)+2 b}{2 \sqrt{a^{2}-b^{2}}}\right)}{\left(a-b\right)^{2} \left(a+b\right)^{2} \sqrt{a^{2}-b^{2}}}-\frac{8}{\left(16 a-16 b\right) \left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{16}{3 \left(\tanh\left(\frac{x}{2}\right)+1\right)^{3} \left(16 a-16 b\right)}-\frac{a}{2 \left(a-b\right)^{2} \left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{}{3 (\tan}$
risch	$\frac{{\mathrm e}^{3 x}}{24 a+24 b}-\frac{3 \,{\mathrm e}^{x} a}{8 \left(a+b\right)^{2}}-\frac{{\mathrm e}^{x} b}{8 \left(a+b\right)^{2}}-\frac{3 \,{\mathrm e}^{-x} a}{8 \left(a-b\right)^{2}}+\frac{{\mathrm e}^{-x} b}{8 \left(a-b\right)^{2}}+\frac{{\mathrm e}^{-3 x}}{24 a-24 b}-\frac{b \,a^{3} \ln\left({\mathrm e}^{x}+\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\, \left(a+b\right)^{2} \left(a-b\right)^{2}}+\frac{b \,a^{3} \ln\left({\mathrm e}^{x}-\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\, \left(a+b\right)^{2} \left(a-b\right)}$

input
```
int(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

3.708. $\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

output
```
-2*a^3*b/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/
(a^2-b^2)^(1/2))-8/(16*a-16*b)/(tanh(1/2*x)+1)^2+16/3/(tanh(1/2*x)+1)^3/(1
6*a-16*b)-1/2*a/(a-b)^2/(tanh(1/2*x)+1)-16/3/(tanh(1/2*x)-1)^3/(16*a+16*b)
-8/(16*a+16*b)/(tanh(1/2*x)-1)^2+1/2*a/(a+b)^2/(tanh(1/2*x)-1)
```

3.708.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(129) = 258$.

Time = 0.29 (sec) , antiderivative size = 1861, normalized size of antiderivative = 13.58

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
[1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b -
 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*
a^2*b^3 - a*b^4 - b^5)*cosh(x)^4 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5
)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4
+ b^5)*cosh(x)^2 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^
5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3
*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2
 - 24*(a^3*b*cosh(x)^3 + 3*a^3*b*cosh(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(
x)^2 + a^3*b*sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh
(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sin
h(x)^2 + a - b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^5 - 2*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh
(x)^3 - (3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*s
inh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4...
```

3.708.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)*sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Timed out
```

3.708.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.708.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{2\,a^3b\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} - \frac{\left(9\,ae^{(2\,x)}-3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\,(a^2-2\,ab+b^2)} + \frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-9\,a^2e^x-12\,abe^x-3\,b^2e^x}{24\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

input
```
integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
-2*a^3*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*
sqrt(a^2 - b^2)) - 1/24*(9*a*e^(2*x) - 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^2
- 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 9*a^2*e
^x - 12*a*b*e^x - 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.708.9 Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.91

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$= \frac{e^{-3x}}{24a-24b} + \frac{e^{3x}}{24a+24b} - \frac{e^{x}(3a+b)}{8(a+b)^2} - \frac{e^{-x}(3a-b)}{8(a-b)^2}$$
$$- \frac{2\,\mathrm{atan}\left(\frac{a^3\,b\,e^x\sqrt{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}}}{a^5\sqrt{a^6b^2}-b^5\sqrt{a^6b^2}+2a^2b^3\sqrt{a^6b^2}-2a^3b^2\sqrt{a^6b^2}+ab^4\sqrt{a^6b^2}-a^4b\sqrt{a^6b^2}}\right)\sqrt{a^6b^2}}{\sqrt{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}}}$$

input
```
int((cosh(x)*sinh(x)^3)/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(-3*x)/(24*a - 24*b) + exp(3*x)/(24*a + 24*b) - (exp(x)*(3*a + b))/(8*(
a + b)^2) - (exp(-x)*(3*a - b))/(8*(a - b)^2) - (2*atan((a^3*b*exp(x)*(a^1
0 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a
^6*b^2)^(1/2) - b^5*(a^6*b^2)^(1/2) + 2*a^2*b^3*(a^6*b^2)^(1/2) - 2*a^3*b^
2*(a^6*b^2)^(1/2) + a*b^4*(a^6*b^2)^(1/2) - a^4*b*(a^6*b^2)^(1/2)))*(a^6*b
^2)^(1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)
^(1/2)
```

3.709 $\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.709.1 Optimal result

Integrand size = 18, antiderivative size = 102

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{a^2bx}{(a^2-b^2)^2}-\frac{bx}{2(a^2-b^2)}-\frac{ab^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}-\frac{b\cosh(x)\sinh(x)}{2(a^2-b^2)}+\frac{a\sinh^2(x)}{2(a^2-b^2)}$$

output
```
a^2*b*x/(a^2-b^2)^2-1/2*b*x/(a^2-b^2)-a*b^2*ln(a*cosh(x)+b*sinh(x))/(a^2-b
^2)^2-1/2*b*cosh(x)*sinh(x)/(a^2-b^2)+1/2*a*sinh(x)^2/(a^2-b^2)
```

3.709.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx =\frac{a(a^2-b^2)\cosh(2x)+b(2(a^2+b^2)x-4ab\log(a\cosh(x)+b\sinh(x))+(-a^2+b^2)\sinh(2x))}{4(a-b)^2(a+b)^2}$$

input
```
Integrate[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(a*(a^2 - b^2)*Cosh[2*x] + b*(2*(a^2 + b^2)*x - 4*a*b*Log[a*Cosh[x] + b*Si
nh[x]] + (-a^2 + b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

3.709.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3042, 26, 3588, 26, 3042, 26, 3044, 15, 3115, 24, 3577, 26, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)\cosh^2(x)}{a\cosh(x)+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3588}$$

$$-i\left(-\frac{ib\int \cosh^2(x)dx}{a^2-b^2}+\frac{a\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{ib\int \cosh^2(x)dx}{a^2-b^2}+\frac{ia\int \cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{ia\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{a\int \cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3044}$$

$$-i\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{ia\int i\sinh(x)d(i\sinh(x))}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)$$

$$\Big\downarrow \text{3115}$$

$$-i\left(\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)$$

$$\Big\downarrow \text{24}$$

$$-i\left(\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3577}$$

$$-i\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3612}$$

$$-i\left(\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}+\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\right)}{a^2-b^2}\right)$$

input
```
Int[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

3.709. $\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

output
```
(-I)*((I*a*b*((a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^
2)))/(a^2 - b^2) + ((I/2)*a*Sinh[x]^2)/(a^2 - b^2) - (I*b*(x/2 + (Cosh[x]*
Sinh[x])/2))/(a^2 - b^2))
```

3.709.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.709. $\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.709.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

method	result
risch	$\frac{xb}{2(a+b)^2}+\frac{e^{2x}}{8a+8b}+\frac{e^{-2x}}{8a-8b}+\frac{2a\,b^2x}{a^4-2a^2b^2+b^4}-\frac{a\,b^2\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$\frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{4}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2(a+b)^2}-\frac{4}{(8a-8b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{2}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+$

input
```
int(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(a+b)^2*b+1/8/(a+b)*exp(2*x)+1/8/(a-b)*exp(-2*x)+2*a*b^2/(a^4-2*a^2*
b^2+b^4)*x-a*b^2/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)+(a-b)/(a+b))
```

3.709.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(96) = 192.

Time = 0.26 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.27

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x)}{}$$

input
```
integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)
*cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^2*b + 2*
a*b^2 + b^3)*x*cosh(x)^2 + a^3 + a^2*b - a*b^2 - b^3 + 2*(3*(a^3 - a^2*b -
 a*b^2 + b^3)*cosh(x)^2 + 2*(a^2*b + 2*a*b^2 + b^3)*x)*sinh(x)^2 - 8*(a*b^
2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2*sinh(x)^2)*log(2*(a*cosh(x)
+ b*sinh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)
^3 + 2*(a^2*b + 2*a*b^2 + b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4
)*cosh(x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2
 + b^4)*sinh(x)^2)
```

3.709.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**2*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Timed out
```

3.709.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{ab^2\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-2a^2b^2+b^4} + \frac{bx}{2(a^2+2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)} + \frac{e^{(-2x)}}{8(a-b)}$$

input
```
integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
-a*b^2*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*b*x/(a
^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) + 1/8*e^(-2*x)/(a - b)
```

3.709.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{ab^2\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4} + \frac{bx}{2(a^2-2ab+b^2)} - \frac{\left(2be^{(2x)}-a+b\right)e^{(-2x)}}{8(a^2-2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)}$$

input
```
integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
-a*b^2*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1
/2*b*x/(a^2 - 2*a*b + b^2) - 1/8*(2*b*e^(2*x) - a + b)*e^(-2*x)/(a^2 - 2*a
*b + b^2) + 1/8*e^(2*x)/(a + b)
```

3.709.9 Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{-2x}}{8a-8b} + \frac{e^{2x}}{8a+8b} + \frac{b\,x}{2(a-b)^2} - \frac{a\,b^2\,\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^4-2a^2\,b^2+b^4}$$

input
```
int((cosh(x)^2*sinh(x))/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(-2*x)/(8*a - 8*b) + exp(2*x)/(8*a + 8*b) + (b*x)/(2*(a - b)^2) - (a*b^
2*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2)
```

3.709. $\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.710 $\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.710.1 Optimal result

Integrand size = 20, antiderivative size = 122

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^2b^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{a^2b\cosh(x)}{(a^2-b^2)^2}-\frac{b\cosh^3(x)}{3(a^2-b^2)}-\frac{ab^2\sinh(x)}{(a^2-b^2)^2}+\frac{a\sinh^3(x)}{3(a^2-b^2)}$$

output
```
a^2*b^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+a^2*
b*cosh(x)/(a^2-b^2)^2-1/3*b*cosh(x)^3/(a^2-b^2)-a*b^2*sinh(x)/(a^2-b^2)^2+
1/3*a*sinh(x)^3/(a^2-b^2)
```

3.710.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.47

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$= \frac{3\sqrt{a-b}b\sqrt{a+b}(3a^2+b^2)\cosh(x)-\sqrt{a-b}b\sqrt{a+b}(a^2-b^2)\cosh(3x)+a\left(24ab^2\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)-\right.}{12(a-b)^{5/2}(a+b)^{5/2}}$$

input
```
Integrate[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(3*Sqrt[a - b]*b*Sqrt[a + b]*(3*a^2 + b^2)*Cosh[x] - Sqrt[a - b]*b*Sqrt[a
+ b]*(a^2 - b^2)*Cosh[3*x] + a*(24*a*b^2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a
- b]*Sqrt[a + b])] - 3*Sqrt[a - b]*Sqrt[a + b]*(a^2 + 3*b^2)*Sinh[x] + Sqr
t[a - b]*Sqrt[a + b]*(a^2 - b^2)*Sinh[3*x]))/(12*(a - b)^(5/2)*(a + b)^(5/
2))
```

3.710.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 25, 3588, 25, 26, 3042, 25, 26, 3044, 15, 3045, 15, 3588, 26, 3042, 26, 3117, 3118, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)\cosh^2(x)}{a\cosh(x)+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ix)^2\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3588}$$

$$\frac{ib\int i\cosh^2(x)\sinh(x)dx}{a^2-b^2}-\frac{a\int -\cosh(x)\sinh^2(x)dx}{a^2-b^2}-\frac{iab\int \frac{i\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{ib\int i\cosh^2(x)\sinh(x)dx}{a^2-b^2}+\frac{a\int \cosh(x)\sinh^2(x)dx}{a^2-b^2}-\frac{iab\int \frac{i\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \cosh^2(x)\sinh(x)dx}{a^2-b^2}+\frac{a\int \cosh(x)\sinh^2(x)dx}{a^2-b^2}+\frac{ab\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int -i\cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{a\int -\cos(ix)\sin(ix)^2dx}{a^2-b^2}+\frac{ab\int -\frac{i\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int -i\cos(ix)^2\sin(ix)dx}{a^2-b^2}-\frac{a\int \cos(ix)\sin(ix)^2dx}{a^2-b^2}+\frac{ab\int -\frac{i\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{ib\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}-\frac{a\int \cos(ix)\sin(ix)^2dx}{a^2-b^2}-\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3044}$$

$$\frac{ia\int -\sinh^2(x)d(i\sinh(x))}{a^2-b^2}+\frac{ib\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}-\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{ib\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}-\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\sinh^3(x)}{3(a^2-b^2)}$$

$$\Big\downarrow \text{3045}$$

$$-\frac{b\int \cosh^2(x)d\cosh(x)}{a^2-b^2}-\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\sinh^3(x)}{3(a^2-b^2)}$$

$$\Big\downarrow \text{15}$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\sinh^3(x)}{3(a^2-b^2)}-\frac{b\cosh^3(x)}{3(a^2-b^2)}$$

$$\Big\downarrow \text{3588}$$

$$-\frac{iab\left(\frac{a\int i\sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\sinh^3(x)}{3(a^2-b^2)}-\frac{b\cosh^3(x)}{3(a^2-b^2)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{iab\left(\frac{ia\int \sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\sinh^3(x)}{3(a^2-b^2)}-\frac{b\cosh^3(x)}{3(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{iab\left(-\frac{ib\int \sin(ix+\frac{\pi}{2})dx}{a^2-b^2}+\frac{ia\int -i\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\sinh^3(x)}{3(a^2-b^2)}-\frac{b\cosh^3(x)}{3(a^2-b^2)}$$

3.710. $\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$$\Big\downarrow \text{26}$$

$$-\frac{iab\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a\int \sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\sinh^3(x)}{3\,(a^2-b^2)}-\frac{b\cosh^3(x)}{3\,(a^2-b^2)}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{iab\left(\frac{a\int \sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\sinh^3(x)}{3\,(a^2-b^2)}-\frac{b\cosh^3(x)}{3\,(a^2-b^2)}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{iab\left(\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\sinh^3(x)}{3\,(a^2-b^2)}-\frac{b\cosh^3(x)}{3\,(a^2-b^2)}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{iab\left(-\frac{ab\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\sinh^3(x)}{3\,(a^2-b^2)}-\frac{b\cosh^3(x)}{3\,(a^2-b^2)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{iab\left(-\frac{ab\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\sinh^3(x)}{3\,(a^2-b^2)}-\frac{b\cosh^3(x)}{3\,(a^2-b^2)}$$

input
```
Int[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
-1/3*(b*Cosh[x]^3)/(a^2 - b^2) + (a*Sinh[x]^3)/(3*(a^2 - b^2)) - (I*a*b*(-
((a*b*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)
^(3/2)) + (I*a*Cosh[x])/(a^2 - b^2) - (I*b*Sinh[x])/(a^2 - b^2)))/(a^2 - b
^2)
```

3.710. $\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.710.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.710. $\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

3.710.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.38

method	result
default	$-\frac{8}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}\left(8a+8b\right)}-\frac{4}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}\left(8a+8b\right)}-\frac{b}{2\left(a+b\right)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{8}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}\left(8a-8b\right)}+\frac{4}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}(\cdots}$
risch	$\frac{e^{3x}}{24a+24b}-\frac{e^{x}a}{8\left(a+b\right)^{2}}+\frac{e^{x}b}{8\left(a+b\right)^{2}}+\frac{a\,e^{-x}}{8a^{2}-16ab+8b^{2}}+\frac{b\,e^{-x}}{8a^{2}-16ab+8b^{2}}-\frac{e^{-3x}}{24\left(a-b\right)}-\frac{b^{2}a^{2}\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,\left(a+b\right)^{2}\left(a-b\right)^{2}}+\frac{b^{2}a^{2}\ln\left(e^{\cdots}\right)}{\sqrt{-a^{2}+b^{2}}\cdots}$

input
```
int(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-8/3/(tanh(1/2*x)-1)^3/(8*a+8*b)-4/(tanh(1/2*x)-1)^2/(8*a+8*b)-1/2*b/(a+b)
^2/(tanh(1/2*x)-1)-8/3/(tanh(1/2*x)+1)^3/(8*a-8*b)+4/(tanh(1/2*x)+1)^2/(8*
a-8*b)+1/2/(a-b)^2*b/(tanh(1/2*x)+1)+2*a^2*b^2/(a+b)^2/(a-b)^2/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

3.710.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. 2(114) = 228.

Time = 0.28 (sec) , antiderivative size = 1847, normalized size of antiderivative = 15.14

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
[1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b +
 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^
2*b^3 - 3*a*b^4 + b^5)*cosh(x)^4 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^
3 - 3*a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5
)*cosh(x))*sinh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
- b^5)*cosh(x)^2 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^
5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(a
^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2
 - 24*(a^2*b^2*cosh(x)^3 + 3*a^2*b^2*cosh(x)^2*sinh(x) + 3*a^2*b^2*cosh(x)
*sinh(x)^2 + a^2*b^2*sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 +
2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x
) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
+ b)*sinh(x)^2 + a - b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4
 - b^5)*cosh(x)^5 - 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b
^5)*cosh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*co
sh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6...
```

3.710.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**2*sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Timed out
```

3.710.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.710.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{2\,a^2b^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{\left(3\,ae^{(2\,x)}+3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\,(a^2-2\,ab+b^2)} + \frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-3\,a^2e^x+3\,b^2e^x}{24\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

input
```
integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
2*a^2*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)
*sqrt(a^2 - b^2)) + 1/24*(3*a*e^(2*x) + 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^2
 - 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 3*a^2*
e^x + 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.710.9 Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.13

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{3x}}{24a+24b}-\frac{e^{-3x}}{24a-24b}-\frac{e^{x}(a-b)}{8(a+b)^2}+\frac{2\,\mathrm{atan}\left(\frac{a^2b^2e^{x}\sqrt{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}}}{a^5\sqrt{a^4b^4}-b^5\sqrt{a^4b^4}+2a^2b^3\sqrt{a^4b^4}-2a^3b^2\sqrt{a^4b^4}+ab^4\sqrt{a^4b^4}-a^4b\sqrt{a^4b^4}}\right)\sqrt{a^4b^4}}{\sqrt{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}}}+\frac{e^{-x}(a+b)}{8(a-b)^2}$$

input
```
int((cosh(x)^2*sinh(x)^2)/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(3*x)/(24*a + 24*b) - exp(-3*x)/(24*a - 24*b) - (exp(x)*(a - b))/(8*(a
+ b)^2) + (2*atan((a^2*b^2*exp(x)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 +
10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^4*b^4)^(1/2) - b^5*(a^4*b^4)^(1/2)
+ 2*a^2*b^3*(a^4*b^4)^(1/2) - 2*a^3*b^2*(a^4*b^4)^(1/2) + a*b^4*(a^4*b^4)^
(1/2) - a^4*b*(a^4*b^4)^(1/2)))*(a^4*b^4)^(1/2))/(a^10 - b^10 + 5*a^2*b^8
- 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + (exp(-x)*(a + b))/(8*(a - b
)^2)
```

3.711 $\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.711.1 Optimal result

Integrand size = 20, antiderivative size = 194

$$\begin{aligned}\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx &= -\frac{a^2b^3x}{(a^2-b^2)^3}-\frac{a^2bx}{2(a^2-b^2)^2}+\frac{bx}{8(a^2-b^2)}\\&\quad+\frac{a^3b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}\\&\quad+\frac{a^2b\cosh(x)\sinh(x)}{2(a^2-b^2)^2}+\frac{b\cosh(x)\sinh(x)}{8(a^2-b^2)}\\&\quad-\frac{b\cosh^3(x)\sinh(x)}{4(a^2-b^2)}-\frac{ab^2\sinh^2(x)}{2(a^2-b^2)^2}+\frac{a\sinh^4(x)}{4(a^2-b^2)}\end{aligned}$$

output
```
-a^2*b^3*x/(a^2-b^2)^3-1/2*a^2*b*x/(a^2-b^2)^2+1/8*b*x/(a^2-b^2)+a^3*b^2*l
n(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3+1/2*a^2*b*cosh(x)*sinh(x)/(a^2-b^2)^2+1
/8*b*cosh(x)*sinh(x)/(a^2-b^2)-1/4*b*cosh(x)^3*sinh(x)/(a^2-b^2)-1/2*a*b^2
*sinh(x)^2/(a^2-b^2)^2+1/4*a*sinh(x)^4/(a^2-b^2)
```

3.711.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.66

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$= \frac{-4a(a^4-b^4)\cosh(2x)+a(a^2-b^2)^2\cosh(4x)-b\Big(4(3a^4x+6a^2b^2x-b^4x-8a^3b\log(a\cosh(x)+b\sinh(x}{32(a-b)^3(a+b)^3}$$

input
```
Integrate[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(-4*a*(a^4 - b^4)*Cosh[2*x] + a*(a^2 - b^2)^2*Cosh[4*x] - b*(4*(3*a^4*x +
6*a^2*b^2*x - b^4*x - 8*a^3*b*Log[a*Cosh[x] + b*Sinh[x]]) - 8*a^2*(a^2 - b
^2)*Sinh[2*x] + (a^2 - b^2)^2*Sinh[4*x]))/(32*(a - b)^3*(a + b)^3)
```

3.711.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.02, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {3042, 26, 3588, 25, 26, 3042, 25, 26, 3044, 15, 3048, 3042, 3115, 24, 3588, 25, 26, 3042, 25, 26, 3044, 15, 3115, 24, 3576, 26, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sin(ix)^3\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\cos(ix)^2\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx$$
$$\Big\downarrow \text{3588}$$

$$i \left(-\frac{ib \int -\cosh^2(x)\sinh^2(x)dx}{a^2-b^2}+\frac{a\int -i\cosh(x)\sinh^3(x)dx}{a^2-b^2}+\frac{iab\int -\frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$i \left(\frac{ib \int \cosh^2(x)\sinh^2(x)dx}{a^2-b^2}+\frac{a\int -i\cosh(x)\sinh^3(x)dx}{a^2-b^2}-\frac{iab\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i \left(\frac{ib \int \cosh^2(x)\sinh^2(x)dx}{a^2-b^2}-\frac{ia\int \cosh(x)\sinh^3(x)dx}{a^2-b^2}-\frac{iab\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i \left(\frac{ib \int -\cos(ix)^2\sin(ix)^2dx}{a^2-b^2}-\frac{ia\int i\cos(ix)\sin(ix)^3dx}{a^2-b^2}-\frac{iab\int -\frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$i \left(-\frac{ib \int \cos(ix)^2\sin(ix)^2dx}{a^2-b^2}-\frac{ia\int i\cos(ix)\sin(ix)^3dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i \left(-\frac{ib \int \cos(ix)^2\sin(ix)^2dx}{a^2-b^2}+\frac{a\int \cos(ix)\sin(ix)^3dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3044}$$

$$i \left(-\frac{ia \int -i\sinh^3(x)d(i\sinh(x))}{a^2-b^2}-\frac{ib\int \cos(ix)^2\sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{15}$$

$$i \left(-\frac{ib \int \cos(ix)^2\sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}\right)$$

$$\Big\downarrow \text{3048}$$

$$i \left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{1}{4}\int \cosh^2(x)dx-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}\right)$$

$$\Big\downarrow \text{3042}$$

3.711. $\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(-\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{1}{4}\int \sin\left(ix+\frac{\pi}{2}\right)^2dx\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}\right)$$

$\downarrow$ 3115

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}\right)$$

$\downarrow$ 24

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}\right)$$

$\downarrow$ 3588

$$i\Bigg(\frac{iab\left(\frac{a\int -\sinh^2(x)dx}{a^2-b^2}-\frac{ib\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\big(\frac{1}{4}\big(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\big)}{a^2-b}$$

$\downarrow$ 25

$$i\Bigg(\frac{iab\left(-\frac{a\int \sinh^2(x)dx}{a^2-b^2}-\frac{ib\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\big(\frac{1}{4}\big(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\big)}{a^2-b}$$

$\downarrow$ 26

$$i\Bigg(\frac{iab\left(-\frac{a\int \sinh^2(x)dx}{a^2-b^2}+\frac{b\int \cosh(x)\sinh(x)dx}{a^2-b^2}-\frac{ab\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\big(\frac{1}{4}\big(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\big)-}{a^2-b^2}$$

$\downarrow$ 3042

$$i\Bigg(\frac{iab\left(-\frac{a\int -\sin(ix)^2dx}{a^2-b^2}+\frac{b\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}-\frac{ab\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\big(\frac{1}{4}\big(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x}{a^2-}$$

$\downarrow$ 25

3.711. $\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$$i\left(\frac{iab\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}+\frac{b\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}-\frac{ab\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{iab\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}-\frac{ib\int \cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \text{3044}$$

$$i\left(\frac{iab\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}-\frac{b\int i\sinh(x)d(i\sinh(x))}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \text{15}$$

$$i\left(\frac{iab\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \text{3115}$$

$$i\left(\frac{iab\left(\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \text{24}$$

$$i\left(\frac{iab\left(\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \text{3576}$$

3.711. $\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$$i\left(\frac{iab\left(\frac{iab\left(-\frac{a\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(\right.\right.}{}\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{iab\left(\frac{iab\left(\frac{ia\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\,\mathrm{c}\right.\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{iab\left(\frac{iab\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\,\mathrm{c}\right.\right.}{}\right.$$

$$\Big\downarrow \text{3612}$$

$$i\left(-\frac{ia\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}+\frac{iab\left(\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}+\frac{ia\right.}{}}{a^2-b^2}\right.$$

input
```
Int[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

3.711. $\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

output
```
I*(((-1/4*I)*a*Sinh[x]^4)/(a^2 - b^2) + (I*a*b*((I*a*b*(((-I)*b*x)/(a^2 -
b^2) + (I*a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)))/(a^2 - b^2) + (b*Sin
h[x]^2)/(2*(a^2 - b^2)) + (a*(x/2 - (Cosh[x]*Sinh[x])/2))/(a^2 - b^2)))/(a
^2 - b^2) - (I*b*(-1/4*(Cosh[x]^3*Sinh[x]) + (x/2 + (Cosh[x]*Sinh[x])/2)/4
))/(a^2 - b^2))
```

3.711.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

3.711. $\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.711.4 Maple [A] (verified)

Time = 10.94 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{3axb}{8(a+b)^3}-\frac{b^2x}{8(a+b)^3}+\frac{e^{4x}}{64a+64b}-\frac{e^{2x}a}{16(a+b)^2}-\frac{e^{-2x}a}{16(a-b)^2}+\frac{e^{-4x}}{64a-64b}-\frac{2a^3b^2x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{a^3b^2\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^6-3a^4b^2+3a^2b^4-b^6}$
default	$\frac{4}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^4(16a+16b)}+\frac{16}{(32a+32b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{-a-3b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{a-b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b(3a+b)\ln(\tanh}{8(a+b)^3}$

input
```
int(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-3/8*a*x/(a+b)^3*b-1/8*b^2*x/(a+b)^3+1/64/(a+b)*exp(4*x)-1/16/(a+b)^2*exp(
2*x)*a-1/16/(a-b)^2*exp(-2*x)*a+1/64/(a-b)*exp(-4*x)-2*a^3*b^2/(a^6-3*a^4*
b^2+3*a^2*b^4-b^6)*x+a^3*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*ln(exp(2*x)+(a-
b)/(a+b))
```

3.711.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(180) = 360$.

Time = 0.27 (sec) , antiderivative size = 1158, normalized size of antiderivative = 5.97

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 - 4*(a^5 - 2*a^4
*b + 2*a^2*b^3 - a*b^4)*cosh(x)^6 - 4*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4 -
 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^
6 - 8*(3*a^4*b + 8*a^3*b^2 + 6*a^2*b^3 - b^5)*x*cosh(x)^4 + 8*(7*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^5 - 2*a^4*b +
2*a^2*b^3 - a*b^4)*cosh(x))*sinh(x)^5 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^
3 + a*b^4 + b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5
)*cosh(x)^4 - 30*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh(x)^2 - 4*(3*a^4*
b + 8*a^3*b^2 + 6*a^2*b^3 - b^5)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*
b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 10*(a^5 - 2*a^4*b + 2*a^2*b^3 -
 a*b^4)*cosh(x)^3 - 4*(3*a^4*b + 8*a^3*b^2 + 6*a^2*b^3 - b^5)*x*cosh(x))*s
inh(x)^3 - 4*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*cosh(x)^2 + 4*(7*(a^5 - a
^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 - a^5 - 2*a^4*b + 2*
a^2*b^3 + a*b^4 - 15*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh(x)^4 - 12*(3
*a^4*b + 8*a^3*b^2 + 6*a^2*b^3 - b^5)*x*cosh(x)^2)*sinh(x)^2 + 64*(a^3*b^2
*cosh(x)^4 + 4*a^3*b^2*cosh(x)^3*sinh(x) + 6*a^3*b^2*cosh(x)^2*sinh(x)^2 +
 4*a^3*b^2*cosh(x)*sinh(x)^3 + a^3*b^2*sinh(x)^4)*log(2*(a*cosh(x) + b*sin
h(x))/(cosh(x) - sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + ...
```

3.711.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**2*sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Timed out
```

3.711.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^3b^2\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{(3ab+b^2)x}{8(a^3+3a^2b+3ab^2+b^3)} - \frac{\left(4ae^{(-2x)}-a-b\right)e^{(4x)}}{64(a^2+2ab+b^2)} - \frac{4ae^{(-2x)}-(a-b)e^{(-4x)}}{64(a^2-2ab+b^2)}$$

input
```
integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
a^3*b^2*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
 - 1/8*(3*a*b + b^2)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/64*(4*a*e^(-2*x
) - a - b)*e^(4*x)/(a^2 + 2*a*b + b^2) - 1/64*(4*a*e^(-2*x) - (a - b)*e^(-
4*x))/(a^2 - 2*a*b + b^2)
```

3.711.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^3b^2\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{(3ab-b^2)x}{8(a^3-3a^2b+3ab^2-b^3)} + \frac{\left(18abe^{(4x)}-6b^2e^{(4x)}-4a^2e^{(2x)}+4abe^{(2x)}+a^2-2ab+b^2\right)e^{(-4x)}}{64(a^3-3a^2b+3ab^2-b^3)} + \frac{ae^{(4x)}+be^{(4x)}-4ae^{(2x)}}{64(a^2+2ab+b^2)}$$

input
```
integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
a^3*b^2*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6) - 1/8*(3*a*b - b^2)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(18
*a*b*e^(4*x) - 6*b^2*e^(4*x) - 4*a^2*e^(2*x) + 4*a*b*e^(2*x) + a^2 - 2*a*b
 + b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(
4*x) - 4*a*e^(2*x))/(a^2 + 2*a*b + b^2)
```

3.711.9 Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{-4x}}{64a-64b}+\frac{e^{4x}}{64a+64b}-\frac{x\,(3ab-b^2)}{8\,(a-b)^3}-\frac{a\,e^{2x}}{16\,(a+b)^2}-\frac{a\,e^{-2x}}{16\,(a-b)^2}+\frac{a^3\,b^2\,\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}$$

input
```
int((cosh(x)^2*sinh(x)^3)/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(-4*x)/(64*a - 64*b) + exp(4*x)/(64*a + 64*b) - (x*(3*a*b - b^2))/(8*(a
 - b)^3) - (a*exp(2*x))/(16*(a + b)^2) - (a*exp(-2*x))/(16*(a - b)^2) + (a
^3*b^2*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)
```

3.712 $\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.712.1 Optimal result

Integrand size = 18, antiderivative size = 137

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{ab^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{ab^2\cosh(x)}{(a^2-b^2)^2} + \frac{a\cosh^3(x)}{3(a^2-b^2)} + \frac{a^2b\sinh(x)}{(a^2-b^2)^2} - \frac{b\sinh(x)}{a^2-b^2} - \frac{b\sinh^3(x)}{3(a^2-b^2)}$$

output
```
-a*b^3*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-a*b^2
*cosh(x)/(a^2-b^2)^2+1/3*a*cosh(x)^3/(a^2-b^2)+a^2*b*sinh(x)/(a^2-b^2)^2-b
*sinh(x)/(a^2-b^2)-1/3*b*sinh(x)^3/(a^2-b^2)
```

3.712.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{1}{12}\left(-\frac{24ab^3\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{3a(a^2-5b^2)\cosh(x)}{(a-b)^2(a+b)^2} + \frac{a\cosh(3x)}{(a-b)(a+b)} + \frac{3b(a^2+3b^2)\sinh(x)}{(a-b)^2(a+b)^2} - \frac{a^2b\sinh(3x)}{(a-b)^2(a+b)^2} + \frac{b^3\sinh(3x)}{(a-b)^2(a+b)^2}\right)$$

input
```
Integrate[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
((-24*a*b^3*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^
(5/2)*(a + b)^(5/2)) + (3*a*(a^2 - 5*b^2)*Cosh[x])/((a - b)^2*(a + b)^2) +
 (a*Cosh[3*x])/((a - b)*(a + b)) + (3*b*(a^2 + 3*b^2)*Sinh[x])/((a - b)^2*
(a + b)^2) - (a^2*b*Sinh[3*x])/((a - b)^2*(a + b)^2) + (b^3*Sinh[3*x])/((a
 - b)^2*(a + b)^2))/12
```

3.712.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.14, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 26, 3588, 26, 3042, 26, 3045, 15, 3113, 2009, 3579, 3042, 3117, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)\cos(ix)^3}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)^3\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3588}$$

$$-i\left(-\frac{ib\int \cosh^3(x)dx}{a^2-b^2}+\frac{a\int i\cosh^2(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{ib\int \cosh^3(x)dx}{a^2-b^2}+\frac{ia\int \cosh^2(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}+\frac{ia\int -i\cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}+\frac{a\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3045}$$

$$-i\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}+\frac{ia\int \cosh^2(x)d\cosh(x)}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)$$

$$\Big\downarrow \text{3113}$$

$$-i\left(\frac{b\int \left(\sinh^2(x)+1\right)d(-i\sinh(x))}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)$$

$$\Big\downarrow \text{3579}$$

$$-i\left(\frac{iab\left(\frac{a\int \cosh(x)dx}{a^2-b^2}-\frac{b^2\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{iab\left(\frac{a\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}-\frac{b^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)$$

$$\Big\downarrow \text{3117}$$

3.712. $\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$$-i\left(\frac{iab\left(-\frac{b^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)$$

$\Big\downarrow$ 3553

$$-i\left(\frac{iab\left(-\frac{ib^2\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}\right.$$

$\Big\downarrow$ 219

$$-i\left(\frac{iab\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)$$

input
```
Int[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(-I)*(((I/3)*a*Cosh[x]^3)/(a^2 - b^2) + (I*a*b*(((-I)*b^2*ArcTanh[((-I)*b*
Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - (b*Cosh[x])/(
a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)))/(a^2 - b^2) + (b*((-I)*Sinh[x] - (I
/3)*Sinh[x]^3))/(a^2 - b^2))
```

3.712.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.712. $\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

3.712. $\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

3.712.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.27

method	result
risch	$\frac{e^{3x}}{24a+24b}+\frac{e^{x}a}{8(a+b)^{2}}+\frac{3\,e^{x}b}{8(a+b)^{2}}+\frac{e^{-x}a}{8(a-b)^{2}}-\frac{3\,e^{-x}b}{8(a-b)^{2}}+\frac{e^{-3x}}{24a-24b}-\frac{b^{3}a\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)^{2}(a-b)^{2}}+\frac{b^{3}a\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)^{2}(a-b)}$
default	$-\frac{4}{3(\tanh(\frac{x}{2})-1)^{3}(4a+4b)}-\frac{2}{(4a+4b)(\tanh(\frac{x}{2})-1)^{2}}-\frac{a+2b}{2(a+b)^{2}(\tanh(\frac{x}{2})-1)}-\frac{2}{(4a-4b)(\tanh(\frac{x}{2})+1)^{2}}+\frac{4}{3(\tanh(\frac{x}{2})+1)^{3}(}$

input
```
int(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/24/(a+b)*exp(x)^3+1/8/(a+b)^2*exp(x)*a+3/8/(a+b)^2*exp(x)*b+1/8/(a-b)^2/
exp(x)*a-3/8/(a-b)^2/exp(x)*b+1/24/(a-b)/exp(x)^3-1/(-a^2+b^2)^(1/2)*b^3*a
/(a+b)^2/(a-b)^2*ln(exp(x)+(a-b)/(-a^2+b^2)^(1/2))+1/(-a^2+b^2)^(1/2)*b^3*
a/(a+b)^2/(a-b)^2*ln(exp(x)-(a-b)/(-a^2+b^2)^(1/2))
```

3.712.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(129) = 258$.

Time = 0.30 (sec) , antiderivative size = 1829, normalized size of antiderivative = 13.35

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
[1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b -
 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*
b^3 + 5*a*b^4 - 3*b^5)*cosh(x)^4 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3
+ 5*a*b^4 - 3*b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5
)*cosh(x))*sinh(x)^3 + 3*(a^5 - a^4*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 +
3*b^5)*cosh(x)^2 + 3*(a^5 - a^4*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^
5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a
^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x)^2)*sinh(x)^2
 - 24*(a*b^3*cosh(x)^3 + 3*a*b^3*cosh(x)^2*sinh(x) + 3*a*b^3*cosh(x)*sinh(
x)^2 + a*b^3*sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh
(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sin
h(x)^2 + a - b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^5 + 2*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh
(x)^3 + (a^5 - a^4*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*cosh(x))*s
inh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4...
```

3.712.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**3*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Timed out
```

3.712.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.712.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{2\,ab^3\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{\left(3\,ae^{(2\,x)}-9\,be^{(2\,x)}+a-b\right)e^{(-3\,x)}}{24\,(a^2-2\,ab+b^2)} + \frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}+3\,a^2e^x+12\,abe^x+9\,b^2e^x}{24\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

input
```
integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
-2*a*b^3*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*
sqrt(a^2 - b^2)) + 1/24*(3*a*e^(2*x) - 9*b*e^(2*x) + a - b)*e^(-3*x)/(a^2
- 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) + 3*a^2*e
^x + 12*a*b*e^x + 9*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.712.9 Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.89

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$= \frac{e^{-3x}}{24a-24b}+\frac{e^{3x}}{24a+24b}+\frac{e^x(a+3b)}{8(a+b)^2}$$
$$-\frac{2\,\mathrm{atan}\left(\frac{a\,b^3\,e^x\sqrt{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}}}{a^5\sqrt{a^2b^6}-b^5\sqrt{a^2b^6}+2a^2b^3\sqrt{a^2b^6}-2a^3b^2\sqrt{a^2b^6}+a\,b^4\sqrt{a^2b^6}-a^4\,b\sqrt{a^2b^6}}\right)\sqrt{a^2b^6}}{\sqrt{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}}}$$
$$+\frac{e^{-x}(a-3b)}{8(a-b)^2}$$

input
```
int((cosh(x)^3*sinh(x))/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(-3*x)/(24*a - 24*b) + exp(3*x)/(24*a + 24*b) + (exp(x)*(a + 3*b))/(8*(
a + b)^2) - (2*atan((a*b^3*exp(x)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 +
10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^2*b^6)^(1/2) - b^5*(a^2*b^6)^(1/2)
+ 2*a^2*b^3*(a^2*b^6)^(1/2) - 2*a^3*b^2*(a^2*b^6)^(1/2) + a*b^4*(a^2*b^6)^
(1/2) - a^4*b*(a^2*b^6)^(1/2)))*(a^2*b^6)^(1/2))/(a^10 - b^10 + 5*a^2*b^8
- 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + (exp(-x)*(a - 3*b))/(8*(a -
 b)^2)
```

3.713 $\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.713.1 Optimal result

Integrand size = 20, antiderivative size = 194

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^3b^2x}{(a^2-b^2)^3} - \frac{ab^2x}{2(a^2-b^2)^2} - \frac{ax}{8(a^2-b^2)} - \frac{b\cosh^4(x)}{4(a^2-b^2)} - \frac{a^2b^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} - \frac{ab^2\cosh(x)\sinh(x)}{2(a^2-b^2)^2} - \frac{a\cosh(x)\sinh(x)}{8(a^2-b^2)} + \frac{a\cosh^3(x)\sinh(x)}{4(a^2-b^2)} + \frac{a^2b\sinh^2(x)}{2(a^2-b^2)^2}$$

output
```
a^3*b^2*x/(a^2-b^2)^3-1/2*a*b^2*x/(a^2-b^2)^2-1/8*a*x/(a^2-b^2)-1/4*b*cosh
(x)^4/(a^2-b^2)-a^2*b^3*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3-1/2*a*b^2*cosh
(x)*sinh(x)/(a^2-b^2)^2-1/8*a*cosh(x)*sinh(x)/(a^2-b^2)+1/4*a*cosh(x)^3*si
nh(x)/(a^2-b^2)+1/2*a^2*b*sinh(x)^2/(a^2-b^2)^2
```

3.713.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.65

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{4b(a^4-b^4)\cosh(2x) - b(a^2-b^2)^2\cosh(4x) + a\Big(-4((a^4-6a^2b^2-3b^4)\,x + 8ab^3\log(a\cosh(x)+b\sinh(x)}{32(a-b)^3(a+b)^3}$$

input
```
Integrate[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
(4*b*(a^4 - b^4)*Cosh[2*x] - b*(a^2 - b^2)^2*Cosh[4*x] + a*(-4*((a^4 - 6*a
^2*b^2 - 3*b^4)*x + 8*a*b^3*Log[a*Cosh[x] + b*Sinh[x]]) + 8*b^2*(-a^2 + b^
2)*Sinh[2*x] + (a^2 - b^2)^2*Sinh[4*x]))/(32*(a - b)^3*(a + b)^3)
```

3.713.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {3042, 25, 3588, 25, 26, 3042, 25, 26, 3045, 15, 3048, 3042, 3115, 24, 3588, 26, 3042, 26, 3044, 15, 3115, 24, 3577, 26, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\sin(ix)^2\cos(ix)^3}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\cos(ix)^3\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \; 3588$$

$$\frac{ib\int i\cosh^3(x)\sinh(x)dx}{a^2-b^2}-\frac{a\int -\cosh^2(x)\sinh^2(x)dx}{a^2-b^2}-\frac{iab\int \frac{i\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{ib\int i\cosh^3(x)\sinh(x)dx}{a^2-b^2}+\frac{a\int \cosh^2(x)\sinh^2(x)dx}{a^2-b^2}-\frac{iab\int \frac{i\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \cosh^3(x)\sinh(x)dx}{a^2-b^2}+\frac{a\int \cosh^2(x)\sinh^2(x)dx}{a^2-b^2}+\frac{ab\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int -i\cos(ix)^3\sin(ix)dx}{a^2-b^2}+\frac{a\int -\cos(ix)^2\sin(ix)^2dx}{a^2-b^2}+\frac{ab\int -\frac{i\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int -i\cos(ix)^3\sin(ix)dx}{a^2-b^2}-\frac{a\int \cos(ix)^2\sin(ix)^2dx}{a^2-b^2}+\frac{ab\int -\frac{i\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{ib\int \cos(ix)^3\sin(ix)dx}{a^2-b^2}-\frac{a\int \cos(ix)^2\sin(ix)^2dx}{a^2-b^2}-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3045}$$

$$-\frac{b\int \cosh^3(x)d\cosh(x)}{a^2-b^2}-\frac{a\int \cos(ix)^2\sin(ix)^2dx}{a^2-b^2}-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{a\int \cos(ix)^2\sin(ix)^2dx}{a^2-b^2}-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\cosh^4(x)}{4\,(a^2-b^2)}$$

$$\Big\downarrow \text{3048}$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\left(\frac{1}{4}\int \cosh^2(x)dx-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4\,(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\left(-\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{1}{4}\int \sin\left(ix+\frac{\pi}{2}\right)^2dx\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4\,(a^2-b^2)}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4\,(a^2-b^2)}$$

$$\Big\downarrow \text{24}$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\cosh^4(x)}{4\,(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3588}$$

3.713. $\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$$-\frac{iab\left(-\frac{ib\int \cosh^2(x)dx}{a^2-b^2}+\frac{a\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{iab\left(-\frac{ib\int \cosh^2(x)dx}{a^2-b^2}+\frac{ia\int \cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{iab\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{ia\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{iab\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{a\int \cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 3044}$$

$$-\frac{iab\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{ia\int i\sinh(x)d(i\sinh(x))}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{iab\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 3115}$$

3.713. $\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$$-\frac{iab\left(\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{iab\left(\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 3577}$$

$$-\frac{iab\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{iab\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{iab\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 3612}$$

3.713. $\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$$-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}-\frac{iab\left(\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}+\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\right)}{a^2-b^2}\right)}{a^2-b^2}$$

input
```
Int[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
-1/4*(b*Cosh[x]^4)/(a^2 - b^2) - (a*(-1/4*(Cosh[x]^3*Sinh[x]) + (x/2 + (Co
sh[x]*Sinh[x])/2)/4))/(a^2 - b^2) - (I*a*b*((I*a*b*((a*x)/(a^2 - b^2) - (b
*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)))/(a^2 - b^2) + ((I/2)*a*Sinh[x]^
2)/(a^2 - b^2) - (I*b*(x/2 + (Cosh[x]*Sinh[x])/2))/(a^2 - b^2)))/(a^2 - b^
2)
```

3.713.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.713. $\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.713. $\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.713.4 Maple [A] (verified)

Time = 7.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{a^{2} x}{8(a+b)^{3}}-\frac{3 a x b}{8(a+b)^{3}}+\frac{\mathrm{e}^{4 x}}{64 a+64 b}+\frac{\mathrm{e}^{2 x} b}{16(a+b)^{2}}+\frac{\mathrm{e}^{-2 x} b}{16(a-b)^{2}}-\frac{\mathrm{e}^{-4 x}}{64(a-b)}+\frac{2 a^{2} b^{3} x}{a^{6}-3 a^{4} b^{2}+3 a^{2} b^{4}-b^{6}}-\frac{a^{2} b^{3} \ln\left(\mathrm{e}^{2 x}+\frac{a-b}{a+b}\right)}{a^{6}-3 a^{4} b^{2}+3 a^{2} b^{4}-b^{6}}$
default	$\frac{2}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}(8 a+8 b)}+\frac{8}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}(16 a+16 b)}-\frac{-3 a-5 b}{8(a+b)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{-a-3 b}{8(a+b)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a(a+3 b) \ln\left(\tanh\left(\frac{x}{2}\right.\right.}{8(a+b)^{3}}$

input
```
int(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^2*x/(a+b)^3-3/8*a*x/(a+b)^3*b+1/64/(a+b)*exp(4*x)+1/16/(a+b)^2*exp(
2*x)*b+1/16/(a-b)^2*exp(-2*x)*b-1/64/(a-b)*exp(-4*x)+2*a^2*b^3/(a^6-3*a^4*
b^2+3*a^2*b^4-b^6)*x-a^2*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*ln(exp(2*x)+(a-
b)/(a+b))
```

3.713.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. 2(180) = 360.

Time = 0.28 (sec) , antiderivative size = 1162, normalized size of antiderivative = 5.99

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 + 4*(a^4*b - 2*a
^3*b^2 + 2*a*b^4 - b^5)*cosh(x)^6 + 4*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5 +
 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^
6 - 8*(a^5 - 6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*x*cosh(x)^4 + 8*(7*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^4*b - 2*a^3*b^
2 + 2*a*b^4 - b^5)*cosh(x))*sinh(x)^5 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^
3 - a*b^4 - b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5
)*cosh(x)^4 + 30*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh(x)^2 - 4*(a^5 -
6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*
b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 10*(a^4*b - 2*a^3*b^2 + 2*a*b^4
 - b^5)*cosh(x)^3 - 4*(a^5 - 6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*x*cosh(x))*s
inh(x)^3 + 4*(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*cosh(x)^2 + 4*(7*(a^5 - a
^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + a^4*b + 2*a^3*b^2
- 2*a*b^4 - b^5 + 15*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh(x)^4 - 12*(a
^5 - 6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*x*cosh(x)^2)*sinh(x)^2 - 64*(a^2*b^3
*cosh(x)^4 + 4*a^2*b^3*cosh(x)^3*sinh(x) + 6*a^2*b^3*cosh(x)^2*sinh(x)^2 +
 4*a^2*b^3*cosh(x)*sinh(x)^3 + a^2*b^3*sinh(x)^4)*log(2*(a*cosh(x) + b*sin
h(x))/(cosh(x) - sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + ...
```

3.713.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**3*sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Timed out
```

3.713.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^2b^3\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{(a^2+3\,ab)x}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)} + \frac{\left(4\,be^{(-2\,x)}+a+b\right)e^{(4\,x)}}{64\,(a^2+2\,ab+b^2)} + \frac{4\,be^{(-2\,x)}-(a-b)e^{(-4\,x)}}{64\,(a^2-2\,ab+b^2)}$$

input
```
integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
-a^2*b^3*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
) - 1/8*(a^2 + 3*a*b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/64*(4*b*e^(-2*
x) + a + b)*e^(4*x)/(a^2 + 2*a*b + b^2) + 1/64*(4*b*e^(-2*x) - (a - b)*e^(
-4*x))/(a^2 - 2*a*b + b^2)
```

3.713.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.04

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^2b^3\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{(a^2-3\,ab)x}{8\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{\left(6\,a^2e^{(4\,x)}-18\,abe^{(4\,x)}+4\,abe^{(2\,x)}-4\,b^2e^{(2\,x)}-a^2+2\,ab-b^2\right)e^{(-4\,x)}}{64\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{ae^{(4\,x)}+be^{(4\,x)}+4\,be^{(2\,x)}}{64\,(a^2+2\,ab+b^2)}$$

input
```
integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
-a^2*b^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6) - 1/8*(a^2 - 3*a*b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(6
*a^2*e^(4*x) - 18*a*b*e^(4*x) + 4*a*b*e^(2*x) - 4*b^2*e^(2*x) - a^2 + 2*a*
b - b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^
(4*x) + 4*b*e^(2*x))/(a^2 + 2*a*b + b^2)
```

3.713.9 Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.66

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{4x}}{64a+64b}-\frac{e^{-4x}}{64a-64b}+\frac{x(3ab-a^2)}{8(a-b)^3}+\frac{b\,e^{2x}}{16(a+b)^2}+\frac{b\,e^{-2x}}{16(a-b)^2}-\frac{a^2b^3\,\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^6-3a^4b^2+3a^2b^4-b^6}$$

input
```
int((cosh(x)^3*sinh(x)^2)/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) + (x*(3*a*b - a^2))/(8*(a
 - b)^3) + (b*exp(2*x))/(16*(a + b)^2) + (b*exp(-2*x))/(16*(a - b)^2) - (a
^2*b^3*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)
```

3.714 $\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.714.1 Optimal result

Integrand size = 20, antiderivative size = 212

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^3b^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}}+\frac{a^3b^2\cosh(x)}{(a^2-b^2)^3} - \frac{ab^2\cosh^3(x)}{3(a^2-b^2)^2}-\frac{a\cosh^3(x)}{3(a^2-b^2)}+\frac{a\cosh^5(x)}{5(a^2-b^2)} - \frac{a^2b^3\sinh(x)}{(a^2-b^2)^3}+\frac{a^2b\sinh^3(x)}{3(a^2-b^2)^2}-\frac{b\sinh^3(x)}{3(a^2-b^2)}-\frac{b\sinh^5(x)}{5(a^2-b^2)}$$

output
```
a^3*b^3*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)+a^3*
b^2*cosh(x)/(a^2-b^2)^3-1/3*a*b^2*cosh(x)^3/(a^2-b^2)^2-1/3*a*cosh(x)^3/(a
^2-b^2)+1/5*a*cosh(x)^5/(a^2-b^2)-a^2*b^3*sinh(x)/(a^2-b^2)^3+1/3*a^2*b*si
nh(x)^3/(a^2-b^2)^2-1/3*b*sinh(x)^3/(a^2-b^2)-1/5*b*sinh(x)^5/(a^2-b^2)
```

3.714.2 Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.53

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$= \frac{1}{32}\left(\frac{4ab(3a^4+10a^2b^2+3b^4)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}}+\frac{2a(a^4+10a^2b^2+5b^4)\cosh(x)}{(a-b)^3(a+b)^3}\right.$$
$$-\frac{2a(a^2+3b^2)\cosh(3x)}{3(a-b)^2(a+b)^2}+\frac{2a\cosh(5x)}{5(a-b)(a+b)}+\frac{2b(5a^4+10a^2b^2+b^4)\sinh(x)}{(-a+b)^3(a+b)^3}$$
$$\left.-3\left(\frac{4ab\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{2a\cosh(x)}{a^2-b^2}+\frac{2b\sinh(x)}{-a^2+b^2}\right)+\frac{2b(3a^2+b^2)\sinh(3x)}{3(a-b)^2(a+b)^2}-\frac{2b\sinh(5x)}{5(a-b)(a+b)}\right)$$

input
```
Integrate[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
((4*a*b*(3*a^4 + 10*a^2*b^2 + 3*b^4)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]
*Sqrt[a + b])])/((a - b)^(7/2)*(a + b)^(7/2)) + (2*a*(a^4 + 10*a^2*b^2 + 5
*b^4)*Cosh[x])/((a - b)^3*(a + b)^3) - (2*a*(a^2 + 3*b^2)*Cosh[3*x])/(3*(a
 - b)^2*(a + b)^2) + (2*a*Cosh[5*x])/(5*(a - b)*(a + b)) + (2*b*(5*a^4 + 1
0*a^2*b^2 + b^4)*Sinh[x])/((-a + b)^3*(a + b)^3) - 3*((4*a*b*ArcTan[(b + a
*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*(a + b)^(3/2)) + (2
*a*Cosh[x])/(a^2 - b^2) + (2*b*Sinh[x])/(-a^2 + b^2)) + (2*b*(3*a^2 + b^2)
*Sinh[3*x])/(3*(a - b)^2*(a + b)^2) - (2*b*Sinh[5*x])/(5*(a - b)*(a + b)))
/32
```

3.714.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{i\sin(ix)^3\cos(ix)^3}{a\cos(ix)-ib\sin(ix)}dx$$
$$\Big\downarrow \; 26$$

$$i\int \frac{\cos(ix)^3\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3588}$$

$$i\left(-\frac{ib\int -\cosh^3(x)\sinh^2(x)dx}{a^2-b^2}+\frac{a\int -i\cosh^2(x)\sinh^3(x)dx}{a^2-b^2}+\frac{iab\int -\frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(\frac{ib\int \cosh^3(x)\sinh^2(x)dx}{a^2-b^2}+\frac{a\int -i\cosh^2(x)\sinh^3(x)dx}{a^2-b^2}-\frac{iab\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{ib\int \cosh^3(x)\sinh^2(x)dx}{a^2-b^2}-\frac{ia\int \cosh^2(x)\sinh^3(x)dx}{a^2-b^2}-\frac{iab\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{ib\int -\cos(ix)^3\sin(ix)^2dx}{a^2-b^2}-\frac{ia\int i\cos(ix)^2\sin(ix)^3dx}{a^2-b^2}-\frac{iab\int -\frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{ib\int \cos(ix)^3\sin(ix)^2dx}{a^2-b^2}-\frac{ia\int i\cos(ix)^2\sin(ix)^3dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{ib\int \cos(ix)^3\sin(ix)^2dx}{a^2-b^2}+\frac{a\int \cos(ix)^2\sin(ix)^3dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3044}$$

$$i\left(-\frac{b\int -\sinh^2(x)\left(\sinh^2(x)+1\right)d(i\sinh(x))}{a^2-b^2}+\frac{a\int \cos(ix)^2\sin(ix)^3dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{244}$$

$$i\left(-\frac{b\int \left(-\sinh^4(x)-\sinh^2(x)\right)d(i\sinh(x))}{a^2-b^2}+\frac{a\int \cos(ix)^2\sin(ix)^3dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

3.714. $\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{a\int\cos(ix)^2\sin(ix)^3dx}{a^2-b^2}+\frac{iab\int\frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3045}$$

$$i\left(\frac{ia\int\cosh^2(x)\left(1-\cosh^2(x)\right)d\cosh(x)}{a^2-b^2}+\frac{iab\int\frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{244}$$

$$i\left(\frac{ia\int\left(\cosh^2(x)-\cosh^4(x)\right)d\cosh(x)}{a^2-b^2}+\frac{iab\int\frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{iab\int\frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\frac{ia\left(\frac{\cosh^3(x)}{3}-\frac{\cosh^5(x)}{5}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3588}$$

$$i\left(\frac{iab\left(-\frac{ib\int i\cosh^2(x)\sinh(x)dx}{a^2-b^2}+\frac{a\int-\cosh(x)\sinh^2(x)dx}{a^2-b^2}+\frac{iab\int\frac{i\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\right.$$

$$\Big\downarrow \text{25}$$

$$i\left(\frac{iab\left(-\frac{ib\int i\cosh^2(x)\sinh(x)dx}{a^2-b^2}-\frac{a\int\cosh(x)\sinh^2(x)dx}{a^2-b^2}+\frac{iab\int\frac{i\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\frac{ia}{\ }\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{iab\left(\frac{b\int\cosh^2(x)\sinh(x)dx}{a^2-b^2}-\frac{a\int\cosh(x)\sinh^2(x)dx}{a^2-b^2}-\frac{ab\int\frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\frac{ia\left(\mathrm{co}\right.}{\ }\right.$$

$$\Big\downarrow \text{3042}$$

3.714. $\int\frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$$i\left(\frac{iab\left(\frac{b\int -i\cos(ix)^2\sin(ix)dx}{a^2-b^2}-\frac{a\int -\cos(ix)\sin(ix)^2dx}{a^2-b^2}-\frac{ab\int -\frac{i\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\frac{ia}{}\right.$$

$$\Big\downarrow \ 25$$

$$i\left(\frac{iab\left(\frac{b\int -i\cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{a\int \cos(ix)\sin(ix)^2dx}{a^2-b^2}-\frac{ab\int -\frac{i\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\frac{ia(}{}\right.$$

$$\Big\downarrow \ 26$$

$$i\left(\frac{iab\left(-\frac{ib\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{a\int \cos(ix)\sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\frac{ia(}{}\right.$$

$$\Big\downarrow \ 3044$$

$$i\left(\frac{iab\left(-\frac{ia\int -\sinh^2(x)d(i\sinh(x))}{a^2-b^2}-\frac{ib\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\right.$$

$$\Big\downarrow \ 15$$

$$i\left(\frac{iab\left(-\frac{ib\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\frac{ia\left(\frac{\cosh^3(x)}{3}-\right.}{a^2-}\right.$$

$$\Big\downarrow \ 3045$$

$$i\left(\frac{iab\left(\frac{b\int \cosh^2(x)d\cosh(x)}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\frac{ia\left(\frac{\cosh^3(x)}{3}-\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \ 15$$

3.714. $\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$$i\left(\frac{iab\left(\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\frac{ia\left(\frac{\cosh^3(x)}{3}-\frac{\cosh^5(x)}{5}\right)}{a^2-b^2}\right)$$

$\downarrow$ 3588

$$i\left(\frac{iab\left(\frac{iab\left(\frac{a\int i\sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}\right.$$

$\downarrow$ 26

$$i\left(\frac{iab\left(\frac{iab\left(\frac{ia\int \sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}\right.$$

$\downarrow$ 3042

$$i\left(\frac{iab\left(\frac{iab\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{ia\int -i\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sin\right.}{a^2-b^2}\right.$$

$\downarrow$ 26

$$i\left(\frac{iab\left(\frac{iab\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a\int \sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3\right.}{a^2-b^2}\right.$$

3.714. $\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

$\downarrow$ 3117

$$i\left(\frac{iab\left(\frac{iab\left(\frac{a\int \sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\right.$$

$\downarrow$ 3118

$$i\left(\frac{iab\left(\frac{iab\left(\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{b\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)\right)}{a^2-b^2}+\right.$$

input
```
Int[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

output
```
$Aborted
```

3.714.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

3.714. $\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

3.714.4 Maple [A] (verified)

Time = 31.60 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.25

method	result
default	$-\frac{16}{5\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}(16a+16b)}-\frac{4}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}(8a+8b)}-\frac{a+3b}{8(a+b)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{5a+7b}{12(a+b)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{a(a+3}{8(a+b)^{3}(\tan}$
risch	$\frac{e^{5x}}{160a+160b}-\frac{e^{3x}a}{96(a+b)^{2}}+\frac{e^{3x}b}{96(a+b)^{2}}-\frac{e^{x}a^{2}}{16(a+b)^{3}}-\frac{e^{x}ab}{4(a+b)^{3}}-\frac{e^{x}b^{2}}{16(a+b)^{3}}-\frac{e^{-x}a^{2}}{16(a^{3}-3a^{2}b+3a\,b^{2}-b^{3})}+\frac{e^{-x}ab}{4a^{3}-12a^{2}b+12a\,b}$

input
```
int(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-16/5/(tanh(1/2*x)-1)^5/(16*a+16*b)-4/(tanh(1/2*x)-1)^4/(8*a+8*b)-1/8*(a+3
*b)/(a+b)^2/(tanh(1/2*x)-1)^2-1/12*(5*a+7*b)/(a+b)^2/(tanh(1/2*x)-1)^3+1/8
*a*(a+3*b)/(a+b)^3/(tanh(1/2*x)-1)-4/(tanh(1/2*x)+1)^4/(8*a-8*b)+16/5/(tan
h(1/2*x)+1)^5/(16*a-16*b)-1/12*(-5*a+7*b)/(a-b)^2/(tanh(1/2*x)+1)^3-1/8*(a
-3*b)/(a-b)^2/(tanh(1/2*x)+1)^2-1/8*a*(a-3*b)/(a-b)^3/(tanh(1/2*x)+1)+2*a^
3*b^3/(a+b)^3/(a-b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^
2-b^2)^(1/2))
```

3.714.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2440 vs. 2(196) = 392.

Time = 0.32 (sec) , antiderivative size = 4935, normalized size of antiderivative = 23.28

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
[1/480*(3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a
*b^6 + b^7)*cosh(x)^10 + 30*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b
^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^9 + 3*(a^7 - a^6*b - 3*a^5*b
^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^10 - 5*(a^7
- 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cos
h(x)^8 - 5*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*
a*b^6 - b^7 - 27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*
b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^8 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - (a^7 - 3*a^6
*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x))*s
inh(x)^7 + 3*a^7 + 3*a^6*b - 9*a^5*b^2 - 9*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5
 - 3*a*b^6 - 3*b^7 - 30*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 -
 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^6 - 10*(3*a^7 + 3*a^6*b - 27*a^5*b^2 + 2
1*a^4*b^3 + 21*a^3*b^4 - 27*a^2*b^5 + 3*a*b^6 + 3*b^7 - 63*(a^7 - a^6*b -
3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 1
4*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b
^7)*cosh(x)^2)*sinh(x)^6 + 4*(189*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3
*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 70*(a^7 - 3*a^6*b + a^5*b^
2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^3 - 45*(a^7 +
 a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*c...
```

3.714.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**3*sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

output
```
Timed out
```

3.714.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.714.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.53

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{2\,a^3b^3\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}} - \frac{\left(30\,a^2e^{(4\,x)}-120\,abe^{(4\,x)}+30\,b^2e^{(4\,x)}+5\,a^2e^{(2\,x)}-5\,b^2e^{(2\,x)}-3\,a^2+6\,ab-3\,b^2\right)e^{(-5\,x)}}{480\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{3\,a^4e^{(5\,x)}+12\,a^3be^{(5\,x)}+18\,a^2b^2e^{(5\,x)}+12\,ab^3e^{(5\,x)}+3\,b^4e^{(5\,x)}-5\,a^4e^{(3\,x)}-10\,a^3be^{(3\,x)}+10\,ab^3e^{(3\,x)}+5}{480\,(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4}$$

input
```
integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
2*a^3*b^3*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*sqrt(a^2 - b^2)) - 1/480*(30*a^2*e^(4*x) - 120*a*b*e^(4*x) +
30*b^2*e^(4*x) + 5*a^2*e^(2*x) - 5*b^2*e^(2*x) - 3*a^2 + 6*a*b - 3*b^2)*e^
(-5*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/480*(3*a^4*e^(5*x) + 12*a^3*b*e
^(5*x) + 18*a^2*b^2*e^(5*x) + 12*a*b^3*e^(5*x) + 3*b^4*e^(5*x) - 5*a^4*e^(
3*x) - 10*a^3*b*e^(3*x) + 10*a*b^3*e^(3*x) + 5*b^4*e^(3*x) - 30*a^4*e^x -
180*a^3*b*e^x - 300*a^2*b^2*e^x - 180*a*b^3*e^x - 30*b^4*e^x)/(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
```

3.714.9 Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.75

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{-5x}}{160a-160b}+\frac{e^{5x}}{160a+160b} + \frac{2\,\mathrm{atan}\left(\frac{a^3\,b^3\,e^x\,\sqrt{a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b^{14}}}{a^7\,\sqrt{a^6\,b^6}+b^7\,\sqrt{a^6\,b^6}-3\,a^2\,b^5\,\sqrt{a^6\,b^6}+3\,a^3\,b^4\,\sqrt{a^6\,b^6}+3\,a^4\,b^3\,\sqrt{a^6\,b^6}-3\,a^5\,b^2\,\sqrt{a^6\,b^6}-a\,b^6\,\sqrt{a^6\,b^6}-a^6\,b\,\sqrt{a^6\,b^6}}\right)\,\sqrt{a^6\,b^6}}{\sqrt{a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b^{14}}} - \frac{e^{-x}\,(a^2-4\,a\,b+b^2)}{16\,(a-b)^3} - \frac{e^{-3x}\,(a+b)}{96\,(a-b)^2} - \frac{e^{3x}\,(a-b)}{96\,(a+b)^2} - \frac{e^{x}\,(a^2+4\,a\,b+b^2)}{16\,(a+b)^3}$$

input
```
int((cosh(x)^3*sinh(x)^3)/(a*cosh(x) + b*sinh(x)),x)
```

output
```
exp(-5*x)/(160*a - 160*b) + exp(5*x)/(160*a + 160*b) + (2*atan((a^3*b^3*ex
p(x)*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 2
1*a^10*b^4 - 7*a^12*b^2)^(1/2))/(a^7*(a^6*b^6)^(1/2) + b^7*(a^6*b^6)^(1/2)
 - 3*a^2*b^5*(a^6*b^6)^(1/2) + 3*a^3*b^4*(a^6*b^6)^(1/2) + 3*a^4*b^3*(a^6*
b^6)^(1/2) - 3*a^5*b^2*(a^6*b^6)^(1/2) - a*b^6*(a^6*b^6)^(1/2) - a^6*b*(a^
6*b^6)^(1/2)))*(a^6*b^6)^(1/2))/(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 +
35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1/2) - (exp(-x)*(a^2
- 4*a*b + b^2))/(16*(a - b)^3) - (exp(-3*x)*(a + b))/(96*(a - b)^2) - (exp
(3*x)*(a - b))/(96*(a + b)^2) - (exp(x)*(4*a*b + a^2 + b^2))/(16*(a + b)^3
)
```

3.715 $\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.715.1 Optimal result

Integrand size = 16, antiderivative size = 93

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2abx}{(a^2-b^2)^2} + \frac{a^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} + \frac{b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} + \frac{b\sinh(x)}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}$$

output
```
-2*a*b*x/(a^2-b^2)^2+a^2*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^2+b^2*ln(a*cosh
(x)+b*sinh(x))/(a^2-b^2)^2+b*sinh(x)/(a^2-b^2)/(a*cosh(x)+b*sinh(x))
```

3.715.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{-2abx+(a^2+b^2)\log(a\cosh(x)+b\sinh(x))+\frac{(a-b)b(a+b)\sinh(x)}{a\cosh(x)+b\sinh(x)}}{(a-b)^2(a+b)^2}$$

input
```
Integrate[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(-2*a*b*x + (a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]] + ((a - b)*b*(a + b)*Si
nh[x])/(a*Cosh[x] + b*Sinh[x]))/((a - b)^2*(a + b)^2)
```

3.715.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.57, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {3042, 26, 3590, 26, 3042, 26, 3554, 3576, 26, 3042, 3577, 26, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3590}$$

$$-i\left(\frac{iab\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{a\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{iab\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{ia\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{iab\int \frac{1}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{iab\int \frac{1}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3554}$$

$$-i\left(-\frac{ib\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ib\sinh(x)}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3576}$$

$$-i\left(-\frac{ib\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(-\frac{a\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh(x)}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{ib\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{ia\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh(x)}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{ib\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh(x)}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3577}$$

$$-i\left(\frac{a\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh(x)}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{a\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh(x)}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3042}$$

3.715. $\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-i\left(\frac{a\left(\frac{ia\int\frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{ax}{a^2-b^2}-\frac{b\int\frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh(x)}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3612}$$

$$-i\left(\frac{ib\sinh(x)}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}+\frac{a\left(\frac{ia\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\right)}{a^2-b^2}\right)$$

input
```
Int[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(-I)*((a*(((-I)*b*x)/(a^2 - b^2) + (I*a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 -
 b^2)))/(a^2 - b^2) - (I*b*((a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[
x]])/(a^2 - b^2)))/(a^2 - b^2) + (I*b*Sinh[x])/((a^2 - b^2)*(a*Cosh[x] + b
*Sinh[x])))
```

3.715.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.715. $\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.715.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{(a^2+b^2)(a+b\tanh(x))\ln(a+b\tanh(x))-(a^2+b^2)(a+b\tanh(x))\ln(1-\tanh(x))-(((x+1)b+a(-1+x))b\tanh(x)+ax(a+b))(a+}{(a-b)^2(a+b)^2(a+b\tanh(x))}$
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{(a+b)^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{(a-b)^2}+\frac{\frac{2b\left(a^2-b^2\right)\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a}+(a^2+b^2)\ln\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a\right)}{(a+b)^2(a-b)^2}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2x\,a^2}{a^4-2a^2b^2+b^4}-\frac{2b^2x}{a^4-2a^2b^2+b^4}-\frac{2ab}{(a-b)(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}+\frac{\ln\left(e^{2x}+\frac{a-b}{a+b}\right)a^2}{a^4-2a^2b^2+b^4}+\frac{b^2\ln\left(e\right.}{a^4-2}$

input
```
int(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
((a^2+b^2)*(a+b*tanh(x))*ln(a+b*tanh(x))-(a^2+b^2)*(a+b*tanh(x))*ln(1-tanh
(x))-(((x+1)*b+a*(-1+x))*b*tanh(x)+a*x*(a+b))*(a+b))/(a-b)^2/(a+b)^2/(a+b*
tanh(x))
```

3.715.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(93) = 186.

Time = 0.27 (sec) , antiderivative size = 376, normalized size of antiderivative = 4.04

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{(a^3+3a^2b+3ab^2+b^3)x\cosh(x)^2+2(a^3+3a^2b+3ab^2+b^3)x\cosh(x)\sinh(x)+(a^3+3a^2b+3ab^2}{a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+(a^5}$$

input
```
integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
-((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*cosh(x)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*x*cosh(x)*sinh(x) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*sinh(x)^2 +
2*a^2*b - 2*a*b^2 + (a^3 + a^2*b - a*b^2 - b^3)*x - (a^3 - a^2*b + a*b^2 -
 b^3 + (a^3 + a^2*b + a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 + a^2*b + a*b^2 + b^
3)*cosh(x)*sinh(x) + (a^3 + a^2*b + a*b^2 + b^3)*sinh(x)^2)*log(2*(a*cosh(
x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3
 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(
x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(
x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2)
```

3.715.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. 2(83) = 166.

Time = 0.84 (sec) , antiderivative size = 962, normalized size of antiderivative = 10.34

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Piecewise((zoo*log(sinh(x)), Eq(a, 0) & Eq(b, 0)), (log(sinh(x))/b**2, Eq(
a, 0)), (-2*x*sinh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*
b**2*cosh(x)**2) + 4*x*sinh(x)*cosh(x)/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x
)*cosh(x) + 8*b**2*cosh(x)**2) - 2*x*cosh(x)**2/(8*b**2*sinh(x)**2 - 16*b*
*2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + sinh(x)**2/(8*b**2*sinh(x)**2 -
16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + cosh(x)**2/(8*b**2*sinh(x)*
*2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2), Eq(a, -b)), (2*x*sinh(x
)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 4
*x*sinh(x)*cosh(x)/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*c
osh(x)**2) + 2*x*cosh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) +
 8*b**2*cosh(x)**2) + sinh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh
(x) + 8*b**2*cosh(x)**2) + cosh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)
*cosh(x) + 8*b**2*cosh(x)**2), Eq(a, b)), (a**3*log(cosh(x) + b*sinh(x)/a)
*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**
3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) - a**3*cosh(x)/(a**5*cosh(x) +
a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x
) + b**5*sinh(x)) - 2*a**2*b*x*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*
a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) +
 a**2*b*log(cosh(x) + b*sinh(x)/a)*sinh(x)/(a**5*cosh(x) + a**4*b*sinh(x)
- 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sin...
```

3.715.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.15

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2\,ab}{a^4-2\,a^2b^2+b^4+(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,x)}} + \frac{(a^2+b^2)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4} + \frac{x}{a^2+2\,ab+b^2}$$

input
```
integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
2*a*b/(a^4 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x)) +
 (a^2 + b^2)*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + x/(a
^2 + 2*a*b + b^2)
```

3.715.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{(a^2+b^2)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4} - \frac{x}{a^2-2ab+b^2} - \frac{a^2e^{(2x)}+b^2e^{(2x)}+a^2-b^2}{(a^3-a^2b-ab^2+b^3)(ae^{(2x)}+be^{(2x)}+a-b)}$$

input
```
integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
(a^2 + b^2)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4
) - x/(a^2 - 2*a*b + b^2) - (a^2*e^(2*x) + b^2*e^(2*x) + a^2 - b^2)/((a^3
- a^2*b - a*b^2 + b^3)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

3.715.9 Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \ln\left(a\cosh(x)+b\sinh(x)\right)\left(\frac{1}{2\,(a+b)^2}+\frac{1}{2\,(a-b)^2}\right) - \frac{\frac{a\cosh(x)}{a^2-b^2}+\frac{2\,a^2\,b\,x\cosh(x)}{(a^2-b^2)^2}+\frac{2\,a\,b^2\,x\sinh(x)}{(a^2-b^2)^2}}{a\cosh\left(x\right)+b\sinh\left(x\right)}$$

input
```
int((cosh(x)*sinh(x))/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
log(a*cosh(x) + b*sinh(x))*(1/(2*(a + b)^2) + 1/(2*(a - b)^2)) - ((a*cosh(
x))/(a^2 - b^2) + (2*a^2*b*x*cosh(x))/(a^2 - b^2)^2 + (2*a*b^2*x*sinh(x))/
(a^2 - b^2)^2)/(a*cosh(x) + b*sinh(x))
```

3.716 $\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.716.1 Optimal result

Integrand size = 18, antiderivative size = 165

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{a^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{2ab^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{2ab\cosh(x)}{(a^2-b^2)^2} + \frac{a^2\sinh(x)}{(a^2-b^2)^2} + \frac{b^2\sinh(x)}{(a^2-b^2)^2} - \frac{a^2b}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}$$

output
```
-a^3*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-2*a*b^2
*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-2*a*b*cosh(
x)/(a^2-b^2)^2+a^2*sinh(x)/(a^2-b^2)^2+b^2*sinh(x)/(a^2-b^2)^2-a^2*b/(a^2-
b^2)^2/(a*cosh(x)+b*sinh(x))
```

3.716.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.35

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{-2a^2\sqrt{a-b}b\sqrt{a+b}\cosh^2(x)+a\cosh(x)\left(-2a(a^2+2b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)+\sqrt{a-b}\sqrt{a+b}(a^2-b^2)\right.}{(a-b)^{5/2}(a+b)^{5}}$$

input
```
Integrate[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(-2*a^2*Sqrt[a - b]*b*Sqrt[a + b]*Cosh[x]^2 + a*Cosh[x]*(-2*a*(a^2 + 2*b^2
)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + Sqrt[a - b]*Sqrt[a
 + b]*(a^2 - b^2)*Sinh[x]) + b*(-(a^2*Sqrt[a - b]*Sqrt[a + b]) - 2*a*(a^2
+ 2*b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x] + Sqr
t[a - b]*Sqrt[a + b]*(a^2 + b^2)*Sinh[x]^2))/((a - b)^(5/2)*(a + b)^(5/2)*
(a*Cosh[x] + b*Sinh[x]))
```

3.716.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.74, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 25, 3590, 25, 26, 3042, 25, 26, 3578, 26, 3042, 26, 3118, 3553, 219, 3588, 26, 3042, 26, 3117, 3118, 3553, 219, 3633, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ix)^2\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3590}$$

$$-\frac{a\int -\frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{iab\int \frac{i\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}+\frac{ib\int \frac{i\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{iab\int \frac{i\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}+\frac{ib\int \frac{i\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{a\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{ab\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}-\frac{b\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{ab\int -\frac{i\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{b\int -\frac{i\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int -\frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \ 25$$

$$\frac{ab\int -\frac{i\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{b\int -\frac{i\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\int \frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \ 26$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\int \frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \ 3578$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\left(-\frac{ib\int i\sinh(x)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \ 26$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\left(\frac{b\int \sinh(x)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\left(\frac{b\int -i\sin(ix)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \ 26$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\left(-\frac{ib\int \sin(ix)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

3.716. $\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$\Big\downarrow \text{3118}$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\left(\frac{a^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\left(\frac{ia^2\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3588}$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{a\int i\sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{ia\int \sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

3.716. $\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{ia\int -i\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\downarrow 26$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a\int \sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\downarrow 3117$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{a\int \sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\downarrow 3118$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\downarrow 3553$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ab\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

3.716. $\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$\Big\downarrow \text{219}$$

$$-\frac{iab\int \frac{\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(-\frac{ab\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3633}$$

$$-\frac{iab\left(-\frac{ib\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ia}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}\right)}{a^2-b^2}-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(-\frac{ab\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{iab\left(-\frac{ib\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}\right)}{a^2-b^2}-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(-\frac{ab\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3553}$$

3.716. $\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx$

$$-\frac{iab\left(\frac{b\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}-\frac{ia}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}\right)}{a^2-b^2}-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(-\frac{ab\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{a\left(\frac{ia^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(-\frac{ab\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{iab\left(\frac{b\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{ia}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}\right)}{a^2-b^2}$$

input
```
Int[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
-((a*((I*a^2*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(3/2) + (b*Cosh[x])/(a^2 - b^2) - (a*Sinh[x])/(a^2 - b^2)))/(a^2 -
 b^2)) + (I*b*(-((a*b*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 - b^
2]])/(a^2 - b^2)^(3/2)) + (I*a*Cosh[x])/(a^2 - b^2) - (I*b*Sinh[x])/(a^2 -
 b^2)))/(a^2 - b^2) - (I*a*b*((b*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sq
rt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - (I*a)/((a^2 - b^2)*(a*Cosh[x] + b*Sinh
[x]))))/(a^2 - b^2)
```

3.716. $\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.716.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

3.716. $\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

rule 3633
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - c*C)/(a^2 - b^2 - c^2)   Int[1/(
a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

3.716.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.78

method	result
default	$-\frac{1}{(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{1}{(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{2a\left(\frac{b^2\tanh\left(\frac{x}{2}\right)+ab}{\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a}+\frac{\left(a^2+2b^2\right)\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2}$
risch	$\frac{e^{x}}{2a^2+4ab+2b^2}-\frac{e^{-x}}{2(a^2-2ab+b^2)}-\frac{2a^2b\,e^{x}}{(a-b)^2(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}-\frac{a^3\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}-\frac{2b^2a\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^2-}}\right.}{\sqrt{-a^2+b^2}\,(a+b)^2(a}$

input
```
int(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a+b)^2/(tanh(1/2*x)-1)-1/(a-b)^2/(tanh(1/2*x)+1)-2*a/(a-b)^2/(a+b)^2*(
(b^2*tanh(1/2*x)+a*b)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+(a^2+2*b^2)/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

3.716.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(157) = 314$.

Time = 0.29 (sec) , antiderivative size = 1819, normalized size of antiderivative = 11.02

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
[-1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^
2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(5*a^4*b - 4*a^2*b^3 - b^5)*cosh(
x)^2 + 2*(5*a^4*b - 4*a^2*b^3 - b^5 - 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b
^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 2*((a^4 + a^3*b + 2*a^2*b^2 + 2*a
*b^3)*cosh(x)^3 + 3*(a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*cosh(x)*sinh(x)^2
+ (a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*sinh(x)^3 + (a^4 - a^3*b + 2*a^2*b^2
 - 2*a*b^3)*cosh(x) + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b^3 + 3*(a^4 + a^3*b
+ 2*a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b
^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*si
nh(x) + (a + b)*sinh(x)^2 + a - b)) - 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*
b^3 + a*b^4 - b^5)*cosh(x)^3 - (5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x))*sinh(x
))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
 b^7)*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a
^2*b^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4
*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a
^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 -
 a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + ...
```

3.716.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)*sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.716.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.716.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,(a^3+2\,ab^2)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{e^x}{2\,(a^2+2\,ab+b^2)} - \frac{a^3e^{(2\,x)}+7\,a^2be^{(2\,x)}+3\,ab^2e^{(2\,x)}+b^3e^{(2\,x)}+a^3+a^2b-ab^2-b^3}{2\,(a^4-2\,a^2b^2+b^4)(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x)}$$

input
```
integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-2*(a^3 + 2*a*b^2)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b
^2 + b^4)*sqrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) - 1/2*(a^3*e^(2*x
) + 7*a^2*b*e^(2*x) + 3*a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*b - a*b^2
- b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

3.716.9 Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.41

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^x}{2\,(a+b)^2} - \frac{e^{-x}}{2\,(a-b)^2} - \frac{2\,\mathrm{atan}\left(\frac{e^x\,\left(a^3\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}+2\,a\,b^2\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-}\right.}{a^5\,\sqrt{a^6+4\,a^4\,b^2+4\,a^2\,b^4}-b^5\,\sqrt{a^6+4\,a^4\,b^2+4\,a^2\,b^4}+2\,a^2\,b^3\,\sqrt{a^6+4\,a^4\,b^2+4\,a^2\,b^4}-2\,a^3\,b^2\,\sqrt{a^6+4\,a^4\,b^2+4\,a^2\,b^4}+a\,b^4\,\sqrt{a^6+4\,a^4\,b^2+}}\right.}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}} - \frac{2\,a^2\,b\,e^x}{(a+b)^2\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)}$$

input
```
int((cosh(x)*sinh(x)^2)/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
exp(x)/(2*(a + b)^2) - exp(-x)/(2*(a - b)^2) - (2*atan((exp(x)*(a^3*(a^10
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 2*a*b^2*
(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))/(a
^5*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - b^5*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)
^(1/2) + 2*a^2*b^3*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - 2*a^3*b^2*(a^6 +
4*a^2*b^4 + 4*a^4*b^2)^(1/2) + a*b^4*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) -
 a^4*b*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2)))*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)
^(1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1
/2) - (2*a^2*b*exp(x))/((a + b)^2*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

3.717 $\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.717.1 Optimal result

Integrand size = 18, antiderivative size = 215

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a^3bx}{(a^2-b^2)^3}+\frac{ab^3x}{(a^2-b^2)^3}+\frac{abx}{(a^2-b^2)^2}+\frac{ab(a^2+b^2)\,x}{(a^2-b^2)^3} - \frac{a^2b}{(a^2-b^2)^2\,(b+a\coth(x))}-\frac{a^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} - \frac{3a^2b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} - \frac{ab\cosh(x)\sinh(x)}{(a^2-b^2)^2}+\frac{a^2\sinh^2(x)}{2\,(a^2-b^2)^2}+\frac{b^2\sinh^2(x)}{2\,(a^2-b^2)^2}$$

output
```
a^3*b*x/(a^2-b^2)^3+a*b^3*x/(a^2-b^2)^3+a*b*x/(a^2-b^2)^2+a*b*(a^2+b^2)*x/
(a^2-b^2)^3-a^2*b/(a^2-b^2)^2/(b+a*coth(x))-a^4*ln(a*cosh(x)+b*sinh(x))/(a
^2-b^2)^3-3*a^2*b^2*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3-a*b*cosh(x)*sinh(x
)/(a^2-b^2)^2+1/2*a^2*sinh(x)^2/(a^2-b^2)^2+1/2*b^2*sinh(x)^2/(a^2-b^2)^2
```

3.717.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.82

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \frac{a(a^2-b^2)^2\cosh(3x)+a\cosh(x)\left(a^4+2a^2b^2-3b^4+24a^3bx+8ab^3x-8a^2(a^2+3b^2)\log(a\cosh(x)+b\sinh(x)\right)\ldots}{8(a-b)^3(a+\ldots}$$

input
```
Integrate[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(a*(a^2 - b^2)^2*Cosh[3*x] + a*Cosh[x]*(a^4 + 2*a^2*b^2 - 3*b^4 + 24*a^3*b
*x + 8*a*b^3*x - 8*a^2*(a^2 + 3*b^2)*Log[a*Cosh[x] + b*Sinh[x]]) - 2*b*((a
^2 - b^2)^2*Cosh[2*x] + 2*a*(3*a^3 - 3*a*b^2 - 6*a^2*b*x - 2*b^3*x + 2*a*(
a^2 + 3*b^2)*Log[a*Cosh[x] + b*Sinh[x]]))*Sinh[x])/(8*(a - b)^3*(a + b)^3*
(a*Cosh[x] + b*Sinh[x]))
```

3.717.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sin(ix)^3\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\cos(ix)\sin(ix)^3}{(a\cos(ix)-ib\sin(ix))^2}dx$$
$$\Big\downarrow \text{3590}$$
$$i\left(\frac{a\int -\frac{i\sinh^3(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{iab\int -\frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}-\frac{ib\int -\frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$
$$\Big\downarrow \text{25}$$

$$i\left(\frac{a\int -\frac{i\sinh^3(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{iab\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}+\frac{ib\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{ia\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{iab\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}+\frac{ib\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{iab\int -\frac{\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int -\frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\int \frac{i\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(\frac{iab\int \frac{\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\int \frac{i\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{iab\int \frac{\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3564}$$

$$i\left(\frac{iab\int \frac{1}{(-ib-ia\coth(x))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3578}$$

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int -\sinh^2(x)dx}{a^2-b^2}+\frac{a^2\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}\right.$$

$$\Big\downarrow \text{25}$$

3.717. $\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{ib\int \sinh^2(x)dx}{a^2-b^2}+\frac{a^2\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{ib\int \sinh^2(x)dx}{a^2-b^2}+\frac{ia^2\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{ib\int -\sin(ix)^2dx}{a^2-b^2}+\frac{ia^2\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 25$$

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int \sin(ix)^2dx}{a^2-b^2}+\frac{ia^2\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int \sin(ix)^2dx}{a^2-b^2}+\frac{a^2\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 3115$$

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{a^2\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \; 24$$

3.717. $\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{a^2\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x\right.}{a^2-b^2}\right.}{a^2-b^2}\right.$$

$\downarrow$ 3576

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{a^2\left(-\frac{a\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib(}{}\right.}{a^2-b^2}\right.$$

$\downarrow$ 26

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{a^2\left(\frac{ia\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}-\frac{1}{2}\right.}{}\right.}{a^2-b^2}\right.$$

$\downarrow$ 3042

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{a^2\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}-\frac{1}{2}\right.}{}\right.}{a^2-b^2}\right.$$

$\downarrow$ 3588

3.717. $\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\left(\frac{a\int -\sinh^2(x)dx}{a^2-b^2}-\frac{ib\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(a^2\left(\frac{ia\int \frac{b\cos(\cdots}{a\cos(\cdots}}{}\cdots\right.\right.}{}\cdots\right.$$

$\downarrow$ 25

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{a\int \sinh^2(x)dx}{a^2-b^2}-\frac{ib\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(a^2\left(\frac{ia\int \frac{b\cos(\cdots}{a\cos(\cdots}}{}\cdots\right.\right.}{}\cdots\right.$$

$\downarrow$ 26

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{a\int \sinh^2(x)dx}{a^2-b^2}+\frac{b\int \cosh(x)\sinh(x)dx}{a^2-b^2}-\frac{ab\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(a^2\left(\frac{ia\int \frac{b\cos(ix)}{a\cos(ix)}\cdots}{a^2-\cdots}\right.\right.}{}\cdots\right.$$

$\downarrow$ 3042

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{a\int -\sin(ix)^2dx}{a^2-b^2}+\frac{b\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}-\frac{ab\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(a^2\left(\frac{ia\int \frac{b\,c\cdots}{a\,c\cdots}}{}\cdots\right.\right.}{}\cdots\right.$$

3.717. $\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$\downarrow$ 25

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}+\frac{b\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}-\frac{ab\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(\frac{a^2\left(\frac{ia\int \frac{b\cos(ix}{a\cos(ix}}{a^2}\right.}{}\right.}{}\right.$$

$\downarrow$ 26

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}-\frac{ib\int \cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(\frac{a^2\left(\frac{ia\int \frac{b\cos(ix)-i}{a\cos(ix)-i}}{a^2-b^2}\right.}{a^2}\right.}{}\right.$$

$\downarrow$ 3044

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}-\frac{b\int i\sinh(x)d(i\sinh(x))}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(\frac{a^2\left(\frac{ia\int \frac{b\cos(ix)}{a\cos(ix)}}{a^2}\right.}{}\right.}{}\right.$$

$\downarrow$ 15

3.717. $\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\left(\frac{a^2\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\right.}{a^2-b^2}\right.}{}\right.$$

$$\Big\downarrow \text{3115}$$

$$i\left(\frac{iab\int \frac{1}{\left(-ib-a\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}-\frac{ib\left(\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\left(\frac{a^2\left(\frac{ia\int \frac{b\cos(i}{a\cos(i}}{a}\right.}{}\right.}{}\right.$$

input
```
Int[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
$Aborted
```

3.717.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3564
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

3.717. $\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

3.717.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.82

method	result
default	$\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)}+\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)^2}+\frac{a\ln(\tanh(\frac{x}{2})-1)}{(a+b)^3}+\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)^2}-\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)}+$
risch	$-\frac{ax}{(a+b)(a^2+2ab+b^2)}+\frac{e^{2x}}{8a^2+16ab+8b^2}+\frac{e^{-2x}}{8a^2-16ab+8b^2}+\frac{2a^4x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{6a^2x\,b^2}{a^6-3a^4b^2+3a^2b^4-b^6}+$

input
```
int(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)^2/(tanh(1/2*x)-1)+1/2/(a+b)^2/(tanh(1/2*x)-1)^2+a/(a+b)^3*ln(tan
h(1/2*x)-1)+1/2/(a-b)^2/(tanh(1/2*x)+1)^2-1/2/(a-b)^2/(tanh(1/2*x)+1)+a/(a
-b)^3*ln(tanh(1/2*x)+1)-2*a^2/(a+b)^3/(a-b)^3*(b*(a^2-b^2)*tanh(1/2*x)/(ta
nh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+1/2*(a^2+3*b^2)*ln(tanh(1/2*x)^2*a+2*b*ta
nh(1/2*x)+a))
```

3.717.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1655 vs. 2(211) = 422.

Time = 0.28 (sec) , antiderivative size = 1655, normalized size of antiderivative = 7.70

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
1/8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2
*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^
3 - 3*a*b^4 + b^5 + 8*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*c
osh(x)^4 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 15*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + 8*(a^5 + 4*a^
4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 - 3*a^4*b + 2*a^3*b^2
+ 2*a^2*b^3 - 3*a*b^4 + b^5 + 8*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a
*b^4)*x)*cosh(x))*sinh(x)^3 + (a^5 + 19*a^4*b - 14*a^3*b^2 - 2*a^2*b^3 - 3
*a*b^4 - b^5 + 8*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*x)*cosh(x)^2 + (a^5 +
 19*a^4*b - 14*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(a^5 - a^4*b - 2*a
^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 - 3*a^4*b + 2*a^3*b^2
 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 8*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*x)*cosh(x)^2 + 8*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*x)*sinh(x)^2 -
 8*((a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^3)*cosh(x)^4 + 4*(a^5 + a^4*b + 3*a
^3*b^2 + 3*a^2*b^3)*cosh(x)*sinh(x)^3 + (a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b
^3)*sinh(x)^4 + (a^5 - a^4*b + 3*a^3*b^2 - 3*a^2*b^3)*cosh(x)^2 + (a^5 - a
^4*b + 3*a^3*b^2 - 3*a^2*b^3 + 6*(a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^3)*...
```

3.717.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)*sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.717.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= -\frac{ax}{a^3+3a^2b+3ab^2+b^3} - \frac{(a^4+3a^2b^2)\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}$$
$$+ \frac{a^4-2a^3b+2ab^3-b^4+(a^4-20a^3b+6a^2b^2-4ab^3+b^4)e^{(-2x)}}{8\left((a^6-3a^4b^2+3a^2b^4-b^6)e^{(-2x)}+(a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6)e^{(-4x)}\right)}$$
$$+ \frac{e^{(-2x)}}{8(a^2-2ab+b^2)}$$

input
```
integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-a*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (a^4 + 3*a^2*b^2)*log(-(a - b)*e^(-
2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(a^4 - 2*a^3*b + 2
*a*b^3 - b^4 + (a^4 - 20*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-2*x))/((a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*e^(-2*x) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a
^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6)*e^(-4*x)) + 1/8*e^(-2*x)/(a^2 - 2*a*b +
b^2)
```

3.717.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \frac{ax}{a^3-3a^2b+3ab^2-b^3} - \frac{(a^4+3a^2b^2)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{e^{(2x)}}{8(a^2+2ab+b^2)}$$
$$+ \frac{2a^3e^{(4x)}-4a^2be^{(4x)}+2ab^2e^{(4x)}+3a^3e^{(2x)}+11a^2be^{(2x)}+ab^2e^{(2x)}+b^3e^{(2x)}+a^3+a^2b-ab^2-b^3}{8(a^4-2a^2b^2+b^4)(ae^{(4x)}+be^{(4x)}+ae^{(2x)}-be^{(2x)})}$$

input
```
integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
a*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (a^4 + 3*a^2*b^2)*log(abs(a*e^(2*x)
+ b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*e^(2*x)/(a
^2 + 2*a*b + b^2) + 1/8*(2*a^3*e^(4*x) - 4*a^2*b*e^(4*x) + 2*a*b^2*e^(4*x)
 + 3*a^3*e^(2*x) + 11*a^2*b*e^(2*x) + a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 +
a^2*b - a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(4*x) + b*e^(4*x) + a*e
^(2*x) - b*e^(2*x)))
```

3.717.9 Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.59

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{2x}}{8(a+b)^2}+\frac{e^{-2x}}{8(a-b)^2}+\frac{ax}{(a-b)^3} - \frac{\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)\left(a^4+3a^2b^2\right)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{2a^3b}{(a+b)^3(a-b)^2\left(a-b+e^{2x}(a+b)\right)}$$

input
```
int((cosh(x)*sinh(x)^3)/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
exp(2*x)/(8*(a + b)^2) + exp(-2*x)/(8*(a - b)^2) + (a*x)/(a - b)^3 - (log(
a - b + a*exp(2*x) + b*exp(2*x))*(a^4 + 3*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2) + (2*a^3*b)/((a + b)^3*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

3.718 $\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.718.1 Optimal result

Integrand size = 18, antiderivative size = 163

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2a^2b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{b^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{a^2\cosh(x)}{(a^2-b^2)^2} + \frac{b^2\cosh(x)}{(a^2-b^2)^2} - \frac{2ab\sinh(x)}{(a^2-b^2)^2} + \frac{ab^2}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}$$

output
```
2*a^2*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+b^3*
arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+a^2*cosh(x)/
(a^2-b^2)^2+b^2*cosh(x)/(a^2-b^2)^2-2*a*b*sinh(x)/(a^2-b^2)^2+a*b^2/(a^2-b
^2)^2/(a*cosh(x)+b*sinh(x))
```

3.718.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{a\sqrt{a-b}(a+b)+2ab\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\cosh(x)+2b^2\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sinh(x)}{4(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))} + \frac{1}{4}\left(\frac{6b(3a^2+b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}+\frac{4(a^2+b^2)\cosh(x)}{(a-b)^2(a+b)^2}-\frac{8ab\sinh(x)}{(a-b)^2(a+b)^2}+\frac{a(a^2+3b^2)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))}\right)$$

input
```
Integrate[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
-1/4*(a*Sqrt[a - b]*(a + b) + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(
Sqrt[a - b]*Sqrt[a + b])]*Cosh[x] + 2*b^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x
/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[
x] + b*Sinh[x])) + ((6*b*(3*a^2 + b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a -
b]*Sqrt[a + b])])/((a - b)^(5/2)*(a + b)^(5/2)) + (4*(a^2 + b^2)*Cosh[x])/
((a - b)^2*(a + b)^2) - (8*a*b*Sinh[x])/((a - b)^2*(a + b)^2) + (a*(a^2 +
3*b^2))/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])))/4
```

3.718.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.78, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.278, Rules used = {3042, 26, 3590, 26, 3042, 26, 3579, 3042, 3117, 3553, 219, 3588, 26, 3042, 26, 3117, 3118, 3553, 219, 3634, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i \sin(ix)\cos(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 3590}$$

$$-i\left(-\frac{ib\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{iab\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}+\frac{a\int \frac{i\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(-\frac{ib\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{iab\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}+\frac{ia\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\int -\frac{i\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3579}$$

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{a\int \cosh(x)dx}{a^2-b^2}-\frac{b^2\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{a\int \sin(ix+\frac{\pi}{2})dx}{a^2-b^2}-\frac{b^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3117}$$

3.718. $\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(-\frac{b^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}\right)$$

$\downarrow$ 3553

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(-\frac{ib^2\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x)}{a^2-b^2}\right.}{a^2-b^2}\right.$$

$\downarrow$ 219

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(}{a^2-b}\right.}{a^2-b^2}\right.$$

$\downarrow$ 3588

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{a\int i\sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\,\mathrm{s}}{}\right.}{(a^2-b^2}\right.}{}\right.$$

$\downarrow$ 26

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{ia\int \sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\,\mathrm{si}}{}\right.}{(a^2-b^2}\right.}{}\right.$$

$\downarrow$ 3042

3.718. $\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{ia\int -i\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{ib^2\text{arctanh}}{}\right.}{}\right.$$

$\downarrow$ 26

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a\int \sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{ib^2\text{arctanh}\left(\right.}{(a}\right.}{}\right.$$

$\downarrow$ 3117

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{a\int \sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-}{\sqrt{a^2-}}\right.}{(a^2-b^2)^{3/2}}\right.}{a}\right.$$

$\downarrow$ 3118

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib}{\sqrt{a^2-b^2}}\right.}{(a^2-b^2)^{3/2}}\right.}{a^2}\right.$$

$\downarrow$ 3553

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(-\frac{ab\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}+\frac{ia\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}-\right.$$

$\downarrow$ 219

3.718. $\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-i\left(\frac{iab\int \frac{\cos(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(-\frac{ab\text{arctanh}\left(\frac{-\right.}{}\right.}{(a^2}\right.}{}\right.$$

$\downarrow$ 3634

$$-i\left(\frac{iab\left(\frac{a\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{b}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}\right)}{a^2-b^2}-\frac{ib\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh}{a^2-}\right.}{a^2-b^2}\right.$$

$\downarrow$ 3042

$$-i\left(\frac{iab\left(\frac{b}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}+\frac{a\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh}{a^2-}\right.}{a^2-b^2}\right.$$

$\downarrow$ 3553

$$-i\left(\frac{iab\left(\frac{b}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}+\frac{ia\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh}{\sqrt{}}\right.}{(a^2-b^2)^{3}}\right.}{}\right.$$

$\downarrow$ 219

$$-i\left(-\frac{ib\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(-\frac{ab\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{ib\sinh(x)}{a^2-b^2}\right.}{a^2-b^2}\right.$$

input
```
Int[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

3.718. $\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

output
```
(-I)*(((-I)*b*(((-I)*b^2*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt[a^2 -
 b^2]])/(a^2 - b^2)^(3/2) - (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b
^2)))/(a^2 - b^2) + (a*(-((a*b*ArcTanh[((-I)*b*Cosh[x] - I*a*Sinh[x])/Sqrt
[a^2 - b^2]])/(a^2 - b^2)^(3/2)) + (I*a*Cosh[x])/(a^2 - b^2) - (I*b*Sinh[x
])/(a^2 - b^2)))/(a^2 - b^2) + (I*a*b*((I*a*ArcTanh[((-I)*b*Cosh[x] - I*a*
Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + b/((a^2 - b^2)*(a*Cosh[x] +
 b*Sinh[x]))))/(a^2 - b^2))
```

3.718.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.718. $\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

rule 3634
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - b*B)/(a^2 - b^2 - c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

3.718. $\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.718.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.80

method	result
default	$-\frac{1}{(a+b)^2(\tanh(\frac{x}{2})-1)}+\frac{1}{(a-b)^2(\tanh(\frac{x}{2})+1)}+\frac{4b\left(\frac{\frac{b^2\tanh(\frac{x}{2})}{2}+\frac{ab}{2}}{\tanh(\frac{x}{2})^2a+2b\tanh(\frac{x}{2})+a}+\frac{(2a^2+b^2)\arctan\left(\frac{2a\tanh(\frac{x}{2})+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{(a+b)^2(a-b)^2}$
risch	$\frac{e^x}{2a^2+4ab+2b^2}+\frac{e^{-x}}{2a^2-4ab+2b^2}+\frac{2a\,b^2e^x}{(a-b)^2(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}-\frac{2b\,a^2\ln\left(e^x-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}-\frac{b^3\ln\left(e^x-\frac{a-b}{\sqrt{-a^2+b}}\right.}{\sqrt{-a^2+b^2}\,(a+b)^2(a}$

input
```
int(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a+b)^2/(tanh(1/2*x)-1)+1/(a-b)^2/(tanh(1/2*x)+1)+4*b/(a+b)^2/(a-b)^2*(
(1/2*b^2*tanh(1/2*x)+1/2*a*b)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+1/2*(2*a
^2+b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

3.718.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(155) = 310$.

Time = 0.29 (sec) , antiderivative size = 1805, normalized size of antiderivative = 11.07

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
[1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2
 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x
)^2 + 2*(a^5 + 4*a^3*b^2 - 5*a*b^4 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^
3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 2*((2*a^3*b + 2*a^2*b^2 + a*b^3 +
b^4)*cosh(x)^3 + 3*(2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*cosh(x)*sinh(x)^2 +
 (2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*sinh(x)^3 + (2*a^3*b - 2*a^2*b^2 + a*
b^3 - b^4)*cosh(x) + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^4 + 3*(2*a^3*b + 2*a
^2*b^2 + a*b^3 + b^4)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*co
sh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^
2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sin
h(x) + (a + b)*sinh(x)^2 + a - b)) + 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b
^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x))*sinh(x)
)/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
b^7)*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^
2*b^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*
b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^
5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 -
a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3...
```

3.718.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**2*sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.718.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.718.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2\,(2\,a^2b+b^3)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{e^x}{2\,(a^2+2\,ab+b^2)} + \frac{a^3e^{(2\,x)}+3\,a^2be^{(2\,x)}+7\,ab^2e^{(2\,x)}+b^3e^{(2\,x)}+a^3+a^2b-ab^2-b^3}{2\,(a^4-2\,a^2b^2+b^4)(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x)}$$

input
```
integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
2*(2*a^2*b + b^3)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^
2 + b^4)*sqrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) + 1/2*(a^3*e^(2*x)
 + 3*a^2*b*e^(2*x) + 7*a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*b - a*b^2 -
 b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

3.718.9 Mupad [B] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.44

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{-x}}{2\,(a-b)^2}+\frac{e^{x}}{2\,(a+b)^2}$$
$$+\frac{2\,\mathrm{atan}\left(\frac{e^{x}\,\left(b^3\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}+2\,a^2\,b\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-}\right.}{a^5\,\sqrt{4\,a^4\,b^2+4\,a^2\,b^4+b^6}-b^5\,\sqrt{4\,a^4\,b^2+4\,a^2\,b^4+b^6}+2\,a^2\,b^3\,\sqrt{4\,a^4\,b^2+4\,a^2\,b^4+b^6}-2\,a^3\,b^2\,\sqrt{4\,a^4\,b^2+4\,a^2\,b^4+b^6}+a\,b^4\,\sqrt{4\,a^4\,b^2+4\,a^2\,b^4}}\right.}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}$$
$$+\frac{2\,a\,b^2\,e^{x}}{(a+b)^2\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)}$$

input
```
int((cosh(x)^2*sinh(x))/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
exp(-x)/(2*(a - b)^2) + exp(x)/(2*(a + b)^2) + (2*atan((exp(x)*(b^3*(a^10
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 2*a^2*b*
(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))/(a
^5*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - b^5*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)
^(1/2) + 2*a^2*b^3*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - 2*a^3*b^2*(b^6 +
4*a^2*b^4 + 4*a^4*b^2)^(1/2) + a*b^4*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) -
 a^4*b*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2)))*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)
^(1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1
/2) + (2*a*b^2*exp(x))/((a + b)^2*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

3.719 $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.719.1 Optimal result

Integrand size = 20, antiderivative size = 205

$$\begin{aligned}\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= -\frac{4a^2b^2x}{(a^2-b^2)^3}-\frac{a^2x}{2(a^2-b^2)^2}+\frac{b^2x}{2(a^2-b^2)^2} \\ &\quad+\frac{2a^3b\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} \\ &\quad+\frac{2ab^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} \\ &\quad+\frac{a^2\cosh(x)\sinh(x)}{2(a^2-b^2)^2}+\frac{b^2\cosh(x)\sinh(x)}{2(a^2-b^2)^2} \\ &\quad-\frac{ab\sinh^2(x)}{(a^2-b^2)^2}+\frac{ab^2\sinh(x)}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}\end{aligned}$$

output
```
-4*a^2*b^2*x/(a^2-b^2)^3-1/2*a^2*x/(a^2-b^2)^2+1/2*b^2*x/(a^2-b^2)^2+2*a^3
*b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3+2*a*b^3*ln(a*cosh(x)+b*sinh(x))/(a^
2-b^2)^3+1/2*a^2*cosh(x)*sinh(x)/(a^2-b^2)^2+1/2*b^2*cosh(x)*sinh(x)/(a^2-
b^2)^2-a*b*sinh(x)^2/(a^2-b^2)^2+a*b^2*sinh(x)/(a^2-b^2)^2/(a*cosh(x)+b*si
nh(x))
```

3.719.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{1}{8}\left(-\frac{4(a^4+6a^2b^2+b^4)\,x}{(a-b)^3(a+b)^3}-\frac{4ab\cosh(2x)}{(a-b)^2(a+b)^2}+\frac{16ab(a^2+b^2)\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}+\frac{(a^4+6a^2b^2+b^4)\sinh(x)}{a(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))}-\frac{\sinh(x)}{a^2\cosh(x)+ab\sinh(x)}+\frac{2(a^2+b^2)\sinh(2x)}{(a-b)^2(a+b)^2}\right)$$

input
```
Integrate[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
((-4*(a^4 + 6*a^2*b^2 + b^4)*x)/((a - b)^3*(a + b)^3) - (4*a*b*Cosh[2*x])/
((a - b)^2*(a + b)^2) + (16*a*b*(a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]])/(a
^2 - b^2)^3 + ((a^4 + 6*a^2*b^2 + b^4)*Sinh[x])/(a*(a - b)^2*(a + b)^2*(a*
Cosh[x] + b*Sinh[x])) - Sinh[x]/(a^2*Cosh[x] + a*b*Sinh[x]) + (2*(a^2 + b^
2)*Sinh[2*x])/((a - b)^2*(a + b)^2))/8
```

3.719.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\sin(ix)^2\cos(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \; 3590$$

$$\frac{ib\int \frac{i\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{a\int -\frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{iab\int \frac{i\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{ib\int \frac{i\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{a\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{iab\int \frac{i\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{a\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{ab\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{ab\int -\frac{i\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{b\int -\frac{i\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int -\frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{ab\int -\frac{i\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{b\int -\frac{i\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3588}$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \cosh^2(x)dx}{a^2-b^2}+\frac{a\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{a\int -\sinh^2(x)dx}{a^2-b^2}-\frac{ib\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \cosh^2(x)dx}{a^2-b^2}+\frac{a\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(-\frac{a\int \sinh^2(x)dx}{a^2-b^2}-\frac{ib\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \cosh^2(x)dx}{a^2-b^2}+\frac{ia\int \cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(-\frac{a\int \sinh^2(x)dx}{a^2-b^2}+\frac{b\int \cosh(x)\sinh(x)dx}{a^2-b^2}-\frac{ab\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

3.719. $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{ia\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(-\frac{a\int -\sin(ix)^2dx}{a^2-b^2}+\frac{b\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}-\frac{ab\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{ia\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}+\frac{b\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}-\frac{ab\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{a\int \cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}-\frac{ib\int \cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3044}$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{ia\int i\sinh(x)d(i\sinh(x))}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}-\frac{b\int i\sinh(x)d(i\sinh(x))}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{15}$$

3.719. $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}-\frac{a\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3576}$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(\frac{iab\left(-\frac{a\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

3.719. $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(\frac{iab\left(\frac{ia\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 3577$$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 26$$

3.719. $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}$$

$\downarrow$ 3042

$$-\frac{iab\int \frac{\cos(ix)\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(\frac{iab\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}$$

$\downarrow$ 3590

$$-\frac{iab\left(\frac{iab\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{a\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}$$

$\downarrow$ 26

3.719. $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-\frac{iab\left(\frac{iab\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{ia\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{iab\left(\frac{iab\int \frac{1}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 26$$

$$-\frac{iab\left(\frac{iab\int \frac{1}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}$$

3.719. $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$\Big\downarrow \text{3554}$$

$$-\frac{a\left(\frac{iab\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{iab\left(-\frac{ib\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ib\sinh(x)}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3576}$$

$$-\frac{a\left(\frac{iab\left(\frac{ia\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{iab\left(-\frac{ib\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(-\frac{a\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh(x)}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}\right)}{a^2-b^2}$$

input
```
Int[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
$Aborted
```

3.719. $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.719.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.719. $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

3.719.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.93

method	result
default	$\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)}+\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)^2}+\frac{(a-b)\ln(\tanh(\frac{x}{2})-1)}{2(a+b)^3}-\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)^2}+\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)}$
risch	$-\frac{ax}{2(a+b)(a^2+2ab+b^2)}+\frac{xb}{2(a+b)(a^2+2ab+b^2)}+\frac{e^{2x}}{8a^2+16ab+8b^2}-\frac{e^{-2x}}{8(a^2-2ab+b^2)}-\frac{4a^3bx}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{4a\,b^3}{a^6-3a^4b^2+3}$

input
```
int(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

3.719. $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

output
```
1/2/(a+b)^2/(tanh(1/2*x)-1)+1/2/(a+b)^2/(tanh(1/2*x)-1)^2+1/2*(a-b)/(a+b)^
3*ln(tanh(1/2*x)-1)-1/2/(a-b)^2/(tanh(1/2*x)+1)^2+1/2/(a-b)^2/(tanh(1/2*x)
+1)+1/2/(a-b)^3*(-a-b)*ln(tanh(1/2*x)+1)+2*a*b/(a+b)^3/(a-b)^3*(b*(a^2-b^2
)*tanh(1/2*x)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+1/2*(2*a^2+2*b^2)*ln(tan
h(1/2*x)^2*a+2*b*tanh(1/2*x)+a))
```

3.719.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1726 vs. 2(197) = 394.

Time = 0.29 (sec) , antiderivative size = 1726, normalized size of antiderivative = 8.42

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas
")
```

output
```
1/8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2
*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^
3 - 3*a*b^4 + b^5 - 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
 b^5)*x)*cosh(x)^4 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^
5 + 15*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 - 4*(
a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x)*sinh(x)^4 + 4*
(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 -
3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 4*(a^5 + 5*a^4*b + 10*a^
3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x)*cosh(x))*sinh(x)^3 - (a^5 + 3*a^4*b
 + 18*a^3*b^2 - 18*a^2*b^3 - 3*a*b^4 - b^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2
- 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*cosh(x)^2 - (a^5 + 3*a^4*b + 18*a^3*b^2 -
18*a^2*b^3 - 3*a*b^4 - b^5 - 15*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b
^4 - b^5)*cosh(x)^4 - 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 +
 b^5 - 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x)*cosh
(x)^2 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*sinh(
x)^2 + 16*((a^4*b + a^3*b^2 + a^2*b^3 + a*b^4)*cosh(x)^4 + 4*(a^4*b + a^3*
b^2 + a^2*b^3 + a*b^4)*cosh(x)*sinh(x)^3 + (a^4*b + a^3*b^2 + a^2*b^3 + a*
b^4)*sinh(x)^4 + (a^4*b - a^3*b^2 + a^2*b^3 - a*b^4)*cosh(x)^2 + (a^4*b...
```

3.719.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**2*sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.719.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.19

$$\begin{aligned}&\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx\\&= -\frac{(a-b)x}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)} + \frac{2\,(a^3b+ab^3)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}\\&\quad+\frac{a^4-2\,a^3b+2\,ab^3-b^4+(a^4-4\,a^3b+22\,a^2b^2-4\,ab^3+b^4)e^{(-2\,x)}}{8\,((a^6-3\,a^4b^2+3\,a^2b^4-b^6)e^{(-2\,x)}+(a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+b^6)e^{(-4\,x)})}\\&\quad-\frac{e^{(-2\,x)}}{8\,(a^2-2\,ab+b^2)}\end{aligned}$$

input
```
integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima
")
```

output
```
-1/2*(a - b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a^3*b + a*b^3)*log(-(a
 - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(a^4 - 2
*a^3*b + 2*a*b^3 - b^4 + (a^4 - 4*a^3*b + 22*a^2*b^2 - 4*a*b^3 + b^4)*e^(-
2*x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*e^(-2*x) + (a^6 - 2*a^5*b - a^4
*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6)*e^(-4*x)) - 1/8*e^(-2*x)/(a^2
- 2*a*b + b^2)
```

3.719.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.13

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{(a+b)x}{2\left(a^3-3\,a^2b+3\,ab^2-b^3\right)} + \frac{2\left(a^3b+ab^3\right)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} + \frac{e^{(2\,x)}}{8\left(a^2+2\,ab+b^2\right)} + \frac{a^3e^{(4\,x)}-3\,a^2be^{(4\,x)}+3\,ab^2e^{(4\,x)}-b^3e^{(4\,x)}-8\,a^2be^{(2\,x)}-8\,ab^2e^{(2\,x)}-a^3-a^2b+ab^2+b^3}{8\left(a^4-2\,a^2b^2+b^4\right)\left(ae^{(4\,x)}+be^{(4\,x)}+ae^{(2\,x)}-be^{(2\,x)}\right)}$$

input
```
integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-1/2*(a + b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 2*(a^3*b + a*b^3)*log(abs
(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8
*e^(2*x)/(a^2 + 2*a*b + b^2) + 1/8*(a^3*e^(4*x) - 3*a^2*b*e^(4*x) + 3*a*b^
2*e^(4*x) - b^3*e^(4*x) - 8*a^2*b*e^(2*x) - 8*a*b^2*e^(2*x) - a^3 - a^2*b
+ a*b^2 + b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(4*x) + b*e^(4*x) + a*e^(2*x)
 - b*e^(2*x)))
```

3.719.9 Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.64

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{2\,x}}{8\,(a+b)^2} - \frac{e^{-2\,x}}{8\,(a-b)^2} + \frac{\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)\,\left(2\,a^3\,b+2\,a\,b^3\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6} - \frac{x\,(a+b)}{2\,(a-b)^3} - \frac{2\,a^2\,b^2}{(a+b)^3\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)}$$

input
```
int((cosh(x)^2*sinh(x)^2)/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
exp(2*x)/(8*(a + b)^2) - exp(-2*x)/(8*(a - b)^2) + (log(a - b + a*exp(2*x)
 + b*exp(2*x))*(2*a*b^3 + 2*a^3*b))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) -
(x*(a + b))/(2*(a - b)^3) - (2*a^2*b^2)/((a + b)^3*(a - b)^2*(a - b + exp(
2*x)*(a + b)))
```

3.720 $\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.720.1 Optimal result

Integrand size = 20, antiderivative size = 261

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2a^4b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{3a^2b^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{4a^2b^2\cosh(x)}{(a^2-b^2)^3} - \frac{a^2\cosh(x)}{(a^2-b^2)^2} + \frac{a^2\cosh^3(x)}{3(a^2-b^2)^2} + \frac{b^2\cosh^3(x)}{3(a^2-b^2)^2} + \frac{2a^3b\sinh(x)}{(a^2-b^2)^3} + \frac{2ab^3\sinh(x)}{(a^2-b^2)^3} - \frac{2ab\sinh^3(x)}{3(a^2-b^2)^2} - \frac{a^3b^2}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))}$$

output
```
-2*a^4*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)-3*a
^2*b^3*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)-4*a^2
*b^2*cosh(x)/(a^2-b^2)^3-a^2*cosh(x)/(a^2-b^2)^2+1/3*a^2*cosh(x)^3/(a^2-b^
2)^2+1/3*b^2*cosh(x)^3/(a^2-b^2)^2+2*a^3*b*sinh(x)/(a^2-b^2)^3+2*a*b^3*sin
h(x)/(a^2-b^2)^3-2/3*a*b*sinh(x)^3/(a^2-b^2)^2-a^3*b^2/(a^2-b^2)^3/(a*cosh
(x)+b*sinh(x))
```

3.720.2 Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.82

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \frac{1}{16}\left(-\frac{6b(3a^2+b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{10b(5a^4+10a^2b^2+b^4)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}} - \frac{4(a^2+b^2)\cosh(x)}{(a-b)^2(a+b)^2} - \frac{8(a^4+6a^2b^2+b^4)\cosh(x)}{(a-b)^3(a+b)^3} + \frac{4(a^2+b^2)\cosh(3x)}{3(a-b)^2(a+b)^2} + \frac{8ab\sinh(x)}{(a-b)^2(a+b)^2} + \frac{32ab(a^2+b^2)\sinh(x)}{(a-b)^3(a+b)^3} - \frac{a(a^2+3b^2)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))} - \frac{a(a^4+10a^2b^2+5b^4)}{(a-b)^3(a+b)^3(a\cosh(x)+b\sinh(x))} + \frac{2\left(a\sqrt{a-b}(a+b)+2ab\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\cosh(x)+2b^2\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sinh(x)\right)}{(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))} - \frac{8ab\sinh(3x)}{3(a-b)^2(a+b)^2}\right)$$

input
```
Integrate[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
((-6*b*(3*a^2 + b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/
((a - b)^(5/2)*(a + b)^(5/2)) - (10*b*(5*a^4 + 10*a^2*b^2 + b^4)*ArcTan[(b
 + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(7/2)*(a + b)^(7/2))
- (4*(a^2 + b^2)*Cosh[x])/((a - b)^2*(a + b)^2) - (8*(a^4 + 6*a^2*b^2 + b^
4)*Cosh[x])/((a - b)^3*(a + b)^3) + (4*(a^2 + b^2)*Cosh[3*x])/(3*(a - b)^2
*(a + b)^2) + (8*a*b*Sinh[x])/((a - b)^2*(a + b)^2) + (32*a*b*(a^2 + b^2)*
Sinh[x])/((a - b)^3*(a + b)^3) - (a*(a^2 + 3*b^2))/((a - b)^2*(a + b)^2*(a
*Cosh[x] + b*Sinh[x])) - (a*(a^4 + 10*a^2*b^2 + 5*b^4))/((a - b)^3*(a + b)
^3*(a*Cosh[x] + b*Sinh[x])) + (2*(a*Sqrt[a - b]*(a + b) + 2*a*b*Sqrt[a + b
]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Cosh[x] + 2*b^2*Sqrt
[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x]))/((a
- b)^(3/2)*(a + b)^2*(a*Cosh[x] + b*Sinh[x])) - (8*a*b*Sinh[3*x])/(3*(a -
b)^2*(a + b)^2))/16
```

3.720.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\sin(ix)^3\cos(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\cos(ix)^2\sin(ix)^3}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 3590}$$

$$i\left(-\frac{ib\int -\frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{a\int -\frac{i\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{iab\int -\frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$i\left(\frac{ib\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{a\int -\frac{i\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{iab\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(\frac{ib\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ia\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{iab\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(-\frac{iab\int -\frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int -\frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\int \frac{i\cos(ix)\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\int \frac{i\cos(ix)\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$\Big\downarrow$ 3588

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{ib\int i\cosh^2(x)\sinh(x)dx}{a^2-b^2}+\frac{a\int -\cosh(x)\sinh^2(x)dx}{a^2-b^2}+\frac{iab\int \frac{i\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(\frac{a}{\ldots}\right.}{\ldots}\right.$$

$\Big\downarrow$ 25

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{ib\int i\cosh^2(x)\sinh(x)dx}{a^2-b^2}-\frac{a\int \cosh(x)\sinh^2(x)dx}{a^2-b^2}+\frac{iab\int \frac{i\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(\frac{a\int \ldots}{\ldots}\right.}{\ldots}\right.$$

$\Big\downarrow$ 26

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(\frac{b\int \cosh^2(x)\sinh(x)dx}{a^2-b^2}-\frac{a\int \cosh(x)\sinh^2(x)dx}{a^2-b^2}-\frac{ab\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(-\frac{ia\int \sin\ldots}{a^2\ldots}\right.}{\ldots}\right.$$

$\Big\downarrow$ 3042

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(\frac{b\int -i\cos(ix)^2\sin(ix)dx}{a^2-b^2}-\frac{a\int -\cos(ix)\sin(ix)^2dx}{a^2-b^2}-\frac{ab\int -\frac{i\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(-\frac{ia\ldots}{\ldots}\right.}{\ldots}\right.$$

$\Big\downarrow$ 25

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(\frac{b\int -i\cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{a\int \cos(ix)\sin(ix)^2dx}{a^2-b^2}-\frac{ab\int -\frac{i\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(-\frac{ia\int\ldots}{\ldots}\right.}{\ldots}\right.$$

$\Big\downarrow$ 26

3.720. $\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{ib\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{a\int \cos(ix)\sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(\frac{a\int \sin(\right.}{a^2-}\right.$$

$\downarrow$ 3044

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{ia\int -\sinh^2(x)d(i\sinh(x))}{a^2-b^2}-\frac{ib\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(\frac{a\int}{}\right.}{}\right.$$

$\downarrow$ 15

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{ib\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\left(\frac{a\int \sin(ix)^3dx}{a^2-b^2}+\right.}{}\right.$$

$\downarrow$ 3045

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{a\int \sin(ix)^3dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ib\sinh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{ib\left(\frac{b\int \cosh^2(x)d\cosh(x)}{a^2-b^2}+\frac{iab}{}\right.}{a}\right.$$

$\downarrow$ 15

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{a\int \sin(ix)^3dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ib\sinh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{ib\left(\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\right.}{a^2-}\right.$$

$\downarrow$ 3113

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\left(\frac{ia\int (1-\cosh^2(x))d\cosh(x)}{a^2-b^2}+\frac{ia}{}\right.}{a^2}\right.$$

3.720. $\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$\Big\downarrow \; 2009$$

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\left(\frac{iab\int \frac{\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ib\,\mathrm{s}}{3(}\right.}{a^2}\right.$$

$$\Big\downarrow \; 3578$$

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(-\frac{ib\int i\sinh(x)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3(x)}{3(a^2-b^2)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x}{3}\right.}{a^2-b^2}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(\frac{b\int \sinh(x)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3(x)}{3(a^2-b^2)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)}{a^2-b^2}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(\frac{b\int -i\sin(ix)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3(x)}{3(a^2-b^2)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right.}{a^2-b^2}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \; 26$$

3.720. $\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$i\left(\frac{iab\int\frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(-\frac{ib\int\sin(ix)dx}{a^2-b^2}+\frac{a^2\int\frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3(x)}{3(a^2-b^2)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\cdots\right)}{a^2-b^2}\cdots\right)}{a^2-b^2}\cdots\right.$$

$$\Big\downarrow \; 3118$$

$$i\left(\frac{iab\int\frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(\frac{a^2\int\frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3(x)}{3(a^2-b^2)}+\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)}{a^2-b^2}\right)}{a^2-b^2}\cdots\right.$$

$$\Big\downarrow \; 3553$$

$$i\left(\frac{iab\int\frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(\frac{ia^2\int\frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}-\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\sinh^3\cdots}{3(a^2-b\cdots}\cdots\right)}{a^2-b^2}\cdots\right.$$

$$\Big\downarrow \; 219$$

$$i\left(\frac{iab\int\frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(\frac{iab\int\frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\left(\frac{iab\left(\frac{ia^2\operatorname{arctanh}\left(\frac{-ia\sinh(x)-i\cdots}{\sqrt{a^2-b\cdots}}\right)}{\left(a^2-b^2\right)^{3/2}}\cdots\right)}{a^2-\cdots}\cdots\right)}{\cdots}\cdots\right.$$

$$\Big\downarrow \; 3588$$

3.720. $\int\frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(\frac{iab\left(\frac{a\int i\sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\right.$$

$$\Big\downarrow \textbf{26}$$

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(\frac{iab\left(\frac{ia\int \sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\right.$$

$$\Big\downarrow \textbf{3042}$$

$$i\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(\frac{iab\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{ia\int -i\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}\right.$$

input
```
Int[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
$Aborted
```

3.720. $\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.720.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.720. $\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

3.720. $\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.720.4 Maple [A] (verified)

Time = 6.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.80

method	result
default	$-\frac{1}{3(a+b)^2(\tanh(\frac{x}{2})-1)^3}-\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)^2}-\frac{-a+b}{2(a+b)^3(\tanh(\frac{x}{2})-1)}-\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)^2}+\frac{1}{3(a-b)^2(\tanh(\frac{x}{2})\cdots}$
risch	$\frac{e^{3x}}{24a^2+48ab+24b^2}-\frac{3\,e^{x}a}{8(a+b)(a^2+2ab+b^2)}+\frac{e^{x}b}{8(a+b)(a^2+2ab+b^2)}-\frac{3\,e^{-x}a}{8(a^3-3a^2b+3a\,b^2-b^3)}-\frac{e^{-x}b}{8(a^3-3a^2b+3a\,b^2-b^3)}+\frac{\cdots}{24a^2\cdots}$

input
```
int(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/(a+b)^2/(tanh(1/2*x)-1)^3-1/2/(a+b)^2/(tanh(1/2*x)-1)^2-1/2/(a+b)^3*(
-a+b)/(tanh(1/2*x)-1)-1/2/(a-b)^2/(tanh(1/2*x)+1)^2+1/3/(a-b)^2/(tanh(1/2*
x)+1)^3-1/2*(a+b)/(a-b)^3/(tanh(1/2*x)+1)-4*a^2*b/(a+b)^3/(a-b)^3*((1/2*b^
2*tanh(1/2*x)+1/2*a*b)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+1/2*(2*a^2+3*b^
2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

3.720.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2503 vs. 2(247) = 494.

Time = 0.33 (sec) , antiderivative size = 5061, normalized size of antiderivative = 19.39

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

3.720.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**2*sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.720.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.720.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= -\frac{2\,a^3b^2e^x}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)(ae^{(2\,x)}+be^{(2\,x)}+a-b)} \\ &\quad - \frac{\left(9\,ae^{(2\,x)}+3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\,(a^3-3\,a^2b+3\,ab^2-b^3)} - \frac{2\,(2\,a^4b+3\,a^2b^3)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}} \\ &\quad + \frac{a^4e^{(3\,x)}+4\,a^3be^{(3\,x)}+6\,a^2b^2e^{(3\,x)}+4\,ab^3e^{(3\,x)}+b^4e^{(3\,x)}-9\,a^4e^x-24\,a^3be^x-18\,a^2b^2e^x+3\,b^4e^x}{24\,(a^6+6\,a^5b+15\,a^4b^2+20\,a^3b^3+15\,a^2b^4+6\,ab^5+b^6)}\end{aligned}$$

input
```
integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-2*a^3*b^2*e^x/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*e^(2*x) + b*e^(2*x)
 + a - b)) - 1/24*(9*a*e^(2*x) + 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^3 - 3*a^
2*b + 3*a*b^2 - b^3) - 2*(2*a^4*b + 3*a^2*b^3)*arctan((a*e^x + b*e^x)/sqrt
(a^2 - b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 1/24*
(a^4*e^(3*x) + 4*a^3*b*e^(3*x) + 6*a^2*b^2*e^(3*x) + 4*a*b^3*e^(3*x) + b^4
*e^(3*x) - 9*a^4*e^x - 24*a^3*b*e^x - 18*a^2*b^2*e^x + 3*b^4*e^x)/(a^6 + 6
*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
```

3.720.9 Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.27

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{3x}}{24(a+b)^2}+\frac{e^{-3x}}{24(a-b)^2}$$
$$-\frac{2\,\mathrm{atan}\left(\frac{e^x\left(2a^4b\sqrt{a^{14}-7a^{12}b^2+21a^{10}b^4-35a^8b^6+35a^6b^8-21a^4b^{10}+7a^2b^{12}-b\right.}{a^7\sqrt{4a^8b^2+12a^6b^4+9a^4b^6}+b^7\sqrt{4a^8b^2+12a^6b^4+9a^4b^6}-3a^2b^5\sqrt{4a^8b^2+12a^6b^4+9a^4b^6}+3a^3b^4\sqrt{4a^8b^2+12a^6b^4+9a^4b^6}}\right.}{\sqrt{a^{14}-7a^{12}b^2+21a^{10}}}$$
$$-\frac{e^x(3a-b)}{8(a+b)^3}-\frac{e^{-x}(3a+b)}{8(a-b)^3}-\frac{2a^3b^2e^x}{(a+b)^3(a-b)^3(a-b+e^{2x}(a+b))}$$

input
```
int((cosh(x)^2*sinh(x)^3)/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
exp(3*x)/(24*(a + b)^2) + exp(-3*x)/(24*(a - b)^2) - (2*atan((exp(x)*(2*a^
4*b*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21
*a^10*b^4 - 7*a^12*b^2)^(1/2) + 3*a^2*b^3*(a^14 - b^14 + 7*a^2*b^12 - 21*a
^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1/2)))/(a^7
*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2) + b^7*(9*a^4*b^6 + 12*a^6*b^4
+ 4*a^8*b^2)^(1/2) - 3*a^2*b^5*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2)
+ 3*a^3*b^4*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2) + 3*a^4*b^3*(9*a^4*
b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2) - 3*a^5*b^2*(9*a^4*b^6 + 12*a^6*b^4 +
4*a^8*b^2)^(1/2) - a*b^6*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2) - a^6*
b*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2)))*(9*a^4*b^6 + 12*a^6*b^4 + 4
*a^8*b^2)^(1/2))/(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35
*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1/2) - (exp(x)*(3*a - b))/(8*(a + b)
^3) - (exp(-x)*(3*a + b))/(8*(a - b)^3) - (2*a^3*b^2*exp(x))/((a + b)^3*(a
 - b)^3*(a - b + exp(2*x)*(a + b)))
```

3.721 $\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.721.1 Optimal result

Integrand size = 18, antiderivative size = 215

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a^3bx}{(a^2-b^2)^3}+\frac{ab^3x}{(a^2-b^2)^3}-\frac{abx}{(a^2-b^2)^2}+\frac{ab(a^2+b^2)\,x}{(a^2-b^2)^3}+\frac{b^2\cosh^2(x)}{2\,(a^2-b^2)^2}-\frac{3a^2b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}-\frac{b^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}-\frac{ab\cosh(x)\sinh(x)}{(a^2-b^2)^2}+\frac{a^2\sinh^2(x)}{2\,(a^2-b^2)^2}+\frac{ab^2}{(a^2-b^2)^2\,(a+b\tanh(x))}$$

output
```
a^3*b*x/(a^2-b^2)^3+a*b^3*x/(a^2-b^2)^3-a*b*x/(a^2-b^2)^2+a*b*(a^2+b^2)*x/
(a^2-b^2)^3+1/2*b^2*cosh(x)^2/(a^2-b^2)^2-3*a^2*b^2*ln(a*cosh(x)+b*sinh(x)
)/(a^2-b^2)^3-b^4*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3-a*b*cosh(x)*sinh(x)/
(a^2-b^2)^2+1/2*a^2*sinh(x)^2/(a^2-b^2)^2+a*b^2/(a^2-b^2)^2/(a+b*tanh(x))
```

3.721.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \frac{a\cosh(x)\left((a^4-b^4)\cosh(2x)-4b(-a(a^2+3b^2)\,x+b(3a^2+b^2)\log(a\cosh(x)+b\sinh(x))+a(a^2-b^2)\,\mathrm{c}\right.}{}$$

input
```
Integrate[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(a*Cosh[x]*((a^4 - b^4)*Cosh[2*x] - 4*b*(-(a*(a^2 + 3*b^2)*x) + b*(3*a^2 +
 b^2)*Log[a*Cosh[x] + b*Sinh[x]] + a*(a^2 - b^2)*Cosh[x]*Sinh[x])) + b*Sin
h[x]*((a^4 - b^4)*Cosh[2*x] + 4*b*(-(a^2*b) + b^3 + a^3*x + 3*a*b^2*x - b*
(3*a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]]) - 2*a*b*(a^2 - b^2)*Sinh[2*x]))/
(4*(a - b)^3*(a + b)^3*(a*Cosh[x] + b*Sinh[x]))
```

3.721.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\sin(ix)\cos(ix)^3}{(a\cos(ix)-ib\sin(ix))^2}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{\cos(ix)^3\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx$$
$$\Big\downarrow \text{3590}$$
$$-i\left(-\frac{ib\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{iab\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}+\frac{a\int \frac{i\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$
$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{ib\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{iab\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}+\frac{ia\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{iab\int \frac{\cos(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\int -\frac{i\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{iab\int \frac{\cos(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3565}$$

$$-i\left(\frac{iab\int \frac{1}{(a+b\tanh(x))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3579}$$

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{a\int \cosh^2(x)dx}{a^2-b^2}-\frac{b^2\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{b\cosh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{a\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b^2\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\cosh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3115}$$

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(-\frac{b^2\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}-\frac{b\cosh^{2}}{2(a^{2}-}\right.}{a^2-b^2}\right.$$

3.721. $\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$\Big\downarrow \ 24$$

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(-\frac{b^2\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \ 3577$$

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\right.}{}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh\right.}{a^2}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \ 3042$$

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh\right.}{a^2}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \ 3588$$

3.721. $\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int \cosh^2(x)dx}{a^2-b^2}+\frac{a\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\int \cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 26

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int \cosh^2(x)dx}{a^2-b^2}+\frac{ia\int \cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\int \cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 3042

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{ia\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\frac{ax}{a^2-b^2}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 26

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{a\int \cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

3.721. $\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$\downarrow$ 3044

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{ia\int i\sinh(x)d(i\sinh(x))}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\frac{ax}{a^2-b^2}\right.}{}\right.}{}\right.$$

$\downarrow$ 15

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}\right)}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\int\frac{b\cos(ix)}{a\cos(ix)}}{a^2}\right.}{a^2-b^2}\right.}{}\right.$$

$\downarrow$ 3115

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(\frac{iab\int \frac{}{a\,c}\right.}{}\right.$$

$\downarrow$ 24

3.721. $\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(\frac{iab\int \frac{}{a\,c}\right.}{}\right.$$

$\downarrow$ 3577

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b\cosh(x)+a\sinh(x))}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\right.}{}\right.$$

$\downarrow$ 26

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cosh(x)+a\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ia\sinh^2(x)}{2(a^2-b^2)}-\frac{ib\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\right.}{}\right.}{}\right.$$

$\downarrow$ 3042

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b\cos(ix)-ia\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(\frac{iab\left(\right.}{}\right.}{}\right.$$

3.721. $\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$\Big\downarrow \text{3612}$$

$$-i\left(\frac{iab\int \frac{1}{(a-ib\tan(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\right)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(\frac{ia\sinh^2(x}{2(a^2-b^2)}\right.}{}\right.$$

$$\Big\downarrow \text{3964}$$

$$-i\left(\frac{iab\left(\frac{\int \frac{a-b\tanh(x)}{a+b\tanh(x)}dx}{a^2-b^2}+\frac{b}{(a^2-b^2)(a+b\tanh(x))}\right)}{a^2-b^2}-\frac{ib\left(-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(}{a^2-b^2}\right.}{a^2-b^2}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{iab\left(\frac{b}{(a^2-b^2)(a+b\tanh(x))}+\frac{\int \frac{a+ib\tan(ix)}{a-ib\tan(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}-\frac{b^2\left(\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(}{a^2-b^2}\right.}{a^2-b^2}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \text{4014}$$

$$-i\left(\frac{iab\left(\frac{b}{(a^2-b^2)(a+b\tanh(x))}+\frac{\frac{x(a^2+b^2)}{a^2-b^2}-\frac{2iab\int -\frac{i(b+a\tanh(x))}{a+b\tanh(x)}dx}{a^2-b^2}}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{a^2-b^2}-\frac{b^2\left(\frac{ax}{a^2-b^2}-\right.}{}\right.}{a^2-b^2}\right.$$

input
```
Int[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
$Aborted
```

3.721. $\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.721.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3565
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.721. $\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

3.721. $\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.721.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.83

method	result
default	$\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)} + \frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)^2} - \frac{b\ln(\tanh(\frac{x}{2})-1)}{(a+b)^3} + \frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)^2} - \frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)} +$
risch	$\frac{xb}{(a+b)(a^2+2ab+b^2)} + \frac{e^{2x}}{8a^2+16ab+8b^2} + \frac{e^{-2x}}{8a^2-16ab+8b^2} + \frac{6a^2x\,b^2}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{2b^4x}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{}{(a-b)^2(a^3+3a^2}$

input
```
int(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)^2/(tanh(1/2*x)-1)+1/2/(a+b)^2/(tanh(1/2*x)-1)^2-b/(a+b)^3*ln(tan
h(1/2*x)-1)+1/2/(a-b)^2/(tanh(1/2*x)+1)^2-1/2/(a-b)^2/(tanh(1/2*x)+1)+1/(a
-b)^3*b*ln(tanh(1/2*x)+1)-2*b^2/(a+b)^3/(a-b)^3*(b*(a^2-b^2)*tanh(1/2*x)/(
tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+1/2*(3*a^2+b^2)*ln(tanh(1/2*x)^2*a+2*b*
tanh(1/2*x)+a))
```

3.721.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1661 vs. 2(211) = 422.

Time = 0.28 (sec) , antiderivative size = 1661, normalized size of antiderivative = 7.73

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
1/8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2
*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^
3 - 3*a*b^4 + b^5 + 8*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*c
osh(x)^4 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 15*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + 8*(a^4*b + 4*
a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 - 3*a^4*b + 2*a^3*b^2
+ 2*a^2*b^3 - 3*a*b^4 + b^5 + 8*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 +
 b^5)*x)*cosh(x))*sinh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 + 14*a^2*b^3 - 19
*a*b^4 - b^5 + 8*(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*x)*cosh(x)^2 + (a^5 +
 3*a^4*b + 2*a^3*b^2 + 14*a^2*b^3 - 19*a*b^4 - b^5 + 15*(a^5 - a^4*b - 2*a
^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 - 3*a^4*b + 2*a^3*b^2
 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 8*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4
+ b^5)*x)*cosh(x)^2 + 8*(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*x)*sinh(x)^2 -
 8*((3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^5)*cosh(x)^4 + 4*(3*a^3*b^2 + 3*a^2
*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x)^3 + (3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b
^5)*sinh(x)^4 + (3*a^3*b^2 - 3*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + (3*a^3*b
^2 - 3*a^2*b^3 + a*b^4 - b^5 + 6*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^5)*...
```

3.721.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**3*sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.721.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{bx}{a^3+3a^2b+3ab^2+b^3} - \frac{(3a^2b^2+b^4)\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{a^4-2a^3b+2ab^3-b^4+(a^4-4a^3b+6a^2b^2-20ab^3+b^4)e^{(-2x)}}{8\left((a^6-3a^4b^2+3a^2b^4-b^6)e^{(-2x)}+(a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6)e^{(-4x)}\right)} + \frac{e^{(-2x)}}{8\,(a^2-2ab+b^2)}$$

input
```
integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
b*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^2*b^2 + b^4)*log(-(a - b)*e^(-2
*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(a^4 - 2*a^3*b + 2*
a*b^3 - b^4 + (a^4 - 4*a^3*b + 6*a^2*b^2 - 20*a*b^3 + b^4)*e^(-2*x))/((a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*e^(-2*x) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^
3*b^3 - a^2*b^4 - 2*a*b^5 + b^6)*e^(-4*x)) + 1/8*e^(-2*x)/(a^2 - 2*a*b + b
^2)
```

3.721.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{bx}{a^3-3a^2b+3ab^2-b^3} - \frac{(3a^2b^2+b^4)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{e^{(2x)}}{8\,(a^2+2ab+b^2)} - \frac{2a^2be^{(4x)}-4ab^2e^{(4x)}+2b^3e^{(4x)}-a^3e^{(2x)}-a^2be^{(2x)}-11ab^2e^{(2x)}-3b^3e^{(2x)}-a^3-a^2b+ab^2+b^3}{8\,(a^4-2a^2b^2+b^4)(ae^{(4x)}+be^{(4x)}+ae^{(2x)}-be^{(2x)})}$$

input
```
integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
b*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a^2*b^2 + b^4)*log(abs(a*e^(2*x)
+ b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*e^(2*x)/(a
^2 + 2*a*b + b^2) - 1/8*(2*a^2*b*e^(4*x) - 4*a*b^2*e^(4*x) + 2*b^3*e^(4*x)
 - a^3*e^(2*x) - a^2*b*e^(2*x) - 11*a*b^2*e^(2*x) - 3*b^3*e^(2*x) - a^3 -
a^2*b + a*b^2 + b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(4*x) + b*e^(4*x) + a*e
^(2*x) - b*e^(2*x)))
```

3.721.9 Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.59

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{2x}}{8\,(a+b)^2}+\frac{e^{-2x}}{8\,(a-b)^2}+\frac{b\,x}{(a-b)^3} - \frac{\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)\,\left(3\,a^2\,b^2+b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6} + \frac{2\,a\,b^3}{(a+b)^3\,(a-b)^2\,\left(a-b+e^{2x}\,(a+b)\right)}$$

input
```
int((cosh(x)^3*sinh(x))/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
exp(2*x)/(8*(a + b)^2) + exp(-2*x)/(8*(a - b)^2) + (b*x)/(a - b)^3 - (log(
a - b + a*exp(2*x) + b*exp(2*x))*(b^4 + 3*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2) + (2*a*b^3)/((a + b)^3*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

3.722 $\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.722.1 Optimal result

Integrand size = 20, antiderivative size = 259

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{3a^3b^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{2ab^4\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{2a^3b\cosh(x)}{(a^2-b^2)^3} + \frac{2ab^3\cosh(x)}{(a^2-b^2)^3} - \frac{2ab\cosh^3(x)}{3(a^2-b^2)^2} - \frac{4a^2b^2\sinh(x)}{(a^2-b^2)^3} + \frac{b^2\sinh(x)}{(a^2-b^2)^2} + \frac{a^2\sinh^3(x)}{3(a^2-b^2)^2} + \frac{b^2\sinh^3(x)}{3(a^2-b^2)^2} + \frac{a^2b^3}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))}$$

output
```
3*a^3*b^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)+2*
a*b^4*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)+2*a^3*
b*cosh(x)/(a^2-b^2)^3+2*a*b^3*cosh(x)/(a^2-b^2)^3-2/3*a*b*cosh(x)^3/(a^2-b
^2)^2-4*a^2*b^2*sinh(x)/(a^2-b^2)^3+b^2*sinh(x)/(a^2-b^2)^2+1/3*a^2*sinh(x
)^3/(a^2-b^2)^2+1/3*b^2*sinh(x)^3/(a^2-b^2)^2+a^2*b^3/(a^2-b^2)^3/(a*cosh(
x)+b*sinh(x))
```

3.722.2 Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.86

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx =$$
$$-\frac{\sqrt{a-b}b(a+b)+2a^2\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\cosh(x)+2ab\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sinh(x)}{8(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))}$$
$$+\frac{1}{16}\left(-\frac{6a(a^2+3b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{8ab\cosh(x)}{(a-b)^2(a+b)^2}+\frac{4(a^2+b^2)\sinh(x)}{(a-b)^2(a+b)^2}\right.$$
$$\left.-\frac{b(3a^2+b^2)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))}\right)+\frac{1}{16}\left(\frac{10a(a^4+10a^2b^2+5b^4)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}}+\frac{32ab(a^2}{(a-}\right.$$

input
```
Integrate[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
-1/8*(Sqrt[a - b]*b*(a + b) + 2*a^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(
Sqrt[a - b]*Sqrt[a + b])]*Cosh[x] + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x
/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[
x] + b*Sinh[x])) + ((-6*a*(a^2 + 3*b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a -
 b]*Sqrt[a + b])])/((a - b)^(5/2)*(a + b)^(5/2)) - (8*a*b*Cosh[x])/((a - b
)^2*(a + b)^2) + (4*(a^2 + b^2)*Sinh[x])/((a - b)^2*(a + b)^2) - (b*(3*a^2
 + b^2))/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])))/16 + ((10*a*(a^4 +
 10*a^2*b^2 + 5*b^4)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/
((a - b)^(7/2)*(a + b)^(7/2)) + (32*a*b*(a^2 + b^2)*Cosh[x])/((a - b)^3*(a
 + b)^3) - (8*a*b*Cosh[3*x])/(3*(a - b)^2*(a + b)^2) - (8*(a^4 + 6*a^2*b^2
 + b^4)*Sinh[x])/((a - b)^3*(a + b)^3) + (b*(5*a^4 + 10*a^2*b^2 + b^4))/((
a - b)^3*(a + b)^3*(a*Cosh[x] + b*Sinh[x])) + (4*(a^2 + b^2)*Sinh[3*x])/(3
*(a - b)^2*(a + b)^2))/16
```

3.722.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ix)^2\cos(ix)^3}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\cos(ix)^3\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx$$

$$\Big\downarrow \ 3590$$

$$\frac{ib\int \frac{i\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{a\int -\frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{iab\int \frac{i\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}$$

$$\Big\downarrow \ 25$$

$$\frac{ib\int \frac{i\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{a\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{iab\int \frac{i\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}$$

$$\Big\downarrow \ 26$$

$$-\frac{b\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{a\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{ab\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{ab\int -\frac{i\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{b\int -\frac{i\cos(ix)^3\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int -\frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \ 25$$

$$\frac{ab\int -\frac{i\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{b\int -\frac{i\cos(ix)^3\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\int \frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \ 26$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int \frac{\cos(ix)^3\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\int \frac{\cos(ix)^2\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \ 3588$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(-\frac{ib\int i\cosh^2(x)\sinh(x)dx}{a^2-b^2}+\frac{a\int -\cosh(x)\sinh^2(x)dx}{a^2-b^2}+\frac{iab\int \frac{i\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \cosh^3(x)dx}{a^2-b^2}+\frac{a\int i\cosh^2(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \ 25$$

3.722. $\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(-\frac{ib\int i\cosh^2(x)\sinh(x)dx}{a^2-b^2}-\frac{a\int \cosh(x)\sinh^2(x)dx}{a^2-b^2}+\frac{iab\int \frac{i\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \cosh^3(x)dx}{a^2-b^2}+\frac{a\int i\cosh^2(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 26$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(\frac{b\int \cosh^2(x)\sinh(x)dx}{a^2-b^2}-\frac{a\int \cosh(x)\sinh^2(x)dx}{a^2-b^2}-\frac{ab\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \cosh^3(x)dx}{a^2-b^2}+\frac{ia\int \cosh^2(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}+\frac{ia\int -i\cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{b\int -i\cos(ix)^2\sin(ix)dx}{a^2-b^2}-\frac{a\int -\cos(ix)\sin(ix)^2dx}{a^2-b^2}-\frac{ab\int -\frac{i\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}+\frac{ia\int -i\cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{b\int -i\cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{a\int \cos(ix)\sin(ix)^2dx}{a^2-b^2}-\frac{ab\int -\frac{i\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 26$$

3.722. $\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}+\frac{a\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(-\frac{ib\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{a\int \cos(ix)\sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$\downarrow$ 3044

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}+\frac{a\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(-\frac{ia\int -\sinh^2(x)d(i\sinh(x))}{a^2-b^2}-\frac{ib\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}$$

$\downarrow$ 15

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}+\frac{a\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(-\frac{ib\int \cos(ix)^2\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$\downarrow$ 3045

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}+\frac{ia\int \cosh^2(x)d\cosh(x)}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(\frac{b\int \cosh^2(x)d\cosh(x)}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$\downarrow$ 15

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$\downarrow$ 3113

3.722. $\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\frac{ib\left(\frac{b\int (\sinh^2(x)+1)d(-i\sinh(x))}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{iab\int \frac{\cos(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3579}$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{iab\left(\frac{a\int \cosh(x)dx}{a^2-b^2}-\frac{b^2\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{iab\left(\frac{a\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}-\frac{b^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3117}$$

3.722. $\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{iab\left(-\frac{b^2\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$\downarrow$ 3553

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\left(\frac{iab\left(-\frac{ib^2\int \frac{1}{a^2-b^2-(-ib\cosh(x)-ia\sinh(x))^2}d(-ib\cosh(x)-ia\sinh(x))}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}-\frac{a\left(\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$\downarrow$ 219

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(\frac{iab\int \frac{\cos(ix)\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$\downarrow$ 3588

3.722. $\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(\frac{iab\left(\frac{a\int i\sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b(-\frac{1}{3}i\sinh^3(x)-i\sinh(x))}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 26$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(\frac{iab\left(\frac{ia\int \sinh(x)dx}{a^2-b^2}-\frac{ib\int \cosh(x)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b(-\frac{1}{3}i\sinh^3(x)-i\sinh(x))}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(\frac{iab\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{ia\int -i\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b(-\frac{1}{3}i\sinh^3(x)-i\sinh(x))}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 26$$

3.722. $\displaystyle\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(\frac{iab\left(-\frac{ib\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a\int \sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

$$\Big\downarrow \; 3117$$

$$-\frac{iab\int \frac{\cos(ix)^2\sin(ix)}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{a\left(\frac{iab\left(\frac{a\int \sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{1}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\sinh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{b\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}+\frac{ib\left(\frac{iab\left(-\frac{ib^2\text{arctanh}\left(\frac{-ia\sinh(x)-ib\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{b\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}+\frac{ia\cosh^3(x)}{3(a^2-b^2)}\right)}{a^2-b^2}$$

input
```
Int[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
$Aborted
```

3.722.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

3.722. $\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.722. $\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

3.722.4 Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.76

method	result
default	$-\frac{1}{3(a+b)^2(\tanh(\frac{x}{2})-1)^3}-\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)^2}-\frac{b}{(a+b)^3(\tanh(\frac{x}{2})-1)}+\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)^2}-\frac{1}{3(a-b)^2(\tanh(\frac{x}{2})+$
risch	$\frac{e^{3x}}{24a^2+48ab+24b^2}-\frac{e^x a}{8(a+b)(a^2+2ab+b^2)}+\frac{3e^x b}{8(a+b)(a^2+2ab+b^2)}+\frac{a e^{-x}}{8a^3-24a^2b+24a b^2-8b^3}+\frac{3e^{-x}b}{8(a^3-3a^2b+3a b^2-b^3)}-24($

input
```
int(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/(a+b)^2/(tanh(1/2*x)-1)^3-1/2/(a+b)^2/(tanh(1/2*x)-1)^2-b/(a+b)^3/(ta
nh(1/2*x)-1)+1/2/(a-b)^2/(tanh(1/2*x)+1)^2-1/3/(a-b)^2/(tanh(1/2*x)+1)^3+1
/(a-b)^3*b/(tanh(1/2*x)+1)+2*a*b^2/(a+b)^3/(a-b)^3*((b^2*tanh(1/2*x)+a*b)/
(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+(3*a^2+2*b^2)/(a^2-b^2)^(1/2)*arctan(1
/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

3.722.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2488 vs. $2(245) = 490$.

Time = 0.36 (sec) , antiderivative size = 5031, normalized size of antiderivative = 19.42

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

3.722.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**3*sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.722.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.722.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.20

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2\,a^2b^3e^x}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)(ae^{(2\,x)}+be^{(2\,x)}+a-b)} + \frac{(3\,ae^{(2\,x)}+9\,be^{(2\,x)}-a+b)e^{(-3\,x)}}{24\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{2\,(3\,a^3b^2+2\,ab^4)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}} + \frac{a^4e^{(3\,x)}+4\,a^3be^{(3\,x)}+6\,a^2b^2e^{(3\,x)}+4\,ab^3e^{(3\,x)}+b^4e^{(3\,x)}-3\,a^4e^x+18\,a^2b^2e^x+24\,ab^3e^x+9\,b^4e^x}{24\,(a^6+6\,a^5b+15\,a^4b^2+20\,a^3b^3+15\,a^2b^4+6\,ab^5+b^6)}$$

input
```
integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
2*a^2*b^3*e^x/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*e^(2*x) + b*e^(2*x)
+ a - b)) + 1/24*(3*a*e^(2*x) + 9*b*e^(2*x) - a + b)*e^(-3*x)/(a^3 - 3*a^2
*b + 3*a*b^2 - b^3) + 2*(3*a^3*b^2 + 2*a*b^4)*arctan((a*e^x + b*e^x)/sqrt(
a^2 - b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 1/24*(
a^4*e^(3*x) + 4*a^3*b*e^(3*x) + 6*a^2*b^2*e^(3*x) + 4*a*b^3*e^(3*x) + b^4*
e^(3*x) - 3*a^4*e^x + 18*a^2*b^2*e^x + 24*a*b^3*e^x + 9*b^4*e^x)/(a^6 + 6*
a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
```

3.722.9 Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.28

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{3x}}{24(a+b)^2} - \frac{e^{-3x}}{24(a-b)^2} - \frac{e^x(a-3b)}{8(a+b)^3}$$
$$+ \frac{2\,\mathrm{atan}\left(\frac{e^x\left(2ab^4\sqrt{a^{14}-7a^{12}b^2+21a^{10}b^4-35a^8b^6+35a^6b^8-21a^4b^{10}+7a^2b^{12}-b}\right.}{a^7\sqrt{9a^6b^4+12a^4b^6+4a^2b^8}+b^7\sqrt{9a^6b^4+12a^4b^6+4a^2b^8}-3a^2b^5\sqrt{9a^6b^4+12a^4b^6+4a^2b^8}+3a^3b^4\sqrt{9a^6b^4+12a^4b^6+4a^2b^8}}}{\sqrt{a^{14}-7a^{12}b^2+21a^{10}}}$$
$$+ \frac{e^{-x}(a+3b)}{8(a-b)^3} + \frac{2a^2b^3e^x}{(a+b)^3(a-b)^3(a-b+e^{2x}(a+b))}$$

input
```
int((cosh(x)^3*sinh(x)^2)/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
exp(3*x)/(24*(a + b)^2) - exp(-3*x)/(24*(a - b)^2) - (exp(x)*(a - 3*b))/(8
*(a + b)^3) + (2*atan((exp(x)*(2*a*b^4*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*
b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1/2) + 3*a^3*b
^2*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*
a^10*b^4 - 7*a^12*b^2)^(1/2)))/(a^7*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(
1/2) + b^7*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2) - 3*a^2*b^5*(4*a^2*b
^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2) + 3*a^3*b^4*(4*a^2*b^8 + 12*a^4*b^6 + 9
*a^6*b^4)^(1/2) + 3*a^4*b^3*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2) - 3
*a^5*b^2*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2) - a*b^6*(4*a^2*b^8 + 1
2*a^4*b^6 + 9*a^6*b^4)^(1/2) - a^6*b*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^
(1/2)))*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2))/(a^14 - b^14 + 7*a^2*b
^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1
/2) + (exp(-x)*(a + 3*b))/(8*(a - b)^3) + (2*a^2*b^3*exp(x))/((a + b)^3*(a
 - b)^3*(a - b + exp(2*x)*(a + b)))
```

3.723 $\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.723.1 Optimal result

Integrand size = 20, antiderivative size = 314

$$\begin{aligned}\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= -\frac{6a^3b^3x}{(a^2-b^2)^4}-\frac{a^3bx}{(a^2-b^2)^3}+\frac{ab^3x}{(a^2-b^2)^3}+\frac{abx}{4(a^2-b^2)^2}\\ &+\frac{b^2\cosh^4(x)}{4(a^2-b^2)^2}+\frac{3a^4b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^4}\\ &+\frac{3a^2b^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^4}+\frac{a^3b\cosh(x)\sinh(x)}{(a^2-b^2)^3}\\ &+\frac{ab^3\cosh(x)\sinh(x)}{(a^2-b^2)^3}+\frac{ab\cosh(x)\sinh(x)}{4(a^2-b^2)^2}\\ &-\frac{ab\cosh^3(x)\sinh(x)}{2(a^2-b^2)^2}-\frac{2a^2b^2\sinh^2(x)}{(a^2-b^2)^3}\\ &+\frac{a^2\sinh^4(x)}{4(a^2-b^2)^2}+\frac{a^2b^3\sinh(x)}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))}\end{aligned}$$

output
```
-6*a^3*b^3*x/(a^2-b^2)^4-a^3*b*x/(a^2-b^2)^3+a*b^3*x/(a^2-b^2)^3+1/4*a*b*x
/(a^2-b^2)^2+1/4*b^2*cosh(x)^4/(a^2-b^2)^2+3*a^4*b^2*ln(a*cosh(x)+b*sinh(x
))/(a^2-b^2)^4+3*a^2*b^4*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^4+a^3*b*cosh(x)
*sinh(x)/(a^2-b^2)^3+a*b^3*cosh(x)*sinh(x)/(a^2-b^2)^3+1/4*a*b*cosh(x)*sin
h(x)/(a^2-b^2)^2-1/2*a*b*cosh(x)^3*sinh(x)/(a^2-b^2)^2-2*a^2*b^2*sinh(x)^2
/(a^2-b^2)^3+1/4*a^2*sinh(x)^4/(a^2-b^2)^2+a^2*b^3*sinh(x)/(a^2-b^2)^3/(a*
cosh(x)+b*sinh(x))
```

3.723.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.17

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \frac{-3a(a^2-b^2)^2(a^2+3b^2)\cosh(3x)+a^7\cosh(5x)-3a^5b^2\cosh(5x)+3a^3b^4\cosh(5x)-ab^6\cosh(5x)-4a\,c}{}$$

input
```
Integrate[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
(-3*a*(a^2 - b^2)^2*(a^2 + 3*b^2)*Cosh[3*x] + a^7*Cosh[5*x] - 3*a^5*b^2*Co
sh[5*x] + 3*a^3*b^4*Cosh[5*x] - a*b^6*Cosh[5*x] - 4*a*Cosh[x]*(a^6 + 9*a^4
*b^2 - 5*a^2*b^4 - 5*b^6 + 12*a^5*b*x + 72*a^3*b^3*x + 12*a*b^5*x - 48*a^2
*b^2*(a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]]) + 20*a^6*b*Sinh[x] + 84*a^4*b
^3*Sinh[x] - 100*a^2*b^5*Sinh[x] - 4*b^7*Sinh[x] - 48*a^5*b^2*x*Sinh[x] -
288*a^3*b^4*x*Sinh[x] - 48*a*b^6*x*Sinh[x] + 192*a^4*b^3*Log[a*Cosh[x] + b
*Sinh[x]]*Sinh[x] + 192*a^2*b^5*Log[a*Cosh[x] + b*Sinh[x]]*Sinh[x] + 9*a^6
*b*Sinh[3*x] - 15*a^4*b^3*Sinh[3*x] + 3*a^2*b^5*Sinh[3*x] + 3*b^7*Sinh[3*x
] - a^6*b*Sinh[5*x] + 3*a^4*b^3*Sinh[5*x] - 3*a^2*b^5*Sinh[5*x] + b^7*Sinh
[5*x])/(64*(a - b)^4*(a + b)^4*(a*Cosh[x] + b*Sinh[x]))
```

3.723.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sin(ix)^3\cos(ix)^3}{(a\cos(ix)-ib\sin(ix))^2}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\cos(ix)^3\sin(ix)^3}{(a\cos(ix)-ib\sin(ix))^2}dx$$
$$\Big\downarrow \text{3590}$$

$$i\left(-\frac{ib\int -\frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{a\int -\frac{i\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{iab\int -\frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$i\left(\frac{ib\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}+\frac{a\int -\frac{i\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{iab\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(\frac{ib\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{ia\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{iab\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(-\frac{iab\int -\frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{ib\int -\frac{\cos(ix)^3\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\int \frac{i\cos(ix)^2\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)^3\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\int \frac{i\cos(ix)^2\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\int \frac{\cos(ix)^3\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{a\int \frac{\cos(ix)^2\sin(ix)^3}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{ 3588}$$

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int -\cosh^2(x)\sinh^2(x)dx}{a^2-b^2}+\frac{a\int -i\cosh(x)\sinh^3(x)dx}{a^2-b^2}+\frac{iab\int -\frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\cdots ib\right.$$

$$\Big\downarrow \text{ 25}$$

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{ib\int \cosh^2(x)\sinh^2(x)dx}{a^2-b^2}+\frac{a\int -i\cosh(x)\sinh^3(x)dx}{a^2-b^2}-\frac{iab\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(-\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \text{ 26}$$

3.723. $\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$i\left(\frac{iab\int\frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{ib\int\cosh^2(x)\sinh^2(x)dx}{a^2-b^2}-\frac{ia\int\cosh(x)\sinh^3(x)dx}{a^2-b^2}-\frac{iab\int\frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{b\int \cdots}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{iab\int\frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(\frac{b\int-i\cos(ix)^3\sin(ix)dx}{a^2-b^2}-\frac{a\int-\cos(ix)^2\sin(ix)^2dx}{a^2-b^2}-\frac{ab\int-\frac{i\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(\frac{ib\cdots}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \text{25}$$

$$i\left(\frac{iab\int\frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(\frac{b\int-i\cos(ix)^3\sin(ix)dx}{a^2-b^2}+\frac{a\int\cos(ix)^2\sin(ix)^2dx}{a^2-b^2}-\frac{ab\int-\frac{i\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(-\frac{ib\cdots}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{iab\int\frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{ib\int\cos(ix)^3\sin(ix)dx}{a^2-b^2}+\frac{a\int\cos(ix)^2\sin(ix)^2dx}{a^2-b^2}+\frac{iab\int\frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(-\frac{ib\int\cdots}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \text{3044}$$

$$i\left(\frac{iab\int\frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{ib\int\cos(ix)^3\sin(ix)dx}{a^2-b^2}+\frac{a\int\cos(ix)^2\sin(ix)^2dx}{a^2-b^2}+\frac{iab\int\frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(-\frac{ia\int\cdots}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \text{15}$$

$$i\left(\frac{iab\int\frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}-\frac{ib\left(-\frac{ib\int\cos(ix)^3\sin(ix)dx}{a^2-b^2}+\frac{a\int\cos(ix)^2\sin(ix)^2dx}{a^2-b^2}+\frac{iab\int\frac{\cos(ix)^2\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(-\frac{ib\int\cdots}{\cdots}\right.}{\cdots}\cdots\right.$$

3.723. $\int\frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$\Big\downarrow 3045$$

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int \cos(ix)^2\sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}\right)}{a^2-b^2}-\frac{ib\left(\frac{b\int \cosh^3(x)d\cos\right.}{a^2-b^2}}{}\right.$$

$$\Big\downarrow 15$$

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(-\frac{ib\int \cos(ix)^2\sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}\right)}{a^2-b^2}-\frac{ib\left(\frac{a\int \cos(ix)^2\sin(i\right.}{a^2-b^2}}{}\right.$$

$$\Big\downarrow 3048$$

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{1}{4}\int \cosh^2(x)dx-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}\right)}{a^2-b^2}-\frac{ib\left(\frac{iab}{}\right.}{}\right.$$

$$\Big\downarrow 3042$$

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(-\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{1}{4}\int \sin\left(ix+\frac{\pi}{2}\right)^2dx\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}\right)}{a^2-b^2}-\frac{ib}{}\right.$$

$$\Big\downarrow 3115$$

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ib\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow 24$$

3.723. $\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\int \frac{\cos(ix)\sin(ix)^2}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)\right)}{a^2-b^2}\right)}{a^2-b^2}-\right.$$

$\downarrow$ 3588

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(\frac{a\int -\sinh^2(x)dx}{a^2-b^2}-\frac{ib\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}\right.\right.}{}\right.}{a^2-b^2}\right.$$

$\downarrow$ 25

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(-\frac{a\int \sinh^2(x)dx}{a^2-b^2}-\frac{ib\int i\cosh(x)\sinh(x)dx}{a^2-b^2}+\frac{iab\int \frac{i\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}\right.\right.}{}\right.}{a^2-b^2}\right.$$

$\downarrow$ 26

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(-\frac{a\int \sinh^2(x)dx}{a^2-b^2}+\frac{b\int \cosh(x)\sinh(x)dx}{a^2-b^2}-\frac{ab\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\right.\right.}{}\right.}{a^2-b^2}\right.$$

$\downarrow$ 3042

3.723. $\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(-\frac{a\int -\sin(ix)^2dx}{a^2-b^2}+\frac{b\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}-\frac{ab\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}-\frac{ib(\frac{1}{4}}{}\right)}{a^2-b^2}\right.$$

$\downarrow$ 25

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}+\frac{b\int -i\cos(ix)\sin(ix)dx}{a^2-b^2}-\frac{ab\int -\frac{i\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}-\frac{ib(\frac{1}{4}(\frac{x}{2}+}{}\right)}{a^2-b^2}\right.$$

$\downarrow$ 26

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}-\frac{ib\int \cos(ix)\sin(ix)dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}-\frac{ib(\frac{1}{4}(\frac{x}{2}+\frac{1}{2}}{}\right)}{a^2-b^2}\right.$$

$\downarrow$ 3044

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}-\frac{b\int i\sinh(x)d(i\sinh(x))}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}-\frac{ib(\frac{1}{4}(\frac{x}{2}+}{}\right)}{a^2-b^2}\right.$$

3.723. $\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

$$\Big\downarrow \text{15}$$

$$i\left(\frac{iab\int \frac{\cos(ix)^2\sin(ix)^2}{(a\cos(ix)-ib\sin(ix))^2}dx}{a^2-b^2}+\frac{a\left(\frac{iab\left(\frac{a\int \sin(ix)^2dx}{a^2-b^2}+\frac{iab\int \frac{\sin(ix)}{a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}+\frac{b\sinh^2(x)}{2\left(a^2-b^2\right)}\right)}{a^2-b^2}-\frac{ia\sinh^4(x)}{4(a^2-b^2)}-\frac{ib\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)\right.}{}\right.}{a^2-b^2}\right.$$

input
```
Int[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

output
```
$Aborted
```

3.723.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.723. $\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

3.723. $\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

3.723.4 Maple [A] (verified)

Time = 17.21 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.88

method	result
default	$\frac{1}{4(a+b)^2(\tanh(\frac{x}{2})-1)^4}+\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)^3}-\frac{-a-5b}{8(a+b)^3(\tanh(\frac{x}{2})-1)^2}-\frac{a-3b}{8(a+b)^3(\tanh(\frac{x}{2})-1)}+\frac{3ab\ln(\tanh(\frac{x}{2})-1)}{4(a+b)^4}$
risch	$-\frac{3abx}{4(a^2+2ab+b^2)(a+b)^2}+\frac{e^{4x}}{64(a+b)^2}-\frac{e^{2x}a}{16(a+b)^3}+\frac{e^{2x}b}{16(a+b)^3}-\frac{e^{-2x}a}{16(a^3-3a^2b+3ab^2-b^3)}-\frac{e^{-2x}b}{16(a^3-3a^2b+3ab^2-b^3)}+$

input
```
int(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/(a+b)^2/(tanh(1/2*x)-1)^4+1/2/(a+b)^2/(tanh(1/2*x)-1)^3-1/8*(-a-5*b)/(
a+b)^3/(tanh(1/2*x)-1)^2-1/8*(a-3*b)/(a+b)^3/(tanh(1/2*x)-1)+3/4*a*b/(a+b)
^4*ln(tanh(1/2*x)-1)+1/4/(a-b)^2/(tanh(1/2*x)+1)^4-1/2/(a-b)^2/(tanh(1/2*x
)+1)^3-1/8*(-a-3*b)/(a-b)^3/(tanh(1/2*x)+1)-1/8*(-a+5*b)/(a-b)^3/(tanh(1/2
*x)+1)^2-3/4*a*b/(a-b)^4*ln(tanh(1/2*x)+1)+2*a^2*b^2/(a+b)^4/(a-b)^4*(b*(a
^2-b^2)*tanh(1/2*x)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+1/2*(3*a^2+3*b^2)*
ln(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a))
```

3.723.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4001 vs. 2(304) = 608.

Time = 0.31 (sec) , antiderivative size = 4001, normalized size of antiderivative = 12.74

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas
")
```

output
```
1/64*((a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
 + b^7)*cosh(x)^10 + 10*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 -
 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^9 + (a^7 - a^6*b - 3*a^5*b^2 + 3
*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^10 - 3*(a^7 - 3*a^
6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^8
 - 3*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6
- b^7 - 15*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 -
a*b^6 + b^7)*cosh(x)^2)*sinh(x)^8 + 24*(5*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4
*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - (a^7 - 3*a^6*b + a
^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x))*sinh(x)
^7 + a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
 b^7 - 4*(a^7 - 5*a^6*b + 9*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 -
5*a*b^6 + b^7 + 12*(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^
5 + a*b^6)*x)*cosh(x)^6 - 2*(2*a^7 - 10*a^6*b + 18*a^5*b^2 - 10*a^4*b^3 -
10*a^3*b^4 + 18*a^2*b^5 - 10*a*b^6 + 2*b^7 - 105*(a^7 - a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 42*(a^7 - 3
*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x
)^2 + 24*(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)
*x)*sinh(x)^6 + 12*(21*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 -
3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 14*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^...
```

3.723.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**3*sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

output
```
Timed out
```

3.723.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{3\,abx}{4\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)} + \frac{3\,(a^4b^2+a^2b^4)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} - \frac{4\,(a+b)e^{(-2\,x)}-(a-b)e^{(-4\,x)}}{64\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+b^6-3\,(a^6-4\,a^5b+5\,a^4b^2-5\,a^2b^4+4\,ab^5-b^6)e^{(-2\,x)}-4\,(}{64\,((a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)e^{(-4\,x)}+(a^8-2\,a^7b-2\,a^6b^2+6\,a^5b^3-}$$

input
```
integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima
")
```

output
```
-3/4*a*b*x/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 3*(a^4*b^2 + a^2*
b^4)*log(-(a - b)*e^(-2*x) - a - b)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b
^6 + b^8) - 1/64*(4*(a + b)*e^(-2*x) - (a - b)*e^(-4*x))/(a^3 - 3*a^2*b +
3*a*b^2 - b^3) + 1/64*(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a
*b^5 + b^6 - 3*(a^6 - 4*a^5*b + 5*a^4*b^2 - 5*a^2*b^4 + 4*a*b^5 - b^6)*e^(
-2*x) - 4*(a^6 - 6*a^5*b + 15*a^4*b^2 - 52*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5
+ b^6)*e^(-4*x))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*e^(-4*x)
 + (a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^
7 - b^8)*e^(-6*x))
```

3.723.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{3\,abx}{4\,(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)} + \frac{\left(36\,abe^{(4\,x)}-4\,a^2e^{(2\,x)}+4\,b^2e^{(2\,x)}+a^2-2\,ab+b^2\right)e^{(-4\,x)}}{64\,(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)} + \frac{3\,(a^4b^2+a^2b^4)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} + \frac{a^2e^{(4\,x)}+2\,abe^{(4\,x)}+b^2e^{(4\,x)}-4\,a^2e^{(2\,x)}+4\,b^2e^{(2\,x)}}{64\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)} - \frac{3\,a^5b^2e^{(2\,x)}+3\,a^4b^3e^{(2\,x)}+3\,a^3b^4e^{(2\,x)}+3\,a^2b^5e^{(2\,x)}+3\,a^5b^2-a^4b^3+a^3b^4-3\,a^2b^5}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)(ae^{(2\,x)}+be^{(2\,x)}+a-b)}$$

input
```
integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-3/4*a*b*x/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 1/64*(36*a*b*e^(4
*x) - 4*a^2*e^(2*x) + 4*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^4 - 4
*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 3*(a^4*b^2 + a^2*b^4)*log(abs(a*e^(2
*x) + b*e^(2*x) + a - b))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)
+ 1/64*(a^2*e^(4*x) + 2*a*b*e^(4*x) + b^2*e^(4*x) - 4*a^2*e^(2*x) + 4*b^2*
e^(2*x))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (3*a^5*b^2*e^(2*x)
+ 3*a^4*b^3*e^(2*x) + 3*a^3*b^4*e^(2*x) + 3*a^2*b^5*e^(2*x) + 3*a^5*b^2 -
a^4*b^3 + a^3*b^4 - 3*a^2*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 +
 b^8)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

3.723.9 Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.55

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{4x}}{64(a+b)^2} + \frac{e^{-4x}}{64(a-b)^2} + \frac{\ln(a-b+a\,e^{2x}+b\,e^{2x})\,(3a^4b^2+3a^2b^4)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8} - \frac{e^{-2x}(a+b)}{16(a-b)^3} - \frac{e^{2x}(a-b)}{16(a+b)^3} - \frac{3abx}{4(a-b)^4} - \frac{2a^3b^3}{(a+b)^4(a-b)^3\,(a-b+e^{2x}\,(a+b))}$$

input
```
int((cosh(x)^3*sinh(x)^3)/(a*cosh(x) + b*sinh(x))^2,x)
```

output
```
exp(4*x)/(64*(a + b)^2) + exp(-4*x)/(64*(a - b)^2) + (log(a - b + a*exp(2*
x) + b*exp(2*x))*(3*a^2*b^4 + 3*a^4*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b
^4 - 4*a^6*b^2) - (exp(-2*x)*(a + b))/(16*(a - b)^3) - (exp(2*x)*(a - b))/
(16*(a + b)^3) - (3*a*b*x)/(4*(a - b)^4) - (2*a^3*b^3)/((a + b)^4*(a - b)^
3*(a - b + exp(2*x)*(a + b)))
```

3.724 $\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

3.724.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx=-\frac{cCx}{b^2-c^2}+\frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

output
```
-c*C*x/(b^2-c^2)+b*C*ln(b*cosh(x)+c*sinh(x))/(b^2-c^2)+A*arctan((c*cosh(x)
+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)
```

3.724.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{2A\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)}{\sqrt{b-c}\sqrt{b+c}}+\frac{C(-cx+b\log(b\cosh(x)+c\sinh(x)))}{b^2-c^2}$$

input
```
Integrate[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

output
```
(2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/(Sqrt[b - c]*Sqr
t[b + c]) + (C*(-(c*x) + b*Log[b*Cosh[x] + c*Sinh[x]]))/(b^2 - c^2)
```

3.724.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 3616, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iC\sin(ix)}{b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{3616}$$

$$A\int \frac{1}{b\cosh(x)+c\sinh(x)}dx-\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{b\cos(ix)-ic\sin(ix)}dx-\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{3553}$$

$$iA\int \frac{1}{b^2-c^2-(-ic\cosh(x)-ib\sinh(x))^2}d(-ic\cosh(x)-ib\sinh(x))-\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{iA\text{arctanh}\left(\frac{-ib\sinh(x)-ic\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

input
```
Int[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

output
```
-((c*C*x)/(b^2 - c^2)) + (I*A*ArcTanh[((-I)*c*Cosh[x] - I*b*Sinh[x])/Sqrt[
b^2 - c^2]])/Sqrt[b^2 - c^2] + (b*C*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2
)
```

3.724.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3616
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (-Simp[b*C*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

3.724.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

method	result
default	$-\frac{2C\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}-\frac{2C\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}+\frac{Cb\ln\left(\tanh\left(\frac{x}{2}\right)^{2}b+2c\tanh\left(\frac{x}{2}\right)+b\right)+\frac{2\left(b^{2}A-A\,c^{2}\right)\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\sqrt{b^{2}-c^{2}}}}{(b-c)(b+c)}$
risch	$\frac{Cx}{b+c}-\frac{2xC\,b^{3}}{b^{4}-2b^{2}c^{2}+c^{4}}+\frac{2xCb\,c^{2}}{b^{4}-2b^{2}c^{2}+c^{4}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{-A^{2}b^{2}+A^{2}c^{2}}}{A(b+c)}\right)bC}{(b+c)(b-c)}+\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{-A^{2}b^{2}+A^{2}c^{2}}}{A(b+c)}\right)\sqrt{-A^{2}b^{2}+A^{2}c^{2}}}{(b+c)(b-c)}+\frac{\ln\left(\mathrm{e}^{x}-\right.}{(}$

input
```
int((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)-2*C/(2*b-2*c)*ln(tanh(1/2*x)+1)+2/(b-c)/(
b+c)*(1/2*C*b*ln(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+(A*b^2-A*c^2)/(b^2-c^2
)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*c)/(b^2-c^2)^(1/2)))
```

3.724.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.91

$$\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \left[\frac{Cb\log\left(\frac{2\,(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\sqrt{-b^2+c^2}A\log\left(\frac{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\sqrt{-b^2+c^2}(\cosh(x}{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+b-}\right.}{b^2-c^2}\right.$$

input
```
integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

output
```
[(C*b*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) - sqrt(-b^2 + c^2
)*A*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2
 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*
(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - (C*b + C*c)*x)/(b^
2 - c^2), (C*b*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) - 2*sqrt
(b^2 - c^2)*A*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh(x)))
- (C*b + C*c)*x)/(b^2 - c^2)]
```

3.724.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(66) = 132.

Time = 26.64 (sec) , antiderivative size = 367, normalized size of antiderivative = 4.59

$$\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Cx\right) \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Cx}{c} \\ -\frac{2A}{-2c\sinh(x)+2c\cosh(x)}-\frac{Cx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)}+\frac{Cx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{C\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} \\ -\frac{2A}{2c\sinh(x)+2c\cosh(x)}+\frac{Cx\sinh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{Cx\cosh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{C\cosh(x)}{2c\sinh(x)+2c\cosh(x)} \\ -\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2}+\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2}+\frac{Cbx}{b^2-c^2}-\frac{2Cb\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2-c^2}+\frac{Cb\log}{} \end{cases}$$

input
```
integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

output
```
Piecewise((zoo*(A*log(tanh(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tan
h(x/2)) + C*x)/c, Eq(b, 0)), (-2*A/(-2*c*sinh(x) + 2*c*cosh(x)) - C*x*sinh
(x)/(-2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)
) - C*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (-2*A/(2*c*sinh(x)
 + 2*c*cosh(x)) + C*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(2
*c*sinh(x) + 2*c*cosh(x)) + C*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)), Eq(b, c
)), (-A*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b*
*2 - c**2) + A*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)
/b)/(b**2 - c**2) + C*b*x/(b**2 - c**2) - 2*C*b*log(tanh(x/2) + 1)/(b**2 -
 c**2) + C*b*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + C
*b*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2 - c**2) - C*c*x/(b**2
 - c**2), True))
```

3.724.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` f
or more de
```

3.724.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{Cb\log\left(be^{(2x)}+ce^{(2x)}+b-c\right)}{b^2-c^2}+\frac{2A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{Cx}{b-c}$$

input
```
integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

output
```
C*b*log(b*e^(2*x) + c*e^(2*x) + b - c)/(b^2 - c^2) + 2*A*arctan((b*e^x + c
*e^x)/sqrt(b^2 - c^2))/sqrt(b^2 - c^2) - C*x/(b - c)
```

3.724.9 Mupad [B] (verification not implemented)

Time = 4.75 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.22

$$\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{A\,\mathrm{e}^x\,\sqrt{b^2-c^2}}{b\,\sqrt{A^2}-c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^2-c^2}} - \frac{C\,x}{b-c} + \frac{C\,b^3\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,\mathrm{e}^{2\,x}+4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4} - \frac{C\,b\,c^2\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,\mathrm{e}^{2\,x}+4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4}$$

input
```
int((A + C*sinh(x))/(b*cosh(x) + c*sinh(x)),x)
```

output
```
(2*atan((A*exp(x)*(b^2 - c^2)^(1/2))/(b*(A^2)^(1/2) - c*(A^2)^(1/2)))*(A^2
)^(1/2))/(b^2 - c^2)^(1/2) - (C*x)/(b - c) + (C*b^3*log(4*A^2*b - 4*A^2*c
+ 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^2) - (C*b*c^2
*log(4*A^2*b - 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4
- 2*b^2*c^2)
```

3.725 $\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

3.725.1 Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{cC\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}} - \frac{bC-Ac\cosh(x)-Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}$$

output
```
-c*C*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+(-b*C+A
*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))
```

3.725.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.89

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{b^2\sqrt{b-c}(b+c)C+2b^2c\sqrt{b+c}C\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)\cosh(x)+\left(-A(b-c)^{3/2}(b+c)^2+2bc^2\sqrt{b+c}C\right.}{b(b-c)^{3/2}(b+c)^2(b\cosh(x)+c\sinh(x))}$$

input
```
Integrate[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
-((b^2*Sqrt[b - c]*(b + c)*C + 2*b^2*c*Sqrt[b + c]*C*ArcTan[(c + b*Tanh[x/
2])/(Sqrt[b - c]*Sqrt[b + c])]*Cosh[x] + (-(A*(b - c)^(3/2)*(b + c)^2) + 2
*b*c^2*Sqrt[b + c]*C*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])*
Sinh[x])/(b*(b - c)^(3/2)*(b + c)^2*(b*Cosh[x] + c*Sinh[x])))
```

3.725.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 3633, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iC\sin(ix)}{(b\cos(ix)-ic\sin(ix))^2}dx$$

$$\Big\downarrow \text{3633}$$

$$-\frac{cC\int \frac{1}{b\cosh(x)+c\sinh(x)}dx}{b^2-c^2}-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}-\frac{cC\int \frac{1}{b\cos(ix)-ic\sin(ix)}dx}{b^2-c^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}-\frac{icC\int \frac{1}{b^2-c^2-(-ic\cosh(x)-ib\sinh(x))^2}d(-ic\cosh(x)-ib\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}-\frac{icC\text{arctanh}\left(\frac{-ib\sinh(x)-ic\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

input
```
Int[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
((-I)*c*C*ArcTanh[((-I)*c*Cosh[x] - I*b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 -
c^2)^(3/2) - (b*C - A*c*Cosh[x] - A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c
*Sinh[x]))
```

3.725.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3633
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - c*C)/(a^2 - b^2 - c^2)   Int[1/(
a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

3.725.4 Maple [A] (verified)

Time = 5.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.40

method	result	size
default	$-\frac{2\left(-\frac{\left(b^{2} A-A\,c^{2}-C c b\right) \tanh \left(\frac{x}{2}\right)}{b\left(b^{2}-c^{2}\right)}+\frac{b C}{b^{2}-c^{2}}\right)}{\tanh \left(\frac{x}{2}\right)^{2} b+2 c \tanh \left(\frac{x}{2}\right)+b}-\frac{2 C c \arctan \left(\frac{2 b \tanh \left(\frac{x}{2}\right)+2 c}{2 \sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$	115
risch	$-\frac{2\left(C b\,{\mathrm e}^{x}+b A-A c\right)}{\left(b-c\right)\left(b+c\right)\left(b\,{\mathrm e}^{2 x}+{\mathrm e}^{2 x} c+b-c\right)}-\frac{c C \ln \left({\mathrm e}^{x}+\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,\left(b+c\right)\left(b-c\right)}+\frac{c C \ln \left({\mathrm e}^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,\left(b+c\right)\left(b-c\right)}$	144

input
```
int((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(-(A*b^2-A*c^2-C*b*c)/b/(b^2-c^2)*tanh(1/2*x)+b*C/(b^2-c^2))/(tanh(1/2*
x)^2*b+2*c*tanh(1/2*x)+b)-2*C*c/(b^2-c^2)^(3/2)*arctan(1/2*(2*b*tanh(1/2*x
)+2*c)/(b^2-c^2)^(1/2))
```

3.725.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(76) = 152$.

Time = 0.27 (sec) , antiderivative size = 679, normalized size of antiderivative = 8.28

$$\int \frac{A + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\, dx$$
$$= \left[-\frac{2\,Ab^3 - 2\,Ab^2c - 2\,Abc^2 + 2\,Ac^3 - \left(Cbc - Cc^2 + (Cbc + Cc^2)\cosh{(x)}^2 + 2\,(Cbc + Cc^2)\cosh{(x)}\sinh\right.}{b^5 - b^4c - 2\,b^3c^2 + 2\,b^2c^3 + bc^4 - c^5 + (b^5 + b^4c - 2\,b^3c^2 - 2\,b^2c}\right.$$
$$- \frac{2\left(Ab^3 - Ab^2c - Abc^2 + Ac^3 - \left(Cbc - Cc^2 + (Cbc + Cc^2)\cosh{(x)}^2 + 2\,(Cbc + Cc^2)\cosh{(x)}\sinh{(x)}\right.\right.}{b^5 - b^4c - 2\,b^3c^2 + 2\,b^2c^3 + bc^4 - c^5 + (b^5 + b^4c - 2\,b^3c^2 - 2\,b^2c^3 + bc^4 + c^5)\cosh{(x)}^2 + 2}$$

input
```
integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

output
```
[-(2*A*b^3 - 2*A*b^2*c - 2*A*b*c^2 + 2*A*c^3 - (C*b*c - C*c^2 + (C*b*c + C
*c^2)*cosh(x)^2 + 2*(C*b*c + C*c^2)*cosh(x)*sinh(x) + (C*b*c + C*c^2)*sinh
(x)^2)*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x)
 + (b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b
 + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c))
+ 2*(C*b^3 - C*b*c^2)*cosh(x) + 2*(C*b^3 - C*b*c^2)*sinh(x))/(b^5 - b^4*c
- 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c
^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c
^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 +
 c^5)*sinh(x)^2), -2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 - (C*b*c - C*c^2 +
 (C*b*c + C*c^2)*cosh(x)^2 + 2*(C*b*c + C*c^2)*cosh(x)*sinh(x) + (C*b*c +
C*c^2)*sinh(x)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x)
+ (b + c)*sinh(x))) + (C*b^3 - C*b*c^2)*cosh(x) + (C*b^3 - C*b*c^2)*sinh(x
))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b
^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 -
 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b
^2*c^3 + b*c^4 + c^5)*sinh(x)^2)]
```

3.725.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2} \, dx = \text{Timed out}$$

input
```
integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

output
```
Timed out
```

3.725.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` f
or more de
```

3.725.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{A + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2} \, dx = -\frac{2\,Cc\arctan\left(\frac{be^x + ce^x}{\sqrt{b^2 - c^2}}\right)}{(b^2 - c^2)^{\frac{3}{2}}} - \frac{2\,(Cbe^x + Ab - Ac)}{(b^2 - c^2)(be^{(2\,x)} + ce^{(2\,x)} + b - c)}$$

input
```
integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

output
```
-2*C*c*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) - 2*(C*b*
e^x + A*b - A*c)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b - c))
```

3.725.9 Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.05

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{C\,c\,\ln\left(\frac{2\,C\,c}{(b+c)^{5/2}\,\sqrt{c-b}}-\frac{2\,C\,c\,\mathrm{e}^x}{-b^3-b^2\,c+b\,c^2+c^3}\right)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{C\,c\,\ln\left(-\frac{2\,C\,c}{(b+c)^{5/2}\,\sqrt{c-b}}-\frac{2\,C\,c\,\mathrm{e}^x}{-b^3-b^2\,c+b\,c^2+c^3}\right)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{\frac{2\,A}{b+c}+\frac{2\,C\,b\,\mathrm{e}^x}{(b+c)\,(b-c)}}{b-c+\mathrm{e}^{2\,x}\,(b+c)}$$

input
```
int((A + C*sinh(x))/(b*cosh(x) + c*sinh(x))^2,x)
```

output
```
(C*c*log((2*C*c)/((b + c)^(5/2)*(c - b)^(1/2)) - (2*C*c*exp(x))/(b*c^2 - b
^2*c - b^3 + c^3)))/((b + c)^(3/2)*(c - b)^(3/2)) - (C*c*log(- (2*C*c)/((b
 + c)^(5/2)*(c - b)^(1/2)) - (2*C*c*exp(x))/(b*c^2 - b^2*c - b^3 + c^3)))/
((b + c)^(3/2)*(c - b)^(3/2)) - ((2*A)/(b + c) + (2*C*b*exp(x))/((b + c)*(
b - c)))/(b - c + exp(2*x)*(b + c))
```

3.726 $\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

3.726.1 Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx=\frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}-\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}-\frac{c^2C\cosh(x)+bcC\sinh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

output
```
1/2*A*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+1/2*(-
b*C+A*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))^2+(-c^2*C*cos
h(x)-b*c*C*sinh(x))/(b^2-c^2)^2/(b*cosh(x)+c*sinh(x))
```

3.726.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx=\frac{1}{2}\left(\frac{2A\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}+\frac{-b^2C+A(b^2-c^2)\sinh(x)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2}+\frac{c(A-2C\sinh(x))}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}\right)$$

input
```
Integrate[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
((2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c)^(3/2)*
(b + c)^(3/2)) + (-(b^2*C) + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)*(b + c)*(b*
Cosh[x] + c*Sinh[x])^2) + (c*(A - 2*C*Sinh[x]))/(b*(b - c)*(b + c)*(b*Cosh
[x] + c*Sinh[x])))/2
```

3.726.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3636, 25, 3042, 3632, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iC\sin(ix)}{(b\cos(ix)-ic\sin(ix))^3}dx$$

$$\Big\downarrow \text{3636}$$

$$\frac{\int -\frac{2cC-Ab\cosh(x)-Ac\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}dx}{2(b^2-c^2)}-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{2cC-Ab\cosh(x)-Ac\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}dx}{2(b^2-c^2)}-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}-\frac{\int \frac{2cC-Ab\cos(ix)+iAc\sin(ix)}{(b\cos(ix)-ic\sin(ix))^2}dx}{2(b^2-c^2)}$$

$$\Big\downarrow \text{3632}$$

$$-\frac{\frac{2(bcC\sinh(x)+c^2C\cosh(x))}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}-A\int \frac{1}{b\cosh(x)+c\sinh(x)}dx}{2(b^2-c^2)}-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}-\frac{\frac{2(bcC\sinh(x)+c^2C\cosh(x))}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}-A\int\frac{1}{b\cos(ix)-ic\sin(ix)}dx}{2(b^2-c^2)}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}-\frac{\frac{2(bcC\sinh(x)+c^2C\cosh(x))}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}-iA\int\frac{1}{b^2-c^2-(-ic\cosh(x)-ib\sinh(x))^2}d(-ic\cosh(x)-ib\sinh(x))}{2(b^2-c^2)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}-\frac{\frac{2(bcC\sinh(x)+c^2C\cosh(x))}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}-\frac{iA\text{arctanh}\left(\frac{-ib\sinh(x)-ic\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}}{2(b^2-c^2)}$$

input
```
Int[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
-1/2*(b*C - A*c*Cosh[x] - A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]
)^2) - (((-I)*A*ArcTanh[((-I)*c*Cosh[x] - I*b*Sinh[x])/Sqrt[b^2 - c^2]])/S
qrt[b^2 - c^2] + (2*(c^2*C*Cosh[x] + b*c*C*Sinh[x]))/((b^2 - c^2)*(b*Cosh[
x] + c*Sinh[x])))/(2*(b^2 - c^2))
```

3.726.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.726. $\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3636
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*C + (a*
C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*((a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b
^2 - c^2))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

3.726.4 Maple [A] (verified)

Time = 31.78 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.52

method	result
default	$\frac{-\frac{A\left(b^{2}-2c^{2}\right)\tanh\left(\frac{x}{2}\right)^{3}}{\left(b^{2}-c^{2}\right)b}+\frac{\left(A\,b^{2}c+2A\,c^{3}+2C\,b^{3}-2Cb\,c^{2}\right)\tanh\left(\frac{x}{2}\right)^{2}}{b^{2}\left(b^{2}-c^{2}\right)}+\frac{A\left(b^{2}+2c^{2}\right)\tanh\left(\frac{x}{2}\right)}{b\left(b^{2}-c^{2}\right)}+\frac{2Ac}{2b^{2}-2c^{2}}}{\left(\tanh\left(\frac{x}{2}\right)^{2}b+2c\tanh\left(\frac{x}{2}\right)+b\right)^{2}}+\frac{A\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{A\,b^{2}\mathrm{e}^{3x}+2Abc\,\mathrm{e}^{3x}+A\,c^{2}\mathrm{e}^{3x}-2C\,b^{2}\mathrm{e}^{2x}+2C\,c^{2}\mathrm{e}^{2x}-A\,\mathrm{e}^{x}b^{2}+A\,\mathrm{e}^{x}c^{2}+2Ccb-2C\,c^{2}}{(b-c)(b\,\mathrm{e}^{2x}+\mathrm{e}^{2x}c+b-c)^{2}(b^{2}+2cb+c^{2})}-\frac{A\ln\left(\mathrm{e}^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{2\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}+\frac{A\ln\left(\mathrm{e}^{x}+\frac{b-c}{\sqrt{-b^{2}+c}}\right.}{2\sqrt{-b^{2}+c^{2}}\,(b+c)(b}$

input
```
int((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/2*A*(b^2-2*c^2)/(b^2-c^2)/b*tanh(1/2*x)^3+1/2*(A*b^2*c+2*A*c^3+2*C*b
^3-2*C*b*c^2)/b^2/(b^2-c^2)*tanh(1/2*x)^2+1/2*A*(b^2+2*c^2)/b/(b^2-c^2)*ta
nh(1/2*x)+1/2*A*c/(b^2-c^2))/(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)^2+A/(b^2-
c^2)^(3/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*c)/(b^2-c^2)^(1/2))
```

3.726.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. 2(114) = 228.

Time = 0.31 (sec) , antiderivative size = 1855, normalized size of antiderivative = 15.08

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

output
```
[1/2*(4*C*b^2*c - 8*C*b*c^2 + 4*C*c^3 + 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c
^3)*cosh(x)^3 + 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 - 4*(C*b^3
 - C*b^2*c - C*b*c^2 + C*c^3)*cosh(x)^2 - 2*(2*C*b^3 - 2*C*b^2*c - 2*C*b*c
^2 + 2*C*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 +
((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)
*sinh(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2
 + 2*(A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A
*c^2)*cosh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b
^2 - A*c^2)*cosh(x))*sinh(x))*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*
(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x)
+ sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b +
c)*sinh(x)^2 + b - c)) - 2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) - 2
*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3
)*cosh(x)^2 + 4*(C*b^3 - C*b^2*c - C*b*c^2 + C*c^3)*cosh(x))*sinh(x))/(b^6
 - 2*b^5*c - b^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*
c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*
b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 +
 (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4
 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 +
3*b^2*c^4 - c^6 + 3*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*...
```

3.726.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

output
```
Timed out
```

3.726.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` f
or more de
```

3.726.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.24

$$\int \frac{A + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{\frac{3}{2}}} + \frac{Ab^2e^{(3\,x)} + 2\,Abce^{(3\,x)} + Ac^2e^{(3\,x)} - 2\,Cb^2e^{(2\,x)} + 2\,Cc^2e^{(2\,x)} - Ab^2e^x + Ac^2e^x + 2\,Cbc - 2\,Cc^2}{(b^3 + b^2c - bc^2 - c^3)\left(be^{(2\,x)} + ce^{(2\,x)} + b - c\right)^2}$$

input
```
integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

output
```
A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + (A*b^2*e^(3*
x) + 2*A*b*c*e^(3*x) + A*c^2*e^(3*x) - 2*C*b^2*e^(2*x) + 2*C*c^2*e^(2*x) -
 A*b^2*e^x + A*c^2*e^x + 2*C*b*c - 2*C*c^2)/((b^3 + b^2*c - b*c^2 - c^3)*(
b*e^(2*x) + c*e^(2*x) + b - c)^2)
```

3.726.9 Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.76

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{\operatorname{atan}\left(\frac{A\,e^x\,\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}}{b^3\,\sqrt{A^2}+c^3\,\sqrt{A^2}-b\,c^2\,\sqrt{A^2}-b^2\,c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}} - \frac{\frac{C}{(b+c)^2}-\frac{A\,e^x}{(b+c)\,(b-c)}}{b-c+e^{2\,x}\,(b+c)} - \frac{\frac{2\,A\,e^x}{b+c}-\frac{C}{b+c}+\frac{C\,e^{2\,x}}{b+c}}{e^{4\,x}\,(b+c)^2+(b-c)^2+2\,e^{2\,x}\,(b+c)\,(b-c)}$$

input
```
int((A + C*sinh(x))/(b*cosh(x) + c*sinh(x))^3,x)
```

output
```
(atan((A*exp(x)*(b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2))/(b^3*(A^2)^(1/2
) + c^3*(A^2)^(1/2) - b*c^2*(A^2)^(1/2) - b^2*c*(A^2)^(1/2)))*(A^2)^(1/2))
/(b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2) - (C/(b + c)^2 - (A*exp(x))/((b
 + c)*(b - c)))/(b - c + exp(2*x)*(b + c)) - ((2*A*exp(x))/(b + c) - C/(b
+ c) + (C*exp(2*x))/(b + c))/(exp(4*x)*(b + c)^2 + (b - c)^2 + 2*exp(2*x)*
(b + c)*(b - c))
```

3.727 $\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

3.727.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{bBx}{b^2-c^2}+\frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

output
```
b*B*x/(b^2-c^2)-B*c*ln(b*cosh(x)+c*sinh(x))/(b^2-c^2)+A*arctan((c*cosh(x)+
b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)
```

3.727.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{bBx+2A\sqrt{b-c}\sqrt{b+c}\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)-Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

input
```
Integrate[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

output
```
(b*B*x + 2*A*Sqrt[b - c]*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]
*Sqrt[b + c])] - B*c*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

3.727.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 3617, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\cos(ix)}{b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{ 3617}$$

$$A\int \frac{1}{b\cosh(x)+c\sinh(x)}dx+\frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$A\int \frac{1}{b\cos(ix)-ic\sin(ix)}dx+\frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{ 3553}$$

$$iA\int \frac{1}{b^2-c^2-(-ic\cosh(x)-ib\sinh(x))^2}d(-ic\cosh(x)-ib\sinh(x))+\frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{iA\text{arctanh}\left(\frac{-ib\sinh(x)-ic\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

input
```
Int[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

output
```
(b*B*x)/(b^2 - c^2) + (I*A*ArcTanh[((-I)*c*Cosh[x] - I*b*Sinh[x])/Sqrt[b^2
 - c^2]])/Sqrt[b^2 - c^2] - (B*c*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

3.727.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3617
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[b*B*((d + e*x)/
(e*(b^2 + c^2))), x] + (Simp[c*B*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/
(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2)   Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

3.727.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

method	result
default	$-\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}+\frac{-Bc\ln\left(\tanh\left(\frac{x}{2}\right)^2b+2c\tanh\left(\frac{x}{2}\right)+b\right)+\frac{2\left(b^2A-A\,c^2\right)\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2c}{2\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}}{(b-c)(b+c)}$
risch	$\frac{Bx}{b+c}+\frac{2xB\,b^2c}{b^4-2b^2c^2+c^4}-\frac{2xB\,c^3}{b^4-2b^2c^2+c^4}-\frac{\ln\left(\mathrm{e}^x+\frac{\sqrt{-A^2b^2+A^2c^2}}{A(b+c)}\right)Bc}{(b+c)(b-c)}+\frac{\ln\left(\mathrm{e}^x+\frac{\sqrt{-A^2b^2+A^2c^2}}{A(b+c)}\right)\sqrt{-A^2b^2+A^2c^2}}{(b+c)(b-c)}-\frac{\ln\left(\mathrm{e}^x-\right.}{(}$

input
```
int((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)+2*B/(2*b-2*c)*ln(tanh(1/2*x)+1)+2/(b-c)/(
b+c)*(-1/2*B*c*ln(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+(A*b^2-A*c^2)/(b^2-c^
2)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*c)/(b^2-c^2)^(1/2)))
```

3.727.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.92

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx$$
$$= \left[-\frac{Bc\log\left(\frac{2\,(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)+\sqrt{-b^2+c^2}A\log\left(\frac{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\sqrt{-b^2+c^2}(\cosh\cdots}{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+\cdots}\right)\cdots}{b^2-c^2}\right.$$
$$\left.-\frac{Bc\log\left(\frac{2\,(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)+2\sqrt{b^2-c^2}A\arctan\left(\frac{\sqrt{b^2-c^2}}{(b+c)\cosh(x)+(b+c)\sinh(x)}\right)-(Bb+Bc)x}{b^2-c^2}\right]$$

input
```
integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

output
```
[-(B*c*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) + sqrt(-b^2 + c^
2)*A*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^
2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2
*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - (B*b + B*c)*x)/(b
^2 - c^2), -(B*c*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) + 2*sq
rt(b^2 - c^2)*A*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh(x))
) - (B*b + B*c)*x)/(b^2 - c^2)]
```

3.727.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(66) = 132$.

Time = 30.01 (sec) , antiderivative size = 697, normalized size of antiderivative = 8.71

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh\left(\frac{x}{2}\right)\right)\right) \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh\left(\frac{x}{2}\right)\right)}{c} \\ -\frac{2A}{-2c\sinh(x)+2c\cosh(x)}+\frac{Bx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{Bx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{B\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} \\ -\frac{2A}{2c\sinh(x)+2c\cosh(x)}+\frac{Bx\sinh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{Bx\cosh(x)}{2c\sinh(x)+2c\cosh(x)}-\frac{B\cosh(x)}{2c\sinh(x)+2c\cosh(x)} \\ \frac{Ab^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2\sqrt{-b^2+c^2}-c^2\sqrt{-b^2+c^2}}-\frac{Ab^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2\sqrt{-b^2+c^2}-c^2\sqrt{-b^2+c^2}}-\frac{Ac^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2\sqrt{-b^2+c^2}-c^2\sqrt{-b^2+c^2}}+\frac{Ac^2\log\left(\tanh\left(\frac{x}{2}\right)+\cdots\right)}{b^2\sqrt{-b^2+c^2}-c^2\cdots} \end{cases}$$

input
```
integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x)
```

output
```
Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(ta
nh(x/2))), Eq(b, 0) & Eq(c, 0)), ((A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x
/2) + 1) + B*log(tanh(x/2)))/c, Eq(b, 0)), (-2*A/(-2*c*sinh(x) + 2*c*cosh(
x)) + B*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*x*cosh(x)/(-2*c*sinh(x)
 + 2*c*cosh(x)) - B*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (-2*
A/(2*c*sinh(x) + 2*c*cosh(x)) + B*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) +
B*x*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(2*c*sinh(x) + 2*c*cos
h(x)), Eq(b, c)), (A*b**2*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**
2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) - A*b**2*log(tanh(x/2) + c
/b + sqrt(-b**2 + c**2)/b)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c*
*2)) - A*c**2*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2*sqrt(-b**2
 + c**2) - c**2*sqrt(-b**2 + c**2)) + A*c**2*log(tanh(x/2) + c/b + sqrt(-b
**2 + c**2)/b)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) + B*b*x
*sqrt(-b**2 + c**2)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) -
B*c*x*sqrt(-b**2 + c**2)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2
)) + 2*B*c*sqrt(-b**2 + c**2)*log(tanh(x/2) + 1)/(b**2*sqrt(-b**2 + c**2)
- c**2*sqrt(-b**2 + c**2)) - B*c*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b -
sqrt(-b**2 + c**2)/b)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2))
- B*c*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2
*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)), True))
```

3.727.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{b\cosh(x) + c\sinh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` f
or more de
```

3.727.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx = -\frac{Bc\log\left(be^{(2x)}+ce^{(2x)}+b-c\right)}{b^2-c^2}+\frac{2A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{Bx}{b-c}$$

input
```
integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

output
```
-B*c*log(b*e^(2*x) + c*e^(2*x) + b - c)/(b^2 - c^2) + 2*A*arctan((b*e^x +
c*e^x)/sqrt(b^2 - c^2))/sqrt(b^2 - c^2) + B*x/(b - c)
```

3.727.9 Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.21

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{A\,e^x\,\sqrt{b^2-c^2}}{b\,\sqrt{A^2}-c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^2-c^2}}+\frac{B\,x}{b-c}+\frac{B\,c^3\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,e^{2\,x}+4\,A^2\,c\,e^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4}-\frac{B\,b^2\,c\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,e^{2\,x}+4\,A^2\,c\,e^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4}$$

input
```
int((A + B*cosh(x))/(b*cosh(x) + c*sinh(x)),x)
```

output
```
(2*atan((A*exp(x)*(b^2 - c^2)^(1/2))/(b*(A^2)^(1/2) - c*(A^2)^(1/2)))*(A^2
)^(1/2))/(b^2 - c^2)^(1/2) + (B*x)/(b - c) + (B*c^3*log(4*A^2*b - 4*A^2*c
+ 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^2) - (B*b^2*c
*log(4*A^2*b - 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4
- 2*b^2*c^2)
```

3.728 $\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

3.728.1 Optimal result

Integrand size = 18, antiderivative size = 78

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{bB\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}} + \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}$$

output
```
b*B*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+(B*c+A*c
*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))
```

3.728.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.94

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{bB\sqrt{b-c}c(b+c)+2b^3B\sqrt{b+c}\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)\cosh(x)+\left(A(b-c)^{3/2}(b+c)^2+2b^2Bc\sqrt{b+c}\,\mathrm{arcta}\right.}{b(b-c)^{3/2}(b+c)^2(b\cosh(x)+c\sinh(x))}$$

input
```
Integrate[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(b*B*Sqrt[b - c]*c*(b + c) + 2*b^3*B*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/
(Sqrt[b - c]*Sqrt[b + c])]*Cosh[x] + (A*(b - c)^(3/2)*(b + c)^2 + 2*b^2*B*
c*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])*Sinh[x]
)/(b*(b - c)^(3/2)*(b + c)^2*(b*Cosh[x] + c*Sinh[x]))
```

3.728.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3634, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)}{(b\cos(ix)-ic\sin(ix))^2}dx$$

$$\Big\downarrow \text{3634}$$

$$\frac{bB\int \frac{1}{b\cosh(x)+c\sinh(x)}dx}{b^2-c^2}+\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{bB\int \frac{1}{b\cos(ix)-ic\sin(ix)}dx}{b^2-c^2}$$

$$\Big\downarrow \text{3553}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{ibB\int \frac{1}{b^2-c^2-(-ic\cosh(x)-ib\sinh(x))^2}d(-ic\cosh(x)-ib\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{ibB\text{arctanh}\left(\frac{-ib\sinh(x)-ic\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

input
```
Int[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(I*b*B*ArcTanh[((-I)*c*Cosh[x] - I*b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2
)^(3/2) + (B*c + A*c*Cosh[x] + A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*Si
nh[x]))
```

3.728.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3634
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - b*B)/(a^2 - b^2 - c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

3.728.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

method	result	size
default	$-\frac{2\left(-\frac{\left(b^{2} A-A\,c^{2}+B\,c^{2}\right) \tanh\left(\frac{x}{2}\right)}{b\left(b^{2}-c^{2}\right)}-\frac{B c}{b^{2}-c^{2}}\right)}{\tanh\left(\frac{x}{2}\right)^{2} b+2 c \tanh\left(\frac{x}{2}\right)+b}+\frac{2 b B \arctan\left(\frac{2 b \tanh\left(\frac{x}{2}\right)+2 c}{2 \sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$	116
risch	$-\frac{2\left(-B c\,{\mathrm e}^{x}+b A-A c\right)}{\left(b-c\right)\left(b+c\right)\left(b\,{\mathrm e}^{2 x}+{\mathrm e}^{2 x} c+b-c\right)}-\frac{b B \ln\left({\mathrm e}^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,\left(b+c\right)\left(b-c\right)}+\frac{b B \ln\left({\mathrm e}^{x}+\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,\left(b+c\right)\left(b-c\right)}$	145

input
```
int((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(-(A*b^2-A*c^2+B*c^2)/b/(b^2-c^2)*tanh(1/2*x)-B*c/(b^2-c^2))/(tanh(1/2*
x)^2*b+2*c*tanh(1/2*x)+b)+2*b*B/(b^2-c^2)^(3/2)*arctan(1/2*(2*b*tanh(1/2*x
)+2*c)/(b^2-c^2)^(1/2))
```

3.728.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(74) = 148$.

Time = 0.27 (sec) , antiderivative size = 680, normalized size of antiderivative = 8.72

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx$$
$$= \left[-\frac{2\,Ab^3 - 2\,Ab^2c - 2\,Abc^2 + 2\,Ac^3 - \left(Bb^2 - Bbc + (Bb^2 + Bbc)\cosh\left(x\right)^2 + 2\,(Bb^2 + Bbc)\cosh\left(x\right)\sinh\right.}{b^5 - b^4c - 2\,b^3c^2 + 2\,b^2c^3 + bc^4 - c^5 + (b^5 + b^4c - 2\,b^3c^2 - 2\,b^2c}\right.$$
$$-\frac{2\left(Ab^3 - Ab^2c - Abc^2 + Ac^3 + \left(Bb^2 - Bbc + (Bb^2 + Bbc)\cosh\left(x\right)^2 + 2\,(Bb^2 + Bbc)\cosh\left(x\right)\sinh\left(x\right)\right.\right.}{b^5 - b^4c - 2\,b^3c^2 + 2\,b^2c^3 + bc^4 - c^5 + (b^5 + b^4c - 2\,b^3c^2 - 2\,b^2c^3 + bc^4 + c^5)\cosh\left(x\right)^2 + 2}$$

input
```
integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

output
```
[-(2*A*b^3 - 2*A*b^2*c - 2*A*b*c^2 + 2*A*c^3 - (B*b^2 - B*b*c + (B*b^2 + B
*b*c)*cosh(x)^2 + 2*(B*b^2 + B*b*c)*cosh(x)*sinh(x) + (B*b^2 + B*b*c)*sinh
(x)^2)*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x)
 + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b
 + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c))
- 2*(B*b^2*c - B*c^3)*cosh(x) - 2*(B*b^2*c - B*c^3)*sinh(x))/(b^5 - b^4*c
- 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c
^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c
^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 +
 c^5)*sinh(x)^2), -2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 + (B*b^2 - B*b*c +
 (B*b^2 + B*b*c)*cosh(x)^2 + 2*(B*b^2 + B*b*c)*cosh(x)*sinh(x) + (B*b^2 +
B*b*c)*sinh(x)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x)
+ (b + c)*sinh(x))) - (B*b^2*c - B*c^3)*cosh(x) - (B*b^2*c - B*c^3)*sinh(x
))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b
^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 -
 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b
^2*c^3 + b*c^4 + c^5)*sinh(x)^2)]
```

3.728.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

output
```
Timed out
```

3.728.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` f
or more de
```

3.728.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \frac{2\,Bb\arctan\left(\frac{be^x + ce^x}{\sqrt{b^2 - c^2}}\right)}{(b^2 - c^2)^{\frac{3}{2}}} + \frac{2\,(Bce^x - Ab + Ac)}{(b^2 - c^2)(be^{(2\,x)} + ce^{(2\,x)} + b - c)}$$

input
```
integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

output
```
2*B*b*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + 2*(B*c*e
^x - A*b + A*c)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b - c))
```

3.728.9 Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.15

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{B\,b\,\ln\left(\frac{2\,B\,b}{(b+c)^{5/2}\,\sqrt{c-b}}+\frac{2\,B\,b\,\mathrm{e}^x}{-b^3-b^2\,c+b\,c^2+c^3}\right)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{B\,b\,\ln\left(\frac{2\,B\,b\,\mathrm{e}^x}{-b^3-b^2\,c+b\,c^2+c^3}-\frac{2\,B\,b}{(b+c)^{5/2}\,\sqrt{c-b}}\right)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{\frac{2\,A}{b+c}-\frac{2\,B\,c\,\mathrm{e}^x}{(b+c)\,(b-c)}}{b-c+\mathrm{e}^{2\,x}\,(b+c)}$$

input
```
int((A + B*cosh(x))/(b*cosh(x) + c*sinh(x))^2,x)
```

output
```
(B*b*log((2*B*b)/((b + c)^(5/2)*(c - b)^(1/2)) + (2*B*b*exp(x))/(b*c^2 - b
^2*c - b^3 + c^3)))/((b + c)^(3/2)*(c - b)^(3/2)) - (B*b*log((2*B*b*exp(x)
)/(b*c^2 - b^2*c - b^3 + c^3) - (2*B*b)/((b + c)^(5/2)*(c - b)^(1/2))))/((
b + c)^(3/2)*(c - b)^(3/2)) - ((2*A)/(b + c) - (2*B*c*exp(x))/((b + c)*(b
- c)))/(b - c + exp(2*x)*(b + c))
```

3.729 $\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

3.729.1 Optimal result

Integrand size = 18, antiderivative size = 120

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}} + \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2} + \frac{bBc\cosh(x)+b^2B\sinh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

output
```
1/2*A*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+1/2*(B
*c+A*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))^2+(b*B*c*cosh(
x)+b^2*B*sinh(x))/(b^2-c^2)^2/(b*cosh(x)+c*sinh(x))
```

3.729.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.12

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{1}{2}\left(\frac{2A\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}} + \frac{Ac+2bB\sinh(x)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))} + \frac{bBc+A\left(b^2-c^2\right)\sinh(x)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2}\right)$$

input
```
Integrate[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
((2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c)^(3/2)*
(b + c)^(3/2)) + (A*c + 2*b*B*Sinh[x])/(b*(b - c)*(b + c)*(b*Cosh[x] + c*S
inh[x])) + (b*B*c + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)*(b + c)*(b*Cosh[x] +
 c*Sinh[x])^2))/2
```

3.729.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 3637, 3042, 3632, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)}{(b\cos(ix)-ic\sin(ix))^3}dx$$

$$\Big\downarrow \text{3637}$$

$$\frac{\int \frac{2bB+Ab\cosh(x)+Ac\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}dx}{2(b^2-c^2)}+\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2bB+Ab\cos(ix)-iAc\sin(ix)}{(b\cos(ix)-ic\sin(ix))^2}dx}{2(b^2-c^2)}$$

$$\Big\downarrow \text{3632}$$

$$\frac{A\int \frac{1}{b\cosh(x)+c\sinh(x)}dx+\frac{2(b^2B\sinh(x)+bBc\cosh(x))}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}}{2(b^2-c^2)}+\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}+\frac{\frac{2(b^2B\sinh(x)+bBc\cosh(x))}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+A\int \frac{1}{b\cos(ix)-ic\sin(ix)}dx}{2(b^2-c^2)}$$

$$\Big\downarrow \text{3553}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{2\left(b^2-c^2\right)\left(b\cosh(x)+c\sinh(x)\right)^2}+\frac{\frac{2\left(b^2B\sinh(x)+bBc\cosh(x)\right)}{\left(b^2-c^2\right)\left(b\cosh(x)+c\sinh(x)\right)}+iA\int\frac{1}{b^2-c^2-\left(-ic\cosh(x)-ib\sinh(x)\right)^2}d(-ic\cosh(x)-ib\sinh(x))}{2\left(b^2-c^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{2\left(b^2-c^2\right)\left(b\cosh(x)+c\sinh(x)\right)^2}+\frac{\frac{2\left(b^2B\sinh(x)+bBc\cosh(x)\right)}{\left(b^2-c^2\right)\left(b\cosh(x)+c\sinh(x)\right)}+\frac{iA\text{arctanh}\left(\frac{-ib\sinh(x)-ic\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}}{2\left(b^2-c^2\right)}$$

input
```
Int[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
(B*c + A*c*Cosh[x] + A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2
) + ((I*A*ArcTanh[((-I)*c*Cosh[x] - I*b*Sinh[x])/Sqrt[b^2 - c^2]])/Sqrt[b^
2 - c^2] + (2*(b*B*c*Cosh[x] + b^2*B*Sinh[x]))/((b^2 - c^2)*(b*Cosh[x] + c
*Sinh[x])))/(2*(b^2 - c^2))
```

3.729.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.729. $\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3637
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d
 + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2
- b^2 - c^2))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n +
 1)*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*
x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2
- b^2 - c^2, 0] && NeQ[n, -2]
```

3.729.4 Maple [A] (verified)

Time = 13.83 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.76

method	result
risch	$\frac{A\,b^{2} {\mathrm e}^{3 x}+2 A b c\,{\mathrm e}^{3 x}+A\,c^{2} {\mathrm e}^{3 x}-2 B\,b^{2} {\mathrm e}^{2 x}+2 B\,c^{2} {\mathrm e}^{2 x}-A\,{\mathrm e}^{x} b^{2}+A\,{\mathrm e}^{x} c^{2}-2 B\,b^{2}+2 b B c}{\left(b -c \right) \left(b\,{\mathrm e}^{2 x}+{\mathrm e}^{2 x} c +b -c \right)^{2} \left(b^{2}+2 c b +c^{2}\right)}-\frac{A \ln \left({\mathrm e}^{x}-\frac{b -c}{\sqrt{-b^{2}+c^{2}}}\right)}{2 \sqrt{-b^{2}+c^{2}}\, \left(b +c \right) \left(b -c \right)}+\frac{A \ln \left({\mathrm e}^{x}+\frac{b -c}{\sqrt{-b^{2}+c^{2}}}\right)}{2 \sqrt{-b^{2}+c^{2}}\, \left(b +c \right) \left(b \right.}$
default	$\frac{-\frac{\left(b^{2} A -2 A\,c^{2}-2 B\,b^{2}+2 B\,c^{2}\right) \tanh \left(\frac{x}{2}\right)^{3}}{\left(b^{2}-c^{2}\right) b}+\frac{c \left(b^{2} A +2 A\,c^{2}+2 B\,b^{2}-2 B\,c^{2}\right) \tanh \left(\frac{x}{2}\right)^{2}}{b^{2} \left(b^{2}-c^{2}\right)}+\frac{\left(b^{2} A +2 A\,c^{2}+2 B\,b^{2}-2 B\,c^{2}\right) \tanh \left(\frac{x}{2}\right)}{b \left(b^{2}-c^{2}\right)}+\frac{2 A c}{2 b^{2}-2 c^{2}}}{\left(\tanh \left(\frac{x}{2}\right)^{2} b +2 c \tanh \left(\frac{x}{2}\right)+b \right)^{2}}+\frac{A\,\mathrm{a}}{}$

input
```
int((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
(A*b^2*exp(x)^3+2*A*b*c*exp(x)^3+A*c^2*exp(x)^3-2*B*b^2*exp(x)^2+2*B*c^2*e
xp(x)^2-A*exp(x)*b^2+A*exp(x)*c^2-2*B*b^2+2*b*B*c)/(b-c)/(b*exp(x)^2+exp(x
)^2*c+b-c)^2/(b^2+2*b*c+c^2)-1/2/(-b^2+c^2)^(1/2)*A/(b+c)/(b-c)*ln(exp(x)-
(b-c)/(-b^2+c^2)^(1/2))+1/2/(-b^2+c^2)^(1/2)*A/(b+c)/(b-c)*ln(exp(x)+(b-c)
/(-b^2+c^2)^(1/2))
```

3.729.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(112) = 224$.

Time = 0.30 (sec) , antiderivative size = 1855, normalized size of antiderivative = 15.46

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

output
```
[-1/2*(4*B*b^3 - 8*B*b^2*c + 4*B*b*c^2 - 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*
c^3)*cosh(x)^3 - 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 + 4*(B*b^
3 - B*b^2*c - B*b*c^2 + B*c^3)*cosh(x)^2 + 2*(2*B*b^3 - 2*B*b^2*c - 2*B*b*
c^2 + 2*B*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 -
 ((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x
)*sinh(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^
2 + 2*(A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c +
A*c^2)*cosh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*
b^2 - A*c^2)*cosh(x))*sinh(x))*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2
*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x)
 + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b +
 c)*sinh(x)^2 + b - c)) + 2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) +
2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^
3)*cosh(x)^2 + 4*(B*b^3 - B*b^2*c - B*b*c^2 + B*c^3)*cosh(x))*sinh(x))/(b^
6 - 2*b^5*c - b^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5
*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2
*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3
+ (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^
4 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 +
 3*b^2*c^4 - c^6 + 3*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2...
```

3.729.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

output
```
Timed out
```

3.729.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` f
or more de
```

3.729.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{be^x + ce^x}{\sqrt{b^2 - c^2}}\right)}{(b^2 - c^2)^{\frac{3}{2}}} + \frac{Ab^2e^{(3x)} + 2\,Abce^{(3x)} + Ac^2e^{(3x)} - 2\,Bb^2e^{(2x)} + 2\,Bc^2e^{(2x)} - Ab^2e^x + Ac^2e^x - 2\,Bb^2 + 2\,Bbc}{(b^3 + b^2c - bc^2 - c^3)(be^{(2x)} + ce^{(2x)} + b - c)^2}$$

input
```
integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

output
```
A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + (A*b^2*e^(3*
x) + 2*A*b*c*e^(3*x) + A*c^2*e^(3*x) - 2*B*b^2*e^(2*x) + 2*B*c^2*e^(2*x) -
 A*b^2*e^x + A*c^2*e^x - 2*B*b^2 + 2*B*b*c)/((b^3 + b^2*c - b*c^2 - c^3)*(
b*e^(2*x) + c*e^(2*x) + b - c)^2)
```

3.729.9 Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.80

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{\operatorname{atan}\left(\frac{A\,e^x\,\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}}{b^3\,\sqrt{A^2}+c^3\,\sqrt{A^2}-b\,c^2\,\sqrt{A^2}-b^2\,c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}} - \frac{\frac{B}{(b+c)^2}-\frac{A\,e^x}{(b+c)\,(b-c)}}{b-c+e^{2\,x}\,(b+c)} - \frac{\frac{B}{b+c}+\frac{2\,A\,e^x}{b+c}+\frac{B\,e^{2\,x}}{b+c}}{e^{4\,x}\,(b+c)^2+(b-c)^2+2\,e^{2\,x}\,(b+c)\,(b-c)}$$

input
```
int((A + B*cosh(x))/(b*cosh(x) + c*sinh(x))^3,x)
```

output
```
(atan((A*exp(x)*(b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2))/(b^3*(A^2)^(1/2
) + c^3*(A^2)^(1/2) - b*c^2*(A^2)^(1/2) - b^2*c*(A^2)^(1/2)))*(A^2)^(1/2))
/(b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2) - (B/(b + c)^2 - (A*exp(x))/((b
 + c)*(b - c)))/(b - c + exp(2*x)*(b + c)) - (B/(b + c) + (2*A*exp(x))/(b
+ c) + (B*exp(2*x))/(b + c))/(exp(4*x)*(b + c)^2 + (b - c)^2 + 2*exp(2*x)*
(b + c)*(b - c))
```

3.730 $\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)} dx$

3.730.1 Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)} dx = \frac{1}{2}(\cosh(x)+\sinh(x))^2$$

output
```
1/2*(cosh(x)+sinh(x))^2
```

3.730.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)} dx = \frac{1}{2}\cosh(2x)+\frac{1}{2}\sinh(2x)$$

input
```
Integrate[(Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

output
```
Cosh[2*x]/2 + Sinh[2*x]/2
```

3.730.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 4885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)+\cosh(x)}{\cosh(x)-\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)-i\sin(ix)}{i\sin(ix)+\cos(ix)}dx$$

$$\Big\downarrow \text{4885}$$

$$\frac{1}{2}(\sinh(x)+\cosh(x))^2$$

input
```
Int[(Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

output
```
(Cosh[x] + Sinh[x])^2/2
```

3.730.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4885
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

3.730.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{e^{2x}}{2}$	7
parallelrisch	$-\frac{1}{\tanh(x)-1}$	9
gosper	$-\frac{\sinh(x)+\cosh(x)}{2(\sinh(x)-\cosh(x))}$	17
default	$\frac{2}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{2}{\tanh\left(\frac{x}{2}\right)-1}$	22

input
```
int((sinh(x)+cosh(x))/(cosh(x)-sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*x)
```

3.730.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=\frac{\cosh\left(x\right)+\sinh\left(x\right)}{2\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

input
```
integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="fricas")
```

output
```
1/2*(cosh(x) + sinh(x))/(cosh(x) - sinh(x))
```

3.730.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=\frac{\cosh\left(x\right)}{-\sinh\left(x\right)+\cosh\left(x\right)}$$

input
```
integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
```

output
```
cosh(x)/(-sinh(x) + cosh(x))
```

3.730.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=\frac{1}{2}\,e^{(2\,x)}$$

input
```
integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="maxima")
```

output
```
1/2*e^(2*x)
```

3.730.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=\frac{1}{2}\,e^{(2\,x)}$$

input
```
integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="giac")
```

output
```
1/2*e^(2*x)
```

3.730.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=\frac{\mathrm{e}^{2\,x}}{2}$$

input
```
int((cosh(x) + sinh(x))/(cosh(x) - sinh(x)),x)
```

output
```
exp(2*x)/2
```

3.731 $\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)}\,dx$

3.731.1 Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)}\,dx = -\frac{1}{2(\cosh(x)+\sinh(x))^2}$$

output
```
-1/2/(cosh(x)+sinh(x))^2
```

3.731.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)}\,dx = -\frac{1}{2}\cosh(2x)+\frac{1}{2}\sinh(2x)$$

input
```
Integrate[(Cosh[x] - Sinh[x])/(Cosh[x] + Sinh[x]),x]
```

output
```
-1/2*Cosh[2*x] + Sinh[2*x]/2
```

3.731.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 4885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)-\sinh(x)}{\sinh(x)+\cosh(x)} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sin(ix)+\cos(ix)}{\cos(ix)-i\sin(ix)}dx$$
$$\Big\downarrow \text{4885}$$
$$-\frac{1}{2(\sinh(x)+\cosh(x))^2}$$

input
```
Int[(Cosh[x] - Sinh[x])/(Cosh[x] + Sinh[x]),x]
```

output
```
-1/2*1/(Cosh[x] + Sinh[x])^2
```

3.731.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4885
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]
```

3.731.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{{\mathrm e}^{-2 x}}{2}$	7
parallelrisch	$-\frac{1}{1+\tanh \left(x \right)}$	9
gosper	$\frac{\sinh \left(x \right)-\cosh \left(x \right)}{2 \sinh \left(x \right)+2 \cosh \left(x \right)}$	17
default	$\frac{2}{\tanh \left(\frac{x}{2}\right)+1}-\frac{2}{\left(\tanh \left(\frac{x}{2}\right)+1\right)^{2}}$	22

input
```
int((cosh(x)-sinh(x))/(sinh(x)+cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-2*x)
```

3.731.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)}\, dx = -\frac{1}{2\,\left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2}\right)}$$

input
```
integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x, algorithm="fricas")
```

output
```
-1/2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

3.731.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)}\, dx = -\frac{\cosh\left(x\right)}{\sinh\left(x\right) + \cosh\left(x\right)}$$

input
```
integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x)
```

output
```
-cosh(x)/(sinh(x) + cosh(x))
```

3.731.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(x) - \sinh(x)}{\cosh(x) + \sinh(x)} \, dx = -\frac{1}{2} \, e^{(-2 \, x)}$$

input
```
integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x, algorithm="maxima")
```

output
```
-1/2*e^(-2*x)
```

3.731.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(x) - \sinh(x)}{\cosh(x) + \sinh(x)} \, dx = -\frac{1}{2} \, e^{(-2 \, x)}$$

input
```
integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x, algorithm="giac")
```

output
```
-1/2*e^(-2*x)
```

3.731.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(x) - \sinh(x)}{\cosh(x) + \sinh(x)} \, dx = -\frac{\mathrm{e}^{-2\,x}}{2}$$

input
```
int((cosh(x) - sinh(x))/(cosh(x) + sinh(x)),x)
```

output
```
-exp(-2*x)/2
```

3.732 $\int \frac{\cosh(x)-i\sinh(x)}{\cosh(x)+i\sinh(x)}\,dx$

3.732.1 Optimal result

Integrand size = 21, antiderivative size = 14

$$\int \frac{\cosh(x)-i\sinh(x)}{\cosh(x)+i\sinh(x)}\,dx = -i\log(\cosh(x)+i\sinh(x))$$

output
```
-I*ln(cosh(x)+I*sinh(x))
```

3.732.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{\cosh(x)-i\sinh(x)}{\cosh(x)+i\sinh(x)}\,dx = \arctan(\tanh(x))-\frac{1}{2}i\log(\cosh(2x))$$

input
```
Integrate[(Cosh[x] - I*Sinh[x])/(Cosh[x] + I*Sinh[x]),x]
```

output
```
ArcTan[Tanh[x]] - (I/2)*Log[Cosh[2*x]]
```

3.732.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x) - i\sinh(x)}{\cosh(x) + i\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix) - \sin(ix)}{\sin(ix) + \cos(ix)}dx$$

$$\Big\downarrow \text{3612}$$

$$-i\log(\cosh(x) + i\sinh(x))$$

input
```
Int[(Cosh[x] - I*Sinh[x])/(Cosh[x] + I*Sinh[x]),x]
```

output
```
(-I)*Log[Cosh[x] + I*Sinh[x]]
```

3.732.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.732.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

method	result	size
risch	$ix - i\ln\left(e^{2x} - i\right)$	17
parallelrisch	$\frac{i(\ln(1-\tanh(x))+\ln(1+\tanh(x))-2\ln(\tanh(x)-i))}{2}$	25
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-i\ln\left(2i\tanh\left(\frac{x}{2}\right)+\tanh\left(\frac{x}{2}\right)^{2}+1\right)$	41

input
```
int((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
I*x-I*ln(exp(2*x)-I)
```

3.732.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\cosh(x) - i\sinh(x)}{\cosh(x) + i\sinh(x)}\,dx = i\,x - i\,\log\left(e^{(2\,x)} - i\right)$$

input
```
integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x, algorithm="fricas")
```

output
```
I*x - I*log(e^(2*x) - I)
```

3.732.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\cosh(x) - i\sinh(x)}{\cosh(x) + i\sinh(x)}\,dx = ix - i\log\left(e^{2x} - i\right)$$

input
```
integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x)
```

output
```
I*x - I*log(exp(2*x) - I)
```

3.732.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\cosh(x) - i\sinh(x)}{\cosh(x) + i\sinh(x)}\,dx = -i\,\log\left(\cosh\left(x\right) + i\,\sinh\left(x\right)\right)$$

input
```
integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x, algorithm="maxima")
```

output
```
-I*log(cosh(x) + I*sinh(x))
```

3.732.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\cosh(x) - i\sinh(x)}{\cosh(x) + i\sinh(x)}\,dx = i\,x - i\,\log\left(e^{(2\,x)} - i\right)$$

input
```
integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x, algorithm="giac")
```

output
```
I*x - I*log(e^(2*x) - I)
```

3.732.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(x) - i\sinh(x)}{\cosh(x) + i\sinh(x)}\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^{2\,x} - \mathrm{i}\right)\,1\mathrm{i}$$

input
```
int((cosh(x) - sinh(x)*1i)/(cosh(x) + sinh(x)*1i),x)
```

output
```
x*1i - log(exp(2*x) - 1i)*1i
```

3.733 $\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

3.733.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{(bB-cC)x}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

output
```
(B*b-C*c)*x/(b^2-c^2)-(B*c-C*b)*ln(b*cosh(x)+c*sinh(x))/(b^2-c^2)
```

3.733.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{(bB-cC)x+(-Bc+bC)\log(b\cosh(x)+c\sinh(x))}{(b-c)(b+c)}$$

input
```
Integrate[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

output
```
((b*B - c*C)*x + (-(B*c) + b*C)*Log[b*Cosh[x] + c*Sinh[x]])/((b - c)*(b +
c))
```

3.733.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(ix)-iC\sin(ix)}{b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{3612}$$

$$\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

input
```
Int[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

output
```
((b*B - c*C)*x)/(b^2 - c^2) - ((B*c - b*C)*Log[b*Cosh[x] + c*Sinh[x]])/(b^
2 - c^2)
```

3.733.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
 + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
 d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.733.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

method	result	size
parallelrisch	$\frac{(-Bc+bC)\ln(c\tanh(x)+b)+(Bc-bC)\ln(1-\tanh(x))+x(b+c)(B-C)}{b^2-c^2}$	56
default	$\frac{2(-B-C)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{2(B-C)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}+\frac{2\left(-\frac{Bc}{2}+\frac{bC}{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)^2b+2c\tanh\left(\frac{x}{2}\right)+b\right)}{(b-c)(b+c)}$	91
risch	$\frac{Bx}{b+c}+\frac{Cx}{b+c}+\frac{2xBc}{b^2-c^2}-\frac{2xbC}{b^2-c^2}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{b-c}{b+c}\right)Bc}{b^2-c^2}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{b-c}{b+c}\right)bC}{b^2-c^2}$	113

input
```
int((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
((-B*c+C*b)*ln(c*tanh(x)+b)+(B*c-C*b)*ln(1-tanh(x))+x*(b+c)*(B-C))/(b^2-c^
2)
```

3.733.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{((B-C)b+(B-C)c)x+(Cb-Bc)\log\left(\frac{2\,(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{b^2-c^2}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas
")
```

output
```
(((B - C)*b + (B - C)*c)*x + (C*b - B*c)*log(2*(b*cosh(x) + c*sinh(x))/(co
sh(x) - sinh(x))))/(b^2 - c^2)
```

3.733.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(39) = 78$.

Time = 0.34 (sec) , antiderivative size = 326, normalized size of antiderivative = 6.15

$$\int \frac{B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}(B\log(\sinh(x)) + Cx) \\ \frac{B\log(\sinh(x))+Cx}{c} \\ \frac{Bx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{Bx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{B\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{Cx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)} + \frac{Cx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} \\ \frac{Bx\sinh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{Bx\cosh(x)}{2c\sinh(x)+2c\cosh(x)} - \frac{B\cosh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{Cx\sinh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{Cx\cosh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{}{2c\sin} \\ \frac{Bbx}{b^2-c^2} - \frac{Bc\log\left(\cosh(x)+\frac{c\sinh(x)}{b}\right)}{b^2-c^2} + \frac{Cb\log\left(\cosh(x)+\frac{c\sinh(x)}{b}\right)}{b^2-c^2} - \frac{Ccx}{b^2-c^2} \end{cases}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

output
```
Piecewise((zoo*(B*log(sinh(x)) + C*x), Eq(b, 0) & Eq(c, 0)), ((B*log(sinh(
x)) + C*x)/c, Eq(b, 0)), (B*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*x*c
osh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x
)) - C*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(-2*c*sinh(x)
+ 2*c*cosh(x)) - C*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (B*x*
sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + B*x*cosh(x)/(2*c*sinh(x) + 2*c*cosh(
x)) - B*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*sinh(x)/(2*c*sinh(x) + 2
*c*cosh(x)) + C*x*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*cosh(x)/(2*c*sin
h(x) + 2*c*cosh(x)), Eq(b, c)), (B*b*x/(b**2 - c**2) - B*c*log(cosh(x) + c
*sinh(x)/b)/(b**2 - c**2) + C*b*log(cosh(x) + c*sinh(x)/b)/(b**2 - c**2) -
 C*c*x/(b**2 - c**2), True))
```

3.733.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.64

$$\int \frac{B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = C\left(\frac{b\log\left(-(b-c)e^{(-2x)} - b - c\right)}{b^2 - c^2} + \frac{x}{b+c}\right) - B\left(\frac{c\log\left(-(b-c)e^{(-2x)} - b - c\right)}{b^2 - c^2} - \frac{x}{b+c}\right)$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxima
")
```

output
```
C*(b*log(-(b - c)*e^(-2*x) - b - c)/(b^2 - c^2) + x/(b + c)) - B*(c*log(-(
b - c)*e^(-2*x) - b - c)/(b^2 - c^2) - x/(b + c))
```

3.733.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{(B-C)x}{b-c}+\frac{(Cb-Bc)\log\left(\left|be^{(2x)}+ce^{(2x)}+b-c\right|\right)}{b^2-c^2}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

output
```
(B - C)*x/(b - c) + (C*b - B*c)*log(abs(b*e^(2*x) + c*e^(2*x) + b - c))/(b
^2 - c^2)
```

3.733.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{x\,(B\,b-C\,c)}{b^2-c^2}-\frac{\ln\left(b\cosh(x)+c\sinh(x)\right)\,(B\,c-C\,b)}{b^2-c^2}$$

input
```
int((B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x)),x)
```

output
```
(x*(B*b - C*c))/(b^2 - c^2) - (log(b*cosh(x) + c*sinh(x))*(B*c - C*b))/(b^
2 - c^2)
```

3.734 $\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

3.734.1 Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx=\frac{(bB-cC)\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}+\frac{Bc-bC}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}$$

output
```
(B*b-C*c)*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+(B
*c-C*b)/(b^2-c^2)/(b*cosh(x)+c*sinh(x))
```

3.734.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx=\frac{2(bB-cC)\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}+\frac{Bc-bC}{(b-c)(b+c)(b\cosh(x)+c\sinh(x))}$$

input
```
Integrate[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(2*(b*B - c*C)*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b -
c)^(3/2)*(b + c)^(3/2)) + (B*c - b*C)/((b - c)*(b + c)*(b*Cosh[x] + c*Sinh
[x]))
```

3.734.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3632, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(ix)-iC\sin(ix)}{(b\cos(ix)-ic\sin(ix))^2}dx$$

$$\Big\downarrow \text{3632}$$

$$\frac{(bB-cC)\int \frac{1}{b\cosh(x)+c\sinh(x)}dx}{b^2-c^2}+\frac{Bc-bC}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Bc-bC}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\int \frac{1}{b\cos(ix)-ic\sin(ix)}dx}{b^2-c^2}$$

$$\Big\downarrow \text{3553}$$

$$\frac{Bc-bC}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{i(bB-cC)\int \frac{1}{b^2-c^2-(-ic\cosh(x)-ib\sinh(x))^2}d(-ic\cosh(x)-ib\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{Bc-bC}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{i(bB-cC)\text{arctanh}\left(\frac{-ib\sinh(x)-ic\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

input
```
Int[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(I*(b*B - c*C)*ArcTanh[((-I)*c*Cosh[x] - I*b*Sinh[x])/Sqrt[b^2 - c^2]])/(b
^2 - c^2)^(3/2) + (B*c - b*C)/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

3.734.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.734.4 Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.51

method	result
default	$\frac{\frac{2c(Bc-bC)\tanh\left(\frac{x}{2}\right)}{\left(b^{2}-c^{2}\right)b}+\frac{2(Bc-bC)}{b^{2}-c^{2}}}{\tanh\left(\frac{x}{2}\right)^{2}b+2c\tanh\left(\frac{x}{2}\right)+b}+\frac{2(Bb-Cc)\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{2\,e^{x}(Bc-bC)}{(b-c)(b+c)(b\,e^{2x}+e^{2x}c+b-c)}-\frac{bB\ln\left(e^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}+\frac{cC\ln\left(e^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}+\frac{bB\ln\left(e^{x}+\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}-\frac{cC\ln\left(e^{x}+\frac{b}{\sqrt{-}}\right.}{\sqrt{-b^{2}+c^{2}}\,(b+c}$

input
```
int((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(c*(B*c-C*b)/(b^2-c^2)/b*tanh(1/2*x)+(B*c-C*b)/(b^2-c^2))/(tanh(1/2*x)^2
*b+2*c*tanh(1/2*x)+b)+2*(B*b-C*c)/(b^2-c^2)^(3/2)*arctan(1/2*(2*b*tanh(1/2
*x)+2*c)/(b^2-c^2)^(1/2))
```

3.734.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(75) = 150$.

Time = 0.29 (sec) , antiderivative size = 749, normalized size of antiderivative = 9.60

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx$$
$$= \Bigg[-\frac{\left(Bb^2 - (B+C)bc + Cc^2 + (Bb^2 + (B-C)bc - Cc^2)\cosh{(x)}^2 + 2\,(Bb^2 + (B-C)bc - Cc^2)\cosh{(x}}{b^5 - b^4c - 2\,b^3c^2 + 2\,b^2c^3 + bc^4 - c^5 + (}$$
$$-\frac{2\left(\left(Bb^2 - (B+C)bc + Cc^2 + (Bb^2 + (B-C)bc - Cc^2)\cosh{(x)}^2 + 2\,(Bb^2 + (B-C)bc - Cc^2)\cosh\right.\right.}{b^5 - b^4c - 2\,b^3c^2 + 2\,b^2c^3 + bc^4 - c^5 + (b^5 + b^4c - 2\,b^3c^2 - 2\,b^2}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fric
as")
```

output
```
[-((B*b^2 - (B + C)*b*c + C*c^2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2
+ 2*(B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c -
 C*c^2)*sinh(x)^2)*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cos
h(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x))
- b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^
2 + b - c)) + 2*(C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*cosh(x) + 2*(C*b^3 - B
*b^2*c - C*b*c^2 + B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 +
b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^
2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x)
+ (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2), -2*((B*b
^2 - (B + C)*b*c + C*c^2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 + 2*(B*
b^2 + (B - C)*b*c - C*c^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C*c^2)
*sinh(x)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b +
 c)*sinh(x))) + (C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*cosh(x) + (C*b^3 - B*b
^2*c - C*b*c^2 + B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*
c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2
+ 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) +
(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2)]
```

3.734.6 Sympy [F(-1)]

Timed out.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

output
```
Timed out
```

3.734.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` f
or more de
```

3.734.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \frac{2\,(Bb - Cc)\arctan\left(\frac{be^x + ce^x}{\sqrt{b^2 - c^2}}\right)}{(b^2 - c^2)^{\frac{3}{2}}} - \frac{2\,(Cbe^x - Bce^x)}{(b^2 - c^2)(be^{(2\,x)} + ce^{(2\,x)} + b - c)}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="giac
")
```

output
```
2*(B*b - C*c)*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) -
2*(C*b*e^x - B*c*e^x)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b - c))
```

3.734.9 Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.55

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \frac{\ln\left(\frac{2\,(B\,b-C\,c)}{(b+c)^{5/2}\,\sqrt{c-b}} + \frac{2\,e^x\,(B\,b-C\,c)}{-b^3-b^2\,c+b\,c^2+c^3}\right)(B\,b-C\,c)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{\ln\left(\frac{2\,e^x\,(B\,b-C\,c)}{-b^3-b^2\,c+b\,c^2+c^3} - \frac{2\,(B\,b-C\,c)}{(b+c)^{5/2}\,\sqrt{c-b}}\right)(B\,b-C\,c)}{(b+c)^{3/2}\,(c-b)^{3/2}} + \frac{2\,e^x\,(B\,c-C\,b)}{(b+c)\,(b-c)\,(b-c+e^{2\,x}\,(b+c))}$$

input
```
int((B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x))^2,x)
```

output
```
(log((2*(B*b - C*c))/((b + c)^(5/2)*(c - b)^(1/2)) + (2*exp(x)*(B*b - C*c)
)/(b*c^2 - b^2*c - b^3 + c^3))*(B*b - C*c))/((b + c)^(3/2)*(c - b)^(3/2))
- (log((2*exp(x)*(B*b - C*c))/(b*c^2 - b^2*c - b^3 + c^3) - (2*(B*b - C*c)
)/((b + c)^(5/2)*(c - b)^(1/2)))*(B*b - C*c))/((b + c)^(3/2)*(c - b)^(3/2)
) + (2*exp(x)*(B*c - C*b))/((b + c)*(b - c)*(b - c + exp(2*x)*(b + c)))
```

3.735 $\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

3.735.1 Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{Bc-bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} + \frac{(bB-cC)\sinh(x)}{b(b^2-c^2)(b\cosh(x)+c\sinh(x))}$$

output
```
1/2*(B*c-C*b)/(b^2-c^2)/(b*cosh(x)+c*sinh(x))^2+(B*b-C*c)*sinh(x)/b/(b^2-c
^2)/(b*cosh(x)+c*sinh(x))
```

3.735.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{(-b^2+c^2)\,C+c(bB-cC)\cosh(2x)+b(bB-cC)\sinh(2x)}{2b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2}$$

input
```
Integrate[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
((-b^2 + c^2)*C + c*(b*B - c*C)*Cosh[2*x] + b*(b*B - c*C)*Sinh[2*x])/(2*b*
(b - c)*(b + c)*(b*Cosh[x] + c*Sinh[x])^2)
```

3.735.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3635, 27, 3042, 3554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(ix)-iC\sin(ix)}{(b\cos(ix)-ic\sin(ix))^3}dx$$

$$\Big\downarrow \text{3635}$$

$$\frac{\int \frac{2(bB-cC)}{(b\cosh(x)+c\sinh(x))^2}dx}{2(b^2-c^2)}+\frac{Bc-bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(bB-cC)\int \frac{1}{(b\cosh(x)+c\sinh(x))^2}dx}{b^2-c^2}+\frac{Bc-bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Bc-bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}+\frac{(bB-cC)\int \frac{1}{(b\cos(ix)-ic\sin(ix))^2}dx}{b^2-c^2}$$

$$\Big\downarrow \text{3554}$$

$$\frac{Bc-bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}+\frac{\sinh(x)(bB-cC)}{b(b^2-c^2)(b\cosh(x)+c\sinh(x))}$$

input
```
Int[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
(B*c - b*C)/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2) + ((b*B - c*C)*Sinh[
x])/(b*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

3.735.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.735.4 Maple [A] (verified)

Time = 14.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2\left(-\frac{B\tanh\left(\frac{x}{2}\right)^{3}}{b}-\frac{(Bc+bC)\tanh\left(\frac{x}{2}\right)^{2}}{b^{2}}-\frac{B\tanh\left(\frac{x}{2}\right)}{b}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}b+2c\tanh\left(\frac{x}{2}\right)+b\right)^{2}}$	63
risch	$-\frac{2\left(Bb\,\mathrm{e}^{2x}+\mathrm{e}^{2x}Bc+\mathrm{e}^{2x}bC+Cc\,\mathrm{e}^{2x}+Bb-Cc\right)}{\left(b\,\mathrm{e}^{2x}+\mathrm{e}^{2x}c+b-c\right)^{2}(b+c)^{2}}$	63

input
```
int((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(-B/b*tanh(1/2*x)^3-(B*c+C*b)/b^2*tanh(1/2*x)^2-B/b*tanh(1/2*x))/(tanh(
1/2*x)^2*b+2*c*tanh(1/2*x)+b)^2
```

3.735.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(69) = 138$.

Time = 0.25 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.27

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx =$$

$$-\frac{}{(b^4 + 4\,b^3c + 6\,b^2c^2 + 4\,bc^3 + c^4)\cosh\left(x\right)^3 + 3\,(b^4 + 4\,b^3c + 6\,b^2c^2 + 4\,bc^3 + c^4)\cosh\left(x\right)\sinh\left(x\right)^2 + (b^4 -}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fric
as")
```

output
```
-2*(((2*B + C)*b + B*c)*cosh(x) + (C*b + (B + 2*C)*c)*sinh(x))/((b^4 + 4*b
^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 + 3*(b^4 + 4*b^3*c + 6*b^2*c^2
 + 4*b*c^3 + c^4)*cosh(x)*sinh(x)^2 + (b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3
 + c^4)*sinh(x)^3 + (3*b^4 + 4*b^3*c - 2*b^2*c^2 - 4*b*c^3 - c^4)*cosh(x)
+ (b^4 + 4*b^3*c + 2*b^2*c^2 - 4*b*c^3 - 3*c^4 + 3*(b^4 + 4*b^3*c + 6*b^2*
c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x))
```

3.735.6 Sympy [F(-1)]

Timed out.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

output
```
Timed out
```

3.735.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(69) = 138$.

Time = 0.22 (sec) , antiderivative size = 337, normalized size of antiderivative = 4.75

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx$$

$$= 2B\left(\frac{(b-c)e^{(-2x)}}{b^4 - 2b^2c^2 + c^4 + 2(b^4 - 2b^3c + 2bc^3 - c^4)e^{(-2x)} + (b^4 - 4b^3c + 6b^2c^2 - 4bc^3 + c^4)e^{(-4x)}} + \frac{}{b^4 - 2}\right.$$

$$-2C\left(\frac{(b-c)e^{(-2x)}}{b^4 - 2b^2c^2 + c^4 + 2(b^4 - 2b^3c + 2bc^3 - c^4)e^{(-2x)} + (b^4 - 4b^3c + 6b^2c^2 - 4bc^3 + c^4)e^{(-4x)}} + \frac{}{b^4 -}\right.$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="maxi
ma")
```

output
```
2*B*((b - c)*e^(-2*x)/(b^4 - 2*b^2*c^2 + c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3
- c^4)*e^(-2*x) + (b^4 - 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*e^(-4*x)) +
b/(b^4 - 2*b^2*c^2 + c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(-2*x) + (b
^4 - 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*e^(-4*x))) - 2*C*((b - c)*e^(-2*
x)/(b^4 - 2*b^2*c^2 + c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(-2*x) + (
b^4 - 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*e^(-4*x)) + c/(b^4 - 2*b^2*c^2
+ c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(-2*x) + (b^4 - 4*b^3*c + 6*b^
2*c^2 - 4*b*c^3 + c^4)*e^(-4*x)))
```

3.735.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = -\frac{2\left(Bbe^{(2x)} + Cbe^{(2x)} + Bce^{(2x)} + Cce^{(2x)} + Bb - Cc\right)}{(b^2 + 2bc + c^2)\left(be^{(2x)} + ce^{(2x)} + b - c\right)^2}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="giac
")
```

output
```
-2*(B*b*e^(2*x) + C*b*e^(2*x) + B*c*e^(2*x) + C*c*e^(2*x) + B*b - C*c)/((b
^2 + 2*b*c + c^2)*(b*e^(2*x) + c*e^(2*x) + b - c)^2)
```

3.735.9 Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{b\,(2\,B+2\,B\,e^{2\,x}+2\,C\,e^{2\,x})+c\,(2\,B\,e^{2\,x}-2\,C+2\,C\,e^{2\,x})}{(b+c)^2\,(b-c+b\,e^{2\,x}+c\,e^{2\,x})^2}$$

input
```
int((B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x))^3,x)
```

output
```
-(b*(2*B + 2*B*exp(2*x) + 2*C*exp(2*x)) + c*(2*B*exp(2*x) - 2*C + 2*C*exp(
2*x)))/((b + c)^2*(b - c + b*exp(2*x) + c*exp(2*x))^2)
```

3.736 $\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

3.736.1 Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x}{b^2-c^2}+\frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}} - \frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

output
```
(B*b-C*c)*x/(b^2-c^2)-(B*c-C*b)*ln(b*cosh(x)+c*sinh(x))/(b^2-c^2)+A*arctan
((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)
```

3.736.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x+2A\sqrt{b-c}\sqrt{b+c}\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)+(-Bc+bC)\log(b\cosh(x)+c\sinh(x))}{(b-c)(b+c)}$$

input
```
Integrate[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

output
```
((b*B - c*C)*x + 2*A*Sqrt[b - c]*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(Sqr
t[b - c]*Sqrt[b + c])] + (-(B*c) + b*C)*Log[b*Cosh[x] + c*Sinh[x]])/((b -
c)*(b + c))
```

3.736.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 3615, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)-iC\sin(ix)}{b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{3615}$$

$$A\int \frac{1}{b\cosh(x)+c\sinh(x)}dx+\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{3042}$$

$$A\int \frac{1}{b\cos(ix)-ic\sin(ix)}dx+\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{3553}$$

$$iA\int \frac{1}{b^2-c^2-(-ic\cosh(x)-ib\sinh(x))^2}d(-ic\cosh(x)-ib\sinh(x))+\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{iA\text{arctanh}\left(\frac{-ib\sinh(x)-ic\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

input
```
Int[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

output
```
((b*B - c*C)*x)/(b^2 - c^2) + (I*A*ArcTanh[((-I)*c*Cosh[x] - I*b*Sinh[x])/
Sqrt[b^2 - c^2]])/Sqrt[b^2 - c^2] - ((B*c - b*C)*Log[b*Cosh[x] + c*Sinh[x]
])/(b^2 - c^2)
```

3.736.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3615
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + (Simp[(c*B - b*C)*(Log[a
 + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c
^2) - a*(b*B + c*C))/(b^2 + c^2)   Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*
x]), x], x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] &&
NeQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

3.736.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.90

method	result
default	$\frac{2(-B-C)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{2(B-C)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}+\frac{\frac{\left(-bBc+C\,b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}b+2c\tanh\left(\frac{x}{2}\right)+b\right)}{b}+\frac{2\left(b^{2}A-A\,c^{2}-B\,c^{2}+Ccb-\left(\right.\right.}{}}{(b-c)(b+c)}$
risch	$\frac{Bx}{b+c}+\frac{Cx}{b+c}+\frac{2xB\,b^{2}c}{b^{4}-2b^{2}c^{2}+c^{4}}-\frac{2xB\,c^{3}}{b^{4}-2b^{2}c^{2}+c^{4}}-\frac{2xC\,b^{3}}{b^{4}-2b^{2}c^{2}+c^{4}}+\frac{2xCb\,c^{2}}{b^{4}-2b^{2}c^{2}+c^{4}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{-A^{2}b^{2}+A^{2}c^{2}}}{A(b+c)}\right)Bc}{(b+c)(b-c)}+\frac{\ln\left(\mathrm{e}^{x}+\right.}{(b}$

input
```
int((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*(-B-C)/(2*b+2*c)*ln(tanh(1/2*x)-1)+2*(B-C)/(2*b-2*c)*ln(tanh(1/2*x)+1)+2
/(b-c)/(b+c)*(1/2*(-B*b*c+C*b^2)/b*ln(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+(
b^2*A-A*c^2-B*c^2+C*c*b-(-B*b*c+C*b^2)*c/b)/(b^2-c^2)^(1/2)*arctan(1/2*(2*
b*tanh(1/2*x)+2*c)/(b^2-c^2)^(1/2)))
```

3.736.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.87

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$$
$$= \left[-\frac{\sqrt{-b^2+c^2}A\log\left(\frac{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\,\sqrt{-b^2+c^2}(\cosh(x)+\sinh(x))-b+c}{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+b-c}\right)-((B-C)b}{b^2-c^2}\right.$$
$$-\frac{2\sqrt{b^2-c^2}A\arctan\left(\frac{\sqrt{b^2-c^2}}{(b+c)\cosh(x)+(b+c)\sinh(x)}\right)-((B-C)b+(B-C)c)x-(Cb-Bc)\log\left(\frac{2\,(b\cosh(x)+c\,s}{\cosh(x)-\sin}\right.}{b^2-c^2}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fric
as")
```

output
```
[-(sqrt(-b^2 + c^2)*A*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) +
 (b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b +
 c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) -
((B - C)*b + (B - C)*c)*x - (C*b - B*c)*log(2*(b*cosh(x) + c*sinh(x))/(cos
h(x) - sinh(x))))/(b^2 - c^2), -(2*sqrt(b^2 - c^2)*A*arctan(sqrt(b^2 - c^2
)/((b + c)*cosh(x) + (b + c)*sinh(x))) - ((B - C)*b + (B - C)*c)*x - (C*b
- B*c)*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))))/(b^2 - c^2)]
```

3.736.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. 2(73) = 146.

Time = 30.67 (sec) , antiderivative size = 643, normalized size of antiderivative = 6.99

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right) + Bx - 2B\log\left(\tanh\left(\frac{x}{2}\right) + 1\right) + B\log\left(\tanh\left(\frac{x}{2}\right)\right) + Cx\right) \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right) + Bx - 2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right) + B\log\left(\tanh\left(\frac{x}{2}\right)\right) + Cx}{c} \\ -\frac{2A}{-2c\sinh(x)+2c\cosh(x)} + \frac{Bx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{Bx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{B\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{Cx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)} \\ -\frac{2A}{2c\sinh(x)+2c\cosh(x)} + \frac{Bx\sinh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{Bx\cosh(x)}{2c\sinh(x)+2c\cosh(x)} - \frac{B\cosh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{Cx\sinh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{}{2c} \\ -\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2} + \frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2} + \frac{Bbx}{b^2-c^2} - \frac{Bcx}{b^2-c^2} + \frac{2Bc\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2-c^2} \end{cases}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

output
```
Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(ta
nh(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tanh(x/2)) + B*x - 2*B*log(
tanh(x/2) + 1) + B*log(tanh(x/2)) + C*x)/c, Eq(b, 0)), (-2*A/(-2*c*sinh(x)
 + 2*c*cosh(x)) + B*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*x*cosh(x)/(
-2*c*sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - C*x
*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(-2*c*sinh(x) + 2*c*co
sh(x)) - C*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (-2*A/(2*c*si
nh(x) + 2*c*cosh(x)) + B*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + B*x*cosh(
x)/(2*c*sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C
*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(2*c*sinh(x) + 2*c*co
sh(x)) + C*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)), Eq(b, c)), (-A*sqrt(-b**2
+ c**2)*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + A*sqrt
(-b**2 + c**2)*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2 - c**2) +
 B*b*x/(b**2 - c**2) - B*c*x/(b**2 - c**2) + 2*B*c*log(tanh(x/2) + 1)/(b**
2 - c**2) - B*c*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2)
- B*c*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + C*b*x/(b
**2 - c**2) - 2*C*b*log(tanh(x/2) + 1)/(b**2 - c**2) + C*b*log(tanh(x/2) +
 c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + C*b*log(tanh(x/2) + c/b + sqr
t(-b**2 + c**2)/b)/(b**2 - c**2) - C*c*x/(b**2 - c**2), True))
```

3.736. $\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

3.736.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` f
or more de
```

3.736.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = \frac{2\,A\arctan\left(\frac{be^{x}+ce^{x}}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}} + \frac{(B-C)x}{b-c} + \frac{(Cb-Bc)\log\left(be^{(2\,x)} + ce^{(2\,x)} + b - c\right)}{b^2-c^2}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac
")
```

output
```
2*A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/sqrt(b^2 - c^2) + (B - C)*x/(b
 - c) + (C*b - B*c)*log(b*e^(2*x) + c*e^(2*x) + b - c)/(b^2 - c^2)
```

3.736.9 Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.28

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{A\,\mathrm{e}^x\,\sqrt{b^2-c^2}}{b\,\sqrt{A^2}-c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^2-c^2}}+\frac{B\,x}{b-c}-\frac{C\,x}{b-c} + \frac{B\,c^3\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,\mathrm{e}^{2\,x}+4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4} + \frac{C\,b^3\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,\mathrm{e}^{2\,x}+4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4} - \frac{B\,b^2\,c\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,\mathrm{e}^{2\,x}+4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4} - \frac{C\,b\,c^2\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,\mathrm{e}^{2\,x}+4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4}$$

input
```
int((A + B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x)),x)
```

output
```
(2*atan((A*exp(x)*(b^2 - c^2)^(1/2))/(b*(A^2)^(1/2) - c*(A^2)^(1/2)))*(A^2
)^(1/2))/(b^2 - c^2)^(1/2) + (B*x)/(b - c) - (C*x)/(b - c) + (B*c^3*log(4*
A^2*b - 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2
*c^2) + (C*b^3*log(4*A^2*b - 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)
))/(b^4 + c^4 - 2*b^2*c^2) - (B*b^2*c*log(4*A^2*b - 4*A^2*c + 4*A^2*b*exp(
2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^2) - (C*b*c^2*log(4*A^2*b -
 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^2)
```

3.737 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

3.737.1 Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx=\frac{(bB-cC)\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}+\frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}$$

output
```
(B*b-C*c)*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+(B
*c-b*C+A*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))
```

3.737.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx=\frac{2(bB-cC)\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}+\frac{b(Bc-bC)+A(b^2-c^2)\sinh(x)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}$$

input
```
Integrate[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(2*(b*B - c*C)*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b -
c)^(3/2)*(b + c)^(3/2)) + (b*(B*c - b*C) + A*(b^2 - c^2)*Sinh[x])/(b*(b -
c)*(b + c)*(b*Cosh[x] + c*Sinh[x]))
```

3.737.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 3632, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)-iC\sin(ix)}{(b\cos(ix)-ic\sin(ix))^2}dx$$

$$\Big\downarrow \text{3632}$$

$$\frac{(bB-cC)\int \frac{1}{b\cosh(x)+c\sinh(x)}dx}{b^2-c^2}+\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\int \frac{1}{b\cos(ix)-ic\sin(ix)}dx}{b^2-c^2}$$

$$\Big\downarrow \text{3553}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{i(bB-cC)\int \frac{1}{b^2-c^2-(-ic\cosh(x)-ib\sinh(x))^2}d(-ic\cosh(x)-ib\sinh(x))}{b^2-c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{i(bB-cC)\text{arctanh}\left(\frac{-ib\sinh(x)-ic\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

input
```
Int[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(I*(b*B - c*C)*ArcTanh[((-I)*c*Cosh[x] - I*b*Sinh[x])/Sqrt[b^2 - c^2]])/(b
^2 - c^2)^(3/2) + (B*c - b*C + A*c*Cosh[x] + A*b*Sinh[x])/((b^2 - c^2)*(b*
Cosh[x] + c*Sinh[x]))
```

3.737.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.737.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.51

method	result
default	$-\frac{2\left(-\frac{\left(b^{2} A-A\,c^{2}+B\,c^{2}-C c b\right)\tanh\left(\frac{x}{2}\right)}{b\left(b^{2}-c^{2}\right)}-\frac{B c-b C}{b^{2}-c^{2}}\right)}{\tanh\left(\frac{x}{2}\right)^{2} b+2 c\tanh\left(\frac{x}{2}\right)+b}+\frac{2(B b-C c)\arctan\left(\frac{2 b\tanh\left(\frac{x}{2}\right)+2 c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$
risch	$-\frac{2(-B c\,\mathrm{e}^{x}+C b\,\mathrm{e}^{x}+b A-A c)}{(b-c)(b+c)(b\,\mathrm{e}^{2x}+\mathrm{e}^{2x} c+b-c)}-\frac{b B\ln\left(\mathrm{e}^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}+\frac{c C\ln\left(\mathrm{e}^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}+\frac{b B\ln\left(\mathrm{e}^{x}+\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}-\frac{c C\ln\left(\mathrm{e}^{x}+ \right.}{\sqrt{-b^{2}+c^{2}}\,(b}$

input
```
int((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOS
E)
```

output
```
-2*(-(A*b^2-A*c^2+B*c^2-C*b*c)/b/(b^2-c^2)*tanh(1/2*x)-(B*c-C*b)/(b^2-c^2)
)/(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+2*(B*b-C*c)/(b^2-c^2)^(3/2)*arctan(1
/2*(2*b*tanh(1/2*x)+2*c)/(b^2-c^2)^(1/2))
```

3.737.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(84) = 168$.

Time = 0.29 (sec) , antiderivative size = 799, normalized size of antiderivative = 9.08

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx$$

$$= \left[-\frac{2\,Ab^3 - 2\,Ab^2c - 2\,Abc^2 + 2\,Ac^3 + \left(Bb^2 - (B+C)bc + Cc^2 + (Bb^2 + (B-C)bc - Cc^2)\cosh{(x)}^2 + \right.}{b^5 - b^4c - 2\,b^3c^2 + 2\,b} \right.$$

$$-\frac{2\left(Ab^3 - Ab^2c - Abc^2 + Ac^3 + \left(Bb^2 - (B+C)bc + Cc^2 + (Bb^2 + (B-C)bc - Cc^2)\cosh{(x)}^2 + 2\,(B\right.\right.}{b^5 - b^4c - 2\,b^3c^2 + 2\,b^2c^3 + bc^4 - c^5 + (b^5 + b^4c}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fr
icas")
```

output
```
[-(2*A*b^3 - 2*A*b^2*c - 2*A*b*c^2 + 2*A*c^3 + (B*b^2 - (B + C)*b*c + C*c^
2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 + 2*(B*b^2 + (B - C)*b*c - C*c
^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C*c^2)*sinh(x)^2)*sqrt(-b^2 +
 c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)
^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 +
2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) + 2*(C*b^3 - B*b^2
*c - C*b*c^2 + B*c^3)*cosh(x) + 2*(C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*sinh
(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2
*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2
 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2
*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2), -2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3
+ (B*b^2 - (B + C)*b*c + C*c^2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 +
 2*(B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c -
C*c^2)*sinh(x)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x)
+ (b + c)*sinh(x))) + (C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*cosh(x) + (C*b^3
 - B*b^2*c - C*b*c^2 + B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^
3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh
(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh
(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2)]
```

3.737.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

output
```
Timed out
```

3.737.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` f
or more de
```

3.737.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{2\,(Bb-Cc)\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{\frac{3}{2}}} - \frac{2\,(Cbe^x-Bce^x+Ab-Ac)}{(b^2-c^2)(be^{(2x)}+ce^{(2x)}+b-c)}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="gi
ac")
```

output
```
2*(B*b - C*c)*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) -
2*(C*b*e^x - B*c*e^x + A*b - A*c)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b
- c))
```

3.737.9 Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.39

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{\ln\left(\frac{2\,(B\,b-C\,c)}{(b+c)^{5/2}\,\sqrt{c-b}}+\frac{2\,e^x\,(B\,b-C\,c)}{-b^3-b^2\,c+b\,c^2+c^3}\right)\,(B\,b-C\,c)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{\ln\left(\frac{2\,e^x\,(B\,b-C\,c)}{-b^3-b^2\,c+b\,c^2+c^3}-\frac{2\,(B\,b-C\,c)}{(b+c)^{5/2}\,\sqrt{c-b}}\right)\,(B\,b-C\,c)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{\frac{2\,A}{b+c}-\frac{2\,e^x\,(B\,c-C\,b)}{(b+c)\,(b-c)}}{b-c+e^{2\,x}\,(b+c)}$$

input
```
int((A + B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x))^2,x)
```

output
```
(log((2*(B*b - C*c))/((b + c)^(5/2)*(c - b)^(1/2)) + (2*exp(x)*(B*b - C*c)
)/(b*c^2 - b^2*c - b^3 + c^3))*(B*b - C*c))/((b + c)^(3/2)*(c - b)^(3/2))
- (log((2*exp(x)*(B*b - C*c))/(b*c^2 - b^2*c - b^3 + c^3) - (2*(B*b - C*c)
)/((b + c)^(5/2)*(c - b)^(1/2)))*(B*b - C*c))/((b + c)^(3/2)*(c - b)^(3/2)
) - ((2*A)/(b + c) - (2*exp(x)*(B*c - C*b))/((b + c)*(b - c)))/(b - c + ex
p(2*x)*(b + c))
```

3.737. $\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

3.738 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

3.738.1 Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2(b^2-c^2)^{3/2}} + \frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} + \frac{c(bB-cC)\cosh(x)+b(bB-cC)\sinh(x)}{(b^2-c^2)^2(b\cosh(x)+c\sinh(x))}$$

output
```
1/2*A*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+1/2*(B
*c-b*C+A*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))^2+(c*(B*b-
C*c)*cosh(x)+b*(B*b-C*c)*sinh(x))/(b^2-c^2)^2/(b*cosh(x)+c*sinh(x))
```

3.738.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{1}{2}\left(\frac{2A\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}} + \frac{b(Bc-bC)+A(b^2-c^2)\sinh(x)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2} + \frac{Ac+2(bB-cC)\sinh(x)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}\right)$$

input
```
Integrate[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
((2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c)^(3/2)*
(b + c)^(3/2)) + (b*(B*c - b*C) + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)*(b + c
)*(b*Cosh[x] + c*Sinh[x])^2) + (A*c + 2*(b*B - c*C)*Sinh[x])/(b*(b - c)*(b
 + c)*(b*Cosh[x] + c*Sinh[x])))/2
```

3.738.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3042, 3635, 3042, 3632, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)-iC\sin(ix)}{(b\cos(ix)-ic\sin(ix))^3}dx$$

$$\Big\downarrow \text{3635}$$

$$\frac{\int \frac{2(bB-cC)+Ab\cosh(x)+Ac\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}dx}{2(b^2-c^2)}+\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2(bB-cC)+Ab\cos(ix)-iAc\sin(ix)}{(b\cos(ix)-ic\sin(ix))^2}dx}{2(b^2-c^2)}$$

$$\Big\downarrow \text{3632}$$

$$\frac{A\int \frac{1}{b\cosh(x)+c\sinh(x)}dx+\frac{2(b\sinh(x)(bB-cC)+c\cosh(x)(bB-cC))}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}}{2(b^2-c^2)}+\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}+\frac{\frac{2(b\sinh(x)(bB-cC)+c\cosh(x)(bB-cC))}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+A\int \frac{1}{b\cos(ix)-ic\sin(ix)}dx}{2(b^2-c^2)}$$

$$\Big\downarrow \text{3553}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}+\frac{\frac{2(b\sinh(x)(bB-cC)+c\cosh(x)(bB-cC))}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+iA\int\frac{1}{b^2-c^2-(-ic\cosh(x)-ib\sinh(x))^2}d(-ic\cosh(x)-ib\sinh(x))}{2(b^2-c^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2}+\frac{\frac{2(b\sinh(x)(bB-cC)+c\cosh(x)(bB-cC))}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{iA\text{arctanh}\left(\frac{-ib\sinh(x)-ic\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}}{2(b^2-c^2)}$$

input
```
Int[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
(B*c - b*C + A*c*Cosh[x] + A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh
[x])^2) + ((I*A*ArcTanh[((-I)*c*Cosh[x] - I*b*Sinh[x])/Sqrt[b^2 - c^2]])/S
qrt[b^2 - c^2] + (2*(c*(b*B - c*C)*Cosh[x] + b*(b*B - c*C)*Sinh[x]))/((b^2
 - c^2)*(b*Cosh[x] + c*Sinh[x])))/(2*(b^2 - c^2))
```

3.738.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.738. $\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.738.4 Maple [A] (verified)

Time = 12.66 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.69

method	result
default	$\frac{-\frac{\left(b^{2} A-2 A c^{2}-2 B b^{2}+2 B c^{2}\right) \tanh \left(\frac{x}{2}\right)^{3}}{\left(b^{2}-c^{2}\right) b}+\frac{\left(A b^{2} c+2 A c^{3}+2 B b^{2} c-2 B c^{3}+2 C b^{3}-2 C b c^{2}\right) \tanh \left(\frac{x}{2}\right)^{2}}{b^{2}\left(b^{2}-c^{2}\right)}+\frac{\left(b^{2} A+2 A c^{2}+2 B b^{2}-2 B c^{2}\right) \tanh \left(\frac{x}{2}\right)}{b\left(b^{2}-c^{2}\right)}+\cdots}{\left(\tanh \left(\frac{x}{2}\right)^{2} b+2 c \tanh \left(\frac{x}{2}\right)+b\right)^{2}}$
risch	$\frac{A b^{2} e^{3 x}+2 A b c\, e^{3 x}+A c^{2} e^{3 x}-2 B b^{2} e^{2 x}+2 B c^{2} e^{2 x}-2 C b^{2} e^{2 x}+2 C c^{2} e^{2 x}-A e^{x} b^{2}+A e^{x} c^{2}-2 B b^{2}+2 b B c+2 C c b-2 C c^{2}}{(b-c)\left(b e^{2 x}+e^{2 x} c+b-c\right)^{2}\left(b^{2}+2 c b+c^{2}\right)}-\frac{A \ln \left(e^{x}-\cdots\right.}{2 \sqrt{-b^{2}+c^{2}} \cdots}$

input
```
int((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOS
E)
```

output
```
2*(-1/2*(A*b^2-2*A*c^2-2*B*b^2+2*B*c^2)/(b^2-c^2)/b*tanh(1/2*x)^3+1/2*(A*b
^2*c+2*A*c^3+2*B*b^2*c-2*B*c^3+2*C*b^3-2*C*b*c^2)/b^2/(b^2-c^2)*tanh(1/2*x
)^2+1/2*(A*b^2+2*A*c^2+2*B*b^2-2*B*c^2)/b/(b^2-c^2)*tanh(1/2*x)+1/2*A*c/(b
^2-c^2))/(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)^2+A/(b^2-c^2)^(3/2)*arctan(1/
2*(2*b*tanh(1/2*x)+2*c)/(b^2-c^2)^(1/2))
```

3.738.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(127) = 254$.

Time = 0.30 (sec) , antiderivative size = 1931, normalized size of antiderivative = 14.30

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fr
icas")
```

output
```
[-1/2*(4*B*b^3 - 4*(2*B + C)*b^2*c + 4*(B + 2*C)*b*c^2 - 4*C*c^3 - 2*(A*b^
3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^3 - 2*(A*b^3 + A*b^2*c - A*b*c^2 -
A*c^3)*sinh(x)^3 + 4*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2 + (B + C
)*c^3)*cosh(x)^2 + 2*(2*(B + C)*b^3 - 2*(B + C)*b^2*c - 2*(B + C)*b*c^2 +
2*(B + C)*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 -
 ((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x
)*sinh(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^
2 + 2*(A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c +
A*c^2)*cosh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*
b^2 - A*c^2)*cosh(x))*sinh(x))*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2
*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x)
 + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b +
 c)*sinh(x)^2 + b - c)) + 2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) +
2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^
3)*cosh(x)^2 + 4*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2 + (B + C)*c^
3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b*
c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6
)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 +
 c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 +
 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(...
```

3.738.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

output
```
Timed out
```

3.738.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` f
or more de
```

3.738.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.36

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{\frac{3}{2}}} + \frac{Ab^2e^{(3x)}+2\,Abce^{(3x)}+Ac^2e^{(3x)}-2\,Bb^2e^{(2x)}-2\,Cb^2e^{(2x)}+2\,Bc^2e^{(2x)}+2\,Cc^2e^{(2x)}-Ab^2e^x+Ac^2e^x}{(b^3+b^2c-bc^2-c^3)(be^{(2x)}+ce^{(2x)}+b-c)^2}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="gi
ac")
```

output
```
A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + (A*b^2*e^(3*
x) + 2*A*b*c*e^(3*x) + A*c^2*e^(3*x) - 2*B*b^2*e^(2*x) - 2*C*b^2*e^(2*x) +
 2*B*c^2*e^(2*x) + 2*C*c^2*e^(2*x) - A*b^2*e^x + A*c^2*e^x - 2*B*b^2 + 2*B
*b*c + 2*C*b*c - 2*C*c^2)/((b^3 + b^2*c - b*c^2 - c^3)*(b*e^(2*x) + c*e^(2
*x) + b - c)^2)
```

3.738.9 Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.66

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{\operatorname{atan}\left(\frac{A\,e^x\,\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}}{b^3\,\sqrt{A^2}+c^3\,\sqrt{A^2}-b\,c^2\,\sqrt{A^2}-b^2\,c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}} - \frac{\frac{B-C}{b+c}+\frac{2\,A\,e^x}{b+c}+\frac{e^{2\,x}\,(B+C)}{b+c}}{e^{4\,x}\,(b+c)^2+(b-c)^2+2\,e^{2\,x}\,(b+c)\,(b-c)} - \frac{\frac{B+C}{(b+c)^2}-\frac{A\,e^x}{(b+c)\,(b-c)}}{b-c+e^{2\,x}\,(b+c)}$$

input
```
int((A + B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x))^3,x)
```

output
```
(atan((A*exp(x)*(b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2))/(b^3*(A^2)^(1/2
) + c^3*(A^2)^(1/2) - b*c^2*(A^2)^(1/2) - b^2*c*(A^2)^(1/2)))*(A^2)^(1/2))
/(b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2) - ((B - C)/(b + c) + (2*A*exp(x
))/(b + c) + (exp(2*x)*(B + C))/(b + c))/(exp(4*x)*(b + c)^2 + (b - c)^2 +
 2*exp(2*x)*(b + c)*(b - c)) - ((B + C)/(b + c)^2 - (A*exp(x))/((b + c)*(b
 - c)))/(b - c + exp(2*x)*(b + c))
```

3.739 $\int (a+b\cosh(x)+c\sinh(x))^3\,dx$

3.739.1 Optimal result

Integrand size = 12, antiderivative size = 119

$$\begin{aligned}\int (a+b\cosh(x)+c\sinh(x))^3\,dx &= \frac{1}{2}a\left(2a^2+3b^2-3c^2\right)x+\frac{1}{6}c\left(11a^2+4b^2-4c^2\right)\cosh(x)\\ &\quad+\frac{1}{6}b\left(11a^2+4b^2-4c^2\right)\sinh(x)\\ &\quad+\frac{5}{6}(ac\cosh(x)+ab\sinh(x))(a+b\cosh(x)+c\sinh(x))\\ &\quad+\frac{1}{3}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^2\end{aligned}$$

output
```
1/2*a*(2*a^2+3*b^2-3*c^2)*x+1/6*c*(11*a^2+4*b^2-4*c^2)*cosh(x)+1/6*b*(11*a
^2+4*b^2-4*c^2)*sinh(x)+5/6*(a*c*cosh(x)+a*b*sinh(x))*(a+b*cosh(x)+c*sinh(
x))+1/3*(c*cosh(x)+b*sinh(x))*(a+b*cosh(x)+c*sinh(x))^2
```

3.739.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\begin{aligned}\int (a+b\cosh(x)+c\sinh(x))^3\,dx &= \frac{1}{12}\left(6a\left(2a^2+3b^2-3c^2\right)x+9c\left(4a^2+b^2-c^2\right)\cosh(x)\right.\\ &\quad+18abc\cosh(2x)+c\left(3b^2+c^2\right)\cosh(3x)\\ &\quad+9b\left(4a^2+b^2-c^2\right)\sinh(x)+9a\left(b^2+c^2\right)\sinh(2x)\\ &\quad\left.+b\left(b^2+3c^2\right)\sinh(3x)\right)\end{aligned}$$

input
```
Integrate[(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
(6*a*(2*a^2 + 3*b^2 - 3*c^2)*x + 9*c*(4*a^2 + b^2 - c^2)*Cosh[x] + 18*a*b*
c*Cosh[2*x] + c*(3*b^2 + c^2)*Cosh[3*x] + 9*b*(4*a^2 + b^2 - c^2)*Sinh[x]
+ 9*a*(b^2 + c^2)*Sinh[2*x] + b*(b^2 + 3*c^2)*Sinh[3*x])/12
```

3.739.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3599, 3042, 3625, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x)+c\sinh(x))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\cos(ix)-ic\sin(ix))^3dx$$

$$\Big\downarrow \text{3599}$$

$$\frac{1}{3}\int (a+b\cosh(x)+c\sinh(x))\left(3a^2+5b\cosh(x)a+5c\sinh(x)a+2b^2-2c^2\right)dx+\frac{1}{3}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^2+\frac{1}{3}\int (a+b\cos(ix)-ic\sin(ix))\left(3a^2+5b\cos(ix)a-5ic\sin(ix)a+2b^2-2c^2\right)dx$$

$$\Big\downarrow \text{3625}$$

$$\frac{1}{3}\left(\frac{\int \left(3\left(2a^2+3b^2-3c^2\right)a^2+b\left(11a^2+4b^2-4c^2\right)\cosh(x)a+c\left(11a^2+4b^2-4c^2\right)\sinh(x)a\right)dx}{2a}+\frac{5}{2}(ab\sinh(x)-\right.$$
$$\frac{1}{3}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3a^2x(2a^2+3b^2-3c^2)+ab\sinh(x)\left(11a^2+4b^2-4c^2\right)+ac\cosh(x)\left(11a^2+4b^2-4c^2\right)}{2a}+\frac{5}{2}(ab\sinh(x)+ac\,\mathrm{c}\right.$$

$$\frac{1}{3}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^2$$

input
```
Int[(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
((c*Cosh[x] + b*Sinh[x])*(a + b*Cosh[x] + c*Sinh[x])^2)/3 + ((5*(a*c*Cosh[
x] + a*b*Sinh[x])*(a + b*Cosh[x] + c*Sinh[x]))/2 + (3*a^2*(2*a^2 + 3*b^2 -
 3*c^2)*x + a*c*(11*a^2 + 4*b^2 - 4*c^2)*Cosh[x] + a*b*(11*a^2 + 4*b^2 - 4
*c^2)*Sinh[x])/(2*a))/3
```

3.739.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

3.739. $\int(a+b\cosh(x)+c\sinh(x))^3\,dx$

3.739.4 Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

method	result
parts	$a^3x+c^3\left(-\frac{2}{3}+\frac{\sinh(x)^2}{3}\right)\cosh(x)+c\,b^2\cosh(x)^3+3a\,b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+\frac{b(c\sinh(x)+a)^3}{c}+b^3\Big($
default	$a^3x+3\sinh(x)\,a^2b+3c\,a^2\cosh(x)+3a\,b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+3cab\cosh(x)^2+3a\,c^2\Big(\frac{\cosh(x)\sin}{2}$
risch	$\frac{3a\,b^2e^{2x}}{8}+a^3x-\frac{3a\,c^2x}{2}+\frac{3a\,b^2x}{2}+\frac{e^{3x}b^3}{24}+\frac{3a^2b\,e^x}{2}+\frac{3b^3e^x}{8}-\frac{3\,e^{-2x}a\,b^2}{8}+\frac{e^{3x}c^2b}{8}+\frac{3\,e^{2x}a\,c^2}{8}+\frac{3\,e^xc\,a^2}{2}+3\,e$

input
```
int((a+b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+c^3*(-2/3+1/3*sinh(x)^2)*cosh(x)+c*b^2*cosh(x)^3+3*a*b^2*(1/2*cosh(x
)*sinh(x)+1/2*x)+b*(c*sinh(x)+a)^3/c+b^3*(2/3+1/3*cosh(x)^2)*sinh(x)+3*c*a
^2*cosh(x)+3*a*c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

3.739.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.34

$$\begin{aligned}\int(a+b\cosh(x)+c\sinh(x))^3\,dx &= \frac{3}{2}\,abc\cosh(x)^2+\frac{1}{12}\left(3\,b^2c+c^3\right)\cosh(x)^3+\frac{1}{12}\left(b^3+3\,bc^2\right)\sinh(x)^3\\&\quad+\frac{1}{4}\left(6\,abc+\left(3\,b^2c+c^3\right)\cosh(x)\right)\sinh(x)^2\\&\quad+\frac{1}{2}\left(2\,a^3+3\,ab^2-3\,ac^2\right)x-\frac{3}{4}\left(c^3-\left(4\,a^2+b^2\right)c\right)\cosh(x)\\&\quad+\frac{1}{4}\left(12\,a^2b+3\,b^3-3\,bc^2+\left(b^3+3\,bc^2\right)\cosh(x)^2+6\left(ab^2+ac^2\right)\cosh(x)\right)\sinh(x)\end{aligned}$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

output
```
3/2*a*b*c*cosh(x)^2 + 1/12*(3*b^2*c + c^3)*cosh(x)^3 + 1/12*(b^3 + 3*b*c^2
)*sinh(x)^3 + 1/4*(6*a*b*c + (3*b^2*c + c^3)*cosh(x))*sinh(x)^2 + 1/2*(2*a
^3 + 3*a*b^2 - 3*a*c^2)*x - 3/4*(c^3 - (4*a^2 + b^2)*c)*cosh(x) + 1/4*(12*
a^2*b + 3*b^3 - 3*b*c^2 + (b^3 + 3*b*c^2)*cosh(x)^2 + 6*(a*b^2 + a*c^2)*co
sh(x))*sinh(x)
```

3.739.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.65

$$\int (a+b\cosh(x)+c\sinh(x))^3\,dx = a^3x+3a^2b\sinh(x)+3a^2c\cosh(x)-\frac{3ab^2x\sinh^2(x)}{2} + \frac{3ab^2x\cosh^2(x)}{2}+\frac{3ab^2\sinh(x)\cosh(x)}{2} + 3abc\cosh^2(x)+\frac{3ac^2x\sinh^2(x)}{2}-\frac{3ac^2x\cosh^2(x)}{2} + \frac{3ac^2\sinh(x)\cosh(x)}{2}-\frac{2b^3\sinh^3(x)}{3} + b^3\sinh(x)\cosh^2(x)+b^2c\cosh^3(x)+bc^2\sinh^3(x) + c^3\sinh^2(x)\cosh(x)-\frac{2c^3\cosh^3(x)}{3}$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))**3,x)
```

output
```
a**3*x + 3*a**2*b*sinh(x) + 3*a**2*c*cosh(x) - 3*a*b**2*x*sinh(x)**2/2 + 3
*a*b**2*x*cosh(x)**2/2 + 3*a*b**2*sinh(x)*cosh(x)/2 + 3*a*b*c*cosh(x)**2 +
 3*a*c**2*x*sinh(x)**2/2 - 3*a*c**2*x*cosh(x)**2/2 + 3*a*c**2*sinh(x)*cosh
(x)/2 - 2*b**3*sinh(x)**3/3 + b**3*sinh(x)*cosh(x)**2 + b**2*c*cosh(x)**3
+ b*c**2*sinh(x)**3 + c**3*sinh(x)**2*cosh(x) - 2*c**3*cosh(x)**3/3
```

3.739.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int (a+b\cosh(x)+c\sinh(x))^3\,dx = b^2c\cosh(x)^3+bc^2\sinh(x)^3+a^3x+\frac{1}{24}c^3\left(e^{(3x)}-9e^{(-x)}+e^{(-3x)}-9e^x\right) + \frac{1}{24}b^3\left(e^{(3x)}-9e^{(-x)}-e^{(-3x)}+9e^x\right)+3(c\cosh(x)+b\sinh(x))a^2 + \frac{3}{8}\left(8bc\cosh(x)^2+b^2\left(4x+e^{(2x)}-e^{(-2x)}\right)-c^2\left(4x-e^{(2x)}+e^{(-2x)}\right)\right)a$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

output
```
b^2*c*cosh(x)^3 + b*c^2*sinh(x)^3 + a^3*x + 1/24*c^3*(e^(3*x) - 9*e^(-x) +
 e^(-3*x) - 9*e^x) + 1/24*b^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x) + 3*
(c*cosh(x) + b*sinh(x))*a^2 + 3/8*(8*b*c*cosh(x)^2 + b^2*(4*x + e^(2*x) -
e^(-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*a
```

3.739.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(109) = 218$.

Time = 0.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.84

$$\int (a + b\cosh(x) + c\sinh(x))^3 \, dx = \frac{1}{24} b^3 e^{(3x)} + \frac{1}{8} b^2 c e^{(3x)} + \frac{1}{8} b c^2 e^{(3x)} + \frac{1}{24} c^3 e^{(3x)} + \frac{3}{8} a b^2 e^{(2x)} + \frac{3}{4} abce^{(2x)} + \frac{3}{8} a c^2 e^{(2x)} + \frac{3}{2} a^2 b e^x + \frac{3}{8} b^3 e^x + \frac{3}{2} a^2 c e^x + \frac{3}{8} b^2 c e^x - \frac{3}{8} b c^2 e^x - \frac{3}{8} c^3 e^x + \frac{1}{2} \left(2 a^3 + 3 ab^2 - 3 ac^2\right) x - \frac{1}{24} \left(b^3 - 3 b^2 c + 3 bc^2 - c^3 + 9 \left(4 a^2 b + b^3 - 4 a^2 c - b^2 c - bc^2 + c^3\right) e^{(2x)} + 9 \left(ab^2 - 2 abc + ac^2\right) e^x\right) e^{(-3}$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

output
```
1/24*b^3*e^(3*x) + 1/8*b^2*c*e^(3*x) + 1/8*b*c^2*e^(3*x) + 1/24*c^3*e^(3*x
) + 3/8*a*b^2*e^(2*x) + 3/4*a*b*c*e^(2*x) + 3/8*a*c^2*e^(2*x) + 3/2*a^2*b*
e^x + 3/8*b^3*e^x + 3/2*a^2*c*e^x + 3/8*b^2*c*e^x - 3/8*b*c^2*e^x - 3/8*c^
3*e^x + 1/2*(2*a^3 + 3*a*b^2 - 3*a*c^2)*x - 1/24*(b^3 - 3*b^2*c + 3*b*c^2
- c^3 + 9*(4*a^2*b + b^3 - 4*a^2*c - b^2*c - b*c^2 + c^3)*e^(2*x) + 9*(a*b
^2 - 2*a*b*c + a*c^2)*e^x)*e^(-3*x)
```

3.739.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10

$$\int (a + b\cosh(x) + c\sinh(x))^3 \, dx = a^3 x + \cosh(x)^3 \left(b^2 c - \frac{2 c^3}{3}\right) + \sinh(x)^3 \left(b c^2 - \frac{2 b^3}{3}\right) + b^3 \cosh(x)^2 \sinh(x) + c^3 \cosh(x) \sinh(x)^2 + 3 a^2 c \cosh(x) + 3 a^2 b \sinh(x) + \frac{3 a \cosh(x) \sinh(x) \left(b^2 + c^2\right)}{2} + \frac{3 a x \cosh(x)^2 \left(b^2 - c^2\right)}{2} + 3 a b c \cosh(x)^2 - \frac{3 a x \sinh(x)^2 \left(b^2 - c^2\right)}{2}$$

input
```
int((a + b*cosh(x) + c*sinh(x))^3,x)
```

output
```
a^3*x + cosh(x)^3*(b^2*c - (2*c^3)/3) + sinh(x)^3*(b*c^2 - (2*b^3)/3) + b^
3*cosh(x)^2*sinh(x) + c^3*cosh(x)*sinh(x)^2 + 3*a^2*c*cosh(x) + 3*a^2*b*si
nh(x) + (3*a*cosh(x)*sinh(x)*(b^2 + c^2))/2 + (3*a*x*cosh(x)^2*(b^2 - c^2)
)/2 + 3*a*b*c*cosh(x)^2 - (3*a*x*sinh(x)^2*(b^2 - c^2))/2
```

3.739. $\int (a + b\cosh(x) + c\sinh(x))^3 \, dx$

3.740 $\int (a + b\cosh(x) + c\sinh(x))^2 \, dx$

3.740.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (a + b\cosh(x) + c\sinh(x))^2 \, dx = \frac{1}{2}\left(2a^2 + b^2 - c^2\right) x + \frac{3}{2}ac\cosh(x) + \frac{3}{2}ab\sinh(x) + \frac{1}{2}(c\cosh(x) + b\sinh(x))(a + b\cosh(x) + c\sinh(x))$$

output
```
1/2*(2*a^2+b^2-c^2)*x+3/2*a*c*cosh(x)+3/2*a*b*sinh(x)+1/2*(c*cosh(x)+b*sin
h(x))*(a+b*cosh(x)+c*sinh(x))
```

3.740.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int (a + b\cosh(x) + c\sinh(x))^2 \, dx = \frac{1}{4}\left(2\left(2a^2 + b^2 - c^2\right) x + 8ac\cosh(x) + 2bc\cosh(2x) + 8ab\sinh(x) + \left(b^2 + c^2\right)\sinh(2x)\right)$$

input
```
Integrate[(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(2*(2*a^2 + b^2 - c^2)*x + 8*a*c*Cosh[x] + 2*b*c*Cosh[2*x] + 8*a*b*Sinh[x]
 + (b^2 + c^2)*Sinh[2*x])/4
```

3.740.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3599, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\cos(ix)-ic\sin(ix))^2dx$$

$$\Big\downarrow \text{3599}$$

$$\frac{1}{2}\int \left(2a^2+3b\cosh(x)a+3c\sinh(x)a+b^2-c^2\right)dx+\frac{1}{2}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(x\left(2a^2+b^2-c^2\right)+3ab\sinh(x)+3ac\cosh(x)\right)+\frac{1}{2}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))$$

input
```
Int[(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
((2*a^2 + b^2 - c^2)*x + 3*a*c*Cosh[x] + 3*a*b*Sinh[x])/2 + ((c*Cosh[x] +
b*Sinh[x])*(a + b*Cosh[x] + c*Sinh[x]))/2
```

3.740.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

3.740.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

method	result
default	$a^2x+2ab\sinh\left(x\right)+2ac\cosh\left(x\right)+b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+cb\cosh\left(x\right)^2+c^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)$
parts	$a^2x+2b\left(\frac{c\sinh(x)^2}{2}+a\sinh\left(x\right)\right)+b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+c^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)+2ac\cosh\left(x\right)$
risch	$a^2x+\frac{b^2x}{2}-\frac{c^2x}{2}+\frac{b^2e^{2x}}{8}+\frac{e^{2x}cb}{4}+\frac{e^{2x}c^2}{8}+b\,e^xa+e^xac-e^{-x}ab+e^{-x}ac-\frac{e^{-2x}b^2}{8}+\frac{e^{-2x}cb}{4}-\frac{e^{-2x}c^2}{8}$

input
```
int((a+b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2*a*b*sinh(x)+2*a*c*cosh(x)+b^2*(1/2*cosh(x)*sinh(x)+1/2*x)+c*b*cosh
(x)^2+c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

3.740.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx=\frac{1}{2}\,bc\cosh\left(x\right)^2+\frac{1}{2}\,bc\sinh\left(x\right)^2+2\,ac\cosh\left(x\right)+\frac{1}{2}\left(2\,a^2+b^2-c^2\right)x+\frac{1}{2}\left(4\,ab+\left(b^2+c^2\right)\cosh\left(x\right)\right)\sinh\left(x\right)$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

output
```
1/2*b*c*cosh(x)^2 + 1/2*b*c*sinh(x)^2 + 2*a*c*cosh(x) + 1/2*(2*a^2 + b^2 -
 c^2)*x + 1/2*(4*a*b + (b^2 + c^2)*cosh(x))*sinh(x)
```

3.740.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx = a^2x + 2ab\sinh{(x)} + 2ac\cosh{(x)} - \frac{b^2x\sinh^2{(x)}}{2} + \frac{b^2x\cosh^2{(x)}}{2} + \frac{b^2\sinh{(x)}\cosh{(x)}}{2} + bc\cosh^2{(x)} + \frac{c^2x\sinh^2{(x)}}{2} - \frac{c^2x\cosh^2{(x)}}{2} + \frac{c^2\sinh{(x)}\cosh{(x)}}{2}$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))**2,x)
```

output
```
a**2*x + 2*a*b*sinh(x) + 2*a*c*cosh(x) - b**2*x*sinh(x)**2/2 + b**2*x*cosh
(x)**2/2 + b**2*sinh(x)*cosh(x)/2 + b*c*cosh(x)**2 + c**2*x*sinh(x)**2/2 -
 c**2*x*cosh(x)**2/2 + c**2*sinh(x)*cosh(x)/2
```

3.740.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx = bc\cosh{(x)}^2 + \frac{1}{8}b^2\left(4x + e^{(2x)} - e^{(-2x)}\right) - \frac{1}{8}c^2\left(4x - e^{(2x)} + e^{(-2x)}\right) + a^2x + 2\left(c\cosh{(x)} + b\sinh{(x)}\right)a$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

output
```
b*c*cosh(x)^2 + 1/8*b^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*c^2*(4*x - e^(2*x
) + e^(-2*x)) + a^2*x + 2*(c*cosh(x) + b*sinh(x))*a
```

3.740.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx = \frac{1}{8}\,b^2e^{(2\,x)}+\frac{1}{4}\,bce^{(2\,x)}+\frac{1}{8}\,c^2e^{(2\,x)} + abe^x+ace^x+\frac{1}{2}\left(2\,a^2+b^2-c^2\right)x - \frac{1}{8}\left(b^2-2\,bc+c^2+8\,(ab-ac)e^x\right)e^{(-2\,x)}$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

output
```
1/8*b^2*e^(2*x) + 1/4*b*c*e^(2*x) + 1/8*c^2*e^(2*x) + a*b*e^x + a*c*e^x +
1/2*(2*a^2 + b^2 - c^2)*x - 1/8*(b^2 - 2*b*c + c^2 + 8*(a*b - a*c)*e^x)*e^
(-2*x)
```

3.740.9 Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx = x\,a^2+2\sinh(x)\,\,a\,b+2\,a\,c\cosh(x)+\frac{\sinh(x)\,\,b^2\cosh(x)}{2} + \frac{x\,b^2}{2}+b\,c\cosh(x)^2+\frac{\sinh(x)\,\,c^2\cosh(x)}{2}-\frac{x\,c^2}{2}$$

input
```
int((a + b*cosh(x) + c*sinh(x))^2,x)
```

output
```
a^2*x + (b^2*x)/2 - (c^2*x)/2 + 2*a*b*sinh(x) + b*c*cosh(x)^2 + (b^2*cosh(
x)*sinh(x))/2 + (c^2*cosh(x)*sinh(x))/2 + 2*a*c*cosh(x)
```

3.741 $\int (a+b\cosh(x)+c\sinh(x))\,dx$

3.741.1 Optimal result

Integrand size = 10, antiderivative size = 12

$$\int (a+b\cosh(x)+c\sinh(x))\,dx=ax+c\cosh(x)+b\sinh(x)$$

output
```
a*x+c*cosh(x)+b*sinh(x)
```

3.741.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+b\cosh(x)+c\sinh(x))\,dx=ax+c\cosh(x)+b\sinh(x)$$

input
```
Integrate[a + b*Cosh[x] + c*Sinh[x],x]
```

output
```
a*x + c*Cosh[x] + b*Sinh[x]
```

3.741.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x)+c\sinh(x))\,dx$$
$$\Big\downarrow \text{ 2009}$$
$$ax+b\sinh(x)+c\cosh(x)$$

input
```
Int[a + b*Cosh[x] + c*Sinh[x],x]
```

output
```
a*x + c*Cosh[x] + b*Sinh[x]
```

3.741.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.741.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$ax+c\cosh\left(x\right)+b\sinh\left(x\right)$	13
parts	$ax+c\cosh\left(x\right)+b\sinh\left(x\right)$	13
risch	$\frac{\left(b\,{\mathrm e}^{2 x}+{\mathrm e}^{2 x} c+2 a x\,{\mathrm e}^{x}-b+c\right) {\mathrm e}^{-x}}{2}$	30

input
```
int(a+b*cosh(x)+c*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
a*x+c*cosh(x)+b*sinh(x)
```

3.741.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+b\cosh(x)+c\sinh(x))\,dx=ax+c\cosh(x)+b\sinh(x)$$

input
```
integrate(a+b*cosh(x)+c*sinh(x),x, algorithm="fricas")
```

output
```
a*x + c*cosh(x) + b*sinh(x)
```

3.741.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+b\cosh(x)+c\sinh(x))\,dx=ax+b\sinh(x)+c\cosh(x)$$

input
```
integrate(a+b*cosh(x)+c*sinh(x),x)
```

output
```
a*x + b*sinh(x) + c*cosh(x)
```

3.741.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+b\cosh(x)+c\sinh(x))\,dx=ax+c\cosh(x)+b\sinh(x)$$

input
```
integrate(a+b*cosh(x)+c*sinh(x),x, algorithm="maxima")
```

output
```
a*x + c*cosh(x) + b*sinh(x)
```

3.741.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int (a + b\cosh(x) + c\sinh(x))\,dx = ax + \frac{1}{2}c\left(e^{(-x)} + e^{x}\right) - \frac{1}{2}b\left(e^{(-x)} - e^{x}\right)$$

input
```
integrate(a+b*cosh(x)+c*sinh(x),x, algorithm="giac")
```

output
```
a*x + 1/2*c*(e^(-x) + e^x) - 1/2*b*(e^(-x) - e^x)
```

3.741.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + b\cosh(x) + c\sinh(x))\,dx = a\,x + c\cosh(x) + b\sinh(x)$$

input
```
int(a + b*cosh(x) + c*sinh(x),x)
```

output
```
a*x + c*cosh(x) + b*sinh(x)
```

3.742 $\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx$

3.742.1 Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{2\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

output
```
-2*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^2)^(1/2)
```

3.742.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{2\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

input
```
Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

output
```
(2*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 + b^
2 - c^2]
```

3.742.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{3603}$$

$$2\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+2c\tanh\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{1083}$$

$$-4\int \frac{1}{4\left(a^2-b^2+c^2\right)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

input
```
Int[(a + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

output
```
(-2*ArcTanh[(2*c - 2*(a - b)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^2])])/Sqrt[a
^2 - b^2 + c^2]
```

3.742.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.742.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{2\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\sqrt{-a^{2}+b^{2}-c^{2}}}$	53
risch	$\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}+c^{2}}-a^{2}+b^{2}-c^{2}}{(b+c)\sqrt{a^{2}-b^{2}+c^{2}}}\right)}{\sqrt{a^{2}-b^{2}+c^{2}}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}+c^{2}}+a^{2}-b^{2}+c^{2}}{(b+c)\sqrt{a^{2}-b^{2}+c^{2}}}\right)}{\sqrt{a^{2}-b^{2}+c^{2}}}$	139

input
```
int(1/(a+b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2
)^(1/2))
```

3.742.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.86

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx$$
$$= \left[\frac{\log\left(\frac{(b^2+2\,bc+c^2)\cosh(x)^2+(b^2+2\,bc+c^2)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,(ab+ac+(b^2+2\,bc+c^2)\cosh(x))\sinh(x)-2\,\sqrt{a^2-b^2}}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b+c)\cosh(x)+a)\sinh(x)+b-c}\right.}{\sqrt{a^2-b^2+c^2}}\right.$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

output
```
[log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^
2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)
*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sin
h(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b +
c)*cosh(x) + a)*sinh(x) + b - c))/sqrt(a^2 - b^2 + c^2), 2*sqrt(-a^2 + b^2
 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) +
 a)/(a^2 - b^2 + c^2))/(a^2 - b^2 + c^2)]
```

3.742.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x)),x)
```

output
```
Timed out
```

3.742.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.742.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{2\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

output
```
2*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/sqrt(-a^2 + b^2 - c^2
)
```

3.742.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a}{\sqrt{-a^2+b^2-c^2}}+\frac{b\,\mathrm{e}^x}{\sqrt{-a^2+b^2-c^2}}+\frac{c\,\mathrm{e}^x}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

input
```
int(1/(a + b*cosh(x) + c*sinh(x)),x)
```

output
```
(2*atan(a/(b^2 - a^2 - c^2)^(1/2) + (b*exp(x))/(b^2 - a^2 - c^2)^(1/2) + (
c*exp(x))/(b^2 - a^2 - c^2)^(1/2)))/(b^2 - a^2 - c^2)^(1/2)
```

3.743 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

3.743.1 Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=-\frac{2a\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}-\frac{c\cosh(x)+b\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

output
```
-2*a*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^2)^(3/2
)+(-c*cosh(x)-b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))
```

3.743.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=-\frac{2a\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{3/2}}+\frac{-ac+(b^2-c^2)\sinh(x)}{b\,(-a^2+b^2-c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

input
```
Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

output
```
(-2*a*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-a^2 + b^2
 - c^2)^(3/2) + (-(a*c) + (b^2 - c^2)*Sinh[x])/(b*(-a^2 + b^2 - c^2)*(a +
b*Cosh[x] + c*Sinh[x]))
```

3.743.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3608, 25, 27, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\cos(ix)-ic\sin(ix))^2}dx$$

$$\Big\downarrow \text{3608}$$

$$-\frac{\int -\frac{a}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}-\frac{b\sinh(x)+c\cosh(x)}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}-\frac{b\sinh(x)+c\cosh(x)}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}-\frac{b\sinh(x)+c\cosh(x)}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(x)+c\cosh(x)}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}+\frac{a\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{a^2-b^2+c^2}$$

$$\Big\downarrow \text{3603}$$

$$\frac{2a\int \frac{1}{-((a-b)\tanh^2(\frac{x}{2}))+2c\tanh(\frac{x}{2})+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2+c^2}-\frac{b\sinh(x)+c\cosh(x)}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{4a\int \frac{1}{4(a^2-b^2+c^2)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}-\frac{b\sinh(x)+c\cosh(x)}{(a^2-b^2+c^2)\left(a+b\cosh(x)+c\sinh(x)\right)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2a\operatorname{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}-\frac{b\sinh(x)+c\cosh(x)}{(a^2-b^2+c^2)\left(a+b\cosh(x)+c\sinh(x)\right)}$$

input
```
Int[(a + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

output
```
(-2*a*ArcTanh[(2*c - 2*(a - b)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^2])])/(a^2
 - b^2 + c^2)^(3/2) - (c*Cosh[x] + b*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Co
sh[x] + c*Sinh[x]))
```

3.743.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.743.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(85) = 170$.

Time = 3.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.12

method	result
default	$-\frac{2\left(-\frac{\left(ab-b^2+c^2\right)\tanh\left(\frac{x}{2}\right)}{a^3-a^2b-a\,b^2+a\,c^2+b^3-c^2b}-\frac{ac}{a^3-a^2b-a\,b^2+a\,c^2+b^3-c^2b}\right)}{\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b-2c\tanh\left(\frac{x}{2}\right)-a-b}-\frac{2a\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^2+b^2-c^2}}\right)}{(a^2-b^2+c^2)\sqrt{-a^2+b^2-c^2}}$
risch	$\frac{2a\,\mathrm{e}^x+2b-2c}{(a^2-b^2+c^2)(b\,\mathrm{e}^{2x}+\mathrm{e}^{2x}c+2a\,\mathrm{e}^x+b-c)}+\frac{a\ln\left(\mathrm{e}^x+\frac{a\left(a^2-b^2+c^2\right)^{\frac{3}{2}}-a^4+2a^2b^2-2c^2a^2-b^4+2b^2c^2-c^4}{(b+c)\left(a^2-b^2+c^2\right)^{\frac{3}{2}}}\right)}{(a^2-b^2+c^2)^{\frac{3}{2}}}-\frac{a\ln\left(\mathrm{e}^x+\frac{a\left(a^2-b^2+c^2\right)^{\frac{3}{2}}}{(b}\right.}{(a^2}$

input
```
int(1/(a+b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(-(a*b-b^2+c^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2)*tanh(1/2*x)-a*c/(a^3-
a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b-2*c*tanh(1/
2*x)-a-b)-2*a/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(
1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

3.743.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(86) = 172$.

Time = 0.27 (sec) , antiderivative size = 1268, normalized size of antiderivative = 14.09

$$\int \frac{1}{(a + b\cosh(x) + c\sinh(x))^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

output
```
[(2*a^2*b - 2*b^3 + 2*b*c^2 - 2*c^3 + (2*a^2*cosh(x) + (a*b + a*c)*cosh(x)
^2 + (a*b + a*c)*sinh(x)^2 + a*b - a*c + 2*(a^2 + (a*b + a*c)*cosh(x))*sin
h(x))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*
b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b
+ a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b
 + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)
^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*(a^2 - b^
2)*c + 2*(a^3 - a*b^2 + a*c^2)*cosh(x) + 2*(a^3 - a*b^2 + a*c^2)*sinh(x))/
(a^4*b - 2*a^2*b^3 + b^5 + b*c^4 - c^5 - 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^
3)*c^2 + (a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a
^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*cosh(x)^2 + (a^4*b - 2*a^2*b^
3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2
*a^2*b^2 + b^4)*c)*sinh(x)^2 - (a^4 - 2*a^2*b^2 + b^4)*c + 2*(a^5 - 2*a^3*
b^2 + a*b^4 + a*c^4 + 2*(a^3 - a*b^2)*c^2)*cosh(x) + 2*(a^5 - 2*a^3*b^2 +
a*b^4 + a*c^4 + 2*(a^3 - a*b^2)*c^2 + (a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c
^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*
cosh(x))*sinh(x)), 2*(a^2*b - b^3 + b*c^2 - c^3 + (2*a^2*cosh(x) + (a*b +
a*c)*cosh(x)^2 + (a*b + a*c)*sinh(x)^2 + a*b - a*c + 2*(a^2 + (a*b + a*c)*
cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b
 + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (a^2 - b^2)*c...
```

3.743.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\cosh(x) + c\sinh(x))^2} \, dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))**2,x)
```

output
```
Timed out
```

3.743.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.743.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \frac{2\,a\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^2-b^2+c^2)\sqrt{-a^2+b^2-c^2}} + \frac{2\,(ae^x+b-c)}{(a^2-b^2+c^2)(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c)}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

output
```
2*a*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^2 + c^2)*
sqrt(-a^2 + b^2 - c^2)) + 2*(a*e^x + b - c)/((a^2 - b^2 + c^2)*(b*e^(2*x)
+ c*e^(2*x) + 2*a*e^x + b - c))
```

3.743.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \int \frac{1}{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)^2}\,dx$$

input
```
int(1/(a + b*cosh(x) + c*sinh(x))^2,x)
```

output
```
int(1/(a + b*cosh(x) + c*sinh(x))^2, x)
```

3.744 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

3.744.1 Optimal result

Integrand size = 12, antiderivative size = 146

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{(2a^2+b^2-c^2)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}} - \frac{c\cosh(x)+b\sinh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+ab\sinh(x))}{2(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))}$$

output
```
-(2*a^2+b^2-c^2)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b
^2+c^2)^(5/2)+1/2*(-c*cosh(x)-b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh
(x))^2-3/2*(a*c*cosh(x)+a*b*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x
))
```

3.744.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \frac{1}{2}\left(\frac{2(2a^2+b^2-c^2)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{5/2}} + \frac{-ac+(b^2-c^2)\sinh(x)}{b(-a^2+b^2-c^2)(a+b\cosh(x)+c\sinh(x))^2} + \frac{c(2a^2+b^2-c^2)-3a(b^2-c^2)\sinh(x)}{b(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))}\right)$$

input
```
Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

output
```
((2*(2*a^2 + b^2 - c^2)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 -
c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (-(a*c) + (b^2 - c^2)*Sinh[x])/(b*(-a^2
+ b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) + (c*(2*a^2 + b^2 - c^2) - 3*a
*(b^2 - c^2)*Sinh[x])/(b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])))
/2
```

3.744.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3608, 25, 3042, 3632, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\cos(ix)-ic\sin(ix))^3}dx$$

$$\Big\downarrow \text{3608}$$

$$-\frac{\int -\frac{2a-b\cosh(x)-c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx}{2(a^2-b^2+c^2)} - \frac{b\sinh(x)+c\cosh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2a-b\cosh(x)-c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx}{2(a^2-b^2+c^2)}-\frac{b\sinh(x)+c\cosh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(x)+c\cosh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2a-b\cos(ix)+ic\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^2}dx}{2(a^2-b^2+c^2)}$$

$$\Big\downarrow \text{3632}$$

$$\frac{\frac{(2a^2+b^2-c^2)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}-\frac{3(ab\sinh(x)+ac\cosh(x))}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}-\frac{b\sinh(x)+c\cosh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(x)+c\cosh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}+\frac{-\frac{3(ab\sinh(x)+ac\cosh(x))}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}+\frac{(2a^2+b^2-c^2)\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{a^2-b^2+c^2}}{2(a^2-b^2+c^2)}$$

$$\Big\downarrow \text{3603}$$

$$\frac{\frac{2(2a^2+b^2-c^2)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+2c\tanh\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2+c^2}-\frac{3(ab\sinh(x)+ac\cosh(x))}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}-\frac{b\sinh(x)+c\cosh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{4(2a^2+b^2-c^2)\int \frac{1}{4\left(a^2-b^2+c^2\right)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}-\frac{3(ab\sinh(x)+ac\cosh(x))}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}-\frac{b\sinh(x)+c\cosh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{2(2a^2+b^2-c^2)\text{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}-\frac{3(ab\sinh(x)+ac\cosh(x))}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}-\frac{b\sinh(x)+c\cosh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

3.744. $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

input
```
Int[(a + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

output
```
-1/2*(c*Cosh[x] + b*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]
)^2) + ((-2*(2*a^2 + b^2 - c^2)*ArcTanh[(2*c - 2*(a - b)*Tanh[x/2])/(2*Sqr
t[a^2 - b^2 + c^2])])/(a^2 - b^2 + c^2)^(3/2) - (3*(a*c*Cosh[x] + a*b*Sinh
[x]))/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])))/(2*(a^2 - b^2 + c^2
))
```

3.744.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.744.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(138) = 276$.

Time = 21.91 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.95

method	result
default	$-\frac{2\left(-\frac{\left(4a^3b-7a^2b^2+5c^2a^2+2a\,b^3-2c^2ab+b^4-3b^2c^2+2c^4\right)\tanh\left(\frac{x}{2}\right)^3}{2(a-b)\left(a^4-2a^2b^2+2c^2a^2+b^4-2b^2c^2+c^4\right)}-\frac{c\left(4a^4-12a^3b+13a^2b^2-7c^2a^2-6a\,b^3+6c^2ab+b^4+b^2c^2-2c^4\right)\tanh\left(\frac{x}{2}\right)}{2\left(a^4-2a^2b^2+2c^2a^2+b^4-2b^2c^2+c^4\right)\left(a^2-2ab+b^2\right)}\right.}{\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\right.}$
risch	$\frac{2\,e^{3x}a^2b+2a^2c\,e^{3x}+e^{3x}b^3+e^{3x}c\,b^2-e^{3x}c^2b-e^{3x}c^3+6a^3e^{2x}+3a\,b^2e^{2x}-3\,e^{2x}a\,c^2+10a^2b\,e^{x}-10\,e^{x}c\,a^2-b^3e^{x}+e^{x}c\,b^2+e^{x}c^2b-e^{x}c^3+3}{(a^2-b^2+c^2)^2(b\,e^{2x}+e^{2x}c+2a\,e^{x}+b-c)^2}$

input
```
int(1/(a+b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*(4*a^3*b-7*a^2*b^2+5*a^2*c^2+2*a*b^3-2*a*b*c^2+b^4-3*b^2*c^2+2*c^
4)/(a-b)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)*tanh(1/2*x)^3-1/2*c*(
4*a^4-12*a^3*b+13*a^2*b^2-7*a^2*c^2-6*a*b^3+6*a*b*c^2+b^4+b^2*c^2-2*c^4)/(
a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1
/2*(4*a^4*b-5*a^3*b^2+11*a^3*c^2-3*a^2*b^3-3*a^2*b*c^2+5*a*b^4-7*a*b^2*c^2
+2*a*c^4-b^5-b^3*c^2+2*b*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/
(a^2-2*a*b+b^2)*tanh(1/2*x)+1/2*c*(4*a^4-3*a^2*b^2+a^2*c^2-b^4+b^2*c^2)/(a
^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(tanh(1/2*x)^2*
a-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2-(2*a^2+b^2-c^2)/(a^4-2*a^2*b^2+2*
a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1
/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

3.744.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3633 vs. 2(138) = 276.

Time = 0.33 (sec) , antiderivative size = 7379, normalized size of antiderivative = 50.54

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.744.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))**3,x)
```

output
```
Timed out
```

3.744.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.744.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(138) = 276$.

Time = 0.27 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.08

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \frac{(2\,a^2+b^2-c^2)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4)\sqrt{-a^2+b^2-c^2}} + \frac{2\,a^2be^{(3\,x)}+b^3e^{(3\,x)}+2\,a^2ce^{(3\,x)}+b^2ce^{(3\,x)}-bc^2e^{(3\,x)}-c^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+3\,ab^2e^{(2\,x)}-3\,ac^2e^{(2\,x)}+10}{(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4)(be^{(2\,x)}+ce^{(2}}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

output
```
(2*a^2 + b^2 - c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a
^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^2 + b^2 - c^2)
) + (2*a^2*b*e^(3*x) + b^3*e^(3*x) + 2*a^2*c*e^(3*x) + b^2*c*e^(3*x) - b*c
^2*e^(3*x) - c^3*e^(3*x) + 6*a^3*e^(2*x) + 3*a*b^2*e^(2*x) - 3*a*c^2*e^(2*
x) + 10*a^2*b*e^x - b^3*e^x - 10*a^2*c*e^x + b^2*c*e^x + b*c^2*e^x - c^3*e
^x + 3*a*b^2 - 6*a*b*c + 3*a*c^2)/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*
b^2*c^2 + c^4)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)^2)
```

3.744.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$$

input
```
int(1/(a + b*cosh(x) + c*sinh(x))^3,x)
```

output
```
int(1/(a + b*cosh(x) + c*sinh(x))^3, x)
```

3.745 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx$

3.745.1 Optimal result

Integrand size = 12, antiderivative size = 220

$$\begin{aligned}\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx &= -\frac{a(2a^2+3b^2-3c^2)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{7/2}} \\ &\quad - \frac{c\cosh(x)+b\sinh(x)}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^3} \\ &\quad - \frac{5(ac\cosh(x)+ab\sinh(x))}{6(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))^2} \\ &\quad - \frac{c(11a^2+4b^2-4c^2)\cosh(x)+b(11a^2+4b^2-4c^2)\sinh(x)}{6(a^2-b^2+c^2)^3(a+b\cosh(x)+c\sinh(x))}\end{aligned}$$

output
```
-a*(2*a^2+3*b^2-3*c^2)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/
(a^2-b^2+c^2)^(7/2)+1/3*(-c*cosh(x)-b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+
c*sinh(x))^3-5/6*(a*c*cosh(x)+a*b*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*
sinh(x))^2+1/6*(-c*(11*a^2+4*b^2-4*c^2)*cosh(x)-b*(11*a^2+4*b^2-4*c^2)*sin
h(x))/(a^2-b^2+c^2)^3/(a+b*cosh(x)+c*sinh(x))
```

3.745.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 488 vs. 2(220) = 440.

Time = 0.63 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.22

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx = -\frac{a(2a^2+3b^2-3c^2)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{7/2}}$$
$$-\frac{-44a^5c-82a^3b^2c-24ab^4c+82a^3c^3+48ab^2c^3-24ac^5-30a^2bc(2a^2+3b^2-3c^2)\cosh(x)-6ac(a^2(-}{}$$

input
```
Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

output
```
-((a*(2*a^2 + 3*b^2 - 3*c^2)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b
^2 - c^2]])/(-a^2 + b^2 - c^2)^(7/2)) - (-44*a^5*c - 82*a^3*b^2*c - 24*a*b
^4*c + 82*a^3*c^3 + 48*a*b^2*c^3 - 24*a*c^5 - 30*a^2*b*c*(2*a^2 + 3*b^2 -
3*c^2)*Cosh[x] - 6*a*c*(a^2*(-7*b^2 + 11*c^2) + 2*(b^4 + b^2*c^2 - 2*c^4))
*Cosh[2*x] + 22*a^2*b^3*c*Cosh[3*x] + 8*b^5*c*Cosh[3*x] - 22*a^2*b*c^3*Cos
h[3*x] - 16*b^3*c^3*Cosh[3*x] + 8*b*c^5*Cosh[3*x] + 72*a^4*b^2*Sinh[x] - 9
*a^2*b^4*Sinh[x] + 12*b^6*Sinh[x] - 132*a^4*c^2*Sinh[x] - 72*a^2*b^2*c^2*S
inh[x] - 36*b^4*c^2*Sinh[x] + 81*a^2*c^4*Sinh[x] + 36*b^2*c^4*Sinh[x] - 12
*c^6*Sinh[x] + 54*a^3*b^3*Sinh[2*x] + 6*a*b^5*Sinh[2*x] - 78*a^3*b*c^2*Sin
h[2*x] - 48*a*b^3*c^2*Sinh[2*x] + 42*a*b*c^4*Sinh[2*x] + 11*a^2*b^4*Sinh[3
*x] + 4*b^6*Sinh[3*x] - 4*b^4*c^2*Sinh[3*x] - 11*a^2*c^4*Sinh[3*x] - 4*b^2
*c^4*Sinh[3*x] + 4*c^6*Sinh[3*x])/(24*b*(a^2 - b^2 + c^2)^3*(a + b*Cosh[x]
 + c*Sinh[x])^3)
```

3.745.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3608, 25, 3042, 3635, 25, 3042, 3632, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx$$
$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a+b\cos(ix)-ic\sin(ix))^4}dx$$

$\Big\downarrow$ 3608

$$-\frac{\int -\frac{3a-2b\cosh(x)-2c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}dx}{3\left(a^2-b^2+c^2\right)}-\frac{b\sinh(x)+c\cosh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{3a-2b\cosh(x)-2c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}dx}{3\left(a^2-b^2+c^2\right)}-\frac{b\sinh(x)+c\cosh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3}$$

$\Big\downarrow$ 3042

$$-\frac{b\sinh(x)+c\cosh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3}+\frac{\int \frac{3a-2b\cos(ix)+2ic\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^3}dx}{3\left(a^2-b^2+c^2\right)}$$

$\Big\downarrow$ 3635

$$\frac{-\frac{\int -\frac{2\left(3a^2+2b^2-2c^2\right)-5ab\cosh(x)-5ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx}{2(a^2-b^2+c^2)}-\frac{5(ab\sinh(x)+ac\cosh(x))}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}}{3\left(a^2-b^2+c^2\right)}-\frac{b\sinh(x)+c\cosh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{2\left(3a^2+2b^2-2c^2\right)-5ab\cosh(x)-5ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx}{2(a^2-b^2+c^2)}-\frac{5(ab\sinh(x)+ac\cosh(x))}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}}{3\left(a^2-b^2+c^2\right)}-\frac{b\sinh(x)+c\cosh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3}$$

$\Big\downarrow$ 3042

$$-\frac{b\sinh(x)+c\cosh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3}+\frac{-\frac{5(ab\sinh(x)+ac\cosh(x))}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2\left(3a^2+2b^2-2c^2\right)-5ab\cos(ix)+5iac\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^2}dx}{2(a^2-b^2+c^2)}}{3\left(a^2-b^2+c^2\right)}$$

$\Big\downarrow$ 3632

$$\frac{\frac{\frac{3a\left(2a^2+3b^2-3c^2\right)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}-\frac{b\sinh(x)\left(11a^2+4b^2-4c^2\right)+c\cosh(x)\left(11a^2+4b^2-4c^2\right)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}-\frac{5(ab\sinh(x)+ac\cosh(x))}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}}{3\left(a^2-b^2+c^2\right)}-\frac{b\sinh(x)+c\cosh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3}$$

3.745. $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(x)+c\cosh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3}+\frac{-\frac{5(ab\sinh(x)+ac\cosh(x))}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}+\frac{-\frac{b\sinh(x)\left(11a^2+4b^2-4c^2\right)+c\cosh(x)\left(11a^2+4b^2-4c^2\right)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{3a\left(2a^2+3b^2-3c^2\right)\int\frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{a^2-b^2+c^2}}{2(a^2-b^2+c^2)}}{3\left(a^2-b^2+c^2\right)}$$

$$\Big\downarrow \text{3603}$$

$$\frac{\frac{\frac{6a\left(2a^2+3b^2-3c^2\right)\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+2c\tanh\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2+c^2}-\frac{b\sinh(x)\left(11a^2+4b^2-4c^2\right)+c\cosh(x)\left(11a^2+4b^2-4c^2\right)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}-\frac{5(ab\sinh(x)+ac\cosh(\ldots}{2(a^2-b^2+c^2)(a+b\cosh(\ldots}}{3\left(a^2-b^2+c^2\right)}$$
$$\frac{b\sinh(x)+c\cosh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{-\frac{12a\left(2a^2+3b^2-3c^2\right)\int\frac{1}{4\left(a^2-b^2+c^2\right)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}-\frac{b\sinh(x)\left(11a^2+4b^2-4c^2\right)+c\cosh(x)\left(11a^2+4b^2-4c^2\right)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}-\frac{5(\ldots}{2(a^2-b^2\ldots}}{3\left(a^2-b^2+c^2\right)}$$
$$\frac{b\sinh(x)+c\cosh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{-\frac{6a\left(2a^2+3b^2-3c^2\right)\text{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{b\sinh(x)\left(11a^2+4b^2-4c^2\right)+c\cosh(x)\left(11a^2+4b^2-4c^2\right)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}-\frac{5(ab\sinh(x)+ac\cosh(x))}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))\ldots}}{3\left(a^2-b^2+c^2\right)}$$
$$\frac{b\sinh(x)+c\cosh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3}$$

input
```
Int[(a + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

3.745. $\int\frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx$

output
```
-1/3*(c*Cosh[x] + b*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]
)^3) + ((-5*(a*c*Cosh[x] + a*b*Sinh[x]))/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[
x] + c*Sinh[x])^2) + ((-6*a*(2*a^2 + 3*b^2 - 3*c^2)*ArcTanh[(2*c - 2*(a -
b)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^2])])/(a^2 - b^2 + c^2)^(3/2) - (c*(11
*a^2 + 4*b^2 - 4*c^2)*Cosh[x] + b*(11*a^2 + 4*b^2 - 4*c^2)*Sinh[x])/((a^2
- b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])))/(2*(a^2 - b^2 + c^2)))/(3*(a^2
- b^2 + c^2))
```

3.745.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.745. $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx$

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.745.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1587 vs. 2(212) = 424.

Time = 118.96 (sec) , antiderivative size = 1588, normalized size of antiderivative = 7.22

method	result	size
risch	Expression too large to display	1588
default	Expression too large to display	1598

input
```
int(1/(a+b*cosh(x)+c*sinh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(33*a^2*b*c^2+102*a^4*b*exp(x)^2+36*a^2*b^3*exp(x)^2+30*a^4*b*exp(x)^4
+45*a^2*b^3*exp(x)^4+8*b^2*c^3+15*exp(x)*a*b^4-12*b*c^4+8*b^3*c^2-11*c^3*a
^2+4*b^5+6*a^3*b^2*exp(x)^5+9*a*b^4*exp(x)^5+82*a^3*b^2*exp(x)^3+24*a*b^4*
exp(x)^3+60*a^3*b^2*exp(x)+11*a^2*b^3+12*b^5*exp(x)^2+4*c^5+44*a^5*exp(x)^
3-12*b^4*c-33*a^2*b^2*c-12*c^5*exp(x)^2-12*b^4*c*exp(x)^2-24*b^3*c^2*exp(x
)^2-102*a^4*c*exp(x)^2-9*a*c^4*exp(x)^5+24*a*c^4*exp(x)^3+6*a^3*c^2*exp(x)
^5+24*b^2*c^3*exp(x)^2+36*a^2*c^3*exp(x)^2+12*b*c^4*exp(x)^2-82*a^3*c^2*ex
p(x)^3-45*a^2*c^3*exp(x)^4+30*a^4*c*exp(x)^4+60*a^3*c^2*exp(x)-15*a*c^4*ex
p(x)+12*a^3*b*c*exp(x)^5+18*a*b^3*c*exp(x)^5-18*a*b*c^3*exp(x)^5+45*a^2*b^
2*c*exp(x)^4-45*a^2*b*c^2*exp(x)^4-48*a*b^2*c^2*exp(x)^3-36*a^2*b^2*c*exp(
x)^2-36*a^2*b*c^2*exp(x)^2-120*a^3*b*c*exp(x)-30*a*b^3*c*exp(x)+30*a*b*c^3
*exp(x))/(a^2-b^2+c^2)^3/(b*exp(x)^2+exp(x)^2*c+2*a*exp(x)+b-c)^3+1/(a^2-b
^2+c^2)^(7/2)*a^3*ln(exp(x)+((a^2-b^2+c^2)^(7/2)*a-a^8+4*a^6*b^2-4*a^6*c^2
-6*a^4*b^4+12*a^4*b^2*c^2-6*a^4*c^4+4*a^2*b^6-12*a^2*b^4*c^2+12*a^2*b^2*c^
4-4*a^2*c^6-b^8+4*c^2*b^6-6*b^4*c^4+4*c^6*b^2-c^8)/(a^2-b^2+c^2)^(7/2)/(b+
c))+3/2/(a^2-b^2+c^2)^(7/2)*a*ln(exp(x)+((a^2-b^2+c^2)^(7/2)*a-a^8+4*a^6*b
^2-4*a^6*c^2-6*a^4*b^4+12*a^4*b^2*c^2-6*a^4*c^4+4*a^2*b^6-12*a^2*b^4*c^2+1
2*a^2*b^2*c^4-4*a^2*c^6-b^8+4*c^2*b^6-6*b^4*c^4+4*c^6*b^2-c^8)/(a^2-b^2+c^
2)^(7/2)/(b+c))*b^2-3/2/(a^2-b^2+c^2)^(7/2)*a*ln(exp(x)+((a^2-b^2+c^2)^(7/
2)*a-a^8+4*a^6*b^2-4*a^6*c^2-6*a^4*b^4+12*a^4*b^2*c^2-6*a^4*c^4+4*a^2*b...
```

3.745.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11492 vs. $2(210) = 420$.

Time = 0.49 (sec) , antiderivative size = 23093, normalized size of antiderivative = 104.97

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.745.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))**4,x)
```

output
```
Timed out
```

3.745.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.745.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(210) = 420.

Time = 0.30 (sec) , antiderivative size = 717, normalized size of antiderivative = 3.26

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx = \frac{(2\,a^3+3\,ab^2-3\,ac^2)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6+3\,a^4c^2-6\,a^2b^2c^2+3\,b^4c^2+3\,a^2c^4-3\,b^2c^4+c^6)\sqrt{-a^2+b^2-c^2}} + \frac{6\,a^3b^2e^{(5\,x)}+9\,ab^4e^{(5\,x)}+12\,a^3bce^{(5\,x)}+18\,ab^3ce^{(5\,x)}+6\,a^3c^2e^{(5\,x)}-18\,abc^3e^{(5\,x)}-9\,ac^4e^{(5\,x)}+30\,a^4be^{(}}{}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="giac")
```

output
```
(2*a^3 + 3*a*b^2 - 3*a*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c
^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 3*a^4*c^2 - 6*a^2*b^2*c^2 + 3*b
^4*c^2 + 3*a^2*c^4 - 3*b^2*c^4 + c^6)*sqrt(-a^2 + b^2 - c^2)) + 1/3*(6*a^3
*b^2*e^(5*x) + 9*a*b^4*e^(5*x) + 12*a^3*b*c*e^(5*x) + 18*a*b^3*c*e^(5*x) +
 6*a^3*c^2*e^(5*x) - 18*a*b*c^3*e^(5*x) - 9*a*c^4*e^(5*x) + 30*a^4*b*e^(4*
x) + 45*a^2*b^3*e^(4*x) + 30*a^4*c*e^(4*x) + 45*a^2*b^2*c*e^(4*x) - 45*a^2
*b*c^2*e^(4*x) - 45*a^2*c^3*e^(4*x) + 44*a^5*e^(3*x) + 82*a^3*b^2*e^(3*x)
+ 24*a*b^4*e^(3*x) - 82*a^3*c^2*e^(3*x) - 48*a*b^2*c^2*e^(3*x) + 24*a*c^4*
e^(3*x) + 102*a^4*b*e^(2*x) + 36*a^2*b^3*e^(2*x) + 12*b^5*e^(2*x) - 102*a^
4*c*e^(2*x) - 36*a^2*b^2*c*e^(2*x) - 12*b^4*c*e^(2*x) - 36*a^2*b*c^2*e^(2*
x) - 24*b^3*c^2*e^(2*x) + 36*a^2*c^3*e^(2*x) + 24*b^2*c^3*e^(2*x) + 12*b*c
^4*e^(2*x) - 12*c^5*e^(2*x) + 60*a^3*b^2*e^x + 15*a*b^4*e^x - 120*a^3*b*c*
e^x - 30*a*b^3*c*e^x + 60*a^3*c^2*e^x + 30*a*b*c^3*e^x - 15*a*c^4*e^x + 11
*a^2*b^3 + 4*b^5 - 33*a^2*b^2*c - 12*b^4*c + 33*a^2*b*c^2 + 8*b^3*c^2 - 11
*a^2*c^3 + 8*b^2*c^3 - 12*b*c^4 + 4*c^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6 + 3*a^4*c^2 - 6*a^2*b^2*c^2 + 3*b^4*c^2 + 3*a^2*c^4 - 3*b^2*c^4 + c^6)*
(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)^3)
```

3.745.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx=\int \frac{1}{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)^4}\,dx$$

input
```
int(1/(a + b*cosh(x) + c*sinh(x))^4,x)
```

output
```
int(1/(a + b*cosh(x) + c*sinh(x))^4, x)
```

3.746 $\int (a+a\cosh(x)+c\sinh(x))^3\,dx$

3.746.1 Optimal result

Integrand size = 12, antiderivative size = 105

$$\begin{aligned}\int (a+a\cosh(x)+c\sinh(x))^3\,dx &= \frac{1}{2}a\left(5a^2-3c^2\right)x+\frac{1}{6}c\left(15a^2-4c^2\right)\cosh(x)\\ &\quad+\frac{1}{6}a\left(15a^2-4c^2\right)\sinh(x)\\ &\quad+\frac{5}{6}\left(ac\cosh(x)+a^2\sinh(x)\right)(a+a\cosh(x)+c\sinh(x))\\ &\quad+\frac{1}{3}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x))^2\end{aligned}$$

output
```
1/2*a*(5*a^2-3*c^2)*x+1/6*c*(15*a^2-4*c^2)*cosh(x)+1/6*a*(15*a^2-4*c^2)*si
nh(x)+5/6*(a*c*cosh(x)+a^2*sinh(x))*(a+a*cosh(x)+c*sinh(x))+1/3*(c*cosh(x)
+a*sinh(x))*(a+a*cosh(x)+c*sinh(x))^2
```

3.746.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\begin{aligned}\int (a+a\cosh(x)+c\sinh(x))^3\,dx &= \frac{1}{12}\big(30a^3x-18ac^2x-9c\left(-5a^2+c^2\right)\cosh(x)\\ &\quad+18a^2c\cosh(2x)+3a^2c\cosh(3x)+c^3\cosh(3x)\\ &\quad+45a^3\sinh(x)-9ac^2\sinh(x)+9a^3\sinh(2x)\\ &\quad+9ac^2\sinh(2x)+a^3\sinh(3x)+3ac^2\sinh(3x)\big)\end{aligned}$$

input
```
Integrate[(a + a*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
(30*a^3*x - 18*a*c^2*x - 9*c*(-5*a^2 + c^2)*Cosh[x] + 18*a^2*c*Cosh[2*x] +
 3*a^2*c*Cosh[3*x] + c^3*Cosh[3*x] + 45*a^3*Sinh[x] - 9*a*c^2*Sinh[x] + 9*
a^3*Sinh[2*x] + 9*a*c^2*Sinh[2*x] + a^3*Sinh[3*x] + 3*a*c^2*Sinh[3*x])/12
```

3.746.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3599, 3042, 3625, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+a+c\sinh(x))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(ix)+a-ic\sin(ix))^3dx$$

$$\Big\downarrow \text{3599}$$

$$\frac{1}{3}\int (\cosh(x)a+a+c\sinh(x))\left(5\cosh(x)a^2+5a^2+5c\sinh(x)a-2c^2\right)dx+\frac{1}{3}(a\sinh(x)+c\cosh(x))(a\cosh(x)+a+c\sinh(x))^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(a\sinh(x)+c\cosh(x))(a\cosh(x)+a+c\sinh(x))^2+\frac{1}{3}\int (\cos(ix)a+a-ic\sin(ix))\left(5\cos(ix)a^2+5a^2-5ic\sin(ix)a-2c^2\right)dx$$

$$\Big\downarrow \text{3625}$$

$$\frac{1}{3}\left(\frac{\int \left(3\left(5a^2-3c^2\right)a^2+\left(15a^2-4c^2\right)\cosh(x)a^2+c\left(15a^2-4c^2\right)\sinh(x)a\right)dx}{2a}+\frac{5}{2}\left(a^2\sinh(x)+ac\cosh(x)\right)(a\text{co}\right.$$
$$\frac{1}{3}(a\sinh(x)+c\cosh(x))(a\cosh(x)+a+c\sinh(x))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3a^2x(5a^2-3c^2)+a^2(15a^2-4c^2)\sinh(x)+ac(15a^2-4c^2)\cosh(x)}{2a}+\frac{5}{2}\left(a^2\sinh(x)+ac\cosh(x)\right)(a\cosh(x)\right.$$
$$\frac{1}{3}(a\sinh(x)+c\cosh(x))(a\cosh(x)+a+c\sinh(x))^2$$

input
```
Int[(a + a*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
((c*Cosh[x] + a*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x])^2)/3 + ((5*(a*c*Cosh[
x] + a^2*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x]))/2 + (3*a^2*(5*a^2 - 3*c^2)*
x + a*c*(15*a^2 - 4*c^2)*Cosh[x] + a^2*(15*a^2 - 4*c^2)*Sinh[x])/(2*a))/3
```

3.746.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

3.746.4 Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

method	result
parts	$a^{3}x+c^{3}\left(-\frac{2}{3}+\frac{\sinh(x)^{2}}{3}\right)\cosh\left(x\right)+c\,a^{2}\cosh\left(x\right)^{3}+3a^{3}\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+\frac{a(c\sinh(x)+a)^{3}}{c}+a^{3}\left(\right.$
default	$a^{3}x+3\sinh\left(x\right)a^{3}+3c\,a^{2}\cosh\left(x\right)+3a^{3}\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+3c\,a^{2}\cosh\left(x\right)^{2}+3a\,c^{2}\left(\frac{\cosh(x)\sinh(x)}{2}\right.$
risch	$\frac{5a^{3}x}{2}-\frac{3a\,c^{2}x}{2}+\frac{e^{3x}a^{3}}{24}+\frac{a^{2}c\,e^{3x}}{8}+\frac{e^{3x}a\,c^{2}}{8}+\frac{e^{3x}c^{3}}{24}+\frac{3a^{3}e^{2x}}{8}+\frac{3\,e^{2x}c\,a^{2}}{4}+\frac{3\,e^{2x}a\,c^{2}}{8}+\frac{15a^{3}e^{x}}{8}+\frac{15\,e^{x}c\,a^{2}}{8}-$

input
```
int((a+a*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+c^3*(-2/3+1/3*sinh(x)^2)*cosh(x)+c*a^2*cosh(x)^3+3*a^3*(1/2*cosh(x)*
sinh(x)+1/2*x)+a*(c*sinh(x)+a)^3/c+a^3*(2/3+1/3*cosh(x)^2)*sinh(x)+3*c*a^2
*cosh(x)+3*a*c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

3.746.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.37

$$\int (a+a\cosh(x)+c\sinh(x))^{3}\,dx$$
$$=\frac{3}{2}\,a^{2}c\cosh\left(x\right)^{2}+\frac{1}{12}\left(3\,a^{2}c+c^{3}\right)\cosh\left(x\right)^{3}+\frac{1}{12}\left(a^{3}+3\,ac^{2}\right)\sinh\left(x\right)^{3}+\frac{1}{4}\left(6\,a^{2}c+\left(3\,a^{2}c+c^{3}\right)\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+\frac{1}{2}\left(5\,a^{3}-3\,ac^{2}\right)x+\frac{3}{4}\left(5\,a^{2}c-c^{3}\right)\cosh\left(x\right)+\frac{1}{4}\left(15\,a^{3}-3\,ac^{2}+\left(a^{3}+3\,ac^{2}\right)\cosh\left(x\right)^{2}+6\left(a^{3}+ac^{2}\right)\cosh\left(x\right)\right)\sinh\left(x\right)$$

input
```
integrate((a+a*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

output
```
3/2*a^2*c*cosh(x)^2 + 1/12*(3*a^2*c + c^3)*cosh(x)^3 + 1/12*(a^3 + 3*a*c^2
)*sinh(x)^3 + 1/4*(6*a^2*c + (3*a^2*c + c^3)*cosh(x))*sinh(x)^2 + 1/2*(5*a
^3 - 3*a*c^2)*x + 3/4*(5*a^2*c - c^3)*cosh(x) + 1/4*(15*a^3 - 3*a*c^2 + (a
^3 + 3*a*c^2)*cosh(x)^2 + 6*(a^3 + a*c^2)*cosh(x))*sinh(x)
```

3.746.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.80

$$\int (a+a\cosh(x)+c\sinh(x))^3\,dx = -\frac{3a^3x\sinh^2(x)}{2}+\frac{3a^3x\cosh^2(x)}{2}+a^3x-\frac{2a^3\sinh^3(x)}{3}+a^3\sinh(x)\cosh^2(x)+\frac{3a^3\sinh(x)\cosh(x)}{2}+3a^3\sinh(x)+a^2c\cosh^3(x)+3a^2c\cosh^2(x)+3a^2c\cosh(x)+\frac{3ac^2x\sinh^2(x)}{2}-\frac{3ac^2x\cosh^2(x)}{2}+ac^2\sinh^3(x)+\frac{3ac^2\sinh(x)\cosh(x)}{2}+c^3\sinh^2(x)\cosh(x)-\frac{2c^3\cosh^3(x)}{3}$$

input
```
integrate((a+a*cosh(x)+c*sinh(x))**3,x)
```

output
```
-3*a**3*x*sinh(x)**2/2 + 3*a**3*x*cosh(x)**2/2 + a**3*x - 2*a**3*sinh(x)**
3/3 + a**3*sinh(x)*cosh(x)**2 + 3*a**3*sinh(x)*cosh(x)/2 + 3*a**3*sinh(x)
+ a**2*c*cosh(x)**3 + 3*a**2*c*cosh(x)**2 + 3*a**2*c*cosh(x) + 3*a*c**2*x*
sinh(x)**2/2 - 3*a*c**2*x*cosh(x)**2/2 + a*c**2*sinh(x)**3 + 3*a*c**2*sinh
(x)*cosh(x)/2 + c**3*sinh(x)**2*cosh(x) - 2*c**3*cosh(x)**3/3
```

3.746.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int (a+a\cosh(x)+c\sinh(x))^3\,dx = a^2c\cosh(x)^3+ac^2\sinh(x)^3+a^3x+\frac{1}{24}c^3\left(e^{(3x)}-9e^{(-x)}+e^{(-3x)}-9e^{x}\right)+\frac{1}{24}a^3\left(e^{(3x)}-9e^{(-x)}-e^{(-3x)}+9e^{x}\right)+3(c\cosh(x)+a\sinh(x))a^2+\frac{3}{8}\left(8ac\cosh(x)^2+a^2\left(4x+e^{(2x)}-e^{(-2x)}\right)-c^2\left(4x-e^{(2x)}+e^{(-2x)}\right)\right)a$$

input
```
integrate((a+a*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

output
```
a^2*c*cosh(x)^3 + a*c^2*sinh(x)^3 + a^3*x + 1/24*c^3*(e^(3*x) - 9*e^(-x) +
 e^(-3*x) - 9*e^x) + 1/24*a^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x) + 3*
(c*cosh(x) + a*sinh(x))*a^2 + 3/8*(8*a*c*cosh(x)^2 + a^2*(4*x + e^(2*x) -
e^(-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*a
```

3.746.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.77

$$\int (a+a\cosh(x)+c\sinh(x))^3\,dx = \frac{1}{24}\,a^3e^{(3\,x)} + \frac{1}{8}\,a^2ce^{(3\,x)} + \frac{1}{8}\,ac^2e^{(3\,x)} + \frac{1}{24}\,c^3e^{(3\,x)} + \frac{3}{8}\,a^3e^{(2\,x)} + \frac{3}{4}\,a^2ce^{(2\,x)} + \frac{3}{8}\,ac^2e^{(2\,x)} + \frac{15}{8}\,a^3e^x + \frac{15}{8}\,a^2ce^x - \frac{3}{8}\,ac^2e^x - \frac{3}{8}\,c^3e^x + \frac{1}{2}\left(5\,a^3 - 3\,ac^2\right)x - \frac{1}{24}\left(a^3 - 3\,a^2c + 3\,ac^2 - c^3 + 9\left(5\,a^3 - 5\,a^2c - ac^2 + c^3\right)e^{(2\,x)} + 9\left(a^3 - 2\,a^2c + ac^2\right)e^x\right)e^{(-3\,x)}$$

input
```
integrate((a+a*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

output
```
1/24*a^3*e^(3*x) + 1/8*a^2*c*e^(3*x) + 1/8*a*c^2*e^(3*x) + 1/24*c^3*e^(3*x
) + 3/8*a^3*e^(2*x) + 3/4*a^2*c*e^(2*x) + 3/8*a*c^2*e^(2*x) + 15/8*a^3*e^x
 + 15/8*a^2*c*e^x - 3/8*a*c^2*e^x - 3/8*c^3*e^x + 1/2*(5*a^3 - 3*a*c^2)*x
- 1/24*(a^3 - 3*a^2*c + 3*a*c^2 - c^3 + 9*(5*a^3 - 5*a^2*c - a*c^2 + c^3)*
e^(2*x) + 9*(a^3 - 2*a^2*c + a*c^2)*e^x)*e^(-3*x)
```

3.746.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int (a+a\cosh(x)+c\sinh(x))^3\,dx = 3\,a^3\sinh(x) + a^3\,x + \cosh(x)^3\left(a^2\,c - \frac{2\,c^3}{3}\right) + \sinh(x)^3\left(a\,c^2 - \frac{2\,a^3}{3}\right) + a^3\cosh(x)^2\sinh(x) + c^3\cosh(x)\,\sinh(x)^2 + 3\,a^2\,c\cosh(x) + 3\,a^2\,c\cosh(x)^2 + \frac{3\,a\cosh(x)\,\sinh(x)\,\left(a^2+c^2\right)}{2} + \frac{3\,a\,x\cosh(x)^2\left(a^2-c^2\right)}{2} - \frac{3\,a\,x\sinh(x)^2\left(a^2-c^2\right)}{2}$$

input
```
int((a + a*cosh(x) + c*sinh(x))^3,x)
```

output
```
3*a^3*sinh(x) + a^3*x + cosh(x)^3*(a^2*c - (2*c^3)/3) + sinh(x)^3*(a*c^2 -
 (2*a^3)/3) + a^3*cosh(x)^2*sinh(x) + c^3*cosh(x)*sinh(x)^2 + 3*a^2*c*cosh
(x) + 3*a^2*c*cosh(x)^2 + (3*a*cosh(x)*sinh(x)*(a^2 + c^2))/2 + (3*a*x*cos
h(x)^2*(a^2 - c^2))/2 - (3*a*x*sinh(x)^2*(a^2 - c^2))/2
```

3.746. $\int (a + a\cosh(x) + c\sinh(x))^3 \, dx$

3.747 $\int (a+a\cosh(x)+c\sinh(x))^2\,dx$

3.747.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (a+a\cosh(x)+c\sinh(x))^2\,dx=\frac{1}{2}\left(3a^2-c^2\right)x+\frac{3}{2}ac\cosh(x)+\frac{3}{2}a^2\sinh(x)+\frac{1}{2}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x))$$

output
```
1/2*(3*a^2-c^2)*x+3/2*a*c*cosh(x)+3/2*a^2*sinh(x)+1/2*(c*cosh(x)+a*sinh(x)
)*(a+a*cosh(x)+c*sinh(x))
```

3.747.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int (a+a\cosh(x)+c\sinh(x))^2\,dx=\frac{1}{2}\left(3a^2-c^2\right)x+2ac\cosh(x)+\frac{1}{2}ac\cosh(2x)+2a^2\sinh(x)+\frac{1}{4}\left(a^2+c^2\right)\sinh(2x)$$

input
```
Integrate[(a + a*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
((3*a^2 - c^2)*x)/2 + 2*a*c*Cosh[x] + (a*c*Cosh[2*x])/2 + 2*a^2*Sinh[x] +
((a^2 + c^2)*Sinh[2*x])/4
```

3.747.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3599, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh (x)+a+c\sinh (x))^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos (ix)+a-ic\sin (ix))^2dx$$

$$\Big\downarrow \text{3599}$$

$$\frac{1}{2}\int \left(3\cosh (x)a^2+3a^2+3c\sinh (x)a-c^2\right)dx+\frac{1}{2}(a\sinh (x)+c\cosh (x))(a\cosh (x)+a+c\sinh (x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(x\left(3a^2-c^2\right)+3a^2\sinh (x)+3ac\cosh (x)\right)+\frac{1}{2}(a\sinh (x)+c\cosh (x))(a\cosh (x)+a+c\sinh (x))$$

input
```
Int[(a + a*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
((3*a^2 - c^2)*x + 3*a*c*Cosh[x] + 3*a^2*Sinh[x])/2 + ((c*Cosh[x] + a*Sinh
[x])*(a + a*Cosh[x] + c*Sinh[x]))/2
```

3.747.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

3.747.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

method	result
default	$a^2x+2a^2\sinh\left(x\right)+2ac\cosh\left(x\right)+a^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+ac\cosh\left(x\right)^2+c^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)$
parts	$a^2x+a^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+2a\left(\frac{c\sinh(x)^2}{2}+a\sinh\left(x\right)\right)+c^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)+2ac\cosh\left(x\right)$
risch	$\frac{3a^2x}{2}-\frac{c^2x}{2}+\frac{a^2\mathrm{e}^{2x}}{8}+\frac{\mathrm{e}^{2x}ac}{4}+\frac{\mathrm{e}^{2x}c^2}{8}+a^2\mathrm{e}^{x}+\mathrm{e}^{x}ac-\mathrm{e}^{-x}a^2+\mathrm{e}^{-x}ac-\frac{\mathrm{e}^{-2x}a^2}{8}+\frac{\mathrm{e}^{-2x}ac}{4}-\frac{\mathrm{e}^{-2x}c^2}{8}$

input
```
int((a+a*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2*a^2*sinh(x)+2*a*c*cosh(x)+a^2*(1/2*cosh(x)*sinh(x)+1/2*x)+a*c*cosh
(x)^2+c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

3.747.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (a+a\cosh(x)+c\sinh(x))^2\,dx=\frac{1}{2}\,ac\cosh\left(x\right)^2+\frac{1}{2}\,ac\sinh\left(x\right)^2+2\,ac\cosh\left(x\right)+\frac{1}{2}\left(3\,a^2-c^2\right)x+\frac{1}{2}\left(4\,a^2+\left(a^2+c^2\right)\cosh\left(x\right)\right)\sinh\left(x\right)$$

input
```
integrate((a+a*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

output
```
1/2*a*c*cosh(x)^2 + 1/2*a*c*sinh(x)^2 + 2*a*c*cosh(x) + 1/2*(3*a^2 - c^2)*
x + 1/2*(4*a^2 + (a^2 + c^2)*cosh(x))*sinh(x)
```

3.747.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int (a + a\cosh(x) + c\sinh(x))^2\,dx = -\frac{a^2x\sinh^2(x)}{2} + \frac{a^2x\cosh^2(x)}{2} + a^2x + \frac{a^2\sinh(x)\cosh(x)}{2} + 2a^2\sinh(x) + ac\cosh^2(x) + 2ac\cosh(x) + \frac{c^2x\sinh^2(x)}{2} - \frac{c^2x\cosh^2(x)}{2} + \frac{c^2\sinh(x)\cosh(x)}{2}$$

input
```
integrate((a+a*cosh(x)+c*sinh(x))**2,x)
```

output
```
-a**2*x*sinh(x)**2/2 + a**2*x*cosh(x)**2/2 + a**2*x + a**2*sinh(x)*cosh(x)
/2 + 2*a**2*sinh(x) + a*c*cosh(x)**2 + 2*a*c*cosh(x) + c**2*x*sinh(x)**2/2
 - c**2*x*cosh(x)**2/2 + c**2*sinh(x)*cosh(x)/2
```

3.747.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int (a + a\cosh(x) + c\sinh(x))^2\,dx = ac\cosh(x)^2 + \frac{1}{8}a^2\left(4x + e^{(2x)} - e^{(-2x)}\right) - \frac{1}{8}c^2\left(4x - e^{(2x)} + e^{(-2x)}\right) + a^2x + 2(c\cosh(x) + a\sinh(x))a$$

input
```
integrate((a+a*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

output
```
a*c*cosh(x)^2 + 1/8*a^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*c^2*(4*x - e^(2*x
) + e^(-2*x)) + a^2*x + 2*(c*cosh(x) + a*sinh(x))*a
```

3.747.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.42

$$\int (a + a\cosh(x) + c\sinh(x))^2\, dx = \frac{1}{8}\, a^2 e^{(2\, x)} + \frac{1}{4}\, ace^{(2\, x)} + \frac{1}{8}\, c^2 e^{(2\, x)} + a^2 e^x + ace^x + \frac{1}{2}\, \left(3\, a^2 - c^2\right)x - \frac{1}{8}\, \left(a^2 - 2\, ac + c^2 + 8\, \left(a^2 - ac\right)e^x\right)e^{(-2\, x)}$$

input
```
integrate((a+a*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

output
```
1/8*a^2*e^(2*x) + 1/4*a*c*e^(2*x) + 1/8*c^2*e^(2*x) + a^2*e^x + a*c*e^x +
1/2*(3*a^2 - c^2)*x - 1/8*(a^2 - 2*a*c + c^2 + 8*(a^2 - a*c)*e^x)*e^(-2*x)
```

3.747.9 Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int (a + a\cosh(x) + c\sinh(x))^2\, dx = 2\, a^2 \sinh(x) + \frac{3\, a^2\, x}{2} - \frac{c^2\, x}{2} + a\, c\cosh(x)^2 + \frac{a^2 \cosh(x)\, \sinh(x)}{2} + \frac{c^2 \cosh(x)\, \sinh(x)}{2} + 2\, a\, c\cosh(x)$$

input
```
int((a + a*cosh(x) + c*sinh(x))^2,x)
```

output
```
2*a^2*sinh(x) + (3*a^2*x)/2 - (c^2*x)/2 + a*c*cosh(x)^2 + (a^2*cosh(x)*sin
h(x))/2 + (c^2*cosh(x)*sinh(x))/2 + 2*a*c*cosh(x)
```

3.748 $\int (a+a\cosh(x)+c\sinh(x))\,dx$

3.748.1 Optimal result

Integrand size = 10, antiderivative size = 12

$$\int (a+a\cosh(x)+c\sinh(x))\,dx=ax+c\cosh(x)+a\sinh(x)$$

output
```
a*x+c*cosh(x)+a*sinh(x)
```

3.748.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+a\cosh(x)+c\sinh(x))\,dx=ax+c\cosh(x)+a\sinh(x)$$

input
```
Integrate[a + a*Cosh[x] + c*Sinh[x],x]
```

output
```
a*x + c*Cosh[x] + a*Sinh[x]
```

3.748.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+a+c\sinh(x))\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+a\sinh(x)+c\cosh(x)$$

input
```
Int[a + a*Cosh[x] + c*Sinh[x],x]
```

output
```
a*x + c*Cosh[x] + a*Sinh[x]
```

3.748.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.748.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$ax+c\cosh\left(x\right)+a\sinh\left(x\right)$	13
parts	$ax+c\cosh\left(x\right)+a\sinh\left(x\right)$	13
risch	$\frac{\left(a\,{\mathrm e}^{2x}+{\mathrm e}^{2x}c+2ax\,{\mathrm e}^{x}-a+c\right){\mathrm e}^{-x}}{2}$	30

input
```
int(a+a*cosh(x)+c*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
a*x+c*cosh(x)+a*sinh(x)
```

3.748.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + a\cosh(x) + c\sinh(x))\,dx = ax + c\cosh(x) + a\sinh(x)$$

input
```
integrate(a+a*cosh(x)+c*sinh(x),x, algorithm="fricas")
```

output
```
a*x + c*cosh(x) + a*sinh(x)
```

3.748.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + a\cosh(x) + c\sinh(x))\,dx = ax + a\sinh(x) + c\cosh(x)$$

input
```
integrate(a+a*cosh(x)+c*sinh(x),x)
```

output
```
a*x + a*sinh(x) + c*cosh(x)
```

3.748.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + a\cosh(x) + c\sinh(x))\,dx = ax + c\cosh(x) + a\sinh(x)$$

input
```
integrate(a+a*cosh(x)+c*sinh(x),x, algorithm="maxima")
```

output
```
a*x + c*cosh(x) + a*sinh(x)
```

3.748.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int (a + a\cosh(x) + c\sinh(x))\, dx = ax + \frac{1}{2}c\left(e^{(-x)} + e^{x}\right) - \frac{1}{2}a\left(e^{(-x)} - e^{x}\right)$$

input
```
integrate(a+a*cosh(x)+c*sinh(x),x, algorithm="giac")
```

output
```
a*x + 1/2*c*(e^(-x) + e^x) - 1/2*a*(e^(-x) - e^x)
```

3.748.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + a\cosh(x) + c\sinh(x))\, dx = a\,x + c\cosh(x) + a\sinh(x)$$

input
```
int(a + a*cosh(x) + c*sinh(x),x)
```

output
```
a*x + c*cosh(x) + a*sinh(x)
```

3.749 $\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx$

3.749.1 Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx=\frac{\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}$$

output
```
ln(a+c*tanh(1/2*x))/c
```

3.749.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx=-\frac{\log\left(\cosh\left(\frac{x}{2}\right)\right)}{c}+\frac{\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)}{c}$$

input
```
Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-1),x]
```

output
```
-(Log[Cosh[x/2]]/c) + Log[a*Cosh[x/2] + c*Sinh[x/2]]/c
```

3.749.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cosh(x)+a+c\sinh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a\cos(ix)+a-ic\sin(ix)}dx$$
$$\Big\downarrow \text{3603}$$
$$2\int \frac{1}{2a+2c\tanh\left(\frac{x}{2}\right)}d\tanh\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{16}$$
$$\frac{\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}$$

input
```
Int[(a + a*Cosh[x] + c*Sinh[x])^(-1),x]
```

output
```
Log[a + c*Tanh[x/2]]/c
```

3.749.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.749.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}$	14
risch	$\frac{\ln\left(e^{x}+\frac{a-c}{a+c}\right)}{c}-\frac{\ln\left(e^{x}+1\right)}{c}$	31

input
```
int(1/(a+a*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+c*tanh(1/2*x))/c
```

3.749.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx = \frac{\log\left((a+c)\cosh\left(x\right)+(a+c)\sinh\left(x\right)+a-c\right)-\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)}{c}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

output
```
(log((a + c)*cosh(x) + (a + c)*sinh(x) + a - c) - log(cosh(x) + sinh(x) +
1))/c
```

3.749.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx = \begin{cases} \frac{\log\left(\frac{a}{c}+\tanh\left(\frac{x}{2}\right)\right)}{c} & \text{for } c \neq 0 \\ \frac{\tanh\left(\frac{x}{2}\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x)),x)
```

output
```
Piecewise((log(a/c + tanh(x/2))/c, Ne(c, 0)), (tanh(x/2)/a, True))
```

3.749.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.40

$$\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx = \frac{\log\left(-(a-c)e^{(-x)}-a-c\right)}{c} - \frac{\log\left(e^{(-x)}+1\right)}{c}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

output
```
log(-(a - c)*e^(-x) - a - c)/c - log(e^(-x) + 1)/c
```

3.749.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx = \frac{(a+c)\log\left(|ae^{x}+ce^{x}+a-c|\right)}{ac+c^{2}} - \frac{\log\left(e^{x}+1\right)}{c}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

output
```
(a + c)*log(abs(a*e^x + c*e^x + a - c))/(a*c + c^2) - log(e^x + 1)/c
```

3.749.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.07

$$\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-c^2}+a\,\mathrm{e}^x\,\sqrt{-c^2}+c\,\mathrm{e}^x\,\sqrt{-c^2}}{c^2}\right)}{\sqrt{-c^2}}$$

input
```
int(1/(a + a*cosh(x) + c*sinh(x)),x)
```

output
```
-(2*atan((a*(-c^2)^(1/2) + a*exp(x)*(-c^2)^(1/2) + c*exp(x)*(-c^2)^(1/2))/
c^2))/(-c^2)^(1/2)
```

3.750 $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx$

3.750.1 Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx=\frac{a\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^3}-\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}$$

output
```
a*ln(a+c*tanh(1/2*x))/c^3+(-c*cosh(x)-a*sinh(x))/c^2/(a+a*cosh(x)+c*sinh(x
))
```

3.750.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(43) = 86.

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx =\frac{2a\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)\right)+\frac{c(-a^2+c^2)\sinh\left(\frac{x}{2}\right)}{a\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)}-c\tanh\left(\frac{x}{2}\right)}{2c^3}$$

input
```
Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-2),x]
```

output
```
(2*a*(-Log[Cosh[x/2]] + Log[a*Cosh[x/2] + c*Sinh[x/2]]) + (c*(-a^2 + c^2)*
Sinh[x/2])/(a*(a*Cosh[x/2] + c*Sinh[x/2])) - c*Tanh[x/2])/(2*c^3)
```

3.750.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3608, 25, 27, 3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x)+a+c\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(ix)+a-ic\sin(ix))^2}dx$$

$$\Big\downarrow \text{3608}$$

$$-\frac{\int -\frac{a}{\cosh(x)a+a+c\sinh(x)}dx}{c^2}-\frac{a\sinh(x)+c\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a}{\cosh(x)a+a+c\sinh(x)}dx}{c^2}-\frac{a\sinh(x)+c\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{1}{\cosh(x)a+a+c\sinh(x)}dx}{c^2}-\frac{a\sinh(x)+c\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\sinh(x)+c\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}+\frac{a\int \frac{1}{\cos(ix)a+a-ic\sin(ix)}dx}{c^2}$$

$$\Big\downarrow \text{3603}$$

$$\frac{2a\int \frac{1}{2a+2c\tanh\left(\frac{x}{2}\right)}d\tanh\left(\frac{x}{2}\right)}{c^2}-\frac{a\sinh(x)+c\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^3}-\frac{a\sinh(x)+c\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}$$

input
```
Int[(a + a*Cosh[x] + c*Sinh[x])^(-2),x]
```

output
```
(a*Log[a + c*Tanh[x/2]])/c^3 - (c*Cosh[x] + a*Sinh[x])/(c^2*(a + a*Cosh[x]
 + c*Sinh[x]))
```

3.750.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.750. $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx$

3.750.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2c^{2}}+\frac{a\ln\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^{3}}-\frac{-a^{2}+c^{2}}{2c^{3}\left(a+c\tanh\left(\frac{x}{2}\right)\right)}$	49
risch	$\frac{2a\,\mathrm{e}^{x}+2a-2c}{c^{2}\left(a\,\mathrm{e}^{2x}+\mathrm{e}^{2x}c+2a\,\mathrm{e}^{x}+a-c\right)}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a-c}{a+c}\right)}{c^{3}}-\frac{a\ln\left(\mathrm{e}^{x}+1\right)}{c^{3}}$	71

input
```
int(1/(a+a*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2*tanh(1/2*x)+a*ln(a+c*tanh(1/2*x))/c^3-1/2/c^3*(-a^2+c^2)/(a+c*tan
h(1/2*x))
```

3.750.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(41) = 82.

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 5.49

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx$$
$$= \frac{2\,ac\cosh\left(x\right)+2\,ac\sinh\left(x\right)+2\,ac-2\,c^{2}+\left(2\,a^{2}\cosh\left(x\right)+\left(a^{2}+ac\right)\cosh\left(x\right)^{2}+\left(a^{2}+ac\right)\sinh\left(x\right)^{2}+a^{2}\right.}{}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

output
```
(2*a*c*cosh(x) + 2*a*c*sinh(x) + 2*a*c - 2*c^2 + (2*a^2*cosh(x) + (a^2 + a
*c)*cosh(x)^2 + (a^2 + a*c)*sinh(x)^2 + a^2 - a*c + 2*(a^2 + (a^2 + a*c)*c
osh(x))*sinh(x))*log((a + c)*cosh(x) + (a + c)*sinh(x) + a - c) - (2*a^2*c
osh(x) + (a^2 + a*c)*cosh(x)^2 + (a^2 + a*c)*sinh(x)^2 + a^2 - a*c + 2*(a^
2 + (a^2 + a*c)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1))/(2*a*c^3*cos
h(x) + a*c^3 - c^4 + (a*c^3 + c^4)*cosh(x)^2 + (a*c^3 + c^4)*sinh(x)^2 + 2
*(a*c^3 + (a*c^3 + c^4)*cosh(x))*sinh(x))
```

3.750.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x))**2,x)
```

output
```
Timed out
```

3.750.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(41) = 82$.

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2\left(ae^{(-x)}+a+c\right)}{2\,ac^2e^{(-x)}+ac^2+c^3+(ac^2-c^3)e^{(-2\,x)}} + \frac{a\log\left(-(a-c)e^{(-x)}-a-c\right)}{c^3} - \frac{a\log\left(e^{(-x)}+1\right)}{c^3}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

output
```
-2*(a*e^(-x) + a + c)/(2*a*c^2*e^(-x) + a*c^2 + c^3 + (a*c^2 - c^3)*e^(-2*
x)) + a*log(-(a - c)*e^(-x) - a - c)/c^3 - a*log(e^(-x) + 1)/c^3
```

3.750.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx = \frac{(a^2+ac)\log\left(|ae^x+ce^x+a-c|\right)}{ac^3+c^4} - \frac{a\log\left(e^x+1\right)}{c^3} + \frac{2\left(ae^x+a-c\right)}{(ae^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+a-c)c^2}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

output
```
(a^2 + a*c)*log(abs(a*e^x + c*e^x + a - c))/(a*c^3 + c^4) - a*log(e^x + 1)
/c^3 + 2*(a*e^x + a - c)/((a*e^(2*x) + c*e^(2*x) + 2*a*e^x + a - c)*c^2)
```

3.750.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^2} \, dx = \int \frac{1}{\left(a + a\cosh\left(x\right) + c\sinh\left(x\right)\right)^2} \, dx$$

input
```
int(1/(a + a*cosh(x) + c*sinh(x))^2,x)
```

output
```
int(1/(a + a*cosh(x) + c*sinh(x))^2, x)
```

3.751 $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx$

3.751.1 Optimal result

Integrand size = 12, antiderivative size = 89

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx = \frac{(3a^2-c^2)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5} - \frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+a^2\sinh(x))}{2c^4(a+a\cosh(x)+c\sinh(x))}$$

output
```
1/2*(3*a^2-c^2)*ln(a+c*tanh(1/2*x))/c^5+1/2*(-c*cosh(x)-a*sinh(x))/c^2/(a+
a*cosh(x)+c*sinh(x))^2-3/2*(a*c*cosh(x)+a^2*sinh(x))/c^4/(a+a*cosh(x)+c*si
nh(x))
```

3.751.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx = \frac{4(-3a^2+c^2)\log\left(\cosh\left(\frac{x}{2}\right)\right)+4(3a^2-c^2)\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)-c^2\operatorname{sech}^2\left(\frac{x}{2}\right)+\frac{(a-c)c^2(a+c)}{\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)}}{8c^5}$$

input
```
Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-3),x]
```

output
```
(4*(-3*a^2 + c^2)*Log[Cosh[x/2]] + 4*(3*a^2 - c^2)*Log[a*Cosh[x/2] + c*Sin
h[x/2]] - c^2*Sech[x/2]^2 + ((a - c)*c^2*(a + c))/(a*Cosh[x/2] + c*Sinh[x/
2])^2 + (6*c*(-a^2 + c^2)*Sinh[x/2])/(a*Cosh[x/2] + c*Sinh[x/2]) - 6*a*c*T
anh[x/2])/(8*c^5)
```

3.751.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3608, 25, 3042, 3632, 3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x)+a+c\sinh(x))^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(a\cos(ix)+a-ic\sin(ix))^3}dx$$
$$\Big\downarrow \text{ 3608}$$
$$-\frac{\int -\frac{-\cosh(x)a+2a-c\sinh(x)}{(\cosh(x)a+a+c\sinh(x))^2}dx}{2c^2}-\frac{a\sinh(x)+c\cosh(x)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}$$
$$\Big\downarrow \text{ 25}$$
$$\frac{\int \frac{-\cosh(x)a+2a-c\sinh(x)}{(\cosh(x)a+a+c\sinh(x))^2}dx}{2c^2}-\frac{a\sinh(x)+c\cosh(x)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}$$
$$\Big\downarrow \text{ 3042}$$
$$-\frac{a\sinh(x)+c\cosh(x)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}+\frac{\int \frac{-\cos(ix)a+2a+ic\sin(ix)}{(\cos(ix)a+a-ic\sin(ix))^2}dx}{2c^2}$$
$$\Big\downarrow \text{ 3632}$$
$$\frac{-\left(1-\frac{3a^2}{c^2}\right)\int \frac{1}{\cosh(x)a+a+c\sinh(x)}dx-\frac{3(a^2\sinh(x)+ac\cosh(x))}{c^2(a\cosh(x)+a+c\sinh(x))}}{2c^2}-\frac{a\sinh(x)+c\cosh(x)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}$$
$$\Big\downarrow \text{ 3042}$$
$$-\frac{a\sinh(x)+c\cosh(x)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}+\frac{-\frac{3(a^2\sinh(x)+ac\cosh(x))}{c^2(a\cosh(x)+a+c\sinh(x))}-\left(1-\frac{3a^2}{c^2}\right)\int \frac{1}{\cos(ix)a+a-ic\sin(ix)}dx}{2c^2}$$

$$\Big\downarrow \text{3603}$$

$$\frac{-2\left(1-\frac{3a^2}{c^2}\right)\int \frac{1}{2a+2c\tanh\left(\frac{x}{2}\right)}d\tanh\left(\frac{x}{2}\right)-\frac{3(a^2\sinh(x)+ac\cosh(x))}{c^2(a\cosh(x)+a+c\sinh(x))}}{2c^2}-\frac{a\sinh(x)+c\cosh(x)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{-\frac{\left(1-\frac{3a^2}{c^2}\right)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}-\frac{3(a^2\sinh(x)+ac\cosh(x))}{c^2(a\cosh(x)+a+c\sinh(x))}}{2c^2}-\frac{a\sinh(x)+c\cosh(x)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}$$

input
```
Int[(a + a*Cosh[x] + c*Sinh[x])^(-3),x]
```

output
```
-1/2*(c*Cosh[x] + a*Sinh[x])/(c^2*(a + a*Cosh[x] + c*Sinh[x])^2) + (-(((1
- (3*a^2)/c^2)*Log[a + c*Tanh[x/2]])/c) - (3*(a*c*Cosh[x] + a^2*Sinh[x]))/
(c^2*(a + a*Cosh[x] + c*Sinh[x])))/(2*c^2)
```

3.751.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.751.4 Maple [A] (verified)

Time = 11.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

method	result
default	$-\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{2} c}{2}+3 a \tanh\left(\frac{x}{2}\right)}{4 c^{4}}+\frac{\left(6 a^{2}-2 c^{2}\right) \ln\left(a+c \tanh\left(\frac{x}{2}\right)\right)}{4 c^{5}}-\frac{a^{4}-2 c^{2} a^{2}+c^{4}}{8 c^{5} \left(a+c \tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{a \left(a^{2}-c^{2}\right)}{c^{5} \left(a+c \tanh\left(\frac{x}{2}\right)\right)}$
risch	$\frac{3 \,\mathrm{e}^{3 x} a^{3}+3 a^{2} c \,\mathrm{e}^{3 x}-\mathrm{e}^{3 x} a \,c^{2}-\mathrm{e}^{3 x} c^{3}+9 a^{3} \mathrm{e}^{2 x}-3 \,\mathrm{e}^{2 x} a \,c^{2}+9 a^{3} \mathrm{e}^{x}-9 \,\mathrm{e}^{x} c \,a^{2}+a \,\mathrm{e}^{x} c^{2}-\mathrm{e}^{x} c^{3}+3 a^{3}-6 c \,a^{2}+3 a \,c^{2}}{\left(a \,\mathrm{e}^{2 x}+\mathrm{e}^{2 x} c+2 a \,\mathrm{e}^{x}+a-c\right)^{2} c^{4}}-\frac{3 \ln\left(\mathrm{e}^{x}+1\right) a^{2}}{2 c^{5}}+\frac{\ln(\mathrm{e}^{x}}{2c}$

input
```
int(1/(a+a*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/c^4*(-1/2*tanh(1/2*x)^2*c+3*a*tanh(1/2*x))+1/4*(6*a^2-2*c^2)/c^5*ln(a
+c*tanh(1/2*x))-1/8/c^5*(a^4-2*a^2*c^2+c^4)/(a+c*tanh(1/2*x))^2+a/c^5*(a^2
-c^2)/(a+c*tanh(1/2*x))
```

3.751.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1504 vs. $2(81) = 162$.

Time = 0.27 (sec) , antiderivative size = 1504, normalized size of antiderivative = 16.90

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

output
```
1/2*(6*a^3*c - 12*a^2*c^2 + 6*a*c^3 + 2*(3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4
)*cosh(x)^3 + 2*(3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4)*sinh(x)^3 + 6*(3*a^3*c
 - a*c^3)*cosh(x)^2 + 6*(3*a^3*c - a*c^3 + (3*a^3*c + 3*a^2*c^2 - a*c^3 -
c^4)*cosh(x))*sinh(x)^2 + 2*(9*a^3*c - 9*a^2*c^2 + a*c^3 - c^4)*cosh(x) +
((3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^4 + (3*a^4 + 6*a^3*
c + 2*a^2*c^2 - 2*a*c^3 - c^4)*sinh(x)^4 + 3*a^4 - 6*a^3*c + 2*a^2*c^2 + 2
*a*c^3 - c^4 + 4*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3)*cosh(x)^3 + 4*(3*a^4
+ 3*a^3*c - a^2*c^2 - a*c^3 + (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4
)*cosh(x))*sinh(x)^3 + 2*(9*a^4 - 6*a^2*c^2 + c^4)*cosh(x)^2 + 2*(9*a^4 -
6*a^2*c^2 + c^4 + 3*(3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^
2 + 6*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3)*cosh(x))*sinh(x)^2 + 4*(3*a^4 -
3*a^3*c - a^2*c^2 + a*c^3)*cosh(x) + 4*(3*a^4 - 3*a^3*c - a^2*c^2 + a*c^3
+ (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^3 + 3*(3*a^4 + 3*a
^3*c - a^2*c^2 - a*c^3)*cosh(x)^2 + (9*a^4 - 6*a^2*c^2 + c^4)*cosh(x))*sin
h(x))*log((a + c)*cosh(x) + (a + c)*sinh(x) + a - c) - ((3*a^4 + 6*a^3*c +
 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^4 + (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a
*c^3 - c^4)*sinh(x)^4 + 3*a^4 - 6*a^3*c + 2*a^2*c^2 + 2*a*c^3 - c^4 + 4*(3
*a^4 + 3*a^3*c - a^2*c^2 - a*c^3)*cosh(x)^3 + 4*(3*a^4 + 3*a^3*c - a^2*c^2
 - a*c^3 + (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x))*sinh(x)^
3 + 2*(9*a^4 - 6*a^2*c^2 + c^4)*cosh(x)^2 + 2*(9*a^4 - 6*a^2*c^2 + c^4 ...
```

3.751.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x))**3,x)
```

output
```
Timed out
```

3.751.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(81) = 162$.

Time = 0.22 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.79

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx =$$
$$-\frac{3a^3+6a^2c+3ac^2+(9a^3+9a^2c+ac^2+c^3)e^{(-x)}+3(3a^3-ac^2)e^{(-2x)}+(3a^3-3a^2c-ac^2+c^3)}{a^2c^4+2ac^5+c^6+4(a^2c^4+ac^5)e^{(-x)}+2(3a^2c^4-c^6)e^{(-2x)}+4(a^2c^4-ac^5)e^{(-3x)}+(a^2c^4-2ac^5+}$$
$$+\frac{(3a^2-c^2)\log\left(-(a-c)e^{(-x)}-a-c\right)}{2c^5}-\frac{(3a^2-c^2)\log\left(e^{(-x)}+1\right)}{2c^5}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

output
```
-(3*a^3 + 6*a^2*c + 3*a*c^2 + (9*a^3 + 9*a^2*c + a*c^2 + c^3)*e^(-x) + 3*(
3*a^3 - a*c^2)*e^(-2*x) + (3*a^3 - 3*a^2*c - a*c^2 + c^3)*e^(-3*x))/(a^2*c
^4 + 2*a*c^5 + c^6 + 4*(a^2*c^4 + a*c^5)*e^(-x) + 2*(3*a^2*c^4 - c^6)*e^(-
2*x) + 4*(a^2*c^4 - a*c^5)*e^(-3*x) + (a^2*c^4 - 2*a*c^5 + c^6)*e^(-4*x))
+ 1/2*(3*a^2 - c^2)*log(-(a - c)*e^(-x) - a - c)/c^5 - 1/2*(3*a^2 - c^2)*l
og(e^(-x) + 1)/c^5
```

3.751.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(81) = 162$.

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.30

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx$$
$$=\frac{(3a^3+3a^2c-ac^2-c^3)\log\left(|ae^x+ce^x+a-c|\right)}{2(ac^5+c^6)}-\frac{(3a^2-c^2)\log\left(e^x+1\right)}{2c^5}$$
$$+\frac{3a^3e^{(3x)}+3a^2ce^{(3x)}-ac^2e^{(3x)}-c^3e^{(3x)}+9a^3e^{(2x)}-3ac^2e^{(2x)}+9a^3e^x-9a^2ce^x+ac^2e^x-c^3e^x+3}{\left(ae^{(2x)}+ce^{(2x)}+2ae^x+a-c\right)^2c^4}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

output
```
1/2*(3*a^3 + 3*a^2*c - a*c^2 - c^3)*log(abs(a*e^x + c*e^x + a - c))/(a*c^5
 + c^6) - 1/2*(3*a^2 - c^2)*log(e^x + 1)/c^5 + (3*a^3*e^(3*x) + 3*a^2*c*e^
(3*x) - a*c^2*e^(3*x) - c^3*e^(3*x) + 9*a^3*e^(2*x) - 3*a*c^2*e^(2*x) + 9*
a^3*e^x - 9*a^2*c*e^x + a*c^2*e^x - c^3*e^x + 3*a^3 - 6*a^2*c + 3*a*c^2)/(
(a*e^(2*x) + c*e^(2*x) + 2*a*e^x + a - c)^2*c^4)
```

3.751.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx=\int \frac{1}{\left(a+a\cosh\left(x\right)+c\sinh\left(x\right)\right)^3}\,dx$$

input
```
int(1/(a + a*cosh(x) + c*sinh(x))^3,x)
```

output
```
int(1/(a + a*cosh(x) + c*sinh(x))^3, x)
```

3.752 $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx$

3.752.1 Optimal result

Integrand size = 12, antiderivative size = 140

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx = \frac{a(5a^2-3c^2)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^7} - \frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3} - \frac{5(ac\cosh(x)+a^2\sinh(x))}{6c^4(a+a\cosh(x)+c\sinh(x))^2} - \frac{c(15a^2-4c^2)\cosh(x)+a(15a^2-4c^2)\sinh(x)}{6c^6(a+a\cosh(x)+c\sinh(x))}$$

output
```
1/2*a*(5*a^2-3*c^2)*ln(a+c*tanh(1/2*x))/c^7+1/3*(-c*cosh(x)-a*sinh(x))/c^2
/(a+a*cosh(x)+c*sinh(x))^3-5/6*(a*c*cosh(x)+a^2*sinh(x))/c^4/(a+a*cosh(x)+
c*sinh(x))^2+1/6*(-c*(15*a^2-4*c^2)*cosh(x)-a*(15*a^2-4*c^2)*sinh(x))/c^6/
(a+a*cosh(x)+c*sinh(x))
```

3.752.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 300 vs. 2(140) = 280.

Time = 0.41 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.14

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx$$
$$= \frac{192(-5a^3+3ac^2)\log\left(\cosh\left(\frac{x}{2}\right)\right)+192a(5a^2-3c^2)\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)-\frac{c\operatorname{sech}^6\left(\frac{x}{2}\right)(-150a^5c+130a^3}{}}{}$$

input
```
Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-4),x]
```

output
```
(192*(-5*a^3 + 3*a*c^2)*Log[Cosh[x/2]] + 192*a*(5*a^2 - 3*c^2)*Log[a*Cosh[
x/2] + c*Sinh[x/2]] - (c*Sech[x/2]^6*(-150*a^5*c + 130*a^3*c^3 - 24*a*c^5
+ (-75*a^5*c + 75*a^3*c^3 + 12*a*c^5)*Cosh[x] + 6*a*c*(25*a^4 - 15*a^2*c^2
 + 4*c^4)*Cosh[2*x] + 75*a^5*c*Cosh[3*x] - 35*a^3*c^3*Cosh[3*x] + 4*a*c^5*
Cosh[3*x] + 150*a^6*Sinh[x] - 255*a^4*c^2*Sinh[x] + 129*a^2*c^4*Sinh[x] -
12*c^6*Sinh[x] + 120*a^6*Sinh[2*x] - 72*a^4*c^2*Sinh[2*x] + 36*a^2*c^4*Sin
h[2*x] + 30*a^6*Sinh[3*x] + 37*a^4*c^2*Sinh[3*x] - 27*a^2*c^4*Sinh[3*x] +
4*c^6*Sinh[3*x]))/(a*(a + c*Tanh[x/2])^3))/(384*c^7)
```

3.752.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3608, 25, 3042, 3635, 25, 3042, 3632, 3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x)+a+c\sinh(x))^4}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(a\cos(ix)+a-ic\sin(ix))^4}dx$$
$$\Big\downarrow \text{ 3608}$$
$$-\frac{\int -\frac{-2\cosh(x)a+3a-2c\sinh(x)}{(\cosh(x)a+a+c\sinh(x))^3}dx}{3c^2}-\frac{a\sinh(x)+c\cosh(x)}{3c^2(a\cosh(x)+a+c\sinh(x))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-2\cosh(x)a+3a-2c\sinh(x)}{(\cosh(x)a+a+c\sinh(x))^3}dx}{3c^2}-\frac{a\sinh(x)+c\cosh(x)}{3c^2(a\cosh(x)+a+c\sinh(x))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\sinh(x)+c\cosh(x)}{3c^2(a\cosh(x)+a+c\sinh(x))^3}+\frac{\int \frac{-2\cos(ix)a+3a+2ic\sin(ix)}{(\cos(ix)a+a-ic\sin(ix))^3}dx}{3c^2}$$

$$\Big\downarrow \text{3635}$$

$$\frac{-\frac{\int -\frac{-5\cosh(x)a^2-5c\sinh(x)a+2\left(5a^2-2c^2\right)}{(\cosh(x)a+a+c\sinh(x))^2}dx}{2c^2}-\frac{5\left(a^2\sinh(x)+ac\cosh(x)\right)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}}{3c^2}-\frac{a\sinh(x)+c\cosh(x)}{3c^2(a\cosh(x)+a+c\sinh(x))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{-5\cosh(x)a^2-5c\sinh(x)a+2\left(5a^2-2c^2\right)}{(\cosh(x)a+a+c\sinh(x))^2}dx}{2c^2}-\frac{5\left(a^2\sinh(x)+ac\cosh(x)\right)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}}{3c^2}-\frac{a\sinh(x)+c\cosh(x)}{3c^2(a\cosh(x)+a+c\sinh(x))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\sinh(x)+c\cosh(x)}{3c^2(a\cosh(x)+a+c\sinh(x))^3}+\frac{-\frac{5\left(a^2\sinh(x)+ac\cosh(x)\right)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}+\frac{\int \frac{-5\cos(ix)a^2+5ic\sin(ix)a+2\left(5a^2-2c^2\right)}{(\cos(ix)a+a-ic\sin(ix))^2}dx}{2c^2}}{3c^2}$$

$$\Big\downarrow \text{3632}$$

$$\frac{\frac{-3a\left(3-\frac{5a^2}{c^2}\right)\int \frac{1}{\cosh(x)a+a+c\sinh(x)}dx-\frac{a\left(15a^2-4c^2\right)\sinh(x)+c\left(15a^2-4c^2\right)\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}}{2c^2}-\frac{5\left(a^2\sinh(x)+ac\cosh(x)\right)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}}{3c^2}-\frac{a\sinh(x)+c\cosh(x)}{3c^2(a\cosh(x)+a+c\sinh(x))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\sinh(x)+c\cosh(x)}{3c^2(a\cosh(x)+a+c\sinh(x))^3}+\frac{-\frac{5\left(a^2\sinh(x)+ac\cosh(x)\right)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}+\frac{-\frac{a\left(15a^2-4c^2\right)\sinh(x)+c\left(15a^2-4c^2\right)\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}-3a\left(3-\frac{5a^2}{c^2}\right)\int \frac{1}{\cos(ix)a+a-ic\sin(ix)}dx}{2c^2}}{3c^2}$$

$$\Big\downarrow \text{3603}$$

3.752. $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx$

$$\frac{\frac{-6a\left(3-\frac{5a^2}{c^2}\right)\int\frac{1}{2a+2c\tanh\left(\frac{x}{2}\right)}d\tanh\left(\frac{x}{2}\right)-\frac{a\left(15a^2-4c^2\right)\sinh(x)+c\left(15a^2-4c^2\right)\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}}{2c^2}-\frac{5(a^2\sinh(x)+ac\cosh(x))}{2c^2(a\cosh(x)+a+c\sinh(x))^2}}{3c^2}-\frac{a\sinh(x)+c\cosh(x)}{3c^2(a\cosh(x)+a+c\sinh(x))^3}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{-\frac{3a\left(3-\frac{5a^2}{c^2}\right)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}-\frac{a\left(15a^2-4c^2\right)\sinh(x)+c\left(15a^2-4c^2\right)\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}}{2c^2}-\frac{5(a^2\sinh(x)+ac\cosh(x))}{2c^2(a\cosh(x)+a+c\sinh(x))^2}}{3c^2}-\frac{a\sinh(x)+c\cosh(x)}{3c^2(a\cosh(x)+a+c\sinh(x))^3}$$

input
```
Int[(a + a*Cosh[x] + c*Sinh[x])^(-4),x]
```

output
```
-1/3*(c*Cosh[x] + a*Sinh[x])/(c^2*(a + a*Cosh[x] + c*Sinh[x])^3) + ((-5*(a
*c*Cosh[x] + a^2*Sinh[x]))/(2*c^2*(a + a*Cosh[x] + c*Sinh[x])^2) + ((-3*a*
(3 - (5*a^2)/c^2)*Log[a + c*Tanh[x/2]])/c - (c*(15*a^2 - 4*c^2)*Cosh[x] +
a*(15*a^2 - 4*c^2)*Sinh[x])/(c^2*(a + a*Cosh[x] + c*Sinh[x])))/(2*c^2))/(3
*c^2)
```

3.752.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.752.4 Maple [A] (verified)

Time = 44.91 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\frac{c^{2} \tanh \left(\frac{x}{2}\right)^{3}}{3}-2 a c \tanh \left(\frac{x}{2}\right)^{2}+10 a^{2} \tanh \left(\frac{x}{2}\right)-3 c^{2} \tanh \left(\frac{x}{2}\right)}{8 c^{6}}+\frac{a \left(5 a^{2}-3 c^{2}\right) \ln \left(a+c \tanh \left(\frac{x}{2}\right)\right)}{2 c^{7}}-\frac{3 a \left(a^{4}-2 c^{2} a^{2}+c^{4}\right)}{8 c^{7} \left(a+c \tanh \left(\frac{x}{2}\right)\right)^{2}}-\frac{-a^{6}+3}{24 c^{7} (}$
risch	$\frac{41 a^{3} c^{2}+15 a^{5}-3 c^{3} a^{2}+4 c^{5}+75 \,{\mathrm e}^{x} a^{5}+15 \,{\mathrm e}^{5 x} a^{5}+150 \,{\mathrm e}^{3 x} a^{5}+75 a^{5} {\mathrm e}^{4 x}+150 a^{5} {\mathrm e}^{2 x}-12 a \,c^{4}-45 a^{4} c+60 a^{2} c^{3} {\mathrm e}^{2 x}-130 a^{3} c^{2} {\mathrm e}^{3 x}-45 a^{2} c^{3} {\mathrm e}^{4 x}}{}$

input
```
int(1/(a+a*cosh(x)+c*sinh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8/c^6*(1/3*c^2*tanh(1/2*x)^3-2*a*c*tanh(1/2*x)^2+10*a^2*tanh(1/2*x)-3*c
^2*tanh(1/2*x))+1/2*a*(5*a^2-3*c^2)*ln(a+c*tanh(1/2*x))/c^7-3/8*a/c^7*(a^4
-2*a^2*c^2+c^4)/(a+c*tanh(1/2*x))^2-1/24/c^7*(-a^6+3*a^4*c^2-3*a^2*c^4+c^6
)/(a+c*tanh(1/2*x))^3-1/8*(-15*a^4+18*a^2*c^2-3*c^4)/c^7/(a+c*tanh(1/2*x))
```

3.752.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4015 vs. $2(130) = 260$.

Time = 0.30 (sec) , antiderivative size = 4015, normalized size of antiderivative = 28.68

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^4} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x))^4,x, algorithm="fricas")
```

output
```
1/6*(30*a^5*c - 90*a^4*c^2 + 82*a^3*c^3 - 6*a^2*c^4 - 24*a*c^5 + 8*c^6 + 6
*(5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*cosh(x)^5 + 6*(5
*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*sinh(x)^5 + 30*(5*a
^5*c + 5*a^4*c^2 - 3*a^3*c^3 - 3*a^2*c^4)*cosh(x)^4 + 30*(5*a^5*c + 5*a^4*
c^2 - 3*a^3*c^3 - 3*a^2*c^4 + (5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^
4 - 3*a*c^5)*cosh(x))*sinh(x)^4 + 4*(75*a^5*c - 65*a^3*c^3 + 12*a*c^5)*cos
h(x)^3 + 4*(75*a^5*c - 65*a^3*c^3 + 12*a*c^5 + 15*(5*a^5*c + 10*a^4*c^2 +
2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*cosh(x)^2 + 30*(5*a^5*c + 5*a^4*c^2 - 3*a
^3*c^3 - 3*a^2*c^4)*cosh(x))*sinh(x)^3 + 12*(25*a^5*c - 25*a^4*c^2 - 10*a^
3*c^3 + 10*a^2*c^4 + 2*a*c^5 - 2*c^6)*cosh(x)^2 + 12*(25*a^5*c - 25*a^4*c^
2 - 10*a^3*c^3 + 10*a^2*c^4 + 2*a*c^5 - 2*c^6 + 5*(5*a^5*c + 10*a^4*c^2 +
2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*cosh(x)^3 + 15*(5*a^5*c + 5*a^4*c^2 - 3*a
^3*c^3 - 3*a^2*c^4)*cosh(x)^2 + (75*a^5*c - 65*a^3*c^3 + 12*a*c^5)*cosh(x)
)*sinh(x)^2 + 30*(5*a^5*c - 10*a^4*c^2 + 4*a^3*c^3 + 2*a^2*c^4 - a*c^5)*co
sh(x) + 3*((5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^
5)*cosh(x)^6 + (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*
a*c^5)*sinh(x)^6 + 5*a^6 - 15*a^5*c + 12*a^4*c^2 + 4*a^3*c^3 - 9*a^2*c^4 +
 3*a*c^5 + 6*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x
)^5 + 6*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4 + (5*a^6 + 1
5*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x))*sinh(x...
```

3.752.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x))**4,x)
```

output
```
Timed out
```

3.752.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(130) = 260$.

Time = 0.23 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.48

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^4}\,dx = -\frac{15\,a^5 + 45\,a^4c + 41\,a^3c^2 + 3\,a^2c^3 - 12\,ac^4 - 4\,c^5 + 15\,(5\,a^5 + 10\,a^4c + 4\,a^3c^2 - 2\,a^2c^3 - ac^4)e^{(-x)} + 6\,(2}{3\,(a^3c^6 + 3\,a^2c^7 + 3\,ac^8 + c^9 + 6\,(a^3c^6 + 2\,a^2c^7 + ac^8)e^{(-x)} + 3\,(5\,a^3c^6 + 5\,c} + \frac{(5\,a^3 - 3\,ac^2)\log\left(-(a-c)e^{(-x)} - a - c\right)}{2\,c^7} - \frac{(5\,a^3 - 3\,ac^2)\log\left(e^{(-x)} + 1\right)}{2\,c^7}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x))^4,x, algorithm="maxima")
```

output
```
-1/3*(15*a^5 + 45*a^4*c + 41*a^3*c^2 + 3*a^2*c^3 - 12*a*c^4 - 4*c^5 + 15*(
5*a^5 + 10*a^4*c + 4*a^3*c^2 - 2*a^2*c^3 - a*c^4)*e^(-x) + 6*(25*a^5 + 25*
a^4*c - 10*a^3*c^2 - 10*a^2*c^3 + 2*a*c^4 + 2*c^5)*e^(-2*x) + 2*(75*a^5 -
65*a^3*c^2 + 12*a*c^4)*e^(-3*x) + 15*(5*a^5 - 5*a^4*c - 3*a^3*c^2 + 3*a^2*
c^3)*e^(-4*x) + 3*(5*a^5 - 10*a^4*c + 2*a^3*c^2 + 6*a^2*c^3 - 3*a*c^4)*e^(
-5*x))/(a^3*c^6 + 3*a^2*c^7 + 3*a*c^8 + c^9 + 6*(a^3*c^6 + 2*a^2*c^7 + a*c
^8)*e^(-x) + 3*(5*a^3*c^6 + 5*a^2*c^7 - a*c^8 - c^9)*e^(-2*x) + 4*(5*a^3*c
^6 - 3*a*c^8)*e^(-3*x) + 3*(5*a^3*c^6 - 5*a^2*c^7 - a*c^8 + c^9)*e^(-4*x)
+ 6*(a^3*c^6 - 2*a^2*c^7 + a*c^8)*e^(-5*x) + (a^3*c^6 - 3*a^2*c^7 + 3*a*c^
8 - c^9)*e^(-6*x)) + 1/2*(5*a^3 - 3*a*c^2)*log(-(a - c)*e^(-x) - a - c)/c^
7 - 1/2*(5*a^3 - 3*a*c^2)*log(e^(-x) + 1)/c^7
```

3.752.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(130) = 260.

Time = 0.27 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.69

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx$$
$$= \frac{(5\,a^4+5\,a^3c-3\,a^2c^2-3\,ac^3)\log(|ae^x+ce^x+a-c|)}{2\,(ac^7+c^8)} - \frac{(5\,a^3-3\,ac^2)\log(e^x+1)}{2\,c^7}$$
$$+ \frac{15\,a^5e^{(5\,x)}+30\,a^4ce^{(5\,x)}+6\,a^3c^2e^{(5\,x)}-18\,a^2c^3e^{(5\,x)}-9\,ac^4e^{(5\,x)}+75\,a^5e^{(4\,x)}+75\,a^4ce^{(4\,x)}-45\,a^3c^2e^{(4\,x}}{}$$

input
```
integrate(1/(a+a*cosh(x)+c*sinh(x))^4,x, algorithm="giac")
```

output
```
1/2*(5*a^4 + 5*a^3*c - 3*a^2*c^2 - 3*a*c^3)*log(abs(a*e^x + c*e^x + a - c)
)/(a*c^7 + c^8) - 1/2*(5*a^3 - 3*a*c^2)*log(e^x + 1)/c^7 + 1/3*(15*a^5*e^(
5*x) + 30*a^4*c*e^(5*x) + 6*a^3*c^2*e^(5*x) - 18*a^2*c^3*e^(5*x) - 9*a*c^4
*e^(5*x) + 75*a^5*e^(4*x) + 75*a^4*c*e^(4*x) - 45*a^3*c^2*e^(4*x) - 45*a^2
*c^3*e^(4*x) + 150*a^5*e^(3*x) - 130*a^3*c^2*e^(3*x) + 24*a*c^4*e^(3*x) +
150*a^5*e^(2*x) - 150*a^4*c*e^(2*x) - 60*a^3*c^2*e^(2*x) + 60*a^2*c^3*e^(2
*x) + 12*a*c^4*e^(2*x) - 12*c^5*e^(2*x) + 75*a^5*e^x - 150*a^4*c*e^x + 60*
a^3*c^2*e^x + 30*a^2*c^3*e^x - 15*a*c^4*e^x + 15*a^5 - 45*a^4*c + 41*a^3*c
^2 - 3*a^2*c^3 - 12*a*c^4 + 4*c^5)/((a*e^(2*x) + c*e^(2*x) + 2*a*e^x + a -
 c)^3*c^6)
```

3.752.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx = \int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx$$

input
```
int(1/(a + a*cosh(x) + c*sinh(x))^4,x)
```

output
```
int(1/(a + a*cosh(x) + c*sinh(x))^4, x)
```

3.753 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 \, dx$

3.753.1 Optimal result

Integrand size = 24, antiderivative size = 188

$$\begin{aligned}\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 dx &= \frac{35}{8}\left(b^2-c^2\right)^2 x+\frac{35}{8}c\left(b^2-c^2\right)^{3/2}\cosh(x)+\frac{35}{8}b\left(b^2-c^2\right)^{3/2}\sinh(x)\\ &\quad+\frac{35}{24}\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\\ &\quad+\frac{7}{12}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\\ &\quad+\frac{1}{4}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\end{aligned}$$

output
```
35/8*(b^2-c^2)^2*x+35/8*c*(b^2-c^2)^(3/2)*cosh(x)+35/8*b*(b^2-c^2)^(3/2)*s
inh(x)+35/24*(b^2-c^2)*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2
)^(1/2))+7/12*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)*(b*cosh(x)+c*sinh(x)+(
b^2-c^2)^(1/2))^2+1/4*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)
^(1/2))^3
```

3.753.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.11

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 dx$$
$$=\frac{35}{8}(b-c)^2(b+c)^2x+7(b-c)c(b+c)\sqrt{b^2-c^2}\cosh(x)+\frac{7}{2}bc(b^2-c^2)\cosh(2x)$$
$$+\frac{1}{3}c\sqrt{b^2-c^2}(3b^2+c^2)\cosh(3x)+\frac{1}{8}bc(b^2+c^2)\cosh(4x)+7b(b-c)(b+c)\sqrt{b^2-c^2}\sinh(x)$$
$$+\frac{7}{4}(b^4-c^4)\sinh(2x)+\frac{1}{3}b\sqrt{b^2-c^2}(b^2+3c^2)\sinh(3x)+\frac{1}{32}(b^4+6b^2c^2+c^4)\sinh(4x)$$

input
```
Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^4,x]
```

output
```
(35*(b - c)^2*(b + c)^2*x)/8 + 7*(b - c)*c*(b + c)*Sqrt[b^2 - c^2]*Cosh[x]
 + (7*b*c*(b^2 - c^2)*Cosh[2*x])/2 + (c*Sqrt[b^2 - c^2]*(3*b^2 + c^2)*Cosh
[3*x])/3 + (b*c*(b^2 + c^2)*Cosh[4*x])/8 + 7*b*(b - c)*(b + c)*Sqrt[b^2 -
c^2]*Sinh[x] + (7*(b^4 - c^4)*Sinh[2*x])/4 + (b*Sqrt[b^2 - c^2]*(b^2 + 3*c
^2)*Sinh[3*x])/3 + ((b^4 + 6*b^2*c^2 + c^4)*Sinh[4*x])/32
```

3.753.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3592, 3042, 3592, 3042, 3592, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^4 dx$$

$$\Big\downarrow \; 3592$$

$$\frac{7}{4}\sqrt{b^2-c^2}\int \left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^3 dx+\frac{1}{4}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3+\frac{7}{4}\sqrt{b^2-c^2}\int\left(b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}\right)^3dx$$

$$\Big\downarrow \text{3592}$$

$$\frac{7}{4}\sqrt{b^2-c^2}\left(\frac{5}{3}\sqrt{b^2-c^2}\int\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^2dx+\frac{1}{3}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh\right.\right.$$
$$\frac{1}{4}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3+$$
$$\frac{7}{4}\sqrt{b^2-c^2}\left(\frac{1}{3}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2+\frac{5}{3}\sqrt{b^2-c^2}\int\left(b\cos(ix)-ic\sin(ix)\right.\right.$$

$$\Big\downarrow \text{3592}$$

$$\frac{7}{4}\sqrt{b^2-c^2}\left(\frac{5}{3}\sqrt{b^2-c^2}\left(\frac{3}{2}\sqrt{b^2-c^2}\int\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)dx+\frac{1}{2}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-\right.\right.\right.$$
$$\frac{1}{4}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3+$$
$$\frac{7}{4}\sqrt{b^2-c^2}\left(\frac{1}{3}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2+\frac{5}{3}\sqrt{b^2-c^2}\left(\frac{3}{2}\sqrt{b^2-c^2}\left(x\sqrt{b^2-c^2}\right.\right.\right.$$

input
```
Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^4,x]
```

output
```
((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3)/4 +
(7*Sqrt[b^2 - c^2]*(((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*Cosh[x]
+ c*Sinh[x])^2)/3 + (5*Sqrt[b^2 - c^2]*((3*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2
]*x + c*Cosh[x] + b*Sinh[x]))/2 + ((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2
] + b*Cosh[x] + c*Sinh[x]))/2))/3))/4
```

3.753. $\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4\,dx$

3.753.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.753.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.71

method	result
parts	$\left(b^{2}-c^{2}\right)^{2} x+c^{4}\left(\left(\frac{\sinh\left(x\right)^{3}}{4}-\frac{3\sinh\left(x\right)}{8}\right)\cosh\left(x\right)+\frac{3x}{8}\right)+4b^{3}\left(\frac{c\sinh\left(x\right)^{4}}{4}+\frac{\sqrt{b^{2}-c^{2}}\,\sinh\left(x\right)^{3}}{3}+\frac{c\sinh\left(x\right)^{2}}{2}+\right.$
default	$b^{4}x+c^{4}x-2b^{2}c^{2}x+4\sqrt{b^{2}-c^{2}}\,b^{3}\left(\frac{2}{3}+\frac{\cosh\left(x\right)^{2}}{3}\right)\sinh\left(x\right)-6b^{2}c^{2}\left(\frac{\cosh\left(x\right)\sinh\left(x\right)}{2}+\frac{x}{2}\right)+4\sqrt{b^{2}-c^{2}}$
risch	$-\frac{35b^{2}c^{2}x}{4}+\frac{{\mathrm e}^{4x}b^{4}}{64}+\frac{7\,{\mathrm e}^{2x}b^{4}}{8}+\frac{35b^{4}x}{8}+\frac{35c^{4}x}{8}+\frac{{\mathrm e}^{3x}\sqrt{b^{2}-c^{2}}\,b^{2}c}{2}+\frac{{\mathrm e}^{3x}\sqrt{b^{2}-c^{2}}\,b\,c^{2}}{2}+\frac{7\,{\mathrm e}^{x}\sqrt{b^{2}-c^{2}}\,b^{2}c}{2}-\frac{7\,{\mathrm e}^{x}\sqrt{b^{2}-c^{2}}\,b}{2}$

input
```
int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x,method=_RETURNVERBOSE)
```

output
```
(b^2-c^2)^2*x+c^4*((1/4*sinh(x)^3-3/8*sinh(x))*cosh(x)+3/8*x)+4*b^3*(1/4*c
*sinh(x)^4+1/3*(b^2-c^2)^(1/2)*sinh(x)^3+1/2*c*sinh(x)^2+sinh(x)*(b^2-c^2)
^(1/2))+6*b^2*c^2*(1/4*cosh(x)^3*sinh(x)-1/8*cosh(x)*sinh(x)-1/8*x)+4*(b^2
-c^2)^(1/2)*b^2*c*cosh(x)^3+6*b^4*(1/2*cosh(x)*sinh(x)+1/2*x)-6*b^2*c^2*(1
/2*cosh(x)*sinh(x)+1/2*x)+4*b*(1/4*sinh(x)^4*c^3+((b-c)*(b+c))^(1/2)*sinh(
x)^3*c^2+3/2*sinh(x)^2*b^2*c-3/2*sinh(x)^2*c^3+((b-c)*(b+c))^(3/2)*sinh(x)
)+b^4*((1/4*cosh(x)^3+3/8*cosh(x))*sinh(x)+3/8*x)+4*c*(b^2-c^2)^(3/2)*cosh
(x)+6*c^2*(b^2-c^2)*(1/2*cosh(x)*sinh(x)-1/2*x)+4*(b^2-c^2)^(1/2)*c^3*(-2/
3+1/3*sinh(x)^2)*cosh(x)
```

3.753.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. $2(164) = 328$.

Time = 0.27 (sec) , antiderivative size = 1293, normalized size of antiderivative = 6.88

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4\,dx=\text{Too large to display}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="fricas")
```

output
```
1/192*(3*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^8 + 24*(b^4 +
 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(x)^7 + 3*(b^4 + 4*b^3*c
 + 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^8 + 168*(b^4 + 2*b^3*c - 2*b*c^3 - c
^4)*cosh(x)^6 + 84*(2*b^4 + 4*b^3*c - 4*b*c^3 - 2*c^4 + (b^4 + 4*b^3*c + 6
*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^6 + 840*(b^4 - 2*b^2*c^2 + c^
4)*x*cosh(x)^4 + 168*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^
3 + 6*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x))*sinh(x)^5 + 210*((b^4 + 4*b
^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^4 + 12*(b^4 + 2*b^3*c - 2*b*c^3
- c^4)*cosh(x)^2 + 4*(b^4 - 2*b^2*c^2 + c^4)*x)*sinh(x)^4 - 3*b^4 + 12*b^3
*c - 18*b^2*c^2 + 12*b*c^3 - 3*c^4 + 168*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b
*c^3 + c^4)*cosh(x)^5 + 20*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3 + 20*
(b^4 - 2*b^2*c^2 + c^4)*x*cosh(x))*sinh(x)^3 - 168*(b^4 - 2*b^3*c + 2*b*c^
3 - c^4)*cosh(x)^2 + 84*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(
x)^6 + 30*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^4 - 2*b^4 + 4*b^3*c - 4*
b*c^3 + 2*c^4 + 60*(b^4 - 2*b^2*c^2 + c^4)*x*cosh(x)^2)*sinh(x)^2 + 24*((b
^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^7 + 42*(b^4 + 2*b^3*c -
2*b*c^3 - c^4)*cosh(x)^5 + 140*(b^4 - 2*b^2*c^2 + c^4)*x*cosh(x)^3 - 14*(b
^4 - 2*b^3*c + 2*b*c^3 - c^4)*cosh(x))*sinh(x) + 32*((b^3 + 3*b^2*c + 3*b*
c^2 + c^3)*cosh(x)^7 + 7*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^6
 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*sinh(x)^7 + 21*(b^3 + b^2*c - b*c^2 ...
```

3.753.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(178) = 356$.

Time = 0.31 (sec) , antiderivative size = 626, normalized size of antiderivative = 3.33

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 dx = \frac{3b^4x\sinh^4(x)}{8} - \frac{3b^4x\sinh^2(x)\cosh^2(x)}{4} - 3b^4x\sinh^2(x) + \frac{3b^4x\cosh^4(x)}{8} + 3b^4x\cosh^2(x) + b^4x - \frac{3b^4\sinh^3(x)\cosh(x)}{8} + \frac{5b^4\sinh(x)\cosh^3(x)}{8} + 3b^4\sinh(x)\cosh(x) + b^3c\cosh^4(x) + 6b^3c\cosh^2(x) - \frac{8b^3\sqrt{b^2-c^2}\sinh^3(x)}{3} + 4b^3\sqrt{b^2-c^2}\sinh(x)\cosh^2(x) + 4b^3\sqrt{b^2-c^2}\sinh(x) - \frac{3b^2c^2x\sinh^4(x)}{4} + \frac{3b^2c^2x\sinh^2(x)\cosh^2(x)}{2} + 6b^2c^2x\sinh^2(x) - \frac{3b^2c^2x\cosh^4(x)}{4} - 6b^2c^2x\cosh^2(x) - 2b^2c^2x + \frac{3b^2c^2\sinh^3(x)\cosh(x)}{4} + \frac{3b^2c^2\sinh(x)\cosh^3(x)}{4} + 4b^2c\sqrt{b^2-c^2}\cosh^3(x) + 4b^2c\sqrt{b^2-c^2}\cosh(x) + bc^3\sinh^4(x) - 6bc^3\cosh^2(x) + 4bc^2\sqrt{b^2-c^2}\sinh^3(x) - 4bc^2\sqrt{b^2-c^2}\sinh(x) + \frac{3c^4x\sinh^4(x)}{8} - \frac{3c^4x\sinh^2(x)\cosh^2(x)}{4} - 3c^4x\sinh^2(x) + \frac{3c^4x\cosh^4(x)}{8} + 3c^4x\cosh^2(x) + c^4x + \frac{5c^4\sinh^3(x)\cosh(x)}{8} - \frac{3c^4\sinh(x)\cosh^3(x)}{8} - 3c^4\sinh(x)\cosh(x) + 4c^3\sqrt{b^2-c^2}\sinh^2(x)\cosh(x) - \frac{8c^3\sqrt{b^2-c^2}\cosh^3(x)}{3} - 4c^3\sqrt{b^2-c^2}\cosh(x)$$

3.753. $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 dx$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**4,x)
```

output
```
3*b**4*x*sinh(x)**4/8 - 3*b**4*x*sinh(x)**2*cosh(x)**2/4 - 3*b**4*x*sinh(x
)**2 + 3*b**4*x*cosh(x)**4/8 + 3*b**4*x*cosh(x)**2 + b**4*x - 3*b**4*sinh(
x)**3*cosh(x)/8 + 5*b**4*sinh(x)*cosh(x)**3/8 + 3*b**4*sinh(x)*cosh(x) + b
**3*c*cosh(x)**4 + 6*b**3*c*cosh(x)**2 - 8*b**3*sqrt(b**2 - c**2)*sinh(x)*
*3/3 + 4*b**3*sqrt(b**2 - c**2)*sinh(x)*cosh(x)**2 + 4*b**3*sqrt(b**2 - c*
*2)*sinh(x) - 3*b**2*c**2*x*sinh(x)**4/4 + 3*b**2*c**2*x*sinh(x)**2*cosh(x
)**2/2 + 6*b**2*c**2*x*sinh(x)**2 - 3*b**2*c**2*x*cosh(x)**4/4 - 6*b**2*c*
*2*x*cosh(x)**2 - 2*b**2*c**2*x + 3*b**2*c**2*sinh(x)**3*cosh(x)/4 + 3*b**
2*c**2*sinh(x)*cosh(x)**3/4 + 4*b**2*c*sqrt(b**2 - c**2)*cosh(x)**3 + 4*b*
*2*c*sqrt(b**2 - c**2)*cosh(x) + b*c**3*sinh(x)**4 - 6*b*c**3*cosh(x)**2 +
 4*b*c**2*sqrt(b**2 - c**2)*sinh(x)**3 - 4*b*c**2*sqrt(b**2 - c**2)*sinh(x
) + 3*c**4*x*sinh(x)**4/8 - 3*c**4*x*sinh(x)**2*cosh(x)**2/4 - 3*c**4*x*si
nh(x)**2 + 3*c**4*x*cosh(x)**4/8 + 3*c**4*x*cosh(x)**2 + c**4*x + 5*c**4*s
inh(x)**3*cosh(x)/8 - 3*c**4*sinh(x)*cosh(x)**3/8 - 3*c**4*sinh(x)*cosh(x)
 + 4*c**3*sqrt(b**2 - c**2)*sinh(x)**2*cosh(x) - 8*c**3*sqrt(b**2 - c**2)*
cosh(x)**3/3 - 4*c**3*sqrt(b**2 - c**2)*cosh(x)
```

3.753.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.47

$$
\begin{aligned}
&\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 dx \\
&= b^3c\cosh(x)^4 + bc^3\sinh(x)^4 + \frac{1}{64}b^4\left(24x+e^{(4x)}+8e^{(2x)}-8e^{(-2x)}-e^{(-4x)}\right) \\
&\quad + \frac{1}{64}c^4\left(24x+e^{(4x)}-8e^{(2x)}+8e^{(-2x)}-e^{(-4x)}\right) - \frac{3}{32}b^2c^2\left(8x-e^{(4x)}+e^{(-4x)}\right) \\
&\quad + \left(b^2-c^2\right)^2x + 4\left(b^2-c^2\right)^{\frac{3}{2}}\left(c\cosh(x)+b\sinh(x)\right) \\
&\quad + \frac{3}{4}\left(8bc\cosh(x)^2 + b^2\left(4x+e^{(2x)}-e^{(-2x)}\right) - c^2\left(4x-e^{(2x)}+e^{(-2x)}\right)\right)\left(b^2-c^2\right) \\
&\quad + \frac{1}{6}\left(24b^2c\cosh(x)^3 + 24bc^2\sinh(x)^3 + c^3\left(e^{(3x)}-9e^{(-x)}+e^{(-3x)}-9e^{x}\right) + b^3\left(e^{(3x)}-9e^{(-x)}-e^{(-3x)}\right.\right.
\end{aligned}
$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="maxima")
```

output
```
b^3*c*cosh(x)^4 + b*c^3*sinh(x)^4 + 1/64*b^4*(24*x + e^(4*x) + 8*e^(2*x) -
 8*e^(-2*x) - e^(-4*x)) + 1/64*c^4*(24*x + e^(4*x) - 8*e^(2*x) + 8*e^(-2*x
) - e^(-4*x)) - 3/32*b^2*c^2*(8*x - e^(4*x) + e^(-4*x)) + (b^2 - c^2)^2*x
+ 4*(b^2 - c^2)^(3/2)*(c*cosh(x) + b*sinh(x)) + 3/4*(8*b*c*cosh(x)^2 + b^2
*(4*x + e^(2*x) - e^(-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*(b^2 - c^2)
+ 1/6*(24*b^2*c*cosh(x)^3 + 24*b*c^2*sinh(x)^3 + c^3*(e^(3*x) - 9*e^(-x) +
 e^(-3*x) - 9*e^x) + b^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x))*sqrt(b^2
 - c^2)
```

3.753.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(164) = 328$.

Time = 0.28 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.07

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 dx = \frac{7}{2}\left(b^3+b^2c-bc^2-c^3\right)\sqrt{b^2-c^2}e^x$$
$$+\frac{35}{8}\left(b^4-2\,b^2c^2+c^4\right)x+\frac{1}{64}\left(b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4\right)e^{(4\,x)}$$
$$+\frac{1}{6}\left(\sqrt{b^2-c^2}b^3+3\sqrt{b^2-c^2}b^2c+3\sqrt{b^2-c^2}bc^2+\sqrt{b^2-c^2}c^3\right)e^{(3\,x)}$$
$$+\frac{7}{8}\left(b^4+2\,b^3c-2\,bc^3-c^4\right)e^{(2\,x)}$$
$$-\frac{1}{192}\left(3\,b^4-12\,b^3c+18\,b^2c^2-12\,bc^3+3\,c^4+672\left(\sqrt{b^2-c^2}b^3-\sqrt{b^2-c^2}b^2c-\sqrt{b^2-c^2}bc^2+\sqrt{b^2-c^2}\right.\right.$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="giac")
```

output
```
7/2*(b^3 + b^2*c - b*c^2 - c^3)*sqrt(b^2 - c^2)*e^x + 35/8*(b^4 - 2*b^2*c^
2 + c^4)*x + 1/64*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*e^(4*x) + 1/
6*(sqrt(b^2 - c^2)*b^3 + 3*sqrt(b^2 - c^2)*b^2*c + 3*sqrt(b^2 - c^2)*b*c^2
 + sqrt(b^2 - c^2)*c^3)*e^(3*x) + 7/8*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*e^(2
*x) - 1/192*(3*b^4 - 12*b^3*c + 18*b^2*c^2 - 12*b*c^3 + 3*c^4 + 672*(sqrt(
b^2 - c^2)*b^3 - sqrt(b^2 - c^2)*b^2*c - sqrt(b^2 - c^2)*b*c^2 + sqrt(b^2
- c^2)*c^3)*e^(3*x) + 168*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(2*x) + 32*(sq
rt(b^2 - c^2)*b^3 - 3*sqrt(b^2 - c^2)*b^2*c + 3*sqrt(b^2 - c^2)*b*c^2 - sq
rt(b^2 - c^2)*c^3)*e^x)*e^(-4*x)
```

3.753.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.92

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4\,dx = x\left(b^2-c^2\right)^2-\cosh(x)^2\left(6\,b\,c^3-6\,b^3\,c\right)$$
$$-\cosh(x)^4\left(b\,c^3-b^3\,c\right)$$
$$+\cosh(x)\,\sinh(x)^3\left(-\frac{3\,b^4}{8}+\frac{3\,b^2\,c^2}{4}+\frac{5\,c^4}{8}\right)$$
$$+\cosh(x)^3\sinh(x)\,\left(\frac{5\,b^4}{8}+\frac{3\,b^2\,c^2}{4}-\frac{3\,c^4}{8}\right)$$
$$+4\,c\cosh(x)\,\left(b^2-c^2\right)^{3/2}$$
$$+4\,b\sinh(x)\,\left(b^2-c^2\right)^{3/2}$$
$$+3\,x\cosh(x)^2\left(b^2-c^2\right)^2$$
$$+\frac{3\,x\cosh(x)^4\left(b^2-c^2\right)^2}{8}$$
$$-3\,x\sinh(x)^2\left(b^2-c^2\right)^2$$
$$+\frac{3\,x\sinh(x)^4\left(b^2-c^2\right)^2}{8}$$
$$+\cosh(x)\,\sinh(x)\,\left(3\,b^4-3\,c^4\right)$$
$$+2\,b\,c^3\cosh(x)^2\sinh(x)^2$$
$$+\frac{4\,c\cosh(x)^3\sqrt{b^2-c^2}\,\left(3\,b^2-2\,c^2\right)}{3}$$
$$-\frac{4\,b\sinh(x)^3\sqrt{b^2-c^2}\,\left(2\,b^2-3\,c^2\right)}{3}$$
$$+4\,b^3\cosh(x)^2\sinh(x)\,\sqrt{b^2-c^2}$$
$$+4\,c^3\cosh(x)\,\sinh(x)^2\sqrt{b^2-c^2}$$
$$-\frac{3\,x\cosh(x)^2\sinh(x)^2\left(b^2-c^2\right)^2}{4}$$

input
```
int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^4,x)
```

output
```
x*(b^2 - c^2)^2 - cosh(x)^2*(6*b*c^3 - 6*b^3*c) - cosh(x)^4*(b*c^3 - b^3*c
) + cosh(x)*sinh(x)^3*((5*c^4)/8 - (3*b^4)/8 + (3*b^2*c^2)/4) + cosh(x)^3*
sinh(x)*((5*b^4)/8 - (3*c^4)/8 + (3*b^2*c^2)/4) + 4*c*cosh(x)*(b^2 - c^2)^
(3/2) + 4*b*sinh(x)*(b^2 - c^2)^(3/2) + 3*x*cosh(x)^2*(b^2 - c^2)^2 + (3*x
*cosh(x)^4*(b^2 - c^2)^2)/8 - 3*x*sinh(x)^2*(b^2 - c^2)^2 + (3*x*sinh(x)^4
*(b^2 - c^2)^2)/8 + cosh(x)*sinh(x)*(3*b^4 - 3*c^4) + 2*b*c^3*cosh(x)^2*si
nh(x)^2 + (4*c*cosh(x)^3*(b^2 - c^2)^(1/2)*(3*b^2 - 2*c^2))/3 - (4*b*sinh(
x)^3*(b^2 - c^2)^(1/2)*(2*b^2 - 3*c^2))/3 + 4*b^3*cosh(x)^2*sinh(x)*(b^2 -
 c^2)^(1/2) + 4*c^3*cosh(x)*sinh(x)^2*(b^2 - c^2)^(1/2) - (3*x*cosh(x)^2*s
inh(x)^2*(b^2 - c^2)^2)/4
```

3.753. $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 \,dx$

3.754 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx$

3.754.1 Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx=\frac{5}{2}\left(b^2-c^2\right)^{3/2}x+\frac{5}{2}c\left(b^2-c^2\right)\cosh(x)+\frac{5}{2}b\left(b^2-c^2\right)\sinh(x)+\frac{5}{6}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)+\frac{1}{3}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2$$

output
```
5/2*(b^2-c^2)^(3/2)*x+5/2*c*(b^2-c^2)*cosh(x)+5/2*b*(b^2-c^2)*sinh(x)+5/6*
(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)
)+1/3*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2
```

3.754.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx=\frac{1}{12}\Big(30(b-c)(b+c)\sqrt{b^2-c^2}x+45c\left(b^2-c^2\right)\cosh(x)+18bc\sqrt{b^2-c^2}\cosh(2x)+c\left(3b^2+c^2\right)\cosh(3x)+45b\left(b^2-c^2\right)\sinh(x)+9\sqrt{b^2-c^2}\left(b^2+c^2\right)\sinh(2x)+b\left(b^2+3c^2\right)\sinh(3x)\Big)$$

input
```
Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
(30*(b - c)*(b + c)*Sqrt[b^2 - c^2]*x + 45*c*(b^2 - c^2)*Cosh[x] + 18*b*c*
Sqrt[b^2 - c^2]*Cosh[2*x] + c*(3*b^2 + c^2)*Cosh[3*x] + 45*b*(b^2 - c^2)*S
inh[x] + 9*Sqrt[b^2 - c^2]*(b^2 + c^2)*Sinh[2*x] + b*(b^2 + 3*c^2)*Sinh[3*
x])/12
```

3.754.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 3592, 3042, 3592, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3 dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^3 dx$$

$$\Big\downarrow \ 3592$$

$$\frac{5}{3}\sqrt{b^2-c^2}\int \left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^2 dx+\frac{1}{3}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2+\frac{5}{3}\sqrt{b^2-c^2}\int \left(b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}\right)^2 dx$$

$$\Big\downarrow \ 3592$$

$$\frac{5}{3}\sqrt{b^2-c^2}\left(\frac{3}{2}\sqrt{b^2-c^2}\int \left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right) dx+\frac{1}{2}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(\right.\right.$$
$$\frac{1}{3}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2+$$
$$\frac{5}{3}\sqrt{b^2-c^2}\left(\frac{3}{2}\sqrt{b^2-c^2}\left(x\sqrt{b^2-c^2}+b\sinh(x)+c\cosh(x)\right)+\frac{1}{2}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+\right.\right.$$

input
```
Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2)/3 +
(5*Sqrt[b^2 - c^2]*((3*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2]*x + c*Cosh[x] + b*
Sinh[x]))/2 + ((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Si
nh[x]))/2))/3
```

3.754.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.754.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

method	result
parts	$(b^2-c^2)^{\frac{3}{2}}x+c^3\left(-\frac{2}{3}+\frac{\sinh(x)^2}{3}\right)\cosh(x)+c\,b^2\cosh(x)^3+3\sqrt{b^2-c^2}\,b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+3b$
default	$b^3\left(\frac{2}{3}+\frac{\cosh(x)^2}{3}\right)\sinh(x)+c\,b^2\cosh(x)^3+3\sqrt{b^2-c^2}\,b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+c^2b\sinh(x)^3+3\sqrt{b}$
risch	$\frac{5(b^2-c^2)^{\frac{3}{2}}x}{2}+\frac{e^{3x}b^3}{24}+\frac{e^{3x}c\,b^2}{8}+\frac{e^{3x}c^2b}{8}+\frac{e^{3x}c^3}{24}+\frac{3\,e^{2x}\sqrt{b^2-c^2}\,b^2}{8}+\frac{3\,e^{2x}\sqrt{b^2-c^2}\,bc}{4}+\frac{3\,e^{2x}\sqrt{b^2-c^2}\,c^2}{8}+\frac{15b^3e^x}{8}+$

input
```
int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
(b^2-c^2)^(3/2)*x+c^3*(-2/3+1/3*sinh(x)^2)*cosh(x)+c*b^2*cosh(x)^3+3*(b^2-
c^2)^(1/2)*b^2*(1/2*cosh(x)*sinh(x)+1/2*x)+3*b*(1/3*sinh(x)^3*c^2+((b-c)*(
b+c))^(1/2)*sinh(x)^2*c+b^2*sinh(x)-sinh(x)*c^2)+b^3*(2/3+1/3*cosh(x)^2)*s
inh(x)+3*c*(b^2-c^2)*cosh(x)+3*(b^2-c^2)^(1/2)*c^2*(1/2*cosh(x)*sinh(x)-1/
2*x)
```

3.754.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(118) = 236$.

Time = 0.27 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.88

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3 dx$$
$$= \frac{(b^3+3\,b^2c+3\,bc^2+c^3)\cosh{(x)}^6+6\,(b^3+3\,b^2c+3\,bc^2+c^3)\cosh{(x)}\sinh{(x)}^5+(b^3+3\,b^2c+3\,bc^2+c^3)}{}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/24*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 6*(b^3 + 3*b^2*c + 3*b*c
^2 + c^3)*cosh(x)*sinh(x)^5 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*sinh(x)^6 +
45*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 15*(3*b^3 + 3*b^2*c - 3*b*c^2 -
 3*c^3 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(x)^4 + 20*((b^3 +
 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 9*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x
))*sinh(x)^3 - b^3 + 3*b^2*c - 3*b*c^2 + c^3 - 45*(b^3 - b^2*c - b*c^2 + c
^3)*cosh(x)^2 + 15*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - 3*b^3 + 3*
b^2*c + 3*b*c^2 - 3*c^3 + 18*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x
)^2 + 6*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 + 30*(b^3 + b^2*c - b*c
^2 - c^3)*cosh(x)^3 - 15*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x) + 3*
(3*(b^2 + 2*b*c + c^2)*cosh(x)^5 + 15*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^
4 + 3*(b^2 + 2*b*c + c^2)*sinh(x)^5 + 20*(b^2 - c^2)*x*cosh(x)^3 + 10*(3*(
b^2 + 2*b*c + c^2)*cosh(x)^2 + 2*(b^2 - c^2)*x)*sinh(x)^3 + 30*((b^2 + 2*b
*c + c^2)*cosh(x)^3 + 2*(b^2 - c^2)*x*cosh(x))*sinh(x)^2 - 3*(b^2 - 2*b*c
+ c^2)*cosh(x) + 3*(5*(b^2 + 2*b*c + c^2)*cosh(x)^4 + 20*(b^2 - c^2)*x*cos
h(x)^2 - b^2 + 2*b*c - c^2)*sinh(x))*sqrt(b^2 - c^2))/(cosh(x)^3 + 3*cosh(
x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

3.754.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(128) = 256.

Time = 0.21 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.19

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3 dx = -\frac{2b^3\sinh^3(x)}{3}+b^3\sinh(x)\cosh^2(x)+3b^3\sinh(x)+b^2c\cosh^3(x)+3b^2c\cosh(x)-\frac{3b^2x\sqrt{b^2-c^2}\sinh^2(x)}{2}+\frac{3b^2x\sqrt{b^2-c^2}\cosh^2(x)}{2}+b^2x\sqrt{b^2-c^2}+\frac{3b^2\sqrt{b^2-c^2}\sinh(x)\cosh(x)}{2}+bc^2\sinh^3(x)-3bc^2\sinh(x)+3bc\sqrt{b^2-c^2}\cosh^2(x)+c^3\sinh^2(x)\cosh(x)-\frac{2c^3\cosh^3(x)}{3}-3c^3\cosh(x)+\frac{3c^2x\sqrt{b^2-c^2}\sinh^2(x)}{2}-\frac{3c^2x\sqrt{b^2-c^2}\cosh^2(x)}{2}-c^2x\sqrt{b^2-c^2}+\frac{3c^2\sqrt{b^2-c^2}\sinh(x)\cosh(x)}{2}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**3,x)
```

output
```
-2*b**3*sinh(x)**3/3 + b**3*sinh(x)*cosh(x)**2 + 3*b**3*sinh(x) + b**2*c*c
osh(x)**3 + 3*b**2*c*cosh(x) - 3*b**2*x*sqrt(b**2 - c**2)*sinh(x)**2/2 + 3
*b**2*x*sqrt(b**2 - c**2)*cosh(x)**2/2 + b**2*x*sqrt(b**2 - c**2) + 3*b**2
*sqrt(b**2 - c**2)*sinh(x)*cosh(x)/2 + b*c**2*sinh(x)**3 - 3*b*c**2*sinh(x
) + 3*b*c*sqrt(b**2 - c**2)*cosh(x)**2 + c**3*sinh(x)**2*cosh(x) - 2*c**3*
cosh(x)**3/3 - 3*c**3*cosh(x) + 3*c**2*x*sqrt(b**2 - c**2)*sinh(x)**2/2 -
3*c**2*x*sqrt(b**2 - c**2)*cosh(x)**2/2 - c**2*x*sqrt(b**2 - c**2) + 3*c**
2*sqrt(b**2 - c**2)*sinh(x)*cosh(x)/2
```

3.754.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx$$
$$= b^2c\cosh(x)^3 + bc^2\sinh(x)^3 + \frac{1}{24}c^3\left(e^{(3x)} - 9e^{(-x)} + e^{(-3x)} - 9e^x\right)$$
$$+ \frac{1}{24}b^3\left(e^{(3x)} - 9e^{(-x)} - e^{(-3x)} + 9e^x\right)$$
$$+ \left(b^2-c^2\right)^{\frac{3}{2}}x + 3\left(b^2-c^2\right)\left(c\cosh(x) + b\sinh(x)\right)$$
$$+ \frac{3}{8}\left(8bc\cosh(x)^2 + b^2\left(4x + e^{(2x)} - e^{(-2x)}\right) - c^2\left(4x - e^{(2x)} + e^{(-2x)}\right)\right)\sqrt{b^2-c^2}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="maxima")
```

output
```
b^2*c*cosh(x)^3 + b*c^2*sinh(x)^3 + 1/24*c^3*(e^(3*x) - 9*e^(-x) + e^(-3*x
) - 9*e^x) + 1/24*b^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x) + (b^2 - c^2
)^(3/2)*x + 3*(b^2 - c^2)*(c*cosh(x) + b*sinh(x)) + 3/8*(8*b*c*cosh(x)^2 +
 b^2*(4*x + e^(2*x) - e^(-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*sqrt(b^2
 - c^2)
```

3.754.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.43

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx$$
$$= \frac{5}{2}\left(b^2-c^2\right)^{\frac{3}{2}}x + \frac{3}{8}\left(b^2+2bc+c^2\right)\sqrt{b^2-c^2}e^{(2x)} + \frac{1}{24}\left(b^3+3b^2c+3bc^2+c^3\right)e^{(3x)}$$
$$- \frac{1}{24}\left(b^3 - 3b^2c + 3bc^2 - c^3 + 45\left(b^3 - b^2c - bc^2 + c^3\right)e^{(2x)} + 9\left(\sqrt{b^2-c^2}b^2 - 2\sqrt{b^2-c^2}bc + \sqrt{b^2-c^2}c\right.\right.$$
$$+ \frac{15}{8}\left(b^3 + b^2c - bc^2 - c^3\right)e^x$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="giac")
```

output
```
5/2*(b^2 - c^2)^(3/2)*x + 3/8*(b^2 + 2*b*c + c^2)*sqrt(b^2 - c^2)*e^(2*x)
+ 1/24*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*e^(3*x) - 1/24*(b^3 - 3*b^2*c + 3*b
*c^2 - c^3 + 45*(b^3 - b^2*c - b*c^2 + c^3)*e^(2*x) + 9*(sqrt(b^2 - c^2)*b
^2 - 2*sqrt(b^2 - c^2)*b*c + sqrt(b^2 - c^2)*c^2)*e^x)*e^(-3*x) + 15/8*(b^
3 + b^2*c - b*c^2 - c^3)*e^x
```

3.754.9 Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.06

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx$$
$$=\frac{11\,b^3\sinh(x)}{3}+\frac{c^3\cosh(x)^3}{3}+\frac{5\,x\,(b^2-c^2)^{3/2}}{2}-4\,c^3\cosh(x)+\frac{b^3\cosh(x)^2\sinh(x)}{3}+3\,b^2\,c\cosh(x)-4\,b\,c^2\sinh(x)+b^2\,c\cosh(x)^3+3\,b\,c\cosh(x)^2\,\sqrt{b^2-c^2}+\frac{3\,b^2\cosh(x)\,\sinh(x)\,\sqrt{b^2-c^2}}{2}+\frac{3\,c^2\cosh(x)\,\sinh(x)\,\sqrt{b^2-c^2}}{2}+b\,c^2\cosh(x)^2\sinh(x)$$

input
```
int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^3,x)
```

output
```
(11*b^3*sinh(x))/3 + (c^3*cosh(x)^3)/3 + (5*x*(b^2 - c^2)^(3/2))/2 - 4*c^3
*cosh(x) + (b^3*cosh(x)^2*sinh(x))/3 + 3*b^2*c*cosh(x) - 4*b*c^2*sinh(x) +
 b^2*c*cosh(x)^3 + 3*b*c*cosh(x)^2*(b^2 - c^2)^(1/2) + (3*b^2*cosh(x)*sinh
(x)*(b^2 - c^2)^(1/2))/2 + (3*c^2*cosh(x)*sinh(x)*(b^2 - c^2)^(1/2))/2 + b
*c^2*cosh(x)^2*sinh(x)
```

3.755 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx$

3.755.1 Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx=\frac{3}{2}\left(b^2-c^2\right)x+\frac{3}{2}c\sqrt{b^2-c^2}\cosh(x)+\frac{3}{2}b\sqrt{b^2-c^2}\sinh(x)+\frac{1}{2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)$$

output
```
3/2*(b^2-c^2)*x+3/2*c*cosh(x)*(b^2-c^2)^(1/2)+3/2*b*sinh(x)*(b^2-c^2)^(1/2
)+1/2*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))
```

3.755.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx=\frac{1}{4}\left(6(b-c)(b+c)x+8c\sqrt{b^2-c^2}\cosh(x)+2bc\cosh(2x)+8b\sqrt{b^2-c^2}\sinh(x)+\left(b^2+c^2\right)\sinh(2x)\right)$$

input
```
Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(6*(b - c)*(b + c)*x + 8*c*Sqrt[b^2 - c^2]*Cosh[x] + 2*b*c*Cosh[2*x] + 8*b
*Sqrt[b^2 - c^2]*Sinh[x] + (b^2 + c^2)*Sinh[2*x])/4
```

3.755.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3592, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^2 dx$$

$$\Big\downarrow \text{3592}$$

$$\frac{3}{2}\sqrt{b^2-c^2}\int \left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)dx+\frac{1}{2}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}\sqrt{b^2-c^2}\left(x\sqrt{b^2-c^2}+b\sinh(x)+c\cosh(x)\right)+\frac{1}{2}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)$$

input
```
Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(3*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2]*x + c*Cosh[x] + b*Sinh[x]))/2 + ((c*Co
sh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]))/2
```

3.755.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.755.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

method	result
default	$c^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)+cb\cosh(x)^2+b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+2c\cosh(x)\sqrt{b^2-c^2}+2b\sinh(x)$
parts	$b^2x+2b\left(\frac{c\sinh(x)^2}{2}+\sinh(x)\sqrt{(b-c)(b+c)}\right)+b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+c^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)-$
risch	$\frac{3b^2x}{2}-\frac{3c^2x}{2}+\frac{b^2e^{2x}}{8}+\frac{e^{2x}cb}{4}+\frac{e^{2x}c^2}{8}+\sqrt{b^2-c^2}\,e^{x}b+\sqrt{b^2-c^2}\,e^{x}c-\sqrt{b^2-c^2}\,e^{-x}b+\sqrt{b^2-c^2}\,e^{-x}$

input
```
int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(1/2*cosh(x)*sinh(x)-1/2*x)+c*b*cosh(x)^2+b^2*(1/2*cosh(x)*sinh(x)+1/2
*x)+2*c*cosh(x)*(b^2-c^2)^(1/2)+2*b*sinh(x)*(b^2-c^2)^(1/2)+b^2*x-c^2*x
```

3.755.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(76) = 152.

Time = 0.26 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.64

$$\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx$$
$$=\frac{(b^2+2\,bc+c^2)\cosh(x)^4+4\,(b^2+2\,bc+c^2)\cosh(x)\sinh(x)^3+(b^2+2\,bc+c^2)\sinh(x)^4+12\,(b^2-c^2)x}{}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/8*((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)
^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 + 12*(b^2 - c^2)*x*cosh(x)^2 + 6*((b^2
+ 2*b*c + c^2)*cosh(x)^2 + 2*(b^2 - c^2)*x)*sinh(x)^2 - b^2 + 2*b*c - c^2
+ 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 + 6*(b^2 - c^2)*x*cosh(x))*sinh(x) + 8*
((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b
- c)*cosh(x) + (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))*sqrt(b^2 - c^2))/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

3.755.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.36

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx = -\frac{b^2x\sinh^2(x)}{2}+\frac{b^2x\cosh^2(x)}{2}+b^2x+\frac{b^2\sinh(x)\cosh(x)}{2}+bc\cosh^2(x)+2b\sqrt{b^2-c^2}\sinh(x)+\frac{c^2x\sinh^2(x)}{2}-\frac{c^2x\cosh^2(x)}{2}-c^2x+\frac{c^2\sinh(x)\cosh(x)}{2}+2c\sqrt{b^2-c^2}\cosh(x)$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**2,x)
```

output
```
-b**2*x*sinh(x)**2/2 + b**2*x*cosh(x)**2/2 + b**2*x + b**2*sinh(x)*cosh(x)
/2 + b*c*cosh(x)**2 + 2*b*sqrt(b**2 - c**2)*sinh(x) + c**2*x*sinh(x)**2/2
- c**2*x*cosh(x)**2/2 - c**2*x + c**2*sinh(x)*cosh(x)/2 + 2*c*sqrt(b**2 -
c**2)*cosh(x)
```

3.755.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx = bc\cosh(x)^2+\frac{1}{8}b^2\left(4x+e^{(2x)}-e^{(-2x)}\right)-\frac{1}{8}c^2\left(4x-e^{(2x)}+e^{(-2x)}\right)+b^2x-c^2x+2\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="maxima")
```

output
```
b*c*cosh(x)^2 + 1/8*b^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*c^2*(4*x - e^(2*x
) + e^(-2*x)) + b^2*x - c^2*x + 2*sqrt(b^2 - c^2)*(c*cosh(x) + b*sinh(x))
```

3.755.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx$$
$$=\sqrt{b^2-c^2}(b+c)e^x+\frac{3}{2}\left(b^2-c^2\right)x+\frac{1}{8}\left(b^2+2\,bc+c^2\right)e^{(2\,x)}$$
$$-\frac{1}{8}\left(b^2-2\,bc+c^2+8\left(\sqrt{b^2-c^2}b-\sqrt{b^2-c^2}c\right)e^x\right)e^{(-2\,x)}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="giac")
```

output
```
sqrt(b^2 - c^2)*(b + c)*e^x + 3/2*(b^2 - c^2)*x + 1/8*(b^2 + 2*b*c + c^2)*
e^(2*x) - 1/8*(b^2 - 2*b*c + c^2 + 8*(sqrt(b^2 - c^2)*b - sqrt(b^2 - c^2)*
c)*e^x)*e^(-2*x)
```

3.755.9 Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx=\frac{3\,b^2\,x}{2}-\frac{3\,c^2\,x}{2}+2\,c\cosh(x)\,\sqrt{b^2-c^2}$$
$$+2\,b\sinh(x)\,\sqrt{b^2-c^2}+b\,c\cosh(x)^2$$
$$+\frac{b^2\cosh(x)\,\sinh(x)}{2}+\frac{c^2\cosh(x)\,\sinh(x)}{2}$$

input
```
int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^2,x)
```

output
```
(3*b^2*x)/2 - (3*c^2*x)/2 + 2*c*cosh(x)*(b^2 - c^2)^(1/2) + 2*b*sinh(x)*(b
^2 - c^2)^(1/2) + b*c*cosh(x)^2 + (b^2*cosh(x)*sinh(x))/2 + (c^2*cosh(x)*s
inh(x))/2
```

3.756 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx$

3.756.1 Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx=\sqrt{b^2-c^2}x+c\cosh(x)+b\sinh(x)$$

output
```
c*cosh(x)+b*sinh(x)+x*(b^2-c^2)^(1/2)
```

3.756.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx=\sqrt{b^2-c^2}x+c\cosh(x)+b\sinh(x)$$

input
```
Integrate[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x],x]
```

output
```
Sqrt[b^2 - c^2]*x + c*Cosh[x] + b*Sinh[x]
```

3.756.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$x\sqrt{b^2-c^2}+b\sinh(x)+c\cosh(x)$$

input
```
Int[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x],x]
```

output
```
Sqrt[b^2 - c^2]*x + c*Cosh[x] + b*Sinh[x]
```

3.756.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.756.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$c\cosh\left(x\right)+b\sinh\left(x\right)+x\sqrt{b^{2}-c^{2}}$	23
parts	$c\cosh\left(x\right)+b\sinh\left(x\right)+x\sqrt{b^{2}-c^{2}}$	23

input
```
int(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c*cosh(x)+b*sinh(x)+x*(b^2-c^2)^(1/2)
```

3.756.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx = \frac{(b+c)\cosh{(x)}^2+2\,(b+c)\cosh{(x)}\sinh{(x)}+(b+c)\sinh{(x)}^2+2\,\sqrt{b^2-c^2}(x\cosh{(x)}+x\sinh{(x)})-b}{2\,(\cosh{(x)}+\sinh{(x)})}$$

input
```
integrate(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2
*sqrt(b^2 - c^2)*(x*cosh(x) + x*sinh(x)) - b + c)/(cosh(x) + sinh(x))
```

3.756.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx = b\sinh{(x)}+c\cosh{(x)}+x\sqrt{b^2-c^2}$$

input
```
integrate(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2),x)
```

output
```
b*sinh(x) + c*cosh(x) + x*sqrt(b**2 - c**2)
```

3.756.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx = c\cosh{(x)}+b\sinh{(x)}+\sqrt{b^2-c^2}x$$

input
```
integrate(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x, algorithm="maxima")
```

output
```
c*cosh(x) + b*sinh(x) + sqrt(b^2 - c^2)*x
```

3.756.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx=\frac{1}{2}\,c\left(e^{(-x)}+e^{x}\right)-\frac{1}{2}\,b\left(e^{(-x)}-e^{x}\right)+\sqrt{b^2-c^2}x$$

input
```
integrate(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*c*(e^(-x) + e^x) - 1/2*b*(e^(-x) - e^x) + sqrt(b^2 - c^2)*x
```

3.756.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx=x\,\sqrt{b^2-c^2}+c\cosh(x)+b\sinh(x)$$

input
```
int(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x),x)
```

output
```
x*(b^2 - c^2)^(1/2) + c*cosh(x) + b*sinh(x)
```

3.757 $\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

3.757.1 Optimal result

Integrand size = 24, antiderivative size = 34

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=-\frac{c+\sqrt{b^2-c^2}\sinh(x)}{c(c\cosh(x)+b\sinh(x))}$$

output
```
(-c-sinh(x)*(b^2-c^2)^(1/2))/c/(c*cosh(x)+b*sinh(x))
```

3.757.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\frac{-c-\sqrt{b^2-c^2}\sinh(x)}{c(c\cosh(x)+b\sinh(x))}$$

input
```
Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

output
```
(-c - Sqrt[b^2 - c^2]*Sinh[x])/(c*(c*Cosh[x] + b*Sinh[x]))
```

3.757.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{3593}$$

$$-\frac{\sqrt{b^2-c^2}\sinh(x)+c}{c(b\sinh(x)+c\cosh(x))}$$

input
```
Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

output
```
-((c + Sqrt[b^2 - c^2]*Sinh[x])/(c*(c*Cosh[x] + b*Sinh[x])))
```

3.757.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3593
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.757.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{2}{\mathrm{e}^{x} b+c\,\mathrm{e}^{x}+\sqrt{b^{2}-c^{2}}}$	25
default	$-\frac{2\left(\sqrt{b^{2}-c^{2}}+b\right)}{c^{2}\left(\tanh\left(\frac{x}{2}\right)+\frac{\sqrt{(b-c)(b+c)}}{c}+\frac{b}{c}\right)}$	46

input
```
int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2/(exp(x)*b+c*exp(x)+(b^2-c^2)^(1/2))
```

3.757.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(32) = 64.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.59

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = -\frac{2\left((b+c)\cosh(x)+(b+c)\sinh(x)-\sqrt{b^2-c^2}\right)}{(b^2+2\,bc+c^2)\cosh(x)^2+2\,(b^2+2\,bc+c^2)\cosh(x)\sinh(x)+(b^2+2\,bc+c^2)\sinh(x)^2-b^2+c^2}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x, algorithm="fricas")
```

output
```
-2*((b + c)*cosh(x) + (b + c)*sinh(x) - sqrt(b^2 - c^2))/((b^2 + 2*b*c + c
^2)*cosh(x)^2 + 2*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x) + (b^2 + 2*b*c + c^2
)*sinh(x)^2 - b^2 + c^2)
```

3.757.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2)),x)
```

output
```
Timed out
```

3.757.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.757.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,0]%%%}+%%%{1,[0,1]%%%},[2]%%%}+%%%{%%{[2,0]:[1,0,
%%%{-1,[2
```

3.757.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\int \frac{1}{b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)}\,dx$$

input
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x)),x)
```

output
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x)), x)
```

3.758 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx$

3.758.1 Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = \frac{c\cosh(x)+b\sinh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2} - \frac{c+\sqrt{b^2-c^2}\sinh(x)}{3c\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))}$$

output
```
1/3*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(
1/2))^2+1/3*(-c-sinh(x)*(b^2-c^2)^(1/2))/c/(c*cosh(x)+b*sinh(x))/(b^2-c^2)
^(1/2)
```

3.758.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = -\frac{-2c\sqrt{b^2-c^2}+2bc\cosh^3(x)+2c^2\sinh(x)+c^2\cosh^2(x)\sinh(x)+b^2\sinh^3(x)}{3c(c\cosh(x)+b\sinh(x))^3}$$

input
```
Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

output
```
-1/3*(-2*c*Sqrt[b^2 - c^2] + 2*b*c*Cosh[x]^3 + 2*c^2*Sinh[x] + c^2*Cosh[x]
^2*Sinh[x] + b^2*Sinh[x]^3)/(c*(c*Cosh[x] + b*Sinh[x])^3)
```

3.758.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3595, 3042, 3593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^2}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{\int \frac{1}{b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}}dx}{3\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{\int \frac{1}{b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}}dx}{3\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3593}$$

$$\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}-\frac{\sqrt{b^2-c^2}\sinh(x)+c}{3c\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}$$

input
```
Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

output
```
(c*Cosh[x] + b*Sinh[x])/(3*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] +
c*Sinh[x])^2) - (c + Sqrt[b^2 - c^2]*Sinh[x])/(3*c*Sqrt[b^2 - c^2]*(c*Cosh
[x] + b*Sinh[x]))
```

3.758.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3593
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.758.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

method	result	size
risch	$-\frac{2\left(3\,{\mathrm e}^{x} b+3 c\,{\mathrm e}^{x}+\sqrt{b^{2}-c^{2}}\right)}{3\left({\mathrm e}^{x} b+c\,{\mathrm e}^{x}+\sqrt{b^{2}-c^{2}}\right)^{3}}$	47
default	$\frac{2\left(\sqrt{b^{2}-c^{2}}+b\right)\left(\frac{\left(\sqrt{b^{2}-c^{2}}+b\right)\tanh\left(\frac{x}{2}\right)^{2}}{c^{2}}+\frac{\left(2b^{2}-c^{2}+2\sqrt{b^{2}-c^{2}}\,b\right)\tanh\left(\frac{x}{2}\right)}{c^{3}}+\frac{\frac{4\sqrt{b^{2}-c^{2}}\,b^{2}}{3}-\frac{2\sqrt{b^{2}-c^{2}}\,c^{2}}{3}+\frac{4b^{3}}{3}-\frac{4c^{2}b}{3}}{c^{4}}\right)}{c^{2}\left(\tanh\left(\frac{x}{2}\right)^{2}+\frac{2\sqrt{(b-c)(b+c)}\,\tanh\left(\frac{x}{2}\right)}{c}+\frac{2\tanh\left(\frac{x}{2}\right)b}{c}+\frac{2\sqrt{(b-c)(b+c)}\,b}{c^{2}}+\frac{2b^{2}}{c^{2}}-1\right)\left(\tanh\left(\frac{x}{2}\right)+\frac{\sqrt{(b-c)(b+c)}}{c}+\frac{b}{c}\right)}$	217

input
```
int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*exp(x)*b+3*c*exp(x)+(b^2-c^2)^(1/2))/(exp(x)*b+c*exp(x)+(b^2-c^2)^
(1/2))^3
```

3.758.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(88) = 176$.

Time = 0.29 (sec) , antiderivative size = 660, normalized size of antiderivative = 6.60

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx =$$

$$-\frac{}{3\left((b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4)\cosh\left(x\right)^6+6\,(b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4)\cosh\left(x\right)\sinh\left(x\right)^5+(b\right.}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*(b^2 + 2*b*c + c^2)*cosh(x)^4 + 12*(b^2 + 2*b*c + c^2)*cosh(x)*sin
h(x)^3 + 3*(b^2 + 2*b*c + c^2)*sinh(x)^4 + 6*(b^2 - c^2)*cosh(x)^2 + 6*(3*
(b^2 + 2*b*c + c^2)*cosh(x)^2 + b^2 - c^2)*sinh(x)^2 - b^2 + 2*b*c - c^2 +
 12*((b^2 + 2*b*c + c^2)*cosh(x)^3 + (b^2 - c^2)*cosh(x))*sinh(x) - 8*((b
+ c)*cosh(x)^3 + 3*(b + c)*cosh(x)^2*sinh(x) + 3*(b + c)*cosh(x)*sinh(x)^2
 + (b + c)*sinh(x)^3)*sqrt(b^2 - c^2))/((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c
^3 + c^4)*cosh(x)^6 + 6*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x
)*sinh(x)^5 + (b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^6 - 3*(b
^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^4 - 3*(b^4 + 2*b^3*c - 2*b*c^3 - c^4
 - 5*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^4 - b^
4 + 2*b^3*c - 2*b*c^3 + c^4 + 4*(5*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 +
c^4)*cosh(x)^3 - 3*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x))*sinh(x)^3 + 3*
(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^2 + 3*(5*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*
c^3 + c^4)*cosh(x)^4 + b^4 - 2*b^2*c^2 + c^4 - 6*(b^4 + 2*b^3*c - 2*b*c^3
- c^4)*cosh(x)^2)*sinh(x)^2 + 6*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^
4)*cosh(x)^5 - 2*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3 + (b^4 - 2*b^2*
c^2 + c^4)*cosh(x))*sinh(x))
```

3.758.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**2,x)
```

output
```
Timed out
```

3.758.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.758.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[2,0]%%%}+%%%{2,[1,1]%%%}+%%%{1,[0,2]%%%},[4]%%%}+%%
%{%%{[%%%
```

3.758.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = \int \frac{1}{\left(b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)\right)^2}\,dx$$

input
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^2,x)
```

output
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^2, x)
```

3.759 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx$

3.759.1 Optimal result

Integrand size = 24, antiderivative size = 146

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \frac{c\cosh(x)+b\sinh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3} + \frac{2(c\cosh(x)+b\sinh(x))}{15(b^2-c^2)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2} - \frac{2\left(c+\sqrt{b^2-c^2}\sinh(x)\right)}{15c(b^2-c^2)(c\cosh(x)+b\sinh(x))}$$

output
```
1/5*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(
1/2))^3+2/15*(c*cosh(x)+b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2
)^(1/2))^2-2/15*(c+sinh(x)*(b^2-c^2)^(1/2))/c/(b^2-c^2)/(c*cosh(x)+b*sinh(
x))
```

3.759.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \frac{12b\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x)) - \frac{b\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))^3}{(b-c)(b+c)} - \frac{2\sqrt{b^2-c^2}\sinh(x)(c\cosh(x)+b\sinh(x))^4}{(b-c)(b+c)} + (c\cosh(x}{15c(c\cosh(x)+b\sinh(x))^5}$$

input
```
Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

output
```
(12*b*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]) - (b*Sqrt[b^2 - c^2]*(c*Cosh
[x] + b*Sinh[x])^3)/((b - c)*(b + c)) - (2*Sqrt[b^2 - c^2]*Sinh[x]*(c*Cosh
[x] + b*Sinh[x])^4)/((b - c)*(b + c)) + (c*Cosh[x] + b*Sinh[x])^2*(-5*c +
Sqrt[b^2 - c^2]*Sinh[x]) - 12*(b^2 - c^2)*(c + Sqrt[b^2 - c^2]*Sinh[x]))/(
15*c*(c*Cosh[x] + b*Sinh[x])^5)
```

3.759.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3595, 3042, 3595, 3042, 3593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^3}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{2\int \frac{1}{\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^2}dx}{5\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2\int \frac{1}{\left(b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}\right)^2}dx}{5\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3595}$$

$$\frac{2\left(\frac{\int \frac{1}{b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}}dx}{3\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\right)}{5\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2\left(\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{\int \frac{1}{b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}}dx}{3\sqrt{b^2-c^2}}\right)}{5\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3593}$$

$$\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2\left(\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}-\frac{\sqrt{b^2-c^2}\sinh(x)+c}{3c\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}\right)}{5\sqrt{b^2-c^2}}$$

input
```
Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

output
```
(c*Cosh[x] + b*Sinh[x])/(5*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] +
c*Sinh[x])^3) + (2*((c*Cosh[x] + b*Sinh[x])/(3*Sqrt[b^2 - c^2]*(Sqrt[b^2 -
 c^2] + b*Cosh[x] + c*Sinh[x])^2) - (c + Sqrt[b^2 - c^2]*Sinh[x])/(3*c*Sqr
t[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))))/(5*Sqrt[b^2 - c^2])
```

3.759.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3593
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.759. $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx$

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.759.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{4\left(10b^2 e^{2x}+20\, e^{2x} c b+10\, e^{2x} c^2+5\sqrt{b^2-c^2}\, e^{x} b+5\sqrt{b^2-c^2}\, e^{x} c+b^2-c^2\right)}{15\left(e^{x} b+c\, e^{x}+\sqrt{b^2-c^2}\right)^5}$
default	$\frac{-\frac{2\left(4\sqrt{b^2-c^2}\, b^2-\sqrt{b^2-c^2}\, c^2+4b^3-3c^2 b\right)\tanh\left(\frac{x}{2}\right)^4}{c^2}-\frac{4\left(8b^4-8b^2c^2+c^4+8\sqrt{b^2-c^2}\, b^3-4\sqrt{b^2-c^2}\, b\, c^2\right)\tanh\left(\frac{x}{2}\right)^3}{c^3}-\frac{8\left(24b^4\sqrt{b^2-c^2}-20b^2c^2\sqrt{b^2\right.}{}}{}}{c^4}$

input
```
int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/15*(10*b^2*exp(x)^2+20*exp(x)^2*c*b+10*c^2*exp(x)^2+5*(b^2-c^2)^(1/2)*e
xp(x)*b+5*(b^2-c^2)^(1/2)*exp(x)*c+b^2-c^2)/(exp(x)*b+c*exp(x)+(b^2-c^2)^(
1/2))^5
```

3.759.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3035 vs. 2(132) = 264.

Time = 0.45 (sec) , antiderivative size = 3035, normalized size of antiderivative = 20.79

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="fricas")
```

output
```
-4/15*(10*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^7 + 70*(b^4
+ 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(x)^6 + 10*(b^4 + 4*b^3
*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^7 + 76*(b^4 + 2*b^3*c - 2*b*c^3 -
c^4)*cosh(x)^5 + 2*(38*b^4 + 76*b^3*c - 76*b*c^3 - 38*c^4 + 105*(b^4 + 4*b
^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^5 + 10*(35*(b^4 + 4*b
^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 + 38*(b^4 + 2*b^3*c - 2*b*c^3
- c^4)*cosh(x))*sinh(x)^4 + 10*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^3 + 10*(35*
(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^4 + b^4 - 2*b^2*c^2 +
c^4 + 76*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^2)*sinh(x)^3 + 10*(21*(b^
4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^5 + 76*(b^4 + 2*b^3*c - 2
*b*c^3 - c^4)*cosh(x)^3 + 3*(b^4 - 2*b^2*c^2 + c^4)*cosh(x))*sinh(x)^2 + 1
0*(7*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^6 + 38*(b^4 + 2*b
^3*c - 2*b*c^3 - c^4)*cosh(x)^4 + 3*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^2)*sin
h(x) - (45*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 270*(b^3 + 3*b^2*c
+ 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + 45*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*si
nh(x)^6 + 55*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 5*(11*b^3 + 11*b^2*c
- 11*b*c^2 - 11*c^3 + 135*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(
x)^4 + 20*(45*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 11*(b^3 + b^2*c
- b*c^2 - c^3)*cosh(x))*sinh(x)^3 + b^3 - 3*b^2*c + 3*b*c^2 - c^3 - 5*(b^3
 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 5*(135*(b^3 + 3*b^2*c + 3*b*c^2 + c...
```

3.759.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**3,x)
```

output
```
Timed out
```

3.759.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.759.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[3,0]%%%}+%%%{3,[2,1]%%%}+%%%{3,[1,2]%%%}+%%%{1,[0,3
]%%%},[6]
```

3.759.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \int \frac{1}{\left(b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)\right)^3}\,dx$$

input
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^3,x)
```

output
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^3, x)
```

3.760 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx$

3.760.1 Optimal result

Integrand size = 24, antiderivative size = 198

$$\begin{aligned}\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx &= \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\\ &+\frac{3(c\cosh(x)+b\sinh(x))}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\\ &+\frac{2(c\cosh(x)+b\sinh(x))}{35\left(b^2-c^2\right)^{3/2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\\ &-\frac{2\left(c+\sqrt{b^2-c^2}\sinh(x)\right)}{35c\left(b^2-c^2\right)^{3/2}(c\cosh(x)+b\sinh(x))}\end{aligned}$$

output
```
1/7*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(
1/2))^4+3/35*(c*cosh(x)+b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2
)^(1/2))^3+2/35*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(3/2)/(b*cosh(x)+c*sinh(x)
+(b^2-c^2)^(1/2))^2-2/35*(c+sinh(x)*(b^2-c^2)^(1/2))/c/(b^2-c^2)^(3/2)/(c*
cosh(x)+b*sinh(x))
```

3.760.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 425 vs. 2(198) = 396.

Time = 0.62 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.15

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx = -\frac{-832b^4c\sqrt{b^2-c^2}+1664b^2c^3\sqrt{b^2-c^2}-832c^5\sqrt{b^2-c^2}+1190bc(b^2-c^2)^2\cosh(x)+448c\sqrt{b^2-c^2}(-b}{}$$

input
```
Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

output
```
-1/1120*(-832*b^4*c*Sqrt[b^2 - c^2] + 1664*b^2*c^3*Sqrt[b^2 - c^2] - 832*c
^5*Sqrt[b^2 - c^2] + 1190*b*c*(b^2 - c^2)^2*Cosh[x] + 448*c*Sqrt[b^2 - c^2
]*(-b^4 + c^4)*Cosh[2*x] + 112*b^5*c*Cosh[3*x] + 56*b^3*c^3*Cosh[3*x] - 16
8*b*c^5*Cosh[3*x] - 28*b^5*c*Cosh[5*x] + 28*b*c^5*Cosh[5*x] + 6*b^5*c*Cosh
[7*x] + 20*b^3*c^3*Cosh[7*x] + 6*b*c^5*Cosh[7*x] - 35*b^6*Sinh[x] + 1295*b
^4*c^2*Sinh[x] - 2485*b^2*c^4*Sinh[x] + 1225*c^6*Sinh[x] - 896*b^3*c^2*Sqr
t[b^2 - c^2]*Sinh[2*x] + 896*b*c^4*Sqrt[b^2 - c^2]*Sinh[2*x] + 21*b^6*Sinh
[3*x] + 189*b^4*c^2*Sinh[3*x] - 161*b^2*c^4*Sinh[3*x] - 49*c^6*Sinh[3*x] -
 7*b^6*Sinh[5*x] - 35*b^4*c^2*Sinh[5*x] + 35*b^2*c^4*Sinh[5*x] + 7*c^6*Sin
h[5*x] + b^6*Sinh[7*x] + 15*b^4*c^2*Sinh[7*x] + 15*b^2*c^4*Sinh[7*x] + c^6
*Sinh[7*x])/((b - c)*c*(b + c)*(c*Cosh[x] + b*Sinh[x])^7)
```

3.760.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3595, 3042, 3595, 3042, 3595, 3042, 3593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^4}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{3\int \frac{1}{\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^3}dx}{7\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(x)+c\cosh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}+\frac{3\int \frac{1}{\left(b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}\right)^3}dx}{7\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3595}$$

$$\frac{3\left(\frac{2\int \frac{1}{\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^2}dx}{5\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\right)}{7\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(x)+c\cosh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}+\frac{3\left(\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2\int \frac{1}{\left(b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}\right)^2}dx}{5\sqrt{b^2-c^2}}\right)}{7\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3595}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{1}{b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}}dx}{3\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\right)}{5\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\right)}{7\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}$$

$$\Big\downarrow \text{3042}$$

3.760. $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}dx$

$$\frac{b\sinh(x)+c\cosh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}+\frac{3\left(\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2\left(\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{\int\frac{1}{b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}}dx}{3\sqrt{b^2-c^2}}\right)}{5\sqrt{b^2-c^2}}\right)}{7\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3593}$$

$$\frac{b\sinh(x)+c\cosh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}+\frac{3\left(\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2\left(\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}-\frac{\sqrt{b^2-c^2}\sinh(x)+c}{3c\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}\right)}{5\sqrt{b^2-c^2}}\right)}{7\sqrt{b^2-c^2}}$$

input
```
Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

output
```
(c*Cosh[x] + b*Sinh[x])/(7*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] +
c*Sinh[x])^4) + (3*((c*Cosh[x] + b*Sinh[x])/(5*Sqrt[b^2 - c^2]*(Sqrt[b^2 -
 c^2] + b*Cosh[x] + c*Sinh[x])^3) + (2*((c*Cosh[x] + b*Sinh[x])/(3*Sqrt[b^
2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2) - (c + Sqrt[b^2 - c^
2]*Sinh[x])/(3*c*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))))/(5*Sqrt[b^2 -
c^2])))/(7*Sqrt[b^2 - c^2])
```

3.760.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3593
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.760. $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx$

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.760.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{4\left(35\,\mathrm{e}^{3x} b^{3}+105\,\mathrm{e}^{3x} c\,b^{2}+105\,\mathrm{e}^{3x} c^{2} b+35\,\mathrm{e}^{3x} c^{3}+21\,\mathrm{e}^{2x} \sqrt{b^{2}-c^{2}}\,b^{2}+42\,\mathrm{e}^{2x} \sqrt{b^{2}-c^{2}}\,bc+21\,\mathrm{e}^{2x} \sqrt{b^{2}-c^{2}}\,c^{2}+7 b^{3} \mathrm{e}^{x}+7\,\mathrm{e}^{x} c\,b^{2}-7\,\mathrm{e}^{x} c^{2} b-\right.}{35\left(\mathrm{e}^{x} b+c\,\mathrm{e}^{x}+\sqrt{b^{2}-c^{2}}\right)^{7}}$
default	$\frac{2\left(8 b^{4}-8 b^{2} c^{2}+c^{4}+8 \sqrt{b^{2}-c^{2}}\,b^{3}-4 \sqrt{b^{2}-c^{2}}\,b\,c^{2}\right) \tanh\left(\frac{x}{2}\right)^{6}}{c^{2}}+\frac{6\left(16 b^{4} \sqrt{b^{2}-c^{2}}-12 b^{2} c^{2} \sqrt{b^{2}-c^{2}}+c^{4} \sqrt{b^{2}-c^{2}}+16 b^{5}-20 b^{3} c^{2}+5 b\,c^{4}\right) \tanh\left(\frac{x}{2}\right)^{5}}{c^{3}}+\frac{4(8}{}$

input
```
int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x,method=_RETURNVERBOSE)
```

output
```
-4/35*(35*b^3*exp(x)^3+105*exp(x)^3*c*b^2+105*exp(x)^3*c^2*b+35*exp(x)^3*c
^3+21*exp(x)^2*(b^2-c^2)^(1/2)*b^2+42*exp(x)^2*(b^2-c^2)^(1/2)*b*c+21*exp(
x)^2*(b^2-c^2)^(1/2)*c^2+7*b^3*exp(x)+7*exp(x)*c*b^2-7*exp(x)*c^2*b-7*exp(
x)*c^3+(b^2-c^2)^(1/2)*b^2-(b^2-c^2)^(1/2)*c^2)/(exp(x)*b+c*exp(x)+(b^2-c^
2)^(1/2))^7
```

3.760.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6590 vs. $2(176) = 352$.

Time = 1.60 (sec) , antiderivative size = 6590, normalized size of antiderivative = 33.28

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.760.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**4,x)
```

output
```
Timed out
```

3.760.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.760.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[4,0]%%%}+%%%{4,[3,1]%%%}+%%%{6,[2,2]%%%}+%%%{4,[1,3
]%%%}+%%%
```

3.760.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx=\int \frac{1}{\left(b\cosh\left(x\right)+\sqrt{b^2-c^2}+c\sinh\left(x\right)\right)^4}\,dx$$

input
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^4,x)
```

output
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^4, x)
```

3.761 $\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx$

3.761.1 Optimal result

Integrand size = 14, antiderivative size = 294

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx = \frac{16}{15}(ac\cosh(x)+ab\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} + \frac{2}{5}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^{3/2} - \frac{2i(23a^2+9b^2-9c^2)\,E\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{15\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} + \frac{16ia(a^2-b^2+c^2)\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{15\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

output
```
2/5*(c*cosh(x)+b*sinh(x))*(a+b*cosh(x)+c*sinh(x))^(3/2)+16/15*(a*c*cosh(x)
+a*b*sinh(x))*(a+b*cosh(x)+c*sinh(x))^(1/2)-2/15*I*(23*a^2+9*b^2-9*c^2)*(c
os(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*El
lipticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c
^2)^(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/((a+b*cosh(x)+c*sinh(x))/
(a+(b^2-c^2)^(1/2)))^(1/2)+16/15*I*a*(a^2-b^2+c^2)*(cos(1/2*I*x-1/2*arctan
(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticF(sin(1/2*I*x-1
/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))^(1/2))*((
a+b*cosh(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)/(a+b*cosh(x)+c*sinh(x))^
(1/2)
```

3.761.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.28 (sec) , antiderivative size = 3775, normalized size of antiderivative = 12.84

$$\int (a + b\cosh(x) + c\sinh(x))^{5/2} \, dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

output
```
Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((2*b*(23*a^2 + 9*b^2 - 9*c^2))/(15*c) + (
22*a*c*Cosh[x])/15 + (2*b*c*Cosh[2*x])/5 + (22*a*b*Sinh[x])/15 + ((b^2 + c
^2)*Sinh[2*x])/5) + (2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1
- b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[
1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]
))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcT
anh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^
2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^
2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2
]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*
Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*Sqrt[
I*(I + Sinh[x + ArcTanh[b/c]])]) + (34*a*b^2*AppellF1[1/2, 1/2, 1/2, 3/2,
((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]
*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh
[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))
*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sq
rt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/
(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqr
t[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/
c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(15*S...
```

3.761.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {3042, 3599, 27, 3042, 3625, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\cos(ix)-ic\sin(ix))^{5/2}dx$$

$$\Big\downarrow \; 3599$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cosh(x)+c\sinh(x)}\left(5a^2+8b\cosh(x)a+8c\sinh(x)a+3b^2-3c^2\right)dx+\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\int \sqrt{a+b\cosh(x)+c\sinh(x)}\left(5a^2+8b\cosh(x)a+8c\sinh(x)a+3b^2-3c^2\right)dx+\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}+\frac{1}{5}\int \sqrt{a+b\cos(ix)-ic\sin(ix)}\left(5a^2+8b\cos(ix)a-8ic\sin(ix)a+3b^2-3c^2\right)dx$$

$$\Big\downarrow \; 3625$$

$$\frac{1}{5}\left(\frac{2\int \frac{(15a^2+17b^2-17c^2)a^2+b(23a^2+9b^2-9c^2)\cosh(x)a+c(23a^2+9b^2-9c^2)\sinh(x)a}{2\sqrt{a+b\cosh(x)+c\sinh(x)}}dx}{3a}+\frac{16}{3}\sqrt{a+b\cosh(x)+c\sinh(x)}(ab\sinh\right.$$

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{\int \frac{(15a^2+17(b^2-c^2))a^2+b(23a^2+9b^2-9c^2)\cosh(x)a+c(23a^2+9b^2-9c^2)\sinh(x)a}{\sqrt{a+b\cosh(x)+c\sinh(x)}}dx}{3a}+\frac{16}{3}\sqrt{a+b\cosh(x)+c\sinh(x)}(ab\sinh(\right.$$

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}+$$

$$\frac{1}{5}\left(\frac{16}{3}\sqrt{a+b\cosh(x)+c\sinh(x)}(ab\sinh(x)+ac\cosh(x))+\frac{\int \frac{(15a^2+17(b^2-c^2))a^2+b(23a^2+9b^2-9c^2)\cos(ix)a-ic(23a^2+9b}{\sqrt{a+b\cos(ix)-ic\sin(ix)}}}{3a}\right.$$

3.761. $\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx$

$$\Big\downarrow \text{3628}$$

$$\frac{1}{5}\left(\frac{a\left(23a^2+9b^2-9c^2\right)\int\sqrt{a+b\cosh(x)+c\sinh(x)}dx-8a^2\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}dx}{3a}+\frac{16}{3}\sqrt{a+\right.$$
$$\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}\sqrt{a+b\cosh(x)+c\sinh(x)}(ab\sinh(x)+ac\cosh(x))+\frac{a\left(23a^2+9b^2-9c^2\right)\int\sqrt{a+b\cos(ix)-ic\sin(ix)}dx}{3a}\right.$$

$$\Big\downarrow \text{3598}$$

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}\sqrt{a+b\cosh(x)+c\sinh(x)}(ab\sinh(x)+ac\cosh(x))+\frac{\frac{a\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-}}{}}}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}\sqrt{a+b\cosh(x)+c\sinh(x)}(ab\sinh(x)+ac\cosh(x))+\frac{\frac{a\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-}}{}}}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}}{}\right.$$

$$\Big\downarrow \text{3132}$$

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}\sqrt{a+b\cosh(x)+c\sinh(x)}(ab\sinh(x)+ac\cosh(x))+\frac{-8a^2\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{a+b\cos(ix)-ic\sin(ix)}}dx-\frac{2ia(2}{}}{}\right.$$

3.761. $\int(a+b\cosh(x)+c\sinh(x))^{5/2}\,dx$

$\downarrow$ 3606

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}+$$

$$\frac{1}{5}\left(\frac{16}{3}\sqrt{a+b\cosh(x)+c\sinh(x)}(ab\sinh(x)+ac\cosh(x))+\frac{-\frac{8a^2(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int\frac{}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}}{}}}}{\sqrt{a+b\cosh(x)+c\sinh(x)}}}{}\right.$$

$\downarrow$ 3042

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}+$$

$$\frac{1}{5}\left(\frac{16}{3}\sqrt{a+b\cosh(x)+c\sinh(x)}(ab\sinh(x)+ac\cosh(x))+\frac{-\frac{8a^2(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int\frac{}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}}{}}}}{\sqrt{a+b\cosh(x)+c\sinh(x)}}}{}\right.$$

$\downarrow$ 3140

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}+$$

$$\frac{1}{5}\left(\frac{16}{3}\sqrt{a+b\cosh(x)+c\sinh(x)}(ab\sinh(x)+ac\cosh(x))+\frac{\frac{16ia^2(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(\right.\right.}{\sqrt{a+b\cosh(x)+c\sinh(x)}}}{}\right.$$

input
```
Int[(a + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

3.761. $\int(a+b\cosh(x)+c\sinh(x))^{5/2}\,dx$

output
```
(2*(c*Cosh[x] + b*Sinh[x])*(a + b*Cosh[x] + c*Sinh[x])^(3/2))/5 + ((16*(a*
c*Cosh[x] + a*b*Sinh[x])*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/3 + (((-2*I)*a*(
23*a^2 + 9*b^2 - 9*c^2)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2
 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a +
 b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])] + ((16*I)*a^2*(a^2 - b^2 +
c^2)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[
b^2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqrt
[a + b*Cosh[x] + c*Sinh[x]])/(3*a))/5
```

3.761.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.761. $\int (a + b\cosh(x) + c\sinh(x))^{5/2} \, dx$

3.761.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(328) = 656$.

Time = 1.74 (sec) , antiderivative size = 899, normalized size of antiderivative = 3.06

method	result
default	$-\frac{\sqrt{(b-c)(b+c)}\left(\frac{\cosh(x)^3 b^2}{3}-\frac{\cosh(x)^3 c^2}{3}+3\cosh(x)a^2-\cosh(x)b^2+\cosh(x)c^2\right)}{\sqrt{\frac{-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{\frac{\left(-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}\right)\sinh(x)^2}{\sqrt{b^2-c^2}}}\left(\ldots\right.}{\ldots}$

input
```
int((a+b*cosh(x)+c*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2)*((
b-c)*(b+c))^(1/2)*(1/3*cosh(x)^3*b^2-1/3*cosh(x)^3*c^2+3*cosh(x)*a^2-cosh(
x)*b^2+cosh(x)*c^2)+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2
)^(1/2)*sinh(x)^2)^(1/2)*(a^3*ln((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/
2))/(b^2-c^2)^(1/2)*cosh(x)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/
(b^2-c^2)^(1/2))^(1/2)+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-
c^2)^(1/2)*sinh(x)^2)^(1/2))/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))
/(b^2-c^2)^(1/2))^(1/2)+1/2*a*c^2*ln((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)
^(1/2))/(b^2-c^2)^(1/2)*cosh(x)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/
2))/(b^2-c^2)^(1/2))^(1/2)+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(
b^2-c^2)^(1/2)*sinh(x)^2)^(1/2))/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1
/2))/(b^2-c^2)^(1/2))^(1/2)-1/2*a*b^2*ln((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-
c^2)^(1/2))/(b^2-c^2)^(1/2)*cosh(x)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)
^(1/2))/(b^2-c^2)^(1/2))^(1/2)+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2
))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2))/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2
)^(1/2))/(b^2-c^2)^(1/2))^(1/2)+1/2*a*cosh(x)/(-b^2*sinh(x)+sinh(x)*c^2+a*
(b^2-c^2)^(1/2))*(b^2-c^2)^(1/2)*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1
/2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*b^2-1/2*a*cosh(x)/(-b^2*sinh(x)+sinh
(x)*c^2+a*(b^2-c^2)^(1/2))*(b^2-c^2)^(1/2)*((-b^2*sinh(x)+sinh(x)*c^2+a*(b
^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*c^2)/sinh(x)/((-b^2*sin...
```

3.761.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 928, normalized size of antiderivative = 3.16

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="fricas")
```

output
```
-1/90*(4*(sqrt(2)*(a^3 - 33*a*b^2 + 33*a*c^2)*cosh(x)^2 + 2*sqrt(2)*(a^3 -
 33*a*b^2 + 33*a*c^2)*cosh(x)*sinh(x) + sqrt(2)*(a^3 - 33*a*b^2 + 33*a*c^2
)*sinh(x)^2)*sqrt(b + c)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(
b^2 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b
*c^2 + c^3), 1/3*(3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c)) +
12*(sqrt(2)*(23*a^2*b + 9*b^3 - 9*b*c^2 - 9*c^3 + (23*a^2 + 9*b^2)*c)*cosh
(x)^2 + 2*sqrt(2)*(23*a^2*b + 9*b^3 - 9*b*c^2 - 9*c^3 + (23*a^2 + 9*b^2)*c
)*cosh(x)*sinh(x) + sqrt(2)*(23*a^2*b + 9*b^3 - 9*b*c^2 - 9*c^3 + (23*a^2
+ 9*b^2)*c)*sinh(x)^2)*sqrt(b + c)*weierstrassZeta(4/3*(4*a^2 - 3*b^2 + 3*
c^2)/(b^2 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c
 + 3*b*c^2 + c^3), weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 +
2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 +
 c^3), 1/3*(3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c))) - 3*(3*
(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 + 3*(b^3 + 3*b^2*c + 3*b*c^2 + c
^3)*sinh(x)^4 + 22*(a*b^2 + 2*a*b*c + a*c^2)*cosh(x)^3 + 2*(11*a*b^2 + 22*
a*b*c + 11*a*c^2 + 6*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x))*sinh(x)^3 -
3*b^3 + 3*b^2*c + 3*b*c^2 - 3*c^3 - 4*(23*a^2*b + 9*b^3 - 9*b*c^2 - 9*c^3
+ (23*a^2 + 9*b^2)*c)*cosh(x)^2 - 2*(46*a^2*b + 18*b^3 - 18*b*c^2 - 18*c^3
 - 9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2 + 2*(23*a^2 + 9*b^2)*c - 33
*(a*b^2 + 2*a*b*c + a*c^2)*cosh(x))*sinh(x)^2 - 22*(a*b^2 - a*c^2)*cosh...
```

3.761.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))**(5/2),x)
```

output
```
Timed out
```

3.761.7 Maxima [F]

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx=\int (b\cosh{(x)}+c\sinh{(x)}+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + c*sinh(x) + a)^(5/2), x)
```

3.761.8 Giac [F]

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx=\int (b\cosh{(x)}+c\sinh{(x)}+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + c*sinh(x) + a)^(5/2), x)
```

3.761.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx=\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx$$

input
```
int((a + b*cosh(x) + c*sinh(x))^(5/2),x)
```

output
```
int((a + b*cosh(x) + c*sinh(x))^(5/2), x)
```

3.762 $\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx$

3.762.1 Optimal result

Integrand size = 14, antiderivative size = 249

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx = \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} - \frac{8iaE\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{3\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} + \frac{2i(a^2-b^2+c^2)\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{3\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

output
```
2/3*(c*cosh(x)+b*sinh(x))*(a+b*cosh(x)+c*sinh(x))^(1/2)-8/3*I*a*(cos(1/2*I
*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticE(
sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2
)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/((a+b*cosh(x)+c*sinh(x))/(a+(b^2-
c^2)^(1/2)))^(1/2)+2/3*I*(a^2-b^2+c^2)*(cos(1/2*I*x-1/2*arctan(b,-I*c))^2)
^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticF(sin(1/2*I*x-1/2*arctan(b,
-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))^(1/2))*((a+b*cosh(x)+
c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)/(a+b*cosh(x)+c*sinh(x))^(1/2)
```

3.762.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.12 (sec) , antiderivative size = 2292, normalized size of antiderivative = 9.20

$$\int (a + b\cosh(x) + c\sinh(x))^{3/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

output
```
((8*a*b)/(3*c) + (2*c*Cosh[x])/3 + (2*b*Sinh[x])/3)*Sqrt[a + b*Cosh[x] + c
*Sinh[x]] + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^
2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/
c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt
[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]
]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c
*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)
/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x
 + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b
^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*Sqrt[I*(I + S
inh[x + ArcTanh[b/c]])]) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a +
Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)
/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTan
h[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x
 + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 +
c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sq
rt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c
^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt
[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*Sqrt[1 - b^2/c^2
]*c*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (2*c*AppellF1[1/2, 1/2, 1/2...
```

3.762.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 3599, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx$$

$\big\downarrow$ 3042

$$\int (a+b\cos(ix)-ic\sin(ix))^{3/2}dx$$

$\big\downarrow$ 3599

$$\frac{2}{3}\int \frac{3a^2+4b\cosh(x)a+4c\sinh(x)a+b^2-c^2}{2\sqrt{a+b\cosh(x)+c\sinh(x)}}dx+\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}$$

$\big\downarrow$ 27

$$\frac{1}{3}\int \frac{3a^2+4b\cosh(x)a+4c\sinh(x)a+b^2-c^2}{\sqrt{a+b\cosh(x)+c\sinh(x)}}dx+\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}$$

$\big\downarrow$ 3042

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}+\frac{1}{3}\int \frac{3a^2+4b\cos(ix)a-4ic\sin(ix)a+b^2-c^2}{\sqrt{a+b\cos(ix)-ic\sin(ix)}}dx$$

$\big\downarrow$ 3628

$$\frac{1}{3}\left(4a\int \sqrt{a+b\cosh(x)+c\sinh(x)}dx-\left(a^2-b^2+c^2\right)\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}dx\right)+\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}$$

$\big\downarrow$ 3042

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}+\frac{1}{3}\left(4a\int \sqrt{a+b\cos(ix)-ic\sin(ix)}dx-\left(a^2-b^2+c^2\right)\int \frac{1}{\sqrt{a+b\cos(ix)-ic\sin(ix)}}dx\right)$$

$\big\downarrow$ 3598

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}+\frac{1}{3}\left(\frac{4a\sqrt{a+b\cosh(x)+c\sinh(x)}\int \sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}{a+\sqrt{b^2-c^2}}}dx}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}-\left(a^2-b^2+c^2\right)\int \frac{1}{\sqrt{a+b\cos(ix}}\right.$$

$\big\downarrow$ 3042

3.762. $\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx$

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}+$$
$$\frac{1}{3}\left(\frac{4a\sqrt{a+b\cosh(x)+c\sinh(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\sin\left(ix-\tan^{-1}(b,-ic)+\frac{\pi}{2}\right)}{a+\sqrt{b^2-c^2}}}dx}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}-(a^2-b^2+c^2)\int\frac{}{\sqrt{a+b\cos(i}}\right.$$

$\big\downarrow$ 3132

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}+$$
$$\frac{1}{3}\left(-(a^2-b^2+c^2)\int\frac{1}{\sqrt{a+b\cos(ix)-ic\sin(ix)}}dx-\frac{8ia\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\right.}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}\right.$$

$\big\downarrow$ 3606

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}+$$
$$\frac{1}{3}\left(-\frac{(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int\frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}}dx}{\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)+c\sinh(}}{\sqrt{a+}}\right.$$

$\big\downarrow$ 3042

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}+$$
$$\frac{1}{3}\left(-\frac{(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int\frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\sin\left(ix-\tan^{-1}(b,-ic)+\frac{\pi}{2}\right)}{a+\sqrt{b^2-c^2}}}}dx}{\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)+c\sinh}}{\sqrt{a}}\right.$$

$\big\downarrow$ 3140

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}+$$
$$\frac{1}{3}\left(\frac{2i(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)+c\,s}}{}\right.$$

input
```
Int[(a + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

3.762. $\int(a+b\cosh(x)+c\sinh(x))^{3/2}\,dx$

output
```
(2*(c*Cosh[x] + b*Sinh[x])*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/3 + (((-8*I)*a
*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2
- c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*Cosh[x] + c*Sinh[x])
/(a + Sqrt[b^2 - c^2])] + ((2*I)*(a^2 - b^2 + c^2)*EllipticF[(I*x - ArcTan
[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Cos
h[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqrt[a + b*Cosh[x] + c*Sinh[x]])
/3
```

3.762.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.762.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.29

method	result
default	$\frac{2a\left(-b^{2}+c^{2}\right)\cosh\left(x\right)}{\sqrt{b^{2}-c^{2}}\,\sqrt{\frac{-b^{2}\sinh\left(x\right)+\sinh\left(x\right)c^{2}+a\sqrt{b^{2}-c^{2}}}{\sqrt{b^{2}-c^{2}}}}}+\frac{\sqrt{\frac{\left(-b^{2}\sinh\left(x\right)+\sinh\left(x\right)c^{2}+a\sqrt{b^{2}-c^{2}}\right)\sinh\left(x\right)^{2}}{\sqrt{b^{2}-c^{2}}}}\,a^{2}\ln\left(\frac{-\sinh\left(x\right)\cosh\left(x\right)b^{2}+\sinh\left(x\right)\cosh\left(x\right)c}{}\right.}{\left(-b^{2}\sinh\left(x\right)+\sinh\right.}$

input
```
int((a+b*cosh(x)+c*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*a/(b^2-c^2)^(1/2)*(-b^2+c^2)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2
))/(b^2-c^2)^(1/2))^(1/2)*cosh(x)+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(
1/2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*a^2*ln((-sinh(x)*cosh(x)*b^2+sinh(x
)*cosh(x)*c^2+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x
)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)
+a)^(1/2))/(b^2-c^2)^(1/2)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(
b^2-c^2)^(1/2))^(1/2))/(-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))*(b^2-c
^2)^(1/2)/sinh(x)
```

3.762.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.86

$$\int (a + b\cosh(x) + c\sinh(x))^{3/2}\,dx = \frac{2\left(\sqrt{2}(a^2+3\,b^2-3\,c^2)\cosh{(x)}+\sqrt{2}(a^2+3\,b^2-3\,c^2)\sinh{(x)}\right)\sqrt{b+c}\text{weierstrassPInve}}{}$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/9*(2*(sqrt(2)*(a^2 + 3*b^2 - 3*c^2)*cosh(x) + sqrt(2)*(a^2 + 3*b^2 - 3*c
^2)*sinh(x))*sqrt(b + c)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(
b^2 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b
*c^2 + c^3), 1/3*(3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c)) -
24*(sqrt(2)*(a*b + a*c)*cosh(x) + sqrt(2)*(a*b + a*c)*sinh(x))*sqrt(b + c)
*weierstrassZeta(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2), -8/27*(8
*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), weierstrassPInv
erse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b
^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), 1/3*(3*(b + c)*cosh(x) + 3*
(b + c)*sinh(x) + 2*a)/(b + c))) + 3*((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2
 + 2*b*c + c^2)*sinh(x)^2 - b^2 + c^2 - 8*(a*b + a*c)*cosh(x) - 2*(4*a*b +
 4*a*c - (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + c*sinh(x)
+ a))/((b + c)*cosh(x) + (b + c)*sinh(x))
```

3.762.6 Sympy [F]

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx=\int (a+b\cosh{(x)}+c\sinh{(x)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))**(3/2),x)
```

output
```
Integral((a + b*cosh(x) + c*sinh(x))**(3/2), x)
```

3.762.7 Maxima [F]

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx=\int (b\cosh{(x)}+c\sinh{(x)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + c*sinh(x) + a)^(3/2), x)
```

3.762.8 Giac [F]

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx=\int (b\cosh{(x)}+c\sinh{(x)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + c*sinh(x) + a)^(3/2), x)
```

3.762.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx=\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx$$

input
```
int((a + b*cosh(x) + c*sinh(x))^(3/2),x)
```

output
```
int((a + b*cosh(x) + c*sinh(x))^(3/2), x)
```

3.763 $\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx$

3.763.1 Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{2iE\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

output
```
-2*I*(cos(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I
*c))*EllipticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a
+(b^2-c^2)^(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/((a+b*cosh(x)+c*si
nh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)
```

3.763.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.09 (sec) , antiderivative size = 1401, normalized size of antiderivative = 13.74

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

output
```
(2*b*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/c + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2
, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^
2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Si
nh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c
))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*
Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]
)/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*S
qrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2
)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[
1 - b^2/c^2]*c*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (b^2*((c*AppellF1[-
1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*
Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]
*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2]
)))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)
/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 -
 c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt
[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]
)/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[
x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c
^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/c + c*...
```

3.763.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3598, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{3598}$$

$$\frac{\sqrt{a+b\cosh(x)+c\sinh(x)}\int \sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}{a+\sqrt{b^2-c^2}}}dx}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cosh(x)+c\sinh(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\sin\left(ix-\tan^{-1}(b,-ic)+\frac{\pi}{2}\right)}{a+\sqrt{b^2-c^2}}}dx}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{2i\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

input
```
Int[Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

output
```
((-2*I)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sq
rt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*Cosh[x] + c*S
inh[x])/(a + Sqrt[b^2 - c^2])]
```

3.763.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

3.763. $\int\sqrt{a+b\cosh(x)+c\sinh(x)}\,dx$

3.763.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(125) = 250$.

Time = 1.82 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.11

method	result
default	$\frac{(-b^2+c^2)\cosh(x)}{\sqrt{b^2-c^2}\sqrt{\frac{-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}}{\sqrt{b^2-c^2}}}} + \frac{\sqrt{\frac{\left(-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}\right)\sinh(x)^2}{\sqrt{b^2-c^2}}}\, a\ln\left(\frac{-\sinh(x)\cosh(x)b^2+\sinh(x)\cosh(x)c^2}{}\right.}{\left(-b^2\sinh(x)+\sinh(\right.}$
risch	Expression too large to display

input
```
int((a+b*cosh(x)+c*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b^2-c^2)^(1/2)*(-b^2+c^2)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))
/(b^2-c^2)^(1/2))^(1/2)*cosh(x)+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/
2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*a*ln((-sinh(x)*cosh(x)*b^2+sinh(x)*co
sh(x)*c^2+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+
a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^
(1/2))/(b^2-c^2)^(1/2)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-
c^2)^(1/2))^(1/2))/(-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))*(b^2-c^2)^
(1/2)/sinh(x)
```

3.763.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.08

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{2\left(\sqrt{2}a\sqrt{b+c}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2+3\,c^2)}{3\,(b^2+2\,bc+c^2)}, -\frac{8\,(8\,a^3-9\,ab^2+9\,ac^2)}{27\,(b^3+3\,b^2c+3\,bc^2+c^3)}, \frac{3\,(b+c)\cosh(x)+3\,(b+c)\sinh(x)+2\,a}{3\,(b+c)}\right) - 3\right.}{}$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(2)*a*sqrt(b + c)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)
/(b^2 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3
*b*c^2 + c^3), 1/3*(3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c))
- 3*sqrt(2)*(b + c)^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2
 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^
2 + c^3), weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c
^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), 1/
3*(3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c))) - 3*sqrt(b*cosh(
x) + c*sinh(x) + a)*(b + c))/(b + c)
```

3.763.6 Sympy [F]

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx=\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*cosh(x) + c*sinh(x)), x)
```

3.763.7 Maxima [F]

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx=\int \sqrt{b\cosh(x)+c\sinh(x)+a}\,dx$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

3.763.8 Giac [F]

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx=\int \sqrt{b\cosh{(x)}+c\sinh{(x)}+a}\,dx$$

input
```
integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

3.763.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx=\int \sqrt{a+b\cosh{(x)}+c\sinh{(x)}}\,dx$$

input
```
int((a + b*cosh(x) + c*sinh(x))^(1/2),x)
```

output
```
int((a + b*cosh(x) + c*sinh(x))^(1/2), x)
```

3.764 $\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx$

3.764.1 Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

output
```
-2*I*(cos(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I
*c))*EllipticF(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a
+(b^2-c^2)^(1/2)))^(1/2))*((a+b*cosh(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1
/2)/(a+b*cosh(x)+c*sinh(x))^(1/2)
```

3.764.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 0.41 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.32

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \frac{2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{a+b\cosh(x)+c\sinh(x)}{a+i\sqrt{1-\frac{b^2}{c^2}}c},\frac{a+b\cosh(x)+c\sinh(x)}{a-i\sqrt{1-\frac{b^2}{c^2}}c}\right)\text{sech}\left(x+\text{arctanh}\left(\frac{b}{c}\right)\right)\sqrt{a+b\cosh(x)+c\sin}}{\sqrt{1-\frac{b^2}{c^2}}c}$$

input
```
Integrate[1/Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

output
```
(2*AppellF1[1/2, 1/2, 1/2, 3/2, (a + b*Cosh[x] + c*Sinh[x])/(a + I*Sqrt[1
- b^2/c^2]*c), (a + b*Cosh[x] + c*Sinh[x])/(a - I*Sqrt[1 - b^2/c^2]*c)]*Se
ch[x + ArcTanh[b/c]]*Sqrt[a + b*Cosh[x] + c*Sinh[x]]*Sqrt[-(((-I)*Sqrt[1 -
 b^2/c^2]*c + b*Cosh[x] + c*Sinh[x])/(a + I*Sqrt[1 - b^2/c^2]*c))]*Sqrt[-(
(I*Sqrt[1 - b^2/c^2]*c + b*Cosh[x] + c*Sinh[x])/(a - I*Sqrt[1 - b^2/c^2]*c
))])/(Sqrt[1 - b^2/c^2]*c)
```

3.764.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\cos(ix)-ic\sin(ix)}}dx$$

$$\Big\downarrow \text{3606}$$

$$\frac{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}}dx}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\sin\left(ix-\tan^{-1}(b,-ic)+\frac{\pi}{2}\right)}{a+\sqrt{b^2-c^2}}}}dx}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

$$\Big\downarrow \text{3140}$$

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

input
```
Int[1/Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

output
```
((-2*I)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sq
rt[b^2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/S
qrt[a + b*Cosh[x] + c*Sinh[x]]
```

3.764.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

3.764.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.46

method	result
default	$\frac{\sqrt{\frac{\left(-b^{2} \sinh(x)+\sinh(x) c^{2}+a \sqrt{b^{2}-c^{2}}\right) \sinh(x)^{2}}{\sqrt{b^{2}-c^{2}}}}\, \ln\left(\frac{-\sinh(x) \cosh(x) b^{2}+\sinh(x) \cosh(x) c^{2}+\cosh(x) \sqrt{b^{2}-c^{2}}\, a+\sqrt{\frac{\left(-b^{2}+c^{2}\right) \sinh(x)^{3}}{\sqrt{b^{2}-c^{2}}}+a \sinh(x)}}{\sqrt{b^{2}-c^{2}} \sqrt{\frac{-b^{2} \sinh(x)+\sinh(x) c^{2}+a \sqrt{b^{2}-c^{2}}}{\sqrt{b^{2}-c^{2}}}}}\right.}{\left(-b^{2} \sinh(x)+\sinh(x) c^{2}+a \sqrt{b^{2}-c^{2}}\right) \sinh(x)}$

input
```
int(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(
1/2)*ln((-sinh(x)*cosh(x)*b^2+sinh(x)*cosh(x)*c^2+cosh(x)*(b^2-c^2)^(1/2)*
a+((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)
*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2)^(1/2)/((-b^2*sinh
(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2))/(-b^2*sinh(x)+s
inh(x)*c^2+a*(b^2-c^2)^(1/2))*(b^2-c^2)^(1/2)/sinh(x)
```

3.764.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2+3\,c^2)}{3\,(b^2+2\,bc+c^2)},-\frac{8\,(8\,a^3-9\,ab^2+9\,ac^2)}{27\,(b^3+3\,b^2c+3\,bc^2+c^3)},\frac{3\,(b+c)\cosh(x)+3\,(b+c)\sinh(x)+2\,a}{3\,(b+c)}\right)}{\sqrt{b+c}}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c
^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), 1/
3*(3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c))/sqrt(b + c)
```

3.764.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a+b\cosh{(x)}+c\sinh{(x)}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*cosh(x) + c*sinh(x)), x)
```

3.764.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh{(x)}+c\sinh{(x)}+a}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

3.764.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh{(x)}+c\sinh{(x)}+a}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

3.764.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a+b\cosh{(x)}+c\sinh{(x)}}}\,dx$$

input
```
int(1/(a + b*cosh(x) + c*sinh(x))^(1/2),x)
```

output
```
int(1/(a + b*cosh(x) + c*sinh(x))^(1/2), x)
```

3.765 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx$

3.765.1 Optimal result

Integrand size = 14, antiderivative size = 156

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx = -\frac{2(c\cosh(x)+b\sinh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}} - \frac{2iE\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

output
```
-2*(c*cosh(x)+b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^(1/2)-2*I*(
cos(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*E
llipticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-
c^2)^(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/(a^2-b^2+c^2)/((a+b*cosh
(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)
```

3.765.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.02 (sec) , antiderivative size = 806, normalized size of antiderivative = 5.17

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx = \frac{2(ab+(b^2-c^2)\cosh(x)) - \frac{2b^3(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} + \frac{2a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},\right.}{}}{}$$

input
```
Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

output
```
(2*(a*b + (b^2 - c^2)*Cosh[x]) - (2*b^3*(a + b*Cosh[x] + c*Sinh[x]))/(b^2
- c^2) + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2, (a + b*Cosh[x] + c*Sinh[x])/(a
+ I*Sqrt[1 - b^2/c^2]*c), (a + b*Cosh[x] + c*Sinh[x])/(a - I*Sqrt[1 - b^2/
c^2]*c)]*Sech[x + ArcTanh[b/c]]*(a + b*Cosh[x] + c*Sinh[x])*Sqrt[-(((-I)*S
qrt[1 - b^2/c^2]*c + b*Cosh[x] + c*Sinh[x])/(a + I*Sqrt[1 - b^2/c^2]*c))]*
Sqrt[-((I*Sqrt[1 - b^2/c^2]*c + b*Cosh[x] + c*Sinh[x])/(a - I*Sqrt[1 - b^2
/c^2]*c))])/Sqrt[1 - b^2/c^2] + (b*c*Sinh[x + ArcTanh[c/b]])/Sqrt[1 - c^2/
b^2] - (b*c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Cosh[x] + c*Sinh[x])/(a
 + b*Sqrt[1 - c^2/b^2]), (a + b*Cosh[x] + c*Sinh[x])/(a - b*Sqrt[1 - c^2/b
^2])]*Sinh[x + ArcTanh[c/b]])/(Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[1 - c^2/b^2]
 - b*Cosh[x] - c*Sinh[x])/(a + b*Sqrt[1 - c^2/b^2])]*Sqrt[(b*Sqrt[1 - c^2/
b^2] + b*Cosh[x] + c*Sinh[x])/(-a + b*Sqrt[1 - c^2/b^2])]) + (c^2*((2*b^2*
(a + b*Cosh[x] + c*Sinh[x]))/(b^2 - c^2) - (c*Sinh[x + ArcTanh[c/b]])/Sqrt
[1 - c^2/b^2] + (c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Cosh[x] + c*Sinh
[x])/(a + b*Sqrt[1 - c^2/b^2]), (a + b*Cosh[x] + c*Sinh[x])/(a - b*Sqrt[1
- c^2/b^2])]*Sinh[x + ArcTanh[c/b]])/(Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[1 - c
^2/b^2] - b*Cosh[x] - c*Sinh[x])/(a + b*Sqrt[1 - c^2/b^2])]*Sqrt[(b*Sqrt[1
 - c^2/b^2] + b*Cosh[x] + c*Sinh[x])/(-a + b*Sqrt[1 - c^2/b^2])])))/b)/(c*
(a^2 - b^2 + c^2)*Sqrt[a + b*Cosh[x] + c*Sinh[x]])
```

3.765.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3607, 3042, 3598, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\cos(ix)-ic\sin(ix))^{3/2}}dx$$

$$\Big\downarrow \text{3607}$$

$$\frac{\int \sqrt{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}-\frac{2(b\sinh(x)+c\cosh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2(b\sinh(x)+c\cosh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\int \sqrt{a+b\cos(ix)-ic\sin(ix)}dx}{a^2-b^2+c^2}$$

$$\Big\downarrow \text{3598}$$

$$-\frac{2(b\sinh(x)+c\cosh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\sqrt{a+b\cosh(x)+c\sinh(x)}\int \sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}dx}{(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2(b\sinh(x)+c\cosh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\sqrt{a+b\cosh(x)+c\sinh(x)}\int \sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\sin\left(ix-\tan^{-1}(b,-ic)+\frac{\pi}{2}\right)}{a+\sqrt{b^2-c^2}}}dx}{(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{2(b\sinh(x)+c\cosh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

input
```
Int[(a + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

output
```
(-2*(c*Cosh[x] + b*Sinh[x]))/((a^2 - b^2 + c^2)*Sqrt[a + b*Cosh[x] + c*Sin
h[x]]) - ((2*I)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])
/(a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/((a^2 - b^2 + c^2
)*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])
```

3.765.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3607
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Simp[1/(a^2 - b^
2 - c^2)   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.765. $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx$

3.765.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(177) = 354$.

Time = 1.02 (sec) , antiderivative size = 888, normalized size of antiderivative = 5.69

method	result
default	$\frac{\sqrt{b^2-c^2}\,\operatorname{arctanh}\left(\frac{(b^2-c^2)\cosh(x)}{\sqrt{(a^2+b^2-c^2)(b^2-c^2)}}\right)}{\sqrt{\frac{-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}}{\sqrt{b^2-c^2}}}\,\sqrt{(a^2+b^2-c^2)(b^2-c^2)}} - \frac{\sqrt{\frac{\left(-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}\right)\sinh(x)^2}{\sqrt{b^2-c^2}}}\,a\left(-\frac{(-b^2+c^2)\ln\left(-\frac{2(\sinh\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}$

input
```
int(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2)*(b^
2-c^2)^(1/2)/((a^2+b^2-c^2)*(b^2-c^2))^(1/2)*arctanh((b^2-c^2)*cosh(x)/((a
^2+b^2-c^2)*(b^2-c^2))^(1/2))-((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2)
)/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*a*(-1/2*(-b^2+c^2)/((a^2+b^2-c^2)*(b-c)
*(b+c))^(1/2)/(b^2-c^2)/((-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-c^2))^(1/2)
*ln((-2*(sinh(x)*(b^2-c^2)^(1/2)-a)*a^2/(b^2-c^2)+2*(b^2*sinh(x)-sinh(x)*c
^2-a*(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*(c
osh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(b^2-c^2))+2*((-sinh(x)*(b^2-c^2)
^(1/2)+a)*a^2/(b^2-c^2))^(1/2)*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2
))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2))/(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^
(1/2)/(b^2-c^2)))+1/2*(b^2-c^2)/((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(-b^2+c^
2)/((-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-c^2))^(1/2)*ln((-2*(sinh(x)*(b^2
-c^2)^(1/2)-a)*a^2/(b^2-c^2)-2*(b^2*sinh(x)-sinh(x)*c^2-a*(b^2-c^2)^(1/2))
/(b^2-c^2)^(3/2)*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*(cosh(x)+((a^2+b^2-c^2)
*(b-c)*(b+c))^(1/2)/(-b^2+c^2))+2*((-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-c
^2))^(1/2)*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*s
inh(x)^2)^(1/2))/(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(-b^2+c^2))))/
sinh(x)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/
2)
```

3.765.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 798, normalized size of antiderivative = 5.12

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="fricas")
```

output
```
2/3*((2*sqrt(2)*a^2*cosh(x) + sqrt(2)*(a*b + a*c)*cosh(x)^2 + sqrt(2)*(a*b
 + a*c)*sinh(x)^2 + 2*(sqrt(2)*a^2 + sqrt(2)*(a*b + a*c)*cosh(x))*sinh(x)
+ sqrt(2)*(a*b - a*c))*sqrt(b + c)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2
+ 3*c^2)/(b^2 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b
^2*c + 3*b*c^2 + c^3), 1/3*(3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(
b + c)) - 3*(sqrt(2)*(b^2 + 2*b*c + c^2)*cosh(x)^2 + sqrt(2)*(b^2 + 2*b*c
+ c^2)*sinh(x)^2 + 2*sqrt(2)*(a*b + a*c)*cosh(x) + 2*(sqrt(2)*(b^2 + 2*b*c
 + c^2)*cosh(x) + sqrt(2)*(a*b + a*c))*sinh(x) + sqrt(2)*(b^2 - c^2))*sqrt
(b + c)*weierstrassZeta(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2), -
8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), weierstr
assPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2), -8/27*(8*a^3
- 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), 1/3*(3*(b + c)*cosh(
x) + 3*(b + c)*sinh(x) + 2*a)/(b + c))) - 6*((b^2 + 2*b*c + c^2)*cosh(x)^2
 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + (a*b + a*c)*cosh(x) + (a*b + a*c + 2*(b
^2 + 2*b*c + c^2)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + c*sinh(x) + a))/(a^2*
b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2 + (a^2*b^2 - b^4 + a^2*c^2 + 2*b*c^3 +
 c^4 + 2*(a^2*b - b^3)*c)*cosh(x)^2 + (a^2*b^2 - b^4 + a^2*c^2 + 2*b*c^3 +
 c^4 + 2*(a^2*b - b^3)*c)*sinh(x)^2 + 2*(a^3*b - a*b^3 + a*b*c^2 + a*c^3 +
 (a^3 - a*b^2)*c)*cosh(x) + 2*(a^3*b - a*b^3 + a*b*c^2 + a*c^3 + (a^3 - a*
b^2)*c + (a^2*b^2 - b^4 + a^2*c^2 + 2*b*c^3 + c^4 + 2*(a^2*b - b^3)*c)*...
```

3.765.6 Sympy [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx = \int \frac{1}{(a+b\cosh{(x)}+c\sinh{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))**(3/2),x)
```

output
```
Integral((a + b*cosh(x) + c*sinh(x))**(-3/2), x)
```

3.765.7 Maxima [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx=\int \frac{1}{(b\cosh\left(x\right)+c\sinh\left(x\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + c*sinh(x) + a)^(-3/2), x)
```

3.765.8 Giac [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx=\int \frac{1}{(b\cosh\left(x\right)+c\sinh\left(x\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + c*sinh(x) + a)^(-3/2), x)
```

3.765.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx=\int \frac{1}{{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)}^{3/2}}\,dx$$

input
```
int(1/(a + b*cosh(x) + c*sinh(x))^(3/2),x)
```

output
```
int(1/(a + b*cosh(x) + c*sinh(x))^(3/2), x)
```

3.766 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx$

3.766.1 Optimal result

Integrand size = 14, antiderivative size = 322

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = -\frac{2(c\cosh(x)+b\sinh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)+ab\sinh(x))}{3(a^2-b^2+c^2)^2\sqrt{a+b\cosh(x)+c\sinh(x)}} - \frac{8iaE\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{3(a^2-b^2+c^2)^2\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} + \frac{2i\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{3(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

output
```
-2/3*(c*cosh(x)+b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^(3/2)-8/3
*(a*c*cosh(x)+a*b*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))^(1/2)-8
/3*I*a*(cos(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,
-I*c))*EllipticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/
(a+(b^2-c^2)^(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/(a^2-b^2+c^2)^2/
((a+b*cosh(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)+2/3*I*(cos(1/2*I*x-1/2
*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticF(sin(1/
2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))^(1
/2))*((a+b*cosh(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)/(a^2-b^2+c^2)/(a+
b*cosh(x)+c*sinh(x))^(1/2)
```

3.766.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.23 (sec) , antiderivative size = 2492, normalized size of antiderivative = 7.74

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

output
```
Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((8*a*(b^2 - c^2))/(3*b*c*(a^2 - b^2 + c^2
)^2) - (2*(a*c - b^2*Sinh[x] + c^2*Sinh[x]))/(3*b*(-a^2 + b^2 - c^2)*(a +
b*Cosh[x] + c*Sinh[x])^2) - (2*(-3*a^2*c - b^2*c + c^3 + 4*a*b^2*Sinh[x] -
 4*a*c^2*Sinh[x]))/(3*b*(-a^2 + b^2 - c^2)^2*(a + b*Cosh[x] + c*Sinh[x])))
 + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh
[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*
c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/
c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1
 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b
^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*S
qrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTan
h[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)
/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*(a^2 - b^2 + c^2)^2*Sq
rt[I*(I + Sinh[x + ArcTanh[b/c]])]) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2,
((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]
*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh
[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))
*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sq
rt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/
(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*...
```

3.766.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {3042, 3608, 27, 3042, 3635, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{(a+b\cos(ix)-ic\sin(ix))^{5/2}}dx$$

$\Big\downarrow$ 3608

$$-\frac{2\int -\frac{3a-b\cosh(x)-c\sinh(x)}{2(a+b\cosh(x)+c\sinh(x))^{3/2}}dx}{3\left(a^2-b^2+c^2\right)}-\frac{2(b\sinh(x)+c\cosh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{3a-b\cosh(x)-c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^{3/2}}dx}{3\left(a^2-b^2+c^2\right)}-\frac{2(b\sinh(x)+c\cosh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{2(b\sinh(x)+c\cosh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{\int \frac{3a-b\cos(ix)+ic\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^{3/2}}dx}{3\left(a^2-b^2+c^2\right)}$$

$\Big\downarrow$ 3635

$$\frac{-\frac{2\int -\frac{3a^2+4b\cosh(x)a+4c\sinh(x)a+b^2-c^2}{2\sqrt{a+b\cosh(x)+c\sinh(x)}}dx}{a^2-b^2+c^2}-\frac{8(ab\sinh(x)+ac\cosh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}}{3\left(a^2-b^2+c^2\right)}-\frac{2(b\sinh(x)+c\cosh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{3a^2+4b\cosh(x)a+4c\sinh(x)a+b^2-c^2}{\sqrt{a+b\cosh(x)+c\sinh(x)}}dx}{a^2-b^2+c^2}-\frac{8(ab\sinh(x)+ac\cosh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}}{3\left(a^2-b^2+c^2\right)}-\frac{2(b\sinh(x)+c\cosh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{2(b\sinh(x)+c\cosh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{8(ab\sinh(x)+ac\cosh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\int \frac{3a^2+4b\cos(ix)a-4ic\sin(ix)a+b^2-c^2}{\sqrt{a+b\cos(ix)-ic\sin(ix)}}dx}{a^2-b^2+c^2}}{3\left(a^2-b^2+c^2\right)}$$

$\Big\downarrow$ 3628

3.766. $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}dx$

$$\frac{\frac{4a\int \sqrt{a+b\cosh(x)+c\sinh(x)}dx-(a^2-b^2+c^2)\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}dx}{a^2-b^2+c^2}-\frac{8(ab\sinh(x)+ac\cosh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}}{3(a^2-b^2+c^2)}-\frac{2(b\sinh(x)+c\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{2(b\sinh(x)+c\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{8(ab\sinh(x)+ac\cosh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{4a\int \sqrt{a+b\cos(ix)-ic\sin(ix)}dx-(a^2-b^2+c^2)\int \frac{1}{\sqrt{a+b\cos(ix)-ic\sin(ix)}}dx}{a^2-b^2+c^2}}{3(a^2-b^2+c^2)}$$

$\big\downarrow$ 3598

$$-\frac{2(b\sinh(x)+c\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{8(ab\sinh(x)+ac\cosh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\frac{4a\sqrt{a+b\cosh(x)+c\sinh(x)}\int \sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}dx}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}-(a^2-b^2+c^2)\int \frac{1}{\sqrt{a+b\cos(i}}}{a^2-b^2+c^2}}{3(a^2-b^2+c^2)}$$

$\big\downarrow$ 3042

$$-\frac{2(b\sinh(x)+c\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{8(ab\sinh(x)+ac\cosh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\frac{4a\sqrt{a+b\cosh(x)+c\sinh(x)}\int \sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\sin\left(ix-\tan^{-1}(b,-ic)+\frac{\pi}{2}\right)}{a+\sqrt{b^2-c^2}}}dx}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}-(a^2-b^2+c^2)\int \frac{1}{\sqrt{a+b\cos(}}}{a^2-b^2+c^2}}{3(a^2-b^2+c^2)}$$

$\big\downarrow$ 3132

$$-\frac{2(b\sinh(x)+c\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{8(ab\sinh(x)+ac\cosh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{-(a^2-b^2+c^2)\int \frac{1}{\sqrt{a+b\cos(ix)-ic\sin(ix)}}dx-\frac{8ia\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right.}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}}{a^2-b^2+c^2}}{3(a^2-b^2+c^2)}$$

$\big\downarrow$ 3606

3.766. $\displaystyle\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx$

$$-\frac{2(b\sinh(x)+c\cosh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{8(ab\sinh(x)+ac\cosh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{-\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int\frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}}dx}{\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(\cdots}{\cdots}}{a^2-b^2+c^2}}{3\left(a^2-b^2+c^2\right)}$$

$\downarrow$ 3042

$$-\frac{2(b\sinh(x)+c\cosh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{8(ab\sinh(x)+ac\cosh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{-\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int\frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\sin\left(ix-\tan^{-1}(b,-ic)+\frac{\pi}{2}\right)}{a+\sqrt{b^2-c^2}}}}dx}{\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cos\cdots}}{\cdots}}{a^2-b^2+c^2}}{3\left(a^2-b^2+c^2\right)}$$

$\downarrow$ 3140

$$-\frac{2(b\sinh(x)+c\cosh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{8(ab\sinh(x)+ac\cosh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\frac{2i\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\operatorname{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)+c\sin\cdots}}{\sqrt{\cdots}}}{a^2-b^2+c^2}}{3\left(a^2-b^2+c^2\right)}$$

input
```
Int[(a + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

output
```
(-2*(c*Cosh[x] + b*Sinh[x]))/(3*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[
x])^(3/2)) + ((-8*(a*c*Cosh[x] + a*b*Sinh[x]))/((a^2 - b^2 + c^2)*Sqrt[a +
 b*Cosh[x] + c*Sinh[x]]) + (((-8*I)*a*EllipticE[(I*x - ArcTan[b, (-I)*c])/
2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[
x]])/Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])] + ((2*I)*(a^2
 - b^2 + c^2)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(
a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2
])])/Sqrt[a + b*Cosh[x] + c*Sinh[x]])/(a^2 - b^2 + c^2))/(3*(a^2 - b^2 + c
^2))
```

3.766. $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx$

3.766.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.766. $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx$

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.766.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2159 vs. $2(356) = 712$.

Time = 2.51 (sec) , antiderivative size = 2160, normalized size of antiderivative = 6.71

method	result	size
default	Expression too large to display	2160

input
```
int(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2)*a*(
b^2-c^2)^(1/2)*(-1/2*cosh(x)/(a^2+b^2-c^2)/(sinh(x)^2*(b^2-c^2)-a^2)+1/2/(
a^2+b^2-c^2)/((a^2+b^2-c^2)*(b^2-c^2))^(1/2)*arctanh((b^2-c^2)*cosh(x)/((a
^2+b^2-c^2)*(b^2-c^2))^(1/2)))+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2
))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*(1/(b^2-c^2)*a^2/(2*a^2+2*b^2-2*c^2)*(
1/(sinh(x)*(b^2-c^2)^(1/2)-a)/a^2*(b^2-c^2)/(cosh(x)+((a^2+b^2-c^2)*(b-c)*
(b+c))^(1/2)/(b^2-c^2))*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2
-c^2)^(1/2)*sinh(x)^2)^(1/2)-(b^2*sinh(x)-sinh(x)*c^2-a*(b^2-c^2)^(1/2))/(
b^2-c^2)^(1/2)*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(sinh(x)*(b^2-c^2)^(1/2)-
a)/a^2/((-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-c^2))^(1/2)*ln((-2*(sinh(x)*
(b^2-c^2)^(1/2)-a)*a^2/(b^2-c^2)+2*(b^2*sinh(x)-sinh(x)*c^2-a*(b^2-c^2)^(1
/2))/(b^2-c^2)^(3/2)*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*(cosh(x)+((a^2+b^2-
c^2)*(b-c)*(b+c))^(1/2)/(b^2-c^2))+2*((-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b^
2-c^2))^(1/2)*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2
)*sinh(x)^2)^(1/2))/(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(b^2-c^2)))
)+(b^2-c^2)*a^2/(2*a^2+2*b^2-2*c^2)/(-b^2+c^2)^2*(1/(sinh(x)*(b^2-c^2)^(1/
2)-a)/a^2*(b^2-c^2)/(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(-b^2+c^2))
*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*sinh(x)^2)^
(1/2)+(b^2*sinh(x)-sinh(x)*c^2-a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*((a^2+b^
2-c^2)*(b-c)*(b+c))^(1/2)/(sinh(x)*(b^2-c^2)^(1/2)-a)/a^2/((-sinh(x)*(b...
```

3.766.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 3730, normalized size of antiderivative = 11.58

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="fricas")
```

output
```
2/9*((sqrt(2)*(a^2*b^2 + 3*b^4 + a^2*c^2 - 6*b*c^3 - 3*c^4 + 2*(a^2*b + 3*
b^3)*c)*cosh(x)^4 + sqrt(2)*(a^2*b^2 + 3*b^4 + a^2*c^2 - 6*b*c^3 - 3*c^4 +
 2*(a^2*b + 3*b^3)*c)*sinh(x)^4 + 4*sqrt(2)*(a^3*b + 3*a*b^3 - 3*a*b*c^2 -
 3*a*c^3 + (a^3 + 3*a*b^2)*c)*cosh(x)^3 + 4*(sqrt(2)*(a^2*b^2 + 3*b^4 + a^
2*c^2 - 6*b*c^3 - 3*c^4 + 2*(a^2*b + 3*b^3)*c)*cosh(x) + sqrt(2)*(a^3*b +
3*a*b^3 - 3*a*b*c^2 - 3*a*c^3 + (a^3 + 3*a*b^2)*c))*sinh(x)^3 + 2*sqrt(2)*
(2*a^4 + 7*a^2*b^2 + 3*b^4 + 3*c^4 - (7*a^2 + 6*b^2)*c^2)*cosh(x)^2 + 2*(3
*sqrt(2)*(a^2*b^2 + 3*b^4 + a^2*c^2 - 6*b*c^3 - 3*c^4 + 2*(a^2*b + 3*b^3)*
c)*cosh(x)^2 + 6*sqrt(2)*(a^3*b + 3*a*b^3 - 3*a*b*c^2 - 3*a*c^3 + (a^3 + 3
*a*b^2)*c)*cosh(x) + sqrt(2)*(2*a^4 + 7*a^2*b^2 + 3*b^4 + 3*c^4 - (7*a^2 +
 6*b^2)*c^2))*sinh(x)^2 + 4*sqrt(2)*(a^3*b + 3*a*b^3 - 3*a*b*c^2 + 3*a*c^3
 - (a^3 + 3*a*b^2)*c)*cosh(x) + 4*(sqrt(2)*(a^2*b^2 + 3*b^4 + a^2*c^2 - 6*
b*c^3 - 3*c^4 + 2*(a^2*b + 3*b^3)*c)*cosh(x)^3 + 3*sqrt(2)*(a^3*b + 3*a*b^
3 - 3*a*b*c^2 - 3*a*c^3 + (a^3 + 3*a*b^2)*c)*cosh(x)^2 + sqrt(2)*(2*a^4 +
7*a^2*b^2 + 3*b^4 + 3*c^4 - (7*a^2 + 6*b^2)*c^2)*cosh(x) + sqrt(2)*(a^3*b
+ 3*a*b^3 - 3*a*b*c^2 + 3*a*c^3 - (a^3 + 3*a*b^2)*c))*sinh(x) + sqrt(2)*(a
^2*b^2 + 3*b^4 + a^2*c^2 + 6*b*c^3 - 3*c^4 - 2*(a^2*b + 3*b^3)*c))*sqrt(b
+ c)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2),
-8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), 1/3*(3*
(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c)) - 12*(sqrt(2)*(a*b^...
```

3.766.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))**(5/2),x)
```

output
```
Timed out
```

3.766.7 Maxima [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \int \frac{1}{(b\cosh(x)+c\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + c*sinh(x) + a)^(-5/2), x)
```

3.766.8 Giac [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \int \frac{1}{(b\cosh(x)+c\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + c*sinh(x) + a)^(-5/2), x)
```

3.766.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx$$

input
```
int(1/(a + b*cosh(x) + c*sinh(x))^(5/2),x)
```

output
```
int(1/(a + b*cosh(x) + c*sinh(x))^(5/2), x)
```

3.767 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx$

3.767.1 Optimal result

Integrand size = 14, antiderivative size = 411

$$\begin{aligned}\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx &= -\frac{2(c\cosh(x)+b\sinh(x))}{5\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^{5/2}} \\ &- \frac{16(ac\cosh(x)+ab\sinh(x))}{15\,(a^2-b^2+c^2)^2\,(a+b\cosh(x)+c\sinh(x))^{3/2}} \\ &- \frac{2i(23a^2+9b^2-9c^2)\,E\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{15\,(a^2-b^2+c^2)^3\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} \\ &+ \frac{16ia\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{15\,(a^2-b^2+c^2)^2\sqrt{a+b\cosh(x)+c\sinh(x)}} \\ &- \frac{2(c(23a^2+9b^2-9c^2)\cosh(x)+b(23a^2+9b^2-9c^2)\sinh(x))}{15\,(a^2-b^2+c^2)^3\sqrt{a+b\cosh(x)+c\sinh(x)}}\end{aligned}$$

output
```
-2/5*(c*cosh(x)+b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^(5/2)-16/
15*(a*c*cosh(x)+a*b*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))^(3/2)
-2/15*(c*(23*a^2+9*b^2-9*c^2)*cosh(x)+b*(23*a^2+9*b^2-9*c^2)*sinh(x))/(a^2
-b^2+c^2)^3/(a+b*cosh(x)+c*sinh(x))^(1/2)-2/15*I*(23*a^2+9*b^2-9*c^2)*(cos
(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*Elli
pticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2
)^(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/(a^2-b^2+c^2)^3/((a+b*cosh(
x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)+16/15*I*a*(cos(1/2*I*x-1/2*arctan
(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticF(sin(1/2*I*x-1
/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))^(1/2))*((
a+b*cosh(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)/(a^2-b^2+c^2)^2/(a+b*cos
h(x)+c*sinh(x))^(1/2)
```

3.767.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.42 (sec) , antiderivative size = 4093, normalized size of antiderivative = 9.96

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-7/2),x]
```

output
```
Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((-2*(23*a^2 + 9*b^2 - 9*c^2)*(b^2 - c^2))
/(15*b*c*(-a^2 + b^2 - c^2)^3) - (2*(a*c - b^2*Sinh[x] + c^2*Sinh[x]))/(5*
b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x])^3) - (2*(-5*a^2*c - 3*b^2
*c + 3*c^3 + 8*a*b^2*Sinh[x] - 8*a*c^2*Sinh[x]))/(15*b*(-a^2 + b^2 - c^2)^
2*(a + b*Cosh[x] + c*Sinh[x])^2) + (2*(-15*a^3*c - 17*a*b^2*c + 17*a*c^3 +
 23*a^2*b^2*Sinh[x] + 9*b^4*Sinh[x] - 23*a^2*c^2*Sinh[x] - 18*b^2*c^2*Sinh
[x] + 9*c^4*Sinh[x]))/(15*b*(-a^2 + b^2 - c^2)^3*(a + b*Cosh[x] + c*Sinh[x
]))) + (2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*
Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*
c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 -
b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqr
t[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt
[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2]
)]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + Ar
cTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 +
c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*(a^2 - b^2 + c^2)^
3*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) + (34*a*b^2*AppellF1[1/2, 1/2, 1/2
, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b
^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]
*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^...
```

3.767.3 Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$, Rules used = {3042, 3608, 27, 3042, 3635, 27, 3042, 3635, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\cos(ix)-ic\sin(ix))^{7/2}}dx$$

$$\Big\downarrow \text{3608}$$

$$-\frac{2\int -\frac{5a-3b\cosh(x)-3c\sinh(x)}{2(a+b\cosh(x)+c\sinh(x))^{5/2}}dx}{5(a^2-b^2+c^2)}-\frac{2(b\sinh(x)+c\cosh(x))}{5(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{5a-3b\cosh(x)-3c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^{5/2}}dx}{5\,(a^2-b^2+c^2)}-\frac{2(b\sinh(x)+c\cosh(x))}{5\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2(b\sinh(x)+c\cosh(x))}{5\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^{5/2}}+\frac{\int \frac{5a-3b\cos(ix)+3ic\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^{5/2}}dx}{5\,(a^2-b^2+c^2)}$$

$$\Big\downarrow \; 3635$$

$$\frac{-\frac{2\int -\frac{3\left(5a^2+3b^2-3c^2\right)-8ab\cosh(x)-8ac\sinh(x)}{2(a+b\cosh(x)+c\sinh(x))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{16(ab\sinh(x)+ac\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}}{5\,(a^2-b^2+c^2)}-\frac{2(b\sinh(x)+c\cosh(x))}{5\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3\left(5a^2+3b^2-3c^2\right)-8ab\cosh(x)-8ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{16(ab\sinh(x)+ac\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}}{5\,(a^2-b^2+c^2)}-\frac{2(b\sinh(x)+c\cosh(x))}{5\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2(b\sinh(x)+c\cosh(x))}{5\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^{5/2}}+\frac{-\frac{16(ab\sinh(x)+ac\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{\int \frac{3\left(5a^2+3b^2-3c^2\right)-8ab\cos(ix)+8iac\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^{3/2}}dx}{3(a^2-b^2+c^2)}}{5\,(a^2-b^2+c^2)}$$

$$\Big\downarrow \; 3635$$

$$\frac{\frac{-\frac{2\int -\frac{a\left(15a^2+17b^2-17c^2\right)+b\left(23a^2+9b^2-9c^2\right)\cosh(x)+c\left(23a^2+9b^2-9c^2\right)\sinh(x)}{2\sqrt{a+b\cosh(x)+c\sinh(x)}}dx}{a^2-b^2+c^2}-\frac{2\left(b\sinh(x)\left(23a^2+9b^2-9c^2\right)+c\cosh(x)\left(23a^2+9b^2-9c^2\right)\right)}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}}{3(a^2-b^2+c^2)}-\frac{}{3(a^2-b}}{5\,(a^2-b^2+c^2)}-\frac{2(b\sinh(x)+c\cosh(x))}{5\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^{5/2}}$$

$$\Big\downarrow \; 27$$

3.767. $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx$

$$\frac{\frac{\frac{\int \frac{a\left(15a^2+17\left(b^2-c^2\right)\right)+b\left(23a^2+9b^2-9c^2\right)\cosh(x)+c\left(23a^2+9b^2-9c^2\right)\sinh(x)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}dx}{a^2-b^2+c^2}-\frac{2\left(b\sinh(x)\left(23a^2+9b^2-9c^2\right)+c\cosh(x)\left(23a^2+9b^2-9c^2\right)\right)}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}}{3(a^2-b^2+c^2)}-\frac{16(a}{3(a^2-b^2+c}}{5\left(a^2-b^2+c^2\right)}$$

$$\frac{2(b\sinh(x)+c\cosh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}}$$

$\downarrow$ 3042

$$-\frac{2(b\sinh(x)+c\cosh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}}+$$

$$\frac{-\frac{16(ab\sinh(x)+ac\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{2\left(b\sinh(x)\left(23a^2+9b^2-9c^2\right)+c\cosh(x)\left(23a^2+9b^2-9c^2\right)\right)}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\int \frac{a\left(15a^2+17\left(b^2-c^2\right)\right)+b\left(23a^2+9b^2-9c^2\right)c}{\sqrt{a+b\cos(ix)-ic}}}{a^2-b^2+c}}{3(a^2-b^2+c^2)}}{5\left(a^2-b^2+c^2\right)}$$

$\downarrow$ 3628

$$\frac{\frac{\frac{\left(23a^2+9b^2-9c^2\right)\int \sqrt{a+b\cosh(x)+c\sinh(x)}dx-8a\left(a^2-b^2+c^2\right)\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}dx}{a^2-b^2+c^2}-\frac{2\left(b\sinh(x)\left(23a^2+9b^2-9c^2\right)+c\cosh(x)\left(23a^2+9b^2-9c^2\right)\right)}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}}{3(a^2-b^2+c^2)}-\frac{}{3(}}{5\left(a^2-b^2+c^2\right)}$$

$$\frac{2(b\sinh(x)+c\cosh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}}$$

$\downarrow$ 3042

$$-\frac{2(b\sinh(x)+c\cosh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}}+$$

$$\frac{-\frac{16(ab\sinh(x)+ac\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{2\left(b\sinh(x)\left(23a^2+9b^2-9c^2\right)+c\cosh(x)\left(23a^2+9b^2-9c^2\right)\right)}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\left(23a^2+9b^2-9c^2\right)\int \sqrt{a+b\cos(ix)-ic\sin(ix)}dx}{a^2-}}{3(a^2-b^2+c^2)}}{5\left(a^2-b^2+c^2\right)}$$

$\downarrow$ 3598

$$-\frac{2(b\sinh(x)+c\cosh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}}+$$

$$\frac{-\frac{16(ab\sinh(x)+ac\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{2\left(b\sinh(x)\left(23a^2+9b^2-9c^2\right)+c\cosh(x)\left(23a^2+9b^2-9c^2\right)\right)}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\frac{\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}\int \sqrt{}}{\sqrt{\frac{a+b\cos}{a}}}}{3(a^2-b^2+c}}{5\left(a^2-b^2+c^2\right)}$$

$\downarrow$ 3042

3.767. $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx$

$$-\frac{2(b\sinh(x)+c\cosh(x))}{5\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)^{5/2}}+$$

$$-\frac{16(ab\sinh(x)+ac\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{2\left(b\sinh(x)\left(23a^2+9b^2-9c^2\right)+c\cosh(x)\left(23a^2+9b^2-9c^2\right)\right)}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}\int \sqrt{\cdots}}{\sqrt{\frac{a+b\,\mathrm{c}\cdots}{\cdots}}}}{3(a^2-b^2+\cdots}$$
$$5\left(a^2-b^2+c^2\right)$$

$\downarrow$ 3132

$$-\frac{2(b\sinh(x)+c\cosh(x))}{5\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)^{5/2}}+$$

$$-\frac{16(ab\sinh(x)+ac\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{2\left(b\sinh(x)\left(23a^2+9b^2-9c^2\right)+c\cosh(x)\left(23a^2+9b^2-9c^2\right)\right)}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{-8a\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{a+b\cos(ix)-ic\sin(ix)}}d\cdots}{\cdots}}{3(a^2-b^2+c^2)\cdots}$$
$$5\left(a^2-b^2+c^2\right)$$

$\downarrow$ 3606

$$-\frac{2(b\sinh(x)+c\cosh(x))}{5\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)^{5/2}}+$$

$$-\frac{16(ab\sinh(x)+ac\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{2\left(b\sinh(x)\left(23a^2+9b^2-9c^2\right)+c\cosh(x)\left(23a^2+9b^2-9c^2\right)\right)}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{-\frac{8a\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int\cdots}{\sqrt{a+b\,\mathrm{c}\cdots}}}{\cdots}}{\cdots}$$
$$5\left(a^2-b^2+c^2\right)$$

$\downarrow$ 3042

$$-\frac{2(b\sinh(x)+c\cosh(x))}{5\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)^{5/2}}+$$

$$-\frac{16(ab\sinh(x)+ac\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{2\left(b\sinh(x)\left(23a^2+9b^2-9c^2\right)+c\cosh(x)\left(23a^2+9b^2-9c^2\right)\right)}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{-\frac{8a\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int\cdots}{\sqrt{a+b\cdots}}}{\cdots}}{\cdots}$$
$$5\left(a^2-b^2+c^2\cdots$$

$\downarrow$ 3140

3.767. $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx$

$$-\frac{2(b\sinh(x)+c\cosh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}}+\frac{-\frac{16(ab\sinh(x)+ac\cosh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}}+\frac{-\frac{2\left(b\sinh(x)\left(23a^2+9b^2-9c^2\right)+c\cosh(x)\left(23a^2+9b^2-9c^2\right)\right)}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{16ia\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,\mathrm{El}}{\sqrt{a+b\cosh(x}}}{}}{5\left(a^2-b^2+c^2\right)}$$

input
```
Int[(a + b*Cosh[x] + c*Sinh[x])^(-7/2),x]
```

output
```
(-2*(c*Cosh[x] + b*Sinh[x]))/(5*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[
x])^(5/2)) + ((-16*(a*c*Cosh[x] + a*b*Sinh[x]))/(3*(a^2 - b^2 + c^2)*(a +
b*Cosh[x] + c*Sinh[x])^(3/2)) + ((-2*(c*(23*a^2 + 9*b^2 - 9*c^2)*Cosh[x] +
 b*(23*a^2 + 9*b^2 - 9*c^2)*Sinh[x]))/((a^2 - b^2 + c^2)*Sqrt[a + b*Cosh[x
] + c*Sinh[x]]) + (((-2*I)*(23*a^2 + 9*b^2 - 9*c^2)*EllipticE[(I*x - ArcTa
n[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cos
h[x] + c*Sinh[x]])/Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])]
 + ((16*I)*a*(a^2 - b^2 + c^2)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*S
qrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a
 + Sqrt[b^2 - c^2])])/Sqrt[a + b*Cosh[x] + c*Sinh[x]])/(a^2 - b^2 + c^2))/
(3*(a^2 - b^2 + c^2)))/(5*(a^2 - b^2 + c^2))
```

3.767.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.767. $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx$

3.767.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 57908 vs. $2(441) = 882$.

Time = 8.22 (sec) , antiderivative size = 57909, normalized size of antiderivative = 140.90

method	result	size
default	Expression too large to display	57909

input
```
int(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.767.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 13897, normalized size of antiderivative = 33.81

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.767.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))**(7/2),x)
```

output
```
Timed out
```

3.767.7 Maxima [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx=\int \frac{1}{(b\cosh{(x)}+c\sinh{(x)}+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + c*sinh(x) + a)^(-7/2), x)
```

3.767.8 Giac [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx=\int \frac{1}{(b\cosh{(x)}+c\sinh{(x)}+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + c*sinh(x) + a)^(-7/2), x)
```

3.767.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx=\int \frac{1}{(a+b\cosh{(x)}+c\sinh{(x)})^{7/2}}\,dx$$

input
```
int(1/(a + b*cosh(x) + c*sinh(x))^(7/2),x)
```

output
```
int(1/(a + b*cosh(x) + c*sinh(x))^(7/2), x)
```

3.768 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx$

3.768.1 Optimal result

Integrand size = 26, antiderivative size = 140

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx=\frac{64(b^2-c^2)\,(c\cosh(x)+b\sinh(x))}{15\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}+\frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}+\frac{2}{5}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}$$

output
```
2/5*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2)+64/1
5*(b^2-c^2)*(c*cosh(x)+b*sinh(x))/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1
/2)+16/15*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)*(b*cosh(x)+c*sinh(x)+(b^2-
c^2)^(1/2))^(1/2)
```

3.768.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 56.31 (sec) , antiderivative size = 4500, normalized size of antiderivative = 32.14

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx=\text{Result too large to show}$$

input
```
Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

output
```
Sqrt[b^2 - c^2]*((4*b*Sqrt[b^2 - c^2])/(3*c) + (4*c*Cosh[x])/3 + (4*b*Sinh
[x])/3)*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]] + Sqrt[Sqrt[b^2 - c^
2] + b*Cosh[x] + c*Sinh[x]]*((44*b*(b^2 - c^2))/(15*c) + (2*c*Sqrt[b^2 - c
^2]*Cosh[x])/15 + (2*b*c*Cosh[2*x])/5 + (2*b*Sqrt[b^2 - c^2]*Sinh[x])/15 +
 ((b^2 + c^2)*Sinh[2*x])/5) + (256*b*(-b + c)*(b + c)^2*Sqrt[b^2 - c^2]*(E
llipticF[ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b +
 c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1] - 2*EllipticPi[-1, ArcSin[S
qrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c
^2])*(-1 + Tanh[x/2])))]], 1])*Sqrt[Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*
Sinh[x]]*(-1 + Tanh[x/2])*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2
]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]*(-c + (-b + Sqrt[b^2 -
 c^2])*Tanh[x/2]))/(15*(b + c - Sqrt[b^2 - c^2])^2*(b + c + Sqrt[b^2 - c^2
])*(1 + Cosh[x])*Sqrt[(Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x])/(1 +
 Cosh[x])^2]*Sqrt[(-1 + Tanh[x/2]^2)*(-2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-1
 + Tanh[x/2]^2) - b*(1 + Tanh[x/2]^2))]) + (128*(b - c)^2*(b + c)^2*Sqrt[S
qrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x]]*(2*b^3*c^2 + 3*b^2*c^3 - c^5
 - 2*b^2*c^2*Sqrt[b^2 - c^2] - 3*b*c^3*Sqrt[b^2 - c^2] - c^4*Sqrt[b^2 - c^
2] + 8*b^4*c*Tanh[x/2] + 12*b^3*c^2*Tanh[x/2] - 2*b^2*c^3*Tanh[x/2] - 8*b*
c^4*Tanh[x/2] - 2*c^5*Tanh[x/2] - 8*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2] - 12*b
^2*c^2*Sqrt[b^2 - c^2]*Tanh[x/2] - 2*b*c^3*Sqrt[b^2 - c^2]*Tanh[x/2] + ...
```

3.768.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3592, 3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^{5/2}dx$$

$$\Big\downarrow \text{ 3592}$$

$$\frac{8}{5}\sqrt{b^2-c^2}\int \left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^{3/2}dx+\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}+\frac{8}{5}\sqrt{b^2-c^2}\int\left(b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3592}$$

$$\frac{8}{5}\sqrt{b^2-c^2}\left(\frac{4}{3}\sqrt{b^2-c^2}\int\sqrt{b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}}dx+\frac{2}{3}\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}(b\sinh(x)\right.$$
$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}+$$
$$\frac{8}{5}\sqrt{b^2-c^2}\left(\frac{2}{3}\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}(b\sinh(x)+c\cosh(x))+\frac{4}{3}\sqrt{b^2-c^2}\int\sqrt{b\cos(ix)-ic\sin(ix)+}\right.$$

$$\Big\downarrow \text{3591}$$

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}+$$
$$\frac{8}{5}\sqrt{b^2-c^2}\left(\frac{2}{3}\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}(b\sinh(x)+c\cosh(x))+\frac{8\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}{3\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\right)$$

input
```
Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

output
```
(2*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2)
)/5 + (8*Sqrt[b^2 - c^2]*((8*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))/(3*S
qrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]) + (2*(c*Cosh[x] + b*Sinh[x])
*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/3))/5
```

3.768. $\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}dx$

3.768.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.768.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(120) = 240$.

Time = 0.60 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.06

method	result
default	$-\frac{(b-c)^2(b+c)^2\left(\frac{\cosh(x)^3}{3}+2\cosh(x)\right)}{\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}\sqrt{(b-c)(b+c)}}-\frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\sinh(x)^2}\sqrt{b^2-c^2}\,(-b^2+c^2)\left(-\frac{\cosh(x)\sqrt{b^2-c^2}\sqrt{-\sqrt{b^2-}}}{2\left(\sinh(x)b^2-\sin\right.}\right.}{\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh}{\sqrt{b^2-}}}}$

input
```
int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)*(b-c)^2*(b+c
)^2/((b-c)*(b+c))^(1/2)*(1/3*cosh(x)^3+2*cosh(x))-(-(b^2-c^2)^(1/2)*(sinh(
x)-1)*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*(-b^2+c^2)*(-1/2*cosh(x)/(sinh(x)*b
^2-sinh(x)*c^2-b^2+c^2)*(b^2-c^2)^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh
(x)^2)^(1/2)+1/2/((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*arctan(((b^2-c^2)^(1/
2)*(sinh(x)-1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/
2)))/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)
```

3.768.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(120) = 240$.

Time = 0.32 (sec) , antiderivative size = 784, normalized size of antiderivative = 5.60

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="fricas
")
```

output
```
1/30*sqrt(1/2)*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 18*(b^3 + 3*
b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + 3*(b^3 + 3*b^2*c + 3*b*c^2 + c^
3)*sinh(x)^6 + 125*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 5*(25*b^3 + 25*
b^2*c - 25*b*c^2 - 25*c^3 + 9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*s
inh(x)^4 + 20*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 25*(b^3 + b^2
*c - b*c^2 - c^3)*cosh(x))*sinh(x)^3 + 3*b^3 - 9*b^2*c + 9*b*c^2 - 3*c^3 +
 125*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 5*(9*(b^3 + 3*b^2*c + 3*b*c^2
 + c^3)*cosh(x)^4 + 25*b^3 - 25*b^2*c - 25*b*c^2 + 25*c^3 + 150*(b^3 + b^2
*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 2*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c
^3)*cosh(x)^5 + 250*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 + 125*(b^3 - b^2
*c - b*c^2 + c^3)*cosh(x))*sinh(x) + 2*(11*(b^2 + 2*b*c + c^2)*cosh(x)^5 +
 55*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^4 + 11*(b^2 + 2*b*c + c^2)*sinh(x)
^5 - 150*(b^2 - c^2)*cosh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^2 - 15
*b^2 + 15*c^2)*sinh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^3 - 45*(b^2
- c^2)*cosh(x))*sinh(x)^2 + 11*(b^2 - 2*b*c + c^2)*cosh(x) + (55*(b^2 + 2*
b*c + c^2)*cosh(x)^4 - 450*(b^2 - c^2)*cosh(x)^2 + 11*b^2 - 22*b*c + 11*c^
2)*sinh(x))*sqrt(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*s
inh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c
)/(cosh(x) + sinh(x)))/((b + c)*cosh(x)^4 + 4*(b + c)*cosh(x)*sinh(x)^3 +
(b + c)*sinh(x)^4 - (b - c)*cosh(x)^2 + (6*(b + c)*cosh(x)^2 - b + c)*s...
```

3.768.6 Sympy [F(-1)]

Timed out.

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(5/2),x)
```

output
```
Timed out
```

3.768.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1783 vs. 2(120) = 240.

Time = 2.15 (sec) , antiderivative size = 1783, normalized size of antiderivative = 12.74

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxima
")
```

output
```
1/20*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c
+ sqrt(b + c)*sqrt(b - c)*c^2)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)
*e^(-2*x) + b + c)^(5/2)*e^(5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b
 + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c
^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b -
c)*c^2)*e^(-2*x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c
)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c
)*c^2)*e^(-4*x) + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/12*sqrt(2)
*(b^3 + b^2*c - b*c^2 - c^3)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e
^(-2*x) + b + c)^(5/2)*e^(3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b +
 c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2
 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)
*c^2)*e^(-2*x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*
sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*
c^2)*e^(-4*x) + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/2*sqrt(2)*(s
qrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*(2*sqrt(b + c)*s
qrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(5/2)*e^(1/2*x)/(sqrt(b + c)
*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)
*c^2 + 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*
b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x) + 10*(b^3 - b^2*c - b*c^2 +...
```

3.768.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(120) = 240.

Time = 0.30 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.25

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx =$$
$$-\frac{\sqrt{2}\Big(150\,(b^2-c^2)^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)}\mathrm{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)+3\left(\sqrt{b^2-c^2}b^2+2\sqrt{b^2-c^2}bc+\sqrt{b^2-c^2}c^2\right)e^{\left(\frac{5}{2}\,x\right)}\mathrm{sgn}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac")
```

output
```
-1/60*sqrt(2)*(150*(b^2 - c^2)^(3/2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x -
b + c) + 3*(sqrt(b^2 - c^2)*b^2 + 2*sqrt(b^2 - c^2)*b*c + sqrt(b^2 - c^2)*
c^2)*e^(5/2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + 25*(b^3 + b^2*c - b*c^2
 - c^3)*e^(3/2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - (25*(b^2 - 2*b*c + c
^2)*sqrt(b^2 - c^2)*e^x*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + 150*(b^3 - b^2
*c - b*c^2 + c^3)*e^(2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + 3*(b^3 - 3*b
^2*c + 3*b*c^2 - c^3)*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(-5/2*x))/sqrt(
b - c)
```

3.768.9 Mupad [F(-1)]

Timed out.

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \int \left(b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)\right)^{5/2}\,dx$$

input
```
int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2),x)
```

output
```
int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2), x)
```

3.769 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx$

3.769.1 Optimal result

Integrand size = 26, antiderivative size = 92

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx = \frac{8\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))}{3\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}} + \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}$$

output
```
8/3*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(
1/2))^(1/2)+2/3*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)
)^(1/2)
```

3.769.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 35.93 (sec) , antiderivative size = 4392, normalized size of antiderivative = 47.74

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

output
```
(2*b*Sqrt[b^2 - c^2]*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/c + ((
2*b*Sqrt[b^2 - c^2])/(3*c) + (2*c*Cosh[x])/3 + (2*b*Sinh[x])/3)*Sqrt[Sqrt[
b^2 - c^2] + b*Cosh[x] + c*Sinh[x]] + (32*b*(-b + c)*(b + c)^2*(EllipticF[
ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt
[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1] - 2*EllipticPi[-1, ArcSin[Sqrt[-(((-
b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1
+ Tanh[x/2])))]], 1])*Sqrt[Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x]]*
(-1 + Tanh[x/2])*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b
+ c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]*(-c + (-b + Sqrt[b^2 - c^2])*Ta
nh[x/2]))/(3*(b + c - Sqrt[b^2 - c^2])^2*(b + c + Sqrt[b^2 - c^2])*(1 + Co
sh[x])*Sqrt[(Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x])/(1 + Cosh[x])^
2]*Sqrt[(-1 + Tanh[x/2]^2)*(-2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-1 + Tanh[x/
2]^2) - b*(1 + Tanh[x/2]^2))]) + (16*(b - c)*(b + c)*Sqrt[Sqrt[(b - c)*(b
+ c)] + b*Cosh[x] + c*Sinh[x]]*(2*b^3*c^2 + 3*b^2*c^3 - c^5 - 2*b^2*c^2*Sq
rt[b^2 - c^2] - 3*b*c^3*Sqrt[b^2 - c^2] - c^4*Sqrt[b^2 - c^2] + 8*b^4*c*Ta
nh[x/2] + 12*b^3*c^2*Tanh[x/2] - 2*b^2*c^3*Tanh[x/2] - 8*b*c^4*Tanh[x/2] -
 2*c^5*Tanh[x/2] - 8*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2] - 12*b^2*c^2*Sqrt[b^2
 - c^2]*Tanh[x/2] - 2*b*c^3*Sqrt[b^2 - c^2]*Tanh[x/2] + 2*c^4*Sqrt[b^2 - c
^2]*Tanh[x/2] + 8*b^5*Tanh[x/2]^2 + 12*b^4*c*Tanh[x/2]^2 - 4*b^3*c^2*Tanh[
x/2]^2 - 11*b^2*c^3*Tanh[x/2]^2 - 2*b*c^4*Tanh[x/2]^2 + c^5*Tanh[x/2]^2...
```

3.769.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^{3/2}dx$$

$$\Big\downarrow \text{3592}$$

$$\frac{4}{3}\sqrt{b^2-c^2}\int \sqrt{b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}}dx+\frac{2}{3}\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}(b\sinh(x)+c\cosh(x))$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}(b\sinh(x)+c\cosh(x))+\frac{4}{3}\sqrt{b^2-c^2}\int\sqrt{b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}}dx$$

$$\Big\downarrow \text{3591}$$

$$\frac{2}{3}\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}(b\sinh(x)+c\cosh(x))+\frac{8\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}{3\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

input
```
Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

output
```
(8*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))/(3*Sqrt[Sqrt[b^2 - c^2] + b*Co
sh[x] + c*Sinh[x]]) + (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[Sqrt[b^2 - c^2] + b*
Cosh[x] + c*Sinh[x]])/3
```

3.769.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.769. $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2} dx$

3.769.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(78) = 156.

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{2(-b^2+c^2)\cosh(x)}{\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\sinh(x)^2}\,(b^2-c^2)\arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)-1)}\,\cosh(x)}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\sinh(x)^2}}\right)}{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)-1)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}}$	189

input
```
int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-b^2+c^2)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)*co
sh(x)+(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(b^2-c^2)/((b^2-c^2)^
(1/2)*(sinh(x)-1))^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*cosh(x
)/(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2))/sinh(x)/(-(sinh(x)*b^2-s
inh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)
```

3.769.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(78) = 156.

Time = 0.30 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.58

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\frac{\sqrt{\frac{1}{2}}\left((b^2+2\,bc+c^2)\cosh\left(x\right)^4+4\,(b^2+2\,bc+c^2)\cosh\left(x\right)\sinh\left(x\right)^3+(b^2+2\,bc+c^2)\,\mathrm{si}\right.}{}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="fricas
")
```

output
```
1/3*sqrt(1/2)*((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(
x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 18*(b^2 - c^2)*cosh(x)^2 +
6*((b^2 + 2*b*c + c^2)*cosh(x)^2 - 3*b^2 + 3*c^2)*sinh(x)^2 + b^2 - 2*b*c
+ c^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - 9*(b^2 - c^2)*cosh(x))*sinh(x)
+ 8*((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 +
 (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b - c)*sinh(x))*sqrt(b^2 - c^2))
*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 +
 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b +
 c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*
cosh(x) + (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))
```

3.769.6 Sympy [F]

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\int \left(b\cosh\left(x\right)+c\sinh\left(x\right)+\sqrt{b^2-c^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(3/2),x)
```

output
```
Integral((b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2))**(3/2), x)
```

3.769.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(78) = 156$.

Time = 0.53 (sec) , antiderivative size = 640, normalized size of antiderivative = 6.96

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\frac{\sqrt{2}\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c\right)\left(2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e\right.}{6\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c+3\left(b^2-c^2\right)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b}\right.\right.}+\frac{3\sqrt{2}(b^2-c^2)\left(2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c\right)^{\frac{3}{2}}e^{\left(\frac{1}{2}x\right)}}{2\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c+3\left(b^2-c^2\right)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)e^{(-2x)}+(b^2\right.}-\frac{3\sqrt{2}\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)\left(2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c\right)^{\frac{3}{2}}e^{(-}}{2\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c+3\left(b^2-c^2\right)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)e^{(-2x)}+(b^2\right.}-\frac{\sqrt{2}(b^2-2bc+c^2)\left(2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c\right)^{\frac{3}{2}}e^{\left(-\frac{3}{2}x\right)}}{6\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c+3\left(b^2-c^2\right)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)e^{(-2x)}+(b^2\right.}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxima
")
```

output
```
1/6*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c)*(2*sqr
t(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(3/2*x)/(s
qrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^2)*e^(-x
) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) + (
b^2 - 2*b*c + c^2)*e^(-3*x)) + 3/2*sqrt(2)*(b^2 - c^2)*(2*sqrt(b + c)*sqrt
(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(1/2*x)/(sqrt(b + c)*sq
rt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^2)*e^(-x) + 3*(sqrt(b
 + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) + (b^2 - 2*b*c +
 c^2)*e^(-3*x)) - 3/2*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqr
t(b - c)*c)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^
(3/2)*e^(-1/2*x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c +
3*(b^2 - c^2)*e^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b -
 c)*c)*e^(-2*x) + (b^2 - 2*b*c + c^2)*e^(-3*x)) - 1/6*sqrt(2)*(b^2 - 2*b*c
 + c^2)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2
)*e^(-3/2*x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b
^2 - c^2)*e^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*
c)*e^(-2*x) + (b^2 - 2*b*c + c^2)*e^(-3*x))
```

3.769.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.99

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx =$$

$$-\frac{\sqrt{2}\Big(\big(\sqrt{b^2-c^2}b+\sqrt{b^2-c^2}c\big)e^{\left(\frac{3}{2}x\right)}\mathrm{sgn}\big(-\sqrt{b^2-c^2}e^x-b+c\big)+9\,(b^2-c^2)e^{\left(\frac{1}{2}x\right)}\mathrm{sgn}\big(-\sqrt{b^2-c^2}e^x-b+c\big)}{6\sqrt{b}}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*((sqrt(b^2 - c^2)*b + sqrt(b^2 - c^2)*c)*e^(3/2*x)*sgn(-sqrt(
b^2 - c^2)*e^x - b + c) + 9*(b^2 - c^2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x
 - b + c) - (9*sqrt(b^2 - c^2)*(b - c)*e^x*sgn(-sqrt(b^2 - c^2)*e^x - b +
c) + (b^2 - 2*b*c + c^2)*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(-3/2*x))/sq
rt(b - c)
```

3.769.9 Mupad [F(-1)]

Timed out.

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\int \left(b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)\right)^{3/2}\,dx$$

input
```
int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2),x)
```

output
```
int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2), x)
```

3.770 $\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

3.770.1 Optimal result

Integrand size = 26, antiderivative size = 37

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\frac{2(c\cosh(x)+b\sinh(x))}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

output
```
2*(c*cosh(x)+b*sinh(x))/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2)
```

3.770.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 31.80 (sec) , antiderivative size = 455, normalized size of antiderivative = 12.30

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$
$$=\frac{4(b-c)(b+c)^2\left(2b^3-2bc^2-2b^2\sqrt{b^2-c^2}+c^2\sqrt{b^2-c^2}+b\left(-2b^2+c^2+2b\sqrt{b^2-c^2}\right)\cosh(x)-2b^2c\sin\right.}{}$$

input
```
Integrate[Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

output
```
(4*(b - c)*(b + c)^2*(2*b^3 - 2*b*c^2 - 2*b^2*Sqrt[b^2 - c^2] + c^2*Sqrt[b
^2 - c^2] + b*(-2*b^2 + c^2 + 2*b*Sqrt[b^2 - c^2])*Cosh[x] - 2*b^2*c*Sinh[
x] + c^3*Sinh[x] + 2*b*c*Sqrt[b^2 - c^2]*Sinh[x] + EllipticE[ArcSin[Sqrt[(
(-b - c + Sqrt[b^2 - c^2])*(Cosh[x] + Sinh[x]))/(-b + c + Sqrt[b^2 - c^2])
]], 1]*(-Cosh[x/2] + Sinh[x/2])*(c*(-2*b^2 + c*(c - Sqrt[b^2 - c^2]) + b*(
c + 2*Sqrt[b^2 - c^2]))*Cosh[x/2] + (-4*b^3 + b*c*(3*c - 2*Sqrt[b^2 - c^2]
) - c^2*(c + Sqrt[b^2 - c^2]) + 2*b^2*(c + 2*Sqrt[b^2 - c^2]))*Sinh[x/2])*
Sqrt[((-b - c + Sqrt[b^2 - c^2])*(Cosh[x] + Sinh[x]))/(-b + c + Sqrt[b^2 -
 c^2])]))/(Sqrt[b^2 - c^2]*(b + c - Sqrt[b^2 - c^2])^2*(-b^2 + c^2 + b*Sqr
t[b^2 - c^2])*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])
```

3.770.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \sqrt{\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)}dx$$
$$\Big\downarrow \ 3591$$
$$\frac{2(b\sinh(x)+c\cosh(x))}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

input
```
Int[Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

output
```
(2*(c*Cosh[x] + b*Sinh[x]))/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]
```

3.770.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

3.770.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(33) = 66.

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.38

method	result
risch	$\frac{\sqrt{2}\,\sqrt{\left(\mathrm{e}^{2x} b+\mathrm{e}^{2x} c+2\sqrt{b^{2}-c^{2}}\,\mathrm{e}^{x}+b-c\right)\mathrm{e}^{-x}}\,\left(\mathrm{e}^{x} b+\mathrm{e}^{x} c-\sqrt{b^{2}-c^{2}}\right)\left(\mathrm{e}^{x} b+\mathrm{e}^{x} c+\sqrt{b^{2}-c^{2}}\right)}{\left(\mathrm{e}^{2x} b+\mathrm{e}^{2x} c+2\sqrt{b^{2}-c^{2}}\,\mathrm{e}^{x}+b-c\right)(b+c)}$
default	$\frac{\left(-b^{2}+c^{2}\right)\cosh(x)}{\sqrt{b^{2}-c^{2}}\,\sqrt{-\frac{\sinh(x)b^{2}-\sinh(x)c^{2}-b^{2}+c^{2}}{\sqrt{b^{2}-c^{2}}}}}+\frac{\sqrt{-\sqrt{b^{2}-c^{2}}\,(\sinh(x)-1)\sinh(x)^{2}}\,\sqrt{b^{2}-c^{2}}\,\arctan\left(\frac{\sqrt{\sqrt{b^{2}-c^{2}}\,(\sinh(x)-1)}\,\cosh(x)}{\sqrt{-\sqrt{b^{2}-c^{2}}\,(\sinh(x)-1)\sinh(x)^{2}}}\right)}{\sqrt{\sqrt{b^{2}-c^{2}}\,(\sinh(x)-1)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^{2}-\sinh(x)c^{2}-b^{2}+c^{2}}{\sqrt{b^{2}-c^{2}}}}}$

input
```
int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*((exp(2*x)*b+exp(2*x)*c+2*(b^2-c^2)^(1/2)*exp(x)+b-c)*exp(-x))^(1/
2)/(exp(2*x)*b+exp(2*x)*c+2*(b^2-c^2)^(1/2)*exp(x)+b-c)*(exp(x)*b+exp(x)*c
-(b^2-c^2)^(1/2))*(exp(x)*b+exp(x)*c+(b^2-c^2)^(1/2))/(b+c)
```

3.770.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(33) = 66.

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.86

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$
$$= \frac{2\sqrt{\frac{1}{2}}\left((b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\sqrt{b^2-c^2}(\cosh(x)+\sinh(x))+\right.}{(b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
2*sqrt(1/2)*((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(
x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)*sqrt(((b + c)*cosh(x
)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2 - c^2)*(c
osh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh(x)^2 + 2*(b
+ c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - b + c)
```

3.770.6 Sympy [F]

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \int \sqrt{b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}}\,dx$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2)), x)
```

3.770.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(33) = 66.

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 4.14

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$
$$=\frac{\sqrt{2}\sqrt{2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c}\sqrt{b+c}\sqrt{b-c}e^{\left(\frac{1}{2}x\right)}}{(b-c)e^{(-x)}+\sqrt{b+c}\sqrt{b-c}}-\frac{\sqrt{2}\sqrt{2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c}(b-c)e^{\left(-\frac{1}{2}x\right)}}{(b-c)e^{(-x)}+\sqrt{b+c}\sqrt{b-c}}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
sqrt(2)*sqrt(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)*
sqrt(b + c)*sqrt(b - c)*e^(1/2*x)/((b - c)*e^(-x) + sqrt(b + c)*sqrt(b - c
)) - sqrt(2)*sqrt(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b
+ c)*(b - c)*e^(-1/2*x)/((b - c)*e^(-x) + sqrt(b + c)*sqrt(b - c))
```

3.770.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(33) = 66.

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$
$$=\frac{\sqrt{2}\left((b-c)e^{\left(-\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)-\sqrt{b^2-c^2}e^{\left(\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)\right)}{\sqrt{b-c}}$$

input
```
integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*((b - c)*e^(-1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - sqrt(b^2 -
 c^2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b + c))/sqrt(b - c)
```

3.770.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \int \sqrt{b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)}\,dx$$

input
```
int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2),x)
```

output
```
int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2), x)
```

3.771 $\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx$

3.771.1 Optimal result

Integrand size = 26, antiderivative size = 99

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx=\frac{\sqrt{2}\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{\sqrt[4]{b^2-c^2}}$$

output
```
arctan(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/((b^2-c^2)^(1/
2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))*2^(1/2)/(b^2-c^2)^(1/4
)
```

3.771.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 49.81 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.13

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = -\frac{\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{\sqrt{b^2-c^2}-b\cosh(x)-c\sinh(x)}{\sqrt{b^2-c^2}}}}{\sqrt{2}}\right),1\right)\left(b^2-c^2+b\sqrt{b^2-c^2}\cosh(x)+c\sqrt{b^2-c^2}\sinh(x)\right.}{\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

input
```
Integrate[1/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

output
```
-((Sqrt[2]*EllipticF[ArcSin[Sqrt[(Sqrt[b^2 - c^2] - b*Cosh[x] - c*Sinh[x])
/Sqrt[b^2 - c^2]]/Sqrt[2]], 1]*(b^2 - c^2 + b*Sqrt[b^2 - c^2]*Cosh[x] + c*
Sqrt[b^2 - c^2]*Sinh[x])*Sqrt[-((-b^2 + c^2 + b*Sqrt[b^2 - c^2]*Cosh[x] +
c*Sqrt[b^2 - c^2]*Sinh[x])/(b^2 - c^2))])/(Sqrt[b^2 - c^2]*(c*Cosh[x] + b*
Sinh[x])*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]))
```

3.771.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 3594, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)}}dx$$

$$\Big\downarrow \text{ 3594}$$

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\sin\left(-\tan^{-1}(b,-ic)+ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3128}$$

$$2i\int \frac{1}{\frac{(b^2-c^2)\sinh^2(x+i\tan^{-1}(b,-ic))}{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))+\sqrt{b^2-c^2}}+2\sqrt{b^2-c^2}}d\left(-\frac{i\sqrt{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)+\sqrt{b^2-c^2}}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{\sqrt[4]{b^2-c^2}}$$

input
```
Int[1/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

output
```
(Sqrt[2]*ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]
*Sqrt[Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/
(b^2 - c^2)^(1/4)
```

3.771.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.771.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{\sqrt{-\sqrt{b^{2}-c^{2}}\,(\sinh(x)-1)\sinh(x)^{2}}\,\arctan\left(\frac{\sqrt{\sqrt{b^{2}-c^{2}}\,(\sinh(x)-1)}\,\cosh(x)}{\sqrt{-\sqrt{b^{2}-c^{2}}\,(\sinh(x)-1)\sinh(x)^{2}}}\right)}{\sqrt{\sqrt{b^{2}-c^{2}}\,(\sinh(x)-1)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^{2}-\sinh(x)c^{2}-b^{2}+c^{2}}{\sqrt{b^{2}-c^{2}}}}}$	129

input
```
int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)-1
))^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1
/2)*(sinh(x)-1)*sinh(x)^2)^(1/2))/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c
^2)/(b^2-c^2)^(1/2))^(1/2)
```

3.771.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 681, normalized size of antiderivative = 6.88

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="fric
as")
```

output
```
[sqrt(2)*sqrt(-1/sqrt(b^2 - c^2))*log(-((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*
(b^2 + 2*b*c + c^2)*cosh(x)^3*sinh(x) + 6*(b^2 + 2*b*c + c^2)*cosh(x)^2*si
nh(x)^2 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*si
nh(x)^4 - 2*sqrt(2)*sqrt(1/2)*(2*(b^2 - c^2)*cosh(x)^2 + 4*(b^2 - c^2)*cos
h(x)*sinh(x) + 2*(b^2 - c^2)*sinh(x)^2 - ((b + c)*cosh(x)^3 + 3*(b + c)*co
sh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 + (b - c)*cosh(x) + (3*(b + c)*cosh(x)
^2 + b - c)*sinh(x))*sqrt(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*
cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)
) + b - c)/(cosh(x) + sinh(x)))*sqrt(-1/sqrt(b^2 - c^2)) - b^2 + 2*b*c - c
^2 - 2*((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^
3 - (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))*sqrt(b^2 - c^
2))/((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)
^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 2*(b^2 - c^2)*cosh(x)^2 + 2*(3*(b^2 +
 2*b*c + c^2)*cosh(x)^2 - b^2 + c^2)*sinh(x)^2 + b^2 - 2*b*c + c^2 + 4*((b
^2 + 2*b*c + c^2)*cosh(x)^3 - (b^2 - c^2)*cosh(x))*sinh(x))), -2*sqrt(2)*a
rctan(sqrt(2)*sqrt(1/2)*(sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) - b + c)*sqrt
(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sq
rt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/(((b + c)*
cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - b + c)*(b^2 -
c^2)^(1/4)))/(b^2 - c^2)^(1/4)]
```

3.771.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh{(x)}+c\sinh{(x)}+\sqrt{b^2-c^2}}}\,dx$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(1/2),x)
```

output
```
Integral(1/sqrt(b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2)), x)
```

3.771.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh{(x)}+c\sinh{(x)}+\sqrt{b^2-c^2}}}\,dx$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/sqrt(b*cosh(x) + c*sinh(x) + sqrt(b^2 - c^2)), x)
```

3.771.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(80) = 160$.

Time = 0.49 (sec) , antiderivative size = 538, normalized size of antiderivative = 5.43

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx =$$

$$-\frac{2\sqrt{2}(b^2-c^2-b+c)\sqrt{b+c}\arctan\left(\frac{}{\sqrt{\sqrt{b^2-c^2}b^5+\sqrt{b^2-c^2}b^4c-2\sqrt{b^2-c^2}b^3c^2-2\sqrt{b^2-}}}\right.}{\sqrt{\sqrt{b^2-c^2}b^5+\sqrt{b^2-c^2}b^4c-2\sqrt{b^2-c^2}b^3c^2-2\sqrt{b^2-c^2}b^2c^3+\sqrt{b^2-c^2}bc^4+\sqrt{b^2-c^2}c^5-2\sqrt{b^2-}}}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac
")
```

output
```
-2*sqrt(2)*(b^2 - c^2 - b + c)*sqrt(b + c)*arctan((b^3*e^(1/2*x) + b^2*c*e
^(1/2*x) - b*c^2*e^(1/2*x) - c^3*e^(1/2*x) - b^2*e^(1/2*x) + c^2*e^(1/2*x)
)/sqrt(sqrt(b^2 - c^2)*b^5 + sqrt(b^2 - c^2)*b^4*c - 2*sqrt(b^2 - c^2)*b^3
*c^2 - 2*sqrt(b^2 - c^2)*b^2*c^3 + sqrt(b^2 - c^2)*b*c^4 + sqrt(b^2 - c^2)
*c^5 - 2*sqrt(b^2 - c^2)*b^4 + 4*sqrt(b^2 - c^2)*b^2*c^2 - 2*sqrt(b^2 - c^
2)*c^4 + sqrt(b^2 - c^2)*b^3 - sqrt(b^2 - c^2)*b^2*c - sqrt(b^2 - c^2)*b*c
^2 + sqrt(b^2 - c^2)*c^3))/(sqrt(sqrt(b^2 - c^2)*b^5 + sqrt(b^2 - c^2)*b^4
*c - 2*sqrt(b^2 - c^2)*b^3*c^2 - 2*sqrt(b^2 - c^2)*b^2*c^3 + sqrt(b^2 - c^
2)*b*c^4 + sqrt(b^2 - c^2)*c^5 - 2*sqrt(b^2 - c^2)*b^4 + 4*sqrt(b^2 - c^2)
*b^2*c^2 - 2*sqrt(b^2 - c^2)*c^4 + sqrt(b^2 - c^2)*b^3 - sqrt(b^2 - c^2)*b
^2*c - sqrt(b^2 - c^2)*b*c^2 + sqrt(b^2 - c^2)*c^3)*sgn(-sqrt(b^2 - c^2)*e
^x - b + c))
```

3.771.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh\left(x\right)+\sqrt{b^2-c^2}+c\sinh\left(x\right)}}\,dx$$

input
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2),x)
```

output
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2), x)
```

3.772 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx$

3.772.1 Optimal result

Integrand size = 26, antiderivative size = 155

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{2\sqrt{2}\,(b^2-c^2)^{3/4}} + \frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}$$

output
```
1/4*arctan(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/((b^2-c^2)
^(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))/(b^2-c^2)^(3/4)*2^
(1/2)+1/2*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-
c^2)^(1/2))^(3/2)
```

3.772.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \$\text{Aborted}$$

input
```
Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

output
```
$Aborted
```

3.772.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 3595, 3042, 3594, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{\int \frac{1}{\sqrt{b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\int \frac{1}{\sqrt{b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3594}$$

$$\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\int \frac{1}{\sqrt{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))+\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\int \frac{1}{\sqrt{\sqrt{b^2-c^2}\sin\left(ix-\tan^{-1}(b,-ic)+\frac{\pi}{2}\right)+\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+$$
$$\frac{i\int\frac{1}{\frac{(b^2-c^2)\sinh^2\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)+\sqrt{b^2-c^2}}+2\sqrt{b^2-c^2}}d\left(-\frac{i\sqrt{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))+\sqrt{b^2-c^2}}}\right)}{2\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+$$
$$\frac{\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{2\sqrt{2}\,(b^2-c^2)^{3/4}}$$

input
```
Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

output
```
ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[Sqr
t[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]/(2*Sqrt[2]
*(b^2 - c^2)^(3/4)) + (c*Cosh[x] + b*Sinh[x])/(2*Sqrt[b^2 - c^2]*(Sqrt[b^2
 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

3.772.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.772.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(128) = 256$.

Time = 0.24 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.69

method	result
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\cosh(x)\sqrt{2}}{2}\right)}{2\sqrt{b^2-c^2}\,\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}} + \frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\sinh(x)^2}\,\sqrt{b^2-c^2}\,\sqrt{2}\left(\ln\left(-\frac{2\left(\cosh(x)\sqrt{b^2-c^2}\,\sqrt{2}\,\sinh(x)-\sinh\right.}{}\right.\right.}{}$

input
```
int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(b^2-c^2)^(1/2)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(
1/2)*2^(1/2)*arctanh(1/2*cosh(x)*2^(1/2))+1/4*(-(b^2-c^2)^(1/2)*(sinh(x)-1
)*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*2^(1/2)*(ln(-2*(cosh(x)*(b^2-c^2)^(1/2)
*2^(1/2)*sinh(x)-sinh(x)*(b^2-c^2)^(1/2)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+(
b^2-c^2)^(1/2)-(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(-(b^2-c^2)^
(1/2)*(sinh(x)-1))^(1/2))/(cosh(x)-2^(1/2)))-ln(2*(cosh(x)*(b^2-c^2)^(1/2)
*2^(1/2)*sinh(x)+sinh(x)*(b^2-c^2)^(1/2)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-(
b^2-c^2)^(1/2)+(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(-(b^2-c^2)^
(1/2)*(sinh(x)-1))^(1/2))/(cosh(x)+2^(1/2))))/(b-c)/(b+c)/(-(b^2-c^2)^(1/2
)*(sinh(x)-1))^(1/2)/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)
^(1/2))^(1/2)
```

3.772.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1801 vs. 2(126) = 252.

Time = 0.43 (sec) , antiderivative size = 1801, normalized size of antiderivative = 11.62

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="fric
as")
```

output
```
1/2*((sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 6*sqrt(2)*(b^3 +
 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + sqrt(2)*(b^3 + 3*b^2*c + 3*b
*c^2 + c^3)*sinh(x)^6 - 3*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 +
3*(5*sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2 - sqrt(2)*(b^3 + b^
2*c - b*c^2 - c^3))*sinh(x)^4 + 4*(5*sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^
3)*cosh(x)^3 - 3*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x))*sinh(x)^3 +
3*sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 3*(5*sqrt(2)*(b^3 + 3*b^
2*c + 3*b*c^2 + c^3)*cosh(x)^4 - 6*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cos
h(x)^2 + sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3))*sinh(x)^2 + 6*(sqrt(2)*(b^3
+ 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 - 2*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^
3)*cosh(x)^3 + sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x) - sqrt
(2)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3))*(b^2 - c^2)^(1/4)*arctan(-sqrt(1/2)*(
sqrt(2)*(b + c)*cosh(x) + sqrt(2)*(b + c)*sinh(x) - sqrt(2)*sqrt(b^2 - c^2
))*(b^2 - c^2)^(1/4)*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) +
 (b + c)*sinh(x)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(
x) + sinh(x)))/((b^2 + 2*b*c + c^2)*cosh(x)^2 + 2*(b^2 + 2*b*c + c^2)*cosh
(x)*sinh(x) + (b^2 + 2*b*c + c^2)*sinh(x)^2 - b^2 + c^2)) - 2*sqrt(1/2)*(4
*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 16*(b^3 + b^2*c - b*c^2 - c^3)*co
sh(x)*sinh(x)^3 + 4*(b^3 + b^2*c - b*c^2 - c^3)*sinh(x)^4 + 4*(b^3 - b^2*c
 - b*c^2 + c^3)*cosh(x)^2 + 4*(b^3 - b^2*c - b*c^2 + c^3 + 6*(b^3 + b^2...
```

3.772.6 Sympy [F]

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\cosh\left(x\right)+c\sinh\left(x\right)+\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(3/2),x)
```

output
```
Integral((b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2))**(-3/2), x)
```

3.772.7 Maxima [F]

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\cosh\left(x\right)+c\sinh\left(x\right)+\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*cosh(x) + c*sinh(x) + sqrt(b^2 - c^2))^(-3/2), x)
```

3.772.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{8,[4,0]%%%}+%%%{16,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{-8,[
2,1]%%%}+
```

3.772.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\cosh\left(x\right)+\sqrt{b^2-c^2}+c\sinh\left(x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2),x)
```

output
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2), x)
```

3.773 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx$

3.773.1 Optimal result

Integrand size = 26, antiderivative size = 205

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{16\sqrt{2}\,(b^2-c^2)^{5/4}} + \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3(c\cosh(x)+b\sinh(x))}{16\,(b^2-c^2)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}$$

output
```
3/32*arctan(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/((b^2-c^2
)^(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))/(b^2-c^2)^(5/4)*2
^(1/2)+1/4*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2
-c^2)^(1/2))^(5/2)+3/16*(c*cosh(x)+b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(
x)+(b^2-c^2)^(1/2))^(3/2)
```

3.773.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx=\$\text{Aborted}$$

input
```
Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

output
```
$Aborted
```

3.773.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3042, 3595, 3042, 3595, 3042, 3594, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{3\int \frac{1}{\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^{3/2}}dx}{8\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}+\frac{3\int \frac{1}{\left(b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}\right)^{3/2}}dx}{8\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3595}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\right)}{8\sqrt{b^2-c^2}}+\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}+\frac{3\left(\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\int \frac{1}{\sqrt{b\cos(ix)-ic\sin(ix)+\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}\right)}{8\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3594}$$

$$\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}+\frac{3\left(\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\int \frac{1}{\sqrt{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)+\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}\right)}{8\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}+\frac{3\left(\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\int \frac{1}{\sqrt{\sqrt{b^2-c^2}\sin\left(ix-\tan^{-1}(b,-ic)+\frac{\pi}{2}\right)+\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}\right)}{8\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}+$$

$$\frac{3\left(\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{i\int \frac{1}{\frac{\left(b^2-c^2\right)\sinh^2\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)+\sqrt{b^2-c^2}}+2\sqrt{b^2-c^2}}d\left(-\frac{i\sqrt{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-i\right.}{\sqrt{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)+}}\right.}{2\sqrt{b^2-c^2}}\right.}{8\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{219}$$

3.773. $\displaystyle\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx$

$$\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}+\frac{3\left(\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}(b^2-c^2)^{3/4}}\right)}{8\sqrt{b^2-c^2}}$$

input
```
Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

output
```
(c*Cosh[x] + b*Sinh[x])/(4*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] +
c*Sinh[x])^(5/2)) + (3*(ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I
)*c]])/(Sqrt[2]*Sqrt[Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b
, (-I)*c]]])]/(2*Sqrt[2]*(b^2 - c^2)^(3/4)) + (c*Cosh[x] + b*Sinh[x])/(2*S
qrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))))/(8*Sqrt[
b^2 - c^2])
```

3.773.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.773. $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx$

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.773.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(172) = 344$.

Time $= 0.27$ (sec) , antiderivative size $= 817$, normalized size of antiderivative $= 3.99$

method	result	size
default	Expression too large to display	817

input
```
int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b^2-c^2)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)/(b^
4-2*b^2*c^2+c^4)*(-1/4*cosh(x)/(cosh(x)^2-2)+1/8*2^(1/2)*arctanh(1/2*cosh(
x)*2^(1/2)))+(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(b^2-c^2)*(1/4
/(b-c)^2/(b+c)^2*(1/(b^2-c^2)^(1/2)/(sinh(x)-1)/(cosh(x)-2^(1/2))*(-(b^2-c
^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)+2^(1/2)/(-(b^2-c^2)^(1/2)*(sinh(x)-
1))^(1/2)*ln((-2*(b^2-c^2)^(1/2)*(sinh(x)-1)-2*(sinh(x)-1)*2^(1/2)*(b^2-c^
2)^(1/2)*(cosh(x)-2^(1/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2
)*(-(b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2))/(cosh(x)-2^(1/2))))+1/4/(b-c)^2/(b
+c)^2*(1/(b^2-c^2)^(1/2)/(sinh(x)-1)/(cosh(x)+2^(1/2))*(-(b^2-c^2)^(1/2)*(
sinh(x)-1)*sinh(x)^2)^(1/2)-2^(1/2)/(-(b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*l
n((-2*(b^2-c^2)^(1/2)*(sinh(x)-1)+2*(sinh(x)-1)*2^(1/2)*(b^2-c^2)^(1/2)*(c
osh(x)+2^(1/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(-(b^2-c^
2)^(1/2)*(sinh(x)-1))^(1/2))/(cosh(x)+2^(1/2))))-1/8/(b-c)^2/(b+c)^2*2^(1/
2)/(-(b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*ln((-2*(b^2-c^2)^(1/2)*(sinh(x)-1)
-2*(sinh(x)-1)*2^(1/2)*(b^2-c^2)^(1/2)*(cosh(x)-2^(1/2))+2*(-(b^2-c^2)^(1/
2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2))/(cos
h(x)-2^(1/2)))+1/8/(b-c)^2/(b+c)^2*2^(1/2)/(-(b^2-c^2)^(1/2)*(sinh(x)-1))^
(1/2)*ln((-2*(b^2-c^2)^(1/2)*(sinh(x)-1)+2*(sinh(x)-1)*2^(1/2)*(b^2-c^2)^(
1/2)*(cosh(x)+2^(1/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(-
(b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2))/(cosh(x)+2^(1/2))))/sinh(x)/(-(sinh...
```

3.773.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5297 vs. 2(170) = 340.

Time = 1.55 (sec) , antiderivative size = 5297, normalized size of antiderivative = 25.84

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="fric
as")
```

output
```
Too large to include
```

3.773.6 Sympy [F]

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(5/2),x)
```

output
```
Integral((b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2))**(-5/2), x)
```

3.773.7 Maxima [F]

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*cosh(x) + c*sinh(x) + sqrt(b^2 - c^2))^(-5/2), x)
```

3.773.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{32,[5,0]%%%}+%%%{96,[4,1]%%%}+%%%{-32,[4,0]%%%}+%%%{64
,[3,2]%%%
```

3.773.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)\right)^{5/2}}\,dx$$

input
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2),x)
```

output
```
int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2), x)
```

3.774 $\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx$

3.774.1 Optimal result

Integrand size = 28, antiderivative size = 146

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx=\frac{64(b^2-c^2)\left(c\cosh(x)+b\sinh(x)\right)}{15\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}-\frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}+\frac{2}{5}(c\cosh(x)+b\sinh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}$$

output
```
2/5*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2)+64/1
5*(b^2-c^2)*(c*cosh(x)+b*sinh(x))/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1
/2)-16/15*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)*(b*cosh(x)+c*sinh(x)-(b^2-
c^2)^(1/2))^(1/2)
```

3.774.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 56.15 (sec) , antiderivative size = 4368, normalized size of antiderivative = 29.92

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx=\text{Result too large to show}$$

input
```
Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

output
```
Sqrt[b^2 - c^2]*((4*b*Sqrt[b^2 - c^2])/(3*c) - (4*c*Cosh[x])/3 - (4*b*Sinh
[x])/3)*Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]] + Sqrt[-Sqrt[b^2 -
c^2] + b*Cosh[x] + c*Sinh[x]]*((44*b*(b^2 - c^2))/(15*c) - (2*c*Sqrt[b^2 -
 c^2]*Cosh[x])/15 + (2*b*c*Cosh[2*x])/5 - (2*b*Sqrt[b^2 - c^2]*Sinh[x])/15
 + ((b^2 + c^2)*Sinh[2*x])/5) + (256*b*c*(-b + c)*(b + c)*Sqrt[b^2 - c^2]*
(-b^2 + c^2)*(EllipticF[ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh
[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1] - 2*EllipticPi
[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + S
qrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1])*Sqrt[-Sqrt[(b - c)*(b + c)] + b*
Cosh[x] + c*Sinh[x]]*(-1 + Tanh[x/2])*(-(((b + c + Sqrt[b^2 - c^2])*(1 + T
anh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))^(3/2)*(c + (b +
Sqrt[b^2 - c^2])*Tanh[x/2])*(-1 + Tanh[x/2]^2))/(15*(b + c + Sqrt[b^2 - c^
2])^3*(-b^2 + c^2 + b*Sqrt[b^2 - c^2])*(1 + Cosh[x])*Sqrt[(-Sqrt[(b - c)*(
b + c)] + b*Cosh[x] + c*Sinh[x])/(1 + Cosh[x])^2]*(1 + Tanh[x/2])^2*Sqrt[-
((-1 + Tanh[x/2]^2)*(2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-1 + Tanh[x/2]^2) +
b*(1 + Tanh[x/2]^2)))]) - (128*(b - c)^2*(b + c)^2*Sqrt[-Sqrt[(b - c)*(b +
 c)] + b*Cosh[x] + c*Sinh[x]]*(2*b^3*c^2 + 3*b^2*c^3 - c^5 + 2*b^2*c^2*Sqr
t[b^2 - c^2] + 3*b*c^3*Sqrt[b^2 - c^2] + c^4*Sqrt[b^2 - c^2] + 8*b^4*c*Tan
h[x/2] + 12*b^3*c^2*Tanh[x/2] - 2*b^2*c^3*Tanh[x/2] - 8*b*c^4*Tanh[x/2] -
2*c^5*Tanh[x/2] + 8*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2] + 12*b^2*c^2*Sqrt[b...
```

3.774.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3592, 3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}dx$$

$\big\downarrow$ 3042

$$\int \left(-\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^{5/2}dx$$

$\big\downarrow$ 3592

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}-\frac{8}{5}\sqrt{b^2-c^2}\int \left(b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}\right)^{3/2}dx$$

$\big\downarrow$ 3042

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}-\frac{8}{5}\sqrt{b^2-c^2}\int \left(b\cos(ix)-ic\sin(ix)-\sqrt{b^2-c^2}\right)^{3/2}dx$$

$\big\downarrow$ 3592

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}-\frac{8}{5}\sqrt{b^2-c^2}\left(\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}-\frac{4}{3}\sqrt{b^2-c^2}\int \sqrt{b\cosh(x)+c\sinh(x)}\right.$$

$\big\downarrow$ 3042

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}-\frac{8}{5}\sqrt{b^2-c^2}\left(\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}-\frac{4}{3}\sqrt{b^2-c^2}\int \sqrt{b\cos(ix)-ic\sin(ix)}\right.$$

$\big\downarrow$ 3591

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}-\frac{8}{5}\sqrt{b^2-c^2}\left(\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}-\frac{8\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\right.$$

input
```
Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

output
```
(2*(c*Cosh[x] + b*Sinh[x])*(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2
))/5 - (8*Sqrt[b^2 - c^2]*((-8*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))/(3
*Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]) + (2*(c*Cosh[x] + b*Sinh[
x])*Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/3))/5
```

3.774. $\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}dx$

3.774.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.774.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(126) = 252$.

Time = 0.60 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.88

method	result
default	$-\frac{\sqrt{(b-c)(b+c)}\,(b-c)(b+c)\left(\frac{\cosh(x)^3}{3}+2\cosh(x)\right)}{\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\sinh(x)^2}\left(\frac{\left(b^2-c^2\right)^2\cosh(x)\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\,\mathrm{s}}}{2\sinh(x)b^2-2\sinh(x)c^2+2b^2-2c^2}\right.}{\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(}{\sqrt{b^2-}}}}$

input
```
int((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)*((b-c)*(b+c)
)^(1/2)*(b-c)*(b+c)*(1/3*cosh(x)^3+2*cosh(x))+(-(b^2-c^2)^(1/2)*(sinh(x)+1
)*sinh(x)^2)^(1/2)*(1/2*(b^2-c^2)^2*cosh(x)/(sinh(x)*b^2-sinh(x)*c^2+b^2-c
^2)*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)-1/2*(b^2-c^2)^(3/2)/((b
^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2
)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)))/sinh(x)/(-(sinh
(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)
```

3.774.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(126) = 252$.

Time = 0.33 (sec) , antiderivative size = 784, normalized size of antiderivative = 5.37

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="fricas
")
```

output
```
1/30*sqrt(1/2)*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 18*(b^3 + 3*
b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + 3*(b^3 + 3*b^2*c + 3*b*c^2 + c^
3)*sinh(x)^6 + 125*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 5*(25*b^3 + 25*
b^2*c - 25*b*c^2 - 25*c^3 + 9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*s
inh(x)^4 + 20*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 25*(b^3 + b^2
*c - b*c^2 - c^3)*cosh(x))*sinh(x)^3 + 3*b^3 - 9*b^2*c + 9*b*c^2 - 3*c^3 +
 125*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 5*(9*(b^3 + 3*b^2*c + 3*b*c^2
 + c^3)*cosh(x)^4 + 25*b^3 - 25*b^2*c - 25*b*c^2 + 25*c^3 + 150*(b^3 + b^2
*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 2*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c
^3)*cosh(x)^5 + 250*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 + 125*(b^3 - b^2
*c - b*c^2 + c^3)*cosh(x))*sinh(x) - 2*(11*(b^2 + 2*b*c + c^2)*cosh(x)^5 +
 55*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^4 + 11*(b^2 + 2*b*c + c^2)*sinh(x)
^5 - 150*(b^2 - c^2)*cosh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^2 - 15
*b^2 + 15*c^2)*sinh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^3 - 45*(b^2
- c^2)*cosh(x))*sinh(x)^2 + 11*(b^2 - 2*b*c + c^2)*cosh(x) + (55*(b^2 + 2*
b*c + c^2)*cosh(x)^4 - 450*(b^2 - c^2)*cosh(x)^2 + 11*b^2 - 22*b*c + 11*c^
2)*sinh(x))*sqrt(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*s
inh(x) + (b + c)*sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c
)/(cosh(x) + sinh(x)))/((b + c)*cosh(x)^4 + 4*(b + c)*cosh(x)*sinh(x)^3 +
(b + c)*sinh(x)^4 - (b - c)*cosh(x)^2 + (6*(b + c)*cosh(x)^2 - b + c)*s...
```

3.774.6 Sympy [F(-1)]

Timed out.

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(5/2),x)
```

output
```
Timed out
```

3.774.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. 2(126) = 252.

Time = 2.28 (sec) , antiderivative size = 1789, normalized size of antiderivative = 12.25

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxima
")
```

output
```
1/20*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c
+ sqrt(b + c)*sqrt(b - c)*c^2)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c
)*e^(-2*x) + b + c)^(5/2)*e^(5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(
b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*
c^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b -
 c)*c^2)*e^(-2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b +
c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b -
c)*c^2)*e^(-4*x) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/12*sqrt(2
)*(b^3 + b^2*c - b*c^2 - c^3)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)
*e^(-2*x) + b + c)^(5/2)*e^(3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b
 + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c
^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b -
c)*c^2)*e^(-2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c
)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c
)*c^2)*e^(-4*x) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/2*sqrt(2)*
(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*(-2*sqrt(b + c
)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(5/2)*e^(1/2*x)/(sqrt(b +
 c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b -
 c)*c^2 - 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b -
c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x) - 10*(b^3 - b^2*c - b*c^...
```

3.774.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(126) = 252.

Time = 0.32 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.16

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2} dx =$$
$$-\frac{\sqrt{2}\Big(150\,(b^2-c^2)^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)}\mathrm{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)+3\,\left(\sqrt{b^2-c^2}b^2+2\,\sqrt{b^2-c^2}bc+\sqrt{b^2-c^2}c^2\right)e^{\left(\frac{5}{2}\,x\right)}\mathrm{sgn}$$

input
```
integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac")
```

output
```
-1/60*sqrt(2)*(150*(b^2 - c^2)^(3/2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x +
b - c) + 3*(sqrt(b^2 - c^2)*b^2 + 2*sqrt(b^2 - c^2)*b*c + sqrt(b^2 - c^2)*
c^2)*e^(5/2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - 25*(b^3 + b^2*c - b*c^2
 - c^3)*e^(3/2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - (25*(b^2 - 2*b*c + c
^2)*sqrt(b^2 - c^2)*e^x*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - 150*(b^3 - b^2
*c - b*c^2 + c^3)*e^(2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - 3*(b^3 - 3*b
^2*c + 3*b*c^2 - c^3)*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(-5/2*x))/sqrt(
b - c)
```

3.774.9 Mupad [F(-1)]

Timed out.

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2} dx = \int \left(b\cosh(x)-\sqrt{b^2-c^2}+c\sinh(x)\right)^{5/2} dx$$

input
```
int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2),x)
```

output
```
int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2), x)
```

3.775 $\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx$

3.775.1 Optimal result

Integrand size = 28, antiderivative size = 96

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=-\frac{8\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}+\frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}$$

output
```
-8/3*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^
(1/2))^(1/2)+2/3*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2
))^(1/2)
```

3.775.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 31.28 (sec) , antiderivative size = 4260, normalized size of antiderivative = 44.38

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\text{Result too large to show}$$

input
```
Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

output
```
(-2*b*Sqrt[b^2 - c^2]*Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/c +
((-2*b*Sqrt[b^2 - c^2])/(3*c) + (2*c*Cosh[x])/3 + (2*b*Sinh[x])/3)*Sqrt[-S
qrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]] - (32*b*c*(-b + c)*(b + c)*(-b^2 +
 c^2)*(EllipticF[ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))
/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1] - 2*EllipticPi[-1, Ar
cSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2
 - c^2])*(-1 + Tanh[x/2])))]], 1])*Sqrt[-Sqrt[(b - c)*(b + c)] + b*Cosh[x]
 + c*Sinh[x]]*(-1 + Tanh[x/2])*(-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2
]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))^(3/2)*(c + (b + Sqrt[b^
2 - c^2])*Tanh[x/2])*(-1 + Tanh[x/2]^2))/(3*(b + c + Sqrt[b^2 - c^2])^3*(-
b^2 + c^2 + b*Sqrt[b^2 - c^2])*(1 + Cosh[x])*Sqrt[(-Sqrt[(b - c)*(b + c)]
+ b*Cosh[x] + c*Sinh[x])/(1 + Cosh[x])^2]*(1 + Tanh[x/2])^2*Sqrt[-((-1 + T
anh[x/2]^2)*(2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-1 + Tanh[x/2]^2) + b*(1 + T
anh[x/2]^2)))]) + (16*(b - c)*(b + c)*Sqrt[-Sqrt[(b - c)*(b + c)] + b*Cosh
[x] + c*Sinh[x]]*(2*b^3*c^2 + 3*b^2*c^3 - c^5 + 2*b^2*c^2*Sqrt[b^2 - c^2]
+ 3*b*c^3*Sqrt[b^2 - c^2] + c^4*Sqrt[b^2 - c^2] + 8*b^4*c*Tanh[x/2] + 12*b
^3*c^2*Tanh[x/2] - 2*b^2*c^3*Tanh[x/2] - 8*b*c^4*Tanh[x/2] - 2*c^5*Tanh[x/
2] + 8*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2] + 12*b^2*c^2*Sqrt[b^2 - c^2]*Tanh[x
/2] + 2*b*c^3*Sqrt[b^2 - c^2]*Tanh[x/2] - 2*c^4*Sqrt[b^2 - c^2]*Tanh[x/2]
+ 8*b^5*Tanh[x/2]^2 + 12*b^4*c*Tanh[x/2]^2 - 4*b^3*c^2*Tanh[x/2]^2 - 11...
```

3.775.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^{3/2}dx$$

$$\Big\downarrow \text{3592}$$

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}-\frac{4}{3}\sqrt{b^2-c^2}\int \sqrt{b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}}dx$$

3.775. $\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}dx$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}-\frac{4}{3}\sqrt{b^2-c^2}\int\sqrt{b\cos(ix)-ic\sin(ix)-\sqrt{b^2-c^2}}dx$$

$$\Big\downarrow \text{3591}$$

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}-\frac{8\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

input
```
Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

output
```
(-8*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))/(3*Sqrt[-Sqrt[b^2 - c^2] + b*
Cosh[x] + c*Sinh[x]]) + (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[-Sqrt[b^2 - c^2] +
 b*Cosh[x] + c*Sinh[x]])/3
```

3.775.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.775. $\int\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}dx$

3.775.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(82) = 164$.

Time = 0.22 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.97

method	result	size
default	$\frac{2(b^2-c^2)\cosh(x)}{\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\sinh(x)^2}\,(b^2-c^2)\arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)+1)}\,\cosh(x)}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\sinh(x)^2}}\right)}{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)+1)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}$	189

input
```
int((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b^2-c^2)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)*cos
h(x)+(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)*(b^2-c^2)/((b^2-c^2)^(
1/2)*(sinh(x)+1))^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*cosh(x)
/(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2))/sinh(x)/(-(sinh(x)*b^2-si
nh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)
```

3.775.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(82) = 164$.

Time = 0.29 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.43

$$\int\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\frac{\sqrt{\frac{1}{2}}\left((b^2+2\,bc+c^2)\cosh(x)^4+4\,(b^2+2\,bc+c^2)\cosh(x)\sinh(x)^3+(b^2+2\,bc+c^2)\,\mathrm{si}\right.}{}$$

input
```
integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="fricas
")
```

output
```
1/3*sqrt(1/2)*((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(
x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 18*(b^2 - c^2)*cosh(x)^2 +
6*((b^2 + 2*b*c + c^2)*cosh(x)^2 - 3*b^2 + 3*c^2)*sinh(x)^2 + b^2 - 2*b*c
+ c^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - 9*(b^2 - c^2)*cosh(x))*sinh(x)
- 8*((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 +
 (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b - c)*sinh(x))*sqrt(b^2 - c^2))
*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 -
 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b +
 c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*
cosh(x) + (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))
```

3.775.6 Sympy [F]

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\int \left(b\cosh{(x)}+c\sinh{(x)}-\sqrt{b^2-c^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(3/2),x)
```

output
```
Integral((b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2))**(3/2), x)
```

3.775.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(82) = 164$.

Time = 0.47 (sec) , antiderivative size = 644, normalized size of antiderivative = 6.71

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx = \frac{\sqrt{2}\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c\right)\left(-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)\right.}{6\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c-3\,(b^2-c^2)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b}\right.\right.} - \frac{3\sqrt{2}(b^2-c^2)\left(-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c\right)^{\frac{3}{2}}e^{\left(\frac{1}{2}x\right)}}{2\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c-3\,(b^2-c^2)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)e^{(-2x)}-(b^2\right.} - \frac{3\sqrt{2}\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)\left(-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c\right)^{\frac{3}{2}}e^{(}}{2\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c-3\,(b^2-c^2)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)e^{(-2x)}-(b^2\right.} + \frac{\sqrt{2}(b^2-2\,bc+c^2)\left(-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c\right)^{\frac{3}{2}}e^{\left(-\frac{3}{2}x\right)}}{6\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c-3\,(b^2-c^2)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)e^{(-2x)}-(b^2\right.}$$

input
```
integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxima
")
```

output
```
1/6*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c)*(-2*sq
rt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(3/2*x)/(
sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 - c^2)*e^(-
x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) -
(b^2 - 2*b*c + c^2)*e^(-3*x)) - 3/2*sqrt(2)*(b^2 - c^2)*(-2*sqrt(b + c)*sq
rt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(1/2*x)/(sqrt(b + c)*
sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 - c^2)*e^(-x) + 3*(sqrt
(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) - (b^2 - 2*b*c
 + c^2)*e^(-3*x)) - 3/2*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*s
qrt(b - c)*c)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b +
c)^(3/2)*e^(-1/2*x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c
 - 3*(b^2 - c^2)*e^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(
b - c)*c)*e^(-2*x) - (b^2 - 2*b*c + c^2)*e^(-3*x)) + 1/6*sqrt(2)*(b^2 - 2*
b*c + c^2)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^
(3/2)*e^(-3/2*x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c -
3*(b^2 - c^2)*e^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b -
 c)*c)*e^(-2*x) - (b^2 - 2*b*c + c^2)*e^(-3*x))
```

3.775.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(82) = 164$.

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.92

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2} dx =$$

$$-\frac{\sqrt{2}\Big(\big(\sqrt{b^2-c^2}b+\sqrt{b^2-c^2}c\big)e^{\left(\frac{3}{2}x\right)}\mathrm{sgn}\big(-\sqrt{b^2-c^2}e^x+b-c\big)-9\,(b^2-c^2)e^{\left(\frac{1}{2}x\right)}\mathrm{sgn}\big(-\sqrt{b^2-c^2}e^x+b-c\big)}{6\,\sqrt{b}}$$

input
```
integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*((sqrt(b^2 - c^2)*b + sqrt(b^2 - c^2)*c)*e^(3/2*x)*sgn(-sqrt(
b^2 - c^2)*e^x + b - c) - 9*(b^2 - c^2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x
 + b - c) - (9*sqrt(b^2 - c^2)*(b - c)*e^x*sgn(-sqrt(b^2 - c^2)*e^x + b -
c) - (b^2 - 2*b*c + c^2)*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(-3/2*x))/sq
rt(b - c)
```

3.775.9 Mupad [F(-1)]

Timed out.

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\int \left(b\cosh(x)-\sqrt{b^2-c^2}+c\sinh(x)\right)^{3/2}\,dx$$

input
```
int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2),x)
```

output
```
int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2), x)
```

3.776 $\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

3.776.1 Optimal result

Integrand size = 28, antiderivative size = 39

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\frac{2(c\cosh(x)+b\sinh(x))}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

output
```
2*(c*cosh(x)+b*sinh(x))/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2)
```

3.776.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 35.97 (sec) , antiderivative size = 4196, normalized size of antiderivative = 107.59

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\text{Result too large to show}$$

input
```
Integrate[Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

output
```
(2*b*Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/c - (8*b*c*Sqrt[b^2 -
 c^2]*(-b^2 + c^2)*(EllipticF[ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1
+ Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1] - 2*Elli
pticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b -
 c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1])*Sqrt[-Sqrt[(b - c)*(b + c)
] + b*Cosh[x] + c*Sinh[x]]*(-1 + Tanh[x/2])*(-(((b + c + Sqrt[b^2 - c^2])*
(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))^(3/2)*(c +
 (b + Sqrt[b^2 - c^2])*Tanh[x/2])*(-1 + Tanh[x/2]^2))/((b + c + Sqrt[b^2 -
 c^2])^3*(-b^2 + c^2 + b*Sqrt[b^2 - c^2])*(1 + Cosh[x])*Sqrt[(-Sqrt[(b - c
)*(b + c)] + b*Cosh[x] + c*Sinh[x])/(1 + Cosh[x])^2]*(1 + Tanh[x/2])^2*Sqr
t[-((-1 + Tanh[x/2]^2)*(2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-1 + Tanh[x/2]^2)
 + b*(1 + Tanh[x/2]^2)))]) - (4*(b - c)*(b + c)*Sqrt[-Sqrt[(b - c)*(b + c)
] + b*Cosh[x] + c*Sinh[x]]*(2*b^3*c^2 + 3*b^2*c^3 - c^5 + 2*b^2*c^2*Sqrt[b
^2 - c^2] + 3*b*c^3*Sqrt[b^2 - c^2] + c^4*Sqrt[b^2 - c^2] + 8*b^4*c*Tanh[x
/2] + 12*b^3*c^2*Tanh[x/2] - 2*b^2*c^3*Tanh[x/2] - 8*b*c^4*Tanh[x/2] - 2*c
^5*Tanh[x/2] + 8*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2] + 12*b^2*c^2*Sqrt[b^2 - c
^2]*Tanh[x/2] + 2*b*c^3*Sqrt[b^2 - c^2]*Tanh[x/2] - 2*c^4*Sqrt[b^2 - c^2]*
Tanh[x/2] + 8*b^5*Tanh[x/2]^2 + 12*b^4*c*Tanh[x/2]^2 - 4*b^3*c^2*Tanh[x/2]
^2 - 11*b^2*c^3*Tanh[x/2]^2 - 2*b*c^4*Tanh[x/2]^2 + c^5*Tanh[x/2]^2 + 8*b^
4*Sqrt[b^2 - c^2]*Tanh[x/2]^2 + 12*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2]^2 - ...
```

3.776.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \ 3591$$

$$\frac{2(b\sinh(x)+c\cosh(x))}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

3.776. $\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

input
```
Int[Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

output
```
(2*(c*Cosh[x] + b*Sinh[x]))/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]
```

3.776.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

3.776.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(35) = 70$.

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.36

method	result
risch	$-\frac{\sqrt{2}\,\sqrt{-\left(-\mathrm{e}^{2x} b-\mathrm{e}^{2x} c+2\sqrt{b^{2}-c^{2}}\,\mathrm{e}^{x}-b+c\right)\mathrm{e}^{-x}}\,\left(\mathrm{e}^{x} b+\mathrm{e}^{x} c+\sqrt{b^{2}-c^{2}}\right)\left(\mathrm{e}^{x} b+\mathrm{e}^{x} c-\sqrt{b^{2}-c^{2}}\right)}{\left(-\mathrm{e}^{2x} b-\mathrm{e}^{2x} c+2\sqrt{b^{2}-c^{2}}\,\mathrm{e}^{x}-b+c\right)(b+c)}$
default	$\frac{\left(-b^{2}+c^{2}\right)\cosh(x)}{\sqrt{b^{2}-c^{2}}\,\sqrt{-\frac{\sinh(x) b^{2}-\sinh(x) c^{2}+b^{2}-c^{2}}{\sqrt{b^{2}-c^{2}}}}}-\frac{\sqrt{-\sqrt{b^{2}-c^{2}}\,(\sinh(x)+1)\sinh(x)^{2}}\,\sqrt{b^{2}-c^{2}}\,\arctan\left(\frac{\sqrt{\sqrt{b^{2}-c^{2}}\,(\sinh(x)+1)}\,\cosh(x)}{\sqrt{-\sqrt{b^{2}-c^{2}}\,(\sinh(x)+1)\sinh(x)^{2}}}\right)}{\sqrt{\sqrt{b^{2}-c^{2}}\,(\sinh(x)+1)}\,\sinh(x)\sqrt{-\frac{\sinh(x) b^{2}-\sinh(x) c^{2}+b^{2}-c^{2}}{\sqrt{b^{2}-c^{2}}}}}$

input
```
int((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2^(1/2)*(-(-exp(2*x)*b-exp(2*x)*c+2*(b^2-c^2)^(1/2)*exp(x)-b+c)*exp(-x))^
(1/2)/(-exp(2*x)*b-exp(2*x)*c+2*(b^2-c^2)^(1/2)*exp(x)-b+c)*(exp(x)*b+exp(
x)*c+(b^2-c^2)^(1/2))*(exp(x)*b+exp(x)*c-(b^2-c^2)^(1/2))/(b+c)
```

3.776.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(35) = 70$.

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.67

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$
$$= \frac{2\sqrt{\frac{1}{2}}\left((b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+2\sqrt{b^2-c^2}(\cosh(x)+\sinh(x))+\right.}{(b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b}$$

input
```
integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
2*sqrt(1/2)*((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(
x)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)*sqrt(((b + c)*cosh(x
)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(b^2 - c^2)*(c
osh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh(x)^2 + 2*(b
+ c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - b + c)
```

3.776.6 Sympy [F]

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \int \sqrt{b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}}\,dx$$

input
```
integrate((b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2)), x)
```

3.776.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(35) = 70$.

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.00

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$
$$= -\frac{\sqrt{2}\sqrt{-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c}\sqrt{b+c}\sqrt{b-c}e^{\left(\frac{1}{2}x\right)}}{(b-c)e^{(-x)}-\sqrt{b+c}\sqrt{b-c}} - \frac{\sqrt{2}\sqrt{-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c}(b-c)e^{\left(-\frac{1}{2}x\right)}}{(b-c)e^{(-x)}-\sqrt{b+c}\sqrt{b-c}}$$

input
```
integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
-sqrt(2)*sqrt(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c
)*sqrt(b + c)*sqrt(b - c)*e^(1/2*x)/((b - c)*e^(-x) - sqrt(b + c)*sqrt(b -
 c)) - sqrt(2)*sqrt(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) +
 b + c)*(b - c)*e^(-1/2*x)/((b - c)*e^(-x) - sqrt(b + c)*sqrt(b - c))
```

3.776.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(35) = 70.

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.08

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = -\frac{\sqrt{2}\left((b-c)e^{\left(-\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)+\sqrt{b^2-c^2}e^{\left(\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)\right)}{\sqrt{b-c}}$$

input
```
integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(2)*((b - c)*e^(-1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + sqrt(b^2
- c^2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b - c))/sqrt(b - c)
```

3.776.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \int \sqrt{b\cosh\left(x\right)-\sqrt{b^2-c^2}+c\sinh\left(x\right)}\,dx$$

input
```
int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2),x)
```

output
```
int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2), x)
```

3.777 $\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx$

3.777.1 Optimal result

Integrand size = 28, antiderivative size = 102

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{\sqrt[4]{b^2-c^2}}$$

output
```
-arctanh(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/(-(b^2-c^2)^
(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))*2^(1/2)/(b^2-c^2)^(
1/4)
```

3.777.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 54.83 (sec) , antiderivative size = 52609, normalized size of antiderivative = 515.77

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

output
```
Result too large to show
```

3.777.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3042, 3594, 3042, 3128, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)}}dx$$

$$\Big\downarrow \text{3594}$$

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\sin\left(-\tan^{-1}(b,-ic)+ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3128}$$

$$2i\int \frac{1}{\frac{(b^2-c^2)\sinh^2(x+i\tan^{-1}(b,-ic))}{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))-\sqrt{b^2-c^2}}-2\sqrt{b^2-c^2}}d\left(-\frac{i\sqrt{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)-\sqrt{b^2-c^2}}}\right)$$

$$\Big\downarrow \text{217}$$

$$-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{\sqrt[4]{b^2-c^2}}$$

input
```
Int[1/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

3.777. $\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}dx$

output
```
-((Sqrt[2]*ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt
[2]*Sqrt[-Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]]
)])/(b^2 - c^2)^(1/4))
```

3.777.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.777.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\sinh(x)^2}\ \arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)+1)}\ \cosh(x)}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\sinh(x)^2}}\right)}{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)+1)}\ \sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}$	129

input
```
int(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)+1
))^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1
/2)*(sinh(x)+1)*sinh(x)^2)^(1/2))/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c
^2)/(b^2-c^2)^(1/2))^(1/2)
```

3.777.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 680, normalized size of antiderivative = 6.67

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="fric
as")
```

output
```
[sqrt(2)*log(-((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(
x)^3*sinh(x) + 6*(b^2 + 2*b*c + c^2)*cosh(x)^2*sinh(x)^2 + 4*(b^2 + 2*b*c
+ c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 2*sqrt(2)*sqrt(
1/2)*(2*(b^2 - c^2)*cosh(x)^2 + 4*(b^2 - c^2)*cosh(x)*sinh(x) + 2*(b^2 - c
^2)*sinh(x)^2 + ((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)
*sinh(x)^3 + (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b - c)*sinh(x))*sqrt
(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)
*sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sin
h(x)))/(b^2 - c^2)^(1/4) - b^2 + 2*b*c - c^2 + 2*((b + c)*cosh(x)^3 + 3*(b
 + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*cosh(x) + (3*(b + c)
*cosh(x)^2 - b + c)*sinh(x))*sqrt(b^2 - c^2))/((b^2 + 2*b*c + c^2)*cosh(x)
^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)
^4 - 2*(b^2 - c^2)*cosh(x)^2 + 2*(3*(b^2 + 2*b*c + c^2)*cosh(x)^2 - b^2 +
c^2)*sinh(x)^2 + b^2 - 2*b*c + c^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - (b
^2 - c^2)*cosh(x))*sinh(x)))/(b^2 - c^2)^(1/4), 2*sqrt(2)*sqrt(-1/sqrt(b^2
 - c^2))*arctan(sqrt(2)*sqrt(1/2)*(sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b
 - c)*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x
)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))*
sqrt(-1/sqrt(b^2 - c^2))/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) +
(b + c)*sinh(x)^2 - b + c))]
```

3.777.6 Sympy [F]

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}}}\,dx$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(1/2),x)
```

output
```
Integral(1/sqrt(b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2)), x)
```

3.777.7 Maxima [F]

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}}}\,dx$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/sqrt(b*cosh(x) + c*sinh(x) - sqrt(b^2 - c^2)), x)
```

3.777.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(83) = 166$.

Time = 0.52 (sec) , antiderivative size = 546, normalized size of antiderivative = 5.35

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx$$

$$= \frac{2\sqrt{2}(b^2-c^2-b+c)\sqrt{b+c}\arctan\left(\frac{\cdots}{\sqrt{-\sqrt{b^2-c^2}b^5-\sqrt{b^2-c^2}b^4c+2\sqrt{b^2-c^2}b^3c^2+2\sqrt{b^2-c^2}\cdots}}\right.}{\sqrt{-\sqrt{b^2-c^2}b^5-\sqrt{b^2-c^2}b^4c+2\sqrt{b^2-c^2}b^3c^2+2\sqrt{b^2-c^2}b^2c^3-\sqrt{b^2-c^2}bc^4-\sqrt{b^2-c^2}c^5+2\sqrt{b^2-\cdots}}}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac
")
```

output
```
2*sqrt(2)*(b^2 - c^2 - b + c)*sqrt(b + c)*arctan((b^3*e^(1/2*x) + b^2*c*e^
(1/2*x) - b*c^2*e^(1/2*x) - c^3*e^(1/2*x) - b^2*e^(1/2*x) + c^2*e^(1/2*x))
/sqrt(-sqrt(b^2 - c^2)*b^5 - sqrt(b^2 - c^2)*b^4*c + 2*sqrt(b^2 - c^2)*b^3
*c^2 + 2*sqrt(b^2 - c^2)*b^2*c^3 - sqrt(b^2 - c^2)*b*c^4 - sqrt(b^2 - c^2)
*c^5 + 2*sqrt(b^2 - c^2)*b^4 - 4*sqrt(b^2 - c^2)*b^2*c^2 + 2*sqrt(b^2 - c^
2)*c^4 - sqrt(b^2 - c^2)*b^3 + sqrt(b^2 - c^2)*b^2*c + sqrt(b^2 - c^2)*b*c
^2 - sqrt(b^2 - c^2)*c^3))/(sqrt(-sqrt(b^2 - c^2)*b^5 - sqrt(b^2 - c^2)*b^
4*c + 2*sqrt(b^2 - c^2)*b^3*c^2 + 2*sqrt(b^2 - c^2)*b^2*c^3 - sqrt(b^2 - c
^2)*b*c^4 - sqrt(b^2 - c^2)*c^5 + 2*sqrt(b^2 - c^2)*b^4 - 4*sqrt(b^2 - c^2
)*b^2*c^2 + 2*sqrt(b^2 - c^2)*c^4 - sqrt(b^2 - c^2)*b^3 + sqrt(b^2 - c^2)*
b^2*c + sqrt(b^2 - c^2)*b*c^2 - sqrt(b^2 - c^2)*c^3)*sgn(-sqrt(b^2 - c^2)*
e^x + b - c))
```

3.777.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx=\int \frac{1}{\sqrt{b\cosh(x)-\sqrt{b^2-c^2}+c\sinh(x)}}\,dx$$

input
```
int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2),x)
```

output
```
int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2), x)
```

3.778 $\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx$

3.778.1 Optimal result

Integrand size = 28, antiderivative size = 159

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}-\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}$$

output
```
1/4*arctanh(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/(-(b^2-c^
2)^(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))/(b^2-c^2)^(3/4)*
2^(1/2)+1/2*(-c*cosh(x)-b*sinh(x))/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(
3/2)/(b^2-c^2)^(1/2)
```

3.778.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx=\$\text{Aborted}$$

input
```
Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

output
```
$Aborted
```

3.778.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3595, 3042, 3594, 3042, 3128, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3595}$$

$$-\frac{\int \frac{1}{\sqrt{b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}-\frac{\int \frac{1}{\sqrt{b\cos(ix)-ic\sin(ix)-\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3594}$$

$$-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))-\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-c^2}\sin\left(ix-\tan^{-1}(b,-ic)+\frac{\pi}{2}\right)-\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}-\frac{i\int\frac{1}{\frac{\left(b^2-c^2\right)\sinh^2\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)-\sqrt{b^2-c^2}}-2\sqrt{b^2-c^2}}d\left(-\frac{i\sqrt{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))-\sqrt{b^2-c^2}}}\right)}{2\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}$$

input
```
Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

output
```
ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[-S
qrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]/(2*Sqrt[
2]*(b^2 - c^2)^(3/4)) - (c*Cosh[x] + b*Sinh[x])/(2*Sqrt[b^2 - c^2]*(-Sqrt[
b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

3.778.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.778. $\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx$

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.778.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(134) = 268$.

Time = 0.23 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.61

method	result
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\cosh(x)\sqrt{2}}{2}\right)}{2\sqrt{b^2-c^2}\,\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}} - \frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\sinh(x)^2}\,\sqrt{b^2-c^2}\,\sqrt{2}\left(\ln\left(-\frac{2\left(\cosh(x)\sqrt{b^2-c^2}\,\sqrt{2}\,\sinh(x)-\sinh\right.}{}\right.\right.}{}$

input
```
int(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(b^2-c^2)^(1/2)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(
1/2)*2^(1/2)*arctanh(1/2*cosh(x)*2^(1/2))-1/4*(-(b^2-c^2)^(1/2)*(sinh(x)+1
)*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*2^(1/2)*(ln(-2*(cosh(x)*(b^2-c^2)^(1/2)
*2^(1/2)*sinh(x)-sinh(x)*(b^2-c^2)^(1/2)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-(
b^2-c^2)^(1/2)-(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1/2)*(sin
h(x)+1)*sinh(x)^2)^(1/2))/(cosh(x)-2^(1/2)))-ln(2*(cosh(x)*(b^2-c^2)^(1/2)
*2^(1/2)*sinh(x)+sinh(x)*(b^2-c^2)^(1/2)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+(
b^2-c^2)^(1/2)+(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1/2)*(sin
h(x)+1)*sinh(x)^2)^(1/2))/(cosh(x)+2^(1/2))))/(b-c)/(b+c)/(-(b^2-c^2)^(1/2
)*(sinh(x)+1))^(1/2)/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)
^(1/2))^(1/2)
```

3.778.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2137 vs. 2(130) = 260.

Time = 0.47 (sec) , antiderivative size = 2137, normalized size of antiderivative = 13.44

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="fric
as")
```

output
```
-1/4*((sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 6*sqrt(2)*(b^3
+ 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + sqrt(2)*(b^3 + 3*b^2*c + 3*
b*c^2 + c^3)*sinh(x)^6 - 3*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 +
 3*(5*sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2 - sqrt(2)*(b^3 + b
^2*c - b*c^2 - c^3))*sinh(x)^4 + 4*(5*sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c
^3)*cosh(x)^3 - 3*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x))*sinh(x)^3 +
 3*sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 3*(5*sqrt(2)*(b^3 + 3*b
^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - 6*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*co
sh(x)^2 + sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3))*sinh(x)^2 + 6*(sqrt(2)*(b^3
 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 - 2*sqrt(2)*(b^3 + b^2*c - b*c^2 - c
^3)*cosh(x)^3 + sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x) - sqr
t(2)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3))*(b^2 - c^2)^(1/4)*log(-((b^2 + 2*b*c
 + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x)^3*sinh(x) + 6*(b^2 + 2*b
*c + c^2)*cosh(x)^2*sinh(x)^2 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^3 +
(b^2 + 2*b*c + c^2)*sinh(x)^4 - 2*sqrt(1/2)*(sqrt(2)*(b + c)*cosh(x)^3 + 3
*sqrt(2)*(b + c)*cosh(x)*sinh(x)^2 + sqrt(2)*(b + c)*sinh(x)^3 + sqrt(2)*(
b - c)*cosh(x) + (3*sqrt(2)*(b + c)*cosh(x)^2 + sqrt(2)*(b - c))*sinh(x) +
 2*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2)*sqr
t(b^2 - c^2))*(b^2 - c^2)^(1/4)*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x
)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) +...
```

3.778.6 Sympy [F]

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\cosh{(x)}+c\sinh{(x)}-\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(3/2),x)
```

output
```
Integral((b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2))**(-3/2), x)
```

3.778.7 Maxima [F]

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\cosh\left(x\right)+c\sinh\left(x\right)-\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*cosh(x) + c*sinh(x) - sqrt(b^2 - c^2))^(-3/2), x)
```

3.778.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{8,[4,0]%%%}+%%%{16,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{-8,[
2,1]%%%}+
```

3.778.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\cosh\left(x\right)-\sqrt{b^2-c^2}+c\sinh\left(x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2),x)
```

output
```
int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2), x)
```

3.779 $\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx$

3.779.1 Optimal result

Integrand size = 28, antiderivative size = 211

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = -\frac{3\operatorname{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{16\sqrt{2}\,(b^2-c^2)^{5/4}} - \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3(c\cosh(x)+b\sinh(x))}{16\,(b^2-c^2)\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}$$

output
```
-3/32*arctanh(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/(-(b^2-
c^2)^(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))/(b^2-c^2)^(5/4
)*2^(1/2)+3/16*(c*cosh(x)+b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x)-(b^2-c
^2)^(1/2))^(3/2)+1/4*(-c*cosh(x)-b*sinh(x))/(b*cosh(x)+c*sinh(x)-(b^2-c^2)
^(1/2))^(5/2)/(b^2-c^2)^(1/2)
```

3.779.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx=\$\text{Aborted}$$

input
```
Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

output
```
$Aborted
```

3.779.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {3042, 3595, 3042, 3595, 3042, 3594, 3042, 3128, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cos(ix)-ic\sin(ix)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3595}$$

$$-\frac{3\int \frac{1}{\left(b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}\right)^{3/2}}dx}{8\sqrt{b^2-c^2}}-\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}-\frac{3\int \frac{1}{\left(b\cos(ix)-ic\sin(ix)-\sqrt{b^2-c^2}\right)^{3/2}}dx}{8\sqrt{b^2-c^2}}$$

$$\Big\downarrow \text{3595}$$

$$-\frac{3\left(-\frac{\int \frac{1}{\sqrt{b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\right)}{8\sqrt{b^2-c^2}}-\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}-\frac{3\left(-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}-\frac{\int \frac{1}{\sqrt{b\cos(ix)-ic\sin(ix)-\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}\right)}{8\sqrt{b^2-c^2}}$$

$$\Big\downarrow \; 3594$$

$$-\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}-\frac{3\left(-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)-\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}\right)}{8\sqrt{b^2-c^2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}-\frac{3\left(-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-c^2}\sin\left(ix-\tan^{-1}(b,-ic)+\frac{\pi}{2}\right)-\sqrt{b^2-c^2}}}dx}{4\sqrt{b^2-c^2}}\right)}{8\sqrt{b^2-c^2}}$$

$$\Big\downarrow \; 3128$$

$$-\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}-\frac{3\left(-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}-\frac{i\int \frac{1}{\frac{\left(b^2-c^2\right)\sinh^2\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)-\sqrt{b^2-c^2}}-2\sqrt{b^2-c^2}}d\left(-\frac{i\sqrt{b^2-c^2}\sinh\left(x+i\tan^{-1}(\cdots\right.}{\sqrt{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-i\cdots\right.}}\cdots\right.}{2\sqrt{b^2-c^2}}\cdots\right.}{8\sqrt{b^2-c^2}}$$

$$\Big\downarrow \; 217$$

3.779. $\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx$

$$-\frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}-\frac{3\left(-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}(b^2-c^2)^{3/4}}\right)}{8\sqrt{b^2-c^2}}$$

input
```
Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

output
```
-1/4*(c*Cosh[x] + b*Sinh[x])/(Sqrt[b^2 - c^2]*(-Sqrt[b^2 - c^2] + b*Cosh[x
] + c*Sinh[x])^(5/2)) - (3*(ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b
, (-I)*c]])/(Sqrt[2]*Sqrt[-Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*Ar
cTan[b, (-I)*c]]])]/(2*Sqrt[2]*(b^2 - c^2)^(3/4)) - (c*Cosh[x] + b*Sinh[x]
)/(2*Sqrt[b^2 - c^2]*(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))))/(
8*Sqrt[b^2 - c^2])
```

3.779.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.779. $\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx$

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.779.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(180) = 360$.

Time $= 0.29$ (sec) , antiderivative size $= 817$, normalized size of antiderivative $= 3.87$

method	result	size
default	Expression too large to display	817

input
```
int(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-b^2+c^2)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)/(b
^4-2*b^2*c^2+c^4)*(-1/4*cosh(x)/(cosh(x)^2-2)+1/8*2^(1/2)*arctanh(1/2*cosh
(x)*2^(1/2)))+(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)*(b^2-c^2)*(1/
4/(b-c)^2/(b+c)^2*(1/(b^2-c^2)^(1/2)/(sinh(x)+1)/(cosh(x)-2^(1/2))*(-(b^2-
c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)+2^(1/2)/(-(b^2-c^2)^(1/2)*(sinh(x)
+1))^(1/2)*ln((-2*(b^2-c^2)^(1/2)*(sinh(x)+1)-2*(sinh(x)+1)*2^(1/2)*(b^2-c
^2)^(1/2)*(cosh(x)-2^(1/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-
c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2))/(cosh(x)-2^(1/2))))+1/4/(b-c)^2/(
b+c)^2*(1/(b^2-c^2)^(1/2)/(sinh(x)+1)/(cosh(x)+2^(1/2))*(-(b^2-c^2)^(1/2)*
(sinh(x)+1)*sinh(x)^2)^(1/2)-2^(1/2)/(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*
ln((-2*(b^2-c^2)^(1/2)*(sinh(x)+1)+2*(sinh(x)+1)*2^(1/2)*(b^2-c^2)^(1/2)*(
cosh(x)+2^(1/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1/2)*
(sinh(x)+1)*sinh(x)^2)^(1/2))/(cosh(x)+2^(1/2))))-1/8/(b-c)^2/(b+c)^2*2^(1
/2)/(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*ln((-2*(b^2-c^2)^(1/2)*(sinh(x)+1
)-2*(sinh(x)+1)*2^(1/2)*(b^2-c^2)^(1/2)*(cosh(x)-2^(1/2))+2*(-(b^2-c^2)^(1
/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2))/(co
sh(x)-2^(1/2)))+1/8/(b-c)^2/(b+c)^2*2^(1/2)/(-(b^2-c^2)^(1/2)*(sinh(x)+1))
^(1/2)*ln((-2*(b^2-c^2)^(1/2)*(sinh(x)+1)+2*(sinh(x)+1)*2^(1/2)*(b^2-c^2)^
(1/2)*(cosh(x)+2^(1/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)
^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2))/(cosh(x)+2^(1/2))))/sinh(x)/(-(sin...
```

3.779.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5675 vs. $2(176) = 352$.

Time = 1.63 (sec) , antiderivative size = 5675, normalized size of antiderivative = 26.90

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="fric
as")
```

output
```
Too large to include
```

3.779.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(5/2),x)
```

output
```
Timed out
```

3.779.7 Maxima [F]

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\cosh\left(x\right)+c\sinh\left(x\right)-\sqrt{b^2-c^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*cosh(x) + c*sinh(x) - sqrt(b^2 - c^2))^(-5/2), x)
```

3.779.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{32,[5,0]%%%}+%%%{96,[4,1]%%%}+%%%{-32,[4,0]%%%}+%%%{64
,[3,2]%%%
```

3.779.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\cosh(x)-\sqrt{b^2-c^2}+c\sinh(x)\right)^{5/2}}\,dx$$

input
```
int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2),x)
```

output
```
int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2), x)
```

3.780 $\int \frac{1}{a+c\,\text{sech}(x)+b\tanh(x)}\,dx$

3.780.1 Optimal result

Integrand size = 12, antiderivative size = 107

$$\int \frac{1}{a+c\,\text{sech}(x)+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{2ac\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2)\sqrt{a^2-b^2-c^2}}-\frac{b\log(c+a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

output
```
a*x/(a^2-b^2)-b*ln(c+a*cosh(x)+b*sinh(x))/(a^2-b^2)-2*a*c*arctan((b+(a-c)*
tanh(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2)/(a^2-b^2-c^2)^(1/2)
```

3.780.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+c\,\text{sech}(x)+b\tanh(x)}\,dx=\frac{ax-\frac{2ac\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}-b\log(c+a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

input
```
Integrate[(a + c*Sech[x] + b*Tanh[x])^(-1),x]
```

output
```
(a*x - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/Sqrt[
a^2 - b^2 - c^2] - b*Log[c + a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

3.780.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3638, 3042, 3617, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\tanh(x)+c\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-ib\tan(ix)+c\sec(ix)}dx$$

$$\Big\downarrow \text{3638}$$

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)+c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)}{a\cos(ix)-ib\sin(ix)+c}dx$$

$$\Big\downarrow \text{3617}$$

$$-\frac{ac\int \frac{1}{c+a\cosh(x)+b\sinh(x)}dx}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x)+c)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ac\int \frac{1}{c+a\cos(ix)-ib\sin(ix)}dx}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x)+c)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

$$\Big\downarrow \text{3603}$$

$$-\frac{2ac\int \frac{1}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x)+c)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4ac\int \frac{1}{-\left(2b+2(a-c)\tanh\left(\frac{x}{2}\right)\right)^2-4(a^2-b^2-c^2)}d\left(2b+2(a-c)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x)+c)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2ac\arctan\left(\frac{2(a-c)\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2)\sqrt{a^2-b^2-c^2}}-\frac{b\log(a\cosh(x)+b\sinh(x)+c)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

input
```
Int[(a + c*Sech[x] + b*Tanh[x])^(-1),x]
```

output
```
(a*x)/(a^2 - b^2) - (2*a*c*ArcTan[(2*b + 2*(a - c)*Tanh[x/2])/(2*Sqrt[a^2
- b^2 - c^2])])/((a^2 - b^2)*Sqrt[a^2 - b^2 - c^2]) - (b*Log[c + a*Cosh[x]
 + b*Sinh[x]])/(a^2 - b^2)
```

3.780.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3617
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[b*B*((d + e*x)/
(e*(b^2 + c^2))), x] + (Simp[c*B*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/
(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2)   Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

3.780. $\int \frac{1}{a+c\,\text{sech}(x)+b\tanh(x)}\,dx$

rule 3638
```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.780.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.65

method	result
default	$\frac{2\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2a-2b}-\frac{2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a+2b}+\frac{\frac{2(-ab+bc)\ln\left(a\tanh\left(\frac{x}{2}\right)^{2}-c\tanh\left(\frac{x}{2}\right)^{2}+2b\tanh\left(\frac{x}{2}\right)+a+c\right)}{2a-2c}+\frac{2\left(-ac-b^{2}-\frac{(-ab+bc)b}{a-c}\right)\arctan\left(\right.}{\sqrt{a^{2}-b^{2}-c}}}{(a-b)(a+b)}$
risch	$\frac{x}{a+b}+\frac{2x\,a^{2}b}{a^{4}-2a^{2}b^{2}-a^{2}c^{2}+b^{4}+b^{2}c^{2}}-\frac{2x\,b^{3}}{a^{4}-2a^{2}b^{2}-a^{2}c^{2}+b^{4}+b^{2}c^{2}}-\frac{2xb\,c^{2}}{a^{4}-2a^{2}b^{2}-a^{2}c^{2}+b^{4}+b^{2}c^{2}}-\frac{\ln\left(\mathrm{e}^{x}-\frac{-a\,c^{2}+\sqrt{-a^{4}c^{2}+a^{2}b^{2}c}}{(a+b)ac}\right.}{a^{4}-2a^{2}b^{2}-a^{2}c^{2}+b^{4}+}$

input
```
int(1/(a+c*sech(x)+b*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(2*a-2*b)*ln(1+tanh(1/2*x))-2/(2*a+2*b)*ln(tanh(1/2*x)-1)+2/(a-b)/(a+b)*
(1/2*(-a*b+b*c)/(a-c)*ln(a*tanh(1/2*x)^2-c*tanh(1/2*x)^2+2*b*tanh(1/2*x)+a
+c)+(-a*c-b^2-(-a*b+b*c)*b/(a-c))/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*
tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(1/2)))
```

3.780.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 429, normalized size of antiderivative = 4.01

$$\int \frac{1}{a+c\text{sech}(x)+b\tanh(x)}\,dx$$
$$=\left[\frac{\sqrt{-a^{2}+b^{2}+c^{2}}ac\log\left(\frac{2\,(a+b)c\cosh(x)+\left(a^{2}+2\,ab+b^{2}\right)\cosh(x)^{2}+\left(a^{2}+2\,ab+b^{2}\right)\sinh(x)^{2}-a^{2}+b^{2}+2\,c^{2}+2\,\left((a+b)c+\left(a^{2}+2\,ab+b^{2}\right)\mathrm{co}\right.}{(a+b)\cosh(x)^{2}+(a+b)\sinh(x)^{2}+2\,c\cosh(x)+2\,\left((a+b)\cosh(x)\right.}\right.}{}\right.$$

input
```
integrate(1/(a+c*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(-a^2 + b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2
)*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 - a^2 + b^2 + 2*c^2 + 2*((a +
b)*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) - 2*sqrt(-a^2 + b^2 + c^2)*((a
 + b)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)
^2 + 2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) + a - b)) + (a^3 + a^2*
b - a*b^2 - b^3 - (a + b)*c^2)*x - (a^2*b - b^3 - b*c^2)*log(2*(a*cosh(x)
+ b*sinh(x) + c)/(cosh(x) - sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 - (a^2 - b^2
)*c^2), (2*sqrt(a^2 - b^2 - c^2)*a*c*arctan(-((a + b)*cosh(x) + (a + b)*si
nh(x) + c)/sqrt(a^2 - b^2 - c^2)) + (a^3 + a^2*b - a*b^2 - b^3 - (a + b)*c
^2)*x - (a^2*b - b^3 - b*c^2)*log(2*(a*cosh(x) + b*sinh(x) + c)/(cosh(x) -
 sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 - (a^2 - b^2)*c^2)]
```

3.780.6 Sympy [F]

$$\int \frac{1}{a + c\text{sech}(x) + b\tanh(x)}\, dx = \int \frac{1}{a + b\tanh(x) + c\,\text{sech}(x)}\, dx$$

input
```
integrate(1/(a+c*sech(x)+b*tanh(x)),x)
```

output
```
Integral(1/(a + b*tanh(x) + c*sech(x)), x)
```

3.780.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + c\text{sech}(x) + b\tanh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.780.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{1}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx = -\frac{2\,ac\arctan\left(\frac{ae^x + be^x + c}{\sqrt{a^2 - b^2 - c^2}}\right)}{\sqrt{a^2 - b^2 - c^2}(a^2 - b^2)} - \frac{b\log\left(ae^{(2\,x)} + be^{(2\,x)} + 2\,ce^x + a - b\right)}{a^2 - b^2} + \frac{x}{a - b}$$

input
```
integrate(1/(a+c*sech(x)+b*tanh(x)),x, algorithm="giac")
```

output
```
-2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(a^2 - b^2 - c^2))/(sqrt(a^2 - b^2 -
 c^2)*(a^2 - b^2)) - b*log(a*e^(2*x) + b*e^(2*x) + 2*c*e^x + a - b)/(a^2 -
 b^2) + x/(a - b)
```

3.780.9 Mupad [B] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 472, normalized size of antiderivative = 4.41

$$\int \frac{1}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx$$
$$= \frac{x}{a-b} + \frac{\ln\left(a - b + 2\,c\,e^x + a\,e^{2\,x} + b\,e^{2\,x}\right)\,\left(-2\,a^2\,b + 2\,b^3 + 2\,b\,c^2\right)}{2\,\left(a^4 - 2\,a^2\,b^2 - a^2\,c^2 + b^4 + b^2\,c^2\right)}$$
$$- \frac{2\,\mathrm{atan}\left(\left(e^x\left(\frac{2\,a\,c}{(a+b)^2\,(a^2-b^2)\,(a-b)^2\,\sqrt{a^2\,c^2}} - \frac{2\left(a^2\,c\,\sqrt{a^2\,c^2} - b^2\,c\,\sqrt{a^2\,c^2}\right)}{a\,(a+b)^2\,(a^2-b^2)^2\,(a-b)^2\,(-a^2+b^2+c^2)}\right) - \frac{2\left(a^3\,\sqrt{a^2\,c^2} + b^3\,\sqrt{a^2\,c^2} - a\,b^2\,\sqrt{a^2\,c^2} - \cdots\right.}{a\,(a+b)^2\,(a^2-b^2)^2\,(a-b)^2\,(-a^2+\cdots}\right.}{\sqrt{-\cdots}}$$

input
```
int(1/(a + b*tanh(x) + c/cosh(x)),x)
```

output
```
x/(a - b) + (log(a - b + 2*c*exp(x) + a*exp(2*x) + b*exp(2*x))*(2*b*c^2 -
2*a^2*b + 2*b^3))/(2*(a^4 + b^4 - 2*a^2*b^2 - a^2*c^2 + b^2*c^2)) - (2*ata
n((exp(x)*((2*a*c)/((a + b)^2*(a^2 - b^2)*(a - b)^2*(a^2*c^2)^(1/2)) - (2*
(a^2*c*(a^2*c^2)^(1/2) - b^2*c*(a^2*c^2)^(1/2)))/(a*(a + b)^2*(a^2 - b^2)^
2*(a - b)^2*(b^2 - a^2 + c^2))) - (2*(a^3*(a^2*c^2)^(1/2) + b^3*(a^2*c^2)^
(1/2) - a*b^2*(a^2*c^2)^(1/2) - a^2*b*(a^2*c^2)^(1/2)))/(a*(a + b)^2*(a^2
- b^2)^2*(a - b)^2*(b^2 - a^2 + c^2)))*((a^3*(-(a^2 - b^2)^2*(b^2 - a^2 +
c^2))^(1/2))/2 - (b^3*(-(a^2 - b^2)^2*(b^2 - a^2 + c^2))^(1/2))/2 - (a*b^2
*(-(a^2 - b^2)^2*(b^2 - a^2 + c^2))^(1/2))/2 + (a^2*b*(-(a^2 - b^2)^2*(b^2
 - a^2 + c^2))^(1/2))/2))*(a^2*c^2)^(1/2))/(-(a^2 - b^2)^2*(b^2 - a^2 + c^
2))^(1/2)
```

3.780. $\int \frac{1}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx$

3.781 $\int \frac{1}{a+b\coth(x)+c\text{csch}(x)}\,dx$

3.781.1 Optimal result

Integrand size = 12, antiderivative size = 113

$$\int \frac{1}{a+b\coth(x)+c\,\text{csch}(x)}\,dx=\frac{ax}{a^2-b^2}+\frac{2ac\,\text{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2)\sqrt{a^2-b^2+c^2}}-\frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2}$$

output
```
a*x/(a^2-b^2)-b*ln(I*c+I*b*cosh(x)+I*a*sinh(x))/(a^2-b^2)+2*a*c*arctanh((a
+(b-c)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2)/(a^2-b^2+c^2)^(1/2)
```

3.781.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int \frac{1}{a+b\coth(x)+c\,\text{csch}(x)}\,dx=\frac{ax-\frac{2ac\arctan\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}-b\log(c+b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

input
```
Integrate[(a + b*Coth[x] + c*Csch[x])^(-1),x]
```

output
```
(a*x - (2*a*c*ArcTan[(a + (b - c)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt
[-a^2 + b^2 - c^2] - b*Log[c + b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

3.781.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3639, 26, 26, 3042, 26, 3616, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\coth(x)+c\text{csch}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+ib\cot(ix)+ic\csc(ix)}dx$$

$$\Big\downarrow \text{3639}$$

$$\int \frac{i\sinh(x)}{ia\sinh(x)+ib\cosh(x)+ic}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\sinh(x)}{c+b\cosh(x)+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sinh(x)}{a\sinh(x)+b\cosh(x)+c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{-ia\sin(ix)+b\cos(ix)+c}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{c+b\cos(ix)-ia\sin(ix)}dx$$

$$\Big\downarrow \text{3616}$$

$$-i\left(-\frac{iac\int \frac{1}{c+b\cosh(x)+a\sinh(x)}dx}{a^2-b^2}-\frac{ib\log(a\sinh(x)+b\cosh(x)+c)}{a^2-b^2}+\frac{iax}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{iac\int \frac{1}{c+b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}-\frac{ib\log(a\sinh(x)+b\cosh(x)+c)}{a^2-b^2}+\frac{iax}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3603}$$

$$-i\left(-\frac{2iac\int \frac{1}{(b-c)\tanh^2\left(\frac{x}{2}\right)+2a\tanh\left(\frac{x}{2}\right)+b+c}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{ib\log(a\sinh(x)+b\cosh(x)+c)}{a^2-b^2}+\frac{iax}{a^2-b^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$-i\left(\frac{4iac\int \frac{1}{4(a^2-b^2+c^2)-\left(2a+2(b-c)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2a+2(b-c)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}-\frac{ib\log(a\sinh(x)+b\cosh(x)+c)}{a^2-b^2}+\frac{iax}{a^2-b^2}\right.$$

$$\Big\downarrow \text{219}$$

$$-i\left(\frac{2iac\text{arctanh}\left(\frac{2a+2(b-c)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2)\sqrt{a^2-b^2+c^2}}-\frac{ib\log(a\sinh(x)+b\cosh(x)+c)}{a^2-b^2}+\frac{iax}{a^2-b^2}\right)$$

input
```
Int[(a + b*Coth[x] + c*Csch[x])^(-1),x]
```

output
```
(-I)*((I*a*x)/(a^2 - b^2) + ((2*I)*a*c*ArcTanh[(2*a + 2*(b - c)*Tanh[x/2])
/(2*Sqrt[a^2 - b^2 + c^2])])/((a^2 - b^2)*Sqrt[a^2 - b^2 + c^2]) - (I*b*Lo
g[c + b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2))
```

3.781.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3616
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (-Simp[b*C*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

rule 3639
```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.781.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.58

method	result
default	$\frac{4\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{4a-4b}+\frac{\frac{2\left(-b^{2}+bc\right)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}b-c\tanh\left(\frac{x}{2}\right)^{2}+2a\tanh\left(\frac{x}{2}\right)+b+c\right)}{2b-2c}+\frac{2\left(-ab-ac-\frac{\left(-b^{2}+bc\right)a}{b-c}\right)\arctan\left(\frac{2(b-c)\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\sqrt{-a^{2}+b^{2}-c^{2}}}}{(a-b)(a+b)}$
risch	$\frac{x}{a+b}+\frac{2x\,a^{2}b}{a^{4}-2a^{2}b^{2}+a^{2}c^{2}+b^{4}-b^{2}c^{2}}-\frac{2x\,b^{3}}{a^{4}-2a^{2}b^{2}+a^{2}c^{2}+b^{4}-b^{2}c^{2}}+\frac{2xb\,c^{2}}{a^{4}-2a^{2}b^{2}+a^{2}c^{2}+b^{4}-b^{2}c^{2}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\,c^{2}+\sqrt{a^{4}c^{2}-a^{2}b^{2}c^{2}+}}{(a+b)ac}\right.}{a^{4}-2a^{2}b^{2}+a^{2}c^{2}+b^{4}-b}$

input
```
int(1/(a+b*coth(x)+c*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
4/(4*a-4*b)*ln(1+tanh(1/2*x))+2/(a-b)/(a+b)*(1/2*(-b^2+b*c)/(b-c)*ln(tanh(
1/2*x)^2*b-c*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b+c)+(-a*b-a*c-(-b^2+b*c)*a/(b-
c))/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2+b^2-c^
2)^(1/2)))-4/(4*a+4*b)*ln(tanh(1/2*x)-1)
```

3.781.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.88

$$\int \frac{1}{a+b\coth(x)+c\text{csch}(x)}\,dx$$
$$= \left[-\frac{\sqrt{a^2-b^2+c^2}ac\log\left(\frac{2\,(a+b)c\cosh(x)+(a^2+2\,ab+b^2)\cosh(x)^2+(a^2+2\,ab+b^2)\sinh(x)^2+a^2-b^2+2\,c^2+2\,((a+b)c+(a^2+2\,ab+b^2)\,\mathrm{c}}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)}\right.}{}\right.$$
$$-\frac{2\sqrt{-a^2+b^2-c^2}ac\arctan\left(\frac{\sqrt{-a^2+b^2-c^2}((a+b)\cosh(x)+(a+b)\sinh(x)+c)}{a^2-b^2+c^2}\right)-(a^3+a^2b-ab^2-b^3+(a+b)c^2)x}{a^4-2\,a^2b^2+b^4+(a^2-b^2)c^2}$$

input
```
integrate(1/(a+b*coth(x)+c*csch(x)),x, algorithm="fricas")
```

output
```
[-(sqrt(a^2 - b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2
)*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 2*c^2 + 2*((a +
b)*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((a
+ b)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^
2 + 2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) - a + b)) - (a^3 + a^2*b
 - a*b^2 - b^3 + (a + b)*c^2)*x + (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) +
 a*sinh(x) + c)/(cosh(x) - sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 + (a^2 - b^2)
*c^2), -(2*sqrt(-a^2 + b^2 - c^2)*a*c*arctan(sqrt(-a^2 + b^2 - c^2)*((a +
b)*cosh(x) + (a + b)*sinh(x) + c)/(a^2 - b^2 + c^2)) - (a^3 + a^2*b - a*b^
2 - b^3 + (a + b)*c^2)*x + (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + a*sinh
(x) + c)/(cosh(x) - sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 + (a^2 - b^2)*c^2)]
```

3.781.6 Sympy [F]

$$\int \frac{1}{a+b\coth(x)+c\text{csch}(x)}\,dx = \int \frac{1}{a+b\coth{(x)}+c\,\text{csch}{(x)}}\,dx$$

input
```
integrate(1/(a+b*coth(x)+c*csch(x)),x)
```

output
```
Integral(1/(a + b*coth(x) + c*csch(x)), x)
```

3.781.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*coth(x)+c*csch(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.781.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = -\frac{2\,ac\arctan\left(\frac{ae^x+be^x+c}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2-c^2}} - \frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^x-a+b\right)}{a^2-b^2} + \frac{x}{a-b}$$

input
```
integrate(1/(a+b*coth(x)+c*csch(x)),x, algorithm="giac")
```

output
```
-2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^2)*sqr
t(-a^2 + b^2 - c^2)) - b*log(a*e^(2*x) + b*e^(2*x) + 2*c*e^x - a + b)/(a^2
 - b^2) + x/(a - b)
```

3.781.9 Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.87

$$\int \frac{1}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = \frac{x}{a-b} - \frac{\ln\left(\frac{2\,(b+c\,e^x)}{(a+b)^2}+\frac{2\,(b-a+c\,e^x)\left(a^2\,b+b\,c^2-b^3+a\,c\,\sqrt{a^2-b^2+c^2}\right)}{(a+b)\,(a^2-b^2)\,(a^2-b^2+c^2)}\right)\left(a^2\,b+b\,c^2-b^3+a\,c\,\sqrt{a^2-b^2+c^2}\right)}{a^4-2\,a^2\,b^2+a^2\,c^2+b^4-b^2\,c^2} - \frac{\ln\left(\frac{2\,(b+c\,e^x)}{(a+b)^2}+\frac{2\,(b-a+c\,e^x)\left(a^2\,b+b\,c^2-b^3-a\,c\,\sqrt{a^2-b^2+c^2}\right)}{(a+b)\,(a^2-b^2)\,(a^2-b^2+c^2)}\right)\left(a^2\,b+b\,c^2-b^3-a\,c\,\sqrt{a^2-b^2+c^2}\right)}{a^4-2\,a^2\,b^2+a^2\,c^2+b^4-b^2\,c^2}$$

input
```
int(1/(a + c/sinh(x) + b*coth(x)),x)
```

output
```
x/(a - b) - (log((2*(b + c*exp(x)))/(a + b)^2 + (2*(b - a + c*exp(x))*(a^2
*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/((a + b)*(a^2 - b^2)*(a^2
 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/(a^4
+ b^4 - 2*a^2*b^2 + a^2*c^2 - b^2*c^2) - (log((2*(b + c*exp(x)))/(a + b)^2
 + (2*(b - a + c*exp(x))*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b^2 + c^2)^(1/2
)))/((a + b)*(a^2 - b^2)*(a^2 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3 - a*c*(a
^2 - b^2 + c^2)^(1/2)))/(a^4 + b^4 - 2*a^2*b^2 + a^2*c^2 - b^2*c^2)
```

3.781. $\int \frac{1}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$

3.782 $\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

3.782.1 Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{cx}{b^2-c^2} - \frac{2a\text{carctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} + \frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

output
```
-c*x/(b^2-c^2)+b*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)-2*a*c*arctanh((c-(a-b
)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-b^2+c^2)^(1/2)
```

3.782.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{-cx+\frac{2ac\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

input
```
Integrate[Sinh[x]/(a + b*Cosh[x] + c*Sinh[x]),x]
```

output
```
(-(c*x) + (2*a*c*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/
Sqrt[-a^2 + b^2 - c^2] + b*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

3.782.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 3616, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{a+b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{a+b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{3616}$$

$$-i\left(\frac{iac\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{b^2-c^2}+\frac{ib\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{icx}{b^2-c^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{iac\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{b^2-c^2}+\frac{ib\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{icx}{b^2-c^2}\right)$$

$$\Big\downarrow \text{3603}$$

$$-i\left(\frac{2iac\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+2c\tanh\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b^2-c^2}+\frac{ib\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{icx}{b^2-c^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$-i\left(-\frac{4iac\int \frac{1}{4(a^2-b^2+c^2)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2}+\frac{ib\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{icx}{b^2-c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(-\frac{2iac\text{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}+\frac{ib\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{icx}{b^2-c^2}\right)$$

input
```
Int[Sinh[x]/(a + b*Cosh[x] + c*Sinh[x]),x]
```

output
```
(-I)*(((-I)*c*x)/(b^2 - c^2) - ((2*I)*a*c*ArcTanh[(2*c - 2*(a - b)*Tanh[x/
2])/(2*Sqrt[a^2 - b^2 + c^2])])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) + (I*b
*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2))
```

3.782.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.782. $\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

rule 3616
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (-Simp[b*C*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

3.782.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.74

method	result
default	$-\frac{4\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{4b-4c}-\frac{4\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4b+4c}+\frac{\frac{2\left(ab-b^{2}\right)\ln\left(a\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{2}b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a-2b}+\frac{2\left(-ac-bc+\frac{\left(ab-b^{2}\right)c}{a-b}\right)\operatorname{arctan}}{\sqrt{-a^{2}+b^{2}}}}{(b-c)(b+c)}$
risch	$\frac{x}{b+c}+\frac{2x\,a^{2}b}{-a^{2}b^{2}+a^{2}c^{2}+b^{4}-2b^{2}c^{2}+c^{4}}-\frac{2x\,b^{3}}{-a^{2}b^{2}+a^{2}c^{2}+b^{4}-2b^{2}c^{2}+c^{4}}+\frac{2xb\,c^{2}}{-a^{2}b^{2}+a^{2}c^{2}+b^{4}-2b^{2}c^{2}+c^{4}}+\frac{\ln\left(\mathrm{e}^{x}-\frac{-a^{2}c+\sqrt{a^{4}c^{2}-a^{2}}}{(b+c)ac}\right)}{a^{2}b^{2}-a^{2}c^{2}-b^{4}+2}$

input
```
int(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-4/(4*b-4*c)*ln(1+tanh(1/2*x))-4/(4*b+4*c)*ln(tanh(1/2*x)-1)+2/(b-c)/(b+c)
*(1/2*(a*b-b^2)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a
-b)+(-a*c-b*c+(a*b-b^2)*c/(a-b))/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*
tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2)))
```

3.782.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 455, normalized size of antiderivative = 4.38

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$$
$$=\left[-\frac{\sqrt{a^2-b^2+c^2}ac\log\left(\frac{(b^2+2\,bc+c^2)\cosh(x)^2+(b^2+2\,bc+c^2)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,(ab+ac+(b^2+2\,bc+c^2)\,\mathrm{co}}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b+c)\cosh(x)-}\right.}{}\right.$$

input
```
integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

output
```
[-(sqrt(a^2 - b^2 + c^2)*a*c*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2
*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b
 + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((
b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x
)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + (a^2*b - b
^3 + b*c^2 + c^3 + (a^2 - b^2)*c)*x - (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(
x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*
b^2)*c^2), (2*sqrt(-a^2 + b^2 - c^2)*a*c*arctan(sqrt(-a^2 + b^2 - c^2)*((b
 + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (a^2*b - b^3 + b
*c^2 + c^3 + (a^2 - b^2)*c)*x + (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + c
*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c
^2)]
```

3.782.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\, dx = \text{Timed out}$$

input
```
integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x)
```

output
```
Timed out
```

3.782.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.782.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{2\,ac\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}(b^2-c^2)} + \frac{b\log\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c\right)}{b^2-c^2} - \frac{x}{b-c}$$

input
```
integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

output
```
2*a*c*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/(sqrt(-a^2 + b^2
- c^2)*(b^2 - c^2)) + b*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)/(b^2
- c^2) - x/(b - c)
```

3.782.9 Mupad [B] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.12

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{x}{b-c} - \frac{\ln\left(\frac{2\,(b+a\,e^x)}{(b+c)^2}-\frac{2\,(b-c+a\,e^x)\,\left(a^2\,b+b\,c^2-b^3+a\,c\,\sqrt{a^2-b^2+c^2}\right)}{(b+c)\,(b^2-c^2)\,(a^2-b^2+c^2)}\right)\,\left(a^2\,b+b\,c^2-b^3+a\,c\,\sqrt{a^2-b^2+c^2}\right)}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4} - \frac{\ln\left(\frac{2\,(b+a\,e^x)}{(b+c)^2}-\frac{2\,(b-c+a\,e^x)\,\left(a^2\,b+b\,c^2-b^3-a\,c\,\sqrt{a^2-b^2+c^2}\right)}{(b+c)\,(b^2-c^2)\,(a^2-b^2+c^2)}\right)\,\left(a^2\,b+b\,c^2-b^3-a\,c\,\sqrt{a^2-b^2+c^2}\right)}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4}$$

input
```
int(sinh(x)/(a + b*cosh(x) + c*sinh(x)),x)
```

output
```
- x/(b - c) - (log((2*(b + a*exp(x)))/(b + c)^2 - (2*(b - c + a*exp(x))*(a
^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/((b + c)*(b^2 - c^2)*(a
^2 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^
4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) - (log((2*(b + a*exp(x)))/(b + c)
^2 - (2*(b - c + a*exp(x))*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b^2 + c^2)^(1
/2)))/((b + c)*(b^2 - c^2)*(a^2 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3 - a*c*
(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2)
```

3.783 $\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx$

3.783.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{x}{2}+\frac{\cosh(x)}{2}-\frac{\sinh(x)}{2}$$

output
```
1/2*x+1/2*cosh(x)-1/2*sinh(x)
```

3.783.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{x}{2}+\frac{\cosh(x)}{2}-\frac{\sinh(x)}{2}$$

input
```
Integrate[Sinh[x]/(1 + Cosh[x] + Sinh[x]),x]
```

output
```
x/2 + Cosh[x]/2 - Sinh[x]/2
```

3.783.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 26, 3610}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{\sinh(x)+\cosh(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{-i\sin(ix)+\cos(ix)+1}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\cos(ix)-i\sin(ix)+1}dx$$

$$\Big\downarrow \text{3610}$$

$$-i\left(\frac{ix}{2}-\frac{1}{2}i\sinh(x)+\frac{1}{2}i\cosh(x)\right)$$

input
```
Int[Sinh[x]/(1 + Cosh[x] + Sinh[x]),x]
```

output
```
(-I)*((I/2)*x + (I/2)*Cosh[x] - (I/2)*Sinh[x])
```

3.783.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3610
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - c*C)*(x
/(2*a^2)), x] + (-Simp[C*(Cos[d + e*x]/(2*a*e)), x] + Simp[c*C*(Sin[d + e*x
]/(2*a*b*e)), x] + Simp[((-a^2)*C + 2*a*c*A + b^2*C)*(Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*b*e)), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

3.783.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{x}{2}+\frac{{\mathrm e}^{-x}}{2}$	11
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{1}{1+\tanh\left(\frac{x}{2}\right)}+\frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2}$	28

input
```
int(sinh(x)/(1+cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*exp(-x)
```

3.783.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{x\cosh\left(x\right)+x\sinh\left(x\right)+1}{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

input
```
integrate(sinh(x)/(1+cosh(x)+sinh(x)),x, algorithm="fricas")
```

output
```
1/2*(x*cosh(x) + x*sinh(x) + 1)/(cosh(x) + sinh(x))
```

3.783.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(12) = 24$.

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{x\tanh\left(\frac{x}{2}\right)}{2\tanh\left(\frac{x}{2}\right)+2} + \frac{x}{2\tanh\left(\frac{x}{2}\right)+2} + \frac{2}{2\tanh\left(\frac{x}{2}\right)+2}$$

input
```
integrate(sinh(x)/(1+cosh(x)+sinh(x)),x)
```

output
```
x*tanh(x/2)/(2*tanh(x/2) + 2) + x/(2*tanh(x/2) + 2) + 2/(2*tanh(x/2) + 2)
```

3.783.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{1}{2}\,x + \frac{1}{2}\,e^{(-x)}$$

input
```
integrate(sinh(x)/(1+cosh(x)+sinh(x)),x, algorithm="maxima")
```

output
```
1/2*x + 1/2*e^(-x)
```

3.783.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{1}{2}\,x + \frac{1}{2}\,e^{(-x)}$$

input
```
integrate(sinh(x)/(1+cosh(x)+sinh(x)),x, algorithm="giac")
```

output
```
1/2*x + 1/2*e^(-x)
```

3.783.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{x}{2}+\frac{e^{-x}}{2}$$

input
```
int(sinh(x)/(cosh(x) + sinh(x) + 1),x)
```

output
```
x/2 + exp(-x)/2
```

3.784 $\int \frac{\operatorname{sech}(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx$

3.784.1 Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{\operatorname{sech}(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx=\frac{2\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

output
```
2*arctan((b+(a-c)*tanh(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^(1/2)
```

3.784.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx=\frac{2\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

input
```
Integrate[Sech[x]/(a + c*Sech[x] + b*Tanh[x]),x]
```

output
```
(2*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/Sqrt[a^2 - b^2 -
 c^2]
```

3.784.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3644, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{a+b\tanh(x)+c\text{sech}(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(ix)}{a-ib\tan(ix)+c\sec(ix)}dx$$
$$\Big\downarrow \text{ 3644}$$
$$\int \frac{1}{a\cosh(x)+b\sinh(x)+c}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{a\cos(ix)-ib\sin(ix)+c}dx$$
$$\Big\downarrow \text{ 3603}$$
$$2\int \frac{1}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{ 1083}$$
$$-4\int \frac{1}{-\left(2b+2(a-c)\tanh\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2-c^2\right)}d\left(2b+2(a-c)\tanh\left(\frac{x}{2}\right)\right)$$
$$\Big\downarrow \text{ 217}$$
$$\frac{2\arctan\left(\frac{2(a-c)\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

input
```
Int[Sech[x]/(a + c*Sech[x] + b*Tanh[x]),x]
```

output
```
(2*ArcTan[(2*b + 2*(a - c)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 - c^2])])/Sqrt[a^2
 - b^2 - c^2]
```

3.784. $\int \frac{\text{sech}(x)}{a+c\text{sech}(x)+b\tanh(x)}dx$

3.784.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3644
```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.784.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{2\arctan\left(\frac{2(a-c)\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$	53
risch	$-\frac{\ln\left(\mathrm{e}^{x}+\frac{c\sqrt{-a^2+b^2+c^2}-a^2+b^2+c^2}{(a+b)\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{c\sqrt{-a^2+b^2+c^2}+a^2-b^2-c^2}{(a+b)\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}$	139

input
```
int(sech(x)/(a+c*sech(x)+b*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(
1/2))
```

3.784.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 4.33

$$\int \frac{\operatorname{sech}(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2+c^2}\log\left(\frac{2\,(a+b)c\cosh(x)+(a^2+2\,ab+b^2)\cosh(x)^2+(a^2+2\,ab+b^2)\sinh(x)^2-a^2+b^2+2\,c^2+2\,((a+b)c+(a^2+2\,ab+b^2)\cosh(x))\sinh(x)-2\sqrt{-a^2+b^2+c^2}((a+b)\cosh(x)+(a+b)\sinh(x)+c)}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh(x)+a-b}\right)}{a^2-b^2-c^2}, -\frac{2\,\arctan\left(-\frac{(a+b)\cosh(x)+(a+b)\sinh(x)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}\right]$$

input
```
integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

output
```
[-sqrt(-a^2 + b^2 + c^2)*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)*co
sh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 - a^2 + b^2 + 2*c^2 + 2*((a + b)*c
 + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) - 2*sqrt(-a^2 + b^2 + c^2)*((a + b
)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) + a - b))/(a^2 - b^2 - c^2)
, -2*arctan(-((a + b)*cosh(x) + (a + b)*sinh(x) + c)/sqrt(a^2 - b^2 - c^2)
)/sqrt(a^2 - b^2 - c^2)]
```

3.784.6 Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = \int \frac{\operatorname{sech}(x)}{a+b\tanh(x)+c\operatorname{sech}(x)}\,dx$$

input
```
integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x)
```

output
```
Integral(sech(x)/(a + b*tanh(x) + c*sech(x)), x)
```

3.784.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(x)}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.784.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{sech}(x)}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx = \frac{2 \arctan\left(\frac{ae^x + be^x + c}{\sqrt{a^2 - b^2 - c^2}}\right)}{\sqrt{a^2 - b^2 - c^2}}$$

input
```
integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x, algorithm="giac")
```

output
```
2*arctan((a*e^x + b*e^x + c)/sqrt(a^2 - b^2 - c^2))/sqrt(a^2 - b^2 - c^2)
```

3.784.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{\operatorname{sech}(x)}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{c}{\sqrt{a^2-b^2-c^2}} + \frac{a\,e^x}{\sqrt{a^2-b^2-c^2}} + \frac{b\,e^x}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2 - b^2 - c^2}}$$

input
```
int(1/(cosh(x)*(a + b*tanh(x) + c/cosh(x))),x)
```

output
```
(2*atan(c/(a^2 - b^2 - c^2)^(1/2) + (a*exp(x))/(a^2 - b^2 - c^2)^(1/2) + (
b*exp(x))/(a^2 - b^2 - c^2)^(1/2)))/(a^2 - b^2 - c^2)^(1/2)
```

3.785 $\int \frac{\text{sech}^2(x)}{a+c\,\text{sech}(x)+b\tanh(x)}\,dx$

3.785.1 Optimal result

Integrand size = 17, antiderivative size = 146

$$\int \frac{\text{sech}^2(x)}{a+c\,\text{sech}(x)+b\tanh(x)}\,dx=\frac{2c\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2}-\frac{2ac\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}-\frac{b\log\left(1+\tanh^2\left(\frac{x}{2}\right)\right)}{b^2+c^2}+\frac{b\log\left(a+c+2b\tanh\left(\frac{x}{2}\right)+(a-c)\tanh^2\left(\frac{x}{2}\right)\right)}{b^2+c^2}$$

output
```
2*c*arctan(tanh(1/2*x))/(b^2+c^2)-b*ln(1+tanh(1/2*x)^2)/(b^2+c^2)+b*ln(a+c
+2*b*tanh(1/2*x)+(a-c)*tanh(1/2*x)^2)/(b^2+c^2)-2*a*c*arctan((b+(a-c)*tanh
(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)
```

3.785.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.66

$$\int \frac{\text{sech}^2(x)}{a+c\,\text{sech}(x)+b\tanh(x)}\,dx=\frac{2c\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2ac\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}+b(-\log(\cosh(x))+\log(c+a\cosh(x)+b\sinh(x)))}{b^2+c^2}$$

input
```
Integrate[Sech[x]^2/(a + c*Sech[x] + b*Tanh[x]),x]
```

output
```
(2*c*ArcTan[Tanh[x/2]] - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 -
b^2 - c^2]])/Sqrt[a^2 - b^2 - c^2] + b*(-Log[Cosh[x]] + Log[c + a*Cosh[x]
+ b*Sinh[x]]))/(b^2 + c^2)
```

3.785.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {3042, 4897, 3042, 4902, 2142, 27, 452, 216, 240, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{a+b\tanh(x)+c\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ix)^2}{a-ib\tan(ix)+c\sec(ix)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\text{sech}(x)}{a\cosh(x)+b\sinh(x)+c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ix)}{a\cos(ix)-ib\sin(ix)+c}dx$$

$$\Big\downarrow \text{4902}$$

$$2\int \frac{1-\tanh^2\left(\frac{x}{2}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)\left((a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c\right)}d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{2142}$$

$$2\left(\frac{\int \frac{4\left(b^2+(a-c)\tanh\left(\frac{x}{2}\right)b-ac\right)}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)}{4\left(b^2+c^2\right)}+\frac{\int \frac{4\left(c-b\tanh\left(\frac{x}{2}\right)\right)}{\tanh^2\left(\frac{x}{2}\right)+1}d\tanh\left(\frac{x}{2}\right)}{4\left(b^2+c^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\int \frac{b^2+(a-c)\tanh\left(\frac{x}{2}\right)b-ac}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)}{b^2+c^2}+\frac{\int \frac{c-b\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}d\tanh\left(\frac{x}{2}\right)}{b^2+c^2}\right)$$

$$\Big\downarrow \textbf{452}$$

$$2\left(\frac{\int \frac{b^2+(a-c)\tanh\left(\frac{x}{2}\right)b-ac}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)}{b^2+c^2}+\frac{c\int \frac{1}{\tanh^2\left(\frac{x}{2}\right)+1}d\tanh\left(\frac{x}{2}\right)-b\int \frac{\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}d\tanh\left(\frac{x}{2}\right)}{b^2+c^2}\right)$$

$$\Big\downarrow \textbf{216}$$

$$2\left(\frac{\int \frac{b^2+(a-c)\tanh\left(\frac{x}{2}\right)b-ac}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)}{b^2+c^2}+\frac{c\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-b\int \frac{\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}d\tanh\left(\frac{x}{2}\right)}{b^2+c^2}\right)$$

$$\Big\downarrow \textbf{240}$$

$$2\left(\frac{\int \frac{b^2+(a-c)\tanh\left(\frac{x}{2}\right)b-ac}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)}{b^2+c^2}+\frac{c\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{2}b\log\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{b^2+c^2}\right)$$

$$\Big\downarrow \textbf{1142}$$

$$2\left(\frac{\frac{1}{2}b\int \frac{2\left(b+(a-c)\tanh\left(\frac{x}{2}\right)\right)}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)-ac\int \frac{1}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)}{b^2+c^2}+\frac{c\arctan\left(\tanh\left(\frac{x}{2}\right)\right.}{}\right.$$

$$\Big\downarrow \textbf{27}$$

$$2\left(\frac{b\int \frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)-ac\int \frac{1}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)}{b^2+c^2}+\frac{c\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b}\right.$$

$$\Big\downarrow \textbf{1083}$$

$$2\left(\frac{2ac\int \frac{1}{-\left(2b+2(a-c)\tanh\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2-c^2\right)}d\left(2b+2(a-c)\tanh\left(\frac{x}{2}\right)\right)+b\int \frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)}{b^2+c^2}+\right.$$

$$\Big\downarrow \textbf{217}$$

3.785. $\int \frac{\operatorname{sech}^2(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx$

$$2\left(\frac{b\int \frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{(a-c)\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a+c}d\tanh\left(\frac{x}{2}\right)-\frac{ac\arctan\left(\frac{2(a-c)\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}}{b^2+c^2}+\frac{c\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{2}b\log\left(\tanh^2\left(\right.\right.}{b^2+c^2}\right.$$

$\downarrow$ 1103

$$2\left(\frac{\frac{1}{2}b\log\left((a-c)\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)+c\right)-\frac{ac\arctan\left(\frac{2(a-c)\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}}{b^2+c^2}+\frac{c\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{2}b\log\left(\mathrm{ta}\right.}{b^2+c^2}\right.$$

input
```
Int[Sech[x]^2/(a + c*Sech[x] + b*Tanh[x]),x]
```

output
```
2*((c*ArcTan[Tanh[x/2]] - (b*Log[1 + Tanh[x/2]^2])/2)/(b^2 + c^2) + (-((a*
c*ArcTan[(2*b + 2*(a - c)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 - c^2])])/Sqrt[a^2
- b^2 - c^2]) + (b*Log[a + c + 2*b*Tanh[x/2] + (a - c)*Tanh[x/2]^2])/2)/(b
^2 + c^2))
```

3.785.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2142
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Sym
bol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2]
, q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a
*c*C*d + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f -
 a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 + b*B*d*f -
A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x
^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f}, x] && PolyQ[Px, x, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.785. $\int \frac{\operatorname{sech}^2(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx$

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

3.785.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.14

method	result
default	$\frac{\frac{2(ab-bc)\ln\left(a\tanh\left(\frac{x}{2}\right)^2-c\tanh\left(\frac{x}{2}\right)^2+2b\tanh\left(\frac{x}{2}\right)+a+c\right)}{2a-2c}+\frac{2\left(-ac+b^2-\frac{(ab-bc)b}{a-c}\right)\arctan\left(\frac{2(a-c)\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}}{b^2+c^2}+\frac{-b\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)\cdots}{b^2\cdots}$
risch	$\frac{2xb}{b^2+c^2}-\frac{2x\,a^2b}{a^2b^2+a^2c^2-b^4-2b^2c^2-c^4}+\frac{2x\,b^3}{a^2b^2+a^2c^2-b^4-2b^2c^2-c^4}+\frac{2xb\,c^2}{a^2b^2+a^2c^2-b^4-2b^2c^2-c^4}-\frac{i\ln(e^x-i)c}{b^2+c^2}-\frac{\ln(e^x+i)b}{b^2+c^2}+\cdots$

input
```
int(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(b^2+c^2)*(1/2*(a*b-b*c)/(a-c)*ln(a*tanh(1/2*x)^2-c*tanh(1/2*x)^2+2*b*ta
nh(1/2*x)+a+c)+(-a*c+b^2-(a*b-b*c)*b/(a-c))/(a^2-b^2-c^2)^(1/2)*arctan(1/2
*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(1/2)))+2/(b^2+c^2)*(-1/2*b*ln(1+
tanh(1/2*x)^2)+c*arctan(tanh(1/2*x)))
```

3.785.5 Fricas [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.33

$$\int \frac{\operatorname{sech}^2(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2+c^2}ac\log\left(\frac{2\,(a+b)c\cosh(x)+(a^2+2\,ab+b^2)\cosh(x)^2+(a^2+2\,ab+b^2)\sinh(x)^2-a^2+b^2+2\,c^2+2\,((a+b)c+(a^2+2\,ab+b^2)\cdots}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh\cdots}\right.}{\cdots}\right.$$

input
```
integrate(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

output
```
[-(sqrt(-a^2 + b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^
2)*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 - a^2 + b^2 + 2*c^2 + 2*((a +
 b)*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) + 2*sqrt(-a^2 + b^2 + c^2)*((
a + b)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x
)^2 + 2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) + a - b)) + 2*(c^3 - (
a^2 - b^2)*c)*arctan(cosh(x) + sinh(x)) - (a^2*b - b^3 - b*c^2)*log(2*(a*c
osh(x) + b*sinh(x) + c)/(cosh(x) - sinh(x))) + (a^2*b - b^3 - b*c^2)*log(2
*cosh(x)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2),
(2*sqrt(a^2 - b^2 - c^2)*a*c*arctan(-((a + b)*cosh(x) + (a + b)*sinh(x) +
c)/sqrt(a^2 - b^2 - c^2)) - 2*(c^3 - (a^2 - b^2)*c)*arctan(cosh(x) + sinh(
x)) + (a^2*b - b^3 - b*c^2)*log(2*(a*cosh(x) + b*sinh(x) + c)/(cosh(x) - s
inh(x))) - (a^2*b - b^3 - b*c^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2*
b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

3.785.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{a + b\tanh(x) + c\operatorname{sech}(x)}\,dx$$

input
```
integrate(sech(x)**2/(a+c*sech(x)+b*tanh(x)),x)
```

output
```
Integral(sech(x)**2/(a + b*tanh(x) + c*sech(x)), x)
```

3.785.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^2(x)}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.785.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}^2(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = -\frac{2\,ac\arctan\left(\frac{ae^x+be^x+c}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}(b^2+c^2)}+\frac{2\,c\arctan\left(e^x\right)}{b^2+c^2}+\frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^x+a-b\right)}{b^2+c^2}-\frac{b\log\left(e^{(2\,x)}+1\right)}{b^2+c^2}$$

input
```
integrate(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x, algorithm="giac")
```

output
```
-2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(a^2 - b^2 - c^2))/(sqrt(a^2 - b^2 -
 c^2)*(b^2 + c^2)) + 2*c*arctan(e^x)/(b^2 + c^2) + b*log(a*e^(2*x) + b*e^(
2*x) + 2*c*e^x + a - b)/(b^2 + c^2) - b*log(e^(2*x) + 1)/(b^2 + c^2)
```

3.785.9 Mupad [B] (verification not implemented)

Time = 30.90 (sec) , antiderivative size = 1069, normalized size of antiderivative = 7.32

$$\int \frac{\operatorname{sech}^2(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx$$

$$= \ln\left(\frac{64\,(a-b+2\,c\,\mathrm{e}^x)}{(a+b)^4}+\frac{\left(\frac{32\,\left(2\,a^3+3\,\mathrm{e}^x\,a^2\,c-2\,a\,b^2+6\,\mathrm{e}^x\,a\,b\,c-2\,a\,c^2+3\,\mathrm{e}^x\,b^2\,c+2\,b\,c^2-4\,\mathrm{e}^x\,c^3\right)}{(a+b)^5}+\left(\frac{32\,(a-b)\,\left(-2\,b^3+6\,\mathrm{e}^x\,b^2\,c-2\,a\,b^2+b\,c^2+6\,a\,\mathrm{e}^x\,b\,c+3\,\mathrm{e}^x \cdots\right.}{(a+b)^5}\cdots\right.}{\cdots}\right.$$

$$-\ln\left(\frac{64\,(a-b+2\,c\,\mathrm{e}^x)}{(a+b)^4}-\frac{\left(\frac{32\,\left(2\,a^3+3\,\mathrm{e}^x\,a^2\,c-2\,a\,b^2+6\,\mathrm{e}^x\,a\,b\,c-2\,a\,c^2+3\,\mathrm{e}^x\,b^2\,c+2\,b\,c^2-4\,\mathrm{e}^x\,c^3\right)}{(a+b)^5}-\left(\frac{32\,(a-b)\,\left(-2\,b^3+6\,\mathrm{e}^x\,b^2\,c-2\,a\,b^2+b\,c^2+6\,a\,\mathrm{e}^x\,b\,c+\cdots\right.}{(a+b)^5}\cdots\right.}{\cdots}\right.$$

$$-\frac{\ln\left(1+\mathrm{e}^x\,1\mathrm{i}\right)}{b-c\,1\mathrm{i}}-\frac{\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{-c+b\,1\mathrm{i}}$$

input
```
int(1/(cosh(x)^2*(a + b*tanh(x) + c/cosh(x))),x)
```

output
```
(log((64*(a - b + 2*c*exp(x)))/(a + b)^4 + (((32*(2*b*c^2 - 2*a*c^2 - 2*a*
b^2 + 2*a^3 - 4*c^3*exp(x) + 3*a^2*c*exp(x) + 3*b^2*c*exp(x) + 6*a*b*c*exp
(x)))/(a + b)^5 + (((32*(a - b)*(2*a*c^2 - 2*a*b^2 + b*c^2 - 2*b^3 + 3*c^3
*exp(x) + 6*b^2*c*exp(x) + 6*a*b*c*exp(x)))/(a + b)^5 - (32*(b*c^2 - a^2*b
 + b^3 + a*c*(b^2 - a^2 + c^2)^(1/2))*(3*a*c^4 - 2*a*b^4 - 3*b*c^4 + 2*a^3
*b^2 - 2*a^3*c^2 - 3*b^3*c^2 + 4*c^5*exp(x) + a*b^2*c^2 + 4*a^2*b*c^2 + b^
4*c*exp(x) - 3*a^2*c^3*exp(x) + 5*b^2*c^3*exp(x) + a^2*b^2*c*exp(x) + 6*a*
b*c^3*exp(x) + 6*a*b^3*c*exp(x) - 4*a^3*b*c*exp(x)))/((a + b)^5*(b^2 + c^2
)*(b^2 - a^2 + c^2)))*(b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 + c^2)^(1/2)))
/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 +
c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(b*c^2 - a^2*b + b^3 + a*c*(
b^2 - a^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 - a^2*c^2 + 2*b^2*c^2) - (lo
g(exp(x) + 1i)*1i)/(b*1i - c) - (log((64*(a - b + 2*c*exp(x)))/(a + b)^4 -
 (((32*(2*b*c^2 - 2*a*c^2 - 2*a*b^2 + 2*a^3 - 4*c^3*exp(x) + 3*a^2*c*exp(x
) + 3*b^2*c*exp(x) + 6*a*b*c*exp(x)))/(a + b)^5 - (((32*(a - b)*(2*a*c^2 -
 2*a*b^2 + b*c^2 - 2*b^3 + 3*c^3*exp(x) + 6*b^2*c*exp(x) + 6*a*b*c*exp(x))
)/(a + b)^5 + (32*(a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c^2)^(1/2))*(3*a
*c^4 - 2*a*b^4 - 3*b*c^4 + 2*a^3*b^2 - 2*a^3*c^2 - 3*b^3*c^2 + 4*c^5*exp(x
) + a*b^2*c^2 + 4*a^2*b*c^2 + b^4*c*exp(x) - 3*a^2*c^3*exp(x) + 5*b^2*c^3*
exp(x) + a^2*b^2*c*exp(x) + 6*a*b*c^3*exp(x) + 6*a*b^3*c*exp(x) - 4*a^3...
```

3.785. $\int \frac{\operatorname{sech}^2(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx$

3.786 $\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx$

3.786.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx = -\frac{2}{3}\operatorname{arctanh}\left(\frac{1}{3}\left(2-\tanh\left(\frac{x}{2}\right)\right)\right)$$

output
```
2/3*arctanh(-2/3+1/3*tanh(1/2*x))
```

3.786.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx = \frac{x}{6}-\frac{1}{3}\log\left(5\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Csch[x]/(2 + 2*Coth[x] + 3*Csch[x]),x]
```

output
```
x/6 - Log[5*Cosh[x/2] - Sinh[x/2]]/3
```

3.786.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 3645, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(x)}{2\coth(x)+3\operatorname{csch}(x)+2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\csc(ix)}{2i\cot(ix)+3i\csc(ix)+2}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\csc(ix)}{2i\cot(ix)+3i\csc(ix)+2}dx$$
$$\Big\downarrow \text{3645}$$
$$i\int \frac{1}{2i\cosh(x)+2i\sinh(x)+3i}dx$$
$$\Big\downarrow \text{3042}$$
$$i\int \frac{1}{2i\cos(ix)+2\sin(ix)+3i}dx$$
$$\Big\downarrow \text{3603}$$
$$2i\int \frac{1}{-i\tanh^2\left(\frac{x}{2}\right)+4i\tanh\left(\frac{x}{2}\right)+5i}d\tanh\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{1083}$$
$$-4i\int \frac{1}{-\left(4i-2i\tanh\left(\frac{x}{2}\right)\right)^2-36}d\left(4i-2i\tanh\left(\frac{x}{2}\right)\right)$$
$$\Big\downarrow \text{217}$$
$$\frac{2}{3}i\arctan\left(\frac{1}{6}\left(4i-2i\tanh\left(\frac{x}{2}\right)\right)\right)$$

input
```
Int[Csch[x]/(2 + 2*Coth[x] + 3*Csch[x]),x]
```

3.786. $\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}dx$

output
```
((2*I)/3)*ArcTan[(4*I - (2*I)*Tanh[x/2])/6]
```

3.786.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3645
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.786. $\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx$

3.786.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{x}{3}-\frac{\ln\left(\mathrm{e}^{x}+\frac{3}{2}\right)}{3}$	12
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-5\right)}{3}+\frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{3}$	20

input
```
int(csch(x)/(2+2*coth(x)+3*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x-1/3*ln(exp(x)+3/2)
```

3.786.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx = \frac{1}{3}\,x-\frac{1}{3}\,\log\left(2\,\cosh\left(x\right)+2\,\sinh\left(x\right)+3\right)$$

input
```
integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x, algorithm="fricas")
```

output
```
1/3*x - 1/3*log(2*cosh(x) + 2*sinh(x) + 3)
```

3.786.6 Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}\left(x\right)}{2\coth\left(x\right)+3\operatorname{csch}\left(x\right)+2}\,dx$$

input
```
integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x)
```

output
```
Integral(csch(x)/(2*coth(x) + 3*csch(x) + 2), x)
```

3.786.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx=-\frac{1}{3}\,\log\left(3\,e^{(-x)}+2\right)$$

input
```
integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x, algorithm="maxima")
```

output
```
-1/3*log(3*e^(-x) + 2)
```

3.786.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx=\frac{1}{3}\,x-\frac{1}{3}\,\log\left(2\,e^{x}+3\right)$$

input
```
integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x, algorithm="giac")
```

output
```
1/3*x - 1/3*log(2*e^x + 3)
```

3.786.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx=\frac{x}{3}-\frac{\ln\left(\mathrm{e}^{x}+\frac{3}{2}\right)}{3}$$

input
```
int(1/(sinh(x)*(2*coth(x) + 3/sinh(x) + 2)),x)
```

output
```
x/3 - log(exp(x) + 3/2)/3
```

3.787 $\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$

3.787.1 Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

output
```
-2*arctanh((a+(b-c)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^2)^(1/2)
```

3.787.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=\frac{2\arctan\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

input
```
Integrate[Csch[x]/(a + b*Coth[x] + c*Csch[x]),x]
```

output
```
(2*ArcTan[(a + (b - c)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 + b^2
 - c^2]
```

3.787.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 3645, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\csc(ix)}{a+ib\cot(ix)+ic\csc(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\csc(ix)}{a+ib\cot(ix)+ic\csc(ix)}dx$$

$$\Big\downarrow \text{3645}$$

$$i\int \frac{1}{ic+ib\cosh(x)+ia\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{1}{ic+ib\cos(ix)+a\sin(ix)}dx$$

$$\Big\downarrow \text{3603}$$

$$2i\int \frac{1}{i(b-c)\tanh^2\left(\frac{x}{2}\right)+2ia\tanh\left(\frac{x}{2}\right)+i(b+c)}d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{1083}$$

$$-4i\int \frac{1}{-\left(2ia+2i(b-c)\tanh\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2+c^2\right)}d\left(2ia+2i(b-c)\tanh\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{2i\arctan\left(\frac{2ia+2i(b-c)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

input
```
Int[Csch[x]/(a + b*Coth[x] + c*Csch[x]),x]
```

output
```
((2*I)*ArcTan[((2*I)*a + (2*I)*(b - c)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^2]
)])/Sqrt[a^2 - b^2 + c^2]
```

3.787.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3645
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.787. $\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$

3.787.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{2\arctan\left(\frac{2(b-c)\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$	53
risch	$\frac{\ln\left(e^x+\frac{c\sqrt{a^2-b^2+c^2}-a^2+b^2-c^2}{(a+b)\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}-\frac{\ln\left(e^x+\frac{c\sqrt{a^2-b^2+c^2}+a^2-b^2+c^2}{(a+b)\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$	139

input
```
int(csch(x)/(a+b*coth(x)+c*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2+b^2-c^2)
^(1/2))
```

3.787.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 244, normalized size of antiderivative = 4.88

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = \left[\frac{\log\left(\frac{2\,(a+b)c\cosh(x)+(a^2+2\,ab+b^2)\cosh(x)^2+(a^2+2\,ab+b^2)\sinh(x)^2+a^2-b^2+2\,c^2+2\,((a+b)c+(a^2+2\,ab+b^2)\cosh(x))\sinh(x)-2\,\sqrt{a^2-b}}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh(x)-a+b}\right.}{\sqrt{a^2-b^2+c^2}}\right.$$

input
```
integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x, algorithm="fricas")
```

output
```
[log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)*cosh(x)^2 + (a^2 + 2*a*b +
 b^2)*sinh(x)^2 + a^2 - b^2 + 2*c^2 + 2*((a + b)*c + (a^2 + 2*a*b + b^2)*c
osh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((a + b)*cosh(x) + (a + b)*sinh(
x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + 2*c*cosh(x) + 2*((a + b)
*cosh(x) + c)*sinh(x) - a + b))/sqrt(a^2 - b^2 + c^2), 2*sqrt(-a^2 + b^2 -
 c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((a + b)*cosh(x) + (a + b)*sinh(x) + c
)/(a^2 - b^2 + c^2))/(a^2 - b^2 + c^2)]
```

3.787.6 Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a + b\coth(x) + c\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}(x)}{a + b\coth(x) + c\operatorname{csch}(x)}\,dx$$

input
```
integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x)
```

output
```
Integral(csch(x)/(a + b*coth(x) + c*csch(x)), x)
```

3.787.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}(x)}{a + b\coth(x) + c\operatorname{csch}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.787.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{csch}(x)}{a + b\coth(x) + c\operatorname{csch}(x)}\,dx = \frac{2\arctan\left(\frac{ae^x + be^x + c}{\sqrt{-a^2 + b^2 - c^2}}\right)}{\sqrt{-a^2 + b^2 - c^2}}$$

input
```
integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x, algorithm="giac")
```

output
```
2*arctan((a*e^x + b*e^x + c)/sqrt(-a^2 + b^2 - c^2))/sqrt(-a^2 + b^2 - c^2
)
```

3.787.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=\frac{2\operatorname{atan}\left(\frac{c}{\sqrt{-a^2+b^2-c^2}}+\frac{a\,\mathrm{e}^x}{\sqrt{-a^2+b^2-c^2}}+\frac{b\,\mathrm{e}^x}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

input
```
int(1/(sinh(x)*(a + c/sinh(x) + b*coth(x))),x)
```

output
```
(2*atan(c/(b^2 - a^2 - c^2)^(1/2) + (a*exp(x))/(b^2 - a^2 - c^2)^(1/2) + (
b*exp(x))/(b^2 - a^2 - c^2)^(1/2)))/(b^2 - a^2 - c^2)^(1/2)
```

3.788 $\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$

3.788.1 Optimal result

Integrand size = 17, antiderivative size = 118

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = -\frac{2ac\operatorname{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}+\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c} - \frac{b\log\left(b+c+2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)\right)}{b^2-c^2}$$

output
```
ln(tanh(1/2*x))/(b+c)-b*ln(b+c+2*a*tanh(1/2*x)+(b-c)*tanh(1/2*x)^2)/(b^2-c
^2)-2*a*c*arctanh((a+(b-c)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^
2-b^2+c^2)^(1/2)
```

3.788.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = \frac{\frac{2ac\arctan\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+(b+c)\log\left(\cosh\left(\frac{x}{2}\right)\right)+(b-c)\log\left(\sinh\left(\frac{x}{2}\right)\right)-b\log(c+b\cosh(x)+a\sinh(x))}{(b-c)(b+c)}$$

input
```
Integrate[Csch[x]^2/(a + b*Coth[x] + c*Csch[x]),x]
```

output
```
((2*a*c*ArcTan[(a + (b - c)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2
+ b^2 - c^2] + (b + c)*Log[Cosh[x/2]] + (b - c)*Log[Sinh[x/2]] - b*Log[c +
 b*Cosh[x] + a*Sinh[x]])/((b - c)*(b + c))
```

3.788.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 25, 4897, 26, 26, 3042, 26, 4902, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\csc(ix)^2}{a+ib\cot(ix)+ic\csc(ix)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\csc(ix)^2}{a+ib\cot(ix)+ic\csc(ix)}dx$$

$$\Big\downarrow \text{4897}$$

$$-\int -\frac{i\operatorname{csch}(x)}{ic+ib\cosh(x)+ia\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\operatorname{csch}(x)}{c+b\cosh(x)+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\operatorname{csch}(x)}{a\sinh(x)+b\cosh(x)+c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\csc(ix)}{-ia\sin(ix)+b\cos(ix)+c}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\csc(ix)}{c+b\cos(ix)-ia\sin(ix)}dx$$

$$\Big\downarrow \text{4902}$$

$$2i\int -\frac{i\coth\left(\frac{x}{2}\right)\left(1-\tanh^2\left(\frac{x}{2}\right)\right)}{2\left((b-c)\tanh^2\left(\frac{x}{2}\right)+2a\tanh\left(\frac{x}{2}\right)+b+c\right)}d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\left(1-\tanh^2\left(\frac{x}{2}\right)\right)\coth\left(\frac{x}{2}\right)}{2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)+b+c}d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{2\left(-a-b\tanh\left(\frac{x}{2}\right)\right)}{(b+c)\left(2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)+b+c\right)}+\frac{\coth\left(\frac{x}{2}\right)}{b+c}\right)d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2ac\,\text{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}-\frac{b\log\left(2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)+b+c\right)}{b^2-c^2}+\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}$$

input
```
Int[Csch[x]^2/(a + b*Coth[x] + c*Csch[x]),x]
```

output
```
(-2*a*c*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^
2)*Sqrt[a^2 - b^2 + c^2]) + Log[Tanh[x/2]]/(b + c) - (b*Log[b + c + 2*a*Ta
nh[x/2] + (b - c)*Tanh[x/2]^2])/(b^2 - c^2)
```

3.788.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.788. $\int \frac{\text{csch}^2(x)}{a+b\coth(x)+c\,\text{csch}(x)}\,dx$

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

3.788.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

method	result
default	$\frac{-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)^{2} b-c\tanh\left(\frac{x}{2}\right)^{2}+2a\tanh\left(\frac{x}{2}\right)+b+c\right)}{b-c}+\frac{\left(-2a+\frac{2ba}{b-c}\right)\arctan\left(\frac{2(b-c)\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\sqrt{-a^{2}+b^{2}-c^{2}}}}{b+c}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}$
risch	$-\frac{x}{b-c}-\frac{x}{b+c}+\frac{2xa^{2}b}{a^{2}b^{2}-a^{2}c^{2}-b^{4}+2b^{2}c^{2}-c^{4}}-\frac{2xb^{3}}{a^{2}b^{2}-a^{2}c^{2}-b^{4}+2b^{2}c^{2}-c^{4}}+\frac{2xbc^{2}}{a^{2}b^{2}-a^{2}c^{2}-b^{4}+2b^{2}c^{2}-c^{4}}+\frac{\ln(1+e^{x})}{b-c}+\frac{\ln(e^{x}}{b+}$

input
```
int(csch(x)^2/(a+b*coth(x)+c*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(b+c)*(-b/(b-c)*ln(tanh(1/2*x)^2*b-c*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b+c)+
(-2*a+2*b*a/(b-c))/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*
a)/(-a^2+b^2-c^2)^(1/2)))+ln(tanh(1/2*x))/(b+c)
```

3.788.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(106) = 212$.

Time = 0.90 (sec) , antiderivative size = 546, normalized size of antiderivative = 4.63

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$$
$$=\left[-\frac{\sqrt{a^2-b^2+c^2}ac\log\left(\frac{2\,(a+b)c\cosh(x)+(a^2+2\,ab+b^2)\cosh(x)^2+(a^2+2\,ab+b^2)\sinh(x)^2+a^2-b^2+2\,c^2+2\,((a+b)c+(a^2+2\,ab+b^2)\,\mathrm{co}}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)}\right.}{}\right.$$

input
```
integrate(csch(x)^2/(a+b*coth(x)+c*csch(x)),x, algorithm="fricas")
```

output
```
[-(sqrt(a^2 - b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2
)*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 2*c^2 + 2*((a +
b)*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((a
+ b)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^
2 + 2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) - a + b)) + (a^2*b - b^3
 + b*c^2)*log(2*(b*cosh(x) + a*sinh(x) + c)/(cosh(x) - sinh(x))) - (a^2*b
- b^3 + b*c^2 + c^3 + (a^2 - b^2)*c)*log(cosh(x) + sinh(x) + 1) - (a^2*b -
 b^3 + b*c^2 - c^3 - (a^2 - b^2)*c)*log(cosh(x) + sinh(x) - 1))/(a^2*b^2 -
 b^4 - c^4 - (a^2 - 2*b^2)*c^2), (2*sqrt(-a^2 + b^2 - c^2)*a*c*arctan(sqrt
(-a^2 + b^2 - c^2)*((a + b)*cosh(x) + (a + b)*sinh(x) + c)/(a^2 - b^2 + c^
2)) - (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + a*sinh(x) + c)/(cosh(x) - s
inh(x))) + (a^2*b - b^3 + b*c^2 + c^3 + (a^2 - b^2)*c)*log(cosh(x) + sinh(
x) + 1) + (a^2*b - b^3 + b*c^2 - c^3 - (a^2 - b^2)*c)*log(cosh(x) + sinh(x
) - 1))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

3.788.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$$

input
```
integrate(csch(x)**2/(a+b*coth(x)+c*csch(x)),x)
```

output
```
Integral(csch(x)**2/(a + b*coth(x) + c*csch(x)), x)
```

3.788.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^2(x)}{a + b\coth(x) + c\operatorname{csch}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(x)^2/(a+b*coth(x)+c*csch(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.788.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^2(x)}{a + b\coth(x) + c\operatorname{csch}(x)}\,dx = \frac{2\,ac\arctan\left(\frac{ae^x + be^x + c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}(b^2-c^2)} - \frac{b\log\left(ae^{(2\,x)} + be^{(2\,x)} + 2\,ce^x - a + b\right)}{b^2-c^2} + \frac{\log\left(e^x+1\right)}{b-c} + \frac{\log\left(|e^x-1|\right)}{b+c}$$

input
```
integrate(csch(x)^2/(a+b*coth(x)+c*csch(x)),x, algorithm="giac")
```

output
```
2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(-a^2 + b^2 - c^2))/(sqrt(-a^2 + b^2
- c^2)*(b^2 - c^2)) - b*log(a*e^(2*x) + b*e^(2*x) + 2*c*e^x - a + b)/(b^2
- c^2) + log(e^x + 1)/(b - c) + log(abs(e^x - 1))/(b + c)
```

3.788.9 Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 1069, normalized size of antiderivative = 9.06

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = \frac{\ln\left(e^x-1\right)}{b+c}+\frac{\ln\left(e^x+1\right)}{b-c}$$

$$+\frac{\ln\left(-\frac{64\,(b-a+2\,c\,e^x)}{(a+b)^4}-\frac{\left(\frac{32\,\left(-2\,a^3+3\,e^x\,a^2\,c+2\,a\,b^2+6\,e^x\,a\,b\,c-2\,a\,c^2+3\,e^x\,b^2\,c+2\,b\,c^2+4\,e^x\,c^3\right)}{(a+b)^5}+\frac{\left(\frac{32\,(a-b)\,\left(2\,b^3+6\,e^x\,b^2\,c+2\,a\,b^2+b\,c^2+6\,a\,e^x\,b\,\cdots\right.}{(a+b)^5}\cdots}{\cdots}\right)}{\cdots}$$

$$+\frac{\ln\left(-\frac{64\,(b-a+2\,c\,e^x)}{(a+b)^4}-\frac{\left(\frac{32\,\left(-2\,a^3+3\,e^x\,a^2\,c+2\,a\,b^2+6\,e^x\,a\,b\,c-2\,a\,c^2+3\,e^x\,b^2\,c+2\,b\,c^2+4\,e^x\,c^3\right)}{(a+b)^5}+\frac{\left(\frac{32\,(a-b)\,\left(2\,b^3+6\,e^x\,b^2\,c+2\,a\,b^2+b\,c^2+6\,a\,e^x\,b\,\cdots\right.}{(a+b)^5}\cdots}{\cdots}\right)}{\cdots}$$

input
```
int(1/(sinh(x)^2*(a + c/sinh(x) + b*coth(x))),x)
```

output
```
log(exp(x) - 1)/(b + c) + log(exp(x) + 1)/(b - c) + (log(- (64*(b - a + 2*
c*exp(x)))/(a + b)^4 - (((32*(2*a*b^2 - 2*a*c^2 + 2*b*c^2 - 2*a^3 + 4*c^3*
exp(x) + 3*a^2*c*exp(x) + 3*b^2*c*exp(x) + 6*a*b*c*exp(x)))/(a + b)^5 + ((
(32*(a - b)*(2*a*b^2 + 2*a*c^2 + b*c^2 + 2*b^3 - 3*c^3*exp(x) + 6*b^2*c*ex
p(x) + 6*a*b*c*exp(x)))/(a + b)^5 - (32*(a^2*b + b*c^2 - b^3 + a*c*(a^2 -
b^2 + c^2)^(1/2))*(2*a*b^4 - 3*a*c^4 + 3*b*c^4 - 2*a^3*b^2 - 2*a^3*c^2 - 3
*b^3*c^2 + 4*c^5*exp(x) + a*b^2*c^2 + 4*a^2*b*c^2 + b^4*c*exp(x) + 3*a^2*c
^3*exp(x) - 5*b^2*c^3*exp(x) + a^2*b^2*c*exp(x) - 6*a*b*c^3*exp(x) + 6*a*b
^3*c*exp(x) - 4*a^3*b*c*exp(x)))/((a + b)^5*(b^2 - c^2)*(a^2 - b^2 + c^2))
)*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/((b^2 - c^2)*(a^2 -
 b^2 + c^2)))*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/((b^2 -
 c^2)*(a^2 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/
2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) + (log(- (64*(b - a + 2*c
*exp(x)))/(a + b)^4 - (((32*(2*a*b^2 - 2*a*c^2 + 2*b*c^2 - 2*a^3 + 4*c^3*e
xp(x) + 3*a^2*c*exp(x) + 3*b^2*c*exp(x) + 6*a*b*c*exp(x)))/(a + b)^5 + (((
32*(a - b)*(2*a*b^2 + 2*a*c^2 + b*c^2 + 2*b^3 - 3*c^3*exp(x) + 6*b^2*c*exp
(x) + 6*a*b*c*exp(x)))/(a + b)^5 - (32*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b
^2 + c^2)^(1/2))*(2*a*b^4 - 3*a*c^4 + 3*b*c^4 - 2*a^3*b^2 - 2*a^3*c^2 - 3*
b^3*c^2 + 4*c^5*exp(x) + a*b^2*c^2 + 4*a^2*b*c^2 + b^4*c*exp(x) + 3*a^2*c^
3*exp(x) - 5*b^2*c^3*exp(x) + a^2*b^2*c*exp(x) - 6*a*b*c^3*exp(x) + 6*a...
```

3.788. $\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$

3.789 $\int \frac{A+C \sinh(x)}{a+b\cosh(x)+c\sinh(x)} \, dx$

3.789.1 Optimal result

Integrand size = 19, antiderivative size = 120

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{cCx}{b^2-c^2} - \frac{2(A(b^2-c^2)+acC)\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} + \frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

output
```
-c*C*x/(b^2-c^2)+b*C*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)-2*(A*(b^2-c^2)+a*
c*C)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-b^2
+c^2)^(1/2)
```

3.789.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{\frac{2(A(b^2-c^2)+acC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}} + C(-cx+b\log(a+b\cosh(x)+c\sinh(x)))}{(b-c)(b+c)}$$

input
```
Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

output
```
((2*(A*(b^2 - c^2) + a*c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^
2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + C*(-(c*x) + b*Log[a + b*Cosh[x] + c*Si
nh[x]]))/((b - c)*(b + c))
```

3.789.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3616, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iC\sin(ix)}{a+b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{3616}$$

$$\left(\frac{acC}{b^2-c^2}+A\right)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cCx}{b^2-c^2}$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{acC}{b^2-c^2}+A\right)\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cCx}{b^2-c^2}$$

$$\Big\downarrow \text{3603}$$

$$2\left(\frac{acC}{b^2-c^2}+A\right)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+2c\tanh\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cCx}{b^2-c^2}$$

$$\Big\downarrow \text{1083}$$

$$-4\left(\frac{acC}{b^2-c^2}+A\right)\int \frac{1}{4\left(a^2-b^2+c^2\right)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cCx}{b^2-c^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(\frac{acC}{b^2-c^2}+A\right)\operatorname{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cCx}{b^2-c^2}$$

input
```
Int[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

output
```
-((c*C*x)/(b^2 - c^2)) - (2*(A + (a*c*C)/(b^2 - c^2))*ArcTanh[(2*c - 2*(a
- b)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^2])])/Sqrt[a^2 - b^2 + c^2] + (b*C*L
og[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

3.789.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3616
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (-Simp[b*C*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

3.789. $\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

3.789.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.67

method	result
default	$-\frac{2C\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2b-2c}+\frac{\frac{2\left(abC-C\,b^{2}\right)\ln\left(a\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{2}b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a-2b}+\frac{2\left(-A\,b^{2}+A\,c^{2}-acC-Ccb+\frac{\left(abC-C\,b^{2}\right)c}{a-b}\right)\operatorname{arcta}}{\sqrt{-a^{2}+b^{2}-c^{2}}}}{(b-c)(b+c)}$
risch	Expression too large to display

input
```
int((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*C/(2*b-2*c)*ln(1+tanh(1/2*x))+2/(b-c)/(b+c)*(1/2*(C*a*b-C*b^2)/(a-b)*ln
(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)+(-A*b^2+A*c^2-a*c*C-
C*c*b+(C*a*b-C*b^2)*c/(a-b))/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh
(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2)))-2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)
```

3.789.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 505, normalized size of antiderivative = 4.21

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$$
$$=\left[-\frac{(Ab^2+Cac-Ac^2)\sqrt{a^2-b^2+c^2}\log\left(\frac{(b^2+2\,bc+c^2)\cosh(x)^2+(b^2+2\,bc+c^2)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,(a}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cos}\right.}{}\right.$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

output
```
[-((A*b^2 + C*a*c - A*c^2)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*
cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a
*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt
(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x
)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) +
b - c)) + (C*a^2*b - C*b^3 + C*b*c^2 + C*c^3 + (C*a^2 - C*b^2)*c)*x - (C*a
^2*b - C*b^3 + C*b*c^2)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(
x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2), (2*(A*b^2 + C*a*c - A*c^2
)*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) +
(b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (C*a^2*b - C*b^3 + C*b*c^2 + C*c
^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 + C*b*c^2)*log(2*(b*cosh(x) +
 c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)
*c^2)]
```

3.789.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

output
```
Timed out
```

3.789.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.789.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{Cb\log\left(be^{(2x)}+ce^{(2x)}+2\,ae^{x}+b-c\right)}{b^2-c^2}-\frac{Cx}{b-c}+\frac{2\left(Ab^2+Cac-Ac^2\right)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}(b^2-c^2)}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

output
```
C*b*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)/(b^2 - c^2) - C*x/(b - c)
 + 2*(A*b^2 + C*a*c - A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 -
c^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

3.789.9 Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.14

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{\ln\left(b\sqrt{a^2-b^2+c^2}-c\sqrt{a^2-b^2+c^2}-a^2\,\mathrm{e}^x+b^2\,\mathrm{e}^x-c^2\,\mathrm{e}^x+a\,\mathrm{e}^x\sqrt{a^2-b^2+c^2}\right)\left(C\,b^3+A\,b^2\sqrt{a^2-b^2}\right.}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4} - \frac{\ln\left(b\sqrt{a^2-b^2+c^2}-c\sqrt{a^2-b^2+c^2}+a^2\,\mathrm{e}^x-b^2\,\mathrm{e}^x+c^2\,\mathrm{e}^x+a\,\mathrm{e}^x\sqrt{a^2-b^2+c^2}\right)\left(A\,b^2\sqrt{a^2-b^2+c^2}\right.}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4} - \frac{C\,x}{b-c}$$

input
```
int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x)),x)
```

output
```
(log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) - a^2*exp(x) +
b^2*exp(x) - c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(C*b^3 + A*b^2
*(a^2 - b^2 + c^2)^(1/2) - C*a^2*b - A*c^2*(a^2 - b^2 + c^2)^(1/2) - C*b*c
^2 + C*a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^
2*c^2) - (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) + a^2*
exp(x) - b^2*exp(x) + c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(A*b^
2*(a^2 - b^2 + c^2)^(1/2) - C*b^3 + C*a^2*b - A*c^2*(a^2 - b^2 + c^2)^(1/2
) + C*b*c^2 + C*a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c
^2 - 2*b^2*c^2) - (C*x)/(b - c)
```

3.790 $\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

3.790.1 Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2(aA+cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}} + \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

output
```
-2*(A*a+C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c
^2)^(3/2)+(b*C-(A*c-C*a)*cosh(x)-A*b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c
*sinh(x))
```

3.790.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2(aA+cC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{3/2}} + \frac{-aAc+a^2C-b^2C+(A(b^2-c^2)+acC)\sinh(x)}{b\,(-a^2+b^2-c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

input
```
Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(-2*(a*A + c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(
-a^2 + b^2 - c^2)^(3/2) + (-(a*A*c) + a^2*C - b^2*C + (A*(b^2 - c^2) + a*c
*C)*Sinh[x])/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

3.790.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3633, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iC\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^2}dx$$

$$\Big\downarrow \text{3633}$$

$$\frac{(aA+cC)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}+\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}+\frac{(aA+cC)\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{a^2-b^2+c^2}$$

$$\Big\downarrow \text{3603}$$

$$\frac{2(aA+cC)\int \frac{1}{-((a-b)\tanh^2(\frac{x}{2}))+2c\tanh(\frac{x}{2})+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2+c^2}+\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}-\frac{4(aA+cC)\int \frac{1}{4(a^2-b^2+c^2)-(2c-2(a-b)\tanh(\frac{x}{2}))^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}-\frac{2(aA+cC)\text{arctanh}\left(\frac{2c-2(a-b)\tanh(\frac{x}{2})}{2\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}$$

input
```
Int[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(-2*(a*A + c*C)*ArcTanh[(2*c - 2*(a - b)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^
2])])/(a^2 - b^2 + c^2)^(3/2) + (b*C - (A*c - a*C)*Cosh[x] - A*b*Sinh[x])/
((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

3.790.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3633
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - c*C)/(a^2 - b^2 - c^2)   Int[1/(
a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

3.790. $\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

3.790.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(102) = 204$.

Time = 1.71 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.07

method	result
default	$-\frac{2\left(-\frac{\left(Aab-A\,b^{2}+A\,c^{2}-acC+Ccb\right)\tanh\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}-\frac{Aac-C\,a^{2}+C\,b^{2}}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}\right)}{a\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{2}b-2c\tanh\left(\frac{x}{2}\right)-a-b}-\frac{2(Aa+Cc)\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{(a^{2}-b^{2}+c^{2})\sqrt{-a^{2}+b^{2}-c^{2}}}$
risch	$\frac{2Aab\,e^{x}+2Aac\,e^{x}-2C\,a^{2}e^{x}+2C\,b^{2}e^{x}+2Cbc\,e^{x}+2A\,b^{2}-2A\,c^{2}+2acC}{(b+c)(a^{2}-b^{2}+c^{2})(e^{2x}b+e^{2x}c+2a\,e^{x}+b-c)}+\frac{\ln\left(e^{x}+\frac{\left(a^{2}-b^{2}+c^{2}\right)^{\frac{3}{2}}a-a^{4}+2a^{2}b^{2}-2a^{2}c^{2}-b^{4}+2b^{2}c^{2}-c^{4}}{\left(a^{2}-b^{2}+c^{2}\right)^{\frac{3}{2}}(b+c)}\right)A}{(a^{2}-b^{2}+c^{2})^{\frac{3}{2}}}$

input
```
int((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(-(A*a*b-A*b^2+A*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2)*tan
h(1/2*x)-(A*a*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/
2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)-2*(A*a+C*c)/(a^2-b^2+c^2)/(-a^
2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2)
)
```

3.790.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(105) = 210$.

Time = 0.31 (sec) , antiderivative size = 2211, normalized size of antiderivative = 20.47

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

output
```
[(2*A*a^2*b^2 - 2*A*b^4 + 2*C*a*c^3 - 2*A*c^4 - 2*(A*a^2 - 2*A*b^2)*c^2 +
(A*a*b^2 + C*b^2*c - A*a*c^2 - C*c^3 + (A*a*b^2 + C*c^3 + (A*a + 2*C*b)*c^
2 + (2*A*a*b + C*b^2)*c)*cosh(x)^2 + (A*a*b^2 + C*c^3 + (A*a + 2*C*b)*c^2
+ (2*A*a*b + C*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b + C*a*c^2 + (A*a^2 + C*a*b)*
c)*cosh(x) + 2*(A*a^2*b + C*a*c^2 + (A*a^2 + C*a*b)*c + (A*a*b^2 + C*c^3 +
 (A*a + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*cosh(x))*sinh(x))*sqrt(a^2 - b^2
 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2
 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c
 + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b +
 c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2
*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + 2*(C*a^3 - C*a*b^2)*c - 2*(C*a^
4 - A*a^3*b - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A
*a*b - C*b^2)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*cosh(x) - 2*(C*
a^4 - A*a^3*b - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 -
 A*a*b - C*b^2)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*sinh(x))/(a^4
*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*
b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4
+ 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*c
osh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4
+ 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c...
```

3.790.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\text{Timed out}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

output
```
Timed out
```

3.790.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C \sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.790.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.66

$$\int \frac{A + C \sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2} \, dx = \frac{2 \, (Aa + Cc) \arctan\left(\frac{be^x + ce^x + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{(a^2 - b^2 + c^2)\sqrt{-a^2 + b^2 - c^2}} - \frac{2 \, (Ca^2 e^x - Aabe^x - Cb^2 e^x - Aace^x - Cbce^x - Ab^2 - Cac + Ac^2)}{(a^2 b - b^3 + a^2 c - b^2 c + bc^2 + c^3)(be^{(2\,x)} + ce^{(2\,x)} + 2\,ae^x + b - c)}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

output
```
2*(A*a + C*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b
^2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(C*a^2*e^x - A*a*b*e^x - C*b^2*e^x -
 A*a*c*e^x - C*b*c*e^x - A*b^2 - C*a*c + A*c^2)/((a^2*b - b^3 + a^2*c - b^
2*c + b*c^2 + c^3)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

3.790.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\int \frac{A+C\sinh(x)}{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)^2}\,dx$$

input
```
int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2,x)
```

output
```
int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2, x)
```

3.791 $\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

3.791.1 Optimal result

Integrand size = 19, antiderivative size = 198

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{(2a^2A+A(b^2-c^2)+3acC)\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}} + \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2} + \frac{abC-(3aAc-a^2C+2c^2C)\cosh(x)-b(3aA+2cC)\sinh(x)}{2(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))}$$

output
```
-(2*a^2*A+A*(b^2-c^2)+3*a*c*C)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)
^(1/2))/(a^2-b^2+c^2)^(5/2)+1/2*(b*C-(A*c-C*a)*cosh(x)-A*b*sinh(x))/(a^2-b
^2+c^2)/(a+b*cosh(x)+c*sinh(x))^2+1/2*(a*b*C-(3*A*a*c-C*a^2+2*C*c^2)*cosh(
x)-b*(3*A*a+2*C*c)*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

3.791.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.88

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \frac{(2a^2A+A(b^2-c^2)+3acC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{5/2}} + \frac{6a^3Ac+3aAb^2c-3aAc^3-2a^4C+4a^2b^2C-2b^4C+5a^2c^2C+4b^2c^2C-2c^4C+2bc(2a^2A+A(b^2-c}{}$$

input
```
Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
((2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[
-a^2 + b^2 - c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (6*a^3*A*c + 3*a*A*b^2*c -
3*a*A*c^3 - 2*a^4*C + 4*a^2*b^2*C - 2*b^4*C + 5*a^2*c^2*C + 4*b^2*c^2*C -
2*c^4*C + 2*b*c*(2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*Cosh[x] + c*(3*a*A*(-b
^2 + c^2) - a^2*c*C + 2*c*(-b^2 + c^2)*C)*Cosh[2*x] - 8*a^2*A*b^2*Sinh[x]
+ 2*A*b^4*Sinh[x] + 12*a^2*A*c^2*Sinh[x] - 2*A*b^2*c^2*Sinh[x] - 4*a^3*c*C
*Sinh[x] - 2*a*b^2*c*C*Sinh[x] + 8*a*c^3*C*Sinh[x] - 3*a*A*b^3*Sinh[2*x] +
 3*a*A*b*c^2*Sinh[2*x] - a^2*b*c*C*Sinh[2*x] - 2*b^3*c*C*Sinh[2*x] + 2*b*c
^3*C*Sinh[2*x])/(4*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])^2)
```

3.791.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3636, 25, 3042, 3632, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iC\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^3}dx$$

$$\Big\downarrow \text{3636}$$

$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}-\frac{\int -\frac{2(aA+cC)-Ab\cosh(x)-(Ac-aC)\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx}{2(a^2-b^2+c^2)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{2(aA+cC)-Ab\cosh(x)-(Ac-aC)\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx}{2\left(a^2-b^2+c^2\right)}+\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2(aA+cC)-Ab\cos(ix)+i(Ac-aC)\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^2}dx}{2\left(a^2-b^2+c^2\right)}$$

$$\Big\downarrow \ 3632$$

$$\frac{\frac{\left(2a^2A+3acC+A\left(b^2-c^2\right)\right)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}+\frac{-\cosh(x)\left(a^2(-C)+3aAc+2c^2C\right)-b\sinh(x)(3aA+2cC)+abC}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}}{2\left(a^2-b^2+c^2\right)}+$$
$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}+$$
$$\frac{\frac{-\cosh(x)\left(a^2(-C)+3aAc+2c^2C\right)-b\sinh(x)(3aA+2cC)+abC}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{\left(2a^2A+3acC+A\left(b^2-c^2\right)\right)\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{a^2-b^2+c^2}}{2\left(a^2-b^2+c^2\right)}$$

$$\Big\downarrow \ 3603$$

$$\frac{\frac{2\left(2a^2A+3acC+A\left(b^2-c^2\right)\right)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+2c\tanh\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2+c^2}+\frac{-\cosh(x)\left(a^2(-C)+3aAc+2c^2C\right)-b\sinh(x)(3aA+2cC)+abC}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}}{2\left(a^2-b^2+c^2\right)}+$$
$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \ 1083$$

$$\frac{\frac{-\cosh(x)\left(a^2(-C)+3aAc+2c^2C\right)-b\sinh(x)(3aA+2cC)+abC}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{4\left(2a^2A+3acC+A\left(b^2-c^2\right)\right)\int \frac{1}{4\left(a^2-b^2+c^2\right)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b\right.}{a^2-b^2+c^2}}{2\left(a^2-b^2+c^2\right)}$$
$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \ 219$$

3.791. $\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

$$\frac{\frac{-\cosh(x)(a^2(-C)+3aAc+2c^2C)-b\sinh(x)(3aA+2cC)+abC}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}-\frac{2(2a^2A+3acC+A(b^2-c^2))\operatorname{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}}{2(a^2-b^2+c^2)}+\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

input
```
Int[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
(b*C - (A*c - a*C)*Cosh[x] - A*b*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh
[x] + c*Sinh[x])^2) + ((-2*(2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*ArcTanh[(2*
c - 2*(a - b)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^2])])/(a^2 - b^2 + c^2)^(3/
2) + (a*b*C - (3*a*A*c - a^2*C + 2*c^2*C)*Cosh[x] - b*(3*a*A + 2*c*C)*Sinh
[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])))/(2*(a^2 - b^2 + c^2)
)
```

3.791.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.791. $\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3636
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*C + (a*
C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*((a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b
^2 - c^2))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

3.791.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(188) = 376$.

Time = 11.62 (sec) , antiderivative size = 836, normalized size of antiderivative = 4.22

method	result
default	$-\frac{2\left(-\frac{\left(4A\,a^3b-7A\,a^2b^2+5A\,a^2c^2+2Aa\,b^3-2Aab\,c^2+A\,b^4-3A\,b^2c^2+2A\,c^4-3C\,a^3c+6C\,a^2bc-3Ca\,b^2c\right)\tanh\left(\frac{x}{2}\right)^3}{2\left(a^4-2a^2b^2+2a^2c^2+b^4-2b^2c^2+c^4\right)(a-b)}-\frac{\left(4A\,a^4c-12A\,a^3bc+13A\,a\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2+5*A*a^2*c^2+2*A*a*b^3-2*A*a*b*c^2+A*b^4-3*
A*b^2*c^2+2*A*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a^4-2*a^2*b^2+2*a^2*
c^2+b^4-2*b^2*c^2+c^4)/(a-b)*tanh(1/2*x)^3-1/2*(4*A*a^4*c-12*A*a^3*b*c+13*
A*a^2*b^2*c-7*A*a^2*c^3-6*A*a*b^3*c+6*A*a*b*c^3+A*b^4*c+A*b^2*c^3-2*A*c^5-
2*C*a^5+2*C*a^4*b+4*C*a^3*b^2+5*C*a^3*c^2-4*C*a^2*b^3-14*C*a^2*b*c^2-2*C*a
*b^4+13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5-4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^
2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(4*A*a^4*
b-5*A*a^3*b^2+11*A*a^3*c^2-3*A*a^2*b^3-3*A*a^2*b*c^2+5*A*a*b^4-7*A*a*b^2*c
^2+2*A*a*c^4-A*b^5-A*b^3*c^2+2*A*b*c^4-5*C*a^4*c+5*C*a^3*b*c+5*C*a^2*b^2*c
+4*C*a^2*c^3-5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c
^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c+A*a^2*c^3
-A*b^4*c+A*b^2*c^3-2*C*a^5+4*C*a^3*b^2+C*a^3*c^2-2*C*a*b^4-C*a*b^2*c^2)/(a
^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tanh(1/2*x)^
2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2-(2*A*a^2+A*b^2-A*c^2+3*C*a*c)/(a^
4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(
2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

3.791.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6084 vs. 2(188) = 376.

Time = 0.59 (sec) , antiderivative size = 12285, normalized size of antiderivative = 62.05

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.791.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\, dx = \text{Timed out}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

output
```
Timed out
```

3.791.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C \sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.791.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. 2(188) = 376.

Time = 0.28 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.16

$$\int \frac{A + C \sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\, dx = \frac{(2\,Aa^2 + Ab^2 + 3\,Cac - Ac^2)\arctan\left(\frac{be^x + ce^x + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{(a^4 - 2\,a^2b^2 + b^4 + 2\,a^2c^2 - 2\,b^2c^2 + c^4)\sqrt{-a^2 + b^2 - c^2}}$$
$$+ \frac{2\,Aa^2b^2e^{(3\,x)} + Ab^4e^{(3\,x)} + 4\,Aa^2bce^{(3\,x)} + 3\,Cab^2ce^{(3\,x)} + 2\,Ab^3ce^{(3\,x)} + 2\,Aa^2c^2e^{(3\,x)} + 6\,Cabc^2e^{(3\,x)} + 3\,C}{}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

output
```
(2*A*a^2 + A*b^2 + 3*C*a*c - A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 +
 b^2 - c^2))/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-
a^2 + b^2 - c^2)) + (2*A*a^2*b^2*e^(3*x) + A*b^4*e^(3*x) + 4*A*a^2*b*c*e^(
3*x) + 3*C*a*b^2*c*e^(3*x) + 2*A*b^3*c*e^(3*x) + 2*A*a^2*c^2*e^(3*x) + 6*C
*a*b*c^2*e^(3*x) + 3*C*a*c^3*e^(3*x) - 2*A*b*c^3*e^(3*x) - A*c^4*e^(3*x) -
 2*C*a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) + 4*C*a^2*b^2*e^(2*x) + 3*A*a*b^3*e^(
2*x) - 2*C*b^4*e^(2*x) + 6*A*a^3*c*e^(2*x) + 9*C*a^2*b*c*e^(2*x) + 3*A*a*b
^2*c*e^(2*x) + 5*C*a^2*c^2*e^(2*x) - 3*A*a*b*c^2*e^(2*x) + 4*C*b^2*c^2*e^(
2*x) - 3*A*a*c^3*e^(2*x) - 2*C*c^4*e^(2*x) + 10*A*a^2*b^2*e^x - A*b^4*e^x
+ 4*C*a^3*c*e^x + 5*C*a*b^2*c*e^x - 10*A*a^2*c^2*e^x + 2*A*b^2*c^2*e^x - 5
*C*a*c^3*e^x - A*c^4*e^x + 3*A*a*b^3 + C*a^2*b*c - 3*A*a*b^2*c + 2*C*b^3*c
 - C*a^2*c^2 - 3*A*a*b*c^2 - 2*C*b^2*c^2 + 3*A*a*c^3 - 2*C*b*c^3 + 2*C*c^4
)/((a^4*b - 2*a^2*b^3 + b^5 + a^4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 -
2*b^3*c^2 + 2*a^2*c^3 - 2*b^2*c^3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) +
2*a*e^x + b - c)^2)
```

3.791.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx$$

input
```
int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3,x)
```

output
```
int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3, x)
```

3.792 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

3.792.1 Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{bBx}{b^2-c^2}+\frac{2(abB-A(b^2-c^2))\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} - \frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

output
```
b*B*x/(b^2-c^2)-B*c*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)+2*(a*b*B-A*(b^2-c^
2))*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-b^2+
c^2)^(1/2)
```

3.792.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{-\frac{2(abB+A(-b^2+c^2))\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+B(bx-c\log(a+b\cosh(x)+c\sinh(x)))}{(b-c)(b+c)}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

output
```
((-2*(a*b*B + A*(-b^2 + c^2))*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 +
b^2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + B*(b*x - c*Log[a + b*Cosh[x] + c*Sin
h[x]]))/((b - c)*(b + c))
```

3.792.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3617, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)}{a+b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{3617}$$

$$\left(A-\frac{abB}{b^2-c^2}\right)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{bBx}{b^2-c^2}$$

$$\Big\downarrow \text{3042}$$

$$\left(A-\frac{abB}{b^2-c^2}\right)\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{bBx}{b^2-c^2}$$

$$\Big\downarrow \text{3603}$$

$$2\left(A-\frac{abB}{b^2-c^2}\right)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+2c\tanh\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{bBx}{b^2-c^2}$$

$$\Big\downarrow \text{1083}$$

$$-4\left(A-\frac{abB}{b^2-c^2}\right)\int \frac{1}{4\left(a^2-b^2+c^2\right)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{bBx}{b^2-c^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(A-\frac{abB}{b^2-c^2}\right)\operatorname{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{bBx}{b^2-c^2}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

output
```
(b*B*x)/(b^2 - c^2) - (2*(A - (a*b*B)/(b^2 - c^2))*ArcTanh[(2*c - 2*(a - b
)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^2])])/Sqrt[a^2 - b^2 + c^2] - (B*c*Log[
a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

3.792.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3617
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[b*B*((d + e*x)/
(e*(b^2 + c^2))), x] + (Simp[c*B*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/
(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2)   Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

3.792. $\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

3.792.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.63

method	result
default	$-\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{\frac{2(-aBc+bBc)\ln\left(a\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)^2b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a-2b}+\frac{2\left(-A\,b^2+A\,c^2+abB+B\,c^2+\frac{(-aBc+bBc)c}{a-b}\right)\operatorname{arct}}{\sqrt{-a^2+b^2-c^2}}}{(b-c)(b+c)}$
risch	Expression too large to display

input
```
int((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)+2/(b-c)/(b+c)*(1/2*(-B*a*c+B*b*c)/(a-b)*l
n(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)+(-A*b^2+A*c^2+a*b*B
+B*c^2+(-B*a*c+B*b*c)*c/(a-b))/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*ta
nh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2)))+2*B/(2*b-2*c)*ln(1+tanh(1/2*x))
```

3.792.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 508, normalized size of antiderivative = 4.20

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$$
$$=\left[-\frac{(Bab-Ab^2+Ac^2)\sqrt{a^2-b^2+c^2}\log\left(\frac{(b^2+2\,bc+c^2)\cosh(x)^2+(b^2+2\,bc+c^2)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,(a}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cos}\right.}{}\right.$$
$$-\frac{2\,(Bab-Ab^2+Ac^2)\sqrt{-a^2+b^2-c^2}\arctan\left(\frac{\sqrt{-a^2+b^2-c^2}((b+c)\cosh(x)+(b+c)\sinh(x)+a)}{a^2-b^2+c^2}\right)-(Ba^2b-Bb^3+B}{a^2b^2-b^4-c^4-(a^2-2\,b}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

output
```
[-((B*a*b - A*b^2 + A*c^2)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*
cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a
*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt
(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x
)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) +
b - c)) - (B*a^2*b - B*b^3 + B*b*c^2 + B*c^3 + (B*a^2 - B*b^2)*c)*x + (B*c
^3 + (B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(
x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2), -(2*(B*a*b - A*b^2 + A*c^
2)*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) +
 (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (B*a^2*b - B*b^3 + B*b*c^2 + B*
c^3 + (B*a^2 - B*b^2)*c)*x + (B*c^3 + (B*a^2 - B*b^2)*c)*log(2*(b*cosh(x)
+ c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2
)*c^2)]
```

3.792.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x) + c\sinh(x)}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

output
```
Timed out
```

3.792.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x) + c\sinh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.792.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{Bc\log\left(be^{(2x)}+ce^{(2x)}+2\,ae^{x}+b-c\right)}{b^2-c^2}+\frac{Bx}{b-c} - \frac{2\left(Bab-Ab^2+Ac^2\right)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}\left(b^2-c^2\right)}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

output
```
-B*c*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)/(b^2 - c^2) + B*x/(b - c
) - 2*(B*a*b - A*b^2 + A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 -
 c^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

3.792.9 Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 375, normalized size of antiderivative = 3.10

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{\ln\left(b\sqrt{a^2-b^2+c^2}-c\sqrt{a^2-b^2+c^2}+a^2\,e^x-b^2\,e^x+c^2\,e^x+a\,e^x\sqrt{a^2-b^2+c^2}\right)\left(B\,c^3-A\,b^2\sqrt{a^2-b^2}\right.}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4} + \frac{\ln\left(b\sqrt{a^2-b^2+c^2}-c\sqrt{a^2-b^2+c^2}-a^2\,e^x+b^2\,e^x-c^2\,e^x+a\,e^x\sqrt{a^2-b^2+c^2}\right)\left(B\,c^3+A\,b^2\sqrt{a^2-}\right.}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4} + \frac{B\,x}{b-c}$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x)),x)
```

output
```
(log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) + a^2*exp(x) -
b^2*exp(x) + c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 - A*b^2
*(a^2 - b^2 + c^2)^(1/2) + B*a^2*c + A*c^2*(a^2 - b^2 + c^2)^(1/2) - B*b^2
*c + B*a*b*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^
2*c^2) + (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) - a^2*
exp(x) + b^2*exp(x) - c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^
3 + A*b^2*(a^2 - b^2 + c^2)^(1/2) + B*a^2*c - A*c^2*(a^2 - b^2 + c^2)^(1/2
) - B*b^2*c - B*a*b*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c
^2 - 2*b^2*c^2) + (B*x)/(b - c)
```

3.793 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

3.793.1 Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2(aA-bB)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}} - \frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

output
```
-2*(A*a-B*b)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c
^2)^(3/2)+(-B*c-A*c*cosh(x)-(A*b-B*a)*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+
c*sinh(x))
```

3.793.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2(aA-bB)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{3/2}} + \frac{-aAc+bBc+(-abB+A(b^2-c^2))\sinh(x)}{b(-a^2+b^2-c^2)(a+b\cosh(x)+c\sinh(x))}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(-2*(a*A - b*B)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(
-a^2 + b^2 - c^2)^(3/2) + (-(a*A*c) + b*B*c + (-(a*b*B) + A*(b^2 - c^2))*S
inh[x])/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

3.793.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3634, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)}{(a+b\cos(ix)-ic\sin(ix))^2}dx$$

$$\Big\downarrow \text{3634}$$

$$\frac{(aA-bB)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}+\frac{(aA-bB)\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{a^2-b^2+c^2}$$

$$\Big\downarrow \text{3603}$$

$$\frac{2(aA-bB)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+2c\tanh\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2+c^2}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{4(aA-bB)\int \frac{1}{4(a^2-b^2+c^2)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2(aA-bB)\text{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(-2*(a*A - b*B)*ArcTanh[(2*c - 2*(a - b)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^
2])])/(a^2 - b^2 + c^2)^(3/2) - (B*c + A*c*Cosh[x] + (A*b - a*B)*Sinh[x])/
((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

3.793.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3634
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - b*B)/(a^2 - b^2 - c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

3.793. $\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

3.793.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(104) = 208$.

Time = 1.70 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.08

method	result
default	$-\frac{2\left(-\frac{\left(Aab-A\,b^{2}+A\,c^{2}-B\,a^{2}+abB-B\,c^{2}\right)\tanh\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}-\frac{(Aa-Bb)c}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}\right)}{a\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{2}b-2c\tanh\left(\frac{x}{2}\right)-a-b}-\frac{2(Aa-Bb)\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{(a^{2}-b^{2}+c^{2})\sqrt{-a^{2}+b^{2}-c^{2}}}$
risch	$\frac{2Aab\,\mathrm{e}^{x}+2Aac\,\mathrm{e}^{x}-2B\,a^{2}\mathrm{e}^{x}-2Bbc\,\mathrm{e}^{x}-2B\,c^{2}\mathrm{e}^{x}+2A\,b^{2}-2A\,c^{2}-2abB}{(b+c)(a^{2}-b^{2}+c^{2})(\mathrm{e}^{2x}b+\mathrm{e}^{2x}c+2a\,\mathrm{e}^{x}+b-c)}+\frac{\ln\left(\mathrm{e}^{x}+\frac{\left(a^{2}-b^{2}+c^{2}\right)^{\frac{3}{2}}a-a^{4}+2a^{2}b^{2}-2a^{2}c^{2}-b^{4}+2b^{2}c^{2}-c^{4}}{\left(a^{2}-b^{2}+c^{2}\right)^{\frac{3}{2}}(b+c)}\right)A}{(a^{2}-b^{2}+c^{2})^{\frac{3}{2}}}$

input
```
int((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(-(A*a*b-A*b^2+A*c^2-B*a^2+B*a*b-B*c^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^
2)*tanh(1/2*x)-(A*a-B*b)*c/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2*
x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)-2*(A*a-B*b)/(a^2-b^2+c^2)/(-a^2+
b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

3.793.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. $2(105) = 210$.

Time = 0.31 (sec) , antiderivative size = 2228, normalized size of antiderivative = 20.63

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

output
```
[-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 + 2*A*c^4 + 2*(A*a^2 + B*
a*b - 2*A*b^2)*c^2 + (A*a*b^2 - B*b^3 - (A*a - B*b)*c^2 + (A*a*b^2 - B*b^3
 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x)^2 + (A*a*b^2 - B*b^3 + (
A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2 + (
A*a^2 - B*a*b)*c)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a^2 - B*a*b)*c + (A*
a*b^2 - B*b^3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x))*sinh(x))*s
qrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c
^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c +
 (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b + c)*c
osh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*
a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + 2*(B*a^4 - A*a^3*b
 - B*a^2*b^2 + A*a*b^3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^
2)*c^2 - (A*a^3 - B*a^2*b - A*a*b^2 + B*b^3)*c)*cosh(x) + 2*(B*a^4 - A*a^3
*b - B*a^2*b^2 + A*a*b^3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*
b^2)*c^2 - (A*a^3 - B*a^2*b - A*a*b^2 + B*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^
2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 +
(a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b
- b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 +
(a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b
- b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2...
```

3.793.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

output
```
Timed out
```

3.793.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.793.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.64

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \frac{2\,(Aa - Bb)\arctan\left(\frac{be^x + ce^x + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{(a^2 - b^2 + c^2)\sqrt{-a^2 + b^2 - c^2}} - \frac{2\,(Ba^2e^x - Aabe^x - Aace^x + Bbce^x + Bc^2e^x + Bab - Ab^2 + Ac^2)}{(a^2b - b^3 + a^2c - b^2c + bc^2 + c^3)(be^{(2\,x)} + ce^{(2\,x)} + 2\,ae^x + b - c)}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

output
```
2*(A*a - B*b)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b
^2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(B*a^2*e^x - A*a*b*e^x - A*a*c*e^x +
 B*b*c*e^x + B*c^2*e^x + B*a*b - A*b^2 + A*c^2)/((a^2*b - b^3 + a^2*c - b^
2*c + b*c^2 + c^3)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

3.793.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\int \frac{A+B\cosh(x)}{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)^2}\,dx$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x))^2,x)
```

output
```
int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x))^2, x)
```

3.794 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

3.794.1 Optimal result

Integrand size = 19, antiderivative size = 194

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx &= -\frac{(2a^2A-3abB+A(b^2-c^2))\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}} \\ &\quad-\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{2\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^2} \\ &\quad-\frac{aBc+(3aA-2bB)c\cosh(x)+(3aAb-a^2B-2b^2B)\sinh(x)}{2\,(a^2-b^2+c^2)^2\,(a+b\cosh(x)+c\sinh(x))}\end{aligned}$$

output
```
-(2*a^2*A-3*a*b*B+A*(b^2-c^2))*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)
^(1/2))/(a^2-b^2+c^2)^(5/2)+1/2*(-B*c-A*c*cosh(x)-(A*b-B*a)*sinh(x))/(a^2-
b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^2+1/2*(-a*B*c-(3*A*a-2*B*b)*c*cosh(x)-(3*
A*a*b-B*a^2-2*B*b^2)*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

3.794.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.73

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \frac{(2a^2A-3abB+A(b^2-c^2))\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{5/2}}$$
$$+\frac{6a^3Ac+3aAb^2c-9a^2bBc-3aAc^3+2bc(2a^2A-3abB+A(b^2-c^2))\cosh(x)+c(a^2bB+2bB(b^2-c^2}{}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
((2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[
-a^2 + b^2 - c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (6*a^3*A*c + 3*a*A*b^2*c -
9*a^2*b*B*c - 3*a*A*c^3 + 2*b*c*(2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*Cosh[x
] + c*(a^2*b*B + 2*b*B*(b^2 - c^2) + 3*a*A*(-b^2 + c^2))*Cosh[2*x] - 8*a^2
*A*b^2*Sinh[x] + 2*A*b^4*Sinh[x] + 4*a^3*b*B*Sinh[x] + 2*a*b^3*B*Sinh[x] +
 12*a^2*A*c^2*Sinh[x] - 2*A*b^2*c^2*Sinh[x] - 8*a*b*B*c^2*Sinh[x] - 3*a*A*
b^3*Sinh[2*x] + a^2*b^2*B*Sinh[2*x] + 2*b^4*B*Sinh[2*x] + 3*a*A*b*c^2*Sinh
[2*x] - 2*b^2*B*c^2*Sinh[2*x])/(4*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c
*Sinh[x])^2)
```

3.794.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3637, 25, 3042, 3632, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{A+B\cos(ix)}{(a+b\cos(ix)-ic\sin(ix))^3}dx$$
$$\Big\downarrow \ 3637$$
$$-\frac{\int -\frac{2(aA-bB)-(Ab-aB)\cosh(x)-Ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx}{2(a^2-b^2+c^2)}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{2(aA-bB)-(Ab-aB)\cosh(x)-Ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx}{2(a^2-b^2+c^2)}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2(aA-bB)-(Ab-aB)\cos(ix)+iAc\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^2}dx}{2(a^2-b^2+c^2)}$$

$\Big\downarrow$ 3632

$$\frac{\frac{(2a^2A-3abB+A(b^2-c^2))\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}-\frac{\sinh(x)(a^2(-B)+3aAb-2b^2B)+c\cosh(x)(3aA-2bB)+aBc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}+\frac{-\frac{\sinh(x)(a^2(-B)+3aAb-2b^2B)+c\cosh(x)(3aA-2bB)+aBc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}+\frac{(2a^2A-3abB+A(b^2-c^2))\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{a^2-b^2+c^2}}{2(a^2-b^2+c^2)}$$

$\Big\downarrow$ 3603

$$\frac{\frac{2(2a^2A-3abB+A(b^2-c^2))\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+2c\tanh\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2+c^2}-\frac{\sinh(x)(a^2(-B)+3aAb-2b^2B)+c\cosh(x)(3aA-2bB)+aBc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$\Big\downarrow$ 1083

$$\frac{-\frac{4(2a^2A-3abB+A(b^2-c^2))\int \frac{1}{4\left(a^2-b^2+c^2\right)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}-\frac{\sinh(x)(a^2(-B)+3aAb-2b^2B)+c\cosh(x)(3aA-}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}$$
$$\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$\Big\downarrow$ 219

3.794. $\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

$$-\frac{-\frac{2\left(2a^2A-3abB+A\left(b^2-c^2\right)\right)\operatorname{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cosh(x)(3aA-2bB)+aBc}{\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)}}{2\left(a^2-b^2+c^2\right)}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{2\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)^2}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
-1/2*(B*c + A*c*Cosh[x] + (A*b - a*B)*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*C
osh[x] + c*Sinh[x])^2) + ((-2*(2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*ArcTanh[
(2*c - 2*(a - b)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^2])])/(a^2 - b^2 + c^2)^
(3/2) - (a*B*c + (3*a*A - 2*b*B)*c*Cosh[x] + (3*a*A*b - a^2*B - 2*b^2*B)*S
inh[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])))/(2*(a^2 - b^2 + c
^2))
```

3.794.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3637
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d
 + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2
- b^2 - c^2))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n +
 1)*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*
x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2
- b^2 - c^2, 0] && NeQ[n, -2]
```

3.794.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 856 vs. $2(190) = 380$.

Time = 11.59 (sec) , antiderivative size = 857, normalized size of antiderivative = 4.42

method	result
default	$-\frac{2\left(-\frac{\left(4Aa^3b-7Aa^2b^2+5Aa^2c^2+2Aab^3-2Aabc^2+Ab^4-3Ab^2c^2+2Ac^4-2Ba^4+3Ba^3b-2Ba^2b^2-4Ba^2c^2+3Bab^3-2Bb^4+4Bb^2c^2-2Bc^4\right.}{2\left(a^4-2a^2b^2+2a^2c^2+b^4-2b^2c^2+c^4\right)(a-b)}\right.}{}$
risch	Expression too large to display

input
```
int((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2+5*A*a^2*c^2+2*A*a*b^3-2*A*a*b*c^2+A*b^4-3*
A*b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2-4*B*a^2*c^2+3*B*a*b^3-2*B*
b^4+4*B*b^2*c^2-2*B*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a-b)
*tanh(1/2*x)^3-1/2*c*(4*A*a^4-12*A*a^3*b+13*A*a^2*b^2-7*A*a^2*c^2-6*A*a*b^
3+6*A*a*b*c^2+A*b^4+A*b^2*c^2-2*A*c^4+2*B*a^4-9*B*a^3*b+14*B*a^2*b^2+4*B*a
^2*c^2-9*B*a*b^3+2*B*b^4-4*B*b^2*c^2+2*B*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4
-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(4*A*a^4*b-5*A*a^3*b^2+1
1*A*a^3*c^2-3*A*a^2*b^3-3*A*a^2*b*c^2+5*A*a*b^4-7*A*a*b^2*c^2+2*A*a*c^4-A*
b^5-A*b^3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3*b^2-4*B*a^3*c^2-B*a^2*b^3-
8*B*a^2*b*c^2+3*B*a*b^4+8*B*a*b^2*c^2-2*B*a*c^4-2*B*b^5+4*B*b^3*c^2-2*B*b*
c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*
x)+1/2*c*(4*A*a^4-3*A*a^2*b^2+A*a^2*c^2-A*b^4+A*b^2*c^2-5*B*a^3*b+5*B*a*b^
3-2*B*a*b*c^2)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)
)/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2-(2*A*a^2+A*b^2-A
*c^2-3*B*a*b)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(
1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

3.794.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6126 vs. 2(187) = 374.

Time = 0.60 (sec) , antiderivative size = 12366, normalized size of antiderivative = 63.74

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.794.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

output
```
Timed out
```

3.794.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.794.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(187) = 374$.

Time = 0.28 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.22

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \frac{(2\,Aa^2-3\,Bab+Ab^2-Ac^2)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4)\sqrt{-a^2+b^2-c^2}}$$
$$+\frac{2\,Aa^2b^2e^{(3\,x)}-3\,Bab^3e^{(3\,x)}+Ab^4e^{(3\,x)}+4\,Aa^2bce^{(3\,x)}-6\,Bab^2ce^{(3\,x)}+2\,Ab^3ce^{(3\,x)}+2\,Aa^2c^2e^{(3\,x)}-3\,B}{}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

output
```
(2*A*a^2 - 3*B*a*b + A*b^2 - A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 +
 b^2 - c^2))/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-
a^2 + b^2 - c^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*x) + A*b^4*e^(3*
x) + 4*A*a^2*b*c*e^(3*x) - 6*B*a*b^2*c*e^(3*x) + 2*A*b^3*c*e^(3*x) + 2*A*a
^2*c^2*e^(3*x) - 3*B*a*b*c^2*e^(3*x) - 2*A*b*c^3*e^(3*x) - A*c^4*e^(3*x) -
 2*B*a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(2*x) + 3*A*a*b^3*e^(
2*x) - 2*B*b^4*e^(2*x) + 6*A*a^3*c*e^(2*x) - 9*B*a^2*b*c*e^(2*x) + 3*A*a*b
^2*c*e^(2*x) - 4*B*a^2*c^2*e^(2*x) - 3*A*a*b*c^2*e^(2*x) + 4*B*b^2*c^2*e^(
2*x) - 3*A*a*c^3*e^(2*x) - 2*B*c^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*
e^x - 5*B*a*b^3*e^x - A*b^4*e^x - 10*A*a^2*c^2*e^x + 5*B*a*b*c^2*e^x + 2*A
*b^2*c^2*e^x - A*c^4*e^x - B*a^2*b^2 + 3*A*a*b^3 - 2*B*b^4 + B*a^2*b*c - 3
*A*a*b^2*c + 2*B*b^3*c - 3*A*a*b*c^2 + 2*B*b^2*c^2 + 3*A*a*c^3 - 2*B*b*c^3
)/((a^4*b - 2*a^2*b^3 + b^5 + a^4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 -
2*b^3*c^2 + 2*a^2*c^3 - 2*b^2*c^3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) +
2*a*e^x + b - c)^2)
```

3.794.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx=\int \frac{A+B\cosh(x)}{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)^3}\,dx$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x))^3,x)
```

output
```
int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x))^3, x)
```

3.795 $\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

3.795.1 Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x}{b^2-c^2}+\frac{2a(bB-cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

output
```
(B*b-C*c)*x/(b^2-c^2)-(B*c-C*b)*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)+2*a*(B
*b-C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-
b^2+c^2)^(1/2)
```

3.795.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x-\frac{2a(bB-cC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+(-Bc+bC)\log(a+b\cosh(x)+c\sinh(x))}{(b-c)(b+c)}$$

input
```
Integrate[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

output
```
((b*B - c*C)*x - (2*a*(b*B - c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^
2 + b^2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + (-(B*c) + b*C)*Log[a + b*Cosh[x]
 + c*Sinh[x]])/((b - c)*(b + c))
```

3.795.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3615, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(ix)-iC\sin(ix)}{a+b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{3615}$$

$$-\frac{a(bB-cC)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{b^2-c^2}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a(bB-cC)\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{b^2-c^2}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

$$\Big\downarrow \text{3603}$$

$$-\frac{2a(bB-cC)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+2c\tanh\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b^2-c^2}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4a(bB-cC)\int \frac{1}{4(a^2-b^2+c^2)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{2a(bB-cC)\text{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

input
```
Int[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

output
```
((b*B - c*C)*x)/(b^2 - c^2) + (2*a*(b*B - c*C)*ArcTanh[(2*c - 2*(a - b)*Ta
nh[x/2])/(2*Sqrt[a^2 - b^2 + c^2])])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) -
 ((B*c - b*C)*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

3.795.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3615
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + (Simp[(c*B - b*C)*(Log[a
 + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c
^2) - a*(b*B + c*C))/(b^2 + c^2)   Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*
x]), x], x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] &&
NeQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

3.795. $\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

3.795.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.81

method	result
default	$\frac{2(-B-C)\ln(\tanh(\frac{x}{2})-1)}{2b+2c}+\frac{2(B-C)\ln(1+\tanh(\frac{x}{2}))}{2b-2c}+\frac{\frac{2\left(-aBc+bBc+abC-C\,b^2\right)\ln\left(a\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)^2b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a-2b}+}{(}$
risch	Expression too large to display

input
```
int((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*(-B-C)/(2*b+2*c)*ln(tanh(1/2*x)-1)+2*(B-C)/(2*b-2*c)*ln(1+tanh(1/2*x))+2
/(b-c)/(b+c)*(1/2*(-B*a*c+B*b*c+C*a*b-C*b^2)/(a-b)*ln(a*tanh(1/2*x)^2-tanh
(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)+(a*b*B+B*c^2-a*c*C-C*c*b+(-B*a*c+B*b*c+C*
a*b-C*b^2)*c/(a-b))/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2
*c)/(-a^2+b^2-c^2)^(1/2)))
```

3.795.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 583, normalized size of antiderivative = 4.66

$$\int\frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$$

$$=\left[\frac{(Bab-Cac)\sqrt{a^2-b^2+c^2}\log\left(\frac{(b^2+2\,bc+c^2)\cosh(x)^2+(b^2+2\,bc+c^2)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,(ab+ac+(b^2+}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b}\right.}{}\right.$$

$$-\frac{2\,(Bab-Cac)\sqrt{-a^2+b^2-c^2}\arctan\left(\frac{\sqrt{-a^2+b^2-c^2}((b+c)\cosh(x)+(b+c)\sinh(x)+a)}{a^2-b^2+c^2}\right)-((B-C)a^2b-(B-C}{}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fric
as")
```

output
```
[((B*a*b - C*a*c)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2
 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(
x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^
2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b
+ c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) +
 ((B - C)*a^2*b - (B - C)*b^3 + (B - C)*b*c^2 + (B - C)*c^3 + ((B - C)*a^2
 - (B - C)*b^2)*c)*x + (C*a^2*b - C*b^3 + C*b*c^2 - B*c^3 - (B*a^2 - B*b^2
)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^
4 - c^4 - (a^2 - 2*b^2)*c^2), -(2*(B*a*b - C*a*c)*sqrt(-a^2 + b^2 - c^2)*a
rctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2
- b^2 + c^2)) - ((B - C)*a^2*b - (B - C)*b^3 + (B - C)*b*c^2 + (B - C)*c^3
 + ((B - C)*a^2 - (B - C)*b^2)*c)*x - (C*a^2*b - C*b^3 + C*b*c^2 - B*c^3 -
 (B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x)))
)/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

3.795.6 Sympy [F(-1)]

Timed out.

$$\int \frac{B\cosh(x) + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

output
```
Timed out
```

3.795.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cosh(x) + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.795.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00

$$\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx=\frac{(B-C)x}{b-c}+\frac{(Cb-Bc)\log\left(be^{(2x)}+ce^{(2x)}+2\,ae^{x}+b-c\right)}{b^2-c^2}-\frac{2\,(Bab-Cac)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}(b^2-c^2)}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac
")
```

output
```
(B - C)*x/(b - c) + (C*b - B*c)*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b -
c)/(b^2 - c^2) - 2*(B*a*b - C*a*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 +
b^2 - c^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

3.795.9 Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.01

$$\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx=\frac{\ln\left(b\sqrt{a^2-b^2+c^2}-c\sqrt{a^2-b^2+c^2}+a^2\,\mathrm{e}^x-b^2\,\mathrm{e}^x+c^2\,\mathrm{e}^x+a\,\mathrm{e}^x\sqrt{a^2-b^2+c^2}\right)\left(B\,c^3+C\,b^3+B\,a^2\,c\cdots\right.}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4\cdots}+\frac{\ln\left(b\sqrt{a^2-b^2+c^2}-c\sqrt{a^2-b^2+c^2}-a^2\,\mathrm{e}^x+b^2\,\mathrm{e}^x-c^2\,\mathrm{e}^x+a\,\mathrm{e}^x\sqrt{a^2-b^2+c^2}\right)\left(B\,c^3+C\,b^3+B\,a^2\cdots\right.}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c\cdots}+\frac{x\,(B-C)}{b-c}$$

input
```
int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x)),x)
```

output
```
(log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) + a^2*exp(x) -
b^2*exp(x) + c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + C*b^3
 + B*a^2*c - C*a^2*b - B*b^2*c - C*b*c^2 + B*a*b*(a^2 - b^2 + c^2)^(1/2) -
 C*a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^
2) + (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) - a^2*exp(
x) + b^2*exp(x) - c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 +
C*b^3 + B*a^2*c - C*a^2*b - B*b^2*c - C*b*c^2 - B*a*b*(a^2 - b^2 + c^2)^(1
/2) + C*a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b
^2*c^2) + (x*(B - C))/(b - c)
```

3.795. $\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

3.796 $\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

3.796.1 Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\frac{2(bB-cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}-\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

output
```
2*(B*b-C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^
2)^(3/2)+(-B*c+b*C+a*C*cosh(x)+a*B*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*s
inh(x))
```

3.796.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\frac{2(bB-cC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{3/2}}+\frac{bBc+a^2C-b^2C+a(-bB+cC)\sinh(x)}{b\,(-a^2+b^2-c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

input
```
Integrate[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(2*(b*B - c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-
a^2 + b^2 - c^2)^(3/2) + (b*B*c + a^2*C - b^2*C + a*(-(b*B) + c*C)*Sinh[x]
)/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

3.796.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3632, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(ix)-iC\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^2}dx$$

$$\Big\downarrow \text{3632}$$

$$-\frac{(bB-cC)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}-\frac{(bB-cC)\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{a^2-b^2+c^2}$$

$$\Big\downarrow \text{3603}$$

$$-\frac{2(bB-cC)\int \frac{1}{-((a-b)\tanh^2(\frac{x}{2}))+2c\tanh(\frac{x}{2})+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2+c^2}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4(bB-cC)\int \frac{1}{4(a^2-b^2+c^2)-(2c-2(a-b)\tanh(\frac{x}{2}))^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{219}$$

$$\frac{2(bB-cC)\text{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)}$$

input
```
Int[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(2*(b*B - c*C)*ArcTanh[(2*c - 2*(a - b)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^2
])])/(a^2 - b^2 + c^2)^(3/2) - (B*c - b*C - a*C*Cosh[x] - a*B*Sinh[x])/((a
^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

3.796.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.796. $\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

3.796.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(99) = 198.

Time = 1.72 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.08

method	result
default	$\frac{-\frac{2\left(B\,a^{2}-abB+B\,c^{2}+acC-Ccb\right)\tanh\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}-\frac{2\left(bBc+C\,a^{2}-C\,b^{2}\right)}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}}{a\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{2}b-2c\tanh\left(\frac{x}{2}\right)-a-b}+\frac{2(Bb-Cc)\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{(a^{2}-b^{2}+c^{2})\sqrt{-a^{2}+b^{2}-c^{2}}}$
risch	$-\frac{2(B\,a^{2}e^{x}+Bbc\,e^{x}+B\,c^{2}e^{x}+C\,a^{2}e^{x}-C\,b^{2}e^{x}-Cbc\,e^{x}+abB-acC)}{(b+c)(a^{2}-b^{2}+c^{2})(e^{2x}b+e^{2x}c+2a\,e^{x}+b-c)}+\frac{\ln\left(e^{x}+\frac{\left(a^{2}-b^{2}+c^{2}\right)^{\frac{3}{2}}a+a^{4}-2a^{2}b^{2}+2a^{2}c^{2}+b^{4}-2b^{2}c^{2}+c^{4}}{\left(a^{2}-b^{2}+c^{2}\right)^{\frac{3}{2}}(b+c)}\right)B}{(a^{2}-b^{2}+c^{2})^{\frac{3}{2}}}$

input
```
int((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOS
E)
```

output
```
2*(-(B*a^2-B*a*b+B*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2)*tanh
(1/2*x)-(B*b*c+C*a^2-C*b^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2
*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)+2*(B*b-C*c)/(a^2-b^2+c^2)/(-a^2
+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

3.796.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. 2(101) = 202.

Time = 0.31 (sec) , antiderivative size = 2119, normalized size of antiderivative = 19.62

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fr
icas")
```

output
```
[-(2*B*a^3*b - 2*B*a*b^3 + 2*B*a*b*c^2 - 2*C*a*c^3 + (B*b^3 - C*b^2*c - B*
b*c^2 + C*c^3 + (B*b^3 + (2*B - C)*b^2*c + (B - 2*C)*b*c^2 - C*c^3)*cosh(x
)^2 + (B*b^3 + (2*B - C)*b^2*c + (B - 2*C)*b*c^2 - C*c^3)*sinh(x)^2 + 2*(B
*a*b^2 + (B - C)*a*b*c - C*a*c^2)*cosh(x) + 2*(B*a*b^2 + (B - C)*a*b*c - C
*a*c^2 + (B*b^3 + (2*B - C)*b^2*c + (B - 2*C)*b*c^2 - C*c^3)*cosh(x))*sinh
(x))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b
*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b +
 a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b
+ c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^
2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*(C*a^3 - C
*a*b^2)*c + 2*((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)*b*c^3 + B
*c^4 + ((2*B + C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B - C)*b^3)*c
)*cosh(x) + 2*((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)*b*c^3 + B
*c^4 + ((2*B + C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B - C)*b^3)*c
)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 -
 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*
a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*
b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*
a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*
b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^...
```

3.796.6 Sympy [F(-1)]

Timed out.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

output
```
Timed out
```

3.796.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.796.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.66

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx$$
$$= -\frac{2\,(Bb - Cc)\arctan\left(\frac{be^x + ce^x + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{(a^2 - b^2 + c^2)\sqrt{-a^2 + b^2 - c^2}}$$
$$- \frac{2\,(Ba^2e^x + Ca^2e^x - Cb^2e^x + Bbce^x - Cbce^x + Bc^2e^x + Bab - Cac)}{(a^2b - b^3 + a^2c - b^2c + bc^2 + c^3)(be^{(2\,x)} + ce^{(2\,x)} + 2\,ae^x + b - c)}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="gi
ac")
```

output
```
-2*(B*b - C*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 -
b^2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(B*a^2*e^x + C*a^2*e^x - C*b^2*e^x
+ B*b*c*e^x - C*b*c*e^x + B*c^2*e^x + B*a*b - C*a*c)/((a^2*b - b^3 + a^2*c
 - b^2*c + b*c^2 + c^3)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

3.796.9 Mupad [F(-1)]

Timed out.

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\int \frac{B\cosh(x)+C\sinh(x)}{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)^2}\,dx$$

input
```
int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2,x)
```

output
```
int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2, x)
```

3.797 $\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

3.797.1 Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \frac{3a(bB-cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}} - \frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)-(2bBc+(a^2-2c^2)C)\cosh(x)-(a^2B+2b(bB-cC))\sinh(x)}{2(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))}$$

output
```
3*a*(B*b-C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+
c^2)^(5/2)+1/2*(-B*c+b*C+a*C*cosh(x)+a*B*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(
x)+c*sinh(x))^2+1/2*(-a*(B*c-C*b)+(2*b*B*c+(a^2-2*c^2)*C)*cosh(x)+(B*a^2+2
*b*(B*b-C*c))*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

3.797.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.64

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{3a(bB-cC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{5/2}} + \frac{-9a^2bBc-2a^4C+4a^2b^2C-2b^4C+5a^2c^2C+4b^2c^2C-2c^4C-6abc(bB-cC)\cosh(x)+c(a^2+2b^2}{}$$

input
```
Integrate[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
(-3*a*(b*B - c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])
/(-a^2 + b^2 - c^2)^(5/2) + (-9*a^2*b*B*c - 2*a^4*C + 4*a^2*b^2*C - 2*b^4*
C + 5*a^2*c^2*C + 4*b^2*c^2*C - 2*c^4*C - 6*a*b*c*(b*B - c*C)*Cosh[x] + c*
(a^2 + 2*b^2 - 2*c^2)*(b*B - c*C)*Cosh[2*x] + 4*a^3*b*B*Sinh[x] + 2*a*b^3*
B*Sinh[x] - 8*a*b*B*c^2*Sinh[x] - 4*a^3*c*C*Sinh[x] - 2*a*b^2*c*C*Sinh[x]
+ 8*a*c^3*C*Sinh[x] + a^2*b^2*B*Sinh[2*x] + 2*b^4*B*Sinh[2*x] - 2*b^2*B*c^
2*Sinh[2*x] - a^2*b*c*C*Sinh[2*x] - 2*b^3*c*C*Sinh[2*x] + 2*b*c^3*C*Sinh[2
*x])/(4*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])^2)
```

3.797.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3635, 3042, 3632, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(ix)-iC\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^3}dx$$

$$\Big\downarrow \text{3635}$$

$$-\frac{\int \frac{2(bB-cC)-aB\cosh(x)-aC\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx}{2(a^2-b^2+c^2)}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}-\frac{\int \frac{2(bB-cC)-aB\cos(ix)+iaC\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^2}dx}{2(a^2-b^2+c^2)}$$

$$\Big\downarrow \text{3632}$$

$$-\frac{\frac{3a(bB-cC)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}+\frac{-\cosh(x)\left(C\left(a^2-2c^2\right)+2bBc\right)-\sinh(x)\left(a^2B+2b(bB-cC)\right)+a(Bc-bC)}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}-\frac{\frac{-\cosh(x)\left(C\left(a^2-2c^2\right)+2bBc\right)-\sinh(x)\left(a^2B+2b(bB-cC)\right)+a(Bc-bC)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{3a(bB-cC)\int\frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{a^2-b^2+c^2}}{2\left(a^2-b^2+c^2\right)}$$

$$\Big\downarrow \text{3603}$$

$$-\frac{\frac{6a(bB-cC)\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+2c\tanh\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2+c^2}+\frac{-\cosh(x)\left(C\left(a^2-2c^2\right)+2bBc\right)-\sinh(x)\left(a^2B+2b(bB-cC)\right)+a(Bc-bC)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}}{2\left(a^2-b^2+c^2\right)}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\frac{-\cosh(x)\left(C\left(a^2-2c^2\right)+2bBc\right)-\sinh(x)\left(a^2B+2b(bB-cC)\right)+a(Bc-bC)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{12a(bB-cC)\int\frac{1}{4\left(a^2-b^2+c^2\right)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d(2c-2(a-b)\text{ t}}{a^2-b^2+c^2}}{2\left(a^2-b^2+c^2\right)}$$
$$\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{-\cosh(x)\left(C\left(a^2-2c^2\right)+2bBc\right)-\sinh(x)\left(a^2B+2b(bB-cC)\right)+a(Bc-bC)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{6a(bB-cC)\text{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}}{2\left(a^2-b^2+c^2\right)}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}$$

input
```
Int[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
-1/2*(B*c - b*C - a*C*Cosh[x] - a*B*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Cos
h[x] + c*Sinh[x])^2) - ((-6*a*(b*B - c*C)*ArcTanh[(2*c - 2*(a - b)*Tanh[x/
2])/(2*Sqrt[a^2 - b^2 + c^2])])/(a^2 - b^2 + c^2)^(3/2) + (a*(B*c - b*C) -
 (2*b*B*c + (a^2 - 2*c^2)*C)*Cosh[x] - (a^2*B + 2*b*(b*B - c*C))*Sinh[x])/
((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])))/(2*(a^2 - b^2 + c^2))
```

3.797. $\int\frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

3.797.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.797. $\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

3.797.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(181) = 362$.

Time = 11.48 (sec) , antiderivative size = 800, normalized size of antiderivative = 4.12

method	result
default	$-\frac{\left(2B\,a^4-3B\,a^3b+2B\,a^2b^2+4B\,a^2c^2-3Ba\,b^3+2B\,b^4-4B\,b^2c^2+2B\,c^4+3C\,a^3c-6C\,a^2bc+3Ca\,b^2c\right)\tanh\left(\frac{x}{2}\right)^3}{\left(a^4-2a^2b^2+2a^2c^2+b^4-2b^2c^2+c^4\right)(a-b)}+\frac{\left(2B\,a^4c-9B\,a^3bc+14B\,a^2b^2c+4 \right.}{}$
risch	$-\frac{-C\,a^2bc-B\,a^2bc+2Cb\,c^3-2C\,c^4+2Bb\,c^3-2C\,b^3c+C\,a^2c^2+2C\,b^2c^2+B\,a^2b^2-2B\,b^3c-2B\,b^2c^2+2B\,b^4-4C\,a^2b^2\mathrm{e}^{2x}-5C\,a^2c^2\mathrm{e}^{2x}-}{}$

input
```
int((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOS
E)
```

output
```
2*(-1/2*(2*B*a^4-3*B*a^3*b+2*B*a^2*b^2+4*B*a^2*c^2-3*B*a*b^3+2*B*b^4-4*B*b
^2*c^2+2*B*c^4+3*C*a^3*c-6*C*a^2*b*c+3*C*a*b^2*c)/(a^4-2*a^2*b^2+2*a^2*c^2
+b^4-2*b^2*c^2+c^4)/(a-b)*tanh(1/2*x)^3+1/2*(2*B*a^4*c-9*B*a^3*b*c+14*B*a^
2*b^2*c+4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c-4*B*b^2*c^3+2*B*c^5-2*C*a^5+2*C*
a^4*b+4*C*a^3*b^2+5*C*a^3*c^2-4*C*a^2*b^3-14*C*a^2*b*c^2-2*C*a*b^4+13*C*a*
b^2*c^2-2*C*a*c^4+2*C*b^5-4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+
b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(2*B*a^5-3*B*a^4*b+B*
a^3*b^2+4*B*a^3*c^2+B*a^2*b^3+8*B*a^2*b*c^2-3*B*a*b^4-8*B*a*b^2*c^2+2*B*a*
c^4+2*B*b^5-4*B*b^3*c^2+2*B*b*c^4+5*C*a^4*c-5*C*a^3*b*c-5*C*a^2*b^2*c-4*C*
a^2*c^3+5*C*a*b^3*c+4*C*a*b*c^3)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^
4)/(a^2-2*a*b+b^2)*tanh(1/2*x)+1/2*a*(5*B*a^2*b*c-5*B*b^3*c+2*B*b*c^3+2*C*
a^4-4*C*a^2*b^2-C*a^2*c^2+2*C*b^4+C*b^2*c^2)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-
2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(
1/2*x)-a-b)^2+3*a*(B*b-C*c)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-
a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/
2))
```

3.797.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4996 vs. $2(182) = 364$.

Time = 0.59 (sec) , antiderivative size = 10107, normalized size of antiderivative = 52.10

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.797.6 Sympy [F(-1)]

Timed out.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

output
```
Timed out
```

3.797.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.797.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(182) = 364$.

Time = 0.27 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.97

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx$$
$$= -\frac{3\,(Bab - Cac)\arctan\left(\frac{be^x + ce^x + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{(a^4 - 2\,a^2b^2 + b^4 + 2\,a^2c^2 - 2\,b^2c^2 + c^4)\sqrt{-a^2 + b^2 - c^2}}$$
$$- \frac{3\,Bab^3e^{(3\,x)} + 6\,Bab^2ce^{(3\,x)} - 3\,Cab^2ce^{(3\,x)} + 3\,Babc^2e^{(3\,x)} - 6\,Cabc^2e^{(3\,x)} - 3\,Cac^3e^{(3\,x)} + 2\,Ba^4e^{(2\,x)} +}{}$$

input
```
integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="gi
ac")
```

output
```
-3*(B*a*b - C*a*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^
4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^2 + b^2 - c^2))
 - (3*B*a*b^3*e^(3*x) + 6*B*a*b^2*c*e^(3*x) - 3*C*a*b^2*c*e^(3*x) + 3*B*a*
b*c^2*e^(3*x) - 6*C*a*b*c^2*e^(3*x) - 3*C*a*c^3*e^(3*x) + 2*B*a^4*e^(2*x)
+ 2*C*a^4*e^(2*x) + 5*B*a^2*b^2*e^(2*x) - 4*C*a^2*b^2*e^(2*x) + 2*B*b^4*e^
(2*x) + 2*C*b^4*e^(2*x) + 9*B*a^2*b*c*e^(2*x) - 9*C*a^2*b*c*e^(2*x) + 4*B*
a^2*c^2*e^(2*x) - 5*C*a^2*c^2*e^(2*x) - 4*B*b^2*c^2*e^(2*x) - 4*C*b^2*c^2*
e^(2*x) + 2*B*c^4*e^(2*x) + 2*C*c^4*e^(2*x) + 4*B*a^3*b*e^x + 5*B*a*b^3*e^
x - 4*C*a^3*c*e^x - 5*C*a*b^2*c*e^x - 5*B*a*b*c^2*e^x + 5*C*a*c^3*e^x + B*
a^2*b^2 + 2*B*b^4 - B*a^2*b*c - C*a^2*b*c - 2*B*b^3*c - 2*C*b^3*c + C*a^2*
c^2 - 2*B*b^2*c^2 + 2*C*b^2*c^2 + 2*B*b*c^3 + 2*C*b*c^3 - 2*C*c^4)/((a^4*b
 - 2*a^2*b^3 + b^5 + a^4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2
 + 2*a^2*c^3 - 2*b^2*c^3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x +
 b - c)^2)
```

3.797.9 Mupad [F(-1)]

Timed out.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx$$

input
```
int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3,x)
```

output
```
int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3, x)
```

3.797. $\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

3.798 $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

3.798.1 Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x}{b^2-c^2} - \frac{2(Ab^2-abB-Ac^2+acC)\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

output
```
(B*b-C*c)*x/(b^2-c^2)-(B*c-C*b)*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)-2*(A*b
^2-A*c^2-B*a*b+C*a*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(
b^2-c^2)/(a^2-b^2+c^2)^(1/2)
```

3.798.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x + \frac{2(Ab^2-abB-Ac^2+acC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}} + (-Bc+bC)\log(a+b\cosh(x)+c\sinh(x))}{(b-c)(b+c)}$$

input
```
Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

output
```
((b*B - c*C)*x + (2*(A*b^2 - a*b*B - A*c^2 + a*c*C)*ArcTan[(c + (-a + b)*T
anh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + (-(B*c) + b*C)
*Log[a + b*Cosh[x] + c*Sinh[x]])/((b - c)*(b + c))
```

3.798.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3615, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)-iC\sin(ix)}{a+b\cos(ix)-ic\sin(ix)}dx$$

$$\Big\downarrow \text{3615}$$

$$\frac{(-abB+acC+Ab^2-Ac^2)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{b^2-c^2}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-abB+acC+Ab^2-Ac^2)\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{b^2-c^2}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

$$\Big\downarrow \text{3603}$$

$$\frac{2(-abB+acC+Ab^2-Ac^2)\int \frac{1}{-((a-b)\tanh^2(\frac{x}{2}))+2c\tanh(\frac{x}{2})+a+b}d\tanh\left(\frac{x}{2}\right)}{b^2-c^2}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{4\left(-abB+acC+Ab^2-Ac^2\right)\int \frac{1}{4(a^2-b^2+c^2)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(-abB+acC+Ab^2-Ac^2\right)\operatorname{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

input
```
Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

output
```
((b*B - c*C)*x)/(b^2 - c^2) - (2*(A*b^2 - a*b*B - A*c^2 + a*c*C)*ArcTanh[(
2*c - 2*(a - b)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^2])])/((b^2 - c^2)*Sqrt[a
^2 - b^2 + c^2]) - ((B*c - b*C)*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2
)
```

3.798.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3615
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + (Simp[(c*B - b*C)*(Log[a
 + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c
^2) - a*(b*B + c*C))/(b^2 + c^2)   Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*
x]), x], x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] &&
NeQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

3.798.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.73

method	result
default	$\frac{2(B-C)\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2b-2c}+\frac{\frac{2\left(-aBc+bBc+abC-C\,b^{2}\right)\ln\left(a\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{2}b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a-2b}+\frac{2\left(-A\,b^{2}+A\,c^{2}+abB+B\,c^{2}-acC\right.}{}}{(b-c)(b+c)}$
risch	Expression too large to display

input
```
int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOS
E)
```

output
```
2*(B-C)/(2*b-2*c)*ln(1+tanh(1/2*x))+2/(b-c)/(b+c)*(1/2*(-B*a*c+B*b*c+C*a*b
-C*b^2)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)+(-A*
b^2+A*c^2+a*b*B+B*c^2-a*c*C-C*c*b+(-B*a*c+B*b*c+C*a*b-C*b^2)*c/(a-b))/(-a^
2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2)
))+2*(-B-C)/(2*b+2*c)*ln(tanh(1/2*x)-1)
```

3.798.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.42

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$$
$$=\left[\frac{(Bab-Ab^2-Cac+Ac^2)\sqrt{a^2-b^2+c^2}\log\left(\frac{(b^2+2\,bc+c^2)\cosh(x)^2+(b^2+2\,bc+c^2)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2-}\right.}{}\right.$$
$$-\frac{2\,(Bab-Ab^2-Cac+Ac^2)\sqrt{-a^2+b^2-c^2}\arctan\left(\frac{\sqrt{-a^2+b^2-c^2}((b+c)\cosh(x)+(b+c)\sinh(x)+a)}{a^2-b^2+c^2}\right)-((B-C)}{}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fr
icas")
```

output
```
[((B*a*b - A*b^2 - C*a*c + A*c^2)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c
+ c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(
a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) +
 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)
*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sin
h(x) + b - c)) + ((B - C)*a^2*b - (B - C)*b^3 + (B - C)*b*c^2 + (B - C)*c^
3 + ((B - C)*a^2 - (B - C)*b^2)*c)*x + (C*a^2*b - C*b^3 + C*b*c^2 - B*c^3
- (B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))
))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2), -(2*(B*a*b - A*b^2 - C*a*c +
 A*c^2)*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh
(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - ((B - C)*a^2*b - (B - C)*b
^3 + (B - C)*b*c^2 + (B - C)*c^3 + ((B - C)*a^2 - (B - C)*b^2)*c)*x - (C*a
^2*b - C*b^3 + C*b*c^2 - B*c^3 - (B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*s
inh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2
)]
```

3.798.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

output
```
Timed out
```

3.798.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.798.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(B-C)x}{b-c} + \frac{(Cb-Bc)\log\left(be^{(2x)}+ce^{(2x)}+2\,ae^{x}+b-c\right)}{b^2-c^2} - \frac{2\left(Bab-Ab^2-Cac+Ac^2\right)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}\,(b^2-c^2)}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="gi
ac")
```

output
```
(B - C)*x/(b - c) + (C*b - B*c)*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b -
c)/(b^2 - c^2) - 2*(B*a*b - A*b^2 - C*a*c + A*c^2)*arctan((b*e^x + c*e^x +
 a)/sqrt(-a^2 + b^2 - c^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

3.798.9 Mupad [B] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.31

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{\ln\left(b\sqrt{a^2-b^2+c^2}-c\sqrt{a^2-b^2+c^2}+a^2\,\mathrm{e}^{x}-b^2\,\mathrm{e}^{x}+c^2\,\mathrm{e}^{x}+a\,\mathrm{e}^{x}\sqrt{a^2-b^2+c^2}\right)\left(B\,c^3+C\,b^3-A\,b^2\sqrt{c}\right.}{-a^2\,b^2+a^2} + \frac{\ln\left(b\sqrt{a^2-b^2+c^2}-c\sqrt{a^2-b^2+c^2}-a^2\,\mathrm{e}^{x}+b^2\,\mathrm{e}^{x}-c^2\,\mathrm{e}^{x}+a\,\mathrm{e}^{x}\sqrt{a^2-b^2+c^2}\right)\left(B\,c^3+C\,b^3+A\,b^2\right.}{-a^2\,b^2+} + \frac{x\,(B-C)}{b-c}$$

input
```
int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x)),x)
```

output
```
(log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) + a^2*exp(x) -
b^2*exp(x) + c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + C*b^3
 - A*b^2*(a^2 - b^2 + c^2)^(1/2) + B*a^2*c - C*a^2*b + A*c^2*(a^2 - b^2 +
c^2)^(1/2) - B*b^2*c - C*b*c^2 + B*a*b*(a^2 - b^2 + c^2)^(1/2) - C*a*c*(a^
2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) + (log(
b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) - a^2*exp(x) + b^2*e
xp(x) - c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + C*b^3 + A*
b^2*(a^2 - b^2 + c^2)^(1/2) + B*a^2*c - C*a^2*b - A*c^2*(a^2 - b^2 + c^2)^
(1/2) - B*b^2*c - C*b*c^2 - B*a*b*(a^2 - b^2 + c^2)^(1/2) + C*a*c*(a^2 - b
^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) + (x*(B - C)
)/(b - c)
```

3.798. $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

3.799 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

3.799.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2(aA-bB+cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}} - \frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

output
```
-2*(A*a-B*b+C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b
^2+c^2)^(3/2)+(-B*c+b*C-(A*c-C*a)*cosh(x)-(A*b-B*a)*sinh(x))/(a^2-b^2+c^2)
/(a+b*cosh(x)+c*sinh(x))
```

3.799.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$$
$$= -\frac{2(aA-bB+cC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{3/2}} + \frac{-aAc+a^2C+b(Bc-bC)+\left(-abB+A\left(b^2-c^2\right)+acC\right)\sinh(x)}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

input
```
Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(-2*(a*A - b*B + c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^
2]])/(-a^2 + b^2 - c^2)^(3/2) + (-(a*A*c) + a^2*C + b*(B*c - b*C) + (-(a*b
*B) + A*(b^2 - c^2) + a*c*C)*Sinh[x])/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x]
 + c*Sinh[x]))
```

3.799.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3632, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)-iC\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^2}dx$$

$$\Big\downarrow \text{3632}$$

$$\frac{(aA-bB+cC)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}+\frac{(aA-bB+cC)\int \frac{1}{a+b\cos(ix)-ic\sin(ix)}dx}{a^2-b^2+c^2}$$

$$\Big\downarrow \text{3603}$$

$$\frac{2(aA-bB+cC)\int \frac{1}{-((a-b)\tanh^2(\frac{x}{2}))+2c\tanh(\frac{x}{2})+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2+c^2}-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{4(aA-bB+cC)\int \frac{1}{4(a^2-b^2+c^2)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

$$\Big\downarrow \; 219$$

$$-\frac{2(aA-bB+cC)\text{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

input
```
Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

output
```
(-2*(a*A - b*B + c*C)*ArcTanh[(2*c - 2*(a - b)*Tanh[x/2])/(2*Sqrt[a^2 - b^
2 + c^2])])/(a^2 - b^2 + c^2)^(3/2) - (B*c - b*C + (A*c - a*C)*Cosh[x] + (
A*b - a*B)*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

3.799.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.799. $\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.799.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(116) = 232$.

Time = 1.74 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.06

method	result
default	$-\frac{2\left(-\frac{\left(Aab-A\,b^{2}+A\,c^{2}-B\,a^{2}+abB-B\,c^{2}-acC+Ccb\right)\tanh\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}-\frac{Aac-bBc-C\,a^{2}+C\,b^{2}}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}\right)}{a\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{2}b-2c\tanh\left(\frac{x}{2}\right)-a-b}-\frac{2(Aa-Bb+Cc)\arctan\left(\frac{2(a-b)\tanh(\ldots}{2\sqrt{-a^{2}+b^{2}\ldots}}\right.}{(a^{2}-b^{2}+c^{2})\sqrt{-a^{2}+b^{2}-c^{2}}\ldots}$
risch	$\frac{2Aab\,\mathrm{e}^{x}+2Aac\,\mathrm{e}^{x}-2B\,a^{2}\mathrm{e}^{x}-2Bbc\,\mathrm{e}^{x}-2B\,c^{2}\mathrm{e}^{x}-2C\,a^{2}\mathrm{e}^{x}+2C\,b^{2}\mathrm{e}^{x}+2Cbc\,\mathrm{e}^{x}+2A\,b^{2}-2A\,c^{2}-2abB+2acC}{(b+c)(a^{2}-b^{2}+c^{2})(\mathrm{e}^{2x}b+\mathrm{e}^{2x}c+2a\,\mathrm{e}^{x}+b-c)}+\frac{\ln\left(\mathrm{e}^{x}+\frac{\left(a^{2}-b^{2}+c^{2}\right)^{\frac{3}{2}}\ldots}{\ldots}\right.}{\ldots}$

input
```
int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERB
OSE)
```

output
```
-2*(-(A*a*b-A*b^2+A*c^2-B*a^2+B*a*b-B*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2+a*
c^2+b^3-b*c^2)*tanh(1/2*x)-(A*a*c-B*b*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2+a*c^
2+b^3-b*c^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)-2*(A*a
-B*b+C*c)/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*
x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

3.799.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. $2(116) = 232$.

Time = 0.31 (sec) , antiderivative size = 2541, normalized size of antiderivative = 21.00

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="
fricas")
```

output
```
[-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - 2*C*a*c^3 + 2*A*c^4 + 2
*(A*a^2 + B*a*b - 2*A*b^2)*c^2 - (A*a*b^2 - B*b^3 + C*b^2*c - C*c^3 - (A*a
 - B*b)*c^2 + (A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*
b - (2*B - C)*b^2)*c)*cosh(x)^2 + (A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2
*C)*b)*c^2 + (2*A*a*b - (2*B - C)*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2
 + C*a*c^2 + (A*a^2 - (B - C)*a*b)*c)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + C*a
*c^2 + (A*a^2 - (B - C)*a*b)*c + (A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*
C)*b)*c^2 + (2*A*a*b - (2*B - C)*b^2)*c)*cosh(x))*sinh(x))*sqrt(a^2 - b^2
+ c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2
+ 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c
+ c^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b +
c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*
((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*(C*a^3 - C*a*b^2)*c + 2*((B +
C)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4 + B*c^4 - (A*a - (B
 - C)*b)*c^3 + ((2*B + C)*a^2 - A*a*b - (B + C)*b^2)*c^2 - (A*a^3 - (B - C
)*a^2*b - A*a*b^2 + (B - C)*b^3)*c)*cosh(x) + 2*((B + C)*a^4 - A*a^3*b - (
B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4 + B*c^4 - (A*a - (B - C)*b)*c^3 + ((2*B
 + C)*a^2 - A*a*b - (B + C)*b^2)*c^2 - (A*a^3 - (B - C)*a^2*b - A*a*b^2 +
(B - C)*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2
)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*...
```

3.799.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

output
```
Timed out
```

3.799.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.799.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.71

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \frac{2\,(Aa - Bb + Cc)\arctan\left(\frac{be^x + ce^x + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{(a^2 - b^2 + c^2)\sqrt{-a^2 + b^2 - c^2}} - \frac{2\,(Ba^2e^x + Ca^2e^x - Aabe^x - Cb^2e^x - Aace^x + Bbce^x - Cbce^x + Bc^2e^x + Bab - Ab^2 - Cac + Ac^2)}{(a^2b - b^3 + a^2c - b^2c + bc^2 + c^3)(be^{(2\,x)} + ce^{(2\,x)} + 2\,ae^x + b - c)}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="
giac")
```

output
```
2*(A*a - B*b + C*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a
^2 - b^2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(B*a^2*e^x + C*a^2*e^x - A*a*b
*e^x - C*b^2*e^x - A*a*c*e^x + B*b*c*e^x - C*b*c*e^x + B*c^2*e^x + B*a*b -
 A*b^2 - C*a*c + A*c^2)/((a^2*b - b^3 + a^2*c - b^2*c + b*c^2 + c^3)*(b*e^
(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

3.799.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh\left(x\right)+c\sinh\left(x\right))^2}\,dx$$

input
```
int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2,x)
```

output
```
int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2, x)
```

3.800 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

3.800.1 Optimal result

Integrand size = 23, antiderivative size = 233

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{(2a^2A+Ab^2-3abB-Ac^2+3acC)\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}} - \frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)+(3aAc-a^2C-2c(bB-cC))\cosh(x)+(3aAb-a^2B-2b(bB-cC))\sinh(x)}{2(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))}$$

output
```
-(2*A*a^2+A*b^2-A*c^2-3*B*a*b+3*C*a*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-
b^2+c^2)^(1/2))/(a^2-b^2+c^2)^(5/2)+1/2*(-B*c+b*C-(A*c-C*a)*cosh(x)-(A*b-B
*a)*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^2+1/2*(-a*(B*c-C*b)-(3*
A*a*c-C*a^2-2*c*(B*b-C*c))*cosh(x)-(3*A*a*b-B*a^2-2*b*(B*b-C*c))*sinh(x))/
(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

3.800.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.00

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$$
$$= \frac{(2a^2A+Ab^2-3abB-Ac^2+3acC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{5/2}}$$
$$+\frac{6a^3Ac+3aAb^2c-9a^2bBc-3aAc^3-2a^4C+4a^2b^2C-2b^4C+5a^2c^2C+4b^2c^2C-2c^4C+2bc(2a^2A}{}$$

input
```
Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
((2*a^2*A + A*b^2 - 3*a*b*B - A*c^2 + 3*a*c*C)*ArcTan[(c + (-a + b)*Tanh[x
/2])/Sqrt[-a^2 + b^2 - c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (6*a^3*A*c + 3*a*
A*b^2*c - 9*a^2*b*B*c - 3*a*A*c^3 - 2*a^4*C + 4*a^2*b^2*C - 2*b^4*C + 5*a^
2*c^2*C + 4*b^2*c^2*C - 2*c^4*C + 2*b*c*(2*a^2*A + A*b^2 - 3*a*b*B - A*c^2
 + 3*a*c*C)*Cosh[x] + c*(3*a*A*(-b^2 + c^2) + a^2*(b*B - c*C) + 2*(b^2 - c
^2)*(b*B - c*C))*Cosh[2*x] - 8*a^2*A*b^2*Sinh[x] + 2*A*b^4*Sinh[x] + 4*a^3
*b*B*Sinh[x] + 2*a*b^3*B*Sinh[x] + 12*a^2*A*c^2*Sinh[x] - 2*A*b^2*c^2*Sinh
[x] - 8*a*b*B*c^2*Sinh[x] - 4*a^3*c*C*Sinh[x] - 2*a*b^2*c*C*Sinh[x] + 8*a*
c^3*C*Sinh[x] - 3*a*A*b^3*Sinh[2*x] + a^2*b^2*B*Sinh[2*x] + 2*b^4*B*Sinh[2
*x] + 3*a*A*b*c^2*Sinh[2*x] - 2*b^2*B*c^2*Sinh[2*x] - a^2*b*c*C*Sinh[2*x]
- 2*b^3*c*C*Sinh[2*x] + 2*b*c^3*C*Sinh[2*x])/(4*b*(a^2 - b^2 + c^2)^2*(a +
 b*Cosh[x] + c*Sinh[x])^2)
```

3.800.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 3635, 25, 3042, 3632, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)-iC\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^3}dx$$

$$\Big\downarrow \text{ 3635}$$

$$-\frac{\int -\frac{2(aA-bB+cC)-(Ab-aB)\cosh(x)-(Ac-aC)\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx}{2(a^2-b^2+c^2)}-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2(aA-bB+cC)-(Ab-aB)\cosh(x)-(Ac-aC)\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}dx}{2(a^2-b^2+c^2)}-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2(aA-bB+cC)-(Ab-aB)\cos(ix)+i(Ac-aC)\sin(ix)}{(a+b\cos(ix)-ic\sin(ix))^2}dx}{2(a^2-b^2+c^2)}$$

$$\Big\downarrow \text{ 3632}$$

$$\frac{\frac{(2a^2A-3abB+3acC+Ab^2-Ac^2)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}dx}{a^2-b^2+c^2}-\frac{\sinh(x)(a^2(-B)+3aAb-2b(bB-cC))+\cosh(x)(a^2(-C)+3aAc-2c(bB-cC))+a(B}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}}{2(a^2-b^2+c^2)}$$
$$\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}+\frac{-\frac{\sinh(x)(a^2(-B)+3aAb-2b(bB-cC))+\cosh(x)(a^2(-C)+3aAc-2c(bB-cC))+a(Bc-bC)}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}+\frac{(2a^2A-3abB+3acC+Ab^2-Ac^2)\int \frac{1}{a+b\cos(ix)-i}}{a^2-b^2+c^2}}{2(a^2-b^2+c^2)}$$

$$\Big\downarrow \text{ 3603}$$

$$\frac{\frac{2(2a^2A-3abB+3acC+Ab^2-Ac^2)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+2c\tanh\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2+c^2}-\frac{\sinh(x)(a^2(-B)+3aAb-2b(bB-cC))+\cosh(x)(a^2(-C)+}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sin}}{2(a^2-b^2+c^2)}$$
$$\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}$$

$$\Big\downarrow \text{ 1083}$$

3.800. $\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

$$-\frac{-\frac{4(2a^2A-3abB+3acC+Ab^2-Ac^2)\int \frac{1}{4\left(a^2-b^2+c^2\right)-\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2c-2(a-b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b(bB-cC)\right)+\text{co}}{(a^2-b^2+c^2)(a\text{-}}}{2\left(a^2-b^2+c^2\right)}$$
$$\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{2(2a^2A-3abB+3acC+Ab^2-Ac^2)\text{arctanh}\left(\frac{2c-2(a-b)\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b(bB-cC)\right)+\cosh(x)\left(a^2(-C)+3aAc-2c(bB\text{-}}{(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))}}{2\left(a^2-b^2+c^2\right)}$$
$$\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}$$

input
```
Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

output
```
-1/2*(B*c - b*C + (A*c - a*C)*Cosh[x] + (A*b - a*B)*Sinh[x])/((a^2 - b^2 +
 c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) + ((-2*(2*a^2*A + A*b^2 - 3*a*b*B - A
*c^2 + 3*a*c*C)*ArcTanh[(2*c - 2*(a - b)*Tanh[x/2])/(2*Sqrt[a^2 - b^2 + c^
2])])/(a^2 - b^2 + c^2)^(3/2) - (a*(B*c - b*C) + (3*a*A*c - a^2*C - 2*c*(b
*B - c*C))*Cosh[x] + (3*a*A*b - a^2*B - 2*b*(b*B - c*C))*Sinh[x])/((a^2 -
b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])))/(2*(a^2 - b^2 + c^2))
```

3.800.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.800.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. $2(228) = 456$.

Time = 11.80 (sec) , antiderivative size = 1084, normalized size of antiderivative = 4.65

method	result	size
default	Expression too large to display	1084
risch	Expression too large to display	1966

input
```
int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERB
OSE)
```

output
```
-2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2+5*A*a^2*c^2+2*A*a*b^3-2*A*a*b*c^2+A*b^4-3*
A*b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2-4*B*a^2*c^2+3*B*a*b^3-2*B*
b^4+4*B*b^2*c^2-2*B*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a-b)/(a^4-2*a^
2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)*tanh(1/2*x)^3-1/2*(4*A*a^4*c-12*A*a^3*b
*c+13*A*a^2*b^2*c-7*A*a^2*c^3-6*A*a*b^3*c+6*A*a*b*c^3+A*b^4*c+A*b^2*c^3-2*
A*c^5+2*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b^2*c+4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4
*c-4*B*b^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*b+4*C*a^3*b^2+5*C*a^3*c^2-4*C*a^2*b
^3-14*C*a^2*b*c^2-2*C*a*b^4+13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5-4*C*b^3*c^2+2
*C*b*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh
(1/2*x)^2+1/2*(4*A*a^4*b-5*A*a^3*b^2+11*A*a^3*c^2-3*A*a^2*b^3-3*A*a^2*b*c^
2+5*A*a*b^4-7*A*a*b^2*c^2+2*A*a*c^4-A*b^5-A*b^3*c^2+2*A*b*c^4-2*B*a^5+3*B*
a^4*b-B*a^3*b^2-4*B*a^3*c^2-B*a^2*b^3-8*B*a^2*b*c^2+3*B*a*b^4+8*B*a*b^2*c^
2-2*B*a*c^4-2*B*b^5+4*B*b^3*c^2-2*B*b*c^4-5*C*a^4*c+5*C*a^3*b*c+5*C*a^2*b^
2*c+4*C*a^2*c^3-5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^
2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c+A*a^2*
c^3-A*b^4*c+A*b^2*c^3-5*B*a^3*b*c+5*B*a*b^3*c-2*B*a*b*c^3-2*C*a^5+4*C*a^3*
b^2+C*a^3*c^2-2*C*a*b^4-C*a*b^2*c^2)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^
2+c^4)/(a^2-2*a*b+b^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a
-b)^2-(2*A*a^2+A*b^2-A*c^2-3*B*a*b+3*C*a*c)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2
*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)...
```

3.800.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6850 vs. $2(223) = 446$.

Time = 0.73 (sec) , antiderivative size = 13813, normalized size of antiderivative = 59.28

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="
fricas")
```

output
```
Too large to include
```

3.800.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

output
```
Timed out
```

3.800.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.800.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. 2(223) = 446.

Time = 0.30 (sec) , antiderivative size = 819, normalized size of antiderivative = 3.52

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$$
$$= \frac{(2\,Aa^2-3\,Bab+Ab^2+3\,Cac-Ac^2)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4)\sqrt{-a^2+b^2-c^2}}$$
$$+\frac{2\,Aa^2b^2e^{(3\,x)}-3\,Bab^3e^{(3\,x)}+Ab^4e^{(3\,x)}+4\,Aa^2bce^{(3\,x)}-6\,Bab^2ce^{(3\,x)}+3\,Cab^2ce^{(3\,x)}+2\,Ab^3ce^{(3\,x)}+2\,A}{}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="
giac")
```

output
```
(2*A*a^2 - 3*B*a*b + A*b^2 + 3*C*a*c - A*c^2)*arctan((b*e^x + c*e^x + a)/s
qrt(-a^2 + b^2 - c^2))/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c
^4)*sqrt(-a^2 + b^2 - c^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*x) + A
*b^4*e^(3*x) + 4*A*a^2*b*c*e^(3*x) - 6*B*a*b^2*c*e^(3*x) + 3*C*a*b^2*c*e^(
3*x) + 2*A*b^3*c*e^(3*x) + 2*A*a^2*c^2*e^(3*x) - 3*B*a*b*c^2*e^(3*x) + 6*C
*a*b*c^2*e^(3*x) + 3*C*a*c^3*e^(3*x) - 2*A*b*c^3*e^(3*x) - A*c^4*e^(3*x) -
 2*B*a^4*e^(2*x) - 2*C*a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(2*
x) + 4*C*a^2*b^2*e^(2*x) + 3*A*a*b^3*e^(2*x) - 2*B*b^4*e^(2*x) - 2*C*b^4*e
^(2*x) + 6*A*a^3*c*e^(2*x) - 9*B*a^2*b*c*e^(2*x) + 9*C*a^2*b*c*e^(2*x) + 3
*A*a*b^2*c*e^(2*x) - 4*B*a^2*c^2*e^(2*x) + 5*C*a^2*c^2*e^(2*x) - 3*A*a*b*c
^2*e^(2*x) + 4*B*b^2*c^2*e^(2*x) + 4*C*b^2*c^2*e^(2*x) - 3*A*a*c^3*e^(2*x)
 - 2*B*c^4*e^(2*x) - 2*C*c^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*e^x -
5*B*a*b^3*e^x - A*b^4*e^x + 4*C*a^3*c*e^x + 5*C*a*b^2*c*e^x - 10*A*a^2*c^2
*e^x + 5*B*a*b*c^2*e^x + 2*A*b^2*c^2*e^x - 5*C*a*c^3*e^x - A*c^4*e^x - B*a
^2*b^2 + 3*A*a*b^3 - 2*B*b^4 + B*a^2*b*c + C*a^2*b*c - 3*A*a*b^2*c + 2*B*b
^3*c + 2*C*b^3*c - C*a^2*c^2 - 3*A*a*b*c^2 + 2*B*b^2*c^2 - 2*C*b^2*c^2 + 3
*A*a*c^3 - 2*B*b*c^3 - 2*C*b*c^3 + 2*C*c^4)/((a^4*b - 2*a^2*b^3 + b^5 + a^
4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2*a^2*c^3 - 2*b^2*c^
3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)^2)
```

3.800.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \int \frac{A+B\cosh(x)+C\sinh(x)}{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)^3}\,dx$$

input
```
int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3,x)
```

output
```
int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3, x)
```

3.801 $\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

3.801.1 Optimal result

Integrand size = 32, antiderivative size = 22

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\frac{c\cosh(x)+b\sinh(x)}{a+b\cosh(x)+c\sinh(x)}$$

output
```
(c*cosh(x)+b*sinh(x))/(a+b*cosh(x)+c*sinh(x))
```

3.801.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\frac{-ac+b^2\sinh(x)-c^2\sinh(x)}{b(a+b\cosh(x)+c\sinh(x))}$$

input
```
Integrate[(b^2 - c^2 + a*b*Cosh[x] + a*c*Sinh[x])/(a + b*Cosh[x] + c*Sinh[
x])^2,x]
```

output
```
(-(a*c) + b^2*Sinh[x] - c^2*Sinh[x])/(b*(a + b*Cosh[x] + c*Sinh[x]))
```

3.801.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3042, 3629}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ab\cosh(x)+ac\sinh(x)+b^2-c^2}{(a+b\cosh(x)+c\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{ab\cos(ix)-iac\sin(ix)+b^2-c^2}{(a+b\cos(ix)-ic\sin(ix))^2}dx$$

$$\Big\downarrow \text{3629}$$

$$\frac{b\sinh(x)+c\cosh(x)}{a+b\cosh(x)+c\sinh(x)}$$

input
```
Int[(b^2 - c^2 + a*b*Cosh[x] + a*c*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,
x]
```

output
```
(c*Cosh[x] + b*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])
```

3.801.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3629
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && EqQ[a*A
- b*B - c*C, 0]
```

3.801.4 Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

method	result	size
risch	$-\frac{2(a\,\mathrm{e}^{x}+b-c)}{\mathrm{e}^{2x}b+\mathrm{e}^{2x}c+2a\,\mathrm{e}^{x}+b-c}$	36
default	$\frac{-\frac{2\left(a b-b^{2}+c^{2}\right)\tanh\left(\frac{x}{2}\right)}{a-b}-\frac{2ac}{a-b}}{a\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{2}b-2c\tanh\left(\frac{x}{2}\right)-a-b}$	73

input
```
int((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x,method=_
RETURNVERBOSE)
```

output
```
-2*(a*exp(x)+b-c)/(exp(2*x)*b+exp(2*x)*c+2*a*exp(x)+b-c)
```

3.801.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(22) = 44$.

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2\,(a\cosh\left(x\right)+a\sinh\left(x\right)+b-c)}{(b+c)\cosh\left(x\right)^{2}+(b+c)\sinh\left(x\right)^{2}+2\,a\cosh\left(x\right)+2\,((b+c)\cosh\left(x\right)+a)\sinh\left(x\right)+b-c}$$

input
```
integrate((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, a
lgorithm="fricas")
```

output
```
-2*(a*cosh(x) + a*sinh(x) + b - c)/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2
+ 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)
```

3.801.6 Sympy [F(-1)]

Timed out.

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((b**2-c**2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x
)
```

output
```
Timed out
```

3.801.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, a
lgorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` f
or more de
```

3.801.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2\,(ae^x+b-c)}{be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c}$$

input
```
integrate((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, a
lgorithm="giac")
```

output
```
-2*(a*e^x + b - c)/(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)
```

3.801.9 Mupad [F(-1)]

Timed out.

$$\int \frac{b^2 - c^2 + ab\cosh(x) + ac\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \int \frac{b^2 + a\cosh(x)\;b - c^2 + a\sinh(x)\;c}{(a + b\cosh(x) + c\sinh(x))^2}\,dx$$

input
```
int((b^2 - c^2 + a*c*sinh(x) + a*b*cosh(x))/(a + b*cosh(x) + c*sinh(x))^2,
x)
```

output
```
int((b^2 - c^2 + a*c*sinh(x) + a*b*cosh(x))/(a + b*cosh(x) + c*sinh(x))^2,
 x)
```

3.802 $\int \frac{A+C \sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$

3.802.1 Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{(2aA+bC)x}{2a^2}+\frac{C\cosh(x)}{2a} - \frac{1}{2}\left(\frac{2A}{a}-\frac{C}{b}+\frac{bC}{a^2}\right)\log(a+b\cosh(x)+b\sinh(x)) - \frac{C\sinh(x)}{2a}$$

output
```
1/2*(2*A*a+C*b)*x/a^2+1/2*C*cosh(x)/a-1/2*(2*A*a*b-C*a^2+C*b^2)*ln(a+b*cos
h(x)+b*sinh(x))/a^2/b-1/2*C*sinh(x)/a
```

3.802.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{(2aAb+a^2C+b^2C)\,x+2abC\cosh(x)+2(-2aAb+a^2C-b^2C)\log\left((a+b)\cosh\left(\frac{x}{2}\right)+(-a+b)\sinh\left(\frac{x}{2}\right)\right.}{4a^2b}$$

input
```
Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

output
```
((2*a*A*b + a^2*C + b^2*C)*x + 2*a*b*C*Cosh[x] + 2*(-2*a*A*b + a^2*C - b^2
*C)*Log[(a + b)*Cosh[x/2] + (-a + b)*Sinh[x/2]] - 2*a*b*C*Sinh[x])/(4*a^2*
b)
```

3.802.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3610}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sinh(x)}{a+b\sinh(x)+b\cosh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A-iC\sin(ix)}{a-ib\sin(ix)+b\cos(ix)}dx$$

$$\Big\downarrow \text{ 3610}$$

$$\frac{x(2aA+bC)}{2a^2}-\frac{1}{2}\left(\frac{bC}{a^2}+\frac{2A}{a}-\frac{C}{b}\right)\log(a+b\sinh(x)+b\cosh(x))-\frac{C\sinh(x)}{2a}+\frac{C\cosh(x)}{2a}$$

input
```
Int[(A + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

output
```
((2*a*A + b*C)*x)/(2*a^2) + (C*Cosh[x])/(2*a) - (((2*A)/a - C/b + (b*C)/a^
2)*Log[a + b*Cosh[x] + b*Sinh[x]])/2 - (C*Sinh[x])/(2*a)
```

3.802.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3610
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - c*C)*(x
/(2*a^2)), x] + (-Simp[C*(Cos[d + e*x]/(2*a*e)), x] + Simp[c*C*(Sin[d + e*x
]/(2*a*b*e)), x] + Simp[((-a^2)*C + 2*a*c*A + b^2*C)*(Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*b*e)), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

3.802.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

method	result
risch	$\frac{C\,\mathrm{e}^{-x}}{2a}+\frac{xA}{a}+\frac{bxC}{2a^{2}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)A}{a}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)C}{2b}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)C}{2a^{2}}$
default	$-\frac{\left(2Aab-C\,a^{2}+C\,b^{2}\right)\ln\left(a\tanh\left(\frac{x}{2}\right)-b\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a^{2}b}+\frac{C}{a\left(1+\tanh\left(\frac{x}{2}\right)\right)}+\frac{\left(2Aa+bC\right)\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2a^{2}}-\frac{C\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}$

input
```
int((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*C/a/exp(x)+1/a*x*A+1/2/a^2*b*x*C-1/a*ln(exp(x)+1/b*a)*A+1/2/b*ln(exp(x
)+1/b*a)*C-1/2/a^2*b*ln(exp(x)+1/b*a)*C
```

3.802.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.51

$$\int\frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$$
$$=\frac{Cab+(2\,Aab+Cb^{2})x\cosh{(x)}+(2\,Aab+Cb^{2})x\sinh{(x)}+((Ca^{2}-2\,Aab-Cb^{2})\cosh{(x)}+(Ca^{2}-2\,A}{2\,(a^{2}b\cosh{(x)}+a^{2}b\sinh{(x)})}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/2*(C*a*b + (2*A*a*b + C*b^2)*x*cosh(x) + (2*A*a*b + C*b^2)*x*sinh(x) + (
(C*a^2 - 2*A*a*b - C*b^2)*cosh(x) + (C*a^2 - 2*A*a*b - C*b^2)*sinh(x))*log
(b*cosh(x) + b*sinh(x) + a))/(a^2*b*cosh(x) + a^2*b*sinh(x))
```

3.802.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(66) = 132$.

Time = 2.21 (sec) , antiderivative size = 753, normalized size of antiderivative = 10.61

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) + b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

output
```
Piecewise((zoo*(A*x + C*cosh(x)), Eq(a, 0) & Eq(b, 0)), (2*A*log(tanh(x/2)
 + 1)*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x
/2) + 2*b) + C*x*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + C*x/(2*b*tanh(x/2) + 2*
b) + 2*C/(2*b*tanh(x/2) + 2*b), Eq(a, b)), (-2*A/(2*b*sinh(x) + 2*b*cosh(x
)) + C*x*sinh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + C*x*cosh(x)/(2*b*sinh(x) +
2*b*cosh(x)) + C*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)), Eq(a, 0)), ((A*x + C
*cosh(x))/a, Eq(b, 0)), (2*A*a*b*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*ta
nh(x/2) + 2*a**2*b) + 2*A*a*b*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a
**2*b) - 2*A*a*b*log(-a/(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2
*b*tanh(x/2) + 2*a**2*b) - 2*A*a*b*log(-a/(a - b) - b/(a - b) + tanh(x/2))
/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*a**2*x*tanh(x/2)/(2*a**2*b*tanh(x/2)
+ 2*a**2*b) + C*a**2*x/(2*a**2*b*tanh(x/2) + 2*a**2*b) - C*a**2*log(tanh(x
/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - C*a**2*log(tanh(x/2)
+ 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*a**2*log(-a/(a - b) - b/(a - b) +
 tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*a**2*log(-a/(a -
 b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b) + 2*C*a*b/(2*
a**2*b*tanh(x/2) + 2*a**2*b) + C*b**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2
*b*tanh(x/2) + 2*a**2*b) + C*b**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) +
 2*a**2*b) - C*b**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a
**2*b*tanh(x/2) + 2*a**2*b) - C*b**2*log(-a/(a - b) - b/(a - b) + tanh(...
```

3.802.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) + b\sinh(x)}\,dx = \frac{1}{2}\,C\left(\frac{x}{b} + \frac{e^{(-x)}}{a} + \frac{(a^2 - b^2)\log\left(ae^{(-x)} + b\right)}{a^2 b}\right) - \frac{A\log\left(ae^{(-x)} + b\right)}{a}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
1/2*C*(x/b + e^(-x)/a + (a^2 - b^2)*log(a*e^(-x) + b)/(a^2*b)) - A*log(a*e
^(-x) + b)/a
```

3.802.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{Ce^{(-x)}}{2\,a} + \frac{(2\,Aa+Cb)x}{2\,a^2} + \frac{(Ca^2-2\,Aab-Cb^2)\log\left(|be^x+a|\right)}{2\,a^2b}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
1/2*C*e^(-x)/a + 1/2*(2*A*a + C*b)*x/a^2 + 1/2*(C*a^2 - 2*A*a*b - C*b^2)*l
og(abs(b*e^x + a))/(a^2*b)
```

3.802.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{C\,\mathrm{e}^{-x}}{2\,a} + \frac{x\,(2\,A\,a+C\,b)}{2\,a^2} - \frac{\ln\left(a+b\,\mathrm{e}^x\right)\,\left(-C\,a^2+2\,A\,a\,b+C\,b^2\right)}{2\,a^2\,b}$$

input
```
int((A + C*sinh(x))/(a + b*cosh(x) + b*sinh(x)),x)
```

output
```
(C*exp(-x))/(2*a) + (x*(2*A*a + C*b))/(2*a^2) - (log(a + b*exp(x))*(C*b^2
- C*a^2 + 2*A*a*b))/(2*a^2*b)
```

3.803 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$

3.803.1 Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{(2aA-bB)x}{2a^2}-\frac{B\cosh(x)}{2a}-\frac{(2aAb-a^2B-b^2B)\log(a+b\cosh(x)+b\sinh(x))}{2a^2b}+\frac{B\sinh(x)}{2a}$$

output
```
1/2*(2*A*a-B*b)*x/a^2-1/2*B*cosh(x)/a-1/2*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*cos
h(x)+b*sinh(x))/a^2/b+1/2*B*sinh(x)/a
```

3.803.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{\left(2aA+\frac{a^2B}{b}-bB\right)x-2aB\cosh(x)+\frac{2(-2aAb+a^2B+b^2B)\log\left((a+b)\cosh\left(\frac{x}{2}\right)+(-a+b)\sinh\left(\frac{x}{2}\right)\right)}{b}+2aB\sinh(x)}{4a^2}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

output
```
((2*a*A + (a^2*B)/b - b*B)*x - 2*a*B*Cosh[x] + (2*(-2*a*A*b + a^2*B + b^2*
B)*Log[(a + b)*Cosh[x/2] + (-a + b)*Sinh[x/2]])/b + 2*a*B*Sinh[x])/(4*a^2)
```

3.803.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3611}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{a+b\sinh(x)+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)}{a-ib\sin(ix)+b\cos(ix)}dx$$

$$\Big\downarrow \text{3611}$$

$$-\frac{(a^2(-B)+2aAb-b^2B)\log(a+b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sinh(x)}{2a}-\frac{B\cosh(x)}{2a}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

output
```
((2*a*A - b*B)*x)/(2*a^2) - (B*Cosh[x])/(2*a) - ((2*a*A*b - a^2*B - b^2*B)
*Log[a + b*Cosh[x] + b*Sinh[x]])/(2*a^2*b) + (B*Sinh[x])/(2*a)
```

3.803.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3611
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - b*B)*(x
/(2*a^2)), x] + (Simp[B*(Sin[d + e*x]/(2*a*e)), x] - Simp[b*B*(Cos[d + e*x]
/(2*a*c*e)), x] + Simp[(a^2*B - 2*a*b*A + b^2*B)*(Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*c*e)), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

3.803.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{B\,\mathrm{e}^{-x}}{2a}+\frac{xA}{a}-\frac{bxB}{2a^2}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)A}{a}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)B}{2b}+\frac{b\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)B}{2a^2}$
default	$-\frac{B}{a\left(1+\tanh\left(\frac{x}{2}\right)\right)}+\frac{(2Aa-Bb)\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2a^2}-\frac{(2Aab-B\,a^2-B\,b^2)\ln\left(a\tanh\left(\frac{x}{2}\right)-b\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a^2b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}$

input
```
int((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*B/a/exp(x)+1/a*x*A-1/2/a^2*b*x*B-1/a*ln(exp(x)+1/b*a)*A+1/2/b*ln(exp(
x)+1/b*a)*B+1/2/a^2*b*ln(exp(x)+1/b*a)*B
```

3.803.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.43

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = -\frac{Bab-(2\,Aab-Bb^2)x\cosh(x)-(2\,Aab-Bb^2)x\sinh(x)-((Ba^2-2\,Aab+Bb^2)\cosh(x)+(Ba^2-}{2\,(a^2b\cosh(x)+a^2b\sinh(x))}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

output
```
-1/2*(B*a*b - (2*A*a*b - B*b^2)*x*cosh(x) - (2*A*a*b - B*b^2)*x*sinh(x) -
((B*a^2 - 2*A*a*b + B*b^2)*cosh(x) + (B*a^2 - 2*A*a*b + B*b^2)*sinh(x))*lo
g(b*cosh(x) + b*sinh(x) + a))/(a^2*b*cosh(x) + a^2*b*sinh(x))
```

3.803.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(66) = 132.

Time = 2.26 (sec) , antiderivative size = 806, normalized size of antiderivative = 10.47

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

output
```
Piecewise((zoo*(A*x + B*sinh(x)), Eq(a, 0) & Eq(b, 0)), (2*A*log(tanh(x/2)
 + 1)*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x
/2) + 2*b) + B*x*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + B*x/(2*b*tanh(x/2) + 2*
b) - 2*B*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh(x/2) + 2*b) - 2*B*log(tanh
(x/2) + 1)/(2*b*tanh(x/2) + 2*b) - 2*B/(2*b*tanh(x/2) + 2*b), Eq(a, b)), (
-2*A/(2*b*sinh(x) + 2*b*cosh(x)) + B*x*sinh(x)/(2*b*sinh(x) + 2*b*cosh(x))
 + B*x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) - B*cosh(x)/(2*b*sinh(x) + 2*b*
cosh(x)), Eq(a, 0)), ((A*x + B*sinh(x))/a, Eq(b, 0)), (2*A*a*b*log(tanh(x/
2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + 2*A*a*b*log(tanh(x/2)
+ 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - 2*A*a*b*log(-a/(a - b) - b/(a - b)
+ tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - 2*A*a*b*log(-a/(a
 - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**2*x*
tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**2*x/(2*a**2*b*tanh(x/2) +
 2*a**2*b) - B*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a
**2*b) - B*a**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**
2*log(-a/(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) +
2*a**2*b) + B*a**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(
x/2) + 2*a**2*b) - 2*B*a*b/(2*a**2*b*tanh(x/2) + 2*a**2*b) - B*b**2*log(ta
nh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - B*b**2*log(tanh(x
/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*b**2*log(-a/(a - b) - b/(a...
```

3.803.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{1}{2}\,B\left(\frac{x}{b}-\frac{e^{(-x)}}{a}+\frac{(a^2+b^2)\log\left(ae^{(-x)}+b\right)}{a^2b}\right) - \frac{A\log\left(ae^{(-x)}+b\right)}{a}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

output
```
1/2*B*(x/b - e^(-x)/a + (a^2 + b^2)*log(a*e^(-x) + b)/(a^2*b)) - A*log(a*e
^(-x) + b)/a
```

3.803.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = -\frac{Be^{(-x)}}{2\,a}+\frac{(2\,Aa-Bb)x}{2\,a^2}+\frac{(Ba^2-2\,Aab+Bb^2)\log\left(|be^x+a|\right)}{2\,a^2b}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

output
```
-1/2*B*e^(-x)/a + 1/2*(2*A*a - B*b)*x/a^2 + 1/2*(B*a^2 - 2*A*a*b + B*b^2)*
log(abs(b*e^x + a))/(a^2*b)
```

3.803.9 Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{x\,(2\,A\,a-B\,b)}{2\,a^2}-\frac{B\,\mathrm{e}^{-x}}{2\,a}+\frac{\ln\left(a+b\,\mathrm{e}^x\right)\,\left(B\,a^2-2\,A\,a\,b+B\,b^2\right)}{2\,a^2\,b}$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x) + b*sinh(x)),x)
```

output
```
(x*(2*A*a - B*b))/(2*a^2) - (B*exp(-x))/(2*a) + (log(a + b*exp(x))*(B*a^2
+ B*b^2 - 2*A*a*b))/(2*a^2*b)
```

3.804 $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$

3.804.1 Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{(2aA-b(B-C))x}{2a^2} - \frac{(2aAb-b^2(B-C)-a^2(B+C))\log(a+b\cosh(x)+b\sinh(x))}{2a^2b} - \frac{(B-C)(\cosh(x)-\sinh(x))}{2a}$$

output
```
1/2*(2*A*a-b*(B-C))*x/a^2-1/2*(2*A*a*b-b^2*(B-C)-a^2*(B+C))*ln(a+b*cosh(x)
+b*sinh(x))/a^2/b-1/2*(B-C)*(cosh(x)-sinh(x))/a
```

3.804.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.20

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{\left(2aA+b(-B+C)+\frac{a^2(B+C)}{b}\right)x-2a(B-C)\cosh(x)+\frac{2(-2aAb+b^2(B-C)+a^2(B+C))\log\left((a+b)\cosh\left(\frac{x}{2}\right)+(-a+b)\sinh\left(\frac{x}{2}\right)\right)}{b}}{4a^2}$$

input
```
Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

output
```
((2*a*A + b*(-B + C) + (a^2*(B + C))/b)*x - 2*a*(B - C)*Cosh[x] + (2*(-2*a
*A*b + b^2*(B - C) + a^2*(B + C))*Log[(a + b)*Cosh[x/2] + (-a + b)*Sinh[x/
2]])/b + 2*a*(B - C)*Sinh[x])/(4*a^2)
```

3.804.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3042, 3609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\sinh(x)+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)-iC\sin(ix)}{a-ib\sin(ix)+b\cos(ix)}dx$$

$$\Big\downarrow \text{3609}$$

$$-\frac{\left(-\left(a^2(B+C)\right)+2aAb-b^2(B-C)\right)\log(a+b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-b(B-C))}{2a^2}-\frac{(B-C)(\cosh(x)-\sinh(x))}{2a}$$

input
```
Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

output
```
((2*a*A - b*(B - C))*x)/(2*a^2) - ((2*a*A*b - b^2*(B - C) - a^2*(B + C))*L
og[a + b*Cosh[x] + b*Sinh[x]])/(2*a^2*b) - ((B - C)*(Cosh[x] - Sinh[x]))/(
2*a)
```

3.804.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3609
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[(2*a*A - b*B - c*C)*(x/(2*a^2)), x] + (-Simp[(b*B + c*C)*((
b*Cos[d + e*x] - c*Sin[d + e*x])/(2*a*b*c*e)), x] + Simp[(a^2*(b*B - c*C) -
 2*a*A*b^2 + b^2*(b*B + c*C))*(Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]]/(2*a^2*b*c*e)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

3.804.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.41

method	result
default	$\frac{(-B-C)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}-\frac{B-C}{a\left(1+\tanh\left(\frac{x}{2}\right)\right)}+\frac{(2Aa-Bb+bC)\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2a^2}-\frac{(2Aab-B\,a^2-B\,b^2-C\,a^2+C\,b^2)\ln\left(a\tanh\left(\frac{x}{2}\right.\right.}{2a^2b}$
risch	$-\frac{B\,e^{-x}}{2a}+\frac{C\,e^{-x}}{2a}+\frac{xA}{a}-\frac{bxB}{2a^2}+\frac{bxC}{2a^2}-\frac{\ln\left(e^x+\frac{a}{b}\right)A}{a}+\frac{\ln\left(e^x+\frac{a}{b}\right)B}{2b}+\frac{b\ln\left(e^x+\frac{a}{b}\right)B}{2a^2}+\frac{\ln\left(e^x+\frac{a}{b}\right)C}{2b}-\frac{b\ln\left(e^x+\frac{a}{b}\right)C}{2a^2}$

input
```
int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOS
E)
```

output
```
1/2*(-B-C)/b*ln(tanh(1/2*x)-1)-(B-C)/a/(1+tanh(1/2*x))+1/2*(2*A*a-B*b+C*b)
/a^2*ln(1+tanh(1/2*x))-1/2*(2*A*a*b-B*a^2-B*b^2-C*a^2+C*b^2)/a^2/b*ln(a*ta
nh(1/2*x)-b*tanh(1/2*x)-a-b)
```

3.804.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = -\frac{(B-C)ab-(2\,Aab-(B-C)b^2)x\cosh\left(x\right)-(2\,Aab-(B-C)b^2)x\sinh\left(x\right)-(((B+C)a^2-2\,Aab}{2\left(a^2b\cosh\left(x\right)+\right.}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="fr
icas")
```

output
```
-1/2*((B - C)*a*b - (2*A*a*b - (B - C)*b^2)*x*cosh(x) - (2*A*a*b - (B - C)
*b^2)*x*sinh(x) - (((B + C)*a^2 - 2*A*a*b + (B - C)*b^2)*cosh(x) + ((B + C
)*a^2 - 2*A*a*b + (B - C)*b^2)*sinh(x))*log(b*cosh(x) + b*sinh(x) + a))/(a
^2*b*cosh(x) + a^2*b*sinh(x))
```

3.804.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1321 vs. $2(70) = 140$.

Time = 2.60 (sec) , antiderivative size = 1321, normalized size of antiderivative = 15.36

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{a + b\cosh(x) + b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

output
```
Piecewise((zoo*(A*x + B*sinh(x) + C*cosh(x)), Eq(a, 0) & Eq(b, 0)), (2*A*l
og(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + 2*A*log(tanh(x/2) + 1)
/(2*b*tanh(x/2) + 2*b) + B*x*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + B*x/(2*b*ta
nh(x/2) + 2*b) - 2*B*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh(x/2) + 2*b) -
2*B*log(tanh(x/2) + 1)/(2*b*tanh(x/2) + 2*b) - 2*B/(2*b*tanh(x/2) + 2*b) +
 C*x*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + C*x/(2*b*tanh(x/2) + 2*b) + 2*C/(2*
b*tanh(x/2) + 2*b), Eq(a, b)), (-2*A/(2*b*sinh(x) + 2*b*cosh(x)) + B*x*sin
h(x)/(2*b*sinh(x) + 2*b*cosh(x)) + B*x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x))
 - B*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + C*x*sinh(x)/(2*b*sinh(x) + 2*b*
cosh(x)) + C*x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + C*cosh(x)/(2*b*sinh(x
) + 2*b*cosh(x)), Eq(a, 0)), ((A*x + B*sinh(x) + C*cosh(x))/a, Eq(b, 0)),
(2*A*a*b*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + 2*
A*a*b*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - 2*A*a*b*log(-a/
(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b)
 - 2*A*a*b*log(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2
*a**2*b) + B*a**2*x*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**2*x/(
2*a**2*b*tanh(x/2) + 2*a**2*b) - B*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a*
*2*b*tanh(x/2) + 2*a**2*b) - B*a**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2)
 + 2*a**2*b) + B*a**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2
*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**2*log(-a/(a - b) - b/(a - b) + tan...
```

3.804.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{1}{2}\,C\left(\frac{x}{b}+\frac{e^{(-x)}}{a}+\frac{(a^2-b^2)\log\left(ae^{(-x)}+b\right)}{a^2b}\right) + \frac{1}{2}\,B\left(\frac{x}{b}-\frac{e^{(-x)}}{a}+\frac{(a^2+b^2)\log\left(ae^{(-x)}+b\right)}{a^2b}\right) - \frac{A\log\left(ae^{(-x)}+b\right)}{a}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="ma
xima")
```

output
```
1/2*C*(x/b + e^(-x)/a + (a^2 - b^2)*log(a*e^(-x) + b)/(a^2*b)) + 1/2*B*(x/
b - e^(-x)/a + (a^2 + b^2)*log(a*e^(-x) + b)/(a^2*b)) - A*log(a*e^(-x) + b
)/a
```

3.804.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{(2\,Aa-Bb+Cb)x}{2\,a^2} - \frac{(Ba-Ca)e^{(-x)}}{2\,a^2} + \frac{(Ba^2+Ca^2-2\,Aab+Bb^2-Cb^2)\log\left(|be^x+a|\right)}{2\,a^2b}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="gi
ac")
```

output
```
1/2*(2*A*a - B*b + C*b)*x/a^2 - 1/2*(B*a - C*a)*e^(-x)/a^2 + 1/2*(B*a^2 +
C*a^2 - 2*A*a*b + B*b^2 - C*b^2)*log(abs(b*e^x + a))/(a^2*b)
```

3.804.9 Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{x\,(2\,A\,a-B\,b+C\,b)}{2\,a^2}-\frac{\mathrm{e}^{-x}\,(B-C)}{2\,a}+\frac{\ln\left(a+b\,\mathrm{e}^{x}\right)\,\left(B\,a^2+B\,b^2+C\,a^2-C\,b^2-2\,A\,a\,b\right)}{2\,a^2\,b}$$

input
```
int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + b*sinh(x)),x)
```

output
```
(x*(2*A*a - B*b + C*b))/(2*a^2) - (exp(-x)*(B - C))/(2*a) + (log(a + b*exp
(x))*(B*a^2 + B*b^2 + C*a^2 - C*b^2 - 2*A*a*b))/(2*a^2*b)
```

3.805 $\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$

3.805.1 Optimal result

Integrand size = 20, antiderivative size = 77

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(2aA-bC)x}{2a^2}+\frac{C\cosh(x)}{2a} + \frac{(2aAb+a^2C-b^2C)\log(a+b\cosh(x)-b\sinh(x))}{2a^2b} + \frac{C\sinh(x)}{2a}$$

output
```
1/2*(2*A*a-C*b)*x/a^2+1/2*C*cosh(x)/a+1/2*(2*A*a*b+C*a^2-C*b^2)*ln(a+b*cos
h(x)-b*sinh(x))/a^2/b+1/2*C*sinh(x)/a
```

3.805.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{\left(2aA-\frac{a^2C}{b}-bC\right)x+2aC\cosh(x)+\frac{2(2aAb+a^2C-b^2C)\log\left((a+b)\cosh\left(\frac{x}{2}\right)+(a-b)\sinh\left(\frac{x}{2}\right)\right)}{b}+2aC\sinh(x)}{4a^2}$$

input
```
Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

output
```
((2*a*A - (a^2*C)/b - b*C)*x + 2*a*C*Cosh[x] + (2*(2*a*A*b + a^2*C - b^2*C
)*Log[(a + b)*Cosh[x/2] + (a - b)*Sinh[x/2]])/b + 2*a*C*Sinh[x])/(4*a^2)
```

3.805.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3610}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sinh(x)}{a-b\sinh(x)+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iC\sin(ix)}{a+ib\sin(ix)+b\cos(ix)}dx$$

$$\Big\downarrow \text{3610}$$

$$\frac{(a^2C+2aAb-b^2C)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bC)}{2a^2}+\frac{C\sinh(x)}{2a}+\frac{C\cosh(x)}{2a}$$

input
```
Int[(A + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

output
```
((2*a*A - b*C)*x)/(2*a^2) + (C*Cosh[x])/(2*a) + ((2*a*A*b + a^2*C - b^2*C)
*Log[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + (C*Sinh[x])/(2*a)
```

3.805.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3610
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - c*C)*(x
/(2*a^2)), x] + (-Simp[C*(Cos[d + e*x]/(2*a*e)), x] + Simp[c*C*(Sin[d + e*x
]/(2*a*b*e)), x] + Simp[((-a^2)*C + 2*a*c*A + b^2*C)*(Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*b*e)), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

3.805.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

method	result
risch	$\frac{C\,\mathrm{e}^{x}}{2a}-\frac{Cx}{2b}+\frac{\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)A}{a}+\frac{\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)C}{2b}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)C}{2a^{2}}$
default	$\frac{\left(2Aab+C\,a^{2}-C\,b^{2}\right)\ln\left(a\tanh\left(\frac{x}{2}\right)-b\tanh\left(\frac{x}{2}\right)+a+b\right)}{2a^{2}b}-\frac{C}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\left(-2Aa+bC\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^{2}}-\frac{C\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2b}$

input
```
int((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*C/a*exp(x)-1/2*C*x/b+1/a*ln(exp(x)+1/a*b)*A+1/2/b*ln(exp(x)+1/a*b)*C-1
/2/a^2*b*ln(exp(x)+1/a*b)*C
```

3.805.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = -\frac{Ca^2x-Cab\cosh\left(x\right)-Cab\sinh\left(x\right)-\left(Ca^2+2\,Aab-Cb^2\right)\log\left(a\cosh\left(x\right)+a\sinh\left(x\right)+b\right)}{2\,a^2b}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="fricas")
```

output
```
-1/2*(C*a^2*x - C*a*b*cosh(x) - C*a*b*sinh(x) - (C*a^2 + 2*A*a*b - C*b^2)*
log(a*cosh(x) + a*sinh(x) + b))/(a^2*b)
```

3.805.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 852 vs. 2(66) = 132.

Time = 2.28 (sec) , antiderivative size = 852, normalized size of antiderivative = 11.06

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

output
```
Piecewise((zoo*(A*x + C*cosh(x)), Eq(a, 0) & Eq(b, 0)), (2*A*x*tanh(x/2)/(
2*b*tanh(x/2) - 2*b) - 2*A*x/(2*b*tanh(x/2) - 2*b) - 2*A*log(tanh(x/2) + 1
)*tanh(x/2)/(2*b*tanh(x/2) - 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x/2)
- 2*b) - C*x*tanh(x/2)/(2*b*tanh(x/2) - 2*b) + C*x/(2*b*tanh(x/2) - 2*b) -
 2*C/(2*b*tanh(x/2) - 2*b), Eq(a, b)), (2*A/(-2*b*sinh(x) + 2*b*cosh(x)) +
 C*x*sinh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) - C*x*cosh(x)/(-2*b*sinh(x) + 2*
b*cosh(x)) + C*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, 0)), ((A*x + C*
cosh(x))/a, Eq(b, 0)), (2*A*a*b*x*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b
) - 2*A*a*b*x/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*log(tanh(x/2) + 1)
*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + 2*A*a*b*log(tanh(x/2) + 1)/(2
*a**2*b*tanh(x/2) - 2*a**2*b) + 2*A*a*b*log(a/(a - b) + b/(a - b) + tanh(x
/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*log(a/(a - b) + b
/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*a**2*log(tanh(x/
2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*a**2*log(tanh(x/2) +
 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*a**2*log(a/(a - b) + b/(a - b) + t
anh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*a**2*log(a/(a - b)
 + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*C*a*b/(2*a**
2*b*tanh(x/2) - 2*a**2*b) - C*b**2*x*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**
2*b) + C*b**2*x/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*b**2*log(tanh(x/2) + 1
)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*b**2*log(tanh(x/2) + 1)...
```

3.805.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) - b\sinh(x)}\,dx = A\left(\frac{x}{a} + \frac{\log\left(be^{(-x)} + a\right)}{a}\right) - \frac{1}{2}C\left(\frac{bx}{a^2} - \frac{e^x}{a} - \frac{(a^2 - b^2)\log\left(be^{(-x)} + a\right)}{a^2b}\right)$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="maxima")
```

output
```
A*(x/a + log(b*e^(-x) + a)/a) - 1/2*C*(b*x/a^2 - e^x/a - (a^2 - b^2)*log(b
*e^(-x) + a)/(a^2*b))
```

3.805.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = -\frac{Cx}{2\,b}+\frac{Ce^x}{2\,a}+\frac{(Ca^2+2\,Aab-Cb^2)\log\left(|ae^x+b|\right)}{2\,a^2b}$$

input
```
integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="giac")
```

output
```
-1/2*C*x/b + 1/2*C*e^x/a + 1/2*(C*a^2 + 2*A*a*b - C*b^2)*log(abs(a*e^x + b
))/(a^2*b)
```

3.805.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{C\,\mathrm{e}^x}{2\,a}-\frac{C\,x}{2\,b}+\frac{\ln\left(b+a\,\mathrm{e}^x\right)\,\left(C\,a^2+2\,A\,a\,b-C\,b^2\right)}{2\,a^2\,b}$$

input
```
int((A + C*sinh(x))/(a + b*cosh(x) - b*sinh(x)),x)
```

output
```
(C*exp(x))/(2*a) - (C*x)/(2*b) + (log(b + a*exp(x))*(C*a^2 - C*b^2 + 2*A*a
*b))/(2*a^2*b)
```

3.806 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$

3.806.1 Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(2aA-bB)x}{2a^2}+\frac{B\cosh(x)}{2a}+\frac{(2aAb-a^2B-b^2B)\log(a+b\cosh(x)-b\sinh(x))}{2a^2b}+\frac{B\sinh(x)}{2a}$$

output
```
1/2*(2*A*a-B*b)*x/a^2+1/2*B*cosh(x)/a+1/2*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*cos
h(x)-b*sinh(x))/a^2/b+1/2*B*sinh(x)/a
```

3.806.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(2aAb+a^2B-b^2B)\,x+2abB\cosh(x)-2(-2aAb+a^2B+b^2B)\log\left((a+b)\cosh\left(\frac{x}{2}\right)+(a-b)\sinh\left(\frac{x}{2}\right)\right.}{4a^2b}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

output
```
((2*a*A*b + a^2*B - b^2*B)*x + 2*a*b*B*Cosh[x] - 2*(-2*a*A*b + a^2*B + b^2
*B)*Log[(a + b)*Cosh[x/2] + (a - b)*Sinh[x/2]] + 2*a*b*B*Sinh[x])/(4*a^2*b
)
```

3.806.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3611}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{a-b\sinh(x)+b\cosh(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\cos(ix)}{a+ib\sin(ix)+b\cos(ix)}dx$$

$$\Big\downarrow \ 3611$$

$$\frac{(a^2(-B)+2aAb-b^2B)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sinh(x)}{2a}+\frac{B\cosh(x)}{2a}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

output
```
((2*a*A - b*B)*x)/(2*a^2) + (B*Cosh[x])/(2*a) + ((2*a*A*b - a^2*B - b^2*B)
*Log[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + (B*Sinh[x])/(2*a)
```

3.806.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3611
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - b*B)*(x
/(2*a^2)), x] + (Simp[B*(Sin[d + e*x]/(2*a*e)), x] - Simp[b*B*(Cos[d + e*x]
/(2*a*c*e)), x] + Simp[(a^2*B - 2*a*b*A + b^2*B)*(Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*c*e)), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

3.806.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

method	result
risch	$\frac{B\,\mathrm{e}^{x}}{2a}+\frac{Bx}{2b}+\frac{\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)A}{a}-\frac{\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)B}{2b}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)B}{2a^{2}}$
default	$\frac{B\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2b}-\frac{B}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\left(-2Aa+Bb\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^{2}}+\frac{\left(2Aab-B\,a^{2}-B\,b^{2}\right)\ln\left(a\tanh\left(\frac{x}{2}\right)-b\tanh\left(\frac{x}{2}\right)+a+b\right)}{2a^{2}b}$

input
```
int((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*B/a*exp(x)+1/2*B*x/b+1/a*ln(exp(x)+1/a*b)*A-1/2/b*ln(exp(x)+1/a*b)*B-1
/2/a^2*b*ln(exp(x)+1/a*b)*B
```

3.806.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{Ba^{2}x+Bab\cosh\left(x\right)+Bab\sinh\left(x\right)-\left(Ba^{2}-2\,Aab+Bb^{2}\right)\log\left(a\cosh\left(x\right)+a\sinh\left(x\right)+b\right)}{2\,a^{2}b}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="fricas")
```

output
```
1/2*(B*a^2*x + B*a*b*cosh(x) + B*a*b*sinh(x) - (B*a^2 - 2*A*a*b + B*b^2)*l
og(a*cosh(x) + a*sinh(x) + b))/(a^2*b)
```

3.806.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 904 vs. $2(66) = 132$.

Time = 2.35 (sec) , antiderivative size = 904, normalized size of antiderivative = 11.59

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x) - b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

output
```
Piecewise((zoo*(A*x + B*sinh(x)), Eq(a, 0) & Eq(b, 0)), (2*A*x*tanh(x/2)/(
2*b*tanh(x/2) - 2*b) - 2*A*x/(2*b*tanh(x/2) - 2*b) - 2*A*log(tanh(x/2) + 1
)*tanh(x/2)/(2*b*tanh(x/2) - 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x/2)
- 2*b) - B*x*tanh(x/2)/(2*b*tanh(x/2) - 2*b) + B*x/(2*b*tanh(x/2) - 2*b) +
 2*B*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh(x/2) - 2*b) - 2*B*log(tanh(x/2
) + 1)/(2*b*tanh(x/2) - 2*b) - 2*B/(2*b*tanh(x/2) - 2*b), Eq(a, b)), (2*A/
(-2*b*sinh(x) + 2*b*cosh(x)) - B*x*sinh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) +
B*x*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) + B*cosh(x)/(-2*b*sinh(x) + 2*b*c
osh(x)), Eq(a, 0)), ((A*x + B*sinh(x))/a, Eq(b, 0)), (2*A*a*b*x*tanh(x/2)/
(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*x/(2*a**2*b*tanh(x/2) - 2*a**2*b
) - 2*A*a*b*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) +
 2*A*a*b*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + 2*A*a*b*log(
a/(a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*
b) - 2*A*a*b*log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) -
2*a**2*b) + B*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a*
*2*b) - B*a**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*a**2
*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*
a**2*b) + B*a**2*log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2
) - 2*a**2*b) - 2*B*a*b/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*b**2*x*tanh(x/
2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + B*b**2*x/(2*a**2*b*tanh(x/2) - 2*a...
```

3.806.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x) - b\sinh(x)}\,dx = A\left(\frac{x}{a} + \frac{\log\left(be^{(-x)} + a\right)}{a}\right) - \frac{1}{2}\,B\left(\frac{bx}{a^2} - \frac{e^x}{a} + \frac{(a^2 + b^2)\log\left(be^{(-x)} + a\right)}{a^2 b}\right)$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="maxima")
```

output
```
A*(x/a + log(b*e^(-x) + a)/a) - 1/2*B*(b*x/a^2 - e^x/a + (a^2 + b^2)*log(b
*e^(-x) + a)/(a^2*b))
```

3.806.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{Bx}{2\,b}+\frac{Be^x}{2\,a}-\frac{(Ba^2-2\,Aab+Bb^2)\log\left(|ae^x+b|\right)}{2\,a^2b}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="giac")
```

output
```
1/2*B*x/b + 1/2*B*e^x/a - 1/2*(B*a^2 - 2*A*a*b + B*b^2)*log(abs(a*e^x + b)
)/(a^2*b)
```

3.806.9 Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{B\,\mathrm{e}^x}{2\,a}+\frac{B\,x}{2\,b}-\frac{\ln\left(b+a\,\mathrm{e}^x\right)\,\left(B\,a^2-2\,A\,a\,b+B\,b^2\right)}{2\,a^2\,b}$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x) - b*sinh(x)),x)
```

output
```
(B*exp(x))/(2*a) + (B*x)/(2*b) - (log(b + a*exp(x))*(B*a^2 + B*b^2 - 2*A*a
*b))/(2*a^2*b)
```

3.807 $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$

3.807.1 Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(2aA-b(B+C))x}{2a^2}+\frac{(2aAb-a^2(B-C)-b^2(B+C))\log(a+b\cosh(x)-b\sinh(x))}{2a^2b} + \frac{(B+C)(\cosh(x)+\sinh(x))}{2a}$$

output
```
1/2*(2*A*a-b*(B+C))*x/a^2+1/2*(2*A*a*b-a^2*(B-C)-b^2*(B+C))*ln(a+b*cosh(x)
-b*sinh(x))/a^2/b+1/2*(B+C)*(cosh(x)+sinh(x))/a
```

3.807.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(2aAb+a^2(B-C)-b^2(B+C))\,x+2ab(B+C)\cosh(x)-2(-2aAb+a^2(B-C)+b^2(B+C))\log\left((\right.}{4a^2b}$$

input
```
Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

output
```
((2*a*A*b + a^2*(B - C) - b^2*(B + C))*x + 2*a*b*(B + C)*Cosh[x] - 2*(-2*a
*A*b + a^2*(B - C) + b^2*(B + C))*Log[(a + b)*Cosh[x/2] + (a - b)*Sinh[x/2
]] + 2*a*b*(B + C)*Sinh[x])/(4*a^2*b)
```

3.807.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a-b\sinh(x)+b\cosh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\cos(ix)-iC\sin(ix)}{a+ib\sin(ix)+b\cos(ix)}dx$$

$$\Big\downarrow \text{ 3609}$$

$$\frac{(-(a^2(B-C))+2aAb-b^2(B+C))\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-b(B+C))}{2a^2}+\frac{(B+C)(\sinh(x)+\cosh(x))}{2a}$$

input
```
Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

output
```
((2*a*A - b*(B + C))*x)/(2*a^2) + ((2*a*A*b - a^2*(B - C) - b^2*(B + C))*L
og[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + ((B + C)*(Cosh[x] + Sinh[x]))/(
2*a)
```

3.807.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3609
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[(2*a*A - b*B - c*C)*(x/(2*a^2)), x] + (-Simp[(b*B + c*C)*((
b*Cos[d + e*x] - c*Sin[d + e*x])/(2*a*b*c*e)), x] + Simp[(a^2*(b*B - c*C) -
 2*a*A*b^2 + b^2*(b*B + c*C))*(Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]]/(2*a^2*b*c*e)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

3.807.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

method	result
risch	$\frac{B\,e^{x}}{2a}+\frac{C\,e^{x}}{2a}+\frac{Bx}{2b}-\frac{Cx}{2b}+\frac{\ln\left(e^{x}+\frac{b}{a}\right)A}{a}-\frac{\ln\left(e^{x}+\frac{b}{a}\right)B}{2b}-\frac{b\ln\left(e^{x}+\frac{b}{a}\right)B}{2a^{2}}+\frac{\ln\left(e^{x}+\frac{b}{a}\right)C}{2b}-\frac{b\ln\left(e^{x}+\frac{b}{a}\right)C}{2a^{2}}$
default	$-\frac{B+C}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{(-2Aa+Bb+bC)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^{2}}+\frac{(2Aab-B\,a^{2}-B\,b^{2}+C\,a^{2}-C\,b^{2})\ln\left(a\tanh\left(\frac{x}{2}\right)-b\tanh\left(\frac{x}{2}\right)+a+b\right)}{2a^{2}b}+$

input
```
int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x,method=_RETURNVERBOS
E)
```

output
```
1/2*B/a*exp(x)+1/2*C/a*exp(x)+1/2*B*x/b-1/2*C*x/b+1/a*ln(exp(x)+1/a*b)*A-1
/2/b*ln(exp(x)+1/a*b)*B-1/2/a^2*b*ln(exp(x)+1/a*b)*B+1/2/b*ln(exp(x)+1/a*b
)*C-1/2/a^2*b*ln(exp(x)+1/a*b)*C
```

3.807.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$$
$$=\frac{(B-C)a^{2}x+(B+C)ab\cosh{(x)}+(B+C)ab\sinh{(x)}-((B-C)a^{2}-2\,Aab+(B+C)b^{2})\log{(a\cosh}}{2\,a^{2}b}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="fr
icas")
```

output
```
1/2*((B - C)*a^2*x + (B + C)*a*b*cosh(x) + (B + C)*a*b*sinh(x) - ((B - C)*
a^2 - 2*A*a*b + (B + C)*b^2)*log(a*cosh(x) + a*sinh(x) + b))/(a^2*b)
```

3.807.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. $2(70) = 140$.

Time = 2.72 (sec) , antiderivative size = 1420, normalized size of antiderivative = 17.53

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{a + b\cosh(x) - b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

output
```
Piecewise((zoo*(A*x + B*sinh(x) + C*cosh(x)), Eq(a, 0) & Eq(b, 0)), (2*A*x
*tanh(x/2)/(2*b*tanh(x/2) - 2*b) - 2*A*x/(2*b*tanh(x/2) - 2*b) - 2*A*log(t
anh(x/2) + 1)*tanh(x/2)/(2*b*tanh(x/2) - 2*b) + 2*A*log(tanh(x/2) + 1)/(2*
b*tanh(x/2) - 2*b) - B*x*tanh(x/2)/(2*b*tanh(x/2) - 2*b) + B*x/(2*b*tanh(x
/2) - 2*b) + 2*B*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh(x/2) - 2*b) - 2*B*
log(tanh(x/2) + 1)/(2*b*tanh(x/2) - 2*b) - 2*B/(2*b*tanh(x/2) - 2*b) - C*x
*tanh(x/2)/(2*b*tanh(x/2) - 2*b) + C*x/(2*b*tanh(x/2) - 2*b) - 2*C/(2*b*ta
nh(x/2) - 2*b), Eq(a, b)), (2*A/(-2*b*sinh(x) + 2*b*cosh(x)) - B*x*sinh(x)
/(-2*b*sinh(x) + 2*b*cosh(x)) + B*x*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) +
 B*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) + C*x*sinh(x)/(-2*b*sinh(x) + 2*b*
cosh(x)) - C*x*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) + C*cosh(x)/(-2*b*sinh
(x) + 2*b*cosh(x)), Eq(a, 0)), ((A*x + B*sinh(x) + C*cosh(x))/a, Eq(b, 0))
, (2*A*a*b*x*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*x/(2*a**2
*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*
tanh(x/2) - 2*a**2*b) + 2*A*a*b*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2
*a**2*b) + 2*A*a*b*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**
2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*log(a/(a - b) + b/(a - b) + tanh(x/2))
/(2*a**2*b*tanh(x/2) - 2*a**2*b) + B*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*
a**2*b*tanh(x/2) - 2*a**2*b) - B*a**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/
2) - 2*a**2*b) - B*a**2*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(x/2...
```

3.807.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = A\left(\frac{x}{a}+\frac{\log\left(be^{(-x)}+a\right)}{a}\right) - \frac{1}{2}\,B\left(\frac{bx}{a^2}-\frac{e^x}{a}+\frac{(a^2+b^2)\log\left(be^{(-x)}+a\right)}{a^2b}\right) - \frac{1}{2}\,C\left(\frac{bx}{a^2}-\frac{e^x}{a}-\frac{(a^2-b^2)\log\left(be^{(-x)}+a\right)}{a^2b}\right)$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="ma
xima")
```

output
```
A*(x/a + log(b*e^(-x) + a)/a) - 1/2*B*(b*x/a^2 - e^x/a + (a^2 + b^2)*log(b
*e^(-x) + a)/(a^2*b)) - 1/2*C*(b*x/a^2 - e^x/a - (a^2 - b^2)*log(b*e^(-x)
+ a)/(a^2*b))
```

3.807.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(B-C)x}{2\,b}+\frac{Be^x+Ce^x}{2\,a} - \frac{(Ba^2-Ca^2-2\,Aab+Bb^2+Cb^2)\log\left(|ae^x+b|\right)}{2\,a^2b}$$

input
```
integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="gi
ac")
```

output
```
1/2*(B - C)*x/b + 1/2*(B*e^x + C*e^x)/a - 1/2*(B*a^2 - C*a^2 - 2*A*a*b + B
*b^2 + C*b^2)*log(abs(a*e^x + b))/(a^2*b)
```

3.807.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{x\,(B-C)}{2\,b}+\frac{\mathrm{e}^x\,(B+C)}{2\,a} - \frac{\ln\left(b+a\,\mathrm{e}^x\right)\,\left(B\,a^2+B\,b^2-C\,a^2+C\,b^2-2\,A\,a\,b\right)}{2\,a^2\,b}$$

input
```
int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) - b*sinh(x)),x)
```

output
```
(x*(B - C))/(2*b) + (exp(x)*(B + C))/(2*a) - (log(b + a*exp(x))*(B*a^2 + B
*b^2 - C*a^2 + C*b^2 - 2*A*a*b))/(2*a^2*b)
```

3.808 $\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx$

3.808.1 Optimal result

Integrand size = 11, antiderivative size = 3

$$\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx = \arctan(\tanh(x))$$

output
```
arctan(tanh(x))
```

3.808.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 9 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx = \frac{1}{2}\arctan(\sinh(2x))$$

input
```
Integrate[(Cosh[x]^2 + Sinh[x]^2)^(-1),x]
```

output
```
ArcTan[Sinh[2*x]]/2
```

3.808.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sinh^2(x)+\cosh^2(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(ix)^2-\sin(ix)^2}dx$$
$$\Big\downarrow \text{4889}$$
$$\int \frac{1}{\tanh^2(x)+1}d\tanh(x)$$
$$\Big\downarrow \text{216}$$
$$\arctan(\tanh(x))$$

input
```
Int[(Cosh[x]^2 + Sinh[x]^2)^(-1),x]
```

output
```
ArcTan[Tanh[x]]
```

3.808.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.808.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 8.00

method	result	size
risch	$\frac{i \ln\left(\mathrm{e}^{2x}+i\right)}{2}-\frac{i \ln\left(\mathrm{e}^{2x}-i\right)}{2}$	24
default	$-\frac{\sqrt{2}\,\left(2+\sqrt{2}\right) \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}-\frac{\left(-2+\sqrt{2}\right)\sqrt{2}\,\arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}$	72

input
```
int(1/(cosh(x)^2+sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*ln(exp(2*x)+I)-1/2*I*ln(exp(2*x)-I)
```

3.808.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(3) = 6.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 6.33

$$\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx = -\arctan\left(-\frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(1/(cosh(x)^2+sinh(x)^2),x, algorithm="fricas")
```

output
```
-arctan(-(cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
```

3.808.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(3) = 6$.

Time = 3.77 (sec) , antiderivative size = 172, normalized size of antiderivative = 57.33

$$\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx = \frac{47321\sqrt{3-2\sqrt{2}}\operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)}{13860\sqrt{2}+19601} + \frac{33461\sqrt{2}\sqrt{3-2\sqrt{2}}\operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)}{13860\sqrt{2}+19601} - \frac{5741\sqrt{2}\sqrt{2\sqrt{2}+3}\operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)}{13860\sqrt{2}+19601} - \frac{8119\sqrt{2\sqrt{2}+3}\operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)}{13860\sqrt{2}+19601}$$

input
```
integrate(1/(cosh(x)**2+sinh(x)**2),x)
```

output
```
47321*sqrt(3 - 2*sqrt(2))*atan(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(13860*sqrt(
2) + 19601) + 33461*sqrt(2)*sqrt(3 - 2*sqrt(2))*atan(tanh(x/2)/sqrt(3 - 2*
sqrt(2)))/(13860*sqrt(2) + 19601) - 5741*sqrt(2)*sqrt(2*sqrt(2) + 3)*atan(
tanh(x/2)/sqrt(2*sqrt(2) + 3))/(13860*sqrt(2) + 19601) - 8119*sqrt(2*sqrt(
2) + 3)*atan(tanh(x/2)/sqrt(2*sqrt(2) + 3))/(13860*sqrt(2) + 19601)
```

3.808.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(3) = 6$.

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 11.67

$$\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx = \arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right) - \arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)$$

input
```
integrate(1/(cosh(x)^2+sinh(x)^2),x, algorithm="maxima")
```

output
```
arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - arctan(-1/2*sqrt(2)*(sqrt(2) -
2*e^(-x)))
```

3.808.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \frac{1}{\cosh^2(x) + \sinh^2(x)}\, dx = \arctan\left(e^{(2\,x)}\right)$$

input
```
integrate(1/(cosh(x)^2+sinh(x)^2),x, algorithm="giac")
```

output
```
arctan(e^(2*x))
```

3.808.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \frac{1}{\cosh^2(x) + \sinh^2(x)}\, dx = \mathrm{atan}\left(\mathrm{e}^{2\,x}\right)$$

input
```
int(1/(cosh(x)^2 + sinh(x)^2),x)
```

output
```
atan(exp(2*x))
```

3.809 $\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2} dx$

3.809.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2} dx = \frac{\tanh(x)}{1+\tanh^2(x)}$$

output
```
tanh(x)/(1+tanh(x)^2)
```

3.809.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2} dx = \frac{1}{2}\tanh(2x)$$

input
```
Integrate[(Cosh[x]^2 + Sinh[x]^2)^(-2),x]
```

output
```
Tanh[2*x]/2
```

3.809.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4889, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sinh^2(x)+\cosh^2(x)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(\cos(ix)^2-\sin(ix)^2\right)^2}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{1-\tanh^2(x)}{\left(\tanh^2(x)+1\right)^2}d\tanh(x)$$

$$\Big\downarrow \text{ 297}$$

$$\frac{\tanh(x)}{\tanh^2(x)+1}$$

input
```
Int[(Cosh[x]^2 + Sinh[x]^2)^(-2),x]
```

output
```
Tanh[x]/(1 + Tanh[x]^2)
```

3.809.3.1 Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.809.4 Maple [A] (verified)

Time = 30.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{1}{1+e^{4x}}$	11
default	$-\frac{2\left(-\tanh\left(\frac{x}{2}\right)^{3}-\tanh\left(\frac{x}{2}\right)\right)}{\tanh\left(\frac{x}{2}\right)^{4}+6\tanh\left(\frac{x}{2}\right)^{2}+1}$	36

input
```
int(1/(cosh(x)^2+sinh(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(1+exp(4*x))
```

3.809.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.64

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2}\,dx = -\frac{1}{\cosh(x)^4+4\cosh(x)^3\sinh(x)+6\cosh(x)^2\sinh(x)^2+4\cosh(x)\sinh(x)^3+\sinh(x)^4+1}$$

input
```
integrate(1/(cosh(x)^2+sinh(x)^2)^2,x, algorithm="fricas")
```

output
```
-1/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*si
nh(x)^3 + sinh(x)^4 + 1)
```

3.809.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(8) = 16.

Time = 0.68 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.36

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2}\,dx=\frac{2\tanh^3\left(\frac{x}{2}\right)}{\tanh^4\left(\frac{x}{2}\right)+6\tanh^2\left(\frac{x}{2}\right)+1}+\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^4\left(\frac{x}{2}\right)+6\tanh^2\left(\frac{x}{2}\right)+1}$$

input
```
integrate(1/(cosh(x)**2+sinh(x)**2)**2,x)
```

output
```
2*tanh(x/2)**3/(tanh(x/2)**4 + 6*tanh(x/2)**2 + 1) + 2*tanh(x/2)/(tanh(x/2
)**4 + 6*tanh(x/2)**2 + 1)
```

3.809.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2}\,dx=\frac{1}{e^{(-4x)}+1}$$

input
```
integrate(1/(cosh(x)^2+sinh(x)^2)^2,x, algorithm="maxima")
```

output
```
1/(e^(-4*x) + 1)
```

3.809.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2}\,dx=-\frac{1}{e^{(4x)}+1}$$

input
```
integrate(1/(cosh(x)^2+sinh(x)^2)^2,x, algorithm="giac")
```

output
```
-1/(e^(4*x) + 1)
```

3.809.9 Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2}\,dx = -\frac{1}{e^{4x}+1}$$

input
```
int(1/(cosh(x)^2 + sinh(x)^2)^2,x)
```

output
```
-1/(exp(4*x) + 1)
```

3.810 $\int \frac{1}{(\cosh^2(x)+\sinh^2(x))^3}\,dx$

3.810.1 Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx=\frac{1}{2}\arctan(\tanh(x))+\frac{\operatorname{sech}^2(x)\tanh(x)}{2\left(1+\tanh^2(x)\right)^2}$$

output
```
1/2*arctan(tanh(x))+1/2*sech(x)^2*tanh(x)/(1+tanh(x)^2)^2
```

3.810.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx=\frac{1}{4}\arctan(\sinh(2x))+\frac{1}{4}\operatorname{sech}(2x)\tanh(2x)$$

input
```
Integrate[(Cosh[x]^2 + Sinh[x]^2)^(-3),x]
```

output
```
ArcTan[Sinh[2*x]]/4 + (Sech[2*x]*Tanh[2*x])/4
```

3.810.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4889, 315, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sinh^2(x)+\cosh^2(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\cos(ix)^2-\sin(ix)^2\right)^3}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\left(1-\tanh^2(x)\right)^2}{\left(\tanh^2(x)+1\right)^3}d\tanh(x)$$

$$\Big\downarrow \text{315}$$

$$\frac{1}{4}\int \frac{2}{\tanh^2(x)+1}d\tanh(x)+\frac{\tanh(x)\left(1-\tanh^2(x)\right)}{2\left(\tanh^2(x)+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1}{\tanh^2(x)+1}d\tanh(x)+\frac{\tanh(x)\left(1-\tanh^2(x)\right)}{2\left(\tanh^2(x)+1\right)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\arctan(\tanh(x))+\frac{\tanh(x)\left(1-\tanh^2(x)\right)}{2\left(\tanh^2(x)+1\right)^2}$$

input
```
Int[(Cosh[x]^2 + Sinh[x]^2)^(-3),x]
```

output
```
ArcTan[Tanh[x]]/2 + (Tanh[x]*(1 - Tanh[x]^2))/(2*(1 + Tanh[x]^2)^2)
```

3.810.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.810.4 Maple [A] (verified)

Time = 29.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.08

method	result
parallelrisch	0
risch	$\frac{e^{2x}(e^{4x}-1)}{2(1+e^{4x})^2}+\frac{i\ln(e^{2x}+i)}{4}-\frac{i\ln(e^{2x}-i)}{4}$
default	$-\frac{2\left(-\frac{\tanh(\frac{x}{2})^7}{2}+\frac{\tanh(\frac{x}{2})^5}{2}+\frac{\tanh(\frac{x}{2})^3}{2}-\frac{\tanh(\frac{x}{2})}{2}\right)}{\left(\tanh(\frac{x}{2})^4+6\tanh(\frac{x}{2})^2+1\right)^2}-\frac{\sqrt{2}\left(2+\sqrt{2}\right)\arctan\left(\frac{2\tanh(\frac{x}{2})}{2+2\sqrt{2}}\right)}{2\left(2+2\sqrt{2}\right)}-\frac{\left(-2+\sqrt{2}\right)\sqrt{2}\arctan\left(\frac{2\tanh}{2\sqrt{2}}\right.}{2\left(2\sqrt{2}-2\right)}$

input
```
int(1/(cosh(x)^2+sinh(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
0
```

3.810.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(22) = 44$.

Time = 0.27 (sec) , antiderivative size = 304, normalized size of antiderivative = 11.69

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx = \frac{\cosh(x)^6+20\cosh(x)^3\sinh(x)^3+15\cosh(x)^2\sinh(x)^4+6\cosh(x)\sinh(x)^5+\sinh(x)^6+(15\cosh(x}{2(\cosh}$$

input
```
integrate(1/(cosh(x)^2+sinh(x)^2)^3,x, algorithm="fricas")
```

output
```
1/2*(cosh(x)^6 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(
x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^4 - 1)*sinh(x)^2 - (cosh(x)^8 + 56*
cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(
x)^8 + 2*(35*cosh(x)^4 + 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(7*cosh(x)^5 + cos
h(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 +
 cosh(x)^3)*sinh(x) + 1)*arctan(-(cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
- cosh(x)^2 + 2*(3*cosh(x)^5 - cosh(x))*sinh(x))/(cosh(x)^8 + 56*cosh(x)^3
*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*
(35*cosh(x)^4 + 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(7*cosh(x)^5 + cosh(x))*sin
h(x)^3 + 4*(7*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 + cosh(x)^
3)*sinh(x) + 1)
```

3.810.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3602 vs. $2(24) = 48$.

Time = 131.07 (sec) , antiderivative size = 3602, normalized size of antiderivative = 138.54

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cosh(x)**2+sinh(x)**2)**3,x)
```

output
```
1939450125521*sqrt(3 - 2*sqrt(2))*tanh(x/2)**8*atan(tanh(x/2)/sqrt(3 - 2*s
qrt(2)))/(136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8
+ 13633242959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 4317
1936039392*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 1363324295
9808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sq
rt(2) + 1606693091074) + 1371398335529*sqrt(2)*sqrt(3 - 2*sqrt(2))*tanh(x/
2)**8*atan(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(1136103579984*sqrt(2)*tanh(x/2)
**8 + 1606693091074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 1
9280317092888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4 + 6105433
7460812*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 1928031709288
8*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1606693091074) - 235294755545*sqr
t(2)*sqrt(2*sqrt(2) + 3)*tanh(x/2)**8*atan(tanh(x/2)/sqrt(2*sqrt(2) + 3))/
(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 1363324
2959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 4317193603939
2*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808*sqrt
(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1
606693091074) - 332757034447*sqrt(2*sqrt(2) + 3)*tanh(x/2)**8*atan(tanh(x/
2)/sqrt(2*sqrt(2) + 3))/(1136103579984*sqrt(2)*tanh(x/2)**8 + 160669309107
4*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh
(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/...
```

3.810.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(22) = 44$.

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx = \frac{e^{(-2\,x)}-e^{(-6\,x)}}{2\left(2\,e^{(-4\,x)}+e^{(-8\,x)}+1\right)}+\frac{1}{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,e^{(-x)}\right)\right) - \frac{1}{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,e^{(-x)}\right)\right)$$

input
```
integrate(1/(cosh(x)^2+sinh(x)^2)^3,x, algorithm="maxima")
```

output
```
1/2*(e^(-2*x) - e^(-6*x))/(2*e^(-4*x) + e^(-8*x) + 1) + 1/2*arctan(1/2*sqr
t(2)*(sqrt(2) + 2*e^(-x))) - 1/2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x)))
```

3.810.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx = \frac{e^{(2x)}-e^{(-2x)}}{2\left(\left(e^{(2x)}-e^{(-2x)}\right)^2+4\right)} + \frac{1}{4}\arctan\left(\frac{1}{2}\left(e^{(4x)}-1\right)e^{(-2x)}\right)$$

input
```
integrate(1/(cosh(x)^2+sinh(x)^2)^3,x, algorithm="giac")
```

output
```
1/2*(e^(2*x) - e^(-2*x))/((e^(2*x) - e^(-2*x))^2 + 4) + 1/4*arctan(1/2*(e^
(4*x) - 1)*e^(-2*x))
```

3.810.9 Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx = \frac{\operatorname{atan}\left(e^{2x}\right)}{2} - \frac{e^{-2x}}{4\cosh(2x)^2} + \frac{1}{4\cosh(2x)}$$

input
```
int(1/(cosh(x)^2 + sinh(x)^2)^3,x)
```

output
```
atan(exp(2*x))/2 - exp(-2*x)/(4*cosh(2*x)^2) + 1/(4*cosh(2*x))
```

3.811 $\int \frac{1}{\cosh^2(x)-\sinh^2(x)}\,dx$

3.811.1 Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\cosh^2(x)-\sinh^2(x)}\,dx = x$$

output
```
x
```

3.811.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^2(x)-\sinh^2(x)}\,dx = x$$

input
```
Integrate[(Cosh[x]^2 - Sinh[x]^2)^(-1),x]
```

output
```
x
```

3.811.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4880, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^2(x)-\sinh^2(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin(ix)^2+\cos(ix)^2}dx$$
$$\Big\downarrow \; 4880$$
$$\int 1dx$$
$$\Big\downarrow \; 24$$
$$x$$

input
```
Int[(Cosh[x]^2 - Sinh[x]^2)^(-1),x]
```

output
```
x
```

3.811.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4880
```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

3.811.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

input
```
int(1/(cosh(x)^2-sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x
```

3.811.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)}\, dx = x$$

input
```
integrate(1/(cosh(x)^2-sinh(x)^2),x, algorithm="fricas")
```

output
```
x
```

3.811.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. 2(0) = 0.

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 10.00

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)}\, dx = \frac{x}{-\sinh^2(x) + \cosh^2(x)}$$

input
```
integrate(1/(cosh(x)**2-sinh(x)**2),x)
```

output
```
x/(-sinh(x)**2 + cosh(x)**2)
```

3.811.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)} \, dx = x$$

input
```
integrate(1/(cosh(x)^2-sinh(x)^2),x, algorithm="maxima")
```

output
```
x
```

3.811.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)} \, dx = x$$

input
```
integrate(1/(cosh(x)^2-sinh(x)^2),x, algorithm="giac")
```

output
```
x
```

3.811.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)} \, dx = x$$

input
```
int(1/(cosh(x)^2 - sinh(x)^2),x)
```

output
```
x
```

3.812 $\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2}\,dx$

3.812.1 Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2}\,dx = x$$

output
```
x
```

3.812.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2}\,dx = x$$

input
```
Integrate[(Cosh[x]^2 - Sinh[x]^2)^(-2),x]
```

output
```
x
```

3.812.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4880, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sin(ix)^2+\cos(ix)^2\right)^2}dx$$

$$\Big\downarrow \text{4880}$$

$$\int 1dx$$

$$\Big\downarrow \text{24}$$

$$x$$

input
```
Int[(Cosh[x]^2 - Sinh[x]^2)^(-2),x]
```

output
```
x
```

3.812.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4880
```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

3.812.4 Maple [A] (verified)

Time = 11.54 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2\,\mathrm{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

input
```
int(1/(cosh(x)^2-sinh(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.812.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2}\,dx = x$$

input
```
integrate(1/(cosh(x)^2-sinh(x)^2)^2,x, algorithm="fricas")
```

output
```
x
```

3.812.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(0) = 0.

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 22.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2}\,dx = \frac{x}{\sinh^4(x)-2\sinh^2(x)\cosh^2(x)+\cosh^4(x)}$$

input
```
integrate(1/(cosh(x)**2-sinh(x)**2)**2,x)
```

output
```
x/(sinh(x)**4 - 2*sinh(x)**2*cosh(x)**2 + cosh(x)**4)
```

3.812.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x) - \sinh^2(x)\right)^2} \, dx = x$$

input
```
integrate(1/(cosh(x)^2-sinh(x)^2)^2,x, algorithm="maxima")
```

output
```
x
```

3.812.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x) - \sinh^2(x)\right)^2} \, dx = x$$

input
```
integrate(1/(cosh(x)^2-sinh(x)^2)^2,x, algorithm="giac")
```

output
```
x
```

3.812.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x) - \sinh^2(x)\right)^2} \, dx = x$$

input
```
int(1/(cosh(x)^2 - sinh(x)^2)^2,x)
```

output
```
x
```

3.813 $\int \frac{1}{(\cosh^2(x)-\sinh^2(x))^3} dx$

3.813.1 Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{(\cosh^2(x) - \sinh^2(x))^3} dx = x$$

output
```
x
```

3.813.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\cosh^2(x) - \sinh^2(x))^3} dx = x$$

input
```
Integrate[(Cosh[x]^2 - Sinh[x]^2)^(-3),x]
```

output
```
x
```

3.813.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4880, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(\sin(ix)^2+\cos(ix)^2\right)^3}dx$$

$$\Big\downarrow \text{ 4880}$$

$$\int 1dx$$

$$\Big\downarrow \text{ 24}$$

$$x$$

input
```
Int[(Cosh[x]^2 - Sinh[x]^2)^(-3),x]
```

output
```
x
```

3.813.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4880
```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

3.813.4 Maple [A] (verified)

Time = 98.96 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2\,\text{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

input
```
int(1/(cosh(x)^2-sinh(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.813.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3}\,dx = x$$

input
```
integrate(1/(cosh(x)^2-sinh(x)^2)^3,x, algorithm="fricas")
```

output
```
x
```

3.813.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(0) = 0.

Time = 0.74 (sec) , antiderivative size = 34, normalized size of antiderivative = 34.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3}\,dx = \frac{x}{-\sinh^6(x)+3\sinh^4(x)\cosh^2(x)-3\sinh^2(x)\cosh^4(x)+\cosh^6(x)}$$

input
```
integrate(1/(cosh(x)**2-sinh(x)**2)**3,x)
```

output
```
x/(-sinh(x)**6 + 3*sinh(x)**4*cosh(x)**2 - 3*sinh(x)**2*cosh(x)**4 + cosh(
x)**6)
```

3.813.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x) - \sinh^2(x)\right)^3} \, dx = x$$

input
```
integrate(1/(cosh(x)^2-sinh(x)^2)^3,x, algorithm="maxima")
```

output
```
x
```

3.813.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x) - \sinh^2(x)\right)^3} \, dx = x$$

input
```
integrate(1/(cosh(x)^2-sinh(x)^2)^3,x, algorithm="giac")
```

output
```
x
```

3.813.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x) - \sinh^2(x)\right)^3} \, dx = x$$

input
```
int(1/(cosh(x)^2 - sinh(x)^2)^3,x)
```

output
```
x
```

3.814 $\int \frac{1}{\text{sech}^2(x)+\tanh^2(x)} dx$

3.814.1 Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{1}{\text{sech}^2(x)+\tanh^2(x)} dx = x$$

output
```
x
```

3.814.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\text{sech}^2(x)+\tanh^2(x)} dx = x$$

input
```
Integrate[(Sech[x]^2 + Tanh[x]^2)^(-1),x]
```

output
```
x
```

3.814.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4881, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tanh^2(x)+\operatorname{sech}^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ix)^2-\tan(ix)^2}dx$$

$$\Big\downarrow \text{4881}$$

$$\int 1dx$$

$$\Big\downarrow \text{24}$$

$$x$$

input
```
Int[(Sech[x]^2 + Tanh[x]^2)^(-1),x]
```

output
```
x
```

3.814.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4881
```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

3.814.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

input
```
int(1/(sech(x)^2+tanh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x
```

3.814.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}^2(x) + \tanh^2(x)} \, dx = x$$

input
```
integrate(1/(sech(x)^2+tanh(x)^2),x, algorithm="fricas")
```

output
```
x
```

3.814.6 Sympy [F]

$$\int \frac{1}{\operatorname{sech}^2(x) + \tanh^2(x)} \, dx = \int \frac{1}{\tanh^2(x) + \operatorname{sech}^2(x)} \, dx$$

input
```
integrate(1/(sech(x)**2+tanh(x)**2),x)
```

output
```
Integral(1/(tanh(x)**2 + sech(x)**2), x)
```

3.814.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}^2(x) + \tanh^2(x)}\, dx = x$$

input
```
integrate(1/(sech(x)^2+tanh(x)^2),x, algorithm="maxima")
```

output
```
x
```

3.814.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}^2(x) + \tanh^2(x)}\, dx = x$$

input
```
integrate(1/(sech(x)^2+tanh(x)^2),x, algorithm="giac")
```

output
```
x
```

3.814.9 Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}^2(x) + \tanh^2(x)}\, dx = x$$

input
```
int(1/(1/cosh(x)^2 + tanh(x)^2),x)
```

output
```
x
```

3.815 $\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2} dx$

3.815.1 Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2} dx = x$$

output
```
x
```

3.815.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2} dx = x$$

input
```
Integrate[(Sech[x]^2 + Tanh[x]^2)^(-2),x]
```

output
```
x
```

3.815.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4881, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\tanh^2(x)+\operatorname{sech}^2(x)\right)^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\left(\sec(ix)^2-\tan(ix)^2\right)^2}dx$$
$$\Big\downarrow \text{ 4881}$$
$$\int 1dx$$
$$\Big\downarrow \text{ 24}$$
$$x$$

input
```
Int[(Sech[x]^2 + Tanh[x]^2)^(-2),x]
```

output
```
x
```

3.815.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4881
```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

3.815.4 Maple [A] (verified)

Time = 38.34 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

input
```
int(1/(sech(x)^2+tanh(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.815.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\, dx = x$$

input
```
integrate(1/(sech(x)^2+tanh(x)^2)^2,x, algorithm="fricas")
```

output
```
x
```

3.815.6 Sympy [F]

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\, dx = \int \frac{1}{\left(\tanh^2(x)+\operatorname{sech}^2(x)\right)^2}\, dx$$

input
```
integrate(1/(sech(x)**2+tanh(x)**2)**2,x)
```

output
```
Integral((tanh(x)**2 + sech(x)**2)**(-2), x)
```

3.815.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\,dx = x$$

input
```
integrate(1/(sech(x)^2+tanh(x)^2)^2,x, algorithm="maxima")
```

output
```
x
```

3.815.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\,dx = x$$

input
```
integrate(1/(sech(x)^2+tanh(x)^2)^2,x, algorithm="giac")
```

output
```
x
```

3.815.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\,dx = x$$

input
```
int(1/(1/cosh(x)^2 + tanh(x)^2)^2,x)
```

output
```
x
```

3.816 $\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3} dx$

3.816.1 Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3} dx = x$$

output
```
x
```

3.816.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3} dx = x$$

input
```
Integrate[(Sech[x]^2 + Tanh[x]^2)^(-3),x]
```

output
```
x
```

3.816.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4881, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\tanh^2(x)+\operatorname{sech}^2(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sec(ix)^2-\tan(ix)^2\right)^3}dx$$

$$\Big\downarrow \text{4881}$$

$$\int 1dx$$

$$\Big\downarrow \text{24}$$

$$x$$

input
```
Int[(Sech[x]^2 + Tanh[x]^2)^(-3),x]
```

output
```
x
```

3.816.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4881
```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

3.816.4 Maple [A] (verified)

Time = 50.52 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
parallelrisch	0	2
default	$2\,\mathrm{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

input
```
int(1/(sech(x)^2+tanh(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.816.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\mathrm{sech}^2(x)+\tanh^2(x)\right)^3}\,dx = x$$

input
```
integrate(1/(sech(x)^2+tanh(x)^2)^3,x, algorithm="fricas")
```

output
```
x
```

3.816.6 Sympy [F]

$$\int \frac{1}{\left(\mathrm{sech}^2(x)+\tanh^2(x)\right)^3}\,dx = \int \frac{1}{\left(\tanh^2(x)+\mathrm{sech}^2(x)\right)^3}\,dx$$

input
```
integrate(1/(sech(x)**2+tanh(x)**2)**3,x)
```

output
```
Integral((tanh(x)**2 + sech(x)**2)**(-3), x)
```

3.816.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3}\,dx=x$$

input
```
integrate(1/(sech(x)^2+tanh(x)^2)^3,x, algorithm="maxima")
```

output
```
x
```

3.816.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3}\,dx=x$$

input
```
integrate(1/(sech(x)^2+tanh(x)^2)^3,x, algorithm="giac")
```

output
```
x
```

3.816.9 Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3}\,dx=x$$

input
```
int(1/(1/cosh(x)^2 + tanh(x)^2)^3,x)
```

output
```
x
```

3.817 $\int \frac{1}{\operatorname{sech}^2(x)-\tanh^2(x)}\,dx$

3.817.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{\operatorname{sech}^2(x)-\tanh^2(x)}\,dx=-x+\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)$$

output
```
-x+arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

3.817.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 529, normalized size of antiderivative = 27.84

$$\int \frac{1}{\operatorname{sech}^2(x)-\tanh^2(x)}\,dx = \frac{\operatorname{sech}(x)\left(\sqrt{2+2i}\arcsin\left(\frac{1}{2}\sqrt{1+i}\sqrt{(-1-i)(i+\sinh(x))}\right)\sqrt{(-1-i)(i+\sinh(x))}-2\operatorname{arctanh}\left(\frac{\sqrt{(-1-i)}}{\sqrt{(1+i)-(}}\right.\right.}{}$$

input
```
Integrate[(Sech[x]^2 - Tanh[x]^2)^(-1),x]
```

output
```
(Sech[x]*(Sqrt[2 + 2*I]*ArcSin[(Sqrt[1 + I]*Sqrt[(-1 - I)*(I + Sinh[x])])/
2]*Sqrt[(-1 - I)*(I + Sinh[x])] - 2*ArcTanh[Sqrt[(-1 - I)*(I + Sinh[x])]/S
qrt[(1 + I) - (1 - I)*Sinh[x]]]*Sqrt[(1 + I) - (1 - I)*Sinh[x]]*Sqrt[1 + I
*Sinh[x]]*Sqrt[(-1 - I)*(I + Sinh[x])] + 2*(-1)^(3/4)*ArcTanh[((-1)^(3/4)*
Sqrt[(-1 - I)*(I + Sinh[x])])/Sqrt[(1 + I) - (1 - I)*Sinh[x]]]*Sqrt[(1 + I
) - (1 - I)*Sinh[x]]*Sqrt[1 + I*Sinh[x]]*Sqrt[(-1 - I)*(I + Sinh[x])] + I*
Sqrt[2 + 2*I]*ArcSin[(Sqrt[1 + I]*Sqrt[(-1 - I)*(I + Sinh[x])])/2]*Sinh[x]
*Sqrt[(-1 - I)*(I + Sinh[x])] + Sqrt[-2 + 2*I]*ArcSinh[(Sqrt[-1 + I]*Sqrt[
(1 - I)*(I + Sinh[x])])/2]*Sqrt[(1 - I)*(I + Sinh[x])] + I*Sqrt[-2 + 2*I]*
ArcSinh[(Sqrt[-1 + I]*Sqrt[(1 - I)*(I + Sinh[x])])/2]*Sinh[x]*Sqrt[(1 - I)
*(I + Sinh[x])] + 2*(-1)^(3/4)*ArcTan[((-1)^(3/4)*Sqrt[(1 - I)*(I + Sinh[x
])])/Sqrt[(1 + I)*(-I + Sinh[x])]]*Sqrt[1 + I*Sinh[x]]*Sqrt[(1 + I)*(-I +
Sinh[x])]*Sqrt[(1 - I)*(I + Sinh[x])] + 2*ArcTanh[Sqrt[(1 - I)*(I + Sinh[x
])]/Sqrt[(1 + I)*(-I + Sinh[x])]]*Sqrt[1 + I*Sinh[x]]*Sqrt[(1 + I)*(-I + S
inh[x])]*Sqrt[(1 - I)*(I + Sinh[x])]))/(2*Sqrt[1 + I*Sinh[x]])
```

3.817.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 4889, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{sech}^2(x)-\tanh^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\tan(ix)^2+\sec(ix)^2}dx$$
$$\Big\downarrow \text{4889}$$
$$\int \frac{1}{2\tanh^4(x)-3\tanh^2(x)+1}d\tanh(x)$$
$$\Big\downarrow \text{1406}$$
$$2\int \frac{1}{2\tanh^2(x)-2}d\tanh(x)-2\int \frac{1}{2\tanh^2(x)-1}d\tanh(x)$$
$$\Big\downarrow \text{220}$$
$$\sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right)-\text{arctanh}(\tanh(x))$$

input
```
Int[(Sech[x]^2 - Tanh[x]^2)^(-1),x]
```

output
```
-ArcTanh[Tanh[x]] + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

3.817.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.817.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(15) = 30$.

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

method	result
risch	$-x+\frac{\sqrt{2}\,\ln\left(e^{2x}+2\sqrt{2}-3\right)}{2}-\frac{\sqrt{2}\,\ln\left(e^{2x}-3-2\sqrt{2}\right)}{2}$
default	$-\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2\tanh(\frac{x}{2}\right.$

input
```
int(1/(sech(x)^2-tanh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-x+1/2*2^(1/2)*ln(exp(2*x)+2*2^(1/2)-3)-1/2*2^(1/2)*ln(exp(2*x)-3-2*2^(1/2
))
```

3.817.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int \frac{1}{\operatorname{sech}^2(x) - \tanh^2(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2 - 4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x) + 3\left(2\sqrt{2}-3\right)\sinh(x)^2 - 2\sqrt{2}+3}{\cosh(x)^2 + \sinh(x)^2 - 3}\right) - x$$

input
```
integrate(1/(sech(x)^2-tanh(x)^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*
sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x
)^2 - 3)) - x
```

3.817.6 Sympy [F]

$$\int \frac{1}{\operatorname{sech}^2(x) - \tanh^2(x)}\,dx = \int \frac{1}{(-\tanh(x) + \operatorname{sech}(x))(\tanh(x) + \operatorname{sech}(x))}\,dx$$

input
```
integrate(1/(sech(x)**2-tanh(x)**2),x)
```

output
```
Integral(1/((-tanh(x) + sech(x))*(tanh(x) + sech(x))), x)
```

3.817.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.37

$$\int \frac{1}{\operatorname{sech}^2(x) - \tanh^2(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2} - e^{(-x)} + 1}{\sqrt{2} + e^{(-x)} - 1}\right) - \frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2} - e^{(-x)} - 1}{\sqrt{2} + e^{(-x)} + 1}\right) - x$$

input
```
integrate(1/(sech(x)^2-tanh(x)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/2*sqrt
(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x
```

3.817.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{1}{\operatorname{sech}^2(x) - \tanh^2(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2} + 2e^{(2x)} - 6\right|}{\left|4\sqrt{2} + 2e^{(2x)} - 6\right|}\right) - x$$

input
```
integrate(1/(sech(x)^2-tanh(x)^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - x
```

3.817.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.95

$$\int \frac{1}{\operatorname{sech}^2(x) - \tanh^2(x)}\,dx = \frac{\sqrt{2}\,\ln\left(8\,e^{2x} + \frac{\sqrt{2}\,(12\,e^{2x}-4)}{2}\right)}{2} - \frac{\sqrt{2}\,\ln\left(8\,e^{2x} - \frac{\sqrt{2}\,(12\,e^{2x}-4)}{2}\right)}{2} - x$$

input
```
int(1/(1/cosh(x)^2 - tanh(x)^2),x)
```

output
```
(2^(1/2)*log(8*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - (2^(1/2)*log
(8*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - x
```

3.817. $\int \frac{1}{\operatorname{sech}^2(x)-\tanh^2(x)}\,dx$

3.818 $\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx$

3.818.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx = x-\frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}+\frac{\tanh(x)}{1-2\tanh^2(x)}$$

output
```
x-1/2*arctanh(2^(1/2)*tanh(x))*2^(1/2)+tanh(x)/(1-2*tanh(x)^2)
```

3.818.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.25 (sec) , antiderivative size = 549, normalized size of antiderivative = 17.71

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx = \frac{\operatorname{sech}(x)\left(-4\sinh(x)-4\sinh^3(x)-(1+i)\sqrt{2}\left(\arctan\left(\frac{(1+i)\sqrt{(-1-i)(i+\sinh(x))}}{\sqrt{(2+2i)-(2-2i)\sinh(x)}}\right)\sqrt{(1+i)-(1-i)\sinh(x)}\right.\right.}{}$$

input
```
Integrate[(Sech[x]^2 - Tanh[x]^2)^(-2),x]
```

output
```
(Sech[x]*(-4*Sinh[x] - 4*Sinh[x]^3 - (1 + I)*Sqrt[2]*(ArcTan[((1 + I)*Sqrt
[(-1 - I)*(I + Sinh[x])])/Sqrt[(2 + 2*I) - (2 - 2*I)*Sinh[x]]]*Sqrt[(1 + I
) - (1 - I)*Sinh[x]] + I*Sqrt[1 + I]*ArcSin[(Sqrt[1 + I]*Sqrt[(-1 - I)*(I
+ Sinh[x])])/2]*Sqrt[1 + I*Sinh[x]])*Sqrt[(-1 - I)*(I + Sinh[x])] + ArcTan
h[Sqrt[(-1 - I)*(I + Sinh[x])]/Sqrt[(1 + I) - (1 - I)*Sinh[x]]]*(-3 + Cosh
[2*x])*Sqrt[(1 + I) - (1 - I)*Sinh[x]]*Sqrt[(-1 - I)*(I + Sinh[x])] + (1 +
 I)*Sqrt[2]*(ArcTan[((1 + I)*Sqrt[(-1 - I)*(I + Sinh[x])])/Sqrt[(2 + 2*I)
- (2 - 2*I)*Sinh[x]]]*Sqrt[(1 + I) - (1 - I)*Sinh[x]] + I*Sqrt[1 + I]*ArcS
in[(Sqrt[1 + I]*Sqrt[(-1 - I)*(I + Sinh[x])])/2]*Sqrt[1 + I*Sinh[x]])*Sinh
[x]^2*Sqrt[(-1 - I)*(I + Sinh[x])] + (ArcSinh[(Sqrt[-1 + I]*Sqrt[(1 - I)*(
I + Sinh[x])])/2]*(-3 + Cosh[2*x])*Sqrt[2 + (2*I)*Sinh[x]]*Sqrt[(1 - I)*(I
 + Sinh[x])])/Sqrt[-1 + I] - ArcTanh[Sqrt[(1 - I)*(I + Sinh[x])]/Sqrt[(1 +
 I)*(-I + Sinh[x])]]*(-3 + Cosh[2*x])*Sqrt[(1 + I)*(-I + Sinh[x])]*Sqrt[(1
 - I)*(I + Sinh[x])] + ((1 + I)*ArcTanh[((1 + I)*Sqrt[(1 - I)*(I + Sinh[x]
)])/(Sqrt[2]*Sqrt[(1 + I)*(-I + Sinh[x])])]*(-3 + Cosh[2*x])*Sqrt[(1 + I)*
(-I + Sinh[x])]*Sqrt[(1 - I)*(I + Sinh[x])])/Sqrt[2]))/(2*(-3 + Cosh[2*x])
)
```

3.818.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 4889, 316, 27, 383, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\tan(ix)^2+\sec(ix)^2\right)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{\left(1-2\tanh^2(x)\right)^2\left(1-\tanh^2(x)\right)}d\tanh(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{1}{2}\int -\frac{2\tanh^2(x)}{\left(1-2\tanh^2(x)\right)\left(1-\tanh^2(x)\right)}d\tanh(x)+\frac{\tanh(x)}{1-2\tanh^2(x)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\tanh(x)}{1-2\tanh^2(x)} - \int \frac{\tanh^2(x)}{(1-2\tanh^2(x))\,(1-\tanh^2(x))}d\tanh(x)$$

$$\Big\downarrow \text{383}$$

$$-\int \frac{1}{1-2\tanh^2(x)}d\tanh(x) + \int \frac{1}{1-\tanh^2(x)}d\tanh(x) + \frac{\tanh(x)}{1-2\tanh^2(x)}$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}(\tanh(x)) - \frac{\text{arctanh}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}} + \frac{\tanh(x)}{1-2\tanh^2(x)}$$

input
```
Int[(Sech[x]^2 - Tanh[x]^2)^(-2),x]
```

output
```
ArcTanh[Tanh[x]] - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2] + Tanh[x]/(1 - 2*Tanh[
x]^2)
```

3.818.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.818.4 Maple [A] (verified)

Time = 16.12 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.06

method	result
parallelrisch	0
risch	$x-\frac{2\left(3\,\mathrm{e}^{2x}-1\right)}{\mathrm{e}^{4x}-6\,\mathrm{e}^{2x}+1}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-3-2\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+2\sqrt{2}-3\right)}{4}$
default	$-\frac{-2\tanh\left(\frac{x}{2}\right)+2}{2\left(\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{2}+\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)+\frac{2\tanh\left(\frac{x}{2}\right)+2}{2\tanh\left(\frac{x}{2}\right)^{2}-4\tanh\left(\frac{x}{2}\right)}$

input
```
int(1/(sech(x)^2-tanh(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
0
```

3.818.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(28) = 56.

Time = 0.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 8.58

$$\int \frac{1}{\left(\operatorname{sech}^2(x) - \tanh^2(x)\right)^2}\,dx$$
$$= \frac{4\,x\cosh\left(x\right)^4 + 16\,x\cosh\left(x\right)\sinh\left(x\right)^3 + 4\,x\sinh\left(x\right)^4 - 24\,(x+1)\cosh\left(x\right)^2 + 24\,\left(x\cosh\left(x\right)^2 - x - 1\right)\sinh}{}$$

input
```
integrate(1/(sech(x)^2-tanh(x)^2)^2,x, algorithm="fricas")
```

output
```
1/4*(4*x*cosh(x)^4 + 16*x*cosh(x)*sinh(x)^3 + 4*x*sinh(x)^4 - 24*(x + 1)*c
osh(x)^2 + 24*(x*cosh(x)^2 - x - 1)*sinh(x)^2 + (sqrt(2)*cosh(x)^4 + 4*sqr
t(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 - sqrt(2
))*sinh(x)^2 - 6*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh
(x))*sinh(x) + sqrt(2))*log((3*(2*sqrt(2) + 3)*cosh(x)^2 - 4*(3*sqrt(2) +
4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 - 2*sqrt(2) - 3)/(cosh(x)
^2 + sinh(x)^2 - 3)) + 16*(x*cosh(x)^3 - 3*(x + 1)*cosh(x))*sinh(x) + 4*x
+ 8)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2 - 1)*sinh
(x)^2 - 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x))*sinh(x) + 1)
```

3.818.6 Sympy [F]

$$\int \frac{1}{\left(\operatorname{sech}^2(x) - \tanh^2(x)\right)^2}\,dx = \int \frac{1}{\left(-\tanh\left(x\right) + \operatorname{sech}\left(x\right)\right)^2\left(\tanh\left(x\right) + \operatorname{sech}\left(x\right)\right)^2}\,dx$$

input
```
integrate(1/(sech(x)**2-tanh(x)**2)**2,x)
```

output
```
Integral(1/((-tanh(x) + sech(x))**2*(tanh(x) + sech(x))**2), x)
```

3.818.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(28) = 56.

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.84

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx = -\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right) + \frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right) + x - \frac{2\left(3\,e^{(-2\,x)}-1\right)}{6\,e^{(-2\,x)}-e^{(-4\,x)}-1}$$

input
```
integrate(1/(sech(x)^2-tanh(x)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) + 1/4*sqr
t(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + x - 2*(3*e^(-2*
x) - 1)/(6*e^(-2*x) - e^(-4*x) - 1)
```

3.818.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2\,e^{(2\,x)}-6\right|}{\left|4\sqrt{2}+2\,e^{(2\,x)}-6\right|}\right) + x - \frac{2\left(3\,e^{(2\,x)}-1\right)}{e^{(4\,x)}-6\,e^{(2\,x)}+1}$$

input
```
integrate(1/(sech(x)^2-tanh(x)^2)^2,x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) + x - 2*(3*e^(2*x) - 1)/(e^(4*x) - 6*e^(2*x) + 1)
```

3.818.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.52

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx = x-\frac{\sqrt{2}\,\ln\left(-4\,e^{2\,x}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{4}\right)}{4}+\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{4}-4\,e^{2\,x}\right)}{4}-\frac{6\,e^{2\,x}-2}{e^{4\,x}-6\,e^{2\,x}+1}$$

input
```
int(1/(1/cosh(x)^2 - tanh(x)^2)^2,x)
```

output
```
x - (2^(1/2)*log(- 4*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/4))/4 + (2^(1/
2)*log((2^(1/2)*(12*exp(2*x) - 4))/4 - 4*exp(2*x)))/4 - (6*exp(2*x) - 2)/(
exp(4*x) - 6*exp(2*x) + 1)
```

3.819 $\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3} dx$

3.819.1 Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3} dx = -x+\frac{7\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}} + \frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2} - \frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}$$

output
```
-x+7/8*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/2*tanh(x)/(1-2*tanh(x)^2)^2-1/4*
tanh(x)/(1-2*tanh(x)^2)
```

3.819.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.75 (sec) , antiderivative size = 765, normalized size of antiderivative = 14.17

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3} dx = \frac{\operatorname{sech}(x)\left(-7\operatorname{arctanh}\left(\frac{\sqrt{(-1-i)(i+\sinh(x))}}{\sqrt{(1+i)-(1-i)\sinh(x)}}\right)(-3+\cosh(2x))^2\sqrt{(2+2i)-(2-2i)\sinh(x)}\sqrt{(-1-i)(i+\sinh(x))}\right.}{}$$

input
```
Integrate[(Sech[x]^2 - Tanh[x]^2)^(-3),x]
```

output
```
(Sech[x]*(-7*ArcTanh[Sqrt[(-1 - I)*(I + Sinh[x])]/Sqrt[(1 + I) - (1 - I)*S
inh[x]]]*(-3 + Cosh[2*x])^2*Sqrt[(2 + 2*I) - (2 - 2*I)*Sinh[x]]*Sqrt[(-1 -
 I)*(I + Sinh[x])] + 7*(-1)^(3/4)*ArcTanh[((-1)^(3/4)*Sqrt[(-1 - I)*(I + S
inh[x])])/Sqrt[(1 + I) - (1 - I)*Sinh[x]]]*(-3 + Cosh[2*x])^2*Sqrt[(2 + 2*
I) - (2 - 2*I)*Sinh[x]]*Sqrt[(-1 - I)*(I + Sinh[x])] + 4*(2*Sqrt[2]*Sinh[x
] + 8*Sqrt[2]*Sinh[x]^3 + 6*Sqrt[2]*Sinh[x]^5 + (7 - I)*Sqrt[1 + I]*ArcSin
[(Sqrt[1 + I]*Sqrt[(-1 - I)*(I + Sinh[x])])/2]*Sqrt[1 + I*Sinh[x]]*Sqrt[(-
1 - I)*(I + Sinh[x])] - (14 - 2*I)*Sqrt[1 + I]*ArcSin[(Sqrt[1 + I]*Sqrt[(-
1 - I)*(I + Sinh[x])])/2]*Sqrt[1 + I*Sinh[x]]*Sinh[x]^2*Sqrt[(-1 - I)*(I +
 Sinh[x])] + (7 - I)*Sqrt[1 + I]*ArcSin[(Sqrt[1 + I]*Sqrt[(-1 - I)*(I + Si
nh[x])])/2]*Sqrt[1 + I*Sinh[x]]*Sinh[x]^4*Sqrt[(-1 - I)*(I + Sinh[x])] + (
7/4 + I/4)*Sqrt[-1 + I]*ArcSinh[(Sqrt[-1 + I]*Sqrt[(1 - I)*(I + Sinh[x])])
/2]*(-3 + Cosh[2*x])^2*Sqrt[1 + I*Sinh[x]]*Sqrt[(1 - I)*(I + Sinh[x])] - (
7 + 7*I)*ArcTanh[((1 + I)*Sqrt[(1 - I)*(I + Sinh[x])])/(Sqrt[2]*Sqrt[(1 +
I)*(-I + Sinh[x])])]*Sqrt[(1 + I)*(-I + Sinh[x])]*Sqrt[(1 - I)*(I + Sinh[x
])] + (7*ArcTanh[Sqrt[(1 - I)*(I + Sinh[x])]/Sqrt[(1 + I)*(-I + Sinh[x])]]
*(-3 + Cosh[2*x])^2*Sqrt[(1 + I)*(-I + Sinh[x])]*Sqrt[(1 - I)*(I + Sinh[x]
)])/(2*Sqrt[2]) + (14 + 14*I)*ArcTanh[((1 + I)*Sqrt[(1 - I)*(I + Sinh[x])]
)/(Sqrt[2]*Sqrt[(1 + I)*(-I + Sinh[x])])]*Sinh[x]^2*Sqrt[(1 + I)*(-I + Sin
h[x])]*Sqrt[(1 - I)*(I + Sinh[x])] - (7 + 7*I)*ArcTanh[((1 + I)*Sqrt[(1...
```

3.819.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 4889, 316, 27, 402, 397, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(\tan(ix)^2+\sec(ix)^2\right)^3}dx$$

$$\Big\downarrow \ 4889$$

$$\int \frac{1}{\left(1-2\tanh^2(x)\right)^3\left(1-\tanh^2(x)\right)}d\tanh(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{1}{4}\int \frac{2\left(1-3\tanh^2(x)\right)}{\left(1-2\tanh^2(x)\right)^2\left(1-\tanh^2(x)\right)}d\tanh(x)+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1-3\tanh^2(x)}{\left(1-2\tanh^2(x)\right)^2\left(1-\tanh^2(x)\right)}d\tanh(x)+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{\tanh^2(x)+3}{\left(1-2\tanh^2(x)\right)\left(1-\tanh^2(x)\right)}d\tanh(x)-\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)}\right)+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}$$

$$\Big\downarrow \text{397}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(7\int \frac{1}{1-2\tanh^2(x)}d\tanh(x)-4\int \frac{1}{1-\tanh^2(x)}d\tanh(x)\right)-\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)}\right)+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{7\text{arctanh}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}-4\text{arctanh}(\tanh(x))\right)-\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)}\right)+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}$$

input
```
Int[(Sech[x]^2 - Tanh[x]^2)^(-3),x]
```

output
```
Tanh[x]/(2*(1 - 2*Tanh[x]^2)^2) + ((-4*ArcTanh[Tanh[x]] + (7*ArcTanh[Sqrt[
2]*Tanh[x]])/Sqrt[2])/2 - Tanh[x]/(2*(1 - 2*Tanh[x]^2)))/2
```

3.819. $\int \frac{1}{\left(\text{sech}^2(x)-\tanh^2(x)\right)^3}\,dx$

3.819.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.819.4 Maple [A] (verified)

Time = 210.08 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

method	result
parallelrisch	0
risch	$-x+\frac{17\,\mathrm{e}^{6x}-57\,\mathrm{e}^{4x}+19\,\mathrm{e}^{2x}-3}{2\left(\mathrm{e}^{4x}-6\,\mathrm{e}^{2x}+1\right)^{2}}+\frac{7\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+2\sqrt{2}-3\right)}{16}-\frac{7\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-3-2\sqrt{2}\right)}{16}$
default	$-\frac{2\left(-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{8}+\frac{\tanh\left(\frac{x}{2}\right)^{2}}{8}-\frac{5\tanh\left(\frac{x}{2}\right)}{8}+\frac{1}{8}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{7\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{8}-\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)-\frac{2\left(-\frac{\mathrm{ta}}{}\right.}{}$

input
```
int(1/(sech(x)^2-tanh(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
0
```

3.819.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(44) = 88.

Time = 0.28 (sec) , antiderivative size = 717, normalized size of antiderivative = 13.28

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(1/(sech(x)^2-tanh(x)^2)^3,x, algorithm="fricas")
```

output
```
-1/16*(16*x*cosh(x)^8 + 128*x*cosh(x)*sinh(x)^7 + 16*x*sinh(x)^8 - 8*(24*x
 + 17)*cosh(x)^6 + 8*(56*x*cosh(x)^2 - 24*x - 17)*sinh(x)^6 + 16*(56*x*cos
h(x)^3 - 3*(24*x + 17)*cosh(x))*sinh(x)^5 + 152*(4*x + 3)*cosh(x)^4 + 8*(1
40*x*cosh(x)^4 - 15*(24*x + 17)*cosh(x)^2 + 76*x + 57)*sinh(x)^4 + 32*(28*
x*cosh(x)^5 - 5*(24*x + 17)*cosh(x)^3 + 19*(4*x + 3)*cosh(x))*sinh(x)^3 -
8*(24*x + 19)*cosh(x)^2 + 8*(56*x*cosh(x)^6 - 15*(24*x + 17)*cosh(x)^4 + 1
14*(4*x + 3)*cosh(x)^2 - 24*x - 19)*sinh(x)^2 - 7*(sqrt(2)*cosh(x)^8 + 8*s
qrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 4*(7*sqrt(2)*cosh(x)^2 - 3*
sqrt(2))*sinh(x)^6 - 12*sqrt(2)*cosh(x)^6 + 8*(7*sqrt(2)*cosh(x)^3 - 9*sqr
t(2)*cosh(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^4 - 90*sqrt(2)*cosh(x)^2 +
 19*sqrt(2))*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 - 3
0*sqrt(2)*cosh(x)^3 + 19*sqrt(2)*cosh(x))*sinh(x)^3 + 4*(7*sqrt(2)*cosh(x)
^6 - 45*sqrt(2)*cosh(x)^4 + 57*sqrt(2)*cosh(x)^2 - 3*sqrt(2))*sinh(x)^2 -
12*sqrt(2)*cosh(x)^2 + 8*(sqrt(2)*cosh(x)^7 - 9*sqrt(2)*cosh(x)^5 + 19*sqr
t(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-(3*(2*sqrt(2)
- 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sin
h(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 - 3)) + 16*(8*x*cosh(x)^7 -
 3*(24*x + 17)*cosh(x)^5 + 38*(4*x + 3)*cosh(x)^3 - (24*x + 19)*cosh(x))*s
inh(x) + 16*x + 24)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*co
sh(x)^2 - 3)*sinh(x)^6 - 12*cosh(x)^6 + 8*(7*cosh(x)^3 - 9*cosh(x))*sin...
```

3.819.6 Sympy [F]

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx=\int \frac{1}{\left(-\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^3\left(\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^3}\,dx$$

input
```
integrate(1/(sech(x)**2-tanh(x)**2)**3,x)
```

output
```
Integral(1/((-tanh(x) + sech(x))**3*(tanh(x) + sech(x))**3), x)
```

3.819.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(44) = 88.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.11

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx = \frac{7}{16}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right) - \frac{7}{16}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right) - x + \frac{19\,e^{(-2\,x)}-57\,e^{(-4\,x)}+17\,e^{(-6\,x)}-3}{2\,\left(12\,e^{(-2\,x)}-38\,e^{(-4\,x)}+12\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}$$

input
```
integrate(1/(sech(x)^2-tanh(x)^2)^3,x, algorithm="maxima")
```

output
```
7/16*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 7/16*sq
rt(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x + 1/2*(19*e^
(-2*x) - 57*e^(-4*x) + 17*e^(-6*x) - 3)/(12*e^(-2*x) - 38*e^(-4*x) + 12*e^
(-6*x) - e^(-8*x) - 1)
```

3.819.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.43

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx = -\frac{7}{16}\,\sqrt{2}\log\left(\frac{\left|-4\,\sqrt{2}+2\,e^{(2\,x)}-6\right|}{\left|4\,\sqrt{2}+2\,e^{(2\,x)}-6\right|}\right) - x + \frac{17\,e^{(6\,x)}-57\,e^{(4\,x)}+19\,e^{(2\,x)}-3}{2\,\left(e^{(4\,x)}-6\,e^{(2\,x)}+1\right)^2}$$

input
```
integrate(1/(sech(x)^2-tanh(x)^2)^3,x, algorithm="giac")
```

output
```
-7/16*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x
) - 6)) - x + 1/2*(17*e^(6*x) - 57*e^(4*x) + 19*e^(2*x) - 3)/(e^(4*x) - 6*
e^(2*x) + 1)^2
```

3.819.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.11

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx = \frac{136\,e^{2\,x}-24}{38\,e^{4\,x}-12\,e^{2\,x}-12\,e^{6\,x}+e^{8\,x}+1} - x - \frac{7\,\sqrt{2}\,\ln\left(7\,e^{2\,x}-\frac{7\,\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{16}\right)}{16} + \frac{7\,\sqrt{2}\,\ln\left(7\,e^{2\,x}+\frac{7\,\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{16}\right)}{16} + \frac{\frac{17\,e^{2\,x}}{2}+\frac{45}{2}}{e^{4\,x}-6\,e^{2\,x}+1}$$

input
```
int(1/(1/cosh(x)^2 - tanh(x)^2)^3,x)
```

output
```
(136*exp(2*x) - 24)/(38*exp(4*x) - 12*exp(2*x) - 12*exp(6*x) + exp(8*x) +
1) - x - (7*2^(1/2)*log(7*exp(2*x) - (7*2^(1/2)*(12*exp(2*x) - 4))/16))/16
 + (7*2^(1/2)*log(7*exp(2*x) + (7*2^(1/2)*(12*exp(2*x) - 4))/16))/16 + ((1
7*exp(2*x))/2 + 45/2)/(exp(4*x) - 6*exp(2*x) + 1)
```

3.820 $\int \frac{1}{\coth^2(x)+\mathrm{csch}^2(x)}\,dx$

3.820.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{\coth^2(x)+\mathrm{csch}^2(x)}\,dx = x-\sqrt{2}\,\mathrm{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

output
```
x-arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

3.820.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^2(x)+\mathrm{csch}^2(x)}\,dx = x-\sqrt{2}\,\mathrm{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

input
```
Integrate[(Coth[x]^2 + Csch[x]^2)^(-1),x]
```

output
```
x - Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

3.820.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4889, 1450, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{-\cot(ix)^2-\csc(ix)^2}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{\tanh^2(x)}{\tanh^4(x)-3\tanh^2(x)+2}d\tanh(x)$$

$$\Big\downarrow \text{ 1450}$$

$$2\int \frac{1}{\tanh^2(x)-2}d\tanh(x)-\int \frac{1}{\tanh^2(x)-1}d\tanh(x)$$

$$\Big\downarrow \text{ 220}$$

$$\operatorname{arctanh}(\tanh(x))-\sqrt{2}\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

input
```
Int[(Coth[x]^2 + Csch[x]^2)^(-1),x]
```

output
```
ArcTanh[Tanh[x]] - Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

3.820.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.820.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.83 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

method	result
risch	$x+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3+2\sqrt{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-2\sqrt{2}+3\right)}{2}$
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)-\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^{2}+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}+1\right)+2\,\mathrm{arcta}\right.}{4}$

input
```
int(1/(coth(x)^2+csch(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x+1/2*2^(1/2)*ln(exp(2*x)+3+2*2^(1/2))-1/2*2^(1/2)*ln(exp(2*x)-2*2^(1/2)+3
)
```

3.820.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.72

$$\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx$$
$$=\frac{1}{2}\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(x)^2-4\left(3\sqrt{2}+4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}+3\right)\sinh(x)^2+2\sqrt{2}+3}{\cosh(x)^2+\sinh(x)^2+3}\right)$$
$$+x$$

input
```
integrate(1/(coth(x)^2+csch(x)^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(x)^2 - 4*(3*sqrt(2) + 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 + 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)
^2 + 3)) + x
```

3.820.6 Sympy [F]

$$\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx=\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx$$

input
```
integrate(1/(coth(x)**2+csch(x)**2),x)
```

output
```
Integral(1/(coth(x)**2 + csch(x)**2), x)
```

3.820.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx=\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)+x$$

input
```
integrate(1/(coth(x)^2+csch(x)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) +
x
```

3.820.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(15) = 30.

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx=-\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)+x$$

input
```
integrate(1/(coth(x)^2+csch(x)^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + x
```

3.820.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.00

$$\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx=x+\frac{\sqrt{2}\ln\left(8\,e^{2x}-\frac{\sqrt{2}\,(12\,e^{2x}+4)}{2}\right)}{2}-\frac{\sqrt{2}\ln\left(8\,e^{2x}+\frac{\sqrt{2}\,(12\,e^{2x}+4)}{2}\right)}{2}$$

input
```
int(1/(coth(x)^2 + 1/sinh(x)^2),x)
```

output
```
x + (2^(1/2)*log(8*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/2))/2 - (2^(1/2)
*log(8*exp(2*x) + (2^(1/2)*(12*exp(2*x) + 4))/2))/2
```

3.821 $\int \frac{1}{\left(\coth^2(x)+\mathbf{csch}^2(x)\right)^2}\,dx$

3.821.1 Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx = x-\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh(x)}{2-\tanh^2(x)}$$

output
```
x-1/2*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)-tanh(x)/(2-tanh(x)^2)
```

3.821.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx = \frac{(3+\cosh(2x))\operatorname{csch}^4(x)\left(6x+2x\cosh(2x)-\sqrt{2}\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)(3+\cosh(2x))-2\sinh(2x)\right)}{8\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}$$

input
```
Integrate[(Coth[x]^2 + Csch[x]^2)^(-2),x]
```

output
```
((3 + Cosh[2*x])*Csch[x]^4*(6*x + 2*x*Cosh[2*x] - Sqrt[2]*ArcTanh[Tanh[x]/
Sqrt[2]]*(3 + Cosh[2*x]) - 2*Sinh[2*x]))/(8*(Coth[x]^2 + Csch[x]^2)^2)
```

3.821.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4889, 372, 27, 303, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-\cot(ix)^2-\csc(ix)^2\right)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tanh^4(x)}{\left(1-\tanh^2(x)\right)\left(2-\tanh^2(x)\right)^2}d\tanh(x)$$

$$\Big\downarrow \text{372}$$

$$\frac{1}{2}\int \frac{2}{\left(1-\tanh^2(x)\right)\left(2-\tanh^2(x)\right)}d\tanh(x)-\frac{\tanh(x)}{2-\tanh^2(x)}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\left(2-\tanh^2(x)\right)}d\tanh(x)-\frac{\tanh(x)}{2-\tanh^2(x)}$$

$$\Big\downarrow \text{303}$$

$$\int \frac{1}{1-\tanh^2(x)}d\tanh(x)-\int \frac{1}{2-\tanh^2(x)}d\tanh(x)-\frac{\tanh(x)}{2-\tanh^2(x)}$$

$$\Big\downarrow \text{219}$$

$$\operatorname{arctanh}(\tanh(x))-\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh(x)}{2-\tanh^2(x)}$$

input
```
Int[(Coth[x]^2 + Csch[x]^2)^(-2),x]
```

output
```
ArcTanh[Tanh[x]] - ArcTanh[Tanh[x]/Sqrt[2]]/Sqrt[2] - Tanh[x]/(2 - Tanh[x]
^2)
```

3.821.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.821. $\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx$

3.821.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.

Time = 26.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

method	result
risch	$x+\frac{6\,\mathrm{e}^{2x}+2}{\mathrm{e}^{4x}+6\,\mathrm{e}^{2x}+1}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3+2\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-2\sqrt{2}+3\right)}{4}$
default	$\frac{-\tanh\left(\frac{x}{2}\right)^{3}-\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{4}+1}-\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^{2}+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}-1\right)\right)}{8}+\frac{\sqrt{2}\,(\ln}{}$

input
```
int(1/(coth(x)^2+csch(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x+2*(3*exp(2*x)+1)/(exp(4*x)+6*exp(2*x)+1)+1/4*2^(1/2)*ln(exp(2*x)+3+2*2^(
1/2))-1/4*2^(1/2)*ln(exp(2*x)-2*2^(1/2)+3)
```

3.821.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 8.19

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx$$

$$=\frac{4\,x\cosh(x)^4+16\,x\cosh(x)\sinh(x)^3+4\,x\sinh(x)^4+24\,(x+1)\cosh(x)^2+24\left(x\cosh(x)^2+x+1\right)\sinh}{}$$

input
```
integrate(1/(coth(x)^2+csch(x)^2)^2,x, algorithm="fricas")
```

output
```
1/4*(4*x*cosh(x)^4 + 16*x*cosh(x)*sinh(x)^3 + 4*x*sinh(x)^4 + 24*(x + 1)*c
osh(x)^2 + 24*(x*cosh(x)^2 + x + 1)*sinh(x)^2 + (sqrt(2)*cosh(x)^4 + 4*sqr
t(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 + sqrt(2
))*sinh(x)^2 + 6*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh
(x))*sinh(x) + sqrt(2))*log((3*(2*sqrt(2) + 3)*cosh(x)^2 - 4*(3*sqrt(2) +
4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 + 2*sqrt(2) + 3)/(cosh(x)
^2 + sinh(x)^2 + 3)) + 16*(x*cosh(x)^3 + 3*(x + 1)*cosh(x))*sinh(x) + 4*x
+ 8)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2 + 1)*sinh
(x)^2 + 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x))*sinh(x) + 1)
```

3.821.6 Sympy [F]

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx=\int \frac{1}{\left(\coth^2{(x)}+\operatorname{csch}^2{(x)}\right)^2}\,dx$$

input
```
integrate(1/(coth(x)**2+csch(x)**2)**2,x)
```

output
```
Integral((coth(x)**2 + csch(x)**2)**(-2), x)
```

3.821.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26)=52$.

Time $=0.30$ (sec) , antiderivative size $=60$, normalized size of antiderivative $=1.88$

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx=\frac{1}{4}\,\sqrt{2}\log\left(-\frac{2\,\sqrt{2}-e^{(-2\,x)}-3}{2\,\sqrt{2}+e^{(-2\,x)}+3}\right)+x-\frac{2\left(3\,e^{(-2\,x)}+1\right)}{6\,e^{(-2\,x)}+e^{(-4\,x)}+1}$$

input
```
integrate(1/(coth(x)^2+csch(x)^2)^2,x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) +
x - 2*(3*e^(-2*x) + 1)/(6*e^(-2*x) + e^(-4*x) + 1)
```

3.821.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26)=52$.

Time $=0.27$ (sec) , antiderivative size $=60$, normalized size of antiderivative $=1.88$

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx=-\frac{1}{4}\,\sqrt{2}\log\left(-\frac{2\,\sqrt{2}-e^{(2\,x)}-3}{2\,\sqrt{2}+e^{(2\,x)}+3}\right)+x+\frac{2\left(3\,e^{(2\,x)}+1\right)}{e^{(4\,x)}+6\,e^{(2\,x)}+1}$$

input
```
integrate(1/(coth(x)^2+csch(x)^2)^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + x
 + 2*(3*e^(2*x) + 1)/(e^(4*x) + 6*e^(2*x) + 1)
```

3.821.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.41

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx = x+\frac{\sqrt{2}\,\ln\left(4\,e^{2\,x}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{4}\right)}{4}-\frac{\sqrt{2}\,\ln\left(4\,e^{2\,x}+\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{4}\right)}{4}+\frac{6\,e^{2\,x}+2}{6\,e^{2\,x}+e^{4\,x}+1}$$

input
```
int(1/(coth(x)^2 + 1/sinh(x)^2)^2,x)
```

output
```
x + (2^(1/2)*log(4*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/4))/4 - (2^(1/2)
*log(4*exp(2*x) + (2^(1/2)*(12*exp(2*x) + 4))/4))/4 + (6*exp(2*x) + 2)/(6*
exp(2*x) + exp(4*x) + 1)
```

3.822 $\int \frac{1}{\left(\coth^2(x)+\text{csch}^2(x)\right)^3}\,dx$

3.822.1 Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{1}{\left(\coth^2(x)+\text{csch}^2(x)\right)^3}\,dx=x-\frac{7\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}$$

output
```
x-7/8*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)-1/2*tanh(x)^3/(2-tanh(x)^2)^2-1
/4*tanh(x)/(2-tanh(x)^2)
```

3.822.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(\coth^2(x)+\text{csch}^2(x)\right)^3}\,dx$$
$$=\frac{76x+48x\cosh(2x)-7\sqrt{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)(3+\cosh(2x))^2+4x\cosh(4x)-2\sinh(2x)-3\sinh(4x)}{8(3+\cosh(2x))^2}$$

input
```
Integrate[(Coth[x]^2 + Csch[x]^2)^(-3),x]
```

output
```
(76*x + 48*x*Cosh[2*x] - 7*Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]*(3 + Cosh[2*x]
)^2 + 4*x*Cosh[4*x] - 2*Sinh[2*x] - 3*Sinh[4*x])/(8*(3 + Cosh[2*x])^2)
```

3.822.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3042, 4889, 372, 27, 440, 25, 397, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-\cot(ix)^2-\csc(ix)^2\right)^3}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tanh^6(x)}{\left(1-\tanh^2(x)\right)\left(2-\tanh^2(x)\right)^3}d\tanh(x)$$

$$\Big\downarrow \text{372}$$

$$\frac{1}{4}\int \frac{2\tanh^2(x)\left(3-\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\left(2-\tanh^2(x)\right)^2}d\tanh(x)-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\tanh^2(x)\left(3-\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\left(2-\tanh^2(x)\right)^2}d\tanh(x)-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}$$

$$\Big\downarrow \text{440}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int -\frac{3\tanh^2(x)+1}{\left(1-\tanh^2(x)\right)\left(2-\tanh^2(x)\right)}d\tanh(x)-\frac{\tanh(x)}{2\left(2-\tanh^2(x)\right)}\right)-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{3\tanh^2(x)+1}{\left(1-\tanh^2(x)\right)\left(2-\tanh^2(x)\right)}d\tanh(x)-\frac{\tanh(x)}{2\left(2-\tanh^2(x)\right)}\right)-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}$$

$$\Big\downarrow \text{397}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4\int \frac{1}{1-\tanh^2(x)}d\tanh(x)-7\int \frac{1}{2-\tanh^2(x)}d\tanh(x)\right)-\frac{\tanh(x)}{2\left(2-\tanh^2(x)\right)}\right)-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4\text{arctanh}(\tanh(x))-\frac{7\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}\right)-\frac{\tanh(x)}{2\left(2-\tanh^2(x)\right)}\right)-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}$$

input
```
Int[(Coth[x]^2 + Csch[x]^2)^(-3),x]
```

output
```
-1/2*Tanh[x]^3/(2 - Tanh[x]^2)^2 + ((4*ArcTanh[Tanh[x]] - (7*ArcTanh[Tanh[
x]/Sqrt[2]])/Sqrt[2])/2 - Tanh[x]/(2*(2 - Tanh[x]^2)))/2
```

3.822.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.822.4 Maple [A] (verified)

Time = 37.53 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

method	result
parallelrisch	0
risch	$x+\frac{17\,{\mathrm e}^{6x}+57\,{\mathrm e}^{4x}+19\,{\mathrm e}^{2x}+3}{2\left({\mathrm e}^{4x}+6\,{\mathrm e}^{2x}+1\right)^{2}}+\frac{7\sqrt{2}\,\ln\left({\mathrm e}^{2x}+3+2\sqrt{2}\right)}{16}-\frac{7\sqrt{2}\,\ln\left({\mathrm e}^{2x}-2\sqrt{2}+3\right)}{16}$
default	$\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)+\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{7}}{4}-\frac{5\tanh\left(\frac{x}{2}\right)^{5}}{4}-\frac{5\tanh\left(\frac{x}{2}\right)^{3}}{4}-\frac{\tanh\left(\frac{x}{2}\right)}{4}}{\left(\tanh\left(\frac{x}{2}\right)^{4}+1\right)^{2}}-\frac{7\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^{2}+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\,\mathrm{arc}$

input
```
int(1/(coth(x)^2+csch(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
0
```

3.822.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. 2(41) = 82.

Time = 0.28 (sec) , antiderivative size = 715, normalized size of antiderivative = 13.24

$$\int \frac{1}{(\coth^2(x)+\operatorname{csch}^2(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(coth(x)^2+csch(x)^2)^3,x, algorithm="fricas")
```

output
```
1/16*(16*x*cosh(x)^8 + 128*x*cosh(x)*sinh(x)^7 + 16*x*sinh(x)^8 + 8*(24*x
+ 17)*cosh(x)^6 + 8*(56*x*cosh(x)^2 + 24*x + 17)*sinh(x)^6 + 16*(56*x*cosh
(x)^3 + 3*(24*x + 17)*cosh(x))*sinh(x)^5 + 152*(4*x + 3)*cosh(x)^4 + 8*(14
0*x*cosh(x)^4 + 15*(24*x + 17)*cosh(x)^2 + 76*x + 57)*sinh(x)^4 + 32*(28*x
*cosh(x)^5 + 5*(24*x + 17)*cosh(x)^3 + 19*(4*x + 3)*cosh(x))*sinh(x)^3 + 8
*(24*x + 19)*cosh(x)^2 + 8*(56*x*cosh(x)^6 + 15*(24*x + 17)*cosh(x)^4 + 11
4*(4*x + 3)*cosh(x)^2 + 24*x + 19)*sinh(x)^2 + 7*(sqrt(2)*cosh(x)^8 + 8*sq
rt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 4*(7*sqrt(2)*cosh(x)^2 + 3*s
qrt(2))*sinh(x)^6 + 12*sqrt(2)*cosh(x)^6 + 8*(7*sqrt(2)*cosh(x)^3 + 9*sqrt
(2)*cosh(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^4 + 90*sqrt(2)*cosh(x)^2 +
19*sqrt(2))*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 + 30
*sqrt(2)*cosh(x)^3 + 19*sqrt(2)*cosh(x))*sinh(x)^3 + 4*(7*sqrt(2)*cosh(x)^
6 + 45*sqrt(2)*cosh(x)^4 + 57*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^2 + 1
2*sqrt(2)*cosh(x)^2 + 8*(sqrt(2)*cosh(x)^7 + 9*sqrt(2)*cosh(x)^5 + 19*sqrt
(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log((3*(2*sqrt(2) +
3)*cosh(x)^2 - 4*(3*sqrt(2) + 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) + 3)*sinh(
x)^2 + 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 + 3)) + 16*(8*x*cosh(x)^7 + 3
*(24*x + 17)*cosh(x)^5 + 38*(4*x + 3)*cosh(x)^3 + (24*x + 19)*cosh(x))*sin
h(x) + 16*x + 24)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh
(x)^2 + 3)*sinh(x)^6 + 12*cosh(x)^6 + 8*(7*cosh(x)^3 + 9*cosh(x))*sinh(...
```

3.822.6 Sympy [F]

$$\int \frac{1}{(\coth^2(x)+\operatorname{csch}^2(x))^3}\,dx = \int \frac{1}{(\coth^2(x)+\operatorname{csch}^2(x))^3}\,dx$$

input
```
integrate(1/(coth(x)**2+csch(x)**2)**3,x)
```

output
```
Integral((coth(x)**2 + csch(x)**2)**(-3), x)
```

3.822.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(41) = 82.

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx = \frac{7}{16}\,\sqrt{2}\log\left(-\frac{2\,\sqrt{2}-e^{(-2\,x)}-3}{2\,\sqrt{2}+e^{(-2\,x)}+3}\right)+x - \frac{19\,e^{(-2\,x)}+57\,e^{(-4\,x)}+17\,e^{(-6\,x)}+3}{2\,\left(12\,e^{(-2\,x)}+38\,e^{(-4\,x)}+12\,e^{(-6\,x)}+e^{(-8\,x)}+1\right)}$$

input
```
integrate(1/(coth(x)^2+csch(x)^2)^3,x, algorithm="maxima")
```

output
```
7/16*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) +
 x - 1/2*(19*e^(-2*x) + 57*e^(-4*x) + 17*e^(-6*x) + 3)/(12*e^(-2*x) + 38*e
^(-4*x) + 12*e^(-6*x) + e^(-8*x) + 1)
```

3.822.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx = -\frac{7}{16}\,\sqrt{2}\log\left(-\frac{2\,\sqrt{2}-e^{(2\,x)}-3}{2\,\sqrt{2}+e^{(2\,x)}+3}\right)+x + \frac{17\,e^{(6\,x)}+57\,e^{(4\,x)}+19\,e^{(2\,x)}+3}{2\,\left(e^{(4\,x)}+6\,e^{(2\,x)}+1\right)^2}$$

input
```
integrate(1/(coth(x)^2+csch(x)^2)^3,x, algorithm="giac")
```

output
```
-7/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) +
x + 1/2*(17*e^(6*x) + 57*e^(4*x) + 19*e^(2*x) + 3)/(e^(4*x) + 6*e^(2*x) +
1)^2
```

3.822.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.07

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx = x+\frac{136\,e^{2\,x}+24}{12\,e^{2\,x}+38\,e^{4\,x}+12\,e^{6\,x}+e^{8\,x}+1}+\frac{7\,\sqrt{2}\,\ln\left(7\,e^{2\,x}-\frac{7\,\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{16}\right)}{16}-\frac{7\,\sqrt{2}\,\ln\left(7\,e^{2\,x}+\frac{7\,\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{16}\right)}{16}+\frac{\frac{17\,e^{2\,x}}{2}-\frac{45}{2}}{6\,e^{2\,x}+e^{4\,x}+1}$$

input
```
int(1/(coth(x)^2 + 1/sinh(x)^2)^3,x)
```

output
```
x + (136*exp(2*x) + 24)/(12*exp(2*x) + 38*exp(4*x) + 12*exp(6*x) + exp(8*x
) + 1) + (7*2^(1/2)*log(7*exp(2*x) - (7*2^(1/2)*(12*exp(2*x) + 4))/16))/16
 - (7*2^(1/2)*log(7*exp(2*x) + (7*2^(1/2)*(12*exp(2*x) + 4))/16))/16 + ((1
7*exp(2*x))/2 - 45/2)/(6*exp(2*x) + exp(4*x) + 1)
```

3.823 $\int \frac{1}{\coth^2(x)-\mathbf{csch}^2(x)} dx$

3.823.1 Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\coth^2(x)-\operatorname{csch}^2(x)} dx = x$$

output
```
x
```

3.823.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^2(x)-\operatorname{csch}^2(x)} dx = x$$

input
```
Integrate[(Coth[x]^2 - Csch[x]^2)^(-1),x]
```

output
```
x
```

3.823.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4882, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\coth^2(x)-\text{csch}^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(ix)^2-\cot(ix)^2}dx$$

$$\Big\downarrow \text{4882}$$

$$\int 1dx$$

$$\Big\downarrow \text{24}$$

$$x$$

input
```
Int[(Coth[x]^2 - Csch[x]^2)^(-1),x]
```

output
```
x
```

3.823.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4882
```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

3.823.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2\,\mathrm{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

input
```
int(1/(coth(x)^2-csch(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x
```

3.823.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^2(x)-\operatorname{csch}^2(x)}\,dx = x$$

input
```
integrate(1/(coth(x)^2-csch(x)^2),x, algorithm="fricas")
```

output
```
x
```

3.823.6 Sympy [F]

$$\int \frac{1}{\coth^2(x)-\operatorname{csch}^2(x)}\,dx = \int \frac{1}{(\coth(x)-\operatorname{csch}(x))\,(\coth(x)+\operatorname{csch}(x))}\,dx$$

input
```
integrate(1/(coth(x)**2-csch(x)**2),x)
```

output
```
Integral(1/((coth(x) - csch(x))*(coth(x) + csch(x))), x)
```

3.823.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^2(x) - \operatorname{csch}^2(x)} \, dx = x$$

input
```
integrate(1/(coth(x)^2-csch(x)^2),x, algorithm="maxima")
```

output
```
x
```

3.823.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^2(x) - \operatorname{csch}^2(x)} \, dx = x$$

input
```
integrate(1/(coth(x)^2-csch(x)^2),x, algorithm="giac")
```

output
```
x
```

3.823.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^2(x) - \operatorname{csch}^2(x)} \, dx = x$$

input
```
int(1/(coth(x)^2 - 1/sinh(x)^2),x)
```

output
```
x
```

3.824 $\int \frac{1}{\left(\coth^2(x)-\mathbf{csch}^2(x)\right)^2}\,dx$

3.824.1 Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^2}\,dx=x$$

output
```
x
```

3.824.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^2}\,dx=x$$

input
```
Integrate[(Coth[x]^2 - Csch[x]^2)^(-2),x]
```

output
```
x
```

3.824.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4882, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\csc(ix)^2-\cot(ix)^2\right)^2}dx$$

$$\Big\downarrow \text{4882}$$

$$\int 1dx$$

$$\Big\downarrow \text{24}$$

$$x$$

input
```
Int[(Coth[x]^2 - Csch[x]^2)^(-2),x]
```

output
```
x
```

3.824.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4882
```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

3.824.4 Maple [A] (verified)

Time = 10.52 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

input
```
int(1/(coth(x)^2-csch(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.824.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^2}\,dx=x$$

input
```
integrate(1/(coth(x)^2-csch(x)^2)^2,x, algorithm="fricas")
```

output
```
x
```

3.824.6 Sympy [F]

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^2}\,dx=\int \frac{1}{\left(\coth(x)-\operatorname{csch}(x)\right)^2\left(\coth(x)+\operatorname{csch}(x)\right)^2}\,dx$$

input
```
integrate(1/(coth(x)**2-csch(x)**2)**2,x)
```

output
```
Integral(1/((coth(x) - csch(x))**2*(coth(x) + csch(x))**2), x)
```

3.824.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^2}\, dx = x$$

input
```
integrate(1/(coth(x)^2-csch(x)^2)^2,x, algorithm="maxima")
```

output
```
x
```

3.824.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^2}\, dx = x$$

input
```
integrate(1/(coth(x)^2-csch(x)^2)^2,x, algorithm="giac")
```

output
```
x
```

3.824.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^2}\, dx = x$$

input
```
int(1/(coth(x)^2 - 1/sinh(x)^2)^2,x)
```

output
```
x
```

3.825 $\int \frac{1}{\left(\coth^2(x) - \text{csch}^2(x)\right)^3} \, dx$

3.825.1 Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\left(\coth^2(x) - \text{csch}^2(x)\right)^3} \, dx = x$$

output
```
x
```

3.825.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \text{csch}^2(x)\right)^3} \, dx = x$$

input
```
Integrate[(Coth[x]^2 - Csch[x]^2)^(-3),x]
```

output
```
x
```

3.825.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4882, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^3} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(\csc(ix)^2-\cot(ix)^2\right)^3}dx$$

$$\Big\downarrow \; 4882$$

$$\int 1dx$$

$$\Big\downarrow \; 24$$

$$x$$

input
```
Int[(Coth[x]^2 - Csch[x]^2)^(-3),x]
```

output
```
x
```

3.825.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4882
```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

3.825.4 Maple [A] (verified)

Time = 116.97 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
parallelrisch	0	2
default	$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

input
```
int(1/(coth(x)^2-csch(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.825.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^3}\,dx=x$$

input
```
integrate(1/(coth(x)^2-csch(x)^2)^3,x, algorithm="fricas")
```

output
```
x
```

3.825.6 Sympy [F]

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^3}\,dx=\int \frac{1}{\left(\coth(x)-\operatorname{csch}(x)\right)^3\left(\coth(x)+\operatorname{csch}(x)\right)^3}\,dx$$

input
```
integrate(1/(coth(x)**2-csch(x)**2)**3,x)
```

output
```
Integral(1/((coth(x) - csch(x))**3*(coth(x) + csch(x))**3), x)
```

3.825.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^3} \, dx = x$$

input
```
integrate(1/(coth(x)^2-csch(x)^2)^3,x, algorithm="maxima")
```

output
```
x
```

3.825.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^3} \, dx = x$$

input
```
integrate(1/(coth(x)^2-csch(x)^2)^3,x, algorithm="giac")
```

output
```
x
```

3.825.9 Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^3} \, dx = x$$

input
```
int(1/(coth(x)^2 - 1/sinh(x)^2)^3,x)
```

output
```
x
```

3.826 $\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

3.826.1 Optimal result

Integrand size = 14, antiderivative size = 271

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\frac{2\sqrt{2}c\arctan\left(\frac{2ic-ib\tanh\left(\frac{x}{2}\right)+\sqrt{-b^2+4ac}\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{-b^2+4ac}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} + \frac{2\sqrt{2}c\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{-b^2+4ac}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}$$

output
```
2*c*arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*
(a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)/(4*a*c-b^2)^(1/2)/(b^2-2*(a-
c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2)-2*c*arctan(1/2*(2*I*c-I*b*tanh(1/2*x)+(4
*a*c-b^2)^(1/2)*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))
^(1/2))*2^(1/2)/(4*a*c-b^2)^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1
/2)
```

3.826.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{2\sqrt{2}c\left(\frac{\arctan\left(\frac{2c+\left(-b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4(a-c)c+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2(a-c)c+b\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{2c-\left(b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[(a + b*Sinh[x] + c*Sinh[x]^2)^(-1),x]
```

output
```
(2*Sqrt[2]*c*(ArcTan[(2*c + (-b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^
2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]]/Sqrt[-b^2 + 2*(a - c)*c + b*Sqrt
[b^2 - 4*a*c]] - ArcTan[(2*c - (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/(Sqrt[2]
*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])]/Sqrt[-b^2 + 2*(a - c)*c
- b*Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

3.826.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3729, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-ib\sin(ix)-c\sin(ix)^2}dx$$

$$\Big\downarrow \text{3729}$$

$$\frac{2c\int \frac{1}{-ib-2ic\sinh(x)+\sqrt{4ac-b^2}}dx}{\sqrt{4ac-b^2}}-\frac{2c\int \frac{1}{-ib-2ic\sinh(x)-\sqrt{4ac-b^2}}dx}{\sqrt{4ac-b^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\int \frac{1}{-ib-2c\sin(ix)+\sqrt{4ac-b^2}}dx}{\sqrt{4ac-b^2}}-\frac{2c\int \frac{1}{-ib-2c\sin(ix)-\sqrt{4ac-b^2}}dx}{\sqrt{4ac-b^2}}$$

$$\Big\downarrow \text{3139}$$

$$\frac{4c\int \frac{1}{\left(ib-\sqrt{4ac-b^2}\right)\tanh^2\left(\frac{x}{2}\right)-4ic\tanh\left(\frac{x}{2}\right)-ib+\sqrt{4ac-b^2}}d\tanh\left(\frac{x}{2}\right)}{\sqrt{4ac-b^2}}-\frac{4c\int \frac{1}{\left(ib+\sqrt{4ac-b^2}\right)\tanh^2\left(\frac{x}{2}\right)-4ic\tanh\left(\frac{x}{2}\right)-ib-\sqrt{4ac-b^2}}d\tanh\left(\frac{x}{2}\right)}{\sqrt{4ac-b^2}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{8c\int \frac{1}{-\left(2\left(ib+\sqrt{4ac-b^2}\right)\tanh\left(\frac{x}{2}\right)-4ic\right)^2-8\left(b^2-i\sqrt{4ac-b^2}b-2(a-c)c\right)}d\left(2\left(ib+\sqrt{4ac-b^2}\right)\tanh\left(\frac{x}{2}\right)-4ic\right)}{\sqrt{4ac-b^2}}-\frac{8c\int \frac{1}{-\left(2\left(ib-\sqrt{4ac-b^2}\right)\tanh\left(\frac{x}{2}\right)-4ic\right)^2-8\left(b^2+i\sqrt{4ac-b^2}b-2(a-c)c\right)}d\left(2\left(ib-\sqrt{4ac-b^2}\right)\tanh\left(\frac{x}{2}\right)-4ic\right)}{\sqrt{4ac-b^2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\sqrt{2}c\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\left(-\sqrt{4ac-b^2}+ib\right)-4ic}{2\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{4ac-b^2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{2\sqrt{2}c\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)-4ic}{2\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{4ac-b^2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

input
```
Int[(a + b*Sinh[x] + c*Sinh[x]^2)^(-1),x]
```

output
```
(2*Sqrt[2]*c*ArcTan[((-4*I)*c + 2*(I*b - Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(2
*Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])])/(Sqrt[-b^2 +
4*a*c]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]]) - (2*Sqrt[2]*c*Ar
cTan[((-4*I)*c + 2*(I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(2*Sqrt[2]*Sqrt[b
^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/(Sqrt[-b^2 + 4*a*c]*Sqrt[b^2
 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])
```

3.826. $\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

3.826.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3729
```
Int[((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n_.) + (c_.)*sin[(d_.) + (e_.)*
(x_)]^(n2_.))^(-1), x_Symbol] :> Module[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c
/q)   Int[1/(b - q + 2*c*Sin[d + e*x]^n), x], x] - Simp[2*(c/q)   Int[1/(b
+ q + 2*c*Sin[d + e*x]^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.826.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.27

method	result
default	$\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)} \frac{\left(-\_R^2+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3\_R^2b-2\_Ra+4\_Rc+b}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(\left(16a^4c^2-8a^3b^2c-32a^3c^3+a^2b^4+32a^2b^2c^2+16a^2c^4-10a\,b^4c-8a\,b^2c^3+b^6+b^4c^2\right)\_Z^4+\left(-8a^2c^2+6a\,b^2c+8a\,c^3-b^4-2b\right.\right.}$

input
```
int(1/(a+b*sinh(x)+c*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
sum((-_R^2+1)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_R),_R=Ro
otOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

3.826.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3313 vs. $2(219) = 438$.

Time = 0.37 (sec) , antiderivative size = 3313, normalized size of antiderivative = 12.23

$$\int \frac{1}{a + b\sinh(x) + c\sinh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*sqrt((b^2 - 2*a*c + 2*c^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2
+ b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 -
4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*
b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c
^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b
*c^2*sinh(x) + 2*b^2*c + sqrt(2)*(b^4 - 4*a*b^2*c - (a^2*b^4 + b^6 - 8*a*c
^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2
+ 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 +
b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*
c + 2*c^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*
b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4
 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*
a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(
2*a^3 + 3*a*b^2)*c)) + 2*(4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2
)*c^2 - (a^2*b^2 + b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 +
 (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4
)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c
 + 2*c^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b
^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c...
```

3.826.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sinh(x)+c*sinh(x)**2),x)
```

output
```
Timed out
```

3.826.7 Maxima [F]

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \int \frac{1}{c\sinh{(x)}^2+b\sinh{(x)}+a}\,dx$$

input
```
integrate(1/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

output
```
integrate(1/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

3.826.8 Giac [A] (verification not implemented)

Time = 62.36 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.00

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = 0$$

input
```
integrate(1/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

output
```
0
```

3.826.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a + b\sinh(x) + c\sinh^2(x)}\,dx = \text{Hanged}$$

input
```
int(1/(a + c*sinh(x)^2 + b*sinh(x)),x)
```

output
```
\text{Hanged}
```

3.827 $\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

3.827.1 Optimal result

Integrand size = 17, antiderivative size = 280

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{\sqrt{2}\left(i+\frac{b}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-ib\tanh\left(\frac{x}{2}\right)+\sqrt{-b^2+4ac}\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} + \frac{\sqrt{2}\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}$$

output
```
arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c
)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(I-b/(4*a*c-b^2)^(1/2))/(b^2-2*(
a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2)+arctan(1/2*(2*I*c-I*b*tanh(1/2*x)+(4*a
*c-b^2)^(1/2)*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(
1/2))*2^(1/2)*(I+b/(4*a*c-b^2)^(1/2))/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2)
)^(1/2)
```

3.827.2 Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.87

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{\sqrt{2}\left(\frac{\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{2c+\left(-b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4(a-c)c+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2(a-c)c+b\sqrt{b^2-4ac}}}+\frac{\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{2c-\left(b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[Sinh[x]/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

output
```
(Sqrt[2]*(((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (-b + Sqrt[b^2 - 4*a*c])
*Tanh[x/2])/Sqrt[-2*b^2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2
 + 2*(a - c)*c + b*Sqrt[b^2 - 4*a*c]] + ((b + Sqrt[b^2 - 4*a*c])*ArcTan[(2
*c - (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[-b^2 + 2*(a - c)*c -
 b*Sqrt[b^2 - 4*a*c]])])/Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]]))/
Sqrt[b^2 - 4*a*c]
```

3.827.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 26, 3737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{a-ib\sin(ix)-c\sin(ix)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{-c\sin(ix)^2-ib\sin(ix)+a}dx$$

$$\Big\downarrow \text{3737}$$

$$-i\int \left(\frac{1-\frac{ib}{\sqrt{4ac-b^2}}}{-ib-2ic\sinh(x)+\sqrt{4ac-b^2}}+\frac{\frac{ib}{\sqrt{4ac-b^2}}+1}{-ib-2ic\sinh(x)-\sqrt{4ac-b^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{\sqrt{2}\left(1-\frac{ib}{\sqrt{4ac-b^2}}\right)\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{\sqrt{2}\left(1+\frac{ib}{\sqrt{4ac-b^2}}\right)\arctan\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b\right.}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a}}\right.}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right.$$

input
```
Int[Sinh[x]/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

output
```
(-I)*(-((Sqrt[2]*(1 - (I*b)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - I*b*Tanh
[x/2] + Sqrt[-b^2 + 4*a*c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*
b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])
 - (Sqrt[2]*(1 + (I*b)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b + Sqrt[-
b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 +
 4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])
```

3.827.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3737
```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

3.827. $\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

3.827.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.25

method	result
default	$2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{\_R\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3\_R^2b-2\_Ra+4\_Rc+b}\right)$
risch	$\sum_{\_R=\mathrm{RootOf}\left((16a^4c^2-8a^3b^2c-32a^3c^3+a^2b^4+32a^2b^2c^2+16a^2c^4-10a\,b^4c-8a\,b^2c^3+b^6+b^4c^2)\_Z^4+(8c\,a^3-2a^2b^2-8a^2c^2+6a\,b^2c-b\right.}$

input
```
int(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_R),_R=RootOf(
a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

3.827.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3309 vs. 2(224) = 448.

Time = 0.36 (sec) , antiderivative size = 3309, normalized size of antiderivative = 11.82

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt((2*a^2 + b^2 - 2*a*c + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2
 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 -
 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2
*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*
c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*
a*b*c*sinh(x) + 2*a*b^2 + sqrt(2)*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c +
 (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^3)*
c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 -
 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2
*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 + b^2 - 2
*a*c + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*
c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12
*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b
^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3
 + 3*a*b^2)*c)) + 2*(a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 + a*b^2)*c^2 - 2
*(2*a^4 + 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (1
6*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c
^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((2*a^2 + b^2 -
2*a*c + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)
*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 ...
```

3.827.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{a + b\sinh(x) + c\sinh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)**2),x)
```

output
```
Timed out
```

3.827.7 Maxima [F]

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \int \frac{\sinh\left(x\right)}{c\sinh\left(x\right)^2+b\sinh\left(x\right)+a}\,dx$$

input
```
integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

output
```
integrate(sinh(x)/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

3.827.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.827.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Hanged}$$

input
```
int(sinh(x)/(a + c*sinh(x)^2 + b*sinh(x)),x)
```

output
```
\text{Hanged}
```

3.828 $\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

3.828.1 Optimal result

Integrand size = 19, antiderivative size = 309

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx=\frac{x}{c}-\frac{\sqrt{2}\left(ib+\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{c\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}-\frac{\sqrt{2}\left(ib-\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{c\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}$$

output
```
x/c-arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*
(a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(I*b+(2*a*c-b^2)/(4*a*c-b^2)
^(1/2))/c/(b^2-2*(a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2)-arctan(1/2*(2*I*c-(I
*b-(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^
(1/2))^(1/2))*2^(1/2)*(I*b+(-2*a*c+b^2)/(4*a*c-b^2)^(1/2))/c/(b^2-2*(a-c)*
c+I*b*(4*a*c-b^2)^(1/2))^(1/2)
```

3.828.2 Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{x-\frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{2c+\left(-b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4(a-c)c+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b^2+2(a-c)c+b\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{2c-\left(b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}}{c}$$

input
```
Integrate[Sinh[x]^2/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

output
```
(x - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (-b + Sqr
t[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*
c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*(a - c)*c + b*Sqrt[b^2 - 4*a*c]]) -
 (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2*c - (b + Sqrt[b^2
- 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c
]])])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]]))/
c
```

3.828.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 25, 3737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\sin(ix)^2}{a-ib\sin(ix)-c\sin(ix)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sin(ix)^2}{-c\sin(ix)^2-ib\sin(ix)+a}dx$$

$$\Big\downarrow \text{3737}$$

$$-\int \left(\frac{-a-b\sinh(x)}{c\left(-c\sinh^2(x)-b\sinh(x)-a\right)}-\frac{1}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{4ac-b^2}}+ib\right)\arctan\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(-\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{\sqrt{2}\left(-\frac{b^2-2ac}{\sqrt{4ac-b^2}}+ib\right)\arctan\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{x}{c}$$

input
```
Int[Sinh[x]^2/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

output
```
x/c - (Sqrt[2]*(I*b + (b^2 - 2*a*c)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c -
(I*b - Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*
b*Sqrt[-b^2 + 4*a*c]])])/(c*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c
]]) - (Sqrt[2]*(I*b - (b^2 - 2*a*c)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c -
(I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*
b*Sqrt[-b^2 + 4*a*c]])])/(c*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c
]])
```

3.828.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.828. $\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

rule 3737
```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

3.828.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.35

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{\left(\_R^2a-2\_Rb-a\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3\_R^2b-2\_Ra+4\_Rc+b}}{c}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{c}+\frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{c}$
risch	Expression too large to display

input
```
int(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/c*sum((_R^2*a-2*_R*b-a)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*
x)-_R),_R=RootOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))-1/c*ln(tanh(1/
2*x)-1)+1/c*ln(1+tanh(1/2*x))
```

3.828.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4943 vs. 2(253) = 506.

Time = 0.60 (sec) , antiderivative size = 4943, normalized size of antiderivative = 16.00

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*c*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c + (4
*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2
)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c
)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a
^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b
^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 -
(a^2*b^2 + b^4)*c^2))*log(-2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c + sqrt(2)*(
8*a^2*b^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^4)*c^2 + (a^2*b^4 + b^6)*c + (8*a^2*c
^7 - 6*(4*a^3 + a*b^2)*c^6 + (24*a^4 + 22*a^2*b^2 + b^4)*c^5 - 2*(4*a^5 +
9*a^3*b^2 + 4*a*b^4)*c^4 + (2*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b
^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (
16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*
c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)
))*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c + (4*a*c^5 - (8*
a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4
*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 -
 (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4
)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^
4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b
^4)*c^2)) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*...
```

3.828.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(x)}{a + b\sinh(x) + c\sinh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**2/(a+b*sinh(x)+c*sinh(x)**2),x)
```

output
```
Timed out
```

3.828.7 Maxima [F]

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx=\int \frac{\sinh(x)^2}{c\sinh(x)^2+b\sinh(x)+a}\,dx$$

input
```
integrate(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

output
```
x/c - 1/4*integrate(8*(b*e^(3*x) + 2*a*e^(2*x) - b*e^x)/(c^2*e^(4*x) + 2*b
*c*e^(3*x) - 2*b*c*e^x + c^2 + 2*(2*a*c - c^2)*e^(2*x)), x)
```

3.828.8 Giac [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.02

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx=\frac{x}{c}$$

input
```
integrate(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

output
```
x/c
```

3.828.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx=\text{Hanged}$$

input
```
int(sinh(x)^2/(a + c*sinh(x)^2 + b*sinh(x)),x)
```

output
```
\text{Hanged}
```

3.829 $\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

3.829.1 Optimal result

Integrand size = 19, antiderivative size = 363

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\frac{bx}{c^2}+\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{-b^2+4ac}}+i\left(b^2-ac+\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\arctan\left(\frac{2ic-ib\tanh\left(\frac{x}{2}\right)+\sqrt{-b^2+4ac}\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{c^2\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} - \frac{\sqrt{2}\left(\frac{b^3}{\sqrt{-b^2+4ac}}-i\left(b^2-ac-\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{c^2\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}+\frac{\cosh(x)}{c}$$

output
```
-b*x/c^2+cosh(x)/c-arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*
2^(1/2)/(b^2-2*(a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(-I*(b^2-a*c-
3*I*a*b*c/(4*a*c-b^2)^(1/2))+b^3/(4*a*c-b^2)^(1/2))/c^2/(b^2-2*(a-c)*c-I*b
*(4*a*c-b^2)^(1/2))^(1/2)+arctan(1/2*(2*I*c-I*b*tanh(1/2*x)+(4*a*c-b^2)^(1
/2)*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1
/2)*(I*(b^2-a*c+3*I*a*b*c/(4*a*c-b^2)^(1/2))+b^3/(4*a*c-b^2)^(1/2))/c^2/(b
^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/2)
```

3.829.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{-bx+\frac{\sqrt{2}\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{2c+\left(-b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4(a-c)c+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b^2+2(a-c)c+b\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^3-3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan}{\sqrt{b^2-4ac}\sqrt{-b^2+2(a-c)c-}}}{c^2}$$

input
```
Integrate[Sinh[x]^3/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

output
```
(-(b*x) + (Sqrt[2]*(-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2
- 4*a*c])*ArcTan[(2*c + (-b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 +
4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*(a
 - c)*c + b*Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 3*a*b*c + b^2*Sqrt[b^2 -
 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(2*c - (b + Sqrt[b^2 - 4*a*c])*Tan
h[x/2])/(Sqrt[2]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])])/(Sqrt[b
^2 - 4*a*c]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]]) + c*Cosh[x])/c
^2
```

3.829.3 Rubi [A] (verified)

Time = 2.99 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 26, 3737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\sin(ix)^3}{a-ib\sin(ix)-c\sin(ix)^2}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\sin(ix)^3}{-c\sin(ix)^2-ib\sin(ix)+a}dx$$

$$\Big\downarrow \text{3737}$$

$$i\int \left(\frac{ib}{c^2}-\frac{i\sinh(x)}{c}+\frac{-i\left(1-\frac{ac}{b^2}\right)\sinh(x)b^2-iab}{c^2\left(c\sinh^2(x)+b\sinh(x)+a\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{\sqrt{2}\left(\frac{3iabc}{\sqrt{4ac-b^2}}-\frac{ib^3}{\sqrt{4ac-b^2}}-ac+b^2\right)\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c^2\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{\sqrt{2}\left(-\frac{3iabc}{\sqrt{4ac-b^2}}+\frac{ib^3}{\sqrt{4ac-b^2}}-ac+b\right.}{c^2\sqrt{-ib\sqrt{4ac}}}\right.$$

input
```
Int[Sinh[x]^3/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

output
```
I*((I*b*x)/c^2 + (Sqrt[2]*(b^2 - a*c - (I*b^3)/Sqrt[-b^2 + 4*a*c] + ((3*I)
*a*b*c)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[-b^2 +
4*a*c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]
])])/(c^2*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]]) + (Sqrt[2]*(b^
2 - a*c + (I*b^3)/Sqrt[-b^2 + 4*a*c] - ((3*I)*a*b*c)/Sqrt[-b^2 + 4*a*c])*A
rcTan[((2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 -
 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/(c^2*Sqrt[b^2 - 2*(a - c)*c - I*
b*Sqrt[-b^2 + 4*a*c]]) - (I*Cosh[x])/c)
```

3.829.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.829. $\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

rule 3737
```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

3.829.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.40

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)} \frac{\left(-ab\_R^2+2\left(-ac+b^2\right)\_R+ab\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3\_R^2b-2\_Ra+4\_Rc+b}}{c^2} - \frac{1}{c\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{b}{}$
risch	Expression too large to display

input
```
int(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*sum((-a*b*_R^2+2*(-a*c+b^2)*_R+a*b)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c
+b)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a)
)-1/c/(tanh(1/2*x)-1)+b/c^2*ln(tanh(1/2*x)-1)+1/c/(1+tanh(1/2*x))-b/c^2*ln
(1+tanh(1/2*x))
```

3.829.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6680 vs. 2(297) = 594.

Time = 1.11 (sec) , antiderivative size = 6680, normalized size of antiderivative = 18.40

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.829.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**3/(a+b*sinh(x)+c*sinh(x)**2),x)
```

output
```
Timed out
```

3.829.7 Maxima [F]

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \int \frac{\sinh(x)^3}{c\sinh(x)^2+b\sinh(x)+a}\,dx$$

input
```
integrate(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*(2*b*x*e^x - c*e^(2*x) - c)*e^(-x)/c^2 - 1/8*integrate(-16*(2*a*b*e^(
2*x) + (b^2 - a*c)*e^(3*x) - (b^2 - a*c)*e^x)/(c^3*e^(4*x) + 2*b*c^2*e^(3*
x) - 2*b*c^2*e^x + c^3 + 2*(2*a*c^2 - c^3)*e^(2*x)), x)
```

3.829.8 Giac [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.07

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\frac{bx}{c^2}+\frac{e^{(-x)}}{2\,c}+\frac{e^x}{2\,c}$$

input
```
integrate(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

output
```
-b*x/c^2 + 1/2*e^(-x)/c + 1/2*e^x/c
```

3.829.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Hanged}$$

input
```
int(sinh(x)^3/(a + c*sinh(x)^2 + b*sinh(x)),x)
```

output
```
\text{Hanged}
```

3.830 $\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx$

3.830.1 Optimal result

Integrand size = 27, antiderivative size = 12

$$\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx=\frac{\cosh(x)}{b-a\sinh(x)}$$

output
```
cosh(x)/(b-a*sinh(x))
```

3.830.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx=-\frac{\cosh(x)}{-b+a\sinh(x)}$$

input
```
Integrate[(a + b*Sinh[x])/(b^2 - 2*a*b*Sinh[x] + a^2*Sinh[x]^2),x]
```

output
```
-(Cosh[x]/(-b + a*Sinh[x]))
```

3.830.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3769, 27, 3042, 3233, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sinh(x)}{a^2\sinh^2(x)-2ab\sinh(x)+b^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-ib\sin(ix)}{-a^2\sin(ix)^2+2iab\sin(ix)+b^2}dx$$

$$\Big\downarrow \text{3769}$$

$$-4a^2\int -\frac{a+b\sinh(x)}{4\left(ab-a^2\sinh(x)\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$a^2\int \frac{a+b\sinh(x)}{\left(ab-a^2\sinh(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \frac{a-ib\sin(ix)}{\left(i\sin(ix)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{3233}$$

$$a^2\left(\frac{\cosh(x)}{a^2b-a^3\sinh(x)}-\frac{\int 0dx}{a^2\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{a^2\cosh(x)}{a^2b-a^3\sinh(x)}$$

input
```
Int[(a + b*Sinh[x])/(b^2 - 2*a*b*Sinh[x] + a^2*Sinh[x]^2),x]
```

output
```
(a^2*Cosh[x])/(a^2*b - a^3*Sinh[x])
```

3.830.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3769
```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[1/(4^n*c^n
)   Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[
{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

3.830.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

method	result	size
parallelrisch	$\frac{a\sinh(x)-b\cosh(x)-b}{(a\sinh(x)-b)b}$	28
risch	$-\frac{2(\mathrm{e}^{x}b+a)}{a(\mathrm{e}^{2x}a-2\,\mathrm{e}^{x}b-a)}$	29
default	$-\frac{2\left(-\frac{a\tanh\left(\frac{x}{2}\right)}{2b}+\frac{1}{2}\right)}{\frac{\tanh\left(\frac{x}{2}\right)^{2}b}{2}+a\tanh\left(\frac{x}{2}\right)-\frac{b}{2}}$	36

3.830. $\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx$

input
```
int((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x,method=_RETURNVERBOS
E)
```

output
```
(a*sinh(x)-b*cosh(x)-b)/(a*sinh(x)-b)/b
```

3.830.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.75

$$\int \frac{a + b\sinh(x)}{b^2 - 2ab\sinh(x) + a^2\sinh^2(x)}\,dx = -\frac{2\,(b\cosh{(x)} + b\sinh{(x)} + a)}{a^2\cosh{(x)}^2 + a^2\sinh{(x)}^2 - 2\,ab\cosh{(x)} - a^2 + 2\,(a^2\cosh{(x)} - ab)\sinh{(x)}}$$

input
```
integrate((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x, algorithm="fr
icas")
```

output
```
-2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 - 2*a*b*cosh
(x) - a^2 + 2*(a^2*cosh(x) - a*b)*sinh(x))
```

3.830.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\sinh(x)}{b^2 - 2ab\sinh(x) + a^2\sinh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(x))/(b**2-2*a*b*sinh(x)+a**2*sinh(x)**2),x)
```

output
```
Timed out
```

3.830.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 18.75

$$\int \frac{a + b\sinh(x)}{b^2 - 2ab\sinh(x) + a^2\sinh^2(x)}\,dx$$
$$= b\left(\frac{a\log\left(\frac{ae^{(-x)}+b-\sqrt{a^2+b^2}}{ae^{(-x)}+b+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\left(b^2e^{(-x)} - ab\right)}{a^4 + a^2b^2 - 2\left(a^3b + ab^3\right)e^{(-x)} - \left(a^4 + a^2b^2\right)e^{(-2x)}}\right)$$
$$- a\left(\frac{b\log\left(\frac{ae^{(-x)}+b-\sqrt{a^2+b^2}}{ae^{(-x)}+b+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(be^{(-x)} - a\right)}{a^3 + ab^2 - 2\left(a^2b + b^3\right)e^{(-x)} - \left(a^3 + ab^2\right)e^{(-2x)}}\right)$$

input
```
integrate((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x, algorithm="ma
xima")
```

output
```
b*(a*log((a*e^(-x) + b - sqrt(a^2 + b^2))/(a*e^(-x) + b + sqrt(a^2 + b^2))
)/(a^2 + b^2)^(3/2) + 2*(b^2*e^(-x) - a*b)/(a^4 + a^2*b^2 - 2*(a^3*b + a*b
^3)*e^(-x) - (a^4 + a^2*b^2)*e^(-2*x))) - a*(b*log((a*e^(-x) + b - sqrt(a^
2 + b^2))/(a*e^(-x) + b + sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^(-x
) - a)/(a^3 + a*b^2 - 2*(a^2*b + b^3)*e^(-x) - (a^3 + a*b^2)*e^(-2*x)))
```

3.830.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{a + b\sinh(x)}{b^2 - 2ab\sinh(x) + a^2\sinh^2(x)}\,dx = -\frac{2\left(be^x + a\right)}{\left(ae^{(2x)} - 2\,be^x - a\right)a}$$

input
```
integrate((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x, algorithm="gi
ac")
```

output
```
-2*(b*e^x + a)/((a*e^(2*x) - 2*b*e^x - a)*a)
```

3.830.9 Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.00

$$\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx = \frac{\frac{2\,e^x\,(a^3\,b+a\,b^3)}{a\,(a^3+a\,b^2)}+2}{a+2\,b\,e^x-a\,e^{2\,x}}$$

input
```
int((a + b*sinh(x))/(a^2*sinh(x)^2 + b^2 - 2*a*b*sinh(x)),x)
```

output
```
((2*exp(x)*(a*b^3 + a^3*b))/(a*(a*b^2 + a^3)) + 2)/(a + 2*b*exp(x) - a*exp
(2*x))
```

3.831 $\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

3.831.1 Optimal result

Integrand size = 21, antiderivative size = 300

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx=\frac{\sqrt{2}\left(ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-ib\tanh\left(\frac{x}{2}\right)+\sqrt{-b^2+4ac}\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}+\frac{\sqrt{2}\left(ie+\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}$$

output
```
arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c
)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(I*e+(-b*e+2*c*d)/(4*a*c-b^2)^(1
/2))/(b^2-2*(a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2)+arctan(1/2*(2*I*c-I*b*tan
h(1/2*x)+(4*a*c-b^2)^(1/2)*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-
b^2)^(1/2))^(1/2))*2^(1/2)*(I*e+(b*e-2*c*d)/(4*a*c-b^2)^(1/2))/(b^2-2*(a-c
)*c+I*b*(4*a*c-b^2)^(1/2))^(1/2)
```

3.831.2 Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.86

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{\sqrt{2}\left(\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{2c+\left(-b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4(a-c)c+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2(a-c)c+b\sqrt{b^2-4ac}}}+\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{2c-\left(b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*Sinh[x])/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

output
```
(Sqrt[2]*(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2*c + (-b + Sqrt[b
^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]
])/Sqrt[-b^2 + 2*(a - c)*c + b*Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b
^2 - 4*a*c])*e)*ArcTan[(2*c - (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/(Sqrt[2]*
Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])])/Sqrt[-b^2 + 2*(a - c)*c
- b*Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

3.831.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3773, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{d-ie\sin(ix)}{a-ib\sin(ix)-c\sin(ix)^2}dx$$

$$\Big\downarrow \text{3773}$$

$$-\left(\left(\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\int \frac{1}{-ib-2ic\sinh(x)-\sqrt{4ac-b^2}}dx\right)-\left(-\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\int \frac{1}{-ib-2ic\sinh(x)+\sqrt{4ac-b^2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-\left(\left(\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\int \frac{1}{-ib-2c\sin(ix)-\sqrt{4ac-b^2}}dx\right)-\left(-\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\int \frac{1}{-ib-2c\sin(ix)+\sqrt{4ac-b^2}}dx$$

$$\Big\downarrow \text{3139}$$

$$-2\left(-\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\int \frac{1}{\left(ib-\sqrt{4ac-b^2}\right)\tanh^2\left(\frac{x}{2}\right)-4ic\tanh\left(\frac{x}{2}\right)-ib+\sqrt{4ac-b^2}}d\tanh\left(\frac{x}{2}\right)-2\left(\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\int \frac{1}{\left(ib+\sqrt{4ac-b^2}\right)\tanh^2\left(\frac{x}{2}\right)-4ic\tanh\left(\frac{x}{2}\right)-ib-\sqrt{4ac-b^2}}d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{1083}$$

$$4\left(-\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\int \frac{1}{-\left(2\left(ib-\sqrt{4ac-b^2}\right)\tanh\left(\frac{x}{2}\right)-4ic\right)^2-8\left(b^2+i\sqrt{4ac-b^2}b-2(a-c)c\right)}d\left(2\left(ib-\sqrt{\right.\right.$$
$$4\left(\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\int \frac{1}{-\left(2\left(ib+\sqrt{4ac-b^2}\right)\tanh\left(\frac{x}{2}\right)-4ic\right)^2-8\left(b^2-i\sqrt{4ac-b^2}b-2(a-c)c\right)}d\left(2\left(ib+\sqrt{4\right.\right.$$

$$\Big\downarrow \text{217}$$

$$-\frac{\sqrt{2}\left(-\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\left(-\sqrt{4ac-b^2}+ib\right)-4ic}{2\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{\sqrt{2}\left(\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)-4ic}{2\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

input
```
Int[(d + e*Sinh[x])/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

output
```
-((Sqrt[2]*(I*e - (2*c*d - b*e)/Sqrt[-b^2 + 4*a*c])*ArcTan[((-4*I)*c + 2*(
I*b - Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(2*Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I
*b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]]
) - (Sqrt[2]*(I*e + (2*c*d - b*e)/Sqrt[-b^2 + 4*a*c])*ArcTan[((-4*I)*c + 2
*(I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(2*Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c -
 I*b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c
]]
```

3.831. $\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

3.831.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3773
```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Simp[(B + (b*B - 2*A*c)/q)   Int[1/(b + q + 2*c*Sin[d + e*x])
, x], x] + Simp[(B - (b*B - 2*A*c)/q)   Int[1/(b - q + 2*c*Sin[d + e*x]), x
], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

3.831.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.67 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.26

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{\left(-\_R^2d+2\_Re+d\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3\_R^2b-2\_Ra+4\_Rc+b}$	79
risch	Expression too large to display	8284

input
```
int((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
sum((-_R^2*d+2*_R*e+d)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-
_R),_R=RootOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

3.831.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6841 vs. 2(244) = 488.

Time = 2.97 (sec) , antiderivative size = 6841, normalized size of antiderivative = 22.80

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.831.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)**2),x)
```

output
```
Timed out
```

3.831.7 Maxima [F]

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \int \frac{e\sinh(x)+d}{c\sinh(x)^2+b\sinh(x)+a}\,dx$$

input
```
integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

output
```
integrate((e*sinh(x) + d)/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

3.831.8 Giac [A] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.00

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx=0$$

input
```
integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

output
```
0
```

3.831.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx=\text{Hanged}$$

input
```
int((d + e*sinh(x))/(a + c*sinh(x)^2 + b*sinh(x)),x)
```

output
```
\text{Hanged}
```

3.832 $\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

3.832.1 Optimal result

Integrand size = 14, antiderivative size = 223

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{4c\,\text{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{4c\,\text{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

output
```
4*c*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c-(-4*a*c+b^
2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*
c-(-4*a*c+b^2)^(1/2))^(1/2)-4*c*arctanh((b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)*t
anh(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(b-2*c+(-4
*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.832.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.89

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{2\sqrt{2}c\left(\frac{\arctan\left(\frac{\left(b-2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)-2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)-b\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\left(-b+2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)+b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[(a + b*Cosh[x] + c*Cosh[x]^2)^(-1),x]
```

output
```
(2*Sqrt[2]*c*(ArcTan[((b - 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2
 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[
b^2 - 4*a*c]] + ArcTan[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*
b^2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]]/Sqrt[-b^2 + 2*c*(a + c) + b*Sq
rt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

3.832.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3730, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\cos(ix)+c\cos(ix)^2}dx$$

$$\Big\downarrow \text{3730}$$

$$\frac{2c\int \frac{1}{b+2c\cosh(x)-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{1}{b+2c\cosh(x)+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\int \frac{1}{b+2c\sin\left(ix+\frac{\pi}{2}\right)-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{1}{b+2c\sin\left(ix+\frac{\pi}{2}\right)+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{3138}$$

$$\frac{4c\int \frac{1}{-\left(\left(b-2c-\sqrt{b^2-4ac}\right)\tanh^2\left(\frac{x}{2}\right)\right)+b+2c-\sqrt{b^2-4ac}}d\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-4ac}}-\frac{4c\int \frac{1}{-\left(\left(b-2c+\sqrt{b^2-4ac}\right)\tanh^2\left(\frac{x}{2}\right)\right)+b+2c+\sqrt{b^2-4ac}}d\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{221}$$

$$\frac{4c\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{4c\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

input
```
Int[(a + b*Cosh[x] + c*Cosh[x]^2)^(-1),x]
```

output
```
(4*c*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*
Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (4*c*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2
 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*
c]*Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

3.832.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3730
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b_.) + cos[(d_.) + (e_.)*(x_)]^
(n2_.)*(c_.))^(-1), x_Symbol] :> Module[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c
/q)   Int[1/(b - q + 2*c*Cos[d + e*x]^n), x], x] - Simp[2*(c/q)   Int[1/(b
+ q + 2*c*Cos[d + e*x]^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.832.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.93

method	result
default	$2(a-b+c)\left(\frac{\left(-b+2c-\sqrt{-4ac+b^2}\right)\operatorname{arctanh}\left(\frac{(-a+b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}\right)}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}+\frac{\left(b-2c-\sqrt{-4ac+b^2}\right)\arctan\left(\frac{(a-b+c)}{\sqrt{\left(\sqrt{-4ac+b^2}\right.}}\right.}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}-a\right.}}\right.$
risch	$\sum_{\_R=\text{RootOf}\left(\left(16a^4c^2-8a^3b^2c+32a^3c^3+a^2b^4-32a^2b^2c^2+16a^2c^4+10a\,b^4c-8a\,b^2c^3-b^6+b^4c^2\right)\_Z^4+\left(-8a^2c^2+6a\,b^2c-8a\,c^3-b^4+2b\right.\right.}$

input
```
int(1/(a+b*cosh(x)+c*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2*(a-b+c)*(1/2*(-b+2*c-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-
4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*
c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))+1/2*(b-2*c-(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*
tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)))
```

3.832.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3485 vs. 2(183) = 366.

Time = 0.38 (sec) , antiderivative size = 3485, normalized size of antiderivative = 15.63

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*sqrt((b^2 - 2*a*c - 2*c^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2
- b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 -
4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*
b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c
^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b
*c^2*sinh(x) + 2*b^2*c + sqrt(2)*(b^4 - 4*a*b^2*c - (a^2*b^4 - b^6 + 8*a*c
^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2
+ 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 +
b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*
c - 2*c^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*
b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4
 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*
a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(
2*a^3 - 3*a*b^2)*c)) + 2*(4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2
)*c^2 - (a^2*b^2 - b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 -
 (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4
)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c
 - 2*c^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b
^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c...
```

3.832.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x)+c*cosh(x)**2),x)
```

output
```
Timed out
```

3.832.7 Maxima [F]

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx=\int \frac{1}{c\cosh{(x)}^2+b\cosh{(x)}+a}\,dx$$

input
```
integrate(1/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

output
```
integrate(1/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

3.832.8 Giac [A] (verification not implemented)

Time = 61.08 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.00

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx=0$$

input
```
integrate(1/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

output
```
0
```

3.832.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx=\text{Hanged}$$

input
```
int(1/(a + b*cosh(x) + c*cosh(x)^2),x)
```

output
```
\text{Hanged}
```

3.833 $\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

3.833.1 Optimal result

Integrand size = 17, antiderivative size = 230

$$\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{2\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

output
```
2*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c-(-4*a*c+b^2)
^(1/2))^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))/(b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(
b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+2*arctanh((b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)
*tanh(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))/(b
-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.833.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.99

$$\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{\sqrt{2}\left(-\frac{\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\left(b-2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)-2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)-b\sqrt{b^2-4ac}}}+\frac{\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\left(-b+2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)+b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[Cosh[x]/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

output
```
(Sqrt[2]*(-(((b + Sqrt[b^2 - 4*a*c])*ArcTan[((b - 2*c + Sqrt[b^2 - 4*a*c])
*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2
 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[
((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b
*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]))/Sqr
t[b^2 - 4*a*c]
```

3.833.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3738, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)}{a+b\cos(ix)+c\cos(ix)^2}dx$$

$$\Big\downarrow \text{3738}$$

$$\int \left(\frac{1-\frac{b}{\sqrt{b^2-4ac}}}{-\sqrt{b^2-4ac}+b+2c\cosh(x)}+\frac{\frac{b}{\sqrt{b^2-4ac}}+1}{\sqrt{b^2-4ac}+b+2c\cosh(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

input
```
Int[Cosh[x]/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

output
```
(2*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Ta
nh[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqrt[b^2 - 4*
a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(1 + b/Sqrt[b^2 - 4*a*c])*Ar
cTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 -
 4*a*c]])
```

3.833.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3738
```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b
_.) + cos[(d_.) + (e_.)*(x_)]^(n2_.)*(c_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[cos[d + e*x]^m*(a + b*cos[d + e*x]^n + c*cos[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

3.833.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.89

method	result
default	$2(a-b+c)\left(\frac{\left(\sqrt{-4ac+b^2}+2a-b\right)\operatorname{arctanh}\left(\frac{(-a+b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}\right)}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}+\frac{\left(-2a+b+\sqrt{-4ac+b^2}\right)\arctan\left(\frac{(a-b+c)}{\sqrt{\left(\sqrt{-4ac+b^2}\right.}}\right.}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}-a+\right.}}\right.$
risch	$\sum_{\_R=\operatorname{RootOf}\left(\left(16a^4c^2-8a^3b^2c+32a^3c^3+a^2b^4-32a^2b^2c^2+16a^2c^4+10a\,b^4c-8a\,b^2c^3-b^6+b^4c^2\right)\_Z^4+\left(8c\,a^3-2a^2b^2+8a^2c^2-6a\,b^2c+b\right.\right.}$

input
```
int(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2*(a-b+c)*(1/2*((-4*a*c+b^2)^(1/2)+2*a-b)/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4
*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c
+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))+1/2*(-2*a+b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*
tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)))
```

3.833.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3505 vs. $2(189) = 378$.

Time = 0.38 (sec) , antiderivative size = 3505, normalized size of antiderivative = 15.24

$$\int \frac{\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\, dx = \text{Too large to display}$$

input
```
integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt((2*a^2 - b^2 + 2*a*c + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2
 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 -
 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2
*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*
c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*
a*b*c*sinh(x) + 2*a*b^2 + sqrt(2)*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c -
 (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*
c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 -
 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2
*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 - b^2 + 2
*a*c + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*
c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12
*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b
^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3
 - 3*a*b^2)*c)) - 2*(a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2
*(2*a^4 - 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (1
6*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c
^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((2*a^2 - b^2 +
2*a*c + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)
*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 ...
```

3.833.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)**2),x)
```

output
```
Timed out
```

3.833.7 Maxima [F]

$$\int \frac{\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \int \frac{\cosh(x)}{c\cosh(x)^2 + b\cosh(x) + a}\,dx$$

input
```
integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

output
```
integrate(cosh(x)/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

3.833.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.833.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Hanged}$$

input
```
int(cosh(x)/(a + b*cosh(x) + c*cosh(x)^2),x)
```

output
```
\text{Hanged}
```

3.834 $\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

3.834.1 Optimal result

Integrand size = 19, antiderivative size = 255

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{x}{c} - \frac{2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{2\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

output
```
x/c-2*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c-(-4*a*c+
b^2)^(1/2))^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c/(b-2*c-(-4*a*c+b^2
)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-2*arctanh((b-2*c+(-4*a*c+b
^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+
b^2)/(-4*a*c+b^2)^(1/2))/c/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c
+b^2)^(1/2))^(1/2)
```

3.834.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.04

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{x+\frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\left(b-2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)-2b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b^2+2c(a+c)-b\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\left(-b+2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b^2+2c(a+c)+b\sqrt{b^2-4ac}}}}{c}$$

input
```
Integrate[Cosh[x]^2/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

output
```
(x + (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[((b - 2*c + Sqrt[
b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) - (
Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[((-b + 2*c + Sqrt[b^2
- 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]))/c
```

3.834.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3738, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{a+b\cos(ix)+c\cos(ix)^2}dx$$

$$\Big\downarrow \text{3738}$$

$$\int \left(\frac{-a-b\cosh(x)}{c\left(a+b\cosh(x)+c\cosh^2(x)\right)}+\frac{1}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{c\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{2\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{x}{c}$$

input
```
Int[Cosh[x]^2/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

output
```
x/c - (2*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqr
t[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(c*Sqrt[b -
 2*c - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (2*(b + (b^
2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*T
anh[x/2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(c*Sqrt[b - 2*c + Sqrt[b^2 -
 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

3.834.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3738
```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b
_.) + cos[(d_.) + (e_.)*(x_)]^(n2_.)*(c_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[cos[d + e*x]^m*(a + b*cos[d + e*x]^n + c*cos[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

3.834. $\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

3.834.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.07

method	result
default	$-\frac{\ln(\tanh(\frac{x}{2})-1)}{c}+\frac{2(a-b+c)\left(\frac{\left(a\sqrt{-4ac+b^2}-b\sqrt{-4ac+b^2}-ab-2ac+b^2\right)\operatorname{arctanh}\left(\frac{(-a+b-c)\tanh(\frac{x}{2})}{\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}\right)}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}+\frac{\left(a\sqrt{-4ac+b^2}-b\sqrt{\ldots}\right.}{2\sqrt{\ldots}}\ldots\right.}{c}$
risch	Expression too large to display

input
```
int(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/c*ln(tanh(1/2*x)-1)+2/c*(a-b+c)*(1/2*(a*(-4*a*c+b^2)^(1/2)-b*(-4*a*c+b^
2)^(1/2)-a*b-2*a*c+b^2)/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-
c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(
a-b+c))^(1/2))+1/2*(a*(-4*a*c+b^2)^(1/2)-b*(-4*a*c+b^2)^(1/2)+a*b+2*a*c-b^
2)/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arc
tan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)))+1/c*ln(
1+tanh(1/2*x))
```

3.834.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5079 vs. 2(215) = 430.

Time = 0.58 (sec) , antiderivative size = 5079, normalized size of antiderivative = 19.92

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.834.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**2/(a+b*cosh(x)+c*cosh(x)**2),x)
```

output
```
Timed out
```

3.834.7 Maxima [F]

$$\int \frac{\cosh^2(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \int \frac{\cosh{(x)}^2}{c\cosh{(x)}^2 + b\cosh{(x)} + a}\,dx$$

input
```
integrate(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

output
```
x/c - 1/4*integrate(8*(b*e^(3*x) + 2*a*e^(2*x) + b*e^x)/(c^2*e^(4*x) + 2*b
*c*e^(3*x) + 2*b*c*e^x + c^2 + 2*(2*a*c + c^2)*e^(2*x)), x)
```

3.834.8 Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.02

$$\int \frac{\cosh^2(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \frac{x}{c}$$

input
```
integrate(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

output
```
x/c
```

3.834.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \text{Hanged}$$

input
```
int(cosh(x)^2/(a + b*cosh(x) + c*cosh(x)^2),x)
```

output
```
\text{Hanged}
```

3.835 $\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

3.835.1 Optimal result

Integrand size = 19, antiderivative size = 299

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = -\frac{bx}{c^2}+\frac{2\left(b^2-ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{3abc}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{2\left(b^2-ac+\frac{b^3}{\sqrt{b^2-4ac}}-\frac{3abc}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}+\frac{\sinh(x)}{c}$$

output
```
-b*x/c^2+sinh(x)/c+2*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/
(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c-b^3/(-4*a*c+b^2)^(1/2)+3*a*b*c/
(-4*a*c+b^2)^(1/2))/c^2/(b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^
2)^(1/2))^(1/2)+2*arctanh((b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+
2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b^3/(-4*a*c+b^2)^(1/2)-3*a*b*c/(-4
*a*c+b^2)^(1/2))/c^2/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^
(1/2))^(1/2)
```

3.835.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.03

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{-bx-\frac{\sqrt{2}\left(b^3-3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\left(b-2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)-2b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b^2+2c(a+c)-b\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan}{\sqrt{b^2-4ac}\sqrt{-b^2+2c(a+c)+b}}}{c^2}$$

input
```
Integrate[Cosh[x]^3/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

output
```
(-(b*x) - (Sqrt[2]*(b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 -
 4*a*c])*ArcTan[((b - 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*
c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a
 + c) - b*Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-b^3 + 3*a*b*c + b^2*Sqrt[b^2 -
4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tan
h[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]) + c*Sinh[x])/c^2
```

3.835.3 Rubi [A] (verified)

Time = 3.53 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3738, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3}{a+b\cos(ix)+c\cos(ix)^2}dx$$

$$\Big\downarrow \text{3738}$$

$$\int \left(\frac{b^2\cosh(x)\left(1-\frac{ac}{b^2}\right)+ab}{c^2\left(a+b\cosh(x)+c\cosh^2(x)\right)}-\frac{b}{c^2}+\frac{\cosh(x)}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{3abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{c^2\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(-\frac{3abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{bx}{c^2}+\frac{\sinh(x)}{c}$$

input
```
Int[Cosh[x]^3/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

output
```
-((b*x)/c^2) + (2*(b^2 - a*c - b^3/Sqrt[b^2 - 4*a*c] + (3*a*b*c)/Sqrt[b^2
- 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2
*c - Sqrt[b^2 - 4*a*c]]])/(c^2*Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Sqrt[b +
2*c - Sqrt[b^2 - 4*a*c]]) + (2*(b^2 - a*c + b^3/Sqrt[b^2 - 4*a*c] - (3*a*b
*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2
])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(c^2*Sqrt[b - 2*c + Sqrt[b^2 - 4*a*
c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]) + Sinh[x]/c
```

3.835.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3738
```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b
_.) + cos[(d_.) + (e_.)*(x_)]^(n2_.)*(c_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[cos[d + e*x]^m*(a + b*cos[d + e*x]^n + c*cos[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

3.835.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.18

method	result
default	$-\frac{1}{c(1+\tanh(\frac{x}{2}))}-\frac{b\ln(1+\tanh(\frac{x}{2}))}{c^2}-\frac{1}{c(\tanh(\frac{x}{2})-1)}+\frac{b\ln(\tanh(\frac{x}{2})-1)}{c^2}+\frac{2(a-b+c)\left(\frac{\left(-ab\sqrt{-4ac+b^2}-ac\sqrt{-4ac+b^2}+b^2\sqrt{\ldots}\right.}{2\sqrt{-4a\ldots}}\right.}{\ldots}$
risch	Expression too large to display

input
```
int(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/c/(1+tanh(1/2*x))-b/c^2*ln(1+tanh(1/2*x))-1/c/(tanh(1/2*x)-1)+b/c^2*ln(
tanh(1/2*x)-1)+2/c^2*(a-b+c)*(1/2*(-a*b*(-4*a*c+b^2)^(1/2)-a*c*(-4*a*c+b^2
)^(1/2)+b^2*(-4*a*c+b^2)^(1/2)-2*a^2*c+a*b^2+3*b*c*a-b^3)/(-4*a*c+b^2)^(1/
2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(
1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))+1/2*(-a*b*(-4*a*c+b^2)^(1
/2)-a*c*(-4*a*c+b^2)^(1/2)+b^2*(-4*a*c+b^2)^(1/2)+2*a^2*c-a*b^2-3*b*c*a+b^
3)/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arc
tan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)))
```

3.835.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6794 vs. 2(255) = 510.

Time = 1.08 (sec) , antiderivative size = 6794, normalized size of antiderivative = 22.72

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.835.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)}{a + b\cosh(x) + c\cosh^2(x)}\, dx = \text{Timed out}$$

input
```
integrate(cosh(x)**3/(a+b*cosh(x)+c*cosh(x)**2),x)
```

output
```
Timed out
```

3.835.7 Maxima [F]

$$\int \frac{\cosh^3(x)}{a + b\cosh(x) + c\cosh^2(x)}\, dx = \int \frac{\cosh(x)^3}{c\cosh(x)^2 + b\cosh(x) + a}\, dx$$

input
```
integrate(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*(2*b*x*e^x - c*e^(2*x) + c)*e^(-x)/c^2 - 1/8*integrate(-16*(2*a*b*e^(
2*x) + (b^2 - a*c)*e^(3*x) + (b^2 - a*c)*e^x)/(c^3*e^(4*x) + 2*b*c^2*e^(3*
x) + 2*b*c^2*e^x + c^3 + 2*(2*a*c^2 + c^3)*e^(2*x)), x)
```

3.835.8 Giac [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.08

$$\int \frac{\cosh^3(x)}{a + b\cosh(x) + c\cosh^2(x)}\, dx = -\frac{bx}{c^2} - \frac{e^{(-x)}}{2c} + \frac{e^x}{2c}$$

input
```
integrate(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

output
```
-b*x/c^2 - 1/2*e^(-x)/c + 1/2*e^x/c
```

3.835.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)}{a + b\cosh(x) + c\cosh^2(x)}\, dx = \text{Hanged}$$

input
```
int(cosh(x)^3/(a + b*cosh(x) + c*cosh(x)^2),x)
```

output
```
\text{Hanged}
```

3.836 $\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx$

3.836.1 Optimal result

Integrand size = 27, antiderivative size = 11

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx=\frac{\sinh(x)}{b+a\cosh(x)}$$

output
```
sinh(x)/(b+a*cosh(x))
```

3.836.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx=\frac{\sinh(x)}{b+a\cosh(x)}$$

input
```
Integrate[(a + b*Cosh[x])/(b^2 + 2*a*b*Cosh[x] + a^2*Cosh[x]^2),x]
```

output
```
Sinh[x]/(b + a*Cosh[x])
```

3.836.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 3770, 27, 3042, 3233, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cosh(x)}{a^2\cosh^2(x)+2ab\cosh(x)+b^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\cos(ix)}{a^2\cos(ix)^2+2ab\cos(ix)+b^2}dx$$

$$\Big\downarrow \text{ 3770}$$

$$4a^2\int \frac{a+b\cosh(x)}{4\left(\cosh(x)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$a^2\int \frac{a+b\cosh(x)}{\left(\cosh(x)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$a^2\int \frac{a+b\sin\left(ix+\frac{\pi}{2}\right)}{\left(\sin\left(ix+\frac{\pi}{2}\right)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{ 3233}$$

$$a^2\left(\frac{\int 0dx}{a^2\left(a^2-b^2\right)}+\frac{\sinh(x)}{a^3\cosh(x)+a^2b}\right)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{a^2\sinh(x)}{a^3\cosh(x)+a^2b}$$

input
```
Int[(a + b*Cosh[x])/(b^2 + 2*a*b*Cosh[x] + a^2*Cosh[x]^2),x]
```

output
```
(a^2*Sinh[x])/(a^2*b + a^3*Cosh[x])
```

3.836.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3770
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.) + (a_)
)^(n_)*(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[1/(4^n*c^n
)   Int[(A + B*Cos[d + e*x])*(b + 2*c*Cos[d + e*x])^(2*n), x], x] /; FreeQ[
{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

3.836.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$\frac{\sinh(x)}{b+a\cosh(x)}$	12
risch	$-\frac{2(\mathrm{e}^{x}b+a)}{a(\mathrm{e}^{2x}a+2\,\mathrm{e}^{x}b+a)}$	27
default	$\frac{2\tanh\left(\frac{x}{2}\right)}{a\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{2}b+a+b}$	29

input
```
int((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x,method=_RETURNVERBOS
E)
```

output
```
sinh(x)/(b+a*cosh(x))
```

3.836.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.91

$$\int \frac{a + b\cosh(x)}{b^2 + 2ab\cosh(x) + a^2\cosh^2(x)}\,dx = -\frac{2\,(b\cosh{(x)} + b\sinh{(x)} + a)}{a^2\cosh{(x)}^2 + a^2\sinh{(x)}^2 + 2\,ab\cosh{(x)} + a^2 + 2\,(a^2\cosh{(x)} + ab)\sinh{(x)}}$$

input
```
integrate((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x, algorithm="fr
icas")
```

output
```
-2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh
(x) + a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

3.836.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\cosh(x)}{b^2 + 2ab\cosh(x) + a^2\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cosh(x))/(b**2+2*a*b*cosh(x)+a**2*cosh(x)**2),x)
```

output
```
Timed out
```

3.836.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.836.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx = -\frac{2\,(be^x+a)}{(ae^{(2\,x)}+2\,be^x+a)a}$$

input
```
integrate((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x, algorithm="gi
ac")
```

output
```
-2*(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a)
```

3.836.9 Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 51, normalized size of antiderivative = 4.64

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx = -\frac{\frac{2\,\mathrm{e}^x\,(a\,b^3-a^3\,b)}{a\,(a\,b^2-a^3)}+2}{a+2\,b\,\mathrm{e}^x+a\,\mathrm{e}^{2\,x}}$$

input
```
int((a + b*cosh(x))/(a^2*cosh(x)^2 + b^2 + 2*a*b*cosh(x)),x)
```

output
```
-((2*exp(x)*(a*b^3 - a^3*b))/(a*(a*b^2 - a^3)) + 2)/(a + 2*b*exp(x) + a*ex
p(2*x))
```

3.837 $\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

3.837.1 Optimal result

Integrand size = 21, antiderivative size = 246

$$\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

output
```
2*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c-(-4*a*c+b^2)
^(1/2))^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(b-2*c-(-4*a*c+b^2)^(1/
2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+2*arctanh((b-2*c+(-4*a*c+b^2)^(
1/2))^(1/2)*tanh(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(
-4*a*c+b^2)^(1/2))/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1
/2))^(1/2)
```

3.837.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98

$$\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{\sqrt{2}\left(-\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\left(b-2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)-2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)-b\sqrt{b^2-4ac}}}+\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\left(-b+2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)+b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*Cosh[x])/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

output
```
(Sqrt[2]*(-(((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[((b - 2*c + Sqrt[
b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]
]])/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + ((2*c*d + (-b + Sqrt
[b^2 - 4*a*c])*e)*ArcTan[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-
2*b^2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b
*Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

3.837.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3774, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{d+e\cos(ix)}{a+b\cos(ix)+c\cos(ix)^2}dx$$

$$\Big\downarrow \ 3774$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{b+2c\cosh(x)-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+2c\cosh(x)+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{b+2c\sin\left(ix+\frac{\pi}{2}\right)-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+2c\sin\left(ix+\frac{\pi}{2}\right)+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{3138}$$

$$2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{-\left(\left(b-2c-\sqrt{b^2-4ac}\right)\tanh^2\left(\frac{x}{2}\right)\right)+b+2c-\sqrt{b^2-4ac}}d\tanh\left(\frac{x}{2}\right)+2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\left(\left(b-2c+\sqrt{b^2-4ac}\right)\tanh^2\left(\frac{x}{2}\right)\right)+b+2c+\sqrt{b^2-4ac}}d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

input
```
Int[(d + e*Cosh[x])/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

output
```
(2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2
- 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - S
qrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(e - (2*c*d - b*
e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2]
)/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*S
qrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

3.837.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3774
```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Simp[(B + (b*B - 2*A*c)/q)   Int[1/(b + q + 2*c*Cos[d + e*x])
, x], x] + Simp[(B - (b*B - 2*A*c)/q)   Int[1/(b - q + 2*c*Cos[d + e*x]), x
], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

3.837.4 Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.03

method	result
default	$2(a-b+c)\left(\frac{\left(-d\sqrt{-4ac+b^2}+e\sqrt{-4ac+b^2}-2ae+bd+be-2cd\right)\arctan\left(\frac{(a-b+c)\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(\sqrt{-4ac+b^2}-a+c\right)(a-b+c)}}\right)}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}-a+c\right)(a-b+c)}}+\frac{\left(-d\sqrt{-4ac+b^2}+e\right.}{}\right.$
risch	Expression too large to display

input
```
int((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2*(a-b+c)*(1/2*(-d*(-4*a*c+b^2)^(1/2)+e*(-4*a*c+b^2)^(1/2)-2*a*e+b*d+b*e-2
*c*d)/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*
arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))+1/2*(
-d*(-4*a*c+b^2)^(1/2)+e*(-4*a*c+b^2)^(1/2)+2*a*e-b*d-b*e+2*c*d)/(-4*a*c+b^
2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)
*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)))
```

3.837.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6997 vs. $2(206) = 412$.

Time = 3.06 (sec) , antiderivative size = 6997, normalized size of antiderivative = 28.44

$$\int \frac{d + e\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.837.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d + e\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)**2),x)
```

output
```
Timed out
```

3.837.7 Maxima [F]

$$\int \frac{d + e\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \int \frac{e\cosh(x) + d}{c\cosh(x)^2 + b\cosh(x) + a}\,dx$$

input
```
integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

output
```
integrate((e*cosh(x) + d)/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

3.837.8 Giac [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.00

$$\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx=0$$

input
```
integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

output
```
0
```

3.837.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx=\text{Hanged}$$

input
```
int((d + e*cosh(x))/(a + b*cosh(x) + c*cosh(x)^2),x)
```

output
```
\text{Hanged}
```

3.838 $\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$

3.838.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{x}{a+b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}$$

output
```
x/(a+b)-arctan(b^(1/2)*tanh(x)/a^(1/2))*a^(1/2)/(a+b)/b^(1/2)
```

3.838.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{x-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}}}{a+b}$$

input
```
Integrate[Sinh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

output
```
(x - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/Sqrt[b])/(a + b)
```

3.838.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 25, 4889, 25, 383, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sin(ix)^2}{a\cos(ix)^2-b\sin(ix)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sin(ix)^2}{a\cos(ix)^2-b\sin(ix)^2}dx$$

$$\Big\downarrow \text{ 4889}$$

$$-\int -\frac{\tanh^2(x)}{(1-\tanh^2(x))\,(b\tanh^2(x)+a)}d\tanh(x)$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{\tanh^2(x)}{(1-\tanh^2(x))\,(a+b\tanh^2(x))}d\tanh(x)$$

$$\Big\downarrow \text{ 383}$$

$$\frac{\int \frac{1}{1-\tanh^2(x)}d\tanh(x)}{a+b}-\frac{a\int \frac{1}{b\tanh^2(x)+a}d\tanh(x)}{a+b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\int \frac{1}{1-\tanh^2(x)}d\tanh(x)}{a+b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}(\tanh(x))}{a+b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}$$

input
```
Int[Sinh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

output
```
-((Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/(Sqrt[b]*(a + b))) + ArcTanh
[Tanh[x]]/(a + b)
```

3.838.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.838. $\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$

3.838.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(31) = 62$.

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.36

method	result
risch	$\frac{x}{a+b}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2x}-\frac{-a+2\sqrt{-ab}+b}{a+b}\right)}{2b(a+b)}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2x}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2b(a+b)}$
default	$\frac{8a^2\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(a+\sqrt{b(a+b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{4a+4b}+\frac{8\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{8a+8b}-$

input
```
int(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x/(a+b)+1/2/b*(-a*b)^(1/2)/(a+b)*ln(exp(2*x)-(-a+2*(-a*b)^(1/2)+b)/(a+b))-
1/2/b*(-a*b)^(1/2)/(a+b)*ln(exp(2*x)+(a+2*(-a*b)^(1/2)-b)/(a+b))
```

3.838.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 367, normalized size of antiderivative = 9.41

$$\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$$
$$=\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{(a^2+2\,ab+b^2)\cosh(x)^4+4\,(a^2+2\,ab+b^2)\cosh(x)\sinh(x)^3+(a^2+2\,ab+b^2)\sinh(x)^4+2\,(a^2-b^2)\cosh(x)^2+2\,\left(3\,(a^2+2\,ab+b^2)\cosh\right.}{(a+b)\cosh(x)^4+4\,(a+b)\cosh(x)\sinh(x)^3+(a+b)\sinh(x}\right.}{}\right.$$
$$\left.-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{\left((a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+a-b\right)\sqrt{\frac{a}{b}}}{2\,a}\right)-x}{a+b}\right]$$

input
```
integrate(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2
)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x
)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 6*
a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x
) - 4*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)
*sinh(x)^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)
*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x
)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) +
 a + b)) + 2*x)/(a + b), -(sqrt(a/b)*arctan(1/2*((a + b)*cosh(x)^2 + 2*(a
+ b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)*sqrt(a/b)/a) - x)/(a + b
)]
```

3.838.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(32) = 64$.

Time = 0.59 (sec) , antiderivative size = 202, normalized size of antiderivative = 5.18

$$\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \\ \frac{x}{b} & \text{for } a=0 \\ -\frac{x\sinh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} + \frac{x\cosh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} - \frac{\sinh(x)\cosh(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} & \text{for } a=-b \\ \frac{x-\frac{\sinh(x)}{\cosh(x)}}{a} & \text{for } b=0 \\ \frac{2x\sqrt{-\frac{b}{a}}}{2a\sqrt{-\frac{b}{a}}+2b\sqrt{-\frac{b}{a}}} + \frac{\log\left(-\sqrt{-\frac{b}{a}}\sinh(x)+\cosh(x)\right)}{2a\sqrt{-\frac{b}{a}}+2b\sqrt{-\frac{b}{a}}} - \frac{\log\left(\sqrt{-\frac{b}{a}}\sinh(x)+\cosh(x)\right)}{2a\sqrt{-\frac{b}{a}}+2b\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(x)**2/(a*cosh(x)**2+b*sinh(x)**2),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (-x*sinh(x)**2/(-
2*b*sinh(x)**2 + 2*b*cosh(x)**2) + x*cosh(x)**2/(-2*b*sinh(x)**2 + 2*b*cos
h(x)**2) - sinh(x)*cosh(x)/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2), Eq(a, -b)),
 ((x - sinh(x)/cosh(x))/a, Eq(b, 0)), (2*x*sqrt(-b/a)/(2*a*sqrt(-b/a) + 2*
b*sqrt(-b/a)) + log(-sqrt(-b/a)*sinh(x) + cosh(x))/(2*a*sqrt(-b/a) + 2*b*s
qrt(-b/a)) - log(sqrt(-b/a)*sinh(x) + cosh(x))/(2*a*sqrt(-b/a) + 2*b*sqrt(
-b/a)), True))
```

3.838.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.90

$$\int \frac{\sinh^2(x)}{a\cosh^2(x) + b\sinh^2(x)}\,dx = -\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2x)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}(a+b)} + \frac{\arctan\left(\frac{(a+b)e^{(-2x)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}} + \frac{x}{a+b}$$

input
```
integrate(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*(a - b)*arctan(1/2*((a + b)*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a
 + b)) + 1/2*arctan(1/2*((a + b)*e^(-2*x) + a - b)/sqrt(a*b))/sqrt(a*b) +
x/(a + b)
```

3.838.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{\sinh^2(x)}{a\cosh^2(x) + b\sinh^2(x)}\,dx = -\frac{a\arctan\left(\frac{ae^{(2x)}+be^{(2x)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} + \frac{x}{a+b}$$

input
```
integrate(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="giac")
```

output
```
-a*arctan(1/2*(a*e^(2*x) + b*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b
)) + x/(a + b)
```

3.838.9 Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 209, normalized size of antiderivative = 5.36

$$\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx = \frac{x}{a+b} - \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\left(\mathrm{e}^{2x}\left(\frac{4a}{(a+b)^4}+\frac{\left(a^2-b^2\right)(a-b)}{(a+b)^3\sqrt{b\,(a+b)^2}\sqrt{a^2\,b+2\,a\,b^2+b^3}}\right)+\frac{(a-b)\left(a^2+2\,a\,b+b^2\right)}{(a+b)^3\sqrt{b\,(a+b)^2}\sqrt{a^2\,b+2\,a\,b^2+b^3}}\right)\left(a^2\sqrt{a^2\,b+2\,a\,b^2+b^3}+b^2\sqrt{a^2\,b+2\,a\,b^2+\cdots}\right.}{2\sqrt{a}}\right.}{\sqrt{a^2\,b+2\,a\,b^2+b^3}}$$

input
```
int(sinh(x)^2/(b*sinh(x)^2 + a*cosh(x)^2),x)
```

output
```
x/(a + b) - (a^(1/2)*atan(((exp(2*x)*((4*a)/(a + b)^4 + ((a^2 - b^2)*(a -
b))/((a + b)^3*(b*(a + b)^2)^(1/2)*(2*a*b^2 + a^2*b + b^3)^(1/2))) + ((a -
 b)*(2*a*b + a^2 + b^2))/((a + b)^3*(b*(a + b)^2)^(1/2)*(2*a*b^2 + a^2*b +
 b^3)^(1/2)))*(a^2*(2*a*b^2 + a^2*b + b^3)^(1/2) + b^2*(2*a*b^2 + a^2*b +
b^3)^(1/2) + 2*a*b*(2*a*b^2 + a^2*b + b^3)^(1/2)))/(2*a^(1/2))))/(2*a*b^2
+ a^2*b + b^3)^(1/2)
```

3.839 $\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$

3.839.1 Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{x}{a+b}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}$$

output
```
x/(a+b)+arctan(b^(1/2)*tanh(x)/a^(1/2))*b^(1/2)/(a+b)/a^(1/2)
```

3.839.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{x+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}}}{a+b}$$

input
```
Integrate[Cosh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

output
```
(x + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/Sqrt[a])/(a + b)
```

3.839.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4889, 303, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{a\cos(ix)^2-b\sin(ix)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{(1-\tanh^2(x))\,(a+b\tanh^2(x))}d\tanh(x)$$

$$\Big\downarrow \text{303}$$

$$\frac{\int \frac{1}{1-\tanh^2(x)}d\tanh(x)}{a+b}+\frac{b\int \frac{1}{b\tanh^2(x)+a}d\tanh(x)}{a+b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\int \frac{1}{1-\tanh^2(x)}d\tanh(x)}{a+b}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{\text{arctanh}(\tanh(x))}{a+b}$$

input
```
Int[Cosh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

output
```
(Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ArcTanh[Ta
nh[x]]/(a + b)
```

3.839.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.839.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(30) = 60$.

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.42

method	result
risch	$\frac{x}{a+b}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2x}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2a(a+b)}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2x}-\frac{-a+2\sqrt{-ab}+b}{a+b}\right)}{2a(a+b)}$
default	$-\frac{2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a+2b}-\frac{2ba\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\,\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(a+\sqrt{b(a+b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{2a\sqrt{b(a+b)}\,\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{a+b}$

3.839. $\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$

input
```
int(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x/(a+b)+1/2/a*(-a*b)^(1/2)/(a+b)*ln(exp(2*x)+(a+2*(-a*b)^(1/2)-b)/(a+b))-1
/2/a*(-a*b)^(1/2)/(a+b)*ln(exp(2*x)-(-a+2*(-a*b)^(1/2)+b)/(a+b))
```

3.839.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 363, normalized size of antiderivative = 9.55

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$$
$$= \left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+2\,ab+b^2)\cosh(x)^4+4\,(a^2+2\,ab+b^2)\cosh(x)\sinh(x)^3+(a^2+2\,ab+b^2)\sinh(x)^4+2\,(a^2-b^2)\cosh(x)^2+2\left(3\,(a^2+2\,ab+b^2)\,\mathrm{co}\right.}{(a+b)\cosh(x)^4+4\,(a+b)\cosh(x)\sinh(x)^3+(a+b)\sinh(x}\right.}{}\right.$$

input
```
integrate(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2
)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x
)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 6*
a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x
) + 4*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)
*sinh(x)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)
*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x
)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) +
 a + b)) + 2*x)/(a + b), (sqrt(b/a)*arctan(1/2*((a + b)*cosh(x)^2 + 2*(a +
 b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)*sqrt(b/a)/b) + x)/(a + b)
]
```

3.839.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(32) = 64$.

Time = 0.58 (sec) , antiderivative size = 224, normalized size of antiderivative = 5.89

$$\int \frac{\cosh^2(x)}{a\cosh^2(x) + b\sinh^2(x)}\,dx = \begin{cases} \tilde{\infty}\left(x - \frac{\cosh(x)}{\sinh(x)}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{x - \frac{\cosh(x)}{\sinh(x)}}{b} & \text{for } a = 0 \\ \frac{x\sinh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} - \frac{x\cosh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} - \frac{\sinh(x)\cosh(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} & \text{for } a = -b \\ \frac{x}{a} & \text{for } b = 0 \\ \frac{2ax\sqrt{-\frac{b}{a}}}{2a^2\sqrt{-\frac{b}{a}}+2ab\sqrt{-\frac{b}{a}}} - \frac{b\log\left(-\sqrt{-\frac{b}{a}}\sinh(x)+\cosh(x)\right)}{2a^2\sqrt{-\frac{b}{a}}+2ab\sqrt{-\frac{b}{a}}} + \frac{b\log\left(\sqrt{-\frac{b}{a}}\sinh(x)+\cosh(x)\right)}{2a^2\sqrt{-\frac{b}{a}}+2ab\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(x)**2/(a*cosh(x)**2+b*sinh(x)**2),x)
```

output
```
Piecewise((zoo*(x - cosh(x)/sinh(x)), Eq(a, 0) & Eq(b, 0)), ((x - cosh(x)/
sinh(x))/b, Eq(a, 0)), (x*sinh(x)**2/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2) -
x*cosh(x)**2/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2) - sinh(x)*cosh(x)/(-2*b*si
nh(x)**2 + 2*b*cosh(x)**2), Eq(a, -b)), (x/a, Eq(b, 0)), (2*a*x*sqrt(-b/a)
/(2*a**2*sqrt(-b/a) + 2*a*b*sqrt(-b/a)) - b*log(-sqrt(-b/a)*sinh(x) + cosh
(x))/(2*a**2*sqrt(-b/a) + 2*a*b*sqrt(-b/a)) + b*log(sqrt(-b/a)*sinh(x) + c
osh(x))/(2*a**2*sqrt(-b/a) + 2*a*b*sqrt(-b/a)), True))
```

3.839.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(30) = 60$.

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.95

$$\int \frac{\cosh^2(x)}{a\cosh^2(x) + b\sinh^2(x)}\,dx = -\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2x)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}(a+b)} - \frac{\arctan\left(\frac{(a+b)e^{(-2x)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}} + \frac{x}{a+b}$$

input
```
integrate(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*(a - b)*arctan(1/2*((a + b)*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a
 + b)) - 1/2*arctan(1/2*((a + b)*e^(-2*x) + a - b)/sqrt(a*b))/sqrt(a*b) +
x/(a + b)
```

3.839.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{b\arctan\left(\frac{ae^{(2\,x)}+be^{(2\,x)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}(a+b)}+\frac{x}{a+b}$$

input
```
integrate(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="giac")
```

output
```
b*arctan(1/2*(a*e^(2*x) + b*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)
) + x/(a + b)
```

3.839.9 Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 208, normalized size of antiderivative = 5.47

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{x}{a+b}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\left(e^{2\,x}\left(\frac{4\,b}{(a+b)^4}+\frac{\left(a^2-b^2\right)(a-b)}{(a+b)^3\,\sqrt{a\,(a+b)^2}\,\sqrt{a^3+2\,a^2\,b+a\,b^2}}\right)+\frac{(a-b)\left(a^2+2\,a\,b+b^2\right)}{(a+b)^3\,\sqrt{a\,(a+b)^2}\,\sqrt{a^3+2\,a^2\,b+a\,b^2}}\right)\left(a^2\,\sqrt{a^3+2\,a^2\,b+a\,b^2}+b^2\,\sqrt{a^3+2\,a^2\,b+\right.}{2\,\sqrt{b}}\right.}{\sqrt{a^3+2\,a^2\,b+a\,b^2}}$$

input
```
int(cosh(x)^2/(b*sinh(x)^2 + a*cosh(x)^2),x)
```

output
```
x/(a + b) + (b^(1/2)*atan(((exp(2*x)*((4*b)/(a + b)^4 + ((a^2 - b^2)*(a -
b))/((a + b)^3*(a*(a + b)^2)^(1/2)*(a*b^2 + 2*a^2*b + a^3)^(1/2))) + ((a -
 b)*(2*a*b + a^2 + b^2))/((a + b)^3*(a*(a + b)^2)^(1/2)*(a*b^2 + 2*a^2*b +
 a^3)^(1/2)))*(a^2*(a*b^2 + 2*a^2*b + a^3)^(1/2) + b^2*(a*b^2 + 2*a^2*b +
a^3)^(1/2) + 2*a*b*(a*b^2 + 2*a^2*b + a^3)^(1/2)))/(2*b^(1/2))))/(a*b^2 +
2*a^2*b + a^3)^(1/2)
```

3.840 $\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)} \, dx$

3.840.1 Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)} \, dx = \frac{x}{2}+\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6(1+\tanh(x))}$$

output
```
1/2*x+2/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)+1/6/(1+tanh(x))
```

3.840.2 Mathematica [A] (verified)

Time = 5.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)} \, dx = \frac{1}{36}\left(18x-8\sqrt{3}\arctan\left(\frac{-1+2\tanh(x)}{\sqrt{3}}\right)+3\cosh(2x) - 3\sinh(2x)\right)$$

input
```
Integrate[Sinh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

output
```
(18*x - 8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] + 3*Cosh[2*x] - 3*Sinh[
2*x])/36
```

3.840.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 26, 4889, 26, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{\sinh^3(x)+\cosh^3(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ix)^3}{i\sin(ix)^3+\cos(ix)^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ix)^3}{\cos(ix)^3+i\sin(ix)^3}dx$$

$$\Big\downarrow \text{4889}$$

$$i\int -\frac{i\tanh^3(x)}{-\tanh^5(x)+\tanh^3(x)-\tanh^2(x)+1}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh^3(x)}{-\tanh^5(x)+\tanh^3(x)-\tanh^2(x)+1}d\tanh(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(-\frac{1}{3\left(\tanh^2(x)-\tanh(x)+1\right)}-\frac{1}{2\left(\tanh^2(x)-1\right)}-\frac{1}{6(\tanh(x)+1)^2}\right)d\tanh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{2}\text{arctanh}(\tanh(x))+\frac{1}{6(\tanh(x)+1)}$$

input
```
Int[Sinh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

output
```
(2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + ArcTanh[Tanh[x]]/2 + 1/(
6*(1 + Tanh[x]))
```

3.840.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.840.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

method	result
risch	$\frac{x}{2}+\frac{{\rm e}^{-2 x}}{12}+\frac{i \sqrt{3}\, \ln \left({\rm e}^{2 x}-i \sqrt{3}\right)}{9}-\frac{i \sqrt{3}\, \ln \left({\rm e}^{2 x}+i \sqrt{3}\right)}{9}$
default	$\frac{i \sqrt{3}\, \ln \left(\tanh \left(\frac{x}{2}\right)^{2}+\left(i \sqrt{3}-1\right) \tanh \left(\frac{x}{2}\right)+1\right)}{9}-\frac{i \sqrt{3}\, \ln \left(\tanh \left(\frac{x}{2}\right)^{2}+\left(-i \sqrt{3}-1\right) \tanh \left(\frac{x}{2}\right)+1\right)}{9}-\frac{\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)}{2}+\frac{1}{3\left(1+\tanh \left(\frac{x}{2}\right)\right.}$

input
```
int(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/12*exp(-2*x)+1/9*I*3^(1/2)*ln(exp(2*x)-I*3^(1/2))-1/9*I*3^(1/2)*ln
(exp(2*x)+I*3^(1/2))
```

3.840.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(29) = 58$.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{18\,x\cosh(x)^2+36\,x\cosh(x)\sinh(x)+18\,x\sinh(x)^2+8\left(\sqrt{3}\cosh(x)^2+2\sqrt{3}\cosh(x)\sinh(x)+\sqrt{3}\sin\right.}{36\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

input
```
integrate(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas")
```

output
```
1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 + 8*(sqrt(3)*
cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sq
rt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) + 3)/(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2)
```

3.840.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(36) = 72$.

Time = 0.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.58

$$\begin{aligned}\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx &= \frac{9x\sinh(x)}{18\sinh(x)+18\cosh(x)}+\frac{9x\cosh(x)}{18\sinh(x)+18\cosh(x)}\\ &\quad-\frac{4\sqrt{3}\sinh(x)\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\cosh(x)}{3\sinh(x)}\right)}{18\sinh(x)+18\cosh(x)}\\ &\quad-\frac{4\sqrt{3}\cosh(x)\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\cosh(x)}{3\sinh(x)}\right)}{18\sinh(x)+18\cosh(x)}\\ &\quad+\frac{3\cosh(x)}{18\sinh(x)+18\cosh(x)}\end{aligned}$$

input
```
integrate(sinh(x)**3/(cosh(x)**3+sinh(x)**3),x)
```

output
```
9*x*sinh(x)/(18*sinh(x) + 18*cosh(x)) + 9*x*cosh(x)/(18*sinh(x) + 18*cosh(
x)) - 4*sqrt(3)*sinh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x)/(3*sinh(x)))/(1
8*sinh(x) + 18*cosh(x)) - 4*sqrt(3)*cosh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cos
h(x)/(3*sinh(x)))/(18*sinh(x) + 18*cosh(x)) + 3*cosh(x)/(18*sinh(x) + 18*c
osh(x))
```

3.840.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(29) = 58$.

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{2}x + \frac{1}{12}e^{(-2x)}$$

input
```
integrate(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2
))) + 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*s
qrt(2))) + 1/2*x + 1/12*e^(-2*x)
```

3.840.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = -\frac{1}{12}\left(3e^{(2x)}-1\right)e^{(-2x)} - \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right) + \frac{1}{2}x$$

input
```
integrate(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
```

output
```
-1/12*(3*e^(2*x) - 1)*e^(-2*x) - 2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) +
 1/2*x
```

3.840.9 Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx=\frac{x}{2}+\frac{e^{-2x}}{12}-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,e^{2x}}{3}\right)}{9}$$

input
```
int(sinh(x)^3/(cosh(x)^3 + sinh(x)^3),x)
```

output
```
x/2 + exp(-2*x)/12 - (2*3^(1/2)*atan((3^(1/2)*exp(2*x))/3))/9
```

3.841 $\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$

3.841.1 Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx=\frac{x}{2}-\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6(1+\tanh(x))}$$

output
```
1/2*x-2/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)-1/6/(1+tanh(x))
```

3.841.2 Mathematica [A] (verified)

Time = 5.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx=\frac{1}{36}\left(18x+8\sqrt{3}\arctan\left(\frac{-1+2\tanh(x)}{\sqrt{3}}\right)-3\cosh(2x)+3\sinh(2x)\right)$$

input
```
Integrate[Cosh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

output
```
(18*x + 8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] - 3*Cosh[2*x] + 3*Sinh[
2*x])/36
```

3.841.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4889, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{\sinh^3(x)+\cosh^3(x)} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ix)^3}{i\sin(ix)^3+\cos(ix)^3}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{1}{-\tanh^5(x)+\tanh^3(x)-\tanh^2(x)+1}d\tanh(x)$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{1}{3\left(\tanh^2(x)-\tanh(x)+1\right)}-\frac{1}{2\left(\tanh^2(x)-1\right)}+\frac{1}{6(\tanh(x)+1)^2}\right)d\tanh(x)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{2}\text{arctanh}(\tanh(x))-\frac{1}{6(\tanh(x)+1)}$$

input
```
Int[Cosh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

output
```
(-2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + ArcTanh[Tanh[x]]/2 - 1/
(6*(1 + Tanh[x]))
```

3.841.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.841.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

method	result
risch	$\frac{x}{2}-\frac{\mathrm{e}^{-2x}}{12}+\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}+i\sqrt{3}\right)}{9}-\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}-i\sqrt{3}\right)}{9}$
default	$\frac{i\sqrt{3}\,\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+\left(-i\sqrt{3}-1\right)\tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{i\sqrt{3}\,\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+\left(i\sqrt{3}-1\right)\tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{1}{3\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{1}{3+3\tanh\left(\frac{x}{2}\right)}$

input
```
int(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/12*exp(-2*x)+1/9*I*3^(1/2)*ln(exp(2*x)+I*3^(1/2))-1/9*I*3^(1/2)*ln
(exp(2*x)-I*3^(1/2))
```

3.841.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{18\,x\cosh(x)^2+36\,x\cosh(x)\sinh(x)+18\,x\sinh(x)^2-8\left(\sqrt{3}\cosh(x)^2+2\sqrt{3}\cosh(x)\sinh(x)+\sqrt{3}\sin\right.}{36\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

input
```
integrate(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas")
```

output
```
1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 - 8*(sqrt(3)*
cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sq
rt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) - 3)/(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2)
```

3.841.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(36) = 72.

Time = 0.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.58

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{9x\sinh(x)}{18\sinh(x)+18\cosh(x)}+\frac{9x\cosh(x)}{18\sinh(x)+18\cosh(x)} + \frac{4\sqrt{3}\sinh(x)\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\cosh(x)}{3\sinh(x)}\right)}{18\sinh(x)+18\cosh(x)} + \frac{4\sqrt{3}\cosh(x)\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\cosh(x)}{3\sinh(x)}\right)}{18\sinh(x)+18\cosh(x)} - \frac{3\cosh(x)}{18\sinh(x)+18\cosh(x)}$$

input
```
integrate(cosh(x)**3/(cosh(x)**3+sinh(x)**3),x)
```

output
```
9*x*sinh(x)/(18*sinh(x) + 18*cosh(x)) + 9*x*cosh(x)/(18*sinh(x) + 18*cosh(
x)) + 4*sqrt(3)*sinh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x)/(3*sinh(x)))/(1
8*sinh(x) + 18*cosh(x)) + 4*sqrt(3)*cosh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cos
h(x)/(3*sinh(x)))/(18*sinh(x) + 18*cosh(x)) - 3*cosh(x)/(18*sinh(x) + 18*c
osh(x))
```

3.841.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(29) = 58$.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{2}x - \frac{1}{12}e^{(-2x)}$$

input
```
integrate(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2)
)) - 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sq
rt(2))) + 1/2*x - 1/12*e^(-2*x)
```

3.841.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = -\frac{1}{12}\left(3e^{(2x)}+1\right)e^{(-2x)} + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right) + \frac{1}{2}x$$

input
```
integrate(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
```

output
```
-1/12*(3*e^(2*x) + 1)*e^(-2*x) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) +
 1/2*x
```

3.841.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{x}{2}-\frac{e^{-2x}}{12}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,e^{2x}}{3}\right)}{9}$$

input
```
int(cosh(x)^3/(cosh(x)^3 + sinh(x)^3),x)
```

output
```
x/2 - exp(-2*x)/12 + (2*3^(1/2)*atan((3^(1/2)*exp(2*x))/3))/9
```

3.842 $\int \frac{x\,\text{csch}(x)\text{sech}(x)}{\sqrt{a\,\text{sech}^2(x)}}\,dx$

3.842.1 Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{2x\text{arctanh}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{\text{PolyLog}\,(2,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{\text{PolyLog}\,(2,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

output
```
-2*x*arctanh(exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-polylog(2,-exp(x))*sech(x
)/(a*sech(x)^2)^(1/2)+polylog(2,exp(x))*sech(x)/(a*sech(x)^2)^(1/2)
```

3.842.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{(x(\log(1-e^x)-\log(1+e^x))-\text{PolyLog}\,(2,-e^x)+\text{PolyLog}\,(2,e^x))\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

input
```
Integrate[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

output
```
((x*(Log[1 - E^x] - Log[1 + E^x]) - PolyLog[2, -E^x] + PolyLog[2, E^x])*Se
ch[x])/Sqrt[a*Sech[x]^2]
```

3.842.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {7271, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\text{sech}(x)\int x\text{csch}(x)dx}{\sqrt{a\text{sech}^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}(x)\int ix\csc(ix)dx}{\sqrt{a\text{sech}^2(x)}}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\text{sech}(x)\int x\csc(ix)dx}{\sqrt{a\text{sech}^2(x)}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{i\text{sech}(x)\left(i\int \log(1-e^x)\,dx-i\int \log(1+e^x)\,dx+2ix\text{arctanh}(e^x)\right)}{\sqrt{a\text{sech}^2(x)}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{i\text{sech}(x)\left(i\int e^{-x}\log(1-e^x)\,de^x-i\int e^{-x}\log(1+e^x)\,de^x+2ix\text{arctanh}(e^x)\right)}{\sqrt{a\text{sech}^2(x)}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{i\text{sech}(x)\left(2ix\text{arctanh}(e^x)+i\,\text{PolyLog}(2,-e^x)-i\,\text{PolyLog}(2,e^x)\right)}{\sqrt{a\text{sech}^2(x)}}$$

input
```
Int[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

output
```
(I*((2*I)*x*ArcTanh[E^x] + I*PolyLog[2, -E^x] - I*PolyLog[2, E^x])*Sech[x]
)/Sqrt[a*Sech[x]^2]
```

3.842.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.842.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(50) = 100$.

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.31

method	result	size
risch	$-\frac{e^{x} x \ln\left(1+e^{x}\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}\,\left(1+e^{2x}\right)}-\frac{e^{x} \operatorname{polylog}\left(2,-e^{x}\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}\,\left(1+e^{2x}\right)}+\frac{e^{x} x \ln\left(1-e^{x}\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}\,\left(1+e^{2x}\right)}+\frac{e^{x} \operatorname{polylog}\left(2,e^{x}\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}\,\left(1+e^{2x}\right)}$	136

input
```
int(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*ln(1+exp(x))-1/
(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(2,-exp(x))+1
/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*ln(1-exp(x))+1/(a
*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(2,exp(x))
```

3.842.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.54

$$\int \frac{x\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = \frac{\left(\left(e^{(2x)}+1\right)\operatorname{Li}_2(\cosh(x)+\sinh(x))-\left(e^{(2x)}+1\right)\operatorname{Li}_2(-\cosh(x)-\sinh(x))-\left(xe^{(2x)}+x\right)\log(\cosh(x)+\cdots}{a}$$

input
```
integrate(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
((e^(2*x) + 1)*dilog(cosh(x) + sinh(x)) - (e^(2*x) + 1)*dilog(-cosh(x) - s
inh(x)) - (x*e^(2*x) + x)*log(cosh(x) + sinh(x) + 1) + (x*e^(2*x) + x)*log
(-cosh(x) - sinh(x) + 1))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))/a
```

3.842.6 Sympy [F]

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x\,\text{csch}\,(x)\,\text{sech}\,(x)}{\sqrt{a\,\text{sech}^2\,(x)}}\,dx$$

input
```
integrate(x*csch(x)*sech(x)/(a*sech(x)**2)**(1/2),x)
```

output
```
Integral(x*csch(x)*sech(x)/sqrt(a*sech(x)**2), x)
```

3.842.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{x\log\left(e^x+1\right)+\text{Li}_2(-e^x)}{\sqrt{a}}+\frac{x\log\left(-e^x+1\right)+\text{Li}_2(e^x)}{\sqrt{a}}$$

input
```
integrate(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(x*log(e^x + 1) + dilog(-e^x))/sqrt(a) + (x*log(-e^x + 1) + dilog(e^x))/s
qrt(a)
```

3.842.8 Giac [F]

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x\,\text{csch}\,(x)\,\text{sech}\,(x)}{\sqrt{a\,\text{sech}\,(x)^2}}\,dx$$

input
```
integrate(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*csch(x)*sech(x)/sqrt(a*sech(x)^2), x)
```

3.842.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x}{\cosh(x)\,\sinh(x)\,\sqrt{\frac{a}{\cosh(x)^2}}}\,dx$$

input
```
int(x/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)),x)
```

output
```
int(x/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)), x)
```

3.843 $\int \frac{x^2 \operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx$

3.843.1 Optimal result

Integrand size = 18, antiderivative size = 104

$$\int \frac{x^2\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = -\frac{2x^2\operatorname{arctanh}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} - \frac{2x\operatorname{PolyLog}(2,-e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} + \frac{2x\operatorname{PolyLog}(2,e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} + \frac{2\operatorname{PolyLog}(3,-e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} - \frac{2\operatorname{PolyLog}(3,e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}$$

output
```
-2*x^2*arctanh(exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-2*x*polylog(2,-exp(x))*
sech(x)/(a*sech(x)^2)^(1/2)+2*x*polylog(2,exp(x))*sech(x)/(a*sech(x)^2)^(1
/2)+2*polylog(3,-exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-2*polylog(3,exp(x))*s
ech(x)/(a*sech(x)^2)^(1/2)
```

3.843.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{x^2\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = \frac{(x^2\log(1-e^x) - x^2\log(1+e^x) - 2x\operatorname{PolyLog}(2,-e^x) + 2x\operatorname{PolyLog}(2,e^x) + 2\operatorname{PolyLog}(3,-e^x) - 2\operatorname{Po}}{\sqrt{a\operatorname{sech}^2(x)}}$$

input
```
Integrate[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

output
```
((x^2*Log[1 - E^x] - x^2*Log[1 + E^x] - 2*x*PolyLog[2, -E^x] + 2*x*PolyLog
[2, E^x] + 2*PolyLog[3, -E^x] - 2*PolyLog[3, E^x])*Sech[x])/Sqrt[a*Sech[x]
^2]
```

3.843.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {7271, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\text{sech}(x)\int x^2\text{csch}(x)dx}{\sqrt{a\text{sech}^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}(x)\int ix^2\csc(ix)dx}{\sqrt{a\text{sech}^2(x)}}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\text{sech}(x)\int x^2\csc(ix)dx}{\sqrt{a\text{sech}^2(x)}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{i\text{sech}(x)\left(2i\int x\log\left(1-e^x\right)dx-2i\int x\log\left(1+e^x\right)dx+2ix^2\text{arctanh}(e^x)\right)}{\sqrt{a\text{sech}^2(x)}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{i\text{sech}(x)\left(-2i\left(\int \text{PolyLog}(2,-e^x)\,dx - x\,\text{PolyLog}(2,-e^x)\right) + 2i\left(\int \text{PolyLog}(2,e^x)\,dx - x\,\text{PolyLog}(2,e^x)\right) + 2ix^2\right.}{\sqrt{a\text{sech}^2(x)}}$$

$\big\downarrow$ 2720

$$\frac{i\text{sech}(x)\left(-2i\left(\int e^{-x}\,\text{PolyLog}(2,-e^x)\,de^x - x\,\text{PolyLog}(2,-e^x)\right) + 2i\left(\int e^{-x}\,\text{PolyLog}(2,e^x)\,de^x - x\,\text{PolyLog}(2,e\right.\right.}{\sqrt{a\text{sech}^2(x)}}$$

$\big\downarrow$ 7143

$$\frac{i\text{sech}(x)\left(2ix^2\text{arctanh}(e^x) - 2i(\text{PolyLog}(3,-e^x) - x\,\text{PolyLog}(2,-e^x)) + 2i(\text{PolyLog}(3,e^x) - x\,\text{PolyLog}(2,e^x))\right)}{\sqrt{a\text{sech}^2(x)}}$$

input
```
Int[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

output
```
(I*((2*I)*x^2*ArcTanh[E^x] - (2*I)*(-(x*PolyLog[2, -E^x]) + PolyLog[3, -E^
x]) + (2*I)*(-(x*PolyLog[2, E^x]) + PolyLog[3, E^x]))*Sech[x])/Sqrt[a*Sech
[x]^2]
```

3.843.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.843. $\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.843.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(89) = 178$.

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.01

method	result
risch	$-\frac{e^{x} x^{2} \ln(1+e^{x})}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^{2}}}\,(1+e^{2x})}-\frac{2\,e^{x} x \,\mathrm{polylog}(2,-e^{x})}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^{2}}}\,(1+e^{2x})}+\frac{2\,e^{x} \mathrm{polylog}(3,-e^{x})}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^{2}}}\,(1+e^{2x})}+\frac{e^{x} x^{2} \ln(1-e^{x})}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^{2}}}\,(1+e^{2x})}+\frac{2\,e^{x} x \,\mathrm{polylog}(2,e^{x})}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^{2}}}\,(1+e^{2x})}-$

input
```
int(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^2*ln(1+exp(x))-
2/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*polylog(2,-exp(x
))+2/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(3,-exp(
x))+1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^2*ln(1-exp(x
))+2/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*polylog(2,exp
(x))-2/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(3,exp
(x))
```

3.843. $\int \frac{x^2\mathrm{csch}(x)\mathrm{sech}(x)}{\sqrt{a\mathrm{sech}^2(x)}}\,dx$

3.843.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(87) = 174$.

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.81

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{\left(2\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^x\text{polylog}(3,\cosh(x)+\sinh(x)) - 2\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^x\text{polylog}(\right.}{}$$

input
```
integrate(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(3, cosh(x)
 + sinh(x)) - 2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylo
g(3, -cosh(x) - sinh(x)) - (2*(x*e^(2*x) + x)*dilog(cosh(x) + sinh(x)) - 2
*(x*e^(2*x) + x)*dilog(-cosh(x) - sinh(x)) - (x^2*e^(2*x) + x^2)*log(cosh(
x) + sinh(x) + 1) + (x^2*e^(2*x) + x^2)*log(-cosh(x) - sinh(x) + 1))*sqrt(
a/(e^(4*x) + 2*e^(2*x) + 1))*e^x)*e^(-x)/a
```

3.843.6 Sympy [F]

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x^2\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx$$

input
```
integrate(x**2*csch(x)*sech(x)/(a*sech(x)**2)**(1/2),x)
```

output
```
Integral(x**2*csch(x)*sech(x)/sqrt(a*sech(x)**2), x)
```

3.843.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int \frac{x^2 \operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = -\frac{x^2\log\left(e^x+1\right)+2\,x\operatorname{Li}_2(-e^x)-2\operatorname{Li}_3(-e^x)}{\sqrt{a}} + \frac{x^2\log\left(-e^x+1\right)+2\,x\operatorname{Li}_2(e^x)-2\operatorname{Li}_3(e^x)}{\sqrt{a}}$$

input
```
integrate(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*polylog(3, -e^x))/sqrt(a) + (x^2*
log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(3, e^x))/sqrt(a)
```

3.843.8 Giac [F]

$$\int \frac{x^2 \operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = \int \frac{x^2\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}(x)^2}}\,dx$$

input
```
integrate(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*csch(x)*sech(x)/sqrt(a*sech(x)^2), x)
```

3.843.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = \int \frac{x^2}{\cosh(x)\,\sinh(x)\,\sqrt{\frac{a}{\cosh(x)^2}}}\,dx$$

input
```
int(x^2/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)),x)
```

output
```
int(x^2/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)), x)
```

3.844 $\int \frac{x^3\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx$

3.844.1 Optimal result

Integrand size = 18, antiderivative size = 150

$$\int \frac{x^3\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = -\frac{2x^3\operatorname{arctanh}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} - \frac{3x^2\operatorname{PolyLog}(2,-e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} + \frac{3x^2\operatorname{PolyLog}(2,e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} + \frac{6x\operatorname{PolyLog}(3,-e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} - \frac{6x\operatorname{PolyLog}(3,e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} - \frac{6\operatorname{PolyLog}(4,-e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} + \frac{6\operatorname{PolyLog}(4,e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}$$

output
```
-2*x^3*arctanh(exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-3*x^2*polylog(2,-exp(x)
)*sech(x)/(a*sech(x)^2)^(1/2)+3*x^2*polylog(2,exp(x))*sech(x)/(a*sech(x)^2
)^(1/2)+6*x*polylog(3,-exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-6*x*polylog(3,e
xp(x))*sech(x)/(a*sech(x)^2)^(1/2)-6*polylog(4,-exp(x))*sech(x)/(a*sech(x)
^2)^(1/2)+6*polylog(4,exp(x))*sech(x)/(a*sech(x)^2)^(1/2)
```

3.844.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.62

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx$$
$$= \frac{(x^3\log(1-e^x) - x^3\log(1+e^x) - 3x^2\,\text{PolyLog}(2,-e^x) + 3x^2\,\text{PolyLog}(2,e^x) + 6x\,\text{PolyLog}(3,-e^x) - 6}{\sqrt{a\text{sech}^2(x)}}$$

input
```
Integrate[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

output
```
((x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLog[2, -E^x] + 3*x^2*Pol
yLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3, E^x] - 6*PolyLog[4, -
E^x] + 6*PolyLog[4, E^x])*Sech[x])/Sqrt[a*Sech[x]^2]
```

3.844.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.65, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {7271, 3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\text{sech}(x)\int x^3\text{csch}(x)dx}{\sqrt{a\text{sech}^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}(x)\int ix^3\csc(ix)dx}{\sqrt{a\text{sech}^2(x)}}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\text{sech}(x)\int x^3\csc(ix)dx}{\sqrt{a\text{sech}^2(x)}}$$

$\big\downarrow$ 4670

$$\frac{i\text{sech}(x)\left(3i\int x^2\log\left(1-e^x\right)dx-3i\int x^2\log\left(1+e^x\right)dx+2ix^3\text{arctanh}(e^x)\right)}{\sqrt{a\text{sech}^2(x)}}$$

$\big\downarrow$ 3011

$$\frac{i\text{sech}(x)\left(-3i\left(2\int x\,\text{PolyLog}\left(2,-e^x\right)dx-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\int x\,\text{PolyLog}\left(2,e^x\right)dx-x^2\,\text{PolyLog}\left(2,e^x\right)\right.\right.}{\sqrt{a\text{sech}^2(x)}}$$

$\big\downarrow$ 7163

$$\frac{i\text{sech}(x)\left(-3i\left(2\left(x\,\text{PolyLog}\left(3,-e^x\right)-\int\text{PolyLog}\left(3,-e^x\right)dx\right)-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\left(x\,\text{PolyLog}\left(3,e^x\right)-\right.\right.\right.}{\sqrt{a\text{sech}^2(x)}}$$

$\big\downarrow$ 2720

$$\frac{i\text{sech}(x)\left(-3i\left(2\left(x\,\text{PolyLog}\left(3,-e^x\right)-\int e^{-x}\,\text{PolyLog}\left(3,-e^x\right)de^x\right)-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\left(x\,\text{PolyLog}\left(3,e\right.\right.\right.\right.}{\sqrt{a\text{sech}^2(x)}}$$

$\big\downarrow$ 7143

$$\frac{i\text{sech}(x)\left(2ix^3\text{arctanh}(e^x)-3i\left(2\left(x\,\text{PolyLog}\left(3,-e^x\right)-\text{PolyLog}\left(4,-e^x\right)\right)-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\left(x\,\text{PolyL}\right.\right.\right.}{\sqrt{a\text{sech}^2(x)}}$$

input
```
Int[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

output
```
(I*((2*I)*x^3*ArcTanh[E^x] - (3*I)*(-(x^2*PolyLog[2, -E^x]) + 2*(x*PolyLog
[3, -E^x] - PolyLog[4, -E^x])) + (3*I)*(-(x^2*PolyLog[2, E^x]) + 2*(x*Poly
Log[3, E^x] - PolyLog[4, E^x])))*Sech[x])/Sqrt[a*Sech[x]^2]
```

3.844. $\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx$

3.844.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.844. $\int \frac{x^3 \text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx$

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

3.844.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(129) = 258$.

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.87

method	result
risch	$-\frac{e^{x} x^{3} \ln(1+e^{x})}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^{2}}}\,(1+e^{2x})}-\frac{3\,e^{x} x^{2} \operatorname{polylog}(2,-e^{x})}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^{2}}}\,(1+e^{2x})}+\frac{6\,e^{x} x \operatorname{polylog}(3,-e^{x})}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^{2}}}\,(1+e^{2x})}-\frac{6\,e^{x} \operatorname{polylog}(4,-e^{x})}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^{2}}}\,(1+e^{2x})}+\frac{e^{x} x^{3} \ln(1-e^{x})}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^{2}}}\,(1+e^{2x})}+$

input
```
int(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^3*ln(1+exp(x))-
3/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^2*polylog(2,-exp
(x))+6/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*polylog(3,-
exp(x))-6/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(4,
-exp(x))+1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^3*ln(1-
exp(x))+3/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^2*polylo
g(2,exp(x))-6/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*poly
log(3,exp(x))+6/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*poly
log(4,exp(x))
```

3.844.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(127) = 254$.

Time = 0.26 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.81

$$\int \frac{x^3\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = \frac{\left(6\sqrt{\frac{a}{e^{(4x)}+2\,e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^{x}\operatorname{polylog}(4,\cosh(x)+\sinh(x))-6\sqrt{\frac{a}{e^{(4x)}+2\,e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^{x}\operatorname{polylog}(4,\right.}{}$$

input
```
integrate(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
(6*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(4, cosh(x)
+ sinh(x)) - 6*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog
(4, -cosh(x) - sinh(x)) - 6*(x*e^(2*x) + x)*sqrt(a/(e^(4*x) + 2*e^(2*x) +
1))*e^x*polylog(3, cosh(x) + sinh(x)) + 6*(x*e^(2*x) + x)*sqrt(a/(e^(4*x)
+ 2*e^(2*x) + 1))*e^x*polylog(3, -cosh(x) - sinh(x)) + (3*(x^2*e^(2*x) + x
^2)*dilog(cosh(x) + sinh(x)) - 3*(x^2*e^(2*x) + x^2)*dilog(-cosh(x) - sinh
(x)) - (x^3*e^(2*x) + x^3)*log(cosh(x) + sinh(x) + 1) + (x^3*e^(2*x) + x^3
)*log(-cosh(x) - sinh(x) + 1))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x)*e^(-
x)/a
```

3.844.6 Sympy [F]

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x^3\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx$$

input
```
integrate(x**3*csch(x)*sech(x)/(a*sech(x)**2)**(1/2),x)
```

output
```
Integral(x**3*csch(x)*sech(x)/sqrt(a*sech(x)**2), x)
```

3.844.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.53

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{x^3\log(e^x+1) + 3\,x^2\text{Li}_2(-e^x) - 6\,x\text{Li}_3(-e^x) + 6\,\text{Li}_4(-e^x)}{\sqrt{a}} + \frac{x^3\log(-e^x+1) + 3\,x^2\text{Li}_2(e^x) - 6\,x\text{Li}_3(e^x) + 6\,\text{Li}_4(e^x)}{\sqrt{a}}$$

input
```
integrate(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(x^3*log(e^x + 1) + 3*x^2*dilog(-e^x) - 6*x*polylog(3, -e^x) + 6*polylog(
4, -e^x))/sqrt(a) + (x^3*log(-e^x + 1) + 3*x^2*dilog(e^x) - 6*x*polylog(3,
 e^x) + 6*polylog(4, e^x))/sqrt(a)
```

3.844.8 Giac [F]

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x^3\,\text{csch}\,(x)\,\text{sech}\,(x)}{\sqrt{a\,\text{sech}\,(x)^2}}\,dx$$

input
```
integrate(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3*csch(x)*sech(x)/sqrt(a*sech(x)^2), x)
```

3.844.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x^3}{\cosh\,(x)\,\sinh\,(x)\,\sqrt{\frac{a}{\cosh(x)^2}}}\,dx$$

input
```
int(x^3/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)),x)
```

output
```
int(x^3/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)), x)
```

3.845 $\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx$

3.845.1 Optimal result

Integrand size = 16, antiderivative size = 73

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx=-\frac{x^2\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}+\frac{x\log(1-e^{2x})\,\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}+\frac{\text{PolyLog}(2,e^{2x})\,\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}$$

output
```
-1/2*x^2*sech(x)^2/(a*sech(x)^4)^(1/2)+x*ln(1-exp(2*x))*sech(x)^2/(a*sech(
x)^4)^(1/2)+1/2*polylog(2,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)
```

3.845.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.59

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx=\frac{(-x(x-2\log(1-e^{2x}))+\text{PolyLog}(2,e^{2x}))\,\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}$$

input
```
Integrate[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

output
```
((-(x*(x - 2*Log[1 - E^(2*x)])) + PolyLog[2, E^(2*x)])*Sech[x]^2)/(2*Sqrt[
a*Sech[x]^4])
```

3.845.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7271, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\text{sech}^2(x)\int x\coth(x)dx}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\text{sech}^2(x)\int -ix\tan\left(ix+\frac{\pi}{2}\right)dx}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\text{sech}^2(x)\int x\tan\left(ix+\frac{\pi}{2}\right)dx}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 4199}$$

$$-\frac{i\text{sech}^2(x)\left(2i\int -\frac{e^{2x}x}{1-e^{2x}}dx-\frac{ix^2}{2}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\int \frac{e^{2x}x}{1-e^{2x}}dx-\frac{ix^2}{2}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\left(\frac{1}{2}\int\log\left(1-e^{2x}\right)dx-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\left(\frac{1}{4}\int e^{-2x}\log\left(1-e^{2x}\right)de^{2x}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\left(-\frac{\text{PolyLog}(2,e^{2x})}{4}-\frac{1}{2}x\log\left(1-e^{2x}\right)\right)-\frac{ix^2}{2}\right)}{\sqrt{a\text{sech}^4(x)}}$$

input
```
Int[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

output
```
((-I)*((-1/2*I)*x^2 - (2*I)*(-1/2*(x*Log[1 - E^(2*x)]) - PolyLog[2, E^(2*x
)]/4))*Sech[x]^2)/Sqrt[a*Sech[x]^4]
```

3.845.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

3.845.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(61) = 122$.

Time = 0.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.40

method	result
risch	$-\frac{e^{2x}x^{2}}{2\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^{4}}}\,\left(1+e^{2x}\right)^{2}}+\frac{e^{2x}x\ln\left(1+e^{x}\right)}{\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^{4}}}\,\left(1+e^{2x}\right)^{2}}+\frac{e^{2x}\operatorname{polylog}\left(2,-e^{x}\right)}{\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^{4}}}\,\left(1+e^{2x}\right)^{2}}+\frac{e^{2x}x\ln\left(1-e^{x}\right)}{\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^{4}}}\,\left(1+e^{2x}\right)^{2}}+\frac{e^{2x}\operatorname{polylog}\left(2,e^{x}\right)}{\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^{4}}}\,\left(1+e^{2x}\right)^{2}}$

input
```
int(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^2+1/(a*ex
p(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*ln(1+exp(x))+1/(a*e
xp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*polylog(2,-exp(x))+1
/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*ln(1-exp(x))+
1/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*polylog(2,exp(
x))
```

3.845.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(60) = 120.

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.08

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{\left(x^2e^{(4x)} + 2x^2e^{(2x)} + x^2 - 2\left(e^{(4x)} + 2e^{(2x)} + 1\right)\text{Li}_2(\cosh(x) + \sinh(x)) - 2\left(e^{(4x)} + 2e^{(2x)} + 1\right)\text{Li}_2(-$$

input
```
integrate(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2 - 2*(e^(4*x) + 2*e^(2*x) + 1)*dilo
g(cosh(x) + sinh(x)) - 2*(e^(4*x) + 2*e^(2*x) + 1)*dilog(-cosh(x) - sinh(x
)) - 2*(x*e^(4*x) + 2*x*e^(2*x) + x)*log(cosh(x) + sinh(x) + 1) - 2*(x*e^(
4*x) + 2*x*e^(2*x) + x)*log(-cosh(x) - sinh(x) + 1))*sqrt(a/(e^(8*x) + 4*e
^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))/a
```

3.845.6 Sympy [F]

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x\,\text{csch}\,(x)\,\text{sech}\,(x)}{\sqrt{a\,\text{sech}^4\,(x)}}\,dx$$

input
```
integrate(x*csch(x)*sech(x)/(a*sech(x)**4)**(1/2),x)
```

output
```
Integral(x*csch(x)*sech(x)/sqrt(a*sech(x)**4), x)
```

3.845.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.59

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{x^2}{2\sqrt{a}} + \frac{x\log(e^x+1) + \text{Li}_2(-e^x)}{\sqrt{a}} + \frac{x\log(-e^x+1) + \text{Li}_2(e^x)}{\sqrt{a}}$$

input
```
integrate(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*x^2/sqrt(a) + (x*log(e^x + 1) + dilog(-e^x))/sqrt(a) + (x*log(-e^x +
1) + dilog(e^x))/sqrt(a)
```

3.845.8 Giac [F]

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\, dx = \int \frac{x \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^4}}\, dx$$

input
```
integrate(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*csch(x)*sech(x)/sqrt(a*sech(x)^4), x)
```

3.845.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\, dx = \int \frac{x}{\cosh(x)\,\sinh(x)\,\sqrt{\frac{a}{\cosh(x)^4}}}\, dx$$

input
```
int(x/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)),x)
```

output
```
int(x/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)), x)
```

3.846 $\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx$

3.846.1 Optimal result

Integrand size = 18, antiderivative size = 98

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}}+\frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}+\frac{x\,\text{PolyLog}\left(2,e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}-\frac{\text{PolyLog}\left(3,e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}$$

output
```
-1/3*x^3*sech(x)^2/(a*sech(x)^4)^(1/2)+x^2*ln(1-exp(2*x))*sech(x)^2/(a*sec
h(x)^4)^(1/2)+x*polylog(2,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)-1/2*poly
log(3,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)
```

3.846.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{\left(-2x^2\left(x-3\log\left(1-e^{2x}\right)\right)+6x\,\text{PolyLog}\left(2,e^{2x}\right)-3\,\text{PolyLog}\left(3,e^{2x}\right)\right)\text{sech}^2(x)}{6\sqrt{a\text{sech}^4(x)}}$$

input
```
Integrate[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

output
```
((-2*x^2*(x - 3*Log[1 - E^(2*x)]) + 6*x*PolyLog[2, E^(2*x)] - 3*PolyLog[3,
 E^(2*x)])*Sech[x]^2)/(6*Sqrt[a*Sech[x]^4])
```

3.846.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7271, 3042, 26, 4199, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\text{sech}^2(x)\int x^2\coth(x)dx}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\text{sech}^2(x)\int -ix^2\tan\left(ix+\frac{\pi}{2}\right)dx}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\text{sech}^2(x)\int x^2\tan\left(ix+\frac{\pi}{2}\right)dx}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 4199}$$

$$-\frac{i\text{sech}^2(x)\left(2i\int -\frac{e^{2x}x^2}{1-e^{2x}}dx-\frac{ix^3}{3}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\int \frac{e^{2x}x^2}{1-e^{2x}}dx-\frac{ix^3}{3}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\left(\int x\log\left(1-e^{2x}\right)dx-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\left(\frac{1}{2}\int \text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\left(\frac{1}{4}\int e^{-2x}\text{PolyLog}\left(2,e^{2x}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\left(-\frac{1}{2}x\,\text{PolyLog}\left(2,e^{2x}\right)+\frac{\text{PolyLog}(3,e^{2x})}{4}-\frac{1}{2}x^2\log\left(1-e^{2x}\right)\right)-\frac{ix^3}{3}\right)}{\sqrt{a\text{sech}^4(x)}}$$

input
```
Int[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

output
```
((-I)*((-1/3*I)*x^3 - (2*I)*(-1/2*(x^2*Log[1 - E^(2*x)]) - (x*PolyLog[2, E
^(2*x)])/2 + PolyLog[3, E^(2*x)]/4))*Sech[x]^2)/Sqrt[a*Sech[x]^4]
```

3.846.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.846. $\int \frac{x^2 \operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx$

3.846.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(83) = 166$.

Time = 0.10 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.58

method	result
risch	$-\frac{e^{2x}x^3}{3\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{e^{2x}x^2\ln(1+e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{2\,e^{2x}x\,\mathrm{polylog}(2,-e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}-\frac{2\,e^{2x}\,\mathrm{polylog}(3,-e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{e^{2x}x^2\ln(1-e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}$

input
```
int(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^3+1/(a*ex
p(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^2*ln(1+exp(x))+2/(a
*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*polylog(2,-exp(x
))-2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*polylog(3,-
exp(x))+1/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^2*ln
(1-exp(x))+2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*p
olylog(2,exp(x))-2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*
x)*polylog(3,exp(x))
```

3.846.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(82) = 164$.

Time = 0.28 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.00

$$\int \frac{x^2\mathrm{csch}(x)\mathrm{sech}(x)}{\sqrt{a\mathrm{sech}^4(x)}}\,dx =$$
$$-\frac{\left(6\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x)}+2e^{(2x)}+1\right)e^{(2x)}\mathrm{polylog}(3,\cosh(x)+\sinh(x))+6\sqrt{\frac{a}{e^{(8x)}+4e^{(6x}}}\right.}{}$$

input
```
integrate(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(6*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x)
 + 2*e^(2*x) + 1)*e^(2*x)*polylog(3, cosh(x) + sinh(x)) + 6*sqrt(a/(e^(8*x
) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*e^(2*x) + 1)*e^(2
*x)*polylog(3, -cosh(x) - sinh(x)) + (x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3 -
6*(x*e^(4*x) + 2*x*e^(2*x) + x)*dilog(cosh(x) + sinh(x)) - 6*(x*e^(4*x) +
2*x*e^(2*x) + x)*dilog(-cosh(x) - sinh(x)) - 3*(x^2*e^(4*x) + 2*x^2*e^(2*x
) + x^2)*log(cosh(x) + sinh(x) + 1) - 3*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2
)*log(-cosh(x) - sinh(x) + 1))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4
*e^(2*x) + 1))*e^(2*x))*e^(-2*x)/a
```

3.846.6 Sympy [F]

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x^2\,\text{csch}\,(x)\,\text{sech}\,(x)}{\sqrt{a\,\text{sech}^4\,(x)}}\,dx$$

input
```
integrate(x**2*csch(x)*sech(x)/(a*sech(x)**4)**(1/2),x)
```

output
```
Integral(x**2*csch(x)*sech(x)/sqrt(a*sech(x)**4), x)
```

3.846.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{x^3}{3\sqrt{a}} + \frac{x^2\log(e^x+1) + 2\,x\text{Li}_2(-e^x) - 2\,\text{Li}_3(-e^x)}{\sqrt{a}} + \frac{x^2\log(-e^x+1) + 2\,x\text{Li}_2(e^x) - 2\,\text{Li}_3(e^x)}{\sqrt{a}}$$

input
```
integrate(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*x^3/sqrt(a) + (x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*polylog(3, -e^x
))/sqrt(a) + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(3, e^x))/sqrt
(a)
```

3.846.8 Giac [F]

$$\int \frac{x^2 \text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x^2 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^4}}\,dx$$

input
```
integrate(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*csch(x)*sech(x)/sqrt(a*sech(x)^4), x)
```

3.846.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x^2}{\cosh(x)\,\sinh(x)\,\sqrt{\frac{a}{\cosh(x)^4}}}\,dx$$

input
```
int(x^2/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)),x)
```

output
```
int(x^2/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)), x)
```

3.847 $\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx$

3.847.1 Optimal result

Integrand size = 18, antiderivative size = 129

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}+\frac{x^3\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}+\frac{3x^2\,\text{PolyLog}\left(2,e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}-\frac{3x\,\text{PolyLog}\left(3,e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}+\frac{3\,\text{PolyLog}\left(4,e^{2x}\right)\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}$$

output
```
-1/4*x^4*sech(x)^2/(a*sech(x)^4)^(1/2)+x^3*ln(1-exp(2*x))*sech(x)^2/(a*sec
h(x)^4)^(1/2)+3/2*x^2*polylog(2,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)-3/
2*x*polylog(3,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)+3/4*polylog(4,exp(2*
x))*sech(x)^2/(a*sech(x)^4)^(1/2)
```

3.847.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.53

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{\left(x^4-4x^3\log\left(1-e^{2x}\right)-6x^2\,\text{PolyLog}\left(2,e^{2x}\right)+6x\,\text{PolyLog}\left(3,e^{2x}\right)-3\,\text{PolyLog}\left(4,e^{2x}\right)\right)\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}$$

input
```
Integrate[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

output
```
-1/4*((x^4 - 4*x^3*Log[1 - E^(2*x)] - 6*x^2*PolyLog[2, E^(2*x)] + 6*x*Poly
Log[3, E^(2*x)] - 3*PolyLog[4, E^(2*x)])*Sech[x]^2)/Sqrt[a*Sech[x]^4]
```

3.847.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.71, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {7271, 3042, 26, 4199, 25, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\text{sech}^2(x)\int x^3\coth(x)dx}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}^2(x)\int -ix^3\tan\left(ix+\frac{\pi}{2}\right)dx}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\text{sech}^2(x)\int x^3\tan\left(ix+\frac{\pi}{2}\right)dx}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{4199}$$

$$-\frac{i\text{sech}^2(x)\left(2i\int -\frac{e^{2x}x^3}{1-e^{2x}}dx-\frac{ix^4}{4}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\int\frac{e^{2x}x^3}{1-e^{2x}}dx-\frac{ix^4}{4}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\left(\frac{3}{2}\int x^2\log\left(1-e^{2x}\right)dx-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)-\frac{ix^4}{4}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\left(\frac{3}{2}\left(\int x\,\text{PolyLog}\left(2,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)-\frac{ix^4}{4}\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{7163}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\left(\frac{3}{2}\left(-\frac{1}{2}\int\text{PolyLog}\left(3,e^{2x}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)+\frac{1}{2}x\,\text{PolyLog}\left(3,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)\right.}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\left(\frac{3}{2}\left(-\frac{1}{4}\int e^{-2x}\text{PolyLog}\left(3,e^{2x}\right)de^{2x}-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)+\frac{1}{2}x\,\text{PolyLog}\left(3,e^{2x}\right)\right)-\frac{1}{2}x^3\log\left(1-\right.\right.\right.}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{i\text{sech}^2(x)\left(-2i\left(\frac{3}{2}\left(-\frac{1}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)+\frac{1}{2}x\,\text{PolyLog}\left(3,e^{2x}\right)-\frac{\text{PolyLog}(4,e^{2x})}{4}\right)-\frac{1}{2}x^3\log\left(1-e^{2x}\right)\right)-\frac{ix^4}{4}\right)}{\sqrt{a\text{sech}^4(x)}}$$

input
```
Int[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

output
```
((-I)*((-1/4*I)*x^4 - (2*I)*(-1/2*(x^3*Log[1 - E^(2*x)]) + (3*(-1/2*(x^2*P
olyLog[2, E^(2*x)]) + (x*PolyLog[3, E^(2*x)])/2 - PolyLog[4, E^(2*x)]/4))/
2))*Sech[x]^2)/Sqrt[a*Sech[x]^4]
```

3.847. $\int\frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx$

3.847.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

3.847. $\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

3.847.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(107) = 214$.

Time = 0.11 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.55

method	result
risch	$-\frac{e^{2x}x^4}{4\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{e^{2x}x^3\ln(1+e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{3\,e^{2x}x^2\,\mathrm{polylog}(2,-e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}-\frac{6\,e^{2x}x\,\mathrm{polylog}(3,-e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{6\,e^{2x}\,\mathrm{polylog}(4,-e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}$

input
```
int(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^4+1/(a*ex
p(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^3*ln(1+exp(x))+3/(a
*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^2*polylog(2,-exp
(x))-6/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*polylog
(3,-exp(x))+6/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*po
lylog(4,-exp(x))+1/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*
x)*x^3*ln(1-exp(x))+3/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp
(2*x)*x^2*polylog(2,exp(x))-6/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x
))^2*exp(2*x)*x*polylog(3,exp(x))+6/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+e
xp(2*x))^2*exp(2*x)*polylog(4,exp(x))
```

3.847.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(106) = 212$.

Time = 0.28 (sec) , antiderivative size = 427, normalized size of antiderivative = 3.31

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{\left(24\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x)}+2e^{(2x)}+1\right)e^{(2x)}\text{polylog}(4,\cosh(x)+\sinh(x))+24\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}}}\right.}{}$$

input
```
integrate(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(24*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x)
 + 2*e^(2*x) + 1)*e^(2*x)*polylog(4, cosh(x) + sinh(x)) + 24*sqrt(a/(e^(8*
x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*e^(2*x) + 1)*e^(
2*x)*polylog(4, -cosh(x) - sinh(x)) - 24*(x*e^(4*x) + 2*x*e^(2*x) + x)*sqr
t(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3,
cosh(x) + sinh(x)) - 24*(x*e^(4*x) + 2*x*e^(2*x) + x)*sqrt(a/(e^(8*x) + 4*
e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -cosh(x) - sinh(x
)) - (x^4*e^(4*x) + 2*x^4*e^(2*x) + x^4 - 12*(x^2*e^(4*x) + 2*x^2*e^(2*x)
+ x^2)*dilog(cosh(x) + sinh(x)) - 12*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*d
ilog(-cosh(x) - sinh(x)) - 4*(x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3)*log(cosh(
x) + sinh(x) + 1) - 4*(x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3)*log(-cosh(x) - s
inh(x) + 1))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(
2*x))*e^(-2*x)/a
```

3.847.6 Sympy [F]

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x^3\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx$$

input
```
integrate(x**3*csch(x)*sech(x)/(a*sech(x)**4)**(1/2),x)
```

output
```
Integral(x**3*csch(x)*sech(x)/sqrt(a*sech(x)**4), x)
```

3.847.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.67

$$\int \frac{x^3\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx = -\frac{x^4}{4\sqrt{a}} + \frac{x^3\log\left(e^x+1\right)+3\,x^2\operatorname{Li}_2(-e^x)-6\,x\operatorname{Li}_3(-e^x)+6\operatorname{Li}_4(-e^x)}{\sqrt{a}} + \frac{x^3\log\left(-e^x+1\right)+3\,x^2\operatorname{Li}_2(e^x)-6\,x\operatorname{Li}_3(e^x)+6\operatorname{Li}_4(e^x)}{\sqrt{a}}$$

input
```
integrate(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*x^4/sqrt(a) + (x^3*log(e^x + 1) + 3*x^2*dilog(-e^x) - 6*x*polylog(3,
-e^x) + 6*polylog(4, -e^x))/sqrt(a) + (x^3*log(-e^x + 1) + 3*x^2*dilog(e^x
) - 6*x*polylog(3, e^x) + 6*polylog(4, e^x))/sqrt(a)
```

3.847.8 Giac [F]

$$\int \frac{x^3\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx = \int \frac{x^3\operatorname{csch}\left(x\right)\operatorname{sech}\left(x\right)}{\sqrt{a\operatorname{sech}\left(x\right)^4}}\,dx$$

input
```
integrate(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3*csch(x)*sech(x)/sqrt(a*sech(x)^4), x)
```

3.847.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx = \int \frac{x^3}{\cosh\left(x\right)\,\sinh\left(x\right)\,\sqrt{\frac{a}{{\cosh\left(x\right)}^4}}}\,dx$$

input
```
int(x^3/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)),x)
```

output
```
int(x^3/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)), x)
```

3.848 $\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx$

3.848.1 Optimal result

Integrand size = 16, antiderivative size = 88

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = x\sqrt{a\text{sech}^2(x)} - \arctan(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - \cosh(x)\,\text{PolyLog}\left(2,-e^x\right)\sqrt{a\text{sech}^2(x)} + \cosh(x)\,\text{PolyLog}\left(2,e^x\right)\sqrt{a\text{sech}^2(x)}$$

output
```
x*(a*sech(x)^2)^(1/2)-arctan(sinh(x))*cosh(x)*(a*sech(x)^2)^(1/2)-2*x*arct
anh(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-cosh(x)*polylog(2,-exp(x))*(a*sech
(x)^2)^(1/2)+cosh(x)*polylog(2,exp(x))*(a*sech(x)^2)^(1/2)
```

3.848.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \left(x - 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\cosh(x) + x\cosh(x)\log\left(1-e^x\right) - x\cosh(x)\log\left(1+e^x\right) - \cosh(x)\,\text{PolyLog}\left(2,-e^x\right) + \cosh(x)\,\text{PolyLog}\left(2,e^x\right)\right)\sqrt{a\text{sech}^2(x)}$$

input
```
Integrate[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

output
```
(x - 2*ArcTan[Tanh[x/2]]*Cosh[x] + x*Cosh[x]*Log[1 - E^x] - x*Cosh[x]*Log[
1 + E^x] - Cosh[x]*PolyLog[2, -E^x] + Cosh[x]*PolyLog[2, E^x])*Sqrt[a*Sech
[x]^2]
```

3.848.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.50, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {7271, 5985, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\int x\text{csch}(x)\text{sech}^2(x)dx$$

$$\Big\downarrow \text{5985}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\left(-\int(\text{sech}(x)-\text{arctanh}(\cosh(x)))dx-x\text{arctanh}(\cosh(x))+x\text{sech}(x)\right)$$

$$\Big\downarrow \text{2009}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\left(-\arctan(\sinh(x))-2x\text{arctanh}(e^x)-\text{PolyLog}(2,-e^x)+\text{PolyLog}(2,e^x)+x\text{sech}(x)\right)$$

input
```
Int[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

output
```
Cosh[x]*Sqrt[a*Sech[x]^2]*(-ArcTan[Sinh[x]] - 2*x*ArcTanh[E^x] - PolyLog[2
, -E^x] + PolyLog[2, E^x] + x*Sech[x])
```

3.848.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.848.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.70

method	result
risch	$2\sqrt{\frac{a\,\mathrm{e}^{2x}}{(1+\mathrm{e}^{2x})^2}}\,x-2\sqrt{\frac{a\,\mathrm{e}^{2x}}{(1+\mathrm{e}^{2x})^2}}\,\mathrm{e}^{-x}(1+\mathrm{e}^{2x})\arctan\left(\mathrm{e}^{x}\right)-\sqrt{\frac{a\,\mathrm{e}^{2x}}{(1+\mathrm{e}^{2x})^2}}\,\mathrm{e}^{-x}(1+\mathrm{e}^{2x})\,\mathrm{dilog}\left(1+\mathrm{e}^{x}\right)-\sqrt{\cdots}$

input
```
int(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*x-2*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*
exp(-x)*(1+exp(2*x))*arctan(exp(x))-(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(
-x)*(1+exp(2*x))*dilog(1+exp(x))-(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(-x)
*(1+exp(2*x))*x*ln(1+exp(x))-(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(-x)*(1+
exp(2*x))*dilog(exp(x))
```

3.848.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(73) = 146.

Time = 0.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.99

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx$$
$$= \frac{\left(2\,x\cosh\left(x\right)e^{\left(2\,x\right)} - 2\left(\left(e^{\left(2\,x\right)}+1\right)\sinh\left(x\right)^{2}+\cosh\left(x\right)^{2}+\left(\cosh\left(x\right)^{2}+1\right)e^{\left(2\,x\right)}+2\left(\cosh\left(x\right)e^{\left(2\,x\right)}+\cosh\left(\right.\right.\right.}{}$$

input
```
integrate(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
(2*x*cosh(x)*e^(2*x) - 2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2
 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*arctan(cosh(x)
+ sinh(x)) + 2*x*cosh(x) + ((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)
^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(cosh(x)
 + sinh(x)) - ((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*
x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(-cosh(x) - sinh(x))
- (x*cosh(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2
*(x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x) + x)*log(cosh(x) + sinh(x) + 1) +
 (x*cosh(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*
(x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x) + x)*log(-cosh(x) - sinh(x) + 1) +
 2*(x*e^(2*x) + x)*sinh(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(2*cosh(
x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)
```

3.848.6 Sympy [F]

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \int x\sqrt{a\,\text{sech}^{2}\left(x\right)}\,\text{csch}\left(x\right)\text{sech}\left(x\right)\,dx$$

input
```
integrate(x*csch(x)*sech(x)*(a*sech(x)**2)**(1/2),x)
```

output
```
Integral(x*sqrt(a*sech(x)**2)*csch(x)*sech(x), x)
```

3.848.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

$$\int x\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^2(x)}\,dx = -(x\log(e^x+1) + \mathrm{Li}_2(-e^x))\sqrt{a} + (x\log(-e^x+1) + \mathrm{Li}_2(e^x))\sqrt{a} - 2\sqrt{a}\arctan(e^x) + \frac{2\sqrt{a}xe^x}{e^{(2x)}+1}$$

input
```
integrate(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(x*log(e^x + 1) + dilog(-e^x))*sqrt(a) + (x*log(-e^x + 1) + dilog(e^x))*s
qrt(a) - 2*sqrt(a)*arctan(e^x) + 2*sqrt(a)*x*e^x/(e^(2*x) + 1)
```

3.848.8 Giac [F]

$$\int x\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^2(x)}\,dx = \int \sqrt{a\,\mathrm{sech}(x)^2}x\,\mathrm{csch}(x)\,\mathrm{sech}(x)\,dx$$

input
```
integrate(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sech(x)^2)*x*csch(x)*sech(x), x)
```

3.848.9 Mupad [F(-1)]

Timed out.

$$\int x\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^2(x)}\,dx = \int \frac{x\sqrt{\frac{a}{\cosh(x)^2}}}{\cosh(x)\,\sinh(x)}\,dx$$

input
```
int((x*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)),x)
```

output
```
int((x*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)), x)
```

3.849 $\int x^2\operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx$

3.849.1 Optimal result

Integrand size = 18, antiderivative size = 187

$$\begin{aligned}\int x^2\operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx &= x^2\sqrt{a\operatorname{sech}^2(x)}-4x\arctan\left(e^x\right)\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\\ &\quad-2x^2\operatorname{arctanh}(e^x)\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\\ &\quad-2x\cosh(x)\operatorname{PolyLog}\left(2,-e^x\right)\sqrt{a\operatorname{sech}^2(x)}\\ &\quad+2i\cosh(x)\operatorname{PolyLog}\left(2,-ie^x\right)\sqrt{a\operatorname{sech}^2(x)}\\ &\quad-2i\cosh(x)\operatorname{PolyLog}\left(2,ie^x\right)\sqrt{a\operatorname{sech}^2(x)}\\ &\quad+2x\cosh(x)\operatorname{PolyLog}\left(2,e^x\right)\sqrt{a\operatorname{sech}^2(x)}\\ &\quad+2\cosh(x)\operatorname{PolyLog}\left(3,-e^x\right)\sqrt{a\operatorname{sech}^2(x)}\\ &\quad-2\cosh(x)\operatorname{PolyLog}\left(3,e^x\right)\sqrt{a\operatorname{sech}^2(x)}\end{aligned}$$

output
```
x^2*(a*sech(x)^2)^(1/2)-4*x*arctan(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-2*x
^2*arctanh(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-2*x*cosh(x)*polylog(2,-exp(
x))*(a*sech(x)^2)^(1/2)+2*I*cosh(x)*polylog(2,-I*exp(x))*(a*sech(x)^2)^(1/
2)-2*I*cosh(x)*polylog(2,I*exp(x))*(a*sech(x)^2)^(1/2)+2*x*cosh(x)*polylog
(2,exp(x))*(a*sech(x)^2)^(1/2)+2*cosh(x)*polylog(3,-exp(x))*(a*sech(x)^2)^
(1/2)-2*cosh(x)*polylog(3,exp(x))*(a*sech(x)^2)^(1/2)
```

3.849.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.68

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \left(x^2-2i\cosh(x)\left(x(\log\left(1-ie^x\right)-\log\left(1+ie^x\right)\right)\right.$$
$$-\text{PolyLog}\left(2,-ie^x\right)+\text{PolyLog}\left(2,ie^x\right))$$
$$+\cosh(x)\left(x^2\log\left(1-e^x\right)-x^2\log\left(1+e^x\right)\right.$$
$$-2x\,\text{PolyLog}\left(2,-e^x\right)+2x\,\text{PolyLog}\left(2,e^x\right)$$
$$\left.\left.+2\,\text{PolyLog}\left(3,-e^x\right)-2\,\text{PolyLog}\left(3,e^x\right)\right)\right)\sqrt{a\text{sech}^2(x)}$$

input
```
Integrate[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

output
```
(x^2 - (2*I)*Cosh[x]*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I
)*E^x] + PolyLog[2, I*E^x]) + Cosh[x]*(x^2*Log[1 - E^x] - x^2*Log[1 + E^x]
 - 2*x*PolyLog[2, -E^x] + 2*x*PolyLog[2, E^x] + 2*PolyLog[3, -E^x] - 2*Pol
yLog[3, E^x]))*Sqrt[a*Sech[x]^2]
```

3.849.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.61, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {7271, 5985, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\int x^2\text{csch}(x)\text{sech}^2(x)dx$$

$$\Big\downarrow \text{5985}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\left(-2\int -x(\text{arctanh}(\cosh(x))-\text{sech}(x))dx+x^2(-\text{arctanh}(\cosh(x)))+x^2\text{sech}(x)\right)$$

$$\Big\downarrow \text{25}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\left(2\int x(\text{arctanh}(\cosh(x))-\text{sech}(x))dx+x^2(-\text{arctanh}(\cosh(x)))+x^2\text{sech}(x)\right)$$

$$\Big\downarrow \text{2010}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\left(2\int (x\text{arctanh}(\cosh(x))-x\text{sech}(x))dx+x^2(-\text{arctanh}(\cosh(x)))+x^2\text{sech}(x)\right)$$

$$\Big\downarrow \text{2009}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\left(2\left(-2x\arctan\left(e^x\right)+x^2(-\text{arctanh}(e^x))+\frac{1}{2}x^2\text{arctanh}(\cosh(x))-x\,\text{PolyLog}\left(2,-e^x\right)+x\,\text{Po}\right.\right.$$

input
```
Int[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

output
```
Cosh[x]*Sqrt[a*Sech[x]^2]*(-(x^2*ArcTanh[Cosh[x]]) + 2*(-2*x*ArcTan[E^x] -
 x^2*ArcTanh[E^x] + (x^2*ArcTanh[Cosh[x]])/2 - x*PolyLog[2, -E^x] + I*Poly
Log[2, (-I)*E^x] - I*PolyLog[2, I*E^x] + x*PolyLog[2, E^x] + PolyLog[3, -E
^x] - PolyLog[3, E^x]) + x^2*Sech[x])
```

3.849.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.849. $\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx$

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.849.4 Maple [F]

$$\int x^2 \operatorname{csch}(x) \operatorname{sech}(x) \sqrt{a \operatorname{sech}(x)^2} dx$$

input
```
int(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

output
```
int(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

3.849.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 786 vs. 2(149) = 298.

Time = 0.28 (sec) , antiderivative size = 786, normalized size of antiderivative = 4.20

$$\int x^2 \operatorname{csch}(x) \operatorname{sech}(x) \sqrt{a \operatorname{sech}^2(x)} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(co
sh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^
x*polylog(3, cosh(x) + sinh(x)) - 2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 +
 (cosh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt
(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(3, -cosh(x) - sinh(x)) - (2*x^2*
cosh(x)*e^(2*x) + 2*x^2*cosh(x) + 2*(x*cosh(x)^2 + (x*e^(2*x) + x)*sinh(x)
^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)
 + x)*dilog(cosh(x) + sinh(x)) - 2*((I*e^(2*x) + I)*sinh(x)^2 + I*cosh(x)^
2 + (I*cosh(x)^2 + I)*e^(2*x) + 2*(I*cosh(x)*e^(2*x) + I*cosh(x))*sinh(x)
+ I)*dilog(I*cosh(x) + I*sinh(x)) - 2*((-I*e^(2*x) - I)*sinh(x)^2 - I*cosh
(x)^2 + (-I*cosh(x)^2 - I)*e^(2*x) + 2*(-I*cosh(x)*e^(2*x) - I*cosh(x))*si
nh(x) - I)*dilog(-I*cosh(x) - I*sinh(x)) - 2*(x*cosh(x)^2 + (x*e^(2*x) + x
)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^(2*x) + x*cosh(x)
)*sinh(x) + x)*dilog(-cosh(x) - sinh(x)) - (x^2*cosh(x)^2 + (x^2*e^(2*x) +
 x^2)*sinh(x)^2 + x^2 + (x^2*cosh(x)^2 + x^2)*e^(2*x) + 2*(x^2*cosh(x)*e^(
2*x) + x^2*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 2*(-I*x*cosh(x)^
2 + (-I*x*e^(2*x) - I*x)*sinh(x)^2 + (-I*x*cosh(x)^2 - I*x)*e^(2*x) + 2*(-
I*x*cosh(x)*e^(2*x) - I*x*cosh(x))*sinh(x) - I*x)*log(I*cosh(x) + I*sinh(x
) + 1) - 2*(I*x*cosh(x)^2 + (I*x*e^(2*x) + I*x)*sinh(x)^2 + (I*x*cosh(x)^2
 + I*x)*e^(2*x) + 2*(I*x*cosh(x)*e^(2*x) + I*x*cosh(x))*sinh(x) + I*x)*...
```

3.849.6 Sympy [F]

$$\int x^2 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx = \int x^2\sqrt{a\operatorname{sech}^2(x)}\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

input
```
integrate(x**2*csch(x)*sech(x)*(a*sech(x)**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*sech(x)**2)*csch(x)*sech(x), x)
```

3.849.7 Maxima [F]

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx=\int \sqrt{a\,\text{sech}\left(x\right)^2}x^2\,\text{csch}\left(x\right)\text{sech}\left(x\right)\,dx$$

input
```
integrate(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(a)*x^2*e^x/(e^(2*x) + 1) - (x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*
polylog(3, -e^x))*sqrt(a) + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylo
g(3, e^x))*sqrt(a) - 4*sqrt(a)*integrate(x*e^x/(e^(2*x) + 1), x)
```

3.849.8 Giac [F]

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx=\int \sqrt{a\,\text{sech}\left(x\right)^2}x^2\,\text{csch}\left(x\right)\text{sech}\left(x\right)\,dx$$

input
```
integrate(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sech(x)^2)*x^2*csch(x)*sech(x), x)
```

3.849.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx=\int \frac{x^2\,\sqrt{\frac{a}{{\mathrm{cosh}\left(x\right)}^2}}}{\mathrm{cosh}\left(x\right)\,\mathrm{sinh}\left(x\right)}\,dx$$

input
```
int((x^2*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)),x)
```

output
```
int((x^2*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)), x)
```

3.850 $\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx$

3.850.1 Optimal result

Integrand size = 18, antiderivative size = 287

$$\begin{aligned}\int x^3\operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx &= x^3\sqrt{a\operatorname{sech}^2(x)} - 6x^2\arctan\left(e^x\right)\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\\
&\quad - 2x^3\operatorname{arctanh}(e^x)\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\\
&\quad - 3x^2\cosh(x)\operatorname{PolyLog}\left(2,-e^x\right)\sqrt{a\operatorname{sech}^2(x)}\\
&\quad + 6ix\cosh(x)\operatorname{PolyLog}\left(2,-ie^x\right)\sqrt{a\operatorname{sech}^2(x)}\\
&\quad - 6ix\cosh(x)\operatorname{PolyLog}\left(2,ie^x\right)\sqrt{a\operatorname{sech}^2(x)}\\
&\quad + 3x^2\cosh(x)\operatorname{PolyLog}\left(2,e^x\right)\sqrt{a\operatorname{sech}^2(x)}\\
&\quad + 6x\cosh(x)\operatorname{PolyLog}\left(3,-e^x\right)\sqrt{a\operatorname{sech}^2(x)}\\
&\quad - 6i\cosh(x)\operatorname{PolyLog}\left(3,-ie^x\right)\sqrt{a\operatorname{sech}^2(x)}\\
&\quad + 6i\cosh(x)\operatorname{PolyLog}\left(3,ie^x\right)\sqrt{a\operatorname{sech}^2(x)}\\
&\quad - 6x\cosh(x)\operatorname{PolyLog}\left(3,e^x\right)\sqrt{a\operatorname{sech}^2(x)}\\
&\quad - 6\cosh(x)\operatorname{PolyLog}\left(4,-e^x\right)\sqrt{a\operatorname{sech}^2(x)}\\
&\quad + 6\cosh(x)\operatorname{PolyLog}\left(4,e^x\right)\sqrt{a\operatorname{sech}^2(x)}\end{aligned}$$

output
```
x^3*(a*sech(x)^2)^(1/2)-6*x^2*arctan(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-2
*x^3*arctanh(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-3*x^2*cosh(x)*polylog(2,-
exp(x))*(a*sech(x)^2)^(1/2)+6*I*x*cosh(x)*polylog(2,-I*exp(x))*(a*sech(x)^
2)^(1/2)-6*I*x*cosh(x)*polylog(2,I*exp(x))*(a*sech(x)^2)^(1/2)+3*x^2*cosh(
x)*polylog(2,exp(x))*(a*sech(x)^2)^(1/2)+6*x*cosh(x)*polylog(3,-exp(x))*(a
*sech(x)^2)^(1/2)-6*I*cosh(x)*polylog(3,-I*exp(x))*(a*sech(x)^2)^(1/2)+6*I
*cosh(x)*polylog(3,I*exp(x))*(a*sech(x)^2)^(1/2)-6*x*cosh(x)*polylog(3,exp
(x))*(a*sech(x)^2)^(1/2)-6*cosh(x)*polylog(4,-exp(x))*(a*sech(x)^2)^(1/2)+
6*cosh(x)*polylog(4,exp(x))*(a*sech(x)^2)^(1/2)
```

3.850.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.63

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \left(x^3-3i\cosh(x)\left(x^2\log\left(1-ie^x\right)-x^2\log\left(1+ie^x\right)-2x\,\text{PolyLog}\left(2,-ie^x\right)+2x\,\text{PolyLog}\left(2,ie^x\right)+2\,\text{PolyLog}\left(3,-ie^x\right)-2\,\text{PolyLog}\left(3,ie^x\right)\right)+\cosh(x)\left(x^3\log\left(1-e^x\right)-x^3\log\left(1+e^x\right)-3x^2\,\text{PolyLog}\left(2,-e^x\right)+3x^2\,\text{PolyLog}\left(2,e^x\right)+6x\,\text{PolyLog}\left(3,-e^x\right)-6x\,\text{PolyLog}\left(3,e^x\right)-6\,\text{PolyLog}\left(4,-e^x\right)+6\,\text{PolyLog}\left(4,e^x\right)\right)\right)\sqrt{a\text{sech}^2(x)}$$

input
```
Integrate[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

output
```
(x^3 - (3*I)*Cosh[x]*(x^2*Log[1 - I*E^x] - x^2*Log[1 + I*E^x] - 2*x*PolyLo
g[2, (-I)*E^x] + 2*x*PolyLog[2, I*E^x] + 2*PolyLog[3, (-I)*E^x] - 2*PolyLo
g[3, I*E^x]) + Cosh[x]*(x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLo
g[2, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3,
 E^x] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x]))*Sqrt[a*Sech[x]^2]
```

3.850.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.59, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {7271, 5985, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\int x^3\text{csch}(x)\text{sech}^2(x)dx$$

$$\Big\downarrow \text{5985}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\left(-3\int -x^2(\text{arctanh}(\cosh(x))-\text{sech}(x))dx+x^3(-\text{arctanh}(\cosh(x)))+x^3\text{sech}(x)\right)$$

$$\Big\downarrow \text{25}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\left(3\int x^2(\text{arctanh}(\cosh(x))-\text{sech}(x))dx+x^3(-\text{arctanh}(\cosh(x)))+x^3\text{sech}(x)\right)$$

$$\Big\downarrow \text{2010}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\left(3\int \left(x^2\text{arctanh}(\cosh(x))-x^2\text{sech}(x)\right)dx+x^3(-\text{arctanh}(\cosh(x)))+x^3\text{sech}(x)\right)$$

$$\Big\downarrow \text{2009}$$

$$\cosh(x)\sqrt{a\text{sech}^2(x)}\left(3\left(-2x^2\arctan\left(e^x\right)-\frac{2}{3}x^3\text{arctanh}(e^x)+\frac{1}{3}x^3\text{arctanh}(\cosh(x))-x^2\,\text{PolyLog}\left(2,-e^x\right)+x^2\,\text{P}\right.\right.$$

input
```
Int[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

output
```
Cosh[x]*Sqrt[a*Sech[x]^2]*(-(x^3*ArcTanh[Cosh[x]]) + 3*(-2*x^2*ArcTan[E^x]
 - (2*x^3*ArcTanh[E^x])/3 + (x^3*ArcTanh[Cosh[x]])/3 - x^2*PolyLog[2, -E^x
] + (2*I)*x*PolyLog[2, (-I)*E^x] - (2*I)*x*PolyLog[2, I*E^x] + x^2*PolyLog
[2, E^x] + 2*x*PolyLog[3, -E^x] - (2*I)*PolyLog[3, (-I)*E^x] + (2*I)*PolyL
og[3, I*E^x] - 2*x*PolyLog[3, E^x] - 2*PolyLog[4, -E^x] + 2*PolyLog[4, E^x
]) + x^3*Sech[x])
```

3.850.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.850.4 Maple [F]

$$\int x^3 \operatorname{csch}(x) \operatorname{sech}(x) \sqrt{a \operatorname{sech}(x)^2} dx$$

input
```
int(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

output
```
int(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

3.850.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1202 vs. 2(229) = 458.

Time = 0.29 (sec) , antiderivative size = 1202, normalized size of antiderivative = 4.19

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
(6*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cos
h(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x
*polylog(4, cosh(x) + sinh(x)) - 6*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 +
(cosh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(
a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(4, -cosh(x) - sinh(x)) - 6*(x*cos
h(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh
(x)*e^(2*x) + x*cosh(x))*sinh(x) + x)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^
x*polylog(3, cosh(x) + sinh(x)) - 6*((-I*e^(2*x) - I)*sinh(x)^2 - I*cosh(x
)^2 + (-I*cosh(x)^2 - I)*e^(2*x) + 2*(-I*cosh(x)*e^(2*x) - I*cosh(x))*sinh
(x) - I)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(3, I*cosh(x) + I*si
nh(x)) - 6*((I*e^(2*x) + I)*sinh(x)^2 + I*cosh(x)^2 + (I*cosh(x)^2 + I)*e^
(2*x) + 2*(I*cosh(x)*e^(2*x) + I*cosh(x))*sinh(x) + I)*sqrt(a/(e^(4*x) + 2
*e^(2*x) + 1))*e^x*polylog(3, -I*cosh(x) - I*sinh(x)) + 6*(x*cosh(x)^2 + (
x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^(2*x
) + x*cosh(x))*sinh(x) + x)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(
3, -cosh(x) - sinh(x)) + (2*x^3*cosh(x)*e^(2*x) + 2*x^3*cosh(x) + 3*(x^2*c
osh(x)^2 + (x^2*e^(2*x) + x^2)*sinh(x)^2 + x^2 + (x^2*cosh(x)^2 + x^2)*e^(
2*x) + 2*(x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x))*dilog(cosh(x) + sinh
(x)) - 6*(I*x*cosh(x)^2 + (I*x*e^(2*x) + I*x)*sinh(x)^2 + (I*x*cosh(x)^2 +
 I*x)*e^(2*x) + 2*(I*x*cosh(x)*e^(2*x) + I*x*cosh(x))*sinh(x) + I*x)*di...
```

3.850.6 Sympy [F]

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \int x^3\sqrt{a\,\text{sech}^2(x)}\,\text{csch}(x)\,\text{sech}(x)\,dx$$

input
```
integrate(x**3*csch(x)*sech(x)*(a*sech(x)**2)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*sech(x)**2)*csch(x)*sech(x), x)
```

3.850.7 Maxima [F]

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \int \sqrt{a\,\text{sech}\,(x)^2}x^3\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

input
```
integrate(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(a)*x^3*e^x/(e^(2*x) + 1) - (x^3*log(e^x + 1) + 3*x^2*dilog(-e^x) -
6*x*polylog(3, -e^x) + 6*polylog(4, -e^x))*sqrt(a) + (x^3*log(-e^x + 1) +
3*x^2*dilog(e^x) - 6*x*polylog(3, e^x) + 6*polylog(4, e^x))*sqrt(a) - 12*s
qrt(a)*integrate(1/2*x^2*e^x/(e^(2*x) + 1), x)
```

3.850.8 Giac [F]

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \int \sqrt{a\,\text{sech}\,(x)^2}x^3\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

input
```
integrate(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sech(x)^2)*x^3*csch(x)*sech(x), x)
```

3.850.9 Mupad [F(-1)]

Timed out.

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \int \frac{x^3\,\sqrt{\frac{a}{\cosh(x)^2}}}{\cosh\,(x)\,\sinh\,(x)}\,dx$$

input
```
int((x^3*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)),x)
```

output
```
int((x^3*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)), x)
```

3.851 $\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx$

3.851.1 Optimal result

Integrand size = 16, antiderivative size = 132

$$\begin{aligned}\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx =& \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\ &- 2x\text{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\ &- \frac{1}{2}\cosh^2(x)\,\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}\\ &+ \frac{1}{2}\cosh^2(x)\,\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)}\\ &- \frac{1}{2}\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x)\\ &- \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2(x)\end{aligned}$$

output
```
1/2*x*cosh(x)^2*(a*sech(x)^4)^(1/2)-2*x*arctanh(exp(2*x))*cosh(x)^2*(a*sec
h(x)^4)^(1/2)-1/2*cosh(x)^2*polylog(2,-exp(2*x))*(a*sech(x)^4)^(1/2)+1/2*c
osh(x)^2*polylog(2,exp(2*x))*(a*sech(x)^4)^(1/2)-1/2*cosh(x)*sinh(x)*(a*se
ch(x)^4)^(1/2)-1/2*x*sinh(x)^2*(a*sech(x)^4)^(1/2)
```

3.851.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.54

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \frac{1}{2}\cosh^2(x)\sqrt{a\text{sech}^4(x)}\big(2x\log\left(1-e^{-2x}\right) - 2x\log\left(1+e^{-2x}\right)+\text{PolyLog}\left(2,-e^{-2x}\right) - \text{PolyLog}\left(2,e^{-2x}\right)+x\text{sech}^2(x)-\tanh(x)\big)$$

input
```
Integrate[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

output
```
(Cosh[x]^2*Sqrt[a*Sech[x]^4]*(2*x*Log[1 - E^(-2*x)] - 2*x*Log[1 + E^(-2*x)
] + PolyLog[2, -E^(-2*x)] - PolyLog[2, E^(-2*x)] + x*Sech[x]^2 - Tanh[x]))
/2
```

3.851.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {7271, 5985, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cosh^2(x)\sqrt{a\text{sech}^4(x)}\int x\text{csch}(x)\text{sech}^3(x)dx$$

$$\Big\downarrow \text{5985}$$

$$\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(-\int\left(\log(\tanh(x))-\frac{\tanh^2(x)}{2}\right)dx-\frac{1}{2}x\tanh^2(x)+x\log(\tanh(x))\right)$$

$$\Big\downarrow \text{2009}$$

$$\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(-2x\text{arctanh}\left(e^{2x}\right)-\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)+\frac{\text{PolyLog}\left(2,e^{2x}\right)}{2}+\frac{x}{2}-\frac{1}{2}x\tanh^2(x)-\frac{\tanh(x)}{2}\right.$$

input
```
Int[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

output
```
Cosh[x]^2*Sqrt[a*Sech[x]^4]*(x/2 - 2*x*ArcTanh[E^(2*x)] - PolyLog[2, -E^(2
*x)]/2 + PolyLog[2, E^(2*x)]/2 - Tanh[x]/2 - (x*Tanh[x]^2)/2)
```

3.851.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.851.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(107) = 214$.

Time = 0.13 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.91

method	result
risch	$\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,e^{-2x}(2x\,e^{2x}+e^{2x}+1)+\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,e^{-2x}(1+e^{2x})^2\,x\ln(1+e^{x})+\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,e^{-2x}(1+e^{2}$

input
```
int(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(2*x*exp(2*x)+exp(2*x)+1)+(a*e
xp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*ln(1+exp(x))+(a*e
xp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(2,-exp(x))-
(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*ln(1+exp(2*x)
)-1/2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(2
,-exp(2*x))+(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*l
n(1-exp(x))+(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*pol
ylog(2,exp(x))
```

3.851.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1757, normalized size of antiderivative = 13.31

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
((2*x + 1)*cosh(x)^2 + ((2*x + 1)*e^(4*x) + 2*(2*x + 1)*e^(2*x) + 2*x + 1)
*sinh(x)^2 + ((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)
*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*co
sh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh
(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2
 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^
3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(cosh(x) + sinh(x)) - ((
e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*co
sh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^
(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x
)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) +
 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^
(2*x) + cosh(x))*sinh(x) + 1)*dilog(I*cosh(x) + I*sinh(x)) - ((e^(4*x) + 2
*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*
x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(
3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cos
h(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)
^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cos
h(x))*sinh(x) + 1)*dilog(-I*cosh(x) - I*sinh(x)) + ((e^(4*x) + 2*e^(2*x) +
 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + co...
```

3.851.6 Sympy [F]

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx=\int x\sqrt{a\,\text{sech}^4\,(x)}\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

input
```
integrate(x*csch(x)*sech(x)*(a*sech(x)**4)**(1/2),x)
```

output
```
Integral(x*sqrt(a*sech(x)**4)*csch(x)*sech(x), x)
```

3.851.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70

$$\begin{aligned}\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx=&-\frac{1}{2}\left(2\,x\log\left(e^{(2\,x)}+1\right)+\text{Li}_2\left(-e^{(2\,x)}\right)\right)\sqrt{a}\\&+\left(x\log\left(e^{x}+1\right)+\text{Li}_2(-e^{x})\right)\sqrt{a}\\&+\left(x\log\left(-e^{x}+1\right)+\text{Li}_2(e^{x})\right)\sqrt{a}\\&+\frac{\left(2\,\sqrt{a}x+\sqrt{a}\right)e^{(2\,x)}+\sqrt{a}}{e^{(4\,x)}+2\,e^{(2\,x)}+1}\end{aligned}$$

input
```
integrate(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*x*log(e^(2*x) + 1) + dilog(-e^(2*x)))*sqrt(a) + (x*log(e^x + 1) +
dilog(-e^x))*sqrt(a) + (x*log(-e^x + 1) + dilog(e^x))*sqrt(a) + ((2*sqrt(a
)*x + sqrt(a))*e^(2*x) + sqrt(a))/(e^(4*x) + 2*e^(2*x) + 1)
```

3.851.8 Giac [F]

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx=\int\sqrt{a\,\text{sech}\,(x)^4}x\,\text{csch}\,(x)\,\text{sech}\,(x)\;dx$$

input
```
integrate(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sech(x)^4)*x*csch(x)*sech(x), x)
```

3.851.9 Mupad [F(-1)]

Timed out.

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx=\int\frac{x\,\sqrt{\frac{a}{{\cosh\left(x\right)}^4}}}{\cosh\left(x\right)\,\sinh\left(x\right)}\,dx$$

input
```
int((x*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)),x)
```

output
```
int((x*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)), x)
```

3.852 $\int x^2\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^4(x)}\,dx$

3.852.1 Optimal result

Integrand size = 18, antiderivative size = 204

$$\begin{aligned}\int x^2\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^4(x)}\,dx =& \frac{1}{2}x^2\cosh^2(x)\sqrt{a\mathrm{sech}^4(x)}\\ &- 2x^2\mathrm{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\mathrm{sech}^4(x)}\\ &+ \cosh^2(x)\log(\cosh(x))\sqrt{a\mathrm{sech}^4(x)}\\ &- x\cosh^2(x)\,\mathrm{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\mathrm{sech}^4(x)}\\ &+ x\cosh^2(x)\,\mathrm{PolyLog}\left(2,e^{2x}\right)\sqrt{a\mathrm{sech}^4(x)}\\ &+ \frac{1}{2}\cosh^2(x)\,\mathrm{PolyLog}\left(3,-e^{2x}\right)\sqrt{a\mathrm{sech}^4(x)}\\ &- \frac{1}{2}\cosh^2(x)\,\mathrm{PolyLog}\left(3,e^{2x}\right)\sqrt{a\mathrm{sech}^4(x)}\\ &- x\cosh(x)\sqrt{a\mathrm{sech}^4(x)}\sinh(x)\\ &- \frac{1}{2}x^2\sqrt{a\mathrm{sech}^4(x)}\sinh^2(x)\end{aligned}$$

output
```
1/2*x^2*cosh(x)^2*(a*sech(x)^4)^(1/2)-2*x^2*arctanh(exp(2*x))*cosh(x)^2*(a
*sech(x)^4)^(1/2)+cosh(x)^2*ln(cosh(x))*(a*sech(x)^4)^(1/2)-x*cosh(x)^2*po
lylog(2,-exp(2*x))*(a*sech(x)^4)^(1/2)+x*cosh(x)^2*polylog(2,exp(2*x))*(a*
sech(x)^4)^(1/2)+1/2*cosh(x)^2*polylog(3,-exp(2*x))*(a*sech(x)^4)^(1/2)-1/
2*cosh(x)^2*polylog(3,exp(2*x))*(a*sech(x)^4)^(1/2)-x*cosh(x)*sinh(x)*(a*s
ech(x)^4)^(1/2)-1/2*x^2*sinh(x)^2*(a*sech(x)^4)^(1/2)
```

3.852.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.55

$$\int x^2 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx = \frac{1}{2}\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\left(-2x+2x^2\log\left(1-e^{-2x}\right) - 2x^2\log\left(1+e^{-2x}\right)+2\log\left(1+e^{2x}\right) + 2x\operatorname{PolyLog}\left(2,-e^{-2x}\right)-2x\operatorname{PolyLog}\left(2,e^{-2x}\right) + \operatorname{PolyLog}\left(3,-e^{-2x}\right)-\operatorname{PolyLog}\left(3,e^{-2x}\right) + x^2\operatorname{sech}^2(x)-2x\tanh(x)\right)$$

input
```
Integrate[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

output
```
(Cosh[x]^2*Sqrt[a*Sech[x]^4]*(-2*x + 2*x^2*Log[1 - E^(-2*x)] - 2*x^2*Log[1
 + E^(-2*x)] + 2*Log[1 + E^(2*x)] + 2*x*PolyLog[2, -E^(-2*x)] - 2*x*PolyLo
g[2, E^(-2*x)] + PolyLog[3, -E^(-2*x)] - PolyLog[3, E^(-2*x)] + x^2*Sech[x
]^2 - 2*x*Tanh[x]))/2
```

3.852.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.48, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {7271, 5985, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\int x^2\operatorname{csch}(x)\operatorname{sech}^3(x)dx$$

$$\Big\downarrow \text{5985}$$

$$\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\left(-2\int \frac{1}{2}x\left(2\log(\tanh(x))-\tanh^2(x)\right)dx-\frac{1}{2}x^2\tanh^2(x)+x^2\log(\tanh(x))\right)$$

$$\Big\downarrow \text{27}$$

$$\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\left(-\int x\left(2\log(\tanh(x))-\tanh^2(x)\right)dx-\frac{1}{2}x^2\tanh^2(x)+x^2\log(\tanh(x))\right)$$

$$\Big\downarrow \text{2010}$$

$$\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\left(-\int\left(2x\log(\tanh(x))-x\tanh^2(x)\right)dx-\frac{1}{2}x^2\tanh^2(x)+x^2\log(\tanh(x))\right)$$

$$\Big\downarrow \text{2009}$$

$$\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\left(-2x^2\operatorname{arctanh}\left(e^{2x}\right)-x\operatorname{PolyLog}\left(2,-e^{2x}\right)+x\operatorname{PolyLog}\left(2,e^{2x}\right)+\frac{1}{2}\operatorname{PolyLog}\left(3,-e^{2x}\right)-\frac{\text{Po}}{}\right.$$

input
```
Int[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

output
```
Cosh[x]^2*Sqrt[a*Sech[x]^4]*(x^2/2 - 2*x^2*ArcTanh[E^(2*x)] + Log[Cosh[x]]
 - x*PolyLog[2, -E^(2*x)] + x*PolyLog[2, E^(2*x)] + PolyLog[3, -E^(2*x)]/2
 - PolyLog[3, E^(2*x)]/2 - x*Tanh[x] - (x^2*Tanh[x]^2)/2)
```

3.852.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.852.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(173) = 346$.

Time = 0.13 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.16

method	result
risch	$2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}\,\mathrm{e}^{-2x}x(x\,\mathrm{e}^{2x}+\mathrm{e}^{2x}+1)-2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}\,\mathrm{e}^{-2x}(1+\mathrm{e}^{2x})^2\ln(\mathrm{e}^{x})+\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}\,\mathrm{e}^{-2x}(1+\mathrm{e}^{2x})^2$

input
```
int(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*x*(x*exp(2*x)+exp(2*x)+1)-2*
(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*ln(exp(x))+(a*e
xp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*ln(1+exp(2*x))+(a*e
xp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2*ln(1+exp(x))+2*
(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*polylog(2,-ex
p(x))-2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog
(3,-exp(x))-(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2
*ln(1+exp(2*x))-(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2
*x*polylog(2,-exp(2*x))+1/2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1
+exp(2*x))^2*polylog(3,-exp(2*x))+(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2
*x)*(1+exp(2*x))^2*x^2*ln(1-exp(x))+2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*ex
p(-2*x)*(1+exp(2*x))^2*x*polylog(2,exp(x))-2*(a*exp(4*x)/(1+exp(2*x))^4)^(
1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(3,exp(x))
```

3.852.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 3431, normalized size of antiderivative = 16.82

$$\int x^2 \text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
-(2*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x)
+ 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 +
 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (
cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(
2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(
x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x
) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, cosh(x) + sinh(x)) - 2*((e^(4*x) +
2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2
*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*
(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*co
sh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x
)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + co
sh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) +
1))*e^(2*x)*polylog(3, I*cosh(x) + I*sinh(x)) - 2*((e^(4*x) + 2*e^(2*x) +
1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x
))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2
 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)
*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(
x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(
x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*...
```

3.852.6 Sympy [F]

$$\int x^2 \text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \int x^2\sqrt{a\,\text{sech}^4(x)}\,\text{csch}(x)\,\text{sech}(x)\,dx$$

input
```
integrate(x**2*csch(x)*sech(x)*(a*sech(x)**4)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*sech(x)**4)*csch(x)*sech(x), x)
```

3.852.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.75

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx$$
$$= -\frac{1}{2}\left(2\,x^2\log\left(e^{(2\,x)}+1\right)+2\,x\text{Li}_2(-e^{(2\,x)})-\text{Li}_3(-e^{(2\,x)})\right)\sqrt{a}$$
$$+\left(x^2\log\left(e^x+1\right)+2\,x\text{Li}_2(-e^x)-2\,\text{Li}_3(-e^x)\right)\sqrt{a}$$
$$+\left(x^2\log\left(-e^x+1\right)+2\,x\text{Li}_2(e^x)-2\,\text{Li}_3(e^x)\right)\sqrt{a}-2\,\sqrt{a}x$$
$$+\sqrt{a}\log\left(e^{(2\,x)}+1\right)+\frac{2\left((\sqrt{a}x^2+\sqrt{a}x)e^{(2\,x)}+\sqrt{a}x\right)}{e^{(4\,x)}+2\,e^{(2\,x)}+1}$$

input
```
integrate(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*x^2*log(e^(2*x) + 1) + 2*x*dilog(-e^(2*x)) - polylog(3, -e^(2*x)))
*sqrt(a) + (x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*polylog(3, -e^x))*sqrt(
a) + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(3, e^x))*sqrt(a) - 2*
sqrt(a)*x + sqrt(a)*log(e^(2*x) + 1) + 2*((sqrt(a)*x^2 + sqrt(a)*x)*e^(2*x
) + sqrt(a)*x)/(e^(4*x) + 2*e^(2*x) + 1)
```

3.852.8 Giac [F]

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx=\int\sqrt{a\,\text{sech}\,(x)^4}x^2\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

input
```
integrate(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sech(x)^4)*x^2*csch(x)*sech(x), x)
```

3.852.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx = \int \frac{x^2\sqrt{\frac{a}{\cosh(x)^4}}}{\cosh(x)\sinh(x)}\,dx$$

input
```
int((x^2*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)),x)
```

output
```
int((x^2*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)), x)
```

3.853 $\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx$

3.853.1 Optimal result

Integrand size = 18, antiderivative size = 326

$$
\begin{aligned}
\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx = & -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)} \\
& - 2x^3\operatorname{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)} \\
& + 3x\cosh^2(x)\log\left(1+e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
& + \frac{3}{2}\cosh^2(x)\operatorname{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
& - \frac{3}{2}x^2\cosh^2(x)\operatorname{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
& + \frac{3}{2}x^2\cosh^2(x)\operatorname{PolyLog}\left(2,e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
& + \frac{3}{2}x\cosh^2(x)\operatorname{PolyLog}\left(3,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
& - \frac{3}{2}x\cosh^2(x)\operatorname{PolyLog}\left(3,e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
& - \frac{3}{4}\cosh^2(x)\operatorname{PolyLog}\left(4,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
& + \frac{3}{4}\cosh^2(x)\operatorname{PolyLog}\left(4,e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
& - \frac{3}{2}x^2\cosh(x)\sqrt{a\operatorname{sech}^4(x)}\sinh(x) \\
& - \frac{1}{2}x^3\sqrt{a\operatorname{sech}^4(x)}\sinh^2(x)
\end{aligned}
$$

output
```
-3/2*x^2*cosh(x)^2*(a*sech(x)^4)^(1/2)+1/2*x^3*cosh(x)^2*(a*sech(x)^4)^(1/
2)-2*x^3*arctanh(exp(2*x))*cosh(x)^2*(a*sech(x)^4)^(1/2)+3*x*cosh(x)^2*ln(
1+exp(2*x))*(a*sech(x)^4)^(1/2)+3/2*cosh(x)^2*polylog(2,-exp(2*x))*(a*sech
(x)^4)^(1/2)-3/2*x^2*cosh(x)^2*polylog(2,-exp(2*x))*(a*sech(x)^4)^(1/2)+3/
2*x^2*cosh(x)^2*polylog(2,exp(2*x))*(a*sech(x)^4)^(1/2)+3/2*x*cosh(x)^2*po
lylog(3,-exp(2*x))*(a*sech(x)^4)^(1/2)-3/2*x*cosh(x)^2*polylog(3,exp(2*x))
*(a*sech(x)^4)^(1/2)-3/4*cosh(x)^2*polylog(4,-exp(2*x))*(a*sech(x)^4)^(1/2
)+3/4*cosh(x)^2*polylog(4,exp(2*x))*(a*sech(x)^4)^(1/2)-3/2*x^2*cosh(x)*si
nh(x)*(a*sech(x)^4)^(1/2)-1/2*x^3*sinh(x)^2*(a*sech(x)^4)^(1/2)
```

3.853.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.46

$$\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx = \frac{1}{4}\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\left(6x^2+4x^3\log\left(1-e^{-2x}\right)+12x\log\left(1+e^{-2x}\right)-4x^3\log\left(1+e^{-2x}\right)+6\left(-1+x^2\right)\operatorname{PolyLog}\left(2,-e^{-2x}\right)-6x^2\operatorname{PolyLog}\left(2,e^{-2x}\right)+6x\operatorname{PolyLog}\left(3,-e^{-2x}\right)-6x\operatorname{PolyLog}\left(3,e^{-2x}\right)+3\operatorname{PolyLog}\left(4,-e^{-2x}\right)-3\operatorname{PolyLog}\left(4,e^{-2x}\right)+2x^3\operatorname{sech}^2(x)-6x^2\tanh(x)\right)$$

input
```
Integrate[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

output
```
(Cosh[x]^2*Sqrt[a*Sech[x]^4]*(6*x^2 + 4*x^3*Log[1 - E^(-2*x)] + 12*x*Log[1
 + E^(-2*x)] - 4*x^3*Log[1 + E^(-2*x)] + 6*(-1 + x^2)*PolyLog[2, -E^(-2*x)
] - 6*x^2*PolyLog[2, E^(-2*x)] + 6*x*PolyLog[3, -E^(-2*x)] - 6*x*PolyLog[3
, E^(-2*x)] + 3*PolyLog[4, -E^(-2*x)] - 3*PolyLog[4, E^(-2*x)] + 2*x^3*Sec
h[x]^2 - 6*x^2*Tanh[x]))/4
```

3.853.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.52, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {7271, 5985, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cosh^2(x)\sqrt{a\text{sech}^4(x)}\int x^3\text{csch}(x)\text{sech}^3(x)dx$$

$$\Big\downarrow \text{5985}$$

$$\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(-3\int \frac{1}{2}x^2\left(2\log(\tanh(x))-\tanh^2(x)\right)dx-\frac{1}{2}x^3\tanh^2(x)+x^3\log(\tanh(x))\right)$$

$$\Big\downarrow \text{27}$$

$$\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(-\frac{3}{2}\int x^2\left(2\log(\tanh(x))-\tanh^2(x)\right)dx-\frac{1}{2}x^3\tanh^2(x)+x^3\log(\tanh(x))\right)$$

$$\Big\downarrow \text{2010}$$

$$\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(-\frac{3}{2}\int \left(2x^2\log(\tanh(x))-x^2\tanh^2(x)\right)dx-\frac{1}{2}x^3\tanh^2(x)+x^3\log(\tanh(x))\right)$$

$$\Big\downarrow \text{2009}$$

$$\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(-\frac{3}{2}\left(\frac{4}{3}x^3\text{arctanh}\left(e^{2x}\right)+x^2\,\text{PolyLog}\left(2,-e^{2x}\right)-x^2\,\text{PolyLog}\left(2,e^{2x}\right)-x\,\text{PolyLog}\left(3,-e^{2x}\right)\right.\right.$$

input
```
Int[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

output
```
Cosh[x]^2*Sqrt[a*Sech[x]^4]*(x^3*Log[Tanh[x]] - (x^3*Tanh[x]^2)/2 - (3*(x^
2 - x^3/3 + (4*x^3*ArcTanh[E^(2*x)])/3 - 2*x*Log[1 + E^(2*x)] + (2*x^3*Log
[Tanh[x]])/3 - PolyLog[2, -E^(2*x)] + x^2*PolyLog[2, -E^(2*x)] - x^2*PolyL
og[2, E^(2*x)] - x*PolyLog[3, -E^(2*x)] + x*PolyLog[3, E^(2*x)] + PolyLog[
4, -E^(2*x)]/2 - PolyLog[4, E^(2*x)]/2 + x^2*Tanh[x]))/2)
```

3.853.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.853.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(269) = 538$.

Time = 0.12 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.85

method	result
risch	$\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}\,\mathrm{e}^{-2x}x^2(2x\,\mathrm{e}^{2x}+3\,\mathrm{e}^{2x}+3)-3\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}\,\mathrm{e}^{-2x}(1+\mathrm{e}^{2x})^2\,x^2+3\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}\,\mathrm{e}^{-2x}(1+\mathrm{e}^{2x})^2$

input
```
int(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*x^2*(2*x*exp(2*x)+3*exp(2*x)+3
)-3*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2+3*(a*ex
p(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*ln(1+exp(2*x))+3/2
*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(2,-exp
(2*x))+(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^3*ln(1
+exp(x))+3*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2*
polylog(2,-exp(x))-6*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*
x))^2*x*polylog(3,-exp(x))+6*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(
1+exp(2*x))^2*polylog(4,-exp(x))-(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*
x)*(1+exp(2*x))^2*x^3*ln(1+exp(2*x))-3/2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)
*exp(-2*x)*(1+exp(2*x))^2*x^2*polylog(2,-exp(2*x))+3/2*(a*exp(4*x)/(1+exp(
2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*polylog(3,-exp(2*x))-3/4*(a*exp(
4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(4,-exp(2*x))+(
a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^3*ln(1-exp(x))
+3*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2*polylog(
2,exp(x))-6*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*p
olylog(3,exp(x))+6*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x)
)^2*polylog(4,exp(x))
```

3.853.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 4629, normalized size of antiderivative = 14.20

$$\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
(6*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) +
 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 +
1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (c
osh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2
*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x
))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x)
 + 4*e^(2*x) + 1))*e^(2*x)*polylog(4, cosh(x) + sinh(x)) - 6*((e^(4*x) + 2
*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*
x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(
3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cos
h(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)
^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cos
h(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1
))*e^(2*x)*polylog(4, I*cosh(x) + I*sinh(x)) - 6*((e^(4*x) + 2*e^(2*x) + 1
)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x)
)*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2
+ 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*
e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x
)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x
) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x...
```

3.853.6 Sympy [F]

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \int x^3\sqrt{a\,\text{sech}^4(x)}\,\text{csch}(x)\,\text{sech}(x)\,dx$$

input
```
integrate(x**3*csch(x)*sech(x)*(a*sech(x)**4)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*sech(x)**4)*csch(x)*sech(x), x)
```

3.853.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.63

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx$$
$$= -3\sqrt{a}x^2 - \frac{1}{3}\left(4x^3\log\left(e^{(2x)}+1\right)+6x^2\text{Li}_2\left(-e^{(2x)}\right)-6x\text{Li}_3(-e^{(2x)})+3\text{Li}_4(-e^{(2x)})\right)\sqrt{a} + \left(x^3\log\left(e^x+1\right)+3x^2\text{Li}_2(-e^x)-6x\text{Li}_3(-e^x)+6\text{Li}_4(-e^x)\right)\sqrt{a} + \left(x^3\log\left(-e^x+1\right)+3x^2\text{Li}_2(e^x)-6x\text{Li}_3(e^x)+6\text{Li}_4(e^x)\right)\sqrt{a} + \frac{3}{2}\left(2x\log\left(e^{(2x)}+1\right)+\text{Li}_2\left(-e^{(2x)}\right)\right)\sqrt{a} + \frac{3\sqrt{a}x^2+\left(2\sqrt{a}x^3+3\sqrt{a}x^2\right)e^{(2x)}}{e^{(4x)}+2e^{(2x)}+1}$$

input
```
integrate(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-3*sqrt(a)*x^2 - 1/3*(4*x^3*log(e^(2*x) + 1) + 6*x^2*dilog(-e^(2*x)) - 6*x
*polylog(3, -e^(2*x)) + 3*polylog(4, -e^(2*x)))*sqrt(a) + (x^3*log(e^x + 1
) + 3*x^2*dilog(-e^x) - 6*x*polylog(3, -e^x) + 6*polylog(4, -e^x))*sqrt(a)
 + (x^3*log(-e^x + 1) + 3*x^2*dilog(e^x) - 6*x*polylog(3, e^x) + 6*polylog
(4, e^x))*sqrt(a) + 3/2*(2*x*log(e^(2*x) + 1) + dilog(-e^(2*x)))*sqrt(a) +
 (3*sqrt(a)*x^2 + (2*sqrt(a)*x^3 + 3*sqrt(a)*x^2)*e^(2*x))/(e^(4*x) + 2*e^
(2*x) + 1)
```

3.853.8 Giac [F]

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \int \sqrt{a\,\text{sech}\,(x)^4}x^3\,\text{csch}\,(x)\,\text{sech}\,(x)\,\,dx$$

input
```
integrate(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sech(x)^4)*x^3*csch(x)*sech(x), x)
```

3.853.9 Mupad [F(-1)]

Timed out.

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx=\int \frac{x^3\,\sqrt{\frac{a}{{\mathrm{cosh}\left(x\right)}^4}}}{\mathrm{cosh}\left(x\right)\,\mathrm{sinh}\left(x\right)}\,dx$$

input
```
int((x^3*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)),x)
```

output
```
int((x^3*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)), x)
```

3.854 $\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx$

3.854.1 Optimal result

Integrand size = 18, antiderivative size = 147

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx$$
$$=\frac{i\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-i\sinh(2c+2dx)),\frac{b(1-i\sinh(2c+2dx))}{2ia+b}\right)\cosh(2c+2dx)\left(a+\frac{1}{2}b\sinh(2c+2dx)\right.}{\sqrt{2}d\sqrt{1+i\sinh(2c+2dx)}}$$

output
```
1/2*I*AppellF1(1/2,-m,1/2,3/2,b*(1-I*sinh(2*d*x+2*c))/(2*I*a+b),1/2-1/2*I*
sinh(2*d*x+2*c))*cosh(2*d*x+2*c)*(a+1/2*b*sinh(2*d*x+2*c))^m/d/(((2*a+b*si
nh(2*d*x+2*c))/(2*a-I*b))^m)*2^(1/2)/(1+I*sinh(2*d*x+2*c))^(1/2)
```

3.854.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.10

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx$$
$$=\frac{\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{2a+b\sinh(2(c+dx))}{2a+ib},\frac{2a+b\sinh(2(c+dx))}{2a-ib}\right)\text{sech}(2(c+dx))\sqrt{\frac{b(1-i\sinh(2(c+dx)))}{2ia+b}}\sqrt{\frac{b(1+i}{}}}{bd(1+m)}$$

input
```
Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^m,x]
```

output
```
(AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*a + b*Sinh[2*(c + d*x)])/(2*a + I*b),
 (2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)]*Sech[2*(c + d*x)]*Sqrt[(b*(1 - I
*Sinh[2*(c + d*x)]))/((2*I)*a + b)]*Sqrt[(b*(1 + I*Sinh[2*(c + d*x)]))/((-
2*I)*a + b)]*(a + (b*Sinh[2*(c + d*x)])/2)^(1 + m))/(b*d*(1 + m))
```

3.854.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3145, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(c+dx)\cosh(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-ib\sin(ic+idx)\cos(ic+idx))^m dx$$

$$\Big\downarrow \text{3145}$$

$$\int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^m dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-\frac{1}{2}ib\sin(2ic+2idx)\right)^m dx$$

$$\Big\downarrow \text{3144}$$

$$-\frac{i\cosh(2c+2dx)\int \frac{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^m}{\sqrt{1-i\sinh(2c+2dx)}\sqrt{i\sinh(2c+2dx)+1}}d(i\sinh(2c+2dx))}{2d\sqrt{1-i\sinh(2c+2dx)}\sqrt{1+i\sinh(2c+2dx)}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{i\cosh(2c+2dx)\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^m\left(\frac{2a+b\sinh(2c+2dx)}{2a-ib}\right)^{-m}\int \frac{\left(\frac{2a}{2a-ib}+\frac{ib\sinh(2c+2dx)}{2ia+b}\right)^m}{\sqrt{1-i\sinh(2c+2dx)}\sqrt{i\sinh(2c+2dx)+1}}d(i\sinh(2c+2dx))}{2d\sqrt{1-i\sinh(2c+2dx)}\sqrt{1+i\sinh(2c+2dx)}}$$

$$\Big\downarrow \text{155}$$

$$\frac{i\cosh(2c+2dx)\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^m\left(\frac{2a+b\sinh(2c+2dx)}{2a-ib}\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-i\sinh(2c+2dx)),\right.}{\sqrt{2}d\sqrt{1+i\sinh(2c+2dx)}}$$

input
```
Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^m,x]
```

output
```
(I*AppellF1[1/2, 1/2, -m, 3/2, (1 - I*Sinh[2*c + 2*d*x])/2, (b*(1 - I*Sinh
[2*c + 2*d*x]))/((2*I)*a + b)]*Cosh[2*c + 2*d*x]*(a + (b*Sinh[2*c + 2*d*x]
)/2)^m)/(Sqrt[2]*d*Sqrt[1 + I*Sinh[2*c + 2*d*x]]*((2*a + b*Sinh[2*c + 2*d*
x])/(2*a - I*b))^m)
```

3.854.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.854.4 Maple [F]

$$\int (a+b\cosh (dx+c)\sinh (dx+c))^m dx$$

input
```
int((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x)
```

output
```
int((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x)
```

3.854.5 Fricas [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx=\int (b\cosh (dx+c)\sinh (dx+c)+a)^m\,dx$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((b*cosh(d*x + c)*sinh(d*x + c) + a)^m, x)
```

3.854.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx=\text{Timed out}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**m,x)
```

output
```
Timed out
```

3.854.7 Maxima [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx = \int (b\cosh(dx+c)\sinh(dx+c)+a)^m\,dx$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^m, x)
```

3.854.8 Giac [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx = \int (b\cosh(dx+c)\sinh(dx+c)+a)^m\,dx$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^m, x)
```

3.854.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx = \int (a+b\cosh(c+d\,x)\,\sinh(c+d\,x))^m\,dx$$

input
```
int((a + b*cosh(c + d*x)*sinh(c + d*x))^m,x)
```

output
```
int((a + b*cosh(c + d*x)*sinh(c + d*x))^m, x)
```

3.855 $\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx$

3.855.1 Optimal result

Integrand size = 18, antiderivative size = 109

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx=\frac{1}{8}a\left(8a^2-3b^2\right)x+\frac{b(16a^2-b^2)\cosh(2c+2dx)}{24d}+\frac{5ab^2\cosh(2c+2dx)\sinh(2c+2dx)}{48d}+\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^2}{48d}$$

output
```
1/8*a*(8*a^2-3*b^2)*x+1/24*b*(16*a^2-b^2)*cosh(2*d*x+2*c)/d+5/48*a*b^2*cos
h(2*d*x+2*c)*sinh(2*d*x+2*c)/d+1/48*b*cosh(2*d*x+2*c)*(2*a+b*sinh(2*d*x+2*
c))^2/d
```

3.855.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx=\frac{9(16a^2b-b^3)\cosh(2(c+dx))+b^3\cosh(6(c+dx))+6a(4(8a^2-3b^2)(c+dx)+3b^2\sinh(4(c+dx)))}{192d}$$

input
```
Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^3,x]
```

output
```
(9*(16*a^2*b - b^3)*Cosh[2*(c + d*x)] + b^3*Cosh[6*(c + d*x)] + 6*a*(4*(8*
a^2 - 3*b^2)*(c + d*x) + 3*b^2*Sinh[4*(c + d*x)]))/(192*d)
```

3.855.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3042, 3145, 3042, 3135, 27, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(c+dx)\cosh(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-ib\sin(ic+idx)\cos(ic+idx))^3dx$$

$$\Big\downarrow \text{3145}$$

$$\int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-\frac{1}{2}ib\sin(2ic+2idx)\right)^3dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{1}{3}\int \frac{1}{4}(2a+b\sinh(2c+2dx))\left(6a^2+5b\sinh(2c+2dx)a-b^2\right)dx+\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^2}{48d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}\int (2a+b\sinh(2c+2dx))\left(6a^2+5b\sinh(2c+2dx)a-b^2\right)dx+\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^2}{48d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^2}{48d}+\frac{1}{12}\int (2a-ib\sin(2ic+2idx))\left(6a^2-5ib\sin(2ic+2idx)a-b^2\right)dx$$

$$\Big\downarrow \text{3213}$$

$$\frac{1}{12}\left(\frac{b(16a^2-b^2)\cosh(2c+2dx)}{2d}+\frac{3}{2}ax(8a^2-3b^2)+\frac{5ab^2\sinh(2c+2dx)\cosh(2c+2dx)}{4d}\right)+\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^2}{48d}$$

input
```
Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^3,x]
```

output
```
(b*Cosh[2*c + 2*d*x]*(2*a + b*Sinh[2*c + 2*d*x])^2)/(48*d) + ((3*a*(8*a^2
- 3*b^2)*x)/2 + (b*(16*a^2 - b^2)*Cosh[2*c + 2*d*x])/(2*d) + (5*a*b^2*Cosh
[2*c + 2*d*x]*Sinh[2*c + 2*d*x])/(4*d))/12
```

3.855.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.855.4 Maple [A] (verified)

Time = 88.70 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

method	result
parts	$a^{3} x +\frac{b^{3} \left(\frac{\cosh\left(d x +c \right)^{6}}{6}-\frac{\cosh\left(d x +c \right)^{4}}{4}\right)}{d}+\frac{3 a^{2} b \cosh\left(d x +c \right)^{2}}{2 d}+\frac{3 a \,b^{2} \left(\frac{\sinh\left(d x +c \right) \cosh\left(d x +c \right)^{3}}{4}-\frac{\cosh\left(d x +c \right) \sinh\left(d x +c \right)}{8}\right.}{d}$
derivativedivides	$\frac{a^{3} \left(d x +c \right)+\frac{3 a^{2} b \cosh\left(d x +c \right)^{2}}{2}+3 a \,b^{2} \left(\frac{\sinh\left(d x +c \right) \cosh\left(d x +c \right)^{3}}{4}-\frac{\cosh\left(d x +c \right) \sinh\left(d x +c \right)}{8}-\frac{d x}{8}-\frac{c}{8}\right)+b^{3} \left(\frac{\sinh\left(d x +c \right)^{2} \cosh\left(d x +c \right.}{6}\right.}{d}$
default	$\frac{a^{3} \left(d x +c \right)+\frac{3 a^{2} b \cosh\left(d x +c \right)^{2}}{2}+3 a \,b^{2} \left(\frac{\sinh\left(d x +c \right) \cosh\left(d x +c \right)^{3}}{4}-\frac{\cosh\left(d x +c \right) \sinh\left(d x +c \right)}{8}-\frac{d x}{8}-\frac{c}{8}\right)+b^{3} \left(\frac{\sinh\left(d x +c \right)^{2} \cosh\left(d x +c \right.}{6}\right.}{d}$
risch	$a^{3} x -\frac{3 a \,b^{2} x}{8}+\frac{b^{3} {\mathrm e}^{6 d x +6 c}}{384 d}+\frac{3 a \,b^{2} {\mathrm e}^{4 d x +4 c}}{64 d}+\frac{3 b \,{\mathrm e}^{2 d x +2 c} a^{2}}{8 d}-\frac{3 b^{3} {\mathrm e}^{2 d x +2 c}}{128 d}+\frac{3 b \,{\mathrm e}^{-2 d x -2 c} a^{2}}{8 d}-\frac{3 b^{3} {\mathrm e}^{-2 d x -2 c}}{128 d}$

input
```
int((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+b^3/d*(1/6*cosh(d*x+c)^6-1/4*cosh(d*x+c)^4)+3/2*a^2*b/d*cosh(d*x+c)^
2+3*a*b^2/d*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8
*d*x-1/8*c)
```

3.855.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.50

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx$$
$$= \frac{b^3\cosh\left(dx+c\right)^6+15\,b^3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^4+b^3\sinh\left(dx+c\right)^6+72\,ab^2\cosh\left(dx+c\right)^3\sinh\left(dx+\right.}{}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/192*(b^3*cosh(d*x + c)^6 + 15*b^3*cosh(d*x + c)^2*sinh(d*x + c)^4 + b^3*
sinh(d*x + c)^6 + 72*a*b^2*cosh(d*x + c)^3*sinh(d*x + c) + 72*a*b^2*cosh(d
*x + c)*sinh(d*x + c)^3 + 24*(8*a^3 - 3*a*b^2)*d*x + 9*(16*a^2*b - b^3)*co
sh(d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 + 48*a^2*b - 3*b^3)*sinh(d*x + c)
^2)/d
```

3.855.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.74

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx$$
$$= \begin{cases} a^3x + \frac{3a^2b\sinh^2(c+dx)}{2d} - \frac{3ab^2x\sinh^4(c+dx)}{8} + \frac{3ab^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} - \frac{3ab^2x\cosh^4(c+dx)}{8} + \frac{3ab^2\sinh^3(c+dx)\cosh(c}{8d} \\ x(a+b\sinh(c)\cosh(c))^3 \end{cases}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)**2/(2*d) - 3*a*b**2*x*sinh(c +
d*x)**4/8 + 3*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - 3*a*b**2*x*co
sh(c + d*x)**4/8 + 3*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + 3*a*b**
2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + b**3*sinh(c + d*x)**2*cosh(c + d*
x)**4/(4*d) - b**3*cosh(c + d*x)**6/(12*d), Ne(d, 0)), (x*(a + b*sinh(c)*c
osh(c))**3, True))
```

3.855.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx$$
$$= a^3x - \frac{1}{384}b^3\left(\frac{\left(9e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d} + \frac{9e^{(-2dx-2c)}-e^{(-6dx-6c)}}{d}\right) - \frac{3}{64}ab^2\left(\frac{8(dx+c)}{d} - \frac{e^{(4dx+4c)}}{d} + \frac{e^{(-4dx-4c)}}{d}\right) + \frac{3a^2b\cosh(dx+c)^2}{2d}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x - 1/384*b^3*((9*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + (9*e^(-2*d
*x - 2*c) - e^(-6*d*x - 6*c))/d) - 3/64*a*b^2*(8*(d*x + c)/d - e^(4*d*x +
4*c)/d + e^(-4*d*x - 4*c)/d) + 3/2*a^2*b*cosh(d*x + c)^2/d
```

3.855.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx = \frac{b^3e^{(6\,dx+6\,c)}}{384\,d} + \frac{3\,ab^2e^{(4\,dx+4\,c)}}{64\,d} - \frac{3\,ab^2e^{(-4\,dx-4\,c)}}{64\,d} + \frac{b^3e^{(-6\,dx-6\,c)}}{384\,d} + \frac{1}{8}\left(8\,a^3-3\,ab^2\right)x + \frac{3\,(16\,a^2b-b^3)e^{(2\,dx+2\,c)}}{128\,d} + \frac{3\,(16\,a^2b-b^3)e^{(-2\,dx-2\,c)}}{128\,d}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
1/384*b^3*e^(6*d*x + 6*c)/d + 3/64*a*b^2*e^(4*d*x + 4*c)/d - 3/64*a*b^2*e^
(-4*d*x - 4*c)/d + 1/384*b^3*e^(-6*d*x - 6*c)/d + 1/8*(8*a^3 - 3*a*b^2)*x
+ 3/128*(16*a^2*b - b^3)*e^(2*d*x + 2*c)/d + 3/128*(16*a^2*b - b^3)*e^(-2*
d*x - 2*c)/d
```

3.855.9 Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx = \frac{\frac{b^3\cosh(6\,c+6\,d\,x)}{8} - \frac{9\,b^3\cosh(2\,c+2\,d\,x)}{8} + 18\,a^2\,b\cosh(2\,c+2\,d\,x) + \frac{9\,a\,b^2\sinh(4\,c+4\,d\,x)}{4} + 24\,a^3\,d\,x - 9\,a\,b^2\,d\,x}{24\,d}$$

input
```
int((a + b*cosh(c + d*x)*sinh(c + d*x))^3,x)
```

output
```
((b^3*cosh(6*c + 6*d*x))/8 - (9*b^3*cosh(2*c + 2*d*x))/8 + 18*a^2*b*cosh(2
*c + 2*d*x) + (9*a*b^2*sinh(4*c + 4*d*x))/4 + 24*a^3*d*x - 9*a*b^2*d*x)/(2
4*d)
```

3.856 $\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx$

3.856.1 Optimal result

Integrand size = 18, antiderivative size = 63

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx=\frac{1}{8}\left(8a^2-b^2\right)x+\frac{ab\cosh(2c+2dx)}{2d}+\frac{b^2\cosh(2c+2dx)\sinh(2c+2dx)}{16d}$$

output
```
1/8*(8*a^2-b^2)*x+1/2*a*b*cosh(2*d*x+2*c)/d+1/16*b^2*cosh(2*d*x+2*c)*sinh(
2*d*x+2*c)/d
```

3.856.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx=\frac{4(8a^2-b^2)\,(c+dx)+16ab\cosh(2(c+dx))+b^2\sinh(4(c+dx))}{32d}$$

input
```
Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^2,x]
```

output
```
(4*(8*a^2 - b^2)*(c + d*x) + 16*a*b*Cosh[2*(c + d*x)] + b^2*Sinh[4*(c + d*
x)])/(32*d)
```

3.856.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 3145, 3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(c+dx)\cosh(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-ib\sin(ic+idx)\cos(ic+idx))^2dx$$

$$\Big\downarrow \text{3145}$$

$$\int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-\frac{1}{2}ib\sin(2ic+2idx)\right)^2dx$$

$$\Big\downarrow \text{3123}$$

$$\frac{1}{8}x(8a^2-b^2)+\frac{ab\cosh(2c+2dx)}{2d}+\frac{b^2\sinh(2c+2dx)\cosh(2c+2dx)}{16d}$$

input
```
Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^2,x]
```

output
```
((8*a^2 - b^2)*x)/8 + (a*b*Cosh[2*c + 2*d*x])/(2*d) + (b^2*Cosh[2*c + 2*d*
x]*Sinh[2*c + 2*d*x])/(16*d)
```

3.856.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.856.4 Maple [A] (verified)

Time = 7.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

method	result	size
parts	$a^{2} x +\frac{b^{2} \left(\frac{\sinh \left(d x +c \right) \cosh \left(d x +c \right)^{3}}{4}-\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{8}-\frac{d x}{8}-\frac{c}{8}\right)}{d}+\frac{a b \cosh \left(d x +c \right)^{2}}{d}$	66
derivativedivides	$\frac{a^{2} \left(d x +c \right)+a b \cosh \left(d x +c \right)^{2}+b^{2} \left(\frac{\sinh \left(d x +c \right) \cosh \left(d x +c \right)^{3}}{4}-\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{8}-\frac{d x}{8}-\frac{c}{8}\right)}{d}$	68
default	$\frac{a^{2} \left(d x +c \right)+a b \cosh \left(d x +c \right)^{2}+b^{2} \left(\frac{\sinh \left(d x +c \right) \cosh \left(d x +c \right)^{3}}{4}-\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{8}-\frac{d x}{8}-\frac{c}{8}\right)}{d}$	68
risch	$a^{2} x -\frac{b^{2} x}{8}+\frac{b^{2} {\mathrm e}^{4 d x +4 c}}{64 d}+\frac{a b \,{\mathrm e}^{2 d x +2 c}}{4 d}+\frac{a b \,{\mathrm e}^{-2 d x -2 c}}{4 d}-\frac{b^{2} {\mathrm e}^{-4 d x -4 c}}{64 d}$	79

input
```
int((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+b^2/d*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8
*d*x-1/8*c)+a*b/d*cosh(d*x+c)^2
```

3.856.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx$$
$$=\frac{b^{2} \cosh \left(d x+c\right)^{3} \sinh \left(d x+c\right)+b^{2} \cosh \left(d x+c\right) \sinh \left(d x+c\right)^{3}+4 \, a b \cosh \left(d x+c\right)^{2}+4 \, a b \sinh \left(d x+c\right)^{2}}{8 \, d}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/8*(b^2*cosh(d*x + c)^3*sinh(d*x + c) + b^2*cosh(d*x + c)*sinh(d*x + c)^3
 + 4*a*b*cosh(d*x + c)^2 + 4*a*b*sinh(d*x + c)^2 + (8*a^2 - b^2)*d*x)/d
```

3.856.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(53) = 106$.

Time = 0.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.05

$$\int (a + b\cosh(c+dx)\sinh(c+dx))^2\,dx$$
$$= \begin{cases} a^2x + \frac{ab\sinh^2(c+dx)}{d} - \frac{b^2x\sinh^4(c+dx)}{8} + \frac{b^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} - \frac{b^2x\cosh^4(c+dx)}{8} + \frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{8d} + \frac{b^2 \\ x(a+b\sinh(c)\cosh(c))^2 \end{cases}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x + a*b*sinh(c + d*x)**2/d - b**2*x*sinh(c + d*x)**4/8 + b
**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - b**2*x*cosh(c + d*x)**4/8 + b*
*2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + b**2*sinh(c + d*x)*cosh(c + d*x)
**3/(8*d), Ne(d, 0)), (x*(a + b*sinh(c)*cosh(c))**2, True))
```

3.856.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int (a + b\cosh(c+dx)\sinh(c+dx))^2\,dx = a^2x - \frac{1}{64}b^2\left(\frac{8(dx+c)}{d} - \frac{e^{(4dx+4c)}}{d} + \frac{e^{(-4dx-4c)}}{d}\right) + \frac{ab\cosh(dx+c)^2}{d}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x - 1/64*b^2*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
+ a*b*cosh(d*x + c)^2/d
```

3.856.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.29

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx = \frac{1}{8}\left(8\,a^2-b^2\right)x+\frac{b^2e^{(4\,dx+4\,c)}}{64\,d}+\frac{abe^{(2\,dx+2\,c)}}{4\,d}+\frac{abe^{(-2\,dx-2\,c)}}{4\,d}-\frac{b^2e^{(-4\,dx-4\,c)}}{64\,d}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*(8*a^2 - b^2)*x + 1/64*b^2*e^(4*d*x + 4*c)/d + 1/4*a*b*e^(2*d*x + 2*c)
/d + 1/4*a*b*e^(-2*d*x - 2*c)/d - 1/64*b^2*e^(-4*d*x - 4*c)/d
```

3.856.9 Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx = \frac{\frac{\sinh(4\,c+4\,d\,x)\,b^2}{32}+\frac{a\cosh(2\,c+2\,d\,x)\,b}{2}}{d}+a^2\,x-\frac{b^2\,x}{8}$$

input
```
int((a + b*cosh(c + d*x)*sinh(c + d*x))^2,x)
```

output
```
((b^2*sinh(4*c + 4*d*x))/32 + (a*b*cosh(2*c + 2*d*x))/2)/d + a^2*x - (b^2*
x)/8
```

3.857 $\int (a+b\cosh(c+dx)\sinh(c+dx))\,dx$

3.857.1 Optimal result

Integrand size = 16, antiderivative size = 20

$$\int (a+b\cosh(c+dx)\sinh(c+dx))\,dx = ax+\frac{b\sinh^2(c+dx)}{2d}$$

output
```
a*x+1/2*b*sinh(d*x+c)^2/d
```

3.857.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int (a+b\cosh(c+dx)\sinh(c+dx))\,dx = ax+\frac{b\cosh(2c)\cosh(2dx)}{4d}+\frac{b\sinh(2c)\sinh(2dx)}{4d}$$

input
```
Integrate[a + b*Cosh[c + d*x]*Sinh[c + d*x],x]
```

output
```
a*x + (b*Cosh[2*c]*Cosh[2*d*x])/(4*d) + (b*Sinh[2*c]*Sinh[2*d*x])/(4*d)
```

3.857.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sinh(c+dx)\cosh(c+dx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax + \frac{b\sinh^2(c+dx)}{2d}$$

input
```
Int[a + b*Cosh[c + d*x]*Sinh[c + d*x],x]
```

output
```
a*x + (b*Sinh[c + d*x]^2)/(2*d)
```

3.857.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.857.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$ax+\frac{b\cosh(dx+c)^2}{2d}$	19
parts	$ax+\frac{b\cosh(dx+c)^2}{2d}$	19
derivativedivides	$\frac{(dx+c)a+\frac{b\cosh(dx+c)^2}{2}}{d}$	24
risch	$ax+\frac{b\,e^{2dx+2c}}{8d}+\frac{b\,e^{-2dx-2c}}{8d}$	35

input
```
int(a+b*cosh(d*x+c)*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b/d*cosh(d*x+c)^2
```

3.857.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int (a + b\cosh(c + dx)\sinh(c + dx))\, dx = \frac{4\, adx + b\cosh{(dx + c)}^2 + b\sinh{(dx + c)}^2}{4\, d}$$

input
```
integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x, algorithm="fricas")
```

output
```
1/4*(4*a*d*x + b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2)/d
```

3.857.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (a + b\cosh(c + dx)\sinh(c + dx))\, dx = ax + b\left(\begin{cases} \frac{\sinh^2{(c+dx)}}{2d} & \text{for}\ d \neq 0 \\ x\sinh{(c)}\cosh{(c)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x)
```

output
```
a*x + b*Piecewise((sinh(c + d*x)**2/(2*d), Ne(d, 0)), (x*sinh(c)*cosh(c),
True))
```

3.857.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (a + b\cosh(c + dx)\sinh(c + dx))\, dx = ax + \frac{b\cosh{(dx + c)}^2}{2\, d}$$

input
```
integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x, algorithm="maxima")
```

output
```
a*x + 1/2*b*cosh(d*x + c)^2/d
```

3.857.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int (a+b\cosh(c+dx)\sinh(c+dx))\,dx = ax + \frac{1}{8}\,b\left(\frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right)$$

input
```
integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x, algorithm="giac")
```

output
```
a*x + 1/8*b*(e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d)
```

3.857.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (a+b\cosh(c+dx)\sinh(c+dx))\,dx = a\,x + \frac{b\cosh(c+d\,x)^2}{2\,d}$$

input
```
int(a + b*cosh(c + d*x)*sinh(c + d*x),x)
```

output
```
a*x + (b*cosh(c + d*x)^2)/(2*d)
```

3.858 $\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx$

3.858.1 Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = -\frac{2\text{arctanh}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}d}$$

output
```
-2*arctanh((b-2*a*tanh(d*x+c))/(4*a^2+b^2)^(1/2))/d/(4*a^2+b^2)^(1/2)
```

3.858.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \frac{2\arctan\left(\frac{b-2a\tanh(c+dx)}{\sqrt{-4a^2-b^2}}\right)}{\sqrt{-4a^2-b^2}d}$$

input
```
Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-1),x]
```

output
```
(2*ArcTan[(b - 2*a*Tanh[c + d*x])/Sqrt[-4*a^2 - b^2]])/(Sqrt[-4*a^2 - b^2]
*d)
```

3.858.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3145, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sinh(c+dx)\cosh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-ib\sin(ic+idx)\cos(ic+idx)}dx$$

$$\Big\downarrow \text{3145}$$

$$\int \frac{1}{a+\frac{1}{2}b\sinh(2c+2dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-\frac{1}{2}ib\sin(2ic+2idx)}dx$$

$$\Big\downarrow \text{3139}$$

$$-\frac{i\int \frac{1}{-a\tanh^2(c+dx)+b\tanh(c+dx)+a}d(i\tanh(c+dx))}{d}$$

$$\Big\downarrow \text{1083}$$

$$\frac{2i\int \frac{1}{-4a^2-b^2+\tanh^2(c+dx)}d(2ia\tanh(c+dx)-ib)}{d}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\text{arctanh}\left(\frac{\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\sqrt{4a^2+b^2}}$$

input
```
Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-1),x]
```

output
```
(2*ArcTanh[Tanh[c + d*x]/Sqrt[4*a^2 + b^2]])/(Sqrt[4*a^2 + b^2]*d)
```

3.858.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.858.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(40) = 80$.

Time = 1.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.02

method	result
risch	$\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\sqrt{4a^2+b^2}-4a^2-b^2}{\sqrt{4a^2+b^2}\,b}\right)}{\sqrt{4a^2+b^2}\,d}-\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\sqrt{4a^2+b^2}+4a^2+b^2}{\sqrt{4a^2+b^2}\,b}\right)}{\sqrt{4a^2+b^2}\,d}$
derivativedivides	$\frac{2a\left(\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+\sqrt{4a^2+b^2}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{2\sqrt{4a^2+b^2}\,a}+\frac{\left(-4a^2-b^2\right)\ln\left(-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+\sqrt{4a^2+b^2}\,\tanh\left(\frac{dx}{2}\right.\right.}{2\left(4a^2+b^2\right)^{\frac{3}{2}}a}\right.}{d}$
default	$\frac{2a\left(\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+\sqrt{4a^2+b^2}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{2\sqrt{4a^2+b^2}\,a}+\frac{\left(-4a^2-b^2\right)\ln\left(-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+\sqrt{4a^2+b^2}\,\tanh\left(\frac{dx}{2}\right.\right.}{2\left(4a^2+b^2\right)^{\frac{3}{2}}a}\right.}{d}$

input
```
int(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/(4*a^2+b^2)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*a*(4*a^2+b^2)^(1/2)-4*a^2-b^2)/
(4*a^2+b^2)^(1/2)/b)-1/(4*a^2+b^2)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*a*(4*a^2+b
^2)^(1/2)+4*a^2+b^2)/(4*a^2+b^2)^(1/2)/b)
```

3.858.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(43) = 86$.

Time = 0.26 (sec) , antiderivative size = 299, normalized size of antiderivative = 6.80

$$\int \frac{1}{a + b\cosh(c + dx)\sinh(c + dx)}\,dx$$
$$= \frac{\log\left(\frac{b^2\cosh(dx+c)^4+4\,b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4+4\,ab\cosh(dx+c)^2+2\left(3\,b^2\cosh(dx+c)^2+2\,ab\right)\sinh(dx+c)^2+8\,a^2+b^2+4}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+4\,a\cosh(dx+c)^2+2\left(3\,b\cosh\right.}\right.}{\sqrt{4\,a^2+b^2}d}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(
d*x + c)^4 + 4*a*b*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b)*sin
h(d*x + c)^2 + 8*a^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + 2*a*b*cosh(d*x + c))
*sinh(d*x + c) - 2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) +
b*sinh(d*x + c)^2 + 2*a)*sqrt(4*a^2 + b^2))/(b*cosh(d*x + c)^4 + 4*b*cosh(
d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 4*a*cosh(d*x + c)^2 + 2*(3*
b*cosh(d*x + c)^2 + 2*a)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + 2*a*cosh
(d*x + c))*sinh(d*x + c) - b))/(sqrt(4*a^2 + b^2)*d)
```

3.858.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a + b\cosh(c + dx)\sinh(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.858.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.66

$$\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \frac{\log\left(\frac{be^{(-2\,dx-2\,c)}-2\,a-\sqrt{4\,a^2+b^2}}{be^{(-2\,dx-2\,c)}-2\,a+\sqrt{4\,a^2+b^2}}\right)}{\sqrt{4\,a^2+b^2}d}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
log((b*e^(-2*d*x - 2*c) - 2*a - sqrt(4*a^2 + b^2))/(b*e^(-2*d*x - 2*c) - 2
*a + sqrt(4*a^2 + b^2)))/(sqrt(4*a^2 + b^2)*d)
```

3.858.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.80

$$\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \frac{\log\left(\frac{\left|2\,be^{(2\,dx+2\,c)}+4\,a-2\,\sqrt{4\,a^2+b^2}\right|}{\left|2\,be^{(2\,dx+2\,c)}+4\,a+2\,\sqrt{4\,a^2+b^2}\right|}\right)}{\sqrt{4\,a^2+b^2}d}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(2*b*e^(2*d*x + 2*c) + 4*a - 2*sqrt(4*a^2 + b^2))/abs(2*b*e^(2*d*x
+ 2*c) + 4*a + 2*sqrt(4*a^2 + b^2)))/(sqrt(4*a^2 + b^2)*d)
```

3.858.9 Mupad [B] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 343, normalized size of antiderivative = 7.80

$$\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx$$
$$= \frac{2\,\mathrm{atan}\left(\left(\frac{b^4\,\sqrt{-4\,a^2\,d^2-b^2\,d^2}}{16}+\frac{a^2\,b^2\,\sqrt{-4\,a^2\,d^2-b^2\,d^2}}{4}\right)\left(\frac{32\,a\,\left(8\,a^2+b^2\right)}{b^4\,d\,\left(4\,a^2+b^2\right)^2}-\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\left(\frac{64\,a\,\left(16\,d\,a^3+4\,d\,a\,b^2\right)}{b^5\,\sqrt{-4\,a^2\,d^2-b^2\,d^2}\,\left(4\,a^2+b^2\right)\,\sqrt{-d^2\,\left(4\,a^2+b\right.}}\right.\right.\right.}{\sqrt{-4\,a^2\,d^2-b^2}}$$

input
```
int(1/(a + b*cosh(c + d*x)*sinh(c + d*x)),x)
```

output
```
(2*atan(((b^4*(- 4*a^2*d^2 - b^2*d^2)^(1/2))/16 + (a^2*b^2*(- 4*a^2*d^2 -
b^2*d^2)^(1/2))/4)*((32*a*(8*a^2 + b^2))/(b^4*d*(4*a^2 + b^2)^2) - exp(2*c
)*exp(2*d*x)*((64*a*(16*a^3*d + 4*a*b^2*d))/(b^5*(- 4*a^2*d^2 - b^2*d^2)^(
1/2)*(4*a^2 + b^2)*(-d^2*(4*a^2 + b^2))^(1/2)) + (16*(8*a^2 + b^2)*(8*a^2*
(- 4*a^2*d^2 - b^2*d^2)^(1/2) + b^2*(- 4*a^2*d^2 - b^2*d^2)^(1/2)))/(b^5*d
*(- 4*a^2*d^2 - b^2*d^2)^(1/2)*(4*a^2 + b^2)^2)) + (64*a*(b^3*d + 4*a^2*b*
d))/(b^5*(- 4*a^2*d^2 - b^2*d^2)^(1/2)*(4*a^2 + b^2)*(-d^2*(4*a^2 + b^2))^
(1/2)))))/(- 4*a^2*d^2 - b^2*d^2)^(1/2)
```

3.858. $\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx$

3.859 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx$

3.859.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx = -\frac{8a\operatorname{arctanh}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{(4a^2+b^2)^{3/2}\,d} - \frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))}$$

output
```
-8*a*arctanh((b-2*a*tanh(d*x+c))/(4*a^2+b^2)^(1/2))/(4*a^2+b^2)^(3/2)/d-2*
b*cosh(2*d*x+2*c)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))
```

3.859.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx = \frac{2\left(-\frac{4a\arctan\left(\frac{b-2a\tanh(c+dx)}{\sqrt{-4a^2-b^2}}\right)}{(-4a^2-b^2)^{3/2}} - \frac{b\cosh(2(c+dx))}{(4a^2+b^2)(2a+b\sinh(2(c+dx)))}\right)}{d}$$

input
```
Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-2),x]
```

output
```
(2*((-4*a*ArcTan[(b - 2*a*Tanh[c + d*x])/Sqrt[-4*a^2 - b^2]])/(-4*a^2 - b^
2)^(3/2) - (b*Cosh[2*(c + d*x)])/((4*a^2 + b^2)*(2*a + b*Sinh[2*(c + d*x)]
))))/d
```

3.859.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3145, 3042, 3143, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sinh(c+dx)\cosh(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-ib\sin(ic+idx)\cos(ic+idx))^2}dx$$

$$\Big\downarrow \text{3145}$$

$$\int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-\frac{1}{2}ib\sin(2ic+2idx)\right)^2}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{4\int -\frac{2a}{2a+b\sinh(2c+2dx)}dx}{4a^2+b^2}-\frac{2b\cosh(2c+2dx)}{d\,(4a^2+b^2)\,(2a+b\sinh(2c+2dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{8a\int \frac{1}{2a+b\sinh(2c+2dx)}dx}{4a^2+b^2}-\frac{2b\cosh(2c+2dx)}{d\,(4a^2+b^2)\,(2a+b\sinh(2c+2dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\cosh(2c+2dx)}{d\,(4a^2+b^2)\,(2a+b\sinh(2c+2dx))}+\frac{8a\int \frac{1}{2a-ib\sin(2ic+2idx)}dx}{4a^2+b^2}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{2b\cosh(2c+2dx)}{d(4a^2+b^2)(2a+b\sinh(2c+2dx))}-\frac{8ia\int\frac{1}{-2a\tanh^2(c+dx)+2b\tanh(c+dx)+2a}d(i\tanh(c+dx))}{d(4a^2+b^2)}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{2b\cosh(2c+2dx)}{d(4a^2+b^2)(2a+b\sinh(2c+2dx))}+\frac{16ia\int\frac{1}{\tanh^2(c+dx)-4(4a^2+b^2)}d(4ia\tanh(c+dx)-2ib)}{d(4a^2+b^2)}$$

$$\Big\downarrow \text{217}$$

$$\frac{8a\text{arctanh}\left(\frac{\tanh(c+dx)}{2\sqrt{4a^2+b^2}}\right)}{d(4a^2+b^2)^{3/2}}-\frac{2b\cosh(2c+2dx)}{d(4a^2+b^2)(2a+b\sinh(2c+2dx))}$$

input
```
Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-2),x]
```

output
```
(8*a*ArcTanh[Tanh[c + d*x]/(2*Sqrt[4*a^2 + b^2])])/((4*a^2 + b^2)^(3/2)*d)
 - (2*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*(2*a + b*Sinh[2*c + 2*d*x]))
```

3.859.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.859.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(85) = 170$.

Time = 15.76 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.39

method	result
risch	$\frac{8a\,\mathrm{e}^{2dx+2c}-4b}{(4a^2+b^2)d(4a\,\mathrm{e}^{2dx+2c}+\mathrm{e}^{4dx+4c}b-b)}+\frac{4a\ln\left(\mathrm{e}^{2dx+2c}+\frac{2(4a^2+b^2)^{\frac{3}{2}}a-16a^4-8a^2b^2-b^4}{b(4a^2+b^2)^{\frac{3}{2}}}\right)}{(4a^2+b^2)^{\frac{3}{2}}d}-\frac{4a\ln\left(\mathrm{e}^{2dx+2c}+\frac{2(4a^2+\cdots}{\cdots}\right)}{(4a^2\cdots}$
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a(4a^2+b^2)}+\frac{4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4a^2+b^2}-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(4a^2+b^2)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+a}-\frac{8a^2\left(\frac{(-4a^2-b^2)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+\sqrt{4a^2+b^2}\cdots\right.}{2(4a^2+b^2)\cdots}\cdots\right.}{\cdots}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a(4a^2+b^2)}+\frac{4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4a^2+b^2}-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(4a^2+b^2)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+a}-\frac{8a^2\left(\frac{(-4a^2-b^2)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+\sqrt{4a^2+b^2}\cdots\right.}{2(4a^2+b^2)\cdots}\cdots\right.}{\cdots}}{d}$

input
```
int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
4*(2*a*exp(2*d*x+2*c)-b)/(4*a^2+b^2)/d/(4*a*exp(2*d*x+2*c)+exp(4*d*x+4*c)*
b-b)+4/(4*a^2+b^2)^(3/2)*a/d*ln(exp(2*d*x+2*c)+(2*(4*a^2+b^2)^(3/2)*a-16*a
^4-8*a^2*b^2-b^4)/b/(4*a^2+b^2)^(3/2))-4/(4*a^2+b^2)^(3/2)*a/d*ln(exp(2*d*
x+2*c)+(2*(4*a^2+b^2)^(3/2)*a+16*a^4+8*a^2*b^2+b^4)/b/(4*a^2+b^2)^(3/2))
```

3.859.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(88) = 176$.

Time = 0.26 (sec) , antiderivative size = 765, normalized size of antiderivative = 8.60

$$\int \frac{1}{(a + b\cosh(c + dx)\sinh(c + dx))^2}\,dx = -\frac{4\left(4\,a^2b + b^3 - 2\,(4\,a^3 + ab^2)\cosh\left(dx + c\right)^2 - 4\,(4\,a^3 + ab^2)\cosh\left(dx + c\right)\sinh\left(dx + c\right) - 2\,(4\,a^3 + ab^2\right.}{(16\,a^4b + 8\,a^2b^3 + b^5)d\cosh}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-4*(4*a^2*b + b^3 - 2*(4*a^3 + a*b^2)*cosh(d*x + c)^2 - 4*(4*a^3 + a*b^2)*
cosh(d*x + c)*sinh(d*x + c) - 2*(4*a^3 + a*b^2)*sinh(d*x + c)^2 - (a*b*cos
h(d*x + c)^4 + 4*a*b*cosh(d*x + c)*sinh(d*x + c)^3 + a*b*sinh(d*x + c)^4 +
 4*a^2*cosh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c)^2 + 2*a^2)*sinh(d*x + c)^2
 - a*b + 4*(a*b*cosh(d*x + c)^3 + 2*a^2*cosh(d*x + c))*sinh(d*x + c))*sqrt
(4*a^2 + b^2)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)
^3 + b^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^
2 + 2*a*b)*sinh(d*x + c)^2 + 8*a^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + 2*a*b*
cosh(d*x + c))*sinh(d*x + c) - 2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*si
nh(d*x + c) + b*sinh(d*x + c)^2 + 2*a)*sqrt(4*a^2 + b^2))/(b*cosh(d*x + c)
^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 4*a*cosh(d*x
+ c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c
)^3 + 2*a*cosh(d*x + c))*sinh(d*x + c) - b)))/((16*a^4*b + 8*a^2*b^3 + b^5
)*d*cosh(d*x + c)^4 + 4*(16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x + c)*sinh(
d*x + c)^3 + (16*a^4*b + 8*a^2*b^3 + b^5)*d*sinh(d*x + c)^4 + 4*(16*a^5 +
8*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(16*a^4*b + 8*a^2*b^3 + b^5)*d
*cosh(d*x + c)^2 + 2*(16*a^5 + 8*a^3*b^2 + a*b^4)*d)*sinh(d*x + c)^2 - (16
*a^4*b + 8*a^2*b^3 + b^5)*d + 4*((16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x +
 c)^3 + 2*(16*a^5 + 8*a^3*b^2 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

3.859.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

3.859.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.69

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx = \frac{4\,a\log\left(\frac{be^{(-2\,dx-2\,c)}-2\,a-\sqrt{4\,a^2+b^2}}{be^{(-2\,dx-2\,c)}-2\,a+\sqrt{4\,a^2+b^2}}\right)}{(4\,a^2+b^2)^{\frac{3}{2}}d} - \frac{4\left(2\,ae^{(-2\,dx-2\,c)}+b\right)}{(4\,a^2b+b^3+4\,(4\,a^3+ab^2)e^{(-2\,dx-2\,c)}-(4\,a^2b+b^3)e^{(-4\,dx-4\,c)})d}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
4*a*log((b*e^(-2*d*x - 2*c) - 2*a - sqrt(4*a^2 + b^2))/(b*e^(-2*d*x - 2*c)
 - 2*a + sqrt(4*a^2 + b^2)))/((4*a^2 + b^2)^(3/2)*d) - 4*(2*a*e^(-2*d*x -
2*c) + b)/((4*a^2*b + b^3 + 4*(4*a^3 + a*b^2)*e^(-2*d*x - 2*c) - (4*a^2*b
+ b^3)*e^(-4*d*x - 4*c))*d)
```

3.859.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.57

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx = \frac{4\left(\frac{a\log\left(\frac{\left|2\,be^{(2\,dx+2\,c)}+4\,a-2\,\sqrt{4\,a^2+b^2}\right|}{\left|2\,be^{(2\,dx+2\,c)}+4\,a+2\,\sqrt{4\,a^2+b^2}\right|}\right)}{(4\,a^2+b^2)^{\frac{3}{2}}} + \frac{2\,ae^{(2\,dx+2\,c)}-b}{(4\,a^2+b^2)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-b\right)}\right)}{d}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
4*(a*log(abs(2*b*e^(2*d*x + 2*c) + 4*a - 2*sqrt(4*a^2 + b^2))/abs(2*b*e^(2
*d*x + 2*c) + 4*a + 2*sqrt(4*a^2 + b^2)))/(4*a^2 + b^2)^(3/2) + (2*a*e^(2*
d*x + 2*c) - b)/((4*a^2 + b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) -
b)))/d
```

3.859.9 Mupad [B] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.57

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx = \frac{4\,a\,\ln\left(\frac{16\,a\,\left(b-2\,a\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{b\,{\left(4\,a^2+b^2\right)}^{3/2}}-\frac{16\,a\,\mathrm{e}^{2\,c+2\,d\,x}}{4\,a^2\,b+b^3}\right)}{d\,{\left(4\,a^2+b^2\right)}^{3/2}} - \frac{4\,a\,\ln\left(-\frac{16\,a\,\mathrm{e}^{2\,c+2\,d\,x}}{4\,a^2\,b+b^3}-\frac{16\,a\,\left(b-2\,a\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{b\,{\left(4\,a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(4\,a^2+b^2\right)}^{3/2}} - \frac{\frac{4\,b^2}{d\,\left(4\,a^2\,b+b^3\right)}-\frac{8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{d\,\left(4\,a^2\,b+b^3\right)}}{4\,a\,\mathrm{e}^{2\,c+2\,d\,x}-b+b\,\mathrm{e}^{4\,c+4\,d\,x}}$$

input
```
int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^2,x)
```

output
```
(4*a*log((16*a*(b - 2*a*exp(2*c + 2*d*x)))/(b*(4*a^2 + b^2)^(3/2)) - (16*a
*exp(2*c + 2*d*x))/(4*a^2*b + b^3)))/(d*(4*a^2 + b^2)^(3/2)) - (4*a*log(-
(16*a*exp(2*c + 2*d*x))/(4*a^2*b + b^3) - (16*a*(b - 2*a*exp(2*c + 2*d*x))
)/(b*(4*a^2 + b^2)^(3/2))))/(d*(4*a^2 + b^2)^(3/2)) - ((4*b^2)/(d*(4*a^2*b
 + b^3)) - (8*a*b*exp(2*c + 2*d*x))/(d*(4*a^2*b + b^3)))/(4*a*exp(2*c + 2*
d*x) - b + b*exp(4*c + 4*d*x))
```

3.860 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx$

3.860.1 Optimal result

Integrand size = 18, antiderivative size = 143

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = -\frac{4(8a^2-b^2)\,\text{arctanh}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{(4a^2+b^2)^{5/2}\,d} - \frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))^2} - \frac{12ab\cosh(2c+2dx)}{(4a^2+b^2)^2\,d(2a+b\sinh(2c+2dx))}$$

output
```
-4*(8*a^2-b^2)*arctanh((b-2*a*tanh(d*x+c))/(4*a^2+b^2)^(1/2))/(4*a^2+b^2)^
(5/2)/d-2*b*cosh(2*d*x+2*c)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))^2-12*a*b
*cosh(2*d*x+2*c)/(4*a^2+b^2)^2/d/(2*a+b*sinh(2*d*x+2*c))
```

3.860.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = \frac{2\left(\frac{2(8a^2-b^2)\arctan\left(\frac{b-2a\tanh(c+dx)}{\sqrt{-4a^2-b^2}}\right)}{\sqrt{-4a^2-b^2}} - \frac{b\cosh(2(c+dx))(16a^2+b^2+6ab\sinh(2(c+dx)))}{(2a+b\sinh(2(c+dx)))^2}\right)}{(4a^2+b^2)^2\,d}$$

input
```
Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3),x]
```

output
```
(2*((2*(8*a^2 - b^2)*ArcTan[(b - 2*a*Tanh[c + d*x])/Sqrt[-4*a^2 - b^2]])/S
qrt[-4*a^2 - b^2] - (b*Cosh[2*(c + d*x)]*(16*a^2 + b^2 + 6*a*b*Sinh[2*(c +
 d*x)]))/(2*a + b*Sinh[2*(c + d*x)])^2))/((4*a^2 + b^2)^2*d)
```

3.860.3 Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {3042, 3145, 3042, 3143, 27, 3042, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sinh(c+dx)\cosh(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-ib\sin(ic+idx)\cos(ic+idx))^3}dx$$

$$\Big\downarrow \text{3145}$$

$$\int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-\frac{1}{2}ib\sin(2ic+2idx)\right)^3}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{2\int -\frac{2(4a-b\sinh(2c+2dx))}{(2a+b\sinh(2c+2dx))^2}dx}{4a^2+b^2}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{4a-b\sinh(2c+2dx)}{(2a+b\sinh(2c+2dx))^2}dx}{4a^2+b^2}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}+\frac{4\int\frac{4a+ib\sin(2ic+2idx)}{(2a-ib\sin(2ic+2idx))^2}dx}{4a^2+b^2}$$

$$\Big\downarrow \text{3233}$$

$$\frac{4\left(-\frac{\int-\frac{8a^2-b^2}{2a+b\sinh(2c+2dx)}dx}{4a^2+b^2}-\frac{3ab\cosh(2c+2dx)}{d(4a^2+b^2)(2a+b\sinh(2c+2dx))}\right)}{4a^2+b^2}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{4\left(\frac{\int\frac{8a^2-b^2}{2a+b\sinh(2c+2dx)}dx}{4a^2+b^2}-\frac{3ab\cosh(2c+2dx)}{d(4a^2+b^2)(2a+b\sinh(2c+2dx))}\right)}{4a^2+b^2}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\left(\frac{(8a^2-b^2)\int\frac{1}{2a+b\sinh(2c+2dx)}dx}{4a^2+b^2}-\frac{3ab\cosh(2c+2dx)}{d(4a^2+b^2)(2a+b\sinh(2c+2dx))}\right)}{4a^2+b^2}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}+\frac{4\left(-\frac{3ab\cosh(2c+2dx)}{d(4a^2+b^2)(2a+b\sinh(2c+2dx))}+\frac{(8a^2-b^2)\int\frac{1}{2a-ib\sin(2ic+2idx)}dx}{4a^2+b^2}\right)}{4a^2+b^2}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}+\frac{4\left(-\frac{3ab\cosh(2c+2dx)}{d(4a^2+b^2)(2a+b\sinh(2c+2dx))}-\frac{i(8a^2-b^2)\int\frac{1}{-2a\tanh^2(c+dx)+2b\tanh(c+dx)+2a}d(i\tanh(c+dx))}{d(4a^2+b^2)}\right)}{4a^2+b^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}+\frac{4\left(-\frac{3ab\cosh(2c+2dx)}{d(4a^2+b^2)(2a+b\sinh(2c+2dx))}+\frac{2i(8a^2-b^2)\int\frac{1}{\tanh^2(c+dx)-4\left(4a^2+b^2\right)}d(4ia\tanh(c+dx)-2ib)}{d(4a^2+b^2)}\right)}{4a^2+b^2}$$

$$\Big\downarrow \text{217}$$

3.860. $\int\frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx$

$$\frac{4\left(\frac{(8a^2-b^2)\operatorname{arctanh}\left(\frac{\tanh(c+dx)}{2\sqrt{4a^2+b^2}}\right)}{d(4a^2+b^2)^{3/2}}-\frac{3ab\cosh(2c+2dx)}{d(4a^2+b^2)(2a+b\sinh(2c+2dx))}\right)}{4a^2+b^2}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)\left(2a+b\sinh(2c+2dx)\right)^2}$$

input
```
Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3),x]
```

output
```
(-2*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*(2*a + b*Sinh[2*c + 2*d*x])^2) +
 (4*(((8*a^2 - b^2)*ArcTanh[Tanh[c + d*x]/(2*Sqrt[4*a^2 + b^2])])/((4*a^2
+ b^2)^(3/2)*d) - (3*a*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*(2*a + b*Sinh
[2*c + 2*d*x]))))/(4*a^2 + b^2)
```

3.860.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.860.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(139) = 278$.

Time = 211.64 (sec) , antiderivative size = 481, normalized size of antiderivative = 3.36

method	result
risch	$\frac{32a^2 b\,e^{6dx+6c}-4b^3 e^{6dx+6c}+192a^3 e^{4dx+4c}-24a\,b^2 e^{4dx+4c}-160a^2 b\,e^{2dx+2c}-4b^3 e^{2dx+2c}+24a\,b^2}{d(4a^2+b^2)^2\left(4a\,e^{2dx+2c}+e^{4dx+4c}b-b\right)^2}+\frac{16\ln\left(e^{2dx+2c}+\cdots\right.}{\cdots}$
derivativedivides	$-\frac{2\left(-\frac{b^2(10a^2+b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{a(16a^4+8a^2b^2+b^4)}+\frac{(32a^4-14a^2b^2-b^4)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{a^2(16a^4+8a^2b^2+b^4)}+\frac{(58a^2+b^2)b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{a(16a^4+8a^2b^2+b^4)}-\frac{2b(32a^4+18a^2b^2+b^4)\cdots}{a^2(16a^4+8a^2\cdots}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-2\tanh\cdots\right.}$
default	$-\frac{2\left(-\frac{b^2(10a^2+b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{a(16a^4+8a^2b^2+b^4)}+\frac{(32a^4-14a^2b^2-b^4)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{a^2(16a^4+8a^2b^2+b^4)}+\frac{(58a^2+b^2)b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{a(16a^4+8a^2b^2+b^4)}-\frac{2b(32a^4+18a^2b^2+b^4)\cdots}{a^2(16a^4+8a^2\cdots}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-2\tanh\cdots\right.}$

input
```
int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
4*(8*a^2*b*exp(6*d*x+6*c)-b^3*exp(6*d*x+6*c)+48*a^3*exp(4*d*x+4*c)-6*a*b^2
*exp(4*d*x+4*c)-40*a^2*b*exp(2*d*x+2*c)-b^3*exp(2*d*x+2*c)+6*a*b^2)/d/(4*a
^2+b^2)^2/(4*a*exp(2*d*x+2*c)+exp(4*d*x+4*c)*b-b)^2+16/(4*a^2+b^2)^(5/2)/d
*ln(exp(2*d*x+2*c)+(2*(4*a^2+b^2)^(5/2)*a-64*a^6-48*a^4*b^2-12*a^2*b^4-b^6
)/(4*a^2+b^2)^(5/2)/b)*a^2-2/(4*a^2+b^2)^(5/2)/d*ln(exp(2*d*x+2*c)+(2*(4*a
^2+b^2)^(5/2)*a-64*a^6-48*a^4*b^2-12*a^2*b^4-b^6)/(4*a^2+b^2)^(5/2)/b)*b^2
-16/(4*a^2+b^2)^(5/2)/d*ln(exp(2*d*x+2*c)+(2*(4*a^2+b^2)^(5/2)*a+64*a^6+48
*a^4*b^2+12*a^2*b^4+b^6)/(4*a^2+b^2)^(5/2)/b)*a^2+2/(4*a^2+b^2)^(5/2)/d*ln
(exp(2*d*x+2*c)+(2*(4*a^2+b^2)^(5/2)*a+64*a^6+48*a^4*b^2+12*a^2*b^4+b^6)/(
4*a^2+b^2)^(5/2)/b)*b^2
```

3.860.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2439 vs. $2(142) = 284$.

Time = 0.29 (sec) , antiderivative size = 2439, normalized size of antiderivative = 17.06

$$\int \frac{1}{(a + b\cosh(c + dx)\sinh(c + dx))^3}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
2*(2*(32*a^4*b + 4*a^2*b^3 - b^5)*cosh(d*x + c)^6 + 12*(32*a^4*b + 4*a^2*b
^3 - b^5)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(32*a^4*b + 4*a^2*b^3 - b^5)*s
inh(d*x + c)^6 + 48*a^3*b^2 + 12*a*b^4 + 12*(32*a^5 + 4*a^3*b^2 - a*b^4)*c
osh(d*x + c)^4 + 6*(64*a^5 + 8*a^3*b^2 - 2*a*b^4 + 5*(32*a^4*b + 4*a^2*b^3
 - b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(32*a^4*b + 4*a^2*b^3 - b^
5)*cosh(d*x + c)^3 + 6*(32*a^5 + 4*a^3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*
x + c)^3 - 2*(160*a^4*b + 44*a^2*b^3 + b^5)*cosh(d*x + c)^2 - 2*(160*a^4*b
 + 44*a^2*b^3 + b^5 - 15*(32*a^4*b + 4*a^2*b^3 - b^5)*cosh(d*x + c)^4 - 36
*(32*a^5 + 4*a^3*b^2 - a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((8*a^2*b
^2 - b^4)*cosh(d*x + c)^8 + 8*(8*a^2*b^2 - b^4)*cosh(d*x + c)*sinh(d*x + c
)^7 + (8*a^2*b^2 - b^4)*sinh(d*x + c)^8 + 8*(8*a^3*b - a*b^3)*cosh(d*x + c
)^6 + 4*(16*a^3*b - 2*a*b^3 + 7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^2)*sinh(d*
x + c)^6 + 8*(7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^3 + 6*(8*a^3*b - a*b^3)*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(64*a^4 - 16*a^2*b^2 + b^4)*cosh(d*x + c)
^4 + 2*(35*(8*a^2*b^2 - b^4)*cosh(d*x + c)^4 + 64*a^4 - 16*a^2*b^2 + b^4 +
 60*(8*a^3*b - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 - b^4 +
 8*(7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^5 + 20*(8*a^3*b - a*b^3)*cosh(d*x +
c)^3 + (64*a^4 - 16*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(8*a
^3*b - a*b^3)*cosh(d*x + c)^2 + 4*(7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^6 + 3
0*(8*a^3*b - a*b^3)*cosh(d*x + c)^4 - 16*a^3*b + 2*a*b^3 + 3*(64*a^4 - ...
```

3.860.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

3.860.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(142) = 284$.

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.29

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = \frac{2\,(8\,a^2-b^2)\log\left(\frac{be^{(-2\,dx-2\,c)}-2\,a-\sqrt{4\,a^2+b^2}}{be^{(-2\,dx-2\,c)}-2\,a+\sqrt{4\,a^2+b^2}}\right)}{(16\,a^4+8\,a^2b^2+b^4)\sqrt{4\,a^2+b^2}d} - \frac{4\,(6\,ab^2+(40\,a^2b+b^3)e^{(-2\,dx-2\,c)}+6\,(8\,a^3-ab^2)e^{(-4\,dx-4\,c)}}{(16\,a^4b^2+8\,a^2b^4+b^6+8\,(16\,a^5b+8\,a^3b^3+ab^5)e^{(-2\,dx-2\,c)}+2\,(128\,a^6+48\,a^4b^2-b^6)e^{(-4\,dx-4\,c)}-8\,(16}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
2*(8*a^2 - b^2)*log((b*e^(-2*d*x - 2*c) - 2*a - sqrt(4*a^2 + b^2))/(b*e^(-
2*d*x - 2*c) - 2*a + sqrt(4*a^2 + b^2)))/((16*a^4 + 8*a^2*b^2 + b^4)*sqrt(
4*a^2 + b^2)*d) - 4*(6*a*b^2 + (40*a^2*b + b^3)*e^(-2*d*x - 2*c) + 6*(8*a^
3 - a*b^2)*e^(-4*d*x - 4*c) - (8*a^2*b - b^3)*e^(-6*d*x - 6*c))/((16*a^4*b
^2 + 8*a^2*b^4 + b^6 + 8*(16*a^5*b + 8*a^3*b^3 + a*b^5)*e^(-2*d*x - 2*c) +
 2*(128*a^6 + 48*a^4*b^2 - b^6)*e^(-4*d*x - 4*c) - 8*(16*a^5*b + 8*a^3*b^3
 + a*b^5)*e^(-6*d*x - 6*c) + (16*a^4*b^2 + 8*a^2*b^4 + b^6)*e^(-8*d*x - 8*
c))*d)
```

3.860.8 Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.79

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = \frac{2\left(\frac{(8\,a^2-b^2)\log\left(\frac{\left|2\,be^{(2\,dx+2\,c)}+4\,a-2\,\sqrt{4\,a^2+b^2}\right|}{\left|2\,be^{(2\,dx+2\,c)}+4\,a+2\,\sqrt{4\,a^2+b^2}\right|}\right)}{(16\,a^4+8\,a^2b^2+b^4)\sqrt{4\,a^2+b^2}} + \frac{2\,(8\,a^2be^{(6\,dx+6\,c)}-b^3e^{(6\,dx+6\,c)}+48\,a^3e^{(4\,dx+4\,c)}-6\,ab^2e^{(4\,dx+4\,c)}-40\,a^2be^{(2\,dx+2\,c)}-}{(16\,a^4+8\,a^2b^2+b^4)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-b\right)^2}\right.}{d}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
2*((8*a^2 - b^2)*log(abs(2*b*e^(2*d*x + 2*c) + 4*a - 2*sqrt(4*a^2 + b^2))/
abs(2*b*e^(2*d*x + 2*c) + 4*a + 2*sqrt(4*a^2 + b^2)))/((16*a^4 + 8*a^2*b^2
 + b^4)*sqrt(4*a^2 + b^2)) + 2*(8*a^2*b*e^(6*d*x + 6*c) - b^3*e^(6*d*x + 6
*c) + 48*a^3*e^(4*d*x + 4*c) - 6*a*b^2*e^(4*d*x + 4*c) - 40*a^2*b*e^(2*d*x
 + 2*c) - b^3*e^(2*d*x + 2*c) + 6*a*b^2)/((16*a^4 + 8*a^2*b^2 + b^4)*(b*e^
(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - b)^2))/d
```

3.860.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = \int \frac{1}{\left(a+b\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^3,x)
```

output
```
int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^3, x)
```

3.861 $\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx$

3.861.1 Optimal result

Integrand size = 20, antiderivative size = 301

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx = \frac{2\sqrt{2}ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d} + \frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d} - \frac{i(92a^2-9b^2)\,E\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)\middle|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{60\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}} + \frac{2i\sqrt{2}a(4a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right),\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{15d\sqrt{2a+b\sinh(2c+2dx)}}$$

output
```
1/40*b*cosh(2*d*x+2*c)*(2*a+b*sinh(2*d*x+2*c))^(3/2)/d*2^(1/2)+2/15*a*b*co
sh(2*d*x+2*c)*2^(1/2)*(2*a+b*sinh(2*d*x+2*c))^(1/2)/d+1/120*I*(92*a^2-9*b^
2)*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticE(cos(I*c
+1/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*(2*a+b*sinh(2*d*x+2*c))^(1/2)/
d*2^(1/2)/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)-2/15*I*a*(4*a^2+b^2)*(
sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticF(cos(I*c+1/4
*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*((2*a+b*sinh(2*d*x+2*c))/(
2*a-I*b))^(1/2)/d/(2*a+b*sinh(2*d*x+2*c))^(1/2)
```

3.861.2 Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.79

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx = \frac{2(184ia^3+92a^2b-18iab^2-9b^3)\,E\left(\frac{1}{4}(-4ic+\pi-4idx)\middle|-\frac{2ib}{2a-ib}\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}-32ia(4a}{}$$

input
```
Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(5/2),x]
```

output
```
(2*((184*I)*a^3 + 92*a^2*b - (18*I)*a*b^2 - 9*b^3)*EllipticE[((-4*I)*c + P
i - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])
/(2*a - I*b)] - (32*I)*a*(4*a^2 + b^2)*EllipticF[((-4*I)*c + Pi - (4*I)*d*
x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)
] + b*(88*a^2*Cosh[2*(c + d*x)] + b*(28*a + 3*b*Sinh[2*(c + d*x)])*Sinh[4*
(c + d*x)]))/(120*d*Sqrt[4*a + 2*b*Sinh[2*(c + d*x)]])
```

3.861.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.99, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.850, Rules used = {3042, 3145, 3042, 3135, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(c+dx)\cosh(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a-ib\sin(ic+idx)\cos(ic+idx))^{5/2}dx$$

$$\Big\downarrow \; 3145$$

$$\int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{5/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a-\frac{1}{2} i b \sin(2 i c+2 i d x)\right)^{5/2}dx$$

$$\Big\downarrow \text{ 3135}$$

$$\frac{2}{5}\int \frac{\sqrt{2a+b\sinh(2c+2dx)}\left(20a^2+16b\sinh(2c+2dx)a-3b^2\right)}{8\sqrt{2}}dx+\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \sqrt{2a+b\sinh(2c+2dx)}\left(20a^2+16b\sinh(2c+2dx)a-3b^2\right)dx}{20\sqrt{2}}+\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}+\frac{\int \sqrt{2a-ib\sin(2ic+2idx)}\left(20a^2-16ib\sin(2ic+2idx)a-3b^2\right)dx}{20\sqrt{2}}$$

$$\Big\downarrow \text{ 3232}$$

$$\frac{\frac{2}{3}\int \frac{2a(60a^2-17b^2)+b(92a^2-9b^2)\sinh(2c+2dx)}{2\sqrt{2a+b\sinh(2c+2dx)}}dx+\frac{16ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{3d}}{20\sqrt{2}}+\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{3}\int \frac{2a(60a^2-17b^2)+b(92a^2-9b^2)\sinh(2c+2dx)}{\sqrt{2a+b\sinh(2c+2dx)}}dx+\frac{16ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{3d}}{20\sqrt{2}}+\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}+\frac{\frac{16ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{3d}+\frac{1}{3}\int \frac{2a(60a^2-17b^2)-ib(92a^2-9b^2)\sin(2ic+2idx)}{\sqrt{2a-ib\sin(2ic+2idx)}}dx}{20\sqrt{2}}$$

$$\Big\downarrow \text{ 3231}$$

3.861. $\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx$

$$\frac{\frac{1}{3}\left(\left(92a^2-9b^2\right)\int\sqrt{2a+b\sinh(2c+2dx)}dx-16a\left(4a^2+b^2\right)\int\frac{1}{\sqrt{2a+b\sinh(2c+2dx)}}dx\right)+\frac{16ab\cosh(2c+2dx)\sqrt{2a+b\sinh(}}{3d}}{20\sqrt{2}}$$

$$\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$\downarrow$ 3042

$$\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}+$$

$$\frac{\frac{16ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{3d}+\frac{1}{3}\left(\left(92a^2-9b^2\right)\int\sqrt{2a-ib\sin(2ic+2idx)}dx-16a\left(4a^2+b^2\right)\int\frac{1}{\sqrt{2a-ib\sin(2ic+}}\right.}{20\sqrt{2}}$$

$\downarrow$ 3134

$$\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}+$$

$$\frac{\frac{16ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{3d}+\frac{1}{3}\left(\frac{\left(92a^2-9b^2\right)\sqrt{2a+b\sinh(2c+2dx)}\int\sqrt{\frac{2a}{2a-ib}+\frac{b\sinh(2c+2dx)}{2a-ib}}dx}{\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}-16a\left(4a^2+b^2\right)\int\frac{}{\sqrt{2a-ib}}\right.}{20\sqrt{2}}$$

$\downarrow$ 3042

$$\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}+$$

$$\frac{\frac{16ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{3d}+\frac{1}{3}\left(\frac{\left(92a^2-9b^2\right)\sqrt{2a+b\sinh(2c+2dx)}\int\sqrt{\frac{2a}{2a-ib}-\frac{ib\sin(2ic+2idx)}{2a-ib}}dx}{\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}-16a\left(4a^2+b^2\right)\int\frac{}{\sqrt{2a-}}\right.}{20\sqrt{2}}$$

$\downarrow$ 3132

$$\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}+$$

$$\frac{\frac{16ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{3d}+\frac{1}{3}\left(-16a\left(4a^2+b^2\right)\int\frac{1}{\sqrt{2a-ib\sin(2ic+2idx)}}dx-\frac{i\left(92a^2-9b^2\right)\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}(2\right.}{d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}\right.}{20\sqrt{2}}$$

$\downarrow$ 3142

$$\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}+$$

$$\frac{\frac{16ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{3d}+\frac{1}{3}\left(-\frac{16a\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\int\frac{1}{\sqrt{\frac{2a}{2a-ib}+\frac{b\sinh(2c+2dx)}{2a-ib}}}dx}{\sqrt{2a+b\sinh(2c+2dx)}}-\frac{i\left(92a^2-9b^2\right)\sqrt{2a+b\sinh(2}}{d\sqrt{\frac{2a}{}}}\right.}{20\sqrt{2}}$$

$\downarrow$ 3042

3.861. $\int(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx$

$$\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}+$$
$$\frac{\frac{16ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{3d}+\frac{1}{3}\left(-\frac{16a(4a^2+b^2)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\int\frac{1}{\sqrt{\frac{2a}{2a-ib}-\frac{ib\sin(2ic+2idx)}{2a-ib}}}dx}{\sqrt{2a+b\sinh(2c+2dx)}}-\frac{i(92a^2-9b^2)\sqrt{2a+b\sinh(}}{d\sqrt{2}}\right.}{20\sqrt{2}}$$

$\downarrow$ 3140

$$\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d}+$$
$$\frac{\frac{16ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{3d}+\frac{1}{3}\left(\frac{16ia(4a^2+b^2)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\text{EllipticF}\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right),\frac{2b}{2ia+b}\right)}{d\sqrt{2a+b\sinh(2c+2dx)}}-\frac{i(92a^2-9b^2)\sqrt{2a+b\,\mathrm{si}}}{d}\right.}{20\sqrt{2}}$$

input
```
Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(5/2),x]
```

output
```
(b*Cosh[2*c + 2*d*x]*(2*a + b*Sinh[2*c + 2*d*x])^(3/2))/(20*Sqrt[2]*d) + (
(16*a*b*Cosh[2*c + 2*d*x]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(3*d) + (((-I)*
(92*a^2 - 9*b^2)*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a
+ b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])
/(2*a - I*b)]) + ((16*I)*a*(4*a^2 + b^2)*EllipticF[((2*I)*c - Pi/2 + (2*I)
*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)
])/(d*Sqrt[2*a + b*Sinh[2*c + 2*d*x]]))/3)/(20*Sqrt[2])
```

3.861.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.861. $\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx$

3.861.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1259 vs. 2(331) = 662.

Time = 6.30 (sec) , antiderivative size = 1260, normalized size of antiderivative = 4.19

method	result	size
default	Expression too large to display	1260

input
```
int((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(64*I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I
)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(
2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^3*
b+16*I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/
(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+
b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a*b^3+24
0*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+
2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sin
h(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^4+24*(-(2*a
+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1
/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+
2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2*b^2-9*(-(2*a+b*si
nh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*(
(sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))
/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^4-368*(-(2*a+b*sinh(2*d*
x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2
*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2
*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^4-56*(-(2*a+b*sinh(2*d*x+2*c))/
(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c
)+I)*b/(I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^...
```

3.861.5 Fricas [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx = \int (b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d
*x + c) + a^2)*sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

3.861.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.861.7 Maxima [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx = \int (b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(5/2), x)
```

3.861.8 Giac [F(-2)]

Exception generated.

$$\int (a + b \cosh(c + dx) \sinh(c + dx))^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.861.9 Mupad [F(-1)]

Timed out.

$$\int (a + b \cosh(c + dx) \sinh(c + dx))^{5/2} \, dx = \int (a + b \cosh(c + d\,x)\,\sinh(c + d\,x))^{5/2} \, dx$$

input
```
int((a + b*cosh(c + d*x)*sinh(c + d*x))^(5/2),x)
```

output
```
int((a + b*cosh(c + d*x)*sinh(c + d*x))^(5/2), x)
```

3.862 $\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx$

3.862.1 Optimal result

Integrand size = 20, antiderivative size = 248

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx = \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d} - \frac{2i\sqrt{2}aE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{3d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}} + \frac{i(4a^2+b^2)\,\mathrm{EllipticF}\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right),\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{6\sqrt{2}d\sqrt{2a+b\sinh(2c+2dx)}}$$

output
```
1/12*b*cosh(2*d*x+2*c)*(2*a+b*sinh(2*d*x+2*c))^(1/2)/d*2^(1/2)+2/3*I*a*(si
n(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticE(cos(I*c+1/4*P
i+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*(2*a+b*sinh(2*d*x+2*c))^(1/2
)/d/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)-1/12*I*(4*a^2+b^2)*(sin(I*c+
1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticF(cos(I*c+1/4*Pi+I*d*
x),2^(1/2)*(b/(2*I*a+b))^(1/2))*((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)/
d*2^(1/2)/(2*a+b*sinh(2*d*x+2*c))^(1/2)
```

3.862.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.81

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx = \frac{16a(2ia+b)E\left(\frac{1}{4}(-4ic+\pi-4idx)\middle|-\frac{2ib}{2a-ib}\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}} - 2i(4a^2+b^2)\operatorname{EllipticF}\left(\frac{1}{4}(-4\right.}{12d\sqrt{4a+2b\sinh(}}$$

input
```
Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(3/2),x]
```

output
```
(16*a*((2*I)*a + b)*EllipticE[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2
*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)] - (2*I)*(4*a^2 +
b^2)*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt
[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)] + b*(4*a*Cosh[2*(c + d*x)] + b*S
inh[4*(c + d*x)]))/(12*d*Sqrt[4*a + 2*b*Sinh[2*(c + d*x)]])
```

3.862.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 3145, 3042, 3135, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(c+dx)\cosh(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a-ib\sin(ic+idx)\cos(ic+idx))^{3/2}dx$$

$$\Big\downarrow \; 3145$$

$$\int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a-\frac{1}{2}ib\sin(2ic+2idx)\right)^{3/2}dx$$

$$\Big\downarrow \; 3135$$

$$\frac{2}{3}\int \frac{12a^2+8b\sinh(2c+2dx)a-b^2}{4\sqrt{2}\sqrt{2a+b\sinh(2c+2dx)}}dx+\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{12a^2+8b\sinh(2c+2dx)a-b^2}{\sqrt{2a+b\sinh(2c+2dx)}}dx}{6\sqrt{2}}+\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}+\frac{\int \frac{12a^2-8ib\sin(2ic+2idx)a-b^2}{\sqrt{2a-ib\sin(2ic+2idx)}}dx}{6\sqrt{2}}$$

$$\Big\downarrow \; 3231$$

$$\frac{8a\int \sqrt{2a+b\sinh(2c+2dx)}dx-\left(4a^2+b^2\right)\int \frac{1}{\sqrt{2a+b\sinh(2c+2dx)}}dx}{6\sqrt{2}}+\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}+\frac{8a\int \sqrt{2a-ib\sin(2ic+2idx)}dx-\left(4a^2+b^2\right)\int \frac{1}{\sqrt{2a-ib\sin(2ic+2idx)}}dx}{6\sqrt{2}}$$

$$\Big\downarrow \; 3134$$

$$\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}+\frac{\frac{8a\sqrt{2a+b\sinh(2c+2dx)}\int \sqrt{\frac{2a}{2a-ib}+\frac{b\sinh(2c+2dx)}{2a-ib}}dx}{\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}-\left(4a^2+b^2\right)\int \frac{1}{\sqrt{2a-ib\sin(2ic+2idx)}}dx}{6\sqrt{2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}+\frac{\frac{8a\sqrt{2a+b\sinh(2c+2dx)}\int \sqrt{\frac{2a}{2a-ib}-\frac{ib\sin(2ic+2idx)}{2a-ib}}dx}{\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}-\left(4a^2+b^2\right)\int \frac{1}{\sqrt{2a-ib\sin(2ic+2idx)}}dx}{6\sqrt{2}}$$

$$\Big\downarrow \; 3132$$

3.862. $\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx$

$$\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}+\frac{-\left(4a^2+b^2\right)\int\frac{1}{\sqrt{2a-ib\sin(2ic+2idx)}}dx-\frac{8ia\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}}{6\sqrt{2}}$$

$\downarrow$ 3142

$$\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}+\frac{-\frac{\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\int\frac{1}{\sqrt{\frac{2a}{2a-ib}+\frac{b\sinh(2c+2dx)}{2a-ib}}}dx}{\sqrt{2a+b\sinh(2c+2dx)}}-\frac{8ia\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}}{6\sqrt{2}}$$

$\downarrow$ 3042

$$\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}+\frac{-\frac{\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\int\frac{1}{\sqrt{\frac{2a}{2a-ib}-\frac{ib\sin(2ic+2idx)}{2a-ib}}}dx}{\sqrt{2a+b\sinh(2c+2dx)}}-\frac{8ia\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}}{6\sqrt{2}}$$

$\downarrow$ 3140

$$\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}+\frac{\frac{i\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\text{EllipticF}\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right),\frac{2b}{2ia+b}\right)}{d\sqrt{2a+b\sinh(2c+2dx)}}-\frac{8ia\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}}{6\sqrt{2}}$$

input
```
Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(3/2),x]
```

output
```
(b*Cosh[2*c + 2*d*x]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(6*Sqrt[2]*d) + (((-
8*I)*a*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt
[2*a + b*Sinh[2*c + 2*d*x]])/(d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*
b)]) + (I*(4*a^2 + b^2)*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((
2*I)*a + b)]*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/(d*Sqrt[2*a +
b*Sinh[2*c + 2*d*x]]))/(6*Sqrt[2])
```

3.862. $\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx$

3.862.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.862.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 934 vs. 2(284) = 568.

Time = 0.64 (sec) , antiderivative size = 935, normalized size of antiderivative = 3.77

method	result
default	$4i\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}}\sqrt{\frac{(-\sinh(2dx+2c)+i)b}{ib+2a}}\sqrt{\frac{(\sinh(2dx+2c)+i)b}{ib-2a}}\ \operatorname{EllipticF}\left(\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}},\sqrt{-\frac{ib-2a}{ib+2a}}\right)a^2b+i\sqrt{-\frac{2a+b\sinh(}{ib-}}$

input
```
int((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(4*I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*
b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*
a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2*b+
I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+
2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sin
h(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^3+24*(-(2*a
+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1
/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+
2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^3+6*(-(2*a+b*sinh(2
*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sin
h(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*
b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a*b^2-32*(-(2*a+b*sinh(2*d*x+2
*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*
x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a)
)^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^3-8*(-(2*a+b*sinh(2*d*x+2*c))/(I*b
-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)
*b/(I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(
-(I*b-2*a)/(I*b+2*a))^(1/2))*a*b^2+sinh(2*d*x+2*c)^3*b^3+2*sinh(2*d*x+2*c)
^2*a*b^2+b^3*sinh(2*d*x+2*c)+2*a*b^2)/b/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*
x+2*c))^(1/2)/d
```

3.862.5 Fricas [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx=\int (b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cosh(d*x + c)*sinh(d*x + c) + a)^(3/2), x)
```

3.862.6 Sympy [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx=\int (a+b\sinh(c+dx)\cosh(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sinh(c + d*x)*cosh(c + d*x))**(3/2), x)
```

3.862.7 Maxima [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx=\int (b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(3/2), x)
```

3.862.8 Giac [F(-2)]

Exception generated.

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.862.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx = \int (a+b\cosh(c+d\,x)\,\sinh(c+d\,x))^{3/2}\,dx$$

input
```
int((a + b*cosh(c + d*x)*sinh(c + d*x))^(3/2),x)
```

output
```
int((a + b*cosh(c + d*x)*sinh(c + d*x))^(3/2), x)
```

3.863 $\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx$

3.863.1 Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = -\frac{iE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

output
```
1/2*I*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticE(cos(
I*c+1/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*(2*a+b*sinh(2*d*x+2*c))^(1/
2)/d*2^(1/2)/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)
```

3.863.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \frac{(2ia+b)E\left(\frac{1}{4}(-4ic+\pi-4idx)|-\frac{2ib}{2a-ib}\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}}{d\sqrt{4a+2b\sinh(2(c+dx))}}$$

input
```
Integrate[Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

output
```
(((2*I)*a + b)*EllipticE[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*a -
I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)])/(d*Sqrt[4*a + 2*b*Sin
h[2*(c + d*x)]])
```

3.863.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3145, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sinh(c+dx)\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-ib\sin(ic+idx)\cos(ic+idx)}dx$$

$$\Big\downarrow \text{3145}$$

$$\int \sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-\frac{1}{2}ib\sin(2ic+2idx)}dx$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{2a+b\sinh(2c+2dx)}\int \sqrt{\frac{2a}{2a-ib}+\frac{b\sinh(2c+2dx)}{2a-ib}}dx}{\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{2a+b\sinh(2c+2dx)}\int \sqrt{\frac{2a}{2a-ib}-\frac{ib\sin(2ic+2idx)}{2a-ib}}dx}{\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{i\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

input
```
Int[Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

output
```
((-I)*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[
2*a + b*Sinh[2*c + 2*d*x]])/(Sqrt[2]*d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2
*a - I*b)])
```

3.863.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.863.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(119) = 238$.

Time = 0.46 (sec) , antiderivative size = 352, normalized size of antiderivative = 3.67

method	result
default	$-\frac{(ib-2a)\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}}\,\sqrt{\frac{(-\sinh(2dx+2c)+i)b}{ib+2a}}\,\sqrt{\frac{(\sinh(2dx+2c)+i)b}{ib-2a}}\,\left(i\,\mathrm{EllipticF}\left(\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}},\sqrt{-\frac{ib-2a}{ib+2a}}\right)b-i\,\mathrm{Ellip}}{b\cosh(2dx+2}$

input
```
int((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(I*b-2*a)*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I
)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)/b*(I*Elliptic
F((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))
*b-I*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b
+2*a))^(1/2))*b+2*a*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(
-(I*b-2*a)/(I*b+2*a))^(1/2))-2*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*
a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a)/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2
*d*x+2*c))^(1/2)/d
```

3.863.5 Fricas [F]

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \int \sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

3.863.6 Sympy [F]

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \int \sqrt{a+b\sinh\left(c+dx\right)\cosh\left(c+dx\right)}\,dx$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(c + d*x)*cosh(c + d*x)), x)
```

3.863.7 Maxima [F]

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \int \sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

3.863.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.863.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \int \sqrt{a+b\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((a + b*cosh(c + d*x)*sinh(c + d*x))^(1/2),x)
```

output
```
int((a + b*cosh(c + d*x)*sinh(c + d*x))^(1/2), x)
```

3.864 $\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx$

3.864.1 Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = -\frac{i\sqrt{2}\,\text{EllipticF}\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right),\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{d\sqrt{2a+b\sinh(2c+2dx)}}$$

output
```
I*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticF(cos(I*c+
1/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*((2*a+b*sinh(2*d*x+2*c)
)/(2*a-I*b))^(1/2)/d/(2*a+b*sinh(2*d*x+2*c))^(1/2)
```

3.864.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = \frac{i\,\text{EllipticF}\left(\frac{1}{4}(-4ic+\pi-4idx),-\frac{2ib}{2a-ib}\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}}{d\sqrt{a+\frac{1}{2}b\sinh(2(c+dx))}}$$

input
```
Integrate[1/Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

output
```
(I*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(
2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)])/(d*Sqrt[a + (b*Sinh[2*(c + d*x)])
/2])
```

3.864.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3145, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sinh(c+dx)\cosh(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a-ib\sin(ic+idx)\cos(ic+idx)}}dx$$

$$\Big\downarrow \text{ 3145}$$

$$\int \frac{1}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a-\frac{1}{2}ib\sin(2ic+2idx)}}dx$$

$$\Big\downarrow \text{ 3142}$$

$$\frac{\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\int \frac{1}{\sqrt{\frac{2a}{2a-ib}+\frac{b\sinh(2c+2dx)}{2a-ib}}}dx}{\sqrt{2a+b\sinh(2c+2dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\int \frac{1}{\sqrt{\frac{2a}{2a-ib}-\frac{ib\sin(2ic+2idx)}{2a-ib}}}dx}{\sqrt{2a+b\sinh(2c+2dx)}}$$

$$\Big\downarrow \text{ 3140}$$

$$-\frac{i\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,\text{EllipticF}\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right),\frac{2b}{2ia+b}\right)}{d\sqrt{2a+b\sinh(2c+2dx)}}$$

input
```
Int[1/Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

output
```
((-I)*Sqrt[2]*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b
)]*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/(d*Sqrt[2*a + b*Sinh[2*c
 + 2*d*x]])
```

3.864.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.864.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.89

method	result	size
default	$-\frac{2(ib-2a)\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}}\,\sqrt{\frac{(-\sinh(2dx+2c)+i)b}{ib+2a}}\,\sqrt{\frac{(\sinh(2dx+2c)+i)b}{ib-2a}}\,\operatorname{EllipticF}\left(\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}},\sqrt{-\frac{ib-2a}{ib+2a}}\right)}{b\cosh(2dx+2c)\sqrt{4a+2b\sinh(2dx+2c)}\,d}$	181

input
```
int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(I*b-2*a)*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)
+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((
-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))/b/
cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

3.864.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.49

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx =$$

$$-\frac{2\left(\sqrt{-b}b\sqrt{\frac{4a^2+b^2}{b^2}}-2a\sqrt{-b}\right)\sqrt{\frac{b\sqrt{\frac{4a^2+b^2}{b^2}}+2a}{b}}F(\arcsin\left(\sqrt{\frac{b\sqrt{\frac{4a^2+b^2}{b^2}}+2a}{b}}(\cosh(dx+c)+\sinh(dx+c))\right)}{b^2d}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(-b)*b*sqrt((4*a^2 + b^2)/b^2) - 2*a*sqrt(-b))*sqrt((b*sqrt((4*a^2
 + b^2)/b^2) + 2*a)/b)*elliptic_f(arcsin(sqrt((b*sqrt((4*a^2 + b^2)/b^2) +
 2*a)/b)*(cosh(d*x + c) + sinh(d*x + c))), (4*a*b*sqrt((4*a^2 + b^2)/b^2)
- 8*a^2 - b^2)/b^2)/(b^2*d)
```

3.864.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sinh{(c+dx)}\cosh{(c+dx)}}}\,dx$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sinh(c + d*x)*cosh(c + d*x)), x)
```

3.864.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cosh{(dx+c)}\sinh{(dx+c)}+a}}\,dx$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

3.864.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.864.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(1/2),x)
```

output
```
int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(1/2), x)
```

3.865 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx$

3.865.1 Optimal result

Integrand size = 20, antiderivative size = 158

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx=-\frac{2\sqrt{2}b\cosh(2c+2dx)}{(4a^2+b^2)\,d\sqrt{2a+b\sinh(2c+2dx)}}-\frac{2i\sqrt{2}E\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{(4a^2+b^2)\,d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

output
```
-2*b*cosh(2*d*x+2*c)*2^(1/2)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))^(1/2)+2
*I*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticE(cos(I*c
+1/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*(2*a+b*sinh(2*d*x+2*c)
)^(1/2)/(4*a^2+b^2)/d/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)
```

3.865.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx=\frac{2\Big(-b\cosh(2(c+dx))+(2ia+b)E\left(\frac{1}{4}(-4ic+\pi-4idx)|-\frac{2ib}{2a-\cdots}\right.\cdots}{(4a^2+b^2)\,d\sqrt{a+\frac{1}{2}b\sinh(2(c+dx))}}$$

input
```
Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3/2),x]
```

output
```
(2*(-(b*Cosh[2*(c + d*x)]) + ((2*I)*a + b)*EllipticE[((-4*I)*c + Pi - (4*I
)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a -
I*b)]))/((4*a^2 + b^2)*d*Sqrt[a + (b*Sinh[2*(c + d*x)])/2])
```

3.865.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3042, 3145, 3042, 3143, 27, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sinh(c+dx)\cosh(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-ib\sin(ic+idx)\cos(ic+idx))^{3/2}}dx$$

$$\Big\downarrow \text{3145}$$

$$\int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-\frac{1}{2}ib\sin(2ic+2idx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{8\int -\frac{\sqrt{2a+b\sinh(2c+2dx)}}{2\sqrt{2}}dx}{4a^2+b^2}-\frac{2\sqrt{2}b\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{2}\int \sqrt{2a+b\sinh(2c+2dx)}dx}{4a^2+b^2}-\frac{2\sqrt{2}b\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{2}b\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}+\frac{2\sqrt{2}\int \sqrt{2a-ib\sin(2ic+2idx)}dx}{4a^2+b^2}$$

$$\Big\downarrow \text{3134}$$

$$-\frac{2\sqrt{2}b\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}+\frac{2\sqrt{2}\sqrt{2a+b\sinh(2c+2dx)}\int\sqrt{\frac{2a}{2a-ib}+\frac{b\sinh(2c+2dx)}{2a-ib}}dx}{(4a^2+b^2)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{2}b\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}+\frac{2\sqrt{2}\sqrt{2a+b\sinh(2c+2dx)}\int\sqrt{\frac{2a}{2a-ib}-\frac{ib\sin(2ic+2idx)}{2a-ib}}dx}{(4a^2+b^2)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{2\sqrt{2}b\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}-\frac{2i\sqrt{2}\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d(4a^2+b^2)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

input
```
Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3/2),x]
```

output
```
(-2*Sqrt[2]*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*Sqrt[2*a + b*Sinh[2*c +
2*d*x]]) - ((2*I)*Sqrt[2]*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/
((2*I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/((4*a^2 + b^2)*d*Sqrt[(2*a
 + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])
```

3.865.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.865.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(177) = 354$.

Time = 0.41 (sec) , antiderivative size = 630, normalized size of antiderivative = 3.99

method	result
default	$16\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}}\,\sqrt{\frac{(-\sinh(2dx+2c)+i)b}{ib+2a}}\,\sqrt{\frac{(\sinh(2dx+2c)+i)b}{ib-2a}}\,\text{EllipticF}\left(\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}},\sqrt{-\frac{ib-2a}{ib+2a}}\right)a^2+4\sqrt{-\frac{2a+b\sinh(}{ib-2}}$

input
```
int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4*(4*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I
*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*
sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2+(-(2*a
+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1
/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+
2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^2-4*(-(2*a+b*sinh(2
*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sin
h(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*
b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2-(-(2*a+b*sinh(2*d*x+2*c))/
(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c
)+I)*b/(I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/
2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^2-b^2*sinh(2*d*x+2*c)^2-b^2)/(4*a^2+b^2
)/b/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

3.865.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1283 vs. 2(168) = 336.

Time = 0.11 (sec) , antiderivative size = 1283, normalized size of antiderivative = 8.12

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-4*(((b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh
(d*x + c)^4 + 4*a*b^2*cosh(d*x + c)^2 - b^3 + 2*(3*b^3*cosh(d*x + c)^2 + 2
*a*b^2)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + 2*a*b^2*cosh(d*x + c))*
sinh(d*x + c))*sqrt(-b)*sqrt((4*a^2 + b^2)/b^2) + 2*(a*b^2*cosh(d*x + c)^4
 + 4*a*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^2*sinh(d*x + c)^4 + 4*a^2*b
*cosh(d*x + c)^2 - a*b^2 + 2*(3*a*b^2*cosh(d*x + c)^2 + 2*a^2*b)*sinh(d*x
+ c)^2 + 4*(a*b^2*cosh(d*x + c)^3 + 2*a^2*b*cosh(d*x + c))*sinh(d*x + c))*
sqrt(-b))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)*elliptic_e(arcsin(sqrt
((b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)*(cosh(d*x + c) + sinh(d*x + c))), (4
*a*b*sqrt((4*a^2 + b^2)/b^2) - 8*a^2 - b^2)/b^2) + (((2*a*b^2 - b^3)*cosh(
d*x + c)^4 + 4*(2*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b^2 -
b^3)*sinh(d*x + c)^4 - 2*a*b^2 + b^3 + 4*(2*a^2*b - a*b^2)*cosh(d*x + c)^2
 + 2*(4*a^2*b - 2*a*b^2 + 3*(2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 4*((2*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(2*a^2*b - a*b^2)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-b)*sqrt((4*a^2 + b^2)/b^2) - 2*((2*a^2*b + a*b^2)*
cosh(d*x + c)^4 + 4*(2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a
^2*b + a*b^2)*sinh(d*x + c)^4 - 2*a^2*b - a*b^2 + 4*(2*a^3 + a^2*b)*cosh(d
*x + c)^2 + 2*(4*a^3 + 2*a^2*b + 3*(2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 4*((2*a^2*b + a*b^2)*cosh(d*x + c)^3 + 2*(2*a^3 + a^2*b)*cos
h(d*x + c))*sinh(d*x + c))*sqrt(-b))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) + ...
```

3.865.6 Sympy [F]

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sinh(c+dx)\cosh(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sinh(c + d*x)*cosh(c + d*x))**(-3/2), x)
```

3.865.7 Maxima [F]

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(-3/2), x)
```

3.865.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Error: Bad Argument Type
```

3.865.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\cosh(c+d\,x)\,\sinh(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(3/2),x)
```

output
```
int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(3/2), x)
```

3.866 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx$

3.866.1 Optimal result

Integrand size = 20, antiderivative size = 325

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = -\frac{4\sqrt{2}b\cosh(2c+2dx)}{3(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}ab\cosh(2c+2dx)}{3(4a^2+b^2)^2\,d\sqrt{2a+b\sinh(2c+2dx)}} - \frac{32i\sqrt{2}aE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{3(4a^2+b^2)^2\,d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}} + \frac{4i\sqrt{2}\,\text{EllipticF}\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right),\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{3(4a^2+b^2)\,d\sqrt{2a+b\sinh(2c+2dx)}}$$

output
```
-4/3*b*cosh(2*d*x+2*c)*2^(1/2)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))^(3/2)
-32/3*a*b*cosh(2*d*x+2*c)*2^(1/2)/(4*a^2+b^2)^2/d/(2*a+b*sinh(2*d*x+2*c))^
(1/2)+32/3*I*a*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*Ellip
ticE(cos(I*c+1/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*(2*a+b*sin
h(2*d*x+2*c))^(1/2)/(4*a^2+b^2)^2/d/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1
/2)-4/3*I*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticF(
cos(I*c+1/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*((2*a+b*sinh(2*
d*x+2*c))/(2*a-I*b))^(1/2)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))^(1/2)
```

3.866.2 Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \frac{4\sqrt{2}\left(8ia(2a-ib)^2E\left(\frac{1}{4}(-4ic+\pi-4idx)\middle|-\frac{2ib}{2a-ib}\right)\left(\frac{2a+b\sinh(2(c}{2a-ib}\right.\right.}{}$$

input
```
Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-5/2),x]
```

output
```
(4*Sqrt[2]*((8*I)*a*(2*a - I*b)^2*EllipticE[((-4*I)*c + Pi - (4*I)*d*x)/4,
 ((-2*I)*b)/(2*a - I*b)]*((2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b))^(3/2) +
 (2*a - I*b)^2*((-2*I)*a + b)*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-
2*I)*b)/(2*a - I*b)]*((2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b))^(3/2) - b*C
osh[2*(c + d*x)]*(20*a^2 + b^2 + 8*a*b*Sinh[2*(c + d*x)])))/(3*(4*a^2 + b^
2)^2*d*(2*a + b*Sinh[2*(c + d*x)])^(3/2))
```

3.866.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3145, 3042, 3143, 25, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sinh(c+dx)\cosh(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-ib\sin(ic+idx)\cos(ic+idx))^{5/2}}dx$$

$$\Big\downarrow \text{3145}$$

$$\int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-\frac{1}{2}ib\sin(2ic+2idx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{8\int -\frac{6a-b\sinh(2c+2dx)}{\sqrt{2}(2a+b\sinh(2c+2dx))^{3/2}}dx}{3\left(4a^2+b^2\right)}-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{8\int \frac{6a-b\sinh(2c+2dx)}{\sqrt{2}(2a+b\sinh(2c+2dx))^{3/2}}dx}{3\left(4a^2+b^2\right)}-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\sqrt{2}\int \frac{6a-b\sinh(2c+2dx)}{(2a+b\sinh(2c+2dx))^{3/2}}dx}{3\left(4a^2+b^2\right)}-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}+\frac{4\sqrt{2}\int \frac{6a+ib\sin(2ic+2idx)}{(2a-ib\sin(2ic+2idx))^{3/2}}dx}{3\left(4a^2+b^2\right)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{4\sqrt{2}\left(-\frac{2\int -\frac{12a^2+8b\sinh(2c+2dx)a-b^2}{2\sqrt{2a+b\sinh(2c+2dx)}}dx}{4a^2+b^2}-\frac{8ab\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}\right)}{3\left(4a^2+b^2\right)}-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\sqrt{2}\left(\frac{\int \frac{12a^2+8b\sinh(2c+2dx)a-b^2}{\sqrt{2a+b\sinh(2c+2dx)}}dx}{4a^2+b^2}-\frac{8ab\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}\right)}{3\left(4a^2+b^2\right)}-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}+\frac{4\sqrt{2}\left(-\frac{8ab\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}+\frac{\int \frac{12a^2-8ib\sin(2ic+2idx)a-b^2}{\sqrt{2a-ib\sin(2ic+2idx)}}dx}{4a^2+b^2}\right)}{3\left(4a^2+b^2\right)}$$

$$\Big\downarrow \text{3231}$$

3.866. $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx$

$$\frac{4\sqrt{2}\left(\frac{8a\int\sqrt{2a+b\sinh(2c+2dx)}dx-(4a^2+b^2)\int\frac{1}{\sqrt{2a+b\sinh(2c+2dx)}}dx}{4a^2+b^2}-\frac{8ab\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}\right)}{3\left(4a^2+b^2\right)}-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}+\frac{4\sqrt{2}\left(-\frac{8ab\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}+\frac{8a\int\sqrt{2a-ib\sin(2ic+2idx)}dx-(4a^2+b^2)\int\frac{1}{\sqrt{2a-ib\sin(2ic+2idx)}}dx}{4a^2+b^2}\right)}{3\left(4a^2+b^2\right)}$$

$$\Big\downarrow \; 3134$$

$$-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}+\frac{4\sqrt{2}\left(-\frac{8ab\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}+\frac{\frac{8a\sqrt{2a+b\sinh(2c+2dx)}\int\sqrt{\frac{2a}{2a-ib}+\frac{b\sinh(2c+2dx)}{2a-ib}}dx}{\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}-(4a^2+b^2)\int\frac{1}{\sqrt{2a-ib\sin(2ic+2idx)}}dx}{4a^2+b^2}\right)}{3\left(4a^2+b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}+\frac{4\sqrt{2}\left(-\frac{8ab\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}+\frac{\frac{8a\sqrt{2a+b\sinh(2c+2dx)}\int\sqrt{\frac{2a}{2a-ib}-\frac{ib\sin(2ic+2idx)}{2a-ib}}dx}{\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}-(4a^2+b^2)\int\frac{1}{\sqrt{2a-ib\sin(2ic+2idx)}}dx}{4a^2+b^2}\right)}{3\left(4a^2+b^2\right)}$$

$$\Big\downarrow \; 3132$$

$$-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}+\frac{4\sqrt{2}\left(-\frac{8ab\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}+\frac{-(4a^2+b^2)\int\frac{1}{\sqrt{2a-ib\sin(2ic+2idx)}}dx-\frac{8ia\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}}{4a^2+b^2}\right)}{3\left(4a^2+b^2\right)}$$

$$\Big\downarrow \; 3142$$

3.866. $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx$

$$-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)\left(2a+b\sinh(2c+2dx)\right)^{3/2}}+\frac{4\sqrt{2}\left(-\frac{8ab\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}+\frac{-\frac{\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\int\frac{1}{\sqrt{\frac{2a}{2a-ib}+\frac{b\sinh(2c+2dx)}{2a-ib}}}dx}{\sqrt{2a+b\sinh(2c+2dx)}}-\frac{8ia\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\right.\right.}{d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}}{4a^2+b^2}\right)}{3\left(4a^2+b^2\right)}$$

$\downarrow$ 3042

$$-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)\left(2a+b\sinh(2c+2dx)\right)^{3/2}}+\frac{4\sqrt{2}\left(-\frac{8ab\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}+\frac{-\frac{\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\int\frac{1}{\sqrt{\frac{2a}{2a-ib}-\frac{ib\sin(2ic+2idx)}{2a-ib}}}dx}{\sqrt{2a+b\sinh(2c+2dx)}}-\frac{8ia\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx\right.\right.}{d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}}{4a^2+b^2}\right)}{3\left(4a^2+b^2\right)}$$

$\downarrow$ 3140

$$-\frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)\left(2a+b\sinh(2c+2dx)\right)^{3/2}}+\frac{4\sqrt{2}\left(-\frac{8ab\cosh(2c+2dx)}{d(4a^2+b^2)\sqrt{2a+b\sinh(2c+2dx)}}+\frac{\frac{i\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\operatorname{EllipticF}\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right),\frac{2b}{2ia+b}\right)}{d\sqrt{2a+b\sinh(2c+2dx)}}-\frac{8ia\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2i\right.\right.}{d\sqrt{\frac{2a+b\sinh(2c+2dx}{2a-ib}}}}{4a^2+b^2}\right)}{3\left(4a^2+b^2\right)}$$

input
```
Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-5/2),x]
```

output
```
(-4*Sqrt[2]*b*Cosh[2*c + 2*d*x])/(3*(4*a^2 + b^2)*d*(2*a + b*Sinh[2*c + 2*
d*x])^(3/2)) + (4*Sqrt[2]*((-8*a*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*Sqr
t[2*a + b*Sinh[2*c + 2*d*x]]) + (((-8*I)*a*EllipticE[((2*I)*c - Pi/2 + (2*
I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(d*Sqrt[(
2*a + b*Sinh[2*c + 2*d*x]])/(2*a - I*b)]) + (I*(4*a^2 + b^2)*EllipticF[((2*
I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[(2*a + b*Sinh[2*c +
2*d*x])/(2*a - I*b)])/(d*Sqrt[2*a + b*Sinh[2*c + 2*d*x]]))/(4*a^2 + b^2)))
/(3*(4*a^2 + b^2))
```

3.866. $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx$

3.866.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.866. $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx$

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.866.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.97

method	result
default	$\frac{4\sqrt{\cosh(2dx+2c)^2(2a+b\sinh(2dx+2c))}\left(-\frac{2\sqrt{\cosh(2dx+2c)^2(2a+b\sinh(2dx+2c))}}{3b\left(4a^2+b^2\right)\left(\sinh(2dx+2c)+\frac{2a}{b}\right)^2}-\frac{16b\cosh(2dx+2c)^2a}{3\left(4a^2+b^2\right)^2\sqrt{\cosh(2dx+2c)^2(2a+b\sinh(2dx+2c))}}+\frac{2(}{}\right.}{}$
risch	Expression too large to display

input
```
int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
4*(cosh(2*d*x+2*c)^2*(2*a+b*sinh(2*d*x+2*c)))^(1/2)*(-2/3/b/(4*a^2+b^2)*(c
osh(2*d*x+2*c)^2*(2*a+b*sinh(2*d*x+2*c)))^(1/2)/(sinh(2*d*x+2*c)+2/b*a)^2-
16/3*b*cosh(2*d*x+2*c)^2/(4*a^2+b^2)^2*a/(cosh(2*d*x+2*c)^2*(2*a+b*sinh(2*
d*x+2*c)))^(1/2)+2*(12*a^2-b^2)/(48*a^4+24*a^2*b^2+3*b^4)*(2/b*a-I)*((-b*s
inh(2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1
/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)/(cosh(2*d*x+2*c)^2*(2*a+b*sinh
(2*d*x+2*c)))^(1/2)*EllipticF(((-b*sinh(2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2),(
(2*a-I*b)/(I*b+2*a))^(1/2))+16/3*a*b/(4*a^2+b^2)^2*(2/b*a-I)*((-b*sinh(2*d
*x+2*c)-2*a)/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((s
inh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)/(cosh(2*d*x+2*c)^2*(2*a+b*sinh(2*d*x+
2*c)))^(1/2)*((-2/b*a-I)*EllipticE(((-b*sinh(2*d*x+2*c)-2*a)/(I*b-2*a))^(1
/2),((2*a-I*b)/(I*b+2*a))^(1/2))+I*EllipticF(((-b*sinh(2*d*x+2*c)-2*a)/(I*
b-2*a))^(1/2),((2*a-I*b)/(I*b+2*a))^(1/2))))/cosh(2*d*x+2*c)/(4*a+2*b*sinh
(2*d*x+2*c))^(1/2)/d
```

3.866.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4231 vs. $2(337) = 674$.

Time = 0.19 (sec) , antiderivative size = 4231, normalized size of antiderivative = 13.02

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-8/3*(8*((a*b^4*cosh(d*x + c)^8 + 8*a*b^4*cosh(d*x + c)*sinh(d*x + c)^7 +
a*b^4*sinh(d*x + c)^8 + 8*a^2*b^3*cosh(d*x + c)^6 - 8*a^2*b^3*cosh(d*x + c
)^2 + 4*(7*a*b^4*cosh(d*x + c)^2 + 2*a^2*b^3)*sinh(d*x + c)^6 + 8*(7*a*b^4
*cosh(d*x + c)^3 + 6*a^2*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + a*b^4 + 2*(8
*a^3*b^2 - a*b^4)*cosh(d*x + c)^4 + 2*(35*a*b^4*cosh(d*x + c)^4 + 60*a^2*b
^3*cosh(d*x + c)^2 + 8*a^3*b^2 - a*b^4)*sinh(d*x + c)^4 + 8*(7*a*b^4*cosh(
d*x + c)^5 + 20*a^2*b^3*cosh(d*x + c)^3 + (8*a^3*b^2 - a*b^4)*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(7*a*b^4*cosh(d*x + c)^6 + 30*a^2*b^3*cosh(d*x + c)
^4 - 2*a^2*b^3 + 3*(8*a^3*b^2 - a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
8*(a*b^4*cosh(d*x + c)^7 + 6*a^2*b^3*cosh(d*x + c)^5 - 2*a^2*b^3*cosh(d*x
+ c) + (8*a^3*b^2 - a*b^4)*cosh(d*x + c)^3)*sinh(d*x + c))*sqrt(-b)*sqrt((
4*a^2 + b^2)/b^2) + 2*(a^2*b^3*cosh(d*x + c)^8 + 8*a^2*b^3*cosh(d*x + c)*s
inh(d*x + c)^7 + a^2*b^3*sinh(d*x + c)^8 + 8*a^3*b^2*cosh(d*x + c)^6 - 8*a
^3*b^2*cosh(d*x + c)^2 + 4*(7*a^2*b^3*cosh(d*x + c)^2 + 2*a^3*b^2)*sinh(d*
x + c)^6 + 8*(7*a^2*b^3*cosh(d*x + c)^3 + 6*a^3*b^2*cosh(d*x + c))*sinh(d*
x + c)^5 + a^2*b^3 + 2*(8*a^4*b - a^2*b^3)*cosh(d*x + c)^4 + 2*(35*a^2*b^3
*cosh(d*x + c)^4 + 60*a^3*b^2*cosh(d*x + c)^2 + 8*a^4*b - a^2*b^3)*sinh(d*
x + c)^4 + 8*(7*a^2*b^3*cosh(d*x + c)^5 + 20*a^3*b^2*cosh(d*x + c)^3 + (8*
a^4*b - a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^2*b^3*cosh(d*x +
c)^6 + 30*a^3*b^2*cosh(d*x + c)^4 - 2*a^3*b^2 + 3*(8*a^4*b - a^2*b^3)*c...
```

3.866.6 Sympy [F]

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\sinh{(c+dx)}\cosh{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sinh(c + d*x)*cosh(c + d*x))**(-5/2), x)
```

3.866.7 Maxima [F]

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(-5/2), x)
```

3.866.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Error: Bad Argument Type
```

3.866.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\cosh(c+d\,x)\,\sinh(c+d\,x))^{5/2}}\,dx$$

input
```
int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(5/2),x)
```

output
```
int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(5/2), x)
```

3.867 $\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx$

3.867.1 Optimal result

Integrand size = 14, antiderivative size = 386

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{4\sqrt{4a^2+b^2}} - \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{4\sqrt{4a^2+b^2}}$$

output
```
x^3*ln(1+b*exp(2*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-x^3*ln(1+b*
exp(2*x)/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+3/2*x^2*polylog(2,-b*e
xp(2*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-3/2*x^2*polylog(2,-b*ex
p(2*x)/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-3/2*x*polylog(3,-b*exp(2
*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+3/2*x*polylog(3,-b*exp(2*x)
/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+3/4*polylog(4,-b*exp(2*x)/(2*a
-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-3/4*polylog(4,-b*exp(2*x)/(2*a+(4*a
^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)
```

3.867.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \frac{4x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)-4x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)+6x^2\operatorname{PolyLog}\left(2,\frac{be^{2x}}{-2a+\sqrt{4a^2+b^2}}\right)-6x^2\operatorname{PolyLog}\left(2,\right.}{}$$

input
```
Integrate[x^3/(a + b*Cosh[x]*Sinh[x]),x]
```

output
```
(4*x^3*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])] - 4*x^3*Log[1 + (b*E
^(2*x))/(2*a + Sqrt[4*a^2 + b^2])] + 6*x^2*PolyLog[2, (b*E^(2*x))/(-2*a +
Sqrt[4*a^2 + b^2])] - 6*x^2*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b
^2]))] - 6*x*PolyLog[3, (b*E^(2*x))/(-2*a + Sqrt[4*a^2 + b^2])] + 6*x*Poly
Log[3, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))] + 3*PolyLog[4, (b*E^(2*x)
)/(-2*a + Sqrt[4*a^2 + b^2])] - 3*PolyLog[4, -((b*E^(2*x))/(2*a + Sqrt[4*a
^2 + b^2]))])/(4*Sqrt[4*a^2 + b^2])
```

3.867.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6162, 3042, 3803, 27, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\sinh(x)\cosh(x)}\,dx$$

$$\Big\downarrow \text{ 6162}$$

$$\int \frac{x^3}{a+\frac{1}{2}b\sinh(2x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{x^3}{a-\frac{1}{2}ib\sin(2ix)}dx$$

$$\Big\downarrow \text{ 3803}$$

$$2\int -\frac{2e^{2x}x^3}{-4e^{2x}a-be^{4x}+b}dx$$

$$\Big\downarrow \text{ 27}$$

$$-4\int \frac{e^{2x}x^3}{-4e^{2x}a-be^{4x}+b}dx$$

$$\Big\downarrow \text{ 2694}$$

$$-4\left(\frac{b\int -\frac{e^{2x}x^3}{2\left(2a+be^{2x}-\sqrt{4a^2+b^2}\right)}dx}{\sqrt{4a^2+b^2}}-\frac{b\int -\frac{e^{2x}x^3}{2\left(2a+be^{2x}+\sqrt{4a^2+b^2}\right)}dx}{\sqrt{4a^2+b^2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-4\left(\frac{b\int \frac{e^{2x}x^3}{2a+be^{2x}+\sqrt{4a^2+b^2}}dx}{2\sqrt{4a^2+b^2}}-\frac{b\int \frac{e^{2x}x^3}{2a+be^{2x}-\sqrt{4a^2+b^2}}dx}{2\sqrt{4a^2+b^2}}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$-4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{2b}-\frac{3\int x^2\log\left(\frac{e^{2x}b}{2a+\sqrt{4a^2+b^2}}+1\right)dx}{2b}\right)}{2\sqrt{4a^2+b^2}}-\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{2b}-\frac{3\int x^2\log\left(\frac{e^{2x}b}{2a-\sqrt{4a^2+b^2}}+1\right)dx}{2b}\right)}{2\sqrt{4a^2+b^2}}\right)$$

$$\Big\downarrow \text{ 3011}$$

3.867. $\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx$

$$-4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{2b}-\frac{3\left(\int x\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)dx-\frac{1}{2}x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)\right)}{2b}\right)}{2\sqrt{4a^2+b^2}}-\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b}}\right.}{2b}\right.}{}\right.$$

$\downarrow$ 7163

$$-4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{2b}-\frac{3\left(-\frac{1}{2}\int\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)dx-\frac{1}{2}x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)+\frac{1}{2}x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b}}\right.\right.}{2b}\right.}{2\sqrt{4a^2+b^2}}\right.$$

$\downarrow$ 2720

$$-4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{2b}-\frac{3\left(-\frac{1}{4}\int e^{-2x}\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)de^{2x}-\frac{1}{2}x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)+\frac{1}{2}x\operatorname{PolyLog}\left(3,-\frac{}{2a+}\right.\right.}{2b}\right.}{2\sqrt{4a^2+b^2}}\right.$$

$\downarrow$ 7143

$$-4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{2b}-\frac{3\left(-\frac{1}{2}x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)+\frac{1}{2}x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)-\frac{1}{4}\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)\right)}{2b}\right.}{2\sqrt{4a^2+b^2}}\right.$$

input
```
Int[x^3/(a + b*Cosh[x]*Sinh[x]),x]
```

3.867. $\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx$

output
```
-4*(-1/2*(b*((x^3*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/(2*b) -
(3*(-1/2*(x^2*PolyLog[2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]) + (x*P
olyLog[3, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/2 - PolyLog[4, -((b*E
^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]/4))/(2*b)))/Sqrt[4*a^2 + b^2] + (b*((x
^3*Log[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/(2*b) - (3*(-1/2*(x^2*P
olyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]) + (x*PolyLog[3, -((b*
E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))])/2 - PolyLog[4, -((b*E^(2*x))/(2*a +
Sqrt[4*a^2 + b^2]))]/4))/(2*b)))/(2*Sqrt[4*a^2 + b^2]))
```

3.867.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6162
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sinh[2*c +
 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.867.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(334) = 668$.

Time = 1.55 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.78

method	result
risch	$\frac{\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)x^3}{-2a-\sqrt{4a^2+b^2}}+\frac{2\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)a\,x^3}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2}\right)}-\frac{x^4}{2\left(-2a-\sqrt{4a^2+b^2}\right)}-\frac{a\,x^4}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2}\right)}+\frac{3\,\text{polylog}\left(\ldots\right.}{2\left(-\ldots\right.}$

input
```
int(x^3/(a+b*cosh(x)*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x^3+2
/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b
^2)^(1/2)))*a*x^3-1/2/(-2*a-(4*a^2+b^2)^(1/2))*x^4-1/(4*a^2+b^2)^(1/2)/(-2
*a-(4*a^2+b^2)^(1/2))*a*x^4+3/2/(-2*a-(4*a^2+b^2)^(1/2))*polylog(2,b*exp(2
*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x^2+3/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1
/2))*polylog(2,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a*x^2-3/2/(-2*a-(4*a^2
+b^2)^(1/2))*polylog(3,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x-3/(4*a^2+b^2
)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*polylog(3,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1
/2)))*a*x+3/4/(-2*a-(4*a^2+b^2)^(1/2))*polylog(4,b*exp(2*x)/(-2*a-(4*a^2+b
^2)^(1/2)))+3/2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*polylog(4,b*exp
(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a+1/(4*a^2+b^2)^(1/2)*x^3*ln(1-b*exp(2*x)/
((4*a^2+b^2)^(1/2)-2*a))-1/2/(4*a^2+b^2)^(1/2)*x^4+3/2/(4*a^2+b^2)^(1/2)*x
^2*polylog(2,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))-3/2/(4*a^2+b^2)^(1/2)*x*p
olylog(3,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))+3/4/(4*a^2+b^2)^(1/2)*polylog
(4,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))
```

3.867.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1488 vs. $2(332) = 664$.

Time = 0.29 (sec) , antiderivative size = 1488, normalized size of antiderivative = 3.85

$$\int \frac{x^3}{a + b\cosh(x)\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

output
```
-(b*x^3*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(
x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2)
+ 2*a)/b) + b)/b) + b*x^3*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a
*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt(
(4*a^2 + b^2)/b^2) + 2*a)/b) - b)/b) - b*x^3*sqrt((4*a^2 + b^2)/b^2)*log((
(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^
2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b) - b*x^3*sqrt((4*a^2
+ b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqr
t((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b) +
3*b*x^2*sqrt((4*a^2 + b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) - (b*co
sh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^
2) + 2*a)/b) + b)/b + 1) + 3*b*x^2*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cos
h(x) + 2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt
(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b) - b)/b + 1) - 3*b*x^2*sqrt((4*a^2 +
 b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sq
rt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b +
1) - 3*b*x^2*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) + (
b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)
/b^2) - 2*a)/b) - b)/b + 1) - 6*b*x*sqrt((4*a^2 + b^2)/b^2)*polylog(3, (2*
a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^...
```

3.867.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{a + b\cosh(x)\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(a+b*cosh(x)*sinh(x)),x)
```

output
```
Timed out
```

3.867.7 Maxima [F]

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x^3}{b\cosh\left(x\right)\sinh\left(x\right)+a}\,dx$$

input
```
integrate(x^3/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
```

output
```
integrate(x^3/(b*cosh(x)*sinh(x) + a), x)
```

3.867.8 Giac [F]

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x^3}{b\cosh\left(x\right)\sinh\left(x\right)+a}\,dx$$

input
```
integrate(x^3/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
```

output
```
integrate(x^3/(b*cosh(x)*sinh(x) + a), x)
```

3.867.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x^3}{a+b\cosh\left(x\right)\,\sinh\left(x\right)}\,dx$$

input
```
int(x^3/(a + b*cosh(x)*sinh(x)),x)
```

output
```
int(x^3/(a + b*cosh(x)*sinh(x)), x)
```

3.868 $\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx$

3.868.1 Optimal result

Integrand size = 14, antiderivative size = 281

$$\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx = \frac{x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}$$

output
```
x^2*ln(1+b*exp(2*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-x^2*ln(1+b*
exp(2*x)/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+x*polylog(2,-b*exp(2*x
)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-x*polylog(2,-b*exp(2*x)/(2*a+
(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-1/2*polylog(3,-b*exp(2*x)/(2*a-(4*a^
2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+1/2*polylog(3,-b*exp(2*x)/(2*a+(4*a^2+b^2
)^(1/2)))/(4*a^2+b^2)^(1/2)
```

3.868.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx = \frac{2x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)-2x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)+2x\,\text{PolyLog}\left(2,\frac{be^{2x}}{-2a+\sqrt{4a^2+b^2}}\right)-2x\,\text{PolyLog}\left(2,-\right.}{2\sqrt{4a^2+b^2}}$$

input
```
Integrate[x^2/(a + b*Cosh[x]*Sinh[x]),x]
```

output
```
(2*x^2*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])] - 2*x^2*Log[1 + (b*E
^(2*x))/(2*a + Sqrt[4*a^2 + b^2])] + 2*x*PolyLog[2, (b*E^(2*x))/(-2*a + Sq
rt[4*a^2 + b^2])] - 2*x*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])
)] - PolyLog[3, (b*E^(2*x))/(-2*a + Sqrt[4*a^2 + b^2])] + PolyLog[3, -((b*
E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))])/(2*Sqrt[4*a^2 + b^2])
```

3.868.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6162, 3042, 3803, 27, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\sinh(x)\cosh(x)}\,dx$$

$$\Big\downarrow \; 6162$$

$$\int \frac{x^2}{a+\frac{1}{2}b\sinh(2x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{x^2}{a-\frac{1}{2}ib\sin(2ix)}dx$$

$$\Big\downarrow \; 3803$$

$$2\int -\frac{2e^{2x}x^2}{-4e^{2x}a-be^{4x}+b}dx$$

$\Big\downarrow$ 27

$$-4\int \frac{e^{2x}x^2}{-4e^{2x}a-be^{4x}+b}dx$$

$\Big\downarrow$ 2694

$$-4\left(\frac{b\int -\frac{e^{2x}x^2}{2\left(2a+be^{2x}-\sqrt{4a^2+b^2}\right)}dx}{\sqrt{4a^2+b^2}}-\frac{b\int -\frac{e^{2x}x^2}{2\left(2a+be^{2x}+\sqrt{4a^2+b^2}\right)}dx}{\sqrt{4a^2+b^2}}\right)$$

$\Big\downarrow$ 27

$$-4\left(\frac{b\int \frac{e^{2x}x^2}{2a+be^{2x}+\sqrt{4a^2+b^2}}dx}{2\sqrt{4a^2+b^2}}-\frac{b\int \frac{e^{2x}x^2}{2a+be^{2x}-\sqrt{4a^2+b^2}}dx}{2\sqrt{4a^2+b^2}}\right)$$

$\Big\downarrow$ 2620

$$-4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{2b}-\frac{\int x\log\left(\frac{e^{2x}b}{2a+\sqrt{4a^2+b^2}}+1\right)dx}{b}\right)}{2\sqrt{4a^2+b^2}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{2b}-\frac{\int x\log\left(\frac{e^{2x}b}{2a-\sqrt{4a^2+b^2}}+1\right)dx}{b}\right)}{2\sqrt{4a^2+b^2}}\right)$$

$\Big\downarrow$ 3011

$$-4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{2b}-\frac{\frac{1}{2}\int \text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)dx-\frac{1}{2}x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{b}\right)}{2\sqrt{4a^2+b^2}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right.}{2b}\right.}{}\right.$$

$\Big\downarrow$ 2720

$$-4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{2b}-\frac{\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{b}\right)}{2\sqrt{4a^2+b^2}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2\ldots}}\right.}{2b}\right.}{}\right.$$

$\Big\downarrow$ 7143

3.868. $\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx$

$$-4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{2b}-\frac{\frac{1}{4}\,\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)-\frac{1}{2}x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{b}\right)}{2\sqrt{4a^2+b^2}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{2b}-\right.}{}\right.$$

input
```
Int[x^2/(a + b*Cosh[x]*Sinh[x]),x]
```

output
```
-4*(-1/2*(b*((x^2*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/(2*b) -
(-1/2*(x*PolyLog[2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]) + PolyLog[3
, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]/4)/b))/Sqrt[4*a^2 + b^2] + (b*
((x^2*Log[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/(2*b) - (-1/2*(x*Pol
yLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]) + PolyLog[3, -((b*E^(2*
x))/(2*a + Sqrt[4*a^2 + b^2]))]/4)/b))/(2*Sqrt[4*a^2 + b^2]))
```

3.868.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6162
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sinh[2*c +
 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.868.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(247) = 494.

Time = 0.94 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.89

method	result
risch	$-\frac{2x^3}{3\left(-2a-\sqrt{4a^2+b^2}\right)}+\frac{x^2\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)}{-2a-\sqrt{4a^2+b^2}}+\frac{x\,\mathrm{polylog}\left(2,\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)}{-2a-\sqrt{4a^2+b^2}}-\frac{\mathrm{polylog}\left(3,\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)}{2\left(-2a-\sqrt{4a^2+b^2}\right)}-\frac{}{3\sqrt{4a^2+}}$

input
```
int(x^2/(a+b*cosh(x)*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(-2*a-(4*a^2+b^2)^(1/2))*x^3+1/(-2*a-(4*a^2+b^2)^(1/2))*x^2*ln(1-b*ex
p(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))+1/(-2*a-(4*a^2+b^2)^(1/2))*x*polylog(2,b*
exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-1/2/(-2*a-(4*a^2+b^2)^(1/2))*polylog(3,
b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-4/3/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^
2)^(1/2))*a*x^3+2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*a*x^2*ln(1-b*
exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))+2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(
1/2))*a*x*polylog(2,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-1/(4*a^2+b^2)^(1/
2)/(-2*a-(4*a^2+b^2)^(1/2))*a*polylog(3,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)
))-2/3/(4*a^2+b^2)^(1/2)*x^3+1/(4*a^2+b^2)^(1/2)*x^2*ln(1-b*exp(2*x)/((4*a
^2+b^2)^(1/2)-2*a))+1/(4*a^2+b^2)^(1/2)*x*polylog(2,b*exp(2*x)/((4*a^2+b^2
)^(1/2)-2*a))-1/2/(4*a^2+b^2)^(1/2)*polylog(3,b*exp(2*x)/((4*a^2+b^2)^(1/2
)-2*a))
```

3.868.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1122 vs. 2(245) = 490.

Time = 0.28 (sec) , antiderivative size = 1122, normalized size of antiderivative = 3.99

$$\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

output
```
-(b*x^2*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(
x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2)
+ 2*a)/b) + b)/b) + b*x^2*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a
*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt(
(4*a^2 + b^2)/b^2) + 2*a)/b) - b)/b) - b*x^2*sqrt((4*a^2 + b^2)/b^2)*log((
(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^
2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b) - b*x^2*sqrt((4*a^2
+ b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqr
t((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b) +
2*b*x*sqrt((4*a^2 + b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh
(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2)
 + 2*a)/b) + b)/b + 1) + 2*b*x*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x)
 + 2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b
*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b) - b)/b + 1) - 2*b*x*sqrt((4*a^2 + b^2)/
b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*
a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b + 1) - 2
*b*x*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x
) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) -
2*a)/b) - b)/b + 1) - 2*b*sqrt((4*a^2 + b^2)/b^2)*polylog(3, (2*a*cosh(x)
+ 2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-...
```

3.868.6 Sympy [F]

$$\int \frac{x^2}{a + b\cosh(x)\sinh(x)}\,dx = \int \frac{x^2}{a + b\sinh(x)\cosh(x)}\,dx$$

input
```
integrate(x**2/(a+b*cosh(x)*sinh(x)),x)
```

output
```
Integral(x**2/(a + b*sinh(x)*cosh(x)), x)
```

3.868.7 Maxima [F]

$$\int \frac{x^2}{a + b\cosh(x)\sinh(x)}\,dx = \int \frac{x^2}{b\cosh(x)\sinh(x) + a}\,dx$$

input
```
integrate(x^2/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*cosh(x)*sinh(x) + a), x)
```

3.868.8 Giac [F]

$$\int \frac{x^2}{a + b\cosh(x)\sinh(x)}\,dx = \int \frac{x^2}{b\cosh(x)\sinh(x) + a}\,dx$$

input
```
integrate(x^2/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
```

output
```
integrate(x^2/(b*cosh(x)*sinh(x) + a), x)
```

3.868.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + b\cosh(x)\sinh(x)}\,dx = \int \frac{x^2}{a + b\cosh(x)\,\sinh(x)}\,dx$$

input
```
int(x^2/(a + b*cosh(x)*sinh(x)),x)
```

output
```
int(x^2/(a + b*cosh(x)*sinh(x)), x)
```

3.869 $\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx$

3.869.1 Optimal result

Integrand size = 12, antiderivative size = 186

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx = \frac{x\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}$$

output
```
x*ln(1+b*exp(2*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-x*ln(1+b*exp(
2*x)/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+1/2*polylog(2,-b*exp(2*x)/
(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-1/2*polylog(2,-b*exp(2*x)/(2*a+
(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)
```

3.869.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.77

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx = \frac{2x\left(\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)-\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)\right)+\operatorname{PolyLog}\left(2,\frac{be^{2x}}{-2a+\sqrt{4a^2+b^2}}\right)-\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+}}\right.}{2\sqrt{4a^2+b^2}}$$

input
```
Integrate[x/(a + b*Cosh[x]*Sinh[x]),x]
```

output
```
(2*x*(Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])] - Log[1 + (b*E^(2*x))
/(2*a + Sqrt[4*a^2 + b^2])]) + PolyLog[2, (b*E^(2*x))/(-2*a + Sqrt[4*a^2 +
 b^2])] - PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))])/(2*Sqrt[4*
a^2 + b^2])
```

3.869.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6162, 3042, 3803, 27, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\sinh(x)\cosh(x)}dx$$

$$\Big\downarrow \text{6162}$$

$$\int \frac{x}{a+\frac{1}{2}b\sinh(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x}{a-\frac{1}{2}ib\sin(2ix)}dx$$

$$\Big\downarrow \text{3803}$$

$$2\int -\frac{2e^{2x}x}{-4e^{2x}a-be^{4x}+b}dx$$

$$\Big\downarrow \text{27}$$

$$-4\int \frac{e^{2x}x}{-4e^{2x}a-be^{4x}+b}dx$$

$$\Big\downarrow \text{2694}$$

$$-4\left(\frac{b\int -\frac{e^{2x}x}{2\left(2a+be^{2x}-\sqrt{4a^2+b^2}\right)}dx}{\sqrt{4a^2+b^2}}-\frac{b\int -\frac{e^{2x}x}{2\left(2a+be^{2x}+\sqrt{4a^2+b^2}\right)}dx}{\sqrt{4a^2+b^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$-4\left(\frac{b\int \frac{e^{2x}x}{2a+be^{2x}+\sqrt{4a^2+b^2}}dx}{2\sqrt{4a^2+b^2}}-\frac{b\int \frac{e^{2x}x}{2a+be^{2x}-\sqrt{4a^2+b^2}}dx}{2\sqrt{4a^2+b^2}}\right)$$

$\downarrow$ 2620

$$-4\left(\frac{b\left(\frac{x\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{2b}-\frac{\int \log\left(\frac{e^{2x}b}{2a+\sqrt{4a^2+b^2}}+1\right)dx}{2b}\right)}{2\sqrt{4a^2+b^2}}-\frac{b\left(\frac{x\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{2b}-\frac{\int \log\left(\frac{e^{2x}b}{2a-\sqrt{4a^2+b^2}}+1\right)dx}{2b}\right)}{2\sqrt{4a^2+b^2}}\right)$$

$\downarrow$ 2715

$$-4\left(\frac{b\left(\frac{x\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{2b}-\frac{\int e^{-2x}\log\left(\frac{e^{2x}b}{2a+\sqrt{4a^2+b^2}}+1\right)de^{2x}}{4b}\right)}{2\sqrt{4a^2+b^2}}-\frac{b\left(\frac{x\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{2b}-\frac{\int e^{-2x}\log\left(\frac{e^{2x}b}{2a-\sqrt{4a^2+b^2}}+1\right)de^{2x}}{4b}\right)}{2\sqrt{4a^2+b^2}}\right)$$

$\downarrow$ 2838

$$-4\left(\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{4b}+\frac{x\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{2b}\right)}{2\sqrt{4a^2+b^2}}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{4b}+\frac{x\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{2b}\right)}{2\sqrt{4a^2+b^2}}\right)$$

input
```
Int[x/(a + b*Cosh[x]*Sinh[x]),x]
```

output
```
-4*(-1/2*(b*((x*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/(2*b) + Po
lyLog[2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]/(4*b)))/Sqrt[4*a^2 + b^
2] + (b*((x*Log[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/(2*b) + PolyLo
g[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]/(4*b)))/(2*Sqrt[4*a^2 + b^2
]))
```

3.869. $\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx$

3.869.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6162
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sinh[2*c +
 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.869. $\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx$

3.869.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(162) = 324$.

Time = 0.91 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.02

method	result
risch	$\frac{\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)x}{-2a-\sqrt{4a^2+b^2}} - \frac{x^2}{-2a-\sqrt{4a^2+b^2}} + \frac{2\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)ax}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2}\right)} - \frac{2a\,x^2}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2}\right)} + \frac{\text{polylog}\left(2, \frac{\ldots}{-2a-\ldots}\right.}{-4a-2\sqrt{4\ldots}}$

input
```
int(x/(a+b*cosh(x)*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x-1/(
-2*a-(4*a^2+b^2)^(1/2))*x^2+2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*l
n(1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a*x-2/(4*a^2+b^2)^(1/2)/(-2*a-(4*
a^2+b^2)^(1/2))*a*x^2+1/2/(-2*a-(4*a^2+b^2)^(1/2))*polylog(2,b*exp(2*x)/(-
2*a-(4*a^2+b^2)^(1/2)))+1/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*polyl
og(2,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a+1/(4*a^2+b^2)^(1/2)*x*ln(1-b*e
xp(2*x)/((4*a^2+b^2)^(1/2)-2*a))-1/(4*a^2+b^2)^(1/2)*x^2+1/2/(4*a^2+b^2)^(
1/2)*polylog(2,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))
```

3.869.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(160) = 320$.

Time = 0.28 (sec) , antiderivative size = 754, normalized size of antiderivative = 4.05

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx = -\frac{bx\sqrt{\frac{4a^2+b^2}{b^2}}\log\left(\frac{\left(2a\cosh(x)+2a\sinh(x)-(b\cosh(x)+b\sinh(x))\sqrt{\frac{4a^2+b^2}{b^2}}\right)\sqrt{-\frac{b\sqrt{\frac{4a^2+b^2}{b^2}}+2a}{b}}+b}{b}\right)+bx\sqrt{\frac{4a^2+b^2}{b^2}}\log\left(-\ldots\right.}{\ldots}$$

input
```
integrate(x/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

output
```
-(b*x*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x)
 + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) +
2*a)/b) + b)/b) + b*x*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*sin
h(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a
^2 + b^2)/b^2) + 2*a)/b) - b)/b) - b*x*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*c
osh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sq
rt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b) - b*x*sqrt((4*a^2 + b^2)/b
^2)*log(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2
 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b) + b*sqrt((
4*a^2 + b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*sinh
(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b) +
 b)/b + 1) + b*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) -
 (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b
^2)/b^2) + 2*a)/b) - b)/b + 1) - b*sqrt((4*a^2 + b^2)/b^2)*dilog(-((2*a*co
sh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqr
t((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b + 1) - b*sqrt((4*a^2 + b^2)/
b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a
^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b + 1))/(4*
a^2 + b^2)
```

3.869.6 Sympy [F]

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx=\int \frac{x}{a+b\sinh(x)\cosh(x)}\,dx$$

input
```
integrate(x/(a+b*cosh(x)*sinh(x)),x)
```

output
```
Integral(x/(a + b*sinh(x)*cosh(x)), x)
```

3.869.7 Maxima [F]

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx=\int \frac{x}{b\cosh(x)\sinh(x)+a}\,dx$$

input
```
integrate(x/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
```

output
```
integrate(x/(b*cosh(x)*sinh(x) + a), x)
```

3.869.8 Giac [F]

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx=\int \frac{x}{b\cosh{(x)}\sinh{(x)}+a}\,dx$$

input
```
integrate(x/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
```

output
```
integrate(x/(b*cosh(x)*sinh(x) + a), x)
```

3.869.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx=\int \frac{x}{a+b\cosh{(x)}\,\sinh{(x)}}\,dx$$

input
```
int(x/(a + b*cosh(x)*sinh(x)),x)
```

output
```
int(x/(a + b*cosh(x)*sinh(x)), x)
```

3.870 $\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx$

3.870.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx = \operatorname{Int}\left(\frac{1}{x\left(a+\frac{1}{2}b\sinh(2x)\right)},x\right)$$

output
```
Unintegrable(1/x/(a+1/2*b*sinh(2*x)),x)
```

3.870.2 Mathematica [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx = \int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx$$

input
```
Integrate[1/(x*(a + b*Cosh[x]*Sinh[x])),x]
```

output
```
Integrate[1/(x*(a + b*Cosh[x]*Sinh[x])), x]
```

3.870.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6162, 3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b\sinh(x)\cosh(x))}dx$$
$$\Big\downarrow \text{6162}$$
$$\int \frac{1}{x\left(a+\frac{1}{2}b\sinh(2x)\right)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x\left(a-\frac{1}{2}ib\sin(2ix)\right)}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x\left(a+\frac{1}{2}b\sinh(2x)\right)}dx$$

input
```
Int[1/(x*(a + b*Cosh[x]*Sinh[x])),x]
```

output
```
$Aborted
```

3.870.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

rule 6162
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sinh[2*c +
 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.870.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\cosh\left(x\right)\sinh\left(x\right)\right)}dx$$

input
```
int(1/x/(a+b*cosh(x)*sinh(x)),x)
```

output
```
int(1/x/(a+b*cosh(x)*sinh(x)),x)
```

3.870.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx=\int \frac{1}{(b\cosh\left(x\right)\sinh\left(x\right)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*cosh(x)*sinh(x) + a*x), x)
```

3.870.6 Sympy [N/A]

Not integrable

Time = 66.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx=\int \frac{1}{x\left(a+b\sinh\left(x\right)\cosh\left(x\right)\right)}\,dx$$

input
```
integrate(1/x/(a+b*cosh(x)*sinh(x)),x)
```

output
```
Integral(1/(x*(a + b*sinh(x)*cosh(x))), x)
```

3.870.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx=\int \frac{1}{(b\cosh{(x)}\sinh{(x)}+a)x}\,dx$$

input
```
integrate(1/x/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*cosh(x)*sinh(x) + a)*x), x)
```

3.870.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx=\int \frac{1}{(b\cosh{(x)}\sinh{(x)}+a)x}\,dx$$

input
```
integrate(1/x/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
```

output
```
integrate(1/((b*cosh(x)*sinh(x) + a)*x), x)
```

3.870.9 Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx = \int \frac{1}{x\ (a+b\cosh{(x)}\ \sinh{(x)})}\,dx$$

input
```
int(1/(x*(a + b*cosh(x)*sinh(x))),x)
```

output
```
int(1/(x*(a + b*cosh(x)*sinh(x))), x)
```

3.871 $\int F^{c(a+bx)} \sinh^n(d+ex) \, dx$

3.871.1 Optimal result

Integrand size = 18, antiderivative size = 95

$$\int F^{c(a+bx)} \sinh^n(d+ex) \, dx = -\frac{\left(1-e^{2(d+ex)}\right)^{-n} F^{c(a+bx)} \text{Hypergeometric2F1}\left(-n,-\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(2-n+\frac{bc\log(F)}{e}\right),e^{2(d+ex)}\right) \sinh^n(d}{en-bc\log(F)}$$

output
```
-F^(c*(b*x+a))*hypergeom([-n, 1/2*(-e*n+b*c*ln(F))/e],[1-1/2*n+1/2*b*c*ln(
F)/e],exp(2*e*x+2*d))*sinh(e*x+d)^n/((1-exp(2*e*x+2*d))^n)/(e*n-b*c*ln(F))
```

3.871.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)} \sinh^n(d+ex) \, dx = \frac{\left(1-e^{2(d+ex)}\right)^{-n} F^{c(a+bx)} \text{Hypergeometric2F1}\left(-n,\frac{-en+bc\log(F)}{2e},1+\frac{-en+bc\log(F)}{2e},e^{2(d+ex)}\right) \sinh^n(d+ex)}{-en+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^n,x]
```

output
```
(F^(c*(a + b*x))*Hypergeometric2F1[-n, (-(e*n) + b*c*Log[F])/(2*e), 1 + (-
(e*n) + b*c*Log[F])/(2*e), E^(2*(d + e*x))]*Sinh[d + e*x]^n)/((1 - E^(2*(d
 + e*x)))^n*(-(e*n) + b*c*Log[F]))
```

3.871.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6005, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)} \sinh^n(d+ex) \, dx$$

$$\Big\downarrow \text{6005}$$

$$e^{n(d+ex)} \left(e^{2(d+ex)}-1\right)^{-n} \sinh^n(d+ex) \int e^{-n(d+ex)} \left(-1+e^{2(d+ex)}\right)^n F^{c(a+bx)}dx$$

$$\Big\downarrow \text{2689}$$

$$-\frac{\left(1-e^{2(d+ex)}\right)^{-n} F^{c(a+bx)} \sinh^n(d+ex) \text{Hypergeometric2F1}\left(-n,-\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right),e^{2(d+ex)}\right)}{en-bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Sinh[d + e*x]^n,x]
```

output
```
-((F^(c*(a + b*x))*Hypergeometric2F1[-n, -1/2*(e*n - b*c*Log[F])/e, (2 - n
 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*Sinh[d + e*x]^n)/((1 - E^(2*(d + e*
x)))^n*(e*n - b*c*Log[F])))
```

3.871.3.1 Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 6005
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[E^(n*(d + e*x))*(Sinh[d + e*x]^n/(-1 + E^(2*(d + e*x)))^n)   In
t[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/E^(n*(d + e*x))), x], x] /; Fre
eQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

3.871.4 Maple [F]

$$\int F^{c(bx+a)} \sinh\left(ex+d\right)^{n} dx$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^n,x)
```

output
```
int(F^(c*(b*x+a))*sinh(e*x+d)^n,x)
```

3.871.5 Fricas [F]

$$\int F^{c(a+bx)} \sinh^n(d+ex)\,dx = \int F^{(bx+a)c} \sinh\left(ex+d\right)^{n}\,dx$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^n,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sinh(e*x + d)^n, x)
```

3.871.6 Sympy [F]

$$\int F^{c(a+bx)} \sinh^n(d+ex)\,dx = \int F^{c(a+bx)} \sinh^{n}\left(d+ex\right) dx$$

input
```
integrate(F**(c*(b*x+a))*sinh(e*x+d)**n,x)
```

output
```
Integral(F**(c*(a + b*x))*sinh(d + e*x)**n, x)
```

3.871.7 Maxima [F]

$$\int F^{c(a+bx)}\sinh^n(d+ex)\,dx=\int F^{(bx+a)c}\sinh\left(ex+d\right)^n\,dx$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^n,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)*c)*sinh(e*x + d)^n, x)
```

3.871.8 Giac [F]

$$\int F^{c(a+bx)}\sinh^n(d+ex)\,dx=\int F^{(bx+a)c}\sinh\left(ex+d\right)^n\,dx$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^n,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sinh(e*x + d)^n, x)
```

3.871.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sinh^n(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,\sinh(d+e\,x)^n\,dx$$

input
```
int(F^(c*(a + b*x))*sinh(d + e*x)^n,x)
```

output
```
int(F^(c*(a + b*x))*sinh(d + e*x)^n, x)
```

3.872 $\int e^{2(a+bx)}\sinh^3(a+bx)\,dx$

3.872.1 Optimal result

Integrand size = 18, antiderivative size = 66

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx=\frac{e^{-a-bx}}{8b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{8b}+\frac{e^{5a+5bx}}{40b}$$

output
```
1/8*exp(-b*x-a)/b+3/8*exp(b*x+a)/b-1/8*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*a
)/b
```

3.872.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx=\frac{e^{-a-bx}\left(5+15e^{2(a+bx)}-5e^{4(a+bx)}+e^{6(a+bx)}\right)}{40b}$$

input
```
Integrate[E^(2*(a + b*x))*Sinh[a + b*x]^3,x]
```

output
```
(E^(-a - b*x)*(5 + 15*E^(2*(a + b*x)) - 5*E^(4*(a + b*x)) + E^(6*(a + b*x)
)))/(40*b)
```

3.872.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{8}e^{-2a-2bx}\left(1-e^{2a+2bx}\right)^3de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int e^{-2a-2bx}\left(1-e^{2a+2bx}\right)^3de^{a+bx}}{8b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(-3+e^{-2a-2bx}+3e^{2a+2bx}-e^{4a+4bx}\right)de^{a+bx}}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{-a-bx}+3e^{a+bx}-e^{3a+3bx}+\frac{1}{5}e^{5a+5bx}}{8b}$$

input
```
Int[E^(2*(a + b*x))*Sinh[a + b*x]^3,x]
```

output
```
(E^(-a - b*x) + 3*E^(a + b*x) - E^(3*a + 3*b*x) + E^(5*a + 5*b*x)/5)/(8*b)
```

3.872.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.872.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{e^{-bx-a}}{8b}+\frac{3\,e^{bx+a}}{8b}-\frac{e^{3bx+3a}}{8b}+\frac{e^{5bx+5a}}{40b}$	55
parallelrisch	$-\frac{e^{2bx+2a}\left(8\sinh(2bx+2a)+2\sinh(3bx+3a)+10\sinh(bx+a)-8\cosh(2bx+2a)-3\cosh(3bx+3a)-5\cosh(bx+a)\right)}{20b}$	76
default	$\frac{\sinh(bx+a)}{4b}-\frac{\sinh(3bx+3a)}{8b}+\frac{\sinh(5bx+5a)}{40b}+\frac{\cosh(bx+a)}{2b}-\frac{\cosh(3bx+3a)}{8b}+\frac{\cosh(5bx+5a)}{40b}$	80

input
```
int(exp(2*b*x+2*a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*exp(-b*x-a)/b+3/8*exp(b*x+a)/b-1/8*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*a
)/b
```

3.872.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)^3+9\cosh(bx+a)\sinh(bx+a)^2-2\sinh(bx+a)^3-2\left(3\cosh(bx+a)^2+5\right)\sinh(bx+}{20\left(b\cosh(bx+a)^2-2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/20*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - 2*sinh(b*x + a
)^3 - 2*(3*cosh(b*x + a)^2 + 5)*sinh(b*x + a) + 5*cosh(b*x + a))/(b*cosh(b
*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.872.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(49) = 98.

Time = 0.88 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.88

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx$$
$$=\begin{cases} \frac{2e^{2a}e^{2bx}\sinh^3(a+bx)}{5b}+\frac{e^{2a}e^{2bx}\sinh^2(a+bx)\cosh(a+bx)}{5b}-\frac{4e^{2a}e^{2bx}\sinh(a+bx)\cosh^2(a+bx)}{5b}+\frac{2e^{2a}e^{2bx}\cosh^3(a+bx)}{5b} & \text{for } b\neq 0 \\ xe^{2a}\sinh^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+a)**3,x)
```

output
```
Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3/(5*b) + exp(2*a)*exp(2*b
*x)*sinh(a + b*x)**2*cosh(a + b*x)/(5*b) - 4*exp(2*a)*exp(2*b*x)*sinh(a +
b*x)*cosh(a + b*x)**2/(5*b) + 2*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**3/(5*b)
, Ne(b, 0)), (x*exp(2*a)*sinh(a)**3, True))
```

3.872.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx=-\frac{\left(5\,e^{(-2\,bx-2\,a)}-15\,e^{(-4\,bx-4\,a)}-1\right)e^{(5\,bx+5\,a)}}{40\,b}+\frac{e^{(-bx-a)}}{8\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/40*(5*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) - 1)*e^(5*b*x + 5*a)/b + 1
/8*e^(-b*x - a)/b
```

3.872.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx=\frac{e^{(5\,bx+5\,a)}}{40\,b}-\frac{e^{(3\,bx+3\,a)}}{8\,b}+\frac{3\,e^{(bx+a)}}{8\,b}+\frac{e^{(-bx-a)}}{8\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/40*e^(5*b*x + 5*a)/b - 1/8*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b + 1/8*e
^(-b*x - a)/b
```

3.872.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.68

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx = \frac{15\,e^{a+b\,x}+5\,e^{-a-b\,x}-5\,e^{3\,a+3\,b\,x}+e^{5\,a+5\,b\,x}}{40\,b}$$

input
```
int(exp(2*a + 2*b*x)*sinh(a + b*x)^3,x)
```

output
```
(15*exp(a + b*x) + 5*exp(- a - b*x) - 5*exp(3*a + 3*b*x) + exp(5*a + 5*b*x
))/(40*b)
```

3.873 $\int e^{2(a+bx)} \sinh^2(a+bx)\,dx$

3.873.1 Optimal result

Integrand size = 18, antiderivative size = 40

$$\int e^{2(a+bx)} \sinh^2(a+bx)\,dx = -\frac{e^{2a+2bx}}{4b} + \frac{e^{4a+4bx}}{16b} + \frac{x}{4}$$

output
```
-1/4*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+4*a)/b+1/4*x
```

3.873.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)} \sinh^2(a+bx)\,dx = \frac{-4e^{2(a+bx)} + e^{4(a+bx)} + 4bx}{16b}$$

input
```
Integrate[E^(2*(a + b*x))*Sinh[a + b*x]^2,x]
```

output
```
(-4*E^(2*(a + b*x)) + E^(4*(a + b*x)) + 4*b*x)/(16*b)
```

3.873.3 Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{1}{4}e^{-a-bx}\left(1-e^{2a+2bx}\right)^2de^{a+bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int e^{-a-bx}\left(1-e^{2a+2bx}\right)^2de^{a+bx}}{4b}$$

$$\Big\downarrow \; 243$$

$$\frac{\int e^{-a-bx}\left(1-e^{2a+2bx}\right)^2de^{2a+2bx}}{8b}$$

$$\Big\downarrow \; 49$$

$$\frac{\int \left(-2+e^{-a-bx}+e^{2a+2bx}\right)de^{2a+2bx}}{8b}$$

$$\Big\downarrow \; 2009$$

$$\frac{\log\left(e^{2a+2bx}\right)-\frac{3}{2}e^{2a+2bx}}{8b}$$

input
```
Int[E^(2*(a + b*x))*Sinh[a + b*x]^2,x]
```

output
```
((-3*E^(2*a + 2*b*x))/2 + Log[E^(2*a + 2*b*x)])/(8*b)
```

3.873.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.873.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{{\mathrm e}^{2 b x +2 a}}{4 b}+\frac{{\mathrm e}^{4 b x +4 a}}{16 b}+\frac{x}{4}$	33
default	$\frac{x}{4}-\frac{\sinh \left(2 b x +2 a \right)}{4 b}+\frac{\sinh \left(4 b x +4 a \right)}{16 b}-\frac{\cosh \left(2 b x +2 a \right)}{4 b}+\frac{\cosh \left(4 b x +4 a \right)}{16 b}$	61

input
```
int(exp(2*b*x+2*a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+4*a)/b+1/4*x
```

3.873.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(32) = 64.

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int e^{2(a+bx)}\sinh^2(a+bx)\,dx$$
$$=\frac{(4\,bx+1)\cosh\left(bx+a\right)^2-2\,(4\,bx-1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+(4\,bx+1)\sinh\left(bx+a\right)^2-4}{16\,\left(b\cosh\left(bx+a\right)^2-2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2\right)}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/16*((4*b*x + 1)*cosh(b*x + a)^2 - 2*(4*b*x - 1)*cosh(b*x + a)*sinh(b*x +
 a) + (4*b*x + 1)*sinh(b*x + a)^2 - 4)/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x +
 a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.873.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(29) = 58.

Time = 0.44 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.48

$$\int e^{2(a+bx)}\sinh^2(a+bx)\,dx$$
$$=\begin{cases}\frac{xe^{2a}e^{2bx}\sinh^2\left(a+bx\right)}{4}-\frac{xe^{2a}e^{2bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2}+\frac{xe^{2a}e^{2bx}\cosh^2\left(a+bx\right)}{4}+\frac{e^{2a}e^{2bx}\sinh^2\left(a+bx\right)}{2b}-\frac{e^{2a}e^{2bx}\sinh\left(a+bx\right)\cos}{4b}\\ xe^{2a}\sinh^2\left(a\right)\end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2/4 - x*exp(2*a)*exp(2*b*x
)*sinh(a + b*x)*cosh(a + b*x)/2 + x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**2/4
 + exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2/(2*b) - exp(2*a)*exp(2*b*x)*sinh(a
 + b*x)*cosh(a + b*x)/(4*b), Ne(b, 0)), (x*exp(2*a)*sinh(a)**2, True))
```

3.873.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{2(a+bx)}\sinh^2(a+bx)\,dx = \frac{1}{4}\,x - \frac{\left(4\,e^{(-2\,bx-2\,a)}-1\right)e^{(4\,bx+4\,a)}}{16\,b} + \frac{a}{4\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*x - 1/16*(4*e^(-2*b*x - 2*a) - 1)*e^(4*b*x + 4*a)/b + 1/4*a/b
```

3.873.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\sinh^2(a+bx)\,dx = \frac{1}{4}\,x + \frac{e^{(4\,bx+4\,a)}}{16\,b} - \frac{e^{(2\,bx+2\,a)}}{4\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*x + 1/16*e^(4*b*x + 4*a)/b - 1/4*e^(2*b*x + 2*a)/b
```

3.873.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\sinh^2(a+bx)\,dx = \frac{x}{4} - \frac{\frac{e^{2\,a+2\,b\,x}}{4} - \frac{e^{4\,a+4\,b\,x}}{16}}{b}$$

input
```
int(exp(2*a + 2*b*x)*sinh(a + b*x)^2,x)
```

output
```
x/4 - (exp(2*a + 2*b*x)/4 - exp(4*a + 4*b*x)/16)/b
```

3.874 $\int e^{2(a+bx)} \sinh(a+bx)\,dx$

3.874.1 Optimal result

Integrand size = 16, antiderivative size = 32

$$\int e^{2(a+bx)} \sinh(a+bx)\,dx = -\frac{e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{6b}$$

output
```
-1/2*exp(b*x+a)/b+1/6*exp(3*b*x+3*a)/b
```

3.874.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)} \sinh(a+bx)\,dx = \frac{e^{a+bx}\left(-3+e^{2(a+bx)}\right)}{6b}$$

input
```
Integrate[E^(2*(a + b*x))*Sinh[a + b*x],x]
```

output
```
(E^(a + b*x)*(-3 + E^(2*(a + b*x))))/(6*b)
```

3.874.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2720, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \sinh(a+bx) \, dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{1}{2}\left(-1+e^{2a+2bx}\right) de^{a+bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \left(-1+e^{2a+2bx}\right) de^{a+bx}}{2b}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{3}e^{3a+3bx}-e^{a+bx}}{2b}$$

input
```
Int[E^(2*(a + b*x))*Sinh[a + b*x],x]
```

output
```
(-E^(a + b*x) + E^(3*a + 3*b*x)/3)/(2*b)
```

3.874.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.874.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{e^{bx+a}}{2b}+\frac{e^{3bx+3a}}{6b}$	27
parallelrisch	$-\frac{e^{2bx+2a}(\cosh(bx+a)-2\sinh(bx+a))}{3b}$	30
default	$-\frac{\sinh(bx+a)}{2b}+\frac{\sinh(3bx+3a)}{6b}-\frac{\cosh(bx+a)}{2b}+\frac{\cosh(3bx+3a)}{6b}$	52

input
```
int(exp(2*b*x+2*a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(b*x+a)/b+1/6*exp(3*b*x+3*a)/b
```

3.874.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int e^{2(a+bx)}\sinh(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-3}{6(b\cosh(bx+a)-b\sinh(bx+a))}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 3
)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

3.874.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(22) = 44$.

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int e^{2(a+bx)}\sinh(a+bx)\,dx=\begin{cases}\frac{2e^{2a}e^{2bx}\sinh(a+bx)}{3b}-\frac{e^{2a}e^{2bx}\cosh(a+bx)}{3b} & \text{for } b\neq 0\\ xe^{2a}\sinh(a) & \text{otherwise}\end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+a),x)
```

output
```
Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)/(3*b) - exp(2*a)*exp(2*b*x)
*cosh(a + b*x)/(3*b), Ne(b, 0)), (x*exp(2*a)*sinh(a), True))
```

3.874.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\sinh(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{6\,b} - \frac{e^{(bx+a)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/6*e^(3*b*x + 3*a)/b - 1/2*e^(b*x + a)/b
```

3.874.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\sinh(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{6\,b} - \frac{e^{(bx+a)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/6*e^(3*b*x + 3*a)/b - 1/2*e^(b*x + a)/b
```

3.874.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\sinh(a+bx)\,dx = -\frac{3\,\mathrm{e}^{a+b\,x} - \mathrm{e}^{3\,a+3\,b\,x}}{6\,b}$$

input
```
int(exp(2*a + 2*b*x)*sinh(a + b*x),x)
```

output
```
-(3*exp(a + b*x) - exp(3*a + 3*b*x))/(6*b)
```

3.875 $\int e^{2(a+bx)}\mathrm{csch}(a+bx)\,dx$

3.875.1 Optimal result

Integrand size = 16, antiderivative size = 26

$$\int e^{2(a+bx)}\mathrm{csch}(a+bx)\,dx=\frac{2e^{a+bx}}{b}-\frac{2\mathrm{arctanh}\left(e^{a+bx}\right)}{b}$$

output
```
2*exp(b*x+a)/b-2*arctanh(exp(b*x+a))/b
```

3.875.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int e^{2(a+bx)}\mathrm{csch}(a+bx)\,dx=\frac{2e^{a+bx}-2\mathrm{arctanh}\left(e^{a+bx}\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[a + b*x],x]
```

output
```
(2*E^(a + b*x) - 2*ArcTanh[E^(a + b*x)])/b
```

3.875.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{2a+2bx}}{1-e^{2a+2bx}}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int \frac{e^{2a+2bx}}{1-e^{2a+2bx}}de^{a+bx}}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{2\left(\int \frac{1}{1-e^{2a+2bx}}de^{a+bx}-e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(\text{arctanh}\left(e^{a+bx}\right)-e^{a+bx}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[a + b*x],x]
```

output
```
(-2*(-E^(a + b*x) + ArcTanh[E^(a + b*x)]))/b
```

3.875.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.875.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

method	result	size
risch	$\frac{2\,\mathrm{e}^{bx+a}}{b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}$	40

input
```
int(exp(2*b*x+2*a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*exp(b*x+a)/b+1/b*ln(exp(b*x+a)-1)-1/b*ln(exp(b*x+a)+1)
```

3.875.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.04

$$\int e^{2(a+bx)}\mathrm{csch}(a+bx)\,dx$$
$$=\frac{2\cosh(bx+a)-\log(\cosh(bx+a)+\sinh(bx+a)+1)+\log(\cosh(bx+a)+\sinh(bx+a)-1)+2\sinh(bx+a)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+a),x, algorithm="fricas")
```

output
```
(2*cosh(b*x + a) - log(cosh(b*x + a) + sinh(b*x + a) + 1) + log(cosh(b*x +
 a) + sinh(b*x + a) - 1) + 2*sinh(b*x + a))/b
```

3.875.6 Sympy [F]

$$\int e^{2(a+bx)}\text{csch}(a+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}\,(a+bx)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+a),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(a + b*x), x)
```

3.875.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int e^{2(a+bx)}\text{csch}(a+bx)\,dx = \frac{2\,e^{(bx+a)}}{b} - \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+a),x, algorithm="maxima")
```

output
```
2*e^(b*x + a)/b - log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

3.875.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int e^{2(a+bx)}\text{csch}(a+bx)\,dx = \frac{2\,e^{(bx+a)} - \log\left(e^{(bx+a)}+1\right) + \log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+a),x, algorithm="giac")
```

output
```
(2*e^(b*x + a) - log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) - 1)))/b
```

3.875.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int e^{2(a+bx)}\operatorname{csch}(a+bx)\,dx = \frac{2\,\mathrm{e}^{a+b\,x}}{b} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

input
```
int(exp(2*a + 2*b*x)/sinh(a + b*x),x)
```

output
```
(2*exp(a + b*x))/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/
2)
```

3.876 $\int e^{2(a+bx)}\operatorname{csch}^2(a+bx)\,dx$

3.876.1 Optimal result

Integrand size = 18, antiderivative size = 42

$$\int e^{2(a+bx)}\operatorname{csch}^2(a+bx)\,dx=\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

output
```
2/b/(1-exp(2*b*x+2*a))+2*ln(1-exp(2*b*x+2*a))/b
```

3.876.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\operatorname{csch}^2(a+bx)\,dx=\frac{2\left(\frac{1}{1-e^{2a+2bx}}+\log\left(1-e^{2a+2bx}\right)\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[a + b*x]^2,x]
```

output
```
(2*((1 - E^(2*a + 2*b*x))^(-1) + Log[1 - E^(2*a + 2*b*x)]))/b
```

3.876.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{4e^{3a+3bx}}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{e^{3a+3bx}}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{2\int \frac{e^{2a+2bx}}{\left(1-e^{2a+2bx}\right)^2}de^{2a+2bx}}{b}$$

$$\Big\downarrow \text{49}$$

$$\frac{2\int \left(\frac{1}{-1+e^{2a+2bx}}+\frac{1}{\left(-1+e^{2a+2bx}\right)^2}\right)de^{2a+2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{1}{1-e^{2a+2bx}}+\log\left(1-e^{2a+2bx}\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[a + b*x]^2,x]
```

output
```
(2*((1 - E^(2*a + 2*b*x))^(-1) + Log[1 - E^(2*a + 2*b*x)]))/b
```

3.876.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.876.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
risch	$-\frac{4a}{b}-\frac{2}{b\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{2\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	43

input
```
int(exp(2*b*x+2*a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/b*a-2/b/(exp(2*b*x+2*a)-1)+2/b*ln(exp(2*b*x+2*a)-1)
```

3.876.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(38) = 76$.

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.48

$$\int e^{2(a+bx)}\text{csch}^2(a+bx)\,dx = \frac{2\left(\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2-1\right)\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-1\right)}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2-b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)
*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - 1)/(b*cosh(b*x + a
)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

3.876.6 Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^2(a+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}^2\left(a+bx\right)dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+a)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(a + b*x)**2, x)
```

3.876.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int e^{2(a+bx)}\text{csch}^2(a+bx)\,dx = 4\,x+\frac{4\,a}{b}+\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2}{b(e^{(-2\,bx-2\,a)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
4*x + 4*a/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b + 2/(b
*(e^(-2*b*x - 2*a) - 1))
```

3.876.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int e^{2(a+bx)}\text{csch}^2(a+bx)\,dx = -\frac{2\left(\frac{e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1} - \log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-2*(e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1) - log(abs(e^(2*b*x + 2*a) - 1)))
/b
```

3.876.9 Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\text{csch}^2(a+bx)\,dx = \frac{2\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b} - \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(exp(2*a + 2*b*x)/sinh(a + b*x)^2,x)
```

output
```
(2*log(exp(2*a)*exp(2*b*x) - 1))/b - 2/(b*(exp(2*a + 2*b*x) - 1))
```

3.877 $\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx$

3.877.1 Optimal result

Integrand size = 18, antiderivative size = 73

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx=-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{3\text{arctanh}\left(e^{a+bx}\right)}{b}$$

output
```
-2*exp(3*b*x+3*a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-exp(2*b*x+2*a))
-3*arctanh(exp(b*x+a))/b
```

3.877.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx=\frac{3e^{a+bx}-5e^{3(a+bx)}-3\left(-1+e^{2(a+bx)}\right)^2\text{arctanh}\left(e^{a+bx}\right)}{b\left(-1+e^{2(a+bx)}\right)^2}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[a + b*x]^3,x]
```

output
```
(3*E^(a + b*x) - 5*E^(3*(a + b*x)) - 3*(-1 + E^(2*(a + b*x)))^2*ArcTanh[E^
(a + b*x)])/(b*(-1 + E^(2*(a + b*x)))^2)
```

3.877.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2720, 27, 252, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{8e^{4a+4bx}}{\left(1-e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8\int \frac{e^{4a+4bx}}{\left(1-e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{8\left(\frac{e^{3a+3bx}}{4\left(1-e^{2a+2bx}\right)^2}-\frac{3}{4}\int \frac{e^{2a+2bx}}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{8\left(\frac{e^{3a+3bx}}{4\left(1-e^{2a+2bx}\right)^2}-\frac{3}{4}\left(\frac{e^{a+bx}}{2\left(1-e^{2a+2bx}\right)}-\frac{1}{2}\int \frac{1}{1-e^{2a+2bx}}de^{a+bx}\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{8\left(\frac{e^{3a+3bx}}{4\left(1-e^{2a+2bx}\right)^2}-\frac{3}{4}\left(\frac{e^{a+bx}}{2\left(1-e^{2a+2bx}\right)}-\frac{1}{2}\text{arctanh}\left(e^{a+bx}\right)\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[a + b*x]^3,x]
```

output
```
(-8*(E^(3*a + 3*b*x)/(4*(1 - E^(2*a + 2*b*x))^2) - (3*(E^(a + b*x)/(2*(1 -
 E^(2*a + 2*b*x))) - ArcTanh[E^(a + b*x)]/2))/4))/b
```

3.877.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.877.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

method	result	size
risch	$-\frac{{\mathrm e}^{b x +a} \left(5 \,{\mathrm e}^{2 b x +2 a}-3\right)}{b \left({\mathrm e}^{2 b x +2 a}-1\right)^{2}}-\frac{3 \ln \left({\mathrm e}^{b x +a}+1\right)}{2 b}+\frac{3 \ln \left({\mathrm e}^{b x +a}-1\right)}{2 b}$	67

input
```
int(exp(2*b*x+2*a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-exp(b*x+a)*(5*exp(2*b*x+2*a)-3)/b/(exp(2*b*x+2*a)-1)^2-3/2/b*ln(exp(b*x+a
)+1)+3/2/b*ln(exp(b*x+a)-1)
```

3.877.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(64) = 128$.

Time = 0.25 (sec) , antiderivative size = 388, normalized size of antiderivative = 5.32

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx =$$
$$-\frac{10 \cosh(bx+a)^3 + 30 \cosh(bx+a)\sinh(bx+a)^2 + 10 \sinh(bx+a)^3 + 3\left(\cosh(bx+a)^4 + 4 \cosh(b\ldots\right.}{\ldots}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(10*cosh(b*x + a)^3 + 30*cosh(b*x + a)*sinh(b*x + a)^2 + 10*sinh(b*x
+ a)^3 + 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
 a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*
(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + s
inh(b*x + a) + 1) - 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
 sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b
*x + a) + sinh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 6
*cosh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b
*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(
b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

3.877.6 Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}^3(a+bx)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+a)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(a + b*x)**3, x)
```

3.877.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx = -\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{5\,e^{(-bx-a)}-3\,e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-3/2*log(e^(-b*x - a) + 1)/b + 3/2*log(e^(-b*x - a) - 1)/b + (5*e^(-b*x -
a) - 3*e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

3.877.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx = -\frac{\frac{2\left(5\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2}+3\,\log\left(e^{(bx+a)}+1\right)-3\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*(2*(5*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 + 3*lo
g(e^(b*x + a) + 1) - 3*log(abs(e^(b*x + a) - 1)))/b
```

3.877.9 Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx = -\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,\mathrm{e}^{3\,a+3\,b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{3\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(exp(2*a + 2*b*x)/sinh(a + b*x)^3,x)
```

output
```
- (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(3*a + 3
*b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (3*exp(a + b*x))/
(b*(exp(2*a + 2*b*x) - 1))
```

3.877. $\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx$

3.878 $\int e^{a+bx} \sinh^3(c+dx)\,dx$

3.878.1 Optimal result

Integrand size = 16, antiderivative size = 139

$$\int e^{a+bx} \sinh^3(c+dx)\,dx = -\frac{6d^3e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4} + \frac{6bd^2e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4} - \frac{3de^{a+bx}\cosh(c+dx)\sinh^2(c+dx)}{b^2-9d^2} + \frac{be^{a+bx}\sinh^3(c+dx)}{b^2-9d^2}$$

output
```
-6*d^3*exp(b*x+a)*cosh(d*x+c)/(b^4-10*b^2*d^2+9*d^4)+6*b*d^2*exp(b*x+a)*si
nh(d*x+c)/(b^4-10*b^2*d^2+9*d^4)-3*d*exp(b*x+a)*cosh(d*x+c)*sinh(d*x+c)^2/
(b^2-9*d^2)+b*exp(b*x+a)*sinh(d*x+c)^3/(b^2-9*d^2)
```

3.878.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int e^{a+bx} \sinh^3(c+dx)\,dx = \frac{e^{a+bx}(3d(b^2-9d^2)\cosh(c+dx) + (-3b^2d+3d^3)\cosh(3(c+dx)) + 2b(-b^2+13d^2+(b^2-d^2)\cosh(2(c+}{4(b^4-10b^2d^2+9d^4)}$$

input
```
Integrate[E^(a + b*x)*Sinh[c + d*x]^3,x]
```

output
```
(E^(a + b*x)*(3*d*(b^2 - 9*d^2)*Cosh[c + d*x] + (-3*b^2*d + 3*d^3)*Cosh[3*
(c + d*x)] + 2*b*(-b^2 + 13*d^2 + (b^2 - d^2)*Cosh[2*(c + d*x)])*Sinh[c +
d*x]))/(4*(b^4 - 10*b^2*d^2 + 9*d^4))
```

3.878.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5999, 5997}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \sinh^3(c+dx) \, dx$$

$$\Big\downarrow \text{5999}$$

$$\frac{6d^2 \int e^{a+bx} \sinh(c+dx)dx}{b^2-9d^2}+\frac{be^{a+bx} \sinh^3(c+dx)}{b^2-9d^2}-\frac{3de^{a+bx} \sinh^2(c+dx) \cosh(c+dx)}{b^2-9d^2}$$

$$\Big\downarrow \text{5997}$$

$$\frac{be^{a+bx} \sinh^3(c+dx)}{b^2-9d^2}-\frac{3de^{a+bx} \sinh^2(c+dx) \cosh(c+dx)}{b^2-9d^2}+\frac{6d^2 \left(\frac{be^{a+bx} \sinh(c+dx)}{b^2-d^2}-\frac{de^{a+bx} \cosh(c+dx)}{b^2-d^2}\right)}{b^2-9d^2}$$

input
```
Int[E^(a + b*x)*Sinh[c + d*x]^3,x]
```

output
```
(-3*d*E^(a + b*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(b^2 - 9*d^2) + (b*E^(a +
 b*x)*Sinh[c + d*x]^3)/(b^2 - 9*d^2) + (6*d^2*(-((d*E^(a + b*x)*Cosh[c + d
*x])/(b^2 - d^2)) + (b*E^(a + b*x)*Sinh[c + d*x])/(b^2 - d^2)))/(b^2 - 9*d
^2)
```

3.878.3.1 Defintions of rubi rules used

rule 5997
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

rule 5999
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Cosh[d + e*x]*(Sinh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] - Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

3.878.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

method	result
risch	$\frac{e^{bx+3dx+a+3c}}{8b+24d}-\frac{3\,e^{bx+dx+a+c}}{8(b+d)}+\frac{3\,e^{bx-dx+a-c}}{8(b-d)}-\frac{e^{bx-3dx+a-3c}}{8(b-3d)}$
parallelrisch	$\frac{e^{bx+a}\left(\left(-3b^2d+3d^3\right)\cosh(3dx+3c)+\left(b^3-b\,d^2\right)\sinh(3dx+3c)-3(b-3d)(b+3d)(b\sinh(dx+c)-d\cosh(dx+c))\right)}{4b^4-40b^2d^2+36d^4}$
default	$-\frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}-\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}-\frac{\cosh(a-3c+(b-3d)}{8(b-3d)}$

input
```
int(exp(b*x+a)*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/(b+3*d)*exp(b*x+3*d*x+a+3*c)-3/8/(b+d)*exp(b*x+d*x+a+c)+3/8/(b-d)*exp(
b*x-d*x+a-c)-1/8/(b-3*d)*exp(b*x-3*d*x+a-3*c)
```

3.878.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(135) = 270.

Time = 0.25 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.27

$$\int e^{a+bx}\sinh^3(c+dx)\,dx =$$
$$-\frac{3\left(b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)^3-\left(\left(b^3-bd^2\right)\cosh\left(bx+a\right)+\left(b^3-bd^2\right)\sinh\left(bx+a\right)\right)\sinh\left(dx\right.}{}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)^3 - ((b^3 - b*d^2)*cosh(
b*x + a) + (b^3 - b*d^2)*sinh(b*x + a))*sinh(d*x + c)^3 - 3*(b^2*d - 9*d^3
)*cosh(b*x + a)*cosh(d*x + c) + 9*((b^2*d - d^3)*cosh(b*x + a)*cosh(d*x +
c) + (b^2*d - d^3)*cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 + 3*((b^2*
d - d^3)*cosh(d*x + c)^3 - (b^2*d - 9*d^3)*cosh(d*x + c))*sinh(b*x + a) -
3*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)^2 - (b^3 - 9*b*d^2)*cosh(b*x
+ a) - (b^3 - 9*b*d^2 - (b^3 - b*d^2)*cosh(d*x + c)^2)*sinh(b*x + a))*sinh
(d*x + c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

3.878.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(131) = 262$.

Time = 2.35 (sec) , antiderivative size = 976, normalized size of antiderivative = 7.02

$$\int e^{a+bx}\sinh^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c)**3,x)
```

output
```
Piecewise((x*exp(a)*sinh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-3*d*x
)*sinh(c + d*x)**3/8 + 3*x*exp(a)*exp(-3*d*x)*sinh(c + d*x)**2*cosh(c + d*
x)/8 + 3*x*exp(a)*exp(-3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + x*exp(a)*
exp(-3*d*x)*cosh(c + d*x)**3/8 - 3*exp(a)*exp(-3*d*x)*sinh(c + d*x)**3/(8*
d) - exp(a)*exp(-3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + exp(a)*exp(
-3*d*x)*cosh(c + d*x)**3/(24*d), Eq(b, -3*d)), (3*x*exp(a)*exp(-d*x)*sinh(
c + d*x)**3/8 + 3*x*exp(a)*exp(-d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 - 3*
x*exp(a)*exp(-d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3*x*exp(a)*exp(-d*x)
*cosh(c + d*x)**3/8 + exp(a)*exp(-d*x)*sinh(c + d*x)**3/(8*d) + 3*exp(a)*e
xp(-d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) - 3*exp(a)*exp(-d*x)*cosh(c
+ d*x)**3/(8*d), Eq(b, -d)), (3*x*exp(a)*exp(d*x)*sinh(c + d*x)**3/8 - 3*x
*exp(a)*exp(d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 - 3*x*exp(a)*exp(d*x)*si
nh(c + d*x)*cosh(c + d*x)**2/8 + 3*x*exp(a)*exp(d*x)*cosh(c + d*x)**3/8 -
exp(a)*exp(d*x)*sinh(c + d*x)**3/(8*d) + 3*exp(a)*exp(d*x)*sinh(c + d*x)**
2*cosh(c + d*x)/(4*d) - 3*exp(a)*exp(d*x)*cosh(c + d*x)**3/(8*d), Eq(b, d)
), (x*exp(a)*exp(3*d*x)*sinh(c + d*x)**3/8 - 3*x*exp(a)*exp(3*d*x)*sinh(c
+ d*x)**2*cosh(c + d*x)/8 + 3*x*exp(a)*exp(3*d*x)*sinh(c + d*x)*cosh(c + d
*x)**2/8 - x*exp(a)*exp(3*d*x)*cosh(c + d*x)**3/8 + 3*exp(a)*exp(3*d*x)*si
nh(c + d*x)**3/(8*d) - exp(a)*exp(3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4
*d) + exp(a)*exp(3*d*x)*cosh(c + d*x)**3/(24*d), Eq(b, 3*d)), (b**3*exp...
```

3.878.7 Maxima [F(-2)]

Exception generated.

$$\int e^{a+bx} \sinh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for
more detai
```

3.878.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.60

$$\int e^{a+bx} \sinh^3(c+dx)\,dx = \frac{e^{(bx+3\,dx+a+3\,c)}}{8\,(b+3\,d)} - \frac{3\,e^{(bx+dx+a+c)}}{8\,(b+d)} + \frac{3\,e^{(bx-dx+a-c)}}{8\,(b-d)} - \frac{e^{(bx-3\,dx+a-3\,c)}}{8\,(b-3\,d)}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*e^(b*x + d*x + a + c)/(b + d
) + 3/8*e^(b*x - d*x + a - c)/(b - d) - 1/8*e^(b*x - 3*d*x + a - 3*c)/(b -
 3*d)
```

3.878.9 Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int e^{a+bx} \sinh^3(c+dx)\,dx =$$

$$-\frac{e^{a+b\,x}\,\left(-b^3\,\sinh(c+d\,x)^3 + 3\,b^2\,d\cosh(c+d\,x)\,\sinh(c+d\,x)^2 - 6\,b\,d^2\cosh(c+d\,x)^2\sinh(c+d\,x) + 7\,b\right.}{b^4 - 10\,b^2\,d^2 + 9\,d^4}$$

input
```
int(exp(a + b*x)*sinh(c + d*x)^3,x)
```

output
```
-(exp(a + b*x)*(6*d^3*cosh(c + d*x)^3 - b^3*sinh(c + d*x)^3 - 9*d^3*cosh(c
 + d*x)*sinh(c + d*x)^2 + 7*b*d^2*sinh(c + d*x)^3 - 6*b*d^2*cosh(c + d*x)^
2*sinh(c + d*x) + 3*b^2*d*cosh(c + d*x)*sinh(c + d*x)^2))/(b^4 + 9*d^4 - 1
0*b^2*d^2)
```

3.878. $\int e^{a+bx}\sinh^3(c+dx)\,dx$

3.879 $\int e^{a+bx}\sinh^2(c+dx)\,dx$

3.879.1 Optimal result

Integrand size = 16, antiderivative size = 88

$$\int e^{a+bx}\sinh^2(c+dx)\,dx=\frac{2d^2e^{a+bx}}{b(b^2-4d^2)}-\frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2}+\frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}$$

output
```
2*d^2*exp(b*x+a)/b/(b^2-4*d^2)-2*d*exp(b*x+a)*cosh(d*x+c)*sinh(d*x+c)/(b^2
-4*d^2)+b*exp(b*x+a)*sinh(d*x+c)^2/(b^2-4*d^2)
```

3.879.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int e^{a+bx}\sinh^2(c+dx)\,dx=\frac{e^{a+bx}(-b^2+4d^2+b^2\cosh(2(c+dx))-2bd\sinh(2(c+dx)))}{2(b^3-4bd^2)}$$

input
```
Integrate[E^(a + b*x)*Sinh[c + d*x]^2,x]
```

output
```
(E^(a + b*x)*(-b^2 + 4*d^2 + b^2*Cosh[2*(c + d*x)] - 2*b*d*Sinh[2*(c + d*x
)]))/(2*(b^3 - 4*b*d^2))
```

3.879.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5999, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^2(c+dx)\,dx$$

$$\Big\downarrow \; 5999$$

$$\frac{2d^2\int e^{a+bx}dx}{b^2-4d^2}+\frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2}$$

$$\Big\downarrow \; 2624$$

$$\frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2}+\frac{2d^2e^{a+bx}}{b\,(b^2-4d^2)}$$

input
```
Int[E^(a + b*x)*Sinh[c + d*x]^2,x]
```

output
```
(2*d^2*E^(a + b*x))/(b*(b^2 - 4*d^2)) - (2*d*E^(a + b*x)*Cosh[c + d*x]*Sin
h[c + d*x])/(b^2 - 4*d^2) + (b*E^(a + b*x)*Sinh[c + d*x]^2)/(b^2 - 4*d^2)
```

3.879.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 5999
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Cosh[d + e*x]*(Sinh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] - Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

3.879.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{e^{bx+a}}{2b}+\frac{e^{bx+2dx+a+2c}}{4b+8d}+\frac{e^{bx-2dx+a-2c}}{4b-8d}$
parallelrisch	$\frac{e^{bx+a}(\cosh(2dx+2c)b^2-2bd\sinh(2dx+2c)-b^2+4d^2)}{2b^3-8b\,d^2}$
default	$-\frac{\sinh(bx+a)}{2b}+\frac{\sinh(a-2c+(b-2d)x)}{4b-8d}+\frac{\sinh(a+2c+(b+2d)x)}{4b+8d}-\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4b-8d}+\frac{\cosh(a+2c\cdots}{4b+\cdots}$

input
```
int(exp(b*x+a)*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(b*x+a)/b+1/4/(b+2*d)*exp(b*x+2*d*x+a+2*c)+1/4/(b-2*d)*exp(b*x-2*d
*x+a-2*c)
```

3.879.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.69

$$\int e^{a+bx}\sinh^2(c+dx)\,dx$$
$$=\frac{b^2\cosh(bx+a)\cosh(dx+c)^2+(b^2\cosh(bx+a)+b^2\sinh(bx+a))\sinh(dx+c)^2-(b^2-4\,d^2)\cosh(bx\cdots}{\cdots}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*cosh(b*x + a) + b^2*sinh(b*x
 + a))*sinh(d*x + c)^2 - (b^2 - 4*d^2)*cosh(b*x + a) + (b^2*cosh(d*x + c)^
2 - b^2 + 4*d^2)*sinh(b*x + a) - 4*(b*d*cosh(b*x + a)*cosh(d*x + c) + b*d*
cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c))/(b^3 - 4*b*d^2)
```

3.879.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(78) = 156$.

Time = 0.84 (sec) , antiderivative size = 428, normalized size of antiderivative = 4.86

$$\int e^{a+bx}\sinh^2(c+dx)\,dx$$
$$= \begin{cases} xe^a\sinh^2(c) \\ \left(\frac{x\sinh^2(c+dx)}{2} - \frac{x\cosh^2(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)e^a \\ \frac{xe^ae^{-2dx}\sinh^2(c+dx)}{4} + \frac{xe^ae^{-2dx}\sinh(c+dx)\cosh(c+dx)}{2} + \frac{xe^ae^{-2dx}\cosh^2(c+dx)}{4} - \frac{e^ae^{-2dx}\sinh^2(c+dx)}{2d} - \frac{e^ae^{-2dx}\sinh(c+dx)}{4d} \\ \frac{xe^ae^{2dx}\sinh^2(c+dx)}{4} - \frac{xe^ae^{2dx}\sinh(c+dx)\cosh(c+dx)}{2} + \frac{xe^ae^{2dx}\cosh^2(c+dx)}{4} + \frac{e^ae^{2dx}\sinh^2(c+dx)}{2d} - \frac{e^ae^{2dx}\sinh(c+dx)\cosh(c}{4d} \\ \frac{b^2e^ae^{bx}\sinh^2(c+dx)}{b^3-4bd^2} - \frac{2bde^ae^{bx}\sinh(c+dx)\cosh(c+dx)}{b^3-4bd^2} - \frac{2d^2e^ae^{bx}\sinh^2(c+dx)}{b^3-4bd^2} + \frac{2d^2e^ae^{bx}\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c)**2,x)
```

output
```
Piecewise((x*exp(a)*sinh(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sinh(c + d*x)**2
/2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*exp(a), Eq(
b, 0)), (x*exp(a)*exp(-2*d*x)*sinh(c + d*x)**2/4 + x*exp(a)*exp(-2*d*x)*si
nh(c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(-2*d*x)*cosh(c + d*x)**2/4 - ex
p(a)*exp(-2*d*x)*sinh(c + d*x)**2/(2*d) - exp(a)*exp(-2*d*x)*sinh(c + d*x)
*cosh(c + d*x)/(4*d), Eq(b, -2*d)), (x*exp(a)*exp(2*d*x)*sinh(c + d*x)**2/
4 - x*exp(a)*exp(2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(2*d*x
)*cosh(c + d*x)**2/4 + exp(a)*exp(2*d*x)*sinh(c + d*x)**2/(2*d) - exp(a)*e
xp(2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d), Eq(b, 2*d)), (b**2*exp(a)*exp
(b*x)*sinh(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*exp(a)*exp(b*x)*sinh(c +
d*x)*cosh(c + d*x)/(b**3 - 4*b*d**2) - 2*d**2*exp(a)*exp(b*x)*sinh(c + d*x
)**2/(b**3 - 4*b*d**2) + 2*d**2*exp(a)*exp(b*x)*cosh(c + d*x)**2/(b**3 - 4
*b*d**2), True))
```

3.879.7 Maxima [F(-2)]

Exception generated.

$$\int e^{a+bx}\sinh^2(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for
more detai
```

3.879.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\sinh^2(c+dx)\,dx = \frac{e^{(bx+2\,dx+a+2\,c)}}{4\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{4\,(b-2\,d)} - \frac{e^{(bx+a)}}{2\,b}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*e^(b*x - 2*d*x + a - 2*c)/(b
 - 2*d) - 1/2*e^(b*x + a)/b
```

3.879.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int e^{a+bx}\sinh^2(c+dx)\,dx$$
$$= -\frac{2\,d^2\,e^{a+b\,x} - b^2\left(\frac{e^{a+b\,x}}{2} - e^{a+b\,x}\left(\frac{e^{-2\,c-2\,d\,x}}{4} + \frac{e^{2\,c+2\,d\,x}}{4}\right)\right) + b\,d\,e^{a+b\,x}\left(\frac{e^{-2\,c-2\,d\,x}}{2} - \frac{e^{2\,c+2\,d\,x}}{2}\right)}{4\,b\,d^2 - b^3}$$

input
```
int(exp(a + b*x)*sinh(c + d*x)^2,x)
```

output
```
-(2*d^2*exp(a + b*x) - b^2*(exp(a + b*x)/2 - exp(a + b*x)*(exp(- 2*c - 2*d
*x)/4 + exp(2*c + 2*d*x)/4)) + b*d*exp(a + b*x)*(exp(- 2*c - 2*d*x)/2 - ex
p(2*c + 2*d*x)/2))/(4*b*d^2 - b^3)
```

3.880 $\int e^{a+bx}\sinh(c+dx)\,dx$

3.880.1 Optimal result

Integrand size = 14, antiderivative size = 54

$$\int e^{a+bx}\sinh(c+dx)\,dx=-\frac{de^{a+bx}\cosh(c+dx)}{b^2-d^2}+\frac{be^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

output
```
-d*exp(b*x+a)*cosh(d*x+c)/(b^2-d^2)+b*exp(b*x+a)*sinh(d*x+c)/(b^2-d^2)
```

3.880.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\sinh(c+dx)\,dx=\frac{e^{a+bx}(-d\cosh(c+dx)+b\sinh(c+dx))}{(b-d)(b+d)}$$

input
```
Integrate[E^(a + b*x)*Sinh[c + d*x],x]
```

output
```
(E^(a + b*x)*(-(d*Cosh[c + d*x]) + b*Sinh[c + d*x]))/((b - d)*(b + d))
```

3.880.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5997}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh(c+dx)\,dx$$

$$\Big\downarrow \text{5997}$$

$$\frac{be^{a+bx}\sinh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\cosh(c+dx)}{b^2-d^2}$$

input
```
Int[E^(a + b*x)*Sinh[c + d*x],x]
```

output
```
-((d*E^(a + b*x)*Cosh[c + d*x])/(b^2 - d^2)) + (b*E^(a + b*x)*Sinh[c + d*x
])/(b^2 - d^2)
```

3.880.3.1 Defintions of rubi rules used

rule 5997
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

3.880.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{e^{bx+a}(b\sinh(dx+c)-d\cosh(dx+c))}{b^2-d^2}$	37
risch	$\frac{e^{bx+dx+a+c}}{2b+2d}-\frac{e^{bx-dx+a-c}}{2(b-d)}$	41
default	$-\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}-\frac{\cosh(a-c+(b-d)x)}{2(b-d)}+\frac{\cosh(a+c+(b+d)x)}{2b+2d}$	78

input
```
int(exp(b*x+a)*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/(b^2-d^2)*(b*sinh(d*x+c)-d*cosh(d*x+c))
```

3.880.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int e^{a+bx}\sinh(c+dx)\,dx = -\frac{d\cosh(bx+a)\cosh(dx+c)+d\cosh(dx+c)\sinh(bx+a)-(b\cosh(bx+a)+b\sinh(bx+a))\sinh(\cdots}{b^2-d^2}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c),x, algorithm="fricas")
```

output
```
-(d*cosh(b*x + a)*cosh(d*x + c) + d*cosh(d*x + c)*sinh(b*x + a) - (b*cosh(
b*x + a) + b*sinh(b*x + a))*sinh(d*x + c))/(b^2 - d^2)
```

3.880.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(42) = 84$.

Time = 0.38 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.72

$$\int e^{a+bx}\sinh(c+dx)\,dx$$
$$= \begin{cases} xe^{a}\sinh(c) & \text{for } b=0\wedge d=0 \\ \frac{xe^{a}e^{-dx}\sinh(c+dx)}{2}+\frac{xe^{a}e^{-dx}\cosh(c+dx)}{2}+\frac{e^{a}e^{-dx}\sinh(c+dx)}{2d}+\frac{e^{a}e^{-dx}\cosh(c+dx)}{d} & \text{for } b=-d \\ \frac{xe^{a}e^{dx}\sinh(c+dx)}{2}-\frac{xe^{a}e^{dx}\cosh(c+dx)}{2}-\frac{e^{a}e^{dx}\sinh(c+dx)}{2d}+\frac{e^{a}e^{dx}\cosh(c+dx)}{d} & \text{for } b=d \\ \frac{be^{a}e^{bx}\sinh(c+dx)}{b^2-d^2}-\frac{de^{a}e^{bx}\cosh(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c),x)
```

output
```
Piecewise((x*exp(a)*sinh(c), Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-d*x)*sin
h(c + d*x)/2 + x*exp(a)*exp(-d*x)*cosh(c + d*x)/2 + exp(a)*exp(-d*x)*sinh(
c + d*x)/(2*d) + exp(a)*exp(-d*x)*cosh(c + d*x)/d, Eq(b, -d)), (x*exp(a)*e
xp(d*x)*sinh(c + d*x)/2 - x*exp(a)*exp(d*x)*cosh(c + d*x)/2 - exp(a)*exp(d
*x)*sinh(c + d*x)/(2*d) + exp(a)*exp(d*x)*cosh(c + d*x)/d, Eq(b, d)), (b*e
xp(a)*exp(b*x)*sinh(c + d*x)/(b**2 - d**2) - d*exp(a)*exp(b*x)*cosh(c + d*
x)/(b**2 - d**2), True))
```

3.880.7 Maxima [F(-2)]

Exception generated.

$$\int e^{a+bx} \sinh(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more
 details)I
```

3.880.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int e^{a+bx} \sinh(c+dx)\,dx = \frac{e^{(bx+dx+a+c)}}{2(b+d)} - \frac{e^{(bx-dx+a-c)}}{2(b-d)}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c),x, algorithm="giac")
```

output
```
1/2*e^(b*x + d*x + a + c)/(b + d) - 1/2*e^(b*x - d*x + a - c)/(b - d)
```

3.880.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int e^{a+bx} \sinh(c+dx)\,dx = -\frac{e^{a-c+b\,x-d\,x}\,\left(b+d-b\,e^{2\,c+2\,d\,x}+d\,e^{2\,c+2\,d\,x}\right)}{2\,\left(b^2-d^2\right)}$$

input
```
int(exp(a + b*x)*sinh(c + d*x),x)
```

output
```
-(exp(a - c + b*x - d*x)*(b + d - b*exp(2*c + 2*d*x) + d*exp(2*c + 2*d*x))
)/(2*(b^2 - d^2))
```

3.881 $\int e^{a+bx}\operatorname{csch}(c+dx)\,dx$

3.881.1 Optimal result

Integrand size = 14, antiderivative size = 50

$$\int e^{a+bx}\operatorname{csch}(c+dx)\,dx=-\frac{2e^{a+c+bx+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),e^{2(c+dx)}\right)}{b+d}$$

output
```
-2*exp(b*x+d*x+a+c)*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],exp(2*d*x+2*c
))/(b+d)
```

3.881.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\operatorname{csch}(c+dx)\,dx=-\frac{2e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),e^{2(c+dx)}\right)}{b+d}$$

input
```
Integrate[E^(a + b*x)*Csch[c + d*x],x]
```

output
```
(-2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2,
 E^(2*(c + d*x))])/(b + d)
```

3.881.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {6016}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}(c+dx)\,dx$$

$$\Big\downarrow \text{ 6016}$$

$$-\frac{2e^{a+bx+c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(\frac{b}{d}+3\right),e^{2(c+dx)}\right)}{b+d}$$

input
```
Int[E^(a + b*x)*Csch[c + d*x],x]
```

output
```
(-2*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2,
 E^(2*(c + d*x))])/(b + d)
```

3.881.3.1 Defintions of rubi rules used

rule 6016
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.881.4 Maple [F]

$$\int \mathrm{e}^{bx+a}\,\text{csch}\left(dx+c\right)dx$$

input
```
int(exp(b*x+a)*csch(d*x+c),x)
```

output
```
int(exp(b*x+a)*csch(d*x+c),x)
```

3.881.5 Fricas [F]

$$\int e^{a+bx}\text{csch}(c+dx)\,dx = \int \text{csch}\,(dx+c)\,e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*csch(d*x+c),x, algorithm="fricas")
```

output
```
integral(csch(d*x + c)*e^(b*x + a), x)
```

3.881.6 Sympy [F]

$$\int e^{a+bx}\text{csch}(c+dx)\,dx = e^{a}\int e^{bx}\,\text{csch}\,(c+dx)\,dx$$

input
```
integrate(exp(b*x+a)*csch(d*x+c),x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(c + d*x), x)
```

3.881.7 Maxima [F]

$$\int e^{a+bx}\text{csch}(c+dx)\,dx = \int \text{csch}\,(dx+c)\,e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*csch(d*x+c),x, algorithm="maxima")
```

output
```
integrate(csch(d*x + c)*e^(b*x + a), x)
```

3.881.8 Giac [F]

$$\int e^{a+bx}\text{csch}(c+dx)\,dx = \int \text{csch}\left(dx+c\right)e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*csch(d*x+c),x, algorithm="giac")
```

output
```
integrate(csch(d*x + c)*e^(b*x + a), x)
```

3.881.9 Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{csch}(c+dx)\,dx = \int \frac{\mathrm{e}^{a+b\,x}}{\sinh\left(c+d\,x\right)}\,dx$$

input
```
int(exp(a + b*x)/sinh(c + d*x),x)
```

output
```
int(exp(a + b*x)/sinh(c + d*x), x)
```

3.882 $\int e^{c+dx}\text{csch}^2(a+bx)\,dx$

3.882.1 Optimal result

Integrand size = 16, antiderivative size = 54

$$\int e^{c+dx}\text{csch}^2(a+bx)\,dx=\frac{4e^{c+dx+2(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{d}{2b},2+\frac{d}{2b},e^{2(a+bx)}\right)}{2b+d}$$

output
```
4*exp(2*b*x+d*x+2*a+c)*hypergeom([2, 1+1/2*d/b],[2+1/2*d/b],exp(2*b*x+2*a)
)/(2*b+d)
```

3.882.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 137 vs. 2(54) = 108.

Time = 1.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.54

$$\int e^{c+dx}\text{csch}^2(a+bx)\,dx=-\frac{2d\left(\frac{e^{2a+c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}-\frac{e^{2a+c+(2b+d)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{d}{2b},2+\frac{d}{2b},e^{2(a+bx)}\right)}{2b+d}\right)}{b\left(-1+e^{2a}\right)}+\frac{e^{c+dx}\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b}$$

input
```
Integrate[E^(c + d*x)*Csch[a + b*x]^2,x]
```

output
```
(-2*d*((E^(2*a + c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E^(2*
(a + b*x))])/d - (E^(2*a + c + (2*b + d)*x)*Hypergeometric2F1[1, 1 + d/(2*
b), 2 + d/(2*b), E^(2*(a + b*x))])/(2*b + d)))/(b*(-1 + E^(2*a))) + (E^(c
+ d*x)*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

3.882.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6016}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 6016}$$

$$\frac{4e^{2(a+bx)+c+dx}\,\text{Hypergeometric2F1}\left(2,\frac{d}{2b}+1,\frac{d}{2b}+2,e^{2(a+bx)}\right)}{2b+d}$$

input
```
Int[E^(c + d*x)*Csch[a + b*x]^2,x]
```

output
```
(4*E^(c + d*x + 2*(a + b*x))*Hypergeometric2F1[2, 1 + d/(2*b), 2 + d/(2*b)
, E^(2*(a + b*x))])/(2*b + d)
```

3.882.3.1 Defintions of rubi rules used

rule 6016
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.882.4 Maple [F]

$$\int e^{dx+c} \operatorname{csch}(bx+a)^2 \, dx$$

input
```
int(exp(d*x+c)*csch(b*x+a)^2,x)
```

output
```
int(exp(d*x+c)*csch(b*x+a)^2,x)
```

3.882.5 Fricas [F]

$$\int e^{c+dx} \operatorname{csch}^2(a+bx) \, dx = \int \operatorname{csch}(bx+a)^2 e^{(dx+c)} \, dx$$

input
```
integrate(exp(d*x+c)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^2*e^(d*x + c), x)
```

3.882.6 Sympy [F]

$$\int e^{c+dx} \operatorname{csch}^2(a+bx) \, dx = e^c \int e^{dx} \operatorname{csch}^2(a+bx) \, dx$$

input
```
integrate(exp(d*x+c)*csch(b*x+a)**2,x)
```

output
```
exp(c)*Integral(exp(d*x)*csch(a + b*x)**2, x)
```

3.882.7 Maxima [F]

$$\int e^{c+dx}\operatorname{csch}^2(a+bx)\,dx=\int \operatorname{csch}\left(bx+a\right)^2e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
16*b*d*integrate(-e^(d*x + c)/(8*b^2 - 6*b*d + d^2 - (8*b^2 - 6*b*d + d^2)
*e^(6*b*x + 6*a) + 3*(8*b^2 - 6*b*d + d^2)*e^(4*b*x + 4*a) - 3*(8*b^2 - 6*
b*d + d^2)*e^(2*b*x + 2*a)), x) - 4*((4*b*e^c - d*e^c)*e^(2*b*x + 2*a) - 4
*b*e^c)*e^(d*x)/(8*b^2 - 6*b*d + d^2 + (8*b^2 - 6*b*d + d^2)*e^(4*b*x + 4*
a) - 2*(8*b^2 - 6*b*d + d^2)*e^(2*b*x + 2*a))
```

3.882.8 Giac [F]

$$\int e^{c+dx}\operatorname{csch}^2(a+bx)\,dx=\int \operatorname{csch}\left(bx+a\right)^2e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^2*e^(d*x + c), x)
```

3.882.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\operatorname{csch}^2(a+bx)\,dx=\int \frac{e^{c+d\,x}}{\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int(exp(c + d*x)/sinh(a + b*x)^2,x)
```

output
```
int(exp(c + d*x)/sinh(a + b*x)^2, x)
```

3.883 $\int e^{c+dx}\text{csch}^3(a+bx)\,dx$

3.883.1 Optimal result

Integrand size = 16, antiderivative size = 100

$$\int e^{c+dx}\text{csch}^3(a+bx)\,dx = -\frac{de^{c+dx}\text{csch}(a+bx)}{2b^2}-\frac{e^{c+dx}\coth(a+bx)\text{csch}(a+bx)}{2b} + \frac{(b-d)e^{a+c+bx+dx}\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{1}{2}\left(3+\frac{d}{b}\right),e^{2(a+bx)}\right)}{b^2}$$

output
```
-1/2*d*exp(d*x+c)*csch(b*x+a)/b^2-1/2*exp(d*x+c)*coth(b*x+a)*csch(b*x+a)/b
+(b-d)*exp(b*x+d*x+a+c)*hypergeom([1, 1/2*(b+d)/b],[3/2+1/2*d/b],exp(2*b*x
+2*a))/b^2
```

3.883.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int e^{c+dx}\text{csch}^3(a+bx)\,dx = \frac{-4e^{c+dx}(d+b\coth(a+bx))\text{csch}(a+bx)+8(b-d)e^{a+c+(b+d)x}\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{1}{2}\left(3+\frac{d}{b}\right),e^{2(}\right.}{8b^2}$$

input
```
Integrate[E^(c + d*x)*Csch[a + b*x]^3,x]
```

output
```
(-4*E^(c + d*x)*(d + b*Coth[a + b*x])*Csch[a + b*x] + 8*(b - d)*E^(a + c +
 (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3 + d/b)/2, E^(2*(a + b*x
))])/(8*b^2)
```

3.883.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6014, 6016}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{6014}$$

$$-\frac{1}{2}\left(1-\frac{d^2}{b^2}\right)\int e^{c+dx}\text{csch}(a+bx)dx-\frac{de^{c+dx}\text{csch}(a+bx)}{2b^2}-\frac{e^{c+dx}\coth(a+bx)\text{csch}(a+bx)}{2b}$$

$$\Big\downarrow \text{6016}$$

$$\frac{\left(1-\frac{d^2}{b^2}\right)e^{a+bx+c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{1}{2}\left(\frac{d}{b}+3\right),e^{2(a+bx)}\right)}{b+d}-\frac{de^{c+dx}\text{csch}(a+bx)}{2b^2}-\frac{e^{c+dx}\coth(a+bx)\text{csch}(a+bx)}{2b}$$

input
```
Int[E^(c + d*x)*Csch[a + b*x]^3,x]
```

output
```
-1/2*(d*E^(c + d*x)*Csch[a + b*x])/b^2 - (E^(c + d*x)*Coth[a + b*x]*Csch[a
 + b*x])/(2*b) + ((1 - d^2/b^2)*E^(a + c + b*x + d*x)*Hypergeometric2F1[1,
 (b + d)/(2*b), (3 + d/b)/2, E^(2*(a + b*x))])/(b + d)
```

3.883.3.1 Defintions of rubi rules used

rule 6014
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csch[d + e*x]^(n - 2)/(e^2*(n -
1)*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csch[d + e*x]^(n - 1)*(Cosh[d + e
*x]/(e*(n - 1))), x] - Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)
*(n - 2))   Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
] && NeQ[n, 2]
```

rule 6016
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.883.4 Maple [F]

$$\int \mathrm{e}^{dx+c} \operatorname{csch}(bx+a)^{3} dx$$

input
```
int(exp(d*x+c)*csch(b*x+a)^3,x)
```

output
```
int(exp(d*x+c)*csch(b*x+a)^3,x)
```

3.883.5 Fricas [F]

$$\int e^{c+dx}\operatorname{csch}^3(a+bx)\,dx = \int \operatorname{csch}(bx+a)^{3} e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^3*e^(d*x + c), x)
```

3.883.6 Sympy [F]

$$\int e^{c+dx}\operatorname{csch}^3(a+bx)\,dx = e^c\int e^{dx}\operatorname{csch}^3{(a+bx)}\,dx$$

input
```
integrate(exp(d*x+c)*csch(b*x+a)**3,x)
```

output
```
exp(c)*Integral(exp(d*x)*csch(a + b*x)**3, x)
```

3.883.7 Maxima [F]

$$\int e^{c+dx}\operatorname{csch}^3(a+bx)\,dx = \int \operatorname{csch}{(bx+a)}^3\,e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
48*(b^2*e^c + b*d*e^c)*integrate(e^(b*x + d*x + a)/(15*b^2 - 8*b*d + d^2 +
 (15*b^2 - 8*b*d + d^2)*e^(8*b*x + 8*a) - 4*(15*b^2 - 8*b*d + d^2)*e^(6*b*
x + 6*a) + 6*(15*b^2 - 8*b*d + d^2)*e^(4*b*x + 4*a) - 4*(15*b^2 - 8*b*d +
d^2)*e^(2*b*x + 2*a)), x) + 8*((5*b*e^c - d*e^c)*e^(3*b*x + 3*a) - 6*b*e^(
b*x + a + c))*e^(d*x)/(15*b^2 - 8*b*d + d^2 - (15*b^2 - 8*b*d + d^2)*e^(6*
b*x + 6*a) + 3*(15*b^2 - 8*b*d + d^2)*e^(4*b*x + 4*a) - 3*(15*b^2 - 8*b*d
+ d^2)*e^(2*b*x + 2*a))
```

3.883.8 Giac [F]

$$\int e^{c+dx}\operatorname{csch}^3(a+bx)\,dx = \int \operatorname{csch}{(bx+a)}^3\,e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^3*e^(d*x + c), x)
```

3.883.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\operatorname{csch}^3(a+bx)\,dx=\int \frac{\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int(exp(c + d*x)/sinh(a + b*x)^3,x)
```

output
```
int(exp(c + d*x)/sinh(a + b*x)^3, x)
```

3.884 $\int F^{c(a+bx)} \cosh^n(d+ex)\,dx$

3.884.1 Optimal result

Integrand size = 18, antiderivative size = 95

$$\int F^{c(a+bx)} \cosh^n(d+ex)\,dx = -\frac{\left(1+e^{2(d+ex)}\right)^{-n} F^{c(a+bx)} \cosh^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,-\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(2-n+\frac{bc\log(F)}{e}\right),-e\right.}{en-bc\log(F)}$$

output
```
-F^(c*(b*x+a))*cosh(e*x+d)^n*hypergeom([-n, 1/2*(-e*n+b*c*ln(F))/e],[1-1/2
*n+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))/((1+exp(2*e*x+2*d))^n)/(e*n-b*c*ln(F)
)
```

3.884.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)} \cosh^n(d+ex)\,dx = \frac{\left(1+e^{2(d+ex)}\right)^{-n} F^{c(a+bx)} \cosh^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,\frac{-en+bc\log(F)}{2e},1+\frac{-en+bc\log(F)}{2e},-e^{2(d+ex)}\right)}{-en+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^n,x]
```

output
```
(F^(c*(a + b*x))*Cosh[d + e*x]^n*Hypergeometric2F1[-n, (-(e*n) + b*c*Log[F
])/(2*e), 1 + (-(e*n) + b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/((1 + E^(2*(
d + e*x)))^n*(-(e*n) + b*c*Log[F]))
```

3.884.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6006, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\cosh^n(d+ex)\,dx$$

$$\Big\downarrow \text{6006}$$

$$e^{n(d+ex)}\left(e^{2(d+ex)}+1\right)^{-n}\cosh^n(d+ex)\int e^{-n(d+ex)}\left(1+e^{2(d+ex)}\right)^nF^{c(a+bx)}dx$$

$$\Big\downarrow \text{2689}$$

$$-\frac{\left(e^{2(d+ex)}+1\right)^{-n}F^{c(a+bx)}\cosh^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,-\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right),-e^{2(d+e}\right.}{en-bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^n,x]
```

output
```
-((F^(c*(a + b*x))*Cosh[d + e*x]^n*Hypergeometric2F1[-n, -1/2*(e*n - b*c*L
og[F])/e, (2 - n + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/((1 + E^(2*(d + e
*x)))^n*(e*n - b*c*Log[F])))
```

3.884.3.1 Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 6006
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[E^(n*(d + e*x))*(Cosh[d + e*x]^n/(1 + E^(2*(d + e*x)))^n)   Int
[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/E^(n*(d + e*x))), x], x] /; FreeQ
[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

3.884.4 Maple [F]

$$\int F^{c(bx+a)} \cosh (ex+d)^n \, dx$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^n,x)
```

output
```
int(F^(c*(b*x+a))*cosh(e*x+d)^n,x)
```

3.884.5 Fricas [F]

$$\int F^{c(a+bx)} \cosh^n(d+ex) \, dx = \int F^{(bx+a)c} \cosh (ex+d)^n \, dx$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^n,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*cosh(e*x + d)^n, x)
```

3.884.6 Sympy [F]

$$\int F^{c(a+bx)}\cosh^n(d+ex)\,dx = \int F^{c(a+bx)}\cosh^n{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**n,x)
```

output
```
Integral(F**(c*(a + b*x))*cosh(d + e*x)**n, x)
```

3.884.7 Maxima [F]

$$\int F^{c(a+bx)}\cosh^n(d+ex)\,dx = \int F^{(bx+a)c}\cosh{(ex+d)}^n\,dx$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^n,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)*c)*cosh(e*x + d)^n, x)
```

3.884.8 Giac [F]

$$\int F^{c(a+bx)}\cosh^n(d+ex)\,dx = \int F^{(bx+a)c}\cosh{(ex+d)}^n\,dx$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^n,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*cosh(e*x + d)^n, x)
```

3.884.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\cosh^n(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,\cosh(d+e\,x)^n\,dx$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^n,x)
```

output
```
int(F^(c*(a + b*x))*cosh(d + e*x)^n, x)
```

3.885 $\int e^{a+bx}\cosh^3(c+dx)\,dx$

3.885.1 Optimal result

Integrand size = 16, antiderivative size = 139

$$\int e^{a+bx}\cosh^3(c+dx)\,dx=-\frac{6bd^2e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4}+\frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2}+\frac{6d^3e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4}-\frac{3de^{a+bx}\cosh^2(c+dx)\sinh(c+dx)}{b^2-9d^2}$$

output
```
-6*b*d^2*exp(b*x+a)*cosh(d*x+c)/(b^4-10*b^2*d^2+9*d^4)+b*exp(b*x+a)*cosh(d
*x+c)^3/(b^2-9*d^2)+6*d^3*exp(b*x+a)*sinh(d*x+c)/(b^4-10*b^2*d^2+9*d^4)-3*
d*exp(b*x+a)*cosh(d*x+c)^2*sinh(d*x+c)/(b^2-9*d^2)
```

3.885.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76

$$\int e^{a+bx}\cosh^3(c+dx)\,dx$$
$$=\frac{e^{a+bx}(3b(b^2-9d^2)\cosh(c+dx)+(b^3-bd^2)\cosh(3(c+dx))+6d(-b^2+5d^2+(-b^2+d^2)\cosh(2(c+dx}{4\,(b^4-10b^2d^2+9d^4)}$$

input
```
Integrate[E^(a + b*x)*Cosh[c + d*x]^3,x]
```

output
```
(E^(a + b*x)*(3*b*(b^2 - 9*d^2)*Cosh[c + d*x] + (b^3 - b*d^2)*Cosh[3*(c +
d*x)] + 6*d*(-b^2 + 5*d^2 + (-b^2 + d^2)*Cosh[2*(c + d*x)])*Sinh[c + d*x])
)/(4*(b^4 - 10*b^2*d^2 + 9*d^4))
```

3.885.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6000, 5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \cosh^3(c+dx)\,dx$$

$$\Big\downarrow \; 6000$$

$$-\frac{6d^2\int e^{a+bx}\cosh(c+dx)dx}{b^2-9d^2}+\frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2}-\frac{3de^{a+bx}\sinh(c+dx)\cosh^2(c+dx)}{b^2-9d^2}$$

$$\Big\downarrow \; 5998$$

$$\frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2}-\frac{3de^{a+bx}\sinh(c+dx)\cosh^2(c+dx)}{b^2-9d^2}-\frac{6d^2\left(\frac{be^{a+bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\sinh(c+dx)}{b^2-d^2}\right)}{b^2-9d^2}$$

input
```
Int[E^(a + b*x)*Cosh[c + d*x]^3,x]
```

output
```
(b*E^(a + b*x)*Cosh[c + d*x]^3)/(b^2 - 9*d^2) - (3*d*E^(a + b*x)*Cosh[c +
d*x]^2*Sinh[c + d*x])/(b^2 - 9*d^2) - (6*d^2*((b*E^(a + b*x)*Cosh[c + d*x]
)/(b^2 - d^2) - (d*E^(a + b*x)*Sinh[c + d*x])/(b^2 - d^2)))/(b^2 - 9*d^2)
```

3.885.3.1 Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

3.885.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

method	result
risch	$\frac{e^{bx+3dx+a+3c}}{8b+24d}+\frac{3\,e^{bx+dx+a+c}}{8(b+d)}+\frac{3\,e^{bx-dx+a-c}}{8(b-d)}+\frac{e^{bx-3dx+a-3c}}{8b-24d}$
parallelrisch	$-\frac{3\,e^{bx+a}\left(\left(-\frac{1}{3}b^3+\frac{1}{3}b\,d^2\right)\cosh(3dx+3c)+\left(b^2d-d^3\right)\sinh(3dx+3c)-(b-3d)(b+3d)(b\cosh(dx+c)-d\sinh(dx+c))\right)}{4b^4-40b^2d^2+36d^4}$
default	$\frac{\sinh(a-3c+(b-3d)x)}{8b-24d}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}+\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}+\frac{\cosh(a-3c+(b-3d)x)}{8b-24d}$

input
```
int(exp(b*x+a)*cosh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/(b+3*d)*exp(b*x+3*d*x+a+3*c)+3/8/(b+d)*exp(b*x+d*x+a+c)+3/8/(b-d)*exp(
b*x-d*x+a-c)+1/8/(b-3*d)*exp(b*x-3*d*x+a-3*c)
```

3.885.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(135) = 270.

Time = 0.27 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.25

$$\int e^{a+bx}\cosh^3(c+dx)\,dx$$
$$= \frac{(b^3-bd^2)\cosh(bx+a)\cosh(dx+c)^3 - 3\left((b^2d-d^3)\cosh(bx+a)+(b^2d-d^3)\sinh(bx+a)\right)\sinh(dx+$$

input
```
integrate(exp(b*x+a)*cosh(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)^3 - 3*((b^2*d - d^3)*cosh(b
*x + a) + (b^2*d - d^3)*sinh(b*x + a))*sinh(d*x + c)^3 + 3*(b^3 - 9*b*d^2)
*cosh(b*x + a)*cosh(d*x + c) + 3*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c
) + (b^3 - b*d^2)*cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 + ((b^3 - b
*d^2)*cosh(d*x + c)^3 + 3*(b^3 - 9*b*d^2)*cosh(d*x + c))*sinh(b*x + a) - 3
*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - 9*d^3)*cosh(b*x
 + a) + (b^2*d - 9*d^3 + 3*(b^2*d - d^3)*cosh(d*x + c)^2)*sinh(b*x + a))*s
inh(d*x + c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

3.885.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. 2(131) = 262.

Time = 2.29 (sec) , antiderivative size = 1085, normalized size of antiderivative = 7.81

$$\int e^{a+bx}\cosh^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*cosh(d*x+c)**3,x)
```

output
```
Piecewise((x*exp(a)*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-3*d*x
)*sinh(c + d*x)**3/8 + 3*x*exp(a)*exp(-3*d*x)*sinh(c + d*x)**2*cosh(c + d*
x)/8 + 3*x*exp(a)*exp(-3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + x*exp(a)*
exp(-3*d*x)*cosh(c + d*x)**3/8 + 11*exp(a)*exp(-3*d*x)*sinh(c + d*x)**3/(2
4*d) + 5*exp(a)*exp(-3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + exp(a)*
exp(-3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/d + exp(a)*exp(-3*d*x)*cosh(c +
 d*x)**3/(24*d), Eq(b, -3*d)), (-3*x*exp(a)*exp(-d*x)*sinh(c + d*x)**3/8 -
 3*x*exp(a)*exp(-d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 + 3*x*exp(a)*exp(-d
*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + 3*x*exp(a)*exp(-d*x)*cosh(c + d*x)*
*3/8 - 5*exp(a)*exp(-d*x)*sinh(c + d*x)**3/(8*d) - exp(a)*exp(-d*x)*sinh(c
 + d*x)**2*cosh(c + d*x)/(4*d) + exp(a)*exp(-d*x)*sinh(c + d*x)*cosh(c + d
*x)**2/d + 3*exp(a)*exp(-d*x)*cosh(c + d*x)**3/(8*d), Eq(b, -d)), (3*x*exp
(a)*exp(d*x)*sinh(c + d*x)**3/8 - 3*x*exp(a)*exp(d*x)*sinh(c + d*x)**2*cos
h(c + d*x)/8 - 3*x*exp(a)*exp(d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + 3*x*
exp(a)*exp(d*x)*cosh(c + d*x)**3/8 - 5*exp(a)*exp(d*x)*sinh(c + d*x)**3/(8
*d) + exp(a)*exp(d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + exp(a)*exp(d*
x)*sinh(c + d*x)*cosh(c + d*x)**2/d - 3*exp(a)*exp(d*x)*cosh(c + d*x)**3/(
8*d), Eq(b, d)), (-x*exp(a)*exp(3*d*x)*sinh(c + d*x)**3/8 + 3*x*exp(a)*exp
(3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 - 3*x*exp(a)*exp(3*d*x)*sinh(c +
d*x)*cosh(c + d*x)**2/8 + x*exp(a)*exp(3*d*x)*cosh(c + d*x)**3/8 + 11*e...
```

3.885.7 Maxima [F(-2)]

Exception generated.

$$\int e^{a+bx} \cosh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(b*x+a)*cosh(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for
more detai
```

3.885.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.60

$$\int e^{a+bx}\cosh^3(c+dx)\,dx=\frac{e^{(bx+3\,dx+a+3\,c)}}{8\,(b+3\,d)}+\frac{3\,e^{(bx+dx+a+c)}}{8\,(b+d)}+\frac{3\,e^{(bx-dx+a-c)}}{8\,(b-d)}+\frac{e^{(bx-3\,dx+a-3\,c)}}{8\,(b-3\,d)}$$

input
```
integrate(exp(b*x+a)*cosh(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/8*e^(b*x + d*x + a + c)/(b + d
) + 3/8*e^(b*x - d*x + a - c)/(b - d) + 1/8*e^(b*x - 3*d*x + a - 3*c)/(b -
 3*d)
```

3.885.9 Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.90

$$\int e^{a+bx}\cosh^3(c+dx)\,dx$$
$$=\frac{\mathrm{e}^{a+b\,x}\,\left(b^3\,\cosh(c+d\,x)^3-3\,b^2\,d\,\cosh(c+d\,x)^2\,\sinh(c+d\,x)-7\,b\,d^2\,\cosh(c+d\,x)^3+6\,b\,d^2\,\cosh(c+d\,x)\,\mathrm{s}\right.}{b^4-10\,b^2\,d^2+9\,d^4}$$

input
```
int(cosh(c + d*x)^3*exp(a + b*x),x)
```

output
```
(exp(a + b*x)*(b^3*cosh(c + d*x)^3 - 6*d^3*sinh(c + d*x)^3 - 7*b*d^2*cosh(
c + d*x)^3 + 9*d^3*cosh(c + d*x)^2*sinh(c + d*x) + 6*b*d^2*cosh(c + d*x)*s
inh(c + d*x)^2 - 3*b^2*d*cosh(c + d*x)^2*sinh(c + d*x)))/(b^4 + 9*d^4 - 10
*b^2*d^2)
```

3.886 $\int e^{a+bx}\cosh^2(c+dx)\,dx$

3.886.1 Optimal result

Integrand size = 16, antiderivative size = 88

$$\int e^{a+bx}\cosh^2(c+dx)\,dx=-\frac{2d^2e^{a+bx}}{b(b^2-4d^2)}+\frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2}$$

output
```
-2*d^2*exp(b*x+a)/b/(b^2-4*d^2)+b*exp(b*x+a)*cosh(d*x+c)^2/(b^2-4*d^2)-2*d
*exp(b*x+a)*cosh(d*x+c)*sinh(d*x+c)/(b^2-4*d^2)
```

3.886.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\cosh^2(c+dx)\,dx=\frac{e^{a+bx}(b^2-4d^2+b^2\cosh(2(c+dx))-2bd\sinh(2(c+dx)))}{2(b^3-4bd^2)}$$

input
```
Integrate[E^(a + b*x)*Cosh[c + d*x]^2,x]
```

output
```
(E^(a + b*x)*(b^2 - 4*d^2 + b^2*Cosh[2*(c + d*x)] - 2*b*d*Sinh[2*(c + d*x)
]))/(2*(b^3 - 4*b*d^2))
```

3.886.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6000, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^2(c+dx)\,dx$$

$$\Big\downarrow \ 6000$$

$$-\frac{2d^2\int e^{a+bx}dx}{b^2-4d^2}+\frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2}$$

$$\Big\downarrow \ 2624$$

$$\frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2}-\frac{2d^2e^{a+bx}}{b(b^2-4d^2)}$$

input
```
Int[E^(a + b*x)*Cosh[c + d*x]^2,x]
```

output
```
(-2*d^2*E^(a + b*x))/(b*(b^2 - 4*d^2)) + (b*E^(a + b*x)*Cosh[c + d*x]^2)/(
b^2 - 4*d^2) - (2*d*E^(a + b*x)*Cosh[c + d*x]*Sinh[c + d*x])/(b^2 - 4*d^2)
```

3.886.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

3.886.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65

method	result
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{bx+2dx+a+2c}}{4b+8d}+\frac{e^{bx-2dx+a-2c}}{4b-8d}$
parallelrisch	$\frac{e^{bx+a}\left(\cosh(2dx+2c)b^{2}-2bd\sinh(2dx+2c)+b^{2}-4d^{2}\right)}{2b^{3}-8b\,d^{2}}$
default	$\frac{\sinh(bx+a)}{2b}+\frac{\sinh(a-2c+(b-2d)x)}{4b-8d}+\frac{\sinh(a+2c+(b+2d)x)}{4b+8d}+\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4b-8d}+\frac{\cosh(a+2c+(b}{4b+8d}$

input
```
int(exp(b*x+a)*cosh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(b*x+a)/b+1/4/(b+2*d)*exp(b*x+2*d*x+a+2*c)+1/4/(b-2*d)*exp(b*x-2*d*
x+a-2*c)
```

3.886.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.66

$$\int e^{a+bx}\cosh^2(c+dx)\,dx$$
$$=\frac{b^2\cosh\left(bx+a\right)\cosh\left(dx+c\right)^2+\left(b^2\cosh\left(bx+a\right)+b^2\sinh\left(bx+a\right)\right)\sinh\left(dx+c\right)^2+\left(b^2-4\,d^2\right)\cosh\left(bx\right.}{}$$

input
```
integrate(exp(b*x+a)*cosh(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*cosh(b*x + a) + b^2*sinh(b*x
 + a))*sinh(d*x + c)^2 + (b^2 - 4*d^2)*cosh(b*x + a) + (b^2*cosh(d*x + c)^
2 + b^2 - 4*d^2)*sinh(b*x + a) - 4*(b*d*cosh(b*x + a)*cosh(d*x + c) + b*d*
cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c))/(b^3 - 4*b*d^2)
```

3.886.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(78) = 156$.

Time = 0.84 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.91

$$\int e^{a+bx}\cosh^2(c+dx)\,dx$$
$$= \begin{cases} xe^{a}\cosh^{2}(c) \\ \left(-\frac{x\sinh^{2}(c+dx)}{2}+\frac{x\cosh^{2}(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)e^{a} \\ \frac{xe^{a}e^{-2dx}\sinh^{2}(c+dx)}{4}+\frac{xe^{a}e^{-2dx}\sinh(c+dx)\cosh(c+dx)}{2}+\frac{xe^{a}e^{-2dx}\cosh^{2}(c+dx)}{4}+\frac{e^{a}e^{-2dx}\sinh^{2}(c+dx)}{2d}+\frac{3e^{a}e^{-2dx}\sinh(c+da}{4d} \\ \frac{xe^{a}e^{2dx}\sinh^{2}(c+dx)}{4}-\frac{xe^{a}e^{2dx}\sinh(c+dx)\cosh(c+dx)}{2}+\frac{xe^{a}e^{2dx}\cosh^{2}(c+dx)}{4}-\frac{e^{a}e^{2dx}\sinh^{2}(c+dx)}{2d}+\frac{3e^{a}e^{2dx}\sinh(c+dx)\cosh(}{4d} \\ \frac{b^{2}e^{a}e^{bx}\cosh^{2}(c+dx)}{b^{3}-4bd^{2}}-\frac{2bde^{a}e^{bx}\sinh(c+dx)\cosh(c+dx)}{b^{3}-4bd^{2}}+\frac{2d^{2}e^{a}e^{bx}\sinh^{2}(c+dx)}{b^{3}-4bd^{2}}-\frac{2d^{2}e^{a}e^{bx}\cosh^{2}(c+dx)}{b^{3}-4bd^{2}} \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(d*x+c)**2,x)
```

output
```
Piecewise((x*exp(a)*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x)**
2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*exp(a), Eq
(b, 0)), (x*exp(a)*exp(-2*d*x)*sinh(c + d*x)**2/4 + x*exp(a)*exp(-2*d*x)*s
inh(c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(-2*d*x)*cosh(c + d*x)**2/4 + e
xp(a)*exp(-2*d*x)*sinh(c + d*x)**2/(2*d) + 3*exp(a)*exp(-2*d*x)*sinh(c + d
*x)*cosh(c + d*x)/(4*d), Eq(b, -2*d)), (x*exp(a)*exp(2*d*x)*sinh(c + d*x)*
*2/4 - x*exp(a)*exp(2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(2*
d*x)*cosh(c + d*x)**2/4 - exp(a)*exp(2*d*x)*sinh(c + d*x)**2/(2*d) + 3*exp
(a)*exp(2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d), Eq(b, 2*d)), (b**2*exp(a
)*exp(b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*exp(a)*exp(b*x)*sinh
(c + d*x)*cosh(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*exp(a)*exp(b*x)*sinh(c
+ d*x)**2/(b**3 - 4*b*d**2) - 2*d**2*exp(a)*exp(b*x)*cosh(c + d*x)**2/(b**
3 - 4*b*d**2), True))
```

3.886.7 Maxima [F(-2)]

Exception generated.

$$\int e^{a+bx}\cosh^2(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(b*x+a)*cosh(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for
more detai
```

3.886.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\cosh^2(c+dx)\,dx = \frac{e^{(bx+2\,dx+a+2\,c)}}{4\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{4\,(b-2\,d)} + \frac{e^{(bx+a)}}{2\,b}$$

input
```
integrate(exp(b*x+a)*cosh(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*e^(b*x - 2*d*x + a - 2*c)/(b
 - 2*d) + 1/2*e^(b*x + a)/b
```

3.886.9 Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int e^{a+bx}\cosh^2(c+dx)\,dx = \frac{2\,d^2\,\mathrm{e}^{a+b\,x} - b^2\cosh(c+d\,x)^2\,\mathrm{e}^{a+b\,x} + 2\,b\,d\cosh(c+d\,x)\,\mathrm{e}^{a+b\,x}\sinh(c+d\,x)}{4\,b\,d^2 - b^3}$$

input
```
int(cosh(c + d*x)^2*exp(a + b*x),x)
```

output
```
(2*d^2*exp(a + b*x) - b^2*cosh(c + d*x)^2*exp(a + b*x) + 2*b*d*cosh(c + d*
x)*exp(a + b*x)*sinh(c + d*x))/(4*b*d^2 - b^3)
```

3.887 $\int e^{a+bx}\cosh(c+dx)\,dx$

3.887.1 Optimal result

Integrand size = 14, antiderivative size = 54

$$\int e^{a+bx}\cosh(c+dx)\,dx=\frac{be^{a+bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

output
```
b*exp(b*x+a)*cosh(d*x+c)/(b^2-d^2)-d*exp(b*x+a)*sinh(d*x+c)/(b^2-d^2)
```

3.887.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\cosh(c+dx)\,dx=\frac{e^{a+bx}(b\cosh(c+dx)-d\sinh(c+dx))}{(b-d)(b+d)}$$

input
```
Integrate[E^(a + b*x)*Cosh[c + d*x],x]
```

output
```
(E^(a + b*x)*(b*Cosh[c + d*x] - d*Sinh[c + d*x]))/((b - d)*(b + d))
```

3.887.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh(c+dx)\,dx$$

$$\Big\downarrow \ 5998$$

$$\frac{be^{a+bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

input
```
Int[E^(a + b*x)*Cosh[c + d*x],x]
```

output
```
(b*E^(a + b*x)*Cosh[c + d*x])/(b^2 - d^2) - (d*E^(a + b*x)*Sinh[c + d*x])/
(b^2 - d^2)
```

3.887.3.1 Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

3.887.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{e^{bx+a}(b\cosh(dx+c)-d\sinh(dx+c))}{b^2-d^2}$	37
risch	$\frac{e^{bx+dx+a+c}}{2b+2d}+\frac{e^{bx-dx+a-c}}{2b-2d}$	41
default	$\frac{\sinh(a-c+(b-d)x)}{2b-2d}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}+\frac{\cosh(a-c+(b-d)x)}{2b-2d}+\frac{\cosh(a+c+(b+d)x)}{2b+2d}$	78

input
```
int(exp(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)*(b*cosh(d*x+c)-d*sinh(d*x+c))/(b^2-d^2)
```

3.887.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int e^{a+bx}\cosh(c+dx)\,dx = \frac{b\cosh(bx+a)\cosh(dx+c)+b\cosh(dx+c)\sinh(bx+a)-(d\cosh(bx+a)+d\sinh(bx+a))\sinh(dx}{b^2-d^2}$$

input
```
integrate(exp(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
(b*cosh(b*x + a)*cosh(d*x + c) + b*cosh(d*x + c)*sinh(b*x + a) - (d*cosh(b
*x + a) + d*sinh(b*x + a))*sinh(d*x + c))/(b^2 - d^2)
```

3.887.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(42) = 84$.

Time = 0.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.09

$$\int e^{a+bx}\cosh(c+dx)\,dx = \begin{cases} xe^a\cosh(c) & \text{for } b=0 \wedge d=0 \\ \frac{xe^ae^{-dx}\sinh(c+dx)}{2}+\frac{xe^ae^{-dx}\cosh(c+dx)}{2}+\frac{e^ae^{-dx}\sinh(c+dx)}{2d} & \text{for } b=-d \\ -\frac{xe^ae^{dx}\sinh(c+dx)}{2}+\frac{xe^ae^{dx}\cosh(c+dx)}{2}+\frac{e^ae^{dx}\sinh(c+dx)}{2d} & \text{for } b=d \\ \frac{be^ae^{bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^ae^{bx}\sinh(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(d*x+c),x)
```

output
```
Piecewise((x*exp(a)*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-d*x)*sin
h(c + d*x)/2 + x*exp(a)*exp(-d*x)*cosh(c + d*x)/2 + exp(a)*exp(-d*x)*sinh(
c + d*x)/(2*d), Eq(b, -d)), (-x*exp(a)*exp(d*x)*sinh(c + d*x)/2 + x*exp(a)
*exp(d*x)*cosh(c + d*x)/2 + exp(a)*exp(d*x)*sinh(c + d*x)/(2*d), Eq(b, d))
, (b*exp(a)*exp(b*x)*cosh(c + d*x)/(b**2 - d**2) - d*exp(a)*exp(b*x)*sinh(
c + d*x)/(b**2 - d**2), True))
```

3.887.7 Maxima [F(-2)]

Exception generated.

$$\int e^{a+bx}\cosh(c+dx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more
 details)I
```

3.887.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh(c+dx)\,dx = \frac{e^{(bx+dx+a+c)}}{2\,(b+d)} + \frac{e^{(bx-dx+a-c)}}{2\,(b-d)}$$

input
```
integrate(exp(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*e^(b*x + d*x + a + c)/(b + d) + 1/2*e^(b*x - d*x + a - c)/(b - d)
```

3.887.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\cosh(c+dx)\,dx = \frac{\mathrm{e}^{a-c+b\,x-d\,x}\,\left(b+d+b\,\mathrm{e}^{2\,c+2\,d\,x}-d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{2\,\left(b^2-d^2\right)}$$

input
```
int(cosh(c + d*x)*exp(a + b*x),x)
```

output
```
(exp(a - c + b*x - d*x)*(b + d + b*exp(2*c + 2*d*x) - d*exp(2*c + 2*d*x)))
/(2*(b^2 - d^2))
```

3.888 $\int e^{a+bx}\text{sech}(c+dx)\,dx$

3.888.1 Optimal result

Integrand size = 14, antiderivative size = 52

$$\int e^{a+bx}\text{sech}(c+dx)\,dx=\frac{2e^{a+c+bx+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2(c+dx)}\right)}{b+d}$$

output
```
2*exp(b*x+d*x+a+c)*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],-exp(2*d*x+2*c
))/(b+d)
```

3.888.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\text{sech}(c+dx)\,dx=\frac{2e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2(c+dx)}\right)}{b+d}$$

input
```
Integrate[E^(a + b*x)*Sech[c + d*x],x]
```

output
```
(2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2,
-E^(2*(c + d*x))])/(b + d)
```

3.888.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}(c+dx)\,dx$$

$$\Big\downarrow \text{6015}$$

$$\frac{2e^{a+bx+c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(\frac{b}{d}+3\right),-e^{2(c+dx)}\right)}{b+d}$$

input
```
Int[E^(a + b*x)*Sech[c + d*x],x]
```

output
```
(2*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2,
-E^(2*(c + d*x))])/(b + d)
```

3.888.3.1 Defintions of rubi rules used

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.888.4 Maple [F]

$$\int e^{bx+a}\,\text{sech}\left(dx+c\right)dx$$

input
```
int(exp(b*x+a)*sech(d*x+c),x)
```

output
```
int(exp(b*x+a)*sech(d*x+c),x)
```

3.888.5 Fricas [F]

$$\int e^{a+bx}\text{sech}(c+dx)\,dx = \int e^{(bx+a)}\,\text{sech}\,(dx+c)\;dx$$

input
```
integrate(exp(b*x+a)*sech(d*x+c),x, algorithm="fricas")
```

output
```
integral(e^(b*x + a)*sech(d*x + c), x)
```

3.888.6 Sympy [F]

$$\int e^{a+bx}\text{sech}(c+dx)\,dx = e^{a}\int e^{bx}\,\text{sech}\,(c+dx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(d*x+c),x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(c + d*x), x)
```

3.888.7 Maxima [F]

$$\int e^{a+bx}\text{sech}(c+dx)\,dx = \int e^{(bx+a)}\,\text{sech}\,(dx+c)\;dx$$

input
```
integrate(exp(b*x+a)*sech(d*x+c),x, algorithm="maxima")
```

output
```
integrate(e^(b*x + a)*sech(d*x + c), x)
```

3.888.8 Giac [F]

$$\int e^{a+bx}\text{sech}(c+dx)\,dx=\int e^{(bx+a)}\,\text{sech}\,(dx+c)\,\,dx$$

input
```
integrate(exp(b*x+a)*sech(d*x+c),x, algorithm="giac")
```

output
```
integrate(e^(b*x + a)*sech(d*x + c), x)
```

3.888.9 Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{sech}(c+dx)\,dx=\int \frac{e^{a+b\,x}}{\cosh\left(c+d\,x\right)}\,dx$$

input
```
int(exp(a + b*x)/cosh(c + d*x),x)
```

output
```
int(exp(a + b*x)/cosh(c + d*x), x)
```

3.889 $\int e^{a+bx}\mathbf{sech}^2(c+dx)\,dx$

3.889.1 Optimal result

Integrand size = 16, antiderivative size = 56

$$\int e^{a+bx}\text{sech}^2(c+dx)\,dx=\frac{4e^{a+bx+2(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},-e^{2(c+dx)}\right)}{b+2d}$$

output
```
4*exp(b*x+2*d*x+a+2*c)*hypergeom([2, 1+1/2*b/d],[2+1/2*b/d],-exp(2*d*x+2*c
))/(b+2*d)
```

3.889.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\text{sech}^2(c+dx)\,dx=\frac{4e^{a+bx+2(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},-e^{2(c+dx)}\right)}{b+2d}$$

input
```
Integrate[E^(a + b*x)*Sech[c + d*x]^2,x]
```

output
```
(4*E^(a + b*x + 2*(c + d*x))*Hypergeometric2F1[2, 1 + b/(2*d), 2 + b/(2*d)
, -E^(2*(c + d*x))])/(b + 2*d)
```

3.889.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^2(c+dx)\,dx$$

$$\Big\downarrow \text{6015}$$

$$\frac{4e^{a+bx+2(c+dx)}\text{Hypergeometric2F1}\left(2,\frac{b}{2d}+1,\frac{b}{2d}+2,-e^{2(c+dx)}\right)}{b+2d}$$

input
```
Int[E^(a + b*x)*Sech[c + d*x]^2,x]
```

output
```
(4*E^(a + b*x + 2*(c + d*x))*Hypergeometric2F1[2, 1 + b/(2*d), 2 + b/(2*d)
, -E^(2*(c + d*x))])/(b + 2*d)
```

3.889.3.1 Defintions of rubi rules used

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.889.4 Maple [F]

$$\int e^{bx+a}\,\text{sech}\,(dx+c)^2\,dx$$

input
```
int(exp(b*x+a)*sech(d*x+c)^2,x)
```

output
```
int(exp(b*x+a)*sech(d*x+c)^2,x)
```

3.889.5 Fricas [F]

$$\int e^{a+bx}\text{sech}^2(c+dx)\,dx=\int e^{(bx+a)}\,\text{sech}\,(dx+c)^2\,dx$$

input
```
integrate(exp(b*x+a)*sech(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(e^(b*x + a)*sech(d*x + c)^2, x)
```

3.889.6 Sympy [F]

$$\int e^{a+bx}\text{sech}^2(c+dx)\,dx=e^a\int e^{bx}\,\text{sech}^2\,(c+dx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(d*x+c)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(c + d*x)**2, x)
```

3.889.7 Maxima [F]

$$\int e^{a+bx}\text{sech}^2(c+dx)\,dx=\int e^{(bx+a)}\,\text{sech}\,(dx+c)^2\,dx$$

input
```
integrate(exp(b*x+a)*sech(d*x+c)^2,x, algorithm="maxima")
```

output
```
4*b*integrate(1/2*e^(b*x + a)/(d*e^(2*d*x + 2*c) + d), x) - 2*e^(b*x + a)/
(d*e^(2*d*x + 2*c) + d)
```

3.889.8 Giac [F]

$$\int e^{a+bx}\operatorname{sech}^2(c+dx)\,dx=\int e^{(bx+a)}\operatorname{sech}(dx+c)^2\,dx$$

input
```
integrate(exp(b*x+a)*sech(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(e^(b*x + a)*sech(d*x + c)^2, x)
```

3.889.9 Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\operatorname{sech}^2(c+dx)\,dx=\int \frac{e^{a+b\,x}}{\cosh(c+d\,x)^2}\,dx$$

input
```
int(exp(a + b*x)/cosh(c + d*x)^2,x)
```

output
```
int(exp(a + b*x)/cosh(c + d*x)^2, x)
```

3.890 $\int e^{a+bx}\text{sech}^3(c+dx)\,dx$

3.890.1 Optimal result

Integrand size = 16, antiderivative size = 103

$$\int e^{a+bx}\text{sech}^3(c+dx)\,dx = -\frac{(b-d)e^{a+c+bx+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2(c+dx)}\right)}{d^2} + \frac{be^{a+bx}\text{sech}(c+dx)}{2d^2} + \frac{e^{a+bx}\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
-(b-d)*exp(b*x+d*x+a+c)*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],-exp(2*d*
x+2*c))/d^2+1/2*b*exp(b*x+a)*sech(d*x+c)/d^2+1/2*exp(b*x+a)*sech(d*x+c)*ta
nh(d*x+c)/d
```

3.890.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\text{sech}^3(c+dx)\,dx = \frac{e^{a+bx}\left(-2(b-d)e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2(c+dx)}\right)+\text{sech}(c+dx)(b+d\tanh(c+dx\right.}{2d^2}$$

input
```
Integrate[E^(a + b*x)*Sech[c + d*x]^3,x]
```

output
```
(E^(a + b*x)*(-2*(b - d)*E^(c + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (
3 + b/d)/2, -E^(2*(c + d*x))] + Sech[c + d*x]*(b + d*Tanh[c + d*x])))/(2*d
^2)
```

3.890.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6013, 6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^3(c+dx)\,dx$$

$$\Big\downarrow \text{6013}$$

$$\frac{1}{2}\left(1-\frac{b^2}{d^2}\right)\int e^{a+bx}\text{sech}(c+dx)dx+\frac{be^{a+bx}\text{sech}(c+dx)}{2d^2}+\frac{e^{a+bx}\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

$$\Big\downarrow \text{6015}$$

$$\frac{\left(1-\frac{b^2}{d^2}\right)e^{a+bx+c+dx}\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(\frac{b}{d}+3\right),-e^{2(c+dx)}\right)}{b+d}+\frac{be^{a+bx}\text{sech}(c+dx)}{2d^2}+\frac{e^{a+bx}\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

input
```
Int[E^(a + b*x)*Sech[c + d*x]^3,x]
```

output
```
((1 - b^2/d^2)*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (
3 + b/d)/2, -E^(2*(c + d*x))])/(b + d) + (b*E^(a + b*x)*Sech[c + d*x])/(2*
d^2) + (E^(a + b*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

3.890.3.1 Defintions of rubi rules used

rule 6013
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a,
b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
 NeQ[n, 2]
```

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.890.4 Maple [F]

$$\int \mathrm{e}^{bx+a} \operatorname{sech}\left(dx+c\right)^{3} dx$$

input
```
int(exp(b*x+a)*sech(d*x+c)^3,x)
```

output
```
int(exp(b*x+a)*sech(d*x+c)^3,x)
```

3.890.5 Fricas [F]

$$\int e^{a+bx}\operatorname{sech}^3(c+dx)\,dx = \int e^{(bx+a)} \operatorname{sech}\left(dx+c\right)^{3}\,dx$$

input
```
integrate(exp(b*x+a)*sech(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(e^(b*x + a)*sech(d*x + c)^3, x)
```

3.890.6 Sympy [F]

$$\int e^{a+bx}\text{sech}^3(c+dx)\,dx = e^{a}\int e^{bx}\,\text{sech}^3\,(c+dx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(d*x+c)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(c + d*x)**3, x)
```

3.890.7 Maxima [F]

$$\int e^{a+bx}\text{sech}^3(c+dx)\,dx = \int e^{(bx+a)}\,\text{sech}\,(dx+c)^3\,dx$$

input
```
integrate(exp(b*x+a)*sech(d*x+c)^3,x, algorithm="maxima")
```

output
```
-8*(b^2*e^c - d^2*e^c)*integrate(1/8*e^(b*x + d*x + a)/(d^2*e^(2*d*x + 2*c
) + d^2), x) + ((b*e^(3*c) + d*e^(3*c))*e^(b*x + 3*d*x + a) + (b*e^c - d*e
^c)*e^(b*x + d*x + a))/(d^2*e^(4*d*x + 4*c) + 2*d^2*e^(2*d*x + 2*c) + d^2)
```

3.890.8 Giac [F]

$$\int e^{a+bx}\text{sech}^3(c+dx)\,dx = \int e^{(bx+a)}\,\text{sech}\,(dx+c)^3\,dx$$

input
```
integrate(exp(b*x+a)*sech(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(e^(b*x + a)*sech(d*x + c)^3, x)
```

3.890.9 Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\operatorname{sech}^3(c+dx)\,dx=\int \frac{e^{a+b\,x}}{\cosh\left(c+d\,x\right)^3}\,dx$$

input
```
int(exp(a + b*x)/cosh(c + d*x)^3,x)
```

output
```
int(exp(a + b*x)/cosh(c + d*x)^3, x)
```

3.891 $\int F^{c(a+bx)}\text{sech}^n(d+ex)\,dx$

3.891.1 Optimal result

Integrand size = 18, antiderivative size = 90

$$\int F^{c(a+bx)}\text{sech}^n(d+ex)\,dx = \frac{\left(1+e^{2(d+ex)}\right)^n F^{ac+bcx}\,\text{Hypergeometric2F1}\left(n,\frac{en+bc\log(F)}{2e},1+\frac{en+bc\log(F)}{2e},-e^{2(d+ex)}\right)\text{sech}^n(d+ex)}{en+bc\log(F)}$$

output
```
(1+exp(2*e*x+2*d))^n*F^(b*c*x+a*c)*hypergeom([n, 1/2*(e*n+b*c*ln(F))/e],[1
+1/2*(e*n+b*c*ln(F))/e],-exp(2*e*x+2*d))*sech(e*x+d)^n/(e*n+b*c*ln(F))
```

3.891.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int F^{c(a+bx)}\text{sech}^n(d+ex)\,dx = \frac{\left(1+e^{2(d+ex)}\right)^n F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(n,\frac{en+bc\log(F)}{2e},1+\frac{en+bc\log(F)}{2e},-e^{2(d+ex)}\right)\text{sech}^n(d+ex)}{en+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]^n,x]
```

output
```
((1 + E^(2*(d + e*x)))^n*F^(c*(a + b*x))*Hypergeometric2F1[n, (e*n + b*c*L
og[F])/(2*e), 1 + (e*n + b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*Sech[d + e*x
]^n)/(e*n + b*c*Log[F])
```

3.891.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6017, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\text{sech}^n(d+ex)\,dx$$

$$\Big\downarrow \text{6017}$$

$$e^{-n(d+ex)}\left(e^{2(d+ex)}+1\right)^n\text{sech}^n(d+ex)\int e^{dn+exn}\left(1+e^{2(d+ex)}\right)^{-n}F^{ac+bxc}dx$$

$$\Big\downarrow \text{2689}$$

$$\frac{e^{-n(d+ex)+dn+enx}\left(e^{2(d+ex)}+1\right)^n F^{ac+bcx}\text{sech}^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en+bc\log(F)}{2e},\frac{en+bc\log(F)}{2e}+1,-e^{2(d-}\right.}{bc\log(F)+en}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]^n,x]
```

output
```
(E^(d*n + e*n*x - n*(d + e*x))*(1 + E^(2*(d + e*x)))^n*F^(a*c + b*c*x)*Hyp
ergeometric2F1[n, (e*n + b*c*Log[F])/(2*e), 1 + (e*n + b*c*Log[F])/(2*e),
-E^(2*(d + e*x))]*Sech[d + e*x]^n)/(e*n + b*c*Log[F])
```

3.891.3.1 Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 6017
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(1 + E^(2*(d + e*x)))^n*(Sech[d + e*x]^n/E^(n*(d + e*x)))   In
t[SimplifyIntegrand[F^(c*(a + b*x))*(E^(n*(d + e*x))/(1 + E^(2*(d + e*x)))^
n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

3.891.4 Maple [F]

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)^n \, dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^n,x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)^n,x)
```

3.891.5 Fricas [F]

$$\int F^{c(a+bx)} \operatorname{sech}^n(d+ex) \, dx = \int F^{(bx+a)c} \operatorname{sech}(ex+d)^n \, dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^n,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)^n, x)
```

3.891.6 Sympy [F]

$$\int F^{c(a+bx)} \operatorname{sech}^n(d+ex) \, dx = \int F^{c(a+bx)} \operatorname{sech}^n(d+ex) \, dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)**n,x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x)**n, x)
```

3.891.7 Maxima [F]

$$\int F^{c(a+bx)}\operatorname{sech}^n(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}(ex+d)^n\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^n,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^n, x)
```

3.891.8 Giac [F]

$$\int F^{c(a+bx)}\operatorname{sech}^n(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}(ex+d)^n\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^n,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^n, x)
```

3.891.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{sech}^n(d+ex)\,dx = \int F^{c\,(a+b\,x)}\left(\frac{1}{\cosh(d+e\,x)}\right)^n dx$$

input
```
int(F^(c*(a + b*x))*(1/cosh(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(1/cosh(d + e*x))^n, x)
```

3.892 $\int F^{c(a+bx)}\operatorname{csch}^n(d+ex)\,dx$

3.892.1 Optimal result

Integrand size = 18, antiderivative size = 91

$$\int F^{c(a+bx)}\operatorname{csch}^n(d+ex)\,dx = -\frac{\left(1-e^{-2(d+ex)}\right)^n F^{ac+bcx}\operatorname{csch}^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(2+n-\frac{bc\log(F)}{e}\right),e^{-2(d+ex)}\right)}{en-bc\log(F)}$$

output
```
-(1-1/exp(2*e*x+2*d))^n*F^(b*c*x+a*c)*csch(e*x+d)^n*hypergeom([n, 1/2*(e*n
-b*c*ln(F))/e],[1+1/2*n-1/2*b*c*ln(F)/e],exp(-2*e*x-2*d))/(e*n-b*c*ln(F))
```

3.892.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int F^{c(a+bx)}\operatorname{csch}^n(d+ex)\,dx = -\frac{\left(1-e^{-2(d+ex)}\right)^n F^{c(a+bx)}\operatorname{csch}^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(2+n-\frac{bc\log(F)}{e}\right),e^{-2(d+ex)}\right)}{en-bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]^n,x]
```

output
```
-(((1 - E^(-2*(d + e*x)))^n*F^(c*(a + b*x))*Csch[d + e*x]^n*Hypergeometric
2F1[n, (e*n - b*c*Log[F])/(2*e), (2 + n - (b*c*Log[F])/e)/2, E^(-2*(d + e*
x))])/(e*n - b*c*Log[F]))
```

3.892.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6018, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\text{csch}^n(d+ex)\,dx$$

$$\Big\downarrow \text{6018}$$

$$e^{n(d+ex)}\left(1-e^{-2(d+ex)}\right)^n\text{csch}^n(d+ex)\int e^{-dn-exn}\left(1-e^{-2(d+ex)}\right)^{-n}F^{ac+bxc}dx$$

$$\Big\downarrow \text{2689}$$

$$-\frac{e^{n(d+ex)-dn-enx}\left(1-e^{-2(d+ex)}\right)^n F^{ac+bcx}\text{csch}^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(n-\frac{bc\log(F)}{e}+2\right)\right.}{en-bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]^n,x]
```

output
```
-((E^(-(d*n) - e*n*x + n*(d + e*x))*(1 - E^(-2*(d + e*x)))^n*F^(a*c + b*c*
x)*Csch[d + e*x]^n*Hypergeometric2F1[n, (e*n - b*c*Log[F])/(2*e), (2 + n -
 (b*c*Log[F])/e)/2, E^(-2*(d + e*x))])/(e*n - b*c*Log[F]))
```

3.892.3.1 Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 6018
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(1 - E^(-2*(d + e*x)))^n*(Csch[d + e*x]^n/E^((-n)*(d + e*x)))
  Int[SimplifyIntegrand[F^(c*(a + b*x))*(1/(E^(n*(d + e*x))*(1 - E^(-2*(d +
 e*x)))^n)), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

3.892.4 Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}\left(ex+d\right)^{n} dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^n,x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)^n,x)
```

3.892.5 Fricas [F]

$$\int F^{c(a+bx)}\operatorname{csch}^n(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}\left(ex+d\right)^{n}\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^n,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)^n, x)
```

3.892.6 Sympy [F]

$$\int F^{c(a+bx)}\operatorname{csch}^n(d+ex)\,dx=\int F^{c(a+bx)}\operatorname{csch}^{n}\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)**n,x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)**n, x)
```

3.892.7 Maxima [F]

$$\int F^{c(a+bx)}\text{csch}^n(d+ex)\,dx = \int F^{(bx+a)c}\,\text{csch}\,(ex+d)^n\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^n,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)^n, x)
```

3.892.8 Giac [F]

$$\int F^{c(a+bx)}\text{csch}^n(d+ex)\,dx = \int F^{(bx+a)c}\,\text{csch}\,(ex+d)^n\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^n,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)^n, x)
```

3.892.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{csch}^n(d+ex)\,dx = \int F^{c\,(a+b\,x)}\left(\frac{1}{\sinh\left(d+e\,x\right)}\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(1/sinh(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(1/sinh(d + e*x))^n, x)
```

3.893 $\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx$

3.893.1 Optimal result

Integrand size = 25, antiderivative size = 254

$$\begin{aligned}\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx &= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2ief^2F^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}\\ &\quad+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}\\ &\quad-\frac{2ibcf^2F^{ac+bcx}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}\\ &\quad-\frac{2ef^2F^{ac+bcx}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)}\\ &\quad+\frac{bcf^2F^{ac+bcx}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)}\end{aligned}$$

output
```
f^2*F^(b*c*x+a*c)/b/c/ln(F)+2*I*e*f^2*F^(b*c*x+a*c)*cosh(e*x+d)/(e^2-b^2*c
^2*ln(F)^2)+2*e^2*f^2*F^(b*c*x+a*c)/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)-2*I*
b*c*f^2*F^(b*c*x+a*c)*ln(F)*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)-2*e*f^2*F^(b
*c*x+a*c)*cosh(e*x+d)*sinh(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)+b*c*f^2*F^(b*c*x
+a*c)*ln(F)*sinh(e*x+d)^2/(4*e^2-b^2*c^2*ln(F)^2)
```

3.893.2 Mathematica [A] (verified)

Time = 7.34 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.77

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx$$
$$=\frac{F^{c(a+bx)}(f+if\sinh(d+ex))^2\left(\frac{3}{bc\log(F)}+\frac{4ie\cosh(d+ex)}{(e-bc\log(F))(e+bc\log(F))}-\frac{bc\cosh(2(d+ex))\log(F)}{-4e^2+b^2c^2\log^2(F)}+\frac{4ibc\log(F)\sinh(d+ex}{(-e+bc\log(F))(e+bc\log}\right.}{2\left(\cosh\left(\frac{1}{2}(d+ex)\right)+i\sinh\left(\frac{1}{2}(d+ex)\right)\right)^4}$$

input
```
Integrate[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x])^2,x]
```

output
```
(F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x])^2*(3/(b*c*Log[F]) + ((4*I)*e*Cosh
[d + e*x])/((e - b*c*Log[F])*(e + b*c*Log[F])) - (b*c*Cosh[2*(d + e*x)]*Lo
g[F])/(-4*e^2 + b^2*c^2*Log[F]^2) + ((4*I)*b*c*Log[F]*Sinh[d + e*x])/((-e
+ b*c*Log[F])*(e + b*c*Log[F])) - (2*e*Sinh[2*(d + e*x)])/(4*e^2 - b^2*c^2
*Log[F]^2)))/(2*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2])^4)
```

3.893.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx$$
$$\Big\downarrow \text{7292}$$
$$\int f^2(1+i\sinh(d+ex))^2F^{ac+bcx}dx$$
$$\Big\downarrow \text{27}$$
$$f^2\int F^{ac+bxc}(i\sinh(d+ex)+1)^2dx$$
$$\Big\downarrow \text{7293}$$
$$f^2\int\left(-\sinh^2(d+ex)F^{ac+bxc}+2i\sinh(d+ex)F^{ac+bxc}+F^{ac+bxc}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$f^2\left(\frac{bc\log(F)\sinh^2(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)}-\frac{2ibc\log(F)\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{2ie\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{2e\sinh(d+e}{4e^2}\right.$$

input
```
Int[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x])^2,x]
```

output
```
f^2*(F^(a*c + b*c*x)/(b*c*Log[F]) + ((2*I)*e*F^(a*c + b*c*x)*Cosh[d + e*x]
)/(e^2 - b^2*c^2*Log[F]^2) + (2*e^2*F^(a*c + b*c*x))/(b*c*Log[F]*(4*e^2 -
b^2*c^2*Log[F]^2)) - ((2*I)*b*c*F^(a*c + b*c*x)*Log[F]*Sinh[d + e*x])/(e^2
 - b^2*c^2*Log[F]^2) - (2*e*F^(a*c + b*c*x)*Cosh[d + e*x]*Sinh[d + e*x])/(
4*e^2 - b^2*c^2*Log[F]^2) + (b*c*F^(a*c + b*c*x)*Log[F]*Sinh[d + e*x]^2)/(
4*e^2 - b^2*c^2*Log[F]^2))
```

3.893.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.893.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.77

method	result
parallelrisch	$-\frac{2F^{c(bx+a)}\left(\frac{\left(\ln(F)^4b^4c^4-\ln(F)^2b^2c^2e^2\right)\cosh(2ex+2d)}{4}+\frac{\left(-\ln(F)^3b^3c^3e+\ln(F)bc\,e^3\right)\sinh(2ex+2d)}{2}+(bc\ln(F)+2e)(bc\ln(F)-2e)\right.}{c^5b^5\ln(F)^5-5c^3b^3\ln(F)^3e^2+4\ln(F)bc\,e^4}$
risch	$\frac{f^2\left(16i\ln(F)bc\,e^3\mathrm{e}^{3ex+3d}-\ln(F)^4b^4c^4\mathrm{e}^{4ex+4d}-4i\ln(F)^3b^3c^3e\,\mathrm{e}^{ex+d}+6\ln(F)^4b^4c^4\mathrm{e}^{2ex+2d}-16i\ln(F)^2b^2c^2e^2\mathrm{e}^{3ex+3d}+2\ln(\right.}{}$

input
```
int(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*F^(c*(b*x+a))*(1/4*(ln(F)^4*b^4*c^4-ln(F)^2*b^2*c^2*e^2)*cosh(2*e*x+2*d
)+1/2*(-ln(F)^3*b^3*c^3*e+ln(F)*b*c*e^3)*sinh(2*e*x+2*d)+(b*c*ln(F)+2*e)*(
b*c*ln(F)-2*e)*(-I*sinh(e*x+d)*b^2*c^2*ln(F)^2+I*cosh(e*x+d)*b*c*e*ln(F)-3
/4*b^2*c^2*ln(F)^2+3/4*e^2))*f^2/(c^5*b^5*ln(F)^5-5*c^3*b^3*ln(F)^3*e^2+4*
ln(F)*b*c*e^4)
```

3.893.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.74

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx$$
$$=\frac{\left(24\,e^4f^2e^{(2\,ex+2\,d)}-\left(b^4c^4f^2e^{(4\,ex+4\,d)}-4i\,b^4c^4f^2e^{(3\,ex+3\,d)}-6\,b^4c^4f^2e^{(2\,ex+2\,d)}+4i\,b^4c^4f^2e^{(ex+d)}+b^4c^4f^2\right)\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x, algorithm="fricas")
```

output
```
1/4*(24*e^4*f^2*e^(2*e*x + 2*d) - (b^4*c^4*f^2*e^(4*e*x + 4*d) - 4*I*b^4*c
^4*f^2*e^(3*e*x + 3*d) - 6*b^4*c^4*f^2*e^(2*e*x + 2*d) + 4*I*b^4*c^4*f^2*e
^(e*x + d) + b^4*c^4*f^2)*log(F)^4 + 2*(b^3*c^3*e*f^2*e^(4*e*x + 4*d) - 2*
I*b^3*c^3*e*f^2*e^(3*e*x + 3*d) - 2*I*b^3*c^3*e*f^2*e^(e*x + d) - b^3*c^3*
e*f^2)*log(F)^3 + (b^2*c^2*e^2*f^2*e^(4*e*x + 4*d) - 16*I*b^2*c^2*e^2*f^2*
e^(3*e*x + 3*d) - 30*b^2*c^2*e^2*f^2*e^(2*e*x + 2*d) + 16*I*b^2*c^2*e^2*f^
2*e^(e*x + d) + b^2*c^2*e^2*f^2)*log(F)^2 - 2*(b*c*e^3*f^2*e^(4*e*x + 4*d)
 - 8*I*b*c*e^3*f^2*e^(3*e*x + 3*d) - 8*I*b*c*e^3*f^2*e^(e*x + d) - b*c*e^3
*f^2)*log(F))*F^(b*c*x + a*c)/(b^5*c^5*e^(2*e*x + 2*d)*log(F)^5 - 5*b^3*c^
3*e^2*e^(2*e*x + 2*d)*log(F)^3 + 4*b*c*e^4*e^(2*e*x + 2*d)*log(F))
```

3.893.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2377 vs. 2(241) = 482.

Time = 35.87 (sec) , antiderivative size = 2377, normalized size of antiderivative = 9.36

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*(f+I*f*sinh(e*x+d))**2,x)
```

output
```
Piecewise((x*(I*f*sinh(d) + f)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e,
0)), (-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x - f*
*2*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*I*f**2*cosh(d + e*x)/e, Eq(F, 1))
, (F**(a*c)*(-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2
*x - f**2*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*I*f**2*cosh(d + e*x)/e), E
q(b, 0)), (-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x
 - f**2*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*I*f**2*cosh(d + e*x)/e, Eq(c
, 0)), (-I*F**(a*c + b*c*x)*f**2*x*sinh(b*c*x*log(F) - d) + I*F**(a*c + b*
c*x)*f**2*x*cosh(b*c*x*log(F) - d) - F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(
F) - d)**2/(3*b*c*log(F)) - 2*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) - d)
*cosh(b*c*x*log(F) - d)/(3*b*c*log(F)) + I*F**(a*c + b*c*x)*f**2*sinh(b*c*
x*log(F) - d)/(b*c*log(F)) + 2*F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F) - d
)**2/(3*b*c*log(F)) - 2*I*F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F) - d)/(b*
c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, -b*c*log(F))), (-F**
(a*c + b*c*x)*f**2*x*sinh(b*c*x*log(F)/2 - d)**2/4 + F**(a*c + b*c*x)*f**2
*x*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)/2 - F**(a*c + b*c*x)*
f**2*x*cosh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*f**2*sinh(b*c*x*lo
g(F)/2 - d)**2/(b*c*log(F)) + F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F)/2 -
d)*cosh(b*c*x*log(F)/2 - d)/(2*b*c*log(F)) - 8*I*F**(a*c + b*c*x)*f**2*sin
h(b*c*x*log(F)/2 - d)/(3*b*c*log(F)) + 4*I*F**(a*c + b*c*x)*f**2*cosh(b...
```

3.893.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.74

$$\begin{aligned}\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx &= -\frac{1}{4}f^2\left(\frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{bc\log(F)+2\,e}+\frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)}}-\frac{2\,F^{bcx+ac}}{bc\log(F)}\right)\\ &\quad+i\,f^2\left(\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{bc\log(F)+e}-\frac{F^{ac}e^{(bcx\log(F)-ex)}}{bce^{d}\log(F)-ee^{d}}\right)+\frac{F^{bcx+ac}f^2}{bc\log(F)}\end{aligned}$$

input
```
integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x, algorithm="maxima")
```

output
```
-1/4*f^2*(F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + F^(a
*c)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) - 2*F^(b*c
*x + a*c)/(b*c*log(F))) + I*f^2*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*l
og(F) + e) - F^(a*c)*e^(b*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^
(b*c*x + a*c)*f^2/(b*c*log(F))
```

3.893.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1546 vs. $2(250) = 500$.

Time = 0.33 (sec) , antiderivative size = 1546, normalized size of antiderivative = 6.09

$$\int F^{c(a+bx)}(f + if \sinh(d + ex))^2 \, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x, algorithm="giac")
```

output
```
3*(2*b*c*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*
c)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c
*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sg
n(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*I*(I*f^2*e^
(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a
*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*f^2*e^(-1/2*I
*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-
2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F))
+ a*c*log(abs(F))) - 1/2*(2*(b*c*log(abs(F)) + 2*e)*f^2*cos(-1/2*pi*b*c*x*
sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) -
pi*b*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*si
n(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((
pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F
)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + I*(-I*f^2*e^(1/2*I*pi*b*c*x*sgn(F)
 - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F)
 - 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e) + I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F
) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(
F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e))*e^(a*c*log(abs(F)) + (b*c*log
(abs(F)) + 2*e)*x + 2*d) - 2*((pi*b*c*sgn(F) - pi*b*c)*f^2*cos(-1/2*pi*...
```

3.893.9 Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.99

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx$$
$$=\frac{F^{c\,(a+b\,x)}\,f^2\,\left(12\,e^4+3\,b^4\,c^4\ln\left(F\right)^4+b^4\,c^4\sinh(d+e\,x)\,\ln\left(F\right)^4\,4\mathrm{i}-b^4\,c^4\ln\left(F\right)^4\cosh(2\,d+2\,e\,x)-15\,b^2\,c}{}$$

input
```
int(F^(c*(a + b*x))*(f + f*sinh(d + e*x)*1i)^2,x)
```

output
```
(F^(c*(a + b*x))*f^2*(12*e^4 + 3*b^4*c^4*log(F)^4 + b^4*c^4*sinh(d + e*x)*
log(F)^4*4i - b^4*c^4*log(F)^4*cosh(2*d + 2*e*x) - 15*b^2*c^2*e^2*log(F)^2
 + 2*b^3*c^3*e*log(F)^3*sinh(2*d + 2*e*x) - b^2*c^2*e^2*sinh(d + e*x)*log(
F)^2*16i - 2*b*c*e^3*log(F)*sinh(2*d + 2*e*x) + b^2*c^2*e^2*log(F)^2*cosh(
2*d + 2*e*x) - b^3*c^3*e*cosh(d + e*x)*log(F)^3*4i + b*c*e^3*cosh(d + e*x)
*log(F)*16i))/(2*b*c*log(F)*(4*e^4 + b^4*c^4*log(F)^4 - 5*b^2*c^2*e^2*log(
F)^2))
```

3.894 $\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx$

3.894.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx = \frac{fF^{ac+bcx}}{bc\log(F)}+\frac{iefF^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}-\frac{ibcfF^{ac+bcx}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

output
```
f*F^(b*c*x+a*c)/b/c/ln(F)+I*e*f*F^(b*c*x+a*c)*cosh(e*x+d)/(e^2-b^2*c^2*ln(
F)^2)-I*b*c*f*F^(b*c*x+a*c)*ln(F)*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

3.894.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx = \frac{fF^{c(a+bx)}\left(-e^2-ibce\cosh(d+ex)\log(F)+b^2c^2\log^2(F)+ib^2c^2\log^2(F)\sinh(d+ex)\right)}{bc\log(F)(-e+bc\log(F))(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x]),x]
```

output
```
(f*F^(c*(a + b*x))*(-e^2 - I*b*c*e*Cosh[d + e*x]*Log[F] + b^2*c^2*Log[F]^2
 + I*b^2*c^2*Log[F]^2*Sinh[d + e*x]))/(b*c*Log[F]*(-e + b*c*Log[F])*(e + b
*c*Log[F]))
```

3.894.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx$$

$$\Big\downarrow \text{7292}$$

$$\int f(1+i\sinh(d+ex))F^{ac+bcx}dx$$

$$\Big\downarrow \text{27}$$

$$f\int F^{ac+bxc}(i\sinh(d+ex)+1)dx$$

$$\Big\downarrow \text{7293}$$

$$f\int \left(i\sinh(d+ex)F^{ac+bxc}+F^{ac+bxc}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$f\left(-\frac{ibc\log(F)\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{ie\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{F^{ac+bcx}}{bc\log(F)}\right)$$

input
```
Int[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x]),x]
```

output
```
f*(F^(a*c + b*c*x)/(b*c*Log[F]) + (I*e*F^(a*c + b*c*x)*Cosh[d + e*x])/(e^2
 - b^2*c^2*Log[F]^2) - (I*b*c*F^(a*c + b*c*x)*Log[F]*Sinh[d + e*x])/(e^2 -
 b^2*c^2*Log[F]^2))
```

3.894.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.894.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

method	result	s
parallelrisch	$\frac{f\,F^{c(bx+a)}\left(i\sinh(ex+d)b^{2}c^{2}\ln(F)^{2}+b^{2}c^{2}\ln(F)^{2}-i\cosh(ex+d)bce\ln(F)-e^{2}\right)}{bc\ln(F)\left(b^{2}c^{2}\ln(F)^{2}-e^{2}\right)}$	9
risch	$\frac{f\left(-i\ln(F)^{2}b^{2}c^{2}\mathrm{e}^{2ex+2d}+i\ln(F)^{2}b^{2}c^{2}-2\ln(F)^{2}b^{2}c^{2}\mathrm{e}^{ex+d}+i\ln(F)bce\,\mathrm{e}^{2ex+2d}+i\ln(F)bce+2e^{2}\mathrm{e}^{ex+d}\right)\mathrm{e}^{-ex-d}F^{c(bx+a)}}{2bc\ln(F)(e-bc\ln(F))(e+bc\ln(F))}$	1

input
```
int(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x,method=_RETURNVERBOSE)
```

output
```
f*F^(c*(b*x+a))/b/c/ln(F)/(b^2*c^2*ln(F)^2-e^2)*(I*ln(F)^2*b^2*c^2*sinh(e*
x+d)+b^2*c^2*ln(F)^2-I*cosh(e*x+d)*b*c*e*ln(F)-e^2)
```

3.894.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.27

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx =$$
$$-\frac{\left(2\,e^{2}fe^{(ex+d)}-\left(i\,b^{2}c^{2}fe^{(2\,ex+2\,d)}+2\,b^{2}c^{2}fe^{(ex+d)}-i\,b^{2}c^{2}f\right)\log\left(F\right)^{2}-\left(-i\,bcefe^{(2\,ex+2\,d)}-i\,bcef\right)\log\left(\right.}{2\left(b^{3}c^{3}e^{(ex+d)}\log\left(F\right)^{3}-bce^{2}e^{(ex+d)}\log\left(F\right)\right)}$$

input
```
integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x, algorithm="fricas")
```

output
```
-1/2*(2*e^2*f*e^(e*x + d) - (I*b^2*c^2*f*e^(2*e*x + 2*d) + 2*b^2*c^2*f*e^(
e*x + d) - I*b^2*c^2*f)*log(F)^2 - (-I*b*c*e*f*e^(2*e*x + 2*d) - I*b*c*e*f
)*log(F))*F^(b*c*x + a*c)/(b^3*c^3*e^(e*x + d)*log(F)^3 - b*c*e^2*e^(e*x +
 d)*log(F))
```

3.894.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(97) = 194.

Time = 1.19 (sec) , antiderivative size = 510, normalized size of antiderivative = 4.81

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx$$
$$=\begin{cases} x(if\sinh\left(d\right)+f) \\ fx+\frac{if\cosh\left(d+ex\right)}{e} \\ F^{ac}\left(fx+\frac{if\cosh\left(d+ex\right)}{e}\right) \\ fx+\frac{if\cosh\left(d+ex\right)}{e} \\ -\frac{iF^{ac+bcx}fx\sinh\left(bcx\log\left(F\right)-d\right)}{2}+\frac{iF^{ac+bcx}fx\cosh\left(bcx\log\left(F\right)-d\right)}{2}+\frac{iF^{ac+bcx}f\sinh\left(bcx\log\left(F\right)-d\right)}{2bc\log\left(F\right)}-\frac{iF^{ac+bcx}f\cosh\left(bcx\log\left(F\right)\right.}{bc\log\left(F\right)} \\ \frac{iF^{ac+bcx}fx\sinh\left(bcx\log\left(F\right)+d\right)}{2}-\frac{iF^{ac+bcx}fx\cosh\left(bcx\log\left(F\right)+d\right)}{2}+\frac{iF^{ac+bcx}f\cosh\left(bcx\log\left(F\right)+d\right)}{2bc\log\left(F\right)}+\frac{F^{ac+bcx}f}{bc\log\left(F\right)} \\ \frac{iF^{ac+bcx}b^{2}c^{2}f\log\left(F\right)^{2}\sinh\left(d+ex\right)}{b^{3}c^{3}\log\left(F\right)^{3}-bce^{2}\log\left(F\right)}+\frac{F^{ac+bcx}b^{2}c^{2}f\log\left(F\right)^{2}}{b^{3}c^{3}\log\left(F\right)^{3}-bce^{2}\log\left(F\right)}-\frac{iF^{ac+bcx}bcef\log\left(F\right)\cosh\left(d+ex\right)}{b^{3}c^{3}\log\left(F\right)^{3}-bce^{2}\log\left(F\right)}-\frac{F^{ac+bcx}e^{2}f}{b^{3}c^{3}\log\left(F\right)^{3}-bce^{2}\log\left(F\right)} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x)
```

output
```
Piecewise((x*(I*f*sinh(d) + f), Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0))
, (f*x + I*f*cosh(d + e*x)/e, Eq(F, 1)), (F**(a*c)*(f*x + I*f*cosh(d + e*x
)/e), Eq(b, 0)), (f*x + I*f*cosh(d + e*x)/e, Eq(c, 0)), (-I*F**(a*c + b*c*
x)*f*x*sinh(b*c*x*log(F) - d)/2 + I*F**(a*c + b*c*x)*f*x*cosh(b*c*x*log(F)
 - d)/2 + I*F**(a*c + b*c*x)*f*sinh(b*c*x*log(F) - d)/(2*b*c*log(F)) - I*F
**(a*c + b*c*x)*f*cosh(b*c*x*log(F) - d)/(b*c*log(F)) + F**(a*c + b*c*x)*f
/(b*c*log(F)), Eq(e, -b*c*log(F))), (I*F**(a*c + b*c*x)*f*x*sinh(b*c*x*log
(F) + d)/2 - I*F**(a*c + b*c*x)*f*x*cosh(b*c*x*log(F) + d)/2 + I*F**(a*c +
 b*c*x)*f*cosh(b*c*x*log(F) + d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*f/(b*c*
log(F)), Eq(e, b*c*log(F))), (I*F**(a*c + b*c*x)*b**2*c**2*f*log(F)**2*sin
h(d + e*x)/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)) + F**(a*c + b*c*x)*b**2
*c**2*f*log(F)**2/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)) - I*F**(a*c + b*
c*x)*b*c*e*f*log(F)*cosh(d + e*x)/(b**3*c**3*log(F)**3 - b*c*e**2*log(F))
- F**(a*c + b*c*x)*e**2*f/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)), True))
```

3.894.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx=\frac{1}{2}if\left(\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{bc\log\left(F\right)+e}-\frac{F^{ac}e^{(bcx\log(F)-ex)}}{bce^{d}\log\left(F\right)-ee^{d}}\right)+\frac{F^{bcx+ac}f}{bc\log\left(F\right)}$$

input
```
integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x, algorithm="maxima")
```

output
```
1/2*I*f*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) - F^(a*c)*e^(
b*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c*x + a*c)*f/(b*c*log
(F))
```

3.894.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 885 vs. 2(104) = 208.

Time = 0.29 (sec) , antiderivative size = 885, normalized size of antiderivative = 8.35

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx=\text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x, algorithm="giac")
```

output
```
2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1
/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F)
 - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f*e^(1/2*I*p
i*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*p
i*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) - I*f*e^(-1/2*I*pi*b*c*x*sgn(
F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F
) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
 - ((pi*b*c*sgn(F) - pi*b*c)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + e)^2) + 2*(b*c*log(abs(F)) + e)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*p
i*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*
(b*c*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d
) - (-I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F)
- 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e)
 - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) +
1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e))
*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + ((pi*b*c*sgn(F) - pi*
b*c)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/...
```

3.894.9 Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx = \frac{F^{c\,(a+b\,x)}\,f\,\left(e^2-b^2\,c^2\ln\left(F\right)^2-b^2\,c^2\sinh(d+e\,x)\,\ln\left(F\right)^2\,1\mathrm{i}+b\,c\,e\cosh(d+e\,x)\,\ln\left(F\right)\,1\mathrm{i}\right)}{b\,c\,\ln\left(F\right)\,\left(e^2-b^2\,c^2\ln\left(F\right)^2\right)}$$

input
```
int(F^(c*(a + b*x))*(f + f*sinh(d + e*x)*1i),x)
```

output
```
(F^(c*(a + b*x))*f*(e^2 - b^2*c^2*log(F)^2 - b^2*c^2*sinh(d + e*x)*log(F)^
2*1i + b*c*e*cosh(d + e*x)*log(F)*1i))/(b*c*log(F)*(e^2 - b^2*c^2*log(F)^2
))
```

3.895 $\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx$

3.895.1 Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx = \frac{2e^{\frac{1}{2}(2d+i\pi+2ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{\frac{1}{2}(2d+i\pi+2ex)}\right)}{f(e+bc\log(F))}$$

output
```
2*exp(d+1/2*I*Pi+e*x)*F^(c*(b*x+a))*hypergeom([2, 1+b*c*ln(F)/e],[2+b*c*ln
(F)/e],-exp(d+1/2*I*Pi+e*x))/f/(e+b*c*ln(F))
```

3.895.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx = \frac{2F^{c(a+bx)}\left(\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e},1+\frac{bc\log(F)}{e},-ie^{d+ex}\right)+\frac{\cosh\left(\frac{ex}{2}\right)-\sinh\left(\frac{ex}{2}\right)}{(-1-ie^d)\cosh\left(\frac{ex}{2}\right)+(1-ie^d)\sinh\left(\frac{ex}{2}\right)}\right)}{ef}$$

input
```
Integrate[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x]),x]
```

output
```
(2*F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/
e, (-I)*E^(d + e*x)] + (Cosh[(e*x)/2] - Sinh[(e*x)/2])/((-1 - I*E^d)*Cosh[
(e*x)/2] + (1 - I*E^d)*Sinh[(e*x)/2])))/(e*f)
```

3.895.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6019, 6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}dx$$

$$\Big\downarrow \text{6019}$$

$$\frac{\int F^{c(a+bx)}\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right)dx}{2f}$$

$$\Big\downarrow \text{6015}$$

$$\frac{2e^{\frac{1}{2}(2d+2ex+i\pi)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{e}+1,\frac{bc\log(F)}{e}+2,-e^{\frac{1}{2}(2d+2ex+i\pi)}\right)}{f(bc\log(F)+e)}$$

input
```
Int[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x]),x]
```

output
```
(2*E^((2*d + I*Pi + 2*e*x)/2)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*
c*Log[F])/e, 2 + (b*c*Log[F])/e, -E^((2*d + I*Pi + 2*e*x)/2)])/(f*(e + b*c
*Log[F]))
```

3.895.3.1 Defintions of rubi rules used

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

rule 6019
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sinh[(d_.) + (e_.)*(x_)
])^(n_.), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Cosh[d/2 - f*(Pi/
(4*g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && Eq
Q[f^2 + g^2, 0] && ILtQ[n, 0]
```

3.895.4 Maple [F]

$$\int \frac{F^{c(bx+a)}}{f + if \sinh(ex+d)} dx$$

input
```
int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x)
```

output
```
int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x)
```

3.895.5 Fricas [F]

$$\int \frac{F^{c(a+bx)}}{f + if \sinh(d+ex)} dx = \int \frac{F^{(bx+a)c}}{i f \sinh(ex+d) + f} dx$$

input
```
integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x, algorithm="fricas")
```

output
```
((e*f*e^(e*x + d) - I*e*f)*integral(-2*I*F^(b*c*x + a*c)*b*c*log(F)/(e*f*e
^(e*x + d) - I*e*f), x) + 2*I*F^(b*c*x + a*c))/(e*f*e^(e*x + d) - I*e*f)
```

3.895.6 Sympy [F]

$$\int \frac{F^{c(a+bx)}}{f + if \sinh(d+ex)} dx = -\frac{i \int \frac{F^{ac+bcx}}{\sinh(d+ex)-i} dx}{f}$$

input
```
integrate(F**(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x)
```

output
```
-I*Integral(F**(a*c + b*c*x)/(sinh(d + e*x) - I), x)/f
```

3.895.7 Maxima [F]

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx=\int \frac{F^{(bx+a)c}}{i\,f\sinh\left(ex+d\right)+f}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x, algorithm="maxima")
```

output
```
-4*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(I*b^2*c^2*f*log(F)^2 - 3*I*b*c*e*f*l
og(F) + 2*I*e^2*f + (b^2*c^2*f*e^(3*d)*log(F)^2 - 3*b*c*e*f*e^(3*d)*log(F)
 + 2*e^2*f*e^(3*d))*e^(3*e*x) - 3*(I*b^2*c^2*f*e^(2*d)*log(F)^2 - 3*I*b*c*
e*f*e^(2*d)*log(F) + 2*I*e^2*f*e^(2*d))*e^(2*e*x) - 3*(b^2*c^2*f*e^d*log(F
)^2 - 3*b*c*e*f*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x)), x)*log(F) - 2*(-2*I*F^
(a*c)*e - (F^(a*c)*b*c*e^d*log(F) - 2*F^(a*c)*e*e^d)*e^(e*x))*F^(b*c*x)/(-
I*b^2*c^2*f*log(F)^2 + 3*I*b*c*e*f*log(F) - 2*I*e^2*f + (I*b^2*c^2*f*e^(2*
d)*log(F)^2 - 3*I*b*c*e*f*e^(2*d)*log(F) + 2*I*e^2*f*e^(2*d))*e^(2*e*x) +
2*(b^2*c^2*f*e^d*log(F)^2 - 3*b*c*e*f*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x))
```

3.895.8 Giac [F]

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx=\int \frac{F^{(bx+a)c}}{i\,f\sinh\left(ex+d\right)+f}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)/(I*f*sinh(e*x + d) + f), x)
```

3.895.9 Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx=\int \frac{F^{c\,(a+b\,x)}}{f+f\,\sinh\left(d+e\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int(F^(c*(a + b*x))/(f + f*sinh(d + e*x)*1i),x)
```

output
```
int(F^(c*(a + b*x))/(f + f*sinh(d + e*x)*1i), x)
```

3.896 $\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx$

3.896.1 Optimal result

Integrand size = 25, antiderivative size = 196

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = \frac{2e^{\frac{1}{2}(2d+i\pi+2ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{\frac{1}{2}(2d+i\pi+2ex)}\right)(e-bc\log(F))}{3e^2f^2} + \frac{bcF^{c(a+bx)}\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\text{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)\tanh\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)}{6ef^2}$$

output
```
2/3*exp(d+1/2*I*Pi+e*x)*F^(c*(b*x+a))*hypergeom([2, 1+b*c*ln(F)/e],[2+b*c*
ln(F)/e],-exp(d+1/2*I*Pi+e*x))*(e-b*c*ln(F))/e^2/f^2+1/6*b*c*F^(c*(b*x+a))
*ln(F)*sech(1/2*d+1/4*I*Pi+1/2*e*x)^2/e^2/f^2+1/6*F^(c*(b*x+a))*sech(1/2*d
+1/4*I*Pi+1/2*e*x)^2*tanh(1/2*d+1/4*I*Pi+1/2*e*x)/e/f^2
```

3.896.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.28

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = \frac{F^{c(a+bx)}\left(\cosh\left(\frac{1}{2}(d+ex)\right)+i\sinh\left(\frac{1}{2}(d+ex)\right)\right)\left(e(ie+bc\log(F))\left(\cosh\left(\frac{1}{2}(d+ex)\right)+i\sinh\left(\frac{1}{2}(d+ex)\right)\right.\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x])^2,x]
```

output
```
(F^(c*(a + b*x))*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2])*(e*(I*e + b*c*L
og[F])*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2]) - (1 - I)*(-1 + (1 + I)*H
ypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, (-I)*E^(d + e*x)])
*(-e^2 + b^2*c^2*Log[F]^2)*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2])^3 + 2
*e^2*Sinh[(d + e*x)/2] + 2*(e^2 - b^2*c^2*Log[F]^2)*(Cosh[(d + e*x)/2] + I
*Sinh[(d + e*x)/2])^2*Sinh[(d + e*x)/2]))/(3*e^3*(f + I*f*Sinh[d + e*x])^2
)
```

3.896.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6019, 6013, 6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx$$

$$\Big\downarrow \text{6019}$$

$$\frac{\int F^{c(a+bx)}\text{sech}^4\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right)dx}{4f^2}$$

$$\Big\downarrow \text{6013}$$

$$\frac{\frac{2}{3}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right)dx+\frac{2bc\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right)F^{c(a+bx)}}{3e^2}+\frac{2\tanh\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{d}{2}+\right.}{3e}}{4f^2}$$

$$\Big\downarrow \text{6015}$$

$$\frac{\frac{8e^{\frac{1}{2}(2d+2ex+i\pi)}F^{c(a+bx)}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{e}+1,\frac{bc\log(F)}{e}+2,-e^{\frac{1}{2}(2d+2ex+i\pi)}\right)}{3(bc\log(F)+e)}+\frac{2bc\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right)}{3e^2}}{4f^2}$$

input
```
Int[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x])^2,x]
```

output
```
((8*E^((2*d + I*Pi + 2*e*x)/2)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b
*c*Log[F])/e, 2 + (b*c*Log[F])/e, -E^((2*d + I*Pi + 2*e*x)/2)]*(1 - (b^2*c
^2*Log[F]^2)/e^2))/(3*(e + b*c*Log[F])) + (2*b*c*F^(c*(a + b*x))*Log[F]*Se
ch[d/2 + (I/4)*Pi + (e*x)/2]^2)/(3*e^2) + (2*F^(c*(a + b*x))*Sech[d/2 + (I
/4)*Pi + (e*x)/2]^2*Tanh[d/2 + (I/4)*Pi + (e*x)/2])/(3*e))/(4*f^2)
```

3.896.3.1 Defintions of rubi rules used

rule 6013
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a,
b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
 NeQ[n, 2]
```

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

rule 6019
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sinh[(d_.) + (e_.)*(x_)
])^(n_.), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Cosh[d/2 - f*(Pi/
(4*g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && Eq
Q[f^2 + g^2, 0] && ILtQ[n, 0]
```

3.896.4 Maple [F]

$$\int \frac{F^{c(bx+a)}}{\left(f+if\sinh\left(ex+d\right)\right)^{2}}dx$$

input
```
int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x)
```

output
```
int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x)
```

3.896.5 Fricas [F]

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(i\,f\sinh(ex+d)+f)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x, algorithm="fricas")
```

output
```
1/3*(2*(3*e^2*e^(e*x + d) - (I*b^2*c^2*e^(2*e*x + 2*d) + 2*b^2*c^2*e^(e*x
+ d) - I*b^2*c^2)*log(F)^2 - I*e^2 - (I*b*c*e*e^(2*e*x + 2*d) + b*c*e*e^(e
*x + d))*log(F))*F^(b*c*x + a*c) + 3*(e^3*f^2*e^(3*e*x + 3*d) - 3*I*e^3*f^
2*e^(2*e*x + 2*d) - 3*e^3*f^2*e^(e*x + d) + I*e^3*f^2)*integral(-2/3*(-I*b
^3*c^3*log(F)^3 + I*b*c*e^2*log(F))*F^(b*c*x + a*c)/(e^3*f^2*e^(e*x + d) -
 I*e^3*f^2), x))/(e^3*f^2*e^(3*e*x + 3*d) - 3*I*e^3*f^2*e^(2*e*x + 2*d) -
3*e^3*f^2*e^(e*x + d) + I*e^3*f^2)
```

3.896.6 Sympy [F]

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = -\frac{\int \frac{F^{ac+bcx}}{\sinh^2(d+ex)-2i\sinh(d+ex)-1}\,dx}{f^2}$$

input
```
integrate(F**(c*(b*x+a))/(f+I*f*sinh(e*x+d))**2,x)
```

output
```
-Integral(F**(a*c + b*c*x)/(sinh(d + e*x)**2 - 2*I*sinh(d + e*x) - 1), x)/
f**2
```

3.896.7 Maxima [F]

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(i\,f\sinh(ex+d)+f)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x, algorithm="maxima")
```

output
```
-16*(-I*F^(a*c)*b^2*c^2*e*log(F)^2 - I*F^(a*c)*b*c*e^2*log(F))*integrate(F
^(b*c*x)/(-I*b^3*c^3*f^2*log(F)^3 + 9*I*b^2*c^2*e*f^2*log(F)^2 - 26*I*b*c*
e^2*f^2*log(F) + 24*I*e^3*f^2 + (b^3*c^3*f^2*e^(5*d)*log(F)^3 - 9*b^2*c^2*
e*f^2*e^(5*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(5*d)*log(F) - 24*e^3*f^2*e^(5*d
))*e^(5*e*x) - 5*(I*b^3*c^3*f^2*e^(4*d)*log(F)^3 - 9*I*b^2*c^2*e*f^2*e^(4*
d)*log(F)^2 + 26*I*b*c*e^2*f^2*e^(4*d)*log(F) - 24*I*e^3*f^2*e^(4*d))*e^(4
*e*x) - 10*(b^3*c^3*f^2*e^(3*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(3*d)*log(F)^
2 + 26*b*c*e^2*f^2*e^(3*d)*log(F) - 24*e^3*f^2*e^(3*d))*e^(3*e*x) - 10*(-I
*b^3*c^3*f^2*e^(2*d)*log(F)^3 + 9*I*b^2*c^2*e*f^2*e^(2*d)*log(F)^2 - 26*I*
b*c*e^2*f^2*e^(2*d)*log(F) + 24*I*e^3*f^2*e^(2*d))*e^(2*e*x) + 5*(b^3*c^3*
f^2*e^d*log(F)^3 - 9*b^2*c^2*e*f^2*e^d*log(F)^2 + 26*b*c*e^2*f^2*e^d*log(F
) - 24*e^3*f^2*e^d)*e^(e*x)), x) + 4*(4*F^(a*c)*b*c*e*log(F) + 4*F^(a*c)*e
^2 - (F^(a*c)*b^2*c^2*e^(2*d)*log(F)^2 - 7*F^(a*c)*b*c*e*e^(2*d)*log(F) +
12*F^(a*c)*e^2*e^(2*d))*e^(2*e*x) + 4*(-I*F^(a*c)*b*c*e*e^d*log(F) + 4*I*F
^(a*c)*e^2*e^d)*e^(e*x))*F^(b*c*x)/(b^3*c^3*f^2*log(F)^3 - 9*b^2*c^2*e*f^2
*log(F)^2 + 26*b*c*e^2*f^2*log(F) - 24*e^3*f^2 + (b^3*c^3*f^2*e^(4*d)*log(
F)^3 - 9*b^2*c^2*e*f^2*e^(4*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(4*d)*log(F) -
24*e^3*f^2*e^(4*d))*e^(4*e*x) - 4*(I*b^3*c^3*f^2*e^(3*d)*log(F)^3 - 9*I*b^
2*c^2*e*f^2*e^(3*d)*log(F)^2 + 26*I*b*c*e^2*f^2*e^(3*d)*log(F) - 24*I*e^3*
f^2*e^(3*d))*e^(3*e*x) - 6*(b^3*c^3*f^2*e^(2*d)*log(F)^3 - 9*b^2*c^2*e*...
```

3.896.8 Giac [F]

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(i\,f\sinh(ex+d)+f)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)/(I*f*sinh(e*x + d) + f)^2, x)
```

3.896.9 Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = \int \frac{F^{c\,(a+b\,x)}}{\left(f+f\sinh\left(d+e\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/(f + f*sinh(d + e*x)*1i)^2,x)
```

output
```
int(F^(c*(a + b*x))/(f + f*sinh(d + e*x)*1i)^2, x)
```

3.897 $\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx$

3.897.1 Optimal result

Integrand size = 22, antiderivative size = 251

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx=\frac{f^2F^{ac+bcx}}{bc\log(F)}-\frac{2bcf^2F^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}-\frac{bcf^2F^{ac+bcx}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)}+\frac{2ef^2F^{ac+bcx}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}+\frac{2ef^2F^{ac+bcx}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)}$$

output
```
f^2*F^(b*c*x+a*c)/b/c/ln(F)-2*b*c*f^2*F^(b*c*x+a*c)*cosh(e*x+d)*ln(F)/(e^2
-b^2*c^2*ln(F)^2)+2*e^2*f^2*F^(b*c*x+a*c)/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2
)-b*c*f^2*F^(b*c*x+a*c)*cosh(e*x+d)^2*ln(F)/(4*e^2-b^2*c^2*ln(F)^2)+2*e*f^
2*F^(b*c*x+a*c)*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)+2*e*f^2*F^(b*c*x+a*c)*co
sh(e*x+d)*sinh(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)
```

3.897.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.92

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx$$
$$=\frac{f^2F^{c(a+bx)}\left(12e^4-15b^2c^2e^2\log^2(F)+3b^4c^4\log^4(F)+4\cosh(d+ex)\left(-4b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)\right)+\ldots\right.}{\ldots}$$

input
```
Integrate[F^(c*(a + b*x))*(f + f*Cosh[d + e*x])^2,x]
```

output
```
(f^2*F^(c*(a + b*x))*(12*e^4 - 15*b^2*c^2*e^2*Log[F]^2 + 3*b^4*c^4*Log[F]^
4 + 4*Cosh[d + e*x]*(-4*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) + Cosh[2*
(d + e*x)]*(-(b^2*c^2*e^2*Log[F]^2) + b^4*c^4*Log[F]^4) + 16*b*c*e^3*Log[F
]*Sinh[d + e*x] - 4*b^3*c^3*e*Log[F]^3*Sinh[d + e*x] + 2*b*c*e^3*Log[F]*Si
nh[2*(d + e*x)] - 2*b^3*c^3*e*Log[F]^3*Sinh[2*(d + e*x)]))/(2*(4*b*c*e^4*L
og[F] - 5*b^3*c^3*e^2*Log[F]^3 + b^5*c^5*Log[F]^5))
```

3.897.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\cosh(d+ex)+f)^2\,dx$$
$$\Big\downarrow \text{7292}$$
$$\int f^2(\cosh(d+ex)+1)^2F^{ac+bcx}dx$$
$$\Big\downarrow \text{27}$$
$$f^2\int F^{ac+bxc}(\cosh(d+ex)+1)^2dx$$
$$\Big\downarrow \text{7293}$$
$$f^2\int \left(\cosh^2(d+ex)F^{ac+bxc}+2\cosh(d+ex)F^{ac+bxc}+F^{ac+bxc}\right)dx$$

$$\downarrow \text{2009}$$

$$f^2\left(\frac{2e\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh^2(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)}-\frac{2bc\log(F)\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)}{4e^2-}\right.$$

input
```
Int[F^(c*(a + b*x))*(f + f*Cosh[d + e*x])^2,x]
```

output
```
f^2*(F^(a*c + b*c*x)/(b*c*Log[F]) - (2*b*c*F^(a*c + b*c*x)*Cosh[d + e*x]*L
og[F])/(e^2 - b^2*c^2*Log[F]^2) + (2*e^2*F^(a*c + b*c*x))/(b*c*Log[F]*(4*e
^2 - b^2*c^2*Log[F]^2)) - (b*c*F^(a*c + b*c*x)*Cosh[d + e*x]^2*Log[F])/(4*
e^2 - b^2*c^2*Log[F]^2) + (2*e*F^(a*c + b*c*x)*Sinh[d + e*x])/(e^2 - b^2*c
^2*Log[F]^2) + (2*e*F^(a*c + b*c*x)*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 -
b^2*c^2*Log[F]^2))
```

3.897.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.897.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{2F^{c(bx+a)}\left(\frac{\left(\ln(F)^4b^4c^4-\ln(F)^2b^2c^2e^2\right)\cosh(2ex+2d)}{4}+\frac{\left(-\ln(F)^3b^3c^3e+\ln(F)bc\,e^3\right)\sinh(2ex+2d)}{2}+(bc\ln(F)+2e)\left(\cosh(ex+d)b^2c\right.\right.}{c^5b^5\ln(F)^5-5c^3b^3\ln(F)^3e^2+4\ln(F)bc\,e^4}$
risch	$f^2\left(\ln(F)^4b^4c^4e^{4ex+4d}+4\ln(F)^4b^4c^4e^{3ex+3d}+6\ln(F)^4b^4c^4e^{2ex+2d}-2\ln(F)^3b^3c^3e\,e^{4ex+4d}+4\ln(F)^4b^4c^4e^{ex+d}-4\ln(F)^3b^3c\right.$

input
```
int(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
2*F^(c*(b*x+a))*(1/4*(ln(F)^4*b^4*c^4-ln(F)^2*b^2*c^2*e^2)*cosh(2*e*x+2*d)
+1/2*(-ln(F)^3*b^3*c^3*e+ln(F)*b*c*e^3)*sinh(2*e*x+2*d)+(b*c*ln(F)+2*e)*(c
osh(e*x+d)*b^2*c^2*ln(F)^2+3/4*b^2*c^2*ln(F)^2-sinh(e*x+d)*b*c*e*ln(F)-3/4
*e^2)*(b*c*ln(F)-2*e))*f^2/(c^5*b^5*ln(F)^5-5*c^3*b^3*ln(F)^3*e^2+4*ln(F)*
b*c*e^4)
```

3.897.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2340 vs. 2(249) = 498.

Time = 0.34 (sec) , antiderivative size = 2340, normalized size of antiderivative = 9.32

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x, algorithm="fricas")
```

output
```
1/4*((24*e^4*f^2*cosh(e*x + d)^2 + (b^4*c^4*f^2*cosh(e*x + d)^4 + 4*b^4*c^
4*f^2*cosh(e*x + d)^3 + 6*b^4*c^4*f^2*cosh(e*x + d)^2 + 4*b^4*c^4*f^2*cosh
(e*x + d) + b^4*c^4*f^2)*log(F)^4 + (b^4*c^4*f^2*log(F)^4 - 2*b^3*c^3*e*f^
2*log(F)^3 - b^2*c^2*e^2*f^2*log(F)^2 + 2*b*c*e^3*f^2*log(F))*sinh(e*x + d
)^4 - 2*(b^3*c^3*e*f^2*cosh(e*x + d)^4 + 2*b^3*c^3*e*f^2*cosh(e*x + d)^3 -
 2*b^3*c^3*e*f^2*cosh(e*x + d) - b^3*c^3*e*f^2)*log(F)^3 + 4*((b^4*c^4*f^2
*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 - (2*b^3*c^3*e*f^2*cosh(e*x + d) +
b^3*c^3*e*f^2)*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d) + 4*b^2*c^2*e^2*f
^2)*log(F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d) + 2*b*c*e^3*f^2)*log(F))*sinh(
e*x + d)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^4 + 16*b^2*c^2*e^2*f^2*cosh(e*
x + d)^3 + 30*b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 16*b^2*c^2*e^2*f^2*cosh(e*
x + d) + b^2*c^2*e^2*f^2)*log(F)^2 + 6*(4*e^4*f^2 + (b^4*c^4*f^2*cosh(e*x
+ d)^2 + 2*b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 - 2*(b^3*c^3*
e*f^2*cosh(e*x + d)^2 + b^3*c^3*e*f^2*cosh(e*x + d))*log(F)^3 - (b^2*c^2*e
^2*f^2*cosh(e*x + d)^2 + 8*b^2*c^2*e^2*f^2*cosh(e*x + d) + 5*b^2*c^2*e^2*f
^2)*log(F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d)^2 + 4*b*c*e^3*f^2*cosh(e*x + d
))*log(F))*sinh(e*x + d)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d)^4 + 8*b*c*e^3*f^
2*cosh(e*x + d)^3 - 8*b*c*e^3*f^2*cosh(e*x + d) - b*c*e^3*f^2)*log(F) + 4*
(12*e^4*f^2*cosh(e*x + d) + (b^4*c^4*f^2*cosh(e*x + d)^3 + 3*b^4*c^4*f^2*c
osh(e*x + d)^2 + 3*b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 - ...
```

3.897.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2346 vs. $2(238) = 476$.

Time = 2.14 (sec) , antiderivative size = 2346, normalized size of antiderivative = 9.35

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*(f+f*cosh(e*x+d))**2,x)
```

output
```
Piecewise((x*(f*cosh(d) + f)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)
), (-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x + f**2
*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*f**2*sinh(d + e*x)/e, Eq(F, 1)), (F
**(a*c)*(-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x +
 f**2*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*f**2*sinh(d + e*x)/e), Eq(b, 0
)), (-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x + f**
2*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*f**2*sinh(d + e*x)/e, Eq(c, 0)), (
-F**(a*c + b*c*x)*f**2*x*sinh(b*c*x*log(F) - d) + F**(a*c + b*c*x)*f**2*x*
cosh(b*c*x*log(F) - d) - 2*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) - d)**2
/(3*b*c*log(F)) + 2*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) - d)*cosh(b*c*
x*log(F) - d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) - d
)/(b*c*log(F)) + F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F) - d)**2/(3*b*c*lo
g(F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, -b*c*log(F))), (F**(a*c
+ b*c*x)*f**2*x*sinh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*f**2*x*si
nh(b*c*x*log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*f**2*
x*cosh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F)/
2 - d)*cosh(b*c*x*log(F)/2 - d)/(2*b*c*log(F)) - 4*F**(a*c + b*c*x)*f**2*s
inh(b*c*x*log(F)/2 - d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2*cosh(b*c*x*
log(F)/2 - d)**2/(b*c*log(F)) + 8*F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F)/
2 - d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, -b*c*...
```

3.897.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.75

$$\begin{aligned}\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx &= \frac{1}{4}f^2\left(\frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{bc\log(F)+2\,e}+\frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)}}+\frac{2\,F^{bcx+ac}}{bc\log(F)}\right)\\&\quad+f^2\left(\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{bc\log(F)+e}+\frac{F^{ac}e^{(bcx\log(F)-ex)}}{bce^{d}\log(F)-ee^{d}}\right)+\frac{F^{bcx+ac}f^2}{bc\log(F)}\end{aligned}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x, algorithm="maxima")
```

output
```
1/4*f^2*(F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + F^(a*
c)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) + 2*F^(b*c*
x + a*c)/(b*c*log(F))) + f^2*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(
F) + e) + F^(a*c)*e^(b*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*
c*x + a*c)*f^2/(b*c*log(F))
```

3.897.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 1548, normalized size of antiderivative = 6.17

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x, algorithm="giac")
```

output
```
3*(2*b*c*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*
c)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c
*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sg
n(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*I*(I*f^2*e^
(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a
*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*f^2*e^(-1/2*I
*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-
2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F))
+ a*c*log(abs(F))) + 1/2*(2*(b*c*log(abs(F)) + 2*e)*f^2*cos(-1/2*pi*b*c*x*
sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) -
pi*b*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*si
n(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((
pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F
)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + I*(I*f^2*e^(1/2*I*pi*b*c*x*sgn(F)
- 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F)
- 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F)
 + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F
) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e))*e^(a*c*log(abs(F)) + (b*c*log(
abs(F)) + 2*e)*x + 2*d) + 2*(2*(b*c*log(abs(F)) + e)*f^2*cos(-1/2*pi*b*...
```

3.897.9 Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.15

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx = \frac{2F^{b\,c\,x}\,F^{a\,c}\,e\,f^2\sinh(d+e\,x)}{e^2-b^2\,c^2\ln(F)^2}+\frac{F^{b\,c\,x}\,F^{a\,c}\,f^2}{b\,c\,\ln(F)}+\frac{2F^{b\,c\,x}\,F^{a\,c}\,e\,f^2\cosh(d+e\,x)\,\sinh(d+e\,x)}{4\,e^2-b^2\,c^2\ln(F)^2}-\frac{2F^{b\,c\,x}\,F^{a\,c}\,b\,c\,f^2\cosh(d+e\,x)\,\ln(F)}{e^2-b^2\,c^2\ln(F)^2}-\frac{2F^{b\,c\,x}\,F^{a\,c}\,e^2\,f^2\sinh(d+e\,x)^2}{b\,c\,\ln(F)\,\left(4\,e^2-b^2\,c^2\ln(F)^2\right)}+\frac{F^{b\,c\,x}\,F^{a\,c}\,f^2\cosh(d+e\,x)^2\,\left(2\,e^2-b^2\,c^2\ln(F)^2\right)}{b\,c\,\ln(F)\,\left(4\,e^2-b^2\,c^2\ln(F)^2\right)}$$

input
```
int(F^(c*(a + b*x))*(f + f*cosh(d + e*x))^2,x)
```

output
```
(2*F^(b*c*x)*F^(a*c)*e*f^2*sinh(d + e*x))/(e^2 - b^2*c^2*log(F)^2) + (F^(b
*c*x)*F^(a*c)*f^2)/(b*c*log(F)) + (2*F^(b*c*x)*F^(a*c)*e*f^2*cosh(d + e*x)
*sinh(d + e*x))/(4*e^2 - b^2*c^2*log(F)^2) - (2*F^(b*c*x)*F^(a*c)*b*c*f^2*
cosh(d + e*x)*log(F))/(e^2 - b^2*c^2*log(F)^2) - (2*F^(b*c*x)*F^(a*c)*e^2*
f^2*sinh(d + e*x)^2)/(b*c*log(F)*(4*e^2 - b^2*c^2*log(F)^2)) + (F^(b*c*x)*
F^(a*c)*f^2*cosh(d + e*x)^2*(2*e^2 - b^2*c^2*log(F)^2))/(b*c*log(F)*(4*e^2
 - b^2*c^2*log(F)^2))
```

3.898 $\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx$

3.898.1 Optimal result

Integrand size = 20, antiderivative size = 101

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx=\frac{fF^{ac+bcx}}{bc\log(F)}-\frac{bcfF^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{efF^{ac+bcx}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

output
```
f*F^(b*c*x+a*c)/b/c/ln(F)-b*c*f*F^(b*c*x+a*c)*cosh(e*x+d)*ln(F)/(e^2-b^2*c
^2*ln(F)^2)+e*f*F^(b*c*x+a*c)*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

3.898.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx=\frac{fF^{c(a+bx)}\left(-e^2+b^2c^2\log^2(F)+b^2c^2\cosh(d+ex)\log^2(F)-bce\log(F)\sinh(d+ex)\right)}{bc\log(F)(-e+bc\log(F))(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*(f + f*Cosh[d + e*x]),x]
```

output
```
(f*F^(c*(a + b*x))*(-e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[d + e*x]*Log[F]
^2 - b*c*e*Log[F]*Sinh[d + e*x]))/(b*c*Log[F]*(-e + b*c*Log[F])*(e + b*c*L
og[F]))
```

3.898.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\cosh(d+ex)+f)\,dx$$

$$\Big\downarrow \text{7292}$$

$$\int f(\cosh(d+ex)+1)F^{ac+bcx}dx$$

$$\Big\downarrow \text{27}$$

$$f\int F^{ac+bxc}(\cosh(d+ex)+1)dx$$

$$\Big\downarrow \text{7293}$$

$$f\int \left(\cosh(d+ex)F^{ac+bxc}+F^{ac+bxc}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$f\left(\frac{e\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{F^{ac+bcx}}{bc\log(F)}\right)$$

input
```
Int[F^(c*(a + b*x))*(f + f*Cosh[d + e*x]),x]
```

output
```
f*(F^(a*c + b*c*x)/(b*c*Log[F]) - (b*c*F^(a*c + b*c*x)*Cosh[d + e*x]*Log[F
])/(e^2 - b^2*c^2*Log[F]^2) + (e*F^(a*c + b*c*x)*Sinh[d + e*x])/(e^2 - b^2
*c^2*Log[F]^2))
```

3.898.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.898.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{f\,F^{c(bx+a)}\left(\cosh(ex+d)b^2c^2\ln(F)^2+b^2c^2\ln(F)^2-\sinh(ex+d)bce\ln(F)-e^2\right)}{\left(b^2c^2\ln(F)^2-e^2\right)bc\ln(F)}$	88
risch	$\frac{f\left(\ln(F)^2b^2c^2\mathrm{e}^{2ex+2d}+2\ln(F)^2b^2c^2\mathrm{e}^{ex+d}+b^2c^2\ln(F)^2-\ln(F)bce\,\mathrm{e}^{2ex+2d}+\ln(F)bce-2e^2\mathrm{e}^{ex+d}\right)\mathrm{e}^{-ex-d}F^{c(bx+a)}}{2bc\ln(F)(bc\ln(F)-e)(e+bc\ln(F))}$	135

input
```
int(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x,method=_RETURNVERBOSE)
```

output
```
f*F^(c*(b*x+a))/(b^2*c^2*ln(F)^2-e^2)/b/c/ln(F)*(cosh(e*x+d)*b^2*c^2*ln(F)
^2+b^2*c^2*ln(F)^2-sinh(e*x+d)*b*c*e*ln(F)-e^2)
```

3.898.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(103) = 206.

Time = 0.26 (sec) , antiderivative size = 430, normalized size of antiderivative = 4.26

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx =$$
$$-\frac{\left(2\,e^2f\cosh\left(ex+d\right)-\left(b^2c^2f\cosh\left(ex+d\right)^2+2\,b^2c^2f\cosh\left(ex+d\right)+b^2c^2f\right)\log\left(F\right)^2-\left(b^2c^2f\log\left(F\right)^2\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x, algorithm="fricas")
```

output
```
-1/2*((2*e^2*f*cosh(e*x + d) - (b^2*c^2*f*cosh(e*x + d)^2 + 2*b^2*c^2*f*co
sh(e*x + d) + b^2*c^2*f)*log(F)^2 - (b^2*c^2*f*log(F)^2 - b*c*e*f*log(F))*
sinh(e*x + d)^2 + (b*c*e*f*cosh(e*x + d)^2 - b*c*e*f)*log(F) + 2*(b*c*e*f*
cosh(e*x + d)*log(F) + e^2*f - (b^2*c^2*f*cosh(e*x + d) + b^2*c^2*f)*log(F
)^2)*sinh(e*x + d))*cosh((b*c*x + a*c)*log(F)) + (2*e^2*f*cosh(e*x + d) -
(b^2*c^2*f*cosh(e*x + d)^2 + 2*b^2*c^2*f*cosh(e*x + d) + b^2*c^2*f)*log(F)
^2 - (b^2*c^2*f*log(F)^2 - b*c*e*f*log(F))*sinh(e*x + d)^2 + (b*c*e*f*cosh
(e*x + d)^2 - b*c*e*f)*log(F) + 2*(b*c*e*f*cosh(e*x + d)*log(F) + e^2*f -
(b^2*c^2*f*cosh(e*x + d) + b^2*c^2*f)*log(F)^2)*sinh(e*x + d))*sinh((b*c*x
 + a*c)*log(F)))/(b^3*c^3*cosh(e*x + d)*log(F)^3 - b*c*e^2*cosh(e*x + d)*l
og(F) + (b^3*c^3*log(F)^3 - b*c*e^2*log(F))*sinh(e*x + d))
```

3.898.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(94) = 188.

Time = 0.65 (sec) , antiderivative size = 488, normalized size of antiderivative = 4.83

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx$$

$$= \begin{cases} x(f\cosh(d)+f) \\ fx+\frac{f\sinh(d+ex)}{e} \\ F^{ac}\left(fx+\frac{f\sinh(d+ex)}{e}\right) \\ fx+\frac{f\sinh(d+ex)}{e} \\ -\frac{F^{ac+bcx}fx\sinh(bcx\log(F)-d)}{2}+\frac{F^{ac+bcx}fx\cosh(bcx\log(F)-d)}{2}+\frac{F^{ac+bcx}f\sinh(bcx\log(F)-d)}{2bc\log(F)}+\frac{F^{ac+bcx}f}{bc\log(F)} \\ -\frac{F^{ac+bcx}fx\sinh(bcx\log(F)+d)}{2}+\frac{F^{ac+bcx}fx\cosh(bcx\log(F)+d)}{2}+\frac{F^{ac+bcx}f\sinh(bcx\log(F)+d)}{bc\log(F)}-\frac{F^{ac+bcx}f\cosh(bcx\log(F)+d}{2bc\log(F)} \\ \frac{F^{ac+bcx}b^2c^2f\log(F)^2\cosh(d+ex)}{b^3c^3\log(F)^3-bce^2\log(F)}+\frac{F^{ac+bcx}b^2c^2f\log(F)^2}{b^3c^3\log(F)^3-bce^2\log(F)}-\frac{F^{ac+bcx}bcef\log(F)\sinh(d+ex)}{b^3c^3\log(F)^3-bce^2\log(F)}-\frac{F^{ac+bcx}e^2f}{b^3c^3\log(F)^3-bce^2\log(F)} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*(f+f*cosh(e*x+d)),x)
```

output
```
Piecewise((x*(f*cosh(d) + f), Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)),
(f*x + f*sinh(d + e*x)/e, Eq(F, 1)), (F**(a*c)*(f*x + f*sinh(d + e*x)/e),
Eq(b, 0)), (f*x + f*sinh(d + e*x)/e, Eq(c, 0)), (-F**(a*c + b*c*x)*f*x*sin
h(b*c*x*log(F) - d)/2 + F**(a*c + b*c*x)*f*x*cosh(b*c*x*log(F) - d)/2 + F*
*(a*c + b*c*x)*f*sinh(b*c*x*log(F) - d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*
f/(b*c*log(F)), Eq(e, -b*c*log(F))), (-F**(a*c + b*c*x)*f*x*sinh(b*c*x*log
(F) + d)/2 + F**(a*c + b*c*x)*f*x*cosh(b*c*x*log(F) + d)/2 + F**(a*c + b*c
*x)*f*sinh(b*c*x*log(F) + d)/(b*c*log(F)) - F**(a*c + b*c*x)*f*cosh(b*c*x*
log(F) + d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*f/(b*c*log(F)), Eq(e, b*c*lo
g(F))), (F**(a*c + b*c*x)*b**2*c**2*f*log(F)**2*cosh(d + e*x)/(b**3*c**3*l
og(F)**3 - b*c*e**2*log(F)) + F**(a*c + b*c*x)*b**2*c**2*f*log(F)**2/(b**3
*c**3*log(F)**3 - b*c*e**2*log(F)) - F**(a*c + b*c*x)*b*c*e*f*log(F)*sinh(
d + e*x)/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)) - F**(a*c + b*c*x)*e**2*f
/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)), True))
```

3.898. $\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx$

3.898.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx=\frac{1}{2}\,f\left(\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{bc\log(F)+e}+\frac{F^{ac}e^{(bcx\log(F)-ex)}}{bce^{d}\log(F)-ee^{d}}\right)+\frac{F^{bcx+ac}f}{bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x, algorithm="maxima")
```

output
```
1/2*f*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + F^(a*c)*e^(b*
c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c*x + a*c)*f/(b*c*log(F
))
```

3.898.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 886, normalized size of antiderivative = 8.77

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx=\text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x, algorithm="giac")
```

output
```
2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1
/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F)
 - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f*e^(1/2*I*p
i*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*p
i*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) - I*f*e^(-1/2*I*pi*b*c*x*sgn(
F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F
) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
 + (2*(b*c*log(abs(F)) + e)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*p
i*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*
(b*c*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d
) + I*(I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F)
 - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e
) - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) +
 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e)
)*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + (2*(b*c*log(abs(F))
- e)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/...
```

3.898.9 Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx = -\frac{F^{b\,c\,x}\,F^{a\,c}\,f\,\mathrm{e}^{-d-e\,x}\,\left(b^2\,c^2\ln\left(F\right)^2-2\,e^2\,\mathrm{e}^{d+e\,x}+b\,c\,e\,\ln\left(F\right)+2\,b^2\,c^2\,\mathrm{e}^{d+e\,x}\ln\left(F\right)^2+b^2\,c^2\,\mathrm{e}^{2\,d+2\,e\,x}\ln\left(F\right)^2}{2\,b\,c\,\ln\left(F\right)\,\left(e^2-b^2\,c^2\ln\left(F\right)^2\right)}$$

input
```
int(F^(c*(a + b*x))*(f + f*cosh(d + e*x)),x)
```

output
```
-(F^(b*c*x)*F^(a*c)*f*exp(- d - e*x)*(b^2*c^2*log(F)^2 - 2*e^2*exp(d + e*x
) + b*c*e*log(F) + 2*b^2*c^2*exp(d + e*x)*log(F)^2 + b^2*c^2*exp(2*d + 2*e
*x)*log(F)^2 - b*c*e*exp(2*d + 2*e*x)*log(F)))/(2*b*c*log(F)*(e^2 - b^2*c^
2*log(F)^2))
```

3.899 $\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx$

3.899.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{d+ex}\right)}{f(e+bc\log(F))}$$

output
```
2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([2, 1+b*c*ln(F)/e],[2+b*c*ln(F)/e],-e
xp(e*x+d))/f/(e+b*c*ln(F))
```

3.899.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{d+ex}\right)}{ef+bcf\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))/(f + f*Cosh[d + e*x]),x]
```

output
```
(2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2
+ (b*c*Log[F])/e, -E^(d + e*x)])/(e*f + b*c*f*Log[F])
```

3.899.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6020, 6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{f\cosh(d+ex)+f}dx$$

$$\Big\downarrow \text{6020}$$

$$\frac{\int F^{c(a+bx)}\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)dx}{2f}$$

$$\Big\downarrow \text{6015}$$

$$\frac{2e^{d+ex}F^{c(a+bx)}\text{ Hypergeometric2F1}\left(2,\frac{bc\log(F)}{e}+1,\frac{bc\log(F)}{e}+2,-e^{d+ex}\right)}{f(bc\log(F)+e)}$$

input
```
Int[F^(c*(a + b*x))/(f + f*Cosh[d + e*x]),x]
```

output
```
(2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2
+ (b*c*Log[F])/e, -E^(d + e*x)])/(f*(e + b*c*Log[F]))
```

3.899.3.1 Defintions of rubi rules used

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

rule 6020
```
Int[(Cosh[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.
)*(x_))), x_Symbol] :> Simp[2^n*g^n   Int[F^(c*(a + b*x))*Cosh[d/2 + e*(x/2
)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] &&
ILtQ[n, 0]
```

3.899.4 Maple [F]

$$\int \frac{F^{c(bx+a)}}{f + f\cosh(ex+d)}dx$$

input
```
int(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x)
```

output
```
int(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x)
```

3.899.5 Fricas [F]

$$\int \frac{F^{c(a+bx)}}{f + f\cosh(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\cosh(ex+d) + f}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)/(f*cosh(e*x + d) + f), x)
```

3.899.6 Sympy [F]

$$\int \frac{F^{c(a+bx)}}{f + f\cosh(d+ex)}\,dx = \frac{\int \frac{F^{ac+bcx}}{\cosh(d+ex)+1}\,dx}{f}$$

input
```
integrate(F**(c*(b*x+a))/(f+f*cosh(e*x+d)),x)
```

output
```
Integral(F**(a*c + b*c*x)/(cosh(d + e*x) + 1), x)/f
```

3.899.7 Maxima [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx=\int \frac{F^{(bx+a)c}}{f\cosh{(ex+d)}+f}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x, algorithm="maxima")
```

output
```
4*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(b^2*c^2*f*log(F)^2 - 3*b*c*e*f*log(F)
 + 2*e^2*f + (b^2*c^2*f*e^(3*d)*log(F)^2 - 3*b*c*e*f*e^(3*d)*log(F) + 2*e^
2*f*e^(3*d))*e^(3*e*x) + 3*(b^2*c^2*f*e^(2*d)*log(F)^2 - 3*b*c*e*f*e^(2*d)
*log(F) + 2*e^2*f*e^(2*d))*e^(2*e*x) + 3*(b^2*c^2*f*e^d*log(F)^2 - 3*b*c*e
*f*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x)), x)*log(F) - 2*(2*F^(a*c)*e - (F^(a*
c)*b*c*e^d*log(F) - 2*F^(a*c)*e*e^d)*e^(e*x))*F^(b*c*x)/(b^2*c^2*f*log(F)^
2 - 3*b*c*e*f*log(F) + 2*e^2*f + (b^2*c^2*f*e^(2*d)*log(F)^2 - 3*b*c*e*f*e
^(2*d)*log(F) + 2*e^2*f*e^(2*d))*e^(2*e*x) + 2*(b^2*c^2*f*e^d*log(F)^2 - 3
*b*c*e*f*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x))
```

3.899.8 Giac [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx=\int \frac{F^{(bx+a)c}}{f\cosh{(ex+d)}+f}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)/(f*cosh(e*x + d) + f), x)
```

3.899.9 Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx=\int \frac{F^{c\,(a+b\,x)}}{f+f\cosh{(d+e\,x)}}\,dx$$

input
```
int(F^(c*(a + b*x))/(f + f*cosh(d + e*x)),x)
```

output
```
int(F^(c*(a + b*x))/(f + f*cosh(d + e*x)), x)
```

3.900 $\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx$

3.900.1 Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{d+ex}\right)(e-bc\log(F))}{3e^2f^2} + \frac{bcF^{c(a+bx)}\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{6ef^2}$$

output
```
2/3*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([2, 1+b*c*ln(F)/e],[2+b*c*ln(F)/e],
-exp(e*x+d))*(e-b*c*ln(F))/e^2/f^2+1/6*b*c*F^(c*(b*x+a))*ln(F)*sech(1/2*e*
x+1/2*d)^2/e^2/f^2+1/6*F^(c*(b*x+a))*sech(1/2*e*x+1/2*d)^2*tanh(1/2*e*x+1/
2*d)/e/f^2
```

3.900.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \frac{2F^{c(a+bx)}\cosh\left(\frac{1}{2}(d+ex)\right)\left(bc\cosh\left(\frac{1}{2}(d+ex)\right)\log(F)+4e^{d+ex}\cosh^3\left(\frac{1}{2}(d+ex)\right)\text{Hypergeometric2F1}\left(2\right.\right.}{3e^2f^2(1+\cosh(d+ex))^2}$$

input
```
Integrate[F^(c*(a + b*x))/(f + f*Cosh[d + e*x])^2,x]
```

output
```
(2*F^(c*(a + b*x))*Cosh[(d + e*x)/2]*(b*c*Cosh[(d + e*x)/2]*Log[F] + 4*E^(
d + e*x)*Cosh[(d + e*x)/2]^3*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 +
(b*c*Log[F])/e, -E^(d + e*x)]*(e - b*c*Log[F]) + e*Sinh[(d + e*x)/2]))/(3*
e^2*f^2*(1 + Cosh[d + e*x])^2)
```

3.900.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6020, 6013, 6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{(f\cosh(d+ex)+f)^2}dx$$

$$\Big\downarrow \text{ 6020}$$

$$\frac{\int F^{c(a+bx)}\text{sech}^4\left(\frac{d}{2}+\frac{ex}{2}\right)dx}{4f^2}$$

$$\Big\downarrow \text{ 6013}$$

$$\frac{\frac{2}{3}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)dx+\frac{2bc\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{3e^2}+\frac{2\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{3e}}{4f^2}$$

$$\Big\downarrow \text{ 6015}$$

$$\frac{\frac{8e^{d+ex}F^{c(a+bx)}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{e}+1,\frac{bc\log(F)}{e}+2,-e^{d+ex}\right)}{3(bc\log(F)+e)}+\frac{2bc\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{3e^2}+\frac{2\tanh\left(\frac{d}{2}\right.}{}}{4f^2}$$

input
```
Int[F^(c*(a + b*x))/(f + f*Cosh[d + e*x])^2,x]
```

output
```
((8*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2
 + (b*c*Log[F])/e, -E^(d + e*x)]*(1 - (b^2*c^2*Log[F]^2)/e^2))/(3*(e + b*c
*Log[F])) + (2*b*c*F^(c*(a + b*x))*Log[F]*Sech[d/2 + (e*x)/2]^2)/(3*e^2) +
 (2*F^(c*(a + b*x))*Sech[d/2 + (e*x)/2]^2*Tanh[d/2 + (e*x)/2])/(3*e))/(4*f
^2)
```

3.900.3.1 Defintions of rubi rules used

rule 6013
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a,
b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
 NeQ[n, 2]
```

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

rule 6020
```
Int[(Cosh[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.
)*(x_))), x_Symbol] :> Simp[2^n*g^n   Int[F^(c*(a + b*x))*Cosh[d/2 + e*(x/2
)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] &&
ILtQ[n, 0]
```

3.900.4 Maple [F]

$$\int \frac{F^{c(bx+a)}}{(f+f\cosh(ex+d))^2}dx$$

input
```
int(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x)
```

output
```
int(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x)
```

3.900.5 Fricas [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\cosh(ex+d)+f)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)/(f^2*cosh(e*x + d)^2 + 2*f^2*cosh(e*x + d) + f^2)
, x)
```

3.900.6 Sympy [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \frac{\int \frac{F^{ac+bcx}}{\cosh^2(d+ex)+2\cosh(d+ex)+1}\,dx}{f^2}$$

input
```
integrate(F**(c*(b*x+a))/(f+f*cosh(e*x+d))**2,x)
```

output
```
Integral(F**(a*c + b*c*x)/(cosh(d + e*x)**2 + 2*cosh(d + e*x) + 1), x)/f**
2
```

3.900.7 Maxima [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\cosh(ex+d)+f)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x, algorithm="maxima")
```

output
```
-16*(F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*b*c*e^2*log(F))*integrate(F^(b*c
*x)/(b^3*c^3*f^2*log(F)^3 - 9*b^2*c^2*e*f^2*log(F)^2 + 26*b*c*e^2*f^2*log(
F) - 24*e^3*f^2 + (b^3*c^3*f^2*e^(5*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(5*d)*
log(F)^2 + 26*b*c*e^2*f^2*e^(5*d)*log(F) - 24*e^3*f^2*e^(5*d))*e^(5*e*x) +
 5*(b^3*c^3*f^2*e^(4*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(4*d)*log(F)^2 + 26*b
*c*e^2*f^2*e^(4*d)*log(F) - 24*e^3*f^2*e^(4*d))*e^(4*e*x) + 10*(b^3*c^3*f^
2*e^(3*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(3*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(
3*d)*log(F) - 24*e^3*f^2*e^(3*d))*e^(3*e*x) + 10*(b^3*c^3*f^2*e^(2*d)*log(
F)^3 - 9*b^2*c^2*e*f^2*e^(2*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(2*d)*log(F) -
24*e^3*f^2*e^(2*d))*e^(2*e*x) + 5*(b^3*c^3*f^2*e^d*log(F)^3 - 9*b^2*c^2*e*
f^2*e^d*log(F)^2 + 26*b*c*e^2*f^2*e^d*log(F) - 24*e^3*f^2*e^d)*e^(e*x)), x
) + 4*(4*F^(a*c)*b*c*e*log(F) + 4*F^(a*c)*e^2 + (F^(a*c)*b^2*c^2*e^(2*d)*l
og(F)^2 - 7*F^(a*c)*b*c*e*e^(2*d)*log(F) + 12*F^(a*c)*e^2*e^(2*d))*e^(2*e*
x) - 4*(F^(a*c)*b*c*e*e^d*log(F) - 4*F^(a*c)*e^2*e^d)*e^(e*x))*F^(b*c*x)/(
b^3*c^3*f^2*log(F)^3 - 9*b^2*c^2*e*f^2*log(F)^2 + 26*b*c*e^2*f^2*log(F) -
24*e^3*f^2 + (b^3*c^3*f^2*e^(4*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(4*d)*log(F
)^2 + 26*b*c*e^2*f^2*e^(4*d)*log(F) - 24*e^3*f^2*e^(4*d))*e^(4*e*x) + 4*(b
^3*c^3*f^2*e^(3*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(3*d)*log(F)^2 + 26*b*c*e^
2*f^2*e^(3*d)*log(F) - 24*e^3*f^2*e^(3*d))*e^(3*e*x) + 6*(b^3*c^3*f^2*e^(2
*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(2*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(2*d...
```

3.900.8 Giac [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{\left(f\cosh\left(ex+d\right)+f\right)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)/(f*cosh(e*x + d) + f)^2, x)
```

3.900.9 Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \int \frac{F^{c\,(a+b\,x)}}{\left(f+f\cosh\left(d+e\,x\right)\right)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/(f + f*cosh(d + e*x))^2,x)
```

output
```
int(F^(c*(a + b*x))/(f + f*cosh(d + e*x))^2, x)
```

3.901 $\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx$

3.901.1 Optimal result

Integrand size = 22, antiderivative size = 69

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx=\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

output
```
1/48*exp(-3*b*x-3*a)/b-1/8*exp(-b*x-a)/b-1/24*exp(3*b*x+3*a)/b+1/80*exp(5*
b*x+5*a)/b
```

3.901.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx=\frac{e^{-3(a+bx)}\left(5-30e^{2(a+bx)}-10e^{6(a+bx)}+3e^{8(a+bx)}\right)}{240b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
(5 - 30*E^(2*(a + b*x)) - 10*E^(6*(a + b*x)) + 3*E^(8*(a + b*x)))/(240*b*E
^(3*(a + b*x)))
```

3.901.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2720, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \sinh^3(a+bx) \cosh(a+bx) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{16} e^{-4a-4bx} \left(1-e^{2a+2bx}\right)^3 \left(1+e^{2a+2bx}\right) de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int e^{-4a-4bx} \left(1-e^{2a+2bx}\right)^3 \left(1+e^{2a+2bx}\right) de^{a+bx}}{16b}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left(e^{-4a-4bx}-2e^{-2a-2bx}+2e^{2a+2bx}-e^{4a+4bx}\right) de^{a+bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}e^{-3a-3bx}-2e^{-a-bx}-\frac{2}{3}e^{3a+3bx}+\frac{1}{5}e^{5a+5bx}}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
(E^(-3*a - 3*b*x)/3 - 2*E^(-a - b*x) - (2*E^(3*a + 3*b*x))/3 + E^(5*a + 5*
b*x)/5)/(16*b)
```

3.901.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.901.4 Maple [A] (verified)

Time = 284.72 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^5}{5}+\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}}{b}$	44
default	$\frac{\frac{\sinh(bx+a)^5}{5}+\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}}{b}$	44
risch	$\frac{e^{-3bx-3a}}{48b}-\frac{e^{-bx-a}}{8b}-\frac{e^{3bx+3a}}{24b}+\frac{e^{5bx+5a}}{80b}$	58

input
```
int(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5*sinh(b*x+a)^5+1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)^3)
```

3.901.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.61

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^4-\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+\left(6\cosh(bx+a)^2-5\right)\sinh(bx+a)^2-5}{30\left(b\cosh(bx+a)-b\sinh(bx+a)\right)}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/30*(cosh(b*x + a)^4 - cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
(6*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^2 - 5*cosh(b*x + a)^2 - (cosh(b*x +
a)^3 - 5*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

3.901.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(53) = 106$.

Time = 2.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.01

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} \frac{e^{a}e^{bx}\sinh^{4}(a+bx)}{5b} - \frac{e^{a}e^{bx}\sinh^{3}(a+bx)\cosh(a+bx)}{5b} + \frac{e^{a}e^{bx}\sinh^{2}(a+bx)\cosh^{2}(a+bx)}{5b} + \frac{2e^{a}e^{bx}\sinh(a+bx)\cosh^{3}(a+bx)}{15b} - \frac{2e^{a}e^{bx}}{} \\ xe^{a}\sinh^{3}(a)\cosh(a) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**4/(5*b) - exp(a)*exp(b*x)*sinh(a
 + b*x)**3*cosh(a + b*x)/(5*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a +
 b*x)**2/(5*b) + 2*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(15*b) -
 2*exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(a)**
3*cosh(a), True))
```

3.901.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(6\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}}{48\,b} + \frac{3\,e^{(5\,bx+5\,a)}-10\,e^{(3\,bx+3\,a)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/48*(6*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a)/b + 1/240*(3*e^(5*b*x + 5*a
) - 10*e^(3*b*x + 3*a))/b
```

3.901.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{5\left(6\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}-3\,e^{(5\,bx+5\,a)}+10\,e^{(3\,bx+3\,a)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/240*(5*(6*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) - 3*e^(5*b*x + 5*a) + 1
0*e^(3*b*x + 3*a))/b
```

3.901.9 Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{30\,\mathrm{e}^{-a-b\,x}-5\,\mathrm{e}^{-3\,a-3\,b\,x}+10\,\mathrm{e}^{3\,a+3\,b\,x}-3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

input
```
int(cosh(a + b*x)*exp(a + b*x)*sinh(a + b*x)^3,x)
```

output
```
-(30*exp(- a - b*x) - 5*exp(- 3*a - 3*b*x) + 10*exp(3*a + 3*b*x) - 3*exp(5
*a + 5*b*x))/(240*b)
```

3.902 $\int e^{a+bx} \cosh(a+bx) \sinh^2(a+bx) \, dx$

3.902.1 Optimal result

Integrand size = 22, antiderivative size = 57

$$\int e^{a+bx} \cosh(a+bx) \sinh^2(a+bx) \, dx = -\frac{e^{-2a-2bx}}{16b} - \frac{e^{2a+2bx}}{16b} + \frac{e^{4a+4bx}}{32b} - \frac{x}{8}$$

output
```
-1/16*exp(-2*b*x-2*a)/b-1/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b-1/8*x
```

3.902.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int e^{a+bx} \cosh(a+bx) \sinh^2(a+bx) \, dx = -\frac{2e^{-2(a+bx)} + 2e^{2(a+bx)} - e^{4(a+bx)} + 4bx}{32b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
-1/32*(2/E^(2*(a + b*x)) + 2*E^(2*(a + b*x)) - E^(4*(a + b*x)) + 4*b*x)/b
```

3.902.3 Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2720, 27, 354, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^2(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{8}e^{-3a-3bx}\left(1-e^{2a+2bx}\right)^2\left(1+e^{2a+2bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int e^{-3a-3bx}\left(1-e^{2a+2bx}\right)^2\left(1+e^{2a+2bx}\right)de^{a+bx}}{8b}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int e^{-2a-2bx}\left(1-e^{2a+2bx}\right)^2\left(1+e^{2a+2bx}\right)de^{2a+2bx}}{16b}$$

$$\Big\downarrow \text{84}$$

$$\frac{\int \left(-1+e^{-2a-2bx}-e^{-a-bx}+e^{2a+2bx}\right)de^{2a+2bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-e^{-a-bx}-\frac{1}{2}e^{2a+2bx}-\log\left(e^{2a+2bx}\right)}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(-E^(-a - b*x) - E^(2*a + 2*b*x)/2 - Log[E^(2*a + 2*b*x)])/(16*b)
```

3.902.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.902.4 Maple [A] (verified)

Time = 19.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{{\mathrm e}^{-2 b x -2 a}}{16 b}-\frac{{\mathrm e}^{2 b x +2 a}}{16 b}+\frac{{\mathrm e}^{4 b x +4 a}}{32 b}-\frac{x}{8}$	47
derivativedivides	$\frac{\frac{\sinh \left(b x +a \right)^{4}}{4}+\frac{\cosh \left(b x +a \right)^{3} \sinh \left(b x +a \right)}{4}-\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{8}-\frac{b x}{8}-\frac{a}{8}}{b}$	53
default	$\frac{\frac{\sinh \left(b x +a \right)^{4}}{4}+\frac{\cosh \left(b x +a \right)^{3} \sinh \left(b x +a \right)}{4}-\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{8}-\frac{b x}{8}-\frac{a}{8}}{b}$	53

input
```
int(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*exp(-2*b*x-2*a)/b-1/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b-1/8*x
```

3.902.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.67

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{\cosh(bx+a)^3+3\cosh(bx+a)\sinh(bx+a)^2-3\sinh(bx+a)^3+2(2bx+1)\cosh(bx+a)-(4bx}{32(b\cosh(bx+a)-b\sinh(bx+a))}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/32*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 3*sinh(b*x + a)
^3 + 2*(2*b*x + 1)*cosh(b*x + a) - (4*b*x + 9*cosh(b*x + a)^2 - 2)*sinh(b*
x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

3.902.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(44) = 88$.

Time = 0.93 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.11

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{xe^ae^{bx}\sinh^3(a+bx)}{8}+\frac{xe^ae^{bx}\sinh^2(a+bx)\cosh(a+bx)}{8}+\frac{xe^ae^{bx}\sinh(a+bx)\cosh^2(a+bx)}{8}-\frac{xe^ae^{bx}\cosh^3(a+bx)}{8}+\frac{3e^ae^{bx}\sinh^3(}{8b} \\ xe^a\sinh^2(a)\cosh(a) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 + x*exp(a)*exp(b*x)*sinh(
a + b*x)**2*cosh(a + b*x)/8 + x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x
)**2/8 - x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 + 3*exp(a)*exp(b*x)*sinh(a +
 b*x)**3/(8*b) - exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(4*b) + ex
p(a)*exp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(a)**2*cosh
(a), True))
```

3.902.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{8}\,x - \frac{a}{8\,b} + \frac{e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)}}{32\,b} - \frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*x - 1/8*a/b + 1/32*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a))/b - 1/16*e^(
-2*b*x - 2*a)/b
```

3.902.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{4\,bx - 2\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 4\,a - e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+2\,a)}}{32\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/32*(4*b*x - 2*(e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 4*a - e^(4*b*x +
 4*a) + 2*e^(2*b*x + 2*a))/b
```

3.902.9 Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{x}{8} - \frac{\frac{e^{-2\,a-2\,b\,x}}{16} + \frac{e^{2\,a+2\,b\,x}}{16} - \frac{e^{4\,a+4\,b\,x}}{32}}{b}$$

input
```
int(cosh(a + b*x)*exp(a + b*x)*sinh(a + b*x)^2,x)
```

output
```
- x/8 - (exp(- 2*a - 2*b*x)/16 + exp(2*a + 2*b*x)/16 - exp(4*a + 4*b*x)/32
)/b
```

3.903 $\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx$

3.903.1 Optimal result

Integrand size = 20, antiderivative size = 35

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx=\frac{e^{-a-bx}}{4b}+\frac{e^{3a+3bx}}{12b}$$

output
```
1/4*exp(-b*x-a)/b+1/12*exp(3*b*x+3*a)/b
```

3.903.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx=\frac{e^{-a-bx}\left(3+e^{4(a+bx)}\right)}{12b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
(E^(-a - b*x)*(3 + E^(4*(a + b*x))))/(12*b)
```

3.903.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int -\frac{1}{4}e^{-2a-2bx}\left(1-e^{4a+4bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int e^{-2a-2bx}\left(1-e^{4a+4bx}\right)de^{a+bx}}{4b}$$

$$\Big\downarrow \text{ 802}$$

$$-\frac{\int \left(e^{-2a-2bx}-e^{2a+2bx}\right)de^{a+bx}}{4b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^{-a-bx}+\frac{1}{3}e^{3a+3bx}}{4b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
(E^(-a - b*x) + E^(3*a + 3*b*x)/3)/(4*b)
```

3.903.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.903.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^3}{3}+\frac{\cosh(bx+a)^3}{3}}{b}$	26
default	$\frac{\frac{\sinh(bx+a)^3}{3}+\frac{\cosh(bx+a)^3}{3}}{b}$	26
risch	$\frac{e^{-bx-a}}{4b}+\frac{e^{3bx+3a}}{12b}$	30

input
```
int(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*sinh(b*x+a)^3+1/3*cosh(b*x+a)^3)
```

3.903.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^2-\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2}{3\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/3*(cosh(b*x + a)^2 - cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*c
osh(b*x + a) - b*sinh(b*x + a))
```

3.903.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(24) = 48.

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.17

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{e^a e^{bx}\sinh^2(a+bx)}{3b} - \frac{e^a e^{bx}\sinh(a+bx)\cosh(a+bx)}{3b} + \frac{e^a e^{bx}\cosh^2(a+bx)}{3b} & \text{for } b \neq 0 \\ xe^a\sinh(a)\cosh(a) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x)
```

output
```
Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) - exp(a)*exp(b*x)*sinh(a
 + b*x)*cosh(a + b*x)/(3*b) + exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b), Ne(b
, 0)), (x*exp(a)*sinh(a)*cosh(a), True))
```

3.903.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{12\,b} + \frac{e^{(-bx-a)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/12*e^(3*b*x + 3*a)/b + 1/4*e^(-b*x - a)/b
```

3.903.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)} + 3\,e^{(-bx-a)}}{12\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/12*(e^(3*b*x + 3*a) + 3*e^(-b*x - a))/b
```

3.903.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{3\,\mathrm{e}^{-a-b\,x}+\mathrm{e}^{3\,a+3\,b\,x}}{12\,b}$$

input
```
int(cosh(a + b*x)*exp(a + b*x)*sinh(a + b*x),x)
```

output
```
(3*exp(- a - b*x) + exp(3*a + 3*b*x))/(12*b)
```

3.904 $\int e^{a+bx}\coth(a+bx)\,dx$

3.904.1 Optimal result

Integrand size = 14, antiderivative size = 25

$$\int e^{a+bx}\coth(a+bx)\,dx=\frac{e^{a+bx}}{b}-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

output
```
exp(b*x+a)/b-2*arctanh(exp(b*x+a))/b
```

3.904.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\coth(a+bx)\,dx=\frac{e^{a+bx}-2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Coth[a + b*x],x]
```

output
```
(E^(a + b*x) - 2*ArcTanh[E^(a + b*x)])/b
```

3.904.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2720, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \coth(a+bx) \, dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int -\frac{1+e^{2a+2bx}}{1-e^{2a+2bx}}de^{a+bx}}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{1+e^{2a+2bx}}{1-e^{2a+2bx}}de^{a+bx}}{b}$$

$$\Big\downarrow \; 299$$

$$\frac{e^{a+bx}-2\int \frac{1}{1-e^{2a+2bx}}de^{a+bx}}{b}$$

$$\Big\downarrow \; 219$$

$$\frac{e^{a+bx}-2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[a + b*x],x]
```

output
```
(E^(a + b*x) - 2*ArcTanh[E^(a + b*x)])/b
```

3.904.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.904.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\sinh(bx+a)+\cosh(bx+a)-2\ \mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	27
default	$\frac{\sinh(bx+a)+\cosh(bx+a)-2\ \mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	27
risch	$\frac{\mathrm{e}^{bx+a}}{b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}$	39

input
```
int(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x+a)+cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

3.904.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(23) = 46.

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int e^{a+bx}\coth(a+bx)\,dx$$
$$=\frac{\cosh\left(bx+a\right)-\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)+\sinh\left(b\right.}{b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

output
```
(cosh(b*x + a) - log(cosh(b*x + a) + sinh(b*x + a) + 1) + log(cosh(b*x + a
) + sinh(b*x + a) - 1) + sinh(b*x + a))/b
```

3.904.6 Sympy [F]

$$\int e^{a+bx}\coth(a+bx)\,dx = e^{a}\int e^{bx}\cosh\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x)
```

output
```
exp(a)*Integral(exp(b*x)*cosh(a + b*x)*csch(a + b*x), x)
```

3.904.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int e^{a+bx}\coth(a+bx)\,dx = \frac{e^{(bx+a)}}{b} - \frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

output
```
e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

3.904.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int e^{a+bx}\coth(a+bx)\,dx = \frac{e^{(bx+a)} - \log\left(e^{(bx+a)}+1\right) + \log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

output
```
(e^(b*x + a) - log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) - 1)))/b
```

3.904.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int e^{a+bx}\coth(a+bx)\,dx=\frac{\mathrm{e}^{a+b\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

input
```
int((cosh(a + b*x)*exp(a + b*x))/sinh(a + b*x),x)
```

output
```
exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

3.905 $\int e^{a+bx}\coth(a+bx)\mathbf{csch}(a+bx)\,dx$

3.905.1 Optimal result

Integrand size = 20, antiderivative size = 41

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx=\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

output
```
2/b/(1-exp(2*b*x+2*a))+ln(1-exp(2*b*x+2*a))/b
```

3.905.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx=\frac{-\frac{2}{-1+e^{2(a+bx)}}+\log\left(1-e^{2(a+bx)}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
(-2/(-1 + E^(2*(a + b*x))) + Log[1 - E^(2*(a + b*x))])/b
```

3.905.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{2e^{a+bx}(1+e^{2a+2bx})}{(1-e^{2a+2bx})^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\int \frac{e^{a+bx}(1+e^{2a+2bx})}{(1-e^{2a+2bx})^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{ 353}$$

$$\frac{\int \frac{1+e^{2a+2bx}}{(1-e^{2a+2bx})^2}de^{2a+2bx}}{b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(\frac{1}{-1+e^{2a+2bx}}+\frac{2}{(-1+e^{2a+2bx})^2}\right)de^{2a+2bx}}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2}{1-e^{2a+2bx}}+\log\left(1-e^{2a+2bx}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
(2/(1 - E^(2*a + 2*b*x)) + Log[1 - E^(2*a + 2*b*x)])/b
```

3.905.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.905.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$\frac{\ln(\sinh(bx+a))+bx+a-\coth(bx+a)}{b}$	25
default	$\frac{\ln(\sinh(bx+a))+bx+a-\coth(bx+a)}{b}$	25
risch	$-\frac{2a}{b}-\frac{2}{b(e^{2bx+2a}-1)}+\frac{\ln(e^{2bx+2a}-1)}{b}$	42

input
```
int(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(ln(sinh(b*x+a))+b*x+a-coth(b*x+a))
```

3.905.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(37) = 74.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.51

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx = \frac{\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2-1\right)\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-2}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2-b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*l
og(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - 2)/(b*cosh(b*x + a)^
2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

3.905.6 Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.905.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx = \frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}-\frac{2}{b(e^{(2\,bx+2\,a)}-1)}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2/(b*(e^(2*b*x + 2*a) -
1))
```

3.905.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - log(abs(e^(2*b*x + 2*a) -
1)))/b
```

3.905.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx = \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b}-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int((cosh(a + b*x)*exp(a + b*x))/sinh(a + b*x)^2,x)
```

output
```
log(exp(2*a)*exp(2*b*x) - 1)/b - 2/(b*(exp(2*a + 2*b*x) - 1))
```

3.906 $\int e^{a+bx}\coth(a+bx)\mathrm{csch}^2(a+bx)\,dx$

3.906.1 Optimal result

Integrand size = 22, antiderivative size = 70

$$\int e^{a+bx}\coth(a+bx)\mathrm{csch}^2(a+bx)\,dx=-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{\operatorname{arctanh}\left(e^{a+bx}\right)}{b}$$

output
```
-2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-arc
tanh(exp(b*x+a))/b
```

3.906.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\coth(a+bx)\mathrm{csch}^2(a+bx)\,dx=\frac{e^{a+bx}-3e^{3(a+bx)}-\left(-1+e^{2(a+bx)}\right)^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b\left(-1+e^{2(a+bx)}\right)^2}$$

input
```
Integrate[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
(E^(a + b*x) - 3*E^(3*(a + b*x)) - (-1 + E^(2*(a + b*x)))^2*ArcTanh[E^(a +
 b*x)])/(b*(-1 + E^(2*(a + b*x)))^2)
```

3.906.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2720, 27, 360, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{4e^{2a+2bx}\left(1+e^{2a+2bx}\right)}{\left(1-e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\int \frac{e^{2a+2bx}\left(1+e^{2a+2bx}\right)}{\left(1-e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{360}$$

$$-\frac{4\left(\frac{e^{a+bx}}{2\left(1-e^{2a+2bx}\right)^2}-\frac{1}{4}\int \frac{2\left(1+2e^{2a+2bx}\right)}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\left(\frac{e^{a+bx}}{2\left(1-e^{2a+2bx}\right)^2}-\frac{1}{2}\int \frac{1+2e^{2a+2bx}}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{298}$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-e^{2a+2bx}}de^{a+bx}-\frac{3e^{a+bx}}{2\left(1-e^{2a+2bx}\right)}\right)+\frac{e^{a+bx}}{2\left(1-e^{2a+2bx}\right)^2}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{1}{2}\text{arctanh}\left(e^{a+bx}\right)-\frac{3e^{a+bx}}{2\left(1-e^{2a+2bx}\right)}\right)+\frac{e^{a+bx}}{2\left(1-e^{2a+2bx}\right)^2}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

3.906. $\int e^{a+bx}\coth(a+bx)\text{csch}^2(a+bx)\,dx$

output
```
(-4*(E^(a + b*x)/(2*(1 - E^(2*a + 2*b*x))^2) + ((-3*E^(a + b*x))/(2*(1 - E
^(2*a + 2*b*x))) + ArcTanh[E^(a + b*x)]/2)/2))/b
```

3.906.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.906.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)}-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\operatorname{csch}(bx+a)}{2}-\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	55
default	$\frac{-\frac{1}{\sinh(bx+a)}-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\operatorname{csch}(bx+a)}{2}-\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	55
risch	$-\frac{e^{bx+a}\left(3\,e^{2bx+2a}-1\right)}{b\left(e^{2bx+2a}-1\right)^2}-\frac{\ln\left(e^{bx+a}+1\right)}{2b}+\frac{\ln\left(e^{bx+a}-1\right)}{2b}$	67

input
```
int(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)-cosh(b*x+a)/sinh(b*x+a)^2+1/2*coth(b*x+a)*csch(b*x+a)-
arctanh(exp(b*x+a)))
```

3.906.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(61) = 122.

Time = 0.25 (sec) , antiderivative size = 387, normalized size of antiderivative = 5.53

$$\int e^{a+bx}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx =$$
$$-\frac{6\cosh(bx+a)^3+18\cosh(bx+a)\sinh(bx+a)^2+6\sinh(bx+a)^3+\left(\cosh(bx+a)^4+4\cosh(bx+\right.}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(6*cosh(b*x + a)^3 + 18*cosh(b*x + a)*sinh(b*x + a)^2 + 6*sinh(b*x +
a)^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cos
h(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(
b*x + a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(
b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a
) + sinh(b*x + a) - 1) + 2*(9*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 2*cosh(
b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(
b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x +
a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

3.906.6 Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\coth(a+bx)\text{csch}^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.906.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int e^{a+bx}\coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\log\left(e^{(bx+a)}+1\right)}{2\,b}+\frac{\log\left(e^{(bx+a)}-1\right)}{2\,b} - \frac{3\,e^{(3\,bx+3\,a)}-e^{(bx+a)}}{b\left(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1\right)}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*log(e^(b*x + a) + 1)/b + 1/2*log(e^(b*x + a) - 1)/b - (3*e^(3*b*x + 3
*a) - e^(b*x + a))/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

3.906.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\frac{2\left(3\,e^{(3\,bx+3\,a)}-e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2}+\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*(2*(3*e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 + log(e^
(b*x + a) + 1) - log(abs(e^(b*x + a) - 1)))/b
```

3.906.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.46

$$\int e^{a+bx}\coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\text{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{\frac{e^{a+b\,x}}{b}+\frac{e^{3\,a+3\,b\,x}}{b}}{e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int((cosh(a + b*x)*exp(a + b*x))/sinh(a + b*x)^3,x)
```

output
```
- atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (exp(a + b*x)/b +
exp(3*a + 3*b*x)/b)/(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1) - (2*exp(a
 + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.907 $\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$

3.907.1 Optimal result

Integrand size = 24, antiderivative size = 91

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx=\frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}-\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}+\frac{x}{16}$$

output
```
1/128*exp(-4*b*x-4*a)/b-1/64*exp(-2*b*x-2*a)/b-1/32*exp(2*b*x+2*a)/b-1/128
*exp(4*b*x+4*a)/b+1/192*exp(6*b*x+6*a)/b+1/16*x
```

3.907.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx=\frac{3e^{-4(a+bx)}-6e^{-2(a+bx)}-12e^{2(a+bx)}-3e^{4(a+bx)}+2e^{6(a+bx)}+24bx}{384b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
(3/E^(4*(a + b*x)) - 6/E^(2*(a + b*x)) - 12*E^(2*(a + b*x)) - 3*E^(4*(a +
b*x)) + 2*E^(6*(a + b*x)) + 24*b*x)/(384*b)
```

3.907.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2720, 27, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^3(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int -\frac{1}{32}e^{-5a-5bx}\left(1-e^{2a+2bx}\right)^3\left(1+e^{2a+2bx}\right)^2de^{a+bx}}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int e^{-5a-5bx}\left(1-e^{2a+2bx}\right)^3\left(1+e^{2a+2bx}\right)^2de^{a+bx}}{32b}$$

$$\Big\downarrow \ 354$$

$$-\frac{\int e^{-3a-3bx}\left(1-e^{2a+2bx}\right)^3\left(1+e^{2a+2bx}\right)^2de^{2a+2bx}}{64b}$$

$$\Big\downarrow \ 99$$

$$-\frac{\int \left(2+e^{-3a-3bx}-e^{-2a-2bx}-2e^{-a-bx}\right)de^{2a+2bx}}{64b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{1}{2}e^{-2a-2bx}+e^{-a-bx}+\frac{5}{2}e^{2a+2bx}-\frac{1}{3}e^{3a+3bx}-2\log\left(e^{2a+2bx}\right)}{64b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
-1/64*(-1/2*E^(-2*a - 2*b*x) + E^(-a - b*x) + (5*E^(2*a + 2*b*x))/2 - E^(3
*a + 3*b*x)/3 - 2*Log[E^(2*a + 2*b*x)])/b
```

3.907.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.907.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)^3}{6}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{8}+\frac{\cosh(bx+a)\sinh(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}+\frac{\cosh(bx+a)^4\sinh(bx+a)^2}{6}-\frac{\cosh(bx+a)^4}{12}}{b}$$

input
```
int(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

output
```
1/b*(1/6*cosh(b*x+a)^3*sinh(b*x+a)^3-1/8*cosh(b*x+a)^3*sinh(b*x+a)+1/16*co
sh(b*x+a)*sinh(b*x+a)+1/16*b*x+1/16*a+1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12
*cosh(b*x+a)^4)
```

3.907.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(74) = 148.

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.84

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{5\cosh(bx+a)^5+25\cosh(bx+a)\sinh(bx+a)^4-\sinh(bx+a)^5-\left(10\cosh(bx+a)^2-3\right)\sinh(bx+a)^3}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/384*(5*cosh(b*x + a)^5 + 25*cosh(b*x + a)*sinh(b*x + a)^4 - sinh(b*x + a
)^5 - (10*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^3 - 9*cosh(b*x + a)^3 + (50*c
osh(b*x + a)^3 - 27*cosh(b*x + a))*sinh(b*x + a)^2 + 12*(2*b*x - 1)*cosh(b
*x + a) - (5*cosh(b*x + a)^4 + 24*b*x - 9*cosh(b*x + a)^2 + 12)*sinh(b*x +
 a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

3.907.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(73) = 146.

Time = 5.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.23

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \begin{cases} -\frac{xe^ae^{bx}\sinh^5(a+bx)}{16}+\frac{xe^ae^{bx}\sinh^4(a+bx)\cosh(a+bx)}{16}+\frac{xe^ae^{bx}\sinh^3(a+bx)\cosh^2(a+bx)}{8}-\frac{xe^ae^{bx}\sinh^2(a+bx)\cosh^3(a+bx)}{8}- \\ xe^a\sinh^3(a)\cosh^2(a) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)**5/16 + x*exp(a)*exp(b*x)*sinh
(a + b*x)**4*cosh(a + b*x)/16 + x*exp(a)*exp(b*x)*sinh(a + b*x)**3*cosh(a
+ b*x)**2/8 - x*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/8 - x*ex
p(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**4/16 + x*exp(a)*exp(b*x)*cosh(a
 + b*x)**5/16 - exp(a)*exp(b*x)*sinh(a + b*x)**5/(32*b) + 3*exp(a)*exp(b*x
)*sinh(a + b*x)**4*cosh(a + b*x)/(32*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2
*cosh(a + b*x)**3/(6*b) - exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**4/(
96*b) - 5*exp(a)*exp(b*x)*cosh(a + b*x)**5/(96*b), Ne(b, 0)), (x*exp(a)*si
nh(a)**3*cosh(a)**2, True))
```

3.907.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(2\,e^{(2\,bx+2\,a)}-1\right)e^{(-4\,bx-4\,a)}}{128\,b}+\frac{bx+a}{16\,b}+\frac{2\,e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}}{384\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/128*(2*e^(2*b*x + 2*a) - 1)*e^(-4*b*x - 4*a)/b + 1/16*(b*x + a)/b + 1/3
84*(2*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a))/b
```

3.907.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{24\,bx-3\left(6\,e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}-1\right)e^{(-4\,bx-4\,a)}+24\,a+2\,e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}}{384\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/384*(24*b*x - 3*(6*e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) - 1)*e^(-4*b*x -
4*a) + 24*a + 2*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a))/
b
```

3.907.9 Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = -\frac{6\,\mathrm{e}^{-2\,a-2\,b\,x}+12\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{-4\,a-4\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{6\,a+6\,b\,x}-24\,b\,x}{384\,b}$$

input
```
int(cosh(a + b*x)^2*exp(a + b*x)*sinh(a + b*x)^3,x)
```

output
```
-(6*exp(- 2*a - 2*b*x) + 12*exp(2*a + 2*b*x) - 3*exp(- 4*a - 4*b*x) + 3*ex
p(4*a + 4*b*x) - 2*exp(6*a + 6*b*x) - 24*b*x)/(384*b)
```

3.907. $\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$

3.908 $\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$

3.908.1 Optimal result

Integrand size = 24, antiderivative size = 49

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx=-\frac{e^{-3a-3bx}}{48b}-\frac{e^{a+bx}}{8b}+\frac{e^{5a+5bx}}{80b}$$

output
```
-1/48*exp(-3*b*x-3*a)/b-1/8*exp(b*x+a)/b+1/80*exp(5*b*x+5*a)/b
```

3.908.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx=\frac{e^{-3(a+bx)}\left(-5-30e^{4(a+bx)}+3e^{8(a+bx)}\right)}{240b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(-5 - 30*E^(4*(a + b*x)) + 3*E^(8*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

3.908.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^2(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{1}{16}e^{-4a-4bx}\left(1-e^{4a+4bx}\right)^2de^{a+bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int e^{-4a-4bx}\left(1-e^{4a+4bx}\right)^2de^{a+bx}}{16b}$$

$$\Big\downarrow \; 802$$

$$\frac{\int \left(-2+e^{-4a-4bx}+e^{4a+4bx}\right)de^{a+bx}}{16b}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{3}e^{-3a-3bx}-2e^{a+bx}+\frac{1}{5}e^{5a+5bx}}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(-1/3*E^(-3*a - 3*b*x) - 2*E^(a + b*x) + E^(5*a + 5*b*x)/5)/(16*b)
```

3.908.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.908.4 Maple [A] (verified)

Time = 96.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{e^{-3bx-3a}}{48b}-\frac{e^{bx+a}}{8b}+\frac{e^{5bx+5a}}{80b}$	41
derivativedivides	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}+\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}}{b}$	70
default	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}+\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}}{b}$	70

input
```
int(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/48*exp(-3*b*x-3*a)/b-1/8*exp(b*x+a)/b+1/80*exp(5*b*x+5*a)/b
```

3.908.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(40) = 80.

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.84

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{\cosh(bx+a)^4-16\cosh(bx+a)^3\sinh(bx+a)+6\cosh(bx+a)^2\sinh(bx+a)^2-16\cosh(bx+a)\sin}{120\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x +
a)^2*sinh(b*x + a)^2 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
+ 15)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

3.908.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(37) = 74.

Time = 2.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.94

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$$
$$=\begin{cases}-\frac{2e^ae^{bx}\sinh^4(a+bx)}{15b}+\frac{2e^ae^{bx}\sinh^3(a+bx)\cosh(a+bx)}{15b}+\frac{e^ae^{bx}\sinh^2(a+bx)\cosh^2(a+bx)}{5b}+\frac{2e^ae^{bx}\sinh(a+bx)\cosh^3(a+bx)}{15b}-2e\\ xe^a\sinh^2(a)\cosh^2(a)\end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) + 2*exp(a)*exp(b*x)*
sinh(a + b*x)**3*cosh(a + b*x)/(15*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*c
osh(a + b*x)**2/(5*b) + 2*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(
15*b) - 2*exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*si
nh(a)**2*cosh(a)**2, True))
```

3.908.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx=\frac{e^{(5\,bx+5\,a)}-10\,e^{(bx+a)}}{80\,b}-\frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/80*(e^(5*b*x + 5*a) - 10*e^(b*x + a))/b - 1/48*e^(-3*b*x - 3*a)/b
```

3.908.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{3\,e^{(5\,bx+5\,a)} - 30\,e^{(bx+a)} - 5\,e^{(-3\,bx-3\,a)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/240*(3*e^(5*b*x + 5*a) - 30*e^(b*x + a) - 5*e^(-3*b*x - 3*a))/b
```

3.908.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{30\,\mathrm{e}^{a+b\,x} + 5\,\mathrm{e}^{-3\,a-3\,b\,x} - 3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

input
```
int(cosh(a + b*x)^2*exp(a + b*x)*sinh(a + b*x)^2,x)
```

output
```
-(30*exp(a + b*x) + 5*exp(- 3*a - 3*b*x) - 3*exp(5*a + 5*b*x))/(240*b)
```

3.909 $\int e^{a+bx} \cosh^2(a+bx) \sinh(a+bx) \, dx$

3.909.1 Optimal result

Integrand size = 22, antiderivative size = 57

$$\int e^{a+bx} \cosh^2(a+bx) \sinh(a+bx) \, dx = \frac{e^{-2a-2bx}}{16b} + \frac{e^{2a+2bx}}{16b} + \frac{e^{4a+4bx}}{32b} - \frac{x}{8}$$

output
```
1/16*exp(-2*b*x-2*a)/b+1/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b-1/8*x
```

3.909.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int e^{a+bx} \cosh^2(a+bx) \sinh(a+bx) \, dx = \frac{2e^{-2(a+bx)} + 2e^{2(a+bx)} + e^{4(a+bx)} - 4bx}{32b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
(2/E^(2*(a + b*x)) + 2*E^(2*(a + b*x)) + E^(4*(a + b*x)) - 4*b*x)/(32*b)
```

3.909.3 Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2720, 27, 354, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{8}e^{-3a-3bx}\left(1-e^{2a+2bx}\right)\left(1+e^{2a+2bx}\right)^2de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int e^{-3a-3bx}\left(1-e^{2a+2bx}\right)\left(1+e^{2a+2bx}\right)^2de^{a+bx}}{8b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int e^{-2a-2bx}\left(1-e^{2a+2bx}\right)\left(1+e^{2a+2bx}\right)^2de^{2a+2bx}}{16b}$$

$$\Big\downarrow \text{84}$$

$$-\frac{\int \left(-1+e^{-2a-2bx}+e^{-a-bx}-e^{2a+2bx}\right)de^{2a+2bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-e^{-a-bx}-\frac{3}{2}e^{2a+2bx}+\log\left(e^{2a+2bx}\right)}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
-1/16*(-E^(-a - b*x) - (3*E^(2*a + 2*b*x))/2 + Log[E^(2*a + 2*b*x)])/b
```

3.909.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.909.4 Maple [A] (verified)

Time = 10.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{e^{-2bx-2a}}{16b}+\frac{e^{2bx+2a}}{16b}+\frac{e^{4bx+4a}}{32b}-\frac{x}{8}$	47
derivativedivides	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}+\frac{\cosh(bx+a)^4}{4}}{b}$	53
default	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}+\frac{\cosh(bx+a)^4}{4}}{b}$	53

input
```
int(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/16*exp(-2*b*x-2*a)/b+1/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b-1/8*x
```

3.909.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(46) = 92.

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

$$\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)^3+9\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3-2(2bx-1)\cosh(bx+a)+(4bx-3}{32(b\cosh(bx+a)-b\sinh(bx+a))}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/32*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x + a)^
3 - 2*(2*b*x - 1)*cosh(b*x + a) + (4*b*x - 3*cosh(b*x + a)^2 + 2)*sinh(b*x
 + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

3.909.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(44) = 88.

Time = 0.88 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.07

$$\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$=\begin{cases}-\frac{xe^ae^{bx}\sinh^3(a+bx)}{8}+\frac{xe^ae^{bx}\sinh^2(a+bx)\cosh(a+bx)}{8}+\frac{xe^ae^{bx}\sinh(a+bx)\cosh^2(a+bx)}{8}-\frac{xe^ae^{bx}\cosh^3(a+bx)}{8}-\frac{e^ae^{bx}\sinh^3(a}{8b} \\ xe^a\sinh(a)\cosh^2(a)\end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 + x*exp(a)*exp(b*x)*sinh(
a + b*x)**2*cosh(a + b*x)/8 + x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x
)**2/8 - x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - exp(a)*exp(b*x)*sinh(a + b
*x)**3/(8*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(4*b) + exp(
a)*exp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(a)*cosh(a)**
2, True))
```

3.909.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx = -\frac{1}{8}\,x - \frac{a}{8\,b} + \frac{e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+2\,a)}}{32\,b} + \frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-1/8*x - 1/8*a/b + 1/32*(e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a))/b + 1/16*e^(
-2*b*x - 2*a)/b
```

3.909.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx = -\frac{4\,bx - 2\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)} + 4\,a - e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)}}{32\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
-1/32*(4*b*x - 2*(e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 4*a - e^(4*b*x +
 4*a) - 2*e^(2*b*x + 2*a))/b
```

3.909.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\frac{e^{-2\,a-2\,b\,x}}{16} + \frac{e^{2\,a+2\,b\,x}}{16} + \frac{e^{4\,a+4\,b\,x}}{32}}{b} - \frac{x}{8}$$

input
```
int(cosh(a + b*x)^2*exp(a + b*x)*sinh(a + b*x),x)
```

output
```
(exp(- 2*a - 2*b*x)/16 + exp(2*a + 2*b*x)/16 + exp(4*a + 4*b*x)/32)/b - x/
8
```

3.910 $\int e^{a+bx} \cosh(a+bx) \coth(a+bx) \, dx$

3.910.1 Optimal result

Integrand size = 20, antiderivative size = 42

$$\int e^{a+bx} \cosh(a+bx) \coth(a+bx) \, dx = \frac{e^{2a+2bx}}{4b} - \frac{x}{2} + \frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

output
```
1/4*exp(2*b*x+2*a)/b-1/2*x+ln(1-exp(2*b*x+2*a))/b
```

3.910.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int e^{a+bx} \cosh(a+bx) \coth(a+bx) \, dx = \frac{e^{2(a+bx)} - 2bx + 4\log\left(1-e^{2(a+bx)}\right)}{4b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
(E^(2*(a + b*x)) - 2*b*x + 4*Log[1 - E^(2*(a + b*x))])/(4*b)
```

3.910.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{-a-bx}\left(1+e^{2a+2bx}\right)^2}{2\left(1-e^{2a+2bx}\right)}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-a-bx}\left(1+e^{2a+2bx}\right)^2}{1-e^{2a+2bx}}de^{a+bx}}{2b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{e^{-a-bx}\left(1+e^{2a+2bx}\right)^2}{1-e^{2a+2bx}}de^{2a+2bx}}{4b}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\int \left(e^{-a-bx}-1-\frac{4}{-1+e^{2a+2bx}}\right)de^{2a+2bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-e^{2a+2bx}+\log\left(e^{2a+2bx}\right)-4\log\left(1-e^{2a+2bx}\right)}{4b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
-1/4*(-E^(2*a + 2*b*x) + Log[E^(2*a + 2*b*x)] - 4*Log[1 - E^(2*a + 2*b*x)]
)/b
```

3.910.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.910.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{x}{2}+\frac{{\mathrm e}^{2 b x +2 a}}{4 b}-\frac{2 a}{b}+\frac{\ln \left({\mathrm e}^{2 b x +2 a}-1\right)}{b}$	41
derivativedivides	$\frac{\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{2}+\frac{b x}{2}+\frac{a}{2}+\frac{\cosh \left(b x +a \right)^{2}}{2}+\ln \left(\sinh \left(b x +a \right)\right)}{b}$	44
default	$\frac{\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{2}+\frac{b x}{2}+\frac{a}{2}+\frac{\cosh \left(b x +a \right)^{2}}{2}+\ln \left(\sinh \left(b x +a \right)\right)}{b}$	44

input
```
int(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x+1/4*exp(2*b*x+2*a)/b-2/b*a+1/b*ln(exp(2*b*x+2*a)-1)
```

3.910.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.71

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx = -\frac{2\,bx-\cosh\left(bx+a\right)^{2}-2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)-\sinh\left(bx+a\right)^{2}-4\,\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(2*b*x - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x +
 a)^2 - 4*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

3.910.6 Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**2*csch(b*x+a),x)
```

output
```
Timed out
```

3.910.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx = -\frac{1}{2}\,x-\frac{a}{2\,b}+\frac{e^{(2\,bx+2\,a)}}{4\,b}+\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*x - 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b + log(e^(b*x + a) + 1)/b + log(e^
(b*x + a) - 1)/b
```

3.910.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx = -\frac{2\,bx+2\,a-e^{(2\,bx+2\,a)}-4\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

output
```
-1/4*(2*b*x + 2*a - e^(2*b*x + 2*a) - 4*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.910.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx = \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b}-\frac{x}{2}+\frac{\mathrm{e}^{2\,a+2\,b\,x}}{4\,b}$$

input
```
int((cosh(a + b*x)^2*exp(a + b*x))/sinh(a + b*x),x)
```

output
```
log(exp(2*a)*exp(2*b*x) - 1)/b - x/2 + exp(2*a + 2*b*x)/(4*b)
```

3.911 $\int e^{a+bx}\coth^2(a+bx)\,dx$

3.911.1 Optimal result

Integrand size = 16, antiderivative size = 53

$$\int e^{a+bx}\coth^2(a+bx)\,dx=\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

output
```
exp(b*x+a)/b+2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-2*arctanh(exp(b*x+a))/b
```

3.911.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.38

$$\int e^{a+bx}\coth^2(a+bx)\,dx$$
$$=\frac{e^{a+bx}\left(\frac{1}{48}e^{-4(a+bx)}\left(-375-713e^{2(a+bx)}-181e^{4(a+bx)}+61e^{6(a+bx)}+\frac{3\left(125+196e^{2(a+bx)}-14e^{4(a+bx)}-52e^{6(a+bx)}+e^{8(a}\right.}{\sqrt{e^{2(a+bx)}}}\right.\right.}{b}$$

input
```
Integrate[E^(a + b*x)*Coth[a + b*x]^2,x]
```

output
```
(E^(a + b*x)*((-375 - 713*E^(2*(a + b*x)) - 181*E^(4*(a + b*x)) + 61*E^(6*
(a + b*x)) + (3*(125 + 196*E^(2*(a + b*x)) - 14*E^(4*(a + b*x)) - 52*E^(6*
(a + b*x)) + E^(8*(a + b*x)))*ArcTanh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a
 + b*x))])/(48*E^(4*(a + b*x))) + (4*(E^(a + b*x) + E^(3*(a + b*x)))^2*Hyp
ergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, E^(2*(a + b*x))])/105))/b
```

3.911.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2720, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{(1+e^{2a+2bx})^2}{(1-e^{2a+2bx})^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(1+\frac{4e^{2a+2bx}}{(1-e^{2a+2bx})^2}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-2\text{arctanh}\left(e^{a+bx}\right)+e^{a+bx}+\frac{2e^{a+bx}}{1-e^{2a+2bx}}}{b}$$

input
```
Int[E^(a + b*x)*Coth[a + b*x]^2,x]
```

output
```
(E^(a + b*x) + (2*E^(a + b*x))/(1 - E^(2*a + 2*b*x)) - 2*ArcTanh[E^(a + b*
x)])/b
```

3.911.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.911.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\cosh(bx+a)-2\,\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)+\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	48
default	$\frac{\cosh(bx+a)-2\,\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)+\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	48
risch	$\frac{\mathrm{e}^{bx+a}}{b}-\frac{2\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}$	63

input
```
int(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a))+cosh(b*x+a)^2/sinh(b*x+a)-2/sinh(b*
x+a))
```

3.911.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(47) = 94.

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.74

$$\int e^{a+bx}\coth^2(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^3+3\,\cosh(bx+a)\sinh(bx+a)^2+\sinh(bx+a)^3-\left(\cosh(bx+a)^2+2\,\cosh(bx+a)\sinh\right.}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (co
sh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(c
osh(b*x + a) + sinh(b*x + a) + 1) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sin
h(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) +
 3*(cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)
^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

3.911.6 Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\coth^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.911.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\coth^2(a+bx)\,dx = \frac{e^{(bx+a)}}{b} - \frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b} - \frac{2\,e^{(bx+a)}}{b(e^{(2\,bx+2\,a)}-1)}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*e^(b*x
 + a)/(b*(e^(2*b*x + 2*a) - 1))
```

3.911.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int e^{a+bx}\coth^2(a+bx)\,dx = -\frac{\frac{2\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1} - e^{(bx+a)} + \log\left(e^{(bx+a)}+1\right) - \log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-(2*e^(b*x + a)/(e^(2*b*x + 2*a) - 1) - e^(b*x + a) + log(e^(b*x + a) + 1)
 - log(abs(e^(b*x + a) - 1)))/b
```

3.911.9 Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\coth^2(a+bx)\,dx = \frac{e^{a+b\,x}}{b} - \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int((cosh(a + b*x)^2*exp(a + b*x))/sinh(a + b*x)^2,x)
```

output
```
exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) -
 (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.912 $\int e^{a+bx}\coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

3.912.1 Optimal result

Integrand size = 22, antiderivative size = 62

$$\int e^{a+bx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx=-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{4}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

output
```
-2/b/(1-exp(2*b*x+2*a))^2+4/b/(1-exp(2*b*x+2*a))+ln(1-exp(2*b*x+2*a))/b
```

3.912.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx=\frac{\frac{2-4e^{2(a+bx)}}{\left(-1+e^{2(a+bx)}\right)^2}+\log\left(1-e^{2(a+bx)}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
((2 - 4*E^(2*(a + b*x)))/(-1 + E^(2*(a + b*x)))^2 + Log[1 - E^(2*(a + b*x)
)])/b
```

3.912.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2720, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth^2(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int -\frac{2e^{a+bx}\left(1+e^{2a+2bx}\right)^2}{\left(1-e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{2\int \frac{e^{a+bx}\left(1+e^{2a+2bx}\right)^2}{\left(1-e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \ 353$$

$$-\frac{\int \frac{\left(1+e^{2a+2bx}\right)^2}{\left(1-e^{2a+2bx}\right)^3}de^{2a+2bx}}{b}$$

$$\Big\downarrow \ 49$$

$$-\frac{\int \left(-\frac{4}{\left(-1+e^{2a+2bx}\right)^2}-\frac{4}{\left(-1+e^{2a+2bx}\right)^3}+\frac{1}{1-e^{2a+2bx}}\right)de^{2a+2bx}}{b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{4}{1-e^{2a+2bx}}+\frac{2}{\left(1-e^{2a+2bx}\right)^2}-\log\left(1-e^{2a+2bx}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
-((2/(1 - E^(2*a + 2*b*x))^2 - 4/(1 - E^(2*a + 2*b*x)) - Log[1 - E^(2*a + 
2*b*x)])/b)
```

3.912.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.912.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

method	result	size
derivativedivides	$\frac{bx+a-\coth(bx+a)+\ln(\sinh(bx+a))-\frac{\coth(bx+a)^2}{2}}{b}$	35
default	$\frac{bx+a-\coth(bx+a)+\ln(\sinh(bx+a))-\frac{\coth(bx+a)^2}{2}}{b}$	35
risch	$-\frac{2a}{b}-\frac{2\left(2\,\mathrm{e}^{2bx+2a}-1\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^2}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	55

input
```
int(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(b*x+a-coth(b*x+a)+ln(sinh(b*x+a))-1/2*coth(b*x+a)^2)
```

3.912.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 262, normalized size of antiderivative = 4.23

$$\int e^{a+bx}\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{4\cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+\right.\right.}{b\cosh(bx+a)^4 + 4b\cosh(bx+a)\sinh(bx+a)^3 + b\sin}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(4*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
 sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh
(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 8*cosh(b*x + a)*sinh(b*x + a)
 + 4*sinh(b*x + a)^2 - 2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x
+ a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2
- b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x +
a) + b)
```

3.912.6 Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\coth^2(a+bx)\text{csch}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.912.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int e^{a+bx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b} - \frac{2\left(2\,e^{(2\,bx+2\,a)}-1\right)}{b(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1)}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*(2*e^(2*b*x + 2*a) - 1
)/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

3.912.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int e^{a+bx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\frac{3\,e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}-1}{\left(e^{(2\,bx+2\,a)}-1\right)^2}-2\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{2\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*((3*e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) - 1)^2
- 2*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.912.9 Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int e^{a+bx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b}-\frac{4}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{2}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int((cosh(a + b*x)^2*exp(a + b*x))/sinh(a + b*x)^3,x)
```

output
```
log(exp(2*a)*exp(2*b*x) - 1)/b - 4/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(exp(
4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

3.913 $\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$

3.913.1 Optimal result

Integrand size = 24, antiderivative size = 69

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx=\frac{e^{-5a-5bx}}{320b}-\frac{3e^{-a-bx}}{64b}-\frac{e^{3a+3bx}}{64b}+\frac{e^{7a+7bx}}{448b}$$

output
```
1/320*exp(-5*b*x-5*a)/b-3/64*exp(-b*x-a)/b-1/64*exp(3*b*x+3*a)/b+1/448*exp
(7*b*x+7*a)/b
```

3.913.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx=\frac{e^{-5(a+bx)}\left(7-105e^{4(a+bx)}-35e^{8(a+bx)}+5e^{12(a+bx)}\right)}{2240b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(7 - 105*E^(4*(a + b*x)) - 35*E^(8*(a + b*x)) + 5*E^(12*(a + b*x)))/(2240*
b*E^(5*(a + b*x)))
```

3.913.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \sinh^3(a+bx) \cosh^3(a+bx) \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int -\frac{1}{64} e^{-6a-6bx} \left(1-e^{4a+4bx}\right)^3 de^{a+bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int e^{-6a-6bx} \left(1-e^{4a+4bx}\right)^3 de^{a+bx}}{64b}$$

$$\Big\downarrow \text{ 802}$$

$$-\frac{\int \left(e^{-6a-6bx}-3e^{-2a-2bx}+3e^{2a+2bx}-e^{6a+6bx}\right) de^{a+bx}}{64b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{5}e^{-5a-5bx}-3e^{-a-bx}-e^{3a+3bx}+\frac{1}{7}e^{7a+7bx}}{64b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(E^(-5*a - 5*b*x)/5 - 3*E^(-a - b*x) - E^(3*a + 3*b*x) + E^(7*a + 7*b*x)/7
)/(64*b)
```

3.913.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.913.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\frac{\frac{\cosh(bx+a)^4\sinh(bx+a)^3}{7}-\frac{3\sinh(bx+a)\cosh(bx+a)^4}{35}+\frac{3\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{35}+\frac{\sinh(bx+a)^2\cosh(bx+a)^5}{7}-\frac{2\cosh(bx+a)^5}{35}}{b}$$

input
```
int(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

output
```
1/b*(1/7*cosh(b*x+a)^4*sinh(b*x+a)^3-3/35*sinh(b*x+a)*cosh(b*x+a)^4+3/35*(
2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+1/7*sinh(b*x+a)^2*cosh(b*x+a)^5-2/35*co
sh(b*x+a)^5)
```

3.913.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(57) = 114.

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.23

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)^6-10\cosh(bx+a)^3\sinh(bx+a)^3+45\cosh(bx+a)^2\sinh(bx+a)^4-3\cosh(bx+a)\text{si}}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/560*(3*cosh(b*x + a)^6 - 10*cosh(b*x + a)^3*sinh(b*x + a)^3 + 45*cosh(b*
x + a)^2*sinh(b*x + a)^4 - 3*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x +
a)^6 + 5*(9*cosh(b*x + a)^4 - 7)*sinh(b*x + a)^2 - 35*cosh(b*x + a)^2 - (3
*cosh(b*x + a)^5 - 35*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a) - b*s
inh(b*x + a))
```

3.913.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(54) = 108$.

Time = 11.96 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.93

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{2e^ae^{bx}\sinh^6(a+bx)}{35b} + \frac{2e^ae^{bx}\sinh^5(a+bx)\cosh(a+bx)}{35b} + \frac{e^ae^{bx}\sinh^4(a+bx)\cosh^2(a+bx)}{7b} - \frac{e^ae^{bx}\sinh^3(a+bx)\cosh^3(a+bx)}{7b} + e^a \\ xe^a\sinh^3(a)\cosh^3(a) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**6/(35*b) + 2*exp(a)*exp(b*x)*
sinh(a + b*x)**5*cosh(a + b*x)/(35*b) + exp(a)*exp(b*x)*sinh(a + b*x)**4*c
osh(a + b*x)**2/(7*b) - exp(a)*exp(b*x)*sinh(a + b*x)**3*cosh(a + b*x)**3/
(7*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)**4/(7*b) + 2*exp(a)
*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**5/(35*b) - 2*exp(a)*exp(b*x)*cosh(a
 + b*x)**6/(35*b), Ne(b, 0)), (x*exp(a)*sinh(a)**3*cosh(a)**3, True))
```

3.913.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(15\,e^{(4\,bx+4\,a)}-1\right)e^{(-5\,bx-5\,a)}}{320\,b} + \frac{e^{(7\,bx+7\,a)}-7\,e^{(3\,bx+3\,a)}}{448\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/320*(15*e^(4*b*x + 4*a) - 1)*e^(-5*b*x - 5*a)/b + 1/448*(e^(7*b*x + 7*a
) - 7*e^(3*b*x + 3*a))/b
```

3.913.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$= -\frac{7\left(15\,e^{(4\,bx+4\,a)}-1\right)e^{(-5\,bx-5\,a)}-5\,e^{(7\,bx+7\,a)}+35\,e^{(3\,bx+3\,a)}}{2240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2240*(7*(15*e^(4*b*x + 4*a) - 1)*e^(-5*b*x - 5*a) - 5*e^(7*b*x + 7*a) +
 35*e^(3*b*x + 3*a))/b
```

3.913.9 Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{105\,\mathrm{e}^{-a-b\,x}+35\,\mathrm{e}^{3\,a+3\,b\,x}-7\,\mathrm{e}^{-5\,a-5\,b\,x}-5\,\mathrm{e}^{7\,a+7\,b\,x}}{2240\,b}$$

input
```
int(cosh(a + b*x)^3*exp(a + b*x)*sinh(a + b*x)^3,x)
```

output
```
-(105*exp(- a - b*x) + 35*exp(3*a + 3*b*x) - 7*exp(- 5*a - 5*b*x) - 5*exp(
7*a + 7*b*x))/(2240*b)
```

3.914 $\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$

3.914.1 Optimal result

Integrand size = 24, antiderivative size = 91

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx=-\frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}+\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}-\frac{x}{16}$$

output
```
-1/128*exp(-4*b*x-4*a)/b-1/64*exp(-2*b*x-2*a)/b-1/32*exp(2*b*x+2*a)/b+1/12
8*exp(4*b*x+4*a)/b+1/192*exp(6*b*x+6*a)/b-1/16*x
```

3.914.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx=-\frac{3e^{-4(a+bx)}+6e^{-2(a+bx)}+12e^{2(a+bx)}-3e^{4(a+bx)}-2e^{6(a+bx)}+24bx}{384b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
-1/384*(3/E^(4*(a + b*x)) + 6/E^(2*(a + b*x)) + 12*E^(2*(a + b*x)) - 3*E^(
4*(a + b*x)) - 2*E^(6*(a + b*x)) + 24*b*x)/b
```

3.914.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2720, 27, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^2(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{32}e^{-5a-5bx}\left(1-e^{2a+2bx}\right)^2\left(1+e^{2a+2bx}\right)^3de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int e^{-5a-5bx}\left(1-e^{2a+2bx}\right)^2\left(1+e^{2a+2bx}\right)^3de^{a+bx}}{32b}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int e^{-3a-3bx}\left(1-e^{2a+2bx}\right)^2\left(1+e^{2a+2bx}\right)^3de^{2a+2bx}}{64b}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(-2+e^{-3a-3bx}+e^{-2a-2bx}-2e^{-a-bx}+2e^{2a+2bx}\right)de^{2a+2bx}}{64b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}e^{-2a-2bx}-e^{-a-bx}-\frac{3}{2}e^{2a+2bx}+\frac{1}{3}e^{3a+3bx}-2\log\left(e^{2a+2bx}\right)}{64b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(-1/2*E^(-2*a - 2*b*x) - E^(-a - b*x) - (3*E^(2*a + 2*b*x))/2 + E^(3*a + 3
*b*x)/3 - 2*Log[E^(2*a + 2*b*x)])/(64*b)
```

3.914.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.914.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\frac{\frac{\cosh(bx+a)^4\sinh(bx+a)^2}{6}-\frac{\cosh(bx+a)^4}{12}+\frac{\cosh(bx+a)^5\sinh(bx+a)}{6}-\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{6}-\frac{bx}{16}-\frac{a}{16}}{b}$$

input
```
int(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

output
```
1/b*(1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*cosh(b*x+a)^4+1/6*cosh(b*x+a)^5*
sinh(b*x+a)-1/6*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)-1/16*b*x-1
/16*a)
```

3.914.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(74) = 148.

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.81

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx =$$
$$-\frac{\cosh(bx+a)^5+5\cosh(bx+a)\sinh(bx+a)^4-5\sinh(bx+a)^5-\left(50\cosh(bx+a)^2+9\right)\sinh(bx+$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/384*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 - 5*sinh(b*x + a
)^5 - (50*cosh(b*x + a)^2 + 9)*sinh(b*x + a)^3 + 3*cosh(b*x + a)^3 + (10*c
osh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x + a)^2 + 12*(2*b*x + 1)*cosh(b*
x + a) - (25*cosh(b*x + a)^4 + 24*b*x + 27*cosh(b*x + a)^2 - 12)*sinh(b*x
+ a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

3.914.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(73) = 146.

Time = 5.27 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.57

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\begin{cases}\frac{xe^ae^{bx}\sinh^5(a+bx)}{16}-\frac{xe^ae^{bx}\sinh^4(a+bx)\cosh(a+bx)}{16}-\frac{xe^ae^{bx}\sinh^3(a+bx)\cosh^2(a+bx)}{8}+\frac{xe^ae^{bx}\sinh^2(a+bx)\cosh^3(a+bx)}{8}+\frac{xe}{} \\ xe^a\sinh^2(a)\cosh^3(a)\end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x*exp(a)*exp(b*x)*sinh(a + b*x)**5/16 - x*exp(a)*exp(b*x)*sinh(
a + b*x)**4*cosh(a + b*x)/16 - x*exp(a)*exp(b*x)*sinh(a + b*x)**3*cosh(a +
 b*x)**2/8 + x*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/8 + x*exp
(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**4/16 - x*exp(a)*exp(b*x)*cosh(a
+ b*x)**5/16 - 13*exp(a)*exp(b*x)*sinh(a + b*x)**5/(96*b) + 7*exp(a)*exp(b
*x)*sinh(a + b*x)**4*cosh(a + b*x)/(96*b) + exp(a)*exp(b*x)*sinh(a + b*x)*
*3*cosh(a + b*x)**2/(3*b) - exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)
**3/(6*b) + exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**4/(96*b) + 5*exp(
a)*exp(b*x)*cosh(a + b*x)**5/(96*b), Ne(b, 0)), (x*exp(a)*sinh(a)**2*cosh(
a)**3, True))
```

3.914.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{\left(2\,e^{(2\,bx+2\,a)}+1\right)e^{(-4\,bx-4\,a)}}{128\,b} - \frac{bx+a}{16\,b} + \frac{2\,e^{(6\,bx+6\,a)}+3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}}{384\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/128*(2*e^(2*b*x + 2*a) + 1)*e^(-4*b*x - 4*a)/b - 1/16*(b*x + a)/b + 1/3
84*(2*e^(6*b*x + 6*a) + 3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a))/b
```

3.914.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{24\,bx-3\left(6\,e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}-1\right)e^{(-4\,bx-4\,a)}+24\,a-2\,e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}}{384\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/384*(24*b*x - 3*(6*e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) - 1)*e^(-4*b*x -
 4*a) + 24*a - 2*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a))
/b
```

3.914.9 Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{6\,\mathrm{e}^{-2\,a-2\,b\,x}+12\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{-4\,a-4\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{6\,a+6\,b\,x}+24\,b\,x}{384\,b}$$

input
```
int(cosh(a + b*x)^3*exp(a + b*x)*sinh(a + b*x)^2,x)
```

output
```
-(6*exp(- 2*a - 2*b*x) + 12*exp(2*a + 2*b*x) + 3*exp(- 4*a - 4*b*x) - 3*ex
p(4*a + 4*b*x) - 2*exp(6*a + 6*b*x) + 24*b*x)/(384*b)
```

3.915 $\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx$

3.915.1 Optimal result

Integrand size = 22, antiderivative size = 69

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{8b}+\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

output
```
1/48*exp(-3*b*x-3*a)/b+1/8*exp(-b*x-a)/b+1/24*exp(3*b*x+3*a)/b+1/80*exp(5*
b*x+5*a)/b
```

3.915.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{e^{-3(a+bx)}\left(5+30e^{2(a+bx)}+10e^{6(a+bx)}+3e^{8(a+bx)}\right)}{240b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
(5 + 30*E^(2*(a + b*x)) + 10*E^(6*(a + b*x)) + 3*E^(8*(a + b*x)))/(240*b*E
^(3*(a + b*x)))
```

3.915.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2720, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{16}e^{-4a-4bx}\left(1-e^{2a+2bx}\right)\left(1+e^{2a+2bx}\right)^3de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int e^{-4a-4bx}\left(1-e^{2a+2bx}\right)\left(1+e^{2a+2bx}\right)^3de^{a+bx}}{16b}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left(e^{-4a-4bx}+2e^{-2a-2bx}-2e^{2a+2bx}-e^{4a+4bx}\right)de^{a+bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}e^{-3a-3bx}+2e^{-a-bx}+\frac{2}{3}e^{3a+3bx}+\frac{1}{5}e^{5a+5bx}}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
(E^(-3*a - 3*b*x)/3 + 2*E^(-a - b*x) + (2*E^(3*a + 3*b*x))/3 + E^(5*a + 5*
b*x)/5)/(16*b)
```

3.915.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.915.4 Maple [A] (verified)

Time = 48.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}+\frac{\cosh(bx+a)^5}{5}}{b}$	52
default	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}+\frac{\cosh(bx+a)^5}{5}}{b}$	52
risch	$\frac{e^{-3bx-3a}}{48b}+\frac{e^{-bx-a}}{8b}+\frac{e^{3bx+3a}}{24b}+\frac{e^{5bx+5a}}{80b}$	58

input
```
int(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5*sinh(b*x+a)*cosh(b*x+a)^4-1/5*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)
+1/5*cosh(b*x+a)^5)
```

3.915.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.61

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^4 - \cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + (6\cosh(bx+a)^2 + 5)\sinh(bx+a)^2 + 5}{30\,(b\cosh(bx+a) - b\sinh(bx+a))}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/30*(cosh(b*x + a)^4 - cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
(6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^2 + 5*cosh(b*x + a)^2 - (cosh(b*x +
a)^3 + 5*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

3.915.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(53) = 106.

Time = 2.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.01

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$= \begin{cases} -\frac{2e^ae^{bx}\sinh^4(a+bx)}{15b} + \frac{2e^ae^{bx}\sinh^3(a+bx)\cosh(a+bx)}{15b} + \frac{e^ae^{bx}\sinh^2(a+bx)\cosh^2(a+bx)}{5b} - \frac{e^ae^{bx}\sinh(a+bx)\cosh^3(a+bx)}{5b} + \frac{e^ae \\ xe^a\sinh(a)\cosh^3(a) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) + 2*exp(a)*exp(b*x)*
sinh(a + b*x)**3*cosh(a + b*x)/(15*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*c
osh(a + b*x)**2/(5*b) - exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(5*
b) + exp(a)*exp(b*x)*cosh(a + b*x)**4/(5*b), Ne(b, 0)), (x*exp(a)*sinh(a)*
cosh(a)**3, True))
```

3.915.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\left(6\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}}{48\,b}+\frac{3\,e^{(5\,bx+5\,a)}+10\,e^{(3\,bx+3\,a)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/48*(6*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a)/b + 1/240*(3*e^(5*b*x + 5*a)
 + 10*e^(3*b*x + 3*a))/b
```

3.915.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{5\left(6\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}+3\,e^{(5\,bx+5\,a)}+10\,e^{(3\,bx+3\,a)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/240*(5*(6*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) + 3*e^(5*b*x + 5*a) + 10
*e^(3*b*x + 3*a))/b
```

3.915.9 Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{30\,\mathrm{e}^{-a-b\,x}+5\,\mathrm{e}^{-3\,a-3\,b\,x}+10\,\mathrm{e}^{3\,a+3\,b\,x}+3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

input
```
int(cosh(a + b*x)^3*exp(a + b*x)*sinh(a + b*x),x)
```

output
```
(30*exp(- a - b*x) + 5*exp(- 3*a - 3*b*x) + 10*exp(3*a + 3*b*x) + 3*exp(5*
a + 5*b*x))/(240*b)
```

3.916 $\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx$

3.916.1 Optimal result

Integrand size = 22, antiderivative size = 59

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx=\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

output
```
1/4*exp(-b*x-a)/b+exp(b*x+a)/b+1/12*exp(3*b*x+3*a)/b-2*arctanh(exp(b*x+a))
/b
```

3.916.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{e^{-a-bx}\left(3+12e^{2(a+bx)}+e^{4(a+bx)}-24\sqrt{e^{2(a+bx)}}\text{arctanh}\left(\sqrt{e^{2(a+bx)}}\right)\right)}{12b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(E^(-a - b*x)*(3 + 12*E^(2*(a + b*x)) + E^(4*(a + b*x)) - 24*Sqrt[E^(2*(a
+ b*x))]*ArcTanh[Sqrt[E^(2*(a + b*x))]]))/(12*b)
```

3.916.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2720, 27, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{-2a-2bx}\left(1+e^{2a+2bx}\right)^3}{4\left(1-e^{2a+2bx}\right)}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2a-2bx}\left(1+e^{2a+2bx}\right)^3}{1-e^{2a+2bx}}de^{a+bx}}{4b}$$

$$\Big\downarrow \text{364}$$

$$-\frac{\int \left(e^{-2a-2bx}-e^{2a+2bx}-4-\frac{8}{-1+e^{2a+2bx}}\right)de^{a+bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-8\text{arctanh}\left(e^{a+bx}\right)+e^{-a-bx}+4e^{a+bx}+\frac{1}{3}e^{3a+3bx}}{4b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(E^(-a - b*x) + 4*E^(a + b*x) + E^(3*a + 3*b*x)/3 - 8*ArcTanh[E^(a + b*x)]
)/(4*b)
```

3.916.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.916.4 Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)+\frac{\cosh(bx+a)^3}{3}+\cosh(bx+a)-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	50
default	$\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)+\frac{\cosh(bx+a)^3}{3}+\cosh(bx+a)-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	50
risch	$\frac{\mathrm{e}^{3bx+3a}}{12b}+\frac{\mathrm{e}^{bx+a}}{b}+\frac{\mathrm{e}^{-bx-a}}{4b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}$	67

input
```
int(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+1/3*cosh(b*x+a)^3+cosh(b*x+a)-2*a
rctanh(exp(b*x+a)))
```

3.916.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(51) = 102$.

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.88

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx$$
$$= \frac{\cosh{(bx+a)}^4 + 4\cosh{(bx+a)}\sinh{(bx+a)}^3 + \sinh{(bx+a)}^4 + 6\left(\cosh{(bx+a)}^2 + 2\right)\sinh{(bx+a)}^2 + }{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
+ 6*(cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 12*cosh(b*x + a)^2 - 12*(cosh(
b*x + a) + sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 12*(cos
h(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 4*(co
sh(b*x + a)^3 + 6*cosh(b*x + a))*sinh(b*x + a) + 3)/(b*cosh(b*x + a) + b*s
inh(b*x + a))
```

3.916.6 Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**3*csch(b*x+a),x)
```

output
```
Timed out
```

3.916.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)} + 12\,e^{(bx+a)}}{12\,b} + \frac{e^{(-bx-a)}}{4\,b} - \frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

output
```
1/12*(e^(3*b*x + 3*a) + 12*e^(b*x + a))/b + 1/4*e^(-b*x - a)/b - log(e^(b*
x + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

3.916.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}+12\,e^{(bx+a)}+3\,e^{(-bx-a)}-12\,\log\left(e^{(bx+a)}+1\right)+12\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{12\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

output
```
1/12*(e^(3*b*x + 3*a) + 12*e^(b*x + a) + 3*e^(-b*x - a) - 12*log(e^(b*x +
a) + 1) + 12*log(abs(e^(b*x + a) - 1)))/b
```

3.916.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{e^{a+b\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}+\frac{e^{-a-b\,x}}{4\,b}+\frac{e^{3\,a+3\,b\,x}}{12\,b}$$

input
```
int((cosh(a + b*x)^3*exp(a + b*x))/sinh(a + b*x),x)
```

output
```
exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) +
 exp(- a - b*x)/(4*b) + exp(3*a + 3*b*x)/(12*b)
```

3.917 $\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx$

3.917.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx=\frac{e^{2a+2bx}}{4b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{x}{2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

output
```
1/4*exp(2*b*x+2*a)/b+2/b/(1-exp(2*b*x+2*a))+1/2*x+ln(1-exp(2*b*x+2*a))/b
```

3.917.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx=\frac{\frac{1}{4}\left(e^{2(a+bx)}-\frac{8}{-1+e^{2(a+bx)}}+2bx\right)+\log\left(1-e^{2(a+bx)}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
((E^(2*(a + b*x)) - 8/(-1 + E^(2*(a + b*x))) + 2*b*x)/4 + Log[1 - E^(2*(a
+ b*x))])/b
```

3.917.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2720, 27, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{-a-bx}\left(1+e^{2a+2bx}\right)^3}{2\left(1-e^{2a+2bx}\right)^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{e^{-a-bx}\left(1+e^{2a+2bx}\right)^3}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}}{2b}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{e^{-a-bx}\left(1+e^{2a+2bx}\right)^3}{\left(1-e^{2a+2bx}\right)^2}de^{2a+2bx}}{4b}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(e^{-a-bx}+1+\frac{4}{-1+e^{2a+2bx}}+\frac{8}{\left(-1+e^{2a+2bx}\right)^2}\right)de^{2a+2bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a+2bx}+\frac{8}{1-e^{2a+2bx}}+\log\left(e^{2a+2bx}\right)+4\log\left(1-e^{2a+2bx}\right)}{4b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
(E^(2*a + 2*b*x) + 8/(1 - E^(2*a + 2*b*x)) + Log[E^(2*a + 2*b*x)] + 4*Log[
1 - E^(2*a + 2*b*x)])/(4*b)
```

3.917.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.917.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))+\frac{\cosh(bx+a)^3}{2\sinh(bx+a)}+\frac{3bx}{2}+\frac{3a}{2}-\frac{3\coth(bx+a)}{2}}{b}$	56
default	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))+\frac{\cosh(bx+a)^3}{2\sinh(bx+a)}+\frac{3bx}{2}+\frac{3a}{2}-\frac{3\coth(bx+a)}{2}}{b}$	56
risch	$\frac{x}{2}+\frac{e^{2bx+2a}}{4b}-\frac{2a}{b}-\frac{2}{b(e^{2bx+2a}-1)}+\frac{\ln(e^{2bx+2a}-1)}{b}$	59

input
```
int(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*cosh(b*x+a)^2+ln(sinh(b*x+a))+1/2*cosh(b*x+a)^3/sinh(b*x+a)+3/2*b
*x+3/2*a-3/2*coth(b*x+a))
```

3.917.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(54) = 108$.

Time = 0.25 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.38

$$\int e^{a+bx} \cosh(a+bx) \coth^2(a+bx) \, dx$$
$$= \frac{\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + (2bx-1)\cosh(bx+a)^2 + (2bx+6\,\mathrm{co}}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 (2*b*x - 1)*cosh(b*x + a)^2 + (2*b*x + 6*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - 2*b*x + 4*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b
*x + a)^2 - 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 2*(2
*cosh(b*x + a)^3 + (2*b*x - 1)*cosh(b*x + a))*sinh(b*x + a) - 8)/(b*cosh(b
*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

3.917.6 Sympy [F(-1)]

Timed out.

$$\int e^{a+bx} \cosh(a+bx) \coth^2(a+bx) \, dx = \text{Timed out}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.917.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{1}{2}\,x+\frac{a}{2\,b}+\frac{e^{(2\,bx+2\,a)}}{4\,b}+\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}-\frac{2}{b(e^{(2\,bx+2\,a)}-1)}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b + log(e^(b*x + a) + 1)/b + log(e^(
b*x + a) - 1)/b - 2/(b*(e^(2*b*x + 2*a) - 1))
```

3.917.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{2\,bx+2\,a-\frac{4\,\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(2\,bx+2\,a)}-1}+e^{(2\,bx+2\,a)}+4\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*(2*b*x + 2*a - 4*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + e^(2*b*
x + 2*a) + 4*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.917.9 Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{x}{2}+\frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b}-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}+\frac{\mathrm{e}^{2\,a+2\,b\,x}}{4\,b}$$

input
```
int((cosh(a + b*x)^3*exp(a + b*x))/sinh(a + b*x)^2,x)
```

output
```
x/2 + log(exp(2*a)*exp(2*b*x) - 1)/b - 2/(b*(exp(2*a + 2*b*x) - 1)) + exp(
2*a + 2*b*x)/(4*b)
```

3.918 $\int e^{a+bx} \coth^3(a+bx)\,dx$

3.918.1 Optimal result

Integrand size = 16, antiderivative size = 81

$$\int e^{a+bx}\coth^3(a+bx)\,dx=\frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}$$

output
```
exp(b*x+a)/b-2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-exp(2*b
*x+2*a))-3*arctanh(exp(b*x+a))/b
```

3.918.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.06 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.53

$$\int e^{a+bx}\coth^3(a+bx)\,dx =$$

$$-\frac{e^{-5(a+bx)}\Big(-21\left(252105+507305e^{2(a+bx)}+173916e^{4(a+bx)}-154296e^{6(a+bx)}-73885e^{8(a+bx)}+4887e^{10(a}\right.}{}$$

input
```
Integrate[E^(a + b*x)*Coth[a + b*x]^3,x]
```

output
```
-1/60480*(-21*(252105 + 507305*E^(2*(a + b*x)) + 173916*E^(4*(a + b*x)) -
154296*E^(6*(a + b*x)) - 73885*E^(8*(a + b*x)) + 4887*E^(10*(a + b*x))) -
(315*(-16807 - 28218*E^(2*(a + b*x)) + 1173*E^(4*(a + b*x)) + 17748*E^(6*(
a + b*x)) + 4299*E^(8*(a + b*x)) - 1434*E^(10*(a + b*x)) + 7*E^(12*(a + b*
x)))*ArcTanh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 384*E^(8*(a +
 b*x))*(1 + E^(2*(a + b*x)))^2*(7 + 5*E^(2*(a + b*x)))*HypergeometricPFQ[{
3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, E^(2*(a + b*x))] + 256*E^(8*(a + b*x))*
(1 + E^(2*(a + b*x)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1,
1, 11/2}, E^(2*(a + b*x))])/(b*E^(5*(a + b*x)))
```

3.918.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 25, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth^3(a+bx)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int -\frac{(1+e^{2a+2bx})^3}{(1-e^{2a+2bx})^3}de^{a+bx}}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{(1+e^{2a+2bx})^3}{(1-e^{2a+2bx})^3}de^{a+bx}}{b}$$

$$\Big\downarrow \ 300$$

$$-\frac{\int \left(\frac{2(1+3e^{4a+4bx})}{(1-e^{2a+2bx})^3}-1\right)de^{a+bx}}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{-3\text{arctanh}\left(e^{a+bx}\right)+e^{a+bx}+\frac{3e^{a+bx}}{1-e^{2a+2bx}}-\frac{2e^{a+bx}}{(1-e^{2a+2bx})^2}}{b}$$

input
```
Int[E^(a + b*x)*Coth[a + b*x]^3,x]
```

output
```
(E^(a + b*x) - (2*E^(a + b*x))/(1 - E^(2*a + 2*b*x))^2 + (3*E^(a + b*x))/(
1 - E^(2*a + 2*b*x)) - 3*ArcTanh[E^(a + b*x)])/b
```

3.918.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.918.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{e^{bx+a}}{b}-\frac{e^{bx+a}\left(3\,e^{2bx+2a}-1\right)}{b\left(e^{2bx+2a}-1\right)^{2}}+\frac{3\ln\left(e^{bx+a}-1\right)}{2b}-\frac{3\ln\left(e^{bx+a}+1\right)}{2b}$	77
derivativedivides	$\frac{\frac{\cosh(bx+a)^{2}}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}+\frac{\cosh(bx+a)^{3}}{\sinh(bx+a)^{2}}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^{2}}+\frac{3\coth(bx+a)\operatorname{csch}(bx+a)}{2}-3\,\mathrm{arctanh}\left(e^{bx+a}\right)}{b}$	89
default	$\frac{\frac{\cosh(bx+a)^{2}}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}+\frac{\cosh(bx+a)^{3}}{\sinh(bx+a)^{2}}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^{2}}+\frac{3\coth(bx+a)\operatorname{csch}(bx+a)}{2}-3\,\mathrm{arctanh}\left(e^{bx+a}\right)}{b}$	89

input
```
int(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b-exp(b*x+a)*(3*exp(2*b*x+2*a)-1)/b/(exp(2*b*x+2*a)-1)^2+3/2/b*
ln(exp(b*x+a)-1)-3/2/b*ln(exp(b*x+a)+1)
```

3.918.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.67

$$\int e^{a+bx}\coth^3(a+bx)\,dx$$
$$= \frac{2\cosh(bx+a)^5 + 10\cosh(bx+a)\sinh(bx+a)^4 + 2\sinh(bx+a)^5 + 10\left(2\cosh(bx+a)^2 - 1\right)\sinh(bx$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b*x + a)^4 + 2*sinh(b*x + a
)^5 + 10*(2*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 10*cosh(b*x + a)^3 + 10
*(2*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^
4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)
^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*
x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*co
sh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^
3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) -
1) + 2*(5*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 + 2)*sinh(b*x + a) + 4*cosh
(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh
(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x +
 a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

3.918.6 Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\coth^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.918.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int e^{a+bx}\coth^3(a+bx)\,dx = \frac{e^{(bx+a)}}{b} - \frac{3\,\log\left(e^{(bx+a)}+1\right)}{2\,b} + \frac{3\,\log\left(e^{(bx+a)}-1\right)}{2\,b} - \frac{3\,e^{(3\,bx+3\,a)} - e^{(bx+a)}}{b(e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)} + 1)}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
e^(b*x + a)/b - 3/2*log(e^(b*x + a) + 1)/b + 3/2*log(e^(b*x + a) - 1)/b -
(3*e^(3*b*x + 3*a) - e^(b*x + a))/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a)
+ 1))
```

3.918.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\coth^3(a+bx)\,dx = -\frac{\frac{2\left(3\,e^{(3\,bx+3\,a)}-e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2} - 2\,e^{(bx+a)} + 3\,\log\left(e^{(bx+a)}+1\right) - 3\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*(2*(3*e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 - 2*e^(b
*x + a) + 3*log(e^(b*x + a) + 1) - 3*log(abs(e^(b*x + a) - 1)))/b
```

3.918.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.20

$$\int e^{a+bx}\coth^3(a+bx)\,dx = \frac{\mathrm{e}^{a+b\,x}}{b} - \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{3\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int((cosh(a + b*x)^3*exp(a + b*x))/sinh(a + b*x)^3,x)
```

output
```
exp(a + b*x)/b - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) -
 (2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (3*exp
(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.918. $\int e^{a+bx}\coth^3(a+bx)\,dx$

3.919 $\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx$

3.919.1 Optimal result

Integrand size = 24, antiderivative size = 57

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx=\frac{e^{-2a-2bx}}{32b}-\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}+\frac{x}{8}$$

output
```
1/32*exp(-2*b*x-2*a)/b-1/32*exp(4*b*x+4*a)/b+1/96*exp(6*b*x+6*a)/b+1/8*x
```

3.919.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx=\frac{3e^{-2(a+bx)}-3e^{4(a+bx)}+e^{6(a+bx)}+12bx}{96b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
(3/E^(2*(a + b*x)) - 3*E^(4*(a + b*x)) + E^(6*(a + b*x)) + 12*b*x)/(96*b)
```

3.919.3 Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2720, 27, 354, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \sinh^3(a+bx) \cosh(a+bx) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{16} e^{-3a-3bx} \left(1-e^{2a+2bx}\right)^3 \left(1+e^{2a+2bx}\right) de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int e^{-3a-3bx} \left(1-e^{2a+2bx}\right)^3 \left(1+e^{2a+2bx}\right) de^{a+bx}}{16b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int e^{-2a-2bx} \left(1-e^{2a+2bx}\right)^3 \left(1+e^{2a+2bx}\right) de^{2a+2bx}}{32b}$$

$$\Big\downarrow \text{84}$$

$$-\frac{\int \left(e^{-2a-2bx}-2e^{-a-bx}+e^{2a+2bx}\right) de^{2a+2bx}}{32b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-e^{-a-bx}+e^{2a+2bx}-\frac{1}{3}e^{3a+3bx}-2\log\left(e^{2a+2bx}\right)}{32b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
-1/32*(-E^(-a - b*x) + E^(2*a + 2*b*x) - E^(3*a + 3*b*x)/3 - 2*Log[E^(2*a
+ 2*b*x)])/b
```

3.919.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.919.4 Maple [A] (verified)

Time = 290.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{\mathrm{e}^{-2bx-2a}}{32b}-\frac{\mathrm{e}^{4bx+4a}}{32b}+\frac{\mathrm{e}^{6bx+6a}}{96b}+\frac{x}{8}$	47
default	$\frac{x}{8}-\frac{\sinh(2bx+2a)}{32b}-\frac{\sinh(4bx+4a)}{32b}+\frac{\sinh(6bx+6a)}{96b}+\frac{\cosh(2bx+2a)}{32b}-\frac{\cosh(4bx+4a)}{32b}+\frac{\cosh(6bx+6a)}{96b}$	89

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*exp(-2*b*x-2*a)/b-1/32*exp(4*b*x+4*a)/b+1/96*exp(6*b*x+6*a)/b+1/8*x
```

3.919.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.67

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{4\cosh(bx+a)^4-8\cosh(bx+a)\sinh(bx+a)^3+4\sinh(bx+a)^4+3(4bx-1)\cosh(bx+a)^2+3(4b}{96\left(b\cosh(bx+a)^2-2b\cosh(bx+\right.}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/96*(4*cosh(b*x + a)^4 - 8*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a
)^4 + 3*(4*b*x - 1)*cosh(b*x + a)^2 + 3*(4*b*x + 8*cosh(b*x + a)^2 - 1)*si
nh(b*x + a)^2 - 2*(4*cosh(b*x + a)^3 + 3*(4*b*x + 1)*cosh(b*x + a))*sinh(b
*x + a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x
 + a)^2)
```

3.919.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(44) = 88.

Time = 2.19 (sec) , antiderivative size = 235, normalized size of antiderivative = 4.12

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases}-\frac{xe^{2a}e^{2bx}\sinh^4(a+bx)}{8}+\frac{xe^{2a}e^{2bx}\sinh^3(a+bx)\cosh(a+bx)}{4}-\frac{xe^{2a}e^{2bx}\sinh(a+bx)\cosh^3(a+bx)}{4}+\frac{xe^{2a}e^{2bx}\cosh^4(a+bx)}{8}+\frac{7e^{2a}e}{} \\ xe^{2a}\sinh^3(a)\cosh(a)\end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Piecewise((-x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**4/8 + x*exp(2*a)*exp(2*b*
x)*sinh(a + b*x)**3*cosh(a + b*x)/4 - x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*
cosh(a + b*x)**3/4 + x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/8 + 7*exp(2*a)
*exp(2*b*x)*sinh(a + b*x)**4/(48*b) - exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3
*cosh(a + b*x)/(6*b) + exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2*cosh(a + b*x)*
*2/(4*b) - exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/(16*b), Ne(b, 0)), (x*exp(
2*a)*sinh(a)**3*cosh(a), True))
```

3.919.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(3\,e^{(-2\,bx-2\,a)}-1\right)e^{(6\,bx+6\,a)}}{96\,b}+\frac{bx+a}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/96*(3*e^(-2*b*x - 2*a) - 1)*e^(6*b*x + 6*a)/b + 1/8*(b*x + a)/b + 1/32*
e^(-2*b*x - 2*a)/b
```

3.919.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{1}{8}\,x+\frac{e^{(6\,bx+6\,a)}}{96\,b}-\frac{e^{(4\,bx+4\,a)}}{32\,b}+\frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*x + 1/96*e^(6*b*x + 6*a)/b - 1/32*e^(4*b*x + 4*a)/b + 1/32*e^(-2*b*x -
 2*a)/b
```

3.919.9 Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{x}{8}+\frac{\frac{e^{-2\,a-2\,b\,x}}{32}-\frac{e^{4\,a+4\,b\,x}}{32}+\frac{e^{6\,a+6\,b\,x}}{96}}{b}$$

input
```
int(cosh(a + b*x)*exp(2*a + 2*b*x)*sinh(a + b*x)^3,x)
```

output
```
x/8 + (exp(- 2*a - 2*b*x)/32 - exp(4*a + 4*b*x)/32 + exp(6*a + 6*b*x)/96)/
b
```

3.920 $\int e^{2(a+bx)} \cosh(a+bx) \sinh^2(a+bx) \, dx$

3.920.1 Optimal result

Integrand size = 24, antiderivative size = 66

$$\int e^{2(a+bx)} \cosh(a+bx) \sinh^2(a+bx) \, dx = -\frac{e^{-a-bx}}{8b} - \frac{e^{a+bx}}{8b} - \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{40b}$$

output
```
-1/8*exp(-b*x-a)/b-1/8*exp(b*x+a)/b-1/24*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5
*a)/b
```

3.920.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int e^{2(a+bx)} \cosh(a+bx) \sinh^2(a+bx) \, dx = \frac{-5e^{a+bx}\left(3+e^{2(a+bx)}\right)+3e^{-a-bx}\left(-5+e^{6(a+bx)}\right)}{120b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(-5*E^(a + b*x)*(3 + E^(2*(a + b*x))) + 3*E^(-a - b*x)*(-5 + E^(6*(a + b*x
))))/(120*b)
```

3.920.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2720, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \sinh^2(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{1}{8}e^{-2a-2bx}\left(1-e^{2a+2bx}\right)^2\left(1+e^{2a+2bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int e^{-2a-2bx}\left(1-e^{2a+2bx}\right)^2\left(1+e^{2a+2bx}\right)de^{a+bx}}{8b}$$

$$\Big\downarrow \; 355$$

$$\frac{\int \left(-1+e^{-2a-2bx}-e^{2a+2bx}+e^{4a+4bx}\right)de^{a+bx}}{8b}$$

$$\Big\downarrow \; 2009$$

$$\frac{-e^{-a-bx}-e^{a+bx}-\frac{1}{3}e^{3a+3bx}+\frac{1}{5}e^{5a+5bx}}{8b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(-E^(-a - b*x) - E^(a + b*x) - E^(3*a + 3*b*x)/3 + E^(5*a + 5*b*x)/5)/(8*b
)
```

3.920.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.920.4 Maple [A] (verified)

Time = 21.42 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{e^{-bx-a}}{8b}-\frac{e^{bx+a}}{8b}-\frac{e^{3bx+3a}}{24b}+\frac{e^{5bx+5a}}{40b}$	55
default	$-\frac{\sinh(3bx+3a)}{24b}+\frac{\sinh(5bx+5a)}{40b}-\frac{\cosh(bx+a)}{4b}-\frac{\cosh(3bx+3a)}{24b}+\frac{\cosh(5bx+5a)}{40b}$	69

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*exp(-b*x-a)/b-1/8*exp(b*x+a)/b-1/24*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5
*a)/b
```

3.920.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx =$$
$$-\frac{6\cosh(bx+a)^3+18\cosh(bx+a)\sinh(bx+a)^2-9\sinh(bx+a)^3-\left(27\cosh(bx+a)^2+5\right)\sinh(b\ldots}{60\left(b\cosh(bx+a)^2-2b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/60*(6*cosh(b*x + a)^3 + 18*cosh(b*x + a)*sinh(b*x + a)^2 - 9*sinh(b*x +
 a)^3 - (27*cosh(b*x + a)^2 + 5)*sinh(b*x + a) + 10*cosh(b*x + a))/(b*cosh
(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.920.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(48) = 96$.

Time = 0.87 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.94

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx$$
$$= \begin{cases} \frac{e^{2a}e^{2bx}\sinh^3(a+bx)}{15b} - \frac{2e^{2a}e^{2bx}\sinh^2(a+bx)\cosh(a+bx)}{15b} + \frac{8e^{2a}e^{2bx}\sinh(a+bx)\cosh^2(a+bx)}{15b} - \frac{4e^{2a}e^{2bx}\cosh^3(a+bx)}{15b} & \text{for } b \neq 0 \\ xe^{2a}\sinh^2(a)\cosh(a) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Piecewise((exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3/(15*b) - 2*exp(2*a)*exp(2*
b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(15*b) + 8*exp(2*a)*exp(2*b*x)*sinh(a
+ b*x)*cosh(a + b*x)**2/(15*b) - 4*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**3/(1
5*b), Ne(b, 0)), (x*exp(2*a)*sinh(a)**2*cosh(a), True))
```

3.920.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{\left(5\,e^{(-2\,bx-2\,a)}+15\,e^{(-4\,bx-4\,a)}-3\right)e^{(5\,bx+5\,a)}}{120\,b} - \frac{e^{(-bx-a)}}{8\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/120*(5*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) - 3)*e^(5*b*x + 5*a)/b -
1/8*e^(-b*x - a)/b
```

3.920.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx=\frac{e^{(5\,bx+5\,a)}}{40\,b}-\frac{e^{(3\,bx+3\,a)}}{24\,b}-\frac{e^{(bx+a)}}{8\,b}-\frac{e^{(-bx-a)}}{8\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/40*e^(5*b*x + 5*a)/b - 1/24*e^(3*b*x + 3*a)/b - 1/8*e^(b*x + a)/b - 1/8*
e^(-b*x - a)/b
```

3.920.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx=-\frac{15\,\mathrm{e}^{a+b\,x}+15\,\mathrm{e}^{-a-b\,x}+5\,\mathrm{e}^{3\,a+3\,b\,x}-3\,\mathrm{e}^{5\,a+5\,b\,x}}{120\,b}$$

input
```
int(cosh(a + b*x)*exp(2*a + 2*b*x)*sinh(a + b*x)^2,x)
```

output
```
-(15*exp(a + b*x) + 15*exp(- a - b*x) + 5*exp(3*a + 3*b*x) - 3*exp(5*a + 5
*b*x))/(120*b)
```

3.921 $\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx$

3.921.1 Optimal result

Integrand size = 22, antiderivative size = 23

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx=\frac{e^{4a+4bx}}{16b}-\frac{x}{4}$$

output
```
1/16*exp(4*b*x+4*a)/b-1/4*x
```

3.921.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx=\frac{1}{4}\left(\frac{e^{4a+4bx}}{4b}-x\right)$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
(E^(4*a + 4*b*x)/(4*b) - x)/4
```

3.921.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int -\frac{1}{4}e^{-a-bx}\left(1-e^{4a+4bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int e^{-a-bx}\left(1-e^{4a+4bx}\right)de^{a+bx}}{4b}$$

$$\Big\downarrow \; 802$$

$$-\frac{\int \left(e^{-a-bx}-e^{3a+3bx}\right)de^{a+bx}}{4b}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{4}e^{4a+4bx}-\log\left(e^{a+bx}\right)}{4b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
(E^(4*a + 4*b*x)/4 - Log[E^(a + b*x)])/(4*b)
```

3.921.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.921.4 Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{e^{4bx+4a}}{16b}-\frac{x}{4}$	19
default	$-\frac{x}{4}+\frac{\sinh(4bx+4a)}{16b}+\frac{\cosh(4bx+4a)}{16b}$	33

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/16*exp(4*b*x+4*a)/b-1/4*x
```

3.921.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.96

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx = -\frac{(4\,bx-1)\cosh{(bx+a)}^2-2\,(4\,bx+1)\cosh{(bx+a)}\sinh{(bx+a)}+(4\,bx-1)\sinh{(bx+a)}^2}{16\left(b\cosh{(bx+a)}^2-2\,b\cosh{(bx+a)}\sinh{(bx+a)}+b\sinh{(bx+a)}^2\right)}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/16*((4*b*x - 1)*cosh(b*x + a)^2 - 2*(4*b*x + 1)*cosh(b*x + a)*sinh(b*x
+ a) + (4*b*x - 1)*sinh(b*x + a)^2)/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)
*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.921.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(15) = 30$.

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.09

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} -\frac{xe^{2a}e^{2bx}\sinh^2(a+bx)}{4} + \frac{xe^{2a}e^{2bx}\sinh(a+bx)\cosh(a+bx)}{2} - \frac{xe^{2a}e^{2bx}\cosh^2(a+bx)}{4} + \frac{e^{2a}e^{2bx}\sinh(a+bx)\cosh(a+bx)}{4b} & \text{for } b \neq \\ xe^{2a}\sinh(a)\cosh(a) & \text{otherwi} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x)
```

output
```
Piecewise((-x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2/4 + x*exp(2*a)*exp(2*b*
x)*sinh(a + b*x)*cosh(a + b*x)/2 - x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**2/
4 + exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x)/(4*b), Ne(b, 0)), (x*e
xp(2*a)*sinh(a)*cosh(a), True))
```

3.921.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx = -\frac{1}{4}\,x - \frac{a}{4\,b} + \frac{e^{(4\,bx+4\,a)}}{16\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*x - 1/4*a/b + 1/16*e^(4*b*x + 4*a)/b
```

3.921.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx = -\frac{1}{4}\,x + \frac{e^{(4\,bx+4\,a)}}{16\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
-1/4*x + 1/16*e^(4*b*x + 4*a)/b
```

3.921.9 Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx=\frac{\mathrm{e}^{4\,a+4\,b\,x}}{16\,b}-\frac{x}{4}$$

input
```
int(cosh(a + b*x)*exp(2*a + 2*b*x)*sinh(a + b*x),x)
```

output
```
exp(4*a + 4*b*x)/(16*b) - x/4
```

3.922 $\int e^{2(a+bx)} \coth(a+bx)\,dx$

3.922.1 Optimal result

Integrand size = 16, antiderivative size = 37

$$\int e^{2(a+bx)} \coth(a+bx)\,dx = \frac{e^{2a+2bx}}{2b} + \frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b+ln(1-exp(2*b*x+2*a))/b
```

3.922.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int e^{2(a+bx)} \coth(a+bx)\,dx = \frac{\frac{1}{2}e^{2a+2bx} + \log\left(1-e^{2a+2bx}\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[a + b*x],x]
```

output
```
(E^(2*a + 2*b*x)/2 + Log[1 - E^(2*a + 2*b*x)])/b
```

3.922.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2720, 25, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\coth(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{a+bx}\left(1+e^{2a+2bx}\right)}{1-e^{2a+2bx}}de^{a+bx}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e^{a+bx}\left(1+e^{2a+2bx}\right)}{1-e^{2a+2bx}}de^{a+bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$-\frac{\int \frac{1+e^{2a+2bx}}{1-e^{2a+2bx}}de^{2a+2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(-1-\frac{2}{-1+e^{2a+2bx}}\right)de^{2a+2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-e^{2a+2bx}-2\log\left(1-e^{2a+2bx}\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Coth[a + b*x],x]
```

output
```
-1/2*(-E^(2*a + 2*b*x) - 2*Log[1 - E^(2*a + 2*b*x)])/b
```

3.922.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.922.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}-\frac{2a}{b}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b}$	38

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b-2/b*a+1/b*ln(exp(2*b*x+2*a)-1)
```

3.922.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int e^{2(a+bx)}\coth(a+bx)\,dx = \frac{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+2\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{2b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 2
*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

3.922.6 Sympy [F]

$$\int e^{2(a+bx)}\coth(a+bx)\,dx = e^{2a}\int e^{2bx}\cosh(a+bx)\operatorname{csch}(a+bx)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*cosh(a + b*x)*csch(a + b*x), x)
```

3.922.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int e^{2(a+bx)}\coth(a+bx)\,dx = \frac{2(bx+a)}{b}+\frac{e^{(2bx+2a)}}{2b}+\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

output
```
2*(b*x + a)/b + 1/2*e^(2*b*x + 2*a)/b + log(e^(-b*x - a) + 1)/b + log(e^(-
b*x - a) - 1)/b
```

3.922.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)} \coth(a+bx)\,dx = \frac{e^{(2\,bx+2\,a)} + 2\,\log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

output
```
1/2*(e^(2*b*x + 2*a) + 2*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.922.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)} \coth(a+bx)\,dx = \frac{e^{2\,a+2\,b\,x} + 2\,\ln\left(e^{2\,a}\,e^{2\,b\,x} - 1\right)}{2\,b}$$

input
```
int((cosh(a + b*x)*exp(2*a + 2*b*x))/sinh(a + b*x),x)
```

output
```
(exp(2*a + 2*b*x) + 2*log(exp(2*a)*exp(2*b*x) - 1))/(2*b)
```

3.923 $\int e^{2(a+bx)} \coth(a+bx)\text{csch}(a+bx)\,dx$

3.923.1 Optimal result

Integrand size = 22, antiderivative size = 54

$$\int e^{2(a+bx)} \coth(a+bx)\text{csch}(a+bx)\,dx = \frac{2e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\,(1-e^{2a+2bx})} - \frac{4\text{arctanh}(e^{a+bx})}{b}$$

output
```
2*exp(b*x+a)/b+2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-4*arctanh(exp(b*x+a))/b
```

3.923.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int e^{2(a+bx)} \coth(a+bx)\text{csch}(a+bx)\,dx = \frac{2e^{a+bx} + \frac{1}{1-e^{a+bx}} - \frac{1}{1+e^{a+bx}} + 2\log\left(1-e^{a+bx}\right) - 2\log\left(1+e^{a+bx}\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
(2*E^(a + b*x) + (1 - E^(a + b*x))^(-1) - (1 + E^(a + b*x))^(-1) + 2*Log[1
 - E^(a + b*x)] - 2*Log[1 + E^(a + b*x)])/b
```

3.923.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2720, 27, 360, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{2e^{2a+2bx}\left(1+e^{2a+2bx}\right)}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\int \frac{e^{2a+2bx}\left(1+e^{2a+2bx}\right)}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{ 360}$$

$$\frac{2\left(\frac{e^{a+bx}}{1-e^{2a+2bx}}-\frac{1}{2}\int \frac{2\left(1+e^{2a+2bx}\right)}{1-e^{2a+2bx}}de^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(\frac{e^{a+bx}}{1-e^{2a+2bx}}-\int \frac{1+e^{2a+2bx}}{1-e^{2a+2bx}}de^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{2\left(-2\int \frac{1}{1-e^{2a+2bx}}de^{a+bx}+e^{a+bx}+\frac{e^{a+bx}}{1-e^{2a+2bx}}\right)}{b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\left(-2\text{arctanh}\left(e^{a+bx}\right)+e^{a+bx}+\frac{e^{a+bx}}{1-e^{2a+2bx}}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
(2*(E^(a + b*x) + E^(a + b*x)/(1 - E^(2*a + 2*b*x)) - 2*ArcTanh[E^(a + b*x
)]))/b
```

3.923. $\int e^{2(a+bx)}\coth(a+bx)\text{csch}(a+bx)\,dx$

3.923.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.923.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{2\,\mathrm{e}^{bx+a}}{b}-\frac{2\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}-\frac{2\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}+\frac{2\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}$	65

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*exp(b*x+a)/b-2/b*exp(b*x+a)/(exp(2*b*x+2*a)-1)-2/b*ln(exp(b*x+a)+1)+2/b*
ln(exp(b*x+a)-1)
```

3.923.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(48) = 96$.

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 3.70

$$\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}(a+bx)\,dx$$
$$= \frac{2\left(\cosh\left(bx+a\right)^{3}+3\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{3}-\left(\cosh\left(bx+a\right)^{2}+2\cosh\left(bx+a\right)\sin\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (
cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log
(cosh(b*x + a) + sinh(b*x + a) + 1) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*s
inh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + (3*cosh(b*x + a)^2 - 2)*sinh(b*x + a) - 2*cosh(b*x + a))/(b*cosh(b*x +
a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

3.923.6 Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.923.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{2\left(2\,e^{(-2\,bx-2\,a)}-1\right)}{b\left(e^{(-bx-a)}-e^{(-3\,bx-3\,a)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b - 2*(2*e^(-2*b*x -
2*a) - 1)/(b*(e^(-b*x - a) - e^(-3*b*x - 3*a)))
```

3.923.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\left(\frac{e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1}-e^{(bx+a)}+\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-2*(e^(b*x + a)/(e^(2*b*x + 2*a) - 1) - e^(b*x + a) + log(e^(b*x + a) + 1)
 - log(abs(e^(b*x + a) - 1)))/b
```

3.923.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}(a+bx)\,dx = \frac{2\,\mathrm{e}^{a+b\,x}}{b}-\frac{4\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int((cosh(a + b*x)*exp(2*a + 2*b*x))/sinh(a + b*x)^2,x)
```

output
```
(2*exp(a + b*x))/b - (4*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/
2) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.923. $\int e^{2(a+bx)}\coth(a+bx)\text{csch}(a+bx)\,dx$

3.924 $\int e^{2(a+bx)} \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

3.924.1 Optimal result

Integrand size = 24, antiderivative size = 63

$$\int e^{2(a+bx)} \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2}{b(1-e^{2a+2bx})^2}+\frac{6}{b(1-e^{2a+2bx})}+\frac{2\log(1-e^{2a+2bx})}{b}$$

output
```
-2/b/(1-exp(2*b*x+2*a))^2+6/b/(1-exp(2*b*x+2*a))+2*ln(1-exp(2*b*x+2*a))/b
```

3.924.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)} \coth(a+bx)\text{csch}^2(a+bx)\,dx = \frac{\frac{4-6e^{2(a+bx)}}{\left(-1+e^{2(a+bx)}\right)^2}+2\log\left(1-e^{2(a+bx)}\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
((4 - 6*E^(2*(a + b*x)))/(-1 + E^(2*(a + b*x)))^2 + 2*Log[1 - E^(2*(a + b*
x))])/b
```

3.924.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2720, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{4e^{3a+3bx}\left(1+e^{2a+2bx}\right)}{\left(1-e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\int \frac{e^{3a+3bx}\left(1+e^{2a+2bx}\right)}{\left(1-e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{2\int \frac{e^{2a+2bx}\left(1+e^{2a+2bx}\right)}{\left(1-e^{2a+2bx}\right)^3}de^{2a+2bx}}{b}$$

$$\Big\downarrow \text{86}$$

$$-\frac{2\int \left(-\frac{3}{\left(-1+e^{2a+2bx}\right)^2}-\frac{2}{\left(-1+e^{2a+2bx}\right)^3}+\frac{1}{1-e^{2a+2bx}}\right)de^{2a+2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{3}{1-e^{2a+2bx}}+\frac{1}{\left(e^{2a+2bx}-1\right)^2}-\log\left(1-e^{2a+2bx}\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
(-2*(-3/(1 - E^(2*a + 2*b*x)) + (-1 + E^(2*a + 2*b*x))^(-2) - Log[1 - E^(2
*a + 2*b*x)]))/b
```

3.924.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.924.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{4a}{b}-\frac{2\left(3\,{\mathrm e}^{2bx+2a}-2\right)}{b\left({\mathrm e}^{2bx+2a}-1\right)^{2}}+\frac{2\ln\left({\mathrm e}^{2bx+2a}-1\right)}{b}$	56

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-4/b*a-2*(3*exp(2*b*x+2*a)-2)/b/(exp(2*b*x+2*a)-1)^2+2/b*ln(exp(2*b*x+2*a)
-1)
```

3.924.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(56) = 112$.

Time = 0.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 4.16

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2\left(3\cosh\left(bx+a\right)^2-\left(\cosh\left(bx+a\right)^4+4\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\sinh\left(bx+a\right)^4+2\left(3\cosh\left(bx\right.\right.\right.}{b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+b\,s}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2*(3*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*
x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*si
nh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 6*cosh(b*x + a)*sinh(b*x +
a) + 3*sinh(b*x + a)^2 - 2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*
x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^
2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x
+ a) + b)
```

3.924.6 Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.924.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = 4\,x+\frac{4\,a}{b}+\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{2\,\left(e^{(-2\,bx-2\,a)}-2\right)}{b(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
4*x + 4*a/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b - 2*(e
^(-2*b*x - 2*a) - 2)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

3.924.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\frac{3\,e^{(4\,bx+4\,a)}-1}{\left(e^{(2\,bx+2\,a)}-1\right)^2}-2\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-((3*e^(4*b*x + 4*a) - 1)/(e^(2*b*x + 2*a) - 1)^2 - 2*log(abs(e^(2*b*x + 2
*a) - 1)))/b
```

3.924.9 Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \frac{2\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b}-\frac{6}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}-\frac{2}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int((cosh(a + b*x)*exp(2*a + 2*b*x))/sinh(a + b*x)^3,x)
```

output
```
(2*log(exp(2*a)*exp(2*b*x) - 1))/b - 6/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(
exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

3.924. $\int e^{2(a+bx)}\coth(a+bx)\text{csch}^2(a+bx)\,dx$

3.925 $\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^3(a+bx)\, dx$

3.925.1 Optimal result

Integrand size = 26, antiderivative size = 100

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^3(a+bx)\, dx = \frac{e^{-3a-3bx}}{96b} - \frac{e^{-a-bx}}{32b} + \frac{e^{a+bx}}{16b} - \frac{e^{3a+3bx}}{48b} - \frac{e^{5a+5bx}}{160b} + \frac{e^{7a+7bx}}{224b}$$

output
```
1/96*exp(-3*b*x-3*a)/b-1/32*exp(-b*x-a)/b+1/16*exp(b*x+a)/b-1/48*exp(3*b*x
+3*a)/b-1/160*exp(5*b*x+5*a)/b+1/224*exp(7*b*x+7*a)/b
```

3.925.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^3(a+bx)\, dx = \frac{e^{-3(a+bx)}\left(35 - 105e^{2(a+bx)} + 210e^{4(a+bx)} - 70e^{6(a+bx)} - 21e^{8(a+bx)} + 15e^{10(a+bx)}\right)}{3360b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
(35 - 105*E^(2*(a + b*x)) + 210*E^(4*(a + b*x)) - 70*E^(6*(a + b*x)) - 21*
E^(8*(a + b*x)) + 15*E^(10*(a + b*x)))/(3360*b*E^(3*(a + b*x)))
```

3.925.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2720, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh^3(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{32}e^{-4a-4bx}\left(1-e^{2a+2bx}\right)^3\left(1+e^{2a+2bx}\right)^2de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int e^{-4a-4bx}\left(1-e^{2a+2bx}\right)^3\left(1+e^{2a+2bx}\right)^2de^{a+bx}}{32b}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left(-2+e^{-4a-4bx}-e^{-2a-2bx}+2e^{2a+2bx}+e^{4a+4bx}-e^{6a+6bx}\right)de^{a+bx}}{32b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}e^{-3a-3bx}-e^{-a-bx}+2e^{a+bx}-\frac{2}{3}e^{3a+3bx}-\frac{1}{5}e^{5a+5bx}+\frac{1}{7}e^{7a+7bx}}{32b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
(E^(-3*a - 3*b*x)/3 - E^(-a - b*x) + 2*E^(a + b*x) - (2*E^(3*a + 3*b*x))/3
 - E^(5*a + 5*b*x)/5 + E^(7*a + 7*b*x)/7)/(32*b)
```

3.925.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.925.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08

$$\frac{3\sinh\left(bx+a\right)}{32b}-\frac{\sinh\left(3bx+3a\right)}{32b}-\frac{\sinh\left(5bx+5a\right)}{160b}+\frac{\sinh\left(7bx+7a\right)}{224b}+\frac{\cosh\left(bx+a\right)}{32b}-\frac{\cosh\left(3bx+3a\right)}{96b}-\cosh$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

output
```
3/32*sinh(b*x+a)/b-1/32/b*sinh(3*b*x+3*a)-1/160/b*sinh(5*b*x+5*a)+1/224/b*
sinh(7*b*x+7*a)+1/32*cosh(b*x+a)/b-1/96*cosh(3*b*x+3*a)/b-1/160*cosh(5*b*x
+5*a)/b+1/224*cosh(7*b*x+7*a)/b
```

3.925.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(82) = 164.

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.75

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{25\cosh\left(bx+a\right)^5+125\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-10\sinh\left(bx+a\right)^5-2\left(50\cosh\left(bx+a\right)^2-21\right)\sinh}{168}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas"
)
```

output
```
1/1680*(25*cosh(b*x + a)^5 + 125*cosh(b*x + a)*sinh(b*x + a)^4 - 10*sinh(b
*x + a)^5 - 2*(50*cosh(b*x + a)^2 - 21)*sinh(b*x + a)^3 - 63*cosh(b*x + a)
^3 + (250*cosh(b*x + a)^3 - 189*cosh(b*x + a))*sinh(b*x + a)^2 - 2*(25*cos
h(b*x + a)^4 - 63*cosh(b*x + a)^2 + 70)*sinh(b*x + a) + 70*cosh(b*x + a))/
(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.925.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(76) = 152$.

Time = 5.64 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.97

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{4e^{2a}e^{2bx}\sinh^5(a+bx)}{35b} + \frac{8e^{2a}e^{2bx}\sinh^4(a+bx)\cosh(a+bx)}{35b} + \frac{2e^{2a}e^{2bx}\sinh^3(a+bx)\cosh^2(a+bx)}{35b} - \frac{e^{2a}e^{2bx}\sinh^2(a+bx)\cosh^3(a+b}{105b} \\ xe^{2a}\sinh^3(a)\cosh^2(a) \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Piecewise((-4*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**5/(35*b) + 8*exp(2*a)*exp
(2*b*x)*sinh(a + b*x)**4*cosh(a + b*x)/(35*b) + 2*exp(2*a)*exp(2*b*x)*sinh
(a + b*x)**3*cosh(a + b*x)**2/(35*b) - exp(2*a)*exp(2*b*x)*sinh(a + b*x)**
2*cosh(a + b*x)**3/(105*b) - 4*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a +
b*x)**4/(105*b) + 2*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**5/(105*b), Ne(b, 0)
), (x*exp(2*a)*sinh(a)**3*cosh(a)**2, True))
```

3.925.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$= -\frac{\left(21\,e^{(-2\,bx-2\,a)} + 70\,e^{(-4\,bx-4\,a)} - 210\,e^{(-6\,bx-6\,a)} - 15\right)e^{(7\,bx+7\,a)}}{3360\,b} - \frac{3\,e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{96\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima"
)
```

output
```
-1/3360*(21*e^(-2*b*x - 2*a) + 70*e^(-4*b*x - 4*a) - 210*e^(-6*b*x - 6*a)
- 15)*e^(7*b*x + 7*a)/b - 1/96*(3*e^(-b*x - a) - e^(-3*b*x - 3*a))/b
```

3.925.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(7\,bx+7\,a)}}{224\,b} - \frac{e^{(5\,bx+5\,a)}}{160\,b} - \frac{e^{(3\,bx+3\,a)}}{48\,b} + \frac{e^{(bx+a)}}{16\,b} - \frac{e^{(-bx-a)}}{32\,b} + \frac{e^{(-3\,bx-3\,a)}}{96\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/224*e^(7*b*x + 7*a)/b - 1/160*e^(5*b*x + 5*a)/b - 1/48*e^(3*b*x + 3*a)/b
 + 1/16*e^(b*x + a)/b - 1/32*e^(-b*x - a)/b + 1/96*e^(-3*b*x - 3*a)/b
```

3.925.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{210\,e^{a+b\,x}-105\,e^{-a-b\,x}+35\,e^{-3\,a-3\,b\,x}-70\,e^{3\,a+3\,b\,x}-21\,e^{5\,a+5\,b\,x}+15\,e^{7\,a+7\,b\,x}}{3360\,b}$$

input
```
int(cosh(a + b*x)^2*exp(2*a + 2*b*x)*sinh(a + b*x)^3,x)
```

output
```
(210*exp(a + b*x) - 105*exp(- a - b*x) + 35*exp(- 3*a - 3*b*x) - 70*exp(3*
a + 3*b*x) - 21*exp(5*a + 5*b*x) + 15*exp(7*a + 7*b*x))/(3360*b)
```

3.926 $\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^2(a+bx)\,dx$

3.926.1 Optimal result

Integrand size = 26, antiderivative size = 52

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^2(a+bx)\,dx = -\frac{e^{-2a-2bx}}{32b} - \frac{e^{2a+2bx}}{16b} + \frac{e^{6a+6bx}}{96b}$$

output
```
-1/32*exp(-2*b*x-2*a)/b-1/16*exp(2*b*x+2*a)/b+1/96*exp(6*b*x+6*a)/b
```

3.926.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.73

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^2(a+bx)\,dx = \frac{e^{-2(a+bx)}\left(-3-6e^{4(a+bx)}+e^{8(a+bx)}\right)}{96b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(-3 - 6*E^(4*(a + b*x)) + E^(8*(a + b*x)))/(96*b*E^(2*(a + b*x)))
```

3.926.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \sinh^2(a+bx) \cosh^2(a+bx) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{16} e^{-3a-3bx} \left(1-e^{4a+4bx}\right)^2 de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int e^{-3a-3bx} \left(1-e^{4a+4bx}\right)^2 de^{a+bx}}{16b}$$

$$\Big\downarrow \text{802}$$

$$\frac{\int \left(e^{-3a-3bx}-2e^{a+bx}+e^{5a+5bx}\right) de^{a+bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2} e^{-2a-2bx}-e^{2a+2bx}+\frac{1}{6} e^{6a+6bx}}{16b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(-1/2*E^(-2*a - 2*b*x) - E^(2*a + 2*b*x) + E^(6*a + 6*b*x)/6)/(16*b)
```

3.926.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.926.4 Maple [A] (verified)

Time = 107.88 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{\mathrm{e}^{-2bx-2a}}{32b}-\frac{\mathrm{e}^{2bx+2a}}{16b}+\frac{\mathrm{e}^{6bx+6a}}{96b}$	44
default	$-\frac{\sinh(2bx+2a)}{32b}+\frac{\sinh(6bx+6a)}{96b}-\frac{3\cosh(2bx+2a)}{32b}+\frac{\cosh(6bx+6a)}{96b}$	58

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32*exp(-2*b*x-2*a)/b-1/16*exp(2*b*x+2*a)/b+1/96*exp(6*b*x+6*a)/b
```

3.926.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.08

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{\cosh\left(bx+a\right)^4-8\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)+6\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^2-8\cosh\left(bx+a\right)\sinh}{48\left(b\cosh\left(bx+a\right)^2-2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2\right)}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas"
)
```

output
```
-1/48*(cosh(b*x + a)^4 - 8*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)
^2*sinh(b*x + a)^2 - 8*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 3
)/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2
)
```

3.926.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(41) = 82$.

Time = 2.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.46

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$$
$$= \begin{cases} -\frac{5e^{2a}e^{2bx}\sinh^4(a+bx)}{48b} + \frac{5e^{2a}e^{2bx}\sinh^3(a+bx)\cosh(a+bx)}{24b} + \frac{e^{2a}e^{2bx}\sinh(a+bx)\cosh^3(a+bx)}{8b} - \frac{e^{2a}e^{2bx}\cosh^4(a+bx)}{16b} & \text{for } b \neq \\ xe^{2a}\sinh^2(a)\cosh^2(a) & \text{otherw} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-5*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**4/(48*b) + 5*exp(2*a)*exp
(2*b*x)*sinh(a + b*x)**3*cosh(a + b*x)/(24*b) + exp(2*a)*exp(2*b*x)*sinh(a
 + b*x)*cosh(a + b*x)**3/(8*b) - exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/(16*
b), Ne(b, 0)), (x*exp(2*a)*sinh(a)**2*cosh(a)**2, True))
```

3.926.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{\left(6\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{96\,b} - \frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima"
)
```

output
```
-1/96*(6*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b - 1/32*e^(-2*b*x - 2*a)/b
```

3.926.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(6\,bx+6\,a)}}{96\,b} - \frac{e^{(2\,bx+2\,a)}}{16\,b} - \frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/96*e^(6*b*x + 6*a)/b - 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x - 2*a)/b
```

3.926.9 Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{3\,\mathrm{e}^{-2\,a-2\,b\,x}+6\,\mathrm{e}^{2\,a+2\,b\,x}-\mathrm{e}^{6\,a+6\,b\,x}}{96\,b}$$

input
```
int(cosh(a + b*x)^2*exp(2*a + 2*b*x)*sinh(a + b*x)^2,x)
```

output
```
-(3*exp(- 2*a - 2*b*x) + 6*exp(2*a + 2*b*x) - exp(6*a + 6*b*x))/(96*b)
```

3.927 $\int e^{2(a+bx)} \cosh^2(a+bx) \sinh(a+bx)\,dx$

3.927.1 Optimal result

Integrand size = 24, antiderivative size = 66

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh(a+bx)\,dx = \frac{e^{-a-bx}}{8b} - \frac{e^{a+bx}}{8b} + \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{40b}$$

output
```
1/8*exp(-b*x-a)/b-1/8*exp(b*x+a)/b+1/24*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*
a)/b
```

3.927.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh(a+bx)\,dx = \frac{e^{a+bx}\left(-3+e^{2(a+bx)}\right)}{24b} + \frac{e^{-a-bx}\left(5+e^{6(a+bx)}\right)}{40b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
(E^(a + b*x)*(-3 + E^(2*(a + b*x))))/(24*b) + (E^(-a - b*x)*(5 + E^(6*(a +
 b*x))))/(40*b)
```

3.927.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2720, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int -\frac{1}{8}e^{-2a-2bx}\left(1-e^{2a+2bx}\right)\left(1+e^{2a+2bx}\right)^2de^{a+bx}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int e^{-2a-2bx}\left(1-e^{2a+2bx}\right)\left(1+e^{2a+2bx}\right)^2de^{a+bx}}{8b}$$

$$\Big\downarrow \; 355$$

$$-\frac{\int \left(1+e^{-2a-2bx}-e^{2a+2bx}-e^{4a+4bx}\right)de^{a+bx}}{8b}$$

$$\Big\downarrow \; 2009$$

$$\frac{e^{-a-bx}-e^{a+bx}+\frac{1}{3}e^{3a+3bx}+\frac{1}{5}e^{5a+5bx}}{8b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
(E^(-a - b*x) - E^(a + b*x) + E^(3*a + 3*b*x)/3 + E^(5*a + 5*b*x)/5)/(8*b)
```

3.927.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.927.4 Maple [A] (verified)

Time = 10.72 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{e^{-bx-a}}{8b}-\frac{e^{bx+a}}{8b}+\frac{e^{3bx+3a}}{24b}+\frac{e^{5bx+5a}}{40b}$	55
default	$-\frac{\sinh(bx+a)}{4b}+\frac{\sinh(3bx+3a)}{24b}+\frac{\sinh(5bx+5a)}{40b}+\frac{\cosh(3bx+3a)}{24b}+\frac{\cosh(5bx+5a)}{40b}$	69

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*exp(-b*x-a)/b-1/8*exp(b*x+a)/b+1/24*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*
a)/b
```

3.927.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$=\frac{9\cosh\left(bx+a\right)^3+27\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-6\sinh\left(bx+a\right)^3-2\left(9\cosh\left(bx+a\right)^2-5\right)\sinh\left(bx\right.}{60\left(b\cosh\left(bx+a\right)^2-2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2\right)}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/60*(9*cosh(b*x + a)^3 + 27*cosh(b*x + a)*sinh(b*x + a)^2 - 6*sinh(b*x +
a)^3 - 2*(9*cosh(b*x + a)^2 - 5)*sinh(b*x + a) - 5*cosh(b*x + a))/(b*cosh(
b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.927.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(48) = 96$.

Time = 1.01 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.94

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh(a+bx)\,dx$$
$$= \begin{cases} -\frac{4e^{2a}e^{2bx}\sinh^3(a+bx)}{15b} + \frac{8e^{2a}e^{2bx}\sinh^2(a+bx)\cosh(a+bx)}{15b} - \frac{2e^{2a}e^{2bx}\sinh(a+bx)\cosh^2(a+bx)}{15b} + \frac{e^{2a}e^{2bx}\cosh^3(a+bx)}{15b} & \text{for } b \neq \\ xe^{2a}\sinh(a)\cosh^2(a) & \text{otherw} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Piecewise((-4*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3/(15*b) + 8*exp(2*a)*exp
(2*b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(15*b) - 2*exp(2*a)*exp(2*b*x)*sinh
(a + b*x)*cosh(a + b*x)**2/(15*b) + exp(2*a)*exp(2*b*x)*cosh(a + b*x)**3/(
15*b), Ne(b, 0)), (x*exp(2*a)*sinh(a)*cosh(a)**2, True))
```

3.927.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh(a+bx)\,dx = \frac{\left(5\,e^{(-2\,bx-2\,a)} - 15\,e^{(-4\,bx-4\,a)} + 3\right)e^{(5\,bx+5\,a)}}{120\,b} + \frac{e^{(-bx-a)}}{8\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/120*(5*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) + 3)*e^(5*b*x + 5*a)/b + 1
/8*e^(-b*x - a)/b
```

3.927.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh(a+bx) \, dx = \frac{e^{(5\,bx+5\,a)}}{40\,b} + \frac{e^{(3\,bx+3\,a)}}{24\,b} - \frac{e^{(bx+a)}}{8\,b} + \frac{e^{(-bx-a)}}{8\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/40*e^(5*b*x + 5*a)/b + 1/24*e^(3*b*x + 3*a)/b - 1/8*e^(b*x + a)/b + 1/8*
e^(-b*x - a)/b
```

3.927.9 Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh(a+bx) \, dx = \frac{15\,\mathrm{e}^{-a-b\,x} - 15\,\mathrm{e}^{a+b\,x} + 5\,\mathrm{e}^{3\,a+3\,b\,x} + 3\,\mathrm{e}^{5\,a+5\,b\,x}}{120\,b}$$

input
```
int(cosh(a + b*x)^2*exp(2*a + 2*b*x)*sinh(a + b*x),x)
```

output
```
(15*exp(- a - b*x) - 15*exp(a + b*x) + 5*exp(3*a + 3*b*x) + 3*exp(5*a + 5*
b*x))/(120*b)
```

3.928 $\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx$

3.928.1 Optimal result

Integrand size = 22, antiderivative size = 45

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx=\frac{3e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

output
```
3/2*exp(b*x+a)/b+1/6*exp(3*b*x+3*a)/b-2*arctanh(exp(b*x+a))/b
```

3.928.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx=-\frac{e^{a+bx}\left(-3-\frac{1}{3}e^{2(a+bx)}+\frac{4\text{arctanh}\left(\sqrt{e^{2(a+bx)}}\right)}{\sqrt{e^{2(a+bx)}}}\right)}{2b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
-1/2*(E^(a + b*x)*(-3 - E^(2*(a + b*x))/3 + (4*ArcTanh[Sqrt[E^(2*(a + b*x)
)]])/Sqrt[E^(2*(a + b*x))]))/b
```

3.928.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2720, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{\left(1+e^{2a+2bx}\right)^2}{2\left(1-e^{2a+2bx}\right)}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(1+e^{2a+2bx}\right)^2}{1-e^{2a+2bx}}de^{a+bx}}{2b}$$

$$\Big\downarrow \text{300}$$

$$-\frac{\int \left(-e^{2a+2bx}-3+\frac{4}{1-e^{2a+2bx}}\right)de^{a+bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-4\text{arctanh}\left(e^{a+bx}\right)+3e^{a+bx}+\frac{1}{3}e^{3a+3bx}}{2b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
(3*E^(a + b*x) + E^(3*a + 3*b*x)/3 - 4*ArcTanh[E^(a + b*x)])/(2*b)
```

3.928.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.928.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{e^{3bx+3a}}{6b}+\frac{3\,e^{bx+a}}{2b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}-\frac{\ln\left(e^{bx+a}+1\right)}{b}$	54

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*exp(3*b*x+3*a)/b+3/2*exp(b*x+a)/b+1/b*ln(exp(b*x+a)-1)-1/b*ln(exp(b*x+
a)+1)
```

3.928.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(38) = 76.

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.18

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx$$
$$=\frac{\cosh\left(bx+a\right)^3+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+\sinh\left(bx+a\right)^3+3\,\left(\cosh\left(bx+a\right)^2+3\right)\sinh\left(bx+a\right)+9}{6\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 +
 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 9*cosh(b*x + a) - 6*log(cosh(b*x
+ a) + sinh(b*x + a) + 1) + 6*log(cosh(b*x + a) + sinh(b*x + a) - 1))/b
```

3.928.6 Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*csch(b*x+a),x)
```

output
```
Timed out
```

3.928.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx=\frac{\left(9\,e^{(-2\,bx-2\,a)}+1\right)e^{(3\,bx+3\,a)}}{6\,b}-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

output
```
1/6*(9*e^(-2*b*x - 2*a) + 1)*e^(3*b*x + 3*a)/b - log(e^(-b*x - a) + 1)/b +
 log(e^(-b*x - a) - 1)/b
```

3.928.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx=\frac{e^{(3\,bx+3\,a)}+9\,e^{(bx+a)}-6\,\log\left(e^{(bx+a)}+1\right)+6\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{6\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

output
```
1/6*(e^(3*b*x + 3*a) + 9*e^(b*x + a) - 6*log(e^(b*x + a) + 1) + 6*log(abs(
e^(b*x + a) - 1)))/b
```

3.928.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx=\frac{3\,\mathrm{e}^{a+b\,x}}{2\,b}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}+\frac{\mathrm{e}^{3\,a+3\,b\,x}}{6\,b}$$

input
```
int((cosh(a + b*x)^2*exp(2*a + 2*b*x))/sinh(a + b*x),x)
```

output
```
(3*exp(a + b*x))/(2*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)
^(1/2) + exp(3*a + 3*b*x)/(6*b)
```

3.929 $\int e^{2(a+bx)} \coth^2(a+bx)\,dx$

3.929.1 Optimal result

Integrand size = 18, antiderivative size = 59

$$\int e^{2(a+bx)} \coth^2(a+bx)\,dx = \frac{e^{2a+2bx}}{2b} + \frac{2}{b\left(1-e^{2a+2bx}\right)} + \frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b+2/b/(1-exp(2*b*x+2*a))+2*ln(1-exp(2*b*x+2*a))/b
```

3.929.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int e^{2(a+bx)} \coth^2(a+bx)\,dx = \frac{\frac{1}{2}e^{2(a+bx)} - \frac{2}{-1+e^{2(a+bx)}} + 2\log\left(1-e^{2(a+bx)}\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[a + b*x]^2,x]
```

output
```
(E^(2*(a + b*x))/2 - 2/(-1 + E^(2*(a + b*x))) + 2*Log[1 - E^(2*(a + b*x))]
)/b
```

3.929.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2720, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{a+bx}\left(1+e^{2a+2bx}\right)^2}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{\left(1+e^{2a+2bx}\right)^2}{\left(1-e^{2a+2bx}\right)^2}de^{2a+2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(1+\frac{4}{-1+e^{2a+2bx}}+\frac{4}{\left(-1+e^{2a+2bx}\right)^2}\right)de^{2a+2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a+2bx}+\frac{4}{1-e^{2a+2bx}}+4\log\left(1-e^{2a+2bx}\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Coth[a + b*x]^2,x]
```

output
```
(E^(2*a + 2*b*x) + 4/(1 - E^(2*a + 2*b*x)) + 4*Log[1 - E^(2*a + 2*b*x)])/(
2*b)
```

3.929.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.929.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}-\frac{4a}{b}-\frac{2}{b\left(e^{2bx+2a}-1\right)}+\frac{2\ln\left(e^{2bx+2a}-1\right)}{b}$	57

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b-4/b*a-2/b/(exp(2*b*x+2*a)-1)+2/b*ln(exp(2*b*x+2*a)-1)
```

3.929.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(52) = 104.

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.31

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + \left(6\cosh(bx+a)^2 - 1\right)\sinh(bx+a)^2 -}{2\,(b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas"
)
```

output
```
1/2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 (6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 4*(cosh(b*x +
 a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(2*sinh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 2*(2*cosh(b*x + a)^3 - cosh(b*x
+ a))*sinh(b*x + a) - 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x +
 a) + b*sinh(b*x + a)^2 - b)
```

3.929.6 Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.929.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx = \frac{4\,(bx+a)}{b} + \frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{5\,e^{(-2\,bx-2\,a)}-1}{2\,b(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)})}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima"
)
```

output
```
4*(b*x + a)/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b - 1/
2*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a)))
```

3.929.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx = -\frac{\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1} - e^{(2\,bx+2\,a)} - 4\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1) - e^(2*b*x + 2*a) - 4*log(ab
s(e^(2*b*x + 2*a) - 1)))/b
```

3.929.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx = \frac{2\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b} - \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} + \frac{\mathrm{e}^{2\,a+2\,b\,x}}{2\,b}$$

input
```
int((cosh(a + b*x)^2*exp(2*a + 2*b*x))/sinh(a + b*x)^2,x)
```

output
```
(2*log(exp(2*a)*exp(2*b*x) - 1))/b - 2/(b*(exp(2*a + 2*b*x) - 1)) + exp(2*
a + 2*b*x)/(2*b)
```

3.930 $\int e^{2(a+bx)} \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

3.930.1 Optimal result

Integrand size = 24, antiderivative size = 85

$$\int e^{2(a+bx)} \coth^2(a+bx)\text{csch}(a+bx)\,dx = \frac{2e^{a+bx}}{b} - \frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{5\text{arctanh}\left(e^{a+bx}\right)}{b}$$

output
```
2*exp(b*x+a)/b-2*exp(3*b*x+3*a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-e
xp(2*b*x+2*a))-5*arctanh(exp(b*x+a))/b
```

3.930.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.66 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.91

$$\int e^{2(a+bx)} \coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{e^{-3(a+bx)}\left(-21\left(56595+62725e^{2(a+bx)}-12071e^{4(a+bx)}-19353e^{6(a+bx)}+768e^{8(a+bx)}\right)+\frac{315\left(3773+2924e^{2(a}\right.}{}\right.}{}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
-1/10080*(-21*(56595 + 62725*E^(2*(a + b*x)) - 12071*E^(4*(a + b*x)) - 193
53*E^(6*(a + b*x)) + 768*E^(8*(a + b*x))) + (315*(3773 + 2924*E^(2*(a + b*
x)) - 2534*E^(4*(a + b*x)) - 1548*E^(6*(a + b*x)) + 297*E^(8*(a + b*x)))*A
rcTanh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 128*E^(8*(a + b*x))
*(9 + 16*E^(2*(a + b*x)) + 7*E^(4*(a + b*x)))*HypergeometricPFQ[{2, 2, 2,
5/2}, {1, 1, 11/2}, E^(2*(a + b*x))] + 128*E^(8*(a + b*x))*(1 + E^(2*(a +
b*x)))^2*HypergeometricPFQ[{2, 2, 2, 2, 5/2}, {1, 1, 1, 11/2}, E^(2*(a + b
*x))])/(b*E^(3*(a + b*x)))
```

3.930.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2720, 27, 366, 27, 360, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \coth^2(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{2a+2bx}\left(1+e^{2a+2bx}\right)^2}{\left(1-e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int \frac{e^{2a+2bx}\left(1+e^{2a+2bx}\right)^2}{\left(1-e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{366}$$

$$-\frac{2\left(\frac{e^{3a+3bx}}{\left(1-e^{2a+2bx}\right)^2}-\frac{1}{4}\int \frac{4e^{2a+2bx}\left(2+e^{2a+2bx}\right)}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\left(\frac{e^{3a+3bx}}{\left(1-e^{2a+2bx}\right)^2}-\int \frac{e^{2a+2bx}\left(2+e^{2a+2bx}\right)}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{360}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{3+2e^{2a+2bx}}{1-e^{2a+2bx}}de^{a+bx}-\frac{3e^{a+bx}}{2(1-e^{2a+2bx})}+\frac{e^{3a+3bx}}{(1-e^{2a+2bx})^2}\right)}{b}$$

$$\Big\downarrow \text{299}$$

$$-\frac{2\left(\frac{1}{2}\left(5\int \frac{1}{1-e^{2a+2bx}}de^{a+bx}-2e^{a+bx}\right)-\frac{3e^{a+bx}}{2(1-e^{2a+2bx})}+\frac{e^{3a+3bx}}{(1-e^{2a+2bx})^2}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(\frac{1}{2}\left(5\text{arctanh}\left(e^{a+bx}\right)-2e^{a+bx}\right)-\frac{3e^{a+bx}}{2(1-e^{2a+2bx})}+\frac{e^{3a+3bx}}{(1-e^{2a+2bx})^2}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
(-2*(E^(3*a + 3*b*x)/(1 - E^(2*a + 2*b*x))^2 - (3*E^(a + b*x))/(2*(1 - E^(
2*a + 2*b*x))) + (-2*E^(a + b*x) + 5*ArcTanh[E^(a + b*x)])/2))/b
```

3.930.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.930. $\int e^{2(a+bx)}\coth^2(a+bx)\text{csch}(a+bx)\,dx$

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.930.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{2\,{\mathrm e}^{b x +a}}{b}-\frac{{\mathrm e}^{b x +a} \left(5\,{\mathrm e}^{2 b x +2 a}-3\right)}{b \left({\mathrm e}^{2 b x +2 a}-1\right)^{2}}+\frac{5 \ln \left({\mathrm e}^{b x +a}-1\right)}{2 b}-\frac{5 \ln \left({\mathrm e}^{b x +a}+1\right)}{2 b}$	78

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*exp(b*x+a)/b-exp(b*x+a)*(5*exp(2*b*x+2*a)-3)/b/(exp(2*b*x+2*a)-1)^2+5/2/
b*ln(exp(b*x+a)-1)-5/2/b*ln(exp(b*x+a)+1)
```

3.930. $\int e^{2(a+bx)}\coth^2(a+bx)\text{csch}(a+bx)\,dx$

3.930.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(75) = 150$.

Time = 0.27 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.40

$$\int e^{2(a+bx)}\coth^2(a+bx)\text{csch}(a+bx)\,dx$$
$$= \frac{4\cosh(bx+a)^5 + 20\cosh(bx+a)\sinh(bx+a)^4 + 4\sinh(bx+a)^5 + 2\left(20\cosh(bx+a)^2 - 9\right)\sinh(bx}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas"
)
```

output
```
1/2*(4*cosh(b*x + a)^5 + 20*cosh(b*x + a)*sinh(b*x + a)^4 + 4*sinh(b*x + a
)^5 + 2*(20*cosh(b*x + a)^2 - 9)*sinh(b*x + a)^3 - 18*cosh(b*x + a)^3 + 2*
(20*cosh(b*x + a)^3 - 27*cosh(b*x + a))*sinh(b*x + a)^2 - 5*(cosh(b*x + a)
^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a
)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b
*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 5*(co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*c
osh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)
^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) -
 1) + 2*(10*cosh(b*x + a)^4 - 27*cosh(b*x + a)^2 + 5)*sinh(b*x + a) + 10*c
osh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*s
inh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*
x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

3.930.6 Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\coth^2(a+bx)\text{csch}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.930.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13

$$\int e^{2(a+bx)}\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{5\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{5\log\left(e^{(-bx-a)}-1\right)}{2\,b} - \frac{9\,e^{(-2\,bx-2\,a)}-5\,e^{(-4\,bx-4\,a)}-2}{b\left(e^{(-bx-a)}-2\,e^{(-3\,bx-3\,a)}+e^{(-5\,bx-5\,a)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima"
)
```

output
```
-5/2*log(e^(-b*x - a) + 1)/b + 5/2*log(e^(-b*x - a) - 1)/b - (9*e^(-2*b*x
- 2*a) - 5*e^(-4*b*x - 4*a) - 2)/(b*(e^(-b*x - a) - 2*e^(-3*b*x - 3*a) + e
^(-5*b*x - 5*a)))
```

3.930.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int e^{2(a+bx)}\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{\frac{2\left(5\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2}-4\,e^{(bx+a)}+5\log\left(e^{(bx+a)}+1\right)-5\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*(2*(5*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 - 4*e^
(b*x + a) + 5*log(e^(b*x + a) + 1) - 5*log(abs(e^(b*x + a) - 1)))/b
```

3.930.9 Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15

$$\int e^{2(a+bx)}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \frac{2\,e^{a+b\,x}}{b}-\frac{5\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{5\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input

```
int((cosh(a + b*x)^2*exp(2*a + 2*b*x))/sinh(a + b*x)^3,x)
```

output

```
(2*exp(a + b*x))/b - (5*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/
2) - (2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (5
*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.931 $\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^3(a+bx)\, dx$

3.931.1 Optimal result

Integrand size = 26, antiderivative size = 57

$$\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^3(a+bx)\, dx = \frac{e^{-4a-4bx}}{256b} - \frac{3e^{4a+4bx}}{256b} + \frac{e^{8a+8bx}}{512b} + \frac{3x}{64}$$

output
```
1/256*exp(-4*b*x-4*a)/b-3/256*exp(4*b*x+4*a)/b+1/512*exp(8*b*x+8*a)/b+3/64
*x
```

3.931.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^3(a+bx)\, dx = \frac{e^{-4(a+bx)} - 3e^{4(a+bx)} + \frac{1}{2}e^{8(a+bx)} + 12bx}{256b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(E^(-4*(a + b*x)) - 3*E^(4*(a + b*x)) + E^(8*(a + b*x))/2 + 12*b*x)/(256*b
)
```

3.931.3 Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2720, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh^3(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{64}e^{-5a-5bx}\left(1-e^{4a+4bx}\right)^3de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int e^{-5a-5bx}\left(1-e^{4a+4bx}\right)^3de^{a+bx}}{64b}$$

$$\Big\downarrow \text{798}$$

$$-\frac{\int e^{-2a-2bx}\left(1-e^{4a+4bx}\right)^3de^{4a+4bx}}{256b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(3+e^{-2a-2bx}-3e^{-a-bx}-e^{4a+4bx}\right)de^{4a+4bx}}{256b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-e^{-a-bx}-\frac{1}{2}e^{2a+2bx}+3e^{4a+4bx}-3\log\left(e^{4a+4bx}\right)}{256b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
-1/256*(-E^(-a - b*x) - E^(2*a + 2*b*x)/2 + 3*E^(4*a + 4*b*x) - 3*Log[E^(4
*a + 4*b*x)])/b
```

3.931.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.931.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\frac{3x}{64}-\frac{\sinh\left(4bx+4a\right)}{64b}+\frac{\sinh\left(8bx+8a\right)}{512b}-\frac{\cosh\left(4bx+4a\right)}{128b}+\frac{\cosh\left(8bx+8a\right)}{512b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

output
```
3/64*x-1/64/b*sinh(4*b*x+4*a)+1/512/b*sinh(8*b*x+8*a)-1/128*cosh(4*b*x+4*a
)/b+1/512*cosh(8*b*x+8*a)/b
```

3.931.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.26

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{3\cosh(bx+a)^6 - 20\cosh(bx+a)^3\sinh(bx+a)^3 + 45\cosh(bx+a)^2\sinh(bx+a)^4 - 6\cosh(bx+a)\mathrm{si}}{512}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas"
)
```

output
```
1/512*(3*cosh(b*x + a)^6 - 20*cosh(b*x + a)^3*sinh(b*x + a)^3 + 45*cosh(b*
x + a)^2*sinh(b*x + a)^4 - 6*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x +
a)^6 + 6*(4*b*x - 1)*cosh(b*x + a)^2 + 3*(15*cosh(b*x + a)^4 + 8*b*x - 2)*
sinh(b*x + a)^2 - 6*(cosh(b*x + a)^5 + 2*(4*b*x + 1)*cosh(b*x + a))*sinh(b
*x + a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x
 + a)^2)
```

3.931.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(48) = 96$.

Time = 12.49 (sec) , antiderivative size = 382, normalized size of antiderivative = 6.70

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} \frac{3xe^{2a}e^{2bx}\sinh^6(a+bx)}{64} - \frac{3xe^{2a}e^{2bx}\sinh^5(a+bx)\cosh(a+bx)}{32} - \frac{3xe^{2a}e^{2bx}\sinh^4(a+bx)\cosh^2(a+bx)}{64} + \frac{3xe^{2a}e^{2bx}\sinh^3(a+bx)\cosh^3(}{16} \\ xe^{2a}\sinh^3(a)\cosh^3(a) \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Piecewise((3*x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**6/64 - 3*x*exp(2*a)*exp(
2*b*x)*sinh(a + b*x)**5*cosh(a + b*x)/32 - 3*x*exp(2*a)*exp(2*b*x)*sinh(a
+ b*x)**4*cosh(a + b*x)**2/64 + 3*x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3*c
osh(a + b*x)**3/16 - 3*x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2*cosh(a + b*x
)**4/64 - 3*x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x)**5/32 + 3*x*
exp(2*a)*exp(2*b*x)*cosh(a + b*x)**6/64 + 3*exp(2*a)*exp(2*b*x)*sinh(a + b
*x)**6/(32*b) - 15*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**5*cosh(a + b*x)/(64*
b) + 13*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3*cosh(a + b*x)**3/(32*b) - 15*
exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x)**5/(64*b) + 3*exp(2*a)*exp
(2*b*x)*cosh(a + b*x)**6/(32*b), Ne(b, 0)), (x*exp(2*a)*sinh(a)**3*cosh(a)
**3, True))
```

3.931.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(6\,e^{(-4\,bx-4\,a)}-1\right)e^{(8\,bx+8\,a)}}{512\,b} + \frac{3\,(bx+a)}{64\,b} + \frac{e^{(-4\,bx-4\,a)}}{256\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima"
)
```

output
```
-1/512*(6*e^(-4*b*x - 4*a) - 1)*e^(8*b*x + 8*a)/b + 3/64*(b*x + a)/b + 1/2
56*e^(-4*b*x - 4*a)/b
```

3.931.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{3}{64}\,x + \frac{e^{(8\,bx+8\,a)}}{512\,b} - \frac{3\,e^{(4\,bx+4\,a)}}{256\,b} + \frac{e^{(-4\,bx-4\,a)}}{256\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
3/64*x + 1/512*e^(8*b*x + 8*a)/b - 3/256*e^(4*b*x + 4*a)/b + 1/256*e^(-4*b
*x - 4*a)/b
```

3.931.9 Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{3\,x}{64}+\frac{\mathrm{e}^{-4\,a-4\,b\,x}}{256\,b}-\frac{3\,\mathrm{e}^{4\,a+4\,b\,x}}{256\,b}+\frac{\mathrm{e}^{8\,a+8\,b\,x}}{512\,b}$$

input
```
int(cosh(a + b*x)^3*exp(2*a + 2*b*x)*sinh(a + b*x)^3,x)
```

output
```
(3*x)/64 + exp(- 4*a - 4*b*x)/(256*b) - (3*exp(4*a + 4*b*x))/(256*b) + exp
(8*a + 8*b*x)/(512*b)
```

3.932 $\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$

3.932.1 Optimal result

Integrand size = 26, antiderivative size = 100

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx=-\frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}-\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}+\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}$$

output
```
-1/96*exp(-3*b*x-3*a)/b-1/32*exp(-b*x-a)/b-1/16*exp(b*x+a)/b-1/48*exp(3*b*
x+3*a)/b+1/160*exp(5*b*x+5*a)/b+1/224*exp(7*b*x+7*a)/b
```

3.932.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx=\frac{e^{-3(a+bx)}\left(-35-105e^{2(a+bx)}-210e^{4(a+bx)}-70e^{6(a+bx)}+21e^{8(a+bx)}+15e^{10(a+bx)}\right)}{3360b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(-35 - 105*E^(2*(a + b*x)) - 210*E^(4*(a + b*x)) - 70*E^(6*(a + b*x)) + 21
*E^(8*(a + b*x)) + 15*E^(10*(a + b*x)))/(3360*b*E^(3*(a + b*x)))
```

3.932.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2720, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh^2(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{32}e^{-4a-4bx}\left(1-e^{2a+2bx}\right)^2\left(1+e^{2a+2bx}\right)^3de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int e^{-4a-4bx}\left(1-e^{2a+2bx}\right)^2\left(1+e^{2a+2bx}\right)^3de^{a+bx}}{32b}$$

$$\Big\downarrow \text{355}$$

$$\frac{\int \left(-2+e^{-4a-4bx}+e^{-2a-2bx}-2e^{2a+2bx}+e^{4a+4bx}+e^{6a+6bx}\right)de^{a+bx}}{32b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}e^{-3a-3bx}-e^{-a-bx}-2e^{a+bx}-\frac{2}{3}e^{3a+3bx}+\frac{1}{5}e^{5a+5bx}+\frac{1}{7}e^{7a+7bx}}{32b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(-1/3*E^(-3*a - 3*b*x) - E^(-a - b*x) - 2*E^(a + b*x) - (2*E^(3*a + 3*b*x)
)/3 + E^(5*a + 5*b*x)/5 + E^(7*a + 7*b*x)/7)/(32*b)
```

3.932.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.932.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08

$$-\frac{\sinh(bx+a)}{32b}-\frac{\sinh(3bx+3a)}{96b}+\frac{\sinh(5bx+5a)}{160b}+\frac{\sinh(7bx+7a)}{224b}-\frac{3\cosh(bx+a)}{32b}-\frac{\cosh(3bx+3a)}{32b}+\frac{\text{co}}{}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

output
```
-1/32*sinh(b*x+a)/b-1/96/b*sinh(3*b*x+3*a)+1/160/b*sinh(5*b*x+5*a)+1/224/b
*sinh(7*b*x+7*a)-3/32*cosh(b*x+a)/b-1/32*cosh(3*b*x+3*a)/b+1/160*cosh(5*b*
x+5*a)/b+1/224*cosh(7*b*x+7*a)/b
```

3.932.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(82) = 164.

Time = 0.27 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.76

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{10\cosh(bx+a)^5+50\cosh(bx+a)\sinh(bx+a)^4-25\sinh(bx+a)^5-\left(250\cosh(bx+a)^2+63\right)\sin}{1}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas"
)
```

output
```
-1/1680*(10*cosh(b*x + a)^5 + 50*cosh(b*x + a)*sinh(b*x + a)^4 - 25*sinh(b
*x + a)^5 - (250*cosh(b*x + a)^2 + 63)*sinh(b*x + a)^3 + 42*cosh(b*x + a)^
3 + 2*(50*cosh(b*x + a)^3 + 63*cosh(b*x + a))*sinh(b*x + a)^2 - (125*cosh(
b*x + a)^4 + 189*cosh(b*x + a)^2 + 70)*sinh(b*x + a) + 140*cosh(b*x + a))/
(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.932.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(76) = 152$.

Time = 5.49 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.97

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\begin{cases}\frac{2e^{2a}e^{2bx}\sinh^5(a+bx)}{105b}-\frac{4e^{2a}e^{2bx}\sinh^4(a+bx)\cosh(a+bx)}{105b}-\frac{e^{2a}e^{2bx}\sinh^3(a+bx)\cosh^2(a+bx)}{105b}+\frac{2e^{2a}e^{2bx}\sinh^2(a+bx)\cosh^3(a+bx)}{35b}\\ xe^{2a}\sinh^2(a)\cosh^3(a)\end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**5/(105*b) - 4*exp(2*a)*exp
(2*b*x)*sinh(a + b*x)**4*cosh(a + b*x)/(105*b) - exp(2*a)*exp(2*b*x)*sinh(
a + b*x)**3*cosh(a + b*x)**2/(105*b) + 2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)
**2*cosh(a + b*x)**3/(35*b) + 8*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a +
 b*x)**4/(35*b) - 4*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**5/(35*b), Ne(b, 0))
, (x*exp(2*a)*sinh(a)**2*cosh(a)**3, True))
```

3.932.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{\left(21\,e^{(-2\,bx-2\,a)}-70\,e^{(-4\,bx-4\,a)}-210\,e^{(-6\,bx-6\,a)}+15\right)e^{(7\,bx+7\,a)}}{3360\,b}-\frac{3\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{96\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima"
)
```

output
```
1/3360*(21*e^(-2*b*x - 2*a) - 70*e^(-4*b*x - 4*a) - 210*e^(-6*b*x - 6*a) +
 15)*e^(7*b*x + 7*a)/b - 1/96*(3*e^(-b*x - a) + e^(-3*b*x - 3*a))/b
```

3.932.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(7\,bx+7\,a)}}{224\,b} + \frac{e^{(5\,bx+5\,a)}}{160\,b} - \frac{e^{(3\,bx+3\,a)}}{48\,b} - \frac{e^{(bx+a)}}{16\,b} - \frac{e^{(-bx-a)}}{32\,b} - \frac{e^{(-3\,bx-3\,a)}}{96\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/224*e^(7*b*x + 7*a)/b + 1/160*e^(5*b*x + 5*a)/b - 1/48*e^(3*b*x + 3*a)/b
 - 1/16*e^(b*x + a)/b - 1/32*e^(-b*x - a)/b - 1/96*e^(-3*b*x - 3*a)/b
```

3.932.9 Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{210\,\mathrm{e}^{a+b\,x} + 105\,\mathrm{e}^{-a-b\,x} + 35\,\mathrm{e}^{-3\,a-3\,b\,x} + 70\,\mathrm{e}^{3\,a+3\,b\,x} - 21\,\mathrm{e}^{5\,a+5\,b\,x} - 15\,\mathrm{e}^{7\,a+7\,b\,x}}{3360\,b}$$

input
```
int(cosh(a + b*x)^3*exp(2*a + 2*b*x)*sinh(a + b*x)^2,x)
```

output
```
-(210*exp(a + b*x) + 105*exp(- a - b*x) + 35*exp(- 3*a - 3*b*x) + 70*exp(3
*a + 3*b*x) - 21*exp(5*a + 5*b*x) - 15*exp(7*a + 7*b*x))/(3360*b)
```

3.933 $\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx$

3.933.1 Optimal result

Integrand size = 24, antiderivative size = 57

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{e^{-2a-2bx}}{32b}+\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}-\frac{x}{8}$$

output
```
1/32*exp(-2*b*x-2*a)/b+1/32*exp(4*b*x+4*a)/b+1/96*exp(6*b*x+6*a)/b-1/8*x
```

3.933.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{3e^{-2(a+bx)}+3e^{4(a+bx)}+e^{6(a+bx)}-12bx}{96b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
(3/E^(2*(a + b*x)) + 3*E^(4*(a + b*x)) + E^(6*(a + b*x)) - 12*b*x)/(96*b)
```

3.933.3 Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2720, 27, 354, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{16}e^{-3a-3bx}\left(1-e^{2a+2bx}\right)\left(1+e^{2a+2bx}\right)^3de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int e^{-3a-3bx}\left(1-e^{2a+2bx}\right)\left(1+e^{2a+2bx}\right)^3de^{a+bx}}{16b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int e^{-2a-2bx}\left(1-e^{2a+2bx}\right)\left(1+e^{2a+2bx}\right)^3de^{2a+2bx}}{32b}$$

$$\Big\downarrow \text{84}$$

$$-\frac{\int \left(e^{-2a-2bx}+2e^{-a-bx}-3e^{2a+2bx}\right)de^{2a+2bx}}{32b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-e^{-a-bx}-e^{2a+2bx}-\frac{1}{3}e^{3a+3bx}+2\log\left(e^{2a+2bx}\right)}{32b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
-1/32*(-E^(-a - b*x) - E^(2*a + 2*b*x) - E^(3*a + 3*b*x)/3 + 2*Log[E^(2*a
+ 2*b*x)])/b
```

3.933.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.933.4 Maple [A] (verified)

Time = 48.70 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{e^{-2bx-2a}}{32b}+\frac{e^{4bx+4a}}{32b}+\frac{e^{6bx+6a}}{96b}-\frac{x}{8}$	47
default	$-\frac{x}{8}-\frac{\sinh(2bx+2a)}{32b}+\frac{\sinh(4bx+4a)}{32b}+\frac{\sinh(6bx+6a)}{96b}+\frac{\cosh(2bx+2a)}{32b}+\frac{\cosh(4bx+4a)}{32b}+\frac{\cosh(6bx+6a)}{96b}$	89

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/32*exp(-2*b*x-2*a)/b+1/32*exp(4*b*x+4*a)/b+1/96*exp(6*b*x+6*a)/b-1/8*x
```

3.933.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.67

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$=\frac{4\cosh(bx+a)^4-8\cosh(bx+a)\sinh(bx+a)^3+4\sinh(bx+a)^4-3(4bx-1)\cosh(bx+a)^2-3(4b}{96\left(b\cosh(bx+a)^2-2b\cosh(bx+\right.}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/96*(4*cosh(b*x + a)^4 - 8*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a
)^4 - 3*(4*b*x - 1)*cosh(b*x + a)^2 - 3*(4*b*x - 8*cosh(b*x + a)^2 - 1)*si
nh(b*x + a)^2 - 2*(4*cosh(b*x + a)^3 - 3*(4*b*x + 1)*cosh(b*x + a))*sinh(b
*x + a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x
 + a)^2)
```

3.933.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(44) = 88.

Time = 2.33 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.09

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$=\begin{cases}\frac{xe^{2a}e^{2bx}\sinh^4(a+bx)}{8}-\frac{xe^{2a}e^{2bx}\sinh^3(a+bx)\cosh(a+bx)}{4}+\frac{xe^{2a}e^{2bx}\sinh(a+bx)\cosh^3(a+bx)}{4}-\frac{xe^{2a}e^{2bx}\cosh^4(a+bx)}{8}+\frac{e^{2a}e^{2bx}}{} \\ xe^{2a}\sinh(a)\cosh^3(a)\end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**4/8 - x*exp(2*a)*exp(2*b*x
)*sinh(a + b*x)**3*cosh(a + b*x)/4 + x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*c
osh(a + b*x)**3/4 - x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/8 + exp(2*a)*ex
p(2*b*x)*sinh(a + b*x)**4/(48*b) - exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3*co
sh(a + b*x)/(6*b) + exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/
(4*b) + exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/(16*b), Ne(b, 0)), (x*exp(2*a
)*sinh(a)*cosh(a)**3, True))
```

3.933.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\left(3\,e^{(-2\,bx-2\,a)}+1\right)e^{(6\,bx+6\,a)}}{96\,b} - \frac{bx+a}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/96*(3*e^(-2*b*x - 2*a) + 1)*e^(6*b*x + 6*a)/b - 1/8*(b*x + a)/b + 1/32*e
^(-2*b*x - 2*a)/b
```

3.933.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx = -\frac{1}{8}\,x + \frac{e^{(6\,bx+6\,a)}}{96\,b} + \frac{e^{(4\,bx+4\,a)}}{32\,b} + \frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
-1/8*x + 1/96*e^(6*b*x + 6*a)/b + 1/32*e^(4*b*x + 4*a)/b + 1/32*e^(-2*b*x
- 2*a)/b
```

3.933.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\frac{e^{-2\,a-2\,b\,x}}{32} + \frac{e^{4\,a+4\,b\,x}}{32} + \frac{e^{6\,a+6\,b\,x}}{96}}{b} - \frac{x}{8}$$

input
```
int(cosh(a + b*x)^3*exp(2*a + 2*b*x)*sinh(a + b*x),x)
```

output
```
(exp(- 2*a - 2*b*x)/32 + exp(4*a + 4*b*x)/32 + exp(6*a + 6*b*x)/96)/b - x/
8
```

3.934 $\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx$

3.934.1 Optimal result

Integrand size = 24, antiderivative size = 59

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx=\frac{e^{2a+2bx}}{2b}+\frac{e^{4a+4bx}}{16b}-\frac{x}{4}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+4*a)/b-1/4*x+ln(1-exp(2*b*x+2*a))/b
```

3.934.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx=\frac{8e^{2(a+bx)}+e^{4(a+bx)}-4bx+16\log\left(1-e^{2(a+bx)}\right)}{16b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(8*E^(2*(a + b*x)) + E^(4*(a + b*x)) - 4*b*x + 16*Log[1 - E^(2*(a + b*x))]
)/(16*b)
```

3.934.3 Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2720, 27, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{-a-bx}\left(1+e^{2a+2bx}\right)^3}{4\left(1-e^{2a+2bx}\right)}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-a-bx}\left(1+e^{2a+2bx}\right)^3}{1-e^{2a+2bx}}de^{a+bx}}{4b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{e^{-a-bx}\left(1+e^{2a+2bx}\right)^3}{1-e^{2a+2bx}}de^{2a+2bx}}{8b}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\int \left(e^{-a-bx}-e^{2a+2bx}-4-\frac{8}{-1+e^{2a+2bx}}\right)de^{2a+2bx}}{8b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{9}{2}e^{2a+2bx}+\log\left(e^{2a+2bx}\right)-8\log\left(1-e^{2a+2bx}\right)}{8b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
-1/8*((-9*E^(2*a + 2*b*x))/2 + Log[E^(2*a + 2*b*x)] - 8*Log[1 - E^(2*a + 2
*b*x)])/b
```

3.934.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.934.4 Maple [A] (verified)

Time = 5.52 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{x}{4}+\frac{{\mathrm e}^{4 b x +4 a}}{16 b}+\frac{{\mathrm e}^{2 b x +2 a}}{2 b}-\frac{2 a}{b}+\frac{\ln \left({\mathrm e}^{2 b x +2 a}-1\right)}{b}$	55

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x+1/16*exp(4*b*x+4*a)/b+1/2*exp(2*b*x+2*a)/b-2/b*a+1/b*ln(exp(2*b*x+2
*a)-1)
```

3.934.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(50) = 100$.

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.15

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2+4\right)\sinh(bx+a)^2}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

output
```
1/16*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
+ 2*(3*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^2 - 4*b*x + 8*cosh(b*x + a)^2 +
4*(cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a) + 16*log(2*sinh(b*x +
a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

3.934.6 Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*csch(b*x+a),x)
```

output
```
Timed out
```

3.934.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\left(8\,e^{(-2\,bx-2\,a)}+1\right)e^{(4\,bx+4\,a)}}{16\,b} + \frac{7\,(bx+a)}{4\,b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

output
```
1/16*(8*e^(-2*b*x - 2*a) + 1)*e^(4*b*x + 4*a)/b + 7/4*(b*x + a)/b + log(e^
(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

3.934.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{4\,bx+4\,a-e^{(4\,bx+4\,a)}-8\,e^{(2\,bx+2\,a)}-16\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{16\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

output
```
-1/16*(4*b*x + 4*a - e^(4*b*x + 4*a) - 8*e^(2*b*x + 2*a) - 16*log(abs(e^(2
*b*x + 2*a) - 1)))/b
```

3.934.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\ln\left(e^{2\,a}\,e^{2\,b\,x}-1\right)}{b}-\frac{x}{4}+\frac{e^{2\,a+2\,b\,x}}{2\,b}+\frac{e^{4\,a+4\,b\,x}}{16\,b}$$

input
```
int((cosh(a + b*x)^3*exp(2*a + 2*b*x))/sinh(a + b*x),x)
```

output
```
log(exp(2*a)*exp(2*b*x) - 1)/b - x/4 + exp(2*a + 2*b*x)/(2*b) + exp(4*a +
4*b*x)/(16*b)
```

3.935 $\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx$

3.935.1 Optimal result

Integrand size = 24, antiderivative size = 73

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx=\frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\operatorname{arctanh}\left(e^{a+bx}\right)}{b}$$

output
```
5/2*exp(b*x+a)/b+1/6*exp(3*b*x+3*a)/b+2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-4*
arctanh(exp(b*x+a))/b
```

3.935.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.01 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.01

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx$$
$$=\frac{e^{-5(a+bx)}\left(-21\left(36015+91925e^{2(a+bx)}+61158e^{4(a+bx)}-20166e^{6(a+bx)}-15061e^{8(a+bx)}+753e^{10(a+bx)}\right)-\right.}{}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
(-21*(36015 + 91925*E^(2*(a + b*x)) + 61158*E^(4*(a + b*x)) - 20166*E^(6*(
a + b*x)) - 15061*E^(8*(a + b*x)) + 753*E^(10*(a + b*x))) - (315*(-2401 -
5328*E^(2*(a + b*x)) - 1821*E^(4*(a + b*x)) + 3264*E^(6*(a + b*x)) + 1149*
E^(8*(a + b*x)) - 240*E^(10*(a + b*x)) + E^(12*(a + b*x)))*ArcTanh[Sqrt[E^
(2*(a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 256*E^(8*(a + b*x))*(1 + E^(2*(a
+ b*x)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, E^(2*(a +
 b*x))])/(60480*b*E^(5*(a + b*x)))
```

3.935.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2720, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{\left(1+e^{2a+2bx}\right)^3}{2\left(1-e^{2a+2bx}\right)^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(1+e^{2a+2bx}\right)^3}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}}{2b}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(-\frac{4\left(1-3e^{2a+2bx}\right)}{\left(1-e^{2a+2bx}\right)^2}+e^{2a+2bx}+5\right)de^{a+bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-8\text{arctanh}\left(e^{a+bx}\right)+5e^{a+bx}+\frac{1}{3}e^{3a+3bx}+\frac{4e^{a+bx}}{1-e^{2a+2bx}}}{2b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
(5*E^(a + b*x) + E^(3*a + 3*b*x)/3 + (4*E^(a + b*x))/(1 - E^(2*a + 2*b*x))
 - 8*ArcTanh[E^(a + b*x)])/(2*b)
```

3.935.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.935.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

method	result	size
risch	$\frac{e^{3bx+3a}}{6b}+\frac{5\,e^{bx+a}}{2b}-\frac{2\,e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}-\frac{2\ln\left(e^{bx+a}+1\right)}{b}+\frac{2\ln\left(e^{bx+a}-1\right)}{b}$	79

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*exp(3*b*x+3*a)/b+5/2*exp(b*x+a)/b-2/b*exp(b*x+a)/(exp(2*b*x+2*a)-1)-2/
b*ln(exp(b*x+a)+1)+2/b*ln(exp(b*x+a)-1)
```

3.935.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(62) = 124.

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.73

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^5 + 5\cosh(bx+a)\sinh(bx+a)^4 + \sinh(bx+a)^5 + 2\left(5\cosh(bx+a)^2+7\right)\sinh(bx+a)^3}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas"
)
```

output
```
1/6*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 +
 2*(5*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^3 + 14*cosh(b*x + a)^3 + 2*(5*cos
h(b*x + a)^3 + 21*cosh(b*x + a))*sinh(b*x + a)^2 - 12*(cosh(b*x + a)^2 + 2
*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + si
nh(b*x + a) + 1) + 12*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + s
inh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (5*cosh(b*x +
 a)^4 + 42*cosh(b*x + a)^2 - 27)*sinh(b*x + a) - 27*cosh(b*x + a))/(b*cosh
(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

3.935.6 Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.935.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{2\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{14\,e^{(-2\,bx-2\,a)}-27\,e^{(-4\,bx-4\,a)}+1}{6\,b\left(e^{(-3\,bx-3\,a)}-e^{(-5\,bx-5\,a)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima"
)
```

output
```
-2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b + 1/6*(14*e^(-2*b*x
 - 2*a) - 27*e^(-4*b*x - 4*a) + 1)/(b*(e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a)
))
```

3.935.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{\frac{12\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1}-e^{(3\,bx+3\,a)}-15\,e^{(bx+a)}+12\log\left(e^{(bx+a)}+1\right)-12\log\left(\left|e^{(bx+a)}-1\right|\right)}{6\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/6*(12*e^(b*x + a)/(e^(2*b*x + 2*a) - 1) - e^(3*b*x + 3*a) - 15*e^(b*x +
 a) + 12*log(e^(b*x + a) + 1) - 12*log(abs(e^(b*x + a) - 1)))/b
```

3.935.9 Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{5\,\mathrm{e}^{a+b\,x}}{2\,b}-\frac{4\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}+\frac{\mathrm{e}^{3\,a+3\,b\,x}}{6\,b}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int((cosh(a + b*x)^3*exp(2*a + 2*b*x))/sinh(a + b*x)^2,x)
```

output
```
(5*exp(a + b*x))/(2*b) - (4*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)
^(1/2) + exp(3*a + 3*b*x)/(6*b) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) -
1))
```

3.935. $\int e^{2(a+bx)} \cosh(a+bx) \coth^2(a+bx)\, dx$

3.936 $\int e^{2(a+bx)}\coth^3(a+bx)\,dx$

3.936.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx=\frac{e^{2a+2bx}}{2b}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{6}{b\left(1-e^{2a+2bx}\right)}+\frac{3\log\left(1-e^{2a+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b-2/b/(1-exp(2*b*x+2*a))^2+6/b/(1-exp(2*b*x+2*a))+3*ln(
1-exp(2*b*x+2*a))/b
```

3.936.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx=\frac{\frac{1}{2}e^{2(a+bx)}+\frac{4-6e^{2(a+bx)}}{\left(-1+e^{2(a+bx)}\right)^2}+3\log\left(1-e^{2(a+bx)}\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[a + b*x]^3,x]
```

output
```
(E^(2*(a + b*x))/2 + (4 - 6*E^(2*(a + b*x)))/(-1 + E^(2*(a + b*x)))^2 + 3*
Log[1 - E^(2*(a + b*x))])/b
```

3.936.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2720, 25, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{a+bx}\left(1+e^{2a+2bx}\right)^3}{\left(1-e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e^{a+bx}\left(1+e^{2a+2bx}\right)^3}{\left(1-e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$-\frac{\int \frac{\left(1+e^{2a+2bx}\right)^3}{\left(1-e^{2a+2bx}\right)^3}de^{2a+2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(-1-\frac{6}{-1+e^{2a+2bx}}-\frac{12}{\left(-1+e^{2a+2bx}\right)^2}-\frac{8}{\left(-1+e^{2a+2bx}\right)^3}\right)de^{2a+2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-e^{2a+2bx}-\frac{12}{1-e^{2a+2bx}}+\frac{4}{\left(1-e^{2a+2bx}\right)^2}-6\log\left(1-e^{2a+2bx}\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Coth[a + b*x]^3,x]
```

output
```
-1/2*(-E^(2*a + 2*b*x) + 4/(1 - E^(2*a + 2*b*x))^2 - 12/(1 - E^(2*a + 2*b*
x)) - 6*Log[1 - E^(2*a + 2*b*x)])/b
```

3.936.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.936.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}-\frac{6a}{b}-\frac{2\left(3\,e^{2bx+2a}-2\right)}{b\left(e^{2bx+2a}-1\right)^{2}}+\frac{3\ln\left(e^{2bx+2a}-1\right)}{b}$	70

input
```
int(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b-6/b*a-2*(3*exp(2*b*x+2*a)-2)/b/(exp(2*b*x+2*a)-1)^2+3
/b*ln(exp(2*b*x+2*a)-1)
```

3.936.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.98

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + \left(15\cosh(bx+a)^2 - 2\right)\sinh(bx+a)^4}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas"
)
```

output
```
1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 (15*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 4*(5*cosh(
b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 12*c
osh(b*x + a)^2 - 11)*sinh(b*x + a)^2 - 11*cosh(b*x + a)^2 + 6*(cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
 a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh
(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*
x + a))) + 2*(3*cosh(b*x + a)^5 - 4*cosh(b*x + a)^3 - 11*cosh(b*x + a))*si
nh(b*x + a) + 8)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 +
b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh
(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

3.936.6 Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.936.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.32

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx = \frac{6\,(bx+a)}{b} + \frac{3\,\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{3\,\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{10\,e^{(-2\,bx-2\,a)} - 5\,e^{(-4\,bx-4\,a)} - 1}{2\,b(e^{(-2\,bx-2\,a)} - 2\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)})}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima"
)
```

output
```
6*(b*x + a)/b + 3*log(e^(-b*x - a) + 1)/b + 3*log(e^(-b*x - a) - 1)/b - 1/
2*(10*e^(-2*b*x - 2*a) - 5*e^(-4*b*x - 4*a) - 1)/(b*(e^(-2*b*x - 2*a) - 2*
e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

3.936.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx = -\frac{\frac{9\,e^{(4\,bx+4\,a)}-6\,e^{(2\,bx+2\,a)}+1}{\left(e^{(2\,bx+2\,a)}-1\right)^2} - e^{(2\,bx+2\,a)} - 6\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*((9*e^(4*b*x + 4*a) - 6*e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)^2
- e^(2*b*x + 2*a) - 6*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

3.936.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx = \frac{3\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b} - \frac{6}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{2}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} + \frac{\mathrm{e}^{2\,a+2\,b\,x}}{2\,b}$$

input
```
int((cosh(a + b*x)^3*exp(2*a + 2*b*x))/sinh(a + b*x)^3,x)
```

output
```
(3*log(exp(2*a)*exp(2*b*x) - 1))/b - 6/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(
exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) + exp(2*a + 2*b*x)/(2*b)
```

3.936. $\int e^{2(a+bx)} \coth^3(a+bx) \, dx$

3.937 $\int e^x \mathrm{sech}(2x) \tanh(2x) \, dx$

3.937.1 Optimal result

Integrand size = 12, antiderivative size = 113

$$\int e^x \mathrm{sech}(2x) \tanh(2x) \, dx = -\frac{e^{3x}}{1+e^{4x}} - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}$$

output
```
-exp(3*x)/(1+exp(4*x))+1/4*arctan(-1+exp(x)*2^(1/2))*2^(1/2)+1/4*arctan(1+
exp(x)*2^(1/2))*2^(1/2)+1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/8*ln(1
+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

3.937.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.37

$$\int e^x \mathrm{sech}(2x) \tanh(2x) \, dx = \frac{2}{3} e^{3x} \left(\text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, -e^{4x}\right) - 2\,\text{Hypergeometric2F1}\left(\frac{3}{4}, 2, \frac{7}{4}, -e^{4x}\right) \right)$$

input
```
Integrate[E^x*Sech[2*x]*Tanh[2*x],x]
```

output
```
(2*E^(3*x)*(Hypergeometric2F1[3/4, 1, 7/4, -E^(4*x)] - 2*Hypergeometric2F1
[3/4, 2, 7/4, -E^(4*x)]))/3
```

3.937.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {2720, 27, 957, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \tanh(2x)\text{sech}(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{2e^{2x}(1-e^{4x})}{(e^{4x}+1)^2}de^x$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{e^{2x}(1-e^{4x})}{(1+e^{4x})^2}de^x$$

$$\Big\downarrow \text{957}$$

$$-2\left(\frac{e^{3x}}{2(e^{4x}+1)}-\frac{1}{2}\int \frac{e^{2x}}{1+e^{4x}}de^x\right)$$

$$\Big\downarrow \text{826}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^{3x}}{2(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{1476}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x-\frac{1}{2}\int \frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^{3x}}{2(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{1082}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-e^{2x}}d(1+\sqrt{2}e^x)}{\sqrt{2}}-\frac{\int \frac{1}{-1-e^{2x}}d(1-\sqrt{2}e^x)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^{3x}}{2(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{217}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\frac{e^{3x}}{2\left(e^{4x}+1\right)}\right)$$

$$\Big\downarrow \; 1479$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\frac{}{2}\right.$$

$$\Big\downarrow \; 25$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\frac{e^{3x}}{2\left(e^{4x}-\right.}\right.$$

$$\Big\downarrow \; 27$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{1}{2}\int \frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\right.$$

$$\Big\downarrow \; 1103$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}\right)\right)\right.$$

input
```
Int[E^x*Sech[2*x]*Tanh[2*x],x]
```

output
```
-2*(E^(3*x)/(2*(1 + E^(4*x))) + ((ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2] - ArcTan
[1 + Sqrt[2]*E^x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*E^x + E^(2*x)]/Sqrt[2
] + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]))/2)/2)
```

3.937. $\int e^x \mathrm{sech}(2x)\tanh(2x)\,dx$

3.937.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.937. $\int e^x \operatorname{sech}(2x) \tanh(2x) \, dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.937.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.35

method	result
risch	$-\frac{e^{3x}}{1+e^{4x}}+2\left(\sum_{\_R=\operatorname{RootOf}\left(4096\_Z^{4}+1\right)}\_R\ln\left(512\_R^{3}+e^{x}\right)\right)$
default	$\frac{\tanh\left(\frac{x}{2}\right)^{3}-3\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)-1}{\tanh\left(\frac{x}{2}\right)^{4}+6\tanh\left(\frac{x}{2}\right)^{2}+1}-\frac{\sqrt{2}\,\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+3+2\sqrt{2}\right)}{8}+\frac{\left(2+\sqrt{2}\right)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{4+4\sqrt{2}}+\frac{\sqrt{2}\,\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+3-2\right.}{8}$

input
```
int(exp(x)*sech(2*x)*tanh(2*x),x,method=_RETURNVERBOSE)
```

output
```
-exp(x)^3/(exp(x)^4+1)+2*sum(_R*ln(512*_R^3+exp(x)),_R=RootOf(4096*_Z^4+1)
)
```

3.937.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.30

$$\int e^x \text{sech}(2x) \tanh(2x)\, dx = -\frac{8 \cosh(x)^3 + 24 \cosh(x)^2 \sinh(x) + 24 \cosh(x) \sinh(x)^2 + 8 \sinh(x)^3 - \left((i-1)\sqrt{2}\cosh(x)^4 + (4i - \right.}{}$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="fricas")
```

output
```
-1/8*(8*cosh(x)^3 + 24*cosh(x)^2*sinh(x) + 24*cosh(x)*sinh(x)^2 + 8*sinh(x
)^3 - ((I - 1)*sqrt(2)*cosh(x)^4 + (4*I - 4)*sqrt(2)*cosh(x)^3*sinh(x) + (
6*I - 6)*sqrt(2)*cosh(x)^2*sinh(x)^2 + (4*I - 4)*sqrt(2)*cosh(x)*sinh(x)^3
 + (I - 1)*sqrt(2)*sinh(x)^4 + (I - 1)*sqrt(2))*log((I + 1)*sqrt(2) + 2*co
sh(x) + 2*sinh(x)) - (-(I + 1)*sqrt(2)*cosh(x)^4 - (4*I + 4)*sqrt(2)*cosh(
x)^3*sinh(x) - (6*I + 6)*sqrt(2)*cosh(x)^2*sinh(x)^2 - (4*I + 4)*sqrt(2)*c
osh(x)*sinh(x)^3 - (I + 1)*sqrt(2)*sinh(x)^4 - (I + 1)*sqrt(2))*log(-(I -
1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - ((I + 1)*sqrt(2)*cosh(x)^4 + (4*I +
4)*sqrt(2)*cosh(x)^3*sinh(x) + (6*I + 6)*sqrt(2)*cosh(x)^2*sinh(x)^2 + (4*
I + 4)*sqrt(2)*cosh(x)*sinh(x)^3 + (I + 1)*sqrt(2)*sinh(x)^4 + (I + 1)*sqr
t(2))*log((I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - (-(I - 1)*sqrt(2)*cos
h(x)^4 - (4*I - 4)*sqrt(2)*cosh(x)^3*sinh(x) - (6*I - 6)*sqrt(2)*cosh(x)^2
*sinh(x)^2 - (4*I - 4)*sqrt(2)*cosh(x)*sinh(x)^3 - (I - 1)*sqrt(2)*sinh(x)
^4 - (I - 1)*sqrt(2))*log(-(I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)))/(cosh
(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4 + 1)
```

3.937.6 Sympy [F]

$$\int e^x \text{sech}(2x) \tanh(2x)\, dx = \int e^x \tanh(2x) \,\text{sech}(2x)\, dx$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x),x)
```

output
```
Integral(exp(x)*tanh(2*x)*sech(2*x), x)
```

3.937.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int e^x \operatorname{sech}(2x)\tanh(2x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right) - \frac{e^{(3x)}}{e^{(4x)}+1}$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
+ 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^(3*x)/(e^(4*x) + 1)
```

3.937.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int e^x \operatorname{sech}(2x)\tanh(2x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right) - \frac{e^{(3x)}}{e^{(4x)}+1}$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
+ 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^(3*x)/(e^(4*x) + 1)
```

3.937.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int e^x \operatorname{sech}(2x)\tanh(2x)\,dx = -\frac{e^{3x}}{e^{4x}+1} + \sqrt{2}\ln\left(1+\sqrt{2}\,e^x\left(-\frac{1}{2}-\frac{1}{2}i\right)\right)\left(\frac{1}{8}+\frac{1}{8}i\right) + \sqrt{2}\ln\left(1+\sqrt{2}\,e^x\left(-\frac{1}{2}+\frac{1}{2}i\right)\right)\left(\frac{1}{8}-\frac{1}{8}i\right) + \sqrt{2}\ln\left(1+\sqrt{2}\,e^x\left(\frac{1}{2}-\frac{1}{2}i\right)\right)\left(-\frac{1}{8}+\frac{1}{8}i\right) + \sqrt{2}\ln\left(1+\sqrt{2}\,e^x\left(\frac{1}{2}+\frac{1}{2}i\right)\right)\left(-\frac{1}{8}-\frac{1}{8}i\right)$$

input
```
int((tanh(2*x)*exp(x))/cosh(2*x),x)
```

output
```
2^(1/2)*log(1 - 2^(1/2)*exp(x)*(1/2 + 1i/2))*(1/8 + 1i/8) + 2^(1/2)*log(1
- 2^(1/2)*exp(x)*(1/2 - 1i/2))*(1/8 - 1i/8) - 2^(1/2)*log(2^(1/2)*exp(x)*(
1/2 - 1i/2) + 1)*(1/8 - 1i/8) - 2^(1/2)*log(2^(1/2)*exp(x)*(1/2 + 1i/2) +
1)*(1/8 + 1i/8) - exp(3*x)/(exp(4*x) + 1)
```

3.938 $\int e^x \mathbf{sech}^2(2x) \tanh(2x) \, dx$

3.938.1 Optimal result

Integrand size = 14, antiderivative size = 129

$$\int e^x \text{sech}^2(2x) \tanh(2x) \, dx = -\frac{e^{5x}}{(1+e^{4x})^2} - \frac{e^x}{4(1+e^{4x})} - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}}$$

output
```
-exp(5*x)/(1+exp(4*x))^2-1/4*exp(x)/(1+exp(4*x))+1/16*arctan(-1+exp(x)*2^(
1/2))*2^(1/2)+1/16*arctan(1+exp(x)*2^(1/2))*2^(1/2)-1/32*ln(1+exp(2*x)-exp
(x)*2^(1/2))*2^(1/2)+1/32*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

3.938.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93

$$\int e^x \text{sech}^2(2x) \tanh(2x) \, dx = \frac{1}{32}\left(\frac{32e^x}{(1+e^{4x})^2} - \frac{40e^x}{1+e^{4x}} - 2\sqrt{2}\arctan\left(1-\sqrt{2}e^x\right) + 2\sqrt{2}\arctan\left(1+\sqrt{2}e^x\right) - \sqrt{2}\log\left(1-\sqrt{2}e^x+e^{2x}\right) + \sqrt{2}\log\left(1+\sqrt{2}e^x+e^{2x}\right)\right)$$

input
```
Integrate[E^x*Sech[2*x]^2*Tanh[2*x],x]
```

output
```
((32*E^x)/(1 + E^(4*x))^2 - (40*E^x)/(1 + E^(4*x)) - 2*Sqrt[2]*ArcTan[1 -
Sqrt[2]*E^x] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^x] - Sqrt[2]*Log[1 - Sqrt[2]
*E^x + E^(2*x)] + Sqrt[2]*Log[1 + Sqrt[2]*E^x + E^(2*x)])/32
```

3.938.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2720, 27, 957, 817, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \tanh(2x)\text{sech}^2(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{4e^{4x}\left(1-e^{4x}\right)}{\left(e^{4x}+1\right)^3}de^x$$

$$\Big\downarrow \text{27}$$

$$-4\int \frac{e^{4x}\left(1-e^{4x}\right)}{\left(1+e^{4x}\right)^3}de^x$$

$$\Big\downarrow \text{957}$$

$$-4\left(\frac{e^{5x}}{4\left(e^{4x}+1\right)^2}-\frac{1}{4}\int \frac{e^{4x}}{\left(1+e^{4x}\right)^2}de^x\right)$$

$$\Big\downarrow \text{817}$$

$$-4\left(\frac{1}{4}\left(\frac{e^x}{4\left(e^{4x}+1\right)}-\frac{1}{4}\int \frac{1}{1+e^{4x}}de^x\right)+\frac{e^{5x}}{4\left(e^{4x}+1\right)^2}\right)$$

$$\Big\downarrow \text{755}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^x}{4\left(e^{4x}+1\right)}\right)+\frac{e^{5x}}{4\left(e^{4x}+1\right)^2}\right)$$

$$\Big\downarrow \text{1476}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x-\frac{1}{2}\int\frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^x}{4\left(e^{4x}+1\right)}\right)+\frac{}{4\left(e^{4\right.}}$$

$\downarrow$ 1082

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-e^{2x}}d\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-e^{2x}}d\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^x}{4\left(e^{4x}+1\right)}\right)+\frac{e^{5x}}{4\left(e^{4x}+1\right.}$$

$\downarrow$ 217

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^x}{4\left(e^{4x}+1\right)}\right)+\frac{e^{5x}}{4\left(e^{4x}+1\right)^2}\right)$$

$\downarrow$ 1479

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\right.\right.$$

$\downarrow$ 25

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\frac{}{4}\right.\right.$$

$\downarrow$ 27

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{1}{2}\int\frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right.\right.\right.$$

$\downarrow$ 1103

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}\right)\right.\right.\right.$$

input
```
Int[E^x*Sech[2*x]^2*Tanh[2*x],x]
```

3.938. $\int e^x\text{sech}^2(2x)\tanh(2x)\,dx$

output
```
-4*(E^(5*x)/(4*(1 + E^(4*x))^2) + (E^x/(4*(1 + E^(4*x))) + ((ArcTan[1 - Sq
rt[2]*E^x]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]
*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]))/
2)/4)/4)
```

3.938.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.938. $\int e^x \text{sech}^2(2x)\tanh(2x)\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.938.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.34

method	result
risch	$-\frac{e^{x}\left(5\,e^{4x}+1\right)}{4\left(1+e^{4x}\right)^{2}}+4\left(\sum_{\_R=\operatorname{RootOf}\left(16777216\_Z^{4}+1\right)}\_R\ln\left(e^{x}+64\_R\right)\right)$
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{7}}{4}-\frac{17\tanh\left(\frac{x}{2}\right)^{6}}{4}-\frac{11\tanh\left(\frac{x}{2}\right)^{5}}{4}-\frac{57\tanh\left(\frac{x}{2}\right)^{4}}{4}+\frac{11\tanh\left(\frac{x}{2}\right)^{3}}{4}-\frac{19\tanh\left(\frac{x}{2}\right)^{2}}{4}-\frac{\tanh\left(\frac{x}{2}\right)}{4}-\frac{3}{4}}{\left(\tanh\left(\frac{x}{2}\right)^{4}+6\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}+\frac{\sqrt{2}\,\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+3+2\sqrt{2}\right)}{32}+$

3.938. $\int e^{x}\operatorname{sech}^{2}(2x)\tanh(2x)\,dx$

input
```
int(exp(x)*sech(2*x)^2*tanh(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(x)*(5*exp(4*x)+1)/(1+exp(4*x))^2+4*sum(_R*ln(exp(x)+64*_R),_R=Roo
tOf(16777216*_Z^4+1))
```

3.938.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 882, normalized size of antiderivative = 6.84

$$\int e^x \text{sech}^2(2x) \tanh(2x) \, dx = \text{Too large to display}$$

input
```
integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="fricas")
```

output
```
-1/32*(40*cosh(x)^5 + 400*cosh(x)^3*sinh(x)^2 + 400*cosh(x)^2*sinh(x)^3 +
200*cosh(x)*sinh(x)^4 + 40*sinh(x)^5 - ((I + 1)*sqrt(2)*cosh(x)^8 + (56*I
+ 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 + (28*I + 28)*sqrt(2)*cosh(x)^2*sinh(x)^
6 + (8*I + 8)*sqrt(2)*cosh(x)*sinh(x)^7 + (I + 1)*sqrt(2)*sinh(x)^8 - 2*(-
(35*I + 35)*sqrt(2)*cosh(x)^4 - (I + 1)*sqrt(2))*sinh(x)^4 + (2*I + 2)*sqr
t(2)*cosh(x)^4 - 8*(-(7*I + 7)*sqrt(2)*cosh(x)^5 - (I + 1)*sqrt(2)*cosh(x)
)*sinh(x)^3 - 4*(-(7*I + 7)*sqrt(2)*cosh(x)^6 - (3*I + 3)*sqrt(2)*cosh(x)^
2)*sinh(x)^2 - 8*(-(I + 1)*sqrt(2)*cosh(x)^7 - (I + 1)*sqrt(2)*cosh(x)^3)*
sinh(x) + (I + 1)*sqrt(2))*log((I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) -
(-(I - 1)*sqrt(2)*cosh(x)^8 - (56*I - 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 - (2
8*I - 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 - (8*I - 8)*sqrt(2)*cosh(x)*sinh(x)^
7 - (I - 1)*sqrt(2)*sinh(x)^8 - 2*((35*I - 35)*sqrt(2)*cosh(x)^4 + (I - 1)
*sqrt(2))*sinh(x)^4 - (2*I - 2)*sqrt(2)*cosh(x)^4 - 8*((7*I - 7)*sqrt(2)*c
osh(x)^5 + (I - 1)*sqrt(2)*cosh(x))*sinh(x)^3 - 4*((7*I - 7)*sqrt(2)*cosh(
x)^6 + (3*I - 3)*sqrt(2)*cosh(x)^2)*sinh(x)^2 - 8*((I - 1)*sqrt(2)*cosh(x)
^7 + (I - 1)*sqrt(2)*cosh(x)^3)*sinh(x) - (I - 1)*sqrt(2))*log(-(I - 1)*sq
rt(2) + 2*cosh(x) + 2*sinh(x)) - ((I - 1)*sqrt(2)*cosh(x)^8 + (56*I - 56)*
sqrt(2)*cosh(x)^3*sinh(x)^5 + (28*I - 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 + (8
*I - 8)*sqrt(2)*cosh(x)*sinh(x)^7 + (I - 1)*sqrt(2)*sinh(x)^8 - 2*(-(35*I
- 35)*sqrt(2)*cosh(x)^4 - (I - 1)*sqrt(2))*sinh(x)^4 + (2*I - 2)*sqrt(2...
```

3.938.6 Sympy [F]

$$\int e^x \mathrm{sech}^2(2x)\tanh(2x)\,dx = \int e^x \tanh(2x)\,\mathrm{sech}^2(2x)\,dx$$

input
```
integrate(exp(x)*sech(2*x)**2*tanh(2*x),x)
```

output
```
Integral(exp(x)*tanh(2*x)*sech(2*x)**2, x)
```

3.938.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.78

$$\begin{aligned}\int e^x \mathrm{sech}^2(2x)\tanh(2x)\,dx &= \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)\\ &\quad+\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)\\ &\quad+\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)\\ &\quad-\frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{5e^{(5x)}+e^x}{4\left(e^{(8x)}+2e^{(4x)}+1\right)}\end{aligned}$$

input
```
integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
1) - 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(5*e^(5*x) + e^x)/
(e^(8*x) + 2*e^(4*x) + 1)
```

3.938.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx = \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) + \frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{5e^{(5x)}+e^x}{4\left(e^{(4x)}+1\right)^2}$$

input
```
integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
1) - 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(5*e^(5*x) + e^x)/
(e^(4*x) + 1)^2
```

3.938.9 Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx = -\frac{\frac{e^{5x}}{2}-\frac{e^x}{2}}{2e^{4x}+e^{8x}+1} - \frac{3e^x}{4\left(e^{4x}+1\right)} + \sqrt{2}\ln\left(-\frac{e^x}{4}+\sqrt{2}\left(-\frac{1}{8}-\frac{1}{8}i\right)\right)\left(\frac{1}{32}+\frac{1}{32}i\right) + \sqrt{2}\ln\left(-\frac{e^x}{4}+\sqrt{2}\left(-\frac{1}{8}+\frac{1}{8}i\right)\right)\left(\frac{1}{32}-\frac{1}{32}i\right) + \sqrt{2}\ln\left(-\frac{e^x}{4}+\sqrt{2}\left(\frac{1}{8}-\frac{1}{8}i\right)\right)\left(-\frac{1}{32}+\frac{1}{32}i\right) + \sqrt{2}\ln\left(-\frac{e^x}{4}+\sqrt{2}\left(\frac{1}{8}+\frac{1}{8}i\right)\right)\left(-\frac{1}{32}-\frac{1}{32}i\right)$$

input
```
int((tanh(2*x)*exp(x))/cosh(2*x)^2,x)
```

output
```
2^(1/2)*log(- exp(x)/4 - 2^(1/2)*(1/8 + 1i/8))*(1/32 + 1i/32) - (3*exp(x))
/(4*(exp(4*x) + 1)) - (exp(5*x)/2 - exp(x)/2)/(2*exp(4*x) + exp(8*x) + 1)
+ 2^(1/2)*log(- exp(x)/4 - 2^(1/2)*(1/8 - 1i/8))*(1/32 - 1i/32) - 2^(1/2)*
log(2^(1/2)*(1/8 - 1i/8) - exp(x)/4)*(1/32 - 1i/32) - 2^(1/2)*log(2^(1/2)*
(1/8 + 1i/8) - exp(x)/4)*(1/32 + 1i/32)
```

3.938. $\int e^x \text{sech}^2(2x)\tanh(2x)\,dx$

3.939 $\int e^x \mathbf{sech}(2x) \tanh^2(2x) \, dx$

3.939.1 Optimal result

Integrand size = 14, antiderivative size = 130

$$\int e^x \text{sech}(2x) \tanh^2(2x) \, dx = \frac{e^{3x}}{(1+e^{4x})^2} - \frac{3e^{3x}}{4(1+e^{4x})} - \frac{5\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{5\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{5\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{5\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}}$$

output
```
exp(3*x)/(1+exp(4*x))^2-3/4*exp(3*x)/(1+exp(4*x))+5/16*arctan(-1+exp(x)*2^
(1/2))*2^(1/2)+5/16*arctan(1+exp(x)*2^(1/2))*2^(1/2)+5/32*ln(1+exp(2*x)-ex
p(x)*2^(1/2))*2^(1/2)-5/32*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

3.939.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.45

$$\int e^x \text{sech}(2x) \tanh^2(2x) \, dx = \frac{e^{3x}-3e^{7x}}{4(1+e^{4x})^2} - \frac{5}{16}\text{RootSum}\left[1+\#1^4\&, \frac{x-\log(e^x-\#1)}{\#1}\&\right]$$

input
```
Integrate[E^x*Sech[2*x]*Tanh[2*x]^2,x]
```

output
```
(E^(3*x) - 3*E^(7*x))/(4*(1 + E^(4*x))^2) - (5*RootSum[1 + #1^4 & , (x - L
og[E^x - #1])/#1 & ])/16
```

3.939.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {2720, 27, 963, 27, 957, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \tanh^2(2x)\text{sech}(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{2e^{2x}\left(1-e^{4x}\right)^2}{\left(e^{4x}+1\right)^3}de^x$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{e^{2x}\left(1-e^{4x}\right)^2}{\left(1+e^{4x}\right)^3}de^x$$

$$\Big\downarrow \text{963}$$

$$2\left(\frac{e^{3x}}{2\left(e^{4x}+1\right)^2}-\frac{1}{8}\int \frac{4e^{2x}\left(1-2e^{4x}\right)}{\left(1+e^{4x}\right)^2}de^x\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{e^{3x}}{2\left(e^{4x}+1\right)^2}-\frac{1}{2}\int \frac{e^{2x}\left(1-2e^{4x}\right)}{\left(1+e^{4x}\right)^2}de^x\right)$$

$$\Big\downarrow \text{957}$$

$$2\left(\frac{1}{2}\left(\frac{5}{4}\int \frac{e^{2x}}{1+e^{4x}}de^x-\frac{3e^{3x}}{4\left(e^{4x}+1\right)}\right)+\frac{e^{3x}}{2\left(e^{4x}+1\right)^2}\right)$$

$$\Big\downarrow \text{826}$$

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{3e^{3x}}{4\left(e^{4x}+1\right)}\right)+\frac{e^{3x}}{2\left(e^{4x}+1\right)^2}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x+\frac{1}{2}\int\frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{3e^{3x}}{4\left(e^{4x}+1\right)}\right)+\frac{e^{3x}}{2\left(e^{4x}+\right.}$$

$\downarrow$ 1082

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-e^{2x}}d\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-e^{2x}}d\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{3e^{3x}}{4\left(e^{4x}+1\right)}\right)+\frac{e^{3x}}{2\left(e^{4x}+1\right)^2}\right.$$

$\downarrow$ 217

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{3e^{3x}}{4\left(e^{4x}+1\right)}\right)+\frac{e^{3x}}{2\left(e^{4x}+1\right)^2}\right)$$

$\downarrow$ 1479

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{}{4}\right.\right.$$

$\downarrow$ 25

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{}{4(e}\right.\right.$$

$\downarrow$ 27

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{1}{2}\int\frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)\right.\right.$$

$\downarrow$ 1103

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}\right)\right)\right.\right.$$

input
```
Int[E^x*Sech[2*x]*Tanh[2*x]^2,x]
```

3.939. $\int e^x \text{sech}(2x)\tanh^2(2x)\,dx$

output
```
2*(E^(3*x)/(2*(1 + E^(4*x))^2) + ((-3*E^(3*x))/(4*(1 + E^(4*x))) + (5*((-(
ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2])/2 + (L
og[1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]
/(2*Sqrt[2]))/2))/4)/2)
```

3.939.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

3.939. $\int e^x \text{sech}(2x) \tanh^2(2x) \, dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.939.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.46 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.37

method	result
risch	$-\frac{e^{3x}(3\,e^{4x}-1)}{4(1+e^{4x})^2}+2\left(\sum_{\_R=\operatorname{RootOf}(1048576\_Z^4+625)}\_R\ln\left(e^x+\frac{32768\_R^3}{125}\right)\right)$
default	$\frac{\frac{5\tanh(\frac{x}{2})^7}{4}+\frac{5\tanh(\frac{x}{2})^6}{4}+\frac{9\tanh(\frac{x}{2})^5}{4}-\frac{19\tanh(\frac{x}{2})^4}{4}-\frac{9\tanh(\frac{x}{2})^3}{4}-\frac{17\tanh(\frac{x}{2})^2}{4}-\frac{5\tanh(\frac{x}{2})}{4}-\frac{1}{4}}{\left(\tanh(\frac{x}{2})^4+6\tanh(\frac{x}{2})^2+1\right)^2}+\frac{5\sqrt{2}\,\ln\left(\tanh(\frac{x}{2})^2+3-2\sqrt{2}\right)}{32}-$

3.939. $\int e^x \operatorname{sech}(2x)\tanh^2(2x)\,dx$

input
```
int(exp(x)*sech(2*x)*tanh(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(x)^3*(3*exp(x)^4-1)/(exp(x)^4+1)^2+2*sum(_R*ln(exp(x)+32768/125*_
R^3),_R=RootOf(1048576*_Z^4+625))
```

3.939.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 920, normalized size of antiderivative = 7.08

$$\int e^x \text{sech}(2x) \tanh^2(2x) \, dx = \text{Too large to display}$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="fricas")
```

output
```
-1/32*(24*cosh(x)^7 + 840*cosh(x)^3*sinh(x)^4 + 504*cosh(x)^2*sinh(x)^5 +
168*cosh(x)*sinh(x)^6 + 24*sinh(x)^7 + 8*(105*cosh(x)^4 - 1)*sinh(x)^3 - 8
*cosh(x)^3 + 24*(21*cosh(x)^5 - cosh(x))*sinh(x)^2 + 5*(-(I - 1)*sqrt(2)*c
osh(x)^8 - (56*I - 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 - (28*I - 28)*sqrt(2)*c
osh(x)^2*sinh(x)^6 - (8*I - 8)*sqrt(2)*cosh(x)*sinh(x)^7 - (I - 1)*sqrt(2)
*sinh(x)^8 + 2*(-(35*I - 35)*sqrt(2)*cosh(x)^4 - (I - 1)*sqrt(2))*sinh(x)^
4 - (2*I - 2)*sqrt(2)*cosh(x)^4 + 8*(-(7*I - 7)*sqrt(2)*cosh(x)^5 - (I - 1
)*sqrt(2)*cosh(x))*sinh(x)^3 + 4*(-(7*I - 7)*sqrt(2)*cosh(x)^6 - (3*I - 3)
*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 8*(-(I - 1)*sqrt(2)*cosh(x)^7 - (I - 1)*sq
rt(2)*cosh(x)^3)*sinh(x) - (I - 1)*sqrt(2))*log((I + 1)*sqrt(2) + 2*cosh(x
) + 2*sinh(x)) + 5*((I + 1)*sqrt(2)*cosh(x)^8 + (56*I + 56)*sqrt(2)*cosh(x
)^3*sinh(x)^5 + (28*I + 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 + (8*I + 8)*sqrt(2
)*cosh(x)*sinh(x)^7 + (I + 1)*sqrt(2)*sinh(x)^8 + 2*((35*I + 35)*sqrt(2)*c
osh(x)^4 + (I + 1)*sqrt(2))*sinh(x)^4 + (2*I + 2)*sqrt(2)*cosh(x)^4 + 8*((
7*I + 7)*sqrt(2)*cosh(x)^5 + (I + 1)*sqrt(2)*cosh(x))*sinh(x)^3 + 4*((7*I
+ 7)*sqrt(2)*cosh(x)^6 + (3*I + 3)*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 8*((I +
1)*sqrt(2)*cosh(x)^7 + (I + 1)*sqrt(2)*cosh(x)^3)*sinh(x) + (I + 1)*sqrt(2
))*log(-(I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + 5*(-(I + 1)*sqrt(2)*cos
h(x)^8 - (56*I + 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 - (28*I + 28)*sqrt(2)*cos
h(x)^2*sinh(x)^6 - (8*I + 8)*sqrt(2)*cosh(x)*sinh(x)^7 - (I + 1)*sqrt(2...
```

3.939.6 Sympy [F]

$$\int e^x \mathrm{sech}(2x)\tanh^2(2x)\,dx = \int e^x \tanh^2(2x)\,\mathrm{sech}\,(2x)\,dx$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x)**2,x)
```

output
```
Integral(exp(x)*tanh(2*x)**2*sech(2*x), x)
```

3.939.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\begin{aligned}\int e^x \mathrm{sech}(2x)\tanh^2(2x)\,dx &= \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)\\&\quad+\frac{5}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)\\&\quad-\frac{5}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)\\&\quad+\frac{5}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{3e^{(7x)}-e^{(3x)}}{4\left(e^{(8x)}+2e^{(4x)}+1\right)}\end{aligned}$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="maxima")
```

output
```
5/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 5/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 5/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
1) + 5/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(3*e^(7*x) - e^(3*
x))/(e^(8*x) + 2*e^(4*x) + 1)
```

3.939.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int e^x \operatorname{sech}(2x)\tanh^2(2x)\,dx = \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{5}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{5}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) + \frac{5}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{3e^{(7x)}-e^{(3x)}}{4\left(e^{(4x)}+1\right)^2}$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="giac")
```

output
```
5/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 5/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 5/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
1) + 5/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(3*e^(7*x) - e^(3*
x))/(e^(4*x) + 1)^2
```

3.939.9 Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int e^x \operatorname{sech}(2x)\tanh^2(2x)\,dx = \frac{e^{3x}}{2e^{4x}+e^{8x}+1} - \frac{3e^{3x}}{4(e^{4x}+1)} + \sqrt{2}\ln\left(\frac{25}{16}+\sqrt{2}e^x\left(-\frac{25}{32}-\frac{25}{32}\mathrm{i}\right)\right)\left(\frac{5}{32}+\frac{5}{32}\mathrm{i}\right) + \sqrt{2}\ln\left(\frac{25}{16}+\sqrt{2}e^x\left(-\frac{25}{32}+\frac{25}{32}\mathrm{i}\right)\right)\left(\frac{5}{32}-\frac{5}{32}\mathrm{i}\right) + \sqrt{2}\ln\left(\frac{25}{16}+\sqrt{2}e^x\left(\frac{25}{32}-\frac{25}{32}\mathrm{i}\right)\right)\left(-\frac{5}{32}+\frac{5}{32}\mathrm{i}\right) + \sqrt{2}\ln\left(\frac{25}{16}+\sqrt{2}e^x\left(\frac{25}{32}+\frac{25}{32}\mathrm{i}\right)\right)\left(-\frac{5}{32}-\frac{5}{32}\mathrm{i}\right)$$

input
```
int((tanh(2*x)^2*exp(x))/cosh(2*x),x)
```

output
```
2^(1/2)*log(25/16 - 2^(1/2)*exp(x)*(25/32 + 25i/32))*(5/32 + 5i/32) + 2^(1
/2)*log(25/16 - 2^(1/2)*exp(x)*(25/32 - 25i/32))*(5/32 - 5i/32) - 2^(1/2)*
log(2^(1/2)*exp(x)*(25/32 - 25i/32) + 25/16)*(5/32 - 5i/32) - 2^(1/2)*log(
2^(1/2)*exp(x)*(25/32 + 25i/32) + 25/16)*(5/32 + 5i/32) + exp(3*x)/(2*exp(
4*x) + exp(8*x) + 1) - (3*exp(3*x))/(4*(exp(4*x) + 1))
```

3.939. $\int e^x \operatorname{sech}(2x)\tanh^2(2x)\,dx$

3.940 $\int e^x \mathbf{sech}^2(2x)\tanh^2(2x)\,dx$

3.940.1 Optimal result

Integrand size = 16, antiderivative size = 149

$$\int e^x \text{sech}^2(2x)\tanh^2(2x)\,dx = \frac{4e^{5x}}{3(1+e^{4x})^3} - \frac{5e^{5x}}{6(1+e^{4x})^2} - \frac{3e^x}{8(1+e^{4x})} - \frac{3\arctan\left(1-\sqrt{2}e^x\right)}{16\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}e^x\right)}{16\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{32\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{32\sqrt{2}}$$

output
```
4/3*exp(5*x)/(1+exp(4*x))^3-5/6*exp(5*x)/(1+exp(4*x))^2-3/8*exp(x)/(1+exp(
4*x))+3/32*arctan(-1+exp(x)*2^(1/2))*2^(1/2)+3/32*arctan(1+exp(x)*2^(1/2))
*2^(1/2)-3/64*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)+3/64*ln(1+exp(2*x)+exp
(x)*2^(1/2))*2^(1/2)
```

3.940.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.43

$$\int e^x \text{sech}^2(2x)\tanh^2(2x)\,dx = \frac{1}{96}\left(-\frac{4e^x(9+6e^{4x}+29e^{8x})}{(1+e^{4x})^3} - 9\text{RootSum}\left[1+\#1^4\&,\frac{x-\log(e^x-\#1)}{\#1^3}\&\right]\right)$$

input
```
Integrate[E^x*Sech[2*x]^2*Tanh[2*x]^2,x]
```

output
```
((-4*E^x*(9 + 6*E^(4*x) + 29*E^(8*x)))/(1 + E^(4*x))^3 - 9*RootSum[1 + #1^
4 & , (x - Log[E^x - #1])/#1^3 & ])/96
```

3.940.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.15, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {2720, 27, 963, 27, 957, 817, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \tanh^2(2x)\text{sech}^2(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{4e^{4x}\left(1-e^{4x}\right)^2}{\left(e^{4x}+1\right)^4}de^x$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{4x}\left(1-e^{4x}\right)^2}{\left(1+e^{4x}\right)^4}de^x$$

$$\Big\downarrow \text{963}$$

$$4\left(\frac{e^{5x}}{3\left(e^{4x}+1\right)^3}-\frac{1}{12}\int \frac{4e^{4x}\left(2-3e^{4x}\right)}{\left(1+e^{4x}\right)^3}de^x\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{e^{5x}}{3\left(e^{4x}+1\right)^3}-\frac{1}{3}\int \frac{e^{4x}\left(2-3e^{4x}\right)}{\left(1+e^{4x}\right)^3}de^x\right)$$

$$\Big\downarrow \text{957}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\int \frac{e^{4x}}{\left(1+e^{4x}\right)^2}de^x-\frac{5e^{5x}}{8\left(e^{4x}+1\right)^2}\right)+\frac{e^{5x}}{3\left(e^{4x}+1\right)^3}\right)$$

$$\Big\downarrow \text{817}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\int \frac{1}{1+e^{4x}}de^x-\frac{e^x}{4\left(e^{4x}+1\right)}\right)-\frac{5e^{5x}}{8\left(e^{4x}+1\right)^2}\right)+\frac{e^{5x}}{3\left(e^{4x}+1\right)^3}\right)$$

$$\Big\downarrow \text{755}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x+\frac{1}{2}\int\frac{1+e^{2x}}{1+e^{4x}}de^x\right)-\frac{e^x}{4\left(e^{4x}+1\right)}\right)-\frac{5e^{5x}}{8\left(e^{4x}+1\right)^2}\right)+\frac{e^{5x}}{3\left(e^{4x}+1\right)^3}\right)$$

$$\Big\downarrow \text{1476}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x+\frac{1}{2}\int\frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{e^x}{4\left(e^{4x}+1\right)}\right)-\frac{5}{8\left(e^{4}\right.}\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-e^{2x}}d\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-e^{2x}}d\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{e^x}{4\left(e^{4x}+1\right)}\right)-\frac{5e^{5x}}{8\left(e^{4x}+\right.}\right.\right.$$

$$\Big\downarrow \text{217}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{e^x}{4\left(e^{4x}+1\right)}\right)-\frac{5e^{5x}}{8\left(e^{4x}+1\right)^2}\right)\right.$$

$$\Big\downarrow \text{1479}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{}{4}\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{1}{2}\int\frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right.\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}\right.\right.\right.\right.\right.$$

3.940. $\int e^x \text{sech}^2(2x)\tanh^2(2x)\,dx$

input
```
Int[E^x*Sech[2*x]^2*Tanh[2*x]^2,x]
```

output
```
4*(E^(5*x)/(3*(1 + E^(4*x))^3) + ((-5*E^(5*x))/(8*(1 + E^(4*x))^2) + (9*(-
1/4*E^x/(1 + E^(4*x)) + ((-(ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]) + ArcTan[1 +
Sqrt[2]*E^x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*E^x + E^(2*x)]/Sqrt[2] + L
og[1 + Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]))/2)/4))/8)/3)
```

3.940.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.940. $\int e^x \operatorname{sech}^2(2x) \tanh^2(2x) \, dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.940.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 13.56 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.34

method	result
risch	$-\frac{e^{x} \left(29 e^{8 x}+6 e^{4 x}+9\right)}{24 \left(1+e^{4 x}\right)^{3}}+4 \left(\sum_{\_R =\operatorname{RootOf}\left(268435456 \_Z^{4}+81\right)}\_R \ln \left(e^{x}+\frac{128 \_R}{3}\right)\right)$
default	$\frac{\frac{3 \tanh \left(\frac{x}{2}\right)^{11}}{8}-\frac{3 \tanh \left(\frac{x}{2}\right)^{10}}{8}-\frac{109 \tanh \left(\frac{x}{2}\right)^{9}}{24}-\frac{173 \tanh \left(\frac{x}{2}\right)^{8}}{8}-\frac{49 \tanh \left(\frac{x}{2}\right)^{7}}{4}-\frac{231 \tanh \left(\frac{x}{2}\right)^{6}}{4}+\frac{49 \tanh \left(\frac{x}{2}\right)^{5}}{4}-\frac{117 \tanh \left(\frac{x}{2}\right)^{4}}{4}+\frac{109 \tanh \left(\frac{x}{2}\right)^{3}}{24} -}{\left(\tanh \left(\frac{x}{2}\right)^{4}+6 \tanh \left(\frac{x}{2}\right)^{2}+1\right)^{3}}$

input
```
int(exp(x)*sech(2*x)^2*tanh(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24*exp(x)*(29*exp(8*x)+6*exp(4*x)+9)/(1+exp(4*x))^3+4*sum(_R*ln(exp(x)+
128/3*_R),_R=RootOf(268435456*_Z^4+81))
```

3.940.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1616, normalized size of antiderivative = 10.85

$$\int e^{x} \operatorname{sech}^{2}(2x) \tanh^{2}(2x)\, dx = \text{Too large to display}$$

input
```
integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="fricas")
```

output
```
-1/192*(232*cosh(x)^9 + 19488*cosh(x)^3*sinh(x)^6 + 8352*cosh(x)^2*sinh(x)
^7 + 2088*cosh(x)*sinh(x)^8 + 232*sinh(x)^9 + 48*(609*cosh(x)^4 + 1)*sinh(
x)^5 + 48*cosh(x)^5 + 48*(609*cosh(x)^5 + 5*cosh(x))*sinh(x)^4 + 96*(203*c
osh(x)^6 + 5*cosh(x)^2)*sinh(x)^3 + 96*(87*cosh(x)^7 + 5*cosh(x)^3)*sinh(x
)^2 + 9*(-(I + 1)*sqrt(2)*cosh(x)^12 - (220*I + 220)*sqrt(2)*cosh(x)^3*sin
h(x)^9 - (66*I + 66)*sqrt(2)*cosh(x)^2*sinh(x)^10 - (12*I + 12)*sqrt(2)*co
sh(x)*sinh(x)^11 - (I + 1)*sqrt(2)*sinh(x)^12 + 3*(-(165*I + 165)*sqrt(2)*
cosh(x)^4 - (I + 1)*sqrt(2))*sinh(x)^8 - (3*I + 3)*sqrt(2)*cosh(x)^8 + 24*
(-(33*I + 33)*sqrt(2)*cosh(x)^5 - (I + 1)*sqrt(2)*cosh(x))*sinh(x)^7 + 84*
(-(11*I + 11)*sqrt(2)*cosh(x)^6 - (I + 1)*sqrt(2)*cosh(x)^2)*sinh(x)^6 + 2
4*(-(33*I + 33)*sqrt(2)*cosh(x)^7 - (7*I + 7)*sqrt(2)*cosh(x)^3)*sinh(x)^5
 + 3*(-(165*I + 165)*sqrt(2)*cosh(x)^8 - (70*I + 70)*sqrt(2)*cosh(x)^4 - (
I + 1)*sqrt(2))*sinh(x)^4 - (3*I + 3)*sqrt(2)*cosh(x)^4 + 4*(-(55*I + 55)*
sqrt(2)*cosh(x)^9 - (42*I + 42)*sqrt(2)*cosh(x)^5 - (3*I + 3)*sqrt(2)*cosh
(x))*sinh(x)^3 + 6*(-(11*I + 11)*sqrt(2)*cosh(x)^10 - (14*I + 14)*sqrt(2)*
cosh(x)^6 - (3*I + 3)*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 12*(-(I + 1)*sqrt(2)*
cosh(x)^11 - (2*I + 2)*sqrt(2)*cosh(x)^7 - (I + 1)*sqrt(2)*cosh(x)^3)*sinh
(x) - (I + 1)*sqrt(2))*log((I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + 9*((
I - 1)*sqrt(2)*cosh(x)^12 + (220*I - 220)*sqrt(2)*cosh(x)^3*sinh(x)^9 + (6
6*I - 66)*sqrt(2)*cosh(x)^2*sinh(x)^10 + (12*I - 12)*sqrt(2)*cosh(x)*si...
```

3.940.6 Sympy [F]

$$\int e^x \text{sech}^2(2x) \tanh^2(2x) \, dx = \int e^x \tanh^2(2x) \operatorname{sech}^2(2x) \, dx$$

input
```
integrate(exp(x)*sech(2*x)**2*tanh(2*x)**2,x)
```

output
```
Integral(exp(x)*tanh(2*x)**2*sech(2*x)**2, x)
```

3.940.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

$$\int e^x \operatorname{sech}^2(2x)\tanh^2(2x)\,dx = \frac{3}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{3}{32}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) + \frac{3}{64}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{3}{64}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{29e^{(9x)}+6e^{(5x)}+9e^x}{24\left(e^{(12x)}+3e^{(8x)}+3e^{(4x)}+1\right)}$$

input
```
integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="maxima")
```

output
```
3/32*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 3/32*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 3/64*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
1) - 3/64*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/24*(29*e^(9*x) + 6*e
^(5*x) + 9*e^x)/(e^(12*x) + 3*e^(8*x) + 3*e^(4*x) + 1)
```

3.940.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.69

$$\int e^x \operatorname{sech}^2(2x)\tanh^2(2x)\,dx = \frac{3}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{3}{32}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) + \frac{3}{64}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{3}{64}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{29e^{(9x)}+6e^{(5x)}+9e^x}{24\left(e^{(4x)}+1\right)^3}$$

input
```
integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="giac")
```

output
```
3/32*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 3/32*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 3/64*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
1) - 3/64*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/24*(29*e^(9*x) + 6*e
^(5*x) + 9*e^x)/(e^(4*x) + 1)^3
```

3.940.9 Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03

$$\int e^x \text{sech}^2(2x)\tanh^2(2x)\,dx = \frac{5\,\mathrm{e}^x}{6\,(2\,\mathrm{e}^{4\,x}+\mathrm{e}^{8\,x}+1)} - \frac{\frac{\mathrm{e}^{9\,x}}{3}-\frac{2\,\mathrm{e}^{5\,x}}{3}+\frac{\mathrm{e}^x}{3}}{3\,\mathrm{e}^{4\,x}+3\,\mathrm{e}^{8\,x}+\mathrm{e}^{12\,x}+1} - \frac{7\,\mathrm{e}^x}{8\,(\mathrm{e}^{4\,x}+1)}$$
$$+\sqrt{2}\,\ln\left(-\frac{3\,\mathrm{e}^x}{8}+\sqrt{2}\left(-\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)\right)\left(\frac{3}{64}+\frac{3}{64}\mathrm{i}\right)$$
$$+\sqrt{2}\,\ln\left(-\frac{3\,\mathrm{e}^x}{8}+\sqrt{2}\left(-\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)\right)\left(\frac{3}{64}-\frac{3}{64}\mathrm{i}\right)$$
$$+\sqrt{2}\,\ln\left(-\frac{3\,\mathrm{e}^x}{8}+\sqrt{2}\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)\right)\left(-\frac{3}{64}+\frac{3}{64}\mathrm{i}\right)$$
$$+\sqrt{2}\,\ln\left(-\frac{3\,\mathrm{e}^x}{8}+\sqrt{2}\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)\right)\left(-\frac{3}{64}-\frac{3}{64}\mathrm{i}\right)$$

input
```
int((tanh(2*x)^2*exp(x))/cosh(2*x)^2,x)
```

output
```
2^(1/2)*log(- (3*exp(x))/8 - 2^(1/2)*(3/16 + 3i/16))*(3/64 + 3i/64) - (exp
(9*x)/3 - (2*exp(5*x))/3 + exp(x)/3)/(3*exp(4*x) + 3*exp(8*x) + exp(12*x)
+ 1) - (7*exp(x))/(8*(exp(4*x) + 1)) + 2^(1/2)*log(- (3*exp(x))/8 - 2^(1/2
)*(3/16 - 3i/16))*(3/64 - 3i/64) - 2^(1/2)*log(2^(1/2)*(3/16 - 3i/16) - (3
*exp(x))/8)*(3/64 - 3i/64) - 2^(1/2)*log(2^(1/2)*(3/16 + 3i/16) - (3*exp(x
))/8)*(3/64 + 3i/64) + (5*exp(x))/(6*(2*exp(4*x) + exp(8*x) + 1))
```

3.941 $\int e^x \coth(2x)\text{csch}(2x)\,dx$

3.941.1 Optimal result

Integrand size = 12, antiderivative size = 34

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = \frac{e^{3x}}{1-e^{4x}} + \frac{\arctan(e^x)}{2} - \frac{\text{arctanh}(e^x)}{2}$$

output
```
exp(3*x)/(1-exp(4*x))+1/2*arctan(exp(x))-1/2*arctanh(exp(x))
```

3.941.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = \frac{1}{2}\left(-\frac{2e^{3x}}{-1+e^{4x}} + \arctan(e^x) - \text{arctanh}(e^x)\right)$$

input
```
Integrate[E^x*Coth[2*x]*Csch[2*x],x]
```

output
```
((-2*E^(3*x))/(-1 + E^(4*x)) + ArcTan[E^x] - ArcTanh[E^x])/2
```

3.941.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 27, 957, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \coth(2x)\text{csch}(2x) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{2e^{2x}(e^{4x}+1)}{(1-e^{4x})^2}de^x$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{e^{2x}(1+e^{4x})}{(1-e^{4x})^2}de^x$$

$$\Big\downarrow \text{957}$$

$$2\left(\frac{e^{3x}}{2(1-e^{4x})}-\frac{1}{2}\int \frac{e^{2x}}{1-e^{4x}}de^x\right)$$

$$\Big\downarrow \text{827}$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1+e^{2x}}de^x-\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x\right)+\frac{e^{3x}}{2(1-e^{4x})}\right)$$

$$\Big\downarrow \text{216}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan(e^x)}{2}-\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x\right)+\frac{e^{3x}}{2(1-e^{4x})}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan(e^x)}{2}-\frac{\text{arctanh}(e^x)}{2}\right)+\frac{e^{3x}}{2(1-e^{4x})}\right)$$

input
```
Int[E^x*Coth[2*x]*Csch[2*x],x]
```

output
```
2*(E^(3*x)/(2*(1 - E^(4*x))) + (ArcTan[E^x]/2 - ArcTanh[E^x]/2)/2)
```

3.941.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.941. $\int e^x \coth(2x)\operatorname{csch}(2x)\,dx$

3.941.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{1}{4\sinh(x)}+\frac{\arctan(\mathrm{e}^{x})}{2}-\frac{\operatorname{arctanh}(\mathrm{e}^{x})}{2}-\frac{1}{4\cosh(x)}$	24
risch	$-\frac{\mathrm{e}^{3x}}{\mathrm{e}^{4x}-1}+\frac{\ln(\mathrm{e}^{x}-1)}{4}+\frac{i\ln(\mathrm{e}^{x}+i)}{4}-\frac{i\ln(\mathrm{e}^{x}-i)}{4}-\frac{\ln(1+\mathrm{e}^{x})}{4}$	48

input
```
int(exp(x)*coth(2*x)*csch(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4/sinh(x)+1/2*arctan(exp(x))-1/2*arctanh(exp(x))-1/4/cosh(x)
```

3.941.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 5.94

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = -\frac{4\cosh(x)^3+12\cosh(x)^2\sinh(x)+12\cosh(x)\sinh(x)^2+4\sinh(x)^3-2\left(\cosh(x)^4+4\cosh(x)^3\sin\right.}{}$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="fricas")
```

output
```
-1/4*(4*cosh(x)^3 + 12*cosh(x)^2*sinh(x) + 12*cosh(x)*sinh(x)^2 + 4*sinh(x
)^3 - 2*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(
x)*sinh(x)^3 + sinh(x)^4 - 1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^4 + 4*c
osh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4
 - 1)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*co
sh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*log(cosh(x) + sin
h(x) - 1))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 - 1)
```

3.941.6 Sympy [F]

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = \int e^x \coth(2x)\,\text{csch}(2x)\,dx$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x),x)
```

output
```
Integral(exp(x)*coth(2*x)*csch(2*x), x)
```

3.941.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = -\frac{e^{(3x)}}{e^{(4x)}-1} + \frac{1}{2}\arctan(e^x) - \frac{1}{4}\log(e^x+1) + \frac{1}{4}\log(e^x-1)$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="maxima")
```

output
```
-e^(3*x)/(e^(4*x) - 1) + 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(e^x
- 1)
```

3.941.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = -\frac{e^{(3x)}}{e^{(4x)}-1} + \frac{1}{2}\arctan(e^x) - \frac{1}{4}\log(e^x+1) + \frac{1}{4}\log(|e^x-1|)$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="giac")
```

output
```
-e^(3*x)/(e^(4*x) - 1) + 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(abs(
e^x - 1))
```

3.941.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = \frac{\ln\left(e^x - 1\right)}{4} - \frac{\text{atan}(e^{-x})}{2} - \frac{\ln\left(-e^x - 1\right)}{4} - \frac{e^{3x}}{e^{4x} - 1}$$

input
```
int((coth(2*x)*exp(x))/sinh(2*x),x)
```

output
```
log(exp(x) - 1)/4 - atan(exp(-x))/2 - log(- exp(x) - 1)/4 - exp(3*x)/(exp(
4*x) - 1)
```

3.942 $\int e^x \coth(2x)\mathbf{csch}^2(2x)\,dx$

3.942.1 Optimal result

Integrand size = 14, antiderivative size = 53

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = -\frac{e^{5x}}{(1-e^{4x})^2} + \frac{e^x}{4(1-e^{4x})} - \frac{\arctan(e^x)}{8} - \frac{\text{arctanh}(e^x)}{8}$$

output
```
-exp(5*x)/(1-exp(4*x))^2+1/4*exp(x)/(1-exp(4*x))-1/8*arctan(exp(x))-1/8*ar
ctanh(exp(x))
```

3.942.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = -\frac{-2e^x + 10e^{5x} + (-1+e^{4x})^2 \arctan(e^x) + (-1+e^{4x})^2 \text{arctanh}(e^x)}{8(-1+e^{4x})^2}$$

input
```
Integrate[E^x*Coth[2*x]*Csch[2*x]^2,x]
```

output
```
-1/8*(-2*E^x + 10*E^(5*x) + (-1 + E^(4*x))^2*ArcTan[E^x] + (-1 + E^(4*x))^
2*ArcTanh[E^x])/(-1 + E^(4*x))^2
```

3.942.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 27, 957, 817, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx$$

$$\Big\downarrow \ 2720$$

$$\int -\frac{4e^{4x}\left(e^{4x}+1\right)}{\left(1-e^{4x}\right)^3}de^x$$

$$\Big\downarrow \ 27$$

$$-4\int \frac{e^{4x}\left(1+e^{4x}\right)}{\left(1-e^{4x}\right)^3}de^x$$

$$\Big\downarrow \ 957$$

$$-4\left(\frac{e^{5x}}{4\left(1-e^{4x}\right)^2}-\frac{1}{4}\int \frac{e^{4x}}{\left(1-e^{4x}\right)^2}de^x\right)$$

$$\Big\downarrow \ 817$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\int \frac{1}{1-e^{4x}}de^x-\frac{e^x}{4\left(1-e^{4x}\right)}\right)+\frac{e^{5x}}{4\left(1-e^{4x}\right)^2}\right)$$

$$\Big\downarrow \ 756$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x+\frac{1}{2}\int \frac{1}{1+e^{2x}}de^x\right)-\frac{e^x}{4\left(1-e^{4x}\right)}\right)+\frac{e^{5x}}{4\left(1-e^{4x}\right)^2}\right)$$

$$\Big\downarrow \ 216$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x+\frac{\arctan\left(e^x\right)}{2}\right)-\frac{e^x}{4\left(1-e^{4x}\right)}\right)+\frac{e^{5x}}{4\left(1-e^{4x}\right)^2}\right)$$

$$\Big\downarrow \ 219$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{\arctan\left(e^x\right)}{2}+\frac{\text{arctanh}(e^x)}{2}\right)-\frac{e^x}{4\left(1-e^{4x}\right)}\right)+\frac{e^{5x}}{4\left(1-e^{4x}\right)^2}\right)$$

input
```
Int[E^x*Coth[2*x]*Csch[2*x]^2,x]
```

output
```
-4*(E^(5*x)/(4*(1 - E^(4*x))^2) + (-1/4*E^x/(1 - E^(4*x)) + (ArcTan[E^x]/2
 + ArcTanh[E^x]/2)/4)/4)
```

3.942.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

3.942. $\int e^x \coth(2x)\text{csch}^2(2x)\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.942.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{e^{x}\left(5\,e^{4x}-1\right)}{4\left(e^{4x}-1\right)^{2}}+\frac{i\ln\left(e^{x}-i\right)}{16}-\frac{i\ln\left(e^{x}+i\right)}{16}-\frac{\ln\left(1+e^{x}\right)}{16}+\frac{\ln\left(e^{x}-1\right)}{16}$
default	$-\frac{\coth(x)\operatorname{csch}(x)}{8}-\frac{\operatorname{arctanh}(e^{x})}{8}+\frac{1}{16\sinh(x)^{2}\cosh(x)}+\frac{3}{16\cosh(x)}-\frac{1}{4\sinh(x)}-\frac{\arctan(e^{x})}{8}+\frac{1}{8\sinh(x)\cosh(x)^{2}}+\frac{3}{}$

input
```
int(exp(x)*coth(2*x)*csch(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(x)*(5*exp(4*x)-1)/(exp(4*x)-1)^2+1/16*I*ln(exp(x)-I)-1/16*I*ln(ex
p(x)+I)-1/16*ln(1+exp(x))+1/16*ln(exp(x)-1)
```

3.942.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. 2(37) = 74.

Time = 0.27 (sec) , antiderivative size = 522, normalized size of antiderivative = 9.85

$$\int e^{x}\coth(2x)\operatorname{csch}^{2}(2x)\,dx = \text{Too large to display}$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="fricas")
```

output
```
-1/16*(20*cosh(x)^5 + 200*cosh(x)^3*sinh(x)^2 + 200*cosh(x)^2*sinh(x)^3 +
100*cosh(x)*sinh(x)^4 + 20*sinh(x)^5 + 2*(cosh(x)^8 + 56*cosh(x)^3*sinh(x)
^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh
(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 +
 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(
x) + 1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 +
28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4
 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(7
*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) +
1)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*c
osh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1
)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(7*cos
h(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) + 1)*l
og(cosh(x) + sinh(x) - 1) + 4*(25*cosh(x)^4 - 1)*sinh(x) - 4*cosh(x))/(cos
h(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(
x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cos
h(x)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*
(cosh(x)^7 - cosh(x)^3)*sinh(x) + 1)
```

3.942.6 Sympy [F]

$$\int e^x \coth(2x)\operatorname{csch}^2(2x)\,dx = \int e^x \coth(2x)\operatorname{csch}^2(2x)\,dx$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x)**2,x)
```

output
```
Integral(exp(x)*coth(2*x)*csch(2*x)**2, x)
```

3.942.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int e^x \coth(2x)\operatorname{csch}^2(2x)\,dx = -\frac{5\,e^{(5\,x)} - e^x}{4\,\left(e^{(8\,x)} - 2\,e^{(4\,x)} + 1\right)} - \frac{1}{8}\arctan\left(e^x\right) - \frac{1}{16}\log\left(e^x + 1\right) + \frac{1}{16}\log\left(e^x - 1\right)$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="maxima")
```

output
```
-1/4*(5*e^(5*x) - e^x)/(e^(8*x) - 2*e^(4*x) + 1) - 1/8*arctan(e^x) - 1/16*
log(e^x + 1) + 1/16*log(e^x - 1)
```

3.942.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = -\frac{5\,e^{(5\,x)} - e^x}{4\,(e^{(4\,x)} - 1)^2} - \frac{1}{8}\,\arctan\left(e^x\right) - \frac{1}{16}\,\log\left(e^x + 1\right) + \frac{1}{16}\,\log\left(\left|e^x - 1\right|\right)$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="giac")
```

output
```
-1/4*(5*e^(5*x) - e^x)/(e^(4*x) - 1)^2 - 1/8*arctan(e^x) - 1/16*log(e^x +
1) + 1/16*log(abs(e^x - 1))
```

3.942.9 Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = \frac{\ln\left(\frac{1}{4} - \frac{e^x}{4}\right)}{16} - \frac{\ln\left(\frac{e^x}{4} + \frac{1}{4}\right)}{16} - \frac{\text{atan}(e^x)}{8} - \frac{e^{5\,x}}{2\,\left(e^{8\,x} - 2\,e^{4\,x} + 1\right)} - \frac{3\,e^x}{4\,\left(e^{4\,x} - 1\right)} - \frac{e^x}{2\,\left(e^{8\,x} - 2\,e^{4\,x} + 1\right)}$$

input
```
int((coth(2*x)*exp(x))/sinh(2*x)^2,x)
```

output
```
log(1/4 - exp(x)/4)/16 - log(exp(x)/4 + 1/4)/16 - atan(exp(x))/8 - exp(5*x
)/(2*(exp(8*x) - 2*exp(4*x) + 1)) - (3*exp(x))/(4*(exp(4*x) - 1)) - exp(x)
/(2*(exp(8*x) - 2*exp(4*x) + 1))
```

3.943 $\int e^x \coth^2(2x)\mathbf{csch}(2x)\,dx$

3.943.1 Optimal result

Integrand size = 14, antiderivative size = 55

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = -\frac{e^{3x}}{(1-e^{4x})^2}+\frac{3e^{3x}}{4(1-e^{4x})}+\frac{5\arctan(e^x)}{8}-\frac{5\text{arctanh}(e^x)}{8}$$

output
```
-exp(3*x)/(1-exp(4*x))^2+3/4*exp(3*x)/(1-exp(4*x))+5/8*arctan(exp(x))-5/8*
arctanh(exp(x))
```

3.943.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.62 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.93

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx$$
$$= \frac{e^{-5x}(177023+244931e^{4x}+43161e^{8x}-26091e^{12x}-7(25289+24152e^{4x}-10058e^{8x}-9048e^{12x}+513e^{16}}{10752}$$
$$-\frac{8e^{7x}(15+26e^{4x}+11e^{8x})\ {}_4F_3\left(\frac{7}{4},2,2,2;1,1,\frac{19}{4};e^{4x}\right)}{1155}$$
$$-\frac{16e^{7x}(1+e^{4x})^2\ {}_5F_4\left(\frac{7}{4},2,2,2,2;1,1,1,\frac{19}{4};e^{4x}\right)}{1155}$$

input
```
Integrate[E^x*Coth[2*x]^2*Csch[2*x],x]
```

output
```
(177023 + 244931*E^(4*x) + 43161*E^(8*x) - 26091*E^(12*x) - 7*(25289 + 241
52*E^(4*x) - 10058*E^(8*x) - 9048*E^(12*x) + 513*E^(16*x))*Hypergeometric2
F1[3/4, 1, 7/4, E^(4*x)])/(10752*E^(5*x)) - (8*E^(7*x)*(15 + 26*E^(4*x) +
11*E^(8*x))*HypergeometricPFQ[{7/4, 2, 2, 2}, {1, 1, 19/4}, E^(4*x)])/1155
 - (16*E^(7*x)*(1 + E^(4*x))^2*HypergeometricPFQ[{7/4, 2, 2, 2, 2}, {1, 1,
 1, 19/4}, E^(4*x)])/1155
```

3.943.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2720, 27, 963, 27, 957, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{2e^{2x}\left(e^{4x}+1\right)^2}{\left(1-e^{4x}\right)^3}de^x$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{e^{2x}\left(1+e^{4x}\right)^2}{\left(1-e^{4x}\right)^3}de^x$$

$$\Big\downarrow \text{963}$$

$$-2\left(\frac{e^{3x}}{2\left(1-e^{4x}\right)^2}-\frac{1}{8}\int \frac{4e^{2x}\left(1+2e^{4x}\right)}{\left(1-e^{4x}\right)^2}de^x\right)$$

$$\Big\downarrow \text{27}$$

$$-2\left(\frac{e^{3x}}{2\left(1-e^{4x}\right)^2}-\frac{1}{2}\int \frac{e^{2x}\left(1+2e^{4x}\right)}{\left(1-e^{4x}\right)^2}de^x\right)$$

$$\Big\downarrow \text{957}$$

$$-2\left(\frac{1}{2}\left(\frac{5}{4}\int \frac{e^{2x}}{1-e^{4x}}de^x-\frac{3e^{3x}}{4\left(1-e^{4x}\right)}\right)+\frac{e^{3x}}{2\left(1-e^{4x}\right)^2}\right)$$

$$\Big\downarrow \text{827}$$

$$-2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x-\frac{1}{2}\int \frac{1}{1+e^{2x}}de^x\right)-\frac{3e^{3x}}{4\left(1-e^{4x}\right)}\right)+\frac{e^{3x}}{2\left(1-e^{4x}\right)^2}\right)$$

$$\Big\downarrow \; 216$$

$$-2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x-\frac{\arctan\left(e^x\right)}{2}\right)-\frac{3e^{3x}}{4\left(1-e^{4x}\right)}\right)+\frac{e^{3x}}{2\left(1-e^{4x}\right)^2}\right)$$

$$\Big\downarrow \; 219$$

$$-2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{\text{arctanh}(e^x)}{2}-\frac{\arctan\left(e^x\right)}{2}\right)-\frac{3e^{3x}}{4\left(1-e^{4x}\right)}\right)+\frac{e^{3x}}{2\left(1-e^{4x}\right)^2}\right)$$

input
```
Int[E^x*Coth[2*x]^2*Csch[2*x],x]
```

output
```
-2*(E^(3*x)/(2*(1 - E^(4*x))^2) + ((-3*E^(3*x))/(4*(1 - E^(4*x))) + (5*(-1
/2*ArcTan[E^x] + ArcTanh[E^x]/2))/4)/2)
```

3.943.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

3.943. $\int e^x \coth^2(2x)\text{csch}(2x)\,dx$

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.943.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{e^{3x}(3e^{4x}+1)}{4(e^{4x}-1)^2}+\frac{5i\ln(e^x+i)}{16}-\frac{5i\ln(e^x-i)}{16}+\frac{5\ln(e^x-1)}{16}-\frac{5\ln(1+e^x)}{16}$
default	$-\frac{\cosh(x)}{2\sinh(x)^2}+\frac{\coth(x)\operatorname{csch}(x)}{2}-\frac{5\operatorname{arctanh}(e^x)}{8}+\frac{5\arctan(e^x)}{8}-\frac{1}{8\sinh(x)\cosh(x)^2}-\frac{3\operatorname{sech}(x)\tanh(x)}{16}-\frac{1}{16\sinh(x)^2\cosh(x)^2}$

input
```
int(exp(x)*coth(2*x)^2*csch(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(x)^3*(3*exp(x)^4+1)/(exp(x)^4-1)^2+5/16*I*ln(exp(x)+I)-5/16*I*ln(
exp(x)-I)+5/16*ln(exp(x)-1)-5/16*ln(1+exp(x))
```

3.943.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(39) = 78.

Time = 0.27 (sec) , antiderivative size = 557, normalized size of antiderivative = 10.13

$$\int e^x \coth^2(2x)\operatorname{csch}(2x)\,dx = \text{Too large to display}$$

input
```
integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="fricas")
```

output
```
-1/16*(12*cosh(x)^7 + 420*cosh(x)^3*sinh(x)^4 + 252*cosh(x)^2*sinh(x)^5 +
84*cosh(x)*sinh(x)^6 + 12*sinh(x)^7 + 4*(105*cosh(x)^4 + 1)*sinh(x)^3 + 4*
cosh(x)^3 + 12*(21*cosh(x)^5 + cosh(x))*sinh(x)^2 - 10*(cosh(x)^8 + 56*cos
h(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^
8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x
))*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - co
sh(x)^3)*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*(cosh(x)^8 + 56*cosh(x
)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 +
 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*
sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(
x)^3)*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) - 5*(cosh(x)^8 + 56*cosh(x)^
3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2
*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*si
nh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)
^3)*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 12*(7*cosh(x)^6 + cosh(x)^2)
*sinh(x))/(cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8
*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x
)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*
sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) + 1)
```

3.943.6 Sympy [F]

$$\int e^x \coth^2(2x)\operatorname{csch}(2x)\,dx = \int e^x \coth^2(2x)\operatorname{csch}(2x)\,dx$$

input
```
integrate(exp(x)*coth(2*x)**2*csch(2*x),x)
```

output
```
Integral(exp(x)*coth(2*x)**2*csch(2*x), x)
```

3.943.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = -\frac{3\,e^{(7\,x)} + e^{(3\,x)}}{4\,(e^{(8\,x)} - 2\,e^{(4\,x)} + 1)} + \frac{5}{8}\,\arctan\left(e^x\right) - \frac{5}{16}\,\log\left(e^x + 1\right) + \frac{5}{16}\,\log\left(e^x - 1\right)$$

input
```
integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="maxima")
```

output
```
-1/4*(3*e^(7*x) + e^(3*x))/(e^(8*x) - 2*e^(4*x) + 1) + 5/8*arctan(e^x) - 5
/16*log(e^x + 1) + 5/16*log(e^x - 1)
```

3.943.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = -\frac{3\,e^{(7\,x)} + e^{(3\,x)}}{4\,(e^{(4\,x)} - 1)^2} + \frac{5}{8}\,\arctan\left(e^x\right) - \frac{5}{16}\,\log\left(e^x + 1\right) + \frac{5}{16}\,\log\left(|e^x - 1|\right)$$

input
```
integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="giac")
```

output
```
-1/4*(3*e^(7*x) + e^(3*x))/(e^(4*x) - 1)^2 + 5/8*arctan(e^x) - 5/16*log(e^
x + 1) + 5/16*log(abs(e^x - 1))
```

3.943.9 Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = \frac{5\,\ln\left(\frac{25\,e^x}{16} - \frac{25}{16}\right)}{16} - \frac{5\,\ln\left(\frac{25\,e^x}{16} + \frac{25}{16}\right)}{16} - \frac{5\,\mathrm{atan}(e^{-x})}{8} - \frac{e^{3\,x}}{e^{8\,x} - 2\,e^{4\,x} + 1} - \frac{3\,e^{3\,x}}{4\,\left(e^{4\,x} - 1\right)}$$

input
```
int((coth(2*x)^2*exp(x))/sinh(2*x),x)
```

output
```
(5*log((25*exp(x))/16 - 25/16))/16 - (5*log((25*exp(x))/16 + 25/16))/16 -
(5*atan(exp(-x)))/8 - exp(3*x)/(exp(8*x) - 2*exp(4*x) + 1) - (3*exp(3*x))/
(4*(exp(4*x) - 1))
```

3.943. $\int e^x \coth^2(2x)\text{csch}(2x)\,dx$

3.944 $\int e^x \coth^2(2x)\mathbf{csch}^2(2x)\,dx$

3.944.1 Optimal result

Integrand size = 16, antiderivative size = 75

$$\int e^x \coth^2(2x)\text{csch}^2(2x)\,dx = \frac{4e^{5x}}{3(1-e^{4x})^3} - \frac{5e^{5x}}{6(1-e^{4x})^2} + \frac{3e^x}{8(1-e^{4x})} - \frac{3\arctan(e^x)}{16} - \frac{3\text{arctanh}(e^x)}{16}$$

output
```
4/3*exp(5*x)/(1-exp(4*x))^3-5/6*exp(5*x)/(1-exp(4*x))^2+3/8*exp(x)/(1-exp(
4*x))-3/16*arctan(exp(x))-3/16*arctanh(exp(x))
```

3.944.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.68 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.13

$$\int e^x \coth^2(2x)\text{csch}^2(2x)\,dx = \frac{e^{-7x}\left(-1070609085 - 946471617e^{4x} + 369641285e^{8x} + 351173641e^{12x} - 23818496e^{16x} + 1070609085\,\text{Hyp}\right.}{}$$

input
```
Integrate[E^x*Coth[2*x]^2*Csch[2*x]^2,x]
```

output
```
(-1070609085 - 946471617*E^(4*x) + 369641285*E^(8*x) + 351173641*E^(12*x)
- 23818496*E^(16*x) + 1070609085*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] +
 732349800*E^(4*x)*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] - 635067810*E^(
8*x)*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] - 384831720*E^(12*x)*Hypergeo
metric2F1[1/4, 1, 5/4, E^(4*x)] + 60913125*E^(16*x)*Hypergeometric2F1[1/4,
 1, 5/4, E^(4*x)] + 1280*E^(16*x)*(821 + 1346*E^(4*x) + 557*E^(8*x))*Hyper
geometricPFQ[{2, 2, 2, 9/4}, {1, 1, 21/4}, E^(4*x)] + 10240*E^(16*x)*(23 +
 42*E^(4*x) + 19*E^(8*x))*HypergeometricPFQ[{2, 2, 2, 2, 9/4}, {1, 1, 1, 2
1/4}, E^(4*x)] + 20480*E^(16*x)*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4}, {1
, 1, 1, 1, 21/4}, E^(4*x)] + 40960*E^(20*x)*HypergeometricPFQ[{2, 2, 2, 2,
 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(4*x)] + 20480*E^(24*x)*HypergeometricPFQ[
{2, 2, 2, 2, 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(4*x)])/(3818880*E^(7*x))
```

3.944.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {2720, 27, 963, 27, 957, 817, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \coth^2(2x)\text{csch}^2(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{4e^{4x}\left(e^{4x}+1\right)^2}{\left(1-e^{4x}\right)^4}de^x$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{4x}\left(1+e^{4x}\right)^2}{\left(1-e^{4x}\right)^4}de^x$$

$$\Big\downarrow \text{963}$$

$$4\left(\frac{e^{5x}}{3\left(1-e^{4x}\right)^3}-\frac{1}{12}\int \frac{4e^{4x}\left(2+3e^{4x}\right)}{\left(1-e^{4x}\right)^3}de^x\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{e^{5x}}{3\left(1-e^{4x}\right)^3}-\frac{1}{3}\int \frac{e^{4x}\left(2+3e^{4x}\right)}{\left(1-e^{4x}\right)^3}de^x\right)$$

$$\Big\downarrow \; 957$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\int \frac{e^{4x}}{(1-e^{4x})^2}de^x-\frac{5e^{5x}}{8\,(1-e^{4x})^2}\right)+\frac{e^{5x}}{3\,(1-e^{4x})^3}\right)$$

$$\Big\downarrow \; 817$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{e^x}{4\,(1-e^{4x})}-\frac{1}{4}\int \frac{1}{1-e^{4x}}de^x\right)-\frac{5e^{5x}}{8\,(1-e^{4x})^2}\right)+\frac{e^{5x}}{3\,(1-e^{4x})^3}\right)$$

$$\Big\downarrow \; 756$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x-\frac{1}{2}\int \frac{1}{1+e^{2x}}de^x\right)+\frac{e^x}{4\,(1-e^{4x})}\right)-\frac{5e^{5x}}{8\,(1-e^{4x})^2}\right)+\frac{e^{5x}}{3\,(1-e^{4x})^3}\right)$$

$$\Big\downarrow \; 216$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x-\frac{1}{2}\arctan\left(e^x\right)\right)+\frac{e^x}{4\,(1-e^{4x})}\right)-\frac{5e^{5x}}{8\,(1-e^{4x})^2}\right)+\frac{e^{5x}}{3\,(1-e^{4x})^3}\right)$$

$$\Big\downarrow \; 219$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(-\frac{1}{2}\arctan\left(e^x\right)-\frac{\text{arctanh}(e^x)}{2}\right)+\frac{e^x}{4\,(1-e^{4x})}\right)-\frac{5e^{5x}}{8\,(1-e^{4x})^2}\right)+\frac{e^{5x}}{3\,(1-e^{4x})^3}\right)$$

input
```
Int[E^x*Coth[2*x]^2*Csch[2*x]^2,x]
```

output
```
4*(E^(5*x)/(3*(1 - E^(4*x))^3) + ((-5*E^(5*x))/(8*(1 - E^(4*x))^2) + (9*(E
^x/(4*(1 - E^(4*x))) + (-1/2*ArcTan[E^x] - ArcTanh[E^x]/2)/4))/8)/3)
```

3.944.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.944. $\int e^x \coth^2(2x)\text{csch}^2(2x)\,dx$

3.944.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{e^{x}\left(29\,e^{8x}-6\,e^{4x}+9\right)}{24\left(e^{4x}-1\right)^{3}}+\frac{3i\ln\left(e^{x}-i\right)}{32}-\frac{3i\ln\left(e^{x}+i\right)}{32}-\frac{3\ln\left(1+e^{x}\right)}{32}+\frac{3\ln\left(e^{x}-1\right)}{32}$
default	$-\frac{\coth(x)\operatorname{csch}(x)}{8}-\frac{3\,\operatorname{arctanh}(e^{x})}{16}+\frac{1}{8\sinh(x)^{2}\cosh(x)}+\frac{7}{32\cosh(x)}-\frac{1}{4\sinh(x)}-\frac{3\arctan(e^{x})}{16}-\frac{1}{32\sinh(x)^{2}\cosh(x)^{3}}$

input
```
int(exp(x)*coth(2*x)^2*csch(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24*exp(x)*(29*exp(8*x)-6*exp(4*x)+9)/(exp(4*x)-1)^3+3/32*I*ln(exp(x)-I)
-3/32*I*ln(exp(x)+I)-3/32*ln(1+exp(x))+3/32*ln(exp(x)-1)
```

3.944.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(51) = 102.

Time = 0.28 (sec) , antiderivative size = 992, normalized size of antiderivative = 13.23

$$\int e^{x}\coth^{2}(2x)\operatorname{csch}^{2}(2x)\,dx=\text{Too large to display}$$

input
```
integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="fricas")
```

output
```
-1/96*(116*cosh(x)^9 + 9744*cosh(x)^3*sinh(x)^6 + 4176*cosh(x)^2*sinh(x)^7
 + 1044*cosh(x)*sinh(x)^8 + 116*sinh(x)^9 + 24*(609*cosh(x)^4 - 1)*sinh(x)
^5 - 24*cosh(x)^5 + 24*(609*cosh(x)^5 - 5*cosh(x))*sinh(x)^4 + 48*(203*cos
h(x)^6 - 5*cosh(x)^2)*sinh(x)^3 + 48*(87*cosh(x)^7 - 5*cosh(x)^3)*sinh(x)^
2 + 18*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*cosh(x)^2*sinh(x)^10 + 1
2*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^4 - 1)*sinh(x)^8 - 3*co
sh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84*(11*cosh(x)^6 - cosh(
x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*sinh(x)^5 + 3*(165*cosh(
x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(55*cosh(x)^9 - 42*co
sh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 14*cosh(x)^6 + 3*cosh(
x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(x)^3)*sinh(x) - 1)*a
rctan(cosh(x) + sinh(x)) + 9*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*co
sh(x)^2*sinh(x)^10 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^4
 - 1)*sinh(x)^8 - 3*cosh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84
*(11*cosh(x)^6 - cosh(x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*si
nh(x)^5 + 3*(165*cosh(x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4
*(55*cosh(x)^9 - 42*cosh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 -
14*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cos
h(x)^3)*sinh(x) - 1)*log(cosh(x) + sinh(x) + 1) - 9*(cosh(x)^12 + 220*cosh
(x)^3*sinh(x)^9 + 66*cosh(x)^2*sinh(x)^10 + 12*cosh(x)*sinh(x)^11 + sin...
```

3.944.6 Sympy [F]

$$\int e^x \coth^2(2x)\text{csch}^2(2x)\,dx = \int e^x \coth^2(2x)\,\text{csch}^2(2x)\,dx$$

input
```
integrate(exp(x)*coth(2*x)**2*csch(2*x)**2,x)
```

output
```
Integral(exp(x)*coth(2*x)**2*csch(2*x)**2, x)
```

3.944.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx = -\frac{29\,e^{(9\,x)} - 6\,e^{(5\,x)} + 9\,e^x}{24\,(e^{(12\,x)} - 3\,e^{(8\,x)} + 3\,e^{(4\,x)} - 1)} - \frac{3}{16}\arctan\left(e^x\right) - \frac{3}{32}\log\left(e^x + 1\right) + \frac{3}{32}\log\left(e^x - 1\right)$$

input
```
integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="maxima")
```

output
```
-1/24*(29*e^(9*x) - 6*e^(5*x) + 9*e^x)/(e^(12*x) - 3*e^(8*x) + 3*e^(4*x) -
 1) - 3/16*arctan(e^x) - 3/32*log(e^x + 1) + 3/32*log(e^x - 1)
```

3.944.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx = -\frac{29\,e^{(9\,x)} - 6\,e^{(5\,x)} + 9\,e^x}{24\,(e^{(4\,x)} - 1)^3} - \frac{3}{16}\arctan\left(e^x\right) - \frac{3}{32}\log\left(e^x + 1\right) + \frac{3}{32}\log\left(|e^x - 1|\right)$$

input
```
integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="giac")
```

output
```
-1/24*(29*e^(9*x) - 6*e^(5*x) + 9*e^x)/(e^(4*x) - 1)^3 - 3/16*arctan(e^x)
- 3/32*log(e^x + 1) + 3/32*log(abs(e^x - 1))
```

3.944.9 Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.52

$$\int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx = \frac{3\,\ln\left(\frac{3}{8} - \frac{3\,e^x}{8}\right)}{32} - \frac{3\,\ln\left(-\frac{3\,e^x}{8} - \frac{3}{8}\right)}{32} - \frac{7\,e^x}{8\,(e^{4\,x} - 1)} - \frac{\frac{2\,e^{5\,x}}{3} + \frac{e^{9\,x}}{3} + \frac{e^x}{3}}{3\,e^{4\,x} - 3\,e^{8\,x} + e^{12\,x} - 1} - \frac{5\,e^x}{6\,(e^{8\,x} - 2\,e^{4\,x} + 1)} - \frac{\ln\left(-\frac{3\,e^x}{8} - \frac{3}{8}\mathrm{i}\right)\,3\mathrm{i}}{32} + \frac{\ln\left(-\frac{3\,e^x}{8} + \frac{3}{8}\mathrm{i}\right)\,3\mathrm{i}}{32}$$

input
```
int((coth(2*x)^2*exp(x))/sinh(2*x)^2,x)
```

output
```
(3*log(3/8 - (3*exp(x))/8))/32 - (3*log(- (3*exp(x))/8 - 3/8))/32 - (log(-
 (3*exp(x))/8 - 3i/8)*3i)/32 + (log(3i/8 - (3*exp(x))/8)*3i)/32 - (7*exp(x
))/(8*(exp(4*x) - 1)) - ((2*exp(5*x))/3 + exp(9*x)/3 + exp(x)/3)/(3*exp(4*
x) - 3*exp(8*x) + exp(12*x) - 1) - (5*exp(x))/(6*(exp(8*x) - 2*exp(4*x) +
1))
```

3.944. $\int e^x \coth^2(2x)\text{csch}^2(2x)\,dx$

3.945 $\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx$

3.945.1 Optimal result

Integrand size = 22, antiderivative size = 137

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx=-\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}+\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}$$

output
```
-1/2*b*exp(d*x+c)*cosh(2*b*x+2*a)/(4*b^2-d^2)+1/2*b*exp(d*x+c)*cosh(4*b*x+
4*a)/(16*b^2-d^2)+1/4*d*exp(d*x+c)*sinh(2*b*x+2*a)/(4*b^2-d^2)-1/8*d*exp(d
*x+c)*sinh(4*b*x+4*a)/(16*b^2-d^2)
```

3.945.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx=\frac{1}{8}e^{c+dx}\left(\frac{-4b\cosh(2(a+bx))+2d\sinh(2(a+bx))}{4b^2-d^2}+\frac{4b\cosh(4(a+bx))-d\sinh(4(a+bx))}{16b^2-d^2}\right)$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
(E^(c + d*x)*((-4*b*Cosh[2*(a + b*x)] + 2*d*Sinh[2*(a + b*x)])/(4*b^2 - d^
2) + (4*b*Cosh[4*(a + b*x)] - d*Sinh[4*(a + b*x)])/(16*b^2 - d^2)))/8
```

3.945.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\sinh^3(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{6035}$$

$$\int \left(\frac{1}{8}e^{c+dx}\sinh(4a+4bx)-\frac{1}{4}e^{c+dx}\sinh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}-\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

output
```
-1/2*(b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(4*b^2 - d^2) + (b*E^(c + d*x)*Cosh
[4*a + 4*b*x])/(2*(16*b^2 - d^2)) + (d*E^(c + d*x)*Sinh[2*a + 2*b*x])/(4*(
4*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[4*a + 4*b*x])/(8*(16*b^2 - d^2))
```

3.945.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.945.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.47

$$\frac{\sinh\left(2a-c+\left(2b-d\right)x\right)}{16b-8d}-\frac{\sinh\left(2a+c+\left(2b+d\right)x\right)}{8\left(2b+d\right)}-\frac{\sinh\left(\left(4b-d\right)x+4a-c\right)}{16\left(4b-d\right)}+\frac{\sinh\left(\left(4b+d\right)x+4a+\right.}{64b+16d}$$

input
```
int(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

output
```
1/8*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-1/8*sinh(2*a+c+(2*b+d)*x)/(2*b+d)-1/16/(
4*b-d)*sinh((4*b-d)*x+4*a-c)+1/16/(4*b+d)*sinh((4*b+d)*x+4*a+c)-1/8*cosh(2
*a-c+(2*b-d)*x)/(2*b-d)-1/8*cosh(2*a+c+(2*b+d)*x)/(2*b+d)+1/16*cosh((4*b-d
)*x+4*a-c)/(4*b-d)+1/16*cosh((4*b+d)*x+4*a+c)/(4*b+d)
```

3.945.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(125) = 250$.

Time = 0.27 (sec) , antiderivative size = 505, normalized size of antiderivative = 3.69

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx =$$
$$-\frac{\left(4\,b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-\left(4\,b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^4+\left(16\right.}{}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*((4*b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^3 - (4*b^3
 - b*d^2)*cosh(d*x + c)*sinh(b*x + a)^4 + (16*b^3 - b*d^2 - 6*(4*b^3 - b*d
^2)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a)^2 + ((4*b^2*d - d^3)*cosh
(b*x + a)^3 - (16*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c)*sinh(b*x + a)
- ((4*b^3 - b*d^2)*cosh(b*x + a)^4 - (16*b^3 - b*d^2)*cosh(b*x + a)^2)*cos
h(d*x + c) - ((4*b^3 - b*d^2)*cosh(b*x + a)^4 - (4*b^2*d - d^3)*cosh(b*x +
 a)*sinh(b*x + a)^3 + (4*b^3 - b*d^2)*sinh(b*x + a)^4 - (16*b^3 - b*d^2)*c
osh(b*x + a)^2 - (16*b^3 - b*d^2 - 6*(4*b^3 - b*d^2)*cosh(b*x + a)^2)*sinh
(b*x + a)^2 - ((4*b^2*d - d^3)*cosh(b*x + a)^3 - (16*b^2*d - d^3)*cosh(b*x
 + a))*sinh(b*x + a))*sinh(d*x + c))/((64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x
 + a)^4 - 2*(64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 +
(64*b^4 - 20*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

3.945.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1295 vs. 2(114) = 228.

Time = 7.77 (sec) , antiderivative size = 1295, normalized size of antiderivative = 9.45

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Piecewise((x*exp(c)*sinh(a)**3*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*ex
p(d*x)*sinh(a - d*x/2)**4/8 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)**3*cosh(a
- d*x/2)/4 - x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)**3/4 - x*ex
p(c)*exp(d*x)*cosh(a - d*x/2)**4/8 - 7*exp(c)*exp(d*x)*sinh(a - d*x/2)**4/
(24*d) - exp(c)*exp(d*x)*sinh(a - d*x/2)**3*cosh(a - d*x/2)/(3*d) - exp(c)
*exp(d*x)*sinh(a - d*x/2)**2*cosh(a - d*x/2)**2/(2*d) + exp(c)*exp(d*x)*co
sh(a - d*x/2)**4/(8*d), Eq(b, -d/2)), (x*exp(c)*exp(d*x)*sinh(a - d*x/4)**
4/16 + x*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(a - d*x/4)/4 + 3*x*exp(c)
*exp(d*x)*sinh(a - d*x/4)**2*cosh(a - d*x/4)**2/8 + x*exp(c)*exp(d*x)*sinh
(a - d*x/4)*cosh(a - d*x/4)**3/4 + x*exp(c)*exp(d*x)*cosh(a - d*x/4)**4/16
 + exp(c)*exp(d*x)*sinh(a - d*x/4)**4/(6*d) + 11*exp(c)*exp(d*x)*sinh(a -
d*x/4)**3*cosh(a - d*x/4)/(12*d) - 5*exp(c)*exp(d*x)*sinh(a - d*x/4)*cosh(
a - d*x/4)**3/(12*d) - exp(c)*exp(d*x)*cosh(a - d*x/4)**4/(6*d), Eq(b, -d/
4)), (-x*exp(c)*exp(d*x)*sinh(a + d*x/4)**4/16 + x*exp(c)*exp(d*x)*sinh(a
+ d*x/4)**3*cosh(a + d*x/4)/4 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/4)**2*cos
h(a + d*x/4)**2/8 + x*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/4
 - x*exp(c)*exp(d*x)*cosh(a + d*x/4)**4/16 - exp(c)*exp(d*x)*sinh(a + d*x/
4)**4/(6*d) + 11*exp(c)*exp(d*x)*sinh(a + d*x/4)**3*cosh(a + d*x/4)/(12*d)
 - 5*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/(12*d) + exp(c)*ex
p(d*x)*cosh(a + d*x/4)**4/(6*d), Eq(b, d/4)), (-x*exp(c)*exp(d*x)*sinh(...
```

3.945.7 Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for mor
e details)
```

3.945.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(4\,bx+dx+4\,a+c)}}{16\,(4\,b+d)} - \frac{e^{(2\,bx+dx+2\,a+c)}}{8\,(2\,b+d)} - \frac{e^{(-2\,bx+dx-2\,a+c)}}{8\,(2\,b-d)} + \frac{e^{(-4\,bx+dx-4\,a+c)}}{16\,(4\,b-d)}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/16*e^(4*b*x + d*x + 4*a + c)/(4*b + d) - 1/8*e^(2*b*x + d*x + 2*a + c)/(
2*b + d) - 1/8*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) + 1/16*e^(-4*b*x + d*x
 - 4*a + c)/(4*b - d)
```

3.945.9 Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.66

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{b\,\mathrm{e}^{c+d\,x}\,\sinh(a+b\,x)^4\,(10\,b^2-d^2)}{64\,b^4-20\,b^2\,d^2+d^4} - \frac{6\,b^3\cosh(a+b\,x)^4\,\mathrm{e}^{c+d\,x}}{64\,b^4-20\,b^2\,d^2+d^4} + \frac{3\,b\cosh(a+b\,x)^2\,\mathrm{e}^{c+d\,x}\,\sinh(a+b\,x)^2}{16\,b^2-d^2} - \frac{d\cosh(a+b\,x)\,\mathrm{e}^{c+d\,x}\,\sinh(a+b\,x)^3\,(10\,b^2-d^2)}{64\,b^4-20\,b^2\,d^2+d^4} + \frac{6\,b^2\,d\cosh(a+b\,x)^3\,\mathrm{e}^{c+d\,x}\,\sinh(a+b\,x)}{(4\,b^2-d^2)\,(16\,b^2-d^2)}$$

input
```
int(cosh(a + b*x)*exp(c + d*x)*sinh(a + b*x)^3,x)
```

output
```
(b*exp(c + d*x)*sinh(a + b*x)^4*(10*b^2 - d^2))/(64*b^4 + d^4 - 20*b^2*d^2
) - (6*b^3*cosh(a + b*x)^4*exp(c + d*x))/(64*b^4 + d^4 - 20*b^2*d^2) + (3*
b*cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x)^2)/(16*b^2 - d^2) - (d*cosh(a
 + b*x)*exp(c + d*x)*sinh(a + b*x)^3*(10*b^2 - d^2))/(64*b^4 + d^4 - 20*b^
2*d^2) + (6*b^2*d*cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x))/((4*b^2 - d^
2)*(16*b^2 - d^2))
```

3.945. $\int e^{c+dx} \cosh(a+bx) \sinh^3(a+bx) \, dx$

3.946 $\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx$

3.946.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx=\frac{de^{c+dx}\cosh(a+bx)}{4(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3bx)}{4(9b^2-d^2)}-\frac{be^{c+dx}\sinh(a+bx)}{4(b^2-d^2)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{4(9b^2-d^2)}$$

output
```
1/4*d*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)-1/4*d*exp(d*x+c)*cosh(3*b*x+3*a)/(9
*b^2-d^2)-1/4*b*exp(d*x+c)*sinh(b*x+a)/(b^2-d^2)+3/4*b*exp(d*x+c)*sinh(3*b
*x+3*a)/(9*b^2-d^2)
```

3.946.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx=\frac{1}{4}e^{c+dx}\left(\frac{d\cosh(a+bx)-b\sinh(a+bx)}{(b-d)(b+d)}+\frac{-d\cosh(3(a+bx))+3b\sinh(3(a+bx))}{9b^2-d^2}\right)$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(E^(c + d*x)*((d*Cosh[a + b*x] - b*Sinh[a + b*x])/((b - d)*(b + d)) + (-(d
*Cosh[3*(a + b*x)]) + 3*b*Sinh[3*(a + b*x)])/(9*b^2 - d^2)))/4
```

3.946.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\sinh^2(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{6035}$$

$$\int\left(\frac{1}{4}e^{c+dx}\cosh(3a+3bx)-\frac{1}{4}e^{c+dx}\cosh(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{be^{c+dx}\sinh(a+bx)}{4(b^2-d^2)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{4(9b^2-d^2)}+\frac{de^{c+dx}\cosh(a+bx)}{4(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3bx)}{4(9b^2-d^2)}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

output
```
(d*E^(c + d*x)*Cosh[a + b*x])/(4*(b^2 - d^2)) - (d*E^(c + d*x)*Cosh[3*a +
3*b*x])/(4*(9*b^2 - d^2)) - (b*E^(c + d*x)*Sinh[a + b*x])/(4*(b^2 - d^2))
+ (3*b*E^(c + d*x)*Sinh[3*a + 3*b*x])/(4*(9*b^2 - d^2))
```

3.946.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.946.4 Maple [A] (verified)

Time = 24.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.40

method	result
default	$-\frac{\sinh(a-c+(b-d)x)}{8(b-d)}-\frac{\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(3a-c+(3b-d)x)}{24b-8d}+\frac{\sinh(3a+c+(3b+d)x)}{24b+8d}+\frac{\cosh(a-c+(b-d)x)}{8b-8d}-\cosh($
risch	$\frac{(3b^3 e^{6bx+6a}-b^2 d\, e^{6bx+6a}-3b\, d^2 e^{6bx+6a}+d^3 e^{6bx+6a}-9b^3 e^{4bx+4a}+9b^2 d\, e^{4bx+4a}+b\, d^2 e^{4bx+4a}-d^3 e^{4bx+4a}+9b^3 e^{2bx+2a}+9b^2 d\, e^{2bx+}}{8(3b+d)(b+d)(3b-d)(b-d)}$

input
```
int(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*sinh(a-c+(b-d)*x)/(b-d)-1/8*sinh(a+c+(b+d)*x)/(b+d)+1/8*sinh(3*a-c+(3
*b-d)*x)/(3*b-d)+1/8*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/8*cosh(a-c+(b-d)*x)/(
b-d)-1/8*cosh(a+c+(b+d)*x)/(b+d)-1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/8*cos
h(3*a+c+(3*b+d)*x)/(3*b+d)
```

3.946.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(115) = 230$.

Time = 0.26 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.98

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx =$$
$$-\frac{3\left(b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2-3\left(b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3+\left(9\,b^3\right.}{}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 - 3*(b^3
 - b*d^2)*cosh(d*x + c)*sinh(b*x + a)^3 + (9*b^3 - b*d^2 - 9*(b^3 - b*d^2)
*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a) + ((b^2*d - d^3)*cosh(b*x +
a)^3 - (9*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c) + ((b^2*d - d^3)*cosh(
b*x + a)^3 + 3*(b^2*d - d^3)*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b^3 - b*d^
2)*sinh(b*x + a)^3 - (9*b^2*d - d^3)*cosh(b*x + a) + (9*b^3 - b*d^2 - 9*(b
^3 - b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^
2*d^2 + d^4)*cosh(b*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^
2*sinh(b*x + a)^2 + (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

3.946.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(109) = 218$.

Time = 2.74 (sec) , antiderivative size = 972, normalized size of antiderivative = 7.65

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x*exp(c)*sinh(a)**2*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*ex
p(d*x)*sinh(a - d*x)**3/8 + x*exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*
x)/8 - x*exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/8 - x*exp(c)*exp(d
*x)*cosh(a - d*x)**3/8 - 3*exp(c)*exp(d*x)*sinh(a - d*x)**3/(8*d) - exp(c)
*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/(4*d) + exp(c)*exp(d*x)*cosh(a -
d*x)**3/(8*d), Eq(b, -d)), (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/8 + 3*x*e
xp(c)*exp(d*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3)/8 + 3*x*exp(c)*exp(d*x)*
sinh(a - d*x/3)*cosh(a - d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**
3/8 + exp(c)*exp(d*x)*sinh(a - d*x/3)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a
- d*x/3)**2*cosh(a - d*x/3)/(4*d) - exp(c)*exp(d*x)*cosh(a - d*x/3)**3/(8*
d), Eq(b, -d/3)), (-x*exp(c)*exp(d*x)*sinh(a + d*x/3)**3/8 + 3*x*exp(c)*ex
p(d*x)*sinh(a + d*x/3)**2*cosh(a + d*x/3)/8 - 3*x*exp(c)*exp(d*x)*sinh(a +
 d*x/3)*cosh(a + d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a + d*x/3)**3/8 + ex
p(c)*exp(d*x)*sinh(a + d*x/3)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a + d*x/3)
*cosh(a + d*x/3)**2/(4*d) - 3*exp(c)*exp(d*x)*cosh(a + d*x/3)**3/(8*d), Eq
(b, d/3)), (-x*exp(c)*exp(d*x)*sinh(a + d*x)**3/8 + x*exp(c)*exp(d*x)*sinh
(a + d*x)**2*cosh(a + d*x)/8 + x*exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*
x)**2/8 - x*exp(c)*exp(d*x)*cosh(a + d*x)**3/8 + exp(c)*exp(d*x)*sinh(a +
d*x)**3/(8*d) + exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**2/(4*d) - exp
(c)*exp(d*x)*cosh(a + d*x)**3/(8*d), Eq(b, d)), (3*b**3*exp(c)*exp(d*x)...
```

3.946.7 Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more
 details)I
```

3.946.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(3\,bx+dx+3\,a+c)}}{8\,(3\,b+d)} - \frac{e^{(bx+dx+a+c)}}{8\,(b+d)} + \frac{e^{(-bx+dx-a+c)}}{8\,(b-d)} - \frac{e^{(-3\,bx+dx-3\,a+c)}}{8\,(3\,b-d)}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/8*e^(3*b*x + d*x + 3*a + c)/(3*b + d) - 1/8*e^(b*x + d*x + a + c)/(b + d
) + 1/8*e^(-b*x + d*x - a + c)/(b - d) - 1/8*e^(-3*b*x + d*x - 3*a + c)/(3
*b - d)
```

3.946.9 Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx$$

$$= \frac{\mathrm{e}^{c+d\,x}\,\left(3\,b^3\,\sinh(a+b\,x)^3+2\,b^2\,d\,\cosh(a+b\,x)^3-3\,b^2\,d\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2-2\,b\,d^2\,\cosh(a+b\,x)}{9\,b^4-10\,b^2\,d^2+d^4}$$

input
```
int(cosh(a + b*x)*exp(c + d*x)*sinh(a + b*x)^2,x)
```

output
```
(exp(c + d*x)*(3*b^3*sinh(a + b*x)^3 + 2*b^2*d*cosh(a + b*x)^3 + d^3*cosh(
a + b*x)*sinh(a + b*x)^2 - b*d^2*sinh(a + b*x)^3 - 2*b*d^2*cosh(a + b*x)^2
*sinh(a + b*x) - 3*b^2*d*cosh(a + b*x)*sinh(a + b*x)^2))/(9*b^4 + d^4 - 10
*b^2*d^2)
```

3.947 $\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx$

3.947.1 Optimal result

Integrand size = 20, antiderivative size = 66

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx=\frac{be^{c+dx}\cosh(2a+2bx)}{4b^2-d^2}-\frac{de^{c+dx}\sinh(2a+2bx)}{2\left(4b^2-d^2\right)}$$

output
```
b*exp(d*x+c)*cosh(2*b*x+2*a)/(4*b^2-d^2)-1/2*d*exp(d*x+c)*sinh(2*b*x+2*a)/
(4*b^2-d^2)
```

3.947.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx=\frac{e^{c+dx}(2b\cosh(2(a+bx))-d\sinh(2(a+bx)))}{2\left(4b^2-d^2\right)}$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
(E^(c + d*x)*(2*b*Cosh[2*(a + b*x)] - d*Sinh[2*(a + b*x)]))/(2*(4*b^2 - d^
2))
```

3.947.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6035, 27, 5997}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx} \sinh(a+bx) \cosh(a+bx) \, dx$$

$$\Big\downarrow \text{6035}$$

$$\int \frac{1}{2} e^{c+dx} \sinh(2a+2bx) dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \int e^{c+dx} \sinh(2a+2bx) dx$$

$$\Big\downarrow \text{5997}$$

$$\frac{1}{2} \left(\frac{2be^{c+dx} \cosh(2a+2bx)}{4b^2-d^2} - \frac{de^{c+dx} \sinh(2a+2bx)}{4b^2-d^2} \right)$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
((2*b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(4*b^2 - d^2) - (d*E^(c + d*x)*Sinh[2
*a + 2*b*x])/(4*b^2 - d^2))/2
```

3.947.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5997
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.947.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{(2\,e^{4bx+4a}b-d\,e^{4bx+4a}+2b+d)\,e^{-2bx+dx-2a+c}}{4(2b+d)(2b-d)}$	61
default	$-\frac{\sinh(2a-c+(2b-d)x)}{4(2b-d)}+\frac{\sinh(2a+c+(2b+d)x)}{8b+4d}+\frac{\cosh(2a-c+(2b-d)x)}{8b-4d}+\frac{\cosh(2a+c+(2b+d)x)}{8b+4d}$	102

input
```
int(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/(2*b+d)/(2*b-d)*(2*exp(4*b*x+4*a)*b-d*exp(4*b*x+4*a)+2*b+d)*exp(-2*b*x
+d*x-2*a+c)
```

3.947.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(62) = 124.

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.15

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx$$
$$=\frac{b\cosh(bx+a)^2\cosh(dx+c)-d\cosh(bx+a)\cosh(dx+c)\sinh(bx+a)+b\cosh(dx+c)\sinh(bx+a)^2}{(4\,b^2-d^2)\cosh(bx+a)^2-(4\,b^2-d}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
(b*cosh(b*x + a)^2*cosh(d*x + c) - d*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x
+ a) + b*cosh(d*x + c)*sinh(b*x + a)^2 + (b*cosh(b*x + a)^2 - d*cosh(b*x +
 a)*sinh(b*x + a) + b*sinh(b*x + a)^2)*sinh(d*x + c))/((4*b^2 - d^2)*cosh(
b*x + a)^2 - (4*b^2 - d^2)*sinh(b*x + a)^2)
```

3.947.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(54) = 108$.

Time = 0.97 (sec) , antiderivative size = 304, normalized size of antiderivative = 4.61

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx$$
$$= \begin{cases} xe^c\sinh(a)\cosh(a) & \text{for } b = \\ \frac{xe^ce^{dx}\sinh^2\left(a-\frac{dx}{2}\right)}{4} + \frac{xe^ce^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2} + \frac{xe^ce^{dx}\cosh^2\left(a-\frac{dx}{2}\right)}{4} + \frac{e^ce^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2d} & \text{for } b = \\ -\frac{xe^ce^{dx}\sinh^2\left(a+\frac{dx}{2}\right)}{4} + \frac{xe^ce^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2} - \frac{xe^ce^{dx}\cosh^2\left(a+\frac{dx}{2}\right)}{4} + \frac{e^ce^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2d} & \text{for } b = \\ \frac{be^ce^{dx}\sinh^2(a+bx)}{4b^2-d^2} + \frac{be^ce^{dx}\cosh^2(a+bx)}{4b^2-d^2} - \frac{de^ce^{dx}\sinh(a+bx)\cosh(a+bx)}{4b^2-d^2} & \text{otherw} \end{cases}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x)
```

output
```
Piecewise((x*exp(c)*sinh(a)*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp(d
*x)*sinh(a - d*x/2)**2/4 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/
2)/2 + x*exp(c)*exp(d*x)*cosh(a - d*x/2)**2/4 + exp(c)*exp(d*x)*sinh(a - d
*x/2)*cosh(a - d*x/2)/(2*d), Eq(b, -d/2)), (-x*exp(c)*exp(d*x)*sinh(a + d*
x/2)**2/4 + x*exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a + d*x/2)/2 - x*exp(c)
*exp(d*x)*cosh(a + d*x/2)**2/4 + exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a +
d*x/2)/(2*d), Eq(b, d/2)), (b*exp(c)*exp(d*x)*sinh(a + b*x)**2/(4*b**2 - d
**2) + b*exp(c)*exp(d*x)*cosh(a + b*x)**2/(4*b**2 - d**2) - d*exp(c)*exp(d
*x)*sinh(a + b*x)*cosh(a + b*x)/(4*b**2 - d**2), True))
```

3.947.7 Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for mor
e details)
```

3.947.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{e^{(2\,bx+dx+2\,a+c)}}{4\,(2\,b+d)} + \frac{e^{(-2\,bx+dx-2\,a+c)}}{4\,(2\,b-d)}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/4*e^(2*b*x + d*x + 2*a + c)/(2*b + d) + 1/4*e^(-2*b*x + d*x - 2*a + c)/(
2*b - d)
```

3.947.9 Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\mathrm{e}^{c+d\,x}\,\mathrm{e}^{-2\,a-2\,b\,x}\,\left(2\,b+d+2\,b\,\mathrm{e}^{4\,a+4\,b\,x}-d\,\mathrm{e}^{4\,a+4\,b\,x}\right)}{4\,\left(4\,b^2-d^2\right)}$$

input
```
int(cosh(a + b*x)*exp(c + d*x)*sinh(a + b*x),x)
```

output
```
(exp(c + d*x)*exp(- 2*a - 2*b*x)*(2*b + d + 2*b*exp(4*a + 4*b*x) - d*exp(4
*a + 4*b*x)))/(4*(4*b^2 - d^2))
```

3.948 $\int e^{c+dx}\cosh(a+bx)\,dx$

3.948.1 Optimal result

Integrand size = 14, antiderivative size = 54

$$\int e^{c+dx}\cosh(a+bx)\,dx=-\frac{de^{c+dx}\cosh(a+bx)}{b^2-d^2}+\frac{be^{c+dx}\sinh(a+bx)}{b^2-d^2}$$

output
```
-d*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)+b*exp(d*x+c)*sinh(b*x+a)/(b^2-d^2)
```

3.948.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int e^{c+dx}\cosh(a+bx)\,dx=\frac{e^{c+dx}(-d\cosh(a+bx)+b\sinh(a+bx))}{(b-d)(b+d)}$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x],x]
```

output
```
(E^(c + d*x)*(-(d*Cosh[a + b*x]) + b*Sinh[a + b*x]))/((b - d)*(b + d))
```

3.948.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\cosh(a+bx)\,dx$$

$$\Big\downarrow \; 5998$$

$$\frac{be^{c+dx}\sinh(a+bx)}{b^2-d^2}-\frac{de^{c+dx}\cosh(a+bx)}{b^2-d^2}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x],x]
```

output
```
-((d*E^(c + d*x)*Cosh[a + b*x])/(b^2 - d^2)) + (b*E^(c + d*x)*Sinh[a + b*x
])/(b^2 - d^2)
```

3.948.3.1 Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

3.948.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{{\mathrm e}^{dx+c}\left(-\cosh\left(bx+a\right)d+b\sinh\left(bx+a\right)\right)}{b^{2}-d^{2}}$	37
risch	$\frac{\left(b\,{\mathrm e}^{2bx+2a}-d\,{\mathrm e}^{2bx+2a}-b-d\right){\mathrm e}^{-bx+dx-a+c}}{2\left(b+d\right)\left(b-d\right)}$	58
default	$\frac{\sinh\left(a-c+\left(b-d\right)x\right)}{2b-2d}+\frac{\sinh\left(a+c+\left(b+d\right)x\right)}{2b+2d}-\frac{\cosh\left(a-c+\left(b-d\right)x\right)}{2\left(b-d\right)}+\frac{\cosh\left(a+c+\left(b+d\right)x\right)}{2b+2d}$	78

input
```
int(exp(d*x+c)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
exp(d*x+c)/(b^2-d^2)*(-cosh(b*x+a)*d+b*sinh(b*x+a))
```

3.948.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.80

$$\int e^{c+dx}\cosh(a+bx)\,dx = -\frac{d\cosh(bx+a)\cosh(dx+c)-b\cosh(dx+c)\sinh(bx+a)+(d\cosh(bx+a)-b\sinh(bx+a))\sinh(\ldots}{(b^2-d^2)\cosh(bx+a)^2-(b^2-d^2)\sinh(bx+a)^2}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-(d*cosh(b*x + a)*cosh(d*x + c) - b*cosh(d*x + c)*sinh(b*x + a) + (d*cosh(
b*x + a) - b*sinh(b*x + a))*sinh(d*x + c))/((b^2 - d^2)*cosh(b*x + a)^2 -
(b^2 - d^2)*sinh(b*x + a)^2)
```

3.948.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(42) = 84.

Time = 0.46 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.41

$$\int e^{c+dx}\cosh(a+bx)\,dx = \begin{cases} xe^{c}\cosh(a) & \text{for } b=0\wedge d=0 \\ \frac{xe^{c}e^{dx}\sinh(a-dx)}{2}+\frac{xe^{c}e^{dx}\cosh(a-dx)}{2}-\frac{e^{c}e^{dx}\sinh(a-dx)}{2d} & \text{for } b=-d \\ -\frac{xe^{c}e^{dx}\sinh(a+dx)}{2}+\frac{xe^{c}e^{dx}\cosh(a+dx)}{2}+\frac{e^{c}e^{dx}\sinh(a+dx)}{d}-\frac{e^{c}e^{dx}\cosh(a+dx)}{2d} & \text{for } b=d \\ \frac{be^{c}e^{dx}\sinh(a+bx)}{b^2-d^2}-\frac{de^{c}e^{dx}\cosh(a+bx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a),x)
```

output
```
Piecewise((x*exp(c)*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp(d*x)*sinh
(a - d*x)/2 + x*exp(c)*exp(d*x)*cosh(a - d*x)/2 - exp(c)*exp(d*x)*sinh(a -
 d*x)/(2*d), Eq(b, -d)), (-x*exp(c)*exp(d*x)*sinh(a + d*x)/2 + x*exp(c)*ex
p(d*x)*cosh(a + d*x)/2 + exp(c)*exp(d*x)*sinh(a + d*x)/d - exp(c)*exp(d*x)
*cosh(a + d*x)/(2*d), Eq(b, d)), (b*exp(c)*exp(d*x)*sinh(a + b*x)/(b**2 -
d**2) - d*exp(c)*exp(d*x)*cosh(a + b*x)/(b**2 - d**2), True))
```

3.948.7 Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more
 details)I
```

3.948.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int e^{c+dx}\cosh(a+bx)\,dx = \frac{e^{(bx+dx+a+c)}}{2\,(b+d)} - \frac{e^{(-bx+dx-a+c)}}{2\,(b-d)}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a),x, algorithm="giac")
```

output
```
1/2*e^(b*x + d*x + a + c)/(b + d) - 1/2*e^(-b*x + d*x - a + c)/(b - d)
```

3.948.9 Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int e^{c+dx}\cosh(a+bx)\,dx = -\frac{e^{c-a-b\,x+d\,x}\,\left(b+d-b\,e^{2\,a+2\,b\,x}+d\,e^{2\,a+2\,b\,x}\right)}{2\,\left(b^2-d^2\right)}$$

input
```
int(cosh(a + b*x)*exp(c + d*x),x)
```

output
```
-(exp(c - a - b*x + d*x)*(b + d - b*exp(2*a + 2*b*x) + d*exp(2*a + 2*b*x))
)/(2*(b^2 - d^2))
```

3.949 $\int e^{c+dx}\coth(a+bx)\,dx$

3.949.1 Optimal result

Integrand size = 14, antiderivative size = 53

$$\int e^{c+dx}\coth(a+bx)\,dx=\frac{e^{c+dx}}{d}-\frac{2e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}$$

output
```
exp(d*x+c)/d-2*exp(d*x+c)*hypergeom([1, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a
))/d
```

3.949.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 120 vs. 2(53) = 106.

Time = 0.52 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.26

$$\int e^{c+dx}\coth(a+bx)\,dx=\frac{e^{c+dx}\coth(a)}{d}-\frac{2e^{2a+c}\left(\frac{e^{dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}-\frac{e^{(2b+d)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{d}{2b},2+\frac{d}{2b},e^{2(a+bx)}\right)}{2b+d}\right)}{-1+e^{2a}}$$

input
```
Integrate[E^(c + d*x)*Coth[a + b*x],x]
```

output
```
(E^(c + d*x)*Coth[a])/d - (2*E^(2*a + c)*((E^(d*x)*Hypergeometric2F1[1, d/
(2*b), 1 + d/(2*b), E^(2*(a + b*x))])/d - (E^((2*b + d)*x)*Hypergeometric2
F1[1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))])/(2*b + d)))/(-1 + E^(2*a
))
```

3.949.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\coth(a+bx)\,dx$$

$$\Big\downarrow \; 6008$$

$$\int \left(\frac{2e^{c+dx}}{e^{2(a+bx)}-1}+e^{c+dx}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{e^{c+dx}}{d}-\frac{2e^{c+dx}\text{Hypergeometric2F1}\left(1,\frac{d}{2b},\frac{d}{2b}+1,e^{2(a+bx)}\right)}{d}$$

input
```
Int[E^(c + d*x)*Coth[a + b*x],x]
```

output
```
E^(c + d*x)/d - (2*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), 
E^(2*(a + b*x))])/d
```

3.949.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6008
```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 + 
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.949.4 Maple [F]

$$\int \mathrm{e}^{dx+c} \cosh(bx+a) \operatorname{csch}(bx+a) \, dx$$

input
```
int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x)
```

output
```
int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x)
```

3.949.5 Fricas [F]

$$\int e^{c+dx} \coth(a+bx) \, dx = \int \cosh(bx+a) \operatorname{csch}(bx+a) e^{(dx+c)} \, dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*csch(b*x + a)*e^(d*x + c), x)
```

3.949.6 Sympy [F]

$$\int e^{c+dx} \coth(a+bx) \, dx = e^{c} \int e^{dx} \cosh(a+bx) \operatorname{csch}(a+bx) \, dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x)
```

output
```
exp(c)*Integral(exp(d*x)*cosh(a + b*x)*csch(a + b*x), x)
```

3.949.7 Maxima [F]

$$\int e^{c+dx} \coth(a+bx)\,dx = \int \cosh(bx+a)\operatorname{csch}(bx+a)\,e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

output
```
-4*b*integrate(e^(d*x + c)/((2*b - d)*e^(4*b*x + 4*a) - 2*(2*b - d)*e^(2*b
*x + 2*a) + 2*b - d), x) - ((2*b*e^c - d*e^c)*e^(2*b*x + 2*a) - 2*b*e^c -
d*e^c)*e^(d*x)/(2*b*d - d^2 - (2*b*d - d^2)*e^(2*b*x + 2*a))
```

3.949.8 Giac [F]

$$\int e^{c+dx} \coth(a+bx)\,dx = \int \cosh(bx+a)\operatorname{csch}(bx+a)\,e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)*csch(b*x + a)*e^(d*x + c), x)
```

3.949.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+dx} \coth(a+bx)\,dx = \int \frac{\cosh(a+b\,x)\,e^{c+d\,x}}{\sinh(a+b\,x)}\,dx$$

input
```
int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x),x)
```

output
```
int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x), x)
```

3.950 $\int e^{c+dx} \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

3.950.1 Optimal result

Integrand size = 20, antiderivative size = 101

$$\int e^{c+dx}\coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d} + \frac{4e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(2,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}$$

output
```
-2*exp(a+c+(b+d)*x)*hypergeom([1, 1/2*(b+d)/b],[1/2*(3*b+d)/b],exp(2*b*x+2
*a))/(b+d)+4*exp(a+c+(b+d)*x)*hypergeom([2, 1/2*(b+d)/b],[1/2*(3*b+d)/b],e
xp(2*b*x+2*a))/(b+d)
```

3.950.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int e^{c+dx}\coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{e^c\text{csch}\left(\frac{1}{2}(a+bx)\right)\text{sech}\left(\frac{1}{2}(a+bx)\right)(\cosh(a)+\sinh(a))\left((b+d)e^{dx}(\cosh(a)-\sinh(a))+2de^{(b+d)x}\,\text{Hyp}\right.}{2b(b+d)}$$

input
```
Integrate[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
-1/2*(E^c*Csch[(a + b*x)/2]*Sech[(a + b*x)/2]*(Cosh[a] + Sinh[a])*((b + d)
*E^(d*x)*(Cosh[a] - Sinh[a]) + 2*d*E^((b + d)*x)*Hypergeometric2F1[1, (b +
 d)/(2*b), (3 + d/b)/2, E^(2*(a + b*x))]*Sinh[a + b*x]))/(b*(b + d))
```

3.950.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx} \coth(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6037}$$

$$\int \left(\frac{2e^{a+x(b+d)+c}}{e^{2(a+bx)}-1}+\frac{4e^{a+x(b+d)+c}}{\left(e^{2(a+bx)}-1\right)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4e^{a+x(b+d)+c}\text{Hypergeometric2F1}\left(2,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}-\frac{2e^{a+x(b+d)+c}\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}$$

input
```
Int[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
(-2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2
*b), E^(2*(a + b*x))])/(b + d) + (4*E^(a + c + (b + d)*x)*Hypergeometric2F
1[2, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))])/(b + d)
```

3.950.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

3.950.4 Maple [F]

$$\int e^{dx+c} \cosh(bx+a) \operatorname{csch}(bx+a)^2 dx$$

input
```
int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x)
```

output
```
int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x)
```

3.950.5 Fricas [F]

$$\int e^{c+dx} \coth(a+bx)\operatorname{csch}(a+bx)\, dx = \int \cosh(bx+a) \operatorname{csch}(bx+a)^2 e^{(dx+c)}\, dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*csch(b*x + a)^2*e^(d*x + c), x)
```

3.950.6 Sympy [F(-1)]

Timed out.

$$\int e^{c+dx} \coth(a+bx)\operatorname{csch}(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.950.7 Maxima [F]

$$\int e^{c+dx}\coth(a+bx)\text{csch}(a+bx)\,dx=\int \cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^{2}e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
16*b*d*integrate(-e^(b*x + d*x + a + c)/(3*b^2 - 4*b*d + d^2 - (3*b^2 - 4*
b*d + d^2)*e^(6*b*x + 6*a) + 3*(3*b^2 - 4*b*d + d^2)*e^(4*b*x + 4*a) - 3*(
3*b^2 - 4*b*d + d^2)*e^(2*b*x + 2*a)), x) - 2*((3*b*e^c - d*e^c)*e^(3*b*x
+ 3*a) - (3*b*e^c + d*e^c)*e^(b*x + a))*e^(d*x)/(3*b^2 - 4*b*d + d^2 + (3*
b^2 - 4*b*d + d^2)*e^(4*b*x + 4*a) - 2*(3*b^2 - 4*b*d + d^2)*e^(2*b*x + 2*
a))
```

3.950.8 Giac [F]

$$\int e^{c+dx}\coth(a+bx)\text{csch}(a+bx)\,dx=\int \cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^{2}e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)*csch(b*x + a)^2*e^(d*x + c), x)
```

3.950.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\coth(a+bx)\text{csch}(a+bx)\,dx=\int \frac{\cosh(a+b\,x)\,\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)^{2}}\,dx$$

input
```
int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x)^2,x)
```

output
```
int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x)^2, x)
```

3.951 $\int e^{c+dx}\coth(a+bx)\mathrm{csch}^2(a+bx)\,dx$

3.951.1 Optimal result

Integrand size = 22, antiderivative size = 113

$$\int e^{c+dx}\coth(a+bx)\mathrm{csch}^2(a+bx)\,dx = \frac{4e^{2a+c+(2b+d)x}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{d}{b}\right),\frac{1}{2}\left(4+\frac{d}{b}\right),e^{2(a+bx)}\right)}{2b+d} - \frac{8e^{2a+c+(2b+d)x}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(2+\frac{d}{b}\right),\frac{1}{2}\left(4+\frac{d}{b}\right),e^{2(a+bx)}\right)}{2b+d}$$

output
```
4*exp(2*a+c+(2*b+d)*x)*hypergeom([2, 1+1/2*d/b],[2+1/2*d/b],exp(2*b*x+2*a)
)/(2*b+d)-8*exp(2*a+c+(2*b+d)*x)*hypergeom([3, 1+1/2*d/b],[2+1/2*d/b],exp(
2*b*x+2*a))/(2*b+d)
```

3.951.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.41

$$\int e^{c+dx}\coth(a+bx)\mathrm{csch}^2(a+bx)\,dx = -\frac{e^{c-\frac{ad}{b}}\left((2b+d)e^{d\left(\frac{a}{b}+x\right)}\left(d\coth(a+bx)+b\mathrm{csch}^2(a+bx)\right)+d(2b+d)e^{d\left(\frac{a}{b}+x\right)}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b}\right.\right.}{2b^2(2b+d)}$$

input
```
Integrate[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
-1/2*(E^(c - (a*d)/b)*((2*b + d)*E^(d*(a/b + x))*(d*Coth[a + b*x] + b*Csch
[a + b*x]^2) + d*(2*b + d)*E^(d*(a/b + x))*Hypergeometric2F1[1, d/(2*b), 1
 + d/(2*b), E^(2*(a + b*x))] + d^2*E^((2 + d/b)*(a + b*x))*Hypergeometric2
F1[1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))]))/(b^2*(2*b + d))
```

3.951.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\coth(a+bx)\text{csch}^2(a+bx)\,dx$$
$$\Big\downarrow \text{6037}$$
$$\int \left(\frac{4e^{2a+x(2b+d)+c}}{\left(e^{2(a+bx)}-1\right)^2}+\frac{8e^{2a+x(2b+d)+c}}{\left(e^{2(a+bx)}-1\right)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{4e^{2a+x(2b+d)+c}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{d}{b}+2\right),\frac{1}{2}\left(\frac{d}{b}+4\right),e^{2(a+bx)}\right)}{2b+d}-\frac{8e^{2a+x(2b+d)+c}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(\frac{d}{b}+2\right),\frac{1}{2}\left(\frac{d}{b}+4\right),e^{2(a+bx)}\right)}{2b+d}$$

input
```
Int[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
(4*E^(2*a + c + (2*b + d)*x)*Hypergeometric2F1[2, (2 + d/b)/2, (4 + d/b)/2
, E^(2*(a + b*x))])/(2*b + d) - (8*E^(2*a + c + (2*b + d)*x)*Hypergeometri
c2F1[3, (2 + d/b)/2, (4 + d/b)/2, E^(2*(a + b*x))])/(2*b + d)
```

3.951.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

3.951.4 Maple [F]

$$\int \mathrm{e}^{dx+c} \cosh (bx+a) \operatorname{csch} (bx+a)^3 \, dx$$

input
```
int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x)
```

output
```
int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x)
```

3.951.5 Fricas [F]

$$\int e^{c+dx} \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int \cosh (bx+a) \operatorname{csch} (bx+a)^3 e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)*csch(b*x + a)^3*e^(d*x + c), x)
```

3.951.6 Sympy [F(-1)]

Timed out.

$$\int e^{c+dx} \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.951.7 Maxima [F]

$$\int e^{c+dx}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx=\int \cosh(bx+a)\operatorname{csch}(bx+a)^3e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-48*b*d^2*integrate(e^(d*x + c)/(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3 + (48*
b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(8*b*x + 8*a) - 4*(48*b^3 - 44*b^2*d +
12*b*d^2 - d^3)*e^(6*b*x + 6*a) + 6*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e
^(4*b*x + 4*a) - 4*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(2*b*x + 2*a)),
x) + 4*(12*b*d*e^c + (24*b^2*e^c - 10*b*d*e^c + d^2*e^c)*e^(4*b*x + 4*a) -
 (24*b^2*e^c + 2*b*d*e^c - d^2*e^c)*e^(2*b*x + 2*a))*e^(d*x)/(48*b^3 - 44*
b^2*d + 12*b*d^2 - d^3 - (48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(6*b*x + 6
*a) + 3*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(4*b*x + 4*a) - 3*(48*b^3 -
 44*b^2*d + 12*b*d^2 - d^3)*e^(2*b*x + 2*a))
```

3.951.8 Giac [F]

$$\int e^{c+dx}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx=\int \cosh(bx+a)\operatorname{csch}(bx+a)^3e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)*csch(b*x + a)^3*e^(d*x + c), x)
```

3.951.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx=\int \frac{\cosh(a+b\,x)\,\mathrm{e}^{c+d\,x}}{\sinh(a+b\,x)^3}\,dx$$

input
```
int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x)^3,x)
```

output
```
int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x)^3, x)
```

3.952 $\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$

3.952.1 Optimal result

Integrand size = 24, antiderivative size = 195

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx=-\frac{be^{c+dx}\cosh(a+bx)}{8(b^2-d^2)}-\frac{3be^{c+dx}\cosh(3a+3bx)}{16(9b^2-d^2)}+\frac{5be^{c+dx}\cosh(5a+5bx)}{16(25b^2-d^2)}+\frac{de^{c+dx}\sinh(a+bx)}{8(b^2-d^2)}+\frac{de^{c+dx}\sinh(3a+3bx)}{16(9b^2-d^2)}-\frac{de^{c+dx}\sinh(5a+5bx)}{16(25b^2-d^2)}$$

output
```
-1/8*b*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)-3/16*b*exp(d*x+c)*cosh(3*b*x+3*a)/
(9*b^2-d^2)+5/16*b*exp(d*x+c)*cosh(5*b*x+5*a)/(25*b^2-d^2)+1/8*d*exp(d*x+c
)*sinh(b*x+a)/(b^2-d^2)+1/16*d*exp(d*x+c)*sinh(3*b*x+3*a)/(9*b^2-d^2)-1/16
*d*exp(d*x+c)*sinh(5*b*x+5*a)/(25*b^2-d^2)
```

3.952.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.60

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx=\frac{1}{16}e^{c+dx}\left(\frac{-2b\cosh(a+bx)+2d\sinh(a+bx)}{(b-d)(b+d)}+\frac{-3b\cosh(3(a+bx))+d\sinh(3(a+bx))}{9b^2-d^2}+\frac{5b\cosh(5(a+bx))-d\sinh(5(a+bx))}{25b^2-d^2}\right)$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
(E^(c + d*x)*((-2*b*Cosh[a + b*x] + 2*d*Sinh[a + b*x])/((b - d)*(b + d)) +
 (-3*b*Cosh[3*(a + b*x)] + d*Sinh[3*(a + b*x)])/(9*b^2 - d^2) + (5*b*Cosh[
5*(a + b*x)] - d*Sinh[5*(a + b*x)])/(25*b^2 - d^2)))/16
```

3.952.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\sinh^3(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{6035}$$

$$\int \left(-\frac{1}{8}e^{c+dx}\sinh(a+bx)-\frac{1}{16}e^{c+dx}\sinh(3a+3bx)+\frac{1}{16}e^{c+dx}\sinh(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{de^{c+dx}\sinh(a+bx)}{8(b^2-d^2)}+\frac{de^{c+dx}\sinh(3a+3bx)}{16(9b^2-d^2)}-\frac{de^{c+dx}\sinh(5a+5bx)}{16(25b^2-d^2)}-\frac{be^{c+dx}\cosh(a+bx)}{8(b^2-d^2)}-\frac{3be^{c+dx}\cosh(3a+3bx)}{16(9b^2-d^2)}+\frac{5be^{c+dx}\cosh(5a+5bx)}{16(25b^2-d^2)}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

output
```
-1/8*(b*E^(c + d*x)*Cosh[a + b*x])/(b^2 - d^2) - (3*b*E^(c + d*x)*Cosh[3*a
 + 3*b*x])/(16*(9*b^2 - d^2)) + (5*b*E^(c + d*x)*Cosh[5*a + 5*b*x])/(16*(2
5*b^2 - d^2)) + (d*E^(c + d*x)*Sinh[a + b*x])/(8*(b^2 - d^2)) + (d*E^(c +
d*x)*Sinh[3*a + 3*b*x])/(16*(9*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[5*a + 5*b
*x])/(16*(25*b^2 - d^2))
```

3.952.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.952.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.43

$$\frac{\sinh(a-c+(b-d)x)}{16b-16d}-\frac{\sinh(a+c+(b+d)x)}{16(b+d)}+\frac{\sinh(3a-c+(3b-d)x)}{96b-32d}-\frac{\sinh(3a+c+(3b+d)x)}{32(3b+d)}-$$

input
```
int(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

output
```
1/16*sinh(a-c+(b-d)*x)/(b-d)-1/16*sinh(a+c+(b+d)*x)/(b+d)+1/32*sinh(3*a-c+
(3*b-d)*x)/(3*b-d)-1/32*sinh(3*a+c+(3*b+d)*x)/(3*b+d)-1/32/(5*b-d)*sinh((5
*b-d)*x+5*a-c)+1/32/(5*b+d)*sinh((5*b+d)*x+5*a+c)-1/16*cosh(a-c+(b-d)*x)/(
b-d)-1/16*cosh(a+c+(b+d)*x)/(b+d)-1/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-1/32*
cosh(3*a+c+(3*b+d)*x)/(3*b+d)+1/32*cosh((5*b-d)*x+5*a-c)/(5*b-d)+1/32*cosh
((5*b+d)*x+5*a+c)/(5*b+d)
```

3.952.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(177) = 354$.

Time = 0.28 (sec) , antiderivative size = 919, normalized size of antiderivative = 4.71

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/16*(25*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x
 + a)^4 - (9*b^4*d - 10*b^2*d^3 + d^5)*cosh(d*x + c)*sinh(b*x + a)^5 + (25
*b^4*d - 26*b^2*d^3 + d^5 - 10*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)^
2)*cosh(d*x + c)*sinh(b*x + a)^3 + (50*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b
*x + a)^3 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c)*s
inh(b*x + a)^2 + (450*b^4*d - 68*b^2*d^3 + 2*d^5 - 5*(9*b^4*d - 10*b^2*d^3
 + d^5)*cosh(b*x + a)^4 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a)^2)
*cosh(d*x + c)*sinh(b*x + a) + (5*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x +
a)^5 - 3*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a)^3 - 2*(225*b^5 - 34*b
^3*d^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c) + (5*(9*b^5 - 10*b^3*d^2 + b*
d^4)*cosh(b*x + a)^5 + 25*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)*sinh(
b*x + a)^4 - (9*b^4*d - 10*b^2*d^3 + d^5)*sinh(b*x + a)^5 - 3*(25*b^5 - 26
*b^3*d^2 + b*d^4)*cosh(b*x + a)^3 + (25*b^4*d - 26*b^2*d^3 + d^5 - 10*(9*b
^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)^2)*sinh(b*x + a)^3 + (50*(9*b^5 - 1
0*b^3*d^2 + b*d^4)*cosh(b*x + a)^3 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(
b*x + a))*sinh(b*x + a)^2 - 2*(225*b^5 - 34*b^3*d^2 + b*d^4)*cosh(b*x + a)
 + (450*b^4*d - 68*b^2*d^3 + 2*d^5 - 5*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b
*x + a)^4 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a)^2)*sinh(b*x + a)
)*sinh(d*x + c))/((225*b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)
^6 - 3*(225*b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^4*sinh(...
```

3.952.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2693 vs. $2(168) = 336$.

Time = 26.30 (sec) , antiderivative size = 2693, normalized size of antiderivative = 13.81

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

output
```
Piecewise((x*exp(c)*sinh(a)**3*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (-x*exp(c
)*exp(d*x)*sinh(a - d*x)**5/16 - x*exp(c)*exp(d*x)*sinh(a - d*x)**4*cosh(a
 - d*x)/16 + x*exp(c)*exp(d*x)*sinh(a - d*x)**3*cosh(a - d*x)**2/8 + x*exp
(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)**3/8 - x*exp(c)*exp(d*x)*sinh(
a - d*x)*cosh(a - d*x)**4/16 - x*exp(c)*exp(d*x)*cosh(a - d*x)**5/16 - exp
(c)*exp(d*x)*sinh(a - d*x)**5/(32*d) - 3*exp(c)*exp(d*x)*sinh(a - d*x)**4*
cosh(a - d*x)/(32*d) - exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)**3/(
6*d) - exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**4/(96*d) + 5*exp(c)*ex
p(d*x)*cosh(a - d*x)**5/(96*d), Eq(b, -d)), (x*exp(c)*exp(d*x)*sinh(a - d*
x/3)**5/32 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/3)**4*cosh(a - d*x/3)/32 + x
*exp(c)*exp(d*x)*sinh(a - d*x/3)**3*cosh(a - d*x/3)**2/16 - x*exp(c)*exp(d
*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3)**3/16 - 3*x*exp(c)*exp(d*x)*sinh(a
- d*x/3)*cosh(a - d*x/3)**4/32 - x*exp(c)*exp(d*x)*cosh(a - d*x/3)**5/32 -
 9*exp(c)*exp(d*x)*sinh(a - d*x/3)**5/(64*d) - 21*exp(c)*exp(d*x)*sinh(a -
 d*x/3)**4*cosh(a - d*x/3)/(64*d) - exp(c)*exp(d*x)*sinh(a - d*x/3)**2*cos
h(a - d*x/3)**3/(2*d) - 27*exp(c)*exp(d*x)*sinh(a - d*x/3)*cosh(a - d*x/3)
**4/(64*d) - 7*exp(c)*exp(d*x)*cosh(a - d*x/3)**5/(64*d), Eq(b, -d/3)), (x
*exp(c)*exp(d*x)*sinh(a - d*x/5)**5/32 + 5*x*exp(c)*exp(d*x)*sinh(a - d*x/
5)**4*cosh(a - d*x/5)/32 + 5*x*exp(c)*exp(d*x)*sinh(a - d*x/5)**3*cosh(a -
 d*x/5)**2/16 + 5*x*exp(c)*exp(d*x)*sinh(a - d*x/5)**2*cosh(a - d*x/5)*...
```

3.952.7 Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more
 details)I
```

3.952.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(5\,bx+dx+5\,a+c)}}{32\,(5\,b+d)} - \frac{e^{(3\,bx+dx+3\,a+c)}}{32\,(3\,b+d)} - \frac{e^{(bx+dx+a+c)}}{16\,(b+d)} - \frac{e^{(-bx+dx-a+c)}}{16\,(b-d)} - \frac{e^{(-3\,bx+dx-3\,a+c)}}{32\,(3\,b-d)} + \frac{e^{(-5\,bx+dx-5\,a+c)}}{32\,(5\,b-d)}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/32*e^(5*b*x + d*x + 5*a + c)/(5*b + d) - 1/32*e^(3*b*x + d*x + 3*a + c)/
(3*b + d) - 1/16*e^(b*x + d*x + a + c)/(b + d) - 1/16*e^(-b*x + d*x - a +
c)/(b - d) - 1/32*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 1/32*e^(-5*b*x +
d*x - 5*a + c)/(5*b - d)
```

3.952.9 Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.03

$$\begin{aligned}\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx &= \frac{3\cosh(a+b\,x)^3\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^2\,(25\,b^5-10\,b^3\,d^2+b\,d^4)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6} \\ &- \frac{\cosh(a+b\,x)^5\,\mathrm{e}^{c+d\,x}\,(30\,b^5-6\,b^3\,d^2)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6} \\ &+ \frac{6\cosh(a+b\,x)^4\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)\,\left(5\,b^4\,d-b^2\,d^3\right)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6} \\ &- \frac{\cosh(a+b\,x)^2\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^3\,(65\,b^4\,d-18\,b^2\,d^3+d^5)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6} \\ &+ \frac{2\,b^2\,d\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^5\,(13\,b^2-d^2)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6} \\ &- \frac{2\,b\,d^2\cosh(a+b\,x)\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^4\,(13\,b^2-d^2)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}\end{aligned}$$

input
```
int(cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x)^3,x)
```

output
```
(3*cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x)^2*(b*d^4 + 25*b^5 - 10*b^3*d
^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2) - (cosh(a + b*x)^5*exp(c +
 d*x)*(30*b^5 - 6*b^3*d^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2) + (
6*cosh(a + b*x)^4*exp(c + d*x)*sinh(a + b*x)*(5*b^4*d - b^2*d^3))/(225*b^6
 - d^6 + 35*b^2*d^4 - 259*b^4*d^2) - (cosh(a + b*x)^2*exp(c + d*x)*sinh(a
+ b*x)^3*(65*b^4*d + d^5 - 18*b^2*d^3))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*
b^4*d^2) + (2*b^2*d*exp(c + d*x)*sinh(a + b*x)^5*(13*b^2 - d^2))/(225*b^6
- d^6 + 35*b^2*d^4 - 259*b^4*d^2) - (2*b*d^2*cosh(a + b*x)*exp(c + d*x)*si
nh(a + b*x)^4*(13*b^2 - d^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2)
```

3.952. $\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$

3.953 $\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$

3.953.1 Optimal result

Integrand size = 24, antiderivative size = 83

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx=-\frac{e^{c+dx}}{8d}-\frac{de^{c+dx}\cosh(4a+4bx)}{8(16b^2-d^2)}+\frac{be^{c+dx}\sinh(4a+4bx)}{2(16b^2-d^2)}$$

output
```
-1/8*exp(d*x+c)/d-1/8*d*exp(d*x+c)*cosh(4*b*x+4*a)/(16*b^2-d^2)+1/2*b*exp(
d*x+c)*sinh(4*b*x+4*a)/(16*b^2-d^2)
```

3.953.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx=\frac{e^{c+dx}(16b^2-d^2+d^2\cosh(4(a+bx))-4bd\sinh(4(a+bx)))}{8(-16b^2d+d^3)}$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(E^(c + d*x)*(16*b^2 - d^2 + d^2*Cosh[4*(a + b*x)] - 4*b*d*Sinh[4*(a + b*x
)]))/(8*(-16*b^2*d + d^3))
```

3.953.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\sinh^2(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{6035}$$

$$\int \left(\frac{1}{8}e^{c+dx}\cosh(4a+4bx)-\frac{1}{8}e^{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{be^{c+dx}\sinh(4a+4bx)}{2\left(16b^2-d^2\right)}-\frac{de^{c+dx}\cosh(4a+4bx)}{8\left(16b^2-d^2\right)}-\frac{e^{c+dx}}{8d}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
-1/8*E^(c + d*x)/d - (d*E^(c + d*x)*Cosh[4*a + 4*b*x])/(8*(16*b^2 - d^2))
+ (b*E^(c + d*x)*Sinh[4*a + 4*b*x])/(2*(16*b^2 - d^2))
```

3.953.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.953.4 Maple [A] (verified)

Time = 112.68 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{\left(-4 d\,{\mathrm e}^{8 b x+8 a} b+d^{2} {\mathrm e}^{8 b x+8 a}+32 \,{\mathrm e}^{4 b x+4 a} b^{2}-2 d^{2} {\mathrm e}^{4 b x+4 a}+4 b d+d^{2}\right) {\mathrm e}^{-4 b x+d x-4 a+c}}{16 \left(4 b+d\right) \left(4 b-d\right) d}$
default	$-\frac{\sinh\left(d x+c\right)}{8 d}+\frac{\sinh\left(\left(4 b-d\right) x+4 a-c\right)}{64 b-16 d}+\frac{\sinh\left(\left(4 b+d\right) x+4 a+c\right)}{64 b+16 d}-\frac{\cosh\left(d x+c\right)}{8 d}-\frac{\cosh\left(\left(4 b-d\right) x+4 a-c\right)}{16 \left(4 b-d\right)}+\frac{\cosh\left(\left(4 b+d\right) x+4 a\right.}{64 b+16 d}$

input
```
int(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16/(4*b+d)/(4*b-d)/d*(-4*d*exp(8*b*x+8*a)*b+d^2*exp(8*b*x+8*a)+32*exp(4
*b*x+4*a)*b^2-2*d^2*exp(4*b*x+4*a)+4*b*d+d^2)*exp(-4*b*x+d*x-4*a+c)
```

3.953.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(74) = 148.

Time = 0.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.65

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$$
$$= \frac{16\,bd\cosh\left(bx+a\right)^3\cosh\left(dx+c\right)\sinh\left(bx+a\right)-6\,d^2\cosh\left(bx+a\right)^2\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2+16\,bd\,\mathrm{c}}{}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(16*b*d*cosh(b*x + a)^3*cosh(d*x + c)*sinh(b*x + a) - 6*d^2*cosh(b*x +
 a)^2*cosh(d*x + c)*sinh(b*x + a)^2 + 16*b*d*cosh(b*x + a)*cosh(d*x + c)*s
inh(b*x + a)^3 - d^2*cosh(d*x + c)*sinh(b*x + a)^4 - (d^2*cosh(b*x + a)^4
+ 16*b^2 - d^2)*cosh(d*x + c) - (d^2*cosh(b*x + a)^4 - 16*b*d*cosh(b*x + a
)^3*sinh(b*x + a) + 6*d^2*cosh(b*x + a)^2*sinh(b*x + a)^2 - 16*b*d*cosh(b*
x + a)*sinh(b*x + a)^3 + d^2*sinh(b*x + a)^4 + 16*b^2 - d^2)*sinh(d*x + c)
)/((16*b^2*d - d^3)*cosh(b*x + a)^4 - 2*(16*b^2*d - d^3)*cosh(b*x + a)^2*s
inh(b*x + a)^2 + (16*b^2*d - d^3)*sinh(b*x + a)^4)
```

3.953.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(66) = 132$.

Time = 6.14 (sec) , antiderivative size = 819, normalized size of antiderivative = 9.87

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$$

$$= \begin{cases} xe^c\sinh^2(a)\cosh^2(a) \\ \frac{xe^ce^{dx}\sinh^4\left(a-\frac{dx}{4}\right)}{16} + \frac{xe^ce^{dx}\sinh^3\left(a-\frac{dx}{4}\right)\cosh\left(a-\frac{dx}{4}\right)}{4} + \frac{3xe^ce^{dx}\sinh^2\left(a-\frac{dx}{4}\right)\cosh^2\left(a-\frac{dx}{4}\right)}{8} + \frac{xe^ce^{dx}\sinh\left(a-\frac{dx}{4}\right)\cosh^3\left(a-\right.}{4} \\ \frac{xe^ce^{dx}\sinh^4\left(a+\frac{dx}{4}\right)}{16} - \frac{xe^ce^{dx}\sinh^3\left(a+\frac{dx}{4}\right)\cosh\left(a+\frac{dx}{4}\right)}{4} + \frac{3xe^ce^{dx}\sinh^2\left(a+\frac{dx}{4}\right)\cosh^2\left(a+\frac{dx}{4}\right)}{8} - \frac{xe^ce^{dx}\sinh\left(a+\frac{dx}{4}\right)\cosh^3\left(a+\right.}{4} \\ \left(-\frac{x\sinh^4(a+bx)}{8} + \frac{x\sinh^2(a+bx)\cosh^2(a+bx)}{4} - \frac{x\cosh^4(a+bx)}{8} + \frac{\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{\sinh(a+bx)\cosh^3(a+bx)}{8b}\right)e^c \\ -\frac{2b^2e^ce^{dx}\sinh^4(a+bx)}{16b^2d-d^3} + \frac{4b^2e^ce^{dx}\sinh^2(a+bx)\cosh^2(a+bx)}{16b^2d-d^3} - \frac{2b^2e^ce^{dx}\cosh^4(a+bx)}{16b^2d-d^3} + \frac{2bde^ce^{dx}\sinh^3(a+bx)\cosh(a+bx)}{16b^2d-d^3} + \frac{2bde}{} \end{cases}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x*exp(c)*sinh(a)**2*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (x*exp(c)
*exp(d*x)*sinh(a - d*x/4)**4/16 + x*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cos
h(a - d*x/4)/4 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/4)**2*cosh(a - d*x/4)**2
/8 + x*exp(c)*exp(d*x)*sinh(a - d*x/4)*cosh(a - d*x/4)**3/4 + x*exp(c)*exp
(d*x)*cosh(a - d*x/4)**4/16 - exp(c)*exp(d*x)*sinh(a - d*x/4)**4/(6*d) - 5
*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(a - d*x/4)/(12*d) - 5*exp(c)*exp(
d*x)*sinh(a - d*x/4)*cosh(a - d*x/4)**3/(12*d) - exp(c)*exp(d*x)*cosh(a -
d*x/4)**4/(6*d), Eq(b, -d/4)), (x*exp(c)*exp(d*x)*sinh(a + d*x/4)**4/16 -
x*exp(c)*exp(d*x)*sinh(a + d*x/4)**3*cosh(a + d*x/4)/4 + 3*x*exp(c)*exp(d*
x)*sinh(a + d*x/4)**2*cosh(a + d*x/4)**2/8 - x*exp(c)*exp(d*x)*sinh(a + d*
x/4)*cosh(a + d*x/4)**3/4 + x*exp(c)*exp(d*x)*cosh(a + d*x/4)**4/16 - exp(
c)*exp(d*x)*sinh(a + d*x/4)**4/(6*d) + 5*exp(c)*exp(d*x)*sinh(a + d*x/4)**
3*cosh(a + d*x/4)/(12*d) + 5*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(a + d*x/
4)**3/(12*d) - exp(c)*exp(d*x)*cosh(a + d*x/4)**4/(6*d), Eq(b, d/4)), ((-x
*sinh(a + b*x)**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 - x*cosh(a + b
*x)**4/8 + sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*x)*cosh(a + b
*x)**3/(8*b))*exp(c), Eq(d, 0)), (-2*b**2*exp(c)*exp(d*x)*sinh(a + b*x)**4
/(16*b**2*d - d**3) + 4*b**2*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x
)**2/(16*b**2*d - d**3) - 2*b**2*exp(c)*exp(d*x)*cosh(a + b*x)**4/(16*b**2
*d - d**3) + 2*b*d*exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)/(16*b...
```

3.953.7 Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(3-d/b>0)', see `assume?` for mor
e details)
```

3.953.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(4\,bx+dx+4\,a+c)}}{16\,(4\,b+d)} - \frac{e^{(-4\,bx+dx-4\,a+c)}}{16\,(4\,b-d)} - \frac{e^{(dx+c)}}{8\,d}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/16*e^(4*b*x + d*x + 4*a + c)/(4*b + d) - 1/16*e^(-4*b*x + d*x - 4*a + c)
/(4*b - d) - 1/8*e^(d*x + c)/d
```

3.953.9 Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$$
$$= -\frac{\frac{d^2\,e^{c+d\,x}\,\left(\frac{e^{-4\,a-4\,b\,x}}{2}+\frac{e^{4\,a+4\,b\,x}}{2}\right)}{8}+\frac{b\,d\,e^{c+d\,x}\,\left(\frac{e^{-4\,a-4\,b\,x}}{2}-\frac{e^{4\,a+4\,b\,x}}{2}\right)}{2}}{16\,b^2\,d-d^3} - \frac{e^{c+d\,x}}{8\,d}$$

input
```
int(cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x)^2,x)
```

output
```
- ((d^2*exp(c + d*x)*(exp(- 4*a - 4*b*x)/2 + exp(4*a + 4*b*x)/2))/8 + (b*d
*exp(c + d*x)*(exp(- 4*a - 4*b*x)/2 - exp(4*a + 4*b*x)/2))/2)/(16*b^2*d -
d^3) - exp(c + d*x)/(8*d)
```

3.953. $\int e^{c+dx} \cosh^2(a+bx) \sinh^2(a+bx) \, dx$

3.954 $\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx$

3.954.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx=\frac{be^{c+dx}\cosh(a+bx)}{4(b^2-d^2)}+\frac{3be^{c+dx}\cosh(3a+3bx)}{4(9b^2-d^2)}-\frac{de^{c+dx}\sinh(a+bx)}{4(b^2-d^2)}-\frac{de^{c+dx}\sinh(3a+3bx)}{4(9b^2-d^2)}$$

output
```
1/4*b*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)+3/4*b*exp(d*x+c)*cosh(3*b*x+3*a)/(9
*b^2-d^2)-1/4*d*exp(d*x+c)*sinh(b*x+a)/(b^2-d^2)-1/4*d*exp(d*x+c)*sinh(3*b
*x+3*a)/(9*b^2-d^2)
```

3.954.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx=\frac{1}{4}e^{c+dx}\left(\frac{b\cosh(a+bx)-d\sinh(a+bx)}{(b-d)(b+d)}+\frac{3b\cosh(3(a+bx))-d\sinh(3(a+bx))}{9b^2-d^2}\right)$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
(E^(c + d*x)*((b*Cosh[a + b*x] - d*Sinh[a + b*x])/((b - d)*(b + d)) + (3*b
*Cosh[3*(a + b*x)] - d*Sinh[3*(a + b*x)])/(9*b^2 - d^2)))/4
```

3.954.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx} \sinh(a+bx) \cosh^2(a+bx) \, dx$$

$$\Big\downarrow \text{6035}$$

$$\int \left(\frac{1}{4}e^{c+dx} \sinh(a+bx) + \frac{1}{4}e^{c+dx} \sinh(3a+3bx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{de^{c+dx} \sinh(a+bx)}{4(b^2-d^2)} - \frac{de^{c+dx} \sinh(3a+3bx)}{4(9b^2-d^2)} + \frac{be^{c+dx} \cosh(a+bx)}{4(b^2-d^2)} + \frac{3be^{c+dx} \cosh(3a+3bx)}{4(9b^2-d^2)}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

output
```
(b*E^(c + d*x)*Cosh[a + b*x])/(4*(b^2 - d^2)) + (3*b*E^(c + d*x)*Cosh[3*a
+ 3*b*x])/(4*(9*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[a + b*x])/(4*(b^2 - d^2)
) - (d*E^(c + d*x)*Sinh[3*a + 3*b*x])/(4*(9*b^2 - d^2))
```

3.954.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.954.4 Maple [A] (verified)

Time = 11.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.40

method	result
default	$-\frac{\sinh(a-c+(b-d)x)}{8(b-d)}+\frac{\sinh(a+c+(b+d)x)}{8b+8d}-\frac{\sinh(3a-c+(3b-d)x)}{8(3b-d)}+\frac{\sinh(3a+c+(3b+d)x)}{24b+8d}+\frac{\cosh(a-c+(b-d)x)}{8b-8d}+\cosh($
risch	$\frac{(3b^3e^{6bx+6a}-b^2d\,e^{6bx+6a}-3b\,d^2e^{6bx+6a}+d^3e^{6bx+6a}+9b^3e^{4bx+4a}-9b^2d\,e^{4bx+4a}-b\,d^2e^{4bx+4a}+d^3e^{4bx+4a}+9b^3e^{2bx+2a}+9b^2d\,e^{2bx+}}{8(3b+d)(b+d)(3b-d)(b-d)}$

input
```
int(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8*sinh(a-c+(b-d)*x)/(b-d)+1/8*sinh(a+c+(b+d)*x)/(b+d)-1/8*sinh(3*a-c+(3
*b-d)*x)/(3*b-d)+1/8*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/8*cosh(a-c+(b-d)*x)/(
b-d)+1/8*cosh(a+c+(b+d)*x)/(b+d)+1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/8*cos
h(3*a+c+(3*b+d)*x)/(3*b+d)
```

3.954.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(115) = 230.

Time = 0.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.00

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{9\left(b^3-bd^2\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2-\left(b^2d-d^3\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-\left(9\,b^2d-\right.}{}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(9*(b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 - (b^2*d
- d^3)*cosh(d*x + c)*sinh(b*x + a)^3 - (9*b^2*d - d^3 + 3*(b^2*d - d^3)*co
sh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a) + (3*(b^3 - b*d^2)*cosh(b*x + a
)^3 + (9*b^3 - b*d^2)*cosh(b*x + a))*cosh(d*x + c) + (3*(b^3 - b*d^2)*cosh
(b*x + a)^3 + 9*(b^3 - b*d^2)*cosh(b*x + a)*sinh(b*x + a)^2 - (b^2*d - d^3
)*sinh(b*x + a)^3 + (9*b^3 - b*d^2)*cosh(b*x + a) - (9*b^2*d - d^3 + 3*(b^
2*d - d^3)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^2
*d^2 + d^4)*cosh(b*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^2
*sinh(b*x + a)^2 + (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

3.954.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(109) = 218$.

Time = 2.60 (sec) , antiderivative size = 972, normalized size of antiderivative = 7.65

$$\int e^{c+dx} \cosh^2(a+bx) \sinh(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**2*sinh(b*x+a),x)
```

output
```
Piecewise((x*exp(c)*sinh(a)*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (-x*exp(c)*e
xp(d*x)*sinh(a - d*x)**3/8 - x*exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d
*x)/8 + x*exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/8 + x*exp(c)*exp(
d*x)*cosh(a - d*x)**3/8 - exp(c)*exp(d*x)*sinh(a - d*x)**3/(8*d) - exp(c)*
exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/(4*d) - exp(c)*exp(d*x)*cosh(a - d
*x)**3/(8*d), Eq(b, -d)), (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/8 + 3*x*ex
p(c)*exp(d*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3)/8 + 3*x*exp(c)*exp(d*x)*s
inh(a - d*x/3)*cosh(a - d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**3
/8 - 3*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/(8*d) - 3*exp(c)*exp(d*x)*sinh(a
 - d*x/3)**2*cosh(a - d*x/3)/(4*d) - exp(c)*exp(d*x)*cosh(a - d*x/3)**3/(8
*d), Eq(b, -d/3)), (x*exp(c)*exp(d*x)*sinh(a + d*x/3)**3/8 - 3*x*exp(c)*ex
p(d*x)*sinh(a + d*x/3)**2*cosh(a + d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a +
 d*x/3)*cosh(a + d*x/3)**2/8 - x*exp(c)*exp(d*x)*cosh(a + d*x/3)**3/8 - ex
p(c)*exp(d*x)*sinh(a + d*x/3)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a + d*x/3)
*cosh(a + d*x/3)**2/(4*d) - exp(c)*exp(d*x)*cosh(a + d*x/3)**3/(8*d), Eq(b
, d/3)), (-x*exp(c)*exp(d*x)*sinh(a + d*x)**3/8 + x*exp(c)*exp(d*x)*sinh(a
 + d*x)**2*cosh(a + d*x)/8 + x*exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)
**2/8 - x*exp(c)*exp(d*x)*cosh(a + d*x)**3/8 + exp(c)*exp(d*x)*sinh(a + d*
x)**3/(8*d) - exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**2/(4*d) + 3*exp
(c)*exp(d*x)*cosh(a + d*x)**3/(8*d), Eq(b, d)), (3*b**3*exp(c)*exp(d*x)...
```

3.954.7 Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx} \cosh^2(a+bx) \sinh(a+bx) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more
 details)I
```

3.954.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{e^{(3\,bx+dx+3\,a+c)}}{8\,(3\,b+d)} + \frac{e^{(bx+dx+a+c)}}{8\,(b+d)} + \frac{e^{(-bx+dx-a+c)}}{8\,(b-d)} + \frac{e^{(-3\,bx+dx-3\,a+c)}}{8\,(3\,b-d)}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/8*e^(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/8*e^(b*x + d*x + a + c)/(b + d
) + 1/8*e^(-b*x + d*x - a + c)/(b - d) + 1/8*e^(-3*b*x + d*x - 3*a + c)/(3
*b - d)
```

3.954.9 Mupad [B] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{e^{c+d\,x}\left(3\,b^3\cosh(a+b\,x)^3 - 3\,b^2\,d\cosh(a+b\,x)^2\sinh(a+b\,x) + 2\,b^2\,d\sinh(a+b\,x)^3 - b\,d^2\cosh(a+b\,x)^3\right.}{9\,b^4 - 10\,b^2\,d^2 + d^4}$$

input
```
int(cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x),x)
```

output
```
(exp(c + d*x)*(3*b^3*cosh(a + b*x)^3 - b*d^2*cosh(a + b*x)^3 + d^3*cosh(a
+ b*x)^2*sinh(a + b*x) + 2*b^2*d*sinh(a + b*x)^3 - 2*b*d^2*cosh(a + b*x)*s
inh(a + b*x)^2 - 3*b^2*d*cosh(a + b*x)^2*sinh(a + b*x)))/(9*b^4 + d^4 - 10
*b^2*d^2)
```

3.955 $\int e^{c+dx}\cosh^2(a+bx)\,dx$

3.955.1 Optimal result

Integrand size = 16, antiderivative size = 95

$$\int e^{c+dx}\cosh^2(a+bx)\,dx=\frac{2b^2e^{c+dx}}{d(4b^2-d^2)}-\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2}+\frac{2be^{c+dx}\cosh(a+bx)\sinh(a+bx)}{4b^2-d^2}$$

output
```
2*b^2*exp(d*x+c)/d/(4*b^2-d^2)-d*exp(d*x+c)*cosh(b*x+a)^2/(4*b^2-d^2)+2*b*
exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)/(4*b^2-d^2)
```

3.955.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int e^{c+dx}\cosh^2(a+bx)\,dx=\frac{e^{c+dx}(-4b^2+d^2+d^2\cosh(2(a+bx))-2bd\sinh(2(a+bx)))}{-8b^2d+2d^3}$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]^2,x]
```

output
```
(E^(c + d*x)*(-4*b^2 + d^2 + d^2*Cosh[2*(a + b*x)] - 2*b*d*Sinh[2*(a + b*x
)]))/(-8*b^2*d + 2*d^3)
```

3.955.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6000, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{6000}$$

$$\frac{2b^2\int e^{c+dx}dx}{4b^2-d^2}-\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2}+\frac{2be^{c+dx}\sinh(a+bx)\cosh(a+bx)}{4b^2-d^2}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2}+\frac{2be^{c+dx}\sinh(a+bx)\cosh(a+bx)}{4b^2-d^2}+\frac{2b^2e^{c+dx}}{d\,(4b^2-d^2)}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]^2,x]
```

output
```
(2*b^2*E^(c + d*x))/(d*(4*b^2 - d^2)) - (d*E^(c + d*x)*Cosh[a + b*x]^2)/(4
*b^2 - d^2) + (2*b*E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2 - d^2)
```

3.955.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

3.955.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{e^{dx+c}(-d^2\cosh(2bx+2a)+2bd\sinh(2bx+2a)+4b^2-d^2)}{8b^2d-2d^3}$
risch	$\frac{(2d\,e^{4bx+4a}b-d^2e^{4bx+4a}+8b^2e^{2bx+2a}-2d^2e^{2bx+2a}-2bd-d^2)e^{-2bx+dx-2a+c}}{4(2b+d)(2b-d)d}$
default	$\frac{\sinh(dx+c)}{2d}+\frac{\sinh(2a-c+(2b-d)x)}{8b-4d}+\frac{\sinh(2a+c+(2b+d)x)}{8b+4d}+\frac{\cosh(dx+c)}{2d}-\frac{\cosh(2a-c+(2b-d)x)}{4(2b-d)}+\frac{\cosh(2a+c+(2}{8b+4d}$

input
```
int(exp(d*x+c)*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
exp(d*x+c)*(-d^2*cosh(2*b*x+2*a)+2*b*d*sinh(2*b*x+2*a)+4*b^2-d^2)/(8*b^2*d
-2*d^3)
```

3.955.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.85

$$\int e^{c+dx}\cosh^2(a+bx)\,dx$$
$$=\frac{4\,bd\cosh(bx+a)\cosh(dx+c)\sinh(bx+a)-d^2\cosh(dx+c)\sinh(bx+a)^2-\left(d^2\cosh(bx+a)^2-4\,b^2\right.}{2\left(\left(4\,b^2d-d^3\right)\cosh(bx\right.}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(4*b*d*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a) - d^2*cosh(d*x + c)*s
inh(b*x + a)^2 - (d^2*cosh(b*x + a)^2 - 4*b^2 + d^2)*cosh(d*x + c) - (d^2*
cosh(b*x + a)^2 - 4*b*d*cosh(b*x + a)*sinh(b*x + a) + d^2*sinh(b*x + a)^2
- 4*b^2 + d^2)*sinh(d*x + c))/((4*b^2*d - d^3)*cosh(b*x + a)^2 - (4*b^2*d
- d^3)*sinh(b*x + a)^2)
```

3.955.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(78) = 156$.

Time = 1.05 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.58

$$\int e^{c+dx}\cosh^2(a+bx)\,dx$$

$$= \begin{cases} xe^c\cosh^2(a) \\ \frac{xe^ce^{dx}\sinh^2\left(a-\frac{dx}{2}\right)}{4} + \frac{xe^ce^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2} + \frac{xe^ce^{dx}\cosh^2\left(a-\frac{dx}{2}\right)}{4} - \frac{e^ce^{dx}\sinh^2\left(a-\frac{dx}{2}\right)}{d} - \frac{3e^ce^{dx}\sinh\left(a-\frac{dx}{2}\right)\cos}{2d} \\ \frac{xe^ce^{dx}\sinh^2\left(a+\frac{dx}{2}\right)}{4} - \frac{xe^ce^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2} + \frac{xe^ce^{dx}\cosh^2\left(a+\frac{dx}{2}\right)}{4} - \frac{e^ce^{dx}\sinh^2\left(a+\frac{dx}{2}\right)}{d} + \frac{3e^ce^{dx}\sinh\left(a+\frac{dx}{2}\right)\cos}{2d} \\ \left(-\frac{x\sinh^2(a+bx)}{2} + \frac{x\cosh^2(a+bx)}{2} + \frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)e^c \\ -\frac{2b^2e^ce^{dx}\sinh^2(a+bx)}{4b^2d-d^3} + \frac{2b^2e^ce^{dx}\cosh^2(a+bx)}{4b^2d-d^3} + \frac{2bde^ce^{dx}\sinh(a+bx)\cosh(a+bx)}{4b^2d-d^3} - \frac{d^2e^ce^{dx}\cosh^2(a+bx)}{4b^2d-d^3} \end{cases}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**2,x)
```

output
```
Piecewise((x*exp(c)*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp(d*x)*s
inh(a - d*x/2)**2/4 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)/2
+ x*exp(c)*exp(d*x)*cosh(a - d*x/2)**2/4 - exp(c)*exp(d*x)*sinh(a - d*x/2)
**2/d - 3*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)/(2*d), Eq(b, -d/
2)), (x*exp(c)*exp(d*x)*sinh(a + d*x/2)**2/4 - x*exp(c)*exp(d*x)*sinh(a +
d*x/2)*cosh(a + d*x/2)/2 + x*exp(c)*exp(d*x)*cosh(a + d*x/2)**2/4 - exp(c)
*exp(d*x)*sinh(a + d*x/2)**2/d + 3*exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a
+ d*x/2)/(2*d), Eq(b, d/2)), ((-x*sinh(a + b*x)**2/2 + x*cosh(a + b*x)**2/
2 + sinh(a + b*x)*cosh(a + b*x)/(2*b))*exp(c), Eq(d, 0)), (-2*b**2*exp(c)*
exp(d*x)*sinh(a + b*x)**2/(4*b**2*d - d**3) + 2*b**2*exp(c)*exp(d*x)*cosh(
a + b*x)**2/(4*b**2*d - d**3) + 2*b*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a
 + b*x)/(4*b**2*d - d**3) - d**2*exp(c)*exp(d*x)*cosh(a + b*x)**2/(4*b**2*
d - d**3), True))
```

3.955.7 Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for mor
e details)
```

3.955.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.61

$$\int e^{c+dx}\cosh^2(a+bx)\,dx = \frac{e^{(2\,bx+dx+2\,a+c)}}{4\,(2\,b+d)} - \frac{e^{(-2\,bx+dx-2\,a+c)}}{4\,(2\,b-d)} + \frac{e^{(dx+c)}}{2\,d}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*e^(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*e^(-2*b*x + d*x - 2*a + c)/(
2*b - d) + 1/2*e^(d*x + c)/d
```

3.955.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int e^{c+dx}\cosh^2(a+bx)\,dx = \frac{2\,b^2\,e^{c+d\,x} - d^2\cosh(a+b\,x)^2\,e^{c+d\,x} + 2\,b\,d\cosh(a+b\,x)\,e^{c+d\,x}\sinh(a+b\,x)}{4\,b^2\,d-d^3}$$

input
```
int(cosh(a + b*x)^2*exp(c + d*x),x)
```

output
```
(2*b^2*exp(c + d*x) - d^2*cosh(a + b*x)^2*exp(c + d*x) + 2*b*d*cosh(a + b*
x)*exp(c + d*x)*sinh(a + b*x))/(4*b^2*d - d^3)
```

3.956 $\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx$

3.956.1 Optimal result

Integrand size = 20, antiderivative size = 103

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx = -\frac{3e^{-a+c-(b-d)x}}{2(b-d)}+\frac{e^{a+c+(b+d)x}}{2(b+d)}+\frac{2e^{-a+c-(b-d)x}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d}$$

output
```
-3/2*exp(-a+c-(b-d)*x)/(b-d)+1/2*exp(a+c+(b+d)*x)/(b+d)+2*exp(-a+c-(b-d)*x
)*hypergeom([1, 1/2*(-b+d)/b],[1/2*(b+d)/b],exp(2*b*x+2*a))/(b-d)
```

3.956.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{c+dx}\left(b\cosh(a+bx)-2(b-d)e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)-d\sinh(a+bx)\right)}{(b-d)(b+d)}$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
(E^(c + d*x)*(b*Cosh[a + b*x] - 2*(b - d)*E^(a + b*x)*Hypergeometric2F1[1,
 (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))] - d*Sinh[a + b*x]))/((b
- d)*(b + d))
```

3.956.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{3}{2}e^{-a-x(b-d)+c}+\frac{1}{2}e^{2(a+bx)-a-x(b-d)+c}+\frac{2e^{-a-x(b-d)+c}}{e^{2(a+bx)}-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e^{-a-x(b-d)+c}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d}-\frac{3e^{-a-x(b-d)+c}}{2(b-d)}+\frac{e^{2(a+bx)-a-x(b-d)+c}}{2(b+d)}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
(-3*E^(-a + c - (b - d)*x))/(2*(b - d)) + E^(-a + c - (b - d)*x + 2*(a + b
*x))/(2*(b + d)) + (2*E^(-a + c - (b - d)*x)*Hypergeometric2F1[1, -1/2*(b
- d)/b, (b + d)/(2*b), E^(2*(a + b*x))])/(b - d)
```

3.956.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

3.956.4 Maple [F]

$$\int \mathrm{e}^{dx+c} \cosh\left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right) dx$$

input
```
int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x)
```

output
```
int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x)
```

3.956.5 Fricas [F]

$$\int e^{c+dx} \cosh(a+bx) \coth(a+bx)\, dx = \int \cosh\left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right) e^{(dx+c)}\, dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^2*csch(b*x + a)*e^(d*x + c), x)
```

3.956.6 Sympy [F(-1)]

Timed out.

$$\int e^{c+dx} \cosh(a+bx) \coth(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**2*csch(b*x+a),x)
```

output
```
Timed out
```

3.956.7 Maxima [F]

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx = \int \cosh(bx+a)^2\operatorname{csch}(bx+a)\,e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

output
```
-4*b*integrate(e^(d*x + c)/((3*b - d)*e^(5*b*x + 5*a) - 2*(3*b - d)*e^(3*b
*x + 3*a) + (3*b - d)*e^(b*x + a)), x) + 1/2*(5*b^2*e^c + 6*b*d*e^c + d^2*
e^c + (3*b^2*e^c - 4*b*d*e^c + d^2*e^c)*e^(4*b*x + 4*a) - 2*(6*b^2*e^c + b
*d*e^c - d^2*e^c)*e^(2*b*x + 2*a))*e^(d*x)/((3*b^3 - b^2*d - 3*b*d^2 + d^3
)*e^(3*b*x + 3*a) - (3*b^3 - b^2*d - 3*b*d^2 + d^3)*e^(b*x + a))
```

3.956.8 Giac [F]

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx = \int \cosh(bx+a)^2\operatorname{csch}(bx+a)\,e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2*csch(b*x + a)*e^(d*x + c), x)
```

3.956.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx = \int \frac{\cosh(a+b\,x)^2\,e^{c+d\,x}}{\sinh(a+b\,x)}\,dx$$

input
```
int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x),x)
```

output
```
int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x), x)
```

3.957 $\int e^{c+dx}\coth^2(a+bx)\,dx$

3.957.1 Optimal result

Integrand size = 16, antiderivative size = 94

$$\int e^{c+dx}\coth^2(a+bx)\,dx=\frac{e^{c+dx}}{d}-\frac{4e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}+\frac{4e^{c+dx}\,\text{Hypergeometric2F1}\left(2,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}$$

output
```
exp(d*x+c)/d-4*exp(d*x+c)*hypergeom([1, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a
))/d+4*exp(d*x+c)*hypergeom([2, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a))/d
```

3.957.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.57

$$\int e^{c+dx}\coth^2(a+bx)\,dx=\frac{e^{c+dx}}{d}-\frac{2d\left(\frac{e^{2a+c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}-\frac{e^{2a+c+(2b+d)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{d}{2b},2+\frac{d}{2b},e^{2(a+bx)}\right)}{2b+d}\right)}{b\left(-1+e^{2a}\right)}+\frac{e^{c+dx}\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b}$$

input
```
Integrate[E^(c + d*x)*Coth[a + b*x]^2,x]
```

output
```
E^(c + d*x)/d - (2*d*((E^(2*a + c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 +
 d/(2*b), E^(2*(a + b*x))])/d - (E^(2*a + c + (2*b + d)*x)*Hypergeometric2
F1[1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))])/(2*b + d)))/(b*(-1 + E^(
2*a)) + (E^(c + d*x)*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

3.957.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{6008}$$

$$\int \left(\frac{4e^{c+dx}}{e^{2(a+bx)}-1}+\frac{4e^{c+dx}}{\left(e^{2(a+bx)}-1\right)^2}+e^{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},\frac{d}{2b}+1,e^{2(a+bx)}\right)}{d}+\frac{4e^{c+dx}\,\text{Hypergeometric2F1}\left(2,\frac{d}{2b},\frac{d}{2b}+1,e^{2(a+bx)}\right)}{d}+\frac{e^{c+dx}}{d}$$

input
```
Int[E^(c + d*x)*Coth[a + b*x]^2,x]
```

output
```
E^(c + d*x)/d - (4*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b),
E^(2*(a + b*x))])/d + (4*E^(c + d*x)*Hypergeometric2F1[2, d/(2*b), 1 + d/(
2*b), E^(2*(a + b*x))])/d
```

3.957.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6008
```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.957.4 Maple [F]

$$\int e^{dx+c} \cosh (bx+a)^2 \operatorname{csch} (bx+a)^2 \, dx$$

input
```
int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

output
```
int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

3.957.5 Fricas [F]

$$\int e^{c+dx} \coth^2(a+bx) \, dx = \int \cosh (bx+a)^2 \operatorname{csch} (bx+a)^2 e^{(dx+c)} \, dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^2*csch(b*x + a)^2*e^(d*x + c), x)
```

3.957.6 Sympy [F(-1)]

Timed out.

$$\int e^{c+dx} \coth^2(a+bx) \, dx = \text{Timed out}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.957.7 Maxima [F]

$$\int e^{c+dx}\coth^2(a+bx)\,dx=\int \cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^2e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
16*b*d*integrate(-e^(d*x + c)/(8*b^2 - 6*b*d + d^2 - (8*b^2 - 6*b*d + d^2)
*e^(6*b*x + 6*a) + 3*(8*b^2 - 6*b*d + d^2)*e^(4*b*x + 4*a) - 3*(8*b^2 - 6*
b*d + d^2)*e^(2*b*x + 2*a)), x) + (8*b^2*e^c + 10*b*d*e^c + d^2*e^c + (8*b
^2*e^c - 6*b*d*e^c + d^2*e^c)*e^(4*b*x + 4*a) - 2*(8*b^2*e^c + 2*b*d*e^c -
 d^2*e^c)*e^(2*b*x + 2*a))*e^(d*x)/(8*b^2*d - 6*b*d^2 + d^3 + (8*b^2*d - 6
*b*d^2 + d^3)*e^(4*b*x + 4*a) - 2*(8*b^2*d - 6*b*d^2 + d^3)*e^(2*b*x + 2*a
))
```

3.957.8 Giac [F]

$$\int e^{c+dx}\coth^2(a+bx)\,dx=\int \cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^2e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2*csch(b*x + a)^2*e^(d*x + c), x)
```

3.957.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\coth^2(a+bx)\,dx=\int \frac{\cosh(a+b\,x)^2\,\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x)^2,x)
```

output
```
int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x)^2, x)
```

3.958 $\int e^{c+dx}\coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

3.958.1 Optimal result

Integrand size = 22, antiderivative size = 151

$$\int e^{c+dx}\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{2e^{a+c+(b+d)x}\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d} + \frac{8e^{a+c+(b+d)x}\text{Hypergeometric2F1}\left(2,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d} - \frac{8e^{a+c+(b+d)x}\text{Hypergeometric2F1}\left(3,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}$$

output
```
-2*exp(a+c+(b+d)*x)*hypergeom([1, 1/2*(b+d)/b],[1/2*(3*b+d)/b],exp(2*b*x+2
*a))/(b+d)+8*exp(a+c+(b+d)*x)*hypergeom([2, 1/2*(b+d)/b],[1/2*(3*b+d)/b],e
xp(2*b*x+2*a))/(b+d)-8*exp(a+c+(b+d)*x)*hypergeom([3, 1/2*(b+d)/b],[1/2*(3
*b+d)/b],exp(2*b*x+2*a))/(b+d)
```

3.958.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int e^{c+dx}\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{e^{c-\frac{ad}{b}}\left((b+d)e^{d\left(\frac{a}{b}+x\right)}(d+b\coth(a+bx))\text{csch}(a+bx)+2(b^2+d^2)\,e^{\frac{(b+d)(a+bx)}{b}}\text{Hypergeometric2F1}\left(1,\frac{b}{\cdots}\right.\right.}{2b^2(b+d)}$$

input
```
Integrate[E^(c + d*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
-1/2*(E^(c - (a*d)/b)*((b + d)*E^(d*(a/b + x))*(d + b*Coth[a + b*x])*Csch[
a + b*x] + 2*(b^2 + d^2)*E^(((b + d)*(a + b*x))/b)*Hypergeometric2F1[1, (b
 + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))]))/(b^2*(b + d))
```

3.958.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{2e^{a+x(b+d)+c}}{e^{2(a+bx)}-1}+\frac{8e^{a+x(b+d)+c}}{\left(e^{2(a+bx)}-1\right)^2}+\frac{8e^{a+x(b+d)+c}}{\left(e^{2(a+bx)}-1\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2e^{a+x(b+d)+c}\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}+\frac{8e^{a+x(b+d)+c}\text{Hypergeometric2F1}\left(2,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}-\frac{8e^{a+x(b+d)+c}\text{Hypergeometric2F1}\left(3,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}$$

input
```
Int[E^(c + d*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
(-2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2
*b), E^(2*(a + b*x))])/(b + d) + (8*E^(a + c + (b + d)*x)*Hypergeometric2F
1[2, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))])/(b + d) - (8*E^(a +
 c + (b + d)*x)*Hypergeometric2F1[3, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*
(a + b*x))])/(b + d)
```

3.958.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

3.958.4 Maple [F]

$$\int \mathrm{e}^{dx+c} \cosh{(bx+a)}^2 \operatorname{csch}{(bx+a)}^3\, dx$$

input
```
int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

output
```
int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

3.958.5 Fricas [F]

$$\int e^{c+dx} \coth^2(a+bx)\operatorname{csch}(a+bx)\, dx = \int \cosh{(bx+a)}^2 \operatorname{csch}{(bx+a)}^3 e^{(dx+c)}\, dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^2*csch(b*x + a)^3*e^(d*x + c), x)
```

3.958.6 Sympy [F(-1)]

Timed out.

$$\int e^{c+dx} \coth^2(a+bx)\operatorname{csch}(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.958.7 Maxima [F]

$$\int e^{c+dx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx=\int \cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^3e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-48*(b^3*e^c + b*d^2*e^c)*integrate(e^(b*x + d*x + a)/(15*b^3 - 23*b^2*d +
 9*b*d^2 - d^3 + (15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(8*b*x + 8*a) - 4*(
15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(6*b*x + 6*a) + 6*(15*b^3 - 23*b^2*d
+ 9*b*d^2 - d^3)*e^(4*b*x + 4*a) - 4*(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e
^(2*b*x + 2*a)), x) + 2*((15*b^2*e^c - 8*b*d*e^c + d^2*e^c)*e^(5*b*x + 5*a
) - 2*(10*b^2*e^c + 3*b*d*e^c - d^2*e^c)*e^(3*b*x + 3*a) + (9*b^2*e^c + 14
*b*d*e^c + d^2*e^c)*e^(b*x + a))*e^(d*x)/(15*b^3 - 23*b^2*d + 9*b*d^2 - d^
3 - (15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(6*b*x + 6*a) + 3*(15*b^3 - 23*b
^2*d + 9*b*d^2 - d^3)*e^(4*b*x + 4*a) - 3*(15*b^3 - 23*b^2*d + 9*b*d^2 - d
^3)*e^(2*b*x + 2*a))
```

3.958.8 Giac [F]

$$\int e^{c+dx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx=\int \cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^3e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2*csch(b*x + a)^3*e^(d*x + c), x)
```

3.958.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx=\int \frac{\cosh(a+b\,x)^2\,e^{c+d\,x}}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x)^3,x)
```

output
```
int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x)^3, x)
```

3.959 $\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$

3.959.1 Optimal result

Integrand size = 24, antiderivative size = 137

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx=-\frac{3be^{c+dx}\cosh(2a+2bx)}{16\,(4b^2-d^2)}+\frac{3be^{c+dx}\cosh(6a+6bx)}{16\,(36b^2-d^2)}+\frac{3de^{c+dx}\sinh(2a+2bx)}{32\,(4b^2-d^2)}-\frac{de^{c+dx}\sinh(6a+6bx)}{32\,(36b^2-d^2)}$$

output
```
-3/16*b*exp(d*x+c)*cosh(2*b*x+2*a)/(4*b^2-d^2)+3/16*b*exp(d*x+c)*cosh(6*b*
x+6*a)/(36*b^2-d^2)+3/32*d*exp(d*x+c)*sinh(2*b*x+2*a)/(4*b^2-d^2)-1/32*d*e
xp(d*x+c)*sinh(6*b*x+6*a)/(36*b^2-d^2)
```

3.959.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{e^{c+dx}(6b(-36b^2+d^2)\cosh(2(a+bx))+6(4b^3-bd^2)\cosh(6(a+bx))+2d(52b^2-d^2+(-4b^2+d^2)\cosh(}{32\,(144b^4-40b^2d^2+d^4)}$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(E^(c + d*x)*(6*b*(-36*b^2 + d^2)*Cosh[2*(a + b*x)] + 6*(4*b^3 - b*d^2)*Co
sh[6*(a + b*x)] + 2*d*(52*b^2 - d^2 + (-4*b^2 + d^2)*Cosh[4*(a + b*x)])*Si
nh[2*(a + b*x)]))/(32*(144*b^4 - 40*b^2*d^2 + d^4))
```

3.959.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\sinh^3(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 6035}$$

$$\int \left(\frac{1}{32}e^{c+dx}\sinh(6a+6bx)-\frac{3}{32}e^{c+dx}\sinh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3de^{c+dx}\sinh(2a+2bx)}{32\,(4b^2-d^2)}-\frac{de^{c+dx}\sinh(6a+6bx)}{32\,(36b^2-d^2)}-\frac{3be^{c+dx}\cosh(2a+2bx)}{16\,(4b^2-d^2)}+\frac{3be^{c+dx}\cosh(6a+6bx)}{16\,(36b^2-d^2)}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

output
```
(-3*b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(16*(4*b^2 - d^2)) + (3*b*E^(c + d*x)
*Cosh[6*a + 6*b*x])/(16*(36*b^2 - d^2)) + (3*d*E^(c + d*x)*Sinh[2*a + 2*b*
x])/(32*(4*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[6*a + 6*b*x])/(32*(36*b^2 - d
^2))
```

3.959.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.959.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.47

$$\frac{3\sinh\left(2a-c+\left(2b-d\right)x\right)}{64\left(2b-d\right)}-\frac{3\sinh\left(2a+c+\left(2b+d\right)x\right)}{64\left(2b+d\right)}-\frac{\sinh\left(\left(6b-d\right)x+6a-c\right)}{64\left(6b-d\right)}+\frac{\sinh\left(\left(6b+d\right)x+6a\right.}{384b+64d}$$

input
```
int(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

output
```
3/64*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-3/64*sinh(2*a+c+(2*b+d)*x)/(2*b+d)-1/64
/(6*b-d)*sinh((6*b-d)*x+6*a-c)+1/64/(6*b+d)*sinh((6*b+d)*x+6*a+c)-3/64*cos
h(2*a-c+(2*b-d)*x)/(2*b-d)-3/64*cosh(2*a+c+(2*b+d)*x)/(2*b+d)+1/64*cosh((6
*b-d)*x+6*a-c)/(6*b-d)+1/64*cosh((6*b+d)*x+6*a+c)/(6*b+d)
```

3.959.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(125) = 250$.

Time = 0.27 (sec) , antiderivative size = 676, normalized size of antiderivative = 4.93

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx =$$
$$-\frac{10\left(4b^2d-d^3\right)\cosh\left(bx+a\right)^3\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-45\left(4b^3-bd^2\right)\cosh\left(bx+a\right)^2\cosh\left(dx+c\right)}{}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/16*(10*(4*b^2*d - d^3)*cosh(b*x + a)^3*cosh(d*x + c)*sinh(b*x + a)^3 -
45*(4*b^3 - b*d^2)*cosh(b*x + a)^2*cosh(d*x + c)*sinh(b*x + a)^4 + 3*(4*b^
2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^5 - 3*(4*b^3 - b*d^2)
*cosh(d*x + c)*sinh(b*x + a)^6 - 3*(15*(4*b^3 - b*d^2)*cosh(b*x + a)^4 - 3
6*b^3 + b*d^2)*cosh(d*x + c)*sinh(b*x + a)^2 + 3*((4*b^2*d - d^3)*cosh(b*x
 + a)^5 - (36*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c)*sinh(b*x + a) - 3*
((4*b^3 - b*d^2)*cosh(b*x + a)^6 - (36*b^3 - b*d^2)*cosh(b*x + a)^2)*cosh(
d*x + c) - (3*(4*b^3 - b*d^2)*cosh(b*x + a)^6 - 10*(4*b^2*d - d^3)*cosh(b*
x + a)^3*sinh(b*x + a)^3 + 45*(4*b^3 - b*d^2)*cosh(b*x + a)^2*sinh(b*x + a
)^4 - 3*(4*b^2*d - d^3)*cosh(b*x + a)*sinh(b*x + a)^5 + 3*(4*b^3 - b*d^2)*
sinh(b*x + a)^6 - 3*(36*b^3 - b*d^2)*cosh(b*x + a)^2 + 3*(15*(4*b^3 - b*d^
2)*cosh(b*x + a)^4 - 36*b^3 + b*d^2)*sinh(b*x + a)^2 - 3*((4*b^2*d - d^3)*
cosh(b*x + a)^5 - (36*b^2*d - d^3)*cosh(b*x + a))*sinh(b*x + a))*sinh(d*x
+ c))/((144*b^4 - 40*b^2*d^2 + d^4)*cosh(b*x + a)^6 - 3*(144*b^4 - 40*b^2*
d^2 + d^4)*cosh(b*x + a)^4*sinh(b*x + a)^2 + 3*(144*b^4 - 40*b^2*d^2 + d^4
)*cosh(b*x + a)^2*sinh(b*x + a)^4 - (144*b^4 - 40*b^2*d^2 + d^4)*sinh(b*x
+ a)^6)
```

3.959.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1916 vs. $2(119) = 238$.

Time = 68.04 (sec) , antiderivative size = 1916, normalized size of antiderivative = 13.99

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

output
```
Piecewise((x*exp(c)*sinh(a)**3*cosh(a)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*exp
(c)*exp(d*x)*sinh(a - d*x/2)**6/64 - 3*x*exp(c)*exp(d*x)*sinh(a - d*x/2)**
5*cosh(a - d*x/2)/32 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/2)**4*cosh(a - d*x
/2)**2/64 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/2)**3*cosh(a - d*x/2)**3/16 +
 3*x*exp(c)*exp(d*x)*sinh(a - d*x/2)**2*cosh(a - d*x/2)**4/64 - 3*x*exp(c)
*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)**5/32 - 3*x*exp(c)*exp(d*x)*cosh
(a - d*x/2)**6/64 - 3*exp(c)*exp(d*x)*sinh(a - d*x/2)**6/(16*d) - 15*exp(c
)*exp(d*x)*sinh(a - d*x/2)**5*cosh(a - d*x/2)/(32*d) + 13*exp(c)*exp(d*x)*
sinh(a - d*x/2)**3*cosh(a - d*x/2)**3/(16*d) - 15*exp(c)*exp(d*x)*sinh(a -
 d*x/2)*cosh(a - d*x/2)**5/(32*d) - 3*exp(c)*exp(d*x)*cosh(a - d*x/2)**6/(
16*d), Eq(b, -d/2)), (x*exp(c)*exp(d*x)*sinh(a - d*x/6)**6/64 + 3*x*exp(c)
*exp(d*x)*sinh(a - d*x/6)**5*cosh(a - d*x/6)/32 + 15*x*exp(c)*exp(d*x)*sin
h(a - d*x/6)**4*cosh(a - d*x/6)**2/64 + 5*x*exp(c)*exp(d*x)*sinh(a - d*x/6
)**3*cosh(a - d*x/6)**3/16 + 15*x*exp(c)*exp(d*x)*sinh(a - d*x/6)**2*cosh(
a - d*x/6)**4/64 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/6)*cosh(a - d*x/6)**5/
32 + x*exp(c)*exp(d*x)*cosh(a - d*x/6)**6/64 - 3*exp(c)*exp(d*x)*sinh(a -
d*x/6)**6/(80*d) - 21*exp(c)*exp(d*x)*sinh(a - d*x/6)**5*cosh(a - d*x/6)/(
160*d) + 11*exp(c)*exp(d*x)*sinh(a - d*x/6)**3*cosh(a - d*x/6)**3/(16*d) -
 21*exp(c)*exp(d*x)*sinh(a - d*x/6)*cosh(a - d*x/6)**5/(160*d) - 3*exp(c)*
exp(d*x)*cosh(a - d*x/6)**6/(80*d), Eq(b, -d/6)), (-x*exp(c)*exp(d*x)*s...
```

3.959.7 Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for mor
e details)
```

3.959.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(6\,bx+dx+6\,a+c)}}{64\,(6\,b+d)} - \frac{3\,e^{(2\,bx+dx+2\,a+c)}}{64\,(2\,b+d)} - \frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{64\,(2\,b-d)} + \frac{e^{(-6\,bx+dx-6\,a+c)}}{64\,(6\,b-d)}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/64*e^(6*b*x + d*x + 6*a + c)/(6*b + d) - 3/64*e^(2*b*x + d*x + 2*a + c)/
(2*b + d) - 3/64*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) + 1/64*e^(-6*b*x + d
*x - 6*a + c)/(6*b - d)
```

3.959.9 Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.33

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{b^3\left(\frac{27\,\mathrm{e}^{c+d\,x}\cosh(2\,a+2\,b\,x)}{4}-\frac{3\,\mathrm{e}^{c+d\,x}\cosh(6\,a+6\,b\,x)}{4}\right)+d^3\left(\frac{3\,\mathrm{e}^{c+d\,x}\sinh(2\,a+2\,b\,x)}{32}-\frac{\mathrm{e}^{c+d\,x}\sinh(6\,a+6\,b\,x)}{32}\right)-b^2\,d\left(\frac{27\,\mathrm{e}^{c+d}}{}\right.}{144\,b^4-40\,b^2\,d^2+d^4}$$

input
```
int(cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x)^3,x)
```

output
```
-(b^3*((27*exp(c + d*x)*cosh(2*a + 2*b*x))/4 - (3*exp(c + d*x)*cosh(6*a +
6*b*x))/4) + d^3*((3*exp(c + d*x)*sinh(2*a + 2*b*x))/32 - (exp(c + d*x)*si
nh(6*a + 6*b*x))/32) - b^2*d*((27*exp(c + d*x)*sinh(2*a + 2*b*x))/8 - (exp
(c + d*x)*sinh(6*a + 6*b*x))/8) - b*d^2*((3*exp(c + d*x)*cosh(2*a + 2*b*x)
)/16 - (3*exp(c + d*x)*cosh(6*a + 6*b*x))/16))/(144*b^4 + d^4 - 40*b^2*d^2
)
```

3.960 $\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$

3.960.1 Optimal result

Integrand size = 24, antiderivative size = 195

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx=\frac{de^{c+dx}\cosh(a+bx)}{8(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3bx)}{16(9b^2-d^2)}-\frac{de^{c+dx}\cosh(5a+5bx)}{16(25b^2-d^2)}-\frac{be^{c+dx}\sinh(a+bx)}{8(b^2-d^2)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{16(9b^2-d^2)}+\frac{5be^{c+dx}\sinh(5a+5bx)}{16(25b^2-d^2)}$$

output
```
1/8*d*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)-1/16*d*exp(d*x+c)*cosh(3*b*x+3*a)/(
9*b^2-d^2)-1/16*d*exp(d*x+c)*cosh(5*b*x+5*a)/(25*b^2-d^2)-1/8*b*exp(d*x+c)
*sinh(b*x+a)/(b^2-d^2)+3/16*b*exp(d*x+c)*sinh(3*b*x+3*a)/(9*b^2-d^2)+5/16*
b*exp(d*x+c)*sinh(5*b*x+5*a)/(25*b^2-d^2)
```

3.960.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.61

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx=\frac{1}{16}e^{c+dx}\left(\frac{2d\cosh(a+bx)-2b\sinh(a+bx)}{(b-d)(b+d)}+\frac{-d\cosh(3(a+bx))+3b\sinh(3(a+bx))}{9b^2-d^2}+\frac{-d\cosh(5(a+bx))+5b\sinh(5(a+bx))}{25b^2-d^2}\right)$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(E^(c + d*x)*((2*d*Cosh[a + b*x] - 2*b*Sinh[a + b*x])/((b - d)*(b + d)) +
(-(d*Cosh[3*(a + b*x)]) + 3*b*Sinh[3*(a + b*x)])/(9*b^2 - d^2) + (-(d*Cosh
[5*(a + b*x)]) + 5*b*Sinh[5*(a + b*x)])/(25*b^2 - d^2)))/16
```

3.960.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\sinh^2(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{6035}$$

$$\int \left(-\frac{1}{8}e^{c+dx}\cosh(a+bx)+\frac{1}{16}e^{c+dx}\cosh(3a+3bx)+\frac{1}{16}e^{c+dx}\cosh(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{be^{c+dx}\sinh(a+bx)}{8(b^2-d^2)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{16(9b^2-d^2)}+\frac{5be^{c+dx}\sinh(5a+5bx)}{16(25b^2-d^2)}+\frac{de^{c+dx}\cosh(a+bx)}{8(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3bx)}{16(9b^2-d^2)}-\frac{de^{c+dx}\cosh(5a+5bx)}{16(25b^2-d^2)}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

output
```
(d*E^(c + d*x)*Cosh[a + b*x])/(8*(b^2 - d^2)) - (d*E^(c + d*x)*Cosh[3*a +
3*b*x])/(16*(9*b^2 - d^2)) - (d*E^(c + d*x)*Cosh[5*a + 5*b*x])/(16*(25*b^2
 - d^2)) - (b*E^(c + d*x)*Sinh[a + b*x])/(8*(b^2 - d^2)) + (3*b*E^(c + d*x
)*Sinh[3*a + 3*b*x])/(16*(9*b^2 - d^2)) + (5*b*E^(c + d*x)*Sinh[5*a + 5*b*
x])/(16*(25*b^2 - d^2))
```

3.960.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.960.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.43

$$-\frac{\sinh\left(a-c+(b-d)\,x\right)}{16\,(b-d)}-\frac{\sinh\left(a+c+(b+d)\,x\right)}{16\,(b+d)}+\frac{\sinh\left(3a-c+(3b-d)\,x\right)}{96b-32d}+\frac{\sinh\left(3a+c+(3b+d)\,x\right)}{96b+32d}$$

input
```
int(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

output
```
-1/16*sinh(a-c+(b-d)*x)/(b-d)-1/16*sinh(a+c+(b+d)*x)/(b+d)+1/32*sinh(3*a-c
+(3*b-d)*x)/(3*b-d)+1/32*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/32/(5*b-d)*sinh((
5*b-d)*x+5*a-c)+1/32/(5*b+d)*sinh((5*b+d)*x+5*a+c)+1/16*cosh(a-c+(b-d)*x)/
(b-d)-1/16*cosh(a+c+(b+d)*x)/(b+d)-1/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/32
*cosh(3*a+c+(3*b+d)*x)/(3*b+d)-1/32*cosh((5*b-d)*x+5*a-c)/(5*b-d)+1/32*cos
h((5*b+d)*x+5*a+c)/(5*b+d)
```

3.960.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(177) = 354$.

Time = 0.26 (sec) , antiderivative size = 917, normalized size of antiderivative = 4.70

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/16*(5*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x
 + a)^4 - 5*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(d*x + c)*sinh(b*x + a)^5 - (
75*b^5 - 78*b^3*d^2 + 3*b*d^4 + 50*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x +
 a)^2)*cosh(d*x + c)*sinh(b*x + a)^3 + (10*(9*b^4*d - 10*b^2*d^3 + d^5)*co
sh(b*x + a)^3 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a))*cosh(d*x +
c)*sinh(b*x + a)^2 + (450*b^5 - 68*b^3*d^2 + 2*b*d^4 - 25*(9*b^5 - 10*b^3*
d^2 + b*d^4)*cosh(b*x + a)^4 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x +
a)^2)*cosh(d*x + c)*sinh(b*x + a) + ((9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x
 + a)^5 + (25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a)^3 - 2*(225*b^4*d - 3
4*b^2*d^3 + d^5)*cosh(b*x + a))*cosh(d*x + c) + ((9*b^4*d - 10*b^2*d^3 + d
^5)*cosh(b*x + a)^5 + 5*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)*sinh(b*
x + a)^4 - 5*(9*b^5 - 10*b^3*d^2 + b*d^4)*sinh(b*x + a)^5 + (25*b^4*d - 26
*b^2*d^3 + d^5)*cosh(b*x + a)^3 - (75*b^5 - 78*b^3*d^2 + 3*b*d^4 + 50*(9*b
^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)^2)*sinh(b*x + a)^3 + (10*(9*b^4*d -
 10*b^2*d^3 + d^5)*cosh(b*x + a)^3 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(
b*x + a))*sinh(b*x + a)^2 - 2*(225*b^4*d - 34*b^2*d^3 + d^5)*cosh(b*x + a)
 + (450*b^5 - 68*b^3*d^2 + 2*b*d^4 - 25*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(
b*x + a)^4 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a)^2)*sinh(b*x + a
))*sinh(d*x + c))/((225*b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a
)^6 - 3*(225*b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^4*sinh...
```

3.960.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2761 vs. 2(168) = 336.

Time = 25.24 (sec) , antiderivative size = 2761, normalized size of antiderivative = 14.16

$$\int e^{c+dx} \cosh^3(a+bx) \sinh^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x*exp(c)*sinh(a)**2*cosh(a)**3, Eq(b, 0) & Eq(d, 0)), (-x*exp(c
)*exp(d*x)*sinh(a - d*x)**5/16 - x*exp(c)*exp(d*x)*sinh(a - d*x)**4*cosh(a
 - d*x)/16 + x*exp(c)*exp(d*x)*sinh(a - d*x)**3*cosh(a - d*x)**2/8 + x*exp
(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)**3/8 - x*exp(c)*exp(d*x)*sinh(
a - d*x)*cosh(a - d*x)**4/16 - x*exp(c)*exp(d*x)*cosh(a - d*x)**5/16 + 13*
exp(c)*exp(d*x)*sinh(a - d*x)**5/(96*d) + 7*exp(c)*exp(d*x)*sinh(a - d*x)*
*4*cosh(a - d*x)/(96*d) - exp(c)*exp(d*x)*sinh(a - d*x)**3*cosh(a - d*x)**
2/(3*d) - exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)**3/(6*d) - exp(c)
*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**4/(96*d) + 5*exp(c)*exp(d*x)*cosh(a
 - d*x)**5/(96*d), Eq(b, -d)), (-x*exp(c)*exp(d*x)*sinh(a - d*x/3)**5/32 -
 3*x*exp(c)*exp(d*x)*sinh(a - d*x/3)**4*cosh(a - d*x/3)/32 - x*exp(c)*exp(
d*x)*sinh(a - d*x/3)**3*cosh(a - d*x/3)**2/16 + x*exp(c)*exp(d*x)*sinh(a -
 d*x/3)**2*cosh(a - d*x/3)**3/16 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/3)*cos
h(a - d*x/3)**4/32 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**5/32 - 23*exp(c)*e
xp(d*x)*sinh(a - d*x/3)**5/(64*d) - 75*exp(c)*exp(d*x)*sinh(a - d*x/3)**4*
cosh(a - d*x/3)/(64*d) - exp(c)*exp(d*x)*sinh(a - d*x/3)**3*cosh(a - d*x/3
)**2/d + exp(c)*exp(d*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3)**3/(2*d) + 27*
exp(c)*exp(d*x)*sinh(a - d*x/3)*cosh(a - d*x/3)**4/(64*d) + 7*exp(c)*exp(d
*x)*cosh(a - d*x/3)**5/(64*d), Eq(b, -d/3)), (x*exp(c)*exp(d*x)*sinh(a - d
*x/5)**5/32 + 5*x*exp(c)*exp(d*x)*sinh(a - d*x/5)**4*cosh(a - d*x/5)/32...
```

3.960.7 Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more
 details)I
```

3.960.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx=\frac{e^{(5\,bx+dx+5\,a+c)}}{32\,(5\,b+d)}+\frac{e^{(3\,bx+dx+3\,a+c)}}{32\,(3\,b+d)}-\frac{e^{(bx+dx+a+c)}}{16\,(b+d)}+\frac{e^{(-bx+dx-a+c)}}{16\,(b-d)}-\frac{e^{(-3\,bx+dx-3\,a+c)}}{32\,(3\,b-d)}-\frac{e^{(-5\,bx+dx-5\,a+c)}}{32\,(5\,b-d)}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/32*e^(5*b*x + d*x + 5*a + c)/(5*b + d) + 1/32*e^(3*b*x + d*x + 3*a + c)/
(3*b + d) - 1/16*e^(b*x + d*x + a + c)/(b + d) + 1/16*e^(-b*x + d*x - a +
c)/(b - d) - 1/32*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) - 1/32*e^(-5*b*x +
d*x - 5*a + c)/(5*b - d)
```

3.960.9 Mupad [B] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.02

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{\cosh(a+b\,x)^5\,\mathrm{e}^{c+d\,x}\,(26\,b^4\,d-2\,b^2\,d^3)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}+\frac{3\cosh(a+b\,x)^2\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^3\,(25\,b^5-10\,b^3\,d^2+b\,d^4)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}+\frac{2\cosh(a+b\,x)^4\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)\,\left(b\,d^4-13\,b^3\,d^2\right)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}-\frac{\cosh(a+b\,x)^3\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^2\,(65\,b^4\,d-18\,b^2\,d^3+d^5)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}-\frac{6\,b^3\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^5\,(5\,b^2-d^2)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}+\frac{6\,b^2\,d\cosh(a+b\,x)\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^4\,(5\,b^2-d^2)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}$$

input
```
int(cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x)^2,x)
```

output
```
(cosh(a + b*x)^5*exp(c + d*x)*(26*b^4*d - 2*b^2*d^3))/(225*b^6 - d^6 + 35*
b^2*d^4 - 259*b^4*d^2) + (3*cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x)^3*(
b*d^4 + 25*b^5 - 10*b^3*d^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2) +
 (2*cosh(a + b*x)^4*exp(c + d*x)*sinh(a + b*x)*(b*d^4 - 13*b^3*d^2))/(225*
b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2) - (cosh(a + b*x)^3*exp(c + d*x)*sinh
(a + b*x)^2*(65*b^4*d + d^5 - 18*b^2*d^3))/(225*b^6 - d^6 + 35*b^2*d^4 - 2
59*b^4*d^2) - (6*b^3*exp(c + d*x)*sinh(a + b*x)^5*(5*b^2 - d^2))/(225*b^6
- d^6 + 35*b^2*d^4 - 259*b^4*d^2) + (6*b^2*d*cosh(a + b*x)*exp(c + d*x)*si
nh(a + b*x)^4*(5*b^2 - d^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2)
```

3.960. $\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$

3.961 $\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx$

3.961.1 Optimal result

Integrand size = 22, antiderivative size = 137

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}-\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}$$

output
```
1/2*b*exp(d*x+c)*cosh(2*b*x+2*a)/(4*b^2-d^2)+1/2*b*exp(d*x+c)*cosh(4*b*x+4
*a)/(16*b^2-d^2)-1/4*d*exp(d*x+c)*sinh(2*b*x+2*a)/(4*b^2-d^2)-1/8*d*exp(d*
x+c)*sinh(4*b*x+4*a)/(16*b^2-d^2)
```

3.961.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{1}{8}e^{c+dx}\left(\frac{4b\cosh(2(a+bx))-2d\sinh(2(a+bx))}{4b^2-d^2}+\frac{4b\cosh(4(a+bx))-d\sinh(4(a+bx))}{16b^2-d^2}\right)$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
(E^(c + d*x)*((4*b*Cosh[2*(a + b*x)] - 2*d*Sinh[2*(a + b*x)])/(4*b^2 - d^2
) + (4*b*Cosh[4*(a + b*x)] - d*Sinh[4*(a + b*x)])/(16*b^2 - d^2)))/8
```

3.961.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\sinh(a+bx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{6035}$$

$$\int \left(\frac{1}{4}e^{c+dx}\sinh(2a+2bx)+\frac{1}{8}e^{c+dx}\sinh(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}+\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

output
```
(b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(2*(4*b^2 - d^2)) + (b*E^(c + d*x)*Cosh[
4*a + 4*b*x])/(2*(16*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[2*a + 2*b*x])/(4*(4
*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[4*a + 4*b*x])/(8*(16*b^2 - d^2))
```

3.961.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.961.4 Maple [A] (verified)

Time = 63.86 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.47

method	result
default	$-\frac{\sinh(2a-c+(2b-d)x)}{8(2b-d)}+\frac{\sinh(2a+c+(2b+d)x)}{16b+8d}-\frac{\sinh((4b-d)x+4a-c)}{16(4b-d)}+\frac{\sinh((4b+d)x+4a+c)}{64b+16d}+\frac{\cosh(2a-c+(2b-d)x)}{16b-8d}+$
risch	$\frac{(16b^3e^{8bx+8a}-4d\,e^{8bx+8a}b^2-4d^2e^{8bx+8a}b+d^3e^{8bx+8a}+64b^3e^{6bx+6a}-32b^2d\,e^{6bx+6a}-4b\,d^2e^{6bx+6a}+2d^3e^{6bx+6a}+64b^3e^{2bx+2a}+32}{16(4b+d)(2b+d)(4b-d)(2b-d)}$

input
```
int(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8*sinh(2*a-c+(2*b-d)*x)/(2*b-d)+1/8*sinh(2*a+c+(2*b+d)*x)/(2*b+d)-1/16/
(4*b-d)*sinh((4*b-d)*x+4*a-c)+1/16/(4*b+d)*sinh((4*b+d)*x+4*a+c)+1/8*cosh(
2*a-c+(2*b-d)*x)/(2*b-d)+1/8*cosh(2*a+c+(2*b+d)*x)/(2*b+d)+1/16*cosh((4*b-
d)*x+4*a-c)/(4*b-d)+1/16*cosh((4*b+d)*x+4*a+c)/(4*b+d)
```

3.961.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(125) = 250$.

Time = 0.27 (sec) , antiderivative size = 501, normalized size of antiderivative = 3.66

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx =$$
$$-\frac{(4\,b^2d-d^3)\cosh(bx+a)\cosh(dx+c)\sinh(bx+a)^3-(4\,b^3-bd^2)\cosh(dx+c)\sinh(bx+a)^4-(16}{}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*((4*b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^3 - (4*b^3
 - b*d^2)*cosh(d*x + c)*sinh(b*x + a)^4 - (16*b^3 - b*d^2 + 6*(4*b^3 - b*d
^2)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a)^2 + ((4*b^2*d - d^3)*cosh
(b*x + a)^3 + (16*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c)*sinh(b*x + a)
- ((4*b^3 - b*d^2)*cosh(b*x + a)^4 + (16*b^3 - b*d^2)*cosh(b*x + a)^2)*cos
h(d*x + c) - ((4*b^3 - b*d^2)*cosh(b*x + a)^4 - (4*b^2*d - d^3)*cosh(b*x +
 a)*sinh(b*x + a)^3 + (4*b^3 - b*d^2)*sinh(b*x + a)^4 + (16*b^3 - b*d^2)*c
osh(b*x + a)^2 + (16*b^3 - b*d^2 + 6*(4*b^3 - b*d^2)*cosh(b*x + a)^2)*sinh
(b*x + a)^2 - ((4*b^2*d - d^3)*cosh(b*x + a)^3 + (16*b^2*d - d^3)*cosh(b*x
 + a))*sinh(b*x + a))*sinh(d*x + c))/((64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x
 + a)^4 - 2*(64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 +
(64*b^4 - 20*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

3.961.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. $2(114) = 228$.

Time = 7.79 (sec) , antiderivative size = 1292, normalized size of antiderivative = 9.43

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**3*sinh(b*x+a),x)
```

output
```
Piecewise((x*exp(c)*sinh(a)*cosh(a)**3, Eq(b, 0) & Eq(d, 0)), (-x*exp(c)*e
xp(d*x)*sinh(a - d*x/2)**4/8 - x*exp(c)*exp(d*x)*sinh(a - d*x/2)**3*cosh(a
 - d*x/2)/4 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)**3/4 + x*e
xp(c)*exp(d*x)*cosh(a - d*x/2)**4/8 - exp(c)*exp(d*x)*sinh(a - d*x/2)**4/(
24*d) - exp(c)*exp(d*x)*sinh(a - d*x/2)**3*cosh(a - d*x/2)/(3*d) - exp(c)*
exp(d*x)*sinh(a - d*x/2)**2*cosh(a - d*x/2)**2/(2*d) - exp(c)*exp(d*x)*cos
h(a - d*x/2)**4/(8*d), Eq(b, -d/2)), (x*exp(c)*exp(d*x)*sinh(a - d*x/4)**4
/16 + x*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(a - d*x/4)/4 + 3*x*exp(c)*
exp(d*x)*sinh(a - d*x/4)**2*cosh(a - d*x/4)**2/8 + x*exp(c)*exp(d*x)*sinh(
a - d*x/4)*cosh(a - d*x/4)**3/4 + x*exp(c)*exp(d*x)*cosh(a - d*x/4)**4/16
- exp(c)*exp(d*x)*sinh(a - d*x/4)**4/(6*d) - 5*exp(c)*exp(d*x)*sinh(a - d*
x/4)**3*cosh(a - d*x/4)/(12*d) + 11*exp(c)*exp(d*x)*sinh(a - d*x/4)*cosh(a
 - d*x/4)**3/(12*d) + exp(c)*exp(d*x)*cosh(a - d*x/4)**4/(6*d), Eq(b, -d/4
)), (-x*exp(c)*exp(d*x)*sinh(a + d*x/4)**4/16 + x*exp(c)*exp(d*x)*sinh(a +
 d*x/4)**3*cosh(a + d*x/4)/4 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/4)**2*cosh
(a + d*x/4)**2/8 + x*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/4
- x*exp(c)*exp(d*x)*cosh(a + d*x/4)**4/16 + exp(c)*exp(d*x)*sinh(a + d*x/4
)**4/(6*d) - 5*exp(c)*exp(d*x)*sinh(a + d*x/4)**3*cosh(a + d*x/4)/(12*d) +
 11*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/(12*d) - exp(c)*exp
(d*x)*cosh(a + d*x/4)**4/(6*d), Eq(b, d/4)), (x*exp(c)*exp(d*x)*sinh(a ...
```

3.961.7 Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for mor
e details)
```

3.961.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{e^{(4\,bx+dx+4\,a+c)}}{16\,(4\,b+d)} + \frac{e^{(2\,bx+dx+2\,a+c)}}{8\,(2\,b+d)} + \frac{e^{(-2\,bx+dx-2\,a+c)}}{8\,(2\,b-d)} + \frac{e^{(-4\,bx+dx-4\,a+c)}}{16\,(4\,b-d)}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/16*e^(4*b*x + d*x + 4*a + c)/(4*b + d) + 1/8*e^(2*b*x + d*x + 2*a + c)/(
2*b + d) + 1/8*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) + 1/16*e^(-4*b*x + d*x
 - 4*a + c)/(4*b - d)
```

3.961.9 Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx = -\frac{b^3\left(6\,e^{c+d\,x}-16\cosh(a+b\,x)^4\,e^{c+d\,x}\right)+b^2\,d\left(4\,e^{c+d\,x}\sinh(a+b\,x)\,\cosh(a+b\,x)^3+6\,e^{c+d\,x}\sinh(a+b\,x}{64\,b^4 -}$$

input
```
int(cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x),x)
```

output
```
-(b^3*(6*exp(c + d*x) - 16*cosh(a + b*x)^4*exp(c + d*x)) + b^2*d*(6*cosh(a
 + b*x)*exp(c + d*x)*sinh(a + b*x) + 4*cosh(a + b*x)^3*exp(c + d*x)*sinh(a
 + b*x)) - b*d^2*(3*cosh(a + b*x)^2*exp(c + d*x) - 4*cosh(a + b*x)^4*exp(c
 + d*x)) - d^3*cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x))/(64*b^4 + d^4 -
 20*b^2*d^2)
```

3.962 $\int e^{c+dx} \cosh^3(a+bx)\,dx$

3.962.1 Optimal result

Integrand size = 16, antiderivative size = 144

$$\int e^{c+dx}\cosh^3(a+bx)\,dx=-\frac{6b^2de^{c+dx}\cosh(a+bx)}{9b^4-10b^2d^2+d^4}-\frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2}+\frac{6b^3e^{c+dx}\sinh(a+bx)}{9b^4-10b^2d^2+d^4}+\frac{3be^{c+dx}\cosh^2(a+bx)\sinh(a+bx)}{9b^2-d^2}$$

output
```
-6*b^2*d*exp(d*x+c)*cosh(b*x+a)/(9*b^4-10*b^2*d^2+d^4)-d*exp(d*x+c)*cosh(b
*x+a)^3/(9*b^2-d^2)+6*b^3*exp(d*x+c)*sinh(b*x+a)/(9*b^4-10*b^2*d^2+d^4)+3*
b*exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)/(9*b^2-d^2)
```

3.962.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.74

$$\int e^{c+dx}\cosh^3(a+bx)\,dx$$
$$=\frac{e^{c+dx}(3d(-9b^2+d^2)\cosh(a+bx)+(-b^2d+d^3)\cosh(3(a+bx))+6b(5b^2-d^2+(b^2-d^2)\cosh(2(a+bx}{4\left(9b^4-10b^2d^2+d^4\right)}$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]^3,x]
```

output
```
(E^(c + d*x)*(3*d*(-9*b^2 + d^2)*Cosh[a + b*x] + (-(b^2*d) + d^3)*Cosh[3*(
a + b*x)] + 6*b*(5*b^2 - d^2 + (b^2 - d^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x
]))/(4*(9*b^4 - 10*b^2*d^2 + d^4))
```

3.962.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6000, 5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 6000}$$

$$\frac{6b^2\int e^{c+dx}\cosh(a+bx)dx}{9b^2-d^2}-\frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2}+\frac{3be^{c+dx}\sinh(a+bx)\cosh^2(a+bx)}{9b^2-d^2}$$

$$\Big\downarrow \text{ 5998}$$

$$-\frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2}+\frac{3be^{c+dx}\sinh(a+bx)\cosh^2(a+bx)}{9b^2-d^2}+\frac{6b^2\left(\frac{be^{c+dx}\sinh(a+bx)}{b^2-d^2}-\frac{de^{c+dx}\cosh(a+bx)}{b^2-d^2}\right)}{9b^2-d^2}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]^3,x]
```

output
```
-((d*E^(c + d*x)*Cosh[a + b*x]^3)/(9*b^2 - d^2)) + (3*b*E^(c + d*x)*Cosh[a
 + b*x]^2*Sinh[a + b*x])/(9*b^2 - d^2) + (6*b^2*(-((d*E^(c + d*x)*Cosh[a +
 b*x])/(b^2 - d^2)) + (b*E^(c + d*x)*Sinh[a + b*x])/(b^2 - d^2)))/(9*b^2 -
 d^2)
```

3.962.3.1 Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

3.962.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{e^{dx+c}\left(\left(-b^2d+d^3\right)\cosh(3bx+3a)+3\left(b^3-b\,d^2\right)\sinh(3bx+3a)+27\left(b+\frac{d}{3}\right)\left(-\cosh(bx+a)d+b\sinh(bx+a)\right)\left(b-\frac{d}{3}\right)\right)}{36b^4-40b^2d^2+4d^4}$
default	$\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}+\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(3a-c+(3b-d)x)}{24b-8d}+\frac{\sinh(3a+c+(3b+d)x)}{24b+8d}-\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}$
risch	$\frac{(3b^3e^{6bx+6a}-b^2d\,e^{6bx+6a}-3b\,d^2e^{6bx+6a}+d^3e^{6bx+6a}+27b^3e^{4bx+4a}-27b^2d\,e^{4bx+4a}-3b\,d^2e^{4bx+4a}+3d^3e^{4bx+4a}-27b^3e^{2bx+2a}}{8(3b+d)(b+d)(3b-d)(b-d)}$

input
```
int(exp(d*x+c)*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(d*x+c)*((-b^2*d+d^3)*cosh(3*b*x+3*a)+3*(b^3-b*d^2)*sinh(3*b*x+3*a)
+27*(b+1/3*d)*(-cosh(b*x+a)*d+b*sinh(b*x+a))*(b-1/3*d))/(9*b^4-10*b^2*d^2+
d^4)
```

3.962.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(140) = 280.

Time = 0.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.65

$$\int e^{c+dx}\cosh^3(a+bx)\,dx =$$
$$-\frac{3\left(b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2-3\left(b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-3\left(9\right.}{}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 - 3*(b^3
 - b*d^2)*cosh(d*x + c)*sinh(b*x + a)^3 - 3*(9*b^3 - b*d^2 + 3*(b^3 - b*d^
2)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a) + ((b^2*d - d^3)*cosh(b*x
+ a)^3 + 3*(9*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c) + ((b^2*d - d^3)*c
osh(b*x + a)^3 + 3*(b^2*d - d^3)*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b^3 -
b*d^2)*sinh(b*x + a)^3 + 3*(9*b^2*d - d^3)*cosh(b*x + a) - 3*(9*b^3 - b*d^
2 + 3*(b^3 - b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4
 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b
*x + a)^2*sinh(b*x + a)^2 + (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

3.962.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. 2(131) = 262.

Time = 2.95 (sec) , antiderivative size = 1046, normalized size of antiderivative = 7.26

$$\int e^{c+dx}\cosh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**3,x)
```

output
```
Piecewise((x*exp(c)*cosh(a)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*exp(c)*exp(d*x
)*sinh(a - d*x)**3/8 - 3*x*exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/
8 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/8 + 3*x*exp(c)*exp(
d*x)*cosh(a - d*x)**3/8 + 5*exp(c)*exp(d*x)*sinh(a - d*x)**3/(8*d) + exp(c
)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/(4*d) - exp(c)*exp(d*x)*sinh(a -
 d*x)*cosh(a - d*x)**2/d - 3*exp(c)*exp(d*x)*cosh(a - d*x)**3/(8*d), Eq(b,
 -d)), (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/8 + 3*x*exp(c)*exp(d*x)*sinh(
a - d*x/3)**2*cosh(a - d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/3)*cosh
(a - d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**3/8 - 11*exp(c)*exp(
d*x)*sinh(a - d*x/3)**3/(8*d) - 15*exp(c)*exp(d*x)*sinh(a - d*x/3)**2*cosh
(a - d*x/3)/(4*d) - 3*exp(c)*exp(d*x)*sinh(a - d*x/3)*cosh(a - d*x/3)**2/d
 - exp(c)*exp(d*x)*cosh(a - d*x/3)**3/(8*d), Eq(b, -d/3)), (-x*exp(c)*exp(
d*x)*sinh(a + d*x/3)**3/8 + 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)**2*cosh(a
+ d*x/3)/8 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/3)**2/8 + x*
exp(c)*exp(d*x)*cosh(a + d*x/3)**3/8 + exp(c)*exp(d*x)*sinh(a + d*x/3)**3/
(8*d) - 3*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/3)**2/(4*d) + 9*exp
(c)*exp(d*x)*cosh(a + d*x/3)**3/(8*d), Eq(b, d/3)), (3*x*exp(c)*exp(d*x)*s
inh(a + d*x)**3/8 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x)**2*cosh(a + d*x)/8 -
 3*x*exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**2/8 + 3*x*exp(c)*exp(d*x
)*cosh(a + d*x)**3/8 - 3*exp(c)*exp(d*x)*sinh(a + d*x)**3/(8*d) + 3*exp...
```

3.962.7 Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx} \cosh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more
 details)I
```

3.962.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

$$\int e^{c+dx} \cosh^3(a+bx)\,dx = \frac{e^{(3\,bx+dx+3\,a+c)}}{8\,(3\,b+d)} + \frac{3\,e^{(bx+dx+a+c)}}{8\,(b+d)} - \frac{3\,e^{(-bx+dx-a+c)}}{8\,(b-d)} - \frac{e^{(-3\,bx+dx-3\,a+c)}}{8\,(3\,b-d)}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*e^(3*b*x + d*x + 3*a + c)/(3*b + d) + 3/8*e^(b*x + d*x + a + c)/(b + d
) - 3/8*e^(-b*x + d*x - a + c)/(b - d) - 1/8*e^(-3*b*x + d*x - 3*a + c)/(3
*b - d)
```

3.962.9 Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int e^{c+dx} \cosh^3(a+bx)\,dx$$
$$= \frac{e^{c+d\,x}\,\left(9\,b^3\cosh(a+b\,x)^2\sinh(a+b\,x) - 6\,b^3\sinh(a+b\,x)^3 - 7\,b^2\,d\cosh(a+b\,x)^3 + 6\,b^2\,d\cosh(a+b\,x)\ \mathrm{s}\right.}{9\,b^4 - 10\,b^2\,d^2 + d^4}$$

input
```
int(cosh(a + b*x)^3*exp(c + d*x),x)
```

output
```
(exp(c + d*x)*(d^3*cosh(a + b*x)^3 - 6*b^3*sinh(a + b*x)^3 - 7*b^2*d*cosh(
a + b*x)^3 + 9*b^3*cosh(a + b*x)^2*sinh(a + b*x) - 3*b*d^2*cosh(a + b*x)^2
*sinh(a + b*x) + 6*b^2*d*cosh(a + b*x)*sinh(a + b*x)^2))/(9*b^4 + d^4 - 10
*b^2*d^2)
```

3.962. $\int e^{c+dx}\cosh^3(a+bx)\,dx$

3.963 $\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx$

3.963.1 Optimal result

Integrand size = 22, antiderivative size = 125

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{7e^{-2a+c-(2b-d)x}}{4(2b-d)}+\frac{e^{c+dx}}{d}+\frac{e^{2a+c+(2b+d)x}}{4(2b+d)} + \frac{2e^{-2a+c-(2b-d)x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-2+\frac{d}{b}\right),\frac{d}{2b},e^{2(a+bx)}\right)}{2b-d}$$

output
```
-7/4*exp(-2*a+c-(2*b-d)*x)/(2*b-d)+exp(d*x+c)/d+1/4*exp(2*a+c+(2*b+d)*x)/(
2*b+d)+2*exp(-2*a+c-(2*b-d)*x)*hypergeom([1, -1+1/2*d/b],[1/2*d/b],exp(2*b
*x+2*a))/(2*b-d)
```

3.963.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.38

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{e^{c-\frac{ad}{b}}\left(2(4b^2-d^2)\,e^{d\left(\frac{a}{b}+x\right)}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)+2(2b-d)de^{\left(2+\frac{d}{b}\right)(a+bx)}\,\text{Hyperg}\ldots\right.}{8b^2d-2\ldots}$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
-((E^(c - (a*d)/b)*(2*(4*b^2 - d^2)*E^(d*(a/b + x))*Hypergeometric2F1[1, d
/(2*b), 1 + d/(2*b), E^(2*(a + b*x))] + 2*(2*b - d)*d*E^((2 + d/b)*(a + b*
x))*Hypergeometric2F1[1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))] + d*E^
(d*(a/b + x))*(-2*b*Cosh[2*(a + b*x)] + d*Sinh[2*(a + b*x)])))/(8*b^2*d -
2*d^3))
```

3.963.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx} \cosh^2(a+bx) \coth(a+bx) \, dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{7}{4} e^{-2a-x(2b-d)+c} + e^{2(a+bx)-2a-x(2b-d)+c} + \frac{1}{4} e^{4(a+bx)-2a-x(2b-d)+c} + \frac{2e^{-2a-x(2b-d)+c}}{e^{2(a+bx)}-1}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e^{-2a-x(2b-d)+c} \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(\frac{d}{b}-2\right), \frac{d}{2b}, e^{2(a+bx)}\right)}{2b-d} - \frac{7e^{-2a-x(2b-d)+c}}{4(2b-d)} + \frac{e^{2(a+bx)-2a-x(2b-d)+c}}{d} + \frac{e^{4(a+bx)-2a-x(2b-d)+c}}{4(2b+d)}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(-7*E^(-2*a + c - (2*b - d)*x))/(4*(2*b - d)) + E^(-2*a + c - (2*b - d)*x
+ 2*(a + b*x))/d + E^(-2*a + c - (2*b - d)*x + 4*(a + b*x))/(4*(2*b + d))
+ (2*E^(-2*a + c - (2*b - d)*x)*Hypergeometric2F1[1, (-2 + d/b)/2, d/(2*b)
, E^(2*(a + b*x))])/(2*b - d)
```

3.963.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

3.963.4 Maple [F]

$$\int \mathrm{e}^{dx+c} \cosh{(bx+a)}^3 \operatorname{csch}{(bx+a)}\, dx$$

input
```
int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x)
```

output
```
int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x)
```

3.963.5 Fricas [F]

$$\int e^{c+dx} \cosh^2(a+bx) \coth(a+bx)\, dx = \int \cosh{(bx+a)}^3 \operatorname{csch}{(bx+a)}\, e^{(dx+c)}\, dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^3*csch(b*x + a)*e^(d*x + c), x)
```

3.963.6 Sympy [F(-1)]

Timed out.

$$\int e^{c+dx} \cosh^2(a+bx) \coth(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**3*csch(b*x+a),x)
```

output
```
Timed out
```

3.963.7 Maxima [F]

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx=\int \cosh(bx+a)^3\operatorname{csch}(bx+a)\,e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

output
```
-4*b*integrate(e^(d*x + c)/((4*b - d)*e^(6*b*x + 6*a) - 2*(4*b - d)*e^(4*b
*x + 4*a) + (4*b - d)*e^(2*b*x + 2*a)), x) + 1/4*(24*b^2*d*e^c + 14*b*d^2*
e^c + d^3*e^c + (8*b^2*d*e^c - 6*b*d^2*e^c + d^3*e^c)*e^(6*b*x + 6*a) + (6
4*b^3*e^c - 24*b^2*d*e^c - 10*b*d^2*e^c + 3*d^3*e^c)*e^(4*b*x + 4*a) - (64
*b^3*e^c + 40*b^2*d*e^c - 2*b*d^2*e^c - 3*d^3*e^c)*e^(2*b*x + 2*a))*e^(d*x
)/((16*b^3*d - 4*b^2*d^2 - 4*b*d^3 + d^4)*e^(4*b*x + 4*a) - (16*b^3*d - 4*
b^2*d^2 - 4*b*d^3 + d^4)*e^(2*b*x + 2*a))
```

3.963.8 Giac [F]

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx=\int \cosh(bx+a)^3\operatorname{csch}(bx+a)\,e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3*csch(b*x + a)*e^(d*x + c), x)
```

3.963.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx=\int \frac{\cosh(a+b\,x)^3\,e^{c+d\,x}}{\sinh(a+b\,x)}\,dx$$

input
```
int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x),x)
```

output
```
int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x), x)
```

3.964 $\int e^{c+dx} \cosh(a+bx) \coth^2(a+bx) \, dx$

3.964.1 Optimal result

Integrand size = 22, antiderivative size = 160

$$\int e^{c+dx} \cosh(a+bx) \coth^2(a+bx) \, dx = -\frac{5e^{-a+c-(b-d)x}}{2(b-d)} + \frac{e^{a+c+(b+d)x}}{2(b+d)} + \frac{6e^{-a+c-(b-d)x} \text{Hypergeometric2F1}\left(1,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d} - \frac{4e^{-a+c-(b-d)x} \text{Hypergeometric2F1}\left(2,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d}$$

output
```
-5/2*exp(-a+c-(b-d)*x)/(b-d)+1/2*exp(a+c+(b+d)*x)/(b+d)+6*exp(-a+c-(b-d)*x
)*hypergeom([1, 1/2*(-b+d)/b],[1/2*(b+d)/b],exp(2*b*x+2*a))/(b-d)-4*exp(-a
+c-(b-d)*x)*hypergeom([2, 1/2*(-b+d)/b],[1/2*(b+d)/b],exp(2*b*x+2*a))/(b-d
)
```

3.964.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int e^{c+dx} \cosh(a+bx) \coth^2(a+bx) \, dx = \frac{e^{c-\frac{ad}{b}} \text{csch}(a+bx) \left(-4(b-d)de^{\frac{(b+d)(a+bx)}{b}} \text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right) \sinh(a+bx) + e^{d\left(\frac{a}{b}+\right.}\right.}{2b(b-d)(b+d)}$$

input
```
Integrate[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
(E^(c - (a*d)/b)*Csch[a + b*x]*(-4*(b - d)*d*E^(((b + d)*(a + b*x))/b)*Hyp
ergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))]*Sinh[a
+ b*x] + E^(d*(a/b + x))*(-3*b^2 + 2*d^2 + b^2*Cosh[2*(a + b*x)] - b*d*Sin
h[2*(a + b*x)])))/(2*b*(b - d)*(b + d))
```

3.964.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{5}{2}e^{-a-x(b-d)+c}+\frac{1}{2}e^{2(a+bx)-a-x(b-d)+c}+\frac{6e^{-a-x(b-d)+c}}{e^{2(a+bx)}-1}+\frac{4e^{-a-x(b-d)+c}}{\left(e^{2(a+bx)}-1\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6e^{-a-x(b-d)+c}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d}-\frac{4e^{-a-x(b-d)+c}\,\text{Hypergeometric2F1}\left(2,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d}-\frac{5e^{-a-x(b-d)+c}}{2(b-d)}+\frac{e^{2(a+bx)-a-x(b-d)+c}}{2(b+d)}$$

input
```
Int[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
(-5*E^(-a + c - (b - d)*x))/(2*(b - d)) + E^(-a + c - (b - d)*x + 2*(a + b
*x))/(2*(b + d)) + (6*E^(-a + c - (b - d)*x)*Hypergeometric2F1[1, -1/2*(b
- d)/b, (b + d)/(2*b), E^(2*(a + b*x))])/(b - d) - (4*E^(-a + c - (b - d)*
x)*Hypergeometric2F1[2, -1/2*(b - d)/b, (b + d)/(2*b), E^(2*(a + b*x))])/(
b - d)
```

3.964.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

3.964.4 Maple [F]

$$\int e^{dx+c} \cosh (bx+a)^3 \operatorname{csch} (bx+a)^2 \, dx$$

input
```
int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

output
```
int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

3.964.5 Fricas [F]

$$\int e^{c+dx} \cosh(a+bx) \coth^2(a+bx) \, dx = \int \cosh (bx+a)^3 \operatorname{csch} (bx+a)^2 e^{(dx+c)} \, dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^3*csch(b*x + a)^2*e^(d*x + c), x)
```

3.964.6 Sympy [F(-1)]

Timed out.

$$\int e^{c+dx} \cosh(a+bx) \coth^2(a+bx) \, dx = \text{Timed out}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

output
```
Timed out
```

3.964.7 Maxima [F]

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx = \int \cosh(bx+a)^3\operatorname{csch}(bx+a)^2 e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
16*b*d*integrate(e^(d*x + c)/((15*b^2 - 8*b*d + d^2)*e^(7*b*x + 7*a) - 3*(
15*b^2 - 8*b*d + d^2)*e^(5*b*x + 5*a) + 3*(15*b^2 - 8*b*d + d^2)*e^(3*b*x
+ 3*a) - (15*b^2 - 8*b*d + d^2)*e^(b*x + a)), x) - 1/2*(15*b^3*e^c + 39*b^
2*d*e^c + 25*b*d^2*e^c + d^3*e^c - (15*b^3*e^c - 23*b^2*d*e^c + 9*b*d^2*e^
c - d^3*e^c)*e^(6*b*x + 6*a) + (105*b^3*e^c - 11*b^2*d*e^c - 17*b*d^2*e^c
+ 3*d^3*e^c)*e^(4*b*x + 4*a) - (105*b^3*e^c + 59*b^2*d*e^c - b*d^2*e^c - 3
*d^3*e^c)*e^(2*b*x + 2*a))*e^(d*x)/((15*b^4 - 8*b^3*d - 14*b^2*d^2 + 8*b*d
^3 - d^4)*e^(5*b*x + 5*a) - 2*(15*b^4 - 8*b^3*d - 14*b^2*d^2 + 8*b*d^3 - d
^4)*e^(3*b*x + 3*a) + (15*b^4 - 8*b^3*d - 14*b^2*d^2 + 8*b*d^3 - d^4)*e^(b
*x + a))
```

3.964.8 Giac [F]

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx = \int \cosh(bx+a)^3\operatorname{csch}(bx+a)^2 e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3*csch(b*x + a)^2*e^(d*x + c), x)
```

3.964.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx = \int \frac{\cosh(a+b\,x)^3\,e^{c+d\,x}}{\sinh(a+b\,x)^2}\,dx$$

input
```
int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x)^2,x)
```

output
```
int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x)^2, x)
```

3.965 $\int e^{c+dx}\coth^3(a+bx)\,dx$

3.965.1 Optimal result

Integrand size = 16, antiderivative size = 135

$$\int e^{c+dx}\coth^3(a+bx)\,dx = \frac{e^{c+dx}}{d} - \frac{6e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d} + \frac{12e^{c+dx}\,\text{Hypergeometric2F1}\left(2,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d} - \frac{8e^{c+dx}\,\text{Hypergeometric2F1}\left(3,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}$$

output
```
exp(d*x+c)/d-6*exp(d*x+c)*hypergeom([1, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a
))/d+12*exp(d*x+c)*hypergeom([2, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a))/d-8*
exp(d*x+c)*hypergeom([3, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a))/d
```

3.965.2 Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.30

$$\int e^{c+dx}\coth^3(a+bx)\,dx=\frac{1}{2}e^c\left(\frac{2e^{dx}\coth(a)}{d}-\frac{e^{dx}\text{csch}^2(a+bx)}{b}-\frac{2(2b^2+d^2)\,e^{2a}\left(\frac{e^{dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}-\frac{e^{(2b+d)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{d}{2b},2+\frac{d}{2b},e^{2(a+bx)}\right)}{2b+d}\right)}{b^2\left(-1+e^{2a}\right)}+\frac{de^{dx}\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b^2}\right)$$

input
```
Integrate[E^(c + d*x)*Coth[a + b*x]^3,x]
```

output
```
(E^c*((2*E^(d*x)*Coth[a])/d - (E^(d*x)*Csch[a + b*x]^2)/b - (2*(2*b^2 + d^
2)*E^(2*a)*((E^(d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E^(2*(a +
b*x))])/d - (E^((2*b + d)*x)*Hypergeometric2F1[1, 1 + d/(2*b), 2 + d/(2*b)
, E^(2*(a + b*x))])/(2*b + d)))/(b^2*(-1 + E^(2*a))) + (d*E^(d*x)*Csch[a]*
Csch[a + b*x]*Sinh[b*x])/b^2))/2
```

3.965.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{6008}$$

$$\int \left(\frac{6e^{c+dx}}{e^{2(a+bx)}-1}+\frac{12e^{c+dx}}{\left(e^{2(a+bx)}-1\right)^2}+\frac{8e^{c+dx}}{\left(e^{2(a+bx)}-1\right)^3}+e^{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},\frac{d}{2b}+1,e^{2(a+bx)}\right)}{d}+\frac{12e^{c+dx}\,\text{Hypergeometric2F1}\left(2,\frac{d}{2b},\frac{d}{2b}+1,e^{2(a+bx)}\right)}{d}-\frac{8e^{c+dx}\,\text{Hypergeometric2F1}\left(3,\frac{d}{2b},\frac{d}{2b}+1,e^{2(a+bx)}\right)}{d}+\frac{e^{c+dx}}{d}$$

input
```
Int[E^(c + d*x)*Coth[a + b*x]^3,x]
```

output
```
E^(c + d*x)/d - (6*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b),
E^(2*(a + b*x))])/d + (12*E^(c + d*x)*Hypergeometric2F1[2, d/(2*b), 1 + d/
(2*b), E^(2*(a + b*x))])/d - (8*E^(c + d*x)*Hypergeometric2F1[3, d/(2*b),
1 + d/(2*b), E^(2*(a + b*x))])/d
```

3.965.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6008
```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.965.4 Maple [F]

$$\int \mathrm{e}^{dx+c} \cosh\left(bx+a\right)^{3} \operatorname{csch}\left(bx+a\right)^{3} dx$$

input
```
int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

output
```
int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

3.965.5 Fricas [F]

$$\int e^{c+dx}\coth^3(a+bx)\,dx=\int \cosh{(bx+a)}^3\operatorname{csch}{(bx+a)}^3e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^3*csch(b*x + a)^3*e^(d*x + c), x)
```

3.965.6 Sympy [F(-1)]

Timed out.

$$\int e^{c+dx}\coth^3(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

output
```
Timed out
```

3.965.7 Maxima [F]

$$\int e^{c+dx}\coth^3(a+bx)\,dx=\int \cosh{(bx+a)}^3\operatorname{csch}{(bx+a)}^3e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-48*(2*b^3*e^c + b*d^2*e^c)*integrate(e^(d*x)/(48*b^3 - 44*b^2*d + 12*b*d^
2 - d^3 + (48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(8*b*x + 8*a) - 4*(48*b^3
 - 44*b^2*d + 12*b*d^2 - d^3)*e^(6*b*x + 6*a) + 6*(48*b^3 - 44*b^2*d + 12*
b*d^2 - d^3)*e^(4*b*x + 4*a) - 4*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(2
*b*x + 2*a)), x) + (48*b^3*e^c + 44*b^2*d*e^c + 36*b*d^2*e^c + d^3*e^c - (
48*b^3*e^c - 44*b^2*d*e^c + 12*b*d^2*e^c - d^3*e^c)*e^(6*b*x + 6*a) + 3*(4
8*b^3*e^c + 4*b^2*d*e^c - 8*b*d^2*e^c + d^3*e^c)*e^(4*b*x + 4*a) - 3*(48*b
^3*e^c + 28*b^2*d*e^c - d^3*e^c)*e^(2*b*x + 2*a))*e^(d*x)/(48*b^3*d - 44*b
^2*d^2 + 12*b*d^3 - d^4 - (48*b^3*d - 44*b^2*d^2 + 12*b*d^3 - d^4)*e^(6*b*
x + 6*a) + 3*(48*b^3*d - 44*b^2*d^2 + 12*b*d^3 - d^4)*e^(4*b*x + 4*a) - 3*
(48*b^3*d - 44*b^2*d^2 + 12*b*d^3 - d^4)*e^(2*b*x + 2*a))
```

3.965.8 Giac [F]

$$\int e^{c+dx} \coth^3(a+bx)\,dx = \int \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)^3 e^{(dx+c)}\,dx$$

input
```
integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3*csch(b*x + a)^3*e^(d*x + c), x)
```

3.965.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+dx} \coth^3(a+bx)\,dx = \int \frac{\cosh(a+b\,x)^3\,\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x)^3,x)
```

output
```
int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x)^3, x)
```

3.966 $\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)\,dx$

3.966.1 Optimal result

Integrand size = 56, antiderivative size = 73

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)\,dx = -\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2}+\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2}$$

output
```
4*b*exp(b*x+a)*sinh(d*x+c)^(3/2)/(4*b^2-9*d^2)-6*d*exp(b*x+a)*cosh(d*x+c)*
sinh(d*x+c)^(1/2)/(4*b^2-9*d^2)
```

3.966.2 Mathematica [A] (verified)

Time = 4.80 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)\,dx = \frac{2e^{a+bx}\sqrt{\sinh(c+dx)}(-3d\cosh(c+dx)+2b\sinh(c+dx))}{4b^2-9d^2}$$

input
```
Integrate[(-3*d^2*E^(a + b*x))/(4*(b^2 - (9*d^2)/4)*Sqrt[Sinh[c + d*x]]) +
 E^(a + b*x)*Sinh[c + d*x]^(3/2),x]
```

output
```
(2*E^(a + b*x)*Sqrt[Sinh[c + d*x]]*(-3*d*Cosh[c + d*x] + 2*b*Sinh[c + d*x]
))/(4*b^2 - 9*d^2)
```

3.966.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2}-\frac{6de^{a+bx}\sqrt{\sinh(c+dx)}\cosh(c+dx)}{4b^2-9d^2}$$

input
```
Int[(-3*d^2*E^(a + b*x))/(4*(b^2 - (9*d^2)/4)*Sqrt[Sinh[c + d*x]]) + E^(a
+ b*x)*Sinh[c + d*x]^(3/2),x]
```

output
```
(-6*d*E^(a + b*x)*Cosh[c + d*x]*Sqrt[Sinh[c + d*x]])/(4*b^2 - 9*d^2) + (4*
b*E^(a + b*x)*Sinh[c + d*x]^(3/2))/(4*b^2 - 9*d^2)
```

3.966.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.966.4 Maple [F]

$$\int \left(e^{bx+a}\sinh(dx+c)^{\frac{3}{2}}-\frac{3d^2e^{bx+a}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(dx+c)}}\right)dx$$

input
```
int(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x
+c)^(1/2),x)
```

output
```
int(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x
+c)^(1/2),x)
```

3.966.5 Fricas [F(-2)]

Exception generated.

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/si
nh(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.966.6 Sympy [F]

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)\,dx$$
$$=\frac{\left(\int 4b^2e^{bx}\sinh^{\frac{3}{2}}\left(c+dx\right)dx+\int\left(-\frac{3d^2e^{bx}}{\sqrt{\sinh\left(c+dx\right)}}\right)dx+\int\left(-9d^2e^{bx}\sinh^{\frac{3}{2}}\left(c+dx\right)\right)dx\right)e^a}{\left(2b-3d\right)\left(2b+3d\right)}$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c)**(3/2)-3/4*d**2*exp(b*x+a)/(b**2-9/4*d**2
)/sinh(d*x+c)**(1/2),x)
```

output
```
(Integral(4*b**2*exp(b*x)*sinh(c + d*x)**(3/2), x) + Integral(-3*d**2*exp(
b*x)/sqrt(sinh(c + d*x)), x) + Integral(-9*d**2*exp(b*x)*sinh(c + d*x)**(3
/2), x))*exp(a)/((2*b - 3*d)*(2*b + 3*d))
```

3.966.7 Maxima [F]

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$$
$$=\int e^{(bx+a)}\sinh\left(dx+c\right)^{\frac{3}{2}}-\frac{3\,d^2e^{(bx+a)}}{(4\,b^2-9\,d^2)\sqrt{\sinh\left(dx+c\right)}}\,dx$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/si
nh(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(e^(b*x + a)*sinh(d*x + c)^(3/2) - 3*d^2*e^(b*x + a)/((4*b^2 - 9*
d^2)*sqrt(sinh(d*x + c))), x)
```

3.966.8 Giac [F]

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$$
$$=\int e^{(bx+a)}\sinh\left(dx+c\right)^{\frac{3}{2}}-\frac{3\,d^2e^{(bx+a)}}{(4\,b^2-9\,d^2)\sqrt{\sinh\left(dx+c\right)}}\,dx$$

input
```
integrate(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/si
nh(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(e^(b*x + a)*sinh(d*x + c)^(3/2) - 3*d^2*e^(b*x + a)/((4*b^2 - 9*
d^2)*sqrt(sinh(d*x + c))), x)
```

3.966.9 Mupad [F(-1)]

Timed out.

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$$
$$=\int \mathrm{e}^{a+b\,x}\,\sinh(c+d\,x)^{3/2}-\frac{3\,d^2\,\mathrm{e}^{a+b\,x}}{4\,\sqrt{\sinh\left(c+d\,x\right)}\,\left(b^2-\frac{9\,d^2}{4}\right)}\,dx$$

input
```
int(exp(a + b*x)*sinh(c + d*x)^(3/2) - (3*d^2*exp(a + b*x))/(4*sinh(c + d*
x)^(1/2)*(b^2 - (9*d^2)/4)),x)
```

output
```
int(exp(a + b*x)*sinh(c + d*x)^(3/2) - (3*d^2*exp(a + b*x))/(4*sinh(c + d*
x)^(1/2)*(b^2 - (9*d^2)/4)), x)
```

3.966. $\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)\,dx$

3.967 $\int e^{n \cosh(a+bx)} \sinh(a+bx)\,dx$

3.967.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int e^{n \cosh(a+bx)} \sinh(a+bx)\,dx = \frac{e^{n \cosh(a+bx)}}{bn}$$

output
```
exp(n*cosh(b*x+a))/b/n
```

3.967.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int e^{n \cosh(a+bx)} \sinh(a+bx)\,dx = \frac{e^{n \cosh(a+bx)}}{bn}$$

input
```
Integrate[E^(n*Cosh[a + b*x])*Sinh[a + b*x],x]
```

output
```
E^(n*Cosh[a + b*x])/(b*n)
```

3.967.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4837, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)e^{n\cosh(a+bx)}\,dx$$

$$\Big\downarrow \text{4837}$$

$$\frac{\int e^{n\cosh(a+bx)}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{e^{n\cosh(a+bx)}}{bn}$$

input
```
Int[E^(n*Cosh[a + b*x])*Sinh[a + b*x],x]
```

output
```
E^(n*Cosh[a + b*x])/(b*n)
```

3.967.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4837
```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Cosh[c*(a +
 b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
 b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.967.4 Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{e^{n\cosh(bx+a)}}{bn}$	17
default	$\frac{e^{n\cosh(bx+a)}}{bn}$	17
risch	$\frac{e^{\frac{n\left(e^{bx+a}+e^{-bx-a}\right)}{2}}}{nb}$	28

input
```
int(exp(n*cosh(b*x+a))*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
exp(n*cosh(b*x+a))/b/n
```

3.967.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx = \frac{\cosh\left(n\cosh\left(bx+a\right)\right)+\sinh\left(n\cosh\left(bx+a\right)\right)}{bn}$$

input
```
integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x, algorithm="fricas")
```

output
```
(cosh(n*cosh(b*x + a)) + sinh(n*cosh(b*x + a)))/(b*n)
```

3.967.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(12) = 24.

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx = \begin{cases} x\sinh\left(a\right) & \text{for } b=0\wedge n=0 \\ xe^{n\cosh\left(a\right)}\sinh\left(a\right) & \text{for } b=0 \\ \frac{\cosh\left(a+bx\right)}{b} & \text{for } n=0 \\ \frac{e^{n\cosh\left(a+bx\right)}}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x)
```

output
```
Piecewise((x*sinh(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*cosh(a))*sinh(a), Eq(
b, 0)), (cosh(a + b*x)/b, Eq(n, 0)), (exp(n*cosh(a + b*x))/(b*n), True))
```

3.967.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx=\frac{e^{(n\cosh(bx+a))}}{bn}$$

input
```
integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x, algorithm="maxima")
```

output
```
e^(n*cosh(b*x + a))/(b*n)
```

3.967.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx=\frac{e^{\left(\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn}$$

input
```
integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x, algorithm="giac")
```

output
```
e^(1/2*n*e^(b*x + a) + 1/2*n*e^(-b*x - a))/(b*n)
```

3.967.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx=\frac{e^{n\cosh(a+b\,x)}}{b\,n}$$

input
```
int(exp(n*cosh(a + b*x))*sinh(a + b*x),x)
```

output
```
exp(n*cosh(a + b*x))/(b*n)
```

3.968 $\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx$

3.968.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx=\frac{e^{n\cosh(c(a+bx))}}{bcn}$$

output
```
exp(n*cosh(c*(b*x+a)))/b/c/n
```

3.968.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx=\frac{e^{n\cosh(c(a+bx))}}{bcn}$$

input
```
Integrate[E^(n*Cosh[a*c + b*c*x])*Sinh[c*(a + b*x)],x]
```

output
```
E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

3.968.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4837, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c(a+bx))e^{n\cosh(ac+bcx)}dx$$

$$\Big\downarrow \text{4837}$$

$$\frac{\int e^{n\cosh(c(a+bx))}d\cosh(c(a+bx))}{bc}$$

$$\Big\downarrow \text{2624}$$

$$\frac{e^{n\cosh(c(a+bx))}}{bcn}$$

input
```
Int[E^(n*Cosh[a*c + b*c*x])*Sinh[c*(a + b*x)],x]
```

output
```
E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

3.968.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4837
```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Cosh[c*(a +
 b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
 b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.968.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{e^{n\cosh(c(bx+a))}}{bcn}$	22
derivativedivides	$\frac{e^{n\cosh(bcx+ac)}}{bcn}$	23
default	$\frac{e^{n\cosh(bcx+ac)}}{bcn}$	23
risch	$\frac{e^{\frac{n\left(e^{c(bx+a)}+e^{-c(bx+a)}\right)}{2}}}{nbc}$	33

input
```
int(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
exp(n*cosh(c*(b*x+a)))/b/c/n
```

3.968.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx = \frac{\cosh\left(n\cosh\left(bcx+ac\right)\right)+\sinh\left(n\cosh\left(bcx+ac\right)\right)}{bcn}$$

input
```
integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x, algorithm="fricas")
```

output
```
(cosh(n*cosh(b*c*x + a*c)) + sinh(n*cosh(b*c*x + a*c)))/(b*c*n)
```

3.968.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(15) = 30$.

Time = 1.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx = \begin{cases} xe^{n\cosh(ac)}\sinh(ac) & \text{for } b=0 \\ 0 & \text{for } c=0 \\ \begin{cases} x\sinh(ac) & \text{for } b=0 \\ 0 & \text{for } c=0 \\ \frac{\cosh(c(a+bx))}{bc} & \text{otherwise} \end{cases} & \text{for } n=0 \\ \frac{e^{n\cosh(ac+bcx)}}{bcn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x)
```

output
```
Piecewise((x*exp(n*cosh(a*c))*sinh(a*c), Eq(b, 0)), (0, Eq(c, 0)), (Piecew
ise((x*sinh(a*c), Eq(b, 0)), (0, Eq(c, 0)), (cosh(c*(a + b*x))/(b*c), True
)), Eq(n, 0)), (exp(n*cosh(a*c + b*c*x))/(b*c*n), True))
```

3.968.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx = \frac{e^{(n\cosh(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x, algorithm="maxima")
```

output
```
e^(n*cosh(b*c*x + a*c))/(b*c*n)
```

3.968.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx=\frac{e^{\left(\frac{1}{2}\,ne^{(bcx+ac)}+\frac{1}{2}\,ne^{(-bcx-ac)}\right)}}{bcn}$$

input
```
integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x, algorithm="giac")
```

output
```
e^(1/2*n*e^(b*c*x + a*c) + 1/2*n*e^(-b*c*x - a*c))/(b*c*n)
```

3.968.9 Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx=\frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}}{b\,c\,n}$$

input
```
int(sinh(c*(a + b*x))*exp(n*cosh(a*c + b*c*x)),x)
```

output
```
(exp((n*exp(b*c*x)*exp(a*c))/2)*exp((n*exp(-b*c*x)*exp(-a*c))/2))/(b*c*n)
```

3.969 $\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx$

3.969.1 Optimal result

Integrand size = 22, antiderivative size = 23

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\frac{e^{n\cosh(ac+bcx)}}{bcn}$$

output
```
exp(n*cosh(b*c*x+a*c))/b/c/n
```

3.969.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\frac{e^{n\cosh(c(a+bx))}}{bcn}$$

input
```
Integrate[E^(n*Cosh[c*(a + b*x)])*Sinh[a*c + b*c*x],x]
```

output
```
E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

3.969.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4837, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(ac+bcx)e^{n\cosh(c(a+bx))}\,dx$$

$$\Big\downarrow \text{ 4837}$$

$$\frac{\int e^{n\cosh(ac+bxc)}d\cosh(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{e^{n\cosh(ac+bcx)}}{bcn}$$

input
```
Int[E^(n*Cosh[c*(a + b*x)])*Sinh[a*c + b*c*x],x]
```

output
```
E^(n*Cosh[a*c + b*c*x])/(b*c*n)
```

3.969.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4837
```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Cosh[c*(a +
 b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
 b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.969.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{e^{n\cosh(c(bx+a))}}{bcn}$	22
default	$\frac{e^{n\cosh(c(bx+a))}}{bcn}$	22
parallelrisch	$\frac{e^{n\cosh(c(bx+a))}}{bcn}$	22
risch	$\frac{e^{\frac{n\left(e^{c(bx+a)}+e^{-c(bx+a)}\right)}{2}}}{nbc}$	33

input
```
int(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
exp(n*cosh(c*(b*x+a)))/b/c/n
```

3.969.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\frac{\cosh\left(n\cosh\left(bcx+ac\right)\right)+\sinh\left(n\cosh\left(bcx+ac\right)\right)}{bcn}$$

input
```
integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x, algorithm="fricas")
```

output
```
(cosh(n*cosh(b*c*x + a*c)) + sinh(n*cosh(b*c*x + a*c)))/(b*c*n)
```

3.969.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(17) = 34.

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\begin{cases}0 & \text{for } b=0\wedge c=0\wedge n=0\\ xe^{n\cosh(ac)}\sinh(ac) & \text{for } b=0\\ 0 & \text{for } c=0\\ \frac{\cosh(ac+bcx)}{bc} & \text{for } n=0\\ \frac{e^{n\cosh(ac+bcx)}}{bcn} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*exp(n*cosh(a*c))*sinh(a*
c), Eq(b, 0)), (0, Eq(c, 0)), (cosh(a*c + b*c*x)/(b*c), Eq(n, 0)), (exp(n*
cosh(a*c + b*c*x))/(b*c*n), True))
```

3.969.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\frac{e^{(n\cosh(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x, algorithm="maxima")
```

output
```
e^(n*cosh(b*c*x + a*c))/(b*c*n)
```

3.969.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\frac{e^{\left(\frac{1}{2}\,ne^{(bcx+ac)}+\frac{1}{2}\,ne^{(-bcx-ac)}\right)}}{bcn}$$

input
```
integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x, algorithm="giac")
```

output
```
e^(1/2*n*e^(b*c*x + a*c) + 1/2*n*e^(-b*c*x - a*c))/(b*c*n)
```

3.969.9 Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}}{b\,c\,n}$$

input
```
int(exp(n*cosh(c*(a + b*x)))*sinh(a*c + b*c*x),x)
```

output
```
(exp((n*exp(b*c*x)*exp(a*c))/2)*exp((n*exp(-b*c*x)*exp(-a*c))/2))/(b*c*n)
```

3.969. $\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx$

3.970 $\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx$

3.970.1 Optimal result

Integrand size = 17, antiderivative size = 13

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx=\frac{\text{ExpIntegralEi}(n\cosh(a+bx))}{b}$$

output
```
Ei(n*cosh(b*x+a))/b
```

3.970.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx=\frac{\text{ExpIntegralEi}(n\cosh(a+bx))}{b}$$

input
```
Integrate[E^(n*Cosh[a + b*x])*Tanh[a + b*x],x]
```

output
```
ExpIntegralEi[n*Cosh[a + b*x]]/b
```

3.970.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4841, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(a+bx)e^{n\cosh(a+bx)}\,dx$$

$$\Big\downarrow \text{4841}$$

$$\frac{\int e^{n\cosh(a+bx)}\text{sech}(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\text{ExpIntegralEi}(n\cosh(a+bx))}{b}$$

input
```
Int[E^(n*Cosh[a + b*x])*Tanh[a + b*x],x]
```

output
```
ExpIntegralEi[n*Cosh[a + b*x]]/b
```

3.970.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 4841
```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Cosh[c*(a
 + b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.970.4 Maple [F]

$$\int \mathrm{e}^{n\cosh(bx+a)} \tanh\left(bx+a\right) dx$$

input
```
int(exp(n*cosh(b*x+a))*tanh(b*x+a),x)
```

output
```
int(exp(n*cosh(b*x+a))*tanh(b*x+a),x)
```

3.970.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(a+bx)} \tanh(a+bx)\, dx = \frac{\mathrm{Ei}(n\cosh\left(bx+a\right))}{b}$$

input
```
integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x, algorithm="fricas")
```

output
```
Ei(n*cosh(b*x + a))/b
```

3.970.6 Sympy [F]

$$\int e^{n\cosh(a+bx)} \tanh(a+bx)\, dx = \int e^{n\cosh\left(a+bx\right)} \tanh\left(a+bx\right) dx$$

input
```
integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x)
```

output
```
Integral(exp(n*cosh(a + b*x))*tanh(a + b*x), x)
```

3.970.7 Maxima [F]

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx = \int e^{(n\cosh(bx+a))}\tanh(bx+a)\ dx$$

input
```
integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(e^(n*cosh(b*x + a))*tanh(b*x + a), x)
```

3.970.8 Giac [F]

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx = \int e^{(n\cosh(bx+a))}\tanh(bx+a)\ dx$$

input
```
integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x, algorithm="giac")
```

output
```
integrate(e^(n*cosh(b*x + a))*tanh(b*x + a), x)
```

3.970.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx = \int e^{n\cosh(a+b\,x)}\tanh(a+b\,x)\ dx$$

input
```
int(exp(n*cosh(a + b*x))*tanh(a + b*x),x)
```

output
```
int(exp(n*cosh(a + b*x))*tanh(a + b*x), x)
```

3.971 $\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx$

3.971.1 Optimal result

Integrand size = 22, antiderivative size = 18

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx=\frac{\text{ExpIntegralEi}(n\cosh(c(a+bx)))}{bc}$$

output
```
Ei(n*cosh(c*(b*x+a)))/b/c
```

3.971.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx=\frac{\text{ExpIntegralEi}(n\cosh(c(a+bx)))}{bc}$$

input
```
Integrate[E^(n*Cosh[a*c + b*c*x])*Tanh[c*(a + b*x)],x]
```

output
```
ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

3.971.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4841, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(c(a+bx))e^{n\cosh(ac+bcx)}\,dx$$

$$\Big\downarrow \text{4841}$$

$$\frac{\int e^{n\cosh(c(a+bx))}\text{sech}(c(a+bx))d\cosh(c(a+bx))}{bc}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\text{ExpIntegralEi}(n\cosh(c(a+bx)))}{bc}$$

input
```
Int[E^(n*Cosh[a*c + b*c*x])*Tanh[c*(a + b*x)],x]
```

output
```
ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

3.971.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 4841
```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Cosh[c*(a
 + b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.971.4 Maple [F]

$$\int e^{n\cosh(bcx+ac)}\tanh\left(c(bx+a)\right)dx$$

input
```
int(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x)
```

output
```
int(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x)
```

3.971.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx = \frac{\text{Ei}(n\cosh\left(bcx+ac\right))}{bc}$$

input
```
integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x, algorithm="fricas")
```

output
```
Ei(n*cosh(b*c*x + a*c))/(b*c)
```

3.971.6 Sympy [F]

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx = \int e^{n\cosh\left(ac+bcx\right)}\tanh\left(ac+bcx\right)dx$$

input
```
integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x)
```

output
```
Integral(exp(n*cosh(a*c + b*c*x))*tanh(a*c + b*c*x), x)
```

3.971.7 Maxima [F]

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx = \int e^{(n\cosh(bcx+ac))}\tanh\left((bx+a)c\right)\,dx$$

input
```
integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(e^(n*cosh(b*c*x + a*c))*tanh((b*x + a)*c), x)
```

3.971.8 Giac [F]

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx = \int e^{(n\cosh(bcx+ac))}\tanh\left((bx+a)c\right)\,dx$$

input
```
integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate(e^(n*cosh(b*c*x + a*c))*tanh((b*x + a)*c), x)
```

3.971.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx = \int \tanh(c\,(a+b\,x))\,\mathrm{e}^{n\cosh(a\,c+b\,c\,x)}\,dx$$

input
```
int(tanh(c*(a + b*x))*exp(n*cosh(a*c + b*c*x)),x)
```

output
```
int(tanh(c*(a + b*x))*exp(n*cosh(a*c + b*c*x)), x)
```

3.972 $\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx$

3.972.1 Optimal result

Integrand size = 22, antiderivative size = 19

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx=\frac{\text{ExpIntegralEi}(n\cosh(ac+bcx))}{bc}$$

output
```
Ei(n*cosh(b*c*x+a*c))/b/c
```

3.972.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx=\frac{\text{ExpIntegralEi}(n\cosh(c(a+bx)))}{bc}$$

input
```
Integrate[E^(n*Cosh[c*(a + b*x)])*Tanh[a*c + b*c*x],x]
```

output
```
ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

3.972.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4841, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(ac+bcx)e^{n\cosh(c(a+bx))}\,dx$$

$$\Big\downarrow \text{4841}$$

$$\frac{\int e^{n\cosh(ac+bxc)}\text{sech}(ac+bxc)d\cosh(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\text{ExpIntegralEi}(n\cosh(ac+bxc))}{bc}$$

input
```
Int[E^(n*Cosh[c*(a + b*x)])*Tanh[a*c + b*c*x],x]
```

output
```
ExpIntegralEi[n*Cosh[a*c + b*c*x]]/(b*c)
```

3.972.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 4841
```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Cosh[c*(a
 + b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.972.4 Maple [F]

$$\int e^{n\cosh(c(bx+a))}\tanh\left(bcx+ac\right)dx$$

input
```
int(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x)
```

output
```
int(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x)
```

3.972.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx = \frac{\mathrm{Ei}(n\cosh\left(bcx+ac\right))}{bc}$$

input
```
integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x, algorithm="fricas")
```

output
```
Ei(n*cosh(b*c*x + a*c))/(b*c)
```

3.972.6 Sympy [F]

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx = \int e^{n\cosh\left(ac+bcx\right)}\tanh\left(ac+bcx\right)dx$$

input
```
integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x)
```

output
```
Integral(exp(n*cosh(a*c + b*c*x))*tanh(a*c + b*c*x), x)
```

3.972.7 Maxima [F]

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx=\int e^{(n\cosh((bx+a)c))}\tanh\left(bcx+ac\right)\,dx$$

input
```
integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x, algorithm="maxima")
```

output
```
integrate(e^(n*cosh((b*x + a)*c))*tanh(b*c*x + a*c), x)
```

3.972.8 Giac [F]

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx=\int e^{(n\cosh((bx+a)c))}\tanh\left(bcx+ac\right)\,dx$$

input
```
integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x, algorithm="giac")
```

output
```
integrate(e^(n*cosh((b*x + a)*c))*tanh(b*c*x + a*c), x)
```

3.972.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx=\int \mathrm{e}^{n\cosh\left(c\,(a+b\,x)\right)}\tanh(a\,c+b\,c\,x)\,dx$$

input
```
int(exp(n*cosh(c*(a + b*x)))*tanh(a*c + b*c*x),x)
```

output
```
int(exp(n*cosh(c*(a + b*x)))*tanh(a*c + b*c*x), x)
```

3.973 $\int e^{n \sinh(a+bx)} \cosh(a+bx) \, dx$

3.973.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int e^{n \sinh(a+bx)} \cosh(a+bx) \, dx = \frac{e^{n \sinh(a+bx)}}{bn}$$

output
```
exp(n*sinh(b*x+a))/b/n
```

3.973.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int e^{n \sinh(a+bx)} \cosh(a+bx) \, dx = \frac{e^{n \sinh(a+bx)}}{bn}$$

input
```
Integrate[E^(n*Sinh[a + b*x])*Cosh[a + b*x],x]
```

output
```
E^(n*Sinh[a + b*x])/(b*n)
```

3.973.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4836, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)e^{n\sinh(a+bx)}\,dx$$

$$\Big\downarrow \text{ 4836}$$

$$\frac{\int e^{n\sinh(a+bx)}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{e^{n\sinh(a+bx)}}{bn}$$

input
```
Int[E^(n*Sinh[a + b*x])*Cosh[a + b*x],x]
```

output
```
E^(n*Sinh[a + b*x])/(b*n)
```

3.973.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4836
```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sinh[c*(a +
 b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
 b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.973.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{e^{n\sinh(bx+a)}}{bn}$	17
default	$\frac{e^{n\sinh(bx+a)}}{bn}$	17
risch	$\frac{e^{-\frac{n\left(-e^{bx+a}+e^{-bx-a}\right)}{2}}}{nb}$	30

input
```
int(exp(n*sinh(b*x+a))*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
exp(n*sinh(b*x+a))/b/n
```

3.973.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx = \frac{\cosh\left(n\sinh\left(bx+a\right)\right)+\sinh\left(n\sinh\left(bx+a\right)\right)}{bn}$$

input
```
integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x, algorithm="fricas")
```

output
```
(cosh(n*sinh(b*x + a)) + sinh(n*sinh(b*x + a)))/(b*n)
```

3.973.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(12) = 24.

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx = \begin{cases} x\cosh\left(a\right) & \text{for}\ b=0\wedge n=0 \\ xe^{n\sinh\left(a\right)}\cosh\left(a\right) & \text{for}\ b=0 \\ \frac{\sinh\left(a+bx\right)}{b} & \text{for}\ n=0 \\ \frac{e^{n\sinh\left(a+bx\right)}}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x)
```

output
```
Piecewise((x*cosh(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*sinh(a))*cosh(a), Eq(
b, 0)), (sinh(a + b*x)/b, Eq(n, 0)), (exp(n*sinh(a + b*x))/(b*n), True))
```

3.973.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx = \frac{e^{(n\sinh(bx+a))}}{bn}$$

input
```
integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x, algorithm="maxima")
```

output
```
e^(n*sinh(b*x + a))/(b*n)
```

3.973.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx = \frac{e^{\left(\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn}$$

input
```
integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x, algorithm="giac")
```

output
```
e^(1/2*n*e^(b*x + a) - 1/2*n*e^(-b*x - a))/(b*n)
```

3.973.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx = \frac{\mathrm{e}^{n\,\sinh(a+b\,x)}}{b\,n}$$

input
```
int(cosh(a + b*x)*exp(n*sinh(a + b*x)),x)
```

output
```
exp(n*sinh(a + b*x))/(b*n)
```

3.974 $\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx$

3.974.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx=\frac{e^{n\sinh(c(a+bx))}}{bcn}$$

output
```
exp(n*sinh(c*(b*x+a)))/b/c/n
```

3.974.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx=\frac{e^{n\sinh(c(a+bx))}}{bcn}$$

input
```
Integrate[E^(n*Sinh[a*c + b*c*x])*Cosh[c*(a + b*x)],x]
```

output
```
E^(n*Sinh[c*(a + b*x)])/(b*c*n)
```

3.974.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4836, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c(a+bx))e^{n\sinh(ac+bcx)}\,dx$$

$$\Big\downarrow \text{4836}$$

$$\frac{\int e^{n\sinh(c(a+bx))}d\sinh(c(a+bx))}{bc}$$

$$\Big\downarrow \text{2624}$$

$$\frac{e^{n\sinh(c(a+bx))}}{bcn}$$

input
```
Int[E^(n*Sinh[a*c + b*c*x])*Cosh[c*(a + b*x)],x]
```

output
```
E^(n*Sinh[c*(a + b*x)])/(b*c*n)
```

3.974.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4836
```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sinh[c*(a +
 b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
 b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.974.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{\mathrm{e}^{n \sinh(c(bx+a))}}{bcn}$	22
derivativedivides	$\frac{\mathrm{e}^{n \sinh(bcx+ac)}}{bcn}$	23
default	$\frac{\mathrm{e}^{n \sinh(bcx+ac)}}{bcn}$	23
risch	$\frac{\mathrm{e}^{-\frac{n\left(-\mathrm{e}^{c(bx+a)}+\mathrm{e}^{-c(bx+a)}\right)}{2}}}{nbc}$	35

input
```
int(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
exp(n*sinh(c*(b*x+a)))/b/c/n
```

3.974.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int e^{n \sinh(ac+bcx)} \cosh(c(a+bx))\,dx = \frac{\cosh\left(n \sinh\left(bcx+ac\right)\right)+\sinh\left(n \sinh\left(bcx+ac\right)\right)}{bcn}$$

input
```
integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x, algorithm="fricas")
```

output
```
(cosh(n*sinh(b*c*x + a*c)) + sinh(n*sinh(b*c*x + a*c)))/(b*c*n)
```

3.974.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(15) = 30$.

Time = 1.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx = \begin{cases} xe^{n\sinh{(ac)}}\cosh{(ac)} & \text{for}\ b=0 \\ x & \text{for}\ c=0 \\ \begin{cases} x\cosh{(ac)} & \text{for}\ b=0 \\ x & \text{for}\ c=0 \\ \frac{\sinh{(c(a+bx))}}{bc} & \text{otherwise} \end{cases} & \text{for}\ n=0 \\ \frac{e^{n\sinh{(ac+bcx)}}}{bcn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x)
```

output
```
Piecewise((x*exp(n*sinh(a*c))*cosh(a*c), Eq(b, 0)), (x, Eq(c, 0)), (Piecew
ise((x*cosh(a*c), Eq(b, 0)), (x, Eq(c, 0)), (sinh(c*(a + b*x))/(b*c), True
)), Eq(n, 0)), (exp(n*sinh(a*c + b*c*x))/(b*c*n), True))
```

3.974.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx = \frac{e^{(n\sinh(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x, algorithm="maxima")
```

output
```
e^(n*sinh(b*c*x + a*c))/(b*c*n)
```

3.974.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx = \frac{e^{\left(\frac{1}{2}\,ne^{(bcx+ac)}-\frac{1}{2}\,ne^{(-bcx-ac)}\right)}}{bcn}$$

input
```
integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x, algorithm="giac")
```

output
```
e^(1/2*n*e^(b*c*x + a*c) - 1/2*n*e^(-b*c*x - a*c))/(b*c*n)
```

3.974.9 Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx = \frac{e^{\frac{n\,e^{b\,c\,x}\,e^{a\,c}}{2}}\,e^{-\frac{n\,e^{-b\,c\,x}\,e^{-a\,c}}{2}}}{b\,c\,n}$$

input
```
int(cosh(c*(a + b*x))*exp(n*sinh(a*c + b*c*x)),x)
```

output
```
(exp((n*exp(b*c*x)*exp(a*c))/2)*exp(-(n*exp(-b*c*x)*exp(-a*c))/2))/(b*c*n)
```

3.975 $\int e^{n \sinh(c(a+bx))} \cosh(ac+bcx)\,dx$

3.975.1 Optimal result

Integrand size = 22, antiderivative size = 23

$$\int e^{n \sinh(c(a+bx))} \cosh(ac+bcx)\,dx = \frac{e^{n \sinh(ac+bcx)}}{bcn}$$

output
```
exp(n*sinh(b*c*x+a*c))/b/c/n
```

3.975.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n \sinh(c(a+bx))} \cosh(ac+bcx)\,dx = \frac{e^{n \sinh(c(a+bx))}}{bcn}$$

input
```
Integrate[E^(n*Sinh[c*(a + b*x)])*Cosh[a*c + b*c*x],x]
```

output
```
E^(n*Sinh[c*(a + b*x)])/(b*c*n)
```

3.975.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4836, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(ac+bcx)e^{n\sinh(c(a+bx))}dx$$

$$\Big\downarrow \text{4836}$$

$$\frac{\int e^{n\sinh(ac+bxc)}d\sinh(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2624}$$

$$\frac{e^{n\sinh(ac+bcx)}}{bcn}$$

input
```
Int[E^(n*Sinh[c*(a + b*x)])*Cosh[a*c + b*c*x],x]
```

output
```
E^(n*Sinh[a*c + b*c*x])/(b*c*n)
```

3.975.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4836
```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sinh[c*(a +
 b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
 b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.975.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{e^{n\sinh(c(bx+a))}}{bcn}$	22
default	$\frac{e^{n\sinh(c(bx+a))}}{bcn}$	22
parallelrisch	$\frac{e^{n\sinh(c(bx+a))}}{bcn}$	22
risch	$\frac{e^{-\frac{n\left(-e^{c(bx+a)}+e^{-c(bx+a)}\right)}{2}}}{nbc}$	35

input
```
int(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
exp(n*sinh(c*(b*x+a)))/b/c/n
```

3.975.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx = \frac{\cosh\left(n\sinh\left(bcx+ac\right)\right)+\sinh\left(n\sinh\left(bcx+ac\right)\right)}{bcn}$$

input
```
integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x, algorithm="fricas")
```

output
```
(cosh(n*sinh(b*c*x + a*c)) + sinh(n*sinh(b*c*x + a*c)))/(b*c*n)
```

3.975.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(17) = 34.

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx = \begin{cases} x & \text{for}\ b=0\wedge c=0\wedge n=0 \\ xe^{n\sinh(ac)}\cosh(ac) & \text{for}\ b=0 \\ x & \text{for}\ c=0 \\ \frac{\sinh(ac+bcx)}{bc} & \text{for}\ n=0 \\ \frac{e^{n\sinh(ac+bcx)}}{bcn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x)
```

output
```
Piecewise((x, Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*exp(n*sinh(a*c))*cosh(a*
c), Eq(b, 0)), (x, Eq(c, 0)), (sinh(a*c + b*c*x)/(b*c), Eq(n, 0)), (exp(n*
sinh(a*c + b*c*x))/(b*c*n), True))
```

3.975.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx = \frac{e^{(n\sinh(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x, algorithm="maxima")
```

output
```
e^(n*sinh(b*c*x + a*c))/(b*c*n)
```

3.975.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx = \frac{e^{\left(\frac{1}{2}\,ne^{(bcx+ac)}-\frac{1}{2}\,ne^{(-bcx-ac)}\right)}}{bcn}$$

input
```
integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x, algorithm="giac")
```

output
```
e^(1/2*n*e^(b*c*x + a*c) - 1/2*n*e^(-b*c*x - a*c))/(b*c*n)
```

3.975.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx = \frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}}{b\,c\,n}$$

input
```
int(exp(n*sinh(c*(a + b*x)))*cosh(a*c + b*c*x),x)
```

output
```
(exp((n*exp(b*c*x)*exp(a*c))/2)*exp(-(n*exp(-b*c*x)*exp(-a*c))/2))/(b*c*n)
```

3.975. $\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx$

3.976 $\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx$

3.976.1 Optimal result

Integrand size = 17, antiderivative size = 13

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx=\frac{\text{ExpIntegralEi}(n\sinh(a+bx))}{b}$$

output
```
Ei(n*sinh(b*x+a))/b
```

3.976.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx=\frac{\text{ExpIntegralEi}(n\sinh(a+bx))}{b}$$

input
```
Integrate[E^(n*Sinh[a + b*x])*Coth[a + b*x],x]
```

output
```
ExpIntegralEi[n*Sinh[a + b*x]]/b
```

3.976.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4840, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(a+bx)e^{n\sinh(a+bx)}\,dx$$

$$\Big\downarrow \text{4840}$$

$$\frac{\int e^{n\sinh(a+bx)}\text{csch}(a+bx)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\text{ExpIntegralEi}(n\sinh(a+bx))}{b}$$

input
```
Int[E^(n*Sinh[a + b*x])*Coth[a + b*x],x]
```

output
```
ExpIntegralEi[n*Sinh[a + b*x]]/b
```

3.976.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 4840
```
Int[Coth[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sinh[c*(a
 + b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.976.4 Maple [F]

$$\int e^{n \sinh(bx+a)} \coth(bx+a)\, dx$$

input
```
int(exp(n*sinh(b*x+a))*coth(b*x+a),x)
```

output
```
int(exp(n*sinh(b*x+a))*coth(b*x+a),x)
```

3.976.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{n \sinh(a+bx)} \coth(a+bx)\, dx = \frac{\mathrm{Ei}(n \sinh(bx+a))}{b}$$

input
```
integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x, algorithm="fricas")
```

output
```
Ei(n*sinh(b*x + a))/b
```

3.976.6 Sympy [F]

$$\int e^{n \sinh(a+bx)} \coth(a+bx)\, dx = \int e^{n \sinh(a+bx)} \coth(a+bx)\, dx$$

input
```
integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x)
```

output
```
Integral(exp(n*sinh(a + b*x))*coth(a + b*x), x)
```

3.976.7 Maxima [F]

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx = \int \coth(bx+a)\,e^{(n\sinh(bx+a))}\,dx$$

input
```
integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x, algorithm="maxima")
```

output
```
integrate(coth(b*x + a)*e^(n*sinh(b*x + a)), x)
```

3.976.8 Giac [F]

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx = \int \coth(bx+a)\,e^{(n\sinh(bx+a))}\,dx$$

input
```
integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x, algorithm="giac")
```

output
```
integrate(coth(b*x + a)*e^(n*sinh(b*x + a)), x)
```

3.976.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx = \int \coth(a+b\,x)\,\mathrm{e}^{n\sinh(a+b\,x)}\,dx$$

input
```
int(coth(a + b*x)*exp(n*sinh(a + b*x)),x)
```

output
```
int(coth(a + b*x)*exp(n*sinh(a + b*x)), x)
```

3.977 $\int e^{n \sinh(ac+bcx)} \coth(c(a+bx))\,dx$

3.977.1 Optimal result

Integrand size = 22, antiderivative size = 18

$$\int e^{n \sinh(ac+bcx)} \coth(c(a+bx))\,dx = \frac{\text{ExpIntegralEi}(n \sinh(c(a+bx)))}{bc}$$

output
```
Ei(n*sinh(c*(b*x+a)))/b/c
```

3.977.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^{n \sinh(ac+bcx)} \coth(c(a+bx))\,dx = \frac{\text{ExpIntegralEi}(n \sinh(c(a+bx)))}{bc}$$

input
```
Integrate[E^(n*Sinh[a*c + b*c*x])*Coth[c*(a + b*x)],x]
```

output
```
ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

3.977.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4840, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(c(a+bx))e^{n\sinh(ac+bcx)}\,dx$$

$$\Big\downarrow \text{4840}$$

$$\frac{\int e^{n\sinh(c(a+bx))}\text{csch}(c(a+bx))d\sinh(c(a+bx))}{bc}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\text{ExpIntegralEi}(n\sinh(c(a+bx)))}{bc}$$

input
```
Int[E^(n*Sinh[a*c + b*c*x])*Coth[c*(a + b*x)],x]
```

output
```
ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

3.977.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 4840
```
Int[Coth[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sinh[c*(a
 + b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.977.4 Maple [F]

$$\int e^{n\sinh(bcx+ac)} \coth\left(c(bx+a)\right) dx$$

input
```
int(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x)
```

output
```
int(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x)
```

3.977.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int e^{n\sinh(ac+bcx)} \coth(c(a+bx))\, dx = \frac{\text{Ei}(n\sinh\left(bcx+ac\right))}{bc}$$

input
```
integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x, algorithm="fricas")
```

output
```
Ei(n*sinh(b*c*x + a*c))/(b*c)
```

3.977.6 Sympy [F]

$$\int e^{n\sinh(ac+bcx)} \coth(c(a+bx))\, dx = \int e^{n\sinh\left(ac+bcx\right)} \coth\left(ac+bcx\right) dx$$

input
```
integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x)
```

output
```
Integral(exp(n*sinh(a*c + b*c*x))*coth(a*c + b*c*x), x)
```

3.977.7 Maxima [F]

$$\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx = \int \coth\left((bx+a)c\right)e^{(n\sinh(bcx+ac))}\,dx$$

input
```
integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(coth((b*x + a)*c)*e^(n*sinh(b*c*x + a*c)), x)
```

3.977.8 Giac [F]

$$\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx = \int \coth\left((bx+a)c\right)e^{(n\sinh(bcx+ac))}\,dx$$

input
```
integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate(coth((b*x + a)*c)*e^(n*sinh(b*c*x + a*c)), x)
```

3.977.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx = \int \coth(c\,(a+b\,x))\;\mathrm{e}^{n\sinh(a\,c+b\,c\,x)}\,dx$$

input
```
int(coth(c*(a + b*x))*exp(n*sinh(a*c + b*c*x)),x)
```

output
```
int(coth(c*(a + b*x))*exp(n*sinh(a*c + b*c*x)), x)
```

3.978 $\int e^{n \sinh(c(a+bx))} \coth(ac+bcx) \, dx$

3.978.1 Optimal result

Integrand size = 22, antiderivative size = 19

$$\int e^{n \sinh(c(a+bx))} \coth(ac+bcx) \, dx = \frac{\text{ExpIntegralEi}(n \sinh(ac+bcx))}{bc}$$

output
```
Ei(n*sinh(b*c*x+a*c))/b/c
```

3.978.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^{n \sinh(c(a+bx))} \coth(ac+bcx) \, dx = \frac{\text{ExpIntegralEi}(n \sinh(c(a+bx)))}{bc}$$

input
```
Integrate[E^(n*Sinh[c*(a + b*x)])*Coth[a*c + b*c*x],x]
```

output
```
ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

3.978.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4840, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(ac+bcx)e^{n\sinh(c(a+bx))}\,dx$$

$$\Big\downarrow \text{4840}$$

$$\frac{\int e^{n\sinh(ac+bxc)}\operatorname{csch}(ac+bxc)d\sinh(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\text{ExpIntegralEi}(n\sinh(ac+bxc))}{bc}$$

input
```
Int[E^(n*Sinh[c*(a + b*x)])*Coth[a*c + b*c*x],x]
```

output
```
ExpIntegralEi[n*Sinh[a*c + b*c*x]]/(b*c)
```

3.978.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 4840
```
Int[Coth[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sinh[c*(a
 + b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.978.4 Maple [F]

$$\int e^{n \sinh(c(bx+a))} \coth(bcx+ac)\, dx$$

input
```
int(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x)
```

output
```
int(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x)
```

3.978.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^{n \sinh(c(a+bx))} \coth(ac+bcx)\, dx = \frac{\mathrm{Ei}(n \sinh(bcx+ac))}{bc}$$

input
```
integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x, algorithm="fricas")
```

output
```
Ei(n*sinh(b*c*x + a*c))/(b*c)
```

3.978.6 Sympy [F]

$$\int e^{n \sinh(c(a+bx))} \coth(ac+bcx)\, dx = \int e^{n \sinh(ac+bcx)} \coth(ac+bcx)\, dx$$

input
```
integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x)
```

output
```
Integral(exp(n*sinh(a*c + b*c*x))*coth(a*c + b*c*x), x)
```

3.978.7 Maxima [F]

$$\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx = \int \coth(bcx+ac)\,e^{(n\sinh((bx+a)c))}\,dx$$

input
```
integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x, algorithm="maxima")
```

output
```
integrate(coth(b*c*x + a*c)*e^(n*sinh((b*x + a)*c)), x)
```

3.978.8 Giac [F]

$$\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx = \int \coth(bcx+ac)\,e^{(n\sinh((bx+a)c))}\,dx$$

input
```
integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x, algorithm="giac")
```

output
```
integrate(coth(b*c*x + a*c)*e^(n*sinh((b*x + a)*c)), x)
```

3.978.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx = \int \mathrm{e}^{n\sinh(c\,(a+b\,x))}\coth(a\,c+b\,c\,x)\,dx$$

input
```
int(exp(n*sinh(c*(a + b*x)))*coth(a*c + b*c*x),x)
```

output
```
int(exp(n*sinh(c*(a + b*x)))*coth(a*c + b*c*x), x)
```

3.979 $\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx$

3.979.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = \frac{\log(a+b\tanh(x))}{b}$$

output
```
ln(a+b*tanh(x))/b
```

3.979.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = \frac{\log(a+b\tanh(x))}{b}$$

input
```
Integrate[Sech[x]^2/(a + b*Tanh[x]),x]
```

output
```
Log[a + b*Tanh[x]]/b
```

3.979.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3987, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ix)^2}{a-ib\tan(ix)}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{1}{a+b\tanh(x)}d(b\tanh(x))}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(a+b\tanh(x))}{b}$$

input
```
Int[Sech[x]^2/(a + b*Tanh[x]),x]
```

output
```
Log[a + b*Tanh[x]]/b
```

3.979.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.979.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\tanh(x))}{b}$	12
default	$\frac{\ln(a+b\tanh(x))}{b}$	12
risch	$\frac{\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{b}-\frac{\ln(1+e^{2x})}{b}$	35

input
```
int(sech(x)^2/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*tanh(x))/b
```

3.979.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.82

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = \frac{\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

input
```
integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
```

output
```
(log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - log(2*cosh(x)/(cosh(
x) - sinh(x))))/b
```

3.979.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx$$

input
```
integrate(sech(x)**2/(a+b*tanh(x)),x)
```

output
```
Integral(sech(x)**2/(a + b*tanh(x)), x)
```

3.979.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = \frac{\log(b\tanh(x)+a)}{b}$$

input
```
integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
```

output
```
log(b*tanh(x) + a)/b
```

3.979.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 4.09

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = \frac{(a+b)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{ab+b^2} - \frac{\log\left(e^{(2x)}+1\right)}{b}$$

input
```
integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

output
```
(a + b)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a*b + b^2) - log(e^(2*x)
+ 1)/b
```

3.979.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 4.55

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = -\frac{2\operatorname{atan}\left(\frac{a\sqrt{-b^2}+a\,e^{2x}\,\sqrt{-b^2}+b\,e^{2x}\,\sqrt{-b^2}}{b^2}\right)}{\sqrt{-b^2}}$$

input
```
int(1/(cosh(x)^2*(a + b*tanh(x))),x)
```

output
```
-(2*atan((a*(-b^2)^(1/2) + a*exp(2*x)*(-b^2)^(1/2) + b*exp(2*x)*(-b^2)^(1/
2))/b^2))/(-b^2)^(1/2)
```

3.980 $\int \frac{\text{sech}^2(x)}{1+\tanh^2(x)}\,dx$

3.980.1 Optimal result

Integrand size = 13, antiderivative size = 3

$$\int \frac{\text{sech}^2(x)}{1+\tanh^2(x)}\,dx = \arctan(\tanh(x))$$

output
```
arctan(tanh(x))
```

3.980.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 9 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \frac{\text{sech}^2(x)}{1+\tanh^2(x)}\,dx = \frac{1}{2}\arctan(\sinh(2x))$$

input
```
Integrate[Sech[x]^2/(1 + Tanh[x]^2),x]
```

output
```
ArcTan[Sinh[2*x]]/2
```

3.980.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4158, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+1} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ix)^2}{1-\tan(ix)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\int \frac{1}{\tanh^2(x)+1}d\tanh(x)$$

$$\Big\downarrow \text{216}$$

$$\arctan(\tanh(x))$$

input
```
Int[Sech[x]^2/(1 + Tanh[x]^2),x]
```

output
```
ArcTan[Tanh[x]]
```

3.980.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.980.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 8.00

method	result	size
risch	$\frac{i\ln\left(\mathrm{e}^{2x}+i\right)}{2}-\frac{i\ln\left(\mathrm{e}^{2x}-i\right)}{2}$	24
default	$-\frac{\left(-2+\sqrt{2}\right)\sqrt{2}\,\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}-\frac{\sqrt{2}\left(2+\sqrt{2}\right)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}$	72

input
```
int(sech(x)^2/(1+tanh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*ln(exp(2*x)+I)-1/2*I*ln(exp(2*x)-I)
```

3.980.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(3) = 6.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 6.33

$$\int\frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx=-\arctan\left(-\frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(sech(x)^2/(1+tanh(x)^2),x, algorithm="fricas")
```

output
```
-arctan(-(cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
```

3.980.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+1}\,dx$$

input
```
integrate(sech(x)**2/(1+tanh(x)**2),x)
```

output
```
Integral(sech(x)**2/(tanh(x)**2 + 1), x)
```

3.980.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(3) = 6$.

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 11.67

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = \arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right) - \arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)$$

input
```
integrate(sech(x)^2/(1+tanh(x)^2),x, algorithm="maxima")
```

output
```
arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - arctan(-1/2*sqrt(2)*(sqrt(2) -
2*e^(-x)))
```

3.980.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = \arctan\left(e^{(2x)}\right)$$

input
```
integrate(sech(x)^2/(1+tanh(x)^2),x, algorithm="giac")
```

output
```
arctan(e^(2*x))
```

3.980.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = \operatorname{atan}\left(e^{2x}\right)$$

input
```
int(1/(cosh(x)^2*(tanh(x)^2 + 1)),x)
```

output
```
atan(exp(2*x))
```

3.981 $\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx$

3.981.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx=\frac{1}{3}\arctan\left(\frac{\tanh(x)}{3}\right)$$

output
```
1/3*arctan(1/3*tanh(x))
```

3.981.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx=-\frac{1}{3}\arctan(3\coth(x))$$

input
```
Integrate[Sech[x]^2/(9 + Tanh[x]^2),x]
```

output
```
-1/3*ArcTan[3*Coth[x]]
```

3.981.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4158, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+9} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(ix)^2}{9-\tan(ix)^2}dx$$

$$\Big\downarrow \; 4158$$

$$\int \frac{1}{\tanh^2(x)+9}d\tanh(x)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{3}\arctan\left(\frac{\tanh(x)}{3}\right)$$

input
```
Int[Sech[x]^2/(9 + Tanh[x]^2),x]
```

output
```
ArcTan[Tanh[x]/3]/3
```

3.981.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.981.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

method	result	size
risch	$\frac{i \ln\left(\mathrm{e}^{2x}+\frac{4}{5}+\frac{3i}{5}\right)}{6}-\frac{i \ln\left(\mathrm{e}^{2x}+\frac{4}{5}-\frac{3i}{5}\right)}{6}$	26
default	$-\frac{\sqrt{10}\,\left(-10+\sqrt{10}\right) \arctan\left(\frac{18 \tanh\left(\frac{x}{2}\right)}{6\sqrt{10}-6}\right)}{5\left(6\sqrt{10}-6\right)}-\frac{\left(10+\sqrt{10}\right)\sqrt{10}\, \arctan\left(\frac{18 \tanh\left(\frac{x}{2}\right)}{6\sqrt{10}+6}\right)}{5\left(6\sqrt{10}+6\right)}$	72

input
```
int(sech(x)^2/(9+tanh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*ln(exp(2*x)+4/5+3/5*I)-1/6*I*ln(exp(2*x)+4/5-3/5*I)
```

3.981.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(7) = 14.

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx = -\frac{1}{3}\arctan\left(-\frac{9\cosh(x)+\sinh(x)}{3\left(\cosh(x)-\sinh(x)\right)}\right)$$

input
```
integrate(sech(x)^2/(9+tanh(x)^2),x, algorithm="fricas")
```

output
```
-1/3*arctan(-1/3*(9*cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
```

3.981.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx=\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+9}\,dx$$

input
```
integrate(sech(x)**2/(9+tanh(x)**2),x)
```

output
```
Integral(sech(x)**2/(tanh(x)**2 + 9), x)
```

3.981.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx=-\frac{1}{3}\arctan\left(\frac{5}{3}\,e^{(-2\,x)}+\frac{4}{3}\right)$$

input
```
integrate(sech(x)^2/(9+tanh(x)^2),x, algorithm="maxima")
```

output
```
-1/3*arctan(5/3*e^(-2*x) + 4/3)
```

3.981.8 Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx=\frac{1}{3}\arctan\left(\frac{5}{3}\,e^{(2\,x)}+\frac{4}{3}\right)$$

input
```
integrate(sech(x)^2/(9+tanh(x)^2),x, algorithm="giac")
```

output
```
1/3*arctan(5/3*e^(2*x) + 4/3)
```

3.981.9 Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx=\frac{\operatorname{atan}\left(\frac{5\,e^{2\,x}}{3}+\frac{4}{3}\right)}{3}$$

input
```
int(1/(cosh(x)^2*(tanh(x)^2 + 9)),x)
```

output
```
atan((5*exp(2*x))/3 + 4/3)/3
```

3.982 $\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx$

3.982.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx=\frac{(a+b\tanh(x))^{1+n}}{b(1+n)}$$

output
```
(a+b*tanh(x))^(1+n)/b/(1+n)
```

3.982.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx=\frac{(a+b\tanh(x))^{1+n}}{b(1+n)}$$

input
```
Integrate[Sech[x]^2*(a + b*Tanh[x])^n,x]
```

output
```
(a + b*Tanh[x])^(1 + n)/(b*(1 + n))
```

3.982.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(ix)^2(a-ib\tan(ix))^ndx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int (a+b\tanh(x))^nd(b\tanh(x))}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+b\tanh(x))^{n+1}}{b(n+1)}$$

input
```
Int[Sech[x]^2*(a + b*Tanh[x])^n,x]
```

output
```
(a + b*Tanh[x])^(1 + n)/(b*(1 + n))
```

3.982.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.982.4 Maple [A] (verified)

Time = 30.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{(a+b\tanh(x))^{1+n}}{b(1+n)}$
default	$\frac{(a+b\tanh(x))^{1+n}}{b(1+n)}$
risch	$\frac{(e^{2x}a+e^{2x}b+a-b)\left((1+e^{2x})a+(e^{2x}-1)b\right)^{n}(1+e^{2x})^{-n}e^{-\frac{i\pi\ \mathrm{csgn}\left(\frac{i\left((1+e^{2x})a+(e^{2x}-1)b\right)}{1+e^{2x}}\right)n\left(-\mathrm{csgn}\left(\frac{i\left((1+e^{2x})a+(e^{2x}\right.}{1+e^{2x}}\right.\right.}{}}}{b(1+n)(1+e^{2x})}$

input
```
int(sech(x)^2*(a+b*tanh(x))^n,x,method=_RETURNVERBOSE)
```

output
```
(a+b*tanh(x))^(1+n)/b/(1+n)
```

3.982.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.63

$$\int \mathrm{sech}^2(x)(a+b\tanh(x))^n\,dx$$
$$=\frac{(a\cosh(x)+b\sinh(x))\cosh\left(n\log\left(\frac{a\cosh(x)+b\sinh(x)}{\cosh(x)}\right)\right)+(a\cosh(x)+b\sinh(x))\sinh\left(n\log\left(\frac{a\cosh(x)+b\sinh(x)}{\cosh(x)}\right.\right.}{(bn+b)\cosh(x)}$$

input
```
integrate(sech(x)^2*(a+b*tanh(x))^n,x, algorithm="fricas")
```

output
```
((a*cosh(x) + b*sinh(x))*cosh(n*log((a*cosh(x) + b*sinh(x))/cosh(x))) + (a
*cosh(x) + b*sinh(x))*sinh(n*log((a*cosh(x) + b*sinh(x))/cosh(x))))/((b*n
+ b)*cosh(x))
```

3.982.6 Sympy [F]

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx = \int (a+b\tanh{(x)})^n \operatorname{sech}^2{(x)}\,dx$$

input
```
integrate(sech(x)**2*(a+b*tanh(x))**n,x)
```

output
```
Integral((a + b*tanh(x))**n*sech(x)**2, x)
```

3.982.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx = \frac{(b\tanh{(x)}+a)^{n+1}}{b(n+1)}$$

input
```
integrate(sech(x)^2*(a+b*tanh(x))^n,x, algorithm="maxima")
```

output
```
(b*tanh(x) + a)^(n + 1)/(b*(n + 1))
```

3.982.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx = \frac{\left(\frac{ae^{(2x)}+be^{(2x)}+a-b}{e^{(2x)}+1}\right)^{n+1}}{b(n+1)}$$

input
```
integrate(sech(x)^2*(a+b*tanh(x))^n,x, algorithm="giac")
```

output
```
((a*e^(2*x) + b*e^(2*x) + a - b)/(e^(2*x) + 1))^(n + 1)/(b*(n + 1))
```

3.982.9 Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.84

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx=\frac{\left(a+\frac{b\,\left(\mathrm{e}^{2\,x}-1\right)}{\mathrm{e}^{2\,x}+1}\right)^n\,\left(a-b+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{b\,\left(\mathrm{e}^{2\,x}+1\right)\,\left(n+1\right)}$$

input
```
int((a + b*tanh(x))^n/cosh(x)^2,x)
```

output
```
((a + (b*(exp(2*x) - 1))/(exp(2*x) + 1))^n*(a - b + a*exp(2*x) + b*exp(2*x
)))/(b*(exp(2*x) + 1)*(n + 1))
```

3.983 $\int \operatorname{sech}^2(x) \left(1+\frac{1}{1-\tanh^2(x)}\right) dx$

3.983.1 Optimal result

Integrand size = 17, antiderivative size = 4

$$\int \operatorname{sech}^2(x) \left(1+\frac{1}{1-\tanh^2(x)}\right) dx = x+\tanh(x)$$

output
```
x+tanh(x)
```

3.983.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11 vs. 2(4) = 8.

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.75

$$\int \operatorname{sech}^2(x) \left(1+\frac{1}{1-\tanh^2(x)}\right) dx = 2x-\operatorname{arctanh}(\tanh(x))+\tanh(x)$$

input
```
Integrate[Sech[x]^2*(1 + (1 - Tanh[x]^2)^(-1)),x]
```

output
```
2*x - ArcTanh[Tanh[x]] + Tanh[x]
```

3.983.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4889, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{1-\tanh^2(x)}+1\right)\operatorname{sech}^2(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(1+\frac{1}{1+\tan(ix)^2}\right)\sec(ix)^2dx$$

$$\Big\downarrow \; 4889$$

$$\int \left(\frac{1}{1-\tanh^2(x)}+1\right)d\tanh(x)$$

$$\Big\downarrow \; 2009$$

$$\operatorname{arctanh}(\tanh(x))+\tanh(x)$$

input
```
Int[Sech[x]^2*(1 + (1 - Tanh[x]^2)^(-1)),x]
```

output
```
ArcTanh[Tanh[x]] + Tanh[x]
```

3.983.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.983.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 1.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

method	result	size
risch	$x-\frac{2}{1+{\mathrm e}^{2 x}}$	13
default	$\frac{2 \tanh \left(\frac{x}{2}\right)}{1+\tanh \left(\frac{x}{2}\right)^{2}}+\ln \left(1+\tanh \left(\frac{x}{2}\right)\right)-\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)$	34

input
```
int(sech(x)^2*(1+1/(1-tanh(x)^2)),x,method=_RETURNVERBOSE)
```

output
```
x-2/(1+exp(2*x))
```

3.983.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(4) = 8$.

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \operatorname{sech}^2(x) \left(1+\frac{1}{1-\tanh^2(x)}\right) dx = \frac{(x-1)\cosh(x)+\sinh(x)}{\cosh(x)}$$

input
```
integrate(sech(x)^2*(1+1/(1-tanh(x)^2)),x, algorithm="fricas")
```

output
```
((x - 1)*cosh(x) + sinh(x))/cosh(x)
```

3.983.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(3) = 6$.

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

$$\int \operatorname{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)\,dx = -\frac{x\operatorname{sech}^2(x)}{\tanh^2(x)-1} - \frac{\tanh(x)\operatorname{sech}^2(x)}{\tanh^2(x)-1}$$

input
```
integrate(sech(x)**2*(1+1/(1-tanh(x)**2)),x)
```

output
```
-x*sech(x)**2/(tanh(x)**2 - 1) - tanh(x)*sech(x)**2/(tanh(x)**2 - 1)
```

3.983.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \operatorname{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)\,dx = x + \frac{2}{e^{(-2x)}+1}$$

input
```
integrate(sech(x)^2*(1+1/(1-tanh(x)^2)),x, algorithm="maxima")
```

output
```
x + 2/(e^(-2*x) + 1)
```

3.983.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \operatorname{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)\,dx = x - \frac{2}{e^{(2x)}+1}$$

input
```
integrate(sech(x)^2*(1+1/(1-tanh(x)^2)),x, algorithm="giac")
```

output
```
x - 2/(e^(2*x) + 1)
```

3.983.9 Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \operatorname{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)\,dx=x-\frac{2}{e^{2\,x}+1}$$

input
```
int(-(1/(tanh(x)^2 - 1) - 1)/cosh(x)^2,x)
```

output
```
x - 2/(exp(2*x) + 1)
```

3.984 $\int \frac{\text{sech}^2(x)(2-\tanh^2(x))}{1-\tanh^2(x)}\,dx$

3.984.1 Optimal result

Integrand size = 23, antiderivative size = 4

$$\int \frac{\text{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = x+\tanh(x)$$

output
```
x+tanh(x)
```

3.984.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11 vs. 2(4) = 8.

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.75

$$\int \frac{\text{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = 2x-\text{arctanh}(\tanh(x))+\tanh(x)$$

input
```
Integrate[(Sech[x]^2*(2 - Tanh[x]^2))/(1 - Tanh[x]^2),x]
```

output
```
2*x - ArcTanh[Tanh[x]] + Tanh[x]
```

3.984.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-\tanh^2(x))\operatorname{sech}^2(x)}{1-\tanh^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(2+\tan(ix)^2)\sec(ix)^2}{1+\tan(ix)^2}dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int (2-\tanh^2(x))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$x+\tanh(x)$$

input
```
Int[(Sech[x]^2*(2 - Tanh[x]^2))/(1 - Tanh[x]^2),x]
```

output
```
x + Tanh[x]
```

3.984.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

3.984.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.

Time = 0.90 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

method	result	size
risch	$x-\frac{2}{1+{\mathrm e}^{2 x}}$	13
default	$\frac{2 \tanh \left(\frac{x}{2}\right)}{1+\tanh \left(\frac{x}{2}\right)^{2}}+\ln \left(1+\tanh \left(\frac{x}{2}\right)\right)-\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)$	34

input
```
int(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x-2/(1+exp(2*x))
```

3.984.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(4) = 8.

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \frac{\operatorname{sech}^2(x) \left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\, dx = \frac{(x-1)\cosh(x)+\sinh(x)}{\cosh(x)}$$

input
```
integrate(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x, algorithm="fricas")
```

output
```
((x - 1)*cosh(x) + sinh(x))/cosh(x)
```

3.984.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(3) = 6.

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

$$\int \frac{\operatorname{sech}^2(x) \left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\, dx = -\frac{x\operatorname{sech}^2(x)}{\tanh^2(x)-1} - \frac{\tanh(x)\operatorname{sech}^2(x)}{\tanh^2(x)-1}$$

input
```
integrate(sech(x)**2*(2-tanh(x)**2)/(1-tanh(x)**2),x)
```

output
```
-x*sech(x)**2/(tanh(x)**2 - 1) - tanh(x)*sech(x)**2/(tanh(x)**2 - 1)
```

3.984.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = x + \frac{2}{e^{(-2x)}+1}$$

input
```
integrate(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x, algorithm="maxima")
```

output
```
x + 2/(e^(-2*x) + 1)
```

3.984.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = x - \frac{2}{e^{(2x)}+1}$$

input
```
integrate(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x, algorithm="giac")
```

output
```
x - 2/(e^(2*x) + 1)
```

3.984.9 Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = x - \frac{2}{\mathrm{e}^{2x}+1}$$

input
```
int((tanh(x)^2 - 2)/(cosh(x)^2*(tanh(x)^2 - 1)),x)
```

output
```
x - 2/(exp(2*x) + 1)
```

3.985 $\int \frac{\text{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx$

3.985.1 Optimal result

Integrand size = 17, antiderivative size = 5

$$\int \frac{\text{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx=\arctan(1+\tanh(x))$$

output
```
arctan(1+tanh(x))
```

3.985.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 5.40

$$\int \frac{\text{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx=-\frac{1}{2}i\log((1-i)+\tanh(x))+\frac{1}{2}i\log((1+i)+\tanh(x))$$

input
```
Integrate[Sech[x]^2/(2 + 2*Tanh[x] + Tanh[x]^2),x]
```

output
```
(-1/2*I)*Log[(1 - I) + Tanh[x]] + (I/2)*Log[(1 + I) + Tanh[x]]
```

3.985.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 4842, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+2\tanh(x)+2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(ix)^2}{-\tan(ix)^2-2i\tan(ix)+2}dx$$

$$\Big\downarrow \text{ 4842}$$

$$\int \frac{1}{\tanh^2(x)+2\tanh(x)+2}d\tanh(x)$$

$$\Big\downarrow \text{ 1082}$$

$$-\int \frac{1}{-(\tanh(x)+1)^2-1}d(\tanh(x)+1)$$

$$\Big\downarrow \text{ 217}$$

$$\arctan(\tanh(x)+1)$$

input
```
Int[Sech[x]^2/(2 + 2*Tanh[x] + Tanh[x]^2),x]
```

output
```
ArcTan[1 + Tanh[x]]
```

3.985.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.985.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 5.20

method	result	size
risch	$\frac{i \ln\left(\mathrm{e}^{2x}+\frac{1}{5}+\frac{2i}{5}\right)}{2}-\frac{i \ln\left(\mathrm{e}^{2x}+\frac{1}{5}-\frac{2i}{5}\right)}{2}$	26
default	$\frac{i \ln\left(\tanh\left(\frac{x}{2}\right)^{2}+(1-i) \tanh\left(\frac{x}{2}\right)+1\right)}{2}-\frac{i \ln\left(\tanh\left(\frac{x}{2}\right)^{2}+(1+i) \tanh\left(\frac{x}{2}\right)+1\right)}{2}$	42

input
```
int(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*ln(exp(2*x)+1/5+2/5*I)-1/2*I*ln(exp(2*x)+1/5-2/5*I)
```

3.985.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(5) = 10$.

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 4.60

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = -\arctan\left(-\frac{3\cosh(x)+2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x, algorithm="fricas")
```

output
```
-arctan(-(3*cosh(x) + 2*sinh(x))/(cosh(x) - sinh(x)))
```

3.985.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+2\tanh(x)+2}\,dx$$

input
```
integrate(sech(x)**2/(2+2*tanh(x)+tanh(x)**2),x)
```

output
```
Integral(sech(x)**2/(tanh(x)**2 + 2*tanh(x) + 2), x)
```

3.985.7 Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = \int \frac{\operatorname{sech}(x)^2}{\tanh(x)^2+2\tanh(x)+2}\,dx$$

input
```
integrate(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x, algorithm="maxima")
```

output
```
integrate(sech(x)^2/(tanh(x)^2 + 2*tanh(x) + 2), x)
```

3.985.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = \arctan\left(\frac{5}{2}\,e^{(2\,x)}+\frac{1}{2}\right)$$

input
```
integrate(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x, algorithm="giac")
```

output
```
arctan(5/2*e^(2*x) + 1/2)
```

3.985.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = \operatorname{atan}\left(\frac{5\,e^{2\,x}}{2}+\frac{1}{2}\right)$$

input
```
int(1/(cosh(x)^2*(2*tanh(x) + tanh(x)^2 + 2)),x)
```

output
```
atan((5*exp(2*x))/2 + 1/2)
```

3.986 $\int \frac{\text{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx$

3.986.1 Optimal result

Integrand size = 16, antiderivative size = 15

$$\int \frac{\text{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx=-\coth(x)-\log(\tanh(x))+\log(1+\tanh(x))$$

output
```
-coth(x)-ln(tanh(x))+ln(1+tanh(x))
```

3.986.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\text{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx=x-\coth(x)-\log(\sinh(x))$$

input
```
Integrate[Sech[x]^2/(Tanh[x]^2 + Tanh[x]^3),x]
```

output
```
x - Coth[x] - Log[Sinh[x]]
```

3.986.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^3(x)+\tanh^2(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(ix)^2}{i\tan(ix)^3-\tan(ix)^2}dx$$
$$\Big\downarrow \text{ 4842}$$
$$\int \frac{\coth^2(x)}{\tanh(x)+1}d\tanh(x)$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(\frac{1}{\tanh(x)+1}+\coth^2(x)-\coth(x)\right)d\tanh(x)$$
$$\Big\downarrow \text{ 2009}$$
$$-\coth(x)-\log(\tanh(x))+\log(\tanh(x)+1)$$

input
```
Int[Sech[x]^2/(Tanh[x]^2 + Tanh[x]^3),x]
```

output
```
-Coth[x] - Log[Tanh[x]] + Log[1 + Tanh[x]]
```

3.986.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.986.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

method	result	size
risch	$2x-\frac{2}{e^{2x}-1}-\ln\left(e^{2x}-1\right)$	24
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)+2\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)$	32

input
```
int(sech(x)^2/(tanh(x)^2+tanh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
2*x-2/(exp(2*x)-1)-ln(exp(2*x)-1)
```

3.986.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(15) = 30.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 5.13

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx = \frac{2\,x\cosh\left(x\right)^2+4\,x\cosh\left(x\right)\sinh\left(x\right)+2\,x\sinh\left(x\right)^2-\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2-1\right)\log}{\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2-1}$$

input
```
integrate(sech(x)^2/(tanh(x)^2+tanh(x)^3),x, algorithm="fricas")
```

output
```
(2*x*cosh(x)^2 + 4*x*cosh(x)*sinh(x) + 2*x*sinh(x)^2 - (cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 2*x - 2)
/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

3.986.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx=\int \frac{\operatorname{sech}^2(x)}{(\tanh(x)+1)\tanh^2(x)}\,dx$$

input
```
integrate(sech(x)**2/(tanh(x)**2+tanh(x)**3),x)
```

output
```
Integral(sech(x)**2/((tanh(x) + 1)*tanh(x)**2), x)
```

3.986.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx=\frac{2}{e^{(-2x)}-1}-\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

input
```
integrate(sech(x)^2/(tanh(x)^2+tanh(x)^3),x, algorithm="maxima")
```

output
```
2/(e^(-2*x) - 1) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

3.986.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx=2x+\frac{e^{(2x)}-3}{e^{(2x)}-1}-\log\left(\left|e^{(2x)}-1\right|\right)$$

input
```
integrate(sech(x)^2/(tanh(x)^2+tanh(x)^3),x, algorithm="giac")
```

output
```
2*x + (e^(2*x) - 3)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

3.986.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx = 2\,x-\ln\left(e^{2\,x}-1\right)-\frac{2}{e^{2\,x}-1}$$

input
```
int(1/(cosh(x)^2*(tanh(x)^2 + tanh(x)^3)),x)
```

output
```
2*x - log(exp(2*x) - 1) - 2/(exp(2*x) - 1)
```

3.987 $\int \frac{\text{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)} \, dx$

3.987.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{\text{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)} \, dx = \coth(x)+\log(1-\tanh(x))-\log(\tanh(x))$$

output
```
coth(x)+ln(1-tanh(x))-ln(tanh(x))
```

3.987.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\text{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)} \, dx = -x+\coth(x)-\log(\sinh(x))$$

input
```
Integrate[Sech[x]^2/(-Tanh[x]^2 + Tanh[x]^3),x]
```

output
```
-x + Coth[x] - Log[Sinh[x]]
```

3.987.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4842, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^3(x)-\tanh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ix)^2}{i\tan(ix)^3+\tan(ix)^2}dx$$

$$\Big\downarrow \text{4842}$$

$$\int -\frac{\coth^2(x)}{1-\tanh(x)}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\coth^2(x)}{1-\tanh(x)}d\tanh(x)$$

$$\Big\downarrow \text{54}$$

$$-\int \left(\coth^2(x)+\coth(x)+\frac{1}{1-\tanh(x)}\right)d\tanh(x)$$

$$\Big\downarrow \text{2009}$$

$$\coth(x)+\log(1-\tanh(x))-\log(\tanh(x))$$

input
```
Int[Sech[x]^2/(-Tanh[x]^2 + Tanh[x]^3),x]
```

output
```
Coth[x] + Log[1 - Tanh[x]] - Log[Tanh[x]]
```

3.987.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.987.4 Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

method	result	size
risch	$\frac{2}{{\mathrm e}^{2 x}-1}-\ln \left({\mathrm e}^{2 x}-1\right)$	21
default	$\frac{\tanh \left(\frac{x}{2}\right)}{2}+\frac{1}{2 \tanh \left(\frac{x}{2}\right)}-\ln \left(\tanh \left(\frac{x}{2}\right)\right)+2 \ln \left(\tanh \left(\frac{x}{2}\right)-1\right)$	32

input
```
int(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
2/(exp(2*x)-1)-ln(exp(2*x)-1)
```

3.987.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.53

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx = -\frac{\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)-2}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}$$

input
```
integrate(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x, algorithm="fricas")
```

output
```
-((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) -
 sinh(x))) - 2)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

3.987.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{(\tanh(x)-1)\tanh^2(x)}\,dx$$

input
```
integrate(sech(x)**2/(-tanh(x)**2+tanh(x)**3),x)
```

output
```
Integral(sech(x)**2/((tanh(x) - 1)*tanh(x)**2), x)
```

3.987.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx = -2x - \frac{2}{e^{(-2x)}-1} - \log\left(e^{(-x)}+1\right) - \log\left(e^{(-x)}-1\right)$$

input
```
integrate(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x, algorithm="maxima")
```

output
```
-2*x - 2/(e^(-2*x) - 1) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

3.987.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx = \frac{e^{(2x)}+1}{e^{(2x)}-1} - \log\left(\left|e^{(2x)}-1\right|\right)$$

input
```
integrate(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x, algorithm="giac")
```

output
```
(e^(2*x) + 1)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

3.987.9 Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx = \frac{2}{e^{2x}-1} - \ln\left(e^{2x}-1\right)$$

input
```
int(-1/(cosh(x)^2*(tanh(x)^2 - tanh(x)^3)),x)
```

output
```
2/(exp(2*x) - 1) - log(exp(2*x) - 1)
```

3.988 $\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx$

3.988.1 Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx=\frac{\arctan\left(\frac{\sqrt[3]{3}+2\ 2^{2/3}\tanh(x)}{3^{5/6}}\right)}{3\ 2^{2/3}\sqrt[6]{3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}}+\frac{\log\left(3^{2/3}+2^{2/3}\sqrt[3]{3}\tanh(x)+2\sqrt[3]{2}\tanh^2(x)\right)}{6\ 6^{2/3}}$$

output
```
1/18*arctan(1/3*(3^(1/3)+2*2^(2/3)*tanh(x))*3^(1/6))*2^(1/3)*3^(5/6)-1/18*
ln(3^(1/3)-2^(2/3)*tanh(x))*6^(1/3)+1/36*ln(3^(2/3)+2^(2/3)*3^(1/3)*tanh(x
)+2*2^(1/3)*tanh(x)^2)*6^(1/3)
```

3.988.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{3+2\ 6^{2/3}\tanh(x)}{3\sqrt{3}}\right)-2\log\left(3-6^{2/3}\tanh(x)\right)+\log\left(3+6^{2/3}\tanh(x)+2\sqrt[3]{6}\tanh^2(x)\right)}{6\ 6^{2/3}}$$

input
```
Integrate[Sech[x]^2/(3 - 4*Tanh[x]^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(3 + 2*6^(2/3)*Tanh[x])/(3*Sqrt[3])] - 2*Log[3 - 6^(2/3)
*Tanh[x]] + Log[3 + 6^(2/3)*Tanh[x] + 2*6^(1/3)*Tanh[x]^2])/(6*6^(2/3))
```

3.988.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4158, 750, 16, 27, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(ix)^2}{3-4i\tan(ix)^3}dx$$

$$\Big\downarrow \; 4158$$

$$\int \frac{1}{3-4\tanh^3(x)}d\tanh(x)$$

$$\Big\downarrow \; 750$$

$$\frac{\int \frac{2^{2/3}\left(\tanh(x)+\sqrt[3]{6}\right)}{2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}}d\tanh(x)}{3\ 3^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{3}-2^{2/3}\tanh(x)}d\tanh(x)}{3\ 3^{2/3}}$$

$$\Big\downarrow \; 16$$

$$\frac{\int \frac{2^{2/3}\left(\tanh(x)+\sqrt[3]{6}\right)}{2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}}d\tanh(x)}{3\ 3^{2/3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{2}{3}\right)^{2/3}\int \frac{\tanh(x)+\sqrt[3]{6}}{2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}}d\tanh(x)-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}}$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{3}\left(\frac{2}{3}\right)^{2/3}\left(\frac{3\sqrt[3]{3}\int \frac{1}{2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}}d\tanh(x)}{2\ 2^{2/3}}+\frac{\int \frac{\sqrt[3]{2}\left(4\tanh(x)+\sqrt[3]{6}\right)}{2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}}d\tanh(x)}{4\sqrt[3]{2}}\right)-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{2}{3}\right)^{2/3}\left(\frac{3\sqrt[3]{3}\int \frac{1}{2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}}d\tanh(x)}{2\ 2^{2/3}}+\frac{1}{4}\int \frac{4\tanh(x)+\sqrt[3]{6}}{2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}}d\tanh\right.$$
$$\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}}$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{3}\left(\frac{2}{3}\right)^{2/3}\left(\frac{1}{4}\int \frac{4\tanh(x)+\sqrt[3]{6}}{2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}}d\tanh(x)-\frac{3\int \frac{1}{-\left(\frac{2\ 2^{2/3}\tanh(x)}{\sqrt[3]{3}}+1\right)^2-3}d\left(\frac{2\ 2^{2/3}\tanh(x)}{\sqrt[3]{3}}+1\right.}{2\sqrt[3]{2}}\right.$$
$$\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}}$$

$$\Big\downarrow \ 217$$

$$\frac{1}{3}\left(\frac{2}{3}\right)^{2/3}\left(\frac{1}{4}\int \frac{4\tanh(x)+\sqrt[3]{6}}{2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}}d\tanh(x)+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\ 2^{2/3}\tanh(x)}{\sqrt[3]{3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}\right)-$$
$$\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}}$$

$$\Big\downarrow \ 1103$$

3.988. $\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx$

$$\frac{1}{3}\left(\frac{2}{3}\right)^{2/3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\ 2^{2/3}\tanh(x)}{\sqrt[3]{3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}+\frac{\log\left(2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}\right)}{4\sqrt[3]{2}}\right)-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}}$$

input
```
Int[Sech[x]^2/(3 - 4*Tanh[x]^3),x]
```

output
```
-1/3*Log[3^(1/3) - 2^(2/3)*Tanh[x]]/6^(2/3) + ((2/3)^(2/3)*((Sqrt[3]*ArcTa
n[(1 + (2*2^(2/3)*Tanh[x])/3^(1/3))/Sqrt[3]])/(2*2^(1/3)) + Log[3^(2/3) +
2^(2/3)*3^(1/3)*Tanh[x] + 2*2^(1/3)*Tanh[x]^2]/(4*2^(1/3))))/3
```

3.988.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.988. $\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.988.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.30

method	result
risch	$4\left(\sum_{\_R=\text{RootOf}\left(62208\_Z^3+1\right)}\_R\ln\left(-10368\_R^2+\mathrm{e}^{2x}+288\_R-7\right)\right)$
derivativedivides	$-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tanh(x)-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}}{4}\right)}{36}+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tanh(x)^2+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\tanh(x)}{4}+\frac{3^{\frac{2}{3}}4^{\frac{1}{3}}}{4}\right)}{72}+\frac{3^{\frac{5}{6}}4^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}4^{\frac{1}{3}}\tanh(x)}{3}\right.}{3}\right.}{36}$
default	$-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tanh(x)-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}}{4}\right)}{36}+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tanh(x)^2+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\tanh(x)}{4}+\frac{3^{\frac{2}{3}}4^{\frac{1}{3}}}{4}\right)}{72}+\frac{3^{\frac{5}{6}}4^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}4^{\frac{1}{3}}\tanh(x)}{3}\right.}{3}\right.}{36}$

input
```
int(sech(x)^2/(3-4*tanh(x)^3),x,method=_RETURNVERBOSE)
```

3.988. $\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx$

output
```
4*sum(_R*ln(-10368*_R^2+exp(2*x)+288*_R-7),_R=RootOf(62208*_Z^3+1))
```

3.988.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.03

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx = -\frac{1}{18}\cdot 36^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{54}\right.$$

$$\cdot 36^{\frac{1}{6}}\left(\left(36^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}+3\cdot 36^{\frac{1}{3}}\sqrt{3}-9\sqrt{3}(-1)^{\frac{1}{3}}\right)\cosh(x)^2+2\left(36^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}+3\cdot 36^{\frac{1}{3}}\sqrt{3}-9\sqrt{3}(-1)^{\frac{1}{3}}\right)\right.$$

$$-\frac{1}{216}$$

$$\cdot 36^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{2\left(\left(36^{\frac{2}{3}}(-1)^{\frac{1}{3}}+3\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}+3\right)\cosh(x)^2-2\left(36^{\frac{2}{3}}(-1)^{\frac{1}{3}}+3\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}\right)\cosh(x)\sin\right.}{\cosh(x)^2-2\cosh(x)}\right.$$

$$+\frac{1}{108}$$

$$\cdot 36^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{2\left(\left(36^{\frac{2}{3}}(-1)^{\frac{1}{3}}-3\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}-9\right)\cosh(x)-\left(36^{\frac{2}{3}}(-1)^{\frac{1}{3}}-3\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}-12\right)\sinh(x\right.}{\cosh(x)-\sinh(x)}\right.$$

input
```
integrate(sech(x)^2/(3-4*tanh(x)^3),x, algorithm="fricas")
```

output
```
-1/18*36^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/54*36^(1/6)*((36^(2/3)*sqrt(3)*
(-1)^(2/3) + 3*36^(1/3)*sqrt(3) - 9*sqrt(3)*(-1)^(1/3))*cosh(x)^2 + 2*(36^
(2/3)*sqrt(3)*(-1)^(2/3) + 3*36^(1/3)*sqrt(3) - 9*sqrt(3)*(-1)^(1/3))*cosh
(x)*sinh(x) + (36^(2/3)*sqrt(3)*(-1)^(2/3) + 3*36^(1/3)*sqrt(3) - 9*sqrt(3
)*(-1)^(1/3))*sinh(x)^2 - 36^(2/3)*sqrt(3)*(-1)^(2/3) - 9*sqrt(3)*(-1)^(1/
3))) - 1/216*36^(2/3)*(-1)^(1/3)*log(2*((36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*
(-1)^(2/3) + 3)*cosh(x)^2 - 2*(36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(2/3)
)*cosh(x)*sinh(x) + (36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(2/3) + 3)*sinh
(x)^2 - 36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(2/3) - 21)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 1/108*36^(2/3)*(-1)^(1/3)*log(2*((36^(2/3)*
(-1)^(1/3) - 3*36^(1/3)*(-1)^(2/3) - 9)*cosh(x) - (36^(2/3)*(-1)^(1/3) - 3
*36^(1/3)*(-1)^(2/3) - 12)*sinh(x))/(cosh(x) - sinh(x)))
```

3.988.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx = -\int \frac{\operatorname{sech}^2(x)}{4\tanh^3(x)-3}\,dx$$

input
```
integrate(sech(x)**2/(3-4*tanh(x)**3),x)
```

output
```
-Integral(sech(x)**2/(4*tanh(x)**3 - 3), x)
```

3.988.7 Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx = \int -\frac{\operatorname{sech}(x)^2}{4\tanh(x)^3-3}\,dx$$

input
```
integrate(sech(x)^2/(3-4*tanh(x)^3),x, algorithm="maxima")
```

output
```
-integrate(sech(x)^2/(4*tanh(x)^3 - 3), x)
```

3.988.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx = 0$$

input
```
integrate(sech(x)^2/(3-4*tanh(x)^3),x, algorithm="giac")
```

output
```
0
```

3.988.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.66

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx = -\frac{6^{1/3}\ln\left(\frac{6^{1/3}\left(29856\,e^{2x}-\frac{6^{1/3}\left(109440\,e^{2x}+153216\right)}{18}+672\right)}{18}-\frac{5696\,e^{2x}}{3}+\frac{4480}{3}\right)}{18} - \frac{6^{1/3}\ln\left(\frac{4480}{3}+\frac{6^{1/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(29856\,e^{2x}-\frac{6^{1/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(109440\,e^{2x}+153216\right)}{18}+672\right)}{18}-\frac{5696\,e^{2x}}{3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{18} + \frac{6^{1/3}\ln\left(\frac{4480}{3}-\frac{6^{1/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(29856\,e^{2x}+\frac{6^{1/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(109440\,e^{2x}+153216\right)}{18}+672\right)}{18}-\frac{5696\,e^{2x}}{3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{18}$$

input
```
int(-1/(cosh(x)^2*(4*tanh(x)^3 - 3)),x)
```

output
```
(6^(1/3)*log(4480/3 - (6^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(29856*exp(2*x) + (6
^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(109440*exp(2*x) + 153216))/18 + 672))/18 -
(5696*exp(2*x))/3)*((3^(1/2)*1i)/2 + 1/2))/18 - (6^(1/3)*log((6^(1/3)*((3^
(1/2)*1i)/2 - 1/2)*(29856*exp(2*x) - (6^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(1094
40*exp(2*x) + 153216))/18 + 672))/18 - (5696*exp(2*x))/3 + 4480/3)*((3^(1/
2)*1i)/2 - 1/2))/18 - (6^(1/3)*log((6^(1/3)*(29856*exp(2*x) - (6^(1/3)*(10
9440*exp(2*x) + 153216))/18 + 672))/18 - (5696*exp(2*x))/3 + 4480/3))/18
```

3.989 $\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx$

3.989.1 Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = -\frac{2\arctan\left(\sqrt{\frac{5}{39}}(1-2\tanh(x))\right)}{\sqrt{195}}$$

output
```
-2/195*arctan(1/39*195^(1/2)*(1-2*tanh(x)))*195^(1/2)
```

3.989.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = -\frac{2\arctan\left(\sqrt{\frac{5}{39}}(1-2\tanh(x))\right)}{\sqrt{195}}$$

input
```
Integrate[Sech[x]^2/(11 - 5*Tanh[x] + 5*Tanh[x]^2),x]
```

output
```
(-2*ArcTan[Sqrt[5/39]*(1 - 2*Tanh[x])])/Sqrt[195]
```

3.989.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4842, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{5\tanh^2(x)-5\tanh(x)+11}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(ix)^2}{-5\tan(ix)^2+5i\tan(ix)+11}dx$$
$$\Big\downarrow \text{4842}$$
$$\int \frac{1}{5\tanh^2(x)-5\tanh(x)+11}d\tanh(x)$$
$$\Big\downarrow \text{1083}$$
$$-2\int \frac{1}{-(10\tanh(x)-5)^2-195}d(10\tanh(x)-5)$$
$$\Big\downarrow \text{217}$$
$$\frac{2\arctan\left(\frac{10\tanh(x)-5}{\sqrt{195}}\right)}{\sqrt{195}}$$

input
```
Int[Sech[x]^2/(11 - 5*Tanh[x] + 5*Tanh[x]^2),x]
```

output
```
(2*ArcTan[(-5 + 10*Tanh[x])/Sqrt[195]])/Sqrt[195]
```

3.989.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.989.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

method	result	size
risch	$\frac{i\sqrt{195}\,\ln\left(\mathrm{e}^{2x}+\frac{i\sqrt{195}}{11}+\frac{6}{11}\right)}{195}-\frac{i\sqrt{195}\,\ln\left(\mathrm{e}^{2x}-\frac{i\sqrt{195}}{11}+\frac{6}{11}\right)}{195}$	40
default	$\frac{i\sqrt{195}\,\ln\left(11\tanh\left(\frac{x}{2}\right)^{2}+\left(-i\sqrt{195}-5\right)\tanh\left(\frac{x}{2}\right)+11\right)}{195}-\frac{i\sqrt{195}\,\ln\left(11\tanh\left(\frac{x}{2}\right)^{2}+\left(i\sqrt{195}-5\right)\tanh\left(\frac{x}{2}\right)+11\right)}{195}$	62

input
```
int(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/195*I*195^(1/2)*ln(exp(2*x)+1/11*I*195^(1/2)+6/11)-1/195*I*195^(1/2)*ln(
exp(2*x)-1/11*I*195^(1/2)+6/11)
```

3.989.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = -\frac{2}{195}\sqrt{195}\arctan\left(-\frac{17\sqrt{195}\cosh(x)+5\sqrt{195}\sinh(x)}{195(\cosh(x)-\sinh(x))}\right)$$

input
```
integrate(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x, algorithm="fricas")
```

output
```
-2/195*sqrt(195)*arctan(-1/195*(17*sqrt(195)*cosh(x) + 5*sqrt(195)*sinh(x)
)/(cosh(x) - sinh(x)))
```

3.989.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{5\tanh^2(x)-5\tanh(x)+11}\,dx$$

input
```
integrate(sech(x)**2/(11-5*tanh(x)+5*tanh(x)**2),x)
```

output
```
Integral(sech(x)**2/(5*tanh(x)**2 - 5*tanh(x) + 11), x)
```

3.989.7 Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = \int \frac{\operatorname{sech}(x)^2}{5\tanh(x)^2-5\tanh(x)+11}\,dx$$

input
```
integrate(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x, algorithm="maxima")
```

output
```
integrate(sech(x)^2/(5*tanh(x)^2 - 5*tanh(x) + 11), x)
```

3.989.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = \frac{2}{195}\sqrt{195}\arctan\left(\frac{1}{195}\sqrt{195}\left(11\,e^{(2\,x)}+6\right)\right)$$

input
```
integrate(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x, algorithm="giac")
```

output
```
2/195*sqrt(195)*arctan(1/195*sqrt(195)*(11*e^(2*x) + 6))
```

3.989.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = \frac{2\sqrt{195}\,\mathrm{atan}\left(\frac{\sqrt{195}\,\left(11\,\mathrm{e}^{2\,x}+6\right)}{195}\right)}{195}$$

input
```
int(1/(cosh(x)^2*(5*tanh(x)^2 - 5*tanh(x) + 11)),x)
```

output
```
(2*195^(1/2)*atan((195^(1/2)*(11*exp(2*x) + 6))/195))/195
```

3.990 $\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx$

3.990.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx=-\frac{(bc-ad)\log(c+d\tanh(x))}{d^2}+\frac{b\tanh(x)}{d}$$

output
```
-(-a*d+b*c)*ln(c+d*tanh(x))/d^2+b*tanh(x)/d
```

3.990.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx=\frac{(-bc+ad)\log(c+d\tanh(x))+bd\tanh(x)}{d^2}$$

input
```
Integrate[(Sech[x]^2*(a + b*Tanh[x]))/(c + d*Tanh[x]),x]
```

output
```
((-(b*c) + a*d)*Log[c + d*Tanh[x]] + b*d*Tanh[x])/d^2
```

3.990.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(ix)^2(a-ib\tan(ix))}{c-id\tan(ix)}dx$$

$$\Big\downarrow \text{ 4842}$$

$$\int \frac{a+b\tanh(x)}{c+d\tanh(x)}d\tanh(x)$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{ad-bc}{d(c+d\tanh(x))}+\frac{b}{d}\right)d\tanh(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\tanh(x)}{d}-\frac{(bc-ad)\log(c+d\tanh(x))}{d^2}$$

input
```
Int[(Sech[x]^2*(a + b*Tanh[x]))/(c + d*Tanh[x]),x]
```

output
```
-(((b*c - a*d)*Log[c + d*Tanh[x]])/d^2) + (b*Tanh[x])/d
```

3.990.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.990.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(28) = 56$.

Time = 1.82 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

method	result	size
default	$-\frac{2\left(-\frac{bd\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^{2}}+\frac{(ad-bc)\ln\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)}{2}\right)}{d^{2}}+\frac{(ad-bc)\ln\left(c\tanh\left(\frac{x}{2}\right)^{2}+2d\tanh\left(\frac{x}{2}\right)+c\right)}{d^{2}}$	75
risch	$-\frac{2b}{d(1+e^{2x})}-\frac{\ln(1+e^{2x})a}{d}+\frac{\ln(1+e^{2x})bc}{d^{2}}+\frac{\ln\left(e^{2x}+\frac{c-d}{c+d}\right)a}{d}-\frac{\ln\left(e^{2x}+\frac{c-d}{c+d}\right)bc}{d^{2}}$	88

input
```
int(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/d^2*(-b*d*tanh(1/2*x)/(1+tanh(1/2*x)^2)+1/2*(a*d-b*c)*ln(1+tanh(1/2*x)^
2))+(a*d-b*c)/d^2*ln(c*tanh(1/2*x)^2+2*d*tanh(1/2*x)+c)
```

3.990.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 6.14

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx =$$
$$-\frac{2\,bd+\left((bc-ad)\cosh\left(x\right)^{2}+2\,(bc-ad)\cosh\left(x\right)\sinh\left(x\right)+(bc-ad)\sinh\left(x\right)^{2}+bc-ad\right)\log\left(\frac{2\,(c\cosh(x}{\cosh(x}\right.}{d^{2}\cosh\left(x\right)^{2}+2\,d^{2}\,\mathrm{c}}$$

input
```
integrate(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x, algorithm="fricas")
```

output
```
-(2*b*d + ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c -
a*d)*sinh(x)^2 + b*c - a*d)*log(2*(c*cosh(x) + d*sinh(x))/(cosh(x) - sinh(
x))) - ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a*d
)*sinh(x)^2 + b*c - a*d)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(d^2*cosh(x)^
2 + 2*d^2*cosh(x)*sinh(x) + d^2*sinh(x)^2 + d^2)
```

3.990.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx=\int \frac{(a+b\tanh{(x)})\operatorname{sech}^2{(x)}}{c+d\tanh{(x)}}\,dx$$

input
```
integrate(sech(x)**2*(a+b*tanh(x))/(c+d*tanh(x)),x)
```

output
```
Integral((a + b*tanh(x))*sech(x)**2/(c + d*tanh(x)), x)
```

3.990.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(28) = 56.

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx = -b\left(\frac{c\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^2}-\frac{c\log\left(e^{(-2x)}+1\right)}{d^2}-\frac{2}{de^{(-2x)}+d}\right)+\frac{a\log\left(d\tanh{(x)}+c\right)}{d}$$

input
```
integrate(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x, algorithm="maxima")
```

output
```
-b*(c*log(-(c - d)*e^(-2*x) - c - d)/d^2 - c*log(e^(-2*x) + 1)/d^2 - 2/(d*
e^(-2*x) + d)) + a*log(d*tanh(x) + c)/d
```

3.990.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(28) = 56.

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.04

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx = -\frac{(bc^2-acd+bcd-ad^2)\log\left(\left|ce^{(2x)}+de^{(2x)}+c-d\right|\right)}{cd^2+d^3} + \frac{(bc-ad)\log\left(e^{(2x)}+1\right)}{d^2} - \frac{bce^{(2x)}-ade^{(2x)}+bc-ad+2bd}{d^2(e^{(2x)}+1)}$$

input
```
integrate(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x, algorithm="giac")
```

output
```
-(b*c^2 - a*c*d + b*c*d - a*d^2)*log(abs(c*e^(2*x) + d*e^(2*x) + c - d))/(
c*d^2 + d^3) + (b*c - a*d)*log(e^(2*x) + 1)/d^2 - (b*c*e^(2*x) - a*d*e^(2*
x) + b*c - a*d + 2*b*d)/(d^2*(e^(2*x) + 1))
```

3.990.9 Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 297, normalized size of antiderivative = 10.61

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\mathrm{e}^{2\,x}\,\left(\frac{4\,\left(a\,d\,\sqrt{-d^4}-b\,c\,\sqrt{-d^4}\right)}{d^2\,\sqrt{{\left(a\,d-b\,c\right)}^2}\,\left(c+d\right)\,\left(c-d\right)\,\sqrt{-d^4}}-\frac{4\,c^2\,\sqrt{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{d^4\,\left(c+d\right)\,\left(c-d\right)\,\left(a\,d-b\,c\right)}\right)\,\left(\frac{d^2\,\sqrt{-d^4}}{4}+\frac{c\,d\,\sqrt{-d^4}}{4}\right)+\frac{4\,c\,\left(d^2\,\sqrt{a^2\,d^2-2\,a\,b\,c\,d+\cdots}{\cdots}\right.\right.}{\sqrt{-d^4}} - \frac{2\,b}{d\,\left(\mathrm{e}^{2\,x}+1\right)}$$

input
```
int((a + b*tanh(x))/(cosh(x)^2*(c + d*tanh(x))),x)
```

output
```
(2*atan(exp(2*x)*((4*(a*d*(-d^4)^(1/2) - b*c*(-d^4)^(1/2)))/(d^2*((a*d - b
*c)^2)^(1/2)*(c + d)*(c - d)*(-d^4)^(1/2)) - (4*c^2*(a^2*d^2 + b^2*c^2 - 2
*a*b*c*d)^(1/2))/(d^4*(c + d)*(c - d)*(a*d - b*c)))*((d^2*(-d^4)^(1/2))/4
+ (c*d*(-d^4)^(1/2))/4) + (4*c*(d^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^(1/2)
- c*d*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^(1/2))*((d^2*(-d^4)^(1/2))/4 + (c*d*
(-d^4)^(1/2))/4))/(d^5*(c + d)*(c - d)*(a*d - b*c)))*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d)^(1/2))/(-d^4)^(1/2) - (2*b)/(d*(exp(2*x) + 1))
```

3.991 $\int \frac{\text{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx$

3.991.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\text{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx = \frac{(bc-ad)^2\log(c+d\tanh(x))}{d^3} - \frac{b(bc-ad)\tanh(x)}{d^2} + \frac{(a+b\tanh(x))^2}{2d}$$

output
```
(-a*d+b*c)^2*ln(c+d*tanh(x))/d^3-b*(-a*d+b*c)*tanh(x)/d^2+1/2*(a+b*tanh(x)
)^2/d
```

3.991.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{\text{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx = -\frac{-2(bc-ad)^2\log(c+d\tanh(x)) + b^2d^2\text{sech}^2(x) + 2bd(bc-2ad)\tanh(x)}{2d^3}$$

input
```
Integrate[(Sech[x]^2*(a + b*Tanh[x])^2)/(c + d*Tanh[x]),x]
```

output
```
-1/2*(-2*(b*c - a*d)^2*Log[c + d*Tanh[x]] + b^2*d^2*Sech[x]^2 + 2*b*d*(b*c
 - 2*a*d)*Tanh[x])/d^3
```

3.991.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ix)^2(a-ib\tan(ix))^2}{c-id\tan(ix)}dx$$

$$\Big\downarrow \text{4842}$$

$$\int \frac{(a+b\tanh(x))^2}{c+d\tanh(x)}d\tanh(x)$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(ad-bc)^2}{d^2(c+d\tanh(x))}-\frac{b(bc-ad)}{d^2}+\frac{b(a+b\tanh(x))}{d}\right)d\tanh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bc-ad)^2\log(c+d\tanh(x))}{d^3}-\frac{b\tanh(x)(bc-ad)}{d^2}+\frac{(a+b\tanh(x))^2}{2d}$$

input
```
Int[(Sech[x]^2*(a + b*Tanh[x])^2)/(c + d*Tanh[x]),x]
```

output
```
((b*c - a*d)^2*Log[c + d*Tanh[x]])/d^3 - (b*(b*c - a*d)*Tanh[x])/d^2 + (a
+ b*Tanh[x])^2/(2*d)
```

3.991.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.991.4 Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{b\left(\frac{b\tanh(x)^2 d}{2}+2\tanh(x)ad-\tanh(x)bc\right)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\ln(c+d\tanh(x))}{d^3}$
default	$\frac{b\left(\frac{b\tanh(x)^2 d}{2}+2\tanh(x)ad-\tanh(x)bc\right)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\ln(c+d\tanh(x))}{d^3}$
risch	$-\frac{2b(2ad\,e^{2x}-bc\,e^{2x}+bd\,e^{2x}+2ad-bc)}{(1+e^{2x})^2d^2}-\frac{\ln(1+e^{2x})a^2}{d}+\frac{2\ln(1+e^{2x})abc}{d^2}-\frac{\ln(1+e^{2x})b^2c^2}{d^3}+\frac{\ln\left(e^{2x}+\frac{c-d}{c+d}\right)a^2}{d}$

input
```
int(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
b/d^2*(1/2*b*tanh(x)^2*d+2*tanh(x)*a*d-tanh(x)*b*c)+(a^2*d^2-2*a*b*c*d+b^2
*c^2)/d^3*ln(c+d*tanh(x))
```

3.991.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(51) = 102.

Time = 0.27 (sec) , antiderivative size = 688, normalized size of antiderivative = 12.98

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx$$
$$=\frac{2b^2cd-4abd^2+2(b^2cd-(2ab+b^2)d^2)\cosh(x)^2+4(b^2cd-(2ab+b^2)d^2)\cosh(x)\sinh(x)+2(b^2cd-}{}$$

input
```
integrate(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x, algorithm="fricas")
```

output
```
(2*b^2*c*d - 4*a*b*d^2 + 2*(b^2*c*d - (2*a*b + b^2)*d^2)*cosh(x)^2 + 4*(b^
2*c*d - (2*a*b + b^2)*d^2)*cosh(x)*sinh(x) + 2*(b^2*c*d - (2*a*b + b^2)*d^
2)*sinh(x)^2 + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*si
nh(x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*cosh(x)^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 3*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)
^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*(c*cosh(x) +
d*sinh(x))/(cosh(x) - sinh(x))) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)
^4 + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*sinh(x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 2*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*cosh(x)^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 3*(b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*cosh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*
log(2*cosh(x)/(cosh(x) - sinh(x))))/(d^3*cosh(x)^4 + 4*d^3*cosh(x)*sinh(x)
^3 + d^3*sinh(x)^4 + 2*d^3*cosh(x)^2 + d^3 + 2*(3*d^3*cosh(x)^2 + d^3)*sin
h(x)^2 + 4*(d^3*cosh(x)^3 + d^3*cosh(x))*sinh(x))
```

3.991.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx=\int \frac{(a+b\tanh{(x)})^2\operatorname{sech}^2{(x)}}{c+d\tanh{(x)}}\,dx$$

input
```
integrate(sech(x)**2*(a+b*tanh(x))**2/(c+d*tanh(x)),x)
```

output
```
Integral((a + b*tanh(x))**2*sech(x)**2/(c + d*tanh(x)), x)
```

3.991.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.85

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx$$
$$= -b^2\left(\frac{2\left((c+d)e^{(-2x)}+c\right)}{2\,d^2e^{(-2x)}+d^2e^{(-4x)}+d^2}-\frac{c^2\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^3}+\frac{c^2\log\left(e^{(-2x)}+1\right)}{d^3}\right) - 2\,ab\left(\frac{c\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^2}-\frac{c\log\left(e^{(-2x)}+1\right)}{d^2}-\frac{2}{de^{(-2x)}+d}\right) + \frac{a^2\log\left(d\tanh\left(x\right)+c\right)}{d}$$

input
```
integrate(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x, algorithm="maxima")
```

output
```
-b^2*(2*((c + d)*e^(-2*x) + c)/(2*d^2*e^(-2*x) + d^2*e^(-4*x) + d^2) - c^2
*log(-(c - d)*e^(-2*x) - c - d)/d^3 + c^2*log(e^(-2*x) + 1)/d^3) - 2*a*b*(
c*log(-(c - d)*e^(-2*x) - c - d)/d^2 - c*log(e^(-2*x) + 1)/d^2 - 2/(d*e^(-
2*x) + d)) + a^2*log(d*tanh(x) + c)/d
```

3.991.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(51) = 102.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 4.98

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx$$
$$= \frac{\left(b^2c^3-2\,abc^2d+b^2c^2d+a^2cd^2-2\,abcd^2+a^2d^3\right)\log\left(\left|ce^{(2x)}+de^{(2x)}+c-d\right|\right)}{cd^3+d^4} - \frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(e^{(2x)}+1\right)}{d^3} + \frac{3\,b^2c^2e^{(4x)}-6\,abcde^{(4x)}+3\,a^2d^2e^{(4x)}+6\,b^2c^2e^{(2x)}-12\,abcde^{(2x)}+4\,b^2cde^{(2x)}+6\,a^2d^2e^{(2x)}-8\,abd^2e^{(}}{2\,d^3\left(e^{(2x)}+1\right)^2}$$

input
```
integrate(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x, algorithm="giac")
```

output
```
(b^2*c^3 - 2*a*b*c^2*d + b^2*c^2*d + a^2*c*d^2 - 2*a*b*c*d^2 + a^2*d^3)*lo
g(abs(c*e^(2*x) + d*e^(2*x) + c - d))/(c*d^3 + d^4) - (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*log(e^(2*x) + 1)/d^3 + 1/2*(3*b^2*c^2*e^(4*x) - 6*a*b*c*d*e^(4
*x) + 3*a^2*d^2*e^(4*x) + 6*b^2*c^2*e^(2*x) - 12*a*b*c*d*e^(2*x) + 4*b^2*c
*d*e^(2*x) + 6*a^2*d^2*e^(2*x) - 8*a*b*d^2*e^(2*x) - 4*b^2*d^2*e^(2*x) + 3
*b^2*c^2 - 6*a*b*c*d + 4*b^2*c*d + 3*a^2*d^2 - 8*a*b*d^2)/(d^3*(e^(2*x) +
1)^2)
```

3.991.9 Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx = \frac{\ln\left(c-d+d\,\mathrm{e}^{2\,x}+c\,\mathrm{e}^{2\,x}\right)\,\left(a\,d-b\,c\right)^2}{d^3} - \frac{2\,\left(b^2\,d-b^2\,c+2\,a\,b\,d\right)}{d^2\,\left(\mathrm{e}^{2\,x}+1\right)} - \frac{\ln\left(\mathrm{e}^{2\,x}+1\right)\,\left(a\,d-b\,c\right)^2}{d^3} + \frac{2\,b^2}{d\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)}$$

input
```
int((a + b*tanh(x))^2/(cosh(x)^2*(c + d*tanh(x))),x)
```

output
```
(log(c - d + d*exp(2*x) + c*exp(2*x))*(a*d - b*c)^2)/d^3 - (2*(b^2*d - b^2
*c + 2*a*b*d))/(d^2*(exp(2*x) + 1)) - (log(exp(2*x) + 1)*(a*d - b*c)^2)/d^
3 + (2*b^2)/(d*(2*exp(2*x) + exp(4*x) + 1))
```

3.992 $\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx$

3.992.1 Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx = -\frac{(bc-ad)^3\log(c+d\tanh(x))}{d^4} + \frac{b(bc-ad)^2\tanh(x)}{d^3} - \frac{(bc-ad)(a+b\tanh(x))^2}{2d^2} + \frac{(a+b\tanh(x))^3}{3d}$$

output
```
-(-a*d+b*c)^3*ln(c+d*tanh(x))/d^4+b*(-a*d+b*c)^2*tanh(x)/d^3-1/2*(-a*d+b*c
)*(a+b*tanh(x))^2/d^2+1/3*(a+b*tanh(x))^3/d
```

3.992.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx$$
$$= \frac{(c\cosh(x)+d\sinh(x))(a+b\tanh(x))^3\left(-6(bc-ad)^3\cosh^2(x)\log(c+d\tanh(x))+6b^3c^2d\cosh(x)\sinh(x \ldots\right.}{6d^4(a\cosh(x)+b\sinh(x))^3(c+d\tan \ldots}$$

input
```
Integrate[(Sech[x]^2*(a + b*Tanh[x])^3)/(c + d*Tanh[x]),x]
```

output
```
((c*Cosh[x] + d*Sinh[x])*(a + b*Tanh[x])^3*(-6*(b*c - a*d)^3*Cosh[x]^2*Log
[c + d*Tanh[x]] + 6*b^3*c^2*d*Cosh[x]*Sinh[x] + b*d^2*(9*a*(-(b*c) + a*d)*
Sinh[2*x] + b*(3*b*c - 9*a*d + 2*b*d*Sinh[x]^2*Tanh[x]))))/(6*d^4*(a*Cosh[
x] + b*Sinh[x])^3*(c + d*Tanh[x]))
```

3.992.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(ix)^2(a-ib\tan(ix))^3}{c-id\tan(ix)}dx$$

$$\Big\downarrow \text{ 4842}$$

$$\int \frac{(a+b\tanh(x))^3}{c+d\tanh(x)}d\tanh(x)$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{(ad-bc)^3}{d^3(c+d\tanh(x))}+\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+b\tanh(x))}{d^2}+\frac{b(a+b\tanh(x))^2}{d}\right)d\tanh(x)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(bc-ad)^3\log(c+d\tanh(x))}{d^4}+\frac{b\tanh(x)(bc-ad)^2}{d^3}-\frac{(bc-ad)(a+b\tanh(x))^2}{2d^2}+\frac{(a+b\tanh(x))^3}{3d}$$

input
```
Int[(Sech[x]^2*(a + b*Tanh[x])^3)/(c + d*Tanh[x]),x]
```

output
```
-(((b*c - a*d)^3*Log[c + d*Tanh[x]])/d^4) + (b*(b*c - a*d)^2*Tanh[x])/d^3
- ((b*c - a*d)*(a + b*Tanh[x])^2)/(2*d^2) + (a + b*Tanh[x])^3/(3*d)
```

3.992.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.992.4 Maple [A] (verified)

Time = 11.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{b\left(\frac{b^{2}\tanh(x)^{3}d^{2}}{3}+\frac{3ab\,d^{2}\tanh(x)^{2}}{2}-\frac{b^{2}cd\tanh(x)^{2}}{2}+3\tanh(x)a^{2}d^{2}-3\tanh(x)abcd+\tanh(x)b^{2}c^{2}\right)}{d^{3}}+$ $(a^{3}d^{3}-3a^{2}bc\,d^{2}+3$…
default	$\frac{b\left(\frac{b^{2}\tanh(x)^{3}d^{2}}{3}+\frac{3ab\,d^{2}\tanh(x)^{2}}{2}-\frac{b^{2}cd\tanh(x)^{2}}{2}+3\tanh(x)a^{2}d^{2}-3\tanh(x)abcd+\tanh(x)b^{2}c^{2}\right)}{d^{3}}+$ $(a^{3}d^{3}-3a^{2}bc\,d^{2}+3$…
risch	$-$ $2b(9a^{2}d^{2}e^{4x}-9abcd\,e^{4x}+9ab\,d^{2}e^{4x}+3b^{2}c^{2}e^{4x}-3b^{2}cd\,e^{4x}+3b^{2}d^{2}e^{4x}+18a^{2}d^{2}e^{2x}-18abcd\,e^{2x}+9ab\,d^{2}e^{2x}+6b^{2}c^{2}e^{2x}-$… / $3d^{3}(1+e^{2x})^{3}$

input
```
int(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
b/d^3*(1/3*b^2*tanh(x)^3*d^2+3/2*a*b*d^2*tanh(x)^2-1/2*b^2*c*d*tanh(x)^2+3
*tanh(x)*a^2*d^2-3*tanh(x)*a*b*c*d+tanh(x)*b^2*c^2)+(a^3*d^3-3*a^2*b*c*d^2
+3*a*b^2*c^2*d-b^3*c^3)/d^4*ln(c+d*tanh(x))
```

3.992.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1975 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 1975, normalized size of antiderivative = 25.32

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x, algorithm="fricas")
```

output
```
-1/3*(6*b^3*c^2*d - 18*a*b^2*c*d^2 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2
+ (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)^4 + 24*(b^3*c^2*d - (3*a*b^2 + b^
3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)*sinh(x)^3 + 6*(b^3*c^2*d
 - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*sinh(x)^4 + 2*(9
*a^2*b + b^3)*d^3 + 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b +
a*b^2)*d^3)*cosh(x)^2 + 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*
b + a*b^2)*d^3 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2
 + b^3)*d^3)*cosh(x)^2)*sinh(x)^2 + 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*cosh(x)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sinh(x)^6 + b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 3*(
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 3*(b^3*c^3
- 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*
d^2 - a^3*d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh
(x)^4 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*s
inh(x)^2 + 6*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(...
```

3.992.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx = \int \frac{(a+b\tanh(x))^3\operatorname{sech}^2(x)}{c+d\tanh(x)}\,dx$$

input
```
integrate(sech(x)**2*(a+b*tanh(x))**3/(c+d*tanh(x)),x)
```

output
```
Integral((a + b*tanh(x))**3*sech(x)**2/(c + d*tanh(x)), x)
```

3.992.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(74) = 148.

Time = 0.29 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.54

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx$$
$$=\frac{1}{3}b^3\left(\frac{2\left(3c^2+d^2+3\left(2c^2+cd\right)e^{(-2x)}+3\left(c^2+cd+d^2\right)e^{(-4x)}\right)}{3d^3e^{(-2x)}+3d^3e^{(-4x)}+d^3e^{(-6x)}+d^3}-\frac{3c^3\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^4}+\frac{3\cdots}{\cdots}\right.$$
$$-3ab^2\left(\frac{2\left((c+d)e^{(-2x)}+c\right)}{2d^2e^{(-2x)}+d^2e^{(-4x)}+d^2}-\frac{c^2\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^3}+\frac{c^2\log\left(e^{(-2x)}+1\right)}{d^3}\right)$$
$$-3a^2b\left(\frac{c\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^2}-\frac{c\log\left(e^{(-2x)}+1\right)}{d^2}-\frac{2}{de^{(-2x)}+d}\right)$$
$$+\frac{a^3\log\left(d\tanh(x)+c\right)}{d}$$

input
```
integrate(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x, algorithm="maxima")
```

output
```
1/3*b^3*(2*(3*c^2 + d^2 + 3*(2*c^2 + c*d)*e^(-2*x) + 3*(c^2 + c*d + d^2)*e
^(-4*x))/(3*d^3*e^(-2*x) + 3*d^3*e^(-4*x) + d^3*e^(-6*x) + d^3) - 3*c^3*lo
g(-(c - d)*e^(-2*x) - c - d)/d^4 + 3*c^3*log(e^(-2*x) + 1)/d^4) - 3*a*b^2*
(2*((c + d)*e^(-2*x) + c)/(2*d^2*e^(-2*x) + d^2*e^(-4*x) + d^2) - c^2*log(
-(c - d)*e^(-2*x) - c - d)/d^3 + c^2*log(e^(-2*x) + 1)/d^3) - 3*a^2*b*(c*l
og(-(c - d)*e^(-2*x) - c - d)/d^2 - c*log(e^(-2*x) + 1)/d^2 - 2/(d*e^(-2*x
) + d)) + a^3*log(d*tanh(x) + c)/d
```

3.992.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(74) = 148.

Time = 0.30 (sec) , antiderivative size = 543, normalized size of antiderivative = 6.96

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx =$$
$$-\frac{\left(b^3c^4-3ab^2c^3d+b^3c^3d+3a^2bc^2d^2-3ab^2c^2d^2-a^3cd^3+3a^2bcd^3-a^3d^4\right)\log\left(\left|ce^{(2x)}+de^{(2x)}+c-\cdots\right.\right.}{cd^4+d^5}$$
$$+\frac{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\log\left(e^{(2x)}+1\right)}{d^4}$$
$$-\frac{11b^3c^3e^{(6x)}-33ab^2c^2de^{(6x)}+33a^2bcd^2e^{(6x)}-11a^3d^3e^{(6x)}+33b^3c^3e^{(4x)}-99ab^2c^2de^{(4x)}+12b^3c^2de^{\cdots}}{\cdots}$$

input
```
integrate(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x, algorithm="giac")
```

output
```
-(b^3*c^4 - 3*a*b^2*c^3*d + b^3*c^3*d + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^2*d^2
- a^3*c*d^3 + 3*a^2*b*c*d^3 - a^3*d^4)*log(abs(c*e^(2*x) + d*e^(2*x) + c -
 d))/(c*d^4 + d^5) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*l
og(e^(2*x) + 1)/d^4 - 1/6*(11*b^3*c^3*e^(6*x) - 33*a*b^2*c^2*d*e^(6*x) + 3
3*a^2*b*c*d^2*e^(6*x) - 11*a^3*d^3*e^(6*x) + 33*b^3*c^3*e^(4*x) - 99*a*b^2
*c^2*d*e^(4*x) + 12*b^3*c^2*d*e^(4*x) + 99*a^2*b*c*d^2*e^(4*x) - 36*a*b^2*
c*d^2*e^(4*x) - 12*b^3*c*d^2*e^(4*x) - 33*a^3*d^3*e^(4*x) + 36*a^2*b*d^3*e
^(4*x) + 36*a*b^2*d^3*e^(4*x) + 12*b^3*d^3*e^(4*x) + 33*b^3*c^3*e^(2*x) -
99*a*b^2*c^2*d*e^(2*x) + 24*b^3*c^2*d*e^(2*x) + 99*a^2*b*c*d^2*e^(2*x) - 7
2*a*b^2*c*d^2*e^(2*x) - 12*b^3*c*d^2*e^(2*x) - 33*a^3*d^3*e^(2*x) + 72*a^2
*b*d^3*e^(2*x) + 36*a*b^2*d^3*e^(2*x) + 11*b^3*c^3 - 33*a*b^2*c^2*d + 12*b
^3*c^2*d + 33*a^2*b*c*d^2 - 36*a*b^2*c*d^2 - 11*a^3*d^3 + 36*a^2*b*d^3 + 4
*b^3*d^3)/(d^4*(e^(2*x) + 1)^3)
```

3.992.9 Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 1347, normalized size of antiderivative = 17.27

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx = \frac{2\,\left(2\,b^3\,d-b^3\,c+3\,a\,b^2\,d\right)}{d^2\,\left(2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1\right)} - \frac{2\,\left(3\,a^2\,b\,d^2-3\,a\,b^2\,c\,d+3\,a\,b^2\,d^2+b^3\,c^2-b^3\,c\,d+b^3\,d^2\right)}{d^3\,\left({\mathrm{e}}^{2\,x}+1\right)} - \frac{8\,b^3}{3\,d\,\left(3\,{\mathrm{e}}^{2\,x}+3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{\left(\frac{32\,c\,\sqrt{a^6\,d^6-6\,a^5\,b\,c\,d^5+15\,a^4\,b^2\,c^2\,d^4-20\,a^3\,b^3\,c^3\,d^3+15\,a^2\,b^4\,c^4\,d^2-6\,a\,b^5\,c^5\,d+b^6\,c^6}\,\left(-a^3\,c\,d^8+a^3\,d^9+3\,a^2\,b\,c^2\,d^7-3\,a^2\,b\,c\,d^8-3\,a\,b^2\,c^3\,d^6 \cdots}{d^{16}\,\sqrt{{\left(a\,d-b\,c\right)}^6}\,\left(c+d\right)\,{\left(c-d\right)}^2\,\left(c^2+2\,c\,d+d^2\right)} \cdots}{\cdots}\right)}{\cdots}$$

input
```
int((a + b*tanh(x))^3/(cosh(x)^2*(c + d*tanh(x))),x)
```

output
```
(2*(2*b^3*d - b^3*c + 3*a*b^2*d))/(d^2*(2*exp(2*x) + exp(4*x) + 1)) - (2*(
b^3*c^2 + b^3*d^2 + 3*a*b^2*d^2 + 3*a^2*b*d^2 - b^3*c*d - 3*a*b^2*c*d))/(d
^3*(exp(2*x) + 1)) - (8*b^3)/(3*d*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)
) - (2*atan((((32*c*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c
^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2)*(a^3*d^
9 - a^3*c*d^8 - b^3*c^3*d^6 + b^3*c^4*d^5 + 3*a*b^2*c^2*d^7 - 3*a*b^2*c^3*
d^6 + 3*a^2*b*c^2*d^7 - 3*a^2*b*c*d^8))/(d^16*((a*d - b*c)^6)^(1/2)*(c + d
)*(c - d)^2*(2*c*d + c^2 + d^2)) - exp(2*x)*((32*c*(2*a^3*c*d^8 - 2*b^3*c^
4*d^5 + 6*a*b^2*c^3*d^6 - 6*a^2*b*c^2*d^7)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4
*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5
*b*c*d^5)^(1/2))/(d^16*((a*d - b*c)^6)^(1/2)*(c + d)*(c - d)^2*(2*c*d + c^
2 + d^2)) - (16*(c^2*(-d^8)^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2
- 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)
^(1/2) + d^2*(-d^8)^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3
*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))*
(c^2 + d^2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))/(d^13*(c + d)*
(c - d)^2*(a*d - b*c)^3*(-d^8)^(1/2)*(2*c*d + c^2 + d^2))) + (16*(c^2*(-d^
8)^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15
*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2) - c*d*(-d^8)^(1...
```

3.992. $\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx$

3.993 $\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{(2+\tanh^3(x))^2}\,dx$

3.993.1 Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{(2+\tanh^3(x))^2}\,dx = -\frac{1}{3\,(2+\tanh^3(x))}$$

output
```
-1/3/(2+tanh(x)^3)
```

3.993.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{(2+\tanh^3(x))^2}\,dx = -\frac{1}{3\,(2+\tanh^3(x))}$$

input
```
Integrate[(Sech[x]^2*Tanh[x]^2)/(2 + Tanh[x]^3)^2,x]
```

output
```
-1/3*1/(2 + Tanh[x]^3)
```

3.993.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 25, 4842, 25, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)\text{sech}^2(x)}{\left(\tanh^3(x)+2\right)^2}dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{\tan(ix)^2\sec(ix)^2}{\left(2+i\tan(ix)^3\right)^2}dx$$
$$\Big\downarrow \; 25$$
$$-\int \frac{\sec(ix)^2\tan(ix)^2}{\left(i\tan(ix)^3+2\right)^2}dx$$
$$\Big\downarrow \; 4842$$
$$-\int -\frac{\tanh^2(x)}{\left(\tanh^3(x)+2\right)^2}d\tanh(x)$$
$$\Big\downarrow \; 25$$
$$\int \frac{\tanh^2(x)}{\left(\tanh^3(x)+2\right)^2}d\tanh(x)$$
$$\Big\downarrow \; 793$$
$$-\frac{1}{3\left(\tanh^3(x)+2\right)}$$

input
```
Int[(Sech[x]^2*Tanh[x]^2)/(2 + Tanh[x]^3)^2,x]
```

output
```
-1/3*1/(2 + Tanh[x]^3)
```

3.993.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.993.4 Maple [A] (verified)

Time = 11.84 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{3\left(2+\tanh\left(x\right)^{3}\right)}$	11
default	$-\frac{1}{3\left(2+\tanh\left(x\right)^{3}\right)}$	11
risch	$-\frac{2\left(3\,{\mathrm e}^{4 x}+1\right)}{9\left(3\,{\mathrm e}^{6 x}+3\,{\mathrm e}^{4 x}+9\,{\mathrm e}^{2 x}+1\right)}$	33

input
```
int(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/(2+tanh(x)^3)
```

3.993. $\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx$

3.993.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(10) = 20.

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 6.08

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{(2+\tanh^3(x))^2}\,dx = -\frac{8\left(\cosh(x)^2+\cosh(x)\sinh(x)+\sinh(x)^2\right)}{9\left(3\cosh(x)^4+12\cosh(x)\sinh(x)^3+3\sinh(x)^4+2\left(9\cosh(x)^2+2\right)\sinh(x)^2+4\cosh(x)^2+4\left(3\right.\right.}$$

input
```
integrate(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x, algorithm="fricas")
```

output
```
-8/9*(cosh(x)^2 + cosh(x)*sinh(x) + sinh(x)^2)/(3*cosh(x)^4 + 12*cosh(x)*s
inh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 + 2)*sinh(x)^2 + 4*cosh(x)^2 + 4*(
3*cosh(x)^3 + cosh(x))*sinh(x) + 9)
```

3.993.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{(2+\tanh^3(x))^2}\,dx = \int \frac{\tanh^2(x)\operatorname{sech}^2(x)}{(\tanh^3(x)+2)^2}\,dx$$

input
```
integrate(sech(x)**2*tanh(x)**2/(2+tanh(x)**3)**2,x)
```

output
```
Integral(tanh(x)**2*sech(x)**2/(tanh(x)**3 + 2)**2, x)
```

3.993.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{(2+\tanh^3(x))^2}\,dx = -\frac{1}{3\left(\tanh(x)^3+2\right)}$$

input
```
integrate(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x, algorithm="maxima")
```

output
```
-1/3/(tanh(x)^3 + 2)
```

3.993.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(10) = 20.

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.67

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx = -\frac{2\left(3\,e^{(4\,x)}+1\right)}{9\left(3\,e^{(6\,x)}+3\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)}$$

input
```
integrate(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x, algorithm="giac")
```

output
```
-2/9*(3*e^(4*x) + 1)/(3*e^(6*x) + 3*e^(4*x) + 9*e^(2*x) + 1)
```

3.993.9 Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.67

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx = -\frac{\frac{2\,\mathrm{e}^{4\,x}}{3}+\frac{2}{9}}{9\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+3\,\mathrm{e}^{6\,x}+1}$$

input
```
int(tanh(x)^2/(cosh(x)^2*(tanh(x)^3 + 2)^2),x)
```

output
```
-((2*exp(4*x))/3 + 2/9)/(9*exp(2*x) + 3*exp(4*x) + 3*exp(6*x) + 1)
```

3.994 $\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx$

3.994.1 Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx=\frac{\tanh^7(x)}{7}-\frac{\tanh^9(x)}{3}+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^{13}(x)}{13}$$

output
```
1/7*tanh(x)^7-1/3*tanh(x)^9+3/11*tanh(x)^11-1/13*tanh(x)^13
```

3.994.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(33) = 66.

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\begin{aligned}\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx&=\frac{16\tanh(x)}{3003}+\frac{8\operatorname{sech}^2(x)\tanh(x)}{3003}\\&+\frac{2\operatorname{sech}^4(x)\tanh(x)}{1001}+\frac{5\operatorname{sech}^6(x)\tanh(x)}{3003}\\&-\frac{53}{429}\operatorname{sech}^8(x)\tanh(x)+\frac{27}{143}\operatorname{sech}^{10}(x)\tanh(x)\\&-\frac{1}{13}\operatorname{sech}^{12}(x)\tanh(x)\end{aligned}$$

input
```
Integrate[Sech[x]^2*Tanh[x]^6*(1 - Tanh[x]^2)^3,x]
```

output
```
(16*Tanh[x])/3003 + (8*Sech[x]^2*Tanh[x])/3003 + (2*Sech[x]^4*Tanh[x])/100
1 + (5*Sech[x]^6*Tanh[x])/3003 - (53*Sech[x]^8*Tanh[x])/429 + (27*Sech[x]^
10*Tanh[x])/143 - (Sech[x]^12*Tanh[x])/13
```

3.994.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3042, 25, 4140, 25, 3042, 25, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^6(x)\left(1-\tanh^2(x)\right)^3\operatorname{sech}^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ix)^6\left(1+\tan(ix)^2\right)^3\left(-\sec(ix)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ix)^2\tan(ix)^6\left(\tan(ix)^2+1\right)^3dx$$

$$\Big\downarrow \text{4140}$$

$$-\int -\operatorname{sech}^8(x)\tanh^6(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int \tanh^6(x)\operatorname{sech}^8(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ix)^6\left(-\sec(ix)^8\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ix)^8\tan(ix)^6dx$$

$$\Big\downarrow \text{3087}$$

$$i\int -\tanh^6(x)\left(1-\tanh^2(x)\right)^3d(i\tanh(x))$$

$$\Big\downarrow \text{ 244}$$

$$i\int \left(\tanh^{12}(x)-3\tanh^{10}(x)+3\tanh^8(x)-\tanh^6(x)\right)d(i\tanh(x))$$

$$\Big\downarrow \text{ 2009}$$

$$i\left(\frac{1}{13}i\tanh^{13}(x)-\frac{3}{11}i\tanh^{11}(x)+\frac{1}{3}i\tanh^9(x)-\frac{1}{7}i\tanh^7(x)\right)$$

input
```
Int[Sech[x]^2*Tanh[x]^6*(1 - Tanh[x]^2)^3,x]
```

output
```
I*((-1/7*I)*Tanh[x]^7 + (I/3)*Tanh[x]^9 - ((3*I)/11)*Tanh[x]^11 + (I/13)*T
anh[x]^13)
```

3.994.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.994. $\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx$

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

3.994.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\frac{\tanh(x)^7}{7}-\frac{\tanh(x)^9}{3}+\frac{3\tanh(x)^{11}}{11}-\frac{\tanh(x)^{13}}{13}$$

input
```
int(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x)
```

output
```
1/7*tanh(x)^7-1/3*tanh(x)^9+3/11*tanh(x)^11-1/13*tanh(x)^13
```

3.994.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(25) = 50$.

Time = 0.24 (sec) , antiderivative size = 778, normalized size of antiderivative = 23.58

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x, algorithm="fricas")
```

output
```
-64/3003*(1502*cosh(x)^9 + 13518*cosh(x)*sinh(x)^8 + 1501*sinh(x)^9 + (540
36*cosh(x)^2 - 4511)*sinh(x)^7 - 4498*cosh(x)^7 + 14*(9012*cosh(x)^3 - 224
9*cosh(x))*sinh(x)^6 + 3*(63042*cosh(x)^4 - 31577*cosh(x)^2 + 2990)*sinh(x
)^5 + 9048*cosh(x)^5 + 2*(94626*cosh(x)^5 - 78715*cosh(x)^3 + 22620*cosh(x
))*sinh(x)^4 + (126084*cosh(x)^6 - 157885*cosh(x)^4 + 89700*cosh(x)^2 - 82
94)*sinh(x)^3 - 8008*cosh(x)^3 + 6*(9012*cosh(x)^7 - 15743*cosh(x)^5 + 150
80*cosh(x)^3 - 4004*cosh(x))*sinh(x)^2 + (13509*cosh(x)^8 - 31577*cosh(x)^
6 + 44850*cosh(x)^4 - 24882*cosh(x)^2 + 6292)*sinh(x) + 4004*cosh(x))/(cos
h(x)^17 + 17*cosh(x)*sinh(x)^16 + sinh(x)^17 + (136*cosh(x)^2 + 13)*sinh(x
)^15 + 13*cosh(x)^15 + 5*(136*cosh(x)^3 + 39*cosh(x))*sinh(x)^14 + (2380*c
osh(x)^4 + 1365*cosh(x)^2 + 78)*sinh(x)^13 + 78*cosh(x)^13 + 13*(476*cosh(
x)^5 + 455*cosh(x)^3 + 78*cosh(x))*sinh(x)^12 + 13*(952*cosh(x)^6 + 1365*c
osh(x)^4 + 468*cosh(x)^2 + 22)*sinh(x)^11 + 286*cosh(x)^11 + 143*(136*cosh
(x)^7 + 273*cosh(x)^5 + 156*cosh(x)^3 + 22*cosh(x))*sinh(x)^10 + (24310*co
sh(x)^8 + 65065*cosh(x)^6 + 55770*cosh(x)^4 + 15730*cosh(x)^2 + 714)*sinh(
x)^9 + 716*cosh(x)^9 + (24310*cosh(x)^9 + 83655*cosh(x)^7 + 100386*cosh(x)
^5 + 47190*cosh(x)^3 + 6444*cosh(x))*sinh(x)^8 + (19448*cosh(x)^10 + 83655
*cosh(x)^8 + 133848*cosh(x)^6 + 94380*cosh(x)^4 + 25704*cosh(x)^2 + 1274)*
sinh(x)^7 + 1300*cosh(x)^7 + (12376*cosh(x)^11 + 65065*cosh(x)^9 + 133848*
cosh(x)^7 + 132132*cosh(x)^5 + 60144*cosh(x)^3 + 9100*cosh(x))*sinh(x)^...
```

3.994.6 Sympy [F]

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx = -\int \left(-\tanh^6(x)\operatorname{sech}^2(x)\right)\,dx - \int 3\tanh^8(x)\operatorname{sech}^2(x)\,dx - \int \left(-3\tanh^{10}(x)\operatorname{sech}^2(x)\right)\,dx - \int \tanh^{12}(x)\operatorname{sech}^2(x)\,dx$$

input
```
integrate(sech(x)**2*tanh(x)**6*(1-tanh(x)**2)**3,x)
```

output
```
-Integral(-tanh(x)**6*sech(x)**2, x) - Integral(3*tanh(x)**8*sech(x)**2, x
) - Integral(-3*tanh(x)**10*sech(x)**2, x) - Integral(tanh(x)**12*sech(x)*
*2, x)
```

3.994.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx = -\frac{1}{13}\tanh\left(x\right)^{13}+\frac{3}{11}\tanh\left(x\right)^{11}-\frac{1}{3}\tanh\left(x\right)^{9}+\frac{1}{7}\tanh\left(x\right)^{7}$$

input
```
integrate(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x, algorithm="maxima")
```

output
```
-1/13*tanh(x)^13 + 3/11*tanh(x)^11 - 1/3*tanh(x)^9 + 1/7*tanh(x)^7
```

3.994.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(25) = 50.

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx = -\frac{32\left(3003\,e^{(18\,x)}-9009\,e^{(16\,x)}+18018\,e^{(14\,x)}-16302\,e^{(12\,x)}+10296\,e^{(10\,x)}-2288\,e^{(8\,x)}+286\,e^{(6\,x)}+78\,e\right.}{3003\left(e^{(2\,x)}+1\right)^{13}}$$

input
```
integrate(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x, algorithm="giac")
```

output
```
-32/3003*(3003*e^(18*x) - 9009*e^(16*x) + 18018*e^(14*x) - 16302*e^(12*x)
+ 10296*e^(10*x) - 2288*e^(8*x) + 286*e^(6*x) + 78*e^(4*x) + 13*e^(2*x) +
1)/(e^(2*x) + 1)^13
```

3.994.9 Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 820, normalized size of antiderivative = 24.85

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx = \text{Too large to display}$$

input
```
int(-(tanh(x)^6*(tanh(x)^2 - 1)^3)/cosh(x)^2,x)
```

output
```
- ((64*exp(4*x))/143 - (256*exp(2*x))/429 + 80/429)/(6*exp(2*x) + 15*exp(4
*x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + exp(12*x) + 1) - ((64*exp(
2*x))/143 - (768*exp(4*x))/143 + (3200*exp(6*x))/143 - (6400*exp(8*x))/143
 + (6720*exp(10*x))/143 - (3584*exp(12*x))/143 + (768*exp(14*x))/143)/(11*
exp(2*x) + 55*exp(4*x) + 165*exp(6*x) + 330*exp(8*x) + 462*exp(10*x) + 462
*exp(12*x) + 330*exp(14*x) + 165*exp(16*x) + 55*exp(18*x) + 11*exp(20*x) +
 exp(22*x) + 1) - ((160*exp(2*x))/143 - (256*exp(4*x))/143 + (128*exp(6*x)
)/143 - 640/3003)/(7*exp(2*x) + 21*exp(4*x) + 35*exp(6*x) + 35*exp(8*x) +
21*exp(10*x) + 7*exp(12*x) + exp(14*x) + 1) - ((128*exp(6*x))/13 - (768*ex
p(8*x))/13 + (1920*exp(10*x))/13 - (2560*exp(12*x))/13 + (1920*exp(14*x))/
13 - (768*exp(16*x))/13 + (128*exp(18*x))/13)/(13*exp(2*x) + 78*exp(4*x) +
 286*exp(6*x) + 715*exp(8*x) + 1287*exp(10*x) + 1716*exp(12*x) + 1716*exp(
14*x) + 1287*exp(16*x) + 715*exp(18*x) + 286*exp(20*x) + 78*exp(22*x) + 13
*exp(24*x) + exp(26*x) + 1) - ((560*exp(4*x))/143 - (640*exp(2*x))/429 - (
1792*exp(6*x))/429 + (224*exp(8*x))/143 + 80/429)/(8*exp(2*x) + 28*exp(4*x
) + 56*exp(6*x) + 70*exp(8*x) + 56*exp(10*x) + 28*exp(12*x) + 8*exp(14*x)
+ exp(16*x) + 1) - ((640*exp(2*x))/429 - (2560*exp(4*x))/429 + (4480*exp(6
*x))/429 - (3584*exp(8*x))/429 + (1792*exp(10*x))/715 - 256/2145)/(9*exp(2
*x) + 36*exp(4*x) + 84*exp(6*x) + 126*exp(8*x) + 126*exp(10*x) + 84*exp(12
*x) + 36*exp(14*x) + 9*exp(16*x) + exp(18*x) + 1) - ((32*exp(4*x))/13 -...
```

3.994. $\int \operatorname{sech}^2(x) \tanh^6(x) \left(1-\tanh^2(x)\right)^3 \, dx$

3.995 $\int \frac{\operatorname{sech}^2(x)(2+\tanh^2(x))}{1+\tanh^3(x)}\,dx$

3.995.1 Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx=-\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+\tanh(x))$$

output
```
ln(1+tanh(x))-2/3*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)
```

3.995.2 Mathematica [A] (verified)

Time = 3.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx=x+\frac{2\arctan\left(\frac{-1+2\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\log(\cosh(x))$$

input
```
Integrate[(Sech[x]^2*(2 + Tanh[x]^2))/(1 + Tanh[x]^3),x]
```

output
```
x + (2*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]])/Sqrt[3] - Log[Cosh[x]]
```

3.995.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4842, 2402, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\tanh^2(x)+2)\operatorname{sech}^2(x)}{\tanh^3(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(2-\tan(ix)^2)\sec(ix)^2}{1+i\tan(ix)^3}dx$$

$$\Big\downarrow \text{4842}$$

$$\int \frac{\tanh^2(x)+2}{\tanh^3(x)+1}d\tanh(x)$$

$$\Big\downarrow \text{2402}$$

$$\int \frac{1}{\tanh^2(x)-\tanh(x)+1}d\tanh(x)+\int \frac{1}{\tanh(x)+1}d\tanh(x)$$

$$\Big\downarrow \text{16}$$

$$\int \frac{1}{\tanh^2(x)-\tanh(x)+1}d\tanh(x)+\log(\tanh(x)+1)$$

$$\Big\downarrow \text{1083}$$

$$\log(\tanh(x)+1)-2\int \frac{1}{-(2\tanh(x)-1)^2-3}d(2\tanh(x)-1)$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{2\tanh(x)-1}{\sqrt{3}}\right)}{\sqrt{3}}+\log(\tanh(x)+1)$$

input
```
Int[(Sech[x]^2*(2 + Tanh[x]^2))/(1 + Tanh[x]^3),x]
```

output
```
(2*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]])/Sqrt[3] + Log[1 + Tanh[x]]
```

3.995. $\int \frac{\operatorname{sech}^2(x)(2+\tanh^2(x))}{1+\tanh^3(x)}\,dx$

3.995.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2402
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Simp[C
/b   Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x]
, x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a,
b}, x] && PolyQ[P2, x, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.995. $\int \frac{\operatorname{sech}^2(x)(2+\tanh^2(x))}{1+\tanh^3(x)} \, dx$

3.995.4 Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\ln\left(1+\tanh\left(x\right)\right)+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2\tanh\left(x\right)-1\right)\sqrt{3}}{3}\right)}{3}$	24
default	$\ln\left(1+\tanh\left(x\right)\right)+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2\tanh\left(x\right)-1\right)\sqrt{3}}{3}\right)}{3}$	24
risch	$2x+\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}+i\sqrt{3}\right)}{3}-\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}-i\sqrt{3}\right)}{3}-\ln\left(1+\mathrm{e}^{2x}\right)$	50

input
```
int(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
ln(1+tanh(x))+2/3*3^(1/2)*arctan(1/3*(2*tanh(x)-1)*3^(1/2))
```

3.995.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh\left(x\right)+\sqrt{3}\sinh\left(x\right)}{3\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}\right) + 2\,x - \log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

input
```
integrate(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x, algorithm="fricas")
```

output
```
-2/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - si
nh(x))) + 2*x - log(2*cosh(x)/(cosh(x) - sinh(x)))
```

3.995.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx = \int \frac{\left(\tanh^2(x)+2\right)\operatorname{sech}^2(x)}{\left(\tanh(x)+1\right)\left(\tanh^2(x)-\tanh(x)+1\right)}\,dx$$

input
```
integrate(sech(x)**2*(2+tanh(x)**2)/(1+tanh(x)**3),x)
```

output
```
Integral((tanh(x)**2 + 2)*sech(x)**2/((tanh(x) + 1)*(tanh(x)**2 - tanh(x)
+ 1)), x)
```

3.995.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(23) = 46$.

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 4.69

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx &= \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right) \\ &\quad -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right) \\ &\quad +\frac{1}{3}\log\left(\tanh(x)^3+1\right)-\frac{1}{3}\log\left(3^{\frac{1}{4}}\sqrt{2}e^{(-x)}+\sqrt{3}e^{(-2x)}+1\right) \\ &\quad -\frac{1}{3}\log\left(-3^{\frac{1}{4}}\sqrt{2}e^{(-x)}+\sqrt{3}e^{(-2x)}+1\right)\end{aligned}$$

input
```
integrate(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2)
)) - 2/3*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sq
rt(2))) + 1/3*log(tanh(x)^3 + 1) - 1/3*log(3^(1/4)*sqrt(2)*e^(-x) + sqrt(3
)*e^(-2*x) + 1) - 1/3*log(-3^(1/4)*sqrt(2)*e^(-x) + sqrt(3)*e^(-2*x) + 1)
```

3.995.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx=\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right)+2x-\log\left(e^{(2x)}+1\right)$$

input
```
integrate(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) + 2*x - log(e^(2*x) + 1)
```

3.995.9 Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx=2x-\ln\left(768\,e^{2x}+768\right)-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\frac{640\sqrt{3}}{3}-\frac{128\sqrt{3}\,e^{2x}}{3}}{\frac{640\,e^{2x}}{3}+128}\right)}{3}$$

input
```
int((tanh(x)^2 + 2)/(cosh(x)^2*(tanh(x)^3 + 1)),x)
```

output
```
2*x - log(768*exp(2*x) + 768) - (2*3^(1/2)*atan(((640*3^(1/2))/3 - (128*3^
(1/2)*exp(2*x))/3)/((640*exp(2*x))/3 + 128)))/3
```

3.996 $\int (1+\cosh^2(x)) \operatorname{sech}^2(x) \, dx$

3.996.1 Optimal result

Integrand size = 11, antiderivative size = 4

$$\int (1+\cosh^2(x)) \operatorname{sech}^2(x) \, dx = x+\tanh(x)$$

output
```
x+tanh(x)
```

3.996.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int (1+\cosh^2(x)) \operatorname{sech}^2(x) \, dx = x+\tanh(x)$$

input
```
Integrate[(1 + Cosh[x]^2)*Sech[x]^2,x]
```

output
```
x + Tanh[x]
```

3.996.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cosh^2(x)+1)\operatorname{sech}^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1+\sin\left(\frac{\pi}{2}+ix\right)^2}{\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3491}$$

$$\int 1dx+\tanh(x)$$

$$\Big\downarrow \text{24}$$

$$x+\tanh(x)$$

input
```
Int[(1 + Cosh[x]^2)*Sech[x]^2,x]
```

output
```
x + Tanh[x]
```

3.996.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

3.996.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$x+\tanh(x)$	5
parallelrisch	$x+\tanh(x)$	5
parts	$x+\tanh(x)$	5
risch	$x-\frac{2}{1+e^{2x}}$	13

input
```
int((1+cosh(x)^2)*sech(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x+tanh(x)
```

3.996.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(4) = 8$.

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int (1+\cosh^2(x))\,\mathrm{sech}^2(x)\,dx = \frac{(x-1)\cosh(x)+\sinh(x)}{\cosh(x)}$$

input
```
integrate((1+cosh(x)^2)*sech(x)^2,x, algorithm="fricas")
```

output
```
((x - 1)*cosh(x) + sinh(x))/cosh(x)
```

3.996.6 Sympy [F]

$$\int (1+\cosh^2(x))\,\mathrm{sech}^2(x)\,dx = \int (\cosh^2(x)+1)\,\mathrm{sech}^2(x)\,dx$$

input
```
integrate((1+cosh(x)**2)*sech(x)**2,x)
```

output
```
Integral((cosh(x)**2 + 1)*sech(x)**2, x)
```

3.996.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int (1+\cosh^2(x))\operatorname{sech}^2(x)\,dx = x + \frac{2}{e^{(-2\,x)}+1}$$

input
```
integrate((1+cosh(x)^2)*sech(x)^2,x, algorithm="maxima")
```

output
```
x + 2/(e^(-2*x) + 1)
```

3.996.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int (1+\cosh^2(x))\operatorname{sech}^2(x)\,dx = x - \frac{2}{e^{(2\,x)}+1}$$

input
```
integrate((1+cosh(x)^2)*sech(x)^2,x, algorithm="giac")
```

output
```
x - 2/(e^(2*x) + 1)
```

3.996.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int (1+\cosh^2(x))\operatorname{sech}^2(x)\,dx = x - \frac{2}{\mathrm{e}^{2\,x}+1}$$

input
```
int((cosh(x)^2 + 1)/cosh(x)^2,x)
```

output
```
x - 2/(exp(2*x) + 1)
```

3.997 $\int \frac{\text{sech}^2(x)}{1+\text{sech}^2(x)-3\tanh(x)} \, dx$

3.997.1 Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{\text{sech}^2(x)}{1+\text{sech}^2(x)-3\tanh(x)} \, dx = \frac{2\text{arctanh}\left(\frac{3+2\tanh(x)}{\sqrt{17}}\right)}{\sqrt{17}}$$

output
```
2/17*arctanh(1/17*(3+2*tanh(x))*17^(1/2))*17^(1/2)
```

3.997.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}^2(x)}{1+\text{sech}^2(x)-3\tanh(x)} \, dx = \frac{2\text{arctanh}\left(\frac{3+2\tanh(x)}{\sqrt{17}}\right)}{\sqrt{17}}$$

input
```
Integrate[Sech[x]^2/(1 + Sech[x]^2 - 3*Tanh[x]),x]
```

output
```
(2*ArcTanh[(3 + 2*Tanh[x])/Sqrt[17]])/Sqrt[17]
```

3.997.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 4889, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{-3\tanh(x)+\text{sech}^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ix)^2}{3i\tan(ix)+\sec(ix)^2+1}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{-\tanh^2(x)-3\tanh(x)+2}d\tanh(x)$$

$$\Big\downarrow \text{1081}$$

$$-\int \left(\frac{2}{\sqrt{17}\left(2\tanh(x)-\sqrt{17}+3\right)}-\frac{2}{\sqrt{17}\left(2\tanh(x)+\sqrt{17}+3\right)}\right)d\tanh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log\left(2\tanh(x)+\sqrt{17}+3\right)}{\sqrt{17}}-\frac{\log\left(2\tanh(x)-\sqrt{17}+3\right)}{\sqrt{17}}$$

input
```
Int[Sech[x]^2/(1 + Sech[x]^2 - 3*Tanh[x]),x]
```

output
```
-(Log[3 - Sqrt[17] + 2*Tanh[x]]/Sqrt[17]) + Log[3 + Sqrt[17] + 2*Tanh[x]]/
Sqrt[17]
```

3.997.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.997.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

method	result	size
risch	$\frac{\sqrt{17}\, \ln\left(e^{2x}+\frac{\sqrt{17}}{2}-\frac{3}{2}\right)}{17}-\frac{\sqrt{17}\, \ln\left(e^{2x}-\frac{\sqrt{17}}{2}-\frac{3}{2}\right)}{17}$	36
default	$-\frac{\sqrt{17}\, \ln\left(-\sqrt{17}\, \tanh\left(\frac{x}{2}\right)+2 \tanh\left(\frac{x}{2}\right)^{2}-3 \tanh\left(\frac{x}{2}\right)+2\right)}{17}+\frac{\sqrt{17}\, \ln\left(\sqrt{17}\, \tanh\left(\frac{x}{2}\right)+2 \tanh\left(\frac{x}{2}\right)^{2}-3 \tanh\left(\frac{x}{2}\right)+2\right)}{17}$	63

input
```
int(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/17*17^(1/2)*ln(exp(2*x)+1/2*17^(1/2)-3/2)-1/17*17^(1/2)*ln(exp(2*x)-1/2*
17^(1/2)-3/2)
```

3.997.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.35

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx$$
$$= \frac{1}{17}\sqrt{17}\log\left(\frac{3\left(\sqrt{17}-5\right)\cosh(x)^2-2\left(3\sqrt{17}-11\right)\cosh(x)\sinh(x)+3\left(\sqrt{17}-5\right)\sinh(x)^2-2\sqrt{17}+\right.}{\cosh(x)^2-6\cosh(x)\sinh(x)+\sinh(x)^2+3}$$

input
```
integrate(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x, algorithm="fricas")
```

output
```
1/17*sqrt(17)*log((3*(sqrt(17) - 5)*cosh(x)^2 - 2*(3*sqrt(17) - 11)*cosh(x
)*sinh(x) + 3*(sqrt(17) - 5)*sinh(x)^2 - 2*sqrt(17) + 6)/(cosh(x)^2 - 6*co
sh(x)*sinh(x) + sinh(x)^2 + 3))
```

3.997.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{-3\tanh(x)+\operatorname{sech}^2(x)+1}\,dx$$

input
```
integrate(sech(x)**2/(1+sech(x)**2-3*tanh(x)),x)
```

output
```
Integral(sech(x)**2/(-3*tanh(x) + sech(x)**2 + 1), x)
```

3.997.7 Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx = \int \frac{\operatorname{sech}(x)^2}{\operatorname{sech}(x)^2-3\tanh(x)+1}\,dx$$

input
```
integrate(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x, algorithm="maxima")
```

output
```
integrate(sech(x)^2/(sech(x)^2 - 3*tanh(x) + 1), x)
```

3.997.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx = -\frac{1}{17}\sqrt{17}\log\left(\frac{\left|-\sqrt{17}+2e^{(2x)}-3\right|}{\left|\sqrt{17}+2e^{(2x)}-3\right|}\right)$$

input
```
integrate(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x, algorithm="giac")
```

output
```
-1/17*sqrt(17)*log(abs(-sqrt(17) + 2*e^(2*x) - 3)/abs(sqrt(17) + 2*e^(2*x)
 - 3))
```

3.997.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.50

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx = -\frac{\sqrt{17}\left(\ln\left(2\,e^{2x}-\frac{\sqrt{17}\,(6\,e^{2x}+8)}{17}\right)-\ln\left(2\,e^{2x}+\frac{\sqrt{17}\,(6\,e^{2x}+8)}{17}\right)\right)}{17}$$

input
```
int(1/(cosh(x)^2*(1/cosh(x)^2 - 3*tanh(x) + 1)),x)
```

output
```
-(17^(1/2)*(log(2*exp(2*x) - (17^(1/2)*(6*exp(2*x) + 8))/17) - log(2*exp(2
*x) + (17^(1/2)*(6*exp(2*x) + 8))/17)))/17
```

3.998 $\int \frac{\text{sech}^2(x)}{\sqrt{4-\text{sech}^2(x)}} dx$

3.998.1 Optimal result

Integrand size = 17, antiderivative size = 9

$$\int \frac{\text{sech}^2(x)}{\sqrt{4-\text{sech}^2(x)}} dx = \text{arcsinh}\left(\frac{\tanh(x)}{\sqrt{3}}\right)$$

output
```
arcsinh(1/3*tanh(x)*3^(1/2))
```

3.998.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(9) = 18.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 4.78

$$\int \frac{\text{sech}^2(x)}{\sqrt{4-\text{sech}^2(x)}} dx = \frac{\text{arctanh}\left(\frac{\sinh(x)}{\sqrt{3+4\sinh^2(x)}}\right)\sqrt{1+2\cosh(2x)}\text{sech}(x)}{\sqrt{4-\text{sech}^2(x)}}$$

input
```
Integrate[Sech[x]^2/Sqrt[4 - Sech[x]^2],x]
```

output
```
(ArcTanh[Sinh[x]/Sqrt[3 + 4*Sinh[x]^2]]*Sqrt[1 + 2*Cosh[2*x]]*Sech[x])/Sqr
t[4 - Sech[x]^2]
```

3.998.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4634, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{\sqrt{4-\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ix)^2}{\sqrt{4-\sec(ix)^2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\int \frac{1}{\sqrt{\tanh^2(x)+3}}d\tanh(x)$$

$$\Big\downarrow \text{222}$$

$$\text{arcsinh}\left(\frac{\tanh(x)}{\sqrt{3}}\right)$$

input
```
Int[Sech[x]^2/Sqrt[4 - Sech[x]^2],x]
```

output
```
ArcSinh[Tanh[x]/Sqrt[3]]
```

3.998.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.998.4 Maple [F]

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{4-\operatorname{sech}(x)^2}}dx$$

input
```
int(sech(x)^2/(4-sech(x)^2)^(1/2),x)
```

output
```
int(sech(x)^2/(4-sech(x)^2)^(1/2),x)
```

3.998.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(8) = 16$.

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 12.44

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{4-\operatorname{sech}^2(x)}}\,dx = -\log\left(-\cosh(x)^2 - 2\cosh(x)\sinh(x) - \sinh(x)^2 + \sqrt{\frac{2\cosh(x)^2 + 2\sinh(x)^2 + 1}{\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2}}\right) + \log\left(-\cosh(x)^2 - 2\cosh(x)\sinh(x) - \sinh(x)^2 + \sqrt{\frac{2\cosh(x)^2 + 2\sinh(x)^2 + 1}{\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2}} - 2\right)$$

input
```
integrate(sech(x)^2/(4-sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-log(-cosh(x)^2 - 2*cosh(x)*sinh(x) - sinh(x)^2 + sqrt((2*cosh(x)^2 + 2*si
nh(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))) + log(-cosh(x)^
2 - 2*cosh(x)*sinh(x) - sinh(x)^2 + sqrt((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2)
```

3.998.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{4-\operatorname{sech}^2(x)}}\,dx = \int \frac{\operatorname{sech}^2(x)}{\sqrt{-(\operatorname{sech}(x)-2)(\operatorname{sech}(x)+2)}}\,dx$$

input
```
integrate(sech(x)**2/(4-sech(x)**2)**(1/2),x)
```

output
```
Integral(sech(x)**2/sqrt(-(sech(x) - 2)*(sech(x) + 2)), x)
```

3.998.7 Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{4-\operatorname{sech}^2(x)}}\,dx = \int \frac{\operatorname{sech}(x)^2}{\sqrt{-\operatorname{sech}(x)^2+4}}\,dx$$

input
```
integrate(sech(x)^2/(4-sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sech(x)^2/sqrt(-sech(x)^2 + 4), x)
```

3.998.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 4.89

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{4-\operatorname{sech}^2(x)}}\,dx = -\log\left(\sqrt{e^{(4x)}+e^{(2x)}+1}-e^{(2x)}\right) + \log\left(-\sqrt{e^{(4x)}+e^{(2x)}+1}+e^{(2x)}+2\right)$$

input
```
integrate(sech(x)^2/(4-sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(sqrt(e^(4*x) + e^(2*x) + 1) - e^(2*x)) + log(-sqrt(e^(4*x) + e^(2*x)
+ 1) + e^(2*x) + 2)
```

3.998.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{sech}^2(x)}{\sqrt{4-\mathrm{sech}^2(x)}}\,dx=\int \frac{1}{\cosh\left(x\right)^2\,\sqrt{4-\frac{1}{\cosh\left(x\right)^2}}}\,dx$$

input
```
int(1/(cosh(x)^2*(4 - 1/cosh(x)^2)^(1/2)),x)
```

output
```
int(1/(cosh(x)^2*(4 - 1/cosh(x)^2)^(1/2)), x)
```

3.999 $\int \frac{\text{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx$

3.999.1 Optimal result

Integrand size = 17, antiderivative size = 9

$$\int \frac{\text{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx=\frac{1}{2}\arcsin(2\tanh(x))$$

output
```
1/2*arcsin(2*tanh(x))
```

3.999.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(9) = 18.

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 5.22

$$\int \frac{\text{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx=\frac{\text{arctanh}\left(\frac{2\sinh(x)}{\sqrt{-1+3\sinh^2(x)}}\right)\sqrt{-5+3\cosh(2x)}\,\text{sech}(x)}{2\sqrt{2-8\tanh^2(x)}}$$

input
```
Integrate[Sech[x]^2/Sqrt[1 - 4*Tanh[x]^2],x]
```

output
```
(ArcTanh[(2*Sinh[x])/Sqrt[-1 + 3*Sinh[x]^2]]*Sqrt[-5 + 3*Cosh[2*x]]*Sech[x
])/(2*Sqrt[2 - 8*Tanh[x]^2])
```

3.999.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4158, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ix)^2}{\sqrt{1+4\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4158}$$

$$\int \frac{1}{\sqrt{1-4\tanh^2(x)}}d\tanh(x)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\arcsin(2\tanh(x))$$

input
```
Int[Sech[x]^2/Sqrt[1 - 4*Tanh[x]^2],x]
```

output
```
ArcSin[2*Tanh[x]]/2
```

3.999.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.999.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\arcsin(2\tanh(x))}{2}$	8
default	$\frac{\arcsin(2\tanh(x))}{2}$	8

input
```
int(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(2*tanh(x))
```

3.999.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(7) = 14.

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 13.11

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx =$$

$$-\frac{1}{2}\arctan\left(\frac{2\sqrt{2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\sqrt{-\frac{3\cos}{\cosh(x)^2-}}}{3\cosh(x)^4+12\cosh(x)\sinh(x)^3+3\sinh(x)^4+2\left(9\cosh(x)^2-5\right)\sinh(x)^2-10\cosh(}\right.$$

input
```
integrate(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*arctan(2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt
(-(3*cosh(x)^2 + 3*sinh(x)^2 - 5)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2))/(3*cosh(x)^4 + 12*cosh(x)*sinh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 -
5)*sinh(x)^2 - 10*cosh(x)^2 + 4*(3*cosh(x)^3 - 5*cosh(x))*sinh(x) + 3))
```

3.999.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx = \int \frac{\operatorname{sech}^2(x)}{\sqrt{-(2\tanh(x)-1)(2\tanh(x)+1)}}\,dx$$

input
```
integrate(sech(x)**2/(1-4*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sech(x)**2/sqrt(-(2*tanh(x) - 1)*(2*tanh(x) + 1)), x)
```

3.999.7 Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx = \int \frac{\operatorname{sech}(x)^2}{\sqrt{-4\tanh(x)^2+1}}\,dx$$

input
```
integrate(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sech(x)^2/sqrt(-4*tanh(x)^2 + 1), x)
```

3.999.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(7) = 14$.

Time $= 0.27$ (sec) , antiderivative size $= 44$, normalized size of antiderivative $= 4.89$

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx = -\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{3}\sqrt{-3e^{(4x)}+10e^{(2x)}-3}-4\right)}{3e^{(2x)}-5}-1\right)\right)$$

input
```
integrate(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-arctan(1/3*sqrt(3)*(2*(sqrt(3)*sqrt(-3*e^(4*x) + 10*e^(2*x) - 3) - 4)/(3*
e^(2*x) - 5) - 1))
```

3.999.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx=\int \frac{1}{\cosh(x)^2\sqrt{1-4\tanh(x)^2}}\,dx$$

input
```
int(1/(cosh(x)^2*(1 - 4*tanh(x)^2)^(1/2)),x)
```

output
```
int(1/(cosh(x)^2*(1 - 4*tanh(x)^2)^(1/2)), x)
```

3.1000 $\int \frac{\text{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}} \, dx$

3.1000.1 Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \frac{\text{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}} \, dx = \text{arctanh}\left(\frac{\tanh(x)}{\sqrt{-4+\tanh^2(x)}}\right)$$

output
```
arctanh(tanh(x)/(-4+tanh(x)^2)^(1/2))
```

3.1000.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(14) = 28.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.29

$$\int \frac{\text{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}} \, dx = \frac{\arctan\left(\frac{\sinh(x)}{\sqrt{4+3\sinh^2(x)}}\right)\sqrt{5+3\cosh(2x)}\text{sech}(x)}{\sqrt{2}\sqrt{-4+\tanh^2(x)}}$$

input
```
Integrate[Sech[x]^2/Sqrt[-4 + Tanh[x]^2],x]
```

output
```
(ArcTan[Sinh[x]/Sqrt[4 + 3*Sinh[x]^2]]*Sqrt[5 + 3*Cosh[2*x]]*Sech[x])/(Sqr
t[2]*Sqrt[-4 + Tanh[x]^2])
```

3.1000.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4158, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{\tanh^2(x)-4}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(ix)^2}{\sqrt{-4-\tan(ix)^2}}dx$$
$$\Big\downarrow \text{ 4158}$$
$$\int \frac{1}{\sqrt{\tanh^2(x)-4}}d\tanh(x)$$
$$\Big\downarrow \text{ 224}$$
$$\int \frac{1}{1-\frac{\tanh^2(x)}{\tanh^2(x)-4}}d\frac{\tanh(x)}{\sqrt{\tanh^2(x)-4}}$$
$$\Big\downarrow \text{ 219}$$
$$\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{\tanh^2(x)-4}}\right)$$

input
```
Int[Sech[x]^2/Sqrt[-4 + Tanh[x]^2],x]
```

output
```
ArcTanh[Tanh[x]/Sqrt[-4 + Tanh[x]^2]]
```

3.1000.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.1000.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\ln\left(\tanh\left(x\right)+\sqrt{-4+\tanh\left(x\right)^{2}}\right)$	13
default	$\ln\left(\tanh\left(x\right)+\sqrt{-4+\tanh\left(x\right)^{2}}\right)$	13

input
```
int(sech(x)^2/(-4+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(tanh(x)+(-4+tanh(x)^2)^(1/2))
```

3.1000.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 11.00

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx$$
$$= \frac{1}{2}\log\left(\frac{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+\sqrt{2}\sqrt{-\frac{3\cosh(x)^2+3\sinh(x)^2+5}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}}-1}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2}\right)$$
$$-\frac{1}{2}\log\left(\frac{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-\sqrt{2}\sqrt{-\frac{3\cosh(x)^2+3\sinh(x)^2+5}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}}-1}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2}\right)$$

input
```
integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(2)*sqrt(-(3*cosh
(x)^2 + 3*sinh(x)^2 + 5)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)
/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1/2*log((cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - sqrt(2)*sqrt(-(3*cosh(x)^2 + 3*sinh(x)^2 + 5)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2))
```

3.1000.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx = \int \frac{\operatorname{sech}^2(x)}{\sqrt{(\tanh(x)-2)(\tanh(x)+2)}}\,dx$$

input
```
integrate(sech(x)**2/(-4+tanh(x)**2)**(1/2),x)
```

output
```
Integral(sech(x)**2/sqrt((tanh(x) - 2)*(tanh(x) + 2)), x)
```

3.1000.7 Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx = \int \frac{\operatorname{sech}(x)^2}{\sqrt{\tanh(x)^2-4}}\,dx$$

input
```
integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sech(x)^2/sqrt(tanh(x)^2 - 4), x)
```

3.1000.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 6.07

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx = \log\left(\frac{8}{3}\sqrt{3}\left(2i\sqrt{3}-3\right)-8\sqrt{3}e^{(2x)}+8\sqrt{3e^{(4x)}+10e^{(2x)}+3}\right) - \log\left(\frac{8}{3}\sqrt{3}\left(-2i\sqrt{3}-3\right)-8\sqrt{3}e^{(2x)}+8\sqrt{3e^{(4x)}+10e^{(2x)}+3}\right)$$

input
```
integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
log(8/3*sqrt(3)*(2*I*sqrt(3) - 3) - 8*sqrt(3)*e^(2*x) + 8*sqrt(3*e^(4*x) +
 10*e^(2*x) + 3)) - log(8/3*sqrt(3)*(-2*I*sqrt(3) - 3) - 8*sqrt(3)*e^(2*x)
 + 8*sqrt(3*e^(4*x) + 10*e^(2*x) + 3))
```

3.1000.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx = \int \frac{1}{\cosh(x)^2\sqrt{\tanh(x)^2-4}}\,dx$$

input
```
int(1/(cosh(x)^2*(tanh(x)^2 - 4)^(1/2)),x)
```

output
```
int(1/(cosh(x)^2*(tanh(x)^2 - 4)^(1/2)), x)
```

3.1001 $\int \sqrt{1+\coth^2(x)}\operatorname{sech}^2(x)\,dx$

3.1001.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \sqrt{1+\coth^2(x)}\operatorname{sech}^2(x)\,dx = -\operatorname{arcsinh}(\coth(x))+\sqrt{1+\coth^2(x)}\tanh(x)$$

output
```
-arcsinh(coth(x))+(1+coth(x)^2)^(1/2)*tanh(x)
```

3.1001.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(19) = 38.

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.68

$$\int \sqrt{1+\coth^2(x)}\operatorname{sech}^2(x)\,dx = \sqrt{1+\coth^2(x)}\operatorname{sech}(2x)\sinh(x)\left(\cosh(x) - \arctan\left(\frac{\cosh(x)}{\sqrt{-\cosh(2x)}}\right)\sqrt{-\cosh(2x)} + \sinh(x)\tanh(x)\right)$$

input
```
Integrate[Sqrt[1 + Coth[x]^2]*Sech[x]^2,x]
```

output
```
Sqrt[1 + Coth[x]^2]*Sech[2*x]*Sinh[x]*(Cosh[x] - ArcTan[Cosh[x]/Sqrt[-Cosh
[2*x]]]*Sqrt[-Cosh[2*x]] + Sinh[x]*Tanh[x])
```

3.1001.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4146, 247, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\coth^2(x)+1}\text{sech}^2(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{1-\tan\left(\frac{\pi}{2}+ix\right)^2}}{\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \; 4146$$

$$-\int \sqrt{\coth^2(x)+1}\tanh^2(x)d\coth(x)$$

$$\Big\downarrow \; 247$$

$$\tanh(x)\sqrt{\coth^2(x)+1}-\int \frac{1}{\sqrt{\coth^2(x)+1}}d\coth(x)$$

$$\Big\downarrow \; 222$$

$$\tanh(x)\sqrt{\coth^2(x)+1}-\text{arcsinh}(\coth(x))$$

input
```
Int[Sqrt[1 + Coth[x]^2]*Sech[x]^2,x]
```

output
```
-ArcSinh[Coth[x]] + Sqrt[1 + Coth[x]^2]*Tanh[x]
```

3.1001.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.1001.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

method	result	size
derivativedivides	$\frac{\left(1+\coth(x)^2\right)^{\frac{3}{2}}}{\coth(x)} - \coth(x)\sqrt{1+\coth(x)^2} - \operatorname{arcsinh}(\coth(x))$	32
default	$\frac{\left(1+\coth(x)^2\right)^{\frac{3}{2}}}{\coth(x)} - \coth(x)\sqrt{1+\coth(x)^2} - \operatorname{arcsinh}(\coth(x))$	32

input
```
int(sech(x)^2*(1+coth(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/coth(x)*(1+coth(x)^2)^(3/2)-coth(x)*(1+coth(x)^2)^(1/2)-arcsinh(coth(x))
```

3.1001.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 11.53

$$\int \sqrt{1+\coth^2(x)}\operatorname{sech}^2(x)\,dx = -\frac{\left(\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\log\left(\frac{\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2+2\,\sqrt{\frac{\cosh(x)^2+\sinh(}{\cosh(x)^2-2\,\cosh(x)\sinh(}}}{\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2}\right.}{}$$

input
```
integrate(sech(x)^2*(1+coth(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2)) - 4*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

3.1001.6 Sympy [F]

$$\int \sqrt{1+\coth^2(x)}\operatorname{sech}^2(x)\,dx = \int \sqrt{\coth^2(x)+1}\,\operatorname{sech}^2(x)\,dx$$

input
```
integrate(sech(x)**2*(1+coth(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(coth(x)**2 + 1)*sech(x)**2, x)
```

3.1001.7 Maxima [F]

$$\int \sqrt{1+\coth^2(x)}\text{sech}^2(x)\,dx=\int \sqrt{\coth(x)^2+1}\,\text{sech}(x)^2\,dx$$

input
```
integrate(sech(x)^2*(1+coth(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(coth(x)^2 + 1)*sech(x)^2, x)
```

3.1001.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(17) = 34.

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 6.32

$$\int \sqrt{1+\coth^2(x)}\text{sech}^2(x)\,dx$$
$$=\frac{1}{2}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{e^{(4x)}+1}-2e^{(2x)}+2\right|}{2\left(\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)}\right)-\frac{4\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-2\sqrt{e^{(4x)}+1}+2e^{(2x)}-1}\right)$$

input
```
integrate(sech(x)^2*(1+coth(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(e^(4*x) + 1) - 2*e^(2
*x) + 2)/(sqrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) + 1)) - 4*(sqrt(e^(4*x) +
1) - e^(2*x) + 1)/((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) +
 2*e^(2*x) - 1))*sgn(e^(2*x) - 1)
```

3.1001.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\coth^2(x)}\text{sech}^2(x)\,dx=\int \frac{\sqrt{\coth(x)^2+1}}{\cosh(x)^2}\,dx$$

input
```
int((coth(x)^2 + 1)^(1/2)/cosh(x)^2,x)
```

output
```
int((coth(x)^2 + 1)^(1/2)/cosh(x)^2, x)
```

3.1002 $\int \text{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx$

3.1002.1 Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \text{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx=\frac{1}{2}\text{arcsinh}(\tanh(x))+\frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)}$$

output
```
1/2*arcsinh(tanh(x))+1/2*(1+tanh(x)^2)^(1/2)*tanh(x)
```

3.1002.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 55 vs. 2(24) = 48.

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.29

$$\begin{aligned}&\int \text{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx\\&=\frac{1}{4}\text{sech}(x)\text{sech}(2x)\left(2\text{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\cosh^2(x)\sqrt{\cosh(2x)}-\sinh(x)\right.\\&\left.+\sinh(3x)\right)\sqrt{1+\tanh^2(x)}\end{aligned}$$

input
```
Integrate[Sech[x]^2*Sqrt[1 + Tanh[x]^2],x]
```

output
```
(Sech[x]*Sech[2*x]*(2*ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]]*Cosh[x]^2*Sqrt[Cosh
[2*x]] - Sinh[x] + Sinh[3*x])*Sqrt[1 + Tanh[x]^2])/4
```

3.1002.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4158, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tanh^2(x)+1}\text{sech}^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{1-\tan(ix)^2}\sec(ix)^2dx$$

$$\Big\downarrow \text{4158}$$

$$\int \sqrt{\tanh^2(x)+1}d\tanh(x)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\tanh^2(x)+1}}d\tanh(x)+\frac{1}{2}\sqrt{\tanh^2(x)+1}\tanh(x)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\text{arcsinh}(\tanh(x))+\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

input
```
Int[Sech[x]^2*Sqrt[1 + Tanh[x]^2],x]
```

output
```
ArcSinh[Tanh[x]]/2 + (Tanh[x]*Sqrt[1 + Tanh[x]^2])/2
```

3.1002.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.1002.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(\tanh(x))}{2}+\frac{\sqrt{1+\tanh(x)^2}\,\tanh(x)}{2}$	19
default	$\frac{\operatorname{arcsinh}(\tanh(x))}{2}+\frac{\sqrt{1+\tanh(x)^2}\,\tanh(x)}{2}$	19

input
```
int(sech(x)^2*(1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(tanh(x))+1/2*(1+tanh(x)^2)^(1/2)*tanh(x)
```

3.1002.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 334, normalized size of antiderivative = 13.92

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx$$

$$= \frac{\left(\cosh(x)^4 + 4\,\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\,\cosh(x)^2+1\right)\sinh(x)^2 + 2\,\cosh(x)^2 + 4\left(\cosh(x)^3 + \right.\right.}{}$$

input
```
integrate(sech(x)^2*(1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*((cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*si
nh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log((cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2)) - (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x
)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*lo
g((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)
^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2)) + 4*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 -
1)*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sin
h(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

3.1002.6 Sympy [F]

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx = \int \sqrt{\tanh^2(x)+1}\operatorname{sech}^2(x)\,dx$$

input
```
integrate(sech(x)**2*(1+tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(tanh(x)**2 + 1)*sech(x)**2, x)
```

3.1002.7 Maxima [F]

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx = \int \sqrt{\tanh(x)^2+1}\operatorname{sech}(x)^2\,dx$$

input
```
integrate(sech(x)^2*(1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(tanh(x)^2 + 1)*sech(x)^2, x)
```

3.1002.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 6.04

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx$$
$$=\frac{1}{4}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)-\frac{4\left(3\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^3-\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-\right.}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-2\sqrt{e^{(4x)}+1}+2\right.}\right.$$

input
```
integrate(sech(x)^2*(1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqrt
(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(3*(sqrt(e^(4*x) + 1) - e^(2*x
))^3 - (sqrt(e^(4*x) + 1) - e^(2*x))^2 - sqrt(e^(4*x) + 1) + e^(2*x) - 1)/
((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2*x) - 1)^2)
```

3.1002.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx=\int \frac{\sqrt{\tanh(x)^2+1}}{\cosh(x)^2}\,dx$$

input
```
int((tanh(x)^2 + 1)^(1/2)/cosh(x)^2,x)
```

output
```
int((tanh(x)^2 + 1)^(1/2)/cosh(x)^2, x)
```

3.1003 $\int \mathrm{sech}^4(x) \left(-1+\mathrm{sech}^2(x)\right)^2 \tanh(x) \, dx$

3.1003.1 Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \mathrm{sech}^4(x) \left(-1+\mathrm{sech}^2(x)\right)^2 \tanh(x) \, dx = \frac{\tanh^6(x)}{6} - \frac{\tanh^8(x)}{8}$$

output
```
1/6*tanh(x)^6-1/8*tanh(x)^8
```

3.1003.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \mathrm{sech}^4(x) \left(-1+\mathrm{sech}^2(x)\right)^2 \tanh(x) \, dx = -\frac{1}{4}\mathrm{sech}^4(x) + \frac{\mathrm{sech}^6(x)}{3} - \frac{\mathrm{sech}^8(x)}{8}$$

input
```
Integrate[Sech[x]^4*(-1 + Sech[x]^2)^2*Tanh[x],x]
```

output
```
-1/4*Sech[x]^4 + Sech[x]^6/3 - Sech[x]^8/8
```

3.1003.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 26, 4608, 26, 3042, 26, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\text{sech}^4(x)\left(\text{sech}^2(x)-1\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)\sec(ix)^4\left(-1+\sec(ix)^2\right)^2dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec(ix)^4\left(1-\sec(ix)^2\right)^2\tan(ix)dx$$

$$\Big\downarrow \text{4608}$$

$$-i\int i\text{sech}^4(x)\tanh^5(x)dx$$

$$\Big\downarrow \text{26}$$

$$\int \tanh^5(x)\text{sech}^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)^5\sec(ix)^4dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec(ix)^4\tan(ix)^5dx$$

$$\Big\downarrow \text{3087}$$

$$-\int i\tanh^5(x)\left(1-\tanh^2(x)\right)d(i\tanh(x))$$

$$\Big\downarrow \text{244}$$

$$-\int \left(i\tanh^5(x)-i\tanh^7(x)\right)d(i\tanh(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tanh^6(x)}{6} - \frac{\tanh^8(x)}{8}$$

input
```
Int[Sech[x]^4*(-1 + Sech[x]^2)^2*Tanh[x],x]
```

output
```
Tanh[x]^6/6 - Tanh[x]^8/8
```

3.1003.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.1003.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\frac{\tanh(x)^6}{6} - \frac{\tanh(x)^8}{8}$$

input
```
int(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x)
```

output
```
1/6*tanh(x)^6-1/8*tanh(x)^8
```

3.1003.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 340, normalized size of antiderivative = 20.00

$$\int \text{sech}^4(x) \left(-1 + \text{sech}^2(x)\right)^2 \tanh(x)\,dx =$$

$$-\frac{}{3\left(\cosh(x)^{10} + 10\cosh(x)\sinh(x)^9 + \sinh(x)^{10} + \left(45\cosh(x)^2 + 8\right)\sinh(x)^8 + 8\cosh(x)^8 + 8\,(15\text{ c}\right.}$$

input
```
integrate(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x, algorithm="fricas")
```

output
```
-4/3*(3*cosh(x)^6 + 18*cosh(x)*sinh(x)^5 + 3*sinh(x)^6 + (45*cosh(x)^2 - 4
)*sinh(x)^4 - 4*cosh(x)^4 + 4*(15*cosh(x)^3 - 4*cosh(x))*sinh(x)^3 + (45*c
osh(x)^4 - 24*cosh(x)^2 + 13)*sinh(x)^2 + 13*cosh(x)^2 + 2*(9*cosh(x)^5 -
8*cosh(x)^3 + 7*cosh(x))*sinh(x) - 4)/(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 +
 sinh(x)^10 + (45*cosh(x)^2 + 8)*sinh(x)^8 + 8*cosh(x)^8 + 8*(15*cosh(x)^3
 + 8*cosh(x))*sinh(x)^7 + (210*cosh(x)^4 + 224*cosh(x)^2 + 29)*sinh(x)^6 +
 29*cosh(x)^6 + 2*(126*cosh(x)^5 + 224*cosh(x)^3 + 81*cosh(x))*sinh(x)^5 +
 (210*cosh(x)^6 + 560*cosh(x)^4 + 435*cosh(x)^2 + 64)*sinh(x)^4 + 64*cosh(
x)^4 + 4*(30*cosh(x)^7 + 112*cosh(x)^5 + 135*cosh(x)^3 + 48*cosh(x))*sinh(
x)^3 + (45*cosh(x)^8 + 224*cosh(x)^6 + 435*cosh(x)^4 + 384*cosh(x)^2 + 98)
*sinh(x)^2 + 98*cosh(x)^2 + 2*(5*cosh(x)^9 + 32*cosh(x)^7 + 81*cosh(x)^5 +
 96*cosh(x)^3 + 42*cosh(x))*sinh(x) + 56)
```

3.1003.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \operatorname{sech}^4(x) \left(-1+\operatorname{sech}^2(x)\right)^2 \tanh(x)\, dx = -\frac{\operatorname{sech}^8(x)}{8} + \frac{\operatorname{sech}^6(x)}{3} - \frac{\operatorname{sech}^4(x)}{4}$$

input
```
integrate(sech(x)**4*(-1+sech(x)**2)**2*tanh(x),x)
```

output
```
-sech(x)**8/8 + sech(x)**6/3 - sech(x)**4/4
```

3.1003.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(13) = 26.

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^4(x) \left(-1+\operatorname{sech}^2(x)\right)^2 \tanh(x)\, dx = -\frac{4}{\left(e^{(-x)}+e^x\right)^4} + \frac{64}{3\left(e^{(-x)}+e^x\right)^6} - \frac{32}{\left(e^{(-x)}+e^x\right)^8}$$

input
```
integrate(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x, algorithm="maxima")
```

output
```
-4/(e^(-x) + e^x)^4 + 64/3/(e^(-x) + e^x)^6 - 32/(e^(-x) + e^x)^8
```

3.1003.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.41

$$\int \operatorname{sech}^4(x) \left(-1+\operatorname{sech}^2(x)\right)^2 \tanh(x)\, dx$$
$$= -\frac{4\left(3\,e^{(12\,x)} - 4\,e^{(10\,x)} + 10\,e^{(8\,x)} - 4\,e^{(6\,x)} + 3\,e^{(4\,x)}\right)}{3\left(e^{(2\,x)}+1\right)^8}$$

input
```
integrate(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x, algorithm="giac")
```

output
```
-4/3*(3*e^(12*x) - 4*e^(10*x) + 10*e^(8*x) - 4*e^(6*x) + 3*e^(4*x))/(e^(2*
x) + 1)^8
```

3.1003.9 Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 375, normalized size of antiderivative = 22.06

$$\int \operatorname{sech}^4(x) \left(-1+\operatorname{sech}^2(x)\right)^2 \tanh(x)\,dx$$
$$= \frac{e^{2x}-5e^{4x}+10e^{6x}-10e^{8x}+5e^{10x}-e^{12x}}{8e^{2x}+28e^{4x}+56e^{6x}+70e^{8x}+56e^{10x}+28e^{12x}+8e^{14x}+e^{16x}+1}$$
$$-\frac{\frac{20e^{4x}}{7}-\frac{10e^{2x}}{7}-\frac{50e^{6x}}{21}+\frac{5e^{8x}}{7}+\frac{5}{21}}{6e^{2x}+15e^{4x}+20e^{6x}+15e^{8x}+6e^{10x}+e^{12x}+1}$$
$$-\frac{\frac{8e^{2x}}{7}-\frac{10e^{4x}}{7}+\frac{4e^{6x}}{7}-\frac{2}{7}}{5e^{2x}+10e^{4x}+10e^{6x}+5e^{8x}+e^{10x}+1}$$
$$-\frac{\frac{10e^{2x}}{7}-\frac{30e^{4x}}{7}+\frac{40e^{6x}}{7}-\frac{25e^{8x}}{7}+\frac{6e^{10x}}{7}-\frac{1}{7}}{7e^{2x}+21e^{4x}+35e^{6x}+35e^{8x}+21e^{10x}+7e^{12x}+e^{14x}+1}$$
$$-\frac{\frac{2e^{2x}}{7}-\frac{5}{21}}{3e^{2x}+3e^{4x}+e^{6x}+1}-\frac{1}{7\left(2e^{2x}+e^{4x}+1\right)}-\frac{\frac{3e^{4x}}{7}-\frac{5e^{2x}}{7}+\frac{2}{7}}{4e^{2x}+6e^{4x}+4e^{6x}+e^{8x}+1}$$

input
```
int((tanh(x)*(1/cosh(x)^2 - 1)^2)/cosh(x)^4,x)
```

output
```
(exp(2*x) - 5*exp(4*x) + 10*exp(6*x) - 10*exp(8*x) + 5*exp(10*x) - exp(12*
x))/(8*exp(2*x) + 28*exp(4*x) + 56*exp(6*x) + 70*exp(8*x) + 56*exp(10*x) +
 28*exp(12*x) + 8*exp(14*x) + exp(16*x) + 1) - ((20*exp(4*x))/7 - (10*exp(
2*x))/7 - (50*exp(6*x))/21 + (5*exp(8*x))/7 + 5/21)/(6*exp(2*x) + 15*exp(4
*x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + exp(12*x) + 1) - ((8*exp(2
*x))/7 - (10*exp(4*x))/7 + (4*exp(6*x))/7 - 2/7)/(5*exp(2*x) + 10*exp(4*x)
 + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1) - ((10*exp(2*x))/7 - (30*exp(
4*x))/7 + (40*exp(6*x))/7 - (25*exp(8*x))/7 + (6*exp(10*x))/7 - 1/7)/(7*ex
p(2*x) + 21*exp(4*x) + 35*exp(6*x) + 35*exp(8*x) + 21*exp(10*x) + 7*exp(12
*x) + exp(14*x) + 1) - ((2*exp(2*x))/7 - 5/21)/(3*exp(2*x) + 3*exp(4*x) +
exp(6*x) + 1) - 1/(7*(2*exp(2*x) + exp(4*x) + 1)) - ((3*exp(4*x))/7 - (5*e
xp(2*x))/7 + 2/7)/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)
```

3.1004 $\int e^{n \sinh(a+bx)} \sinh(2a+2bx)\, dx$

3.1004.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int e^{n \sinh(a+bx)} \sinh(2a+2bx)\, dx = -\frac{2e^{n \sinh(a+bx)}}{bn^2} + \frac{2e^{n \sinh(a+bx)} \sinh(a+bx)}{bn}$$

output
```
-2*exp(n*sinh(b*x+a))/b/n^2+2*exp(n*sinh(b*x+a))*sinh(b*x+a)/b/n
```

3.1004.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int e^{n \sinh(a+bx)} \sinh(2a+2bx)\, dx = \frac{2e^{n \sinh(a+bx)}(-1+n \sinh(a+bx))}{bn^2}$$

input
```
Integrate[E^(n*Sinh[a + b*x])*Sinh[2*a + 2*b*x],x]
```

output
```
(2*E^(n*Sinh[a + b*x])*(-1 + n*Sinh[a + b*x]))/(b*n^2)
```

3.1004.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4878, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(2a+2bx)e^{n\sinh(a+bx)}\,dx$$

$$\Big\downarrow \text{4878}$$

$$\frac{\int 2e^{n\sinh(a+bx)}\sinh(a+bx)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int e^{n\sinh(a+bx)}\sinh(a+bx)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{2\left(\frac{\sinh(a+bx)e^{n\sinh(a+bx)}}{n}-\frac{\int e^{n\sinh(a+bx)}d\sinh(a+bx)}{n}\right)}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{2\left(\frac{\sinh(a+bx)e^{n\sinh(a+bx)}}{n}-\frac{e^{n\sinh(a+bx)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Sinh[a + b*x])*Sinh[2*a + 2*b*x],x]
```

output
```
(2*(-(E^(n*Sinh[a + b*x])/n^2) + (E^(n*Sinh[a + b*x])*Sinh[a + b*x])/n))/b
```

3.1004.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.1004.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

method	result	size
risch	$\frac{\left(n\,{\mathrm e}^{2 b x +2 a}-n -2\,{\mathrm e}^{b x +a}\right) {\mathrm e}^{-b x -a -\frac{n\,{\mathrm e}^{-b x -a}}{2}+\frac{n\,{\mathrm e}^{b x +a}}{2}}}{n^{2} b}$	61

input
```
int(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/n^2/b*(n*exp(2*b*x+2*a)-n-2*exp(b*x+a))*exp(-b*x-a-1/2*n*exp(-b*x-a)+1/2
*n*exp(b*x+a))
```

3.1004.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx$$
$$=\frac{2\left(\left(n\sinh\left(bx+a\right)-1\right)\cosh\left(n\sinh\left(bx+a\right)\right)+\left(n\sinh\left(bx+a\right)-1\right)\sinh\left(n\sinh\left(bx+a\right)\right)\right)}{bn^2\cosh\left(bx+a\right)^2-bn^2\sinh\left(bx+a\right)^2}$$

input
```
integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
```

output
```
2*((n*sinh(b*x + a) - 1)*cosh(n*sinh(b*x + a)) + (n*sinh(b*x + a) - 1)*sin
h(n*sinh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

3.1004.6 Sympy [F]

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx=\int e^{n\sinh{(a+bx)}}\sinh{(2a+2bx)}\,dx$$

input
```
integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x)
```

output
```
Integral(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x), x)
```

3.1004.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.42

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx=\frac{e^{\left(bx+\frac{1}{2}ne^{(bx+a)}-\frac{1}{2}ne^{(-bx-a)}+a\right)}}{bn}-\frac{e^{\left(-bx+\frac{1}{2}ne^{(bx+a)}-\frac{1}{2}ne^{(-bx-a)}-a\right)}}{bn}-\frac{2\,e^{\left(\frac{1}{2}ne^{(bx+a)}-\frac{1}{2}ne^{(-bx-a)}\right)}}{bn^2}$$

input
```
integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
```

output
```
e^(b*x + 1/2*n*e^(b*x + a) - 1/2*n*e^(-b*x - a) + a)/(b*n) - e^(-b*x + 1/2
*n*e^(b*x + a) - 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) -
1/2*n*e^(-b*x - a))/(b*n^2)
```

3.1004.8 Giac [F]

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx = \int e^{(n\sinh(bx+a))}\sinh(2\,bx+2\,a)\,dx$$

input
```
integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
```

output
```
integrate(e^(n*sinh(b*x + a))*sinh(2*b*x + 2*a), x)
```

3.1004.9 Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.51

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx = \frac{e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{b\,x}\,e^{a}\,e^{-\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n} - \frac{e^{-a}\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{-b\,x}\,e^{-\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n} - \frac{2\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{-\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n^2}$$

input
```
int(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x),x)
```

output
```
(exp((n*exp(b*x)*exp(a))/2)*exp(b*x)*exp(a)*exp(-(n*exp(-a)*exp(-b*x))/2))
/(b*n) - (exp(-a)*exp((n*exp(b*x)*exp(a))/2)*exp(-b*x)*exp(-(n*exp(-a)*exp
(-b*x))/2))/(b*n) - (2*exp((n*exp(b*x)*exp(a))/2)*exp(-(n*exp(-a)*exp(-b*x
))/2))/(b*n^2)
```

3.1005 $\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx$

3.1005.1 Optimal result

Integrand size = 19, antiderivative size = 43

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx=-\frac{2e^{n\sinh(a+bx)}}{bn^2}+\frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn}$$

output
```
-2*exp(n*sinh(b*x+a))/b/n^2+2*exp(n*sinh(b*x+a))*sinh(b*x+a)/b/n
```

3.1005.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx=\frac{2e^{n\sinh(a+bx)}(-1+n\sinh(a+bx))}{bn^2}$$

input
```
Integrate[E^(n*Sinh[a + b*x])*Sinh[2*(a + b*x)],x]
```

output
```
(2*E^(n*Sinh[a + b*x])*(-1 + n*Sinh[a + b*x]))/(b*n^2)
```

3.1005.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {4878, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(2(a+bx))e^{n\sinh(a+bx)}\,dx$$

$$\Big\downarrow \text{4878}$$

$$\frac{\int 2e^{n\sinh(a+bx)}\sinh(a+bx)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int e^{n\sinh(a+bx)}\sinh(a+bx)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{2\left(\frac{\sinh(a+bx)e^{n\sinh(a+bx)}}{n}-\frac{\int e^{n\sinh(a+bx)}d\sinh(a+bx)}{n}\right)}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{2\left(\frac{\sinh(a+bx)e^{n\sinh(a+bx)}}{n}-\frac{e^{n\sinh(a+bx)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Sinh[a + b*x])*Sinh[2*(a + b*x)],x]
```

output
```
(2*(-(E^(n*Sinh[a + b*x])/n^2) + (E^(n*Sinh[a + b*x])*Sinh[a + b*x])/n))/b
```

3.1005.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.1005.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

method	result	size
risch	$\frac{\left(n\,\mathrm{e}^{2bx+2a}-n-2\,\mathrm{e}^{bx+a}\right)\mathrm{e}^{-bx-a-\frac{n\,\mathrm{e}^{-bx-a}}{2}+\frac{n\,\mathrm{e}^{bx+a}}{2}}}{n^{2}b}$	61

input
```
int(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/n^2/b*(n*exp(2*b*x+2*a)-n-2*exp(b*x+a))*exp(-b*x-a-1/2*n*exp(-b*x-a)+1/2
*n*exp(b*x+a))
```

3.1005.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx$$
$$=\frac{2\left(\left(n\sinh\left(bx+a\right)-1\right)\cosh\left(n\sinh\left(bx+a\right)\right)+\left(n\sinh\left(bx+a\right)-1\right)\sinh\left(n\sinh\left(bx+a\right)\right)\right)}{bn^{2}\cosh\left(bx+a\right)^{2}-bn^{2}\sinh\left(bx+a\right)^{2}}$$

input
```
integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
```

output
```
2*((n*sinh(b*x + a) - 1)*cosh(n*sinh(b*x + a)) + (n*sinh(b*x + a) - 1)*sin
h(n*sinh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

3.1005.6 Sympy [F]

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx = \int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx$$

input
```
integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x)
```

output
```
Integral(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x), x)
```

3.1005.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(41) = 82$.

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.42

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx = \frac{e^{\left(bx+\frac{1}{2}ne^{(bx+a)}-\frac{1}{2}ne^{(-bx-a)}+a\right)}}{bn} - \frac{e^{\left(-bx+\frac{1}{2}ne^{(bx+a)}-\frac{1}{2}ne^{(-bx-a)}-a\right)}}{bn} - \frac{2e^{\left(\frac{1}{2}ne^{(bx+a)}-\frac{1}{2}ne^{(-bx-a)}\right)}}{bn^2}$$

input
```
integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
```

output
```
e^(b*x + 1/2*n*e^(b*x + a) - 1/2*n*e^(-b*x - a) + a)/(b*n) - e^(-b*x + 1/2
*n*e^(b*x + a) - 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) -
1/2*n*e^(-b*x - a))/(b*n^2)
```

3.1005.8 Giac [F]

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx = \int e^{(n\sinh(bx+a))}\sinh\left(2\,bx+2\,a\right)\,dx$$

input
```
integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
```

output
```
integrate(e^(n*sinh(b*x + a))*sinh(2*b*x + 2*a), x)
```

3.1005.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.51

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx = \frac{e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{b\,x}\,e^{a}\,e^{-\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n} - \frac{e^{-a}\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{-b\,x}\,e^{-\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n} - \frac{2\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{-\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n^2}$$

input
```
int(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x),x)
```

output
```
(exp((n*exp(b*x)*exp(a))/2)*exp(b*x)*exp(a)*exp(-(n*exp(-a)*exp(-b*x))/2))
/(b*n) - (exp(-a)*exp((n*exp(b*x)*exp(a))/2)*exp(-b*x)*exp(-(n*exp(-a)*exp
(-b*x))/2))/(b*n) - (2*exp((n*exp(b*x)*exp(a))/2)*exp(-(n*exp(-a)*exp(-b*x
))/2))/(b*n^2)
```

3.1006 $\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$

3.1006.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx=-\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

output
```
-4*exp(n*sinh(1/2*a+1/2*b*x))/b/n^2+4*exp(n*sinh(1/2*a+1/2*b*x))*sinh(1/2*
a+1/2*b*x)/b/n
```

3.1006.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx=\frac{4e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

input
```
Integrate[E^(n*Sinh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

output
```
(4*E^(n*Sinh[(a + b*x)/2])*(-1 + n*Sinh[(a + b*x)/2]))/(b*n^2)
```

3.1006.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4878, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}dx$$

$$\Big\downarrow \text{4878}$$

$$\frac{2\int 2e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)d\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)d\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{4\left(\frac{\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}d\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{n}\right)}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{4\left(\frac{\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Sinh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

output
```
(4*(-(E^(n*Sinh[a/2 + (b*x)/2])/n^2) + (E^(n*Sinh[a/2 + (b*x)/2])*Sinh[a/2
 + (b*x)/2])/n))/b
```

3.1006.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.1006.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{2\left(n\,{\mathrm e}^{b x +a}-n -2\,{\mathrm e}^{\frac{a}{2}+\frac{b x}{2}}\right) {\mathrm e}^{-\frac{a}{2}-\frac{b x}{2}-\frac{n\,{\mathrm e}^{-\frac{a}{2}-\frac{b x}{2}}}{2}+\frac{n\,{\mathrm e}^{\frac{a}{2}+\frac{b x}{2}}}{2}}}{n^{2} b}$	65

input
```
int(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/n^2/b*(n*exp(b*x+a)-n-2*exp(1/2*a+1/2*b*x))*exp(-1/2*a-1/2*b*x-1/2*n*exp
(-1/2*a-1/2*b*x)+1/2*n*exp(1/2*a+1/2*b*x))
```

3.1006.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{4\left(\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right.\right.\right.}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

input
```
integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

output
```
4*((n*sinh(1/2*b*x + 1/2*a) - 1)*cosh(n*sinh(1/2*b*x + 1/2*a)) + (n*sinh(1
/2*b*x + 1/2*a) - 1)*sinh(n*sinh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x +
1/2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

3.1006.6 Sympy [F]

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$$

input
```
integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

output
```
Integral(exp(n*sinh(a/2 + b*x/2))*sinh(a + b*x), x)
```

3.1006.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn} - \frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn} - \frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^2}$$

input
```
integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

output
```
2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/
2*a)/(b*n) - 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x
 - 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*
b*x - 1/2*a))/(b*n^2)
```

3.1006.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(50) = 100$.

Time = 0.33 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.98

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$$
$$= \frac{2\left(ne^{\left(bx+\frac{1}{4}\left(2bxe^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}+ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}bx-\frac{1}{2}a\right)}-\frac{1}{4}\left(2bxe^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}-ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}bx-\frac{1}{2}a\right)}+a\right)}-ne^{\left(\frac{1}{4}\left(2bxe^{\left(\frac{1}{2}bx+\cdots\right.\right.\right.}\right.}{\cdots}$$

input
```
integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

output
```
2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)*e^(-1/2*
b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)*e^(-1/2
*b*x - 1/2*a) + a) - n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) -
 n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a)
+ n)*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a)
 + n*e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a
) - n*e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)
/(b*n^2)
```

3.1006.9 Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{a}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n} - \frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n} - \frac{4\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n^2}$$

input
```
int(exp(n*sinh(a/2 + (b*x)/2))*sinh(a + b*x),x)
```

output
```
(2*exp(-a/2)*exp(b*x)*exp(-(b*x)/2)*exp(a)*exp(-(n*exp(-a/2)*exp(-(b*x)/2)
)/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n) - (2*exp(-a/2)*exp(-(b*x)/2)*
exp(-(n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n
) - (4*exp(-(n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2
))/(b*n^2)
```

3.1007 $\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx$

3.1007.1 Optimal result

Integrand size = 21, antiderivative size = 64

$$\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx=-\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

output
```
-4*exp(n*sinh(1/2*a+1/2*b*x))/b/n^2+4*exp(n*sinh(1/2*a+1/2*b*x))*sinh(1/2*
a+1/2*b*x)/b/n
```

3.1007.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx=\frac{4e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

input
```
Integrate[E^(n*Sinh[(a + b*x)/2])*Sinh[a + b*x],x]
```

output
```
(4*E^(n*Sinh[(a + b*x)/2])*(-1 + n*Sinh[(a + b*x)/2]))/(b*n^2)
```

3.1007.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4878, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}dx$$

$$\Big\downarrow \ 4878$$

$$\frac{2\int 2e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)d\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{4\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)d\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \ 2607$$

$$\frac{4\left(\frac{\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}d\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{n}\right)}{b}$$

$$\Big\downarrow \ 2624$$

$$\frac{4\left(\frac{\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Sinh[(a + b*x)/2])*Sinh[a + b*x],x]
```

output
```
(4*(-(E^(n*Sinh[a/2 + (b*x)/2])/n^2) + (E^(n*Sinh[a/2 + (b*x)/2])*Sinh[a/2
 + (b*x)/2])/n))/b
```

3.1007.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.1007.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{2\left(n\,{\mathrm e}^{b x +a}-n -2\,{\mathrm e}^{\frac{a}{2}+\frac{b x}{2}}\right) {\mathrm e}^{-\frac{a}{2}-\frac{b x}{2}-\frac{n\,{\mathrm e}^{-\frac{a}{2}-\frac{b x}{2}}}{2}+\frac{n\,{\mathrm e}^{\frac{a}{2}+\frac{b x}{2}}}{2}}}{n^{2} b}$	65

input
```
int(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/n^2/b*(n*exp(b*x+a)-n-2*exp(1/2*a+1/2*b*x))*exp(-1/2*a-1/2*b*x-1/2*n*exp
(-1/2*a-1/2*b*x)+1/2*n*exp(1/2*a+1/2*b*x))
```

3.1007.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{4\left(\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right.\right.\right.}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

input
```
integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

output
```
4*((n*sinh(1/2*b*x + 1/2*a) - 1)*cosh(n*sinh(1/2*b*x + 1/2*a)) + (n*sinh(1
/2*b*x + 1/2*a) - 1)*sinh(n*sinh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x +
1/2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

3.1007.6 Sympy [F]

$$\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

input
```
integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

output
```
Integral(exp(n*sinh(a/2 + b*x/2))*sinh(a + b*x), x)
```

3.1007.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83

$$\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn} - \frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn} - \frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^2}$$

input
```
integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

output
```
2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/
2*a)/(b*n) - 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x
 - 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*
b*x - 1/2*a))/(b*n^2)
```

3.1007.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(50) = 100$.

Time = 0.32 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.98

$$\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx$$

$$= \frac{2\left(ne^{\left(bx+\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+a\right)} - ne^{\left(\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\right.}\right.\right.}\right.}{}$$

input
```
integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

output
```
2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)*e^(-1/2*
b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)*e^(-1/2
*b*x - 1/2*a) + a) - n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) -
 n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a)
+ n)*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a)
 + n*e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a
) - n*e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)
/(b*n^2)
```

3.1007.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{a}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n} - \frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n} - \frac{4\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n^2}$$

input
```
int(exp(n*sinh(a/2 + (b*x)/2))*sinh(a + b*x),x)
```

output
```
(2*exp(-a/2)*exp(b*x)*exp(-(b*x)/2)*exp(a)*exp(-(n*exp(-a/2)*exp(-(b*x)/2)
)/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n) - (2*exp(-a/2)*exp(-(b*x)/2)*
exp(-(n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n
) - (4*exp(-(n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2
))/(b*n^2)
```

3.1008 $\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx$

3.1008.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx=-\frac{2e^{n\cosh(a+bx)}}{bn^2}+\frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn}$$

output
```
-2*exp(n*cosh(b*x+a))/b/n^2+2*exp(n*cosh(b*x+a))*cosh(b*x+a)/b/n
```

3.1008.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx=\frac{2e^{n\cosh(a+bx)}(-1+n\cosh(a+bx))}{bn^2}$$

input
```
Integrate[E^(n*Cosh[a + b*x])*Sinh[2*a + 2*b*x],x]
```

output
```
(2*E^(n*Cosh[a + b*x])*(-1 + n*Cosh[a + b*x]))/(b*n^2)
```

3.1008.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4879, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(2a+2bx)e^{n\cosh(a+bx)}\,dx$$

$$\Big\downarrow \text{ 4879}$$

$$\frac{i\int -2ie^{n\cosh(a+bx)}\cosh(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\int e^{n\cosh(a+bx)}\cosh(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{ 2607}$$

$$\frac{2\left(\frac{\cosh(a+bx)e^{n\cosh(a+bx)}}{n}-\frac{\int e^{n\cosh(a+bx)}d\cosh(a+bx)}{n}\right)}{b}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{2\left(\frac{\cosh(a+bx)e^{n\cosh(a+bx)}}{n}-\frac{e^{n\cosh(a+bx)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Cosh[a + b*x])*Sinh[2*a + 2*b*x],x]
```

output
```
(2*(-(E^(n*Cosh[a + b*x])/n^2) + (E^(n*Cosh[a + b*x])*Cosh[a + b*x])/n))/b
```

3.1008.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.1008.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

method	result	size
risch	$\frac{\left(n\,\mathrm{e}^{2bx+2a}+n-2\,\mathrm{e}^{bx+a}\right)\mathrm{e}^{-bx-a+\frac{n\,\mathrm{e}^{-bx-a}}{2}+\frac{n\,\mathrm{e}^{bx+a}}{2}}}{n^2 b}$	59

input
```
int(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/n^2/b*(n*exp(2*b*x+2*a)+n-2*exp(b*x+a))*exp(-b*x-a+1/2*n*exp(-b*x-a)+1/2
*n*exp(b*x+a))
```

3.1008.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx = \frac{2\left((n\cosh(bx+a)-1)\cosh(n\cosh(bx+a))+(n\cosh(bx+a)-1)\sinh(n\cosh(bx+a))\right)}{bn^2\cosh(bx+a)^2-bn^2\sinh(bx+a)^2}$$

input
```
integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
```

output
```
2*((n*cosh(b*x + a) - 1)*cosh(n*cosh(b*x + a)) + (n*cosh(b*x + a) - 1)*sin
h(n*cosh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

3.1008.6 Sympy [F]

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx=\int e^{n\cosh{(a+bx)}}\sinh{(2a+2bx)}\,dx$$

input
```
integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x)
```

output
```
Integral(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x), x)
```

3.1008.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(41) = 82.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx=\frac{e^{\left(bx+\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}+a\right)}}{bn}+\frac{e^{\left(-bx+\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}-a\right)}}{bn}-\frac{2\,e^{\left(\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn^2}$$

input
```
integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
```

output
```
e^(b*x + 1/2*n*e^(b*x + a) + 1/2*n*e^(-b*x - a) + a)/(b*n) + e^(-b*x + 1/2
*n*e^(b*x + a) + 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) +
1/2*n*e^(-b*x - a))/(b*n^2)
```

3.1008.8 Giac [F]

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx=\int e^{(n\cosh(bx+a))}\sinh\left(2\,bx+2\,a\right)\,dx$$

input
```
integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
```

output
```
integrate(e^(n*cosh(b*x + a))*sinh(2*b*x + 2*a), x)
```

3.1008.9 Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.49

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx=\frac{e^{-a}\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{-b\,x}\,e^{\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n}-\frac{2\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n^2}+\frac{e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{b\,x}\,e^{a}\,e^{\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n}$$

input
```
int(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x),x)
```

output
```
(exp(-a)*exp((n*exp(b*x)*exp(a))/2)*exp(-b*x)*exp((n*exp(-a)*exp(-b*x))/2)
)/(b*n) - (2*exp((n*exp(b*x)*exp(a))/2)*exp((n*exp(-a)*exp(-b*x))/2))/(b*n
^2) + (exp((n*exp(b*x)*exp(a))/2)*exp(b*x)*exp(a)*exp((n*exp(-a)*exp(-b*x)
)/2))/(b*n)
```

3.1009 $\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx$

3.1009.1 Optimal result

Integrand size = 19, antiderivative size = 43

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx=-\frac{2e^{n\cosh(a+bx)}}{bn^2}+\frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn}$$

output
```
-2*exp(n*cosh(b*x+a))/b/n^2+2*exp(n*cosh(b*x+a))*cosh(b*x+a)/b/n
```

3.1009.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx=\frac{2e^{n\cosh(a+bx)}(-1+n\cosh(a+bx))}{bn^2}$$

input
```
Integrate[E^(n*Cosh[a + b*x])*Sinh[2*(a + b*x)],x]
```

output
```
(2*E^(n*Cosh[a + b*x])*(-1 + n*Cosh[a + b*x]))/(b*n^2)
```

3.1009.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {4879, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(2(a+bx))e^{n\cosh(a+bx)}\,dx$$

$$\Big\downarrow \text{4879}$$

$$\frac{i\int -2ie^{n\cosh(a+bx)}\cosh(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int e^{n\cosh(a+bx)}\cosh(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{2\left(\frac{\cosh(a+bx)e^{n\cosh(a+bx)}}{n}-\frac{\int e^{n\cosh(a+bx)}d\cosh(a+bx)}{n}\right)}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{2\left(\frac{\cosh(a+bx)e^{n\cosh(a+bx)}}{n}-\frac{e^{n\cosh(a+bx)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Cosh[a + b*x])*Sinh[2*(a + b*x)],x]
```

output
```
(2*(-(E^(n*Cosh[a + b*x])/n^2) + (E^(n*Cosh[a + b*x])*Cosh[a + b*x])/n))/b
```

3.1009.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.1009.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

method	result	size
risch	$\frac{\left(n\,\mathrm{e}^{2bx+2a}+n-2\,\mathrm{e}^{bx+a}\right)\mathrm{e}^{-bx-a+\frac{n\,\mathrm{e}^{-bx-a}}{2}+\frac{n\,\mathrm{e}^{bx+a}}{2}}}{n^{2}b}$	59

input
```
int(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/n^2/b*(n*exp(2*b*x+2*a)+n-2*exp(b*x+a))*exp(-b*x-a+1/2*n*exp(-b*x-a)+1/2
*n*exp(b*x+a))
```

3.1009.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \frac{2\,((n\cosh(bx+a)-1)\cosh(n\cosh(bx+a))+(n\cosh(bx+a)-1)\sinh(n\cosh(bx+a)))}{bn^{2}\cosh(bx+a)^{2}-bn^{2}\sinh(bx+a)^{2}}$$

input
```
integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
```

output
```
2*((n*cosh(b*x + a) - 1)*cosh(n*cosh(b*x + a)) + (n*cosh(b*x + a) - 1)*sin
h(n*cosh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

3.1009.6 Sympy [F]

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx$$

input
```
integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x)
```

output
```
Integral(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x), x)
```

3.1009.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(41) = 82.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \frac{e^{\left(bx+\frac{1}{2}ne^{(bx+a)}+\frac{1}{2}ne^{(-bx-a)}+a\right)}}{bn} + \frac{e^{\left(-bx+\frac{1}{2}ne^{(bx+a)}+\frac{1}{2}ne^{(-bx-a)}-a\right)}}{bn} - \frac{2e^{\left(\frac{1}{2}ne^{(bx+a)}+\frac{1}{2}ne^{(-bx-a)}\right)}}{bn^2}$$

input
```
integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
```

output
```
e^(b*x + 1/2*n*e^(b*x + a) + 1/2*n*e^(-b*x - a) + a)/(b*n) + e^(-b*x + 1/2
*n*e^(b*x + a) + 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) +
1/2*n*e^(-b*x - a))/(b*n^2)
```

3.1009.8 Giac [F]

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \int e^{(n\cosh(bx+a))}\sinh\left(2\,bx+2\,a\right)\,dx$$

input
```
integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
```

output
```
integrate(e^(n*cosh(b*x + a))*sinh(2*b*x + 2*a), x)
```

3.1009.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.49

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \frac{e^{-a}\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{-b\,x}\,e^{\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n} - \frac{2\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n^2} + \frac{e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{b\,x}\,e^{a}\,e^{\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n}$$

input
```
int(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x),x)
```

output
```
(exp(-a)*exp((n*exp(b*x)*exp(a))/2)*exp(-b*x)*exp((n*exp(-a)*exp(-b*x))/2)
)/(b*n) - (2*exp((n*exp(b*x)*exp(a))/2)*exp((n*exp(-a)*exp(-b*x))/2))/(b*n
^2) + (exp((n*exp(b*x)*exp(a))/2)*exp(b*x)*exp(a)*exp((n*exp(-a)*exp(-b*x)
)/2))/(b*n)
```

3.1010 $\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$

3.1010.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx=-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

output
```
-4*exp(n*cosh(1/2*a+1/2*b*x))/b/n^2+4*exp(n*cosh(1/2*a+1/2*b*x))*cosh(1/2*
a+1/2*b*x)/b/n
```

3.1010.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx=\frac{4e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

input
```
Integrate[E^(n*Cosh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

output
```
(4*E^(n*Cosh[(a + b*x)/2])*(-1 + n*Cosh[(a + b*x)/2]))/(b*n^2)
```

3.1010.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4879, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh (a+b x) e^{n \cosh \left(\frac{a}{2}+\frac{b x}{2}\right)} \, dx$$

$$\Big\downarrow \text{4879}$$

$$\frac{2 i \int -2 i e^{n \cosh \left(\frac{a}{2}+\frac{b x}{2}\right)} \cosh \left(\frac{a}{2}+\frac{b x}{2}\right)d\cosh \left(\frac{a}{2}+\frac{b x}{2}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4 \int e^{n \cosh \left(\frac{a}{2}+\frac{b x}{2}\right)} \cosh \left(\frac{a}{2}+\frac{b x}{2}\right)d\cosh \left(\frac{a}{2}+\frac{b x}{2}\right)}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{4 \left(\frac{\cosh \left(\frac{a}{2}+\frac{b x}{2}\right) e^{n \cosh \left(\frac{a}{2}+\frac{b x}{2}\right)}}{n}-\frac{\int e^{n \cosh \left(\frac{a}{2}+\frac{b x}{2}\right)}d\cosh \left(\frac{a}{2}+\frac{b x}{2}\right)}{n}\right)}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{4 \left(\frac{\cosh \left(\frac{a}{2}+\frac{b x}{2}\right) e^{n \cosh \left(\frac{a}{2}+\frac{b x}{2}\right)}}{n}-\frac{e^{n \cosh \left(\frac{a}{2}+\frac{b x}{2}\right)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Cosh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

output
```
(4*(-(E^(n*Cosh[a/2 + (b*x)/2])/n^2) + (E^(n*Cosh[a/2 + (b*x)/2])*Cosh[a/2
 + (b*x)/2])/n))/b
```

3.1010.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.1010.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{2\left(n\,{\mathrm e}^{b x +a}+n -2\,{\mathrm e}^{\frac{a}{2}+\frac{b x}{2}}\right) {\mathrm e}^{-\frac{a}{2}-\frac{b x}{2}+\frac{n\,{\mathrm e}^{-\frac{a}{2}-\frac{b x}{2}}}{2}+\frac{n\,{\mathrm e}^{\frac{a}{2}+\frac{b x}{2}}}{2}}}{n^{2} b}$	63

input
```
int(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/n^2/b*(n*exp(b*x+a)+n-2*exp(1/2*a+1/2*b*x))*exp(-1/2*a-1/2*b*x+1/2*n*exp
(-1/2*a-1/2*b*x)+1/2*n*exp(1/2*a+1/2*b*x))
```

3.1010.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$$
$$=\frac{4\left(\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\right.\right.}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

input
```
integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

output
```
4*((n*cosh(1/2*b*x + 1/2*a) - 1)*cosh(n*cosh(1/2*b*x + 1/2*a)) + (n*cosh(1
/2*b*x + 1/2*a) - 1)*sinh(n*cosh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x +
1/2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

3.1010.6 Sympy [F]

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx=\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

input
```
integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

output
```
Integral(exp(n*cosh(a/2 + b*x/2))*sinh(a + b*x), x)
```

3.1010.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx=\frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn}+\frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn}-\frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^2}$$

input
```
integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

output
```
2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/
2*a)/(b*n) + 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x
 - 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*
b*x - 1/2*a))/(b*n^2)
```

3.1010.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(50) = 100.

Time = 0.30 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.97

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$$
$$= \frac{2\left(ne^{\left(bx+\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+a\right)}+ne^{\left(\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\cdots\right.}\right.\right.}\right.}{}$$

input
```
integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

output
```
2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)*e^(-1/2*
b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)*e^(-1/2
*b*x - 1/2*a) + a) + n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) +
 n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a)
- n)*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a)
 + n*e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a
) - n*e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)
/(b*n^2)
```

3.1010.9 Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n} - \frac{4\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n^2} + \frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{a}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n}$$

input
```
int(exp(n*cosh(a/2 + (b*x)/2))*sinh(a + b*x),x)
```

output
```
(2*exp(-a/2)*exp(-(b*x)/2)*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a
/2)*exp((b*x)/2))/2))/(b*n) - (4*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n
*exp(a/2)*exp((b*x)/2))/2))/(b*n^2) + (2*exp(-a/2)*exp(b*x)*exp(-(b*x)/2)*
exp(a)*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2)
)/(b*n)
```

3.1010. $\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$

3.1011 $\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx$

3.1011.1 Optimal result

Integrand size = 21, antiderivative size = 64

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx=-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

output
```
-4*exp(n*cosh(1/2*a+1/2*b*x))/b/n^2+4*exp(n*cosh(1/2*a+1/2*b*x))*cosh(1/2*
a+1/2*b*x)/b/n
```

3.1011.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx=\frac{4e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

input
```
Integrate[E^(n*Cosh[(a + b*x)/2])*Sinh[a + b*x],x]
```

output
```
(4*E^(n*Cosh[(a + b*x)/2])*(-1 + n*Cosh[(a + b*x)/2]))/(b*n^2)
```

3.1011.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4879, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}dx$$

$$\Big\downarrow \text{4879}$$

$$\frac{2i\int -2ie^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)d\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)d\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{4\left(\frac{\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}d\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{n}\right)}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{4\left(\frac{\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Cosh[(a + b*x)/2])*Sinh[a + b*x],x]
```

output
```
(4*(-(E^(n*Cosh[a/2 + (b*x)/2])/n^2) + (E^(n*Cosh[a/2 + (b*x)/2])*Cosh[a/2
 + (b*x)/2])/n))/b
```

3.1011.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.1011.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{2\left(n\,{\mathrm e}^{b x +a}+n -2\,{\mathrm e}^{\frac{a}{2}+\frac{b x}{2}}\right) {\mathrm e}^{-\frac{a}{2}-\frac{b x}{2}+\frac{n\,{\mathrm e}^{-\frac{a}{2}-\frac{b x}{2}}}{2}+\frac{n\,{\mathrm e}^{\frac{a}{2}+\frac{b x}{2}}}{2}}}{n^{2} b}$	63

input
```
int(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/n^2/b*(n*exp(b*x+a)+n-2*exp(1/2*a+1/2*b*x))*exp(-1/2*a-1/2*b*x+1/2*n*exp
(-1/2*a-1/2*b*x)+1/2*n*exp(1/2*a+1/2*b*x))
```

3.1011.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx$$
$$=\frac{4\left(\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\right.\right.\right.}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

input
```
integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

output
```
4*((n*cosh(1/2*b*x + 1/2*a) - 1)*cosh(n*cosh(1/2*b*x + 1/2*a)) + (n*cosh(1
/2*b*x + 1/2*a) - 1)*sinh(n*cosh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x +
1/2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

3.1011.6 Sympy [F]

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx=\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

input
```
integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

output
```
Integral(exp(n*cosh(a/2 + b*x/2))*sinh(a + b*x), x)
```

3.1011.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx=\frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn}+\frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn}-\frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^2}$$

input
```
integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

output
```
2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/
2*a)/(b*n) + 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x
 - 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*
b*x - 1/2*a))/(b*n^2)
```

3.1011.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(50) = 100$.

Time = 0.29 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.97

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx$$
$$= \frac{2\left(ne^{\left(bx+\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+a\right)}+ne^{\left(\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\right.}\right.\right.}\right.}{}$$

input
```
integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

output
```
2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)*e^(-1/2*
b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)*e^(-1/2
*b*x - 1/2*a) + a) + n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) +
 n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a)
- n)*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a)
 + n*e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a
) - n*e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)
/(b*n^2)
```

3.1011.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{2\,{\mathrm{e}}^{-\frac{a}{2}}\,{\mathrm{e}}^{-\frac{b\,x}{2}}\,{\mathrm{e}}^{\frac{n\,{\mathrm{e}}^{-\frac{a}{2}}\,{\mathrm{e}}^{-\frac{b\,x}{2}}}{2}}\,{\mathrm{e}}^{\frac{n\,{\mathrm{e}}^{a/2}\,{\mathrm{e}}^{\frac{b\,x}{2}}}{2}}}{b\,n}-\frac{4\,{\mathrm{e}}^{\frac{n\,{\mathrm{e}}^{-\frac{a}{2}}\,{\mathrm{e}}^{-\frac{b\,x}{2}}}{2}}\,{\mathrm{e}}^{\frac{n\,{\mathrm{e}}^{a/2}\,{\mathrm{e}}^{\frac{b\,x}{2}}}{2}}}{b\,n^2}+\frac{2\,{\mathrm{e}}^{-\frac{a}{2}}\,{\mathrm{e}}^{b\,x}\,{\mathrm{e}}^{-\frac{b\,x}{2}}\,{\mathrm{e}}^{a}\,{\mathrm{e}}^{\frac{n\,{\mathrm{e}}^{-\frac{a}{2}}\,{\mathrm{e}}^{-\frac{b\,x}{2}}}{2}}\,{\mathrm{e}}^{\frac{n\,{\mathrm{e}}^{a/2}\,{\mathrm{e}}^{\frac{b\,x}{2}}}{2}}}{b\,n}$$

input
```
int(exp(n*cosh(a/2 + (b*x)/2))*sinh(a + b*x),x)
```

output
```
(2*exp(-a/2)*exp(-(b*x)/2)*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a
/2)*exp((b*x)/2))/2))/(b*n) - (4*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n
*exp(a/2)*exp((b*x)/2))/2))/(b*n^2) + (2*exp(-a/2)*exp(b*x)*exp(-(b*x)/2)*
exp(a)*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2)
)/(b*n)
```

3.1011. $\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx$

3.1012 $\int \textbf{csch}(x) \log(\tanh(x))\textbf{sech}(x) \, dx$

3.1012.1 Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \operatorname{csch}(x) \log(\tanh(x))\operatorname{sech}(x) \, dx = \frac{1}{2} \log^2(\tanh(x))$$

output
```
1/2*ln(tanh(x))^2
```

3.1012.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(x) \log(\tanh(x))\operatorname{sech}(x) \, dx = \frac{1}{2} \log^2(\tanh(x))$$

input
```
Integrate[Csch[x]*Log[Tanh[x]]*Sech[x],x]
```

output
```
Log[Tanh[x]]^2/2
```

3.1012.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(x)\operatorname{sech}(x)\log(\tanh(x))\,dx$$

$$\Big\downarrow \; 7237$$

$$\frac{1}{2}\log^2(\tanh(x))$$

input
```
Int[Csch[x]*Log[Tanh[x]]*Sech[x],x]
```

output
```
Log[Tanh[x]]^2/2
```

3.1012.3.1 Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.1012.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\ln(\tanh(x))^2}{2}$
default	$\frac{\ln(\tanh(x))^2}{2}$
risch	$\frac{\ln(1+e^{2x})^2}{2} - \ln(e^{2x}-1)\ln(1+e^{2x}) + \frac{\ln(e^{2x}-1)^2}{2} + \frac{i\ln(1+e^{2x})\pi\,\mathrm{csgn}\left(\frac{i}{1+e^{2x}}\right)\mathrm{csgn}(i(e^{2x}-1))\,\mathrm{csgn}\left(\right.}{2}$

input
```
int(csch(x)*ln(tanh(x))*sech(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(tanh(x))^2
```

3.1012.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx = \frac{1}{2}\log\left(\frac{\sinh(x)}{\cosh(x)}\right)^2$$

input
```
integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="fricas")
```

output
```
1/2*log(sinh(x)/cosh(x))^2
```

3.1012.6 Sympy [F]

$$\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx = \int \log(\tanh(x))\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

input
```
integrate(csch(x)*ln(tanh(x))*sech(x),x)
```

output
```
Integral(log(tanh(x))*csch(x)*sech(x), x)
```

3.1012.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(7) = 14.

Time = 0.96 (sec) , antiderivative size = 95, normalized size of antiderivative = 10.56

$$\begin{aligned}&\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx\\ &= (\log(e^x+1)+\log(-e^x+1))\log\left(e^{(2x)}+1\right)-\frac{1}{2}\log\left(e^{(2x)}+1\right)^2\\ &\quad-\frac{1}{2}\log(e^x+1)^2-\log(e^x+1)\log(-e^x+1)-\frac{1}{2}\log(-e^x+1)^2\\ &\quad+\left(\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)-\log\left(e^{(-2x)}+1\right)\right)\log(\tanh(x))\end{aligned}$$

input
```
integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="maxima")
```

output
```
(log(e^x + 1) + log(-e^x + 1))*log(e^(2*x) + 1) - 1/2*log(e^(2*x) + 1)^2 -
 1/2*log(e^x + 1)^2 - log(e^x + 1)*log(-e^x + 1) - 1/2*log(-e^x + 1)^2 + (
log(e^(-x) + 1) + log(e^(-x) - 1) - log(e^(-2*x) + 1))*log(tanh(x))
```

3.1012.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(7) = 14.

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx = \frac{1}{2}\log\left(\frac{e^{(2x)}-1}{e^{(2x)}+1}\right)^2$$

input
```
integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="giac")
```

output
```
1/2*log((e^(2*x) - 1)/(e^(2*x) + 1))^2
```

3.1012.9 Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx = \frac{\left(\ln\left(e^{2x}-1\right)-\ln\left(e^{2x}+1\right)\right)^2}{2}$$

input
```
int(log(tanh(x))/(cosh(x)*sinh(x)),x)
```

output
```
(log(exp(2*x) - 1) - log(exp(2*x) + 1))^2/2
```

3.1013 $\int \operatorname{csch}(2x)\log(\tanh(x))\,dx$

3.1013.1 Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{1}{4}\log^2(\tanh(x))$$

output
```
1/4*ln(tanh(x))^2
```

3.1013.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{1}{4}\log^2(\tanh(x))$$

input
```
Integrate[Csch[2*x]*Log[Tanh[x]],x]
```

output
```
Log[Tanh[x]]^2/4
```

3.1013.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx$$

$$\Big\downarrow \; 7237$$

$$\frac{1}{4}\log^2(\tanh(x))$$

input
```
Int[Csch[2*x]*Log[Tanh[x]],x]
```

output
```
Log[Tanh[x]]^2/4
```

3.1013.3.1 Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.1013.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\ln(\tanh(x))^2}{4}$
default	$\frac{\ln(\tanh(x))^2}{4}$
parallelrisch	$\frac{\ln(\tanh(x))^2}{4}$
risch	$\frac{\ln(1+e^{2x})^2}{4}-\frac{\ln(e^{2x}-1)\ln(1+e^{2x})}{2}+\frac{\ln(e^{2x}-1)^2}{4}+\frac{i\ln(1+e^{2x})\pi\operatorname{csgn}\left(\frac{i}{1+e^{2x}}\right)\operatorname{csgn}\left(i(e^{2x}-1)\right)\operatorname{csgn}\left(\frac{i(e^{2x}-1)}{1+e^{2x}}\right.}{4}$

input
```
int(csch(2*x)*ln(tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(tanh(x))^2
```

3.1013.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{1}{4}\log\left(\frac{\sinh(x)}{\cosh(x)}\right)^2$$

input
```
integrate(csch(2*x)*log(tanh(x)),x, algorithm="fricas")
```

output
```
1/4*log(sinh(x)/cosh(x))^2
```

3.1013.6 Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \text{Timed out}$$

input
```
integrate(csch(2*x)*ln(tanh(x)),x)
```

output
```
Timed out
```

3.1013.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{1}{4}\log(\tanh(x))^2$$

input
```
integrate(csch(2*x)*log(tanh(x)),x, algorithm="maxima")
```

output
```
1/4*log(tanh(x))^2
```

3.1013.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(7) = 14.

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{1}{4}\log\left(\frac{e^{(2x)}-1}{e^{(2x)}+1}\right)^2$$

input
```
integrate(csch(2*x)*log(tanh(x)),x, algorithm="giac")
```

output
```
1/4*log((e^(2*x) - 1)/(e^(2*x) + 1))^2
```

3.1013.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{\left(\ln\left(\mathrm{e}^{2x}-1\right)-\ln\left(\mathrm{e}^{2x}+1\right)\right)^2}{4}$$

input
```
int(log(tanh(x))/sinh(2*x),x)
```

output
```
(log(exp(2*x) - 1) - log(exp(2*x) + 1))^2/4
```

3.1014 $\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx$

3.1014.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx=\operatorname{Int}(\cosh(a+bx)F(c,d,\sinh(a+bx),r,s),x)$$

output
```
CannotIntegrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
```

3.1014.2 Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx=\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx$$

input
```
Integrate[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s],x]
```

output
```
Integrate[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s], x]
```

3.1014.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4836, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx$$

$$\Big\downarrow \ 4836$$

$$\frac{\int F(c,d,\sinh(a+bx),r,s)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \ 7299$$

$$\frac{\int F(c,d,\sinh(a+bx),r,s)d\sinh(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s],x]
```

output
```
$Aborted
```

3.1014.3.1 Defintions of rubi rules used

rule 4836
```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sinh[c*(a +
 b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
 b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.1014.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \cosh(bx+a)\, F(c,d,\sinh(bx+a)\,,r,s)\, dx$$

input
```
int(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
```

output
```
int(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
```

3.1014.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\, dx = \int F(c,d,\sinh(bx+a)\,,r,s)\cosh(bx+a)\, dx$$

input
```
integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x, algorithm="fricas")
```

output
```
integral(F(c, d, sinh(b*x + a), r, s)*cosh(b*x + a), x)
```

3.1014.6 Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\, dx = \int F(c,d,\sinh(a+bx),r,s)\cosh(a+bx)\, dx$$

input
```
integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
```

output
```
Integral(F(c, d, sinh(a + b*x), r, s)*cosh(a + b*x), x)
```

3.1014.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx = \int F(c,d,\sinh(bx+a),r,s)\cosh(bx+a)\,dx$$

input
```
integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x, algorithm="maxima")
```

output
```
integrate(F(c, d, sinh(b*x + a), r, s)*cosh(b*x + a), x)
```

3.1014.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx = \int F(c,d,\sinh(bx+a),r,s)\cosh(bx+a)\,dx$$

input
```
integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x, algorithm="giac")
```

output
```
integrate(F(c, d, sinh(b*x + a), r, s)*cosh(b*x + a), x)
```

3.1014.9 Mupad [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx = \int \cosh(a+b\,x)\ F(c,d,\sinh(a+b\,x)\,,r,s)\,dx$$

input
```
int(cosh(a + b*x)*F(c, d, sinh(a + b*x), r, s),x)
```

output
```
int(cosh(a + b*x)*F(c, d, sinh(a + b*x), r, s), x)
```

3.1015 $\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx$

3.1015.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx = \text{Int}(F(c,d,\cosh(a+bx),r,s)\sinh(a+bx),x)$$

output
```
CannotIntegrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
```

3.1015.2 Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx = \int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx$$

input
```
Integrate[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x],x]
```

output
```
Integrate[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x], x]
```

3.1015.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4837, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)F(c,d,\cosh(a+bx),r,s)\,dx$$

$$\Big\downarrow \text{4837}$$

$$\frac{\int F(c,d,\cosh(a+bx),r,s)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\int F(c,d,\cosh(a+bx),r,s)d\cosh(a+bx)}{b}$$

input
```
Int[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x],x]
```

output
```
$Aborted
```

3.1015.3.1 Defintions of rubi rules used

rule 4837
```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Cosh[c*(a +
 b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
 b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.1015.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int F(c,d,\cosh (bx+a),r,s)\sinh (bx+a)\,dx$$

input
```
int(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
```

output
```
int(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
```

3.1015.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx=\int F(c,d,\cosh (bx+a),r,s)\sinh (bx+a)\,dx$$

input
```
integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(F(c, d, cosh(b*x + a), r, s)*sinh(b*x + a), x)
```

3.1015.6 Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx=\int F(c,d,\cosh (a+bx),r,s)\sinh (a+bx)\,dx$$

input
```
integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
```

output
```
Integral(F(c, d, cosh(a + b*x), r, s)*sinh(a + b*x), x)
```

3.1015.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx = \int F(c,d,\cosh\left(bx+a\right),r,s)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(F(c, d, cosh(b*x + a), r, s)*sinh(b*x + a), x)
```

3.1015.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx = \int F(c,d,\cosh\left(bx+a\right),r,s)\sinh\left(bx+a\right)\,dx$$

input
```
integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(F(c, d, cosh(b*x + a), r, s)*sinh(b*x + a), x)
```

3.1015.9 Mupad [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx = \int \sinh(a+b\,x)\;F(c,d,\cosh(a+b\,x)\,,r,s)\;dx$$

input
```
int(sinh(a + b*x)*F(c, d, cosh(a + b*x), r, s),x)
```

output
```
int(sinh(a + b*x)*F(c, d, cosh(a + b*x), r, s), x)
```

3.1016 $\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx$

3.1016.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx=\text{Int}\big(F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx),x\big)$$

output
```
CannotIntegrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x)
```

3.1016.2 Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx=\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx$$

input
```
Integrate[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2,x]
```

output
```
Integrate[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2, x]
```

3.1016.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4846, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^2(a+bx)F(c,d,\tanh(a+bx),r,s)\,dx$$

$$\Big\downarrow \text{4846}$$

$$\frac{\int F(c,d,\tanh(a+bx),r,s)d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\int F(c,d,\tanh(a+bx),r,s)d\tanh(a+bx)}{b}$$

input
```
Int[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2,x]
```

output
```
$Aborted
```

3.1016.3.1 Defintions of rubi rules used

rule 4846
```
Int[(u_)*Sech[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tanh[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tanh[c*(a
 + b*x)]/d, u, x], x], x, Tanh[c*(a + b*x)]/d], x] /; FunctionOfQ[Tanh[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.1016.4 Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int F(c,d,\tanh\left(bx+a\right),r,s)\operatorname{sech}\left(bx+a\right)^{2}dx$$

input
```
int(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x)
```

output
```
int(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x)
```

3.1016.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tanh(a+bx),r,s)\operatorname{sech}^2(a+bx)\,dx=\int F(c,d,\tanh\left(bx+a\right),r,s)\operatorname{sech}\left(bx+a\right)^{2}\,dx$$

input
```
integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(F(c, d, tanh(b*x + a), r, s)*sech(b*x + a)^2, x)
```

3.1016.6 Sympy [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int F(c,d,\tanh(a+bx),r,s)\operatorname{sech}^2(a+bx)\,dx=\int F(c,d,\tanh\left(a+bx\right),r,s)\operatorname{sech}^{2}\left(a+bx\right)dx$$

input
```
integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)**2,x)
```

output
```
Integral(F(c, d, tanh(a + b*x), r, s)*sech(a + b*x)**2, x)
```

3.1016.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx=\int F(c,d,\tanh(bx+a),r,s)\,\text{sech}\,(bx+a)^2\,dx$$

input
```
integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(F(c, d, tanh(b*x + a), r, s)*sech(b*x + a)^2, x)
```

3.1016.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx=\int F(c,d,\tanh(bx+a),r,s)\,\text{sech}\,(bx+a)^2\,dx$$

input
```
integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(F(c, d, tanh(b*x + a), r, s)*sech(b*x + a)^2, x)
```

3.1016.9 Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx=\int \frac{F(c,d,\tanh(a+b\,x),r,s)}{\cosh(a+b\,x)^2}\,dx$$

input
```
int(F(c, d, tanh(a + b*x), r, s)/cosh(a + b*x)^2,x)
```

output
```
int(F(c, d, tanh(a + b*x), r, s)/cosh(a + b*x)^2, x)
```

3.1017 $\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx$

3.1017.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx = \operatorname{Int}\left(\operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s),x\right)$$

output
```
CannotIntegrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x)
```

3.1017.2 Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx = \int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx$$

input
```
Integrate[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s],x]
```

output
```
Integrate[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s], x]
```

3.1017.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4847, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx$$
$$\Big\downarrow \text{ 4847}$$
$$-\frac{\int F(c,d,\coth(a+bx),r,s)d\coth(a+bx)}{b}$$
$$\Big\downarrow \text{ 7299}$$
$$-\frac{\int F(c,d,\coth(a+bx),r,s)d\coth(a+bx)}{b}$$

input
```
Int[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s],x]
```

output
```
$Aborted
```

3.1017.3.1 Defintions of rubi rules used

rule 4847
```
Int[Csch[(c_.)*((a_.) + (b_.)*(x_))]^2*(u_), x_Symbol] :> With[{d = FreeFac
tors[Coth[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Coth[c*(
a + b*x)]/d, u, x], x], x, Coth[c*(a + b*x)]/d], x] /; FunctionOfQ[Coth[c*(
a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.1017.4 Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(bx+a)^2 F(c,d,\coth(bx+a),r,s)\,dx$$

input
```
int(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x)
```

output
```
int(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x)
```

3.1017.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx = \int F(c,d,\coth(bx+a),r,s)\operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x, algorithm="fricas")
```

output
```
integral(F(c, d, coth(b*x + a), r, s)*csch(b*x + a)^2, x)
```

3.1017.6 Sympy [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx = \int F(c,d,\coth(a+bx),r,s)\operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**2*F(c,d,coth(b*x+a),r,s),x)
```

output
```
Integral(F(c, d, coth(a + b*x), r, s)*csch(a + b*x)**2, x)
```

3.1017.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx = \int F(c,d,\coth(bx+a),r,s)\operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x, algorithm="maxima")
```

output
```
integrate(F(c, d, coth(b*x + a), r, s)*csch(b*x + a)^2, x)
```

3.1017.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx = \int F(c,d,\coth(bx+a),r,s)\operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x, algorithm="giac")
```

output
```
integrate(F(c, d, coth(b*x + a), r, s)*csch(b*x + a)^2, x)
```

3.1017.9 Mupad [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx = \int \frac{F(c,d,\coth(a+b\,x),r,s)}{\sinh(a+b\,x)^2}\,dx$$

input
```
int(F(c, d, coth(a + b*x), r, s)/sinh(a + b*x)^2,x)
```

output
```
int(F(c, d, coth(a + b*x), r, s)/sinh(a + b*x)^2, x)
```

3.1018 $\int \text{sech}(x) \left(5-11\text{sech}^2(x)\right) \tanh(x) \, dx$

3.1018.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \text{sech}(x) \left(5-11\text{sech}^2(x)\right) \tanh(x) \, dx = -5\text{sech}(x)+\frac{11\text{sech}^3(x)}{3}$$

output
```
-5*sech(x)+11/3*sech(x)^3
```

3.1018.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \text{sech}(x) \left(5-11\text{sech}^2(x)\right) \tanh(x) \, dx = -5\text{sech}(x)+\frac{11\text{sech}^3(x)}{3}$$

input
```
Integrate[Sech[x]*(5 - 11*Sech[x]^2)*Tanh[x],x]
```

output
```
-5*Sech[x] + (11*Sech[x]^3)/3
```

3.1018.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 26, 4839, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\text{sech}(x)\left(5-11\text{sech}^2(x)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)\sec(ix)\left(5-11\sec(ix)^2\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec(ix)\left(5-11\sec(ix)^2\right)\tan(ix)dx$$

$$\Big\downarrow \text{4839}$$

$$\int -\left(\left(11-5\cosh^2(x)\right)\text{sech}^4(x)\right)d\cosh(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \left(11-5\cosh^2(x)\right)\text{sech}^4(x)d\cosh(x)$$

$$\Big\downarrow \text{244}$$

$$-\int \left(11\text{sech}^4(x)-5\text{sech}^2(x)\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{11\text{sech}^3(x)}{3}-5\text{sech}(x)$$

input
```
Int[Sech[x]*(5 - 11*Sech[x]^2)*Tanh[x],x]
```

output
```
-5*Sech[x] + (11*Sech[x]^3)/3
```

3.1018.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.1018.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-5\,\mathrm{sech}\left(x\right)+\frac{11\,\mathrm{sech}\left(x\right)^{3}}{3}$	12
default	$-5\,\mathrm{sech}\left(x\right)+\frac{11\,\mathrm{sech}\left(x\right)^{3}}{3}$	12
parts	$-5\,\mathrm{sech}\left(x\right)+\frac{11\,\mathrm{sech}\left(x\right)^{3}}{3}$	12
risch	$-\frac{2\,\mathrm{e}^{x}\left(15\,\mathrm{e}^{4x}-14\,\mathrm{e}^{2x}+15\right)}{3\left(1+\mathrm{e}^{2x}\right)^{3}}$	27

input
```
int(sech(x)*(5-11*sech(x)^2)*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
-5*sech(x)+11/3*sech(x)^3
```

3.1018.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 6.69

$$\int \operatorname{sech}(x) \left(5-11\operatorname{sech}^2(x)\right) \tanh(x)\, dx =$$
$$-\frac{2\left(15\,\cosh\left(x\right)^{3}+45\,\cosh\left(x\right)\sinh\left(x\right)^{2}+15\,\sinh\left(x\right)^{3}+\left(45\,\cosh\left(x\right)^{2}-29\right)\sinh\left(x\right)+\right.}{3\left(\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\,\cosh\left(x\right)^{2}+2\right)\sinh\left(x\right)^{2}+4\,\cosh\left(x\right)^{2}+4\left(\cosh\left(x\right.\right.\right.}$$

input
```
integrate(sech(x)*(5-11*sech(x)^2)*tanh(x),x, algorithm="fricas")
```

output
```
-2/3*(15*cosh(x)^3 + 45*cosh(x)*sinh(x)^2 + 15*sinh(x)^3 + (45*cosh(x)^2 -
 29)*sinh(x) + cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(
3*cosh(x)^2 + 2)*sinh(x)^2 + 4*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x)
 + 3)
```

3.1018.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \operatorname{sech}(x) \left(5-11\operatorname{sech}^2(x)\right) \tanh(x)\, dx = \frac{11\operatorname{sech}^{3}\left(x\right)}{3} - 5\operatorname{sech}\left(x\right)$$

input
```
integrate(sech(x)*(5-11*sech(x)**2)*tanh(x),x)
```

output
```
11*sech(x)**3/3 - 5*sech(x)
```

3.1018.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \operatorname{sech}(x) \left(5 - 11\operatorname{sech}^2(x)\right) \tanh(x) \, dx = -\frac{10}{e^{(-x)} + e^x} + \frac{88}{3 \left(e^{(-x)} + e^x\right)^3}$$

input
```
integrate(sech(x)*(5-11*sech(x)^2)*tanh(x),x, algorithm="maxima")
```

output
```
-10/(e^(-x) + e^x) + 88/3/(e^(-x) + e^x)^3
```

3.1018.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \operatorname{sech}(x) \left(5 - 11\operatorname{sech}^2(x)\right) \tanh(x) \, dx = -\frac{2 \left(15 \left(e^{(-x)} + e^x\right)^2 - 44\right)}{3 \left(e^{(-x)} + e^x\right)^3}$$

input
```
integrate(sech(x)*(5-11*sech(x)^2)*tanh(x),x, algorithm="giac")
```

output
```
-2/3*(15*(e^(-x) + e^x)^2 - 44)/(e^(-x) + e^x)^3
```

3.1018.9 Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}(x) \left(5 - 11\operatorname{sech}^2(x)\right) \tanh(x) \, dx = -\frac{2 \, \mathrm{e}^x \left(15 \, \mathrm{e}^{4x} - 14 \, \mathrm{e}^{2x} + 15\right)}{3 \left(\mathrm{e}^{2x} + 1\right)^3}$$

input
```
int(-(tanh(x)*(11/cosh(x)^2 - 5))/cosh(x),x)
```

output
```
-(2*exp(x)*(15*exp(4*x) - 14*exp(2*x) + 15))/(3*(exp(2*x) + 1)^3)
```

3.1019 $\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$

3.1019.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx=-\frac{\log(a+b\coth(x))}{b}$$

output
```
-ln(a+b*coth(x))/b
```

3.1019.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx=\frac{\log(\sinh(x))-\log(b\cosh(x)+a\sinh(x))}{b}$$

input
```
Integrate[Csch[x]^2/(a + b*Coth[x]),x]
```

output
```
(Log[Sinh[x]] - Log[b*Cosh[x] + a*Sinh[x]])/b
```

3.1019.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 25, 3987, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\sec\left(-\frac{\pi}{2}+ix\right)^2}{a-ib\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\sec\left(ix-\frac{\pi}{2}\right)^2}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3987$$

$$-\frac{\int \frac{1}{a+b\coth(x)}d(b\coth(x))}{b}$$

$$\Big\downarrow \ 16$$

$$-\frac{\log(a+b\coth(x))}{b}$$

input
```
Int[Csch[x]^2/(a + b*Coth[x]),x]
```

output
```
-(Log[a + b*Coth[x]]/b)
```

3.1019.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.1019.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+b\coth(x))}{b}$	13
default	$-\frac{\ln(a+b\coth(x))}{b}$	13
risch	$-\frac{\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{b}+\frac{\ln(e^{2x}-1)}{b}$	36

input
```
int(csch(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(a+b*coth(x))/b
```

3.1019.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

input
```
integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
-(log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh
(x) - sinh(x))))/b
```

3.1019.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$$

input
```
integrate(csch(x)**2/(a+b*coth(x)),x)
```

output
```
Integral(csch(x)**2/(a + b*coth(x)), x)
```

3.1019.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{\log(b\coth(x)+a)}{b}$$

input
```
integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-log(b*coth(x) + a)/b
```

3.1019.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.83

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{(a+b)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{ab+b^2} + \frac{\log\left(\left|e^{(2x)}-1\right|\right)}{b}$$

input
```
integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-(a + b)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b + b^2) + log(abs(e^(
2*x) - 1))/b
```

3.1019.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 4.25

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{a\,e^{2\,x}\,\sqrt{-b^2}-a\,\sqrt{-b^2}+b\,e^{2\,x}\,\sqrt{-b^2}}{b^2}\right)}{\sqrt{-b^2}}$$

input
```
int(1/(sinh(x)^2*(a + b*coth(x))),x)
```

output
```
-(2*atan((a*exp(2*x)*(-b^2)^(1/2) - a*(-b^2)^(1/2) + b*exp(2*x)*(-b^2)^(1/
2))/b^2))/(-b^2)^(1/2)
```

3.1020 $\int (a+b\coth(x))^n\text{csch}^2(x)\,dx$

3.1020.1 Optimal result

Integrand size = 13, antiderivative size = 20

$$\int (a+b\coth(x))^n\text{csch}^2(x)\,dx=-\frac{(a+b\coth(x))^{1+n}}{b(1+n)}$$

output
```
-(a+b*coth(x))^(1+n)/b/(1+n)
```

3.1020.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+b\coth(x))^n\text{csch}^2(x)\,dx=-\frac{(a+b\coth(x))^{1+n}}{b(1+n)}$$

input
```
Integrate[(a + b*Coth[x])^n*Csch[x]^2,x]
```

output
```
-((a + b*Coth[x])^(1 + n)/(b*(1 + n)))
```

3.1020.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 25, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(x)(a+b\coth(x))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sec\left(-\frac{\pi}{2}+ix\right)^2\left(a-ib\tan\left(-\frac{\pi}{2}+ix\right)\right)^n dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(ix-\frac{\pi}{2}\right)^2\left(a-ib\tan\left(ix-\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3987}$$

$$-\frac{\int (a+b\coth(x))^n d(b\coth(x))}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{(a+b\coth(x))^{n+1}}{b(n+1)}$$

input
```
Int[(a + b*Coth[x])^n*Csch[x]^2,x]
```

output
```
-((a + b*Coth[x])^(1 + n)/(b*(1 + n)))
```

3.1020.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.1020.4 Maple [A] (verified)

Time = 19.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{(a+b\coth(x))^{1+n}}{b(1+n)}$
default	$-\frac{(a+b\coth(x))^{1+n}}{b(1+n)}$
risch	$-\frac{(e^{2x}a+e^{2x}b-a+b)\left(a(e^{2x}-1)+b(1+e^{2x})\right)^{n}(e^{2x}-1)^{-n}e^{-\frac{i\,\mathrm{csgn}\left(\frac{i\left(a\left(e^{2x}-1\right)+b\left(1+e^{2x}\right)\right)}{e^{2x}-1}\right)\pi n\left(-\mathrm{csgn}\left(\frac{i\left(a\left(e^{2x}-1\right)+b\left(\right.\right.}{e^{2x}-1}\right.\right.}{}}}{b(1+n)(e^{2x}-1)}$

input
```
int((a+b*coth(x))^n*csch(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-(a+b*coth(x))^(1+n)/b/(1+n)
```

3.1020.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(20) = 40.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int (a+b\coth(x))^n\operatorname{csch}^2(x)\,dx =$$
$$-\frac{(b\cosh(x)+a\sinh(x))\cosh\left(n\log\left(\frac{b\cosh(x)+a\sinh(x)}{\sinh(x)}\right)\right)+(b\cosh(x)+a\sinh(x))\sinh\left(n\log\left(\frac{b\cosh(x)+}{\sinh}\right.\right.}{(bn+b)\sinh(x)}$$

input
```
integrate((a+b*coth(x))^n*csch(x)^2,x, algorithm="fricas")
```

output
```
-((b*cosh(x) + a*sinh(x))*cosh(n*log((b*cosh(x) + a*sinh(x))/sinh(x))) + (
b*cosh(x) + a*sinh(x))*sinh(n*log((b*cosh(x) + a*sinh(x))/sinh(x))))/((b*n
 + b)*sinh(x))
```

3.1020.6 Sympy [F]

$$\int (a+b\coth(x))^n\text{csch}^2(x)\,dx = \int (a+b\coth{(x)})^n\,\text{csch}^2{(x)}\,dx$$

input
```
integrate((a+b*coth(x))**n*csch(x)**2,x)
```

output
```
Integral((a + b*coth(x))**n*csch(x)**2, x)
```

3.1020.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+b\coth(x))^n\text{csch}^2(x)\,dx = -\frac{(b\coth{(x)}+a)^{n+1}}{b(n+1)}$$

input
```
integrate((a+b*coth(x))^n*csch(x)^2,x, algorithm="maxima")
```

output
```
-(b*coth(x) + a)^(n + 1)/(b*(n + 1))
```

3.1020.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int (a+b\coth(x))^n\text{csch}^2(x)\,dx = -\frac{\left(\frac{ae^{(2x)}+be^{(2x)}-a+b}{e^{(2x)}-1}\right)^{n+1}}{b(n+1)}$$

input
```
integrate((a+b*coth(x))^n*csch(x)^2,x, algorithm="giac")
```

output
```
-((a*e^(2*x) + b*e^(2*x) - a + b)/(e^(2*x) - 1))^(n + 1)/(b*(n + 1))
```

3.1020.9 Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int (a+b\coth(x))^n \operatorname{csch}^2(x)\,dx = -\frac{\left(a+\frac{b\,(\mathrm{e}^{2\,x}+1)}{\mathrm{e}^{2\,x}-1}\right)^n\,(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x})}{b\,(\mathrm{e}^{2\,x}-1)\,(n+1)}$$

input
```
int((a + b*coth(x))^n/sinh(x)^2,x)
```

output
```
-((a + (b*(exp(2*x) + 1))/(exp(2*x) - 1))^n*(b - a + a*exp(2*x) + b*exp(2*
x)))/(b*(exp(2*x) - 1)*(n + 1))
```

3.1021 $\int \operatorname{csch}^2(x) \left(-1+\sinh^2(x)\right) \, dx$

3.1021.1 Optimal result

Integrand size = 11, antiderivative size = 4

$$\int \operatorname{csch}^2(x) \left(-1+\sinh^2(x)\right) \, dx = x+\coth(x)$$

output
```
x+coth(x)
```

3.1021.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(x) \left(-1+\sinh^2(x)\right) \, dx = x+\coth(x)$$

input
```
Integrate[Csch[x]^2*(-1 + Sinh[x]^2),x]
```

output
```
x + Coth[x]
```

3.1021.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 25, 25, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sinh^2(x)-1\right)\text{csch}^2(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{-1-\sin(ix)^2}{\sin(ix)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int -\frac{\sin(ix)^2+1}{\sin(ix)^2}dx$$

$$\Big\downarrow \; 25$$

$$\int \frac{1+\sin(ix)^2}{\sin(ix)^2}dx$$

$$\Big\downarrow \; 3491$$

$$\int 1dx+\coth(x)$$

$$\Big\downarrow \; 24$$

$$x+\coth(x)$$

input
```
Int[Csch[x]^2*(-1 + Sinh[x]^2),x]
```

output
```
x + Coth[x]
```

3.1021.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

3.1021.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$x+\coth\left(x\right)$	5
parallelrisch	$x+\coth\left(x\right)$	5
risch	$x+\frac{2}{{\mathrm e}^{2 x}-1}$	13

input
```
int(csch(x)^2*(-1+sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x+coth(x)
```

3.1021.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(4) = 8$.

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx=\frac{(x-1)\sinh(x)+\cosh(x)}{\sinh(x)}$$

input
```
integrate(csch(x)^2*(-1+sinh(x)^2),x, algorithm="fricas")
```

output
```
((x - 1)*sinh(x) + cosh(x))/sinh(x)
```

3.1021.6 Sympy [F]

$$\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx=\int(\sinh(x)-1)(\sinh(x)+1)\operatorname{csch}^2(x)\,dx$$

input
```
integrate(csch(x)**2*(-1+sinh(x)**2),x)
```

output
```
Integral((sinh(x) - 1)*(sinh(x) + 1)*csch(x)**2, x)
```

3.1021.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx=x-\frac{2}{e^{(-2x)}-1}$$

input
```
integrate(csch(x)^2*(-1+sinh(x)^2),x, algorithm="maxima")
```

output
```
x - 2/(e^(-2*x) - 1)
```

3.1021.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \operatorname{csch}^2(x) \left(-1+\sinh^2(x)\right) \, dx = x + \frac{2}{e^{(2x)}-1}$$

input
```
integrate(csch(x)^2*(-1+sinh(x)^2),x, algorithm="giac")
```

output
```
x + 2/(e^(2*x) - 1)
```

3.1021.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \operatorname{csch}^2(x) \left(-1+\sinh^2(x)\right) \, dx = x + \frac{2}{e^{2x}-1}$$

input
```
int((sinh(x)^2 - 1)/sinh(x)^2,x)
```

output
```
x + 2/(exp(2*x) - 1)
```

3.1022 $\int \left(-1-\frac{1}{1-\coth^2(x)}\right) \operatorname{csch}^2(x)\,dx$

3.1022.1 Optimal result

Integrand size = 19, antiderivative size = 4

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right) \operatorname{csch}^2(x)\,dx = x+\coth(x)$$

output
```
x+coth(x)
```

3.1022.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 4.75

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right) \operatorname{csch}^2(x)\,dx = 2x+\coth(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(x)\right)$$

input
```
Integrate[(-1 - (1 - Coth[x]^2)^(-1))*Csch[x]^2,x]
```

output
```
2*x + Coth[x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x]^2]
```

3.1022.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 25, 25, 4889, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\frac{1}{1-\coth^2(x)}-1\right)\operatorname{csch}^2(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\left(\left(-1-\frac{1}{1+\cot(ix)^2}\right)\csc(ix)^2\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\left(\left(1+\frac{1}{\cot(ix)^2+1}\right)\csc(ix)^2\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \left(1+\frac{1}{1+\cot(ix)^2}\right)\csc(ix)^2dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int -\coth^2(x)\left(\frac{1}{1-\coth^2(x)}+1\right)d\tanh(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \coth^2(x)\left(1+\frac{1}{1-\coth^2(x)}\right)d\tanh(x)$$

$$\Big\downarrow \text{ 2010}$$

$$-\int \left(\coth^2(x)+\frac{1}{\tanh^2(x)-1}\right)d\tanh(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\operatorname{arctanh}(\tanh(x))+\coth(x)$$

input
```
Int[(-1 - (1 - Coth[x]^2)^(-1))*Csch[x]^2,x]
```

output
```
ArcTanh[Tanh[x]] + Coth[x]
```

3.1022.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.1022.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

method	result	size
risch	$x+\frac{2}{\mathrm{e}^{2x}-1}$	13
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2\tanh\left(\frac{x}{2}\right)}$	32

input
```
int((-1-1/(1-coth(x)^2))*csch(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x+2/(exp(2*x)-1)
```

3.1022.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(4) = 8$.

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right) \operatorname{csch}^2(x) \, dx = \frac{(x-1)\sinh(x)+\cosh(x)}{\sinh(x)}$$

input
```
integrate((-1-1/(1-coth(x)^2))*csch(x)^2,x, algorithm="fricas")
```

output
```
((x - 1)*sinh(x) + cosh(x))/sinh(x)
```

3.1022.6 Sympy [F]

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right) \operatorname{csch}^2(x) \, dx = -\int \left(-\frac{2\operatorname{csch}^2(x)}{\coth^2(x)-1}\right) dx - \int \frac{\coth^2(x)\operatorname{csch}^2(x)}{\coth^2(x)-1} \, dx$$

input
```
integrate((-1-1/(1-coth(x)**2))*csch(x)**2,x)
```

output
```
-Integral(-2*csch(x)**2/(coth(x)**2 - 1), x) - Integral(coth(x)**2*csch(x)
**2/(coth(x)**2 - 1), x)
```

3.1022.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right) \operatorname{csch}^2(x) \, dx = x - \frac{2}{e^{(-2x)}-1}$$

input
```
integrate((-1-1/(1-coth(x)^2))*csch(x)^2,x, algorithm="maxima")
```

output
```
x - 2/(e^(-2*x) - 1)
```

3.1022.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right)\operatorname{csch}^2(x)\,dx = x+\frac{2}{e^{(2x)}-1}$$

input
```
integrate((-1-1/(1-coth(x)^2))*csch(x)^2,x, algorithm="giac")
```

output
```
x + 2/(e^(2*x) - 1)
```

3.1022.9 Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right)\operatorname{csch}^2(x)\,dx = x+\frac{2}{e^{2x}-1}$$

input
```
int((1/(coth(x)^2 - 1) - 1)/sinh(x)^2,x)
```

output
```
x + 2/(exp(2*x) - 1)
```

3.1023 $\int \frac{(a+b\coth(x))\text{csch}^2(x)}{c+d\coth(x)}\,dx$

3.1023.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{(a+b\coth(x))\text{csch}^2(x)}{c+d\coth(x)}\,dx=-\frac{b\coth(x)}{d}+\frac{(bc-ad)\log(c+d\coth(x))}{d^2}$$

output
```
-b*coth(x)/d+(-a*d+b*c)*ln(c+d*coth(x))/d^2
```

3.1023.2 Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\coth(x))\text{csch}^2(x)}{c+d\coth(x)}\,dx =\frac{(a+b\coth(x))(-bd\coth(x)-(bc-ad)(\log(\sinh(x))-\log(d\cosh(x)+c\sinh(x))))\sinh(x)}{d^2(b\cosh(x)+a\sinh(x))}$$

input
```
Integrate[((a + b*Coth[x])*Csch[x]^2)/(c + d*Coth[x]),x]
```

output
```
((a + b*Coth[x])*(-(b*d*Coth[x]) - (b*c - a*d)*(Log[Sinh[x]] - Log[d*Cosh[
x] + c*Sinh[x]]))*Sinh[x])/(d^2*(b*Cosh[x] + a*Sinh[x]))
```

3.1023.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 25, 4844, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(x)(a+b\coth(x))}{c+d\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\csc(ix)^2(a+ib\cot(ix))}{c+id\cot(ix)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{(a+ib\cot(ix))\csc(ix)^2}{c+id\cot(ix)}dx$$

$$\Big\downarrow \text{4844}$$

$$-\int \frac{a+b\coth(x)}{c+d\coth(x)}d\coth(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{b}{d}+\frac{ad-bc}{d(c+d\coth(x))}\right)d\coth(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bc-ad)\log(c+d\coth(x))}{d^2}-\frac{b\coth(x)}{d}$$

input
```
Int[((a + b*Coth[x])*Csch[x]^2)/(c + d*Coth[x]),x]
```

output
```
-((b*Coth[x])/d) + ((b*c - a*d)*Log[c + d*Coth[x]])/d^2
```

3.1023.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4844
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cot[c*(a
+ b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc]
|| EqQ[F, csc])
```

3.1023.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(28) = 56$.

Time = 0.52 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

method	result	size
default	$\frac{(-2ad+2bc)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}d+2c\tanh\left(\frac{x}{2}\right)+d\right)}{2d^{2}}-\frac{\tanh\left(\frac{x}{2}\right)b}{2d}-\frac{b}{2d\tanh\left(\frac{x}{2}\right)}+\frac{(2ad-2bc)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2d^{2}}$	75
risch	$-\frac{2b}{d(e^{2x}-1)}-\frac{\ln\left(e^{2x}-\frac{c-d}{c+d}\right)a}{d}+\frac{\ln\left(e^{2x}-\frac{c-d}{c+d}\right)bc}{d^{2}}+\frac{\ln(e^{2x}-1)a}{d}-\frac{\ln(e^{2x}-1)bc}{d^{2}}$	90

input
```
int((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/d^2*(-2*a*d+2*b*c)*ln(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)-1/2*tanh(1/2*
x)*b/d-1/2*b/d/tanh(1/2*x)+1/2/d^2*(2*a*d-2*b*c)*ln(tanh(1/2*x))
```

3.1023.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 6.21

$$\int \frac{(a+b\coth(x))\text{csch}^2(x)}{c+d\coth(x)}\,dx =$$

$$-\frac{2\,bd-\left((bc-ad)\cosh\left(x\right)^2+2\,(bc-ad)\cosh\left(x\right)\sinh\left(x\right)+(bc-ad)\sinh\left(x\right)^2-bc+ad\right)\log\left(\frac{2\,(d\cosh(x}{\cosh(x}\right.}{d^2\cosh\left(x\right)^2+2\,d^2\,\text{co}}$$

input
```
integrate((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x, algorithm="fricas")
```

output
```
-(2*b*d - ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c -
a*d)*sinh(x)^2 - b*c + a*d)*log(2*(d*cosh(x) + c*sinh(x))/(cosh(x) - sinh(
x))) + ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a*d
)*sinh(x)^2 - b*c + a*d)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(d^2*cosh(x)^
2 + 2*d^2*cosh(x)*sinh(x) + d^2*sinh(x)^2 - d^2)
```

3.1023.6 Sympy [F]

$$\int \frac{(a+b\coth(x))\text{csch}^2(x)}{c+d\coth(x)}\,dx = \int \frac{(a+b\coth\left(x\right))\,\text{csch}^2\left(x\right)}{c+d\coth\left(x\right)}\,dx$$

input
```
integrate((a+b*coth(x))*csch(x)**2/(c+d*coth(x)),x)
```

output
```
Integral((a + b*coth(x))*csch(x)**2/(c + d*coth(x)), x)
```

3.1023.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(28) = 56$.

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.75

$$\int \frac{(a+b\coth(x))\text{csch}^2(x)}{c+d\coth(x)}\,dx$$

$$= b\left(\frac{c\log\left(-(c-d)e^{(-2\,x)}+c+d\right)}{d^2}-\frac{c\log\left(e^{(-x)}+1\right)}{d^2}-\frac{c\log\left(e^{(-x)}-1\right)}{d^2}+\frac{2}{de^{(-2\,x)}-d}\right)$$
$$-\frac{a\log\left(d\coth\left(x\right)+c\right)}{d}$$

input
```
integrate((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x, algorithm="maxima")
```

output
```
b*(c*log(-(c - d)*e^(-2*x) + c + d)/d^2 - c*log(e^(-x) + 1)/d^2 - c*log(e^
(-x) - 1)/d^2 + 2/(d*e^(-2*x) - d)) - a*log(d*coth(x) + c)/d
```

3.1023.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.04

$$\int \frac{(a+b\coth(x))\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{(bc^2-acd+bcd-ad^2)\log\left(\left|ce^{(2x)}+de^{(2x)}-c+d\right|\right)}{cd^2+d^3} - \frac{(bc-ad)\log\left(\left|e^{(2x)}-1\right|\right)}{d^2} + \frac{bce^{(2x)}-ade^{(2x)}-bc+ad-2bd}{d^2(e^{(2x)}-1)}$$

input
```
integrate((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x, algorithm="giac")
```

output
```
(b*c^2 - a*c*d + b*c*d - a*d^2)*log(abs(c*e^(2*x) + d*e^(2*x) - c + d))/(c
*d^2 + d^3) - (b*c - a*d)*log(abs(e^(2*x) - 1))/d^2 + (b*c*e^(2*x) - a*d*e
^(2*x) - b*c + a*d - 2*b*d)/(d^2*(e^(2*x) - 1))
```

3.1023.9 Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 297, normalized size of antiderivative = 10.61

$$\int \frac{(a+b\coth(x))\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{2\,\mathrm{atan}\left(\mathrm{e}^{2\,x}\left(\frac{4\,\left(a\,d\,\sqrt{-d^4}-b\,c\,\sqrt{-d^4}\right)}{d^2\,\sqrt{{\left(a\,d-b\,c\right)}^2}\,\left(c+d\right)\,\left(c-d\right)\,\sqrt{-d^4}}-\frac{4\,c^2\,\sqrt{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{d^4\,\left(c+d\right)\,\left(c-d\right)\,\left(a\,d-b\,c\right)}\right)\,\left(\frac{d^2\,\sqrt{-d^4}}{4}+\frac{c\,d\,\sqrt{-d^4}}{4}\right)-\frac{4\,c\,\left(d^2\,\sqrt{a^2\,d^2-2\,a\,b\,c\,d-}\right.}{}\right.}{\sqrt{-d^4}} - \frac{2\,b}{d\,\left(\mathrm{e}^{2\,x}-1\right)}$$

input
```
int((a + b*coth(x))/(sinh(x)^2*(c + d*coth(x))),x)
```

output
```
(2*atan(exp(2*x)*((4*(a*d*(-d^4)^(1/2) - b*c*(-d^4)^(1/2)))/(d^2*((a*d - b
*c)^2)^(1/2)*(c + d)*(c - d)*(-d^4)^(1/2)) - (4*c^2*(a^2*d^2 + b^2*c^2 - 2
*a*b*c*d)^(1/2))/(d^4*(c + d)*(c - d)*(a*d - b*c)))*((d^2*(-d^4)^(1/2))/4
+ (c*d*(-d^4)^(1/2))/4) - (4*c*(d^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^(1/2)
- c*d*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^(1/2))*((d^2*(-d^4)^(1/2))/4 + (c*d*
(-d^4)^(1/2))/4))/(d^5*(c + d)*(c - d)*(a*d - b*c)))*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d)^(1/2))/(-d^4)^(1/2) - (2*b)/(d*(exp(2*x) - 1))
```

3.1023. $\int \frac{(a+b\coth(x))\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx$

3.1024 $\int \frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx$

3.1024.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{b(bc-ad)\coth(x)}{d^2} - \frac{(a+b\coth(x))^2}{2d} - \frac{(bc-ad)^2\log(c+d\coth(x))}{d^3}$$

output
```
b*(-a*d+b*c)*coth(x)/d^2-1/2*(a+b*coth(x))^2/d-(-a*d+b*c)^2*ln(c+d*coth(x)
)/d^3
```

3.1024.2 Mathematica [A] (verified)

Time = 3.51 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{2bd(bc-2ad)\coth(x) - b^2d^2\operatorname{csch}^2(x) + 2(bc-ad)^2(\log(\sinh(x)) - \log(d\cosh(x)+c\sinh(x)))}{2d^3}$$

input
```
Integrate[((a + b*Coth[x])^2*Csch[x]^2)/(c + d*Coth[x]),x]
```

output
```
(2*b*d*(b*c - 2*a*d)*Coth[x] - b^2*d^2*Csch[x]^2 + 2*(b*c - a*d)^2*(Log[Si
nh[x]] - Log[d*Cosh[x] + c*Sinh[x]]))/(2*d^3)
```

3.1024.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 25, 4844, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(x)(a+b\coth(x))^2}{c+d\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\csc(ix)^2(a+ib\cot(ix))^2}{c+id\cot(ix)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{(a+ib\cot(ix))^2\csc(ix)^2}{c+id\cot(ix)}dx$$

$$\Big\downarrow \text{4844}$$

$$-\int \frac{(a+b\coth(x))^2}{c+d\coth(x)}d\coth(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{(ad-bc)^2}{d^2(c+d\coth(x))}-\frac{b(bc-ad)}{d^2}+\frac{b(a+b\coth(x))}{d}\right)d\coth(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^2\log(c+d\coth(x))}{d^3}+\frac{b\coth(x)(bc-ad)}{d^2}-\frac{(a+b\coth(x))^2}{2d}$$

input
```
Int[((a + b*Coth[x])^2*Csch[x]^2)/(c + d*Coth[x]),x]
```

output
```
(b*(b*c - a*d)*Coth[x])/d^2 - (a + b*Coth[x])^2/(2*d) - ((b*c - a*d)^2*Log
[c + d*Coth[x]])/d^3
```

3.1024.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4844
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cot[c*(a
+ b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc]
|| EqQ[F, csc])
```

3.1024.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{b\left(\frac{b\coth(x)^2 d}{2}+2\coth(x)ad-\coth(x)bc\right)}{d^2}-\frac{(a^2d^2-2abcd+b^2c^2)\ln(c+d\coth(x))}{d^3}$
default	$-\frac{b\left(\frac{b\coth(x)^2 d}{2}+2\coth(x)ad-\coth(x)bc\right)}{d^2}-\frac{(a^2d^2-2abcd+b^2c^2)\ln(c+d\coth(x))}{d^3}$
risch	$-\frac{2b(2ad\,e^{2x}-bc\,e^{2x}+bd\,e^{2x}-2ad+bc)}{(e^{2x}-1)^2d^2}+\frac{\ln(e^{2x}-1)a^2}{d}-\frac{2\ln(e^{2x}-1)abc}{d^2}+\frac{\ln(e^{2x}-1)b^2c^2}{d^3}-\frac{\ln\left(e^{2x}-\frac{c-d}{c+d}\right)a^2}{d}$

input
```
int((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-b/d^2*(1/2*b*coth(x)^2*d+2*coth(x)*a*d-coth(x)*b*c)-(a^2*d^2-2*a*b*c*d+b^
2*c^2)/d^3*ln(c+d*coth(x))
```

3.1024.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. 2(51) = 102.

Time = 0.29 (sec) , antiderivative size = 694, normalized size of antiderivative = 13.09

$$\int \frac{(a+b\coth(x))^2\text{csch}^2(x)}{c+d\coth(x)}\,dx =$$

$$-\frac{2\,b^2cd-4\,abd^2-2\,(b^2cd-(2\,ab+b^2)d^2)\cosh{(x)}^2-4\,(b^2cd-(2\,ab+b^2)d^2)\cosh{(x)}\sinh{(x)}-2\,(b^2cd}{}$$

input
```
integrate((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x, algorithm="fricas")
```

output
```
-(2*b^2*c*d - 4*a*b*d^2 - 2*(b^2*c*d - (2*a*b + b^2)*d^2)*cosh(x)^2 - 4*(b
^2*c*d - (2*a*b + b^2)*d^2)*cosh(x)*sinh(x) - 2*(b^2*c*d - (2*a*b + b^2)*d
^2)*sinh(x)^2 + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*s
inh(x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*cosh(x)^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 3*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x
)^3 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*(d*cosh(x) +
 c*sinh(x))/(cosh(x) - sinh(x))) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x
)^4 + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sinh(x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 2*(b^2*c^2 -
 2*a*b*c*d + a^2*d^2)*cosh(x)^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 3*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*cosh(x)^3 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))
*log(2*sinh(x)/(cosh(x) - sinh(x))))/(d^3*cosh(x)^4 + 4*d^3*cosh(x)*sinh(x
)^3 + d^3*sinh(x)^4 - 2*d^3*cosh(x)^2 + d^3 + 2*(3*d^3*cosh(x)^2 - d^3)*si
nh(x)^2 + 4*(d^3*cosh(x)^3 - d^3*cosh(x))*sinh(x))
```

3.1024.6 Sympy [F]

$$\int \frac{(a+b\coth(x))^2\text{csch}^2(x)}{c+d\coth(x)}\,dx = \int \frac{(a+b\coth{(x)})^2\operatorname{csch}^2{(x)}}{c+d\coth{(x)}}\,dx$$

input
```
integrate((a+b*coth(x))**2*csch(x)**2/(c+d*coth(x)),x)
```

output
```
Integral((a + b*coth(x))**2*csch(x)**2/(c + d*coth(x)), x)
```

3.1024.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(51) = 102$.

Time = 0.20 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.34

$$\int \frac{(a+b\coth(x))^2\text{csch}^2(x)}{c+d\coth(x)}\,dx$$
$$= b^2\left(\frac{2\left((c+d)e^{(-2x)}-c\right)}{2d^2e^{(-2x)}-d^2e^{(-4x)}-d^2} - \frac{c^2\log\left(-(c-d)e^{(-2x)}+c+d\right)}{d^3} + \frac{c^2\log\left(e^{(-x)}+1\right)}{d^3} + \frac{c^2\log\left(e^{(-x)}-1\right)}{d^3}\right.$$
$$\left. + 2ab\left(\frac{c\log\left(-(c-d)e^{(-2x)}+c+d\right)}{d^2} - \frac{c\log\left(e^{(-x)}+1\right)}{d^2} - \frac{c\log\left(e^{(-x)}-1\right)}{d^2} + \frac{2}{de^{(-2x)}-d}\right)\right.$$
$$- \frac{a^2\log\left(d\coth(x)+c\right)}{d}$$

input
```
integrate((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x, algorithm="maxima")
```

output
```
b^2*(2*((c + d)*e^(-2*x) - c)/(2*d^2*e^(-2*x) - d^2*e^(-4*x) - d^2) - c^2*
log(-(c - d)*e^(-2*x) + c + d)/d^3 + c^2*log(e^(-x) + 1)/d^3 + c^2*log(e^(
-x) - 1)/d^3) + 2*a*b*(c*log(-(c - d)*e^(-2*x) + c + d)/d^2 - c*log(e^(-x)
 + 1)/d^2 - c*log(e^(-x) - 1)/d^2 + 2/(d*e^(-2*x) - d)) - a^2*log(d*coth(x
) + c)/d
```

3.1024.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(51) = 102$.

Time = 0.28 (sec) , antiderivative size = 265, normalized size of antiderivative = 5.00

$$\int \frac{(a+b\coth(x))^2\text{csch}^2(x)}{c+d\coth(x)}\,dx$$
$$= -\frac{(b^2c^3-2abc^2d+b^2c^2d+a^2cd^2-2abcd^2+a^2d^3)\log\left(\left|ce^{(2x)}+de^{(2x)}-c+d\right|\right)}{cd^3+d^4}$$
$$+ \frac{(b^2c^2-2abcd+a^2d^2)\log\left(\left|e^{(2x)}-1\right|\right)}{d^3}$$
$$- \frac{3b^2c^2e^{(4x)}-6abcde^{(4x)}+3a^2d^2e^{(4x)}-6b^2c^2e^{(2x)}+12abcde^{(2x)}-4b^2cde^{(2x)}-6a^2d^2e^{(2x)}+8abd^2e^{(}}{2d^3\left(e^{(2x)}-1\right)^2}$$

input
```
integrate((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x, algorithm="giac")
```

output
```
-(b^2*c^3 - 2*a*b*c^2*d + b^2*c^2*d + a^2*c*d^2 - 2*a*b*c*d^2 + a^2*d^3)*l
og(abs(c*e^(2*x) + d*e^(2*x) - c + d))/(c*d^3 + d^4) + (b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*log(abs(e^(2*x) - 1))/d^3 - 1/2*(3*b^2*c^2*e^(4*x) - 6*a*b*c*
d*e^(4*x) + 3*a^2*d^2*e^(4*x) - 6*b^2*c^2*e^(2*x) + 12*a*b*c*d*e^(2*x) - 4
*b^2*c*d*e^(2*x) - 6*a^2*d^2*e^(2*x) + 8*a*b*d^2*e^(2*x) + 4*b^2*d^2*e^(2*
x) + 3*b^2*c^2 - 6*a*b*c*d + 4*b^2*c*d + 3*a^2*d^2 - 8*a*b*d^2)/(d^3*(e^(2
*x) - 1)^2)
```

3.1024.9 Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{\ln\left(e^{2x}-1\right)\,\left(a\,d-b\,c\right)^2}{d^3} - \frac{\ln\left(d-c+d\,e^{2x}+c\,e^{2x}\right)\,\left(a\,d-b\,c\right)^2}{d^3} - \frac{2\left(b^2\,d-b^2\,c+2\,a\,b\,d\right)}{d^2\,\left(e^{2x}-1\right)} - \frac{2\,b^2}{d\,\left(e^{4x}-2\,e^{2x}+1\right)}$$

input
```
int((a + b*coth(x))^2/(sinh(x)^2*(c + d*coth(x))),x)
```

output
```
(log(exp(2*x) - 1)*(a*d - b*c)^2)/d^3 - (log(d - c + d*exp(2*x) + c*exp(2*
x))*(a*d - b*c)^2)/d^3 - (2*(b^2*d - b^2*c + 2*a*b*d))/(d^2*(exp(2*x) - 1)
) - (2*b^2)/(d*(exp(4*x) - 2*exp(2*x) + 1))
```

3.1025 $\int \frac{(a+b\coth(x))^3\text{csch}^2(x)}{c+d\coth(x)}\,dx$

3.1025.1 Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{(a+b\coth(x))^3\text{csch}^2(x)}{c+d\coth(x)}\,dx = -\frac{b(bc-ad)^2\coth(x)}{d^3}+\frac{(bc-ad)(a+b\coth(x))^2}{2d^2} -\frac{(a+b\coth(x))^3}{3d}+\frac{(bc-ad)^3\log(c+d\coth(x))}{d^4}$$

output
```
-b*(-a*d+b*c)^2*coth(x)/d^3+1/2*(-a*d+b*c)*(a+b*coth(x))^2/d^2-1/3*(a+b*co
th(x))^3/d+(-a*d+b*c)^3*ln(c+d*coth(x))/d^4
```

3.1025.2 Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.74

$$\int \frac{(a+b\coth(x))^3\text{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{(a+b\coth(x))^3(d\cosh(x)+c\sinh(x))\left(-2b^3d^3\coth(x)-6(bc-ad)^3(\log(\sinh(x))-\log(d\cosh(x)+c\,\text{si}\right.}{6d^4(c+d\coth(x))(b\cosh(x)+}$$

input
```
Integrate[((a + b*Coth[x])^3*Csch[x]^2)/(c + d*Coth[x]),x]
```

output
```
((a + b*Coth[x])^3*(d*Cosh[x] + c*Sinh[x])*(-2*b^3*d^3*Coth[x] - 6*(b*c -
a*d)^3*(Log[Sinh[x]] - Log[d*Cosh[x] + c*Sinh[x]])*Sinh[x]^2 - b*d*(-3*b*d
*(b*c - 3*a*d) + (-9*a*b*c*d + 9*a^2*d^2 + b^2*(3*c^2 + d^2))*Sinh[2*x])))
/(6*d^4*(c + d*Coth[x])*(b*Cosh[x] + a*Sinh[x])^3)
```

3.1025.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4844, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(x)(a+b\coth(x))^3}{c+d\coth(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\csc(ix)^2(a+ib\cot(ix))^3}{c+id\cot(ix)}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{(a+ib\cot(ix))^3\csc(ix)^2}{c+id\cot(ix)}dx$$
$$\Big\downarrow \text{4844}$$
$$-\int \frac{(a+b\coth(x))^3}{c+d\coth(x)}d\coth(x)$$
$$\Big\downarrow \text{49}$$
$$-\int \left(\frac{(ad-bc)^3}{d^3(c+d\coth(x))}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+b\coth(x))^2}{d}-\frac{b(bc-ad)(a+b\coth(x))}{d^2}\right)d\coth(x)$$
$$\Big\downarrow \text{2009}$$
$$\frac{(bc-ad)^3\log(c+d\coth(x))}{d^4}-\frac{b\coth(x)(bc-ad)^2}{d^3}+\frac{(bc-ad)(a+b\coth(x))^2}{2d^2}-\frac{(a+b\coth(x))^3}{3d}$$

input
```
Int[((a + b*Coth[x])^3*Csch[x]^2)/(c + d*Coth[x]),x]
```

output
```
-((b*(b*c - a*d)^2*Coth[x])/d^3) + ((b*c - a*d)*(a + b*Coth[x])^2)/(2*d^2)
 - (a + b*Coth[x])^3/(3*d) + ((b*c - a*d)^3*Log[c + d*Coth[x]])/d^4
```

3.1025.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4844
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cot[c*(a
+ b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc]
|| EqQ[F, csc])
```

3.1025.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.51

method	result
derivativedivides	$-\frac{b\left(\frac{b^{2}\coth(x)^{3}d^{2}}{3}+\frac{3ab\,d^{2}\coth(x)^{2}}{2}-\frac{b^{2}cd\coth(x)^{2}}{2}+3\coth(x)a^{2}d^{2}-3\coth(x)abcd+\coth(x)b^{2}c^{2}\right)}{d^{3}}-\frac{(a^{3}d^{3}-3a^{2}bc\,d^{2}+\cdots}{}$
default	$-\frac{b\left(\frac{b^{2}\coth(x)^{3}d^{2}}{3}+\frac{3ab\,d^{2}\coth(x)^{2}}{2}-\frac{b^{2}cd\coth(x)^{2}}{2}+3\coth(x)a^{2}d^{2}-3\coth(x)abcd+\coth(x)b^{2}c^{2}\right)}{d^{3}}-\frac{(a^{3}d^{3}-3a^{2}bc\,d^{2}+\cdots}{}$
risch	$-\frac{2b(9a^{2}d^{2}e^{4x}-9abcd\,e^{4x}+9ab\,d^{2}e^{4x}+3b^{2}c^{2}e^{4x}-3b^{2}cd\,e^{4x}+3b^{2}d^{2}e^{4x}-18a^{2}d^{2}e^{2x}+18abcd\,e^{2x}-9ab\,d^{2}e^{2x}-6b^{2}c^{2}e^{2x}+\cdots}{3d^{3}(e^{2x}-1)^{3}}$

input
```
int((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-b/d^3*(1/3*b^2*coth(x)^3*d^2+3/2*a*b*d^2*coth(x)^2-1/2*b^2*c*d*coth(x)^2+
3*coth(x)*a^2*d^2-3*coth(x)*a*b*c*d+coth(x)*b^2*c^2)-(a^3*d^3-3*a^2*b*c*d^
2+3*a*b^2*c^2*d-b^3*c^3)/d^4*ln(c+d*coth(x))
```

3.1025.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1980 vs. $2(74) = 148$.

Time = 0.31 (sec) , antiderivative size = 1980, normalized size of antiderivative = 25.38

$$\int \frac{(a + b\coth(x))^3 \text{csch}^2(x)}{c + d\coth(x)}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x, algorithm="fricas")
```

output
```
-1/3*(6*b^3*c^2*d - 18*a*b^2*c*d^2 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2
+ (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)^4 + 24*(b^3*c^2*d - (3*a*b^2 + b^
3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)*sinh(x)^3 + 6*(b^3*c^2*d
 - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*sinh(x)^4 + 2*(9
*a^2*b + b^3)*d^3 - 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b +
a*b^2)*d^3)*cosh(x)^2 - 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*
b + a*b^2)*d^3 - 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2
 + b^3)*d^3)*cosh(x)^2)*sinh(x)^2 - 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*cosh(x)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sinh(x)^6 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - 3*(
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 3*(b^3*c^3
- 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*
d^2 - a^3*d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh
(x)^4 - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*s
inh(x)^2 + 6*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(...
```

3.1025.6 Sympy [F]

$$\int \frac{(a+b\coth(x))^3\text{csch}^2(x)}{c+d\coth(x)}\,dx = \int \frac{(a+b\coth{(x)})^3\,\text{csch}^2\,(x)}{c+d\coth{(x)}}\,dx$$

input
```
integrate((a+b*coth(x))**3*csch(x)**2/(c+d*coth(x)),x)
```

output
```
Integral((a + b*coth(x))**3*csch(x)**2/(c + d*coth(x)), x)
```

3.1025.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(74) = 148.

Time = 0.22 (sec) , antiderivative size = 316, normalized size of antiderivative = 4.05

$$\int \frac{(a+b\coth(x))^3\text{csch}^2(x)}{c+d\coth(x)}\,dx$$
$$= \frac{1}{3}b^3\left(\frac{2\left(3\,c^2+d^2-3\,(2\,c^2+cd)e^{(-2\,x)}+3\,(c^2+cd+d^2)e^{(-4\,x)}\right)}{3\,d^3e^{(-2\,x)}-3\,d^3e^{(-4\,x)}+d^3e^{(-6\,x)}-d^3}+\frac{3\,c^3\log\left(-(c-d)e^{(-2\,x)}+c+d\right)}{d^4}-\frac{3\,c}{\cdots}\right.$$
$$+3\,ab^2\left(\frac{2\left((c+d)e^{(-2\,x)}-c\right)}{2\,d^2e^{(-2\,x)}-d^2e^{(-4\,x)}-d^2}-\frac{c^2\log\left(-(c-d)e^{(-2\,x)}+c+d\right)}{d^3}+\frac{c^2\log\left(e^{(-x)}+1\right)}{d^3}+\frac{c^2\log\left(e^{(-x)}\cdots\right.}{d^3}\right.$$
$$+3\,a^2b\left(\frac{c\log\left(-(c-d)e^{(-2\,x)}+c+d\right)}{d^2}-\frac{c\log\left(e^{(-x)}+1\right)}{d^2}-\frac{c\log\left(e^{(-x)}-1\right)}{d^2}+\frac{2}{de^{(-2\,x)}-d}\right)$$
$$-\frac{a^3\log\left(d\coth{(x)}+c\right)}{d}$$

input
```
integrate((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x, algorithm="maxima")
```

output
```
1/3*b^3*(2*(3*c^2 + d^2 - 3*(2*c^2 + c*d)*e^(-2*x) + 3*(c^2 + c*d + d^2)*e
^(-4*x))/(3*d^3*e^(-2*x) - 3*d^3*e^(-4*x) + d^3*e^(-6*x) - d^3) + 3*c^3*lo
g(-(c - d)*e^(-2*x) + c + d)/d^4 - 3*c^3*log(e^(-x) + 1)/d^4 - 3*c^3*log(e
^(-x) - 1)/d^4) + 3*a*b^2*(2*((c + d)*e^(-2*x) - c)/(2*d^2*e^(-2*x) - d^2*
e^(-4*x) - d^2) - c^2*log(-(c - d)*e^(-2*x) + c + d)/d^3 + c^2*log(e^(-x)
+ 1)/d^3 + c^2*log(e^(-x) - 1)/d^3) + 3*a^2*b*(c*log(-(c - d)*e^(-2*x) + c
 + d)/d^2 - c*log(e^(-x) + 1)/d^2 - c*log(e^(-x) - 1)/d^2 + 2/(d*e^(-2*x)
- d)) - a^3*log(d*coth(x) + c)/d
```

3.1025.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 544, normalized size of antiderivative = 6.97

$$\int \frac{(a+b\coth(x))^3\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx$$
$$= \frac{(b^3c^4 - 3\,ab^2c^3d + b^3c^3d + 3\,a^2bc^2d^2 - 3\,ab^2c^2d^2 - a^3cd^3 + 3\,a^2bcd^3 - a^3d^4)\log\left(\left|ce^{(2\,x)} + de^{(2\,x)} - c + d\right|\right)}{cd^4 + d^5}$$
$$- \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\log\left(\left|e^{(2\,x)} - 1\right|\right)}{d^4}$$
$$+ \frac{11\,b^3c^3e^{(6\,x)} - 33\,ab^2c^2de^{(6\,x)} + 33\,a^2bcd^2e^{(6\,x)} - 11\,a^3d^3e^{(6\,x)} - 33\,b^3c^3e^{(4\,x)} + 99\,ab^2c^2de^{(4\,x)} - 12\,b^3c^2de^{(}}{}$$

input
```
integrate((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x, algorithm="giac")
```

output
```
(b^3*c^4 - 3*a*b^2*c^3*d + b^3*c^3*d + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^2*d^2 -
 a^3*c*d^3 + 3*a^2*b*c*d^3 - a^3*d^4)*log(abs(c*e^(2*x) + d*e^(2*x) - c +
d))/(c*d^4 + d^5) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*lo
g(abs(e^(2*x) - 1))/d^4 + 1/6*(11*b^3*c^3*e^(6*x) - 33*a*b^2*c^2*d*e^(6*x)
 + 33*a^2*b*c*d^2*e^(6*x) - 11*a^3*d^3*e^(6*x) - 33*b^3*c^3*e^(4*x) + 99*a
*b^2*c^2*d*e^(4*x) - 12*b^3*c^2*d*e^(4*x) - 99*a^2*b*c*d^2*e^(4*x) + 36*a*
b^2*c*d^2*e^(4*x) + 12*b^3*c*d^2*e^(4*x) + 33*a^3*d^3*e^(4*x) - 36*a^2*b*d
^3*e^(4*x) - 36*a*b^2*d^3*e^(4*x) - 12*b^3*d^3*e^(4*x) + 33*b^3*c^3*e^(2*x
) - 99*a*b^2*c^2*d*e^(2*x) + 24*b^3*c^2*d*e^(2*x) + 99*a^2*b*c*d^2*e^(2*x)
 - 72*a*b^2*c*d^2*e^(2*x) - 12*b^3*c*d^2*e^(2*x) - 33*a^3*d^3*e^(2*x) + 72
*a^2*b*d^3*e^(2*x) + 36*a*b^2*d^3*e^(2*x) - 11*b^3*c^3 + 33*a*b^2*c^2*d -
12*b^3*c^2*d - 33*a^2*b*c*d^2 + 36*a*b^2*c*d^2 + 11*a^3*d^3 - 36*a^2*b*d^3
 - 4*b^3*d^3)/(d^4*(e^(2*x) - 1)^3)
```

3.1025.9 Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 1346, normalized size of antiderivative = 17.26

$$\int \frac{(a+b\coth(x))^3\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx$$

$$= \frac{2\,\mathrm{atan}\left(\frac{\left(\mathrm{e}^{2\,x}\left(\frac{32\,c\,\left(2\,a^3\,c\,d^8-6\,a^2\,b\,c^2\,d^7+6\,a\,b^2\,c^3\,d^6-2\,b^3\,c^4\,d^5\right)\,\sqrt{a^6\,d^6-6\,a^5\,b\,c\,d^5+15\,a^4\,b^2\,c^2\,d^4-20\,a^3\,b^3\,c^3\,d^3+15\,a^2\,b^4\,c^4\,d^2-6\,a\,b^5\,c^5\,d+b^6\,c^6}}{d^{16}\,\sqrt{{\left(a\,d-b\,c\right)}^6}\,\left(c+d\right)\,{\left(c-d\right)}^2\,\left(c^2+2\,c\,d+d^2\right)} - 1\cdots\right.\right.}{\cdots}\right)}{\cdots}$$

$$-\frac{2\,\left(2\,b^3\,d-b^3\,c+3\,a\,b^2\,d\right)}{d^2\,\left({\mathrm{e}}^{4\,x}-2\,{\mathrm{e}}^{2\,x}+1\right)}-\frac{8\,b^3}{3\,d\,\left(3\,{\mathrm{e}}^{2\,x}-3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}-1\right)}$$
$$-\frac{2\,\left(3\,a^2\,b\,d^2-3\,a\,b^2\,c\,d+3\,a\,b^2\,d^2+b^3\,c^2-b^3\,c\,d+b^3\,d^2\right)}{d^3\,\left({\mathrm{e}}^{2\,x}-1\right)}$$

input
```
int((a + b*coth(x))^3/(sinh(x)^2*(c + d*coth(x))),x)
```

output
```
(2*atan(((exp(2*x)*((32*c*(2*a^3*c*d^8 - 2*b^3*c^4*d^5 + 6*a*b^2*c^3*d^6 -
 6*a^2*b*c^2*d^7)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3
*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))/(d^16*((
a*d - b*c)^6)^(1/2)*(c + d)*(c - d)^2*(2*c*d + c^2 + d^2)) - (16*(c^2*(-d^
8)^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15
*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2) + d^2*(-d^8)^(1/2)
*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))*(c^2 + d^2)*(a^6*d^6 + b^
6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a
*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))/(d^13*(c + d)*(c - d)^2*(a*d - b*c)^3*(
-d^8)^(1/2)*(2*c*d + c^2 + d^2))) + (32*c*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*
c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*
b*c*d^5)^(1/2)*(a^3*d^9 - a^3*c*d^8 - b^3*c^3*d^6 + b^3*c^4*d^5 + 3*a*b^2*
c^2*d^7 - 3*a*b^2*c^3*d^6 + 3*a^2*b*c^2*d^7 - 3*a^2*b*c*d^8))/(d^16*((a*d
- b*c)^6)^(1/2)*(c + d)*(c - d)^2*(2*c*d + c^2 + d^2)) + (16*(c^2*(-d^8)^(
1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2) - c*d*(-d^8)^(1/2)*(a^
6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))*(c^2 + d^2)*(a^6*d^6 + b^6*c^
6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*...
```

3.1026 $\int \cosh^3(x) \left(a+b\cosh^2(x)\right)^3 \sinh(x)\,dx$

3.1026.1 Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \cosh^3(x) \left(a+b\cosh^2(x)\right)^3 \sinh(x)\,dx = -\frac{a\left(a+b\cosh^2(x)\right)^4}{8b^2}+\frac{\left(a+b\cosh^2(x)\right)^5}{10b^2}$$

output
```
-1/8*a*(a+b*cosh(x)^2)^4/b^2+1/10*(a+b*cosh(x)^2)^5/b^2
```

3.1026.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 136 vs. 2(36) = 72.

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.78

$$\begin{aligned}\int \cosh^3(x) \left(a+b\cosh^2(x)\right)^3 \sinh(x)\,dx &= \frac{1}{32}\Big(12a^2b\cosh^4(x)+8ab^2\cosh^6(x)+2b^3\cosh^8(x)\\&\quad+4a^3\cosh(2x)+4a^2b\cosh^3(x)\cosh(3x)\\&\quad+a^3\cosh(4x)+\frac{1}{32}ab^2(48\cosh(2x)+36\cosh(4x)\\&\quad+16\cosh(6x)+3\cosh(8x))\\&\quad+\frac{1}{320}b^3(140\cosh(2x)+100\cosh(4x)\\&\quad+50\cosh(6x)+15\cosh(8x)+2\cosh(10x))\Big)\end{aligned}$$

input
```
Integrate[Cosh[x]^3*(a + b*Cosh[x]^2)^3*Sinh[x],x]
```

output
```
(12*a^2*b*Cosh[x]^4 + 8*a*b^2*Cosh[x]^6 + 2*b^3*Cosh[x]^8 + 4*a^3*Cosh[2*x
] + 4*a^2*b*Cosh[x]^3*Cosh[3*x] + a^3*Cosh[4*x] + (a*b^2*(48*Cosh[2*x] + 3
6*Cosh[4*x] + 16*Cosh[6*x] + 3*Cosh[8*x]))/32 + (b^3*(140*Cosh[2*x] + 100*
Cosh[4*x] + 50*Cosh[6*x] + 15*Cosh[8*x] + 2*Cosh[10*x]))/320)/32
```

3.1026.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 4835, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\cosh^3(x)\left(a+b\cosh^2(x)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ix)\cos(ix)^3\left(a+b\cos(ix)^2\right)^3 dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \cos(ix)^3\left(b\cos(ix)^2+a\right)^3\sin(ix)dx$$

$$\Big\downarrow \text{4835}$$

$$\int \cosh^3(x)\left(a+b\cosh^2(x)\right)^3 d\cosh(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \cosh^2(x)\left(b\cosh^2(x)+a\right)^3 d\cosh^2(x)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{\left(b\cosh^2(x)+a\right)^4}{b}-\frac{a\left(b\cosh^2(x)+a\right)^3}{b}\right)d\cosh^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\left(a+b\cosh^2(x)\right)^5}{5b^2}-\frac{a\left(a+b\cosh^2(x)\right)^4}{4b^2}\right)$$

input
```
Int[Cosh[x]^3*(a + b*Cosh[x]^2)^3*Sinh[x],x]
```

output
```
(-1/4*(a*(a + b*Cosh[x]^2)^4)/b^2 + (a + b*Cosh[x]^2)^5/(5*b^2))/2
```

3.1026.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.1026.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\frac{b^3 \cosh(x)^{10}}{10} + \frac{3a\,b^2 \cosh(x)^8}{8} + \frac{a^2 b \cosh(x)^6}{2} + \frac{a^3 \cosh(x)^4}{4}$$

input
```
int(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x)
```

output
```
1/10*b^3*cosh(x)^10+3/8*a*b^2*cosh(x)^8+1/2*a^2*b*cosh(x)^6+1/4*a^3*cosh(x
)^4
```

3.1026.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(32) = 64.

Time = 0.28 (sec) , antiderivative size = 386, normalized size of antiderivative = 10.72

$$\int \cosh^3(x) \left(a + b\cosh^2(x)\right)^3 \sinh(x)\,dx$$
$$= \frac{1}{5120} b^3 \cosh(x)^{10} + \frac{1}{5120} b^3 \sinh(x)^{10} + \frac{1}{1024} \left(3\,ab^2 + 2\,b^3\right) \cosh(x)^8$$
$$+ \frac{1}{1024} \left(9\,b^3 \cosh(x)^2 + 3\,ab^2 + 2\,b^3\right) \sinh(x)^8 + \frac{1}{1024} \left(16\,a^2 b + 24\,ab^2 + 9\,b^3\right) \cosh(x)^6$$
$$+ \frac{1}{1024} \left(42\,b^3 \cosh(x)^4 + 16\,a^2 b + 24\,ab^2 + 9\,b^3 + 28\left(3\,ab^2 + 2\,b^3\right) \cosh(x)^2\right) \sinh(x)^6$$
$$+ \frac{1}{256} \left(8\,a^3 + 24\,a^2 b + 21\,ab^2 + 6\,b^3\right) \cosh(x)^4$$
$$+ \frac{1}{1024} \left(42\,b^3 \cosh(x)^6 + 70\left(3\,ab^2 + 2\,b^3\right) \cosh(x)^4 + 32\,a^3 + 96\,a^2 b + 84\,ab^2 + 24\,b^3 + 15\left(16\,a^2 b + 24\,ab\right.\right.$$
$$+ \frac{1}{512} \left(64\,a^3 + 120\,a^2 b + 84\,ab^2 + 21\,b^3\right) \cosh(x)^2$$
$$+ \frac{1}{1024} \left(9\,b^3 \cosh(x)^8 + 28\left(3\,ab^2 + 2\,b^3\right) \cosh(x)^6 + 15\left(16\,a^2 b + 24\,ab^2 + 9\,b^3\right) \cosh(x)^4 + 128\,a^3 + 240\right.$$

input
```
integrate(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x, algorithm="fricas")
```

output
```
1/5120*b^3*cosh(x)^10 + 1/5120*b^3*sinh(x)^10 + 1/1024*(3*a*b^2 + 2*b^3)*c
osh(x)^8 + 1/1024*(9*b^3*cosh(x)^2 + 3*a*b^2 + 2*b^3)*sinh(x)^8 + 1/1024*(
16*a^2*b + 24*a*b^2 + 9*b^3)*cosh(x)^6 + 1/1024*(42*b^3*cosh(x)^4 + 16*a^2
*b + 24*a*b^2 + 9*b^3 + 28*(3*a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^6 + 1/256*
(8*a^3 + 24*a^2*b + 21*a*b^2 + 6*b^3)*cosh(x)^4 + 1/1024*(42*b^3*cosh(x)^6
 + 70*(3*a*b^2 + 2*b^3)*cosh(x)^4 + 32*a^3 + 96*a^2*b + 84*a*b^2 + 24*b^3
+ 15*(16*a^2*b + 24*a*b^2 + 9*b^3)*cosh(x)^2)*sinh(x)^4 + 1/512*(64*a^3 +
120*a^2*b + 84*a*b^2 + 21*b^3)*cosh(x)^2 + 1/1024*(9*b^3*cosh(x)^8 + 28*(3
*a*b^2 + 2*b^3)*cosh(x)^6 + 15*(16*a^2*b + 24*a*b^2 + 9*b^3)*cosh(x)^4 + 1
28*a^3 + 240*a^2*b + 168*a*b^2 + 42*b^3 + 24*(8*a^3 + 24*a^2*b + 21*a*b^2
+ 6*b^3)*cosh(x)^2)*sinh(x)^2
```

3.1026.6 Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \cosh^3(x)\left(a+b\cosh^2(x)\right)^3\sinh(x)\,dx = \frac{a^3\cosh^4(x)}{4} + \frac{a^2b\cosh^6(x)}{2} + \frac{3ab^2\cosh^8(x)}{8} + \frac{b^3\cosh^{10}(x)}{10}$$

input
```
integrate(cosh(x)**3*(a+b*cosh(x)**2)**3*sinh(x),x)
```

output
```
a**3*cosh(x)**4/4 + a**2*b*cosh(x)**6/2 + 3*a*b**2*cosh(x)**8/8 + b**3*cos
h(x)**10/10
```

3.1026.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cosh^3(x)\left(a+b\cosh^2(x)\right)^3\sinh(x)\,dx = \frac{1}{10}\,b^3\cosh(x)^{10} + \frac{3}{8}\,ab^2\cosh(x)^8 + \frac{1}{2}\,a^2b\cosh(x)^6 + \frac{1}{4}\,a^3\cosh(x)^4$$

input
```
integrate(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x, algorithm="maxima")
```

output
```
1/10*b^3*cosh(x)^10 + 3/8*a*b^2*cosh(x)^8 + 1/2*a^2*b*cosh(x)^6 + 1/4*a^3*
cosh(x)^4
```

3.1026.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 6.22

$$\int \cosh^3(x)\left(a+b\cosh^2(x)\right)^3\sinh(x)\,dx = \frac{1}{10240}\,b^3\left(e^{(2\,x)}+e^{(-2\,x)}\right)^5 + \frac{3}{2048}\,ab^2\left(e^{(2\,x)}+e^{(-2\,x)}\right)^4 + \frac{1}{1024}\,b^3\left(e^{(2\,x)}+e^{(-2\,x)}\right)^4 + \frac{1}{128}\,a^2b\left(e^{(2\,x)}+e^{(-2\,x)}\right)^3 + \frac{3}{256}\,ab^2\left(e^{(2\,x)}+e^{(-2\,x)}\right)^3 + \frac{1}{256}\,b^3\left(e^{(2\,x)}+e^{(-2\,x)}\right)^3 + \frac{1}{64}\,a^3\left(e^{(2\,x)}+e^{(-2\,x)}\right)^2 + \frac{3}{64}\,a^2b\left(e^{(2\,x)}+e^{(-2\,x)}\right)^2 + \frac{9}{256}\,ab^2\left(e^{(2\,x)}+e^{(-2\,x)}\right)^2 + \frac{1}{128}\,b^3\left(e^{(2\,x)}+e^{(-2\,x)}\right)^2 + \frac{1}{16}\,a^3\left(e^{(2\,x)}+e^{(-2\,x)}\right) + \frac{3}{32}\,a^2b\left(e^{(2\,x)}+e^{(-2\,x)}\right) + \frac{3}{64}\,ab^2\left(e^{(2\,x)}+e^{(-2\,x)}\right) + \frac{1}{128}\,b^3\left(e^{(2\,x)}+e^{(-2\,x)}\right)$$

input
```
integrate(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x, algorithm="giac")
```

output
```
1/10240*b^3*(e^(2*x) + e^(-2*x))^5 + 3/2048*a*b^2*(e^(2*x) + e^(-2*x))^4 +
 1/1024*b^3*(e^(2*x) + e^(-2*x))^4 + 1/128*a^2*b*(e^(2*x) + e^(-2*x))^3 +
3/256*a*b^2*(e^(2*x) + e^(-2*x))^3 + 1/256*b^3*(e^(2*x) + e^(-2*x))^3 + 1/
64*a^3*(e^(2*x) + e^(-2*x))^2 + 3/64*a^2*b*(e^(2*x) + e^(-2*x))^2 + 9/256*
a*b^2*(e^(2*x) + e^(-2*x))^2 + 1/128*b^3*(e^(2*x) + e^(-2*x))^2 + 1/16*a^3
*(e^(2*x) + e^(-2*x)) + 3/32*a^2*b*(e^(2*x) + e^(-2*x)) + 3/64*a*b^2*(e^(2
*x) + e^(-2*x)) + 1/128*b^3*(e^(2*x) + e^(-2*x))
```

3.1026.9 Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cosh^3(x) \left(a + b\cosh^2(x)\right)^3 \sinh(x)\, dx = \frac{a^3 \cosh(x)^4}{4} + \frac{a^2\, b \cosh(x)^6}{2} + \frac{3\, a\, b^2 \cosh(x)^8}{8} + \frac{b^3 \cosh(x)^{10}}{10}$$

input
```
int(cosh(x)^3*sinh(x)*(a + b*cosh(x)^2)^3,x)
```

output
```
(a^3*cosh(x)^4)/4 + (b^3*cosh(x)^10)/10 + (a^2*b*cosh(x)^6)/2 + (3*a*b^2*c
osh(x)^8)/8
```

3.1027 $\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx$

3.1027.1 Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx=-\frac{a\left(a+b\sinh^2(x)\right)^4}{8b^2}+\frac{\left(a+b\sinh^2(x)\right)^5}{10b^2}$$

output
```
-1/8*a*(a+b*sinh(x)^2)^4/b^2+1/10*(a+b*sinh(x)^2)^5/b^2
```

3.1027.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(36) = 72.

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.17

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx$$
$$=\frac{-20(64a^3+24ab^2-7b^3)\cosh(2x)+20(16a^3+18ab^2-5b^3)\cosh(4x)+b\big(-10(16a-5b)b\cosh(6x)+15}{10240}$$

input
```
Integrate[Cosh[x]*Sinh[x]^3*(a + b*Sinh[x]^2)^3,x]
```

output
```
(-20*(64*a^3 + 24*a*b^2 - 7*b^3)*Cosh[2*x] + 20*(16*a^3 + 18*a*b^2 - 5*b^3
)*Cosh[4*x] + b*(-10*(16*a - 5*b)*b*Cosh[6*x] + 15*(2*a - b)*b*Cosh[8*x] +
 2*b^2*Cosh[10*x] + 320*((-4*a + b)^2 - b^2*Cosh[2*x])*Sinh[x]^6))/10240
```

3.1027.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 26, 3677, 26, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(x)\cosh(x)\left(a+b\sinh^2(x)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ix)^3\cos(ix)\left(a-b\sin(ix)^2\right)^3 dx$$

$$\Big\downarrow \text{26}$$

$$i\int \cos(ix)\sin(ix)^3\left(a-b\sin(ix)^2\right)^3 dx$$

$$\Big\downarrow \text{3677}$$

$$i\int -i\sinh^3(x)\left(b\sinh^2(x)+a\right)^3 d\sinh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \sinh^3(x)\left(a+b\sinh^2(x)\right)^3 d\sinh(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \sinh^2(x)\left(b\sinh^2(x)+a\right)^3 d\sinh^2(x)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{\left(b\sinh^2(x)+a\right)^4}{b}-\frac{a\left(b\sinh^2(x)+a\right)^3}{b}\right) d\sinh^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\left(a+b\sinh^2(x)\right)^5}{5b^2}-\frac{a\left(a+b\sinh^2(x)\right)^4}{4b^2}\right)$$

input
```
Int[Cosh[x]*Sinh[x]^3*(a + b*Sinh[x]^2)^3,x]
```

output
```
(-1/4*(a*(a + b*Sinh[x]^2)^4)/b^2 + (a + b*Sinh[x]^2)^5/(5*b^2))/2
```

3.1027.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3677
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((
m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

3.1027.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\frac{b^3 \sinh\left(x\right)^{10}}{10}+\frac{3a\,b^2 \sinh\left(x\right)^{8}}{8}+\frac{a^2 b \sinh\left(x\right)^{6}}{2}+\frac{a^3 \sinh\left(x\right)^{4}}{4}$$

input
```
int(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x)
```

output
```
1/10*b^3*sinh(x)^10+3/8*a*b^2*sinh(x)^8+1/2*a^2*b*sinh(x)^6+1/4*a^3*sinh(x
)^4
```

3.1027.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(32) = 64.

Time = 0.25 (sec) , antiderivative size = 386, normalized size of antiderivative = 10.72

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx$$
$$= \frac{1}{5120}b^3\cosh(x)^{10} + \frac{1}{5120}b^3\sinh(x)^{10} + \frac{1}{1024}\left(3ab^2-2b^3\right)\cosh(x)^8$$
$$+ \frac{1}{1024}\left(9b^3\cosh(x)^2+3ab^2-2b^3\right)\sinh(x)^8 + \frac{1}{1024}\left(16a^2b-24ab^2+9b^3\right)\cosh(x)^6$$
$$+ \frac{1}{1024}\left(42b^3\cosh(x)^4+16a^2b-24ab^2+9b^3+28\left(3ab^2-2b^3\right)\cosh(x)^2\right)\sinh(x)^6$$
$$+ \frac{1}{256}\left(8a^3-24a^2b+21ab^2-6b^3\right)\cosh(x)^4$$
$$+ \frac{1}{1024}\left(42b^3\cosh(x)^6+70\left(3ab^2-2b^3\right)\cosh(x)^4+32a^3-96a^2b+84ab^2-24b^3+15\left(16a^2b-24ab\right.\right.$$
$$- \frac{1}{512}\left(64a^3-120a^2b+84ab^2-21b^3\right)\cosh(x)^2$$
$$+ \frac{1}{1024}\left(9b^3\cosh(x)^8+28\left(3ab^2-2b^3\right)\cosh(x)^6+15\left(16a^2b-24ab^2+9b^3\right)\cosh(x)^4-128a^3+240\right.$$

input
```
integrate(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x, algorithm="fricas")
```

output
```
1/5120*b^3*cosh(x)^10 + 1/5120*b^3*sinh(x)^10 + 1/1024*(3*a*b^2 - 2*b^3)*c
osh(x)^8 + 1/1024*(9*b^3*cosh(x)^2 + 3*a*b^2 - 2*b^3)*sinh(x)^8 + 1/1024*(
16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(x)^6 + 1/1024*(42*b^3*cosh(x)^4 + 16*a^2
*b - 24*a*b^2 + 9*b^3 + 28*(3*a*b^2 - 2*b^3)*cosh(x)^2)*sinh(x)^6 + 1/256*
(8*a^3 - 24*a^2*b + 21*a*b^2 - 6*b^3)*cosh(x)^4 + 1/1024*(42*b^3*cosh(x)^6
 + 70*(3*a*b^2 - 2*b^3)*cosh(x)^4 + 32*a^3 - 96*a^2*b + 84*a*b^2 - 24*b^3
+ 15*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(x)^2)*sinh(x)^4 - 1/512*(64*a^3 -
120*a^2*b + 84*a*b^2 - 21*b^3)*cosh(x)^2 + 1/1024*(9*b^3*cosh(x)^8 + 28*(3
*a*b^2 - 2*b^3)*cosh(x)^6 + 15*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(x)^4 - 1
28*a^3 + 240*a^2*b - 168*a*b^2 + 42*b^3 + 24*(8*a^3 - 24*a^2*b + 21*a*b^2
- 6*b^3)*cosh(x)^2)*sinh(x)^2
```

3.1027.6 Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx=\frac{a^3\sinh^4(x)}{4}+\frac{a^2b\sinh^6(x)}{2}+\frac{3ab^2\sinh^8(x)}{8}+\frac{b^3\sinh^{10}(x)}{10}$$

input
```
integrate(cosh(x)*sinh(x)**3*(a+b*sinh(x)**2)**3,x)
```

output
```
a**3*sinh(x)**4/4 + a**2*b*sinh(x)**6/2 + 3*a*b**2*sinh(x)**8/8 + b**3*sin
h(x)**10/10
```

3.1027.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx=\frac{1}{10}\,b^3\sinh(x)^{10}+\frac{3}{8}\,ab^2\sinh(x)^8+\frac{1}{2}\,a^2b\sinh(x)^6+\frac{1}{4}\,a^3\sinh(x)^4$$

input
```
integrate(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x, algorithm="maxima")
```

output
```
1/10*b^3*sinh(x)^10 + 3/8*a*b^2*sinh(x)^8 + 1/2*a^2*b*sinh(x)^6 + 1/4*a^3*
sinh(x)^4
```

3.1027.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 6.22

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx = \frac{1}{10240}b^3\left(e^{(2x)}+e^{(-2x)}\right)^5 + \frac{3}{2048}ab^2\left(e^{(2x)}+e^{(-2x)}\right)^4 - \frac{1}{1024}b^3\left(e^{(2x)}+e^{(-2x)}\right)^4 + \frac{1}{128}a^2b\left(e^{(2x)}+e^{(-2x)}\right)^3 - \frac{3}{256}ab^2\left(e^{(2x)}+e^{(-2x)}\right)^3 + \frac{1}{256}b^3\left(e^{(2x)}+e^{(-2x)}\right)^3 + \frac{1}{64}a^3\left(e^{(2x)}+e^{(-2x)}\right)^2 - \frac{3}{64}a^2b\left(e^{(2x)}+e^{(-2x)}\right)^2 + \frac{9}{256}ab^2\left(e^{(2x)}+e^{(-2x)}\right)^2 - \frac{1}{128}b^3\left(e^{(2x)}+e^{(-2x)}\right)^2 - \frac{1}{16}a^3\left(e^{(2x)}+e^{(-2x)}\right) + \frac{3}{32}a^2b\left(e^{(2x)}+e^{(-2x)}\right) - \frac{3}{64}ab^2\left(e^{(2x)}+e^{(-2x)}\right) + \frac{1}{128}b^3\left(e^{(2x)}+e^{(-2x)}\right)$$

input
```
integrate(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x, algorithm="giac")
```

output
```
1/10240*b^3*(e^(2*x) + e^(-2*x))^5 + 3/2048*a*b^2*(e^(2*x) + e^(-2*x))^4 -
 1/1024*b^3*(e^(2*x) + e^(-2*x))^4 + 1/128*a^2*b*(e^(2*x) + e^(-2*x))^3 -
3/256*a*b^2*(e^(2*x) + e^(-2*x))^3 + 1/256*b^3*(e^(2*x) + e^(-2*x))^3 + 1/
64*a^3*(e^(2*x) + e^(-2*x))^2 - 3/64*a^2*b*(e^(2*x) + e^(-2*x))^2 + 9/256*
a*b^2*(e^(2*x) + e^(-2*x))^2 - 1/128*b^3*(e^(2*x) + e^(-2*x))^2 - 1/16*a^3
*(e^(2*x) + e^(-2*x)) + 3/32*a^2*b*(e^(2*x) + e^(-2*x)) - 3/64*a*b^2*(e^(2
*x) + e^(-2*x)) + 1/128*b^3*(e^(2*x) + e^(-2*x))
```

3.1027. $\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx$

3.1027.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx=\frac{a^3\sinh(x)^4}{4}+\frac{a^2\,b\sinh(x)^6}{2}+\frac{3\,a\,b^2\sinh(x)^8}{8}+\frac{b^3\sinh(x)^{10}}{10}$$

input
```
int(cosh(x)*sinh(x)^3*(a + b*sinh(x)^2)^3,x)
```

output
```
(a^3*sinh(x)^4)/4 + (b^3*sinh(x)^10)/10 + (a^2*b*sinh(x)^6)/2 + (3*a*b^2*s
inh(x)^8)/8
```

3.1028 $\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx$

3.1028.1 Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx = \frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

output
```
1/3*(a+b*sinh(x)^2)^(3/2)/b
```

3.1028.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx = \frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

input
```
Integrate[Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2],x]
```

output
```
(a + b*Sinh[x]^2)^(3/2)/(3*b)
```

3.1028.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 26, 3677, 26, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\cosh(x)\sqrt{a+b\sinh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ix)\cos(ix)\sqrt{a-b\sin(ix)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \cos(ix)\sin(ix)\sqrt{a-b\sin(ix)^2}dx$$

$$\Big\downarrow \text{3677}$$

$$-i\int i\sinh(x)\sqrt{b\sinh^2(x)+a}d\sinh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \sinh(x)\sqrt{a+b\sinh^2(x)}d\sinh(x)$$

$$\Big\downarrow \text{241}$$

$$\frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

input
```
Int[Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2],x]
```

output
```
(a + b*Sinh[x]^2)^(3/2)/(3*b)
```

3.1028.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3677
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((
m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

3.1028.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\left(a+b\sinh(x)^2\right)^{\frac{3}{2}}}{3b}$	16
default	$\frac{\left(a+b\sinh(x)^2\right)^{\frac{3}{2}}}{3b}$	16

input
```
int(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a+b*sinh(x)^2)^(3/2)/b
```

3.1028.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 8.11

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx$$
$$= \frac{\sqrt{2}\left(b\cosh\left(x\right)^4+4\,b\cosh\left(x\right)\sinh\left(x\right)^3+b\sinh\left(x\right)^4+2\left(2\,a-b\right)\cosh\left(x\right)^2+2\left(3\,b\cosh\left(x\right)^2+2\,a-b\right)\sin\cdots\right.}{24\left(b\cosh\left(x\right)^3+3\,b\cosh\left(x\right)^2\sinh\left(x\right)+3\,b\cos\cdots\right.}$$

input
```
integrate(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a -
 b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a - b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (
2*a - b)*cosh(x))*sinh(x) + b)*sqrt((b*cosh(x)^2 + b*sinh(x)^2 + 2*a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(b*cosh(x)^3 + 3*b*cosh(x)^2*
sinh(x) + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3)
```

3.1028.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx = \begin{cases} \frac{a\sqrt{a+b\sinh^2(x)}}{3b} + \frac{\sqrt{a+b\sinh^2(x)}\sinh^2(x)}{3} & \text{for } b \neq 0 \\ \frac{\sqrt{a}\cosh^2(x)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(x)*sinh(x)*(a+b*sinh(x)**2)**(1/2),x)
```

output
```
Piecewise((a*sqrt(a + b*sinh(x)**2)/(3*b) + sqrt(a + b*sinh(x)**2)*sinh(x)
**2/3, Ne(b, 0)), (sqrt(a)*cosh(x)**2/2, True))
```

3.1028.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx = \frac{\left(b\sinh\left(x\right)^2+a\right)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(b*sinh(x)^2 + a)^(3/2)/b
```

3.1028.8 Giac [F(-2)]

Exception generated.

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.1028.9 Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx = \frac{\left(b\sinh(x)^2+a\right)^{3/2}}{3\,b}$$

input
```
int(cosh(x)*sinh(x)*(a + b*sinh(x)^2)^(1/2),x)
```

output
```
(a + b*sinh(x)^2)^(3/2)/(3*b)
```

3.1029 $\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx$

3.1029.1 Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = -\frac{1}{2}\operatorname{arcsinh}(\log(\coth(x))) - \frac{1}{2}\log(\coth(x))\sqrt{1+\log^2(\coth(x))}$$

output
```
-1/2*arcsinh(ln(coth(x)))-1/2*ln(coth(x))*(1+ln(coth(x))^2)^(1/2)
```

3.1029.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = -\frac{1}{2}\log(\coth(x))\sqrt{1+\log^2(\coth(x))} + \frac{1}{2}\log\left(-\log(\coth(x))+\sqrt{1+\log^2(\coth(x))}\right)$$

input
```
Integrate[Csch[x]*Sqrt[1 + Log[Coth[x]]^2]*Sech[x],x]
```

output
```
-1/2*(Log[Coth[x]]*Sqrt[1 + Log[Coth[x]]^2]) + Log[-Log[Coth[x]] + Sqrt[1
+ Log[Coth[x]]^2]]/2
```

3.1029.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {7247, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{\log^2(\coth(x))+1}\,dx$$

$$\Big\downarrow \; 7247$$

$$-\int \sqrt{\log^2(\coth(x))+1}d\log(\coth(x))$$

$$\Big\downarrow \; 211$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{\log^2(\coth(x))+1}}d\log(\coth(x))-\frac{1}{2}\sqrt{\log^2(\coth(x))+1}\log(\coth(x))$$

$$\Big\downarrow \; 222$$

$$-\frac{1}{2}\text{arcsinh}(\log(\coth(x)))-\frac{1}{2}\log(\coth(x))\sqrt{\log^2(\coth(x))+1}$$

input
```
Int[Csch[x]*Sqrt[1 + Log[Coth[x]]^2]*Sech[x],x]
```

output
```
-1/2*ArcSinh[Log[Coth[x]]] - (Log[Coth[x]]*Sqrt[1 + Log[Coth[x]]^2])/2
```

3.1029.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 7247
```
Int[(u_.)*((a_.) + (b_.)*(y_)^(n_))^(p_), x_Symbol] :> With[{q = Derivative
Divides[y, u, x]}, Simp[q   Subst[Int[(a + b*x^n)^p, x], x, y], x] /;  !Fal
seQ[q]] /; FreeQ[{a, b, n, p}, x]
```

3.1029.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\operatorname{arcsinh}(\ln(\coth(x)))}{2}-\frac{\ln(\coth(x))\sqrt{1+\ln(\coth(x))^{2}}}{2}$	22
default	$-\frac{\operatorname{arcsinh}(\ln(\coth(x)))}{2}-\frac{\ln(\coth(x))\sqrt{1+\ln(\coth(x))^{2}}}{2}$	22

input
```
int(csch(x)*sech(x)*(1+ln(coth(x))^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arcsinh(ln(coth(x)))-1/2*ln(coth(x))*(1+ln(coth(x))^2)^(1/2)
```

3.1029.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(21) = 42.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = -\frac{1}{2}\sqrt{\log\left(\frac{\cosh(x)}{\sinh(x)}\right)^2+1}\log\left(\frac{\cosh(x)}{\sinh(x)}\right) + \frac{1}{2}\log\left(\sqrt{\log\left(\frac{\cosh(x)}{\sinh(x)}\right)^2+1} - \log\left(\frac{\cosh(x)}{\sinh(x)}\right)\right)$$

input
```
integrate(csch(x)*sech(x)*(1+log(coth(x))^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(log(cosh(x)/sinh(x))^2 + 1)*log(cosh(x)/sinh(x)) + 1/2*log(sqrt(
log(cosh(x)/sinh(x))^2 + 1) - log(cosh(x)/sinh(x)))
```

3.1029.6 Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = \text{Timed out}$$

input
```
integrate(csch(x)*sech(x)*(1+ln(coth(x))**2)**(1/2),x)
```

output
```
Timed out
```

3.1029.7 Maxima [F]

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = \int \sqrt{\log\left(\coth\left(x\right)\right)^2+1}\operatorname{csch}\left(x\right)\operatorname{sech}\left(x\right)\,dx$$

input
```
integrate(csch(x)*sech(x)*(1+log(coth(x))^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(log(coth(x))^2 + 1)*csch(x)*sech(x), x)
```

3.1029.8 Giac [F]

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = \int \sqrt{\log\left(\coth\left(x\right)\right)^2+1}\operatorname{csch}\left(x\right)\operatorname{sech}\left(x\right)\,dx$$

input
```
integrate(csch(x)*sech(x)*(1+log(coth(x))^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(log(coth(x))^2 + 1)*csch(x)*sech(x), x)
```

3.1029.9 Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = -\frac{\operatorname{asinh}(\ln(\coth(x)))}{2} - \frac{\ln(\coth(x))\,\sqrt{\ln(\coth(x))^2+1}}{2}$$

input
```
int((log(coth(x))^2 + 1)^(1/2)/(cosh(x)*sinh(x)),x)
```

output
```
- asinh(log(coth(x)))/2 - (log(coth(x))*(log(coth(x))^2 + 1)^(1/2))/2
```

3.1030 $\int \frac{\coth(\sqrt{x})\text{csch}(\sqrt{x})}{\sqrt{x}}\,dx$

3.1030.1 Optimal result

Integrand size = 18, antiderivative size = 8

$$\int \frac{\coth\left(\sqrt{x}\right)\text{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\text{csch}\left(\sqrt{x}\right)$$

output
```
-2*csch(x^(1/2))
```

3.1030.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(\sqrt{x}\right)\text{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\text{csch}\left(\sqrt{x}\right)$$

input
```
Integrate[(Coth[Sqrt[x]]*Csch[Sqrt[x]])/Sqrt[x],x]
```

output
```
-2*Csch[Sqrt[x]]
```

3.1030.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {7266, 3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth\left(\sqrt{x}\right)\text{csch}\left(\sqrt{x}\right)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{7266}$$

$$2\int \coth\left(\sqrt{x}\right)\text{csch}\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \sec\left(i\sqrt{x}-\frac{\pi}{2}\right)\tan\left(i\sqrt{x}-\frac{\pi}{2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3086}$$

$$-2i\int 1d\left(-i\text{csch}\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{24}$$

$$-2\text{csch}\left(\sqrt{x}\right)$$

input
```
Int[(Coth[Sqrt[x]]*Csch[Sqrt[x]])/Sqrt[x],x]
```

output
```
-2*Csch[Sqrt[x]]
```

3.1030.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.1030.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-2\,\mathrm{csch}\left(\sqrt{x}\right)$	7
default	$-2\,\mathrm{csch}\left(\sqrt{x}\right)$	7

input
```
int(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*csch(x^(1/2))
```

3.1030.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(6) = 12.

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 4.62

$$\int \frac{\coth\left(\sqrt{x}\right)\mathrm{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{4\left(\cosh\left(\sqrt{x}\right)+\sinh\left(\sqrt{x}\right)\right)}{\cosh\left(\sqrt{x}\right)^2+2\,\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)+\sinh\left(\sqrt{x}\right)^2-1}$$

input
```
integrate(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
-4*(cosh(sqrt(x)) + sinh(sqrt(x)))/(cosh(sqrt(x))^2 + 2*cosh(sqrt(x))*sinh
(sqrt(x)) + sinh(sqrt(x))^2 - 1)
```

3.1030.6 Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(\sqrt{x}\right)\operatorname{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\operatorname{csch}\left(\sqrt{x}\right)$$

input
```
integrate(coth(x**(1/2))*csch(x**(1/2))/x**(1/2),x)
```

output
```
-2*csch(sqrt(x))
```

3.1030.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{\coth\left(\sqrt{x}\right)\operatorname{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{4}{e^{\left(-\sqrt{x}\right)} - e^{\sqrt{x}}}$$

input
```
integrate(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
4/(e^(-sqrt(x)) - e^sqrt(x))
```

3.1030.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{\coth\left(\sqrt{x}\right)\operatorname{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{4}{e^{\left(-\sqrt{x}\right)} - e^{\sqrt{x}}}$$

input
```
integrate(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
4/(e^(-sqrt(x)) - e^sqrt(x))
```

3.1030.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(\sqrt{x}\right)\operatorname{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{2}{\sinh\left(\sqrt{x}\right)}$$

input
```
int(coth(x^(1/2))/(x^(1/2)*sinh(x^(1/2))),x)
```

output
```
-2/sinh(x^(1/2))
```

3.1031 $\int \frac{\cosh(\sqrt{x})\sinh(\sqrt{x})}{\sqrt{x}}\,dx$

3.1031.1 Optimal result

Integrand size = 18, antiderivative size = 8

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sinh^2\left(\sqrt{x}\right)$$

output
```
sinh(x^(1/2))^2
```

3.1031.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{1}{2}\cosh\left(2\sqrt{x}\right)$$

input
```
Integrate[(Cosh[Sqrt[x]]*Sinh[Sqrt[x]])/Sqrt[x],x]
```

output
```
Cosh[2*Sqrt[x]]/2
```

3.1031.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5893}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh\left(\sqrt{x}\right)\cosh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{5893}$$

$$\sinh^2\left(\sqrt{x}\right)$$

input
```
Int[(Cosh[Sqrt[x]]*Sinh[Sqrt[x]])/Sqrt[x],x]
```

output
```
Sinh[Sqrt[x]]^2
```

3.1031.3.1 Defintions of rubi rules used

rule 5893
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_)]
^(p_.), x_Symbol] :> Simp[Sinh[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; Free
Q[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

3.1031.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\cosh\left(\sqrt{x}\right)^2$	7
default	$\cosh\left(\sqrt{x}\right)^2$	7
meijerg	$-\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(2\sqrt{x})}{\sqrt{\pi}}\right)}{2}$	21

input
```
int(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cosh(x^(1/2))^2
```

3.1031.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{1}{2}\cosh\left(\sqrt{x}\right)^2 + \frac{1}{2}\sinh\left(\sqrt{x}\right)^2$$

input
```
integrate(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
1/2*cosh(sqrt(x))^2 + 1/2*sinh(sqrt(x))^2
```

3.1031.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sinh^2\left(\sqrt{x}\right)$$

input
```
integrate(cosh(x**(1/2))*sinh(x**(1/2))/x**(1/2),x)
```

output
```
sinh(sqrt(x))**2
```

3.1031.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \cosh\left(\sqrt{x}\right)^2$$

input
```
integrate(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
cosh(sqrt(x))^2
```

3.1031.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{1}{4}\,e^{\left(2\,\sqrt{x}\right)} + \frac{1}{4}\,e^{\left(-2\,\sqrt{x}\right)}$$

input
```
integrate(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
1/4*e^(2*sqrt(x)) + 1/4*e^(-2*sqrt(x))
```

3.1031.9 Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \cosh\left(\sqrt{x}\right)^2$$

input
```
int((cosh(x^(1/2))*sinh(x^(1/2)))/x^(1/2),x)
```

output
```
cosh(x^(1/2))^2
```

3.1032 $\int \frac{\mathrm{sech}(\sqrt{x})\tanh(\sqrt{x})}{\sqrt{x}}\,dx$

3.1032.1 Optimal result

Integrand size = 18, antiderivative size = 8

$$\int \frac{\mathrm{sech}(\sqrt{x})\tanh(\sqrt{x})}{\sqrt{x}}\,dx = -2\mathrm{sech}(\sqrt{x})$$

output
```
-2*sech(x^(1/2))
```

3.1032.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{sech}(\sqrt{x})\tanh(\sqrt{x})}{\sqrt{x}}\,dx = -2\mathrm{sech}(\sqrt{x})$$

input
```
Integrate[(Sech[Sqrt[x]]*Tanh[Sqrt[x]])/Sqrt[x],x]
```

output
```
-2*Sech[Sqrt[x]]
```

3.1032.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {7266, 3042, 26, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh\left(\sqrt{x}\right)\text{sech}\left(\sqrt{x}\right)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{7266}$$

$$2\int \text{sech}\left(\sqrt{x}\right)\tanh\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int -i\sec\left(i\sqrt{x}\right)\tan\left(i\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{26}$$

$$-2i\int \sec\left(i\sqrt{x}\right)\tan\left(i\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3086}$$

$$-2\int 1d\text{sech}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{24}$$

$$-2\text{sech}\left(\sqrt{x}\right)$$

input
```
Int[(Sech[Sqrt[x]]*Tanh[Sqrt[x]])/Sqrt[x],x]
```

output
```
-2*Sech[Sqrt[x]]
```

3.1032. $\int \frac{\text{sech}\left(\sqrt{x}\right)\tanh\left(\sqrt{x}\right)}{\sqrt{x}}dx$

3.1032.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.1032.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-2\,\mathrm{sech}\left(\sqrt{x}\right)$	7
default	$-2\,\mathrm{sech}\left(\sqrt{x}\right)$	7

input
```
int(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*sech(x^(1/2))
```

3.1032.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 4.62

$$\int \frac{\operatorname{sech}(\sqrt{x})\tanh(\sqrt{x})}{\sqrt{x}}\,dx = -\frac{4\left(\cosh(\sqrt{x})+\sinh(\sqrt{x})\right)}{\cosh(\sqrt{x})^2+2\cosh(\sqrt{x})\sinh(\sqrt{x})+\sinh(\sqrt{x})^2+1}$$

input
```
integrate(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
-4*(cosh(sqrt(x)) + sinh(sqrt(x)))/(cosh(sqrt(x))^2 + 2*cosh(sqrt(x))*sinh
(sqrt(x)) + sinh(sqrt(x))^2 + 1)
```

3.1032.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(\sqrt{x})\tanh(\sqrt{x})}{\sqrt{x}}\,dx = -2\operatorname{sech}(\sqrt{x})$$

input
```
integrate(sech(x**(1/2))*tanh(x**(1/2))/x**(1/2),x)
```

output
```
-2*sech(sqrt(x))
```

3.1032.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{\operatorname{sech}(\sqrt{x})\tanh(\sqrt{x})}{\sqrt{x}}\,dx = -\frac{4}{e^{(-\sqrt{x})}+e^{\sqrt{x}}}$$

input
```
integrate(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
-4/(e^(-sqrt(x)) + e^sqrt(x))
```

3.1032.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{\operatorname{sech}(\sqrt{x}) \tanh(\sqrt{x})}{\sqrt{x}} \, dx = -\frac{4}{e^{(-\sqrt{x})} + e^{\sqrt{x}}}$$

input
```
integrate(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
-4/(e^(-sqrt(x)) + e^sqrt(x))
```

3.1032.9 Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(\sqrt{x}) \tanh(\sqrt{x})}{\sqrt{x}} \, dx = -\frac{2}{\cosh(\sqrt{x})}$$

input
```
int(tanh(x^(1/2))/(x^(1/2)*cosh(x^(1/2))),x)
```

output
```
-2/cosh(x^(1/2))
```

3.1033 $\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx$

3.1033.1 Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}+\frac{\log(a+b\sinh(2x))}{4b}$$

output
```
1/4*ln(a+b*sinh(2*x))/b+1/2*arctanh((b-a*tanh(x))/(a^2+b^2)^(1/2))/(a^2+b^
2)^(1/2)
```

3.1033.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx=\frac{1}{4}\left(-\frac{2\arctan\left(\frac{b-a\tanh(x)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{\log(a+b\sinh(2x))}{b}\right)$$

input
```
Integrate[Sinh[x]^2/(a + b*Sinh[2*x]),x]
```

output
```
((-2*ArcTan[(b - a*Tanh[x])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[a +
b*Sinh[2*x]]/b)/4
```

3.1033.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.73, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 25, 4889, 25, 2142, 27, 240, 1142, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\sin(ix)^2}{a-ib\sin(2ix)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\sin(ix)^2}{a-ib\sin(2ix)}dx$$

$$\Big\downarrow \ 4889$$

$$-\int -\frac{\tanh^2(x)}{(1-\tanh^2(x))\left(-a\tanh^2(x)+2b\tanh(x)+a\right)}d\tanh(x)$$

$$\Big\downarrow \ 25$$

$$\int \frac{\tanh^2(x)}{(1-\tanh^2(x))\left(-a\tanh^2(x)+a+2b\tanh(x)\right)}d\tanh(x)$$

$$\Big\downarrow \ 2142$$

$$-\frac{\int \frac{2ab\tanh(x)}{-a\tanh^2(x)+2b\tanh(x)+a}d\tanh(x)}{4b^2}-\frac{\int -\frac{2b\tanh(x)}{1-\tanh^2(x)}d\tanh(x)}{4b^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\tanh(x)}{1-\tanh^2(x)}d\tanh(x)}{2b}-\frac{a\int \frac{\tanh(x)}{-a\tanh^2(x)+2b\tanh(x)+a}d\tanh(x)}{2b}$$

$$\Big\downarrow \ 240$$

$$-\frac{a\int \frac{\tanh(x)}{-a\tanh^2(x)+2b\tanh(x)+a}d\tanh(x)}{2b}-\frac{\log\left(1-\tanh^2(x)\right)}{4b}$$

$$\Big\downarrow \ 1142$$

$$-\frac{a\left(\frac{b\int \frac{1}{-a\tanh^2(x)+2b\tanh(x)+a}d\tanh(x)}{a}-\frac{\int \frac{2(b-a\tanh(x))}{-a\tanh^2(x)+2b\tanh(x)+a}d\tanh(x)}{2a}\right)}{2b}-\frac{\log\left(1-\tanh^2(x)\right)}{4b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(\frac{b\int \frac{1}{-a\tanh^2(x)+2b\tanh(x)+a}d\tanh(x)}{a}-\frac{\int \frac{b-a\tanh(x)}{-a\tanh^2(x)+2b\tanh(x)+a}d\tanh(x)}{a}\right)}{2b}-\frac{\log\left(1-\tanh^2(x)\right)}{4b}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{a\left(-\frac{2b\int \frac{1}{4(a^2+b^2)-(2b-2a\tanh(x))^2}d(2b-2a\tanh(x))}{a}-\frac{\int \frac{b-a\tanh(x)}{-a\tanh^2(x)+2b\tanh(x)+a}d\tanh(x)}{a}\right)}{2b}-\frac{\log\left(1-\tanh^2(x)\right)}{4b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a\left(-\frac{\int \frac{b-a\tanh(x)}{-a\tanh^2(x)+2b\tanh(x)+a}d\tanh(x)}{a}-\frac{b\,\text{arctanh}\left(\frac{2b-2a\tanh(x)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{2b}-\frac{\log\left(1-\tanh^2(x)\right)}{4b}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{a\left(-\frac{b\,\text{arctanh}\left(\frac{2b-2a\tanh(x)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{\log\left(-a\tanh^2(x)+a+2b\tanh(x)\right)}{2a}\right)}{2b}-\frac{\log\left(1-\tanh^2(x)\right)}{4b}$$

input
```
Int[Sinh[x]^2/(a + b*Sinh[2*x]),x]
```

output
```
-1/4*Log[1 - Tanh[x]^2]/b - (a*(-((b*ArcTanh[(2*b - 2*a*Tanh[x])/(2*Sqrt[a
^2 + b^2])])/(a*Sqrt[a^2 + b^2])) - Log[a + 2*b*Tanh[x] - a*Tanh[x]^2]/(2*
a)))/(2*b)
```

3.1033. $\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx$

3.1033.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2142
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Sym
bol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2]
, q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a
*c*C*d + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f -
 a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 + b*B*d*f -
A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x
^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f}, x] && PolyQ[Px, x, 2]
```

3.1033. $\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.1033.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.65

method	result
default	$\frac{a\left(\frac{\ln\left(a\tanh(x)^2-2b\tanh(x)-a\right)}{2a}-\frac{b\,\mathrm{arctanh}\left(\frac{2a\tanh(x)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{2b}-\frac{\ln(1+\tanh(x))}{4b}-\frac{\ln(\tanh(x)-1)}{4b}$
risch	$\frac{x}{2b}-\frac{x\,a^2b}{a^2b^2+b^4}-\frac{x\,b^3}{a^2b^2+b^4}+\frac{\ln\left(e^{2x}+\frac{ab+\sqrt{a^2b^2+b^4}}{b^2}\right)a^2}{4(a^2+b^2)b}+\frac{b\ln\left(e^{2x}+\frac{ab+\sqrt{a^2b^2+b^4}}{b^2}\right)}{4a^2+4b^2}+\frac{\ln\left(e^{2x}+\frac{ab+\sqrt{a^2b^2+b^4}}{b^2}\right)\sqrt{a^2b^2+b^4}}{4(a^2+b^2)b}$

input
```
int(sinh(x)^2/(a+b*sinh(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*a*(1/2/a*ln(a*tanh(x)^2-2*b*tanh(x)-a)-1/a*b/(a^2+b^2)^(1/2)*arctanh
(1/2*(2*a*tanh(x)-2*b)/(a^2+b^2)^(1/2)))-1/4/b*ln(1+tanh(x))-1/4/b*ln(tanh
(x)-1)
```

3.1033.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(46) = 92.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 4.83

$$\int\frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx$$
$$=\frac{\sqrt{a^2+b^2}b\log\left(\frac{b^2\cosh(x)^4+4b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2ab\cosh(x)^2+2\left(3b^2\cosh(x)^2+ab\right)\sinh(x)^2+2a^2+b^2+4\left(b^2\cosh(x)^3+a\right.}{b\cosh(x)^4+4b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2a\cosh(x)^2+2\left(3b\cosh(x)^2+a\right)\sinh(x\right.}\right.}{}$$

input
```
integrate(sinh(x)^2/(a+b*sinh(2*x)),x, algorithm="fricas")
```

output
```
1/4*(sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*
sinh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a^2
+ b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) + 2*(b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(a^2 + b^2))/(b*cosh(x)^4 + 4*b*cosh(x
)*sinh(x)^3 + b*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^
2 + 4*(b*cosh(x)^3 + a*cosh(x))*sinh(x) - b)) - 2*(a^2 + b^2)*x + (a^2 + b
^2)*log(2*(2*b*cosh(x)*sinh(x) + a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(a^2*b + b^3)
```

3.1033.6 Sympy [F]

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx=\int \frac{\sinh^2{(x)}}{a+b\sinh{(2x)}}\,dx$$

input
```
integrate(sinh(x)**2/(a+b*sinh(2*x)),x)
```

output
```
Integral(sinh(x)**2/(a + b*sinh(2*x)), x)
```

3.1033.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx=-\frac{\log\left(\frac{be^{(-2x)}-a-\sqrt{a^2+b^2}}{be^{(-2x)}-a+\sqrt{a^2+b^2}}\right)}{4\sqrt{a^2+b^2}}-\frac{x}{2b}+\frac{\log\left(be^{(4x)}+2ae^{(2x)}-b\right)}{4b}$$

input
```
integrate(sinh(x)^2/(a+b*sinh(2*x)),x, algorithm="maxima")
```

output
```
-1/4*log((b*e^(-2*x) - a - sqrt(a^2 + b^2))/(b*e^(-2*x) - a + sqrt(a^2 + b
^2)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(b*e^(4*x) + 2*a*e^(2*x) - b)/b
```

3.1033.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx = -\frac{\log\left(\frac{\left|2\,be^{(2\,x)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(2\,x)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\,\sqrt{a^2+b^2}} - \frac{x}{2\,b} + \frac{\log\left(\left|be^{(4\,x)}+2\,ae^{(2\,x)}-b\right|\right)}{4\,b}$$

input
```
integrate(sinh(x)^2/(a+b*sinh(2*x)),x, algorithm="giac")
```

output
```
-1/4*log(abs(2*b*e^(2*x) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(2*x) + 2*a
+ 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(abs(b*e^(4*x) +
2*a*e^(2*x) - b))/b
```

3.1033.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 273, normalized size of antiderivative = 5.25

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx = \frac{\operatorname{atan}\left(\frac{a^7}{(-a^2-b^2)^{7/2}}+\frac{b^7\,e^{2\,x}}{(-a^2-b^2)^{7/2}}+\frac{a\,b^6}{(-a^2-b^2)^{7/2}}+\frac{3\,a^3\,b^4}{(-a^2-b^2)^{7/2}}+\frac{3\,a^5\,b^2}{(-a^2-b^2)^{7/2}}+\frac{3\,a^2\,b^5\,e^{2\,x}}{(-a^2-b^2)^{7/2}}+\frac{3\,a^4\,b^3\,e^{2\,x}}{(-a^2-b^2)^{7/2}}+\frac{a^6\,b\,e^{2\,x}}{(-a^2-b^2)^{7/2}}\right)}{2\,\sqrt{-a^2-b^2}} - \frac{x}{2\,b} + \frac{4\,b^3\,\ln\left(2\,a\,e^{2\,x}-b+b\,e^{4\,x}\right)}{16\,a^2\,b^2+16\,b^4} + \frac{4\,a^2\,b\,\ln\left(2\,a\,e^{2\,x}-b+b\,e^{4\,x}\right)}{16\,a^2\,b^2+16\,b^4}$$

input
```
int(sinh(x)^2/(a + b*sinh(2*x)),x)
```

output
```
atan(a^7/(- a^2 - b^2)^(7/2) + (b^7*exp(2*x))/(- a^2 - b^2)^(7/2) + (a*b^6
)/(- a^2 - b^2)^(7/2) + (3*a^3*b^4)/(- a^2 - b^2)^(7/2) + (3*a^5*b^2)/(- a
^2 - b^2)^(7/2) + (3*a^2*b^5*exp(2*x))/(- a^2 - b^2)^(7/2) + (3*a^4*b^3*ex
p(2*x))/(- a^2 - b^2)^(7/2) + (a^6*b*exp(2*x))/(- a^2 - b^2)^(7/2))/(2*(-
a^2 - b^2)^(1/2)) - x/(2*b) + (4*b^3*log(2*a*exp(2*x) - b + b*exp(4*x)))/(
16*b^4 + 16*a^2*b^2) + (4*a^2*b*log(2*a*exp(2*x) - b + b*exp(4*x)))/(16*b^
4 + 16*a^2*b^2)
```

3.1034 $\int \frac{\cosh^2(x)}{a+b\sinh(2x)} dx$

3.1034.1 Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx=-\frac{\operatorname{arctanh}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}+\frac{\log(a+b\sinh(2x))}{4b}$$

output
```
1/4*ln(a+b*sinh(2*x))/b-1/2*arctanh((b-a*tanh(x))/(a^2+b^2)^(1/2))/(a^2+b^
2)^(1/2)
```

3.1034.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx=\frac{1}{4}\left(\frac{2\arctan\left(\frac{b-a\tanh(x)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{\log(a+b\sinh(2x))}{b}\right)$$

input
```
Integrate[Cosh[x]^2/(a + b*Sinh[2*x]),x]
```

output
```
((2*ArcTan[(b - a*Tanh[x])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[a + b
*Sinh[2*x]]/b)/4
```

3.1034.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4889, 1301, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{a-ib\sin(2ix)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{(1-\tanh^2(x))\left(-a\tanh^2(x)+a+2b\tanh(x)\right)}d\tanh(x)$$

$$\Big\downarrow \text{1301}$$

$$-\int \left(-\frac{\tanh(x)}{2b\left(1-\tanh^2(x)\right)}-\frac{2b-a\tanh(x)}{2b\left(-a\tanh^2(x)+2b\tanh(x)+a\right)}\right)d\tanh(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}+\frac{\log\left(-a\tanh^2(x)+a+2b\tanh(x)\right)}{4b}-\frac{\log\left(1-\tanh^2(x)\right)}{4b}$$

input
```
Int[Cosh[x]^2/(a + b*Sinh[2*x]),x]
```

output
```
-1/2*ArcTanh[(b - a*Tanh[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2] - Log[1 - Ta
nh[x]^2]/(4*b) + Log[a + 2*b*Tanh[x] - a*Tanh[x]^2]/(4*b)
```

3.1034.3.1 Defintions of rubi rules used

rule 1301
```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> With[{r = Rt[(-a)*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(-r
+ c*x)^p*(r + c*x)^p*(d + e*x + f*x^2)^q, x], x], x] /; EqQ[p, -1] ||  !Fra
ctionalPowerFactorQ[r]] /; FreeQ[{a, c, d, e, f}, x] && ILtQ[p, 0] && Integ
erQ[q] && NiceSqrtQ[(-a)*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.1034.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

method	result
default	$-\frac{\ln(1+\tanh(x))}{4b}+\frac{\frac{\ln\left(a\tanh(x)^2-2b\tanh(x)-a\right)}{2}+\frac{b\,\operatorname{arctanh}\left(\frac{2a\tanh(x)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{2b}-\frac{\ln(\tanh(x)-1)}{4b}$
risch	$\frac{x}{2b}-\frac{x\,a^2b}{a^2b^2+b^4}-\frac{x\,b^3}{a^2b^2+b^4}+\frac{\ln\left(e^{2x}-\frac{-ab+\sqrt{a^2b^2+b^4}}{b^2}\right)a^2}{4(a^2+b^2)b}+\frac{b\ln\left(e^{2x}-\frac{-ab+\sqrt{a^2b^2+b^4}}{b^2}\right)}{4a^2+4b^2}+\frac{\ln\left(e^{2x}-\frac{-ab+\sqrt{a^2b^2+b^4}}{b^2}\right)\sqrt{a^2b^2}}{4(a^2+b^2)b}$

input
```
int(cosh(x)^2/(a+b*sinh(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*ln(1+tanh(x))+1/2/b*(1/2*ln(a*tanh(x)^2-2*b*tanh(x)-a)+b/(a^2+b^2)^
(1/2)*arctanh(1/2*(2*a*tanh(x)-2*b)/(a^2+b^2)^(1/2)))-1/4/b*ln(tanh(x)-1)
```

3.1034.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 4.83

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}b\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,ab\cosh(x)^2+2\,\left(3\,b^2\cosh(x)^2+ab\right)\sinh(x)^2+2\,a^2+b^2+4\,\left(b^2\cosh(x)^3+a\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,a\cosh(x)^2+2\,\left(3\,b\cosh(x)^2+a\right)\sinh(x}\right.}{}$$

input
```
integrate(cosh(x)^2/(a+b*sinh(2*x)),x, algorithm="fricas")
```

output
```
1/4*(sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*
sinh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a^2
+ b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) - 2*(b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(a^2 + b^2))/(b*cosh(x)^4 + 4*b*cosh(x
)*sinh(x)^3 + b*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^
2 + 4*(b*cosh(x)^3 + a*cosh(x))*sinh(x) - b)) - 2*(a^2 + b^2)*x + (a^2 + b
^2)*log(2*(2*b*cosh(x)*sinh(x) + a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(a^2*b + b^3)
```

3.1034.6 Sympy [F]

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx = \int \frac{\cosh^2{(x)}}{a+b\sinh{(2x)}}\,dx$$

input
```
integrate(cosh(x)**2/(a+b*sinh(2*x)),x)
```

output
```
Integral(cosh(x)**2/(a + b*sinh(2*x)), x)
```

3.1034.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx = \frac{\log\left(\frac{be^{(-2x)}-a-\sqrt{a^2+b^2}}{be^{(-2x)}-a+\sqrt{a^2+b^2}}\right)}{4\sqrt{a^2+b^2}} - \frac{x}{2b} + \frac{\log\left(be^{(4x)}+2ae^{(2x)}-b\right)}{4b}$$

input
```
integrate(cosh(x)^2/(a+b*sinh(2*x)),x, algorithm="maxima")
```

output
```
1/4*log((b*e^(-2*x) - a - sqrt(a^2 + b^2))/(b*e^(-2*x) - a + sqrt(a^2 + b^
2)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(b*e^(4*x) + 2*a*e^(2*x) - b)/b
```

3.1034.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx = \frac{\log\left(\frac{\left|2\,be^{(2x)}+2\,a-2\sqrt{a^2+b^2}\right|}{\left|2\,be^{(2x)}+2\,a+2\sqrt{a^2+b^2}\right|}\right)}{4\sqrt{a^2+b^2}} - \frac{x}{2b} + \frac{\log\left(\left|be^{(4x)}+2\,ae^{(2x)}-b\right|\right)}{4b}$$

input
```
integrate(cosh(x)^2/(a+b*sinh(2*x)),x, algorithm="giac")
```

output
```
1/4*log(abs(2*b*e^(2*x) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(2*x) + 2*a +
 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(abs(b*e^(4*x) + 2
*a*e^(2*x) - b))/b
```

3.1034.9 Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.63

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx = \frac{\mathrm{atan}\left(\frac{a}{\sqrt{-a^2-b^2}}+\frac{b\,\mathrm{e}^{2x}}{\sqrt{-a^2-b^2}}\right)}{2\sqrt{-a^2-b^2}} - \frac{x}{2b} + \frac{4\,b^3\,\ln\left(2\,a\,\mathrm{e}^{2x}-b+b\,\mathrm{e}^{4x}\right)}{16\,a^2\,b^2+16\,b^4} + \frac{4\,a^2\,b\,\ln\left(2\,a\,\mathrm{e}^{2x}-b+b\,\mathrm{e}^{4x}\right)}{16\,a^2\,b^2+16\,b^4}$$

input
```
int(cosh(x)^2/(a + b*sinh(2*x)),x)
```

output
```
atan(a/(- a^2 - b^2)^(1/2) + (b*exp(2*x))/(- a^2 - b^2)^(1/2))/(2*(- a^2 -
 b^2)^(1/2)) - x/(2*b) + (4*b^3*log(2*a*exp(2*x) - b + b*exp(4*x)))/(16*b^
4 + 16*a^2*b^2) + (4*a^2*b*log(2*a*exp(2*x) - b + b*exp(4*x)))/(16*b^4 + 1
6*a^2*b^2)
```

3.1034. $\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx$

3.1035 $\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx$

3.1035.1 Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx=\frac{x}{2b}-\frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2\sqrt{a-b}\,b}$$

output
```
1/2*x/b-1/2*arctanh((a-b)^(1/2)*tanh(x)/(a+b)^(1/2))*(a+b)^(1/2)/b/(a-b)^(
1/2)
```

3.1035.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx=\frac{x+\frac{(a+b)\arctan\left(\frac{(a-b)\tanh(x)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{2b}$$

input
```
Integrate[Sinh[x]^2/(a + b*Cosh[2*x]),x]
```

output
```
(x + ((a + b)*ArcTan[((a - b)*Tanh[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
)/(2*b)
```

3.1035.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 25, 4889, 25, 1450, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ix)^2}{a+b\cos(2ix)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ix)^2}{a+b\cos(2ix)}dx$$

$$\Big\downarrow \text{4889}$$

$$-\int -\frac{\tanh^2(x)}{(a-b)\tanh^4(x)-2a\tanh^2(x)+a+b}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\tanh^2(x)}{(a-b)\tanh^4(x)-2a\tanh^2(x)+a+b}d\tanh(x)$$

$$\Big\downarrow \text{1450}$$

$$\frac{(a+b)\int \frac{1}{(a-b)\tanh^2(x)-a-b}d\tanh(x)}{2b}+\frac{1}{2}\left(1-\frac{a}{b}\right)\int \frac{1}{(a-b)\tanh^2(x)-a+b}d\tanh(x)$$

$$\Big\downarrow \text{221}$$

$$-\frac{\left(1-\frac{a}{b}\right)\text{arctanh}(\tanh(x))}{2(a-b)}-\frac{\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}}$$

input
```
Int[Sinh[x]^2/(a + b*Cosh[2*x]),x]
```

output
```
-1/2*((1 - a/b)*ArcTanh[Tanh[x]])/(a - b) - (Sqrt[a + b]*ArcTanh[(Sqrt[a -
 b]*Tanh[x])/Sqrt[a + b]])/(2*Sqrt[a - b]*b)
```

3.1035.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.1035.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{(a+b)\,\operatorname{arctanh}\left(\frac{(a-b)\tanh(x)}{\sqrt{(a+b)(a-b)}}\right)}{2b\sqrt{(a+b)(a-b)}}-\frac{\ln(\tanh(x)-1)}{4b}+\frac{\ln(1+\tanh(x))}{4b}$	61
risch	$\frac{x}{2b}+\frac{\sqrt{(a+b)(a-b)}\,\ln\left(\mathrm{e}^{2x}+\frac{a+\sqrt{(a+b)(a-b)}}{b}\right)}{4(a-b)b}-\frac{\sqrt{(a+b)(a-b)}\,\ln\left(\mathrm{e}^{2x}-\frac{-a+\sqrt{(a+b)(a-b)}}{b}\right)}{4(a-b)b}$	103

input
```
int(sinh(x)^2/(a+b*cosh(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a+b)/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(x)/((a+b)*(a-b))^(1/2)
)-1/4/b*ln(tanh(x)-1)+1/4/b*ln(1+tanh(x))
```

3.1035.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 303, normalized size of antiderivative = 5.83

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx$$
$$= \left[\frac{\sqrt{\frac{a+b}{a-b}}\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,ab\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+ab\right)\sinh(x)^2+2\,a^2-b^2+4\left(b^2\cosh(x)^3+ab\cos\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,a\cosh(x)^2+2\left(3\,b\cosh(x)^2+\right.}\right.}{4\,b}\right.$$
$$\left. -\frac{\sqrt{-\frac{a+b}{a-b}}\arctan\left(\frac{\left(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2+a\right)\sqrt{-\frac{a+b}{a-b}}}{a+b}\right)-x}{2\,b}\right]$$

input
```
integrate(sinh(x)^2/(a+b*cosh(2*x)),x, algorithm="fricas")
```

output
```
[1/4*(sqrt((a + b)/(a - b))*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 +
 b^2*sinh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2
*a^2 - b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) + 2*((a*b - b^2)*cosh
(x)^2 + 2*(a*b - b^2)*cosh(x)*sinh(x) + (a*b - b^2)*sinh(x)^2 + a^2 - a*b)
*sqrt((a + b)/(a - b)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4
 + 2*a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4*(b*cosh(x)^3 + a*co
sh(x))*sinh(x) + b)) + 2*x)/b, -1/2*(sqrt(-(a + b)/(a - b))*arctan((b*cosh
(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-(a + b)/(a - b))/(a +
 b)) - x)/b]
```

3.1035.6 Sympy [F]

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = \int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx$$

input
```
integrate(sinh(x)**2/(a+b*cosh(2*x)),x)
```

output
```
Integral(sinh(x)**2/(a + b*cosh(2*x)), x)
```

3.1035.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(2*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.1035.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = -\frac{(a+b)\arctan\left(\frac{be^{(2x)}+a}{\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}b} + \frac{x}{2b}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(2*x)),x, algorithm="giac")
```

output
```
-1/2*(a + b)*arctan((b*e^(2*x) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b)
 + 1/2*x/b
```

3.1035.9 Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.94

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx$$
$$= \frac{x}{2b} - \frac{\ln\left(a\,b+2\,a^2\,e^{2\,x}-b^2\,e^{2\,x}+b\,\sqrt{a+b}\,\sqrt{a-b}+2\,a\,e^{2\,x}\,\sqrt{a+b}\,\sqrt{a-b}\right)\,\sqrt{a+b}}{4\,b\,\sqrt{a-b}}$$
$$+ \frac{\ln\left(b^2\,e^{2\,x}-2\,a^2\,e^{2\,x}-a\,b+b\,\sqrt{a+b}\,\sqrt{a-b}+2\,a\,e^{2\,x}\,\sqrt{a+b}\,\sqrt{a-b}\right)\,\sqrt{a+b}}{4\,b\,\sqrt{a-b}}$$

input
```
int(sinh(x)^2/(a + b*cosh(2*x)),x)
```

output
```
x/(2*b) - (log(a*b + 2*a^2*exp(2*x) - b^2*exp(2*x) + b*(a + b)^(1/2)*(a -
b)^(1/2) + 2*a*exp(2*x)*(a + b)^(1/2)*(a - b)^(1/2))*(a + b)^(1/2))/(4*b*(
a - b)^(1/2)) + (log(b^2*exp(2*x) - 2*a^2*exp(2*x) - a*b + b*(a + b)^(1/2)
*(a - b)^(1/2) + 2*a*exp(2*x)*(a + b)^(1/2)*(a - b)^(1/2))*(a + b)^(1/2))/
(4*b*(a - b)^(1/2))
```

3.1036 $\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx$

3.1036.1 Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = \frac{x}{2b} - \frac{\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

output
```
1/2*x/b-1/2*arctanh((a-b)^(1/2)*tanh(x)/(a+b)^(1/2))*(a-b)^(1/2)/b/(a+b)^(
1/2)
```

3.1036.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = \frac{x + \frac{(a-b)\arctan\left(\frac{(a-b)\tanh(x)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{2b}$$

input
```
Integrate[Cosh[x]^2/(a + b*Cosh[2*x]),x]
```

output
```
(x + ((a - b)*ArcTan[((a - b)*Tanh[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
)/(2*b)
```

3.1036.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4889, 1406, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\cos(ix)^2}{a+b\cos(2ix)}dx$$
$$\Big\downarrow \; 4889$$
$$\int \frac{1}{(a-b)\tanh^4(x)-2a\tanh^2(x)+a+b}d\tanh(x)$$
$$\Big\downarrow \; 1406$$
$$\frac{(a-b)\int \frac{1}{(a-b)\tanh^2(x)-a-b}d\tanh(x)}{2b}-\frac{(a-b)\int \frac{1}{(a-b)\tanh^2(x)-a+b}d\tanh(x)}{2b}$$
$$\Big\downarrow \; 221$$
$$\frac{\operatorname{arctanh}(\tanh(x))}{2b}-\frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

input
```
Int[Cosh[x]^2/(a + b*Cosh[2*x]),x]
```

output
```
ArcTanh[Tanh[x]]/(2*b) - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[x])/Sqrt[a
 + b]])/(2*b*Sqrt[a + b])
```

3.1036.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.1036.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{(a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh(x)}{\sqrt{(a+b)(a-b)}}\right)}{2b\sqrt{(a+b)(a-b)}}-\frac{\ln(\tanh(x)-1)}{4b}+\frac{\ln(1+\tanh(x))}{4b}$	63
risch	$\frac{x}{2b}+\frac{\sqrt{(a+b)(a-b)}\,\ln\left(\mathrm{e}^{2x}+\frac{a+\sqrt{(a+b)(a-b)}}{b}\right)}{4(a+b)b}-\frac{\sqrt{(a+b)(a-b)}\,\ln\left(\mathrm{e}^{2x}-\frac{-a+\sqrt{(a+b)(a-b)}}{b}\right)}{4(a+b)b}$	99

input
```
int(cosh(x)^2/(a+b*cosh(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a-b)/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(x)/((a+b)*(a-b))^(1/2)
)-1/4/b*ln(tanh(x)-1)+1/4/b*ln(1+tanh(x))
```

3.1036.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 5.71

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx$$
$$= \left[\frac{\sqrt{\frac{a-b}{a+b}}\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,ab\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+ab\right)\sinh(x)^2+2\,a^2-b^2+4\left(b^2\cosh(x)^3+ab\cos\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,a\cosh(x)^2+2\left(3\,b\cosh(x)^2+\right.}\right.}{4\,b}\right.$$

input
```
integrate(cosh(x)^2/(a+b*cosh(2*x)),x, algorithm="fricas")
```

output
```
[1/4*(sqrt((a - b)/(a + b))*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 +
 b^2*sinh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2
*a^2 - b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) + 2*((a*b + b^2)*cosh
(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + a^2 + a*b)
*sqrt((a - b)/(a + b)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4
 + 2*a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4*(b*cosh(x)^3 + a*co
sh(x))*sinh(x) + b)) + 2*x)/b, 1/2*(sqrt(-(a - b)/(a + b))*arctan(-(b*cosh
(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-(a - b)/(a + b))/(a -
 b)) + x)/b]
```

3.1036.6 Sympy [F]

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = \int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx$$

input
```
integrate(cosh(x)**2/(a+b*cosh(2*x)),x)
```

output
```
Integral(cosh(x)**2/(a + b*cosh(2*x)), x)
```

3.1036.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^2/(a+b*cosh(2*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.1036.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = -\frac{(a-b)\arctan\left(\frac{be^{(2x)}+a}{\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}b} + \frac{x}{2b}$$

input
```
integrate(cosh(x)^2/(a+b*cosh(2*x)),x, algorithm="giac")
```

output
```
-1/2*(a - b)*arctan((b*e^(2*x) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b)
 + 1/2*x/b
```

3.1036.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.31

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = \frac{x}{2\,b} - \frac{\ln\left(\frac{e^{2\,x}\,(a-b)}{b^2} + \frac{\sqrt{a-b}\,(b+a\,e^{2\,x})}{b^2\,\sqrt{a+b}}\right)\,\sqrt{a-b}}{4\,b\,\sqrt{a+b}} + \frac{\ln\left(\frac{e^{2\,x}\,(a-b)}{b^2} - \frac{\sqrt{a-b}\,(b+a\,e^{2\,x})}{b^2\,\sqrt{a+b}}\right)\,\sqrt{a-b}}{4\,b\,\sqrt{a+b}}$$

input
```
int(cosh(x)^2/(a + b*cosh(2*x)),x)
```

output
```
x/(2*b) - (log((exp(2*x)*(a - b))/b^2 + ((a - b)^(1/2)*(b + a*exp(2*x)))/(
b^2*(a + b)^(1/2)))*(a - b)^(1/2))/(4*b*(a + b)^(1/2)) + (log((exp(2*x)*(a
 - b))/b^2 - ((a - b)^(1/2)*(b + a*exp(2*x)))/(b^2*(a + b)^(1/2)))*(a - b)
^(1/2))/(4*b*(a + b)^(1/2))
```

3.1036. $\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx$

3.1037 $\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx$

3.1037.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a\sinh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
arctan((a*sinh(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)
```

3.1037.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \frac{\arctan(\sinh(c+dx))\sinh(c+dx)}{d\sqrt{a\sinh^2(c+dx)}}$$

input
```
Integrate[Tanh[c + d*x]/Sqrt[a*Sinh[c + d*x]^2],x]
```

output
```
(ArcTan[Sinh[c + d*x]]*Sinh[c + d*x])/(d*Sqrt[a*Sinh[c + d*x]^2])
```

3.1037.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 26, 3684, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\tan(ic+idx)}{\sqrt{-a\sin(ic+idx)^2}}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\tan(ic+idx)}{\sqrt{-a\sin(ic+idx)^2}}dx$$

$$\Big\downarrow \; 3684$$

$$\frac{\int \frac{1}{\sqrt{a\sinh^2(c+dx)}(\sinh^2(c+dx)+1)}d\sinh^2(c+dx)}{2d}$$

$$\Big\downarrow \; 73$$

$$\frac{\int \frac{1}{\frac{\sinh^4(c+dx)}{a}+1}d\sqrt{a\sinh^2(c+dx)}}{ad}$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan\left(\frac{\sqrt{a\sinh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[Tanh[c + d*x]/Sqrt[a*Sinh[c + d*x]^2],x]
```

output
```
ArcTan[Sqrt[a*Sinh[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d)
```

3.1037.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.1037.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{\text{` int/indef0`}\left(\frac{\sinh(dx+c)}{\cosh(dx+c)^{2}\sqrt{a\sinh(dx+c)^{2}}},\sinh(dx+c)\right)}{d}$	39
risch	$\frac{i\ln\left(\mathrm{e}^{dx}+i\mathrm{e}^{-c}\right)\left(\mathrm{e}^{2dx+2c}-1\right)\mathrm{e}^{-dx-c}}{d\sqrt{\left(\mathrm{e}^{2dx+2c}-1\right)^{2}a\,\mathrm{e}^{-2dx-2c}}}-\frac{i\ln\left(\mathrm{e}^{dx}-i\mathrm{e}^{-c}\right)\left(\mathrm{e}^{2dx+2c}-1\right)\mathrm{e}^{-dx-c}}{d\sqrt{\left(\mathrm{e}^{2dx+2c}-1\right)^{2}a\,\mathrm{e}^{-2dx-2c}}}$	132

input
```
int(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(sinh(d*x+c)/cosh(d*x+c)^2/(a*sinh(d*x+c)^2)^(1/2),sinh(d*x+c)
)/d
```

3.1037.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(24) = 48$.

Time = 0.27 (sec) , antiderivative size = 335, normalized size of antiderivative = 11.17

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \left[-\frac{\sqrt{-a}\log\left(-\frac{a\cosh(dx+c)^2+2\sqrt{ae^{(4dx+4c)}-2ae^{(2dx+2c)}+a}\left(\cosh(dx+c)e^{(dx+c)}+e^{(dx+c)}\sinh(dx+c)\right)\sqrt{-a}e^{(-dx-c)}-\left(ae^{(2dx+2c)}-\cdots\right.}{\left(e^{(2dx+2c)}-1\right)\sinh(dx+c)^2-\cosh(dx+c)^2+\left(\cosh(dx+c)^2+1\right)e^{(2dx+2\cdots}}\right.}{ad}\right.$$

input
```
integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-a)*log(-(a*cosh(d*x + c)^2 + 2*sqrt(a*e^(4*d*x + 4*c) - 2*a*e^(2*d
*x + 2*c) + a)*(cosh(d*x + c)*e^(d*x + c) + e^(d*x + c)*sinh(d*x + c))*sqr
t(-a)*e^(-d*x - c) - (a*e^(2*d*x + 2*c) - a)*sinh(d*x + c)^2 - (a*cosh(d*x
 + c)^2 - a)*e^(2*d*x + 2*c) - 2*(a*cosh(d*x + c)*e^(2*d*x + 2*c) - a*cosh
(d*x + c))*sinh(d*x + c) - a)/((e^(2*d*x + 2*c) - 1)*sinh(d*x + c)^2 - cos
h(d*x + c)^2 + (cosh(d*x + c)^2 + 1)*e^(2*d*x + 2*c) + 2*(cosh(d*x + c)*e^
(2*d*x + 2*c) - cosh(d*x + c))*sinh(d*x + c) - 1))/(a*d), 2*sqrt(a*e^(4*d*
x + 4*c) - 2*a*e^(2*d*x + 2*c) + a)*arctan(cosh(d*x + c) + sinh(d*x + c))/
(a*d*e^(2*d*x + 2*c) - a*d)]
```

3.1037.6 Sympy [F]

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \int \frac{\tanh\left(c+dx\right)}{\sqrt{a\sinh^2\left(c+dx\right)}}\,dx$$

input
```
integrate(tanh(d*x+c)/(a*sinh(d*x+c)**2)**(1/2),x)
```

output
```
Integral(tanh(c + d*x)/sqrt(a*sinh(c + d*x)**2), x)
```

3.1037.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \frac{2\arctan\left(e^{(-dx-c)}\right)}{\sqrt{a}d}$$

input
```
integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(e^(-d*x - c))/(sqrt(a)*d)
```

3.1037.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \frac{2\arctan\left(e^{(dx+c)}\right)}{\sqrt{a}d\mathrm{sgn}\left(e^{(3\,dx+3\,c)}-e^{(dx+c)}\right)}$$

input
```
integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(e^(d*x + c))/(sqrt(a)*d*sgn(e^(3*d*x + 3*c) - e^(d*x + c)))
```

3.1037.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \int \frac{\tanh(c+d\,x)}{\sqrt{a\sinh\left(c+d\,x\right)^2}}\,dx$$

input

```
int(tanh(c + d*x)/(a*sinh(c + d*x)^2)^(1/2),x)
```

output

```
int(tanh(c + d*x)/(a*sinh(c + d*x)^2)^(1/2), x)
```

3.1038 $\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx$

3.1038.1 Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
-arctanh((a*cosh(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)
```

3.1038.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Integrate[Coth[c + d*x]/Sqrt[a*Cosh[c + d*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a*Cosh[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))
```

3.1038.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 26, 3684, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a\sin\left(ic+idx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)}{\sqrt{a\sin\left(\frac{1}{2}(2ic+\pi)+idx\right)^2}}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{1}{\sqrt{a\cosh^2(c+dx)}(1-\cosh^2(c+dx))}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{1-\frac{\cosh^4(c+dx)}{a}}d\sqrt{a\cosh^2(c+dx)}}{ad}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[Coth[c + d*x]/Sqrt[a*Cosh[c + d*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a*Cosh[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))
```

3.1038.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.1038.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\cosh(dx+c)\operatorname{arctanh}(\cosh(dx+c))}{\sqrt{a\cosh(dx+c)^2}\,d}$	31
risch	$\frac{\ln(e^{dx}-e^{-c})\left(e^{2dx+2c}+1\right)e^{-dx-c}}{d\sqrt{\left(e^{2dx+2c}+1\right)^2 a\,e^{-2dx-2c}}}-\frac{\ln(e^{dx}+e^{-c})\left(e^{2dx+2c}+1\right)e^{-dx-c}}{d\sqrt{\left(e^{2dx+2c}+1\right)^2 a\,e^{-2dx-2c}}}$	125

input
```
int(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*cosh(d*x+c)^2)^(1/2)*cosh(d*x+c)*arctanh(cosh(d*x+c))/d
```

3.1038.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 5.61

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+a}\log\left(\frac{\cosh(dx+c)+\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)+1}\right)}{ade^{(2\,dx+2\,c)}+ad}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}}}{a\cosh(dx+c)e^{(2\,dx+2\,c)}+a\cosh(dx+c)}\right.}{ad}\right.$$

input
```
integrate(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + a)*log((cosh(d*x + c) + si
nh(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) + 1))/(a*d*e^(2*d*x + 2*c)
 + a*d), 2*sqrt(-a)*arctan(sqrt(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) +
a)*sqrt(-a)/(a*cosh(d*x + c)*e^(2*d*x + 2*c) + a*cosh(d*x + c) + (a*e^(2*d
*x + 2*c) + a)*sinh(d*x + c)))/(a*d)]
```

3.1038.6 Sympy [F]

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = \int \frac{\coth{(c+dx)}}{\sqrt{a\cosh^2{(c+dx)}}}\,dx$$

input
```
integrate(coth(d*x+c)/(a*cosh(d*x+c)**2)**(1/2),x)
```

output
```
Integral(coth(c + d*x)/sqrt(a*cosh(c + d*x)**2), x)
```

3.1038.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = -\frac{\log\left(e^{(-dx-c)}+1\right)}{\sqrt{a}d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{\sqrt{a}d}$$

input
```
integrate(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
-log(e^(-d*x - c) + 1)/(sqrt(a)*d) + log(e^(-d*x - c) - 1)/(sqrt(a)*d)
```

3.1038.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1038.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = \int \frac{\coth(c+d\,x)}{\sqrt{a\cosh\left(c+d\,x\right)^2}}\,dx$$

input
```
int(coth(c + d*x)/(a*cosh(c + d*x)^2)^(1/2),x)
```

output
```
int(coth(c + d*x)/(a*cosh(c + d*x)^2)^(1/2), x)
```

3.1039 $\int x \cosh(2x)\text{sech}(x)\,dx$

3.1039.1 Optimal result

Integrand size = 8, antiderivative size = 43

$$\int x\cosh(2x)\text{sech}(x)\,dx = -2x\arctan(e^x)-2\cosh(x)+i\,\text{PolyLog}(2,-ie^x)-i\,\text{PolyLog}(2,ie^x)+2x\sinh(x)$$

output
```
-2*x*arctan(exp(x))-2*cosh(x)+I*polylog(2,-I*exp(x))-I*polylog(2,I*exp(x))
+2*x*sinh(x)
```

3.1039.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int x\cosh(2x)\text{sech}(x)\,dx = -2\cosh(x)-i(x(\log(1-ie^x)-\log(1+ie^x))-\text{PolyLog}(2,-ie^x)+\text{PolyLog}(2,ie^x))+2x\sinh(x)$$

input
```
Integrate[x*Cosh[2*x]*Sech[x],x]
```

output
```
-2*Cosh[x] - I*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I)*E^x]
 + PolyLog[2, I*E^x]) + 2*x*Sinh[x]
```

3.1039.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5996, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh(2x)\text{sech}(x)\,dx$$

$$\Big\downarrow \text{5996}$$

$$\int (x\cosh(x)+x\sinh(x)\tanh(x))dx$$

$$\Big\downarrow \text{2009}$$

$$-2x\arctan\left(e^x\right)+i\,\text{PolyLog}\left(2,-ie^x\right)-i\,\text{PolyLog}\left(2,ie^x\right)+2x\sinh(x)-2\cosh(x)$$

input
```
Int[x*Cosh[2*x]*Sech[x],x]
```

output
```
-2*x*ArcTan[E^x] - 2*Cosh[x] + I*PolyLog[2, (-I)*E^x] - I*PolyLog[2, I*E^x
] + 2*x*Sinh[x]
```

3.1039.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5996
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sinh, Cosh}, F] && MemberQ[{Sech, Csch}, G] && IGtQ[p, 0] && IGtQ[q, 0]
&& EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

3.1039.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

method	result
risch	$2\left(-\frac{1}{2}+\frac{x}{2}\right) \mathrm{e}^{x}+2\left(-\frac{1}{2}-\frac{x}{2}\right) \mathrm{e}^{-x}+i x \ln \left(1+i \mathrm{e}^{x}\right)-i x \ln \left(1-i \mathrm{e}^{x}\right)+i \operatorname{dilog}\left(1+i \mathrm{e}^{x}\right)-i \operatorname{dilog}\left(1-\right.$

input
```
int(x*cosh(2*x)*sech(x),x,method=_RETURNVERBOSE)
```

output
```
2*(-1/2+1/2*x)*exp(x)+2*(-1/2-1/2*x)/exp(x)+I*x*ln(1+I*exp(x))-I*x*ln(1-I*
exp(x))+I*dilog(1+I*exp(x))-I*dilog(1-I*exp(x))
```

3.1039.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(30) = 60.

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.88

$$\int x \cosh(2x)\operatorname{sech}(x)\,dx$$
$$= \frac{(x-1)\cosh(x)^2 + 2(x-1)\cosh(x)\sinh(x) + (x-1)\sinh(x)^2 + (-i\cosh(x) - i\sinh(x))\mathrm{Li}_2(i\cosh(}{}$$

input
```
integrate(x*cosh(2*x)*sech(x),x, algorithm="fricas")
```

output
```
((x - 1)*cosh(x)^2 + 2*(x - 1)*cosh(x)*sinh(x) + (x - 1)*sinh(x)^2 + (-I*c
osh(x) - I*sinh(x))*dilog(I*cosh(x) + I*sinh(x)) + (I*cosh(x) + I*sinh(x))
*dilog(-I*cosh(x) - I*sinh(x)) + (I*x*cosh(x) + I*x*sinh(x))*log(I*cosh(x)
 + I*sinh(x) + 1) + (-I*x*cosh(x) - I*x*sinh(x))*log(-I*cosh(x) - I*sinh(x
) + 1) - x - 1)/(cosh(x) + sinh(x))
```

3.1039.6 Sympy [F]

$$\int x\cosh(2x)\text{sech}(x)\,dx = \int x\cosh(2x)\,\text{sech}(x)\,dx$$

input
```
integrate(x*cosh(2*x)*sech(x),x)
```

output
```
Integral(x*cosh(2*x)*sech(x), x)
```

3.1039.7 Maxima [F]

$$\int x\cosh(2x)\text{sech}(x)\,dx = \int x\cosh(2\,x)\,\text{sech}(x)\,dx$$

input
```
integrate(x*cosh(2*x)*sech(x),x, algorithm="maxima")
```

output
```
-(x + 1)*e^(-x) + (x - 1)*e^x - 2*integrate(x*e^x/(e^(2*x) + 1), x)
```

3.1039.8 Giac [F]

$$\int x\cosh(2x)\text{sech}(x)\,dx = \int x\cosh(2\,x)\,\text{sech}(x)\,dx$$

input
```
integrate(x*cosh(2*x)*sech(x),x, algorithm="giac")
```

output
```
integrate(x*cosh(2*x)*sech(x), x)
```

3.1039.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh(2x)\text{sech}(x)\,dx = \int \frac{x\cosh(2\,x)}{\cosh\left(x\right)}\,dx$$

input
```
int((x*cosh(2*x))/cosh(x),x)
```

output
```
int((x*cosh(2*x))/cosh(x), x)
```

3.1040 $\int x \cosh(2x)\mathbf{sech}^2(x)\,dx$

3.1040.1 Optimal result

Integrand size = 10, antiderivative size = 12

$$\int x\cosh(2x)\text{sech}^2(x)\,dx = x^2 + \log(\cosh(x)) - x\tanh(x)$$

output
```
x^2+ln(cosh(x))-x*tanh(x)
```

3.1040.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\cosh(2x)\text{sech}^2(x)\,dx = x^2 + \log(\cosh(x)) - x\tanh(x)$$

input
```
Integrate[x*Cosh[2*x]*Sech[x]^2,x]
```

output
```
x^2 + Log[Cosh[x]] - x*Tanh[x]
```

3.1040.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5996, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cosh(2x)\text{sech}^2(x)\,dx$$

$$\Big\downarrow \text{5996}$$

$$\int \left(x + x\tanh^2(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x^2 - x\tanh(x) + \log(\cosh(x))$$

input
```
Int[x*Cosh[2*x]*Sech[x]^2,x]
```

output
```
x^2 + Log[Cosh[x]] - x*Tanh[x]
```

3.1040.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5996
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sinh, Cosh}, F] && MemberQ[{Sech, Csch}, G] && IGtQ[p, 0] && IGtQ[q, 0]
&& EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

3.1040.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 1.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

method	result	size
risch	$x^2 - 2x + \frac{2x}{1+e^{2x}} + \ln\left(1 + e^{2x}\right)$	26

input
```
int(x*cosh(2*x)*sech(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x^2-2*x+2*x/(1+exp(2*x))+ln(1+exp(2*x))
```

3.1040.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 7.58

$$\int x \cosh(2x)\text{sech}^2(x)\,dx$$
$$= \frac{(x^2 - 2x)\cosh(x)^2 + 2(x^2 - 2x)\cosh(x)\sinh(x) + (x^2 - 2x)\sinh(x)^2 + x^2 + \left(\cosh(x)^2 + 2\cosh(x)\text{s}\right.}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1}$$

input
```
integrate(x*cosh(2*x)*sech(x)^2,x, algorithm="fricas")
```

output
```
((x^2 - 2*x)*cosh(x)^2 + 2*(x^2 - 2*x)*cosh(x)*sinh(x) + (x^2 - 2*x)*sinh(
x)^2 + x^2 + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(2*cosh(x)
/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

3.1040.6 Sympy [F]

$$\int x \cosh(2x)\text{sech}^2(x)\,dx = \int x \cosh(2x)\,\text{sech}^2(x)\,dx$$

input
```
integrate(x*cosh(2*x)*sech(x)**2,x)
```

output
```
Integral(x*cosh(2*x)*sech(x)**2, x)
```

3.1040.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int x\cosh(2x)\operatorname{sech}^2(x)\,dx = \frac{x^2 + (x^2 - 2\,x)e^{(2\,x)}}{e^{(2\,x)} + 1} + \log\left(e^{(2\,x)} + 1\right)$$

input
```
integrate(x*cosh(2*x)*sech(x)^2,x, algorithm="maxima")
```

output
```
(x^2 + (x^2 - 2*x)*e^(2*x))/(e^(2*x) + 1) + log(e^(2*x) + 1)
```

3.1040.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.92

$$\int x\cosh(2x)\operatorname{sech}^2(x)\,dx = \frac{x^2e^{(2\,x)} + x^2 - 2\,xe^{(2\,x)} + e^{(2\,x)}\log\left(e^{(2\,x)} + 1\right) + \log\left(e^{(2\,x)} + 1\right)}{e^{(2\,x)} + 1}$$

input
```
integrate(x*cosh(2*x)*sech(x)^2,x, algorithm="giac")
```

output
```
(x^2*e^(2*x) + x^2 - 2*x*e^(2*x) + e^(2*x)*log(e^(2*x) + 1) + log(e^(2*x)
+ 1))/(e^(2*x) + 1)
```

3.1040.9 Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int x\cosh(2x)\operatorname{sech}^2(x)\,dx = \ln\left(\mathrm{e}^{2\,x} + 1\right) - 2\,x + \frac{2\,x}{\mathrm{e}^{2\,x} + 1} + x^2$$

input
```
int((x*cosh(2*x))/cosh(x)^2,x)
```

output
```
log(exp(2*x) + 1) - 2*x + (2*x)/(exp(2*x) + 1) + x^2
```

3.1041 $\int x \cosh(2x)\mathrm{sech}^3(x)\,dx$

3.1041.1 Optimal result

Integrand size = 10, antiderivative size = 53

$$\int x\cosh(2x)\mathrm{sech}^3(x)\,dx = 3x\arctan\left(e^x\right) - \frac{3}{2}i\,\mathrm{PolyLog}\left(2,-ie^x\right) + \frac{3}{2}i\,\mathrm{PolyLog}\left(2,ie^x\right) - \frac{\mathrm{sech}(x)}{2} - \frac{1}{2}x\mathrm{sech}(x)\tanh(x)$$

output
```
3*x*arctan(exp(x))-3/2*I*polylog(2,-I*exp(x))+3/2*I*polylog(2,I*exp(x))-1/
2*sech(x)-1/2*x*sech(x)*tanh(x)
```

3.1041.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int x\cosh(2x)\mathrm{sech}^3(x)\,dx = \frac{3}{2}i(x(\log\left(1-ie^x\right)-\log\left(1+ie^x\right)) - \mathrm{PolyLog}\left(2,-ie^x\right) + \mathrm{PolyLog}\left(2,ie^x\right)) - \frac{\mathrm{sech}(x)}{2} - \frac{1}{2}x\mathrm{sech}(x)\tanh(x)$$

input
```
Integrate[x*Cosh[2*x]*Sech[x]^3,x]
```

output
```
((3*I)/2)*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I)*E^x] + Po
lyLog[2, I*E^x]) - Sech[x]/2 - (x*Sech[x]*Tanh[x])/2
```

3.1041.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5996, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh(2x)\text{sech}^3(x)\,dx$$

$$\Big\downarrow \text{5996}$$

$$\int \left(x\text{sech}(x)+x\tanh^2(x)\text{sech}(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3x\arctan\left(e^x\right)-\frac{3}{2}i\,\text{PolyLog}\left(2,-ie^x\right)+\frac{3}{2}i\,\text{PolyLog}\left(2,ie^x\right)-\frac{\text{sech}(x)}{2}-\frac{1}{2}x\tanh(x)\text{sech}(x)$$

input
```
Int[x*Cosh[2*x]*Sech[x]^3,x]
```

output
```
3*x*ArcTan[E^x] - ((3*I)/2)*PolyLog[2, (-I)*E^x] + ((3*I)/2)*PolyLog[2, I*
E^x] - Sech[x]/2 - (x*Sech[x]*Tanh[x])/2
```

3.1041.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5996
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sinh, Cosh}, F] && MemberQ[{Sech, Csch}, G] && IGtQ[p, 0] && IGtQ[q, 0]
&& EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

3.1041.4 Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.42

method	result	size
risch	$-\frac{e^{x}\left(x\,e^{2x}+e^{2x}-x+1\right)}{\left(1+e^{2x}\right)^{2}}-\frac{3ix\ln\left(1+ie^{x}\right)}{2}+\frac{3ix\ln\left(1-ie^{x}\right)}{2}-\frac{3i\,\mathrm{dilog}\left(1+ie^{x}\right)}{2}+\frac{3i\,\mathrm{dilog}\left(1-ie^{x}\right)}{2}$	75

input
```
int(x*cosh(2*x)*sech(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-exp(x)*(x*exp(x)^2+exp(x)^2-x+1)/(exp(x)^2+1)^2-3/2*I*x*ln(1+I*exp(x))+3/
2*I*x*ln(1-I*exp(x))-3/2*I*dilog(1+I*exp(x))+3/2*I*dilog(1-I*exp(x))
```

3.1041.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(32) = 64.

Time = 0.27 (sec) , antiderivative size = 408, normalized size of antiderivative = 7.70

$$\int x\cosh(2x)\mathrm{sech}^3(x)\,dx =$$
$$-\frac{2\,(x+1)\cosh(x)^3+6\,(x+1)\cosh(x)\sinh(x)^2+2\,(x+1)\sinh(x)^3-2\,(x-1)\cosh(x)+3\,(-i\,\cosh$$

input
```
integrate(x*cosh(2*x)*sech(x)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(x + 1)*cosh(x)^3 + 6*(x + 1)*cosh(x)*sinh(x)^2 + 2*(x + 1)*sinh(x
)^3 - 2*(x - 1)*cosh(x) + 3*(-I*cosh(x)^4 - 4*I*cosh(x)*sinh(x)^3 - I*sinh
(x)^4 + 2*(-3*I*cosh(x)^2 - I)*sinh(x)^2 - 2*I*cosh(x)^2 + 4*(-I*cosh(x)^3
 - I*cosh(x))*sinh(x) - I)*dilog(I*cosh(x) + I*sinh(x)) + 3*(I*cosh(x)^4 +
 4*I*cosh(x)*sinh(x)^3 + I*sinh(x)^4 + 2*(3*I*cosh(x)^2 + I)*sinh(x)^2 + 2
*I*cosh(x)^2 + 4*(I*cosh(x)^3 + I*cosh(x))*sinh(x) + I)*dilog(-I*cosh(x) -
 I*sinh(x)) + 3*(I*x*cosh(x)^4 + 4*I*x*cosh(x)*sinh(x)^3 + I*x*sinh(x)^4 +
 2*I*x*cosh(x)^2 + 2*(3*I*x*cosh(x)^2 + I*x)*sinh(x)^2 + 4*(I*x*cosh(x)^3
+ I*x*cosh(x))*sinh(x) + I*x)*log(I*cosh(x) + I*sinh(x) + 1) + 3*(-I*x*cos
h(x)^4 - 4*I*x*cosh(x)*sinh(x)^3 - I*x*sinh(x)^4 - 2*I*x*cosh(x)^2 + 2*(-3
*I*x*cosh(x)^2 - I*x)*sinh(x)^2 + 4*(-I*x*cosh(x)^3 - I*x*cosh(x))*sinh(x)
 - I*x)*log(-I*cosh(x) - I*sinh(x) + 1) + 2*(3*(x + 1)*cosh(x)^2 - x + 1)*
sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1
)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

3.1041.6 Sympy [F]

$$\int x\cosh(2x)\text{sech}^3(x)\,dx = \int x\cosh(2x)\,\text{sech}^3(x)\,dx$$

input
```
integrate(x*cosh(2*x)*sech(x)**3,x)
```

output
```
Integral(x*cosh(2*x)*sech(x)**3, x)
```

3.1041.7 Maxima [F]

$$\int x\cosh(2x)\text{sech}^3(x)\,dx = \int x\cosh(2\,x)\,\text{sech}(x)^3\,dx$$

input
```
integrate(x*cosh(2*x)*sech(x)^3,x, algorithm="maxima")
```

output
```
-((x + 1)*e^(3*x) - (x - 1)*e^x)/(e^(4*x) + 2*e^(2*x) + 1) + 12*integrate(
1/4*x*e^x/(e^(2*x) + 1), x)
```

3.1041.8 Giac [F]

$$\int x\cosh(2x)\text{sech}^3(x)\,dx = \int x\cosh(2\,x)\,\text{sech}(x)^3\,dx$$

input
```
integrate(x*cosh(2*x)*sech(x)^3,x, algorithm="giac")
```

output
```
integrate(x*cosh(2*x)*sech(x)^3, x)
```

3.1041.9 Mupad [F(-1)]

Timed out.

$$\int x\cosh(2x)\operatorname{sech}^3(x)\,dx=\int \frac{x\cosh(2\,x)}{{\cosh\left(x\right)}^3}\,dx$$

input
```
int((x*cosh(2*x))/cosh(x)^3,x)
```

output
```
int((x*cosh(2*x))/cosh(x)^3, x)
```

3.1042 $\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx$

3.1042.1 Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx=\frac{2x}{\sqrt{\operatorname{csch}(x)}}-\frac{4\operatorname{sech}(x)}{\operatorname{csch}^{\frac{3}{2}}(x)}$$

output
```
-4*sech(x)/csch(x)^(3/2)+2*x/csch(x)^(1/2)
```

3.1042.2 Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx=\frac{2(x\operatorname{csch}(x)-2\operatorname{sech}(x))}{\operatorname{csch}^{\frac{3}{2}}(x)}$$

input
```
Integrate[Sqrt[Csch[x]]*(x*Cosh[x] - 4*Sech[x]*Tanh[x]),x]
```

output
```
(2*(x*Csch[x] - 2*Sech[x]))/Csch[x]^(3/2)
```

3.1042.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\text{csch}(x)}(x\cosh(x)-4\tanh(x)\text{sech}(x))\,dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(x\cosh(x)\sqrt{\text{csch}(x)}-\frac{4\text{sech}^2(x)}{\sqrt{\text{csch}(x)}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2x}{\sqrt{\text{csch}(x)}}-\frac{4\text{sech}(x)}{\text{csch}^{\frac{3}{2}}(x)}$$

input
```
Int[Sqrt[Csch[x]]*(x*Cosh[x] - 4*Sech[x]*Tanh[x]),x]
```

output
```
(2*x)/Sqrt[Csch[x]] - (4*Sech[x])/Csch[x]^(3/2)
```

3.1042.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.1042.4 Maple [F]

$$\int \sqrt{\operatorname{csch}(x)}\,(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx$$

input
```
int(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x)
```

output
```
int(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x)
```

3.1042.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.1042.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx = \text{Timed out}$$

input
```
integrate(csch(x)**(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x)
```

output
```
Timed out
```

3.1042.7 Maxima [F]

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx = \int (x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\sqrt{\operatorname{csch}(x)}\,dx$$

input
```
integrate(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x, algorithm="maxima
")
```

output
```
integrate((x*cosh(x) - 4*sech(x)*tanh(x))*sqrt(csch(x)), x)
```

3.1042.8 Giac [F]

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx = \int (x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\sqrt{\operatorname{csch}(x)}\,dx$$

input
```
integrate(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x, algorithm="giac")
```

output
```
integrate((x*cosh(x) - 4*sech(x)*tanh(x))*sqrt(csch(x)), x)
```

3.1042.9 Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx = \frac{e^{-x}\sqrt{-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}\left(e^{2x}-1\right)\left(x-2e^{2x}+x\,e^{2x}+2\right)}{e^{2x}+1}$$

input
```
int(-(1/sinh(x))^(1/2)*((4*tanh(x))/cosh(x) - x*cosh(x)),x)
```

output
```
(exp(-x)*(-1/(exp(-x)/2 - exp(x)/2))^(1/2)*(exp(2*x) - 1)*(x - 2*exp(2*x)
+ x*exp(2*x) + 2))/(exp(2*x) + 1)
```

3.1043 $\int \sinh(x)(\cosh(x)+\sinh(x))\,dx$

3.1043.1 Optimal result

Integrand size = 8, antiderivative size = 22

$$\int \sinh(x)(\cosh(x)+\sinh(x))\,dx=-\frac{x}{2}+\frac{1}{2}\cosh(x)\sinh(x)+\frac{\sinh^2(x)}{2}$$

output
```
-1/2*x+1/2*cosh(x)*sinh(x)+1/2*sinh(x)^2
```

3.1043.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sinh(x)(\cosh(x)+\sinh(x))\,dx=-\frac{x}{2}+\frac{\cosh^2(x)}{2}+\frac{1}{4}\sinh(2x)$$

input
```
Integrate[Sinh[x]*(Cosh[x] + Sinh[x]),x]
```

output
```
-1/2*x + Cosh[x]^2/2 + Sinh[2*x]/4
```

3.1043.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 26, 3568, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)(\sinh(x)+\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ix)(\cos(ix)-i\sin(ix))dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (\cos(ix)-i\sin(ix))\sin(ix)dx$$

$$\Big\downarrow \text{3568}$$

$$-i\int \left(i\sinh^2(x)+i\cosh(x)\sinh(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{ix}{2}+\frac{1}{2}i\sinh^2(x)+\frac{1}{2}i\sinh(x)\cosh(x)\right)$$

input
```
Int[Sinh[x]*(Cosh[x] + Sinh[x]),x]
```

output
```
(-I)*((-1/2*I)*x + (I/2)*Cosh[x]*Sinh[x] + (I/2)*Sinh[x]^2)
```

3.1043.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3568
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.1043.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

method	result	size
risch	$-\frac{x}{2}+\frac{{\mathrm e}^{2 x}}{4}$	11
default	$\frac{\cosh\left(x\right)^{2}}{2}+\frac{\cosh\left(x\right) \sinh\left(x\right)}{2}-\frac{x}{2}$	17
parts	$\frac{\cosh\left(x\right)^{2}}{2}+\frac{\cosh\left(x\right) \sinh\left(x\right)}{2}-\frac{x}{2}$	17
meijerg	$-\frac{\sqrt{\pi}\, \left(\frac{1}{\sqrt{\pi}}-\frac{\cosh\left(2 x\right)}{\sqrt{\pi}}\right)}{4}+\frac{i \sqrt{\pi}\, \left(\frac{2 i x}{\sqrt{\pi}}-\frac{i \sinh\left(2 x\right)}{\sqrt{\pi}}\right)}{4}$	44

input
```
int(sinh(x)*(cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x+1/4*exp(2*x)
```

3.1043.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \sinh(x)(\cosh(x)+\sinh(x))\,dx=-\frac{(2\,x-1)\cosh\left(x\right)-(2\,x+1)\sinh\left(x\right)}{4\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

input
```
integrate(sinh(x)*(cosh(x)+sinh(x)),x, algorithm="fricas")
```

output
```
-1/4*((2*x - 1)*cosh(x) - (2*x + 1)*sinh(x))/(cosh(x) - sinh(x))
```

3.1043.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \sinh(x)(\cosh(x)+\sinh(x))\,dx = \frac{x\sinh^2(x)}{2} - \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2} + \frac{\cosh^2(x)}{2}$$

input
```
integrate(sinh(x)*(cosh(x)+sinh(x)),x)
```

output
```
x*sinh(x)**2/2 - x*cosh(x)**2/2 + sinh(x)*cosh(x)/2 + cosh(x)**2/2
```

3.1043.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sinh(x)(\cosh(x)+\sinh(x))\,dx = \frac{1}{2}\cosh(x)^2 - \frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

input
```
integrate(sinh(x)*(cosh(x)+sinh(x)),x, algorithm="maxima")
```

output
```
1/2*cosh(x)^2 - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

3.1043.8 Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int \sinh(x)(\cosh(x)+\sinh(x))\,dx = -\frac{1}{2}x + \frac{1}{4}e^{(2x)}$$

input
```
integrate(sinh(x)*(cosh(x)+sinh(x)),x, algorithm="giac")
```

output
```
-1/2*x + 1/4*e^(2*x)
```

3.1043.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int \sinh(x)(\cosh(x)+\sinh(x))\,dx=\frac{e^{2x}}{4}-\frac{x}{2}$$

input
```
int(sinh(x)*(cosh(x) + sinh(x)),x)
```

output
```
exp(2*x)/4 - x/2
```

3.1044 $\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx$

3.1044.1 Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{1}{4}\log\left(1-\tanh\left(\frac{x}{2}\right)\right)+\frac{3}{4}\log\left(1+\tanh\left(\frac{x}{2}\right)\right) + \frac{1}{2\left(1-\tanh\left(\frac{x}{2}\right)\right)} - \frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)^2} + \frac{1}{1+\tanh\left(\frac{x}{2}\right)}$$

output
```
1/4*ln(1-tanh(1/2*x))+3/4*ln(1+tanh(1/2*x))+1/2/(1-tanh(1/2*x))-1/2/(1+tan
h(1/2*x))^2+1/(1+tanh(1/2*x))
```

3.1044.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{x}{4}+\frac{\cosh(x)}{2}-\frac{1}{8}\cosh(2x)-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{8}\sinh(2x)$$

input
```
Integrate[(1 + Sinh[x]^2)/(1 + Cosh[x] + Sinh[x]),x]
```

output
```
x/4 + Cosh[x]/2 - Cosh[2*x]/8 - Log[Cosh[x/2]] + Sinh[2*x]/8
```

3.1044.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 4897, 3042, 4902, 27, 652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)+1}{\sinh(x)+\cosh(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-\sin(ix)^2}{-i\sin(ix)+\cos(ix)+1}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cosh^2(x)}{\sinh(x)+\cosh(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{-i\sin(ix)+\cos(ix)+1}dx$$

$$\Big\downarrow \text{4902}$$

$$2\int \frac{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}{2\left(1-\tanh\left(\frac{x}{2}\right)\right)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}{\left(1-\tanh\left(\frac{x}{2}\right)\right)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{3}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}\right)d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{4}\log\left(1-\tanh\left(\frac{x}{2}\right)\right)+\frac{3}{4}\log\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

input
```
Int[(1 + Sinh[x]^2)/(1 + Cosh[x] + Sinh[x]),x]
```

output
```
Log[1 - Tanh[x/2]]/4 + (3*Log[1 + Tanh[x/2]])/4 + 1/(2*(1 - Tanh[x/2])) -
1/(2*(1 + Tanh[x/2])^2) + (1 + Tanh[x/2])^(-1)
```

3.1044.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

3.1044.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.41

method	result	size
risch	$\frac{3x}{4}+\frac{e^{x}}{4}+\frac{e^{-x}}{4}-\frac{e^{-2x}}{8}-\ln\left(1+e^{x}\right)$	28
default	$-\frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{1}{1+\tanh\left(\frac{x}{2}\right)}+\frac{3\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{4}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4}$	48

input
```
int((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
3/4*x+1/4*exp(x)+1/4*exp(-x)-1/8*exp(-2*x)-ln(1+exp(x))
```

3.1044.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx$$
$$=\frac{6\,x\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)^{3}+6\left(x+\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+2\,\sinh\left(x\right)^{3}-8\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\right.}{8\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\right.}$$

input
```
integrate((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x, algorithm="fricas")
```

output
```
1/8*(6*x*cosh(x)^2 + 2*cosh(x)^3 + 6*(x + cosh(x))*sinh(x)^2 + 2*sinh(x)^3
 - 8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*log(cosh(x) + sinh(x) + 1
) + 2*(6*x*cosh(x) + 3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x) - 1)/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)
```

3.1044.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(51) = 102.

Time = 0.66 (sec) , antiderivative size = 381, normalized size of antiderivative = 5.52

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx = -\frac{x\tanh^3\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} - \frac{x\tanh^2\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} + \frac{x\tanh\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} + \frac{x}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} + \frac{4\log\left(\tanh\left(\frac{x}{2}\right)+1\right)\tanh^3\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} + \frac{4\log\left(\tanh\left(\frac{x}{2}\right)+1\right)\tanh^2\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} - \frac{4\log\left(\tanh\left(\frac{x}{2}\right)+1\right)\tanh\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} - \frac{4\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} + \frac{2\tanh^2\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} - \frac{6\tanh\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} - \frac{4}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4}$$

input
```
integrate((1+sinh(x)**2)/(1+cosh(x)+sinh(x)),x)
```

output
```
-x*tanh(x/2)**3/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - x*ta
nh(x/2)**2/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + x*tanh(x/
2)/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + x/(4*tanh(x/2)**3
 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + 4*log(tanh(x/2) + 1)*tanh(x/2)**3/(
4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + 4*log(tanh(x/2) + 1)*
tanh(x/2)**2/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 4*log(t
anh(x/2) + 1)*tanh(x/2)/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4
) - 4*log(tanh(x/2) + 1)/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) -
4) + 2*tanh(x/2)**2/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) -
6*tanh(x/2)/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 4/(4*tan
h(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4)
```

3.1044. $\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx$

3.1044.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.42

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx = -\frac{1}{4}\,x+\frac{1}{4}\,e^{(-x)}-\frac{1}{8}\,e^{(-2\,x)}+\frac{1}{4}\,e^{x}-\log\left(e^{(-x)}+1\right)$$

input
```
integrate((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x, algorithm="maxima")
```

output
```
-1/4*x + 1/4*e^(-x) - 1/8*e^(-2*x) + 1/4*e^x - log(e^(-x) + 1)
```

3.1044.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.39

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{1}{8}\,(2\,e^{x}-1)e^{(-2\,x)}+\frac{3}{4}\,x+\frac{1}{4}\,e^{x}-\log\left(e^{x}+1\right)$$

input
```
integrate((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x, algorithm="giac")
```

output
```
1/8*(2*e^x - 1)*e^(-2*x) + 3/4*x + 1/4*e^x - log(e^x + 1)
```

3.1044.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.39

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{3\,x}{4}+\frac{\mathrm{e}^{-x}}{4}-\frac{\mathrm{e}^{-2\,x}}{8}-\ln\left(\mathrm{e}^{x}+1\right)+\frac{\mathrm{e}^{x}}{4}$$

input
```
int((sinh(x)^2 + 1)/(cosh(x) + sinh(x) + 1),x)
```

output
```
(3*x)/4 + exp(-x)/4 - exp(-2*x)/8 - log(exp(x) + 1) + exp(x)/4
```

3.1045 $\int x^5 \cosh^7(a+bx^3) \sinh(a+bx^3)\,dx$

3.1045.1 Optimal result

Integrand size = 22, antiderivative size = 129

$$\int x^5 \cosh^7(a+bx^3) \sinh(a+bx^3)\,dx = -\frac{35x^3}{3072b} + \frac{x^3\cosh^8(a+bx^3)}{24b} - \frac{35\cosh(a+bx^3)\sinh(a+bx^3)}{3072b^2} - \frac{35\cosh^3(a+bx^3)\sinh(a+bx^3)}{4608b^2} - \frac{7\cosh^5(a+bx^3)\sinh(a+bx^3)}{1152b^2} - \frac{\cosh^7(a+bx^3)\sinh(a+bx^3)}{192b^2}$$

output
```
-35/3072*x^3/b+1/24*x^3*cosh(b*x^3+a)^8/b-35/3072*cosh(b*x^3+a)*sinh(b*x^3
+a)/b^2-35/4608*cosh(b*x^3+a)^3*sinh(b*x^3+a)/b^2-7/1152*cosh(b*x^3+a)^5*s
inh(b*x^3+a)/b^2-1/192*cosh(b*x^3+a)^7*sinh(b*x^3+a)/b^2
```

3.1045.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93

$$\int x^5 \cosh^7(a+bx^3)\sinh(a+bx^3)\,dx = \frac{1344bx^3\cosh(2(a+bx^3))+672bx^3\cosh(4(a+bx^3))+192bx^3\cosh(6(a+bx^3))+24bx^3\cosh(8(a+bx^3)}{73728b^2}$$

input
```
Integrate[x^5*Cosh[a + b*x^3]^7*Sinh[a + b*x^3],x]
```

output
```
(1344*b*x^3*Cosh[2*(a + b*x^3)] + 672*b*x^3*Cosh[4*(a + b*x^3)] + 192*b*x^
3*Cosh[6*(a + b*x^3)] + 24*b*x^3*Cosh[8*(a + b*x^3)] - 672*Sinh[2*(a + b*x
^3)] - 168*Sinh[4*(a + b*x^3)] - 32*Sinh[6*(a + b*x^3)] - 3*Sinh[8*(a + b*
x^3)])/(73728*b^2)
```

3.1045.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5896, 5844, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sinh(a+bx^3)\cosh^7(a+bx^3)\,dx$$

$$\Big\downarrow \; 5896$$

$$\frac{x^3\cosh^8(a+bx^3)}{24b}-\frac{\int x^2\cosh^8(bx^3+a)\,dx}{8b}$$

$$\Big\downarrow \; 5844$$

$$\frac{x^3\cosh^8(a+bx^3)}{24b}-\frac{\int \cosh^8(bx^3+a)\,dx^3}{24b}$$

$$\Big\downarrow \; 3042$$

$$\frac{x^3\cosh^8(a+bx^3)}{24b}-\frac{\int \sin\left(ibx^3+ia+\frac{\pi}{2}\right)^8dx^3}{24b}$$

$$\Big\downarrow \; 3115$$

$$\frac{x^3\cosh^8\left(a+bx^3\right)}{24b}-\frac{\frac{7}{8}\int \cosh^6\left(bx^3+a\right)dx^3+\frac{\sinh\left(a+bx^3\right)\cosh^7\left(a+bx^3\right)}{8b}}{24b}$$

$\downarrow$ 3042

$$\frac{x^3\cosh^8\left(a+bx^3\right)}{24b}-\frac{\frac{\sinh\left(a+bx^3\right)\cosh^7\left(a+bx^3\right)}{8b}+\frac{7}{8}\int \sin\left(ibx^3+ia+\frac{\pi}{2}\right)^6dx^3}{24b}$$

$\downarrow$ 3115

$$\frac{x^3\cosh^8\left(a+bx^3\right)}{24b}-\frac{\frac{7}{8}\left(\frac{5}{6}\int \cosh^4\left(bx^3+a\right)dx^3+\frac{\sinh\left(a+bx^3\right)\cosh^5\left(a+bx^3\right)}{6b}\right)+\frac{\sinh\left(a+bx^3\right)\cosh^7\left(a+bx^3\right)}{8b}}{24b}$$

$\downarrow$ 3042

$$\frac{x^3\cosh^8\left(a+bx^3\right)}{24b}-\frac{\frac{\sinh\left(a+bx^3\right)\cosh^7\left(a+bx^3\right)}{8b}+\frac{7}{8}\left(\frac{\sinh\left(a+bx^3\right)\cosh^5\left(a+bx^3\right)}{6b}+\frac{5}{6}\int \sin\left(ibx^3+ia+\frac{\pi}{2}\right)^4dx^3\right)}{24b}$$

$\downarrow$ 3115

$$\frac{x^3\cosh^8\left(a+bx^3\right)}{24b}-\frac{\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cosh^2\left(bx^3+a\right)dx^3+\frac{\sinh\left(a+bx^3\right)\cosh^3\left(a+bx^3\right)}{4b}\right)+\frac{\sinh\left(a+bx^3\right)\cosh^5\left(a+bx^3\right)}{6b}\right)+\frac{\sinh\left(a+bx^3\right)\cosh^7\left(a+bx^3\right)}{8b}}{24b}$$

$\downarrow$ 3042

$$\frac{x^3\cosh^8\left(a+bx^3\right)}{24b}-\frac{\frac{\sinh\left(a+bx^3\right)\cosh^7\left(a+bx^3\right)}{8b}+\frac{7}{8}\left(\frac{\sinh\left(a+bx^3\right)\cosh^5\left(a+bx^3\right)}{6b}+\frac{5}{6}\left(\frac{\sinh\left(a+bx^3\right)\cosh^3\left(a+bx^3\right)}{4b}+\frac{3}{4}\int \sin\left(ibx^3+ia+\frac{\pi}{2}\right)^2dx^3\right)\right)}{24b}$$

$\downarrow$ 3115

$$\frac{x^3\cosh^8\left(a+bx^3\right)}{24b}-\frac{\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx^3}{2}+\frac{\sinh\left(a+bx^3\right)\cosh\left(a+bx^3\right)}{2b}\right)+\frac{\sinh\left(a+bx^3\right)\cosh^3\left(a+bx^3\right)}{4b}\right)+\frac{\sinh\left(a+bx^3\right)\cosh^5\left(a+bx^3\right)}{6b}\right)+\frac{\sinh\left(a+bx^3\right)\cosh^7\left(a+bx^3\right)}{8b}}{24b}$$

$\downarrow$ 24

$$\frac{x^3\cosh^8\left(a+bx^3\right)}{24b}-\frac{\frac{\sinh\left(a+bx^3\right)\cosh^7\left(a+bx^3\right)}{8b}+\frac{7}{8}\left(\frac{\sinh\left(a+bx^3\right)\cosh^5\left(a+bx^3\right)}{6b}+\frac{5}{6}\left(\frac{\sinh\left(a+bx^3\right)\cosh^3\left(a+bx^3\right)}{4b}+\frac{3}{4}\left(\frac{\sinh\left(a+bx^3\right)\cosh\left(a+bx^3\right)}{2b}+\frac{x^3}{2}\right)\right)\right)}{24b}$$

3.1045. $\int x^5\cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)\,dx$

input
```
Int[x^5*Cosh[a + b*x^3]^7*Sinh[a + b*x^3],x]
```

output
```
(x^3*Cosh[a + b*x^3]^8)/(24*b) - ((Cosh[a + b*x^3]^7*Sinh[a + b*x^3])/(8*b
) + (7*((Cosh[a + b*x^3]^5*Sinh[a + b*x^3])/(6*b) + (5*((Cosh[a + b*x^3]^3
*Sinh[a + b*x^3])/(4*b) + (3*(x^3/2 + (Cosh[a + b*x^3]*Sinh[a + b*x^3])/(2
*b)))/4))/6))/8)/(24*b)
```

3.1045.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

rule 5896
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cosh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.1045.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.50

$$\frac{(8b\,x^3-1)\,\mathrm{e}^{8b\,x^3+8a}}{49152b^2}+\frac{(6b\,x^3-1)\,\mathrm{e}^{6b\,x^3+6a}}{4608b^2}+\frac{7(4b\,x^3-1)\,\mathrm{e}^{4b\,x^3+4a}}{6144b^2}+\frac{7(2b\,x^3-1)\,\mathrm{e}^{2b\,x^3+2a}}{1536b^2}+\frac{7(2b\,x^3+1)\,\mathrm{e}^{-2b\,x^3-}}{1536b^2}$$

input
```
int(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x)
```

output
```
1/49152*(8*b*x^3-1)/b^2*exp(8*b*x^3+8*a)+1/4608*(6*b*x^3-1)/b^2*exp(6*b*x^
3+6*a)+7/6144*(4*b*x^3-1)/b^2*exp(4*b*x^3+4*a)+7/1536*(2*b*x^3-1)/b^2*exp(
2*b*x^3+2*a)+7/1536*(2*b*x^3+1)/b^2*exp(-2*b*x^3-2*a)+7/6144*(4*b*x^3+1)/b
^2*exp(-4*b*x^3-4*a)+1/4608*(6*b*x^3+1)/b^2*exp(-6*b*x^3-6*a)+1/49152*(8*b
*x^3+1)/b^2*exp(-8*b*x^3-8*a)
```

3.1045.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(117) = 234.

Time = 0.26 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.07

$$\int x^5\cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)\,dx$$
$$=\frac{3\,bx^3\cosh\left(bx^3+a\right)^8+3\,bx^3\sinh\left(bx^3+a\right)^8+24\,bx^3\cosh\left(bx^3+a\right)^6+84\,bx^3\cosh\left(bx^3+a\right)^4-3\,\cosh\left(bx\right.}{}$$

input
```
integrate(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x, algorithm="fricas")
```

output
```
1/9216*(3*b*x^3*cosh(b*x^3 + a)^8 + 3*b*x^3*sinh(b*x^3 + a)^8 + 24*b*x^3*c
osh(b*x^3 + a)^6 + 84*b*x^3*cosh(b*x^3 + a)^4 - 3*cosh(b*x^3 + a)*sinh(b*x
^3 + a)^7 + 12*(7*b*x^3*cosh(b*x^3 + a)^2 + 2*b*x^3)*sinh(b*x^3 + a)^6 + 1
68*b*x^3*cosh(b*x^3 + a)^2 - 3*(7*cosh(b*x^3 + a)^3 + 8*cosh(b*x^3 + a))*s
inh(b*x^3 + a)^5 + 6*(35*b*x^3*cosh(b*x^3 + a)^4 + 60*b*x^3*cosh(b*x^3 + a
)^2 + 14*b*x^3)*sinh(b*x^3 + a)^4 - (21*cosh(b*x^3 + a)^5 + 80*cosh(b*x^3
+ a)^3 + 84*cosh(b*x^3 + a))*sinh(b*x^3 + a)^3 + 12*(7*b*x^3*cosh(b*x^3 +
a)^6 + 30*b*x^3*cosh(b*x^3 + a)^4 + 42*b*x^3*cosh(b*x^3 + a)^2 + 14*b*x^3)
*sinh(b*x^3 + a)^2 - 3*(cosh(b*x^3 + a)^7 + 8*cosh(b*x^3 + a)^5 + 28*cosh(
b*x^3 + a)^3 + 56*cosh(b*x^3 + a))*sinh(b*x^3 + a))/b^2
```

3.1045.6 Sympy [A] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.87

$$\int x^5 \cosh^7\left(a+bx^3\right) \sinh\left(a+bx^3\right) \, dx = \begin{cases} -\frac{35x^3 \sinh^8(a+bx^3)}{3072b} + \frac{35x^3 \sinh^6(a+bx^3)\cosh^2(a+bx^3)}{768b} - \frac{35x^3 \sinh^4(a+bx^3)\cosh^4(a+bx^3)}{512b} + \frac{35x^3 \sinh^2(a+bx^3)\cosh^6(a+bx^3)}{768b} \\ \frac{x^6 \sinh(a)\cosh^7(a)}{6} \end{cases}$$

input
```
integrate(x**5*cosh(b*x**3+a)**7*sinh(b*x**3+a),x)
```

output
```
Piecewise((-35*x**3*sinh(a + b*x**3)**8/(3072*b) + 35*x**3*sinh(a + b*x**3
)**6*cosh(a + b*x**3)**2/(768*b) - 35*x**3*sinh(a + b*x**3)**4*cosh(a + b*
x**3)**4/(512*b) + 35*x**3*sinh(a + b*x**3)**2*cosh(a + b*x**3)**6/(768*b)
 + 31*x**3*cosh(a + b*x**3)**8/(1024*b) + 35*sinh(a + b*x**3)**7*cosh(a +
b*x**3)/(3072*b**2) - 385*sinh(a + b*x**3)**5*cosh(a + b*x**3)**3/(9216*b*
*2) + 511*sinh(a + b*x**3)**3*cosh(a + b*x**3)**5/(9216*b**2) - 31*sinh(a
+ b*x**3)*cosh(a + b*x**3)**7/(1024*b**2), Ne(b, 0)), (x**6*sinh(a)*cosh(a
)**7/6, True))
```

3.1045.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.65

$$\begin{aligned}\int x^5 \cosh^7\left(a+bx^3\right) \sinh\left(a+bx^3\right) \, dx &= \frac{\left(8\,bx^3 e^{(8\,a)} - e^{(8\,a)}\right) e^{\left(8\,bx^3\right)}}{49152\,b^2} \\ &+ \frac{\left(6\,bx^3 e^{(6\,a)} - e^{(6\,a)}\right) e^{\left(6\,bx^3\right)}}{4608\,b^2} \\ &+ \frac{7\left(4\,bx^3 e^{(4\,a)} - e^{(4\,a)}\right) e^{\left(4\,bx^3\right)}}{6144\,b^2} \\ &+ \frac{7\left(2\,bx^3 e^{(2\,a)} - e^{(2\,a)}\right) e^{\left(2\,bx^3\right)}}{1536\,b^2} \\ &+ \frac{7\left(2\,bx^3+1\right) e^{\left(-2\,bx^3-2\,a\right)}}{1536\,b^2} \\ &+ \frac{7\left(4\,bx^3+1\right) e^{\left(-4\,bx^3-4\,a\right)}}{6144\,b^2} \\ &+ \frac{\left(6\,bx^3+1\right) e^{\left(-6\,bx^3-6\,a\right)}}{4608\,b^2} + \frac{\left(8\,bx^3+1\right) e^{\left(-8\,bx^3-8\,a\right)}}{49152\,b^2}\end{aligned}$$

input
```
integrate(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x, algorithm="maxima")
```

output
```
1/49152*(8*b*x^3*e^(8*a) - e^(8*a))*e^(8*b*x^3)/b^2 + 1/4608*(6*b*x^3*e^(6
*a) - e^(6*a))*e^(6*b*x^3)/b^2 + 7/6144*(4*b*x^3*e^(4*a) - e^(4*a))*e^(4*b
*x^3)/b^2 + 7/1536*(2*b*x^3*e^(2*a) - e^(2*a))*e^(2*b*x^3)/b^2 + 7/1536*(2
*b*x^3 + 1)*e^(-2*b*x^3 - 2*a)/b^2 + 7/6144*(4*b*x^3 + 1)*e^(-4*b*x^3 - 4*
a)/b^2 + 1/4608*(6*b*x^3 + 1)*e^(-6*b*x^3 - 6*a)/b^2 + 1/49152*(8*b*x^3 +
1)*e^(-8*b*x^3 - 8*a)/b^2
```

3.1045.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(117) = 234$.

Time = 0.27 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.99

$$\int x^5 \cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)\,dx =$$
$$-\frac{a\left(e^{(2\,bx^3+2\,a)}+e^{(-2\,bx^3-2\,a)}\right)^4+8\,a\left(e^{(2\,bx^3+2\,a)}+e^{(-2\,bx^3-2\,a)}\right)^3+24\,a\left(e^{(2\,bx^3+2\,a)}+e^{(-2\,bx^3-2\,a)}\right)^2+32}{6144\,b^2}$$
$$+\frac{24\,(bx^3+a)e^{(8\,bx^3+8\,a)}+192\,(bx^3+a)e^{(6\,bx^3+6\,a)}+672\,(bx^3+a)e^{(4\,bx^3+4\,a)}+1344\,(bx^3+a)e^{(2\,bx^3+2\,a)}+}{}$$

input
```
integrate(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x, algorithm="giac")
```

output
```
-1/6144*(a*(e^(2*b*x^3 + 2*a) + e^(-2*b*x^3 - 2*a))^4 + 8*a*(e^(2*b*x^3 +
2*a) + e^(-2*b*x^3 - 2*a))^3 + 24*a*(e^(2*b*x^3 + 2*a) + e^(-2*b*x^3 - 2*a
))^2 + 32*a*(e^(2*b*x^3 + 2*a) + e^(-2*b*x^3 - 2*a)))/b^2 + 1/147456*(24*(
b*x^3 + a)*e^(8*b*x^3 + 8*a) + 192*(b*x^3 + a)*e^(6*b*x^3 + 6*a) + 672*(b*
x^3 + a)*e^(4*b*x^3 + 4*a) + 1344*(b*x^3 + a)*e^(2*b*x^3 + 2*a) + 1344*(b*
x^3 + a)*e^(-2*b*x^3 - 2*a) + 672*(b*x^3 + a)*e^(-4*b*x^3 - 4*a) + 192*(b*
x^3 + a)*e^(-6*b*x^3 - 6*a) + 24*(b*x^3 + a)*e^(-8*b*x^3 - 8*a) - 3*e^(8*b
*x^3 + 8*a) - 32*e^(6*b*x^3 + 6*a) - 168*e^(4*b*x^3 + 4*a) - 672*e^(2*b*x^
3 + 2*a) + 672*e^(-2*b*x^3 - 2*a) + 168*e^(-4*b*x^3 - 4*a) + 32*e^(-6*b*x^
3 - 6*a) + 3*e^(-8*b*x^3 - 8*a))/b^2
```

3.1045.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.65

$$\int x^5 \cosh^7\left(a+bx^3\right) \sinh\left(a+bx^3\right)\,dx = e^{-2\,b\,x^3-2\,a}\left(\frac{7}{1536\,b^2}+\frac{7\,x^3}{768\,b}\right) - e^{2\,b\,x^3+2\,a}\left(\frac{7}{1536\,b^2}-\frac{7\,x^3}{768\,b}\right) + e^{-6\,b\,x^3-6\,a}\left(\frac{1}{4608\,b^2}+\frac{x^3}{768\,b}\right) - e^{6\,b\,x^3+6\,a}\left(\frac{1}{4608\,b^2}-\frac{x^3}{768\,b}\right) + e^{-4\,b\,x^3-4\,a}\left(\frac{7}{6144\,b^2}+\frac{7\,x^3}{1536\,b}\right) - e^{4\,b\,x^3+4\,a}\left(\frac{7}{6144\,b^2}-\frac{7\,x^3}{1536\,b}\right) + e^{-8\,b\,x^3-8\,a}\left(\frac{1}{49152\,b^2}+\frac{x^3}{6144\,b}\right) - e^{8\,b\,x^3+8\,a}\left(\frac{1}{49152\,b^2}-\frac{x^3}{6144\,b}\right)$$

input
```
int(x^5*cosh(a + b*x^3)^7*sinh(a + b*x^3),x)
```

output
```
exp(- 2*a - 2*b*x^3)*(7/(1536*b^2) + (7*x^3)/(768*b)) - exp(2*a + 2*b*x^3)
*(7/(1536*b^2) - (7*x^3)/(768*b)) + exp(- 6*a - 6*b*x^3)*(1/(4608*b^2) + x
^3/(768*b)) - exp(6*a + 6*b*x^3)*(1/(4608*b^2) - x^3/(768*b)) + exp(- 4*a
- 4*b*x^3)*(7/(6144*b^2) + (7*x^3)/(1536*b)) - exp(4*a + 4*b*x^3)*(7/(6144
*b^2) - (7*x^3)/(1536*b)) + exp(- 8*a - 8*b*x^3)*(1/(49152*b^2) + x^3/(614
4*b)) - exp(8*a + 8*b*x^3)*(1/(49152*b^2) - x^3/(6144*b))
```

3.1046 $\int \frac{\cosh^2(x)}{1+e^x} dx$

3.1046.1 Optimal result

Integrand size = 12, antiderivative size = 39

$$\int \frac{\cosh^2(x)}{1+e^x} dx = -\frac{1}{8}e^{-2x} + \frac{e^{-x}}{4} + \frac{e^x}{4} + \frac{3x}{4} - \log(1+e^x)$$

output
```
-1/8/exp(2*x)+1/4/exp(x)+1/4*exp(x)+3/4*x-ln(1+exp(x))
```

3.1046.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^2(x)}{1+e^x} dx = \frac{1}{4}\left(-\frac{1}{2}e^{-2x} + e^{-x} + e^x + 3x - 4\log(1+e^x)\right)$$

input
```
Integrate[Cosh[x]^2/(1 + E^x),x]
```

output
```
(-1/2*1/E^(2*x) + E^(-x) + E^x + 3*x - 4*Log[1 + E^x])/4
```

3.1046.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2720, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{e^x+1} \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{e^{-3x} \left(e^{2x}+1\right)^2}{4 \left(e^x+1\right)}de^x$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \int \frac{e^{-3x} \left(1+e^{2x}\right)^2}{1+e^x}de^x$$

$$\Big\downarrow \text{522}$$

$$\frac{1}{4} \int \left(e^{-3x}-e^{-2x}+3 e^{-x}+1-\frac{4}{1+e^x}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(-\frac{e^{-2x}}{2}+e^{-x}+e^x+3 \log \left(e^x\right)-4 \log \left(e^x+1\right)\right)$$

input
```
Int[Cosh[x]^2/(1 + E^x),x]
```

output
```
(-1/2*1/E^(2*x) + E^(-x) + E^x + 3*Log[E^x] - 4*Log[1 + E^x])/4
```

3.1046.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.1046.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{3x}{4}+\frac{{\mathrm e}^{x}}{4}+\frac{{\mathrm e}^{-x}}{4}-\frac{{\mathrm e}^{-2x}}{8}-\ln\left(1+{\mathrm e}^{x}\right)$	28
default	$-\frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{1}{1+\tanh\left(\frac{x}{2}\right)}+\frac{3\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{4}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4}$	48

input
```
int(cosh(x)^2/(1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
3/4*x+1/4*exp(x)+1/4*exp(-x)-1/8*exp(-2*x)-ln(1+exp(x))
```

3.1046.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(27) = 54.

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \frac{\cosh^2(x)}{1+e^x}\,dx$$
$$=\frac{6\,x\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)^{3}+6\left(x+\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+2\,\sinh\left(x\right)^{3}-8\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\right.}{8\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\right.}$$

input
```
integrate(cosh(x)^2/(1+exp(x)),x, algorithm="fricas")
```

output
```
1/8*(6*x*cosh(x)^2 + 2*cosh(x)^3 + 6*(x + cosh(x))*sinh(x)^2 + 2*sinh(x)^3
 - 8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*log(cosh(x) + sinh(x) + 1
) + 2*(6*x*cosh(x) + 3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x) - 1)/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)
```

3.1046.6 Sympy [F]

$$\int \frac{\cosh^2(x)}{1+e^x}\,dx = \int \frac{\cosh^2(x)}{e^x+1}\,dx$$

input
```
integrate(cosh(x)**2/(1+exp(x)),x)
```

output
```
Integral(cosh(x)**2/(exp(x) + 1), x)
```

3.1046.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{\cosh^2(x)}{1+e^x}\,dx = \frac{1}{8}\,(2\,e^x-1)e^{(-2\,x)} + \frac{3}{4}\,x + \frac{1}{4}\,e^x - \log\left(e^x+1\right)$$

input
```
integrate(cosh(x)^2/(1+exp(x)),x, algorithm="maxima")
```

output
```
1/8*(2*e^x - 1)*e^(-2*x) + 3/4*x + 1/4*e^x - log(e^x + 1)
```

3.1046.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{\cosh^2(x)}{1+e^x}\,dx = \frac{1}{8}\,(2\,e^x-1)e^{(-2\,x)} + \frac{3}{4}\,x + \frac{1}{4}\,e^x - \log\left(e^x+1\right)$$

input
```
integrate(cosh(x)^2/(1+exp(x)),x, algorithm="giac")
```

output
```
1/8*(2*e^x - 1)*e^(-2*x) + 3/4*x + 1/4*e^x - log(e^x + 1)
```

3.1046.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{\cosh^2(x)}{1+e^x}\,dx = \frac{3\,x}{4} + \frac{\mathrm{e}^{-x}}{4} - \frac{\mathrm{e}^{-2\,x}}{8} - \ln\left(\mathrm{e}^x + 1\right) + \frac{\mathrm{e}^x}{4}$$

input
```
int(cosh(x)^2/(exp(x) + 1),x)
```

output
```
(3*x)/4 + exp(-x)/4 - exp(-2*x)/8 - log(exp(x) + 1) + exp(x)/4
```

3.1047 $\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx$

3.1047.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx = -\frac{4}{5}(1+\operatorname{sech}(x))^{5/2}+\frac{2}{7}(1+\operatorname{sech}(x))^{7/2}$$

output
```
-4/5*(1+sech(x))^(5/2)+2/7*(1+sech(x))^(7/2)
```

3.1047.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx = -\frac{8}{35}\cosh^4\left(\frac{x}{2}\right)(-5+9\cosh(x))\operatorname{sech}^3(x)\sqrt{1+\operatorname{sech}(x)}$$

input
```
Integrate[Sech[x]*Sqrt[1 + Sech[x]]*Tanh[x]^3,x]
```

output
```
(-8*Cosh[x/2]^4*(-5 + 9*Cosh[x])*Sech[x]^3*Sqrt[1 + Sech[x]])/35
```

3.1047.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 26, 4873, 1894, 1388, 946, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(x)\text{sech}(x)\sqrt{\text{sech}(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ix)^3\sec(ix)\sqrt{1+\sec(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec(ix)\sqrt{\sec(ix)+1}\tan(ix)^3dx$$

$$\Big\downarrow \text{4873}$$

$$-\int \left(1-\cosh^2(x)\right)\text{sech}^4(x)\sqrt{\text{sech}(x)+1}d\cosh(x)$$

$$\Big\downarrow \text{1894}$$

$$-\int \text{sech}^2(x)\sqrt{\text{sech}(x)+1}\left(\text{sech}^2(x)-1\right)d\cosh(x)$$

$$\Big\downarrow \text{1388}$$

$$-\int (\text{sech}(x)-1)\text{sech}^2(x)(\text{sech}(x)+1)^{3/2}d\cosh(x)$$

$$\Big\downarrow \text{946}$$

$$\int -\left((1-\text{sech}(x))(\text{sech}(x)+1)^{3/2}\right)d\text{sech}(x)$$

$$\Big\downarrow \text{25}$$

$$-\int (1-\text{sech}(x))(\text{sech}(x)+1)^{3/2}d\text{sech}(x)$$

$$\Big\downarrow \text{53}$$

$$-\int \left(2(\text{sech}(x)+1)^{3/2}-(\text{sech}(x)+1)^{5/2}\right)d\text{sech}(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}(\operatorname{sech}(x)+1)^{7/2}-\frac{4}{5}(\operatorname{sech}(x)+1)^{5/2}$$

input
```
Int[Sech[x]*Sqrt[1 + Sech[x]]*Tanh[x]^3,x]
```

output
```
(-4*(1 + Sech[x])^(5/2))/5 + (2*(1 + Sech[x])^(7/2))/7
```

3.1047.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1894
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4873
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-(b*c*d^(n - 1))^(-1)   Subst[Int[Subst
For[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*
(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
 c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan]
)
```

3.1047.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$-\frac{4(1+\operatorname{sech}(x))^{\frac{5}{2}}}{5}+\frac{2(1+\operatorname{sech}(x))^{\frac{7}{2}}}{7}$	18
default	$-\frac{4(1+\operatorname{sech}(x))^{\frac{5}{2}}}{5}+\frac{2(1+\operatorname{sech}(x))^{\frac{7}{2}}}{7}$	18

input
```
int(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-4/5*(1+sech(x))^(5/2)+2/7*(1+sech(x))^(7/2)
```

3.1047.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 431, normalized size of antiderivative = 17.24

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx =$$

$$-\frac{2\left(9\cosh(x)^6+54\cosh(x)\sinh(x)^5+9\sinh(x)^6+27\left(5\cosh(x)^2+1\right)\sinh(x)^4+27\cosh(x)^4+3\right.}{}$$

input
```
integrate(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x, algorithm="fricas")
```

output
```
-2/35*(9*cosh(x)^6 + 54*cosh(x)*sinh(x)^5 + 9*sinh(x)^6 + 27*(5*cosh(x)^2
+ 1)*sinh(x)^4 + 27*cosh(x)^4 + 36*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 2
7*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 27*cosh(x)^2 + 54*(cosh(x)^5
 + 2*cosh(x)^3 + cosh(x))*sinh(x) + (9*cosh(x)^7 + 7*(9*cosh(x) + 5)*sinh(
x)^6 + 9*sinh(x)^7 + 35*cosh(x)^6 + 7*(27*cosh(x)^2 + 30*cosh(x) + 7)*sinh
(x)^5 + 49*cosh(x)^5 + 35*(9*cosh(x)^3 + 15*cosh(x)^2 + 7*cosh(x) + 1)*sin
h(x)^4 + 35*cosh(x)^4 + 35*(9*cosh(x)^4 + 20*cosh(x)^3 + 14*cosh(x)^2 + 4*
cosh(x) + 1)*sinh(x)^3 + 35*cosh(x)^3 + 7*(27*cosh(x)^5 + 75*cosh(x)^4 + 7
0*cosh(x)^3 + 30*cosh(x)^2 + 15*cosh(x) + 7)*sinh(x)^2 + 49*cosh(x)^2 + 7*
(9*cosh(x)^6 + 30*cosh(x)^5 + 35*cosh(x)^4 + 20*cosh(x)^3 + 15*cosh(x)^2 +
 14*cosh(x) + 5)*sinh(x) + 35*cosh(x) + 9)/sqrt(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + 1) + 9)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3
*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*s
inh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(
cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

3.1047.6 Sympy [F]

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx = \int \sqrt{\operatorname{sech}(x)+1}\tanh^3(x)\operatorname{sech}(x)\,dx$$

input
```
integrate(sech(x)*(1+sech(x))**(1/2)*tanh(x)**3,x)
```

output
```
Integral(sqrt(sech(x) + 1)*tanh(x)**3*sech(x), x)
```

3.1047.7 Maxima [F]

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx = \int \sqrt{\operatorname{sech}(x)+1}\operatorname{sech}(x)\tanh(x)^3\,dx$$

input
```
integrate(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(sech(x) + 1)*sech(x)*tanh(x)^3, x)
```

3.1047.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx = -\frac{2\,(((((((9\,e^x+35)e^x+49)e^x+35)e^x+35)e^x+49)e^x+35)e^x+9)}{35\,(e^{(2\,x)}+1)^{\frac{7}{2}}}$$

input
```
integrate(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x, algorithm="giac")
```

output
```
-2/35*(((((((9*e^x + 35)*e^x + 49)*e^x + 35)*e^x + 35)*e^x + 49)*e^x + 35)
*e^x + 9)/(e^(2*x) + 1)^(7/2)
```

3.1047.9 Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 5.92

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx = \frac{\left(\frac{72\,e^x}{35}-\frac{24}{5}\right)\sqrt{\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}+1}}{(e^x+1)\,(e^{2\,x}+1)^2} - \frac{\left(\frac{16\,e^x}{7}-\frac{16}{7}\right)\sqrt{\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}+1}}{(e^x+1)\,(e^{2\,x}+1)^3} - \frac{\left(\frac{44\,e^x}{35}-4\right)\sqrt{\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}+1}}{(e^x+1)\,(e^{2\,x}+1)} - \frac{\left(\frac{18\,e^x}{35}+2\right)\sqrt{\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}+1}}{e^x+1}$$

input
```
int((tanh(x)^3*(1/cosh(x) + 1)^(1/2))/cosh(x),x)
```

output
```
(((72*exp(x))/35 - 24/5)*(1/(exp(-x)/2 + exp(x)/2) + 1)^(1/2))/((exp(x) +
1)*(exp(2*x) + 1)^2) - (((16*exp(x))/7 - 16/7)*(1/(exp(-x)/2 + exp(x)/2) +
 1)^(1/2))/((exp(x) + 1)*(exp(2*x) + 1)^3) - (((44*exp(x))/35 - 4)*(1/(exp
(-x)/2 + exp(x)/2) + 1)^(1/2))/((exp(x) + 1)*(exp(2*x) + 1)) - (((18*exp(x
))/35 + 2)*(1/(exp(-x)/2 + exp(x)/2) + 1)^(1/2))/(exp(x) + 1)
```

3.1048 $\int \coth^3(x)\mathbf{csch}(x)\sqrt{1+\mathbf{csch}(x)}\,dx$

3.1048.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx = -\frac{4}{3}(1+\operatorname{csch}(x))^{3/2} + \frac{4}{5}(1+\operatorname{csch}(x))^{5/2} - \frac{2}{7}(1+\operatorname{csch}(x))^{7/2}$$

output
```
-4/3*(1+csch(x))^(3/2)+4/5*(1+csch(x))^(5/2)-2/7*(1+csch(x))^(7/2)
```

3.1048.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx = -\frac{1}{210}\operatorname{csch}^3(x)\sqrt{1+\operatorname{csch}(x)}(-2+62\cosh(2x) - 117\sinh(x)+43\sinh(3x))$$

input
```
Integrate[Coth[x]^3*Csch[x]*Sqrt[1 + Csch[x]],x]
```

output
```
-1/210*(Csch[x]^3*Sqrt[1 + Csch[x]]*(-2 + 62*Cosh[2*x] - 117*Sinh[x] + 43*
Sinh[3*x]))
```

3.1048.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 4872, 26, 1894, 1799, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(x)\text{csch}(x)\sqrt{\text{csch}(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(ix)^3\sqrt{1+i\csc(ix)}\csc(ix)dx$$

$$\Big\downarrow \text{4872}$$

$$i\int -i\text{csch}^4(x)\sqrt{\text{csch}(x)+1}\left(\sinh^2(x)+1\right)d\sinh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \left(\sinh^2(x)+1\right)\text{csch}^4(x)\sqrt{\text{csch}(x)+1}d\sinh(x)$$

$$\Big\downarrow \text{1894}$$

$$\int \text{csch}^2(x)\sqrt{\text{csch}(x)+1}\left(\text{csch}^2(x)+1\right)d\sinh(x)$$

$$\Big\downarrow \text{1799}$$

$$-\int \sqrt{\text{csch}(x)+1}\left(\text{csch}^2(x)+1\right)d\text{csch}(x)$$

$$\Big\downarrow \text{476}$$

$$-\int \left((\text{csch}(x)+1)^{5/2}-2(\text{csch}(x)+1)^{3/2}+2\sqrt{\text{csch}(x)+1}\right)d\text{csch}(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{7}(\text{csch}(x)+1)^{7/2}+\frac{4}{5}(\text{csch}(x)+1)^{5/2}-\frac{4}{3}(\text{csch}(x)+1)^{3/2}$$

input
```
Int[Coth[x]^3*Csch[x]*Sqrt[1 + Csch[x]],x]
```

output
```
(-4*(1 + Csch[x])^(3/2))/3 + (4*(1 + Csch[x])^(5/2))/5 - (2*(1 + Csch[x])^
(7/2))/7
```

3.1048.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

rule 1894
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4872
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[1/(b*c*d^(n - 1))   Subst[Int[SubstFor[
(1 - d^2*x^2)^((n - 1)/2)/x^n, Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a +
 b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c},
 x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.1048.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{4(1+\operatorname{csch}(x))^{\frac{3}{2}}}{3}+\frac{4(1+\operatorname{csch}(x))^{\frac{5}{2}}}{5}-\frac{2(1+\operatorname{csch}(x))^{\frac{7}{2}}}{7}$	26
default	$-\frac{4(1+\operatorname{csch}(x))^{\frac{3}{2}}}{3}+\frac{4(1+\operatorname{csch}(x))^{\frac{5}{2}}}{5}-\frac{2(1+\operatorname{csch}(x))^{\frac{7}{2}}}{7}$	26

input
```
int(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(1+csch(x))^(3/2)+4/5*(1+csch(x))^(5/2)-2/7*(1+csch(x))^(7/2)
```

3.1048.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(25) = 50.

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 7.32

$$\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx =$$

$$-\frac{2\left(43\cosh(x)^6+2\left(129\cosh(x)+31\right)\sinh(x)^5+43\sinh(x)^6+62\cosh(x)^5+\left(645\cosh(x)^2+310\right.}{10}$$

input
```
integrate(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(43*cosh(x)^6 + 2*(129*cosh(x) + 31)*sinh(x)^5 + 43*sinh(x)^6 + 62*
cosh(x)^5 + (645*cosh(x)^2 + 310*cosh(x) - 117)*sinh(x)^4 - 117*cosh(x)^4
+ 4*(215*cosh(x)^3 + 155*cosh(x)^2 - 117*cosh(x) - 1)*sinh(x)^3 - 4*cosh(x
)^3 + (645*cosh(x)^4 + 620*cosh(x)^3 - 702*cosh(x)^2 - 12*cosh(x) + 117)*s
inh(x)^2 + 117*cosh(x)^2 + 2*(129*cosh(x)^5 + 155*cosh(x)^4 - 234*cosh(x)^
3 - 6*cosh(x)^2 + 117*cosh(x) + 31)*sinh(x) + 62*cosh(x) - 43)*sqrt((sinh(
x) + 1)/sinh(x))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(
x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3
+ 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5
 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)
```

3.1048.6 Sympy [F]

$$\int \coth^3(x)\text{csch}(x)\sqrt{1+\text{csch}(x)}\,dx = \int \sqrt{\text{csch}\,(x)+1}\coth^3\,(x)\,\text{csch}\,(x)\,dx$$

input
```
integrate(coth(x)**3*csch(x)*(1+csch(x))**(1/2),x)
```

output
```
Integral(sqrt(csch(x) + 1)*coth(x)**3*csch(x), x)
```

3.1048.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 389, normalized size of antiderivative = 10.51

$$\begin{aligned}
&\int \coth^3(x)\text{csch}(x)\sqrt{1+\text{csch}(x)}\,dx \\
&= \frac{124\sqrt{-2\,e^{(-x)}+e^{(-2\,x)}-1}e^{(-x)}}{105\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1)} \\
&- \frac{78\sqrt{-2\,e^{(-x)}+e^{(-2\,x)}-1}e^{(-2\,x)}}{35\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1)} \\
&- \frac{8\sqrt{-2\,e^{(-x)}+e^{(-2\,x)}-1}e^{(-3\,x)}}{105\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1)} \\
&+ \frac{78\sqrt{-2\,e^{(-x)}+e^{(-2\,x)}-1}e^{(-4\,x)}}{35\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1)} \\
&+ \frac{124\sqrt{-2\,e^{(-x)}+e^{(-2\,x)}-1}e^{(-5\,x)}}{105\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1)} \\
&- \frac{86\sqrt{-2\,e^{(-x)}+e^{(-2\,x)}-1}e^{(-6\,x)}}{105\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1)} \\
&+ \frac{86\sqrt{-2\,e^{(-x)}+e^{(-2\,x)}-1}}{105\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1)}
\end{aligned}$$

input
```
integrate(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x, algorithm="maxima")
```

output
```
124/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x
) - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) - 78/35*sqrt(-2*e^(-x) +
e^(-2*x) - 1)*e^(-2*x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*
e^(-4*x) + e^(-6*x) - 1)) - 8/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-3*x)/
(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1
)) + 78/35*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-4*x)/(sqrt(e^(-x) + 1)*sqrt(
e^(-x) - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) + 124/105*sqrt(-2*e^
(-x) + e^(-2*x) - 1)*e^(-5*x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*
x) - 3*e^(-4*x) + e^(-6*x) - 1)) - 86/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e
^(-6*x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-
6*x) - 1)) + 86/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)/(sqrt(e^(-x) + 1)*sqrt(
e^(-x) - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1))
```

3.1048.8 Giac [F]

$$\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx = \int \sqrt{\operatorname{csch}(x)+1}\coth(x)^3\operatorname{csch}(x)\,dx$$

input
```
integrate(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(csch(x) + 1)*coth(x)^3*csch(x), x)
```

3.1048.9 Mupad [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 5.59

$$\begin{aligned}\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx = &-\frac{8\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}}{35\,(e^{2x}-1)}-\frac{8\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}}{35\,(e^{4x}-2\,e^{2x}+1)}\\&-\frac{86\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}}{105}-\frac{16\,e^{x}\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}}{7\,(3\,e^{2x}-3\,e^{4x}+e^{6x}-1)}\\&-\frac{16\,e^{x}\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}}{7\,(e^{4x}-2\,e^{2x}+1)}-\frac{124\,e^{x}\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}}{105\,(e^{2x}-1)}\end{aligned}$$

input
```
int((coth(x)^3*(1/sinh(x) + 1)^(1/2))/sinh(x),x)
```

output
```
- (8*(1 - 1/(exp(-x)/2 - exp(x)/2))^(1/2))/(35*(exp(2*x) - 1)) - (8*(1 - 1
/(exp(-x)/2 - exp(x)/2))^(1/2))/(35*(exp(4*x) - 2*exp(2*x) + 1)) - (86*(1
- 1/(exp(-x)/2 - exp(x)/2))^(1/2))/105 - (16*exp(x)*(1 - 1/(exp(-x)/2 - ex
p(x)/2))^(1/2))/(7*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (16*exp(x)*
(1 - 1/(exp(-x)/2 - exp(x)/2))^(1/2))/(7*(exp(4*x) - 2*exp(2*x) + 1)) - (1
24*exp(x)*(1 - 1/(exp(-x)/2 - exp(x)/2))^(1/2))/(105*(exp(2*x) - 1))
```

3.1048. $\int \coth^3(x)\text{csch}(x)\sqrt{1+\text{csch}(x)}\,dx$

3.1049 $\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx$

3.1049.1 Optimal result

Integrand size = 13, antiderivative size = 4

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \cosh^x(x)$$

output
```
cosh(x)^x
```

3.1049.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \cosh^x(x)$$

input
```
Integrate[Cosh[x]^x*(Log[Cosh[x]] + x*Tanh[x]),x]
```

output
```
Cosh[x]^x
```

3.1049.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^x(x)(x\tanh(x)+\log(\cosh(x)))\,dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(x\sinh(x)\cosh^{x-1}(x)+\cosh^x(x)\log(\cosh(x))\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\cosh^x(x)$$

input
```
Int[Cosh[x]^x*(Log[Cosh[x]] + x*Tanh[x]),x]
```

output
```
Cosh[x]^x
```

3.1049.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.1049.4 Maple [A] (verified)

Time = 12.68 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\cosh\left(x\right)^{x}$	5
default	$\cosh\left(x\right)^{x}$	5

input
```
int(cosh(x)^x*(ln(cosh(x))+x*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
cosh(x)^x
```

3.1049.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(4) = 8$.

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \cosh\left(x\log\left(\cosh\left(x\right)\right)\right) + \sinh\left(x\log\left(\cosh\left(x\right)\right)\right)$$

input
```
integrate(cosh(x)^x*(log(cosh(x))+x*tanh(x)),x, algorithm="fricas")
```

output
```
cosh(x*log(cosh(x))) + sinh(x*log(cosh(x)))
```

3.1049.6 Sympy [F]

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \int \left(x\tanh\left(x\right) + \log\left(\cosh\left(x\right)\right)\right)\cosh^{x}\left(x\right)\,dx$$

input
```
integrate(cosh(x)**x*(ln(cosh(x))+x*tanh(x)),x)
```

output
```
Integral((x*tanh(x) + log(cosh(x)))*cosh(x)**x, x)
```

3.1049.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(4) = 8$.

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 5.25

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = e^{\left(-x^2 - x\log(2) + x\log\left(e^{(2x)} + 1\right)\right)}$$

input
```
integrate(cosh(x)^x*(log(cosh(x))+x*tanh(x)),x, algorithm="maxima")
```

output
```
e^(-x^2 - x*log(2) + x*log(e^(2*x) + 1))
```

3.1049.8 Giac [F]

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \int (x\tanh{(x)} + \log{(\cosh{(x)})})\cosh{(x)}^x\,dx$$

input
```
integrate(cosh(x)^x*(log(cosh(x))+x*tanh(x)),x, algorithm="giac")
```

output
```
integrate((x*tanh(x) + log(cosh(x)))*cosh(x)^x, x)
```

3.1049.9 Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \cosh(x)^x$$

input
```
int(cosh(x)^x*(log(cosh(x)) + x*tanh(x)),x)
```

output
```
cosh(x)^x
```

3.1050 $\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx$

3.1050.1 Optimal result

Integrand size = 23, antiderivative size = 27

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx=\frac{\left(e^{c+dx}\right)^n F^{a+bx}}{dn+b\log(F)}$$

output
```
exp(d*x+c)^n*F^(b*x+a)/(d*n+b*ln(F))
```

3.1050.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx=\frac{F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n}{dn+b\log(F)}$$

input
```
Integrate[F^(a + b*x)*(Cosh[c + d*x] + Sinh[c + d*x])^n,x]
```

output
```
(F^(a + b*x)*(Cosh[c + d*x] + Sinh[c + d*x])^n)/(d*n + b*Log[F])
```

3.1050.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6182, 2717, 2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx}(\sinh(c+dx)+\cosh(c+dx))^n\,dx$$

$$\Big\downarrow \; 6182$$

$$\int F^{a+bx}\left(e^{c+dx}\right)^n dx$$

$$\Big\downarrow \; 2717$$

$$e^{-n(c+dx)}\left(e^{c+dx}\right)^n\int e^{n(c+dx)}F^{a+bx}dx$$

$$\Big\downarrow \; 2725$$

$$e^{-n(c+dx)}\left(e^{c+dx}\right)^n\int e^{cn+a\log(F)+x(dn+b\log(F))}dx$$

$$\Big\downarrow \; 2624$$

$$\frac{F^a\left(e^{c+dx}\right)^n\exp(x(b\log(F)+dn)-n(c+dx)+cn)}{b\log(F)+dn}$$

input
```
Int[F^(a + b*x)*(Cosh[c + d*x] + Sinh[c + d*x])^n,x]
```

output
```
(E^(c*n - n*(c + d*x) + x*(d*n + b*Log[F]))*(E^(c + d*x))^n*F^a)/(d*n + b*
Log[F])
```

3.1050.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2717
```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Simp[(a*F^v)^n/F^(n*v)   Int
[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

rule 6182
```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

3.1050.4 Maple [A] (verified)

Time = 5.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\mathrm{e}^{n(dx+c)} F^{bx+a}}{dn+b\ln(F)}$	27
gosper	$\frac{F^{bx+a}(\cosh(dx+c)+\sinh(dx+c))^{n}}{dn+b\ln(F)}$	34

input
```
int(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
1/(d*n+b*ln(F))*exp(n*(d*x+c))*F^(b*x+a)
```

3.1050.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx$$
$$=\frac{(\cosh(dnx+cn)+\sinh(dnx+cn))\cosh((bx+a)\log(F))+(\cosh(dnx+cn)+\sinh(dnx+cn))\sinh(}{dn+b\log(F)}$$

input
```
integrate(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x, algorithm="fricas")
```

output
```
((cosh(d*n*x + c*n) + sinh(d*n*x + c*n))*cosh((b*x + a)*log(F)) + (cosh(d*
n*x + c*n) + sinh(d*n*x + c*n))*sinh((b*x + a)*log(F)))/(d*n + b*log(F))
```

3.1050.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(22) = 44.

Time = 0.43 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx$$
$$=\begin{cases}\frac{F^{a+bx}(\sinh(c+dx)+\cosh(c+dx))^n}{b\log(F)+dn} & \text{for } b\neq-\frac{dn}{\log(F)}\\ F^{a-\frac{dnx}{\log(F)}}x(\sinh(c+dx)+\cosh(c+dx))^n & \text{otherwise}\end{cases}$$

input
```
integrate(F**(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))**n,x)
```

output
```
Piecewise((F**(a + b*x)*(sinh(c + d*x) + cosh(c + d*x))**n/(b*log(F) + d*n
), Ne(b, -d*n/log(F))), (F**(a - d*n*x/log(F))*x*(sinh(c + d*x) + cosh(c +
 d*x))**n, True))
```

3.1050.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx=\frac{F^ae^{(dnx+bx\log(F)+cn)}}{dn+b\log(F)}$$

input
```
integrate(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x, algorithm="maxima")
```

output
```
F^a*e^(d*n*x + b*x*log(F) + c*n)/(d*n + b*log(F))
```

3.1050.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 10.15

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx$$
$$=2\left(\frac{2\,(dn+b\log(|F|))\cos\left(-\frac{1}{2}\pi bx\text{sgn}(F)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\text{sgn}(F)+\frac{1}{2}\pi a\right)}{(\pi b\text{sgn}(F)-\pi b)^2+4\,(dn+b\log(|F|))^2}-\frac{(\pi b\text{sgn}(F)-\pi b)\sin\left(-\frac{1}{2}\pi bx\right.}{(\pi b\text{sgn}(F)-\pi\right.$$
$$+i\left(\frac{i\,e^{\left(\frac{1}{2}i\pi bx\text{sgn}(F)-\frac{1}{2}i\pi bx+\frac{1}{2}i\pi a\text{sgn}(F)-\frac{1}{2}i\pi a\right)}}{i\pi b\text{sgn}(F)-i\pi b+2\,dn+2\,b\log(|F|)}-\frac{i\,e^{\left(-\frac{1}{2}i\pi bx\text{sgn}(F)+\frac{1}{2}i\pi bx-\frac{1}{2}i\pi a\text{sgn}(F)+\frac{1}{2}i\pi a\right)}}{-i\pi b\text{sgn}(F)+i\pi b+2\,dn+2\,b\log(|F|)}\right)e^{(cn+(dn+b\log(|I}$$

input
```
integrate(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x, algorithm="giac")
```

output
```
2*(2*(d*n + b*log(abs(F)))*cos(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*
sgn(F) + 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n + b*log(abs(F)))^2) -
(pi*b*sgn(F) - pi*b)*sin(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*sgn(F)
 + 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n + b*log(abs(F)))^2))*e^(c*n
+ (d*n + b*log(abs(F)))*x + a*log(abs(F))) + I*(I*e^(1/2*I*pi*b*x*sgn(F) -
 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a)/(I*pi*b*sgn(F) - I*pi*b +
2*d*n + 2*b*log(abs(F))) - I*e^(-1/2*I*pi*b*x*sgn(F) + 1/2*I*pi*b*x - 1/2*
I*pi*a*sgn(F) + 1/2*I*pi*a)/(-I*pi*b*sgn(F) + I*pi*b + 2*d*n + 2*b*log(abs
(F))))*e^(c*n + (d*n + b*log(abs(F)))*x + a*log(abs(F)))
```

3.1050.9 Mupad [F(-1)]

Timed out.

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx=\int F^{a+b\,x}\,(\cosh(c+d\,x)+\sinh(c+d\,x))^n\,dx$$

input
```
int(F^(a + b*x)*(cosh(c + d*x) + sinh(c + d*x))^n,x)
```

output
```
int(F^(a + b*x)*(cosh(c + d*x) + sinh(c + d*x))^n, x)
```

3.1051 $\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx$

3.1051.1 Optimal result

Integrand size = 25, antiderivative size = 32

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx=-\frac{\left(e^{-c-dx}\right)^n F^{a+bx}}{dn-b\log(F)}$$

output
```
-exp(-d*x-c)^n*F^(b*x+a)/(d*n-b*ln(F))
```

3.1051.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx=-\frac{F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n}{dn-b\log(F)}$$

input
```
Integrate[F^(a + b*x)*(Cosh[c + d*x] - Sinh[c + d*x])^n,x]
```

output
```
-((F^(a + b*x)*(Cosh[c + d*x] - Sinh[c + d*x])^n)/(d*n - b*Log[F]))
```

3.1051.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6182, 2717, 2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx$$

$$\Big\downarrow \ 6182$$

$$\int F^{a+bx}\left(e^{-c-dx}\right)^n dx$$

$$\Big\downarrow \ 2717$$

$$e^{n(c+dx)}\left(e^{-c-dx}\right)^n\int e^{-n(c+dx)}F^{a+bx}dx$$

$$\Big\downarrow \ 2725$$

$$e^{n(c+dx)}\left(e^{-c-dx}\right)^n\int e^{-cn+a\log(F)-x(dn-b\log(F))}dx$$

$$\Big\downarrow \ 2624$$

$$-\frac{F^a\left(e^{-c-dx}\right)^n\exp(-x(dn-b\log(F))+n(c+dx)-cn)}{dn-b\log(F)}$$

input
```
Int[F^(a + b*x)*(Cosh[c + d*x] - Sinh[c + d*x])^n,x]
```

output
```
-((E^(-(c*n) + n*(c + d*x) - x*(d*n - b*Log[F]))*(E^(-c - d*x))^n*F^a)/(d*
n - b*Log[F]))
```

3.1051.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2717
```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Simp[(a*F^v)^n/F^(n*v)   Int
[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

rule 6182
```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

3.1051.4 Maple [A] (verified)

Time = 5.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{F^{bx+a}(\cosh(dx+c)-\sinh(dx+c))^{n}}{b\ln(F)-dn}$	37

input
```
int(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
1/(b*ln(F)-d*n)*F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n
```

3.1051.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(31) = 62.

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.38

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx =$$
$$-\frac{(\cosh(dnx+cn)-\sinh(dnx+cn))\cosh((bx+a)\log(F))+(\cosh(dnx+cn)-\sinh(dnx+cn))\,\mathrm{sin}}{dn-b\log(F)}$$

input
```
integrate(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x, algorithm="fricas")
```

output
```
-((cosh(d*n*x + c*n) - sinh(d*n*x + c*n))*cosh((b*x + a)*log(F)) + (cosh(d
*n*x + c*n) - sinh(d*n*x + c*n))*sinh((b*x + a)*log(F)))/(d*n - b*log(F))
```

3.1051.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx = \begin{cases} \frac{F^{a+bx}(-\sinh{(c+dx)}+\cosh{(c+dx)})^n}{b\log{(F)}-dn} & \text{for } b\neq\frac{dn}{\log{(F)}} \\ F^{a+\frac{dnx}{\log{(F)}}}x(-\sinh{(c+dx)}+\cosh{(c+dx)})^n & \text{otherwise} \end{cases}$$

input
```
integrate(F**(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))**n,x)
```

output
```
Piecewise((F**(a + b*x)*(-sinh(c + d*x) + cosh(c + d*x))**n/(b*log(F) - d*
n), Ne(b, d*n/log(F))), (F**(a + d*n*x/log(F))*x*(-sinh(c + d*x) + cosh(c
+ d*x))**n, True))
```

3.1051.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx = -\frac{F^a e^{(-dnx+bx\log(F))}}{dne^{(cn)}-be^{(cn)}\log{(F)}}$$

input
```
integrate(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x, algorithm="maxima")
```

output
```
-F^a*e^(-d*n*x + b*x*log(F))/(d*n*e^(c*n) - b*e^(c*n)*log(F))
```

3.1051.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 282, normalized size of antiderivative = 8.81

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx =$$

$$-2\left(\frac{2\,(dn-b\log{(|F|)})\cos{\left(-\frac{1}{2}\,\pi bx\mathrm{sgn}(F)+\frac{1}{2}\,\pi bx-\frac{1}{2}\,\pi a\mathrm{sgn}(F)+\frac{1}{2}\,\pi a\right)}}{(\pi b\mathrm{sgn}\,(F)-\pi b)^2+4\,(dn-b\log{(|F|)})^2}+\frac{(\pi b\mathrm{sgn}(F)-\pi b)\sin{\left(-\frac{1}{2}\,\pi\right.}}{(\pi b\mathrm{sgn}\,(F)-}\right.$$

$$+i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi bx\mathrm{sgn}(F)-\frac{1}{2}i\,\pi bx+\frac{1}{2}i\,\pi a\mathrm{sgn}(F)-\frac{1}{2}i\,\pi a\right)}}{i\,\pi b\mathrm{sgn}\,(F)-i\,\pi b-2\,dn+2\,b\log{(|F|)}}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bx\mathrm{sgn}(F)+\frac{1}{2}i\,\pi bx-\frac{1}{2}i\,\pi a\mathrm{sgn}(F)+\frac{1}{2}i\,\pi a\right)}}{-i\,\pi b\mathrm{sgn}\,(F)+i\,\pi b-2\,dn+2\,b\log{(|F|)}}\right)e^{(-cn-(dn-b\log}$$

input
```
integrate(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x, algorithm="giac")
```

output
```
-2*(2*(d*n - b*log(abs(F)))*cos(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a
*sgn(F) + 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n - b*log(abs(F)))^2) +
 (pi*b*sgn(F) - pi*b)*sin(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*sgn(F
) + 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n - b*log(abs(F)))^2))*e^(-c*
n - (d*n - b*log(abs(F)))*x + a*log(abs(F))) + I*(I*e^(1/2*I*pi*b*x*sgn(F)
 - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a)/(I*pi*b*sgn(F) - I*pi*b
- 2*d*n + 2*b*log(abs(F))) - I*e^(-1/2*I*pi*b*x*sgn(F) + 1/2*I*pi*b*x - 1/
2*I*pi*a*sgn(F) + 1/2*I*pi*a)/(-I*pi*b*sgn(F) + I*pi*b - 2*d*n + 2*b*log(a
bs(F))))*e^(-c*n - (d*n - b*log(abs(F)))*x + a*log(abs(F)))
```

3.1051.9 Mupad [F(-1)]

Timed out.

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx=\int F^{a+b\,x}\,(\cosh(c+d\,x)-\sinh(c+d\,x))^n\,dx$$

input
```
int(F^(a + b*x)*(cosh(c + d*x) - sinh(c + d*x))^n,x)
```

output
```
int(F^(a + b*x)*(cosh(c + d*x) - sinh(c + d*x))^n, x)
```

3.1052 $\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx$

3.1052.1 Optimal result

Integrand size = 39, antiderivative size = 51

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = -\frac{\arctan\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b} + \frac{\arctan\left(1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}$$

output
```
1/2*arctan(-1+2^(1/2)*tanh(b*x+a))/b*2^(1/2)+1/2*arctan(1+2^(1/2)*tanh(b*x
+a))/b*2^(1/2)
```

3.1052.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.49

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = \frac{\arctan\left(\frac{\sinh(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

input
```
Integrate[(Cosh[a + b*x]^4 - Sinh[a + b*x]^4)/(Cosh[a + b*x]^4 + Sinh[a +
b*x]^4),x]
```

output
```
ArcTan[Sinh[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b)
```

3.1052.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3042, 4889, 1476, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\sinh^4(a+bx)+\cosh^4(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ia+ibx)^4-\sin(ia+ibx)^4}{\sin(ia+ibx)^4+\cos(ia+ibx)^4}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int \frac{\tanh^2(a+bx)+1}{\tanh^4(a+bx)+1}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{1}{2}\int \frac{1}{\tanh^2(a+bx)-\sqrt{2}\tanh(a+bx)+1}d\tanh(a+bx)+\frac{1}{2}\int \frac{1}{\tanh^2(a+bx)+\sqrt{2}\tanh(a+bx)+1}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\int \frac{1}{-\left(1-\sqrt{2}\tanh(a+bx)\right)^2-1}d\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}\tanh(a+bx)+1\right)^2-1}d\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}}}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\arctan\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}}}{b}$$

input
```
Int[(Cosh[a + b*x]^4 - Sinh[a + b*x]^4)/(Cosh[a + b*x]^4 + Sinh[a + b*x]^4
),x]
```

output
```
(-(ArcTan[1 - Sqrt[2]*Tanh[a + b*x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Tanh[a
+ b*x]]/Sqrt[2])/b
```

3.1052.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.1052. $\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx$

3.1052.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

method	result
risch	$\frac{i\sqrt{2}\,\ln\left(e^{4bx+4a}+2i\sqrt{2}\,e^{2bx+2a}-1\right)}{4b}-\frac{i\sqrt{2}\,\ln\left(e^{4bx+4a}-2i\sqrt{2}\,e^{2bx+2a}-1\right)}{4b}$
derivativedivides	$\frac{\frac{i\sqrt{2}\,\ln\left(-2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}-2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)-2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\right.\right.}{}}{b}$
default	$\frac{\frac{i\sqrt{2}\,\ln\left(-2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}-2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)-2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\right.\right.}{}}{b}$

input
```
int((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x,method=_
RETURNVERBOSE)
```

output
```
1/4*I*2^(1/2)/b*ln(exp(4*b*x+4*a)+2*I*2^(1/2)*exp(2*b*x+2*a)-1)-1/4*I*2^(1
/2)/b*ln(exp(4*b*x+4*a)-2*I*2^(1/2)*exp(2*b*x+2*a)-1)
```

3.1052.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(43) = 86.

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.76

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx =$$
$$-\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(bx+a)^3+3\sqrt{2}\cosh(bx+a)\sinh(bx+a)^2+\sqrt{2}\sinh(bx+a)^3+\left(3\sqrt{2}\cosh(bx+a)^2-7\sqrt{2}\right)\sinh(bx+a)+7\sqrt{2}\cosh(bx}{4\left(\cosh(bx+a)^3-3\cosh(bx+a)^2\sinh(bx+a)+3\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3\right)}\right.}{2b}$$

input
```
integrate((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x, a
lgorithm="fricas")
```

output
```
-1/2*(sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x +
a)*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3 + (3*sqrt(2)*cosh(b*x + a)^2
- 7*sqrt(2))*sinh(b*x + a) + 7*sqrt(2)*cosh(b*x + a))/(cosh(b*x + a)^3 - 3
*cosh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*
x + a)^3)) + sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x
 + a))/(cosh(b*x + a) - sinh(b*x + a))))/b
```

3.1052.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.90

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = \begin{cases} -x & \text{for } a=\frac{i\pi}{2}\wedge b=0 \\ \frac{x(-\sinh^4(a)+\cosh^4(a))}{\sinh^4(a)+\cosh^4(a)} & \text{for } b=0 \\ -x & \text{for } a=-bx+\frac{i\pi}{2} \\ \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sinh(a+bx)}{\cosh(a+bx)}-1\right)}{2b}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sinh(a+bx)}{\cosh(a+bx)}+1\right)}{2b} & \text{otherwise} \end{cases}$$

input
```
integrate((cosh(b*x+a)**4-sinh(b*x+a)**4)/(cosh(b*x+a)**4+sinh(b*x+a)**4),
x)
```

output
```
Piecewise((-x, Eq(b, 0) & Eq(a, I*pi/2)), (x*(-sinh(a)**4 + cosh(a)**4)/(s
inh(a)**4 + cosh(a)**4), Eq(b, 0)), (-x, Eq(a, -b*x + I*pi/2)), (sqrt(2)*a
tan(sqrt(2)*sinh(a + b*x)/cosh(a + b*x) - 1)/(2*b) + sqrt(2)*atan(sqrt(2)*
sinh(a + b*x)/cosh(a + b*x) + 1)/(2*b), True))
```

3.1052.7 Maxima [F]

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = \int \frac{\cosh(bx+a)^4-\sinh(bx+a)^4}{\cosh(bx+a)^4+\sinh(bx+a)^4}\,dx$$

input
```
integrate((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x, a
lgorithm="maxima")
```

output
```
2*integrate((e^(-b*x - a) + e^(-5*b*x - 5*a))*e^(-b*x - a)/(6*e^(-4*b*x -
4*a) + e^(-8*b*x - 8*a) + 1), x) + 2*integrate(e^(6*b*x + 6*a)/(e^(8*b*x +
 8*a) + 6*e^(4*b*x + 4*a) + 1), x) + 2*integrate(e^(-6*b*x - 6*a)/(6*e^(-4
*b*x - 4*a) + e^(-8*b*x - 8*a) + 1), x)
```

3.1052.8 Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = \frac{\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(e^{(4\,bx+4\,a)}-1\right)e^{(-2\,bx-2\,a)}\right)}{2\,b}$$

input
```
integrate((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x, a
lgorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/4*sqrt(2)*(e^(4*b*x + 4*a) - 1)*e^(-2*b*x - 2*a))/b
```

3.1052.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = \frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{\sqrt{2}\,e^{2\,a}\,e^{2\,b\,x}\,\sqrt{b^2}}{4\,b}\right)+\mathrm{atan}\left(\frac{\sqrt{b^2}\left(\frac{56\,\sqrt{2}\,e^{2\,a}\,e^{2\,b\,x}}{b}+\frac{8\,\sqrt{2}\,e^{6\,a}\,e^{6\,b\,x}}{b}\right)}{32}\right)\right)}{2\,\sqrt{b^2}}$$

input
```
int((cosh(a + b*x)^4 - sinh(a + b*x)^4)/(cosh(a + b*x)^4 + sinh(a + b*x)^4
),x)
```

output
```
(2^(1/2)*(atan((2^(1/2)*exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/(4*b)) + atan(((b
^2)^(1/2)*((56*2^(1/2)*exp(2*a)*exp(2*b*x))/b + (8*2^(1/2)*exp(6*a)*exp(6*
b*x))/b))/32)))/(2*(b^2)^(1/2))
```

3.1053 $\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx$

3.1053.1 Optimal result

Integrand size = 39, antiderivative size = 47

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx=-\frac{4\arctan\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}-\frac{1}{3b(1+\tanh(a+bx))}$$

output
```
-4/9*arctan(1/3*(1-2*tanh(b*x+a))*3^(1/2))/b*3^(1/2)-1/3/b/(1+tanh(b*x+a))
```

3.1053.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(47) = 94.

Time = 6.94 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.45

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx$$
$$=\frac{(-\cosh(a+bx)+\sinh(a+bx))\left(\left(3+8\sqrt{3}\arctan\left(\frac{\operatorname{sech}(bx)(\cosh(2a+bx)-2\sinh(2a+bx))}{\sqrt{3}}\right)\right)\cosh(a+bx)+\left(-\right.\right.}{18b}$$

input
```
Integrate[(Cosh[a + b*x]^3 - Sinh[a + b*x]^3)/(Cosh[a + b*x]^3 + Sinh[a +
b*x]^3),x]
```

output
```
((-Cosh[a + b*x] + Sinh[a + b*x])*((3 + 8*Sqrt[3]*ArcTan[(Sech[b*x]*(Cosh[
2*a + b*x] - 2*Sinh[2*a + b*x]))/Sqrt[3]])*Cosh[a + b*x] + (-3 + 8*Sqrt[3]
*ArcTan[(Sech[b*x]*(Cosh[2*a + b*x] - 2*Sinh[2*a + b*x]))/Sqrt[3]])*Sinh[a
 + b*x]))/(18*b)
```

3.1053.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3042, 4889, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\sinh^3(a+bx)+\cosh^3(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ia+ibx)^3-i\sin(ia+ibx)^3}{i\sin(ia+ibx)^3+\cos(ia+ibx)^3}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int \frac{\tanh^2(a+bx)+\tanh(a+bx)+1}{\tanh^4(a+bx)+\tanh^3(a+bx)+\tanh(a+bx)+1}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{2462}$$

$$\frac{\int \left(\frac{1}{3(\tanh(a+bx)+1)^2}+\frac{2}{3(\tanh^2(a+bx)-\tanh(a+bx)+1)}\right)d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{4\arctan\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3(\tanh(a+bx)+1)}}{b}$$

input
```
Int[(Cosh[a + b*x]^3 - Sinh[a + b*x]^3)/(Cosh[a + b*x]^3 + Sinh[a + b*x]^3
),x]
```

output
```
((-4*ArcTan[(1 - 2*Tanh[a + b*x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(3*(1 + Tanh[a
 + b*x])))/b
```

3.1053. $\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}dx$

3.1053.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.1053.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{{\mathrm e}^{-2 b x -2 a}}{6 b}+\frac{2 i \sqrt{3}\, \ln \left({\mathrm e}^{2 b x +2 a}+i \sqrt{3}\right)}{9 b}-\frac{2 i \sqrt{3}\, \ln \left({\mathrm e}^{2 b x +2 a}-i \sqrt{3}\right)}{9 b}$
derivativedivides	$\frac{\frac{2 i \sqrt{3}\, \ln \left(\tanh \left(\frac{a}{2}+\frac{b x}{2}\right)^{2}+\left(-i \sqrt{3}-1\right) \tanh \left(\frac{a}{2}+\frac{b x}{2}\right)+1\right)}{9}-\frac{2 i \sqrt{3}\, \ln \left(\tanh \left(\frac{a}{2}+\frac{b x}{2}\right)^{2}+\left(i \sqrt{3}-1\right) \tanh \left(\frac{a}{2}+\frac{b x}{2}\right)+1\right)}{9}-\frac{2}{3 \left(\tanh \left(\frac{a}{2}+\frac{b x}{2}\right)\right.}}{b}$
default	$\frac{\frac{2 i \sqrt{3}\, \ln \left(\tanh \left(\frac{a}{2}+\frac{b x}{2}\right)^{2}+\left(-i \sqrt{3}-1\right) \tanh \left(\frac{a}{2}+\frac{b x}{2}\right)+1\right)}{9}-\frac{2 i \sqrt{3}\, \ln \left(\tanh \left(\frac{a}{2}+\frac{b x}{2}\right)^{2}+\left(i \sqrt{3}-1\right) \tanh \left(\frac{a}{2}+\frac{b x}{2}\right)+1\right)}{9}-\frac{2}{3 \left(\tanh \left(\frac{a}{2}+\frac{b x}{2}\right)\right.}}{b}$

input
```
int((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x,method=_
RETURNVERBOSE)
```

output
```
-1/6*exp(-2*b*x-2*a)/b+2/9*I*3^(1/2)/b*ln(exp(2*b*x+2*a)+I*3^(1/2))-2/9*I*
3^(1/2)/b*ln(exp(2*b*x+2*a)-I*3^(1/2))
```

3.1053.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(40) = 80$.

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.70

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx = -\frac{8\left(\sqrt{3}\cosh\left(bx+a\right)^2+2\sqrt{3}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{3}\sinh\left(bx+a\right)^2\right)\arctan\left(-\frac{\sqrt{3}\cosh(bx+a)+\sqrt{3}}{3\,(\cosh(bx+a)-\sin}\right.}{18\left(b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2\right)}$$

input
```
integrate((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x, a
lgorithm="fricas")
```

output
```
-1/18*(8*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a)
+ sqrt(3)*sinh(b*x + a)^2)*arctan(-1/3*(sqrt(3)*cosh(b*x + a) + sqrt(3)*si
nh(b*x + a))/(cosh(b*x + a) - sinh(b*x + a))) + 3)/(b*cosh(b*x + a)^2 + 2*
b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.1053.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.30

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx = \begin{cases} -x & \text{for } (a \\ \frac{x(-\sinh^3(a)+\cosh^3(a))}{\sinh^3(a)+\cosh^3(a)} & \text{for } b = \\ \frac{4\sqrt{3}\sinh(a+bx)\,\mathrm{atan}\left(\frac{2\sqrt{3}\sinh(a+bx)}{3\cosh(a+bx)}-\frac{\sqrt{3}}{3}\right)}{9b\sinh(a+bx)+9b\cosh(a+bx)} + \frac{3\sinh(a+bx)}{9b\sinh(a+bx)+9b\cosh(a+bx)} + \frac{4\sqrt{3}\cosh(a+bx)\,\mathrm{atan}\left(\frac{2\sqrt{3}\sinh(a+bx)}{3\cosh(a+bx)}-\frac{\sqrt{3}}{3}\right)}{9b\sinh(a+bx)+9b\cosh(a+bx)} & \text{otherw} \end{cases}$$

input
```
integrate((cosh(b*x+a)**3-sinh(b*x+a)**3)/(cosh(b*x+a)**3+sinh(b*x+a)**3),
x)
```

output
```
Piecewise((-x, (Eq(b, 0) | Eq(a, -b*x + I*pi/2)) & (Eq(a, I*pi/2) | Eq(a,
-b*x + I*pi/2))), (x*(-sinh(a)**3 + cosh(a)**3)/(sinh(a)**3 + cosh(a)**3),
 Eq(b, 0)), (4*sqrt(3)*sinh(a + b*x)*atan(2*sqrt(3)*sinh(a + b*x)/(3*cosh(
a + b*x)) - sqrt(3)/3)/(9*b*sinh(a + b*x) + 9*b*cosh(a + b*x)) + 3*sinh(a
+ b*x)/(9*b*sinh(a + b*x) + 9*b*cosh(a + b*x)) + 4*sqrt(3)*cosh(a + b*x)*a
tan(2*sqrt(3)*sinh(a + b*x)/(3*cosh(a + b*x)) - sqrt(3)/3)/(9*b*sinh(a + b
*x) + 9*b*cosh(a + b*x)), True))
```

3.1053.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(40) = 80$.

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.98

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx = \frac{4\left(\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)\right)}{9\,b} - \frac{e^{(-2\,bx-2\,a)}}{6\,b}$$

input
```
integrate((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x, a
lgorithm="maxima")
```

output
```
4/9*(sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) + 3^(1/4)*
sqrt(2))) - sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) - 3
^(1/4)*sqrt(2))))/b - 1/6*e^(-2*b*x - 2*a)/b
```

3.1053.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx = \frac{8\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2\,bx+2\,a)}\right)-3\,e^{(-2\,bx-2\,a)}}{18\,b}$$

input
```
integrate((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x, a
lgorithm="giac")
```

output
```
1/18*(8*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*b*x + 2*a)) - 3*e^(-2*b*x - 2*a))/
b
```

3.1053.9 Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx=\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{b^2}}{3\,b}\right)}{9\,\sqrt{b^2}}-\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{6\,b}$$

input
```
int((cosh(a + b*x)^3 - sinh(a + b*x)^3)/(cosh(a + b*x)^3 + sinh(a + b*x)^3
),x)
```

output
```
(4*3^(1/2)*atan((3^(1/2)*exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/(3*b)))/(9*(b^2)
^(1/2)) - exp(- 2*a - 2*b*x)/(6*b)
```

3.1054 $\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx$

3.1054.1 Optimal result

Integrand size = 39, antiderivative size = 11

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx=\frac{\arctan(\tanh(a+bx))}{b}$$

output
```
arctan(tanh(b*x+a))/b
```

3.1054.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx=\frac{\arctan(\sinh(2a+2bx))}{2b}$$

input
```
Integrate[(Cosh[a + b*x]^2 - Sinh[a + b*x]^2)/(Cosh[a + b*x]^2 + Sinh[a + 
b*x]^2),x]
```

output
```
ArcTan[Sinh[2*a + 2*b*x]]/(2*b)
```

3.1054.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3042, 4880, 3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\sinh^2(a+bx)+\cosh^2(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(ia+ibx)^2+\cos(ia+ibx)^2}{\cos(ia+ibx)^2-\sin(ia+ibx)^2}dx$$

$$\Big\downarrow \text{4880}$$

$$\int \frac{1}{\sinh^2(a+bx)+\cosh^2(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ia+ibx)^2-\sin(ia+ibx)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int \frac{1}{\tanh^2(a+bx)+1}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan(\tanh(a+bx))}{b}$$

input
```
Int[(Cosh[a + b*x]^2 - Sinh[a + b*x]^2)/(Cosh[a + b*x]^2 + Sinh[a + b*x]^2
),x]
```

output
```
ArcTan[Tanh[a + b*x]]/b
```

3.1054.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4880
```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.1054.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

method	result	size
parallelrisch	$-\frac{i(\ln(\tanh(bx+a)-i)-\ln(\tanh(bx+a)+i))}{2b}$	30
risch	$\frac{i\ln\left(\mathrm{e}^{2bx+2a}+i\right)}{2b}-\frac{i\ln\left(\mathrm{e}^{2bx+2a}-i\right)}{2b}$	40
derivativedivides	$\frac{-\frac{\left(-2+\sqrt{2}\right)\sqrt{2}\,\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}-\frac{\sqrt{2}\left(2+\sqrt{2}\right)\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}}{b}$	86
default	$\frac{-\frac{\left(-2+\sqrt{2}\right)\sqrt{2}\,\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}-\frac{\sqrt{2}\left(2+\sqrt{2}\right)\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}}{b}$	86

input
```
int((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x,method=_
RETURNVERBOSE)
```

output
```
-1/2*I*(ln(tanh(b*x+a)-I)-ln(tanh(b*x+a)+I))/b
```

3.1054.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(11) = 22$.

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.45

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx = -\frac{\arctan\left(-\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

input
```
integrate((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x, a
lgorithm="fricas")
```

output
```
-arctan(-(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a) - sinh(b*x + a)))/
b
```

3.1054.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 53, normalized size of antiderivative = 4.82

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx = \begin{cases} -x & \text{for } a=\frac{i\pi}{2}\wedge b=0 \\ \frac{x(-\sinh^2(a)+\cosh^2(a))}{\sinh^2(a)+\cosh^2(a)} & \text{for } b=0 \\ -x & \text{for } a=-bx+\frac{i\pi}{2} \\ \frac{\operatorname{atan}\left(\frac{\sinh(a+bx)}{\cosh(a+bx)}\right)}{b} & \text{otherwise} \end{cases}$$

input
```
integrate((cosh(b*x+a)**2-sinh(b*x+a)**2)/(cosh(b*x+a)**2+sinh(b*x+a)**2),
x)
```

output
```
Piecewise((-x, Eq(b, 0) & Eq(a, I*pi/2)), (x*(-sinh(a)**2 + cosh(a)**2)/(s
inh(a)**2 + cosh(a)**2), Eq(b, 0)), (-x, Eq(a, -b*x + I*pi/2)), (atan(sinh
(a + b*x)/cosh(a + b*x))/b, True))
```

3.1054.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(11) = 22$.

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.45

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx = \frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(-bx-a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(-bx-a)}\right)\right)}{b}$$

input
```
integrate((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x, a
lgorithm="maxima")
```

output
```
(arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-b*x - a))) - arctan(-1/2*sqrt(2)*(sqr
t(2) - 2*e^(-b*x - a))))/b
```

3.1054.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 4.00

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx = -\frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(bx+a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(bx+a)}\right)\right)}{b}$$

input
```
integrate((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x, a
lgorithm="giac")
```

output
```
-(arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(b*x + a))) - arctan(-1/2*sqrt(2)*(sqr
t(2) - 2*e^(b*x + a))))/b
```

3.1054.9 Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx=\frac{\operatorname{atan}\left(\frac{e^{2a}\,e^{2bx}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}$$

input
```
int((cosh(a + b*x)^2 - sinh(a + b*x)^2)/(cosh(a + b*x)^2 + sinh(a + b*x)^2
),x)
```

output
```
atan((exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/b)/(b^2)^(1/2)
```

3.1055 $\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx$

3.1055.1 Optimal result

Integrand size = 31, antiderivative size = 22

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{1}{2b(\cosh(a+bx)+\sinh(a+bx))^2}$$

output
```
-1/2/b/(cosh(b*x+a)+sinh(b*x+a))^2
```

3.1055.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(22) = 44.

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{\cosh(2a)\cosh(2bx)}{2b}+\frac{\cosh(2bx)\sinh(2a)}{2b} + \frac{\cosh(2a)\sinh(2bx)}{2b}-\frac{\sinh(2a)\sinh(2bx)}{2b}$$

input
```
Integrate[(Cosh[a + b*x] - Sinh[a + b*x])/(Cosh[a + b*x] + Sinh[a + b*x]),
x]
```

output
```
-1/2*(Cosh[2*a]*Cosh[2*b*x])/b + (Cosh[2*b*x]*Sinh[2*a])/(2*b) + (Cosh[2*a
]*Sinh[2*b*x])/(2*b) - (Sinh[2*a]*Sinh[2*b*x])/(2*b)
```

3.1055.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3042, 4885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\sinh(a+bx)+\cosh(a+bx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sin(ia+ibx)+\cos(ia+ibx)}{\cos(ia+ibx)-i\sin(ia+ibx)}dx$$
$$\Big\downarrow \text{4885}$$
$$-\frac{1}{2b(\sinh(a+bx)+\cosh(a+bx))^2}$$

input
```
Int[(Cosh[a + b*x] - Sinh[a + b*x])/(Cosh[a + b*x] + Sinh[a + b*x]),x]
```

output
```
-1/2*1/(b*(Cosh[a + b*x] + Sinh[a + b*x])^2)
```

3.1055.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4885
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

3.1055.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{e^{-2bx-2a}}{2b}$	15
parallelrisch	$\frac{\tanh(bx+a)}{b(1+\tanh(bx+a))}$	21
gosper	$\frac{\sinh(bx+a)-\cosh(bx+a)}{2b(\cosh(bx+a)+\sinh(bx+a))}$	36
derivativedivides	$\frac{\frac{2}{\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1}-\frac{2}{\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)^2}}{b}$	36
default	$\frac{\frac{2}{\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1}-\frac{2}{\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)^2}}{b}$	36

input
```
int((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x,method=_RETURNVE
RBOSE)
```

output
```
-1/2*exp(-2*b*x-2*a)/b
```

3.1055.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx$$
$$= -\frac{1}{2\left(b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2\right)}$$

input
```
integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x, algorithm
="fricas")
```

output
```
-1/2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a
)^2)
```

3.1055.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = \begin{cases} \frac{\sinh(a+bx)}{b\sinh(a+bx)+b\cosh(a+bx)} & \text{for } b\neq 0 \\ \frac{x(-\sinh(a)+\cosh(a))}{\sinh(a)+\cosh(a)} & \text{otherwise} \end{cases}$$

input
```
integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x)
```

output
```
Piecewise((sinh(a + b*x)/(b*sinh(a + b*x) + b*cosh(a + b*x)), Ne(b, 0)), (
x*(-sinh(a) + cosh(a))/(sinh(a) + cosh(a)), True))
```

3.1055.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{e^{(-2\,bx-2\,a)}}{2\,b}$$

input
```
integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x, algorithm
="maxima")
```

output
```
-1/2*e^(-2*b*x - 2*a)/b
```

3.1055.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{e^{(-2\,bx-2\,a)}}{2\,b}$$

input
```
integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x, algorithm
="giac")
```

output
```
-1/2*e^(-2*b*x - 2*a)/b
```

3.1055.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{e^{-2\,a-2\,b\,x}}{2\,b}$$

input
```
int((cosh(a + b*x) - sinh(a + b*x))/(cosh(a + b*x) + sinh(a + b*x)),x)
```

output
```
-exp(- 2*a - 2*b*x)/(2*b)
```

3.1056 $\int \frac{-\text{csch}(a+bx)+\text{sech}(a+bx)}{\text{csch}(a+bx)+\text{sech}(a+bx)} dx$

3.1056.1 Optimal result

Integrand size = 31, antiderivative size = 14

$$\int \frac{-\text{csch}(a+bx)+\text{sech}(a+bx)}{\text{csch}(a+bx)+\text{sech}(a+bx)} dx = \frac{1}{b(1+\tanh(a+bx))}$$

output
```
1/b/(1+tanh(b*x+a))
```

3.1056.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.64

$$\int \frac{-\text{csch}(a+bx)+\text{sech}(a+bx)}{\text{csch}(a+bx)+\text{sech}(a+bx)} dx = \frac{\cosh(2a)\cosh(2bx)}{2b} - \frac{\cosh(2bx)\sinh(2a)}{2b} - \frac{\cosh(2a)\sinh(2bx)}{2b} + \frac{\sinh(2a)\sinh(2bx)}{2b}$$

input
```
Integrate[(-Csch[a + b*x] + Sech[a + b*x])/(Csch[a + b*x] + Sech[a + b*x])
,x]
```

output
```
(Cosh[2*a]*Cosh[2*b*x])/(2*b) - (Cosh[2*b*x]*Sinh[2*a])/(2*b) - (Cosh[2*a]
*Sinh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b)
```

3.1056.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3042, 4889, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(a+bx)-\operatorname{csch}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(ia+ibx)-i\csc(ia+ibx)}{i\csc(ia+ibx)+\sec(ia+ibx)}dx$$

$$\Big\downarrow \; 4889$$

$$\frac{\int -\frac{1}{(\tanh(a+bx)+1)^2}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \; 17$$

$$\frac{1}{b(\tanh(a+bx)+1)}$$

input
```
Int[(-Csch[a + b*x] + Sech[a + b*x])/(Csch[a + b*x] + Sech[a + b*x]),x]
```

output
```
1/(b*(1 + Tanh[a + b*x]))
```

3.1056.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.1056.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{{\mathrm e}^{-2 b x -2 a}}{2 b}$	15
parallelrisch	$-\frac{\tanh \left(b x +a \right)}{b \left(1+\tanh \left(b x +a \right)\right)}$	22
derivativedivides	$\frac{\frac{2}{\left(\tanh \left(\frac{a}{2}+\frac{b x}{2}\right)+1\right)^{2}}-\frac{2}{\tanh \left(\frac{a}{2}+\frac{b x}{2}\right)+1}}{b}$	36
default	$\frac{\frac{2}{\left(\tanh \left(\frac{a}{2}+\frac{b x}{2}\right)+1\right)^{2}}-\frac{2}{\tanh \left(\frac{a}{2}+\frac{b x}{2}\right)+1}}{b}$	36

input
```
int((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x,method=_RETURNV
ERBOSE)
```

output
```
1/2*exp(-2*b*x-2*a)/b
```

3.1056.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.86

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx = \frac{1}{2\left(b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

input
```
integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x, algorith
m="fricas")
```

output
```
1/2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)
^2)
```

3.1056.6 Sympy [F]

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx = -\int \frac{\operatorname{csch}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx - \int \left(-\frac{\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\right)dx$$

input
```
integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x)
```

output
```
-Integral(csch(a + b*x)/(csch(a + b*x) + sech(a + b*x)), x) - Integral(-se
ch(a + b*x)/(csch(a + b*x) + sech(a + b*x)), x)
```

3.1056.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx = \frac{e^{(-2\,bx-2\,a)}}{2\,b}$$

input
```
integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x, algorith
m="maxima")
```

output
```
1/2*e^(-2*b*x - 2*a)/b
```

3.1056.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx = \frac{e^{(-2\,bx-2\,a)}}{2\,b}$$

input
```
integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x, algorith
m="giac")
```

output
```
1/2*e^(-2*b*x - 2*a)/b
```

3.1056.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx=\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{2\,b}$$

input
```
int((1/cosh(a + b*x) - 1/sinh(a + b*x))/(1/cosh(a + b*x) + 1/sinh(a + b*x)
),x)
```

output
```
exp(- 2*a - 2*b*x)/(2*b)
```

3.1057 $\int \frac{-\text{csch}^2(a+bx)+\text{sech}^2(a+bx)}{\text{csch}^2(a+bx)+\text{sech}^2(a+bx)} \, dx$

3.1057.1 Optimal result

Integrand size = 39, antiderivative size = 12

$$\int \frac{-\text{csch}^2(a+bx)+\text{sech}^2(a+bx)}{\text{csch}^2(a+bx)+\text{sech}^2(a+bx)} \, dx = -\frac{\arctan(\tanh(a+bx))}{b}$$

output
```
-arctan(tanh(b*x+a))/b
```

3.1057.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{-\text{csch}^2(a+bx)+\text{sech}^2(a+bx)}{\text{csch}^2(a+bx)+\text{sech}^2(a+bx)} \, dx = -\frac{\arctan(\sinh(2a+2bx))}{2b}$$

input
```
Integrate[(-Csch[a + b*x]^2 + Sech[a + b*x]^2)/(Csch[a + b*x]^2 + Sech[a + 
 b*x]^2),x]
```

output
```
-1/2*ArcTan[Sinh[2*a + 2*b*x]]/b
```

3.1057.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 4889, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(a+bx)-\text{csch}^2(a+bx)}{\text{csch}^2(a+bx)+\text{sech}^2(a+bx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc(ia+ibx)^2+\sec(ia+ibx)^2}{\sec(ia+ibx)^2-\csc(ia+ibx)^2}dx$$

$$\Big\downarrow \ 4889$$

$$\frac{\int \frac{1}{-\tanh^2(a+bx)-1}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \ 217$$

$$-\frac{\arctan(\tanh(a+bx))}{b}$$

input
```
Int[(-Csch[a + b*x]^2 + Sech[a + b*x]^2)/(Csch[a + b*x]^2 + Sech[a + b*x]^
2),x]
```

output
```
-(ArcTan[Tanh[a + b*x]]/b)
```

3.1057.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.1057.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

method	result	size
parallelrisch	$\frac{i(\ln(\tanh(bx+a)-i)-\ln(\tanh(bx+a)+i))}{2b}$	30
risch	$\frac{i\ln\left(e^{2bx+2a}-i\right)}{2b}-\frac{i\ln\left(e^{2bx+2a}+i\right)}{2b}$	40
derivativedivides	$\frac{\frac{\sqrt{2}\left(2+\sqrt{2}\right)\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}+\frac{\left(-2+\sqrt{2}\right)\sqrt{2}\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}}{b}$	84
default	$\frac{\frac{\sqrt{2}\left(2+\sqrt{2}\right)\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}+\frac{\left(-2+\sqrt{2}\right)\sqrt{2}\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}}{b}$	84

input
```
int((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x,method=
_RETURNVERBOSE)
```

output
```
1/2*I*(ln(tanh(b*x+a)-I)-ln(tanh(b*x+a)+I))/b
```

3.1057.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.08

$$\int \frac{-\text{csch}^2(a+bx)+\text{sech}^2(a+bx)}{\text{csch}^2(a+bx)+\text{sech}^2(a+bx)}\,dx=\frac{\arctan\left(-\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

input
```
integrate((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x,
algorithm="fricas")
```

output
```
arctan(-(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a) - sinh(b*x + a)))/b
```

3.1057.6 Sympy [F]

$$\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx = -\int \frac{\operatorname{csch}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx - \int \left(-\frac{\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\right)dx$$

input
```
integrate((-csch(b*x+a)**2+sech(b*x+a)**2)/(csch(b*x+a)**2+sech(b*x+a)**2)
,x)
```

output
```
-Integral(csch(a + b*x)**2/(csch(a + b*x)**2 + sech(a + b*x)**2), x) - Int
egral(-sech(a + b*x)**2/(csch(a + b*x)**2 + sech(a + b*x)**2), x)
```

3.1057.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(12) = 24$.

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 4.17

$$\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx = -\frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-bx-a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-bx-a)}\right)\right)}{b}$$

input
```
integrate((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x,
algorithm="maxima")
```

output
```
-(arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-b*x - a))) - arctan(-1/2*sqrt(2)*(sq
rt(2) - 2*e^(-b*x - a))))/b
```

3.1057.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

$$\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx = \frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(bx+a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(bx+a)}\right)\right)}{b}$$

input
```
integrate((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x,
algorithm="giac")
```

output
```
(arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(b*x + a))) - arctan(-1/2*sqrt(2)*(sqrt
(2) - 2*e^(b*x + a))))/b
```

3.1057.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx = -\frac{\operatorname{atan}\left(\frac{e^{2a}\,e^{2bx}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}$$

input
```
int((1/cosh(a + b*x)^2 - 1/sinh(a + b*x)^2)/(1/cosh(a + b*x)^2 + 1/sinh(a
+ b*x)^2),x)
```

output
```
-atan((exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/b)/(b^2)^(1/2)
```

3.1058 $\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx$

3.1058.1 Optimal result

Integrand size = 39, antiderivative size = 47

$$\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx=\frac{4\arctan\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}+\frac{1}{3b(1+\tanh(a+bx))}$$

output
```
4/9*arctan(1/3*(1-2*tanh(b*x+a))*3^(1/2))/b*3^(1/2)+1/3/b/(1+tanh(b*x+a))
```

3.1058.2 Mathematica [A] (verified)

Time = 6.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx = \frac{-8\sqrt{3}\arctan\left(\frac{-1+2\tanh(a+bx)}{\sqrt{3}}\right)+3\cosh(2(a+bx))-3\sinh(2(a+bx))}{18b}$$

input
```
Integrate[(-Csch[a + b*x]^3 + Sech[a + b*x]^3)/(Csch[a + b*x]^3 + Sech[a +
 b*x]^3),x]
```

output
```
(-8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[a + b*x])/Sqrt[3]] + 3*Cosh[2*(a + b*x)] -
 3*Sinh[2*(a + b*x)])/(18*b)
```

3.1058.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3042, 4889, 25, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(a+bx)-\text{csch}^3(a+bx)}{\text{csch}^3(a+bx)+\text{sech}^3(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\csc(ia+ibx)^3+\sec(ia+ibx)^3}{\sec(ia+ibx)^3-i\csc(ia+ibx)^3}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int -\frac{\tanh^2(a+bx)+\tanh(a+bx)+1}{\tanh^4(a+bx)+\tanh^3(a+bx)+\tanh(a+bx)+1}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\tanh^2(a+bx)+\tanh(a+bx)+1}{\tanh^4(a+bx)+\tanh^3(a+bx)+\tanh(a+bx)+1}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{2462}$$

$$-\frac{\int \left(\frac{1}{3(\tanh(a+bx)+1)^2}+\frac{2}{3(\tanh^2(a+bx)-\tanh(a+bx)+1)}\right)d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{4\arctan\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3(\tanh(a+bx)+1)}}{b}$$

input
```
Int[(-Csch[a + b*x]^3 + Sech[a + b*x]^3)/(Csch[a + b*x]^3 + Sech[a + b*x]^
3),x]
```

output
```
((4*ArcTan[(1 - 2*Tanh[a + b*x])/Sqrt[3]])/(3*Sqrt[3]) + 1/(3*(1 + Tanh[a
+ b*x])))/b
```

3.1058.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.1058.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

method	result
risch	$\frac{\mathrm{e}^{-2bx-2a}}{6b}+\frac{2i\sqrt{3}\,\ln\left(\mathrm{e}^{2bx+2a}-i\sqrt{3}\right)}{9b}-\frac{2i\sqrt{3}\,\ln\left(\mathrm{e}^{2bx+2a}+i\sqrt{3}\right)}{9b}$
derivativedivides	$\frac{\frac{2i\sqrt{3}\,\ln\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^2+\left(i\sqrt{3}-1\right)\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{9}-\frac{2i\sqrt{3}\,\ln\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^2+\left(-i\sqrt{3}-1\right)\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{9}+\frac{2}{3\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\cdots}}{b}$
default	$\frac{\frac{2i\sqrt{3}\,\ln\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^2+\left(i\sqrt{3}-1\right)\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{9}-\frac{2i\sqrt{3}\,\ln\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^2+\left(-i\sqrt{3}-1\right)\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{9}+\frac{2}{3\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\cdots}}{b}$

input
```
int((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x,method=
_RETURNVERBOSE)
```

output
```
1/6*exp(-2*b*x-2*a)/b+2/9*I*3^(1/2)/b*ln(exp(2*b*x+2*a)-I*3^(1/2))-2/9*I*3
^(1/2)/b*ln(exp(2*b*x+2*a)+I*3^(1/2))
```

3.1058.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(40) = 80$.

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.70

$$\int \frac{-\text{csch}^3(a+bx)+\text{sech}^3(a+bx)}{\text{csch}^3(a+bx)+\text{sech}^3(a+bx)}\,dx = \frac{8\left(\sqrt{3}\cosh(bx+a)^2+2\sqrt{3}\cosh(bx+a)\sinh(bx+a)+\sqrt{3}\sinh(bx+a)^2\right)\arctan\left(-\frac{\sqrt{3}\cosh(bx+a)+\sqrt{3}\sinh(bx+a)}{3(\cosh(bx+a)-\sinh(bx+a))}\right)+3}{18\left(b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

input
```
integrate((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x,
algorithm="fricas")
```

output
```
1/18*(8*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
 sqrt(3)*sinh(b*x + a)^2)*arctan(-1/3*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sin
h(b*x + a))/(cosh(b*x + a) - sinh(b*x + a))) + 3)/(b*cosh(b*x + a)^2 + 2*b
*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

3.1058.6 Sympy [F]

$$\int \frac{-\text{csch}^3(a+bx)+\text{sech}^3(a+bx)}{\text{csch}^3(a+bx)+\text{sech}^3(a+bx)}\,dx = -\int \frac{\text{csch}^3(a+bx)}{\text{csch}^3(a+bx)+\text{sech}^3(a+bx)}\,dx - \int \left(-\frac{\text{sech}^3(a+bx)}{\text{csch}^3(a+bx)+\text{sech}^3(a+bx)}\right)dx$$

input
```
integrate((-csch(b*x+a)**3+sech(b*x+a)**3)/(csch(b*x+a)**3+sech(b*x+a)**3)
,x)
```

output
```
-Integral(csch(a + b*x)**3/(csch(a + b*x)**3 + sech(a + b*x)**3), x) - Int
egral(-sech(a + b*x)**3/(csch(a + b*x)**3 + sech(a + b*x)**3), x)
```

3.1058.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(40) = 80.

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.98

$$\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx = -\frac{4\left(\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)\right)}{9\,b} + \frac{e^{(-2\,bx-2\,a)}}{6\,b}$$

input
```
integrate((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x,
algorithm="maxima")
```

output
```
-4/9*(sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) + 3^(1/4)
*sqrt(2))) - sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) -
3^(1/4)*sqrt(2))))/b + 1/6*e^(-2*b*x - 2*a)/b
```

3.1058.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx = -\frac{8\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2\,bx+2\,a)}\right)-3\,e^{(-2\,bx-2\,a)}}{18\,b}$$

input
```
integrate((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x,
algorithm="giac")
```

output
```
-1/18*(8*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*b*x + 2*a)) - 3*e^(-2*b*x - 2*a))
/b
```

3.1058.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx=\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{6\,b}-\frac{4\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{b^2}}{3\,b}\right)}{9\,\sqrt{b^2}}$$

input
```
int((1/cosh(a + b*x)^3 - 1/sinh(a + b*x)^3)/(1/cosh(a + b*x)^3 + 1/sinh(a
+ b*x)^3),x)
```

output
```
exp(- 2*a - 2*b*x)/(6*b) - (4*3^(1/2)*atan((3^(1/2)*exp(2*a)*exp(2*b*x)*(b
^2)^(1/2))/(3*b)))/(9*(b^2)^(1/2))
```

3.1059 $\int \frac{-\text{csch}^4(a+bx)+\text{sech}^4(a+bx)}{\text{csch}^4(a+bx)+\text{sech}^4(a+bx)}\,dx$

3.1059.1 Optimal result

Integrand size = 39, antiderivative size = 51

$$\int \frac{-\text{csch}^4(a+bx)+\text{sech}^4(a+bx)}{\text{csch}^4(a+bx)+\text{sech}^4(a+bx)}\,dx = \frac{\arctan\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b} - \frac{\arctan\left(1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}$$

output
```
-1/2*arctan(-1+2^(1/2)*tanh(b*x+a))/b*2^(1/2)-1/2*arctan(1+2^(1/2)*tanh(b*
x+a))/b*2^(1/2)
```

3.1059.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

$$\int \frac{-\text{csch}^4(a+bx)+\text{sech}^4(a+bx)}{\text{csch}^4(a+bx)+\text{sech}^4(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sinh(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

input
```
Integrate[(-Csch[a + b*x]^4 + Sech[a + b*x]^4)/(Csch[a + b*x]^4 + Sech[a +
 b*x]^4),x]
```

output
```
-(ArcTan[Sinh[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b))
```

3.1059.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4889, 25, 1476, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(a+bx)-\operatorname{csch}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ia+ibx)^4-\csc(ia+ibx)^4}{\csc(ia+ibx)^4+\sec(ia+ibx)^4}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int -\frac{\tanh^2(a+bx)+1}{\tanh^4(a+bx)+1}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\tanh^2(a+bx)+1}{\tanh^4(a+bx)+1}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{-\frac{1}{2}\int \frac{1}{\tanh^2(a+bx)-\sqrt{2}\tanh(a+bx)+1}d\tanh(a+bx)-\frac{1}{2}\int \frac{1}{\tanh^2(a+bx)+\sqrt{2}\tanh(a+bx)+1}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\int \frac{1}{-\left(\sqrt{2}\tanh(a+bx)+1\right)^2-1}d\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(1-\sqrt{2}\tanh(a+bx)\right)^2-1}d\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}}}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\arctan\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}}}{b}$$

input
```
Int[(-Csch[a + b*x]^4 + Sech[a + b*x]^4)/(Csch[a + b*x]^4 + Sech[a + b*x]^
4),x]
```

output
```
(ArcTan[1 - Sqrt[2]*Tanh[a + b*x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Tanh[a + b
*x]]/Sqrt[2])/b
```

3.1059.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.1059. $\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx$

3.1059.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

method	result
risch	$\frac{i\sqrt{2}\,\ln\left(e^{4bx+4a}-2i\sqrt{2}\,e^{2bx+2a}-1\right)}{4b}-\frac{i\sqrt{2}\,\ln\left(e^{4bx+4a}+2i\sqrt{2}\,e^{2bx+2a}-1\right)}{4b}$
derivativedivides	$\frac{\frac{i\sqrt{2}\,\ln\left(2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}+2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)-2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(-2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\right.\right.}{}}{b}$
default	$\frac{\frac{i\sqrt{2}\,\ln\left(2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}+2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)-2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(-2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\right.\right.}{}}{b}$

input
```
int((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x,method=
_RETURNVERBOSE)
```

output
```
1/4*I*2^(1/2)/b*ln(exp(4*b*x+4*a)-2*I*2^(1/2)*exp(2*b*x+2*a)-1)-1/4*I*2^(1
/2)/b*ln(exp(4*b*x+4*a)+2*I*2^(1/2)*exp(2*b*x+2*a)-1)
```

3.1059.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(43) = 86.

Time = 0.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.76

$$\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx$$
$$=\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(bx+a)^3+3\sqrt{2}\cosh(bx+a)\sinh(bx+a)^2+\sqrt{2}\sinh(bx+a)^3+\left(3\sqrt{2}\cosh(bx+a)^2-7\sqrt{2}\right)\sinh(bx+a)+7\sqrt{2}\cosh(bx+a}{4\left(\cosh(bx+a)^3-3\cosh(bx+a)^2\sinh(bx+a)+3\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3\right)}\right.}{2b}$$

input
```
integrate((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x,
algorithm="fricas")
```

output
```
1/2*(sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a
)*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3 + (3*sqrt(2)*cosh(b*x + a)^2 -
 7*sqrt(2))*sinh(b*x + a) + 7*sqrt(2)*cosh(b*x + a))/(cosh(b*x + a)^3 - 3*
cosh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x
 + a)^3)) + sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x
+ a))/(cosh(b*x + a) - sinh(b*x + a))))/b
```

3.1059.6 Sympy [F]

$$\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx = -\int \frac{\operatorname{csch}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx - \int \left(-\frac{\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\right)dx$$

input
```
integrate((-csch(b*x+a)**4+sech(b*x+a)**4)/(csch(b*x+a)**4+sech(b*x+a)**4)
,x)
```

output
```
-Integral(csch(a + b*x)**4/(csch(a + b*x)**4 + sech(a + b*x)**4), x) - Int
egral(-sech(a + b*x)**4/(csch(a + b*x)**4 + sech(a + b*x)**4), x)
```

3.1059.7 Maxima [F]

$$\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx = \int -\frac{\operatorname{csch}(bx+a)^4-\operatorname{sech}(bx+a)^4}{\operatorname{csch}(bx+a)^4+\operatorname{sech}(bx+a)^4}\,dx$$

input
```
integrate((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x,
algorithm="maxima")
```

output
```
-2*integrate((e^(-b*x - a) + e^(-5*b*x - 5*a))*e^(-b*x - a)/(6*e^(-4*b*x -
 4*a) + e^(-8*b*x - 8*a) + 1), x) - 2*integrate((e^(-4*b*x - 4*a) + 1)*e^(
-2*b*x - 2*a)/(6*e^(-4*b*x - 4*a) + e^(-8*b*x - 8*a) + 1), x)
```

3.1059.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(e^{(4\,bx+4\,a)}-1\right)e^{(-2\,bx-2\,a)}\right)}{2\,b}$$

input
```
integrate((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x,
algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(1/4*sqrt(2)*(e^(4*b*x + 4*a) - 1)*e^(-2*b*x - 2*a))/b
```

3.1059.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx = -\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{b^2}}{4\,b}\right)+\operatorname{atan}\left(\frac{\sqrt{b^2}\,\left(\frac{56\,\sqrt{2}\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{b}+\frac{8\,\sqrt{2}\,\mathrm{e}^{6\,a}\,\mathrm{e}^{6\,b\,x}}{b}\right)}{32}\right)\right)}{2\,\sqrt{b^2}}$$

input
```
int((1/cosh(a + b*x)^4 - 1/sinh(a + b*x)^4)/(1/cosh(a + b*x)^4 + 1/sinh(a
+ b*x)^4),x)
```

output
```
-(2^(1/2)*(atan((2^(1/2)*exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/(4*b)) + atan(((
b^2)^(1/2)*((56*2^(1/2)*exp(2*a)*exp(2*b*x))/b + (8*2^(1/2)*exp(6*a)*exp(6
*b*x))/b))/32)))/(2*(b^2)^(1/2))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```